Preface

Since publication of the third edition, considerable changes have occurred and are still occurring in the industry. The changes include a nuclear renaissance worldwide followed by the unfortunate events in Japan at Fukushima Daiichi and the development and construction of advanced plant designs here and abroad. To keep up to date with these and other changes, Chapter 4 was revised to include a discussion of these new reactor types as well as candidates for the next generation of reactors. Since nuclear engineering is now of worldwide interest, extensive updates to the discussion about the application of nuclear technology outside the US are included as well as additional problems using the SI system of units. Licensing in the US has also advanced since the third edition. The new process now used in the US is included in Chapter 11. Changes in health physics that were in the offing when the third edition was published are included in Chapter 9. Numerous tables were updated to include more recent data and information relevant to nuclear engineering. A discussion of the Fukushima Daiichi accident is also included in Chapter 11.

Yet, even as this book is readied for printing the nuclear industry once again is in a state of flux. New reactors in the United States and overseas are under construction. In the United States, the new reactors being built are overshadowed by the premature shutdown of older generation reactors decreasing the total number of reactors and ultimately the percentage of electricity generated by nuclear energy.

Only a few years ago, US nuclear power was experiencing a renaissance with applications for over 25 new reactors. With the encouragement of the US Government coupled with the high price of natural gas, the future of the industry was optimistic. The proposed reactors were the latest designs capable of maintaining core cooling during accidents for days without any source of electrical power. Also, the final solution for spent fuel disposal looked promising with the submittal of the Yucca Mountain Project license application to the US Nuclear Regulatory Commission (NRC).

The financial crisis of 2007–2008 created a dramatic drop in electricity growth. Coupled with the availability of cheap natural gas from fracking, the ability of nuclear to compete economically caused many prospective licensees to rethink their commitment to building new reactors or keeping existing reactors operating. Today, there are only five new reactors under construction—Watts Bar Unit 2, Vogtle 3 and 4, and Summer 2 and 3. The Vogtle and Summer reactors are both Westinghouse AP1000s, the latest design PWRs. The Watts Bar Unit 2 construction was originally begun in the 1980s and is the last of the earlier generation Westinghouse PWRs built in the United States.

Outside the United States the situation is different, yet similar. Countries that never had a nuclear power program have new reactors under construction. The United Arab Emirates has four Korean-design Advanced Pressurized Water Reactors (APWRs) under construction. In other countries with existing programs, there are major expansions. China has 36 operating reactors and another 20 under construction. In 2016 alone China added five new reactors to the electric grid. The opposite is true in Germany where there once was a robust program and now there is a commitment to shut down all the reactors as a response to the Fukushima Daiichi accident. Worldwide, there are 60 new reactors currently under construction, with most being built in Asia and Central and Eastern Europe.

The Yucca Mountain Project is also on hold. The application for a license was submitted to the US NRC by the Department of Energy in 2008 despite strong objection by the State of Nevada. Because of efforts by Nevada's influential Congressional delegation no funding to complete the licensing process has been appropriated effectively preventing the licensing process from moving forward. However, in 2015, the NRC issued the Safety Evaluation Report (SER) with a finding "that with reasonable assurance, subject to proposed conditions, DOE's application meets the NRC's regulatory requirements" for safety prior to closure. In 2016, the NRC issued the Final Supplement to the Yucca Mountain Environmental Impact Statement (EIS) with the conclusion that the "potential impacts on groundwater and surface groundwater discharges and determines all impacts would be 'small.'"

The combination of the SER conclusion and the EIS would provide sufficient basis to allow a hearing to go forth on technical challenges by the State of Nevada and other parties. Depending on the outcome of the hearings, the Commission could authorize issuance of the license. However, until funding is obtained, the process is held in abeyance and no further work envisioned. Because of the impasse with Congress, efforts are underway to license interim storage facilities to collect spent fuel from where it is stored and place it in a central location. To use funds collected for the Yucca Mountain facility, however, requires a change in the Nuclear Waste Policy Act, something strongly opposed by Congressional supporters of Yucca Mountain. It is unlikely that either the Yucca Mountain licensing or the construction of an interim facility will go forward in the near future because of the political division within Congress. Meanwhile, the US NRC has examined the continued storage of spent fuel at reactor sites and concluded in its EIS, "For at-reactor storage, the unavoidable adverse environmental impacts for each resource area are SMALL for all timeframes with the exception of waste management impacts, which are SMALL to MODERATE for the indefinite storage timeframe, and historic and cultural resource impacts, which are SMALL to LARGE for the long-term and indefinite storage timeframes."

The premature shutdown of some nuclear plants and the low number of new plants in the US is likely to have a very negative impact on the US effort to reduce greenhouse gas emissions by 2025 to 26 to 28% below 2005 levels. Nuclear has a very small carbon footprint when compared to other sources since the nuclear fission process emits no carbon. Table 1 compares the estimated CO2 emissions in grams per kWh of carbon for various energy technologies.

TABLE 1 COMPARISON OF ESTIMATED CARBON EMISSIONS FOR VARIOUS TECHNOLOGIES[1]

Technology	Carbon Emissions (g/kWh)
Coal	900–1000
Combined cycle gas turbine*	500
Photovoltaic	50–100
Wind	5–30
Nuclear	6–26
Hydro*	3–11

*Does not include methane emissions which though small in tonnage are 34 times more potent than carbon dioxide in their impact on global warming.

[1] From "The Role of Nuclear Power in Reducing Greenhouse Gas Emissions" by Anthony Baratta, from Global Climate Change-The Technology Challenge, F. Princiotta, ed. (Springer, 2011)

In the case of nuclear, the studies include the entire lifecycle from mining to utilization in the reactor. Construction emissions are spread out over the lifecycle of the plant assuming an average capacity factor. For combined cycle and hydro, methane leakage emissions from gas pipelines in the case of gas turbines and rotting organic matter in dammed rivers in the case of hydro are not included. If these are included, the amount of greenhouse gas emitted increases significantly per kWh.

Nuclear, which currently accounts for about 60% of the carbon free energy in the United States, is a significant low carbon emitting energy source.

Under the US Environmental Protection Administration's Clean Power plan, guidelines for power plants, the power sector would reduce its emissions by 2030 to 30% below the 2005 level. The plan envisioned that the power sector would achieve this level by assuming all the then existing 100 nuclear power reactors would continue to operate and not be replaced by natural gas–fired facilities. Unfortunately, that has not happened due to unfavorable economics. Since 2012, 14 of 15 reactors have been closed or are scheduled to be closed (as of this writing, the single unit Fitzpatrick facility may remain open under a plan proposed by the current Governor of NY State). Many of these reactors had a decade or more before their licenses would expire. The primary cause for most of these closures is cheap natural gas and in some cases subsidized wind power. These closures are occurring despite climate change caused by greenhouse gas emissions. The market place is not rewarding nuclear for its near zero carbon contribution. Unless a change occurs to recognize the benefit of the only low carbon source to generate power despite wind or cloud cover, it is difficult to understand how the United States will achieve the goals of the Clean Power plan.

A case in point is Germany. All nuclear power plants are to be phased out in response to the Fukushima Daiichi accident. Through a concerted effort, renewable energy generated about 30% of the nation's electricity in 2015 or 194 billion kWh. Yet of the 273 billion kWh generated by soft and hard coal, an amount that has hardly changed despite the growth in renewable energy in Germany. Reliability and distribution problems remain, making it questionable if Germany will reach its goal of a carbon-free energy sector by 2030.

A recent report to the Secretary of Energy by the Secretary's Advisory Board concluded, "Current policies and market designs fail to recognize fully the zero-carbon, base load, nonproliferation, and other values of nuclear power generation in the United States."[2] If nuclear is to remain and grow in the United States despite competition from cheap natural gas and subsidized wind, the market must change to recognize the benefit of this low-carbon energy source

[2] Secretary of Energy Advisory Board Report of the Task Force on the Future of Nuclear Power, U.S. Department of Energy, September 22, 2016.

capable of power generation regardless of weather conditions. Such recognition could include rewarding plant operators for the availability of their facilities when idle because of adequate wind or solar. The low carbon emission of nuclear when compared to natural gas could be encouraged through caps on emissions, a carbon tax, or reward for zero carbon emission. Expanding state Renewable Portfolio Standard programs to become "Low-Carbon Portfolio Standard" programs or equivalent by including nuclear and other zero-carbon technologies might help the most. Such a change would recognize nuclear's unique ability to generate large quantities of energy with zero carbon or other greenhouse gas emissions.

Last of all, people must realize that all sources of energy have their benefits and drawbacks.

New to this edition

Updates to this edition include:

- Coverage of the latest reactor types under construction in the US and elsewhere around the world as well as the next generation of reactors, GEN IV and SMR's is included in Chapter 4.
- Discussion of the Fukushima Daiichi accident and its consequences was added to Chapter 11.
- Throughout the text problems were changed to use the SI system as well as the English system of units.
- Description of the new licensing process now in use in the US is included in Chapter 11.
- Chapter 9 was revised to include the latest changes in health physics regulations and approaches since publication of the Third Edition.
- All of the chapter were updated to include more recent data and information relevant to nuclear engineering both in the US and internationally.

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Dedication

To all those who had faith in this project particularly my family.

Tony

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1

Nuclear Engineering

Nuclear engineering is an endeavor that makes use of radiation and radioactive material for the benefit of mankind. Nuclear engineers, like their counterparts in chemical engineering, endeavor to improve the quality of life by manipulating basic building blocks of matter. Unlike chemical engineers however, nuclear engineers work with reactions that produce millions of times more energy per reaction than any other known material. Originating from the nucleus of an atom, nuclear energy has proved to be a tremendous source of energy.

Despite its association with the atomic bombs dropped during World War II and the arms race of the cold war, nuclear energy today provides a significant amount of energy on a global scale. Many are now heralding it as a source free from the problems of fossil fuels—greenhouse gas emissions. Despite these benefits there is still the association of nuclear power with the tremendous destructive force exhibited by the bombings in Japan.

With the end of the cold war nuclear engineering is largely focused on the use of nuclear reactions to either generate power or on its application in the medical field. Nuclear power applications generally involve the use of the fission reactions in large central power stations and smaller mobile power plants used primarily for ship propulsion. The world demand for electricity is again increasing and with it the need for new generation facilities. For those areas of the world that have little in the way of fossil fuels or have chosen to use these for feedstock in the petrochemical industry, nuclear power is considered the source of choice for electricity generation. In the United States alone nuclear power generates nearly 20% of the electricity. In other countries, notably France, the proportion approaches 80%. Thirteen countries rely on nuclear power for over one quarter of their electricity.

Recent concerns over the emission of nitrous oxides and carbon dioxide have increased the concern about continued use of fossil fuels as a source of energy. The Kyoto accords, developed in 1997, require a reduction in emissions below current values. These targets can be reached in the United States only by lowering the living standards or by continuing use of nuclear power and other renewables for the generation of electricity. A typical 1,000-megawatt coal burning plant may emit as much as 100,000 tons of sulfur dioxide, 75,000 tons of nitrogen oxides, and 5,000 tons of fly ash in 1 year. Nuclear power plants produce none of these air pollutants and emit only trace amounts of radioactive gasses. As a result, in 2014, the use of nuclear power to generate 20% of the electricity in the United States avoided the emission of 595 million metric tonnes of CO_2. This accounts for 62.9% of the U.S. emission free generation.

To date, the widest application of nuclear power in mobile systems has been for the propulsion of naval vessels, especially submarines and aircraft carriers. Here the tremendous advantages of nuclear power are utilized to allow extended operations without support ships. In the case of the submarine, the ability to cruise without large amounts of oxygen for combustion enables the submarine to remain at sea underwater for almost limitless time. In the case of an aircraft carrier, the large quantity of space that was taken up by fuel oil in a conventionally powered aircraft carrier can be devoted to aviation fuel and other supplies on a nuclear-powered aircraft carrier.

