FOREWORD

We are proud of the fact that earlier editions of *Precalculus: Graphical, Numerical, Algebraic* were among the first to recognize the potential of hand-held graphers for helping students understand function behavior. The power of visualization eventually transformed the teaching and learning of calculus at the college level and in the AP® program, then led to reforms in the high school curriculum articulated in the NCTM *Principles and Standards for School Mathematics* and more recently in the Common Core State Standards. All along the way, this text has kept current with the best practices while continuing to pioneer new ideas in exploration and pedagogy that enhance student learning (for example, the study of function behavior based on the Twelve Basic Functions, an idea that has gained widespread acceptance in the text world).

For those students continuing to a calculus course, this precalculus text concludes with a chapter that prepares students for the two central themes of calculus: instantaneous rate of change and continuous accumulation. This intuitively appealing preview of calculus is both more useful and more reasonable than the traditional, unmotivated foray into the computation of limits, and it is more in keeping with the stated goals and objectives of the AP courses and their emphasis on depth of knowledge.

Recognizing that precalculus is a capstone course for many students, we include *quantitative literacy* topics such as probability, statistics, and the mathematics of finance and integrate the use of data and modeling throughout the text. Our goal is to provide students with the critical-thinking skills and mathematical know-how needed to succeed in college, career, or any endeavor.

Continuing in the spirit of the nine earlier editions, we have integrated graphing technology throughout the course, not as an additional topic but as an essential tool for both mathematical discovery and effective problem solving. Graphing technology enables students to study a full catalog of basic functions at the beginning of the course, thereby giving them insights into function properties that are not seen in many texts until later chapters. By connecting the algebra of functions to the visualization of their graphs, we are even able to introduce students to parametric equations, piecewise-defined functions, limit notation, and an intuitive understanding of continuity as early as Chapter 1. However, the advances in technology and increased familiarity with calculators have blurred some of the distinctions between solving problems and supporting solutions that we had once assumed to be apparent. Therefore, we ask that some exercises be solved without calculators. (See the Technology and Exercises section of the Preface.)

Once students are comfortable with the language of functions, the text guides them through a more traditional exploration of twelve basic functions and their algebraic properties, always reinforcing the connections among their algebraic, graphical, and numerical representations. This text uses a consistent approach to modeling, emphasizing the use of particular types of functions to model behavior in the real world. Modeling is a fundamental aspect of our problem-solving process that is introduced in Section 1.1 and used throughout the text. The text has a wealth of data and range of applications to illustrate how mathematics and statistics connect to every facet of modern life. Each chapter, 1–11, concludes with a modeling project to reinforce and extend students' ability to solve modeling problems.

This text has faithfully incorporated not only the teaching strategies that have made *Calculus: Graphical, Numerical, Algebraic* so popular, but also some of the strategies from the popular Pearson high school algebra series, and thus has produced a seamless pedagogical transition from prealgebra through calculus for students. Although this book can certainly be appreciated on its own merits, teachers who seek coherence and vertical alignment in their mathematics sequence might consider this pedagogical approach to be an additional asset of *Precalculus: Graphical, Numerical, Algebraic*.

This text is written to address current and emerging state curriculum standards. In particular, we embrace NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* and its emphasis on the importance of helping students to make sense of mathematics and to reason using mathematics. The NCTM's *Principles and Standards for School Mathematics* identified five "Process Standards" that should be fundamental in mathematics education. The first of these standards was Problem Solving. Since then, the emphasis on problem solving has continued to grow, to the point that it is now integral to the instructional process in many mathematics classrooms. When the Common Core State Standards for Mathematics detailed eight "Standards for Mathematical Practice" that should be fundamental in mathematics education, again the first of these addressed problem solving. Individual states have also released their own standards over the years, and problem solving is invariably front and center as a fundamental objective. Problem solving, reasoning, sense making, and the related processes and practices of mathematics are central to the approach we use in *Precalculus: Graphical, Numerical, Algebraic*.

We embrace the growing importance and wide applicability of Statistics. Because Statistics is increasingly used in college coursework, the workplace, and everyday life, we include a full chapter on Statistics to help students see that statistical analysis is an investigative process that turns loosely formed ideas into scientific studies. Our five sections on data analysis, probability, and statistical literacy are aligned with the *GAISE* Report published by the American Statistical Association, the College Board's AP® Statistics curriculum, and the Common Core State Standards. Chapter 10 is not intended as a course in statistics but rather as an introduction to set the stage for possible further study.

Dedication

We dedicate this text to the memory of
our eminent colleague, dear friend, and inspirational coauthor
***Bert K. Waits** (1940–2014).*
With his passing, the mathematics community lost a uniquely talented leader.
May he rest in peace, and may the power of visualization*,*
which he passionately promoted, live on!

