SEVENTH EDITION

# INTERMEDIATE



Elayn Martin-Gay



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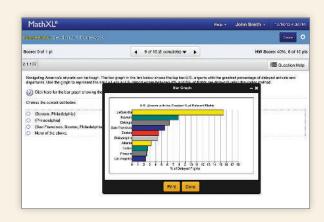


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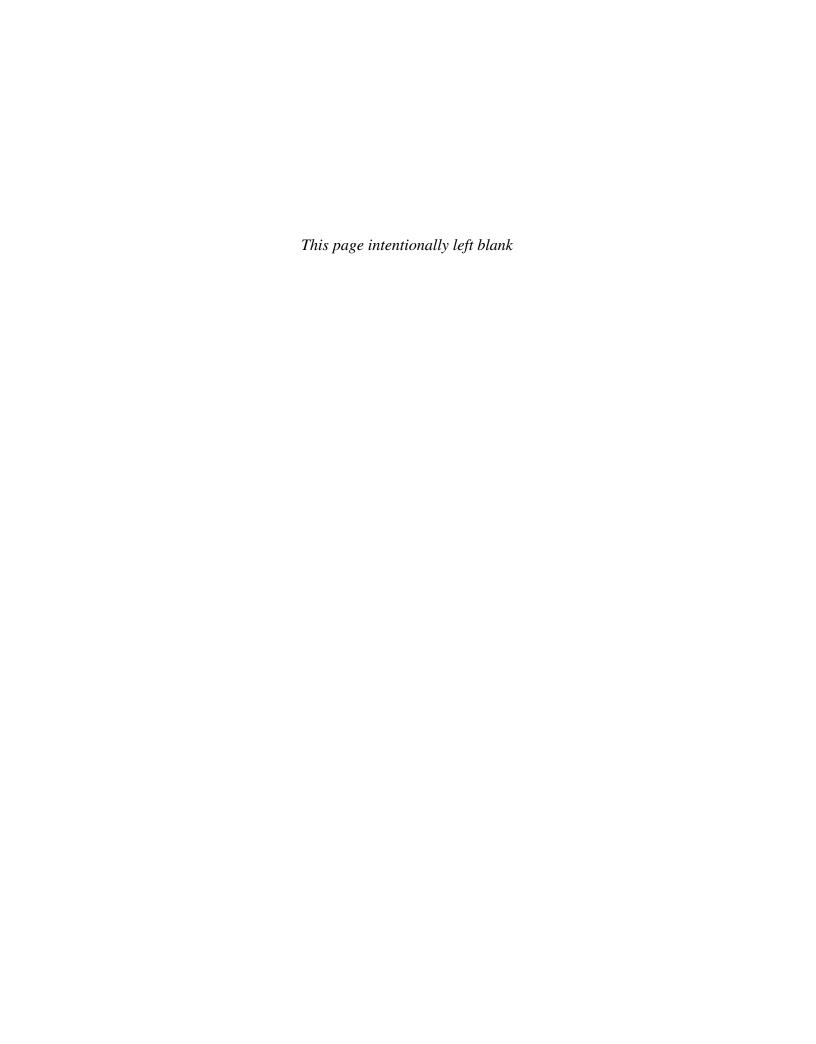
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# Intermediate Algebra



# Intermediate Algebra

Seventh Edition

Elayn Martin-Gay

University of New Orleans

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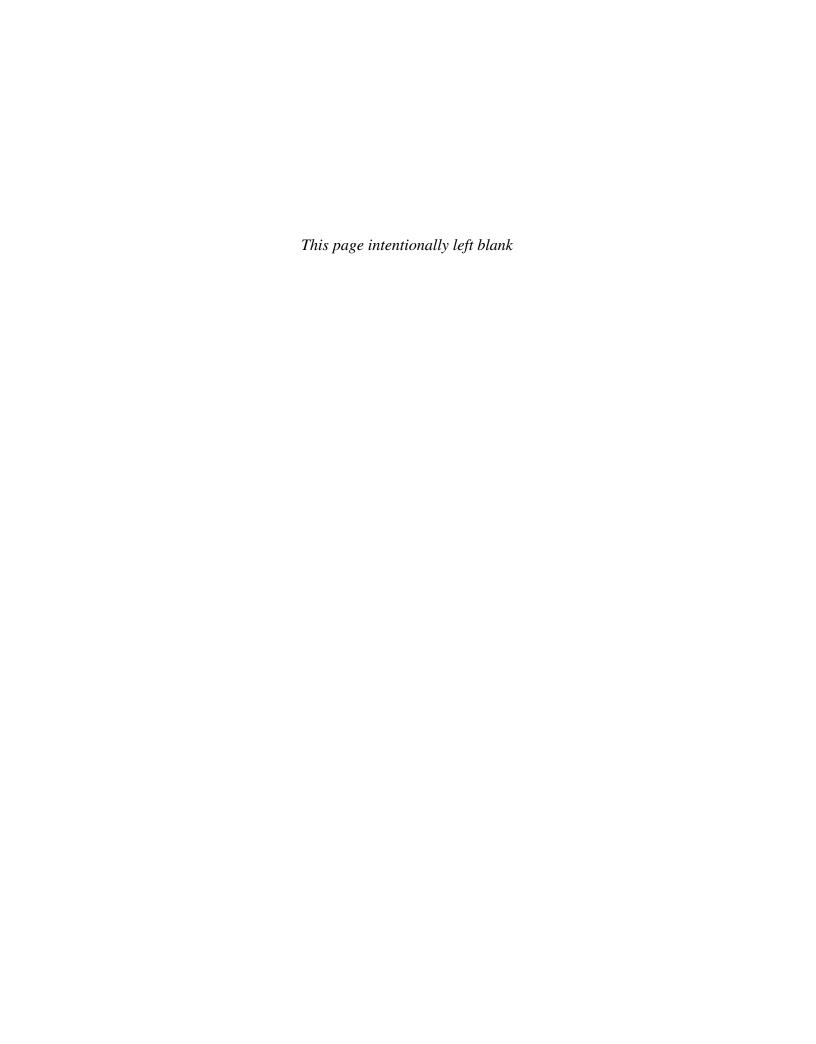
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ISBN-10: 0-13-419617-1 ISBN-13: 978-0-13-419617-6 This book is dedicated to my sister—Karen Martin Callac Pasch

There's not enough space on this page to write how wonderful she was while walking this earth.

She is in a better place now; and for that, I celebrate.



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# **Preface**

Intermediate Algebra, Seventh Edition was written to provide a solid foundation in algebra for students who might not have previous experience in algebra. Specific care was taken to make sure students have the most up-to-date, relevant text preparation for their next mathematics course or for nonmathematical courses that require an understanding of algebraic fundamentals. I have tried to achieve this by writing a user-friendly text that is keyed to objectives and contains many worked-out examples. As suggested by AMATYC and the NCTM Standards (plus Addenda), real-life and real-data applications, data interpretation, conceptual understanding, problem solving, writing, cooperative learning, appropriate use of technology, number sense, estimation, critical thinking, and geometric concepts are emphasized and integrated throughout the book.

The many factors that contributed to the success of the previous editions have been retained. In preparing the Seventh Edition, I considered comments and suggestions of colleagues, students, and many users of the prior edition throughout the country.

#### What's New in the Seventh Edition?

• New Getting Ready for the Test can be found before each Chapter Test. These exercises help increase student success by helping students prepare for their chapter test. The purpose of these exercises is to check students' conceptual understanding of the topics in the chapter as well as common student errors. It is suggested that students complete and check these exercises before taking a practice Chapter Test. All Getting Ready for the Test exercises are either Multiple Choice or Matching, and all answers can be found in the answer section of this text.

**Video Solutions** of all exercises can be found in MyMathLab and on the Interactive DVD Lecture Series. These video solutions contain brief explanations and reminders of material in the chapter. Where applicable, incorrect choices contain explanations.

Getting Ready for the Test exercise numbers marked in blue indicate that the question is available in **Learning Catalytics**.

- New Learning Catalytics is an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking. Generate class discussion, guide your lecture, and promote peer-to-peer learning with real-time analytics. Accessible through MyMathLab, instructors can use Learning Catalytics to:
  - Pose a variety of open-ended questions that help your students develop critical thinking skills.
  - Monitor responses to find out where students are struggling.
  - Use real-time data to adjust your instructional strategy and try other ways of engaging your students during class.
  - Manage student interactions by automatically grouping students for discussion, teamwork, and peer-to-peer learning.

For *Intermediate Algebra*, Seventh Edition, new Getting Ready for the Test exercises marked in blue are available in Learning Catalytics. To find the questions in Learning Catalytics, search the question library for **MGIA7e** Ch and the chapter number. For example, search **MGIA7e** Ch4 for questions from Chapter 4.

 New Student Success Tips Videos are 3- to 5-minute video segments designed to be daily reminders to students to continue practicing and maintaining good organizational and study habits. They are organized in three categories and are available in MyMathLab and the Interactive Lecture Series. The categories are:

- 1. Success Tips that apply to any course in college in general, such as Time Management.
- 2. Success Tips that apply to any mathematics course. One example is based on understanding that mathematics is a course that requires homework to be completed in a timely fashion.
- 3. Section- or Content-specific Success Tips to help students avoid common mistakes or to better understand concepts that often prove challenging. One example of this type of tip is how to apply the order of operations to simplify an expression.
- New Key Concept Activity Lab Workbook includes Extension Exercises, Exploration Activities, Conceptual Exercises, and Group Activities. These activities are a great way to engage students in conceptual projects and exploration as well as group work.
- The Martin-Gay MyMathLab course has been updated and revised to provide more exercise coverage, including assignable video check questions and an expanded video program. There are section lectures videos for every section, which students can also access at the specific objective level; new Getting Ready for the Test video solutions; new Student Success Tips videos; and an increased number of watch clips at the exercise level to help students while doing homework in MathXL.
  - Vocabulary, Readiness & Video Check Questions continue to be available in the text and for assignment in MyMathLab. The Readiness exercises center on a student's understanding of a concept that is necessary in order to continue to the exercise set. The video check questions are included in every section for every learning objective. These exercises are a great way to assess whether students have viewed and understood the key concepts presented in the videos.
- Exercise Sets Revised and Updated The text exercise sets have been carefully examined and revised. Special focus was placed on making sure that even- and odd-numbered exercises are paired and that real-life applications are updated.

#### **Key Continuing Resources and Pedagogical Features**

 Interactive DVD Lecture Series, featuring your text author Elayn Martin-Gay, provides students with active learning at their own pace. The videos offer the following resources and more:

A complete lecture for each section of the text highlights key examples and exercises from the text. Pop-ups reinforce key terms, definitions, and concepts.

An interface with menu navigation features allows students to quickly find and focus on the examples and exercises they need to review.

Interactive Concept Check exercises measure students' understanding of key concepts and common trouble spots.

New Student Success Tips Videos.

 The Interactive DVD Lecture Series also includes the following resources for test prep:

#### **New Getting Ready for the Chapter Test Videos**

The Chapter Test Prep Videos help students during their most teachable moment-when they are preparing for a test. This innovation provides step-by-step solutions for the exercises found in each Chapter Test. For the Seventh Edition, the chapter test prep videos are also available on YouTube<sup>™</sup>. The videos are captioned in English and Spanish.

**The Practice Final Exam Videos** help students prepare for an end-of-course final. Students can watch full video solutions to each exercise in the Practice Final Exam at the end of this text.

- The Video Organizer is designed to help students take notes and work practice exercises while watching the Interactive Lecture Series videos (available in MyMathLab and on DVD). All content in the Video Organizer is presented in the same order as it is presented in the videos, making it easy for students to create a course notebook and build good study habits.
  - Covers all of the video examples in order.
  - Provides ample space for students to write down key definitions and properties.
  - Includes Play and Pause button icons to prompt students to follow along with the author for some exercises while they try others on their own.