In addition to naval vessels, nuclear-powered merchant ships were also developed. The U.S. ship *Savannah*, which operated briefly in the late 1960s and early 1970s, showed that nuclear power for a merchant ship while practical was not economical. Other countries including Japan, Germany, and the former Soviet Union have also used nuclear power for civilian surface ship propulsion. Of these, the German ore carrier, *Otto Hahn*, operated successfully for 10 years but was retired since it too proved uneconomical. The icebreaker *Lenin* of the former Soviet Union demonstrated another useful application of nuclear power. The trial was so successful that the Soviets built additional ships of this type.

Nuclear power has also been developed for aircraft and space applications. From 1949 to 1961 when the project was terminated, the United States spent approximately $1 billion to develop a nuclear-powered airplane. The project, the Aircraft Nuclear Propulsion Project (ANP), was begun at a time when the United States did not have aircraft that could fly roundtrip from the U.S. mainland to a distant adversary. Because of the enormous range that could be expected from a nuclear-powered airplane, the range problem would have been easily resolved. With the advent of long-range ballistic missiles which could be fired from the mainland or from submarines, the need for such an aircraft disappeared and the program was terminated.

Nuclear-powered spacecraft have been developed and are in use today. Typically a nuclear reaction is used to provide electricity for probes that are intended for use in deep space. There, photovoltaic systems cannot provide sufficient energy because of the weak solar radiation found in deep space. Typically, a radioactive source is used and the energy emitted is converted into heat and then electricity using thermocouples. Nuclear-powered rockets are under consideration as well. The long duration of a manned flight to Mars, for example suggests that nuclear power would be useful if not essential. The desirability of a nuclear rocket for such long-distance missions stems from the fact that the total vehicular mass required for a long-distance mission is considerably less if the vehicle is powered by a nuclear rocket rather than by a conventional chemical rocket. The estimated mass of a chemical rocket required for a manned mission from a stationary parking orbit to an orbit around Mars is approximately 4,100,000 kg. The mass of a nuclear rocket for the same mission is estimated to be only 430,000 kg. Nuclear rockets have been under active development in the United States for many years.

The application of radiation and nuclear reactions is not limited to nuclear explosives and nuclear power. Radiation and radioactive isotopes are useful in a wide range of important applications. The production of radioisotopes whether from reactors or accelerators, is a major industry in its own right. The applications of radiation and radioisotopes range from life-saving medical procedures to material characterization to food preservation.

Radioactive tracing is one such method. In this technique, one of the atoms in a molecule is replaced by a radioactive atom of the same element. A radioactive carbon atom may be substituted for a normal carbon atom at a particular location in a molecule when the molecule is synthesized. Later, after the molecule has reacted chemically, either in a laboratory experiment or a biological system, it is possible to determine the disposition of the atom in question by observing the radiation emanating from the radioactive atom. This technique has proved to be of enormous value in studies of chemical reaction processes and in research in the life sciences. A similar procedure is used in industry to measure and sometimes to control the flow and mixing of fluids. A small quantity of radioactive material is placed in the moving fluid and the radiation is monitored downstream. By proper calibration it is possible to relate the downstream radiation level with the fluid's rate of flow or the extent of its dilution. In a similar way radioactive atoms may be incorporated at the time of fabrication into various moving parts of machinery such as pistons and tool bits. The radioactivity observed in the lubricating fluid then becomes an accurate measure of the rate of wear of the part under study.

A related technique known as *activation analysis* is based on the fact that every species of radioactive atom emits its own characteristic radiations. The chemical composition of a substance can therefore be determined by observing the radiation emitted when a small sample of the substance is caused to become radioactive. This may be done by exposing the sample to beams of either neutrons or charged particles. Because it is possible to determine extremely minute concentrations in this way (in some cases, one part in 10^{12}), activation analysis has proved to be a valuable tool in medicine, law enforcement, pollution control, and other fields in which trace concentrations of certain elements play an important role.

2

Atomic and Nuclear Physics

A knowledge of atomic and nuclear physics is essential to the nuclear engineer because these subjects form the scientific foundation on which the nuclear engineering profession is based. The relevant parts of atomic and nuclear physics are reviewed in this chapter and the next.

2.1 FUNDAMENTAL PARTICLES

The physical world is composed of combinations of various subatomic or fundamental particles. A number of fundamental particles have been discovered. This led to the discovery that these fundamental particles are in turn made up of quarks bound together by gluons.

In current theory, particles of interest to the nuclear engineer may be divided into leptons and hadrons. The electron, positron, and neutrino are leptons. Hadrons of interest are the proton and neutron which belong to a subclass of hadrons called *baryons*. The leptons are subject to the weak nuclear forces whereas hadrons and baryons in particular experience both the weak and strong nuclear forces. It is the hadrons that are composed of quarks, and it is the exchange of gluons between collections of quarks that is responsible for the strong nuclear force.

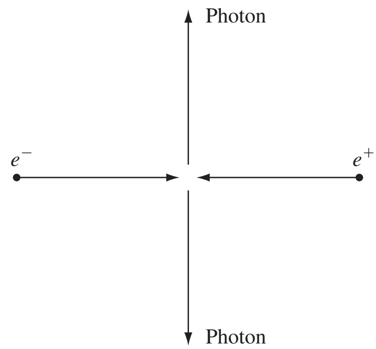

Figure 2.1 The annihilation of a negatron and positron with the release of two photons.

For the understanding of nuclear reactors and the reactions of interest to their operation, it is only important to consider a class of these particles and not explore their structure. Of these particles only the following are important in nuclear engineering.[1]

Electron The electron has a rest mass[2] $m_e = 9.10939 \times 10^{-31}$ kg[3] and carries a charge $e = 1.60219 \times 10^{-19}$ coulombs. There are two types of electrons: one carrying a negative charge $-e$ and the other carrying a positive charge $+e$. Except for the difference in the sign of their charge these two particles are identical. The negative electrons, or *negatrons* as they are sometimes called are the normal electrons encountered in this world. Positive electrons, or *positrons*, are relatively rare. When under the proper circumstances a positron collides with a negatron, the two electrons disappear and two (and occasionally more) photons (particles of electromagnetic radiation) are emitted as shown in Fig. 2.1. This process is known as *electron annihilation*, and the photons that appear are called *annihilation radiation*.

[1]However that is not to say that the internal structure of these particles as described by quark theory does not manifest itself in nuclear reactions. A case in point is the radioactive decay by electron emission to be discussed later. Unfortunately space does not permit a discussion of quark theory in this text. Such discussions may be found in several of the particle physics references at the end of this chapter.

[2]According to the theory of relativity the mass of a particle is a function of its speed relative to the observer. In giving the masses of the fundamental particles it is necessary to specify that the particle is at rest with respect to the observer—hence the term *rest mass*.

[3]A discussion of units, their symbols, and abbreviations, together with tables of conversion factors, are found in Appendix I at the end of this book. Tabulations of fundamental constants and nuclear data are given in Appendix I.

Proton This particle has a rest mass $m_e = 1.6726 \times 10^{-27}$ kg and carries a positive charge equal in magnitude to the charge on the electron. Protons with negative charge have also been discovered but these particles are of no importance in nuclear engineering.

Neutron The mass of the neutron is slightly larger than the mass of the proton—namely, $m_n = 1.674929 \times 10^{-27}$ kg, and it is electrically neutral. The neutron is not a stable particle except when it is bound into an atomic nucleus. A free neutron decays to a proton with the emission of a negative electron (β-decay; see Section 2.8) and an antineutrino, a process that takes, on the average, about 12 minutes.

Photon It is a curious fact that all particles in nature sometimes behave like particles and sometimes like waves. Thus, certain phenomena that are normally thought of as being strictly wavelike in character also appear to have an associated corpuscular or particle like behavior. Electromagnetic waves fall into this category. The particle associated with electromagnetic waves is called the *photon*. This is a particle with zero rest mass and zero charge, which travels in a vacuum at only one speed—namely, the speed of light, $c = 2.9979 \times 10^8$ m/sec.

Neutrino This is another particle with near zero rest mass[4] and no electrical charge that appears in the decay of certain nuclei. There are at least six types of neutrinos, only two of which (called *electron neutrinos* and *electron antineutrinos*) are important in the atomic process and are of interest in nuclear engineering. For most purposes it is not necessary to make a distinction between the two and they are lumped together as neutrinos.

2.2 ATOMIC AND NUCLEAR STRUCTURES

As the reader is doubtless aware atoms are the building blocks of gross matter as it is seen and felt. The atom consists of a small but massive nucleus surrounded by a cloud of rapidly moving (negative) electrons. The nucleus is composed of protons and neutrons. The total number of protons in the nucleus is called the *atomic number* of the atom and is given the symbol Z. The total electrical charge of the nucleus is therefore $+Ze$. In a neutral atom, there are as many electrons as protons—namely Z—moving about the nucleus. The electrons are responsible for

[4]Until recently neutrinos were thought to not have mass. Recent experiments have shown they have a mass many thousands of times smaller than other particles.

the chemical behavior of atoms and identify the various chemical elements. For example hydrogen (H) has one electron, helium (He) has two, lithium (Li) has three, and so on.

The number of neutrons in a nucleus is known as the *neutron number* and is denoted by N. The total number of *nucleons*—protons and neutrons in a nucleus—is equal to $Z + N = A$, where A is called the *atomic mass number* or *nucleon number*.

The various species of atoms whose nuclei contain particular numbers of protons and neutrons are called *nuclides*. Each nuclide is denoted by the chemical symbol of the element (this specifies Z) with the atomic mass number as superscript (this determines N since $N = A - Z$). Thus, the symbol ^{1}H refers to the nuclide of hydrogen ($Z = 1$) with a single proton as nucleus; ^{2}H is the hydrogen nuclide with a neutron as well as a proton in the nucleus (^{2}H is also called *deuterium* or *heavy hydrogen*); ^{4}He is the helium ($Z = 2$) nuclide whose nucleus consists of two protons and two neutrons; and so on. For greater clarity Z is sometimes written as a subscript, as in $^{1}_{1}H$, $^{2}_{1}H$, $^{4}_{2}He$, and so on.

Atoms such as ^{1}H and ^{2}H, whose nuclei contain the same number of protons but different numbers of neutrons (same Z but different N—therefore different A), are known as *isotopes*. Oxygen, for instance, has three stable isotopes, ^{16}O, ^{17}O, and ^{18}O ($Z = 8$; $N = 8, 9, 10$), and five known unstable (i.e., *radioactive*) isotopes, ^{13}O, ^{14}O, ^{15}O, ^{19}O, and ^{20}O ($Z = 8$; $N = 5, 6, 7, 11, 12$).

The stable isotopes (and a few of the unstable ones) are the atoms that are found in the naturally occurring elements. However they are not found in equal amounts; some isotopes of a given element are more abundant than others. For example 99.8% of naturally occurring oxygen atoms are the isotope ^{16}O, 0.037% are the isotope ^{17}O, and 0.204% are ^{18}O. A table of some of the more important isotopes and their abundance is given in Appendix II. It should be noted that isotopic abundances are given in atom percent—that is the percentages of the atoms of an element that are particular isotopes. Atom percent is often abbreviated as *a/o*.

Example 2.1

A glass of water is known to contain 6.6×10^{24} atoms of hydrogen. How many atoms of deuterium (^{2}H) are present?

Solution. According to Table II.2 in Appendix II, the isotopic abundance of ^{2}H is 0.015 *a/o*. The fraction of the hydrogen which is ^{2}H, is therefore 1.5×10^{-4}. The total number of ^{2}H atoms in the glass is then $1.5 \times 10^{-4} \times 6.6 \times 10^{24} = 9.9 \times 10^{20}$. [*Ans.*]

2.3 ATOMIC AND MOLECULAR WEIGHTS

The *atomic weight* of an atom is defined as the mass of the neutral atom relative to the mass of a neutral ^{12}C atom on a scale in which the atomic weight of ^{12}C is arbitrarily taken to be precisely 12. In symbols let $m(^AZ)$ be the mass of the neutral atom denoted by ^AZ and $m(^{12}C)$ be the mass of neutral ^{12}C. Then, the atomic weight of ^AZ, $M(^AZ)$, is given by

$$M(^AZ) = 12 \times \frac{m(^AZ)}{m(^{12}C)}. \tag{2.1}$$

Suppose that some atom was precisely twice as heavy as ^{12}C. According to Eq. (2.1), this atom would have the atomic weight of $12 \times 2 = 24$.