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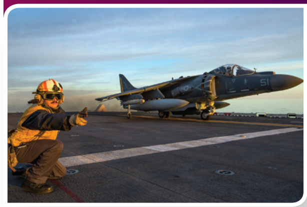

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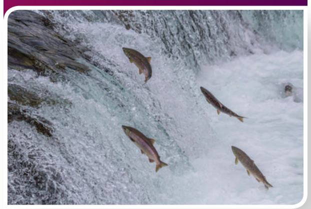

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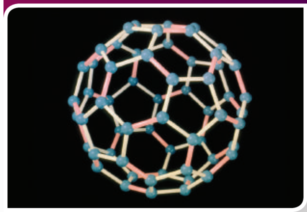

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Dr. Foley coauthored *Precalculus: A Graphing Approach; Precalculus: Functions and Graphs;* and *Advanced Quantitative Reasoning: Mathematics for the World Around Us.*

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Dan Kennedy received his undergraduate degree from the College of the Holy Cross and his master's degree and Ph.D. in mathematics from the University of North Carolina at Chapel Hill. Since 1973 he has taught mathematics at the Baylor School in Chattanooga, Tennessee, where he holds the Cartter Lupton Distinguished Professorship. Dr. Kennedy joined the Advanced Placement® Calculus Test Development Committee in 1986, then in 1990 became the first high school teacher in 35 years to chair that committee. It was during his tenure as chair that the program moved to require graphing calculators and laid the early groundwork for the 1998 reform of the Advanced Placement Calculus curriculum. The author of the 1997 *Teacher's Guide—AP® Calculus*, Dr. Kennedy has conducted more than 50 workshops and institutes for high school calculus teachers. His articles on mathematics teaching have appeared in the *Mathematics Teacher* and the *American Mathematical Monthly*, and he is a frequent speaker on education reform at professional and civic meetings. Dr. Kennedy was named a Tandy Technology Scholar in 1992 and a Presidential Award winner in 1995.

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Mr. Bock coauthored the AP Statistics textbook *Stats: Modeling the World,* the non-AP text *Stats in Your World,* Barron's *AP Calculus* review book, and Barron's *AP Calculus Flash Cards.*

PREFACE

Our Approach

The Rule of Four—A Balanced Approach

A principal feature of this text is the balance among the algebraic, numerical, graphical, and verbal methods of representing problems: the rule of four. For instance, we obtain solutions algebraically when that is the most appropriate technique to use, and we obtain solutions graphically or numerically when algebra is difficult to use. We urge students to solve problems by one method and then to support or confirm their solutions by using another method. We believe that students must learn the value of each of these methods or representations and must learn to choose the one most appropriate for solving the particular problem under consideration. This approach reinforces the idea that to understand a problem fully, students need to understand it algebraically as well as graphically and numerically.

Problem-Solving Approach

Systematic problem solving is emphasized in the examples throughout the text, using the following variation of Polya's problem-solving process that emphasizes modeling with mathematics:

- *understand* the problem,
- *develop* a mathematical model for the problem,
- *solve* the mathematical model and support or confirm the solution, and
- *interpret* the solution within the problem setting.

Students are encouraged to use this process throughout the text.

Twelve Basic Functions

Twelve basic functions are emphasized throughout the text as a major theme and focus. These functions are

- The Identity Function
- The Squaring Function
- The Cubing Function
- The Reciprocal Function
- The Square Root Function
- The Exponential Function
- The Natural Logarithm Function
- The Sine Function
- The Cosine Function
- The Absolute Value Function
- The Greatest Integer Function
- The Logistic Function

One of the most distinctive features of this text is that it introduces students to the full vocabulary of functions early in the course. Students meet the twelve basic functions graphically in Chapter 1 and are able to compare and contrast them as they learn about concepts like domain, range, symmetry, continuity, end behavior, asymptotes, extrema, and even periodicity—concepts that are difficult to appreciate when the only examples a teacher can refer to are polynomials. With this text, students are able to characterize functions by their behavior within the first month of classes. Once students have a comfortable understanding of functions in general, the rest of the course consists of studying the various types of functions in greater depth, particularly with respect to their algebraic properties and modeling applications.

These functions are used to develop the fundamental analytic skills that are needed in calculus and advanced mathematics courses. A complete gallery of basic functions is included in Appendix C and inside the back cover of the text for easy reference.

Applications, Data, and Modeling

The majority of the applications in the text are based on real data from cited sources, and their presentations are self-contained. As they work through the applications, students are exposed to functions as mechanisms for modeling real-life problems. They learn to analyze and model data, represent data graphically, interpret graphs, and fit curves. Additionally, the tabular representation of data presented in this text highlights the concept that a function is a correspondence between numerical variables. This helps students build the connection between numerical quantities and graphs and recognize the importance of a full graphical, numerical, and algebraic understanding of a problem. For a complete listing of applications, please see the Applications Index on page 923.

Technology and Exercises

The authors of this text have encouraged the use of technology in mathematics education for more than three decades. Our approach to problem solving (pages 68–69) distinguishes between **solving** the problem and **supporting** or **confirming** the solution, and emphasizes how technology figures in each of these processes.