The Video Organizer is available in a loose-leaf, notebook-ready format. It is also available for download in MyMathLab. Answers to all video questions are available to instructors in MyMathLab and the Instructor's Resource Center.

#### **Key Pedagogical Features**

The following key features have been retained and/or updated for the Seventh Edition of the text:

**Problem-Solving Process** This is formally introduced in Chapter 2 with a four-step process that is integrated throughout the text. The four steps are **Understand, Translate, Solve,** and **Interpret.** The repeated use of these steps in a variety of examples shows their wide applicability. Reinforcing the steps can increase students' comfort level and confidence in tackling problems.

**Exercise Sets Revised and Updated** The exercise sets have been carefully examined and extensively revised. Special focus was placed on making sure that even- and odd-numbered exercises are paired.

**Examples** Detailed, step-by-step examples were added, deleted, replaced, or updated as needed. Many examples reflect real life. Additional instructional support is provided in the annotated examples.

**Practice Exercises** Throughout the text, each worked-out example has a parallel Practice Exercise. These invite students to be actively involved in the learning process. Students should try each Practice Exercise after finishing the corresponding example. Learning by doing will help students grasp ideas before moving on to other concepts. Answers to the Practice Exercises are provided in the back of the text.

**Helpful Hints** Helpful Hints contain practical advice on applying mathematical concepts. Strategically placed where students are most likely to need immediate reinforcement, Helpful Hints help students avoid common trouble areas and mistakes.

**Concept Checks** This feature allows students to gauge their grasp of an idea as it is being presented in the text. Concept Checks stress conceptual understanding at the point of use and help suppress misconceived notions before they start. Answers appear at the bottom of the page. Exercises related to Concept Checks are included in the exercise sets.

**Mixed Practice Exercises** Found in the section exercise sets, these require students to determine the problem type and strategy needed to solve it just as they would need to do on a test.

**Integrated Reviews** A unique, mid-chapter exercise set that helps students assimilate new skills and concepts that they have learned separately over several sections. These

reviews provide yet another opportunity for students to work with mixed exercises as they master the topics.

**Vocabulary Check** Provides an opportunity for students to become more familiar with the use of mathematical terms as they strengthen their verbal skills. These appear at the end of each chapter before the Chapter Highlights. Vocabulary, Readiness, and Video Check exercises provide practice at the section level.

**Chapter Highlights** Found at the end of every chapter, these contain key definitions and concepts with examples to help students understand and retain what they have learned and help them organize their notes and study for tests.

**Chapter Review** The end of every chapter contains a comprehensive review of topics introduced in the chapter. The Chapter Review offers exercises keyed to every section in the chapter, as well as Mixed Review exercises that are not keyed to sections.

**Chapter Test and Chapter Test Prep Video** The Chapter Test is structured to include those problems that involve common student errors. The **Chapter Test Prep Videos** give students instant author access to a step-by-step video solution of each exercise in the Chapter Test.

**Cumulative Review** Follows every chapter in the text (except Chapter 1). Each odd-numbered exercise contained in the Cumulative Review is an earlier worked example in the text that is referenced in the back of the book along with the answer.

**Writing Exercises** \ These exercises occur in almost every exercise set and require students to provide a written response to explain concepts or justify their thinking.

**Applications** Real-world and real-data applications have been thoroughly updated, and many new applications are included. These exercises occur in almost every exercise set, show the relevance of mathematics, and help students gradually and continuously develop their problem-solving skills.

**Review Exercises** These exercises occur in each exercise set (except in Chapter 1) and are keyed to earlier sections. They review concepts learned earlier in the text that will be needed in the next section or chapter.

**Exercise Set Resource Icons** Located at the opening of each exercise set, these icons remind students of the resources available for extra practice and support:





See Student Resource descriptions page xv for details on the individual resources available.

**Exercise Icons** These icons facilitate the assignment of specialized exercises and let students know what resources can support them.

- ▶ Video icon: exercise worked on the Interactive DVD Lecture Series and in MyMathLab.
- △ Triangle icon: identifies exercises involving geometric concepts.
- Nencil icon: indicates a written response is needed.
- Calculator icon: optional exercises intended to be solved using a scientific or graphing calculator.

**Optional: Calculator Exploration Boxes and Calculator Exercises** The optional Calculator Explorations provide keystrokes and exercises at appropriate points to give an opportunity for students to become familiar with these tools. Section exercises that are best completed by using a calculator are identified by for ease of assignment.

#### **Student and Instructor Resources**

#### STUDENT RESOURCES

# **Interactive DVD Lecture Series** Videos

Provides students with active learning at their own pace. The videos offer:

- A complete lecture for each text section. The interface allows easy navigation to examples and exercises students need to review.
- Interactive Concept Check exercises
- Student Success Tips Videos
- Practice Final Exam
- Getting Ready for the Chapter Test Videos
- Chapter Test Prep Videos

#### Video Organizer

Designed to help students take notes and work practice exercises while watching the Interactive Lecture Series videos.

- Covers all of the video examples in order.
- Provides ample space for students to write down key definitions and rules.
- Includes Play and Pause button icons to prompt students to follow along with the author for some exercises while they try others on their own.

Available in loose-leaf, notebook-ready format and in MyMathLab.

#### **Student Solutions Manual**

Provides completely worked-out solutions to the odd-numbered section exercises; all exercises in the Integrated Reviews, Chapter Reviews, Chapter Tests, and Cumulative Reviews.

**Key Concept Activity Lab Workbook** includes Extension Exercises, Exploration Activities, Conceptual Exercises, and Group Activities.

#### **INSTRUCTOR RESOURCES**

#### **Instructor's Resource Manual with Tests and Annotated Instructor's Edition** Mini-Lectures Contains all the content found in the student edition, plus the following: Mini-lectures for each text section Additional Practice worksheets for each section • Classroom example paired to each example • Several forms of test per chapter—free response • Answers to exercises on the same text page and multiple choice • Teaching Tips throughout the text, placed at key Answers to all items points Video Answer Section **Instructor's Solutions Manual TestGen**<sup>®</sup> (Available for download from the IRC) Instructor-to-Instructor Videos—available in the Online Resources Instructor Resources section of the MyMathLab course. **MyMathLab**<sup>®</sup> (access code required) MathXL<sup>®</sup> (access code required)



# Get the most out of MyMathLab®



MyMathLab is the world's leading online resource for teaching and learning mathematics. MyMathLab helps students and instructors improve results and provides engaging experiences and personalized learning for each student so learning can happen in any environment. Plus, it offers flexible and time-saving course-management features to allow instructors to easily manage their classes while remaining in complete control, regardless of course format.

# Personalized Support for Students

- MyMathLab comes with many learning resources—eText, animations, videos, and more—all
  designed to support your students as they progress through their course.
- The Adaptive Study Plan acts as a personal tutor, updating in real time based on student
  performance to provide personalized recommendations on what to work on next.
   With the new Companion Study Plan assignments, instructors can now assign the
  Study Plan as a prerequisite to a test or quiz, helping to guide students through concepts
  they need to master.
- Personalized Homework allows instructors to create homework assignments tailored to each student's specific needs by focusing on just the topics they have not yet mastered.

Used by nearly 4 million students each year, the MyMathLab and MyStatLab family of products delivers consistent, measurable gains in student learning outcomes, retention, and subsequent course success.

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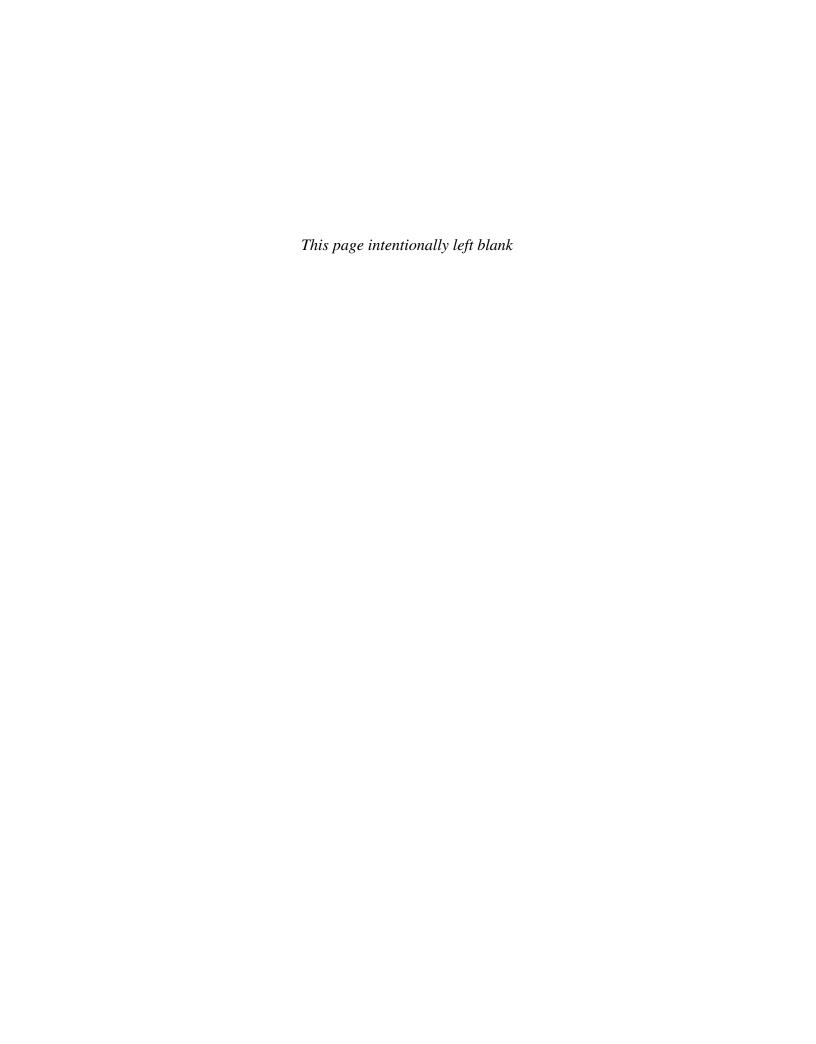
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#### About the Author

Elayn Martin-Gay has taught mathematics at the University of New Orleans for more than 25 years. Her numerous teaching awards include the local University Alumni Association's Award for Excellence in Teaching, and Outstanding Developmental Educator at University of New Orleans, presented by the Louisiana Association of Developmental Educators.

Prior to writing textbooks, Elayn Martin-Gay developed an acclaimed series of lecture videos to support developmental mathematics students in their quest for success. These highly successful videos originally served as the foundation material for her texts. Today, the videos are specific to each book in the Martin-Gay series. The author has also created Chapter Test Prep videos to help students during their most "teachable moment"—as they prepare for a test—along with Instructor-to-Instructor videos that provide teaching tips, hints, and suggestions for each developmental mathematics course, including basic mathematics, prealgebra, beginning algebra, and intermediate algebra. Her most recent innovations are the Algebra Prep Apps for the iPhone and iPod Touch. These Apps embrace the different learning styles, schedules, and paces of students and provide them with quality math tutoring.

Elayn is the author of 12 published textbooks as well as multimedia interactive mathematics, all specializing in developmental mathematics courses. She has participated as an author across the broadest range of educational materials: textbooks, videos, tutorial software, and courseware. This offers an opportunity of various combinations for an integrated teaching and learning package offering great consistency for the student.