As noted in Section 2.2, the elements found in nature often consist of a number of isotopes. The atomic weight of the element is then defined as the *average* atomic weight of the mixture. Thus if γ_i is the isotopic abundance in atom percent of the ith isotope of atomic weight M_i, then the atomic weight of the element is

$$M = \sum_i \gamma_i M_i / 100. \tag{2.2}$$

The total mass of a molecule relative to the mass of a neutral ^{12}C atom is called the *molecular weight*. This is merely the sum of the atomic weights of the constituent atoms. For instance, oxygen gas consists of the molecule O_2 and its molecular weight is therefore $2 \times 15.99938 = 31.99876$.

Example 2.2

Using the data in the following table compute the atomic weight of naturally occurring oxygen.

Isotope	**Abundance (*a/o*)**	**Atomic weight**
^{16}O	99.759	15.99492
^{17}O	0.037	16.99913
^{18}O	0.204	17.99916

Solution. From Eq. (2.2), it follows that

$$M(O) = \frac{1}{100}\left[\gamma(^{16}O)\cdot M(^{16}O) + \gamma(^{17}O)\cdot M(^{17}O) + \gamma(^{18}O)\cdot M(^{18}O)\right]$$

$$= 15.99938. \ [Ans.]$$

It must be emphasized that atomic and molecular weights are unitless numbers, being ratios of the masses of atoms or molecules. By contrast the *gram atomic weight* and *gram molecular weight* are defined as the amount of a substance having a mass, in grams, equal to the atomic or molecular weight of the substance. This amount of material is also called a *mole*. Thus one gram atomic weight or one mole of ^{12}C is exactly 12 g of this isotope, one mole of O_2 gas is 31.99876 g and so on.

Since atomic weight is a ratio of atomic masses and a mole is an atomic weight in grams, it follows that the number of atoms or molecules in a mole of any substance is a constant, independent of the nature of the substance. For instance suppose that a hypothetical nuclide has an atomic weight of 24.0000. It follows that the individual atoms of this substance are exactly twice as massive as ^{12}C. Therefore there must be the same number of atoms in 24.0000 g of this nuclide as in 12.0000 g of ^{12}C. This state of affairs is known as *Avogadro's law*, and the number of atoms or molecules in a mole is called *Avogadro's number*. This number is denoted by N_A and is equal to $N_A = 0.6022045 \times 10^{24}$.[5] In meters kilogram seconds, Avogadro number is the number of atoms in a kilogram-mole or $N_A = 0.6022045 \times 10^{27}$.

Using Avogadro's number it is possible to compute the mass of a single atom or molecule. For example since one gram mole of ^{12}C has a mass of 12 g and contains N_A atoms, it follows that the mass of one atom in ^{12}C is

$$m(^{12}\mathrm{C}) = \frac{12}{0.6022045 \times 10^{24}} = 1.99268 \times 10^{-23} \text{ g.}$$

$$= 1.99268 \times 10^{-26} \text{ kg.}$$

There is a more natural unit in terms of which the masses of individual atoms are usually expressed. This is the *atomic mass unit* (abbreviated amu) which is defined as one twelfth the mass of the neutral ^{12}C atom, that is

$$1 \text{ amu} = \tfrac{1}{12} \times m(^{12}\mathrm{C}).$$

Inverting this equation gives

$$m(^{12}\mathrm{C}) = 12 \text{ amu.}$$

[5]Ordinarily a number of this type would be written as 6.0222045×10^{23}. However in nuclear engineering problems, for reasons given in Chap. 3 (Example 3.1), Avogadro's number should always be written as the numerical factor times 10^{24}.

Introducing $m\,^{12}\text{C}$ from the preceding paragraph gives

$$\begin{aligned}1 \text{ amu} &= \tfrac{1}{12} \times 1.99268 \times 10^{-23} \text{ g} = 1/N_A \text{ g}.\\ &= 1.66057 \times 10^{-24} \text{ g}.\\ &= 1.66057 \times 10^{-27} \text{ kg}.\end{aligned}$$

Also from Eq. (2.1),

$$m(^AZ) = \frac{m(^{12}\text{C})}{12} \times M(^AZ),$$

so that

$$m(^AZ) = M(^AZ) \text{ amu}.$$

The mass of any atom in amu is numerically equal to the atomic weight of the atom in question.

2.4 ATOMIC AND NUCLEAR RADII

The size of an atom is somewhat difficult to define because the atomic electron cloud does not have a well-defined outer edge. Electrons may occasionally move far from the nucleus while at other times they pass close to the nucleus. A reasonable measure of atomic size is given by the average distance from the nucleus that the outermost electron is to be found. Except for a few of the lightest atoms, these average radii are approximately the same for all atoms—namely about 2×10^{-10} m. Since the number of atomic electrons increases with increasing atomic number, it is evident that the average electron density in the electron cloud also increases with atomic number.

The nucleus, like the atom, does not have a sharp outer boundary. Its surface is diffuse, although somewhat less than that of an atom. Measurements in which neutrons are scattered from nuclei (see Section 3.5) show that to a first approximation the nucleus may be considered to be a sphere with a radius given by the following formula:

$$R = 1.25\, fm \times A^{1/3}, \tag{2.3}$$

where R is in femtometers (fm) and A is the atomic mass number. One femtometer is 10^{-13} centimeters or 10^{-16} meters.

Since the volume of a sphere is proportional to the cube of the radius it follows from Eq. (2.3) that the volume V of a nucleus is proportional to A. This also means that the ratio A/V—that is the number of nucleons per unit volume—is a constant for all nuclei. This uniform density of nuclear matter suggests that nuclei are similar to liquid drops which also have the same density whether they are large or small. This liquid-drop model of the nucleus accounts for many of the physical properties of nuclei.

2.5 MASS AND ENERGY

One of the striking results of Einstein's theory of relativity is that mass and energy are equivalent and convertible, one to the other. In particular the complete annihilation of a particle or other body of rest mass m_0 releases an amount of energy, E_{rest}, which is given by Einstein's famous formula

$$E_{\text{rest}} = m_0 c^2, \tag{2.4}$$

where c is the speed of light. For example the annihilation of 1 g of matter would lead to a release of $E = 1 \times (2.9979 \times 10^{10})^2 = 8.9874 \times 10^{20}$ ergs $= 8.9874 \times 10^{13}$ joules. This is a substantial amount of energy, which in more common units is equal to about 25 million kilowatt-hours.

Another unit of energy that is often used in nuclear engineering is the *electron volt*, denoted by eV. This is defined as the increase in the kinetic energy of an electron when it falls through an electrical potential of one volt. This is equal to the charge of the electron multiplied by the potential drop—that is

$$\begin{aligned} 1\text{ eV} &= 1.60219 \times 10^{-19}\text{ coulomb} \times 1\text{ volt} \\ &= 1.60219 \times 10^{-19}\text{ joule.} \end{aligned}$$

Other energy units frequently encountered are the MeV (10^6 eV) and the keV (10^3 eV).

Example 2.3

Calculate the rest-mass energy of the electron in MeV.

Solution. From Eq. (2.4) the rest-mass energy of the electron is

$$\begin{aligned} m_e c^2 &= 9.10929 \times 10^{-23} \times (2.9979 \times 10^{10})^2 \\ &= 8.18689 \times 10^{-7}\text{ ergs} = 8.18689 \times 10^{-14}\text{ joule.} \end{aligned}$$

Expressed in MeV this is

$$8.18689 \times 10^{-14}\text{ joule} \div 1.6022 \times 10^{-13}\text{ MeV/joule} = 0.511\text{ MeV. } [\textit{Ans.}]$$

Example 2.4

Compute the energy equivalent of the atomic mass unit.

Solution. This can most easily be computed using the result of the previous example. Since according to Section 2.3, 1 amu = 1.6606×10^{-24} g, it follows that 1 amu is equivalent to

$$\frac{1.6606 \times 10^{-24}\ \text{g/amu}}{9.10939 \times 10^{-31}\ \text{g/electron}} \times 0.511\ \text{MeV/electron} = 931.5\ \text{MeV. } [\textit{Ans.}]$$

When a body is in motion, its mass increases relative to an observer at rest according to the formula

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}, \tag{2.5}$$

where m_0 is its rest mass and v is its speed. From Eq. (2.5), it is seen that m reduces to m_0 as v goes to zero. However as v approaches c, m increases without limit. The *total energy* of a particle is its rest-mass energy plus its kinetic energy, is given by

$$E_{\text{total}} = mc^2, \tag{2.6}$$

where m is as given in Eq. (2.5). Finally the *kinetic energy* E is the difference between the total energy and the rest-mass energy. That is

$$E = mc^2 - m_0c^2 \tag{2.7}$$

$$= m_0c^2\left[\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right]. \tag{2.8}$$

The radical in the first term in Eq. (2.8) can be expanded in powers of $(v/c)^2$ using the binomial theorem. When $v \ll c$, the series may be truncated after the first term. The resulting expression for E is

$$E = \tfrac{1}{2}m_0v^2, \tag{2.9}$$

which is the familiar formula for kinetic energy in classical mechanics. It should be noted that Eq. (2.9) may be used instead of Eq. (2.8) only when the kinetic energy computed from Eq. (2.9) is small compared with the rest-mass energy. That is Eq. (2.9) is valid provided

$$\tfrac{1}{2}m_0v^2 \ll m_0c^2. \tag{2.10}$$

As a practical matter Eq. (2.9) is usually accurate enough for most purposes provided $v \leq 0.2c$ or

$$E \leq 0.02E_{\text{rest}}. \tag{2.11}$$

According to Example 2.3, the rest-mass energy of an electron is 0.511 MeV. From Eq. (2.11), it follows that the relativistic formula Eq. (2.8) must be used for electrons with kinetic energies greater than about 0.02×0.511 MeV $= 0.010$ MeV $= 10$ KeV. Since many of the electrons encountered in nuclear engineering have kinetic energies greater than this it is often necessary to use Eq. (2.8) for electrons.

By contrast, the rest mass of the neutron is almost 1,000 MeV and $0.02\ E_{\text{rest}} = 20$ MeV. In practice neutrons rarely have kinetic energies in excess of 20 MeV. It is permissible in all nuclear engineering problems to calculate the kinetic energy of neutrons from Eq. (2.9). When the neutron mass is inserted into Eq. (2.9) the following formula is obtained:

$$v = 1.383 \times 10^{6}\sqrt{E}, \tag{2.12}$$

where v is in cm/sec and E is the kinetic energy of the neutron in eV.

It is important to recognize that Eqs. (2.8) and (2.9) are valid only for particles with nonzero rest mass; for example, they do not apply to photons. (It should be understood that photons have no rest-mass energy and it is not proper to use the term *kinetic energy* in referring to such particles.) Photons only travel at the speed of light and their total energy is given by quite a different formula—namely

$$E = h\nu, \tag{2.13}$$

where h is Planck's constant and ν is the frequency of the electromagnetic wave associated with the photon. Planck's constant has units of energy $\times$ time; if E is to be expressed in eV, h is equal to 4.136×10^{-15} eV-sec.

2.6 PARTICLE WAVELENGTHS

It was pointed out in Section 2.1 that all of the particles in nature have an associated wavelength. The wavelength λ associated with a particle having momentum p is

$$\lambda = \frac{h}{p}, \tag{2.14}$$

where h is again Planck's constant. For particles of nonzero rest mass p is given by

$$p = m\ , \tag{2.15}$$

where m is the mass of the particle and v is its speed. At nonrelativistic energies, p can be written as

$$p = \sqrt{2m_0 E},$$

where E is the kinetic energy. When this expression is introduced into Eq. (2.14), the particle wavelength becomes

$$\lambda = \frac{h}{\sqrt{2m_0 E}}. \tag{2.16}$$

This formula is valid for the neutrons encountered in nuclear engineering. Introducing the value of the neutron mass gives the following expression for the neutron wavelength:

$$\lambda = \frac{2.860 \times 10^{-9}}{\sqrt{E}}, \tag{2.17}$$

where λ is in centimeters and E is the kinetic energy of the neutron in eV. For the relativistic case, it is convenient to compute p directly by solving the relativistic equations in the preceding section. This gives

$$p = \frac{1}{c}\sqrt{E_{\text{total}}^2 - E_{\text{rest}}^2}, \tag{2.18}$$

and so

$$\lambda = \frac{hc}{\sqrt{E_{\text{total}} - E_{\text{rest}}}}. \tag{2.19}$$

The momentum of a particle of zero rest mass is not given by Eq. (2.15), but rather by the expression

$$p = \frac{E}{c}, \tag{2.20}$$

in which E is the energy of the particle. When Eq. (2.20) is inserted into Eq. (2.14), the result is

$$\lambda = \frac{hc}{E}. \tag{2.21}$$

Introducing numerical values for h and c in the appropriate units gives finally

$$\lambda = \frac{1.240 \times 10^{-6}}{E}, \tag{2.22}$$

where λ is in meters and E is in eV. Equation (2.22) is valid for photons and all other particles of zero rest mass.

2.7 EXCITED STATES AND RADIATION

The Z atomic electrons that cluster about the nucleus move in more or less well-defined orbits. However some of these electrons are more tightly bound in the atom than others. For example, only 7.38 eV is required to remove the outermost electron from a lead atom ($Z = 82$) whereas 88 keV (88,000 eV) is required to remove the innermost or *K-electron*. The process of removing an electron from an atom is called *ionization*, and the energies 7.38 eV and 88 keV are known as the *ionization energies* for the electrons in question.

In a neutral atom, it is possible for the electrons to be in a variety of different orbits or states. The state of lowest energy is the one in which an atom is normally found, and this is called the *ground state*. When the atom possesses more energy than its ground state energy it is said to be in an *excited state* or an *energy level*. The ground state and the various excited states can conveniently be depicted by an *energy level diagram* like the one shown in Fig. 2.2 for hydrogen. The highest energy state corresponds to the situation in which the electron has been completely removed from the atom and the atom is ionized.

An atom cannot remain in an excited state indefinitely; it eventually decays to one or another of the states at lower energy and in this way the atom eventually returns to the ground state. When such a transition occurs a photon is emitted by the atom with an energy equal to the difference in the energies of the two states. For example when a hydrogen atom in the first excited state at 10.19 eV (see Fig. 2.2) decays to the ground state, a photon with an

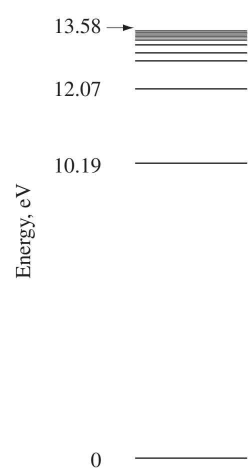

Figure 2.2 The energy levels of the hydrogen atom (not to scale).

energy of 10.19 eV is emitted. From Eq. (2.22) this photon has a wavelength of $\lambda = 1.240 \times 10^{-6}/10.19 = 1.217 \times 10^{-7}$ m. Radiation of this wavelength lies in the ultraviolet region of the electromagnetic spectrum.

Example 2.5

A high-energy electron strikes a lead atom and ejects one of the *K*-electrons from the atom. What wavelength radiation is emitted when an outer electron drops into the vacancy?

Solution. The ionization energy of the *K*-electron is 88 keV and so the atom minus this electron is actually in an excited state 88 keV above the ground state. When the outer electron drops into the *K* position the resulting atom still lacks an electron, but now this is an outer, weakly bound electron. In its final state the atom is excited by only 7.38 eV, much less than its initial 88 keV. Thus, the photon in this transition is emitted with an energy of slightly less than 88 keV. The corresponding wavelength is

$$\lambda = 1.240 \times 10^{-6}/8.8 \times 10^{4} = 1.409 \times 10^{-11} \text{ m. [\textit{Ans.}]}$$

Such a photon is in the x-ray region of the electromagnetic spectrum. This process, the ejection of an inner tightly bound electron followed by the transition of another electron, is one way in which x-rays are produced.

The nucleons in nuclei, like the electrons in atoms, can also be thought of as moving about in various orbits although these are not as well defined and understood as those in atoms. In any case there is a state of lowest energy, the ground state; except for the very lightest nuclei, all nuclei have excited states as well. These states are shown in Fig. 2.3 for ^{12}C. A comparison of Figs. 2.2 and 2.3 shows that the energies of the excited states and the energies between states are considerably greater for nuclei than for atoms. Although this conclusion is based only on the states of hydrogen and ^{12}C it is found to be true in general. This is due to the fact that the nuclear forces acting between nucleons are much stronger than the electrostatic forces acting between electrons and the nucleus.

Nuclei in excited states may decay to a lower lying state, as do atoms, by emitting a photon with an energy equal to the difference between the energies of the initial and final states. The energies of photons emitted in this way from a nucleus are usually much greater than the energies of photons originating in electronic transitions, and such photons are called *γ-rays*.

A nucleus in an excited state can also lose its excitation energy by internal conversion. In this process the excitation energy of the nucleus is transferred into the kinetic energy of one of the innermost atomic electrons. The electron is then

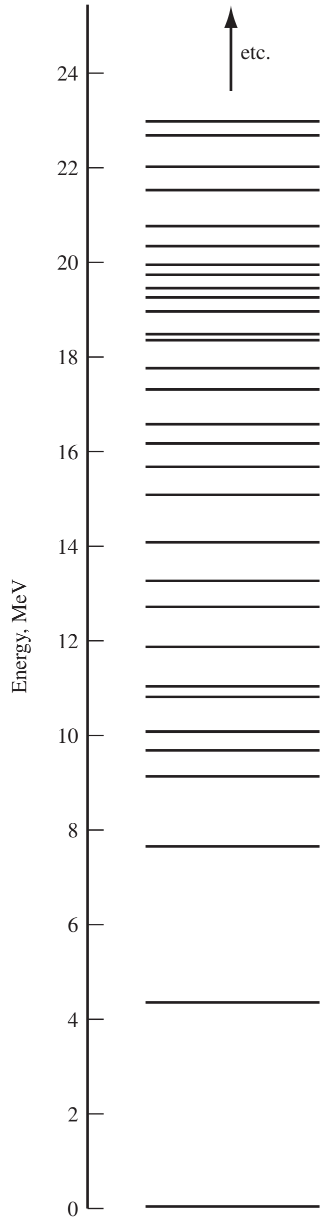

Figure 2.3 The energy levels of carbon 12.

ejected from the atom with an energy equal to that of the nuclear transition less the ionization energy of the electron. Internal conversion thus competes with γ-ray emission in the decay of nuclear-excited states.

The hole remaining in the electron cloud after the departure of the electron in internal conversion is later filled by one of the outer atomic electrons. This transition is accompanied either by the emission of an x-ray or the ejection of another electron in a process similar to internal conversion. Electrons originating in this way are called *Auger electrons*.

2.8 NUCLEAR STABILITY AND RADIOACTIVE DECAY

Figure 2.4 shows a plot of the known nuclides as a function of their atomic and neutron numbers. On a larger scale, with sufficient space provided to tabulate data for each nuclide, Fig. 2.4 is known as a *Segre chart* or the *chart of the nuclides*. The figure depicts that there are more neutrons than protons in nuclides with Z greater than about 20, that is, for atoms beyond calcium in the periodic table. These extra neutrons are necessary for the stability of the heavier nuclei. The excess neutrons act somewhat like nuclear glue, holding the nucleus together by compensating for the repulsive electrical forces between the positively charged protons.

It is clear from Fig. 2.4 that only certain combinations of neutrons and protons lead to stable nuclei. Although generally there are several nuclides

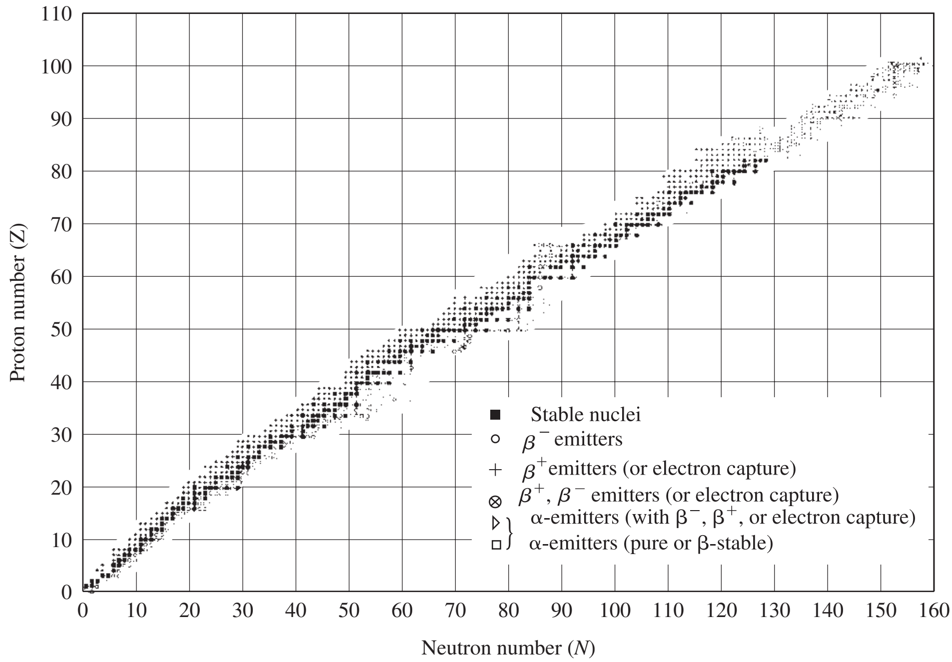

Figure 2.4 The chart of nuclides showing stable and unstable nuclei. (Based on S. E. Liverhant, *Elementary Introduction to Nuclear Reactor Physics*. New York: Wiley, 1960.)

with the same atomic number but different neutron numbers (these are the isotopes of the element), if there are either too many or too few neutrons for a given number of protons, the resulting nucleus is not stable and it undergoes *radioactive decay*. As noted in Section 2.2 the isotopes of oxygen ($A = 8$) with $N = 8$, 9, and 10 are stable, but the isotopes with $N = 5$, 6, 7, 11, and 12 are radioactive. In the case of the isotopes with $N = 5$, 6, and 7, there are not enough neutrons for stability, whereas the isotopes with $N = 11$ and 12 have too many neutrons.

Nuclei such as ^{15}O, which are lacking in neutrons, undergo β^+-*decay*. In this process, one of the protons in the nucleus is transformed into a neutron, and a positron and a neutrino are emitted. The number of protons is thus reduced from 8 to 7 so that the resulting nucleus is an isotope of nitrogen, ^{15}N, which is stable. This transformation is written as

$$^{15}\text{O} \xrightarrow{\beta^+} {}^{15}\text{N} + \nu,$$

where β^+ signifies the emitted positron which in this context is called a β-*ray* and ν denotes the neutrino. By contrast nuclei like ^{19}O, which are excessively neutron-rich, decay by β^- decay, emitting a negative electron and an antineutrino:

$$^{19}\text{O} \xrightarrow{\beta^-} {}^{19}\text{F} + \overline{\nu},$$

where $\overline{\nu}$ stands for the antineutrino. In this case a neutron changes into a proton and the atomic number increases by one unit. It should be noted that in both β^+-decay and β^--decay the atomic mass number remains the same.

In both forms of β-decay, the emitted electrons appear with a continuous energy spectrum like that shown in Fig. 2.5. The ordinate in the figure, $N(E)$, is equal to the number of electrons emitted per unit energy, which have a kinetic energy E. Thus the actual number of electrons emitted with kinetic energies between E and $E + dE$ is $N(E)dE$. It should be noted in the figure that there is a definite maximum energy E_{max} above which no electrons are observed. It has been shown that the average energy of the electrons $\overline{E}$ is approximately equal to $0.3E_{\text{max}}$ in the case of β^--decay. In β^+-decay $\overline{E} \simeq 0.4E_{\text{max}}$.