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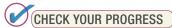


# Real Numbers and Algebraic Expressions

- 1.1 Study Skill Tips for Success in Mathematics
- 1.2 Algebraic Expressions and Sets of Numbers
- 1.3 Operations on Real Numbers and Order of Operations

Integrated Review—Algebraic Expressions and Operations on Whole Numbers

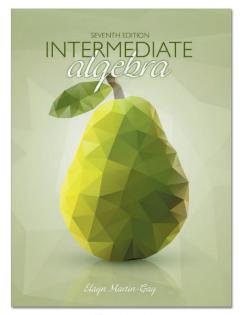
1.4 Properties of Real Numbers and Algebraic Expressions



Vocabulary Check
Chapter Highlights
Chapter Review
Getting Ready for the Test
Chapter Test

In arithmetic, we add, subtract, multiply, divide, raise to powers, and take roots of numbers. In algebra, we add, subtract, multiply, divide, raise to powers, and take roots of variables. Letters, such as x, that represent numbers are called **variables**. Understanding these algebraic expressions depends on your understanding of arithmetic expressions. This chapter reviews the arithmetic operations on real numbers and the corresponding algebraic expressions.

A Selection of Resources for Success in this Mathematics Course





**Textbook** 

Instructor



MyMathLab and MathXL



Video Organizer



Interactive Lecture Series

For more information about the resources illustrated above, read Section 1.1.

# 1.1

## Study Skill Tips for Success in Mathematics

#### OBJECTIVES

- 1 Get Ready for This Course.
- 2 Understand Some General Tips for Success.
- 3 Know How to Use This Text.
- 4 Know How to Use Text Resources.
- 5 Get Help as Soon as You Need It.
- 6 Learn How to Prepare for and Take an Exam.
- 7 Develop Good Time Management.

Before reading Section 1.1, you might want to ask yourself a few questions.

- 1. When you took your last math course, were you organized? Were your notes and materials from that course easy to find, or were they disorganized and hard to find—if you saved them at all?
- 2. Were you satisfied—really satisfied—with your performance in that course? In other words, do you feel that your outcome represented your best effort?

If the answer is "no" to these questions, then it is time to make a change. Changing to or resuming good study skill habits is not a process you can start and stop as you please. It is something that you must remember and practice each and every day. To begin, continue reading this section.

#### **OBJECTIVE**

#### 1 Getting Ready for This Course



Now that you have decided to take this course, remember that a *positive attitude* will make all the difference in the world. Your belief that you can succeed is just as important as your commitment to this course. Make sure you are ready for this course by having the time and positive attitude that it takes to succeed.

Make sure that you are familiar with the way that this course is being taught. Is it a traditional course, in which you have a printed textbook and meet with an instructor? Is it taught totally online, and your textbook is electronic and you e-mail your instructor? Or is your course structured somewhere in between these two methods? (Not all of the tips that follow will apply to all forms of instruction.)

Also make sure that you have scheduled your math course for a time that will give you the best chance for success. For example, if you are also working, you may want to check with your employer to make sure that your work hours will not conflict with your course schedule.

On the day of your first class period, double-check your schedule and allow your-self extra time to arrive on time in case of traffic problems or difficulty locating your classroom. Make sure that you are aware of and bring all necessary class materials.

#### OBJECTIVE

#### 2 General Tips for Success



Below are some general tips that will increase your chance for success in a mathematics class. Many of these tips will also help you in other courses you may be taking.

Most important! Organize your class materials. In the next couple pages, many ideas will be presented to help you organize your class materials—notes, any handouts, completed homework, previous tests, etc. In general, you MUST have these materials organized. All of them will be valuable references throughout your course and when studying for upcoming tests and the final exam. One way to make sure you can locate these materials when you need them is to use a three-ring binder. This binder should be used solely for your mathematics class and should be brought to each and every class or lab. This way, any material can be immediately inserted in a section of this binder and will be there when you need it.

Form study groups and/or exchange names and e-mail addresses. Depending on how your course is taught, you may want to keep in contact with your fellow students. Some ways of doing this are to form a study group—whether in person or through the Internet. Also, you may want to ask if anyone is interested in exchanging e-mail addresses or any other form of contact.

Choose to attend all class periods. If possible, sit near the front of the classroom. This way, you will see and hear the presentation better. It may also be easier for you to participate in classroom activities.

#### Helpful Hint

#### MyMathLab® and MathXL®

When assignments are turned in online, keep a hard copy of your complete written work. You will need to refer to your written work to be able to ask questions and to study for tests later.

#### Helpful Hint

#### MyMathLab® and MathXL®

If you are doing your homework online, you can work and re-work those exercises that you struggle with until you master them. Try working through all the assigned exercises twice before the due date.

#### Helpful Hint

#### MyMathLab® and MathXL®

If you are completing your homework online, it's important to work each exercise on paper before submitting the answer. That way, you can check your work and follow your steps to find and correct any mistakes.

#### **Helpful Hint**

#### MyMathLab® and MathXL®

Be aware of assignments and due dates set by your instructor. Don't wait until the last minute to submit work online.

Do your homework. You've probably heard the phrase "practice makes perfect" in relation to music and sports. It also applies to mathematics. You will find that the more time you spend solving mathematics exercises, the easier the process becomes. Be sure to schedule enough time to complete your assignments before the due date assigned by your instructor.

Check your work. Review the steps you took while working a problem. Learn to check your answers in the original exercises. You may also compare your answers with the "Answers to Selected Exercises" section in the back of the book. If you have made a mistake, try to figure out what went wrong. Then correct your mistake. If you can't find what went wrong, don't erase your work or throw it away. Show your work to your instructor, a tutor in a math lab, or a classmate. It is easier for someone to find where you had trouble if he or she looks at your original work.

Learn from your mistakes and be patient with yourself. Everyone, even your instructor, makes mistakes. (That definitely includes me – Elayn Martin-Gay.) Use your errors to learn and to become a better math student. The key is finding and understanding your errors.

Was your mistake a careless one, or did you make it because you can't read your own math writing? If so, try to work more slowly or write more neatly and make a conscious effort to carefully check your work.

Did you make a mistake because you don't understand a concept? Take the time to review the concept or ask questions to better understand it.

Did you skip too many steps? Skipping steps or trying to do too many steps mentally may lead to preventable mistakes.

Know how to get help if you need it. It's all right to ask for help. In fact, it's a good idea to ask for help whenever there is something that you don't understand. Make sure you know when your instructor has office hours and how to find his or her office. Find out whether math tutoring services are available on your campus. Check on the hours, location, and requirements of the tutoring service.

Don't be afraid to ask questions. You are not the only person in class with questions. Other students are normally grateful that someone has spoken up.

Turn in assignments on time. This way, you can be sure that you will not lose points for being late. Show every step of a problem and be neat and organized. Also be sure that you understand which problems are assigned for homework. If allowed, you can always double-check the assignment with another student in your class.

#### OBJECTIVE

#### Knowing and Using Your Text

Flip through the pages of this text or view the e-text pages on a computer screen. Start noticing examples, exercise sets, end-of-chapter material, and so on. Every text is organized in some manner. Learn the way this text is organized by reading about and then finding an example in your text of each type of resource listed below. Finding and using these resources throughout your course will increase your chance of success.

- Practice Exercises. Each example in every section has a parallel Practice exercise. As you read a section, try each Practice exercise after you've finished the corresponding example. This "learn-by-doing" approach will help you grasp ideas before you move on to other concepts. Answers are at the back of the text.
- Symbols at the Beginning of an Exercise Set. If you need help with a particular section, the symbols listed at the beginning of each exercise set will remind you of the numerous resources available.
- Objectives. The main section of exercises in each exercise set is referenced by an an example(s). There is also often a section of exercises entitled "Mixed Practice," which may be referenced by two or more examples or sections. These are mixed exercises written to prepare you for your next exam. Use all of this referencing if you have trouble completing an assignment from the exercise set.

- *Icons (Symbols)*. Make sure that you understand the meaning of the icons that are beside many exercises. tells you that the corresponding exercise may be viewed on the video segment that corresponds to that section. ▶ tells you that this exercise is a writing exercise in which you should answer in complete sentences. △ tells you that the exercise involves geometry. tells you that this exercise is worked more efficiently with the aid of a calculator. Also, a feature called Graphing Calculator Explorations may be found before select exercise sets.
- Integrated Reviews. Found in the middle of each chapter, these reviews offer you a chance to practice—in one place—the many concepts that you have learned separately over several sections.
- *End-of-Chapter Opportunities*. There are many opportunities at the end of each chapter to help you understand the concepts of the chapter.

**Vocabulary Checks** contain key vocabulary terms introduced in the chapter.

Chapter Highlights contain chapter summaries and examples.

**Chapter Reviews** contain review exercises. The first part is organized section by section and the second part contains a set of mixed exercises.

**Getting Ready for the Tests** contain conceptual exercises written to prepare students for chapter test directions as well as mixed sections of exercises.

**Chapter Tests** are sample tests to help you prepare for an exam. The Chapter Test Prep Videos found in the Interactive Lecture Series, MyMathLab, and YouTube provide the video solution to each question on each Chapter Test.

**Cumulative Reviews** start at Chapter 2 and are reviews consisting of material from the beginning of the book to the end of that particular chapter.

Student Resources in Your Textbook. You will find a Student Resources section at
the back of this textbook. It contains the following to help you study and prepare
for tests:

**Study Skill Builders** contain study skills advice. To increase your chance for success in the course, read these study tips, and answer the questions.

**Bigger Picture—Study Guide Outline** provides you with a study guide outline of the course, with examples.

**Practice Final** provides you with a Practice Final Exam to help you prepare for a final. The video solutions to each question are provided in the Interactive DVD Lecture Series and within MyMathLab<sup>®</sup>.

Resources to Check Your Work. The Answers to Selected Exercises section provides answers to all odd-numbered section exercises and all integrated review and chapter test exercises.

## OBJECTIVE 4

#### Knowing and Using Video and Notebook Organizer Resources



#### Video Resources

Below is a list of video resources that are all made by me—the author of your text, Elayn Martin-Gay. By making these videos, I can be sure that the methods presented are consistent with those in the text.

- Interactive DVD Lecture Series. Exercises marked with a are fully worked out by the author on the DVDs and within MyMathLab. The lecture series provides approximately 20 minutes of instruction per section and is organized by Objective.
- Chapter Test Prep Videos. These videos provide solutions to all of the Chapter Test exercises worked out by the author. They can be found in MyMathLab, the Interactive Lecture series, and YouTube. This supplement is very helpful before a test or exam
- Student Success Tips. These video segments are about 3 minutes long and are daily reminders to help you continue practicing and maintaining good organizational and study habits.
- *Final Exam Videos*. These video segments provide solutions to each question. These videos can be found within MyMathLab and the Interactive Lecture Series.

#### Helpful Hint

#### MyMathLab<sup>®</sup>

In MyMathLab, you have access to the following video resources:

- Lecture Videos for each section
- Chapter Test Prep Videos

Use these videos provided by the author to prepare for class, review, and study for tests.

#### Notebook Organizer Resource

This resource is in three-ring notebook ready form. It is to be inserted in a three-ring binder and completed. This resource is numbered according to the sections in your text to which they refer.

• Video Organizer. This organizer is closely tied to the Interactive Lecture (Video) Series. Each section should be completed while watching the lecture video on the same section. Once completed, you will have a set of notes to accompany the Lecture (Video) Series section by section.

#### **Helpful Hint**

#### MyMathLab® and MathXL®

- Use the **Help Me Solve This** button to get step-by-step help for the exercise you are working. You will need to work an additional exercise of the same type before you can get credit for having worked it correctly.
- Use the **Video** button to view a video clip of the author working a similar exercise.

#### OBJECTIVE

#### **Getting Help** 5



If you have trouble completing assignments or understanding the mathematics, get help as soon as you need it! This tip is presented as an objective on its own because it is so important. In mathematics, usually the material presented in one section builds on your understanding of the previous section. This means that if you don't understand the concepts covered during a class period, there is a good chance that you will not understand the concepts covered during the next class period. If this happens to you, get help as soon as you can.

Where can you get help? Many suggestions have been made in this section on where to get help, and now it is up to you to get it. Try your instructor, a tutoring center, or a math lab, or you may want to form a study group with fellow classmates. If you do decide to see your instructor or go to a tutoring center, make sure that you have a neat notebook and are ready with your questions.

#### OBJECTIVE

#### Preparing for and Taking an Exam



Make sure that you allow yourself plenty of time to prepare for a test. If you think that you are a little "math anxious," it may be that you are not preparing for a test in a way that will ensure success. The way that you prepare for a test in mathematics is important. To prepare for a test:

- 1. Review your previous homework assignments.
- 2. Review any notes from class and section-level quizzes you have taken. (If this is a final exam, also review chapter tests you have taken.)
- 3. Review concepts and definitions by reading the Chapter Highlights at the end of each chapter.
- 4. Practice working out exercises by completing the Chapter Review found at the end of each chapter. (If this is a final exam, go through a Cumulative Review. There is one found at the end of each chapter except Chapter 1. Choose the review found at the end of the latest chapter that you have covered in your course.) Don't stop here!
- 5. It is important that you place yourself in conditions similar to test conditions to find out how you will perform. In other words, as soon as you feel that you know the material, get a few blank sheets of paper and take a sample test. There is a Chapter Test available at the end of each chapter, or you can work selected problems from the Chapter Review. Your instructor may also provide you with a review sheet. During this sample test, do not use your notes or your textbook. Then check your sample test. If your sample test is the Chapter Test in the text, don't forget that the video solutions are in MyMathLab, the Interactive Lecture Series, and YouTube. If you are not satisfied with the results, study the areas that you are weak in and try again.
- 6. On the day of the test, allow yourself plenty of time to arrive at where you will be taking your exam.

#### **Helpful Hint**

#### MyMathLab® and MathXL®

Review your written work for previous assignments. Then, go back and re-work previous assignments. Open a previous assignment, and click Similar Exercise to generate new exercises. Re-work the exercises until you fully understand them and can work them without help features.

When taking your test:

- 1. Read the directions on the test carefully.
- **2.** Read each problem carefully as you take the test. Make sure that you answer the question asked.
- 3. Watch your time and pace yourself so that you can attempt each problem on your test.
- **4.** If you have time, check your work and answers.
- **5.** Do not turn your test in early. If you have extra time, spend it double-checking your work.

## OBJECTIVE 7

#### **Managing Your Time**



As a college student, you know the demands that classes, homework, work, and family place on your time. Some days you probably wonder how you'll ever get everything done. One key to managing your time is developing a schedule. Here are some hints for making a schedule:

- 1. Make a list of all of your weekly commitments for the term. Include classes, work, regular meetings, extracurricular activities, etc. You may also find it helpful to list such things as laundry, regular workouts, grocery shopping, etc.
- 2. Next, estimate the time needed for each item on the list. Also make a note of how often you will need to do each item. Don't forget to include time estimates for the reading, studying, and homework you do outside of your classes. You may want to ask your instructor for help estimating the time needed.
- **3.** In the exercise set that follows, you are asked to block out a typical week on the schedule grid given. Start with items with fixed time slots like classes and work.
- **4.** Next, include the items on your list with flexible time slots. Think carefully about how best to schedule items such as study time.
- 5. Don't fill up every time slot on the schedule. Remember that you need to allow time for eating, sleeping, and relaxing! You should also allow a little extra time in case some items take longer than planned.
- **6.** If you find that your weekly schedule is too full for you to handle, you may need to make some changes in your workload, classload, or other areas of your life. You may want to talk to your advisor, manager or supervisor at work, or someone in your college's academic counseling center for help with such decisions.



### Exercise Set MyMathLab®



- **1.** What is your instructor's name?
- **2.** What are your instructor's office location and office hours?
- **3.** What is the best way to contact your instructor?
- **4.** Do you have the name and contact information of at least one other student in class?
- 5. Will your instructor allow you to use a calculator in this class?
- **6.** Why is it important that you write step-by-step solutions to homework exercises and keep a hard copy of all work submitted?
- **7.** Is there a tutoring service available on campus? If so, what are its hours? What services are available?

- **8.** Have you attempted this course before? If so, write down ways that you might improve your chances of success during this next attempt.
- 9. List some steps that you can take if you begin having trouble understanding the material or completing an assignment. If you are completing your homework in MyMathLab<sup>®</sup> and MathXL<sup>®</sup>, list the resources you can use for help.
- **10.** How many hours of studying does your instructor advise for each hour of instruction?
- 11. What does the \ icon in this text mean?
- 12. What does the  $\triangle$  icon in this text mean?
- 13. What does the con in this text mean?

- 14. What are Practice exercises?
- **15.** When might be the best time to work a Practice exercise?
- **16.** Where are the answers to Practice exercises?
- 17. What answers are contained in this text and where are they?
- **18.** What are Study Skills Builders and where are they?
- 19. What and where are Integrated Reviews?
- 20. How many times is it suggested that you work through the homework exercises in MathXL® before the submission deadline?
- 21. How far in advance of the assigned due date is it suggested that homework be submitted online? Why?
- 22. Chapter Highlights are found at the end of each chapter. Find the Chapter 1 Highlights and explain how you might use it and how it might be helpful.

- 23. Chapter Reviews are found at the end of each chapter. Find the Chapter 1 Review and explain how you might use it and how it might be helpful.
- 24. Chapter Tests are found at the end of each chapter. Find the Chapter 1 Test and explain how you might use it and how it might be helpful when preparing for an exam on Chapter 1. Include how the Chapter Test Prep Videos may help. If you are working in MyMathLab® and MathXL®, how can you use previous homework assignments to study?
- 25. What is the Video Organizer? Explain the contents and how it might be used.
- 26. Explain how the Video Organizer can help you when watching a lecture video.
- 27. Read or reread objective 7 and fill out the following schedule grid.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1:00 a.m.							
2:00 a.m.							
3:00 a.m.							
4:00 a.m.							
5:00 a.m.							
6:00 a.m.							
7:00 a.m.							
8:00 a.m.							
9:00 a.m.							
10:00 a.m.							
11:00 a.m.							
Noon							
1:00 p.m.							
2:00 p.m.							
3:00 p.m.							
4:00 p.m.							
5:00 p.m.							
6:00 p.m.							
7:00 p.m.							
8:00 p.m.							
9:00 p.m.							
10:00 p.m.							
11:00 p.m.							
Midnight							

# Algebraic Expressions and Sets of Numbers 🕞

#### **OBJECTIVES**

- 1 Identify and Evaluate Algebraic Expressions.
- 2 Identify Natural Numbers, Whole Numbers, Integers, and Rational and Irrational Real Numbers.
- 3 Find the Absolute Value of a Number.
- 4 Find the Opposite of a Number.
- 5 Write Phrases as Algebraic Expressions.

OBJECTIVE

#### 1 Evaluating Algebraic Expressions



Recall that letters that represent numbers are called **variables.** An **algebraic expression** (or simply **expression**) is formed by numbers and variables connected by the operations of addition, subtraction, multiplication, division, raising to powers, or taking roots. For example,

$$2x$$
,  $\frac{x+5}{6}$ ,  $\sqrt{y} - 1.6$ , and  $z^3$ 

are algebraic expressions or, more simply, expressions. (Recall that the expression 2x means  $2 \cdot x$ .)

Algebraic expressions occur often during problem solving. For example, the average cost to own and operate a sedan in the United States for 2014 was \$0.592 per mile. The expression 0.592m gives the total cost to operate a car (sedan) annually for m miles. (Source: AAA)

#### **Driving Costs**

Driving 15,000 Miles per Year	Sedan Average	SUV HWD	Minivan
Cost per Mile	\$0.592	\$0.736	\$0.65
Cost per Year	?	\$11,040	\$9750

To find the cost of driving a car for 15,000 miles, for example, we replace the variable m with 15,000 and perform the indicated operation. This process is called **evaluating** an expression, and the result is called the **value** of the expression for the given replacement value.

Helpful Hint

Recall that 0.592m means  $0.592 \times m$ 

In our example, when m = 15,000,

$$\rightarrow 0.592m = 0.592(15,000) = 8880$$

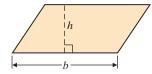
Thus, it costs \$8880 to own and operate a sedan for 15,000 miles of driving.



**EXAMPLE I** 

Finding the Area of a Tile

The research department of a flooring company is considering a new flooring design that contains parallelograms. The area of a parallelogram with base b and height h is bh. Find the area of a parallelogram with base 10 centimeters and height 8.2 centimeters.



**Solution** We replace b with 10 and h with 8.2 in the algebraic expression bh.

$$bh = 10 \cdot 8.2 = 82$$

The area is 82 square centimeters

PRACTICE

The tile edging for a bathroom is in the shape of a triangle. The area of a triangle with base b and height h is  $A = \frac{1}{2}bh$ . Find the area of the tile if the base measures 3.5 cm and the height measures 8 cm.

Algebraic expressions simplify to different values depending on replacement values. (Order of operations is needed for simplifying many expressions. We fully review this in Section 1.3.)

**EXAMPLE 2** 

Evaluate 3x - y when x = 15 and y = 4.

**Solution** We replace x with 15 and y with 4 in the expression.

$$3x - y = 3 \cdot 15 - 4 = 45 - 4 = 41$$

PRACTICE

Evaluate 2p - q when p = 17 and q = 3. 2

When evaluating an expression to solve a problem, we often need to think about the kind of number that is appropriate for the solution. For example, if we are asked to determine the maximum number of parking spaces for a parking lot to be constructed, an answer of  $98\frac{1}{10}$  is not appropriate because  $\frac{1}{10}$  of a parking space is not realistic.

## OBJECTIVE

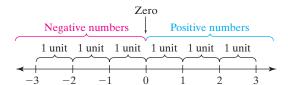
#### **Identifying Common Sets of Numbers** 2



Let's review some common sets of numbers and their graphs on a number line. To construct a number line, we draw a line and label a point 0 with which we associate the number 0. This point is called the **origin.** Choose a point to the right of 0 and label it 1. The distance from 0 to 1 is called the **unit distance** and can be used to locate more points. The **positive numbers** lie to the right of the origin, and the **negative numbers** lie to the left of the origin. The number 0 is neither positive nor negative.

## **Helpful Hint**

0 is neither a positive number nor a negative number.



## **CONCEPT CHECK**

Use the definitions of positive numbers, negative numbers, and zero to describe the meaning of nonnegative numbers.

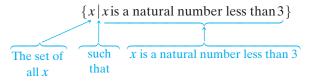
> A number is **graphed** on a number line by shading the point on the number line that corresponds to the number. Some common sets of numbers and their graphs include:



Each listing of three dots, ..., is called an ellipsis and means to continue in the same pattern.

A set is a collection of objects. The objects of a set are called its members or **elements.** When the elements of a set are listed, such as those displayed in the box above, the set is written in **roster** form.

A set can also be written in **set builder notation**, which describes the members of a set but does not list them. The following set is written in set builder notation.



## **Helpful Hint**

Use  $\{ \}$  or  $\emptyset$  to write the empty set.  $\{\emptyset\}$  is **not** the empty set because it has one element:  $\emptyset$ .

This same set written in roster form is  $\{1, 2\}$ .

A set that contains no elements is called the **empty set** (or **null set**), symbolized by  $\{ \}$  or  $\emptyset$ . The set

$$\{x \mid x \text{ is a month with 32 days}\}\ \text{is } \emptyset \text{ or } \{\}\$$

because no month has 32 days. The set has no elements.