Frequently, the *daughter nucleus*, the nucleus formed in β-decay, is also unstable and undergoes β-decay. This leads to *decay chains* like the following:

$$^{20}\text{O} \xrightarrow{\beta^-} {}^{20}\text{F} \xrightarrow{\beta^-} {}^{20}\text{Ne (stable)}.$$

A nucleus which is lacking in neutrons can also increase its neutron number by electron capture. In this process an atomic electron interacts with one of the protons in the nucleus, and a neutron is formed of the union. This leaves a vacancy in the electron cloud which is later filled by another electron which in turn leads to the emission of γ-rays, which are necessarily characteristic of the

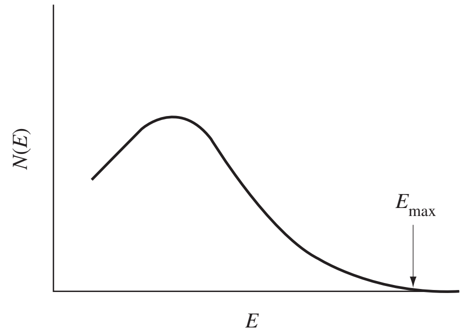

Figure 2.5 A typical energy spectrum of electrons emitted in beta decay.

daughter element or the emission of an Auger electron. Usually the electron that is captured by the nucleus is the innermost or K-electron and so this mode of decay is also called *K-capture*. Since the daughter nucleus produced in electron capture is the same as the nucleus formed in β^+-decay these two processes often compete with one another.

Another way by which unstable nuclei undergo radioactive decay is by the emission of an *α-particle*. This particle is the highly stable nucleus of the isotope ^{4}He, consisting of two protons and two neutrons. The emission of an α-particle reduces the atomic number by two and the mass number by four. Thus the α-decay of $^{238}_{92}$U (uranium-238) leads to $^{234}_{90}$Th (thorium-234) according to the equation

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}.$$

Decay by α-particle emission is comparatively rare in nuclides lighter than lead, but it is common for the heavier nuclei. In marked contrast to β-decay α-particles are emitted in a discrete (line) energy spectrum similar to photon line spectra from excited atoms. This is shown in Table 2.1 where data is given for the four groups of α-particles observed in the decay of ^{226}Ra (radium-226).

The nucleus formed as the result of β-decay (+ or −), electron capture, or α-decay is often left in an excited state following the transformation. The excited (daughter) nucleus usually decays[6] by the emission of one or more γ-rays in the

[6] Strictly speaking the term *decay* should not be used to describe the emission of γ-rays from nuclei in excited states since only the energy and not the character of the nucleus changes in the process. More properly γ-ray emission should be referred to as nuclear *dependent-excitation* not decay. However the use of the term *decay* is well established in the literature.

TABLE 2.1 ALPHA-PARTICLE SPECTRUM OF ^{226}Ra

α-particle energy	Relative number of particles (%)
4.782	94.6
4.599	5.4
4.340	0.0051
4.194	7×10^{-4}

manner explained in Section 2.7. An example of a situation of this kind is shown in Fig. 2.6 for decay of ^{60}Co—a nuclide widely used in nuclear engineering. A diagram like that shown in the figure is known as a *decay scheme*. It should be especially noted that the major γ-rays are emitted by the daughter nucleus, in this case ^{60}Ni although they are frequently attributed to (and arise as the result of) the decay of the parent nucleus ^{60}Co.

Most nuclei in excited states decay by the emission of γ-rays in an immeasurably short time after these states are formed. However owing to peculiarities in their internal structure the decay of certain excited states is delayed to a point where the nuclei in these states appear to be semistable. Such long-lived states are called *isomeric states* of the nuclei in question. The decay by γ-ray emission of one

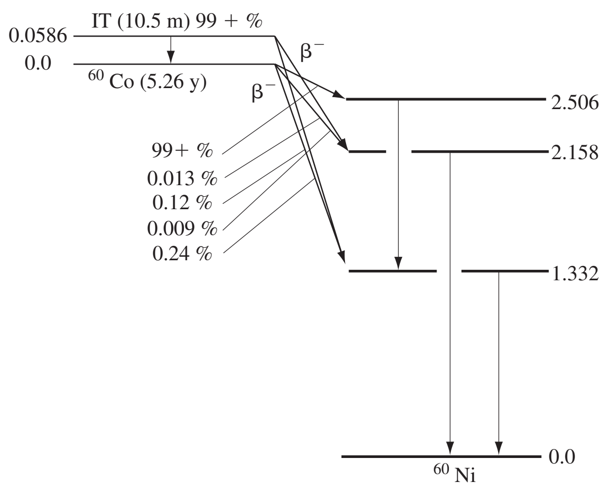

Figure 2.6 Decay scheme of cobalt 60 showing the known radiation emitted. The numbers on the side of the excited states are the energies of these states in MeV above the ground state. The relative occurrence of competing decays is indicated by the various percentages.

of these states is called an *isomeric transition* and is indicated as IT in nuclear data tabulations. In some cases isomeric states may also undergo β-decay. Figure 2.6 shows the isomeric state found at 58 keV above the ground state of ^{60}Co. As a rule, isometric states occur at energies very close to the ground state. This state is formed from the β^--decay of ^{60}Fe, not shown in the figure. It is observed that this isomeric state decays in two ways: either to the ground state of ^{60}Co or by β^--decay to the first two excited states of ^{60}Ni. Of these two decay modes the first is by far the more probable (>99%) and occurs largely by internal conversion.

In summary, a nucleus without the necessary numbers of protons and neutrons for stability will decay by the emission of α-rays or β-rays or undergo electron capture, all of which may be accompanied by the subsequent emission of γ-rays. It must be emphasized that most radioactive nuclei do not ordinarily decay by emitting neutrons or protons. As in the discussion of fission there are exceptions of great importance to reactor control.

2.9 RADIOACTIVITY CALCULATIONS

Calculations of the decay of radioactive nuclei are relatively straightforward, owing to the fact that there is only one fundamental law governing all decay processes. This law states that the probability per unit time that a nucleus will decay is a constant independent of time. This constant is called the *decay constant* and is denoted by λ.[7]

Consider the decay of a sample of radioactive material. If at time t there are $n(t)$ atoms that as yet have not decayed, $\lambda n(t)dt$ of these will decay, on the average, in the time interval dt between t and $t + dt$. The rate at which atoms decay in the sample is therefore $\lambda n(t)$ disintegrations per unit time. This decay rate is called the *activity* of the sample and is denoted by the symbol α. Thus, the activity at time t is given by

$$\alpha(t) = \lambda n(t). \tag{2.23}$$

Activity has traditionally been measured in units of *curies* or the SI unit *becquerels*. One curie denoted as Ci is defined as exactly 3.7×10^{10} disintegrations per second and one becquerel or Bq is defined as exactly one disintegration per second. Units to describe small activities are the *millicurie*, 10^{-3} curie, abbreviated as mCi, the *microcurie*, 10^{-6} curie, denoted by μCi, and the *picocurie*, 10^{-12} curie,

[7] By tradition, the same symbol λ is used for both decay constant and the wavelength defined earlier as well as for mean free path defined later. Because of their very different uses no confusion should arise.

which is written as pCi. One Bq is therfore equal to 2.703×10^{-11} Ci or 27 pCi, one kBq is 2.7×10^{-8} Ci one terabecquerel TBq $= 2.7 \times 10^{12}$ disintegrations per second is 27.027 Ci.

Since $\lambda n(t)dt$ nuclei decay in the time interval dt it follows that the decrease in the number of undecayed nuclei in the sample in the time dt is

$$-dn(t) = \lambda n(t)dt.$$

This equation can be integrated to give

$$n(t) = n_0 e^{-\lambda t}, \tag{2.24}$$

where n_0 is the number of atoms at $t = 0$. Multiplying both sides of Eq. (2.24) by λ gives the activity of the sample at time t—namely,

$$\alpha(t) = \alpha_0 e^{-\lambda t}, \tag{2.25}$$

where α_0 is the activity at $t = 0$. The activity thus decreases exponentially with time.

The time during which the activity falls by a factor of two is known as the *half-life* and is given the symbol $T_{1/2}$ Introducing this definition

$$\alpha(T_{1/2}) = \alpha_0/2$$

into Eq. (2.25) gives

$$\alpha_0/2 = \alpha_0 e^{-\lambda T_{1/2}}.$$

Then taking the logarithm of both sides of this equation and solving for $T_{1/2}$ gives

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}. \tag{2.26}$$

Solving Eq. (2.26) for λ and substituting into Eq. (2.25) gives

$$\alpha(t) = \alpha_0 e^{-0.693t/T_{1/2}}. \tag{2.27}$$

This equation is often easier to use in computations of radioactive decay than Eq. (2.25) especially with the advent of pocket-size electronic calculators and spread sheet programs because half-lives are more widely tabulated than decay constants. Equation (2.27) eliminates the need to compute λ.

It is not difficult to show that the average life expectancy or *mean-life*, $\bar{t}$, of a radioactive nucleus is related to the decay constant by the formula

$$\bar{t} = 1/\lambda.$$

From Eq. (2.25), it can be seen that in one mean-life the activity falls to $1/e$ of its initial value. In view of Eq. (2.26), the mean-life and half-life are related by

$$\bar{t} = \frac{T_{1/2}}{0.693} = 1.44 T_{1/2}. \tag{2.28}$$

The exponential decay of a radioactive sample is shown in Fig. 2.7, where the half-life and mean-life are also indicated.

It is frequently necessary to consider problems in which radioactive nuclides are produced in a nuclear reactor or in the target chamber of an accelerator. Let it be assumed for simplicity that the nuclide is produced at the constant rate of R atoms/sec. As soon as it is formed a radioactive atom may decay. The change in the number of atoms of the nuclide in the time dt is given by a simple rate equation

the time rate of change of the nuclide =

the rate of production − the rate of loss

or symbolically

$$dn/dt = -\lambda n + R.$$

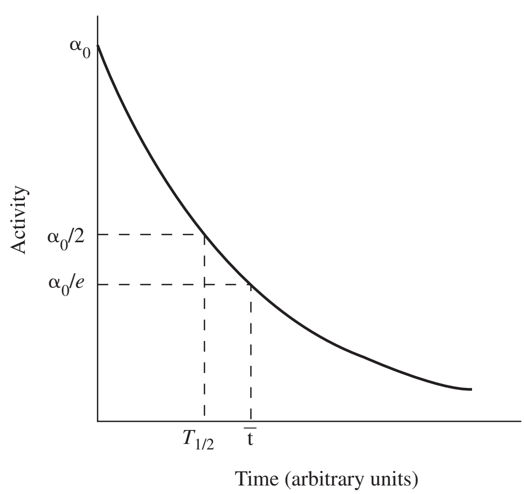

Figure 2.7 The decay of a radioactive sample.

This equation can be integrated with the result

$$n = n_0 e^{-\lambda t} + \frac{R}{\lambda}(1 - e^{-\lambda t}), \tag{2.29}$$

where n_0 is again the number of radioactive atoms present at $t = 0$. Multiplying this equation through by λ gives the activity of the nuclide

$$\alpha = \alpha_0 e^{-\lambda t} + R(1 - e^{-\lambda t}). \tag{2.30}$$

If $\alpha_0 = 0$, then, according to this result, α increases steadily from zero and as $t \rightarrow \infty$, α approaches the maximum value $\alpha_{max} = R$. Similarly n approaches a constant value n_{max} given by R/λ. If $\alpha_0 \neq 0$, then the activity due to decay of the atoms originally present is added to the activity of the newly produced nuclide. In both cases the activity approaches the value $\alpha_{max} = R$ as $t \rightarrow \infty$.

Example 2.6

Gold-198 ($T_{1/2} = 64.8$ hr) can be produced by bombarding stable ^{197}Au with neutrons in a nuclear reactor. Suppose that a ^{197}Au foil weighing 0.1 g is placed in a certain reactor for 12 hrs and its activity is 0.90 Ci when removed.