#### **EXAMPLE 3** Write each set in roster form. (List the elements of each set.)

- **a.**  $\{x \mid x \text{ is a natural number greater than } 100\}$
- **b.**  $\{x \mid x \text{ is a whole number between 1 and 6}\}$

## **Solution**

#### PRACTICE

3 Write each set in roster form. (List the elements of each set.)

- **a.**  $\{x \mid x \text{ is a whole number between 5 and 10}\}$
- **b.**  $\{x | x \text{ is a natural number greater than 40}\}$

The symbol  $\in$  denotes that an element is in a particular set. The symbol  $\in$  is read as "is an element of." For example, the true statement

3 is an element of 
$$\{1, 2, 3, 4, 5\}$$

can be written in symbols as

$$3 \in \{1, 2, 3, 4, 5\}$$

The symbol ∉ is read as "is not an element of." In symbols, we write the true statement "p is not an element of  $\{a, 5, g, j, q\}$ " as

$$p \notin \{a, 5, g, j, q\}$$

#### **EXAMPLE 4** Determine whether each statement is true or false.

**a.** 
$$3 \in \{x | x \text{ is a natural number}\}$$

**b.** 
$$7 \notin \{1, 2, 3\}$$

## **Solution**

- a. True, since 3 is a natural number and therefore an element of the set.
- **b.** True, since 7 is not an element of the set  $\{1, 2, 3\}$ .

### PRACTICE

Determine whether each statement is true or false. 4

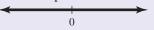
**a.** 
$$7 \in \{x | x \text{ is a natural number}\}$$

**b.** 
$$6 \notin \{1, 3, 5, 7\}$$

We can use set builder notation to describe three other common sets of numbers.

## **Identifying Numbers**

**Real numbers:**  $\{x \mid x \text{ corresponds to a point on a number line}\}$ 



Rational numbers:  $\left\{\frac{a}{b}\middle| a \text{ and } b \text{ are integers and } b \neq 0\right\}$ 

**Irrational numbers:**  $\{x \mid x \text{ is a real number and } x \text{ is not a rational number}\}$ 

Notice from the definition that all real numbers are either rational or irrational.

Every rational number can be written as a decimal that either repeats in a pattern or terminates. For example,

## Rational Numbers

$$\frac{1}{2} = 0.5$$

$$\frac{5}{4} = 1.25$$

$$\frac{2}{3} = 0.66666666... = 0.\overline{6} \quad \frac{1}{11} = 0.090909... = 0.\overline{09}$$

An irrational number written as a decimal neither terminates nor repeats. For example,  $\pi$  and  $\sqrt{2}$  are irrational numbers. Their decimal form neither terminates nor repeats. Decimal approximations of each are below:

## Irrational Numbers

$$\pi \approx 3.141592... \quad \sqrt{2} \approx 1.414213...$$

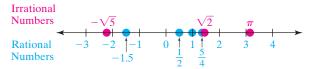
Notice that every integer is also a rational number since each integer can be written as the quotient of itself and 1:

$$3 = \frac{3}{1}$$
,  $0 = \frac{0}{1}$ ,  $-8 = \frac{-8}{1}$ 

Not every rational number, however, is an integer. The rational number  $\frac{2}{3}$ , for example, is not an integer. Some square roots are rational numbers and some are irrational numbers. For example,  $\sqrt{2}$ ,  $\sqrt{3}$ , and  $\sqrt{7}$  are irrational numbers while  $\sqrt{25}$  is a rational number because  $\sqrt{25} = 5 = \frac{5}{1}$ . The set of rational numbers together with the set of irrational numbers make up the set of real numbers. To help you make the distinction between rational and irrational numbers, here are a few examples of each.

Real Numbers				
	Rational N	Irrational Numbers		
Examples Equivalent Quotient of Integers, $\frac{a}{b}$			Examples	
$-\frac{2}{3}$	=	$\frac{-2}{3}$ or $\frac{2}{-3}$	$\sqrt{5}$	
$\sqrt{36}$	=	<u>6</u> 1	$\frac{\sqrt{6}}{7}$	
5	=	$\frac{5}{1}$	$-\sqrt{3}$	
0	=	$\frac{0}{1}$	$\pi$	
1.2	=	$\frac{12}{10}$	$\frac{2}{\sqrt{3}}$	
$3\frac{7}{8}$	=	$\frac{31}{8}$		

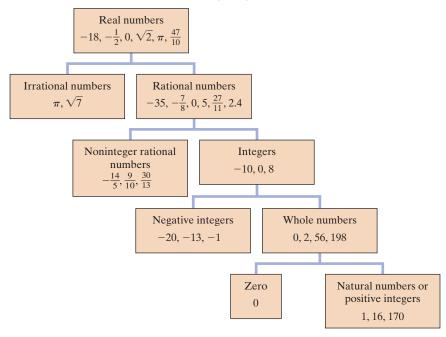
Some rational and irrational numbers are graphed below.



Earlier, we mentioned that every integer is also a rational number. In other words, all the elements of the set of integers are also elements of the set of rational numbers. When this happens, we say that the set of integers, set Z, is a subset of the set of rational numbers, set Q. In symbols,

$$Z \subseteq Q$$
is a subset of

The natural numbers, whole numbers, integers, rational numbers, and irrational numbers are each a subset of the set of real numbers. The relationships among these sets of numbers are shown in the following diagram.



**EXAMPLE 5** Determine whether the following statements are true or false.

**a.** 3 is a real number.

- **b.**  $\frac{1}{5}$  is an irrational number.
- **c.** Every rational number is an integer.
- **d.**  $\{1,5\} \subseteq \{2,3,4,5\}$

## Solution

- **a.** True. Every whole number is a real number.
- **b.** False. The number  $\frac{1}{5}$  is a rational number since it is in the form  $\frac{a}{b}$  with a and b integers and  $b \neq 0$ .
- **c.** False. The number  $\frac{2}{3}$ , for example, is a rational number, but it is not an integer.
- **d.** False. The element 1 in the first set is not an element of the second set.

## PRACTICE

- Determine whether the following statements are true or false. 5
  - **a.** -5 is a real number.

**b.**  $\sqrt{8}$  is a rational number.

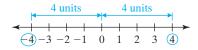
- c. Every whole number is a rational number.
- **d.**  $\{2,4\} \subseteq \{1,3,4,7\}$

OBJECTIVE

#### Finding the Absolute Value of a Number 3



A number line can also be used to visualize distance, which leads to the concept of absolute value. The **absolute value** of a real number a, written as |a|, is the distance between a and 0 on a number line. Since distance is always positive or zero, |a| is always positive or zero.



Using the number line, we see that

$$|4| = 4$$
 and also  $|-4| = 4$ 

Why? Because both 4 and -4 are a distance of 4 units from 0.

An equivalent definition of the absolute value of a real number a is given next.

## **Absolute Value**

The absolute value of a, written as |a|, is

$$|a| = \begin{cases} a \text{ if } a \text{ is 0 or a positive number} \\ \hline a \text{ if } a \text{ is a negative number} \end{cases}$$
the opposite of

## EXAMPLE 6

Find each absolute value.

**a.** 
$$|3|$$
 **b.**  $\left|-\frac{1}{7}\right|$  **c.**  $-|2.7|$  **d.**  $-|-8|$  **e.**  $|0|$ 

**Solution** 

**a.** |3| = 3 since 3 is located 3 units from 0 on a number line.

**b.** 
$$\left| -\frac{1}{7} \right| = \frac{1}{7}$$
 since  $-\frac{1}{7}$  is  $\frac{1}{7}$  units from 0 on a number line.

c. -|2.7| = -2.7. The negative sign outside the absolute value bars means to take the opposite of the absolute value of 2.7.

**d.** 
$$-|-8| = -8$$
. Since  $|-8|$  is  $8, -|-8| = -8$ .

**e.** 
$$|0| = 0$$
 since 0 is located 0 units from 0 on a number line.

PRACTICE

Find each absolute value.

**a.** 
$$|4|$$
 **b.**  $\left|-\frac{1}{2}\right|$  **c.**  $|1|$  **d.**  $-|6.8|$  **e.**  $-|-4|$ 



Explain how you know that |14| = -14 is a false statement.

## Answer to Concept Check:

|14| = 14 since the absolute value of a number is the distance between the number and 0, and distance cannot be negative.

### OBJECTIVE

#### Finding the Opposite of a Number 4



A number line can also help us visualize opposites. Two numbers that are the same distance from 0 on a number line but are on opposite sides of 0 are called **opposites.** 

See the definition illustrated on the number lines below.

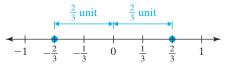
**Helpful Hint** 

The opposite of 0 is 0.

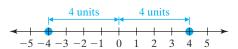
The opposite of 6.5 is -6.5



The opposite of  $\frac{2}{3}$  is  $-\frac{2}{3}$ .



The opposite of -4 is 4.



## **Opposite**

The opposite of a number a is the number -a.

Above, we state that the opposite of a number a is -a. This means that the opposite of -4 is -(-4). But from the number line above, the opposite of -4 is 4. This means that -(-4) = 4, and in general, we have the following property.

## **Double Negative Property**

For every real number a, -(-a) = a.

## **EXAMPLE 7**

Write the opposite of each number.

**b.** 
$$\frac{1}{5}$$

**Solution** 

**a.** The opposite of 8 is -8.

**b.** The opposite of 
$$\frac{1}{5}$$
 is  $-\frac{1}{5}$ .

**c.** The opposite of 
$$-9.6$$
 is  $-(-9.6) = 9.6$ .

PRACTICE

**7** Write the opposite of each number.

**b.** 
$$-\frac{3}{5}$$

OBJECTIVE

## **5** Writing Phrases as Algebraic Expressions



Often, solving problems involves translating a phrase to an algebraic expression. The following is a partial list of key words and phrases and their usual direct translations.

Selected Key Words/Phrases and Their Translations				
Addition	Subtraction	Multiplication	Division	
sum	difference of	product	quotient	
plus	minus	times	divide	
added to	subtracted from	multiply	into	
more than	less than	twice	ratio	
increased by	decreased by	of		
total	less			

**EXAMPLE 8** Translate each phrase to an algebraic expression. Use the variable *x* to represent each unknown number.

- a. Eight times a number
- **b.** Three more than eight times a number
- **c.** The quotient of a number and -7
- **d.** One and six-tenths subtracted from twice a number
- e. Six less than a number
- f. Twice the sum of four and a number

## Solution

- **a.**  $8 \cdot x$  or 8x
- **b.** 8x + 3

**c.** 
$$x \div -7 \text{ or } \frac{x}{-7}$$

**d.** 
$$2x - 1.6$$
 or  $2x - 1\frac{6}{10}$ 

**e.** 
$$x - 6$$

**f.** 
$$2(4 + x)$$

# PRACTICE 8

Translate each phrase to an algebraic expression. Use the variable *x* to represent each unknown number.

- a. The product of 3 and a number
- **b.** Five less than twice a number
- c. Three and five-eighths more than a number
- **d.** The quotient of a number and 2
- e. Fourteen subtracted from a number
- **f.** Five times the sum of a number and ten

## Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

whole numbers integers rational number a natural numbers value irrational number -a absolute value expression variables

- 1. Letters that represent numbers are called
- **2.** Finding the \_\_ of an expression means evaluating the expression.
- **3.** The of a number is that number's distance from 0 on a number line.
- **4.** A(n) \_\_\_\_\_\_ is formed by numbers and variables connected by operations such as addition, subtraction, multiplication, division, raising to powers, and/or taking roots.
- **5.** The \_\_\_\_\_ are  $\{1, 2, 3, \ldots\}$ .
- **6.** The \_\_\_\_\_ are  $\{0, 1, 2, 3, \ldots\}$ .
- 7. The are  $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$ .
- 8. The number  $\sqrt{5}$  is a(n) \_\_\_\_\_.
- 9. The number  $\frac{5}{7}$  is a(n) \_\_\_\_\_\_.
- **10.** The opposite of a is a.

2

OBJECTIVE

3

OBJECTIVE

4

**OBJECTIVE** 5



Watch the section lecture video and answer the following questions.