(a) What is the theoretical maximum activity due to ^{198}Au in the foil in Ci, in Bq?

(b) How long does it take for the activity to reach 80 percent of the maximum?

Solution

1. The value of R in Eq. (2.30) can be found from the data at 12 hrs. From Eq. (2.26), $\lambda = 0.693/64.8 = 1.07 \times 10^{-2}\ \text{hr}^{-1}$. Then substituting into Eq.(2.30) gives

$$0.90 = R[1 - e^{1.07 \times 10^{-2} \times 12}]$$

or from Eq. (2.26)

$$0.90 = R[1 - e^{-0.693 \times 12/64.8}]..$$

Solving either of these equations yields $R = 7.5$ Ci or 0.2775 TBq. (To get R in atoms/sec, which is not required in this problem, it is merely necessary to multiply the prior value of R by 3.7×10^{10}). According to the previous discussion the theoretical maximum activity is also 7.5 Ci or 0.2775 TBq. [*Ans.*]

2. The time to reach 80% of α_{max} can also be found from Eq. (2.30):

$$0.8R = R(1 - e^{\lambda t}).$$

Solving for t gives $t = 150$ hrs. [*Ans.*]

Another problem that is often encountered is the calculation of the activity of the radioactive nuclide B in the decay chain,

$$A \rightarrow B \rightarrow C \rightarrow \ldots .$$

It is clear that an atom of B is formed with each decay of an atom of A. We can write a simple rate equation for this behavior.

the time rate of change of B =

the rate of production from A − the rate of decay of B to C.

Since $\lambda_A n_A$ is the rate atoms of A decay into atoms of B, the rate atoms of B are produced is $\lambda_A n_A$. The rate of decay of B atoms is $\lambda_B n_B$ so the rate of change of B, dn_B/dt is

$$\frac{dn_B}{dt} = -\lambda_B n_B + \lambda_A n_A.$$

Substituting Eq. (2.24) for n_A gives the following differential equation for n_B:

$$\frac{dn_B}{dt} = -\lambda_B n_B + \lambda_A n_{A0} e^{-\lambda_A t}, \tag{2.31}$$

where n_{A0} is the number of atoms of A at $t = 0$. Integrating Eq. (2.31) gives

$$n_B = n_{B0} e^{-\lambda_B t} + \frac{n_{A0}\lambda_A}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}). \tag{2.32}$$

In terms of activity, this equation may be written as

$$\alpha_B = \alpha_{B0} e^{-\lambda_B t} + \frac{\alpha_{A0}\lambda_B}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}), \tag{2.33}$$

where α_{A0} and α_{B0} are the initial activities of A and B respectively. Generalizations of Eq. (2.33) for computing the activity of the nth nuclide in a long decay chain have been derived and are found in the references at the end of the chapter.

2.10 NUCLEAR REACTIONS

A *nuclear reaction* is said to have taken place when two nuclear particles—two nuclei or a nucleus and a nucleon—interact to produce two or more nuclear particles or γ-rays. If the initial nuclei are denoted by a and b and the product nuclei and/or γ-rays (for simplicity it is assumed that there are only two) are denoted by c and d, the reaction can be represented by the equation

$$a + b \rightarrow c + d. \tag{2.34}$$

The detailed theoretical treatment of nuclear reactions is beyond the scope of this book. For present purposes it is sufficient to note four of the fundamental laws governing these reactions:

1. *Conservation of nucleons.* The total number of nucleons before and after a reaction are the same.
2. *Conservation of charge.* The sum of the charges on all the particles before and after a reaction are the same.
3. *Conservation of momentum.* The total momentum of the interacting particles before and after a reaction are the same.
4. *Conservation of energy.* Energy including rest-mass energy is conserved in nuclear reactions.

It is important to note that conservation of nucleons and conservation of charge do not imply conservation of protons and neutrons separately.

The principle of the conservation of energy can be used to predict whether a certain reaction is energetically possible. Consider a reaction of the type given in Eq. (2.34). The total energy before the reaction is the sum of the kinetic energies of the particles a and b plus the rest-mass energy of each particle. Similarly the energy after the reaction is the sum of the kinetic energies of particles c and d plus their rest-mass energies. By conservation of energy it follows that

$$E_a + E_b + M_a c^2 + M_b c^2 = E_c + E_d + M_c c^2 + M_d c^2, \tag{2.35}$$

where E_a, E_b, and so on are the kinetic energies of particles a, b, and so on. Equation (2.35) can be rearranged in the form

$$(E_c + E_d) - (E_a + E_b) = [(M_a + M_b) - (M_c + M_d)]c^2. \tag{2.36}$$

Since the quantities on the left-hand side represent the kinetic energy of the particles, it is evident that the change in the kinetic energies of the particles before and after the reaction is equal to the difference in the rest-mass energies of the particles before and after the reaction.

The right-hand side of Eq. (2.36) is known as the *Q-value* of the reaction; that is

$$Q = [(M_a + M_b) - (M_c + M_d)]c^2. \tag{2.37}$$

In all computations and tabulations, Q is always expressed in MeV. Recalling that 1 amu is approximately 931 MeV, we can write the Q value as

$$Q = [(M_a + M_b) - (M_c + M_d)]931 \text{ MeV}. \tag{2.38}$$

From Eq. (2.36) it is clear that when Q is positive there is a net *increase* in the kinetic energies of the particles. Such reactions are called *exothermic*. When Q is negative there is a net decrease in the energies of the particles and the reaction is said to be *endothermic*. Since in exothermic reactions there is a net decrease in mass, nuclear mass is converted into kinetic energy, while in endothermic reactions there is a net increase, thus kinetic energy is converted into mass.

Equation (2.37) gives Q in terms of the masses of the nuclei a, b, and so on. However the Q value can also be in terms of the masses of the neutral atoms containing these nuclei. Thus, in view of the conservation of charge,

$$Z_a + Z_b = Z_c + Z_d, \tag{2.39}$$

where Z_a, Z_b, and so on, are the atomic numbers of a, b, and so on, and Eq. (2.37) can be put in the form

$$\begin{aligned} Q = \{[(M_a + Z_a m_e) + (M_b + Z_b m_e)] \\ -[(M_c + Z_c m_e) + (M_d + Z_d m_e)]\}931 \text{ MeV}, \end{aligned} \tag{2.40}$$

where m_e is the electron rest mass in amu. But $M_a + Z_a m_e$ is equal to the mass of the neutral atom of a, $M_b + Z_b m_e$ is the mass of the atom of b, and so on. It follows that Eq. (2.37) is a valid formula for Q where M_a, M_b, and so on, are interpreted as the masses (in amu) of the neutral atoms in question although the actual nuclear reaction involves only the atomic nuclei. It is fortunate that Q can be computed from neutral atomic mass data since the masses of most bare nuclei are not accurately known.

Incidentally, in the usual experimental arrangement, one of the particles, say a, is at rest in some sort of target and the particle b is projected against the target. In this case, Eq. (2.34) is often written in the abbreviated form

$$a(b, c)d$$

or

$$a(b, d)c,$$

whichever is the more appropriate. For example when oxygen is bombarded by energetic neutrons one of the reactions that occurs is

$$^{16}\text{O} + n \rightarrow {}^{16}\text{N} + {}^{1}\text{H}.$$

In abbreviated form this is

$$^{16}\text{O}(n, p)^{16}\text{N},$$

where the symbols n and p refer to the incident neutron and emergent proton, respectively.

Example 2.7

Complete the following reaction:

$$^{14}\text{N} + n \rightarrow ? + {}^{1}\text{H}.$$

Solution. The atomic number of ^{14}N is 7, that of the neutron is 0. The sum of the atomic numbers on the left-hand side of the reaction is therefore 7, and the sum on the right must also be 7. Since $Z = 1$ for hydrogen it follows that Z of the unknown nuclide is $7 - 1 = 6$ (carbon). The total number of nucleons on the left is the sum of the atomic mass numbers—namely $14 + 1 = 15$. Since the mass number of ^{1}H is 1 the carbon isotope formed in this reaction must be ^{14}C. Thus the reaction is

$$^{14}\text{N} + n \rightarrow {}^{14}\text{C} + {}^{1}\text{H}.$$

Example 2.8

One of the reactions that occurs when ^{3}H (tritium) is bombarded by deuterons (^{2}H nuclei) is

$$^{3}\text{H}(d, n)^{4}\text{He},$$

where d refers to the bombarding deuteron. Compute the Q value of this reaction.

Solution. The Q value is obtained from the following neutral atomic masses (in amu):

$$\begin{array}{rr} M(^{3}\text{H}) = 3.016049 & M(^{4}\text{He}) = 4.002604 \\ M(^{2}\text{H}) = 2.014102 & M(\text{n}) = 1.008665 \\ \hline M(^{3}\text{H}) + M(^{2}\text{H}) = 5.030151 & M(^{4}\text{He}) + M(\text{n}) = 5.011269 \end{array}$$

Thus from Eq. (2.37), the Q value in amu is $Q = 5.030151 - 5.011269 = 0.018882$ amu. Since 1 amu = 931.502 MeV (see Ex. 2.4), $Q = 0.018882 \times 931.502 = 17.588$ MeV, which is positive and so this reaction is exothermic. This means that when stationary ^{3}H atoms are bombarded by 1-MeV deuterons the sum of the kinetic energies of the emergent α-particle (^{4}He) and neutron is $17.588 + 1 = 18.588$ MeV.

2.11 BINDING ENERGY

When a low-energy neutron and a proton combine to form a deuteron, the nucleus of ^2H, a 2.23-MeV γ-ray is emitted and the deuteron recoils slightly with an energy of about 1.3 keV. The reaction in question is

$$p + n \rightarrow d + \gamma,$$

or, in terms of the neutral atoms

$$^1\mathrm{H} + \mathrm{n} \rightarrow {}^2\mathrm{H} + \gamma.$$

Since the γ-ray escapes from the site of the reaction leaving the deuteron behind, it follows from the conservation of energy that the mass of the deuteron in energy units is approximately 2.23 MeV less than the sum of the masses of the neutron and proton. This difference in mass between the deuteron and its constituent nucleons is called the *mass defect* of the deuteron.

In a similar way the masses of all nuclei are somewhat smaller than the sum of the masses of the neutrons and protons contained in them. The mass defect for an arbitrary nucleus is the difference

$$\Delta = ZM_p + NM_n - M_A, \tag{2.41}$$

where M_A is the mass of the nucleus. Equation (2.41) can also be written as

$$\Delta = Z(M_p + m_e) + NM_n - (M_A + Zm_e), \tag{2.42}$$

where m_e is the mass of an electron. The quantity $M_p + m_e$ is equal to the mass of neutral ^1H while $M_A + Zm_e$ is equal to the mass M of the neutral atom. The mass defect of the nucleus is therefore

$$\Delta = ZM(^1H) + NM_n - M, \tag{2.43}$$

which shows that Δ can be computed from the tabulated masses of neutral atoms. Equations (2.41) and (2.43) are not precisely equivalent owing to slight differences in electronic energies but this is not important for most purposes.

When Δ is expressed in energy units it is equal to the energy that is necessary to break the nucleus into its constituent nucleons. This energy is known as the *binding energy* of the system since it represents the energy with which the nucleus is held together. However when a nucleus is produced from A nucleons Δ is equal to the energy released in the process. Thus in the case of the deuteron the binding energy is 2.23 MeV. This is the energy released when the deuteron is formed and it is also the energy required to split the deuteron into a neutron and proton.

The total binding energy of nuclei is an increasing function of the atomic mass number A. However it does not increase at a constant rate. This can be seen most conveniently by plotting the average binding energy per nucleon, Δ/A, versus A, as shown in Fig. 2.8. It is noted that there are a number of deviations from the curve at low A, while above $A = 50$ the curve is a smooth but decreasing function of A. This behavior of the binding energy curve is important in determining possible sources of nuclear energy.