- OBJECTIVE 11. When the algebraic expression in **Example 2** is evaluated for the 1 variable, why is 10,612.8 not the final answer for this application? **OBJECTIVE** 
  - **12.** Based on the lecture before **Example 3**, what is the relationship among real numbers, rational numbers, and irrational numbers?
  - **13.** Based on the lecture before Example 7, complete the following statements. The absolute value of a number is that number's from zero on a number line. Or, more formally, |a| = a if a is number. Also, |a| = -a if a is a number. or a
  - **14.** Based on the lecture before **Example** 10, the description of opposite using a number line is given. Explain the difference between the absolute value of a number and the opposite of a number using the number line descriptions.
  - **15.** From Example 12, why must we be careful when translating phrases dealing with subtraction?

## **Exercise Set** MyMathLab<sup>®</sup>

Find the value of each algebraic expression at the given replacement values. See Examples 1 and 2.

- **1.** 5x when x = 7
- **2.** 3*y* when y = 45
- 3. 9.8z when z = 3.1
- **4.** 7.1a when a = 1.5
- 5. ab when  $a = \frac{1}{2}$  and  $b = \frac{3}{4}$ 
  - **6.** yz when  $y = \frac{2}{3}$  and  $z = \frac{1}{5}$
  - 7. 3x + y when x = 6 and y = 4
  - **8.** 2a b when a = 12 and b = 7
  - 9. The B737-400 aircraft flies an average speed of 400 miles per



The expression 400t gives the distance traveled by the aircraft in t hours. Find the distance traveled by the B737-400 in 5 hours.

- 10. The algebraic expression 1.5x gives the total length of shelf space needed in inches for x encyclopedias. Find the length of shelf space needed for a set of 30 encyclopedias.
- △ 11. Employees at Walmart constantly reorganize and reshelve merchandise. In doing so, they calculate floor space needed for displays. The algebraic expression  $l \cdot w$  gives the floor space needed in square units for a display that measures length *l* units and width w units. Calculate the floor space needed for a display whose length is 5.1 feet and whose width is 4 feet.

- 12. The algebraic expression  $\frac{x}{5}$  can be used to calculate the distance in miles that you are from a flash of lightning, where x is the number of seconds between the time you see a flash of lightning and the time you hear the thunder. Calculate the distance that you are from the flash of lightning if you hear the thunder 2 seconds after you see the lightning.
- ▶ 13. The B737-400 aircraft costs \$2948 dollars per hour to operate. The algebraic expression 2948t gives the total cost to operate the aircraft for t hours. Find the total cost to operate the B737-400 for 3.6 hours.
  - 14. On November 16, 2004, the NASA X-43A scramjet achieved the designation of fastest airplane in the world, reaching a speed of 7500 miles per hour. At this speed, the algebraic expression 7500t gives the total distance flown in t hours. Find the distance flown by the X-43A scramjet in 1.6 hours.

Write each set in roster form. (List the elements of each set.) See Example 3.

- **D** 15.  $\{x \mid x \text{ is a natural number less than 6}\}$ 
  - **16.**  $\{x \mid x \text{ is a natural number greater than 6}\}$
  - 17.  $\{x \mid x \text{ is a natural number between } 10 \text{ and } 17\}$
  - **18.**  $\{x \mid x \text{ is an odd natural number}\}$
  - **19.**  $\{x \mid x \text{ is a whole number that is not a natural number}\}$
  - **20.**  $\{x \mid x \text{ is a natural number less than } 1\}$
  - **21.**  $\{x \mid x \text{ is an even whole number less than 9}\}$
  - 22.  $\{x \mid x \text{ is an odd whole number less than 9}\}$

Graph each set on a number line.

- **23.** {0, 2, 4, 6}
- **24.** {1, 3, 5, 7}

- **27.** { -2, -6, -10}
- **29.**  $\left\{-\frac{1}{3}, -1\frac{1}{3}\right\}$

List the elements of the set  $\left\{3, 0, \sqrt{7}, \sqrt{36}, \frac{2}{5}, -134\right\}$  that are also elements of the given set. See Example 4.

- **○** 31. Whole numbers
  - **32.** Integers
  - 33. Natural numbers
  - 34. Rational numbers
  - 35. Irrational numbers
  - 36. Real numbers

Place  $\in$  or  $\notin$  in the space provided to make each statement true. See Example 4.

- $\bigcirc$  37.  $-11 \quad \{x \mid x \text{ is an integer}\}\$ 
  - **38.** 0  $\{x \mid x \text{ is a positive integer}\}$
  - **39.** -6 {2, 4, 6, ...}
- **40.** 12 {1, 2, 3, ...}
- **41.** 12 {1, 3, 5, ...}
- **42.** 0 {1, 2, 3, ...}
- $\{x \mid x \text{ is an irrational number}\}$
- $\{x \mid x \text{ is a natural number}\}\$

Determine whether each statement is true or false. See Examples 4 and 5. Use the following sets of numbers.

- N =set of natural numbers
- Z = set of integers
- I = set of irrational numbers
- Q = set of rational numbers
- $\mathbb{R}$  = set of real numbers
- **45.**  $Z \subset \mathbb{R}$

**46.**  $\mathbb{R} \subset N$ 

**47.**  $-1 \in Z$ 

**48.**  $\frac{1}{2} \in Q$ 

- **49.**  $0 \in N$
- **50.**  $Z \subseteq Q$
- **51.**  $\sqrt{5}$  ∉ *I*

52.  $\pi \notin \mathbb{R}$ 

53.  $N \subset Z$ 

**54.**  $I \subset N$ 

55.  $\mathbb{R} \subseteq Q$ 

**56.**  $N \subseteq Q$ 

Find each absolute value. See Example 6.

**○** 57. -|2|

**58.** |8|

**▶ 59.** |-4|

**60.** |-6|

**0 61.** |0|

**62.** |-1|

**63.** - | -3|

**64.** -|-11|

Write the opposite of each number. See Example 7.

**○ 65.** −6.2

**66.** −7.8

**O** 67.

71.

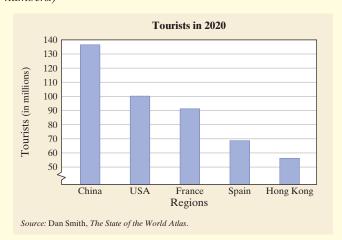
Translating Write each phrase as an algebraic expression. Use the variable x to represent each unknown number. See Example 8.

- 73. Twice a number.
- 74. Six times a number.
- 75. Five more than twice a number.
- **76.** One more than six times a number.

- **77.** Ten less than a number.
  - 78. A number minus seven.
  - 79. The sum of a number and two.
  - 80. The difference of twenty-five and a number.
  - **81.** A number divided by eleven.
  - **82.** The quotient of a number and thirteen.
- **S3.** Twelve, minus three times a number.
  - **84.** Four, subtracted from three times a number.
  - **85.** A number plus two and three-tenths.
  - **86.** Fifteen and seven-tenths plus a number
  - 87. One and one-third less a number.
  - 88. A number less two and three-fourths.
  - 89. The quotient of five and the difference of four and a number.
  - 90. The quotient of four and the sum of a number and one.
  - 91. Twice the sum of a number and three.
  - 92. Eight times the difference of a number and nine.

## **CONCEPT EXTENSIONS**

Use the bar graph below to complete the given table by estimating the millions of tourists predicted for each region. (Use whole numbers.)



93.	China	
94.	France	
95.	Spain	
96.	Hong Kong	

- **▶ 97.** Explain why -(-2) and -|-2| simplify to different numbers.
- **\ 98.** The boxed definition of absolute value states that |a| = -aif a is a negative number. Explain why |a| is always nonnegative, even though |a| = -a for negative values of a.
- **♦ 99.** In your own words, explain why every natural number is also a rational number but not every rational number is a natural number.
- **100.** In your own words, explain why every irrational number is a real number but not every real number is an irrational number.
- **101.** In your own words, explain why the empty set is a subset of every set.
- **102.** In your own words, explain why every set is a subset of itself.

# Operations on Real Numbers and Order of Operations

## **OBJECTIVES**

- 1 Add and Subtract Real Numbers.
- 2 Multiply and Divide Real Numbers.
- 3 Evaluate Expressions Containing Exponents.
- 4 Find Roots of Numbers.
- 5 Use the Order of Operations.
- 6 Evaluate Algebraic Expressions.

**OBJECTIVE** 

#### Adding and Subtracting Real Numbers 1



When solving problems, we often have to add real numbers. For example, if the New Orleans Saints lose 5 yards in one play, then lose another 7 yards in the next play, their total loss may be described by -5 + (-7).

The addition of two real numbers may be summarized by the following.

## **Adding Real Numbers**

- 1. To add two numbers with the same sign, add their absolute values and attach their common sign.
- **2.** To add two numbers with *different signs*, subtract the smaller absolute value from the larger absolute value and attach the sign of the number with the larger absolute value.

For example, to add -5 + (-7), first add their absolute values.

$$|-5| = 5$$
,  $|-7| = 7$ , and  $5 + 7 = 12$ 

Next, attach their common negative sign.

$$-5 + (-7) = -12$$

(This represents a total loss of 12 yards for the New Orleans Saints in the example above.)

To find (-4) + 3, first subtract their absolute values.

$$|-4| = 4$$
,  $|3| = 3$ , and  $4 - 3 = 1$ 

Next, attach the sign of the number with the larger absolute value.

$$(-4) + 3 = -1$$

## EXAMPLE I Add.

$$a. -3 + (-11)$$

**b.** 
$$3 + (-7)$$

$$\mathbf{c.} -10 + 15$$

**d.** 
$$-8.3 + (-1.9)$$
 **e.**  $-\frac{1}{4} + \frac{1}{2}$  **f.**  $-\frac{2}{3} + \frac{3}{7}$ 

$$e. -\frac{1}{4} + \frac{1}{2}$$

**f.** 
$$-\frac{2}{3} + \frac{3}{7}$$

## **Solution**

$$\mathbf{a} \cdot -3 + (-11) = -(3 + 11) = -14$$

**b.** 
$$3 + (-7) = -4$$

$$\mathbf{c} \cdot -10 + 15 = 5$$

**d.** 
$$-8.3 + (-1.9) = -10.2$$

$$\mathbf{e} \cdot -\frac{1}{4} + \frac{1}{2} = -\frac{1}{4} + \frac{1 \cdot 2}{2 \cdot 2} = -\frac{1}{4} + \frac{2}{4} = \frac{1}{4}$$

**f.** 
$$-\frac{2}{3} + \frac{3}{7} = -\frac{14}{21} + \frac{9}{21} = -\frac{5}{21}$$

PRACTICE

Add. 1

$$a. -6 + (-2)$$

**b.** 
$$5 + (-8)$$

c. 
$$-4 + 9$$

**d.** 
$$-3.2 + (-4.9)$$
 **e.**  $-\frac{3}{5} + \frac{2}{3}$  **f.**  $-\frac{5}{11} + \frac{3}{22}$ 

$$e. -\frac{3}{5} + \frac{2}{3}$$

$$\mathbf{f.} - \frac{5}{11} + \frac{3}{22}$$

Subtraction of two real numbers may be defined in terms of addition.

## **Subtracting Real Numbers**

If a and b are real numbers.

$$a - b = a + (-b)$$

In other words, to subtract a real number, we add its opposite.