Those nuclei in which the binding energy per nucleon is high are especially stable or tightly bound and a relatively large amount of energy must be supplied to these systems to break them apart. However when such nuclei are formed from their constituent nucleons, a relatively large amount of energy is released. By contrast, nuclei with low binding energy per nucleon can be more easily disrupted and they release less energy when formed.

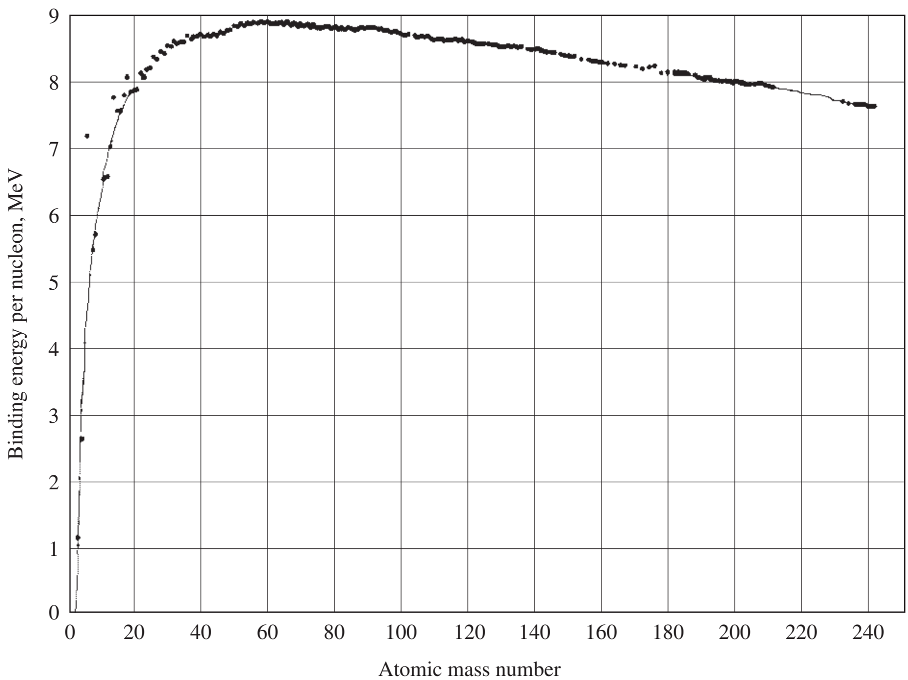

Figure 2.8 Binding energy per nucleon as a function of atomic mass number.

The Q value of a nuclear reaction can be expressed in terms of the binding energies of the reacting particles or nuclei. Consider the reaction given by Eq. (2.34). From Eq. (2.43) the binding energy of a, in units of mass, is

$$\text{BE}(a) = Z_a M(^1\text{H}) + N_a M_n - M_a.$$

The mass of a can then be written as

$$M_a = Z_a M(^1\text{H}) + N_a M_n - \text{BE}(a).$$

Similarly,

$$M_b = Z_b M(^1\text{H}) + N_b M_n - \text{BE}(b),$$

and so on. Substituting these expressions for M_a, M_b, and so on, into Eq. (2.37) and noting that

$$Z_a + Z_b = Z_c + Z_d$$
$$N_a + N_b = N_c + N_d,$$

gives

$$Q = [\text{BE}(c) + \text{BE}(d)] - [\text{BE}(a) - \text{BE}(b)].$$

Here the c^2 has been dropped because Q and the binding energies all have units of energy.

This equation shows that Q is positive—that is the reaction is exothermic—when the total binding energy of the product nuclei is greater than the binding energy of the initial nuclei. Put another way, whenever it is possible to produce a more stable configuration by combining two less stable nuclei energy is released in the process. Such reactions are possible with a great many pairs of nuclides. For instance when two deuterons, each with a binding energy of 2.23 MeV, react to form ^{3}H, having a total binding energy of 8.48 MeV, according to the equation

$$2\,^2\text{H} \rightarrow \,^3\text{H} + \,^1\text{H}, \qquad (2.44)$$

there is a net gain in the binding energy of the system of $8.48 - 2 \times 2.23 = 4.02$ MeV. In this case the energy appears as kinetic energy of the product nuclei ^{3}H and ^{1}H.

Reactions such as Eq. (2.44), in which at least one heavier, more stable nucleus is produced from two lighter, less stable nuclei, are known as *fusion* *reactions*. Reactions of this type are responsible for the enormous release of energy in hydrogen bombs and may some day provide a significant source of thermonuclear power.

In the regions of large A in Fig. 2.8, it is seen that a more stable configuration is formed when a heavy nucleus splits into two parts. The binding energy per nucleon in ^{238}U is about 7.5 MeV whereas it is about 8.4 MeV in the neighborhood of $A = 238/2 = 119$. Thus, if a uranium nucleus divides into two lighter nuclei, each with about half the uranium mass, there is a gain in the binding energy of the system of approximately 0.9 MeV per nucleon which amounts to a total energy release of about $238 \times 0.9 = 214$ MeV. This process is called *nuclear fission* and is the source of energy in nuclear reactors.

It must be emphasized that the binding energy per nucleon shown in Fig. 2.8 is an average overall of the nucleons in the nucleus and does not refer to any one nucleon. Occasionally it is necessary to know the binding energy of a particular nucleon in the nucleus—that is the amount of energy required to extract the nucleon from the nucleus. This binding energy is also called the *separation energy* and is entirely analogous to the ionization energy of an electron in an atom. Consider the separation energy E_s of the least bound neutron—sometimes called the *last* neutron—in the nucleus ^{A}Z. Since the neutron is bound in the nucleus it follows that the mass of the nucleus (and the neutral atom) ^{A}Z is less than the sum of the masses of the neutron and the residual nucleus ^{A-1}Z by an amount, in energy MeV, equal to E_s. In symbols this is

$$E_S = [M_n + M(^{A-1}Z) - M(^{A}Z)]931 \text{ MeV/amu}. \tag{2.45}$$

The energy E_S is just sufficient to remove a neutron from the nucleus without providing it with any kinetic energy. However if this procedure is reversed and a neutron with no kinetic energy is absorbed by the nucleus ^{A-1}Z, the energy E_S is released in the process.

Example 2.9

Calculate the binding energy of the last neutron in ^{13}C.

Solution. If the neutron is removed from ^{13}C the residual nucleus is ^{12}C. The binding energy or separation energy is then computed from Eq. (2.45) as follows:

$$\begin{aligned}
M(^{12}\text{C}) &= 12.00000 \\
\underline{M_n} &= \underline{1.00866} \\
M_n + M(^{12}\text{C}) &= 13.00866 \\
\underline{-M(^{13}\text{C})} &= \underline{13.00335} \\
E_S &= 0.00531 \text{ amu} \quad \times 931 \text{ MeV} = 4.95 \text{ MeV } [\textit{Ans.}]
\end{aligned}$$

Before leaving the discussion of nuclear binding energy it should be noted that nuclei containing 2, 6, 8, 14, 20, 28, 50, 82, or 126 neutrons or protons are especially stable. These nuclei are said to be *magic* and their associated numbers of nucleons are known as *magic numbers*. These correspond to the numbers of neutrons or protons that are required to fill shells (or subshells) of nucleons in the nucleus in much the same way that electron shells are filled in atoms.

The existence of magic nuclei has a number of practical consequences in nuclear engineering. For instance nuclei with a magic neutron number absorb neutrons to only a very small extent and materials of this type can be used where neutron absorption must be avoided. For example Zirconium, whose most abundant isotope contains 50 neutrons, has been widely used as a structural material in reactors for this reason.

2.12 NUCLEAR MODELS

Two models of the nucleus are useful in explaining the various phenomena observed in nuclear physics—the shell model and the liquid drop model. Although neither of these models can completely explain the observed behavior of nuclei, they do provide valuable insight into the nuclear structure and cause many of the nuclear reactions of interest to the nuclear engineer.

Shell Model

The shell model may be thought of as the nuclear analogue to the many electron atom. In this model the collective interaction of the nucleons in the nucleus generate a potential well. One can then think of a single nucleon as if it is moving in the well created by the average effect of the other nucleons. As in the case of other such potential wells such a well can have one or more quantized states. These states are then populated in the same way that the atomic orbitals of an atom are populated by electrons. Just as in the atom there is a maximum number of nucleons that may occupy a shell. When this number is reached a closed shell results.

Although a detailed discussion of this model is beyond the scope of this text, a few remarks are necessary to understand the stability of certain nuclei and the origin of the magic numbers. The neutrons and protons fill each level in a potential well according to the Pauli exclusion principle. Accounting for the angular momentum for each state there are $2j + 1$ possible substates for each level with total angular momentum j.

Since we are dealing with two sets of identical particles—neutrons on the one hand and protons on the other—there are really two such wells, one for each.

They differ by the coulomb interaction of the protons. The levels are then filled according to the exclusion principle. The differing m_j values will split apart in energy because of the spin orbit interaction. As in the case of the many electron atoms this may result in reordering the levels and development of wider gaps in between the energy levels than otherwise would be expected. Since the neutron and proton wells can each have closed shells the nuclei can be extremely stable when both wells have closed shells and less so when neither do. This phenomenon gives rise to the magic numbers discussed earlier.

Liquid Drop Model

From Section 2.11 the binding energy is the mass defect expressed in energy units. The liquid-drop model of the nucleus seeks to explain the mass defect in terms of a balance between the forces binding the nucleons in the nucleus and the coulombic repulsion between the protons.

The nucleus may be thought of as a drop of nuclear liquid. Just as a water droplet experiences a number of forces acting to hold it together, so does the nuclear droplet. To a first approximation the mass of a nuclear droplet is just the mass of the components—the neutrons and protons. These are interacting in the nucleus and are bound by the nuclear forces. The binding of each nucleon to its neighbors means that energy and mass must be added to tear the nucleus apart. The mass may then be approximated by:

$$M = N\,M_n + Z\,M_p - \alpha A. \tag{2.46}$$

Equation 2.46 overestimates the effect of the bonds between the nucleons since those near the surface cannot have the same number of bonds as those deep inside the nucleus. To correct for this a surface correction term must be added:

$$M = N\,M_n + Z\,M_p - \alpha A + 4\pi R^2 T, \tag{2.47}$$

where T denotes the surface tension. Since the radius R of the nucleus is proportional to $A^{1/3}$, we can rewrite this term:

$$M = N\,M_n + Z\,M_p - \alpha A + \beta A^{2/3}. \tag{2.48}$$

The coulombic repulsion tends to increase the energy and hence mass of the nucleus. Using the potential energy associated with the repulsive force the expression becomes:

$$M = N\,M_n + Z\,M_p - \alpha A + \beta A^{2/3} + \gamma Z^2/A^{1/3}. \tag{2.49}$$

There are additional strictly nuclear effects that must be accounted for in the mass equation. These account for a preference for the nucleons to pair together and for the effect of the Pauli exclusion principle.[8]

In the shell model the nucleons were thought of as filling two potential wells. The lowest energy system would then be one in which the number of protons would equal the number of neutrons since the wells would be filled to the same height, with each level filled according to the Pauli exclusion principle. The nucleus having $N = Z$ should then be more stable than the nucleus with $N \neq Z$. To account for this effect a correction term must be added to the mass equation:

$$M = N\,M_n + Z\,M_p - \alpha A + \beta A^{2/3} + \gamma Z^2/A^{1/3} + \zeta(A - 2Z)/A. \qquad (2.50)$$

Finally if one examines the stable nuclides, one finds a preference for nuclei with even numbers of neutrons and protons. The preference reflects that, experimentally, the bond between two neutrons or two protons is stronger than that between a neutron and proton. Nuclei with odd numbers of neutrons and odd numbers of protons would thus be less strongly bound together. When either Z or N is odd and the other even, one would expect the binding to be somewhere in between these two cases. To account for this effect a pairing term denoted by δ is added to the expression:

$$M = N\,M_n + Z\,M_p - \alpha A + \beta A^{2/3} + \gamma Z^2/A^{1/3} + \zeta(A - 2Z)/A + \delta. \qquad (2.51)$$

The term δ is 0 if either N or Z is odd and the other even, positive if both are odd, and negative if both are even. Equation 2.51 is the mass equation.

The coefficients for the mass equation are obtained by fitting the expression to the known nuclei. When this is done the semi-empirical mass equation is obtained. The values for each of the coefficients are typically taken as:

Mass of neutron	939.573 MeV
Mass of proton	938.280 MeV
α	15.56 MeV
β	17.23 MeV
γ	0.697 MeV
ζ	23.285 MeV
δ	12.0 MeV

[8] For a discussion of the Pauli exclusion principle see references on modern physics.

The formula can accurately predict the nuclear masses of many nuclides with $Z > 20$. The ability to predict these masses suggests that there is some truth to the way the liquid-drop models the interactions of the nucleons in the nucleus.

For atomic masses the mass of a hydrogen atom (938.791 MeV) may be substituted for the mass of the proton to account for the mass of the atomic electrons.

Example 2.10

Calculate the mass and binding energy of $^{107}_{47}Ag$ using the mass equation.

Solution. The mass equation may be used to calculate the binding energy by noting that the negative of the sum of the last five terms represents the binding energy of the constituent nucleons. The atomic mass of the $^{107}_{47}Ag$ is first obtained by using the mass formula and noting that N is even and Z is odd. The term involving δ is thus taken as zero.

$N \times m_n$	60×939.573 MeV
$Z \times m_H$	47×938.791 MeV
$-\alpha \times A$	-15.56×107 MeV
$+\beta \times A^{2/3}$	$17.23 \times 107^{2/3}$ MeV
$\gamma \times Z^2/A^{1/3}$	$0.697 \times 47^2/107^{1/3}$ MeV
$\zeta \times (A - 2 \times Z)^2/A$	$23.285 \times (107 - 2 \times 47)^2/107$ MeV
Mass (MeV)	199548.1173 MeV
Mass (amu)	106.8684 amu

The measured mass of $^{107}_{47}Ag$ is 106.905092 amu or within 0.034% of the calculated value. Summing the last four terms gives a total binding energy of 949.44 MeV or 8.9 MeV/nucleon—a value slightly higher than the measured value of approximately 8.6 MeV.

2.13 GASES, LIQUIDS, AND SOLIDS

Before concluding this review of atomic and nuclear physics it is appropriate to consider the nature of gross physical matter since this is the material encountered in all practical problems. Classically there are three so-called states of matter: gas, liquid, and solid. The principal characteristics of these are as follows.

Gases The noble gases—helium, neon, argon, krypton, xenon, and radon—and most metallic vapors are monatomic—that is they are composed of more or less freely moving, independent atoms. Virtually all other gases consist of equally freely moving diatomic or polyatomic molecules. The random, disordered motion of these particles is one of the characteristic features of all gases.

Solids Most of the solids used in nuclear systems—namely metals and ceramics—are crystalline solids. Such solids are composed of large numbers of *microcrystals*, each of which consists of an ordered three-dimensional array or lattice of atoms. Each microcrystal contains an enormous number of individual atoms. Since the regularity in the arrangement of the atoms in the lattice extends over so many atoms (often over the entire microcrystal), such crystals are said to exhibit long-range order. There are a number of other materials that are called *solids* because they are rigid bodies, that do not exhibit long-range order. Examples of such materials are plastics, organic materials, glasses, and various amorphous solids.

Liquids The microscopic structure of liquids is considerably more complicated than is usually assumed. The atoms and/or molecules in a liquid interact strongly with one another; as a result they tend to be ordered as they are in a crystal but not over such long distances. The ordered arrangement breaks down over long distances. For this reason liquids are said to exhibit short-range order.

The Maxwellian Distribution

In a gas the energies of the atoms or molecules are distributed according to the Maxwellian distribution function. If $N(E)$ is the density of particles per unit energy then $N(E)dE$ is the number of particles per unit volume having energies between E and $E + dE$. According to the Maxwellian distribution $N(E)$ is given by the formula

$$N(E) = \frac{2\pi N}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT}. \tag{2.52}$$

In Eq. (2.52) N is the total number of particles per unit volume; that is the particle density; k is Boltzmann's constant, which has units of energy per degree Kelvin:

$$\begin{aligned} k &= 1.3806 \times 10^{-23} \text{ joule}/°\text{K} \\ &= 8.6170 \times 10^{-5} \text{ eV}/\text{K}; \end{aligned}$$

and T is the absolute temperature of the gas in degrees Kelvin. The function $N(E)$ is plotted in Fig. 2.9.

For solids and liquids the energy distribution functions are more complicated than the one given in Eq. (2.52). However it has been shown that to a first approximation $N(E)$ for solids and liquids can also be represented by Eq. (2.52), but the parameter T differs somewhat from the actual temperature

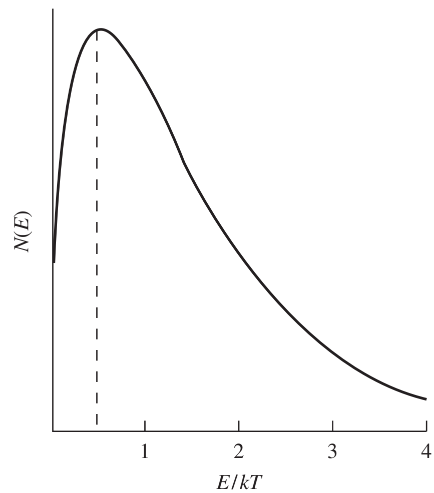

Figure 2.9 The Maxwellian distribution function.

of the substance. The difference is small for temperatures above about 300 K. At these temperatures it can often be assumed that Eq. (2.52) applies to solids, liquids, and gases.

The *most probable energy* in a distribution such as the one given in Eq. (2.52) is defined as the energy corresponding to the maximum of the curve. This can be calculated by placing the derivative of $N(E)$ equal to zero. The most probable energy, E_p in a Maxwellian energy distribution is then easily found to be

$$E_p = \tfrac{1}{2}kT. \tag{2.53}$$

However the *average energy*, $\overline{E}$, is defined by the integral

$$\overline{E} = \frac{1}{N}\int_0^\infty N(E)E\,dE. \tag{2.54}$$

Substituting Eq. (2.52) into Eq. (2.54) and carrying out the integration gives

$$\overline{E} = \tfrac{3}{2}kT. \tag{2.55}$$

The combination of parameters kT in Eqs. (2.53) and (2.55) often appears in the equations of nuclear engineering. Calculations involving these parameters are then expedited by remembering that, for $T_0 = 293.61$ K, kT has the value

$$kT_0 = 0.0253 \text{ eV} \simeq \frac{1}{40}\text{ eV}. \tag{2.56}$$

Example 2.11

What are the most probable and average energies of air molecules in a New York City subway in summertime at 38°C (about 100°F)?

Solution. It is first necessary to compute the temperature in degrees Kelvin. From the formula

$$°\mathrm{K} = °\mathrm{C} + 273.15,$$

it follows that the temperature of the air is 311.15 K. Then using Eqs. (2.53) and (2.55) gives

$$E_p = \frac{1}{2} \times 0.0253 \times \frac{311.15}{293.61} = 0.0134 \text{ eV. } [Ans.]$$

Finally $\overline{E} = 3E_p$ so that $\overline{E} = 0.0402$ eV. [*Ans.*]

The Gas Law

To a first approximation, gases obey the familiar *ideal gas law*

$$PV = n_M RT, \tag{2.57}$$

where P is the gas pressure, V is the volume, n_M is the number of moles of gas contained in V, R is the gas constant, and T is the absolute temperature. Equation (2.57) can also be written as

$$P = \left(\frac{n_M N_A}{V}\right)\left(\frac{R}{N_A}\right)T,$$

where N_A is Avogadro's number. Since there are N_A atoms per mole it follows that there are $n_M N_A$ atoms in V. The first factor is therefore equal to N—the total number of atoms or molecules per unit volume. At the same time the factor R/N_A is the definition of k, Boltzmann's constant. The ideal gas law can thus be put in the convenient form

$$P = NkT. \tag{2.58}$$

From Eq. (2.58) it is seen that gas pressure can be (and in certain applications is) expressed in units of energy per unit volume.

2.14 ATOM DENSITY

In nuclear engineering problems it is often necessary to calculate the number of atoms or molecules contained in 1 cm^3 of a substance. Consider first a material such as sodium, which is composed of only one type of atom. Then if ρ is its physical density in g/cm^3 and M is its gram atomic weight it follows that there are ρ/M gram moles of the substance in 1 cm^3. Since each gram mole contains N_A atoms where N_A is Avogadro's number the atom density N, in atoms per cm^3, is simply

$$N = \frac{\rho N_A}{M}. \tag{2.59}$$

Example 2.12

The density of sodium is 0.97 g/cm^3. Calculate its atom density.

Solution. The atomic weight of sodium is 22.990. Then from Eq. (2.59),

$$N = \frac{0.97 \times 0.6022 \times 10^{24}}{22.990} = 0.0254 \times 10^{24}. \text{ [}Ans.\text{]}$$

(It is usual to express the atom densities as a factor times 10^{24}.)

Equation (2.59) also applies to substances composed of individual molecules except that N is the molecule density (molecules per cm^3) and M is the gram molecular weight. To find the number of atoms of a particular type per cm^3 it is merely necessary to multiply the molecular density by the number, n_i, of those atoms present in the molecule,

$$N_l = n_i \frac{\rho N_A}{M}. \tag{2.60}$$

The computation of atom density for crystalline solids such as NaCl and for liquids is just as easy as for simple atomic and molecular substances but the explanation is more complicated. The problem here is that there are no recognizable molecules—an entire microcrystal of NaCl is a molecule. What should be done in this case is to assume that the material consists of hypothetical molecules containing appropriate numbers of the constituent atoms. (These molecules are in fact unit cells of the crystalline solid and contain the appropriate number of atoms.) Then by using the molecular weight for this pseudomolecule in Eq. (2.59), the computed value of N gives the molecular density of this molecule. The atom densities can then be computed from this number in the usual way as illustrated in the following example.

Example 2.13

The density of a NaCl crystal is 2.17 g/cm^3. Compute the atom densities of Na and Cl.

Solution. The atomic weight of Na and Cl are 22.990 and 35.453 respectively. The molecular weight for a pseudomolecule of NaCl is therefore 58.443. Using Eq. (2.59) gives

$$N = \frac{2.17 \times 0.6022 \times 10^{24}}{58.443} = 0.0224 \times 10^{24} \text{ molecules/cm}^3.$$

Since there is one atom each of Na and Cl per molecule it follows that this is also equal to the atom density of each atom. [*Ans.*]

Frequently it is required to compute the number of atoms of a particular isotope per cm^3. Since, as pointed out in Section 2.2, the abundance of isotopes is always stated in atom percent the atom density of an isotope is just the total atom density of the element as derived earlier, multiplied by the isotopic abundance expressed as a fraction. Thus the atom density N_i for the *i*th isotope is

$$N_l = \frac{\gamma_i \rho N_A}{100M}, \tag{2.61}$$

where γ_i is the isotopic abundance in *atom percent* abbreviated a/o.

The chemical compositions of mixtures of elements such as metallic alloys are usually given in terms of the percent by weight of the various constituents. If ρ is the physical density of the mixture then the average density of the *i*th component is

$$\rho_i = \frac{{}_i\rho}{100}, \tag{2.62}$$

where $_i$ is the *weight percent*, abbreviated w/o, of the component. From Eq. (2.59) it follows that the atom density of this component is

$$N_i = \frac{w_i \rho N_A}{100M_i}, \tag{2.63}$$

where M_i is its gram atomic weight.

With a substance whose composition is specified by a chemical formula, the percent by weight of a particular element is equal to the ratio of its atomic weight in the compound to the total molecular weight of the compound. Thus, with the compound X_mY_n, the molecular weight is $mM_x + nM_y$, where M_x and M_y are the atomic weights of X and Y respectively, and the percent by weight of the element X is

$$w/o(X) = \frac{mM_x}{mM_x + nM_y} \times 100. \tag{2.64}$$