## **EXAMPLE 2** Subtract.

**a.** 
$$2 - 8$$

**b.** 
$$-8 - (-1)$$

**a.** 
$$2-8$$
 **b.**  $-8-(-1)$  **c.**  $-11-5$  **d.**  $10.7-(-9.8)$ 

**e.** 
$$-\frac{2}{3} - \frac{1}{2}$$
 **f.** 1 - 0.06 **g.** Subtract 7 from 4.

**Solution** Add the opposite

**a.** 
$$2 - 8 = 2 + (-8) = -6$$

Add the opposite

a. 
$$2-8=2+(-8)=-6$$

Add the opposite

b.  $-8-(-1)=-8+(1)=-7$ 

**c.** 
$$-11 - 5 = -11 + (-5) = -16$$
 **d.**  $10.7 - (-9.8) = 10.7 + 9.8 = 20.5$ 

$$\mathbf{e.} - \frac{2}{3} - \frac{1}{2} = -\frac{2 \cdot 2}{3 \cdot 2} - \frac{1 \cdot 3}{2 \cdot 3} = -\frac{4}{6} + \left(-\frac{3}{6}\right) = -\frac{7}{6}$$

**f.** 
$$1 - 0.06 = 1 + (-0.06) = 0.94$$
 **g.**  $4 - 7 = 4 + (-7) = -3$ 

$$\mathbf{g} \cdot 4 - 7 = 4 + (-7) = -3$$

#### PRACTICE

2 Subtract.

**b.** 
$$-6 - (-3)$$

**c.** 
$$-7 - 3$$

**a.** 
$$3 - 11$$
 **b.**  $-6 - (-3)$  **c.**  $-7 - 5$  **d.**  $4.2 - (-3.5)$ 

**e.** 
$$-\frac{5}{7} - \frac{1}{3}$$

**e.** 
$$-\frac{5}{7} - \frac{1}{3}$$
 **f.** 3 – 1.2 **g.** Subtract 9 from 2.

To add or subtract three or more real numbers, add or subtract from left to right.

## **EXAMPLE 3** Simplify the following expressions.

**a.** 
$$11 + 2 - 7$$

**b.** 
$$-5 - 4 + 2$$

**Solution** 

a. 
$$11 + 2 - 7 = 13 - 7 = 6$$

**a.** 
$$11 + 2 - 7 = 13 - 7 = 6$$
 **b.**  $-5 - 4 + 2 = -9 + 2 = -7$ 

### PRACTICE

Simplify the following expressions.

**a.** 
$$13 + 5 - 6$$

**b.** 
$$-6 - 2 + 4$$

### **OBJECTIVE**

#### Multiplying and Dividing Real Numbers 2



To discover sign patterns when you multiply real numbers, recall that multiplication by a positive integer is the same as repeated addition. For example,

$$3(2) = 2 + 2 + 2 = 6$$
  
 $3(-2) = (-2) + (-2) + (-2) = -6$ 

Notice here that 3(-2) = -6. This illustrates that the product of two numbers with different signs is negative. We summarize sign patterns for multiplying any two real numbers as follows.

## **Multiplying Two Real Numbers**

The product of two numbers with the *same* sign is positive.

The product of two numbers with *different* signs is negative.

$$0 \cdot a = 0$$

## **Product Property of 0**

$$0 \cdot a = 0$$
 Also  $a \cdot 0 = 0$ 

## **EXAMPLE 4** Multiply.

**a.** 
$$(-8)(-1)$$
 **b.**  $-2\left(\frac{1}{6}\right)$  **c.**  $-1.2(0.3)$  **d.**  $0(-11)$ 

**b.** 
$$-2\left(\frac{1}{6}\right)$$

**e.** 
$$\frac{1}{5} \left( -\frac{10}{11} \right)$$

**e.** 
$$\frac{1}{5} \left( -\frac{10}{11} \right)$$
 **f.**  $(7)(1)(-2)(-3)$  **g.**  $8(-2)(0)$ 

## **Solution**

- a. Since the signs of the two numbers are the same, the product is positive. Thus (-8)(-1) = +8, or 8.
- **b.** Since the signs of the two numbers are different or unlike, the product is negative. Thus  $-2\left(\frac{1}{6}\right) = -\frac{2}{6} = -\frac{1}{3}$ .

$$\mathbf{c} \cdot -1.2(0.3) = -0.36$$

**d.** 
$$0(-11) = 0$$

**e.** 
$$\frac{1}{5}\left(-\frac{10}{11}\right) = -\frac{10}{55} = -\frac{2}{11}$$

**f.** To multiply three or more real numbers, you may multiply from left to right.

$$(7)(1)(-2)(-3) = 7(-2)(-3)$$

$$= -14(-3)$$

$$= 42$$

**g.** Since zero is a factor, the product is zero.

$$(8)(-2)(0) = 0$$

## PRACTICE

#### Multiply. 4

**a.** 
$$(-5)(3)$$
 **b.**  $(-7)\left(-\frac{1}{14}\right)$  **c.**  $5.1(-2)$  **d.**  $14(0)$ 

**e.** 
$$\left(-\frac{1}{4}\right)\left(\frac{8}{13}\right)$$
 **f.**  $6(-1)(-2)(3)$  **g.**  $5(-2.3)$ 

**f.** 
$$6(-1)(-2)(3)$$

The following sign patterns may be helpful when we are multiplying.

- 1. An odd number of negative factors gives a negative product.
- **2.** An even number of negative factors gives a positive product.

Recall that  $\frac{8}{4} = 2$  because  $2 \cdot 4 = 8$ . Likewise,  $\frac{8}{-4} = -2$  because (-2)(-4) = 8.

Also,  $\frac{-8}{4} = -2$  because (-2)4 = -8, and  $\frac{-8}{-4} = 2$  because 2(-4) = -8. From

these examples, we can see that the sign patterns for division are the same as for multiplication.

## **Dividing Two Real Numbers**

The quotient of two numbers with the *same* sign is positive.

The quotient of two numbers with different signs is negative.

Recall from your knowledge of fractions that division by a nonzero real number b is the same as multiplication by  $\frac{1}{b}$ . In other words,

$$\frac{a}{b} = a \div b = a \cdot \frac{1}{b}$$

This means that to simplify  $\frac{a}{b}$ , we can divide by b or multiply by  $\frac{1}{b}$ . The nonzero numbers b and  $\frac{1}{b}$  are called **reciprocals.** 

Notice that b must be a nonzero number. We do not define division by 0. For example,  $5 \div 0$ , or  $\frac{5}{0}$ , is undefined. To see why, recall that if  $5 \div 0 = n$ , a number, then  $n \cdot 0 = 5$ . This is not possible since  $n \cdot 0 = 0$  for any number n and is never 5. Thus far, we have learned that we cannot divide 5 or any other nonzero number by 0.

Can we divide 0 by 0? By the same reasoning, if  $0 \div 0 = n$ , a number, then  $n \cdot 0 = 0$ . This is true for any number n so that the quotient  $0 \div 0$  would not be a single number. To avoid this, we say that

Division by 0 is undefined.

## **EXAMPLE 5**

Divide.

$$\frac{20}{-4}$$
 k

$$-\frac{-9}{-3}$$

**c.** 
$$-\frac{3}{8} \div$$

**d.** 
$$\frac{-40}{10}$$

**a.** 
$$\frac{20}{-4}$$
 **b.**  $\frac{-9}{-3}$  **c.**  $-\frac{3}{8} \div 3$  **d.**  $\frac{-40}{10}$  **e.**  $\frac{-1}{10} \div \frac{-2}{5}$  **f.**  $\frac{8}{0}$ 

**f.** 
$$\frac{8}{0}$$

## Solution

- **a.** Since the signs are different or unlike, the quotient is negative and  $\frac{20}{-4} = -5$ .
- **b.** Since the signs are the same, the quotient is positive and  $\frac{-9}{-3} = 3$ .

**c.** 
$$-\frac{3}{8} \div 3 = -\frac{3}{8} \cdot \frac{1}{3} = -\frac{1}{8}$$
 **d.**  $\frac{-40}{10} = -4$ 

**d.** 
$$\frac{-40}{10} = -4$$

$$\mathbf{e} \cdot \frac{-1}{10} \div \frac{-2}{5} = -\frac{1}{10} \cdot -\frac{5}{2} = \frac{1}{4}$$

**f.** 
$$\frac{8}{0}$$
 is undefined.

PRACTICE

Divide. 5

**a.** 
$$\frac{-16}{8}$$

**b.** 
$$\frac{-15}{-3}$$

**c.** 
$$-\frac{2}{3} \div 4$$

**d.** 
$$\frac{54}{-9}$$

**e.** 
$$-\frac{1}{12} \div \left(-\frac{3}{4}\right)$$
 **f.**  $\frac{0}{-7}$ 

**f.** 
$$\frac{0}{-7}$$

With sign rules for division, we can understand why the positioning of the negative sign in a fraction does not change the value of the fraction. For example,

$$\frac{-12}{3} = -4$$
,  $\frac{12}{-3} = -4$ , and  $-\frac{12}{3} = -4$ 

Since all the fractions equal -4, we can say that

$$\frac{-12}{3} = \frac{12}{-3} = -\frac{12}{3}$$

In general, the following holds true.

If a and b are real numbers and  $b \neq 0$ , then  $\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$ .

#### OBJECTIVE

#### Evaluating Expressions Containing Exponents 3



Recall that when two numbers are multiplied, they are called factors. For example, in  $3 \cdot 5 = 15$ , the 3 and 5 are called factors.

A natural number exponent is a shorthand notation for repeated multiplication of the same factor. This repeated factor is called the **base**, and the number of times it is used as a factor is indicated by the **exponent.** For example,

exponent
$$4^{3} = \underbrace{4 \cdot 4 \cdot 4}_{4 \text{ is a factor 3 times}} = 64$$

## **Exponents**

If a is a real number and n is a natural number, then the nth power of a, or a raised to the *n*th power, written as  $a^n$ , is the product of *n* factors, each of which is *a*.

$$a^{n} = \underbrace{a \cdot a \cdot a \cdot a \cdot \cdots \cdot a}_{a \text{ is a factor } n \text{ times.}}$$

It is not necessary to write an exponent of 1. For Example, 3 is assumed to be  $3^1$ .

#### **EXAMPLE 6** Evaluate each expression.

**a.** 
$$3^2$$

**b.** 
$$\left(\frac{1}{2}\right)^4$$

**d.** 
$$(-5)^2$$

**e.** 
$$-5^3$$

**f.** 
$$(-5)^3$$

## **Solution**

**a.** 
$$3^2 = 3 \cdot 3 = 9$$

**b.** 
$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{16}$$

$$\mathbf{c.} -5^2 = -(5 \cdot 5) = -25$$

**d.** 
$$(-5)^2 = (-5)(-5) = 25$$

**e.** 
$$-5^3 = -(5 \cdot 5 \cdot 5) = -125$$

**c.** 
$$-5^2 = -(5 \cdot 5) = -25$$
 **d.**  $(-5)^2 = (-5)(-5) = 25$  **e.**  $-5^3 = -(5 \cdot 5 \cdot 5) = -125$  **f.**  $(-5)^3 = (-5)(-5)(-5) = -125$ 

## PRACTICE

**6** Evaluate each expression.

**a.** 
$$2^3$$

**b.** 
$$\left(\frac{1}{3}\right)^2$$

**c.** 
$$-12^2$$

**d.** 
$$(-12)^2$$

**e.** 
$$-4^3$$

**f.** 
$$(-4)^3$$

## **CONCEPT CHECK**

When  $(-8.2)^7$  is evaluated, will the value be positive or negative? How can you tell without making any calculations?

Helpful Hint

Be very careful when simplifying expressions such as  $-5^2$  and  $(-5)^2$ .

$$-5^2 = -(5 \cdot 5) = -25$$
 and  $(-5)^2 = (-5)(-5) = 25$ 

Without parentheses, the base to square is 5, not -5.

**OBJECTIVE** 

#### Finding Roots of Numbers 4



The opposite of squaring a number is taking the **square root** of a number. For example, since the square of 4, or  $4^2$ , is 16, we say that a square root of 16 is 4. The notation  $\sqrt{a}$ denotes the **positive**, or **principal**, **square root** of a nonnegative number a. We then have in symbols that  $\sqrt{16} = 4$ . The negative square root of 16 is written  $-\sqrt{16} = -4$ . The square root of a negative number such as  $\sqrt{-16}$  is not a real number. Why? There is no real number that, when squared, gives a negative number.

**EXAMPLE 7** Find the square roots.

**a.** 
$$\sqrt{9}$$

**b.** 
$$\sqrt{25}$$

**c.** 
$$\sqrt{\frac{1}{4}}$$

**a.** 
$$\sqrt{9}$$
 **b.**  $\sqrt{25}$  **c.**  $\sqrt{\frac{1}{4}}$  **d.**  $-\sqrt{36}$  **e.**  $\sqrt{-36}$ 

**e.** 
$$\sqrt{-36}$$

Solution

**a.** 
$$\sqrt{9} = 3$$
 since 3 is positive and  $3^2 = 9$ . **b.**  $\sqrt{25} = 5$  since  $5^2 = 25$ .

**b.** 
$$\sqrt{25} = 5 \text{ since } 5^2$$

**c.** 
$$\sqrt{\frac{1}{4}} = \frac{1}{2} \operatorname{since} \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

**d.** 
$$-\sqrt{36} = -6$$

e. 
$$\sqrt{-36}$$
 is not a real number.

PRACTICE

Find the square roots. 7

**a.** 
$$\sqrt{49}$$
 **b.**  $\sqrt{\frac{1}{16}}$  **c.**  $-\sqrt{64}$  **d.**  $\sqrt{-64}$  **e.**  $\sqrt{100}$ 

**b.** 
$$\sqrt{\frac{1}{16}}$$

**c.** 
$$-\sqrt{64}$$

**d.** 
$$\sqrt{-64}$$

**e.** 
$$\sqrt{100}$$

We can find roots other than square roots. Since  $(-2)^3$  is -8, we say that the **cube root** of -8 is -2. This is written as

$$\sqrt[3]{-8} = -2.$$

Also, since  $3^4 = 81$  and 3 is positive,

$$\sqrt[4]{81} = 3.$$

**EXAMPLE 8** Find the roots.

**a.** 
$$\sqrt[3]{-27}$$

**b.** 
$$\sqrt[5]{1}$$

**c.** 
$$\sqrt[4]{16}$$

Solution

**a.** 
$$\sqrt[3]{-27} = -3$$
 since  $(-3)^3 = -27$ .

**b.** 
$$\sqrt[5]{1} = 1$$
 since  $1^5 = 1$ .

**c.** 
$$\sqrt[4]{16} = 2$$
 since 2 is positive and  $2^4 = 16$ .

PRACTICE

Find the roots.

**a.** 
$$\sqrt[3]{64}$$

**b.** 
$$\sqrt[5]{-1}$$

**c.** 
$$\sqrt[4]{10,000}$$

Of course, as mentioned in Section 1.2, not all roots simplify to rational numbers. We study radicals further in Chapter 7.

#### OBJECTIVE

#### Using the Order of Operations 5



Expressions containing more than one operation are written to follow a particular agreed-upon **order of operations.** For example, when we write  $3 + 2 \cdot 10$ , we mean to multiply first and then add.

## **Order of Operations**

Simplify expressions using the order that follows.

If grouping symbols such as parentheses are present, simplify expressions within those first, starting with the innermost set. If fraction bars are present, simplify the numerator and denominator separately.

- 1. Evaluate exponential expressions, roots, or absolute values in order from left to right.
- 2. Multiply or divide in order from left to right.
- 3. Add or subtract in order from left to right.

## Helpful Hint

Fraction bars, radical signs, and absolute value bars can sometimes be used as grouping symbols. For example,

	Fraction Bar	Radical Sign	Absolute Value Bars
Grouping Symbol	$\frac{-1-7}{6-11}$	$\sqrt{15 + 1}$	$ -7.2 - \sqrt{4} $
Not Grouping Symbol	$-\frac{8}{9}$	$\sqrt{9}$	-3.2

#### **EXAMPLE 9** Simplify.

**a.** 
$$20 \div 2 \cdot 10$$

**b.** 
$$1 + 2(1 - 4)^2$$

**b.** 
$$1 + 2(1-4)^2$$
 **c.**  $\frac{|-2|^3 + 1}{-7 - \sqrt{4}}$ 

### Solution

a. Be careful! Here, we multiply or divide in order from left to right. Thus, divide, then multiply.

$$20 \div 2 \cdot 10 = 10 \cdot 10 = 100$$

**b.** Remember the order of operations so that you are *not* tempted to add 1 and 2 first.

$$1 + 2(1 - 4)^2 = 1 + 2(-3)^2$$
 Simplify inside grouping symbols first.  

$$= 1 + 2(9)$$
 Write  $(-3)^2$  as 9.  

$$= 1 + 18$$
 Multiply.  

$$= 19$$
 Add.

c. Simplify the numerator and the denominator separately; then divide.

$$\frac{|-2|^3 + 1}{-7 - \sqrt{4}} = \frac{2^3 + 1}{-7 - 2}$$
 Write  $|-2|$  as 2 and  $\sqrt{4}$  as 2.  

$$= \frac{8 + 1}{-9}$$
 Write  $2^3$  as 8.  

$$= \frac{9}{-9} = -1$$
 Simplify the numerator, then divide.

PRACTICE

9 Simplify.

**a.** 
$$14 - 3 \cdot 4$$

**b.** 
$$3(5-8)^2$$

c. 
$$\frac{|-5|^2+4}{\sqrt{4}-3}$$

Besides parentheses, other symbols used for grouping expressions are brackets [ ] and braces { }. These other grouping symbols are commonly used when we group expressions that already contain parentheses.

**EXAMPLE 10** Simplify: 
$$3 - [(4 - 6) + 2(5 - 9)]$$

Solution 
$$3 - [(4-6) + 2(5-9)] = 3 - [-2 + 2(-4)]$$
 Simplify within the innermost sets of parentheses.  
=  $3 - [-2 + (-8)]$  parentheses.  
=  $3 - [-10]$  = 13

Helpful Hint

When grouping symbols occur within grouping symbols, remember to perform operations on the innermost set first.

PRACTICE

10 Simplify: 
$$5 - [(3-5) + 6(2-4)]$$

Simplify:  $\frac{-5\sqrt{30-5+(-2)^2}}{4^2+|7-10|}$ **EXAMPLE 11** 

**Solution** Here, the fraction bar, radical sign, and absolute value bars serve as grouping symbols. Thus, we simplify within the radical sign and absolute value bars first, remembering to calculate above and below the fraction bar separately.

$$\frac{-5\sqrt{30-5} + (-2)^2}{4^2 + |7-10|} = \frac{-5\sqrt{25} + (-2)^2}{4^2 + |-3|} = \frac{-5 \cdot 5 + 4}{16+3} = \frac{-25+4}{16+3}$$
$$= \frac{-21}{19} \text{ or } -\frac{21}{19}$$

PRACTICE Simplify: 
$$\frac{-2\sqrt{12+4}-(-3)^2}{6^2+|1-9|}$$

## **CONCEPT CHECK**

True or false? If two people use the order of operations to simplify a numerical expression and neither makes a calculation error, it is not possible that they each obtain a different result. Explain.

## OBJECTIVE

## **Evaluating Algebraic Expressions**



Recall from Section 1.2 that an algebraic expression is formed by numbers and variables connected by the operations of addition, subtraction, multiplication, division, raising to powers, and/or taking roots. Also, if numbers are substituted for the variables in an algebraic expression and the operations performed, the result is called **the value of the** expression for the given replacement values. This entire process is called evaluating an expression.

## **EXAMPLE 12** Evaluate each expression when x = 4 and y = -3.

**a.** 
$$3x - 7y$$
 **b.**  $-2y^2$  **c.**  $\frac{\sqrt{x}}{v} - \frac{y}{x}$ 

**Solution** For each expression, replace x with 4 and y with -3.

**a.** 
$$3x - 7y = 3 \cdot 4 - 7(-3)$$
 Let  $x = 4$  and  $y = -3$ .  
 $= 12 - (-21)$  Multiply.  
 $= 12 + 21$  Write as an addition.  
 $= 33$  Add.

$$b. -2y^2 = \underbrace{-2(-3)^2}$$

$$= -2(9)$$

$$= -18$$
Add.

Helpful Hint

In  $-2(-3)^2$ , the exponent 2 goes with the base of  $-3$  only.

c. 
$$\frac{\sqrt{x}}{y} - \frac{y}{x} = \frac{\sqrt{4}}{-3} - \frac{-3}{4}$$

$$= -\frac{2}{3} + \frac{3}{4} \qquad \text{Write } \sqrt{4} \text{ as } 2.$$

$$= -\frac{2}{3} \cdot \frac{4}{4} + \frac{3}{4} \cdot \frac{3}{3} \quad \text{The LCD is } 12.$$

$$= -\frac{8}{12} + \frac{9}{12} \qquad \text{Write each fraction with a denominator of } 12.$$

$$= \frac{1}{12} \qquad \text{Add.}$$

#### PRACTICE

**12** Evaluate each expression when x = 16 and y = -5

**a.** 
$$2x - 7y$$
 **b.**  $-4y^2$  **c.**  $\frac{\sqrt{x}}{y} - \frac{y}{x}$ 

Sometimes variables such as  $x_1$  and  $x_2$  will be used in this book. The small 1 and 2 are called **subscripts.** The variable  $x_1$  can be read as "x sub 1," and the variable  $x_2$  can be read as "x sub 2." The important thing to remember is that they are two different variables. For example, if  $x_1 = -5$  and  $x_2 = 7$ , then

$$x_1 - x_2 = -5 - 7 = -12.$$

**EXAMPLE 13** The algebraic expression  $\frac{5(x-32)}{9}$  represents the equivalent tempera-

ture in degrees Celsius when x is the temperature in degrees Fahrenheit. Complete the following table by evaluating this expression at the given values of x.

Degrees Fahrenheit	х	-4	10	32
Degrees Celsius	$\frac{5(x-32)}{9}$			

<u>Solution</u> To complete the table, evaluate  $\frac{5(x-32)}{9}$  at each given replacement value.

When x = -4,

$$\frac{5(x-32)}{9} = \frac{5(-4-32)}{9} = \frac{5(-36)}{9} = -20$$

When x = 10,

$$\frac{5(x-32)}{9} = \frac{5(10-32)}{9} = \frac{5(-22)}{9} = -\frac{110}{9} \text{ or } -12\frac{2}{9}$$

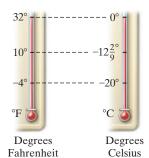
When x = 32,

$$\frac{5(x-32)}{9} = \frac{5(32-32)}{9} = \frac{5 \cdot 0}{9} = 0$$

The completed table is

Degrees Fahrenheit	х	-4	10	32
Degrees Celsius	$\frac{5(x-32)}{9}$	-20	$-\frac{110}{9}$ or $-12\frac{2}{9}$	0

Thus,  $-4^{\circ}$ F is equivalent to  $-20^{\circ}$ C,  $10^{\circ}$ F is equivalent to  $-\frac{110^{\circ}}{9}$ C, and  $32^{\circ}$ F is equivalent to 0°C.



PRACTICE

The algebraic expression  $\frac{9}{5}x + 32$  represents the equivalent temperature in 13 degrees Fahrenheit when x is the temperature in degrees Celsius. Complete the following table by evaluating this expression at the given values of x.

Degrees Celsius	x	-5	10	25
Degrees Fahrenheit	$\frac{9}{5}x + 32$			

## Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may be used more than once and some used not at all.

undefined exponent reciprocal square root

- **1.**  $0 \cdot a =$ \_\_\_\_\_.
- 2.  $\frac{0}{4}$  simplifies to \_\_\_\_\_ while  $\frac{4}{0}$  is \_\_\_\_\_.
- 3. The \_\_\_\_\_ of the nonzero number b is  $\frac{1}{h}$ .
- **4.** The fraction  $-\frac{a}{b} =$ \_\_\_\_\_\_\_ = \_\_\_\_\_.
- **5.** A(n) is a shorthand notation for repeated multiplication of the same number.
- **6.** In  $(-5)^2$ , the 2 is the \_\_\_\_\_ and the -5 is the \_\_\_\_\_.
- 7. The opposite of squaring a number is taking the of a number.
- **8.** Using order of operations,  $9 \div 3 \cdot 3 =$