We have come to realize, however, that advances in technology and increased familiarity with calculators have gradually blurred some of the distinctions between solving and supporting that we had once assumed to be apparent. We do not want to retreat in any way from our support of modern technology, but we now provide specific guidance about the intent of the various exercises in our text.

Therefore, as a service to teachers and students alike, exercises in this text that **should be solved without calculators** are identified with gray ovals around the exercise numbers. These usually are exercises that demonstrate how various functions behave algebraically or how algebraic representations reflect graphical behavior and vice versa. Application problems usually have no restrictions, in keeping with our emphasis on **modeling** and on bringing **all representations** to bear when confronting real-world problems.

Incidentally, we continue to encourage the use of calculators to **support** answers graphically or numerically after the problems have been solved with pencil and paper. Any time students can make connections among the graphical, analytical, and numerical representations, they are doing good mathematics.

As a final note, we will freely admit that different teachers use our text in different ways, and some will probably override our no-calculator recommendations to fit with their pedagogical strategies. In the end, the teachers know what is best for their students, and we are just here to help.

Content Changes to This Edition

Although the table of contents is essentially the same, this edition includes numerous substantial changes. About 5% of the examples have been replaced; another 5% have new data or new contexts. Additionally, 15–20% of the examples have been enhanced or clarified in some way. As for the exercises, again, about 5% have been replaced and another 5% have new data or new contexts. Plus, 5–10% of the exercises have been enhanced or clarified in some way. In particular, to keep the applications of mathematics relevant to our students, we have included the most current data available to us at the time of publication. As an example, look at the Chapter Opener problem on page 142. Not only does this include current data but also an entirely new twist: piecewise modeling.

Several other changes have been made as well. We have updated many of the student and teacher notes. Most significantly, we have added a new teacher note feature—Helping

English Learners—to assist teachers in supporting their students whose first language is not English. The *Annotated Teacher's Edition* now has a Complete Answers section that contains answers for all explorations and exercises even if an answer appears adjacent to the corresponding exploration or exercise. We have updated calculator screenshots to conform to the enhanced capabilities of modern graphing calculators. We have updated and renamed the capstone projects for Chapters 1–11 as Modeling Projects to reflect that they can be used as a bridge to the open-ended modeling recommended in the *GAIMME* report, published in 2016 by the Consortium for Mathematics and Its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM).

Features

Chapter Openers include a general description of an application that can be solved with the concepts learned in the chapter. The application is revisited later in the chapter via a specific problem that is solved.

A **Chapter Overview** begins each chapter to give students a sense of what they are going to learn. This overview provides a roadmap of the chapter and also indicates how the topics in the chapter are connected under one big idea. It is always helpful to remember that mathematics isn't modular, but interconnected, and that the skills and concepts learned throughout the course build on one another to help students understand more complicated processes and relationships. Similarly, the **What you'll learn about . . . and why** feature presents the big ideas in each section and explains their purpose.

Throughout the text, **Vocabulary** is highlighted in yellow for easy reference. Additionally, **Properties**, **Definitions**, and **Theorems** are boxed in purple, and **Procedures** in green, so that they can be easily found. The **Web/Real Data** icon marks the examples and exercises that use data cited from authentic sources.

Each example ends with a suggestion to **Now try** a related exercise. Working the suggested exercise is an easy way for students to check their comprehension of the material while reading each section. In the *Annotated Teacher's Edition*, various examples are marked for the teacher with the PPT icon. Alternatives are provided for these examples in the PowerPoint Slides.

Explorations appear throughout the text and provide students with the perfect opportunity to become active learners and to discover mathematics on their own. This will help hone critical-thinking and problem-solving skills. Some are technology-based; others involve exploring mathematical ideas and connections.

Margin Notes on various topics appear throughout the text. Some of these offer practical advice on using a grapher to obtain the best, most accurate results. Other notes include historical information, give hints about examples, or provide insight to help students avoid common pitfalls and errors.

The **Looking Ahead to Calculus** icon is found throughout the text next to many examples and topics to point out concepts that students will encounter again in calculus. Ideas that foreshadow calculus, such as limits, maximum and minimum, asymptotes, and continuity, are highlighted. Some calculus notation and language are introduced in the early chapters and used throughout the text to establish familiarity.

The review material at the end of each chapter consists of sections dedicated to helping students review the chapter concepts. **Key Ideas** are broken into parts: Properties, Theorems, and Formulas; Procedures; and Gallery of Functions. The **Review Exercises** represent the full range of exercises covered in the chapter and give additional practice with the ideas developed in the chapter. The exercises with red numbers indicate problems that would make up a good chapter test. A **Modeling Project** concludes each chapter and requires students to analyze data. It can be assigned as either individual or group work. Each project expands upon concepts and ideas taught in the chapter and engages students in modeling with mathematics.

Exercise Sets

Each exercise set begins with a **Quick Review** to help students review skills needed in the exercise set and refers them to other sections they can go to for help. Some exercises are designed to be solved *without* a *calculator*; the numbers of these exercises are printed within a gray oval. Students are urged to **support** the answers to these (and all) exercises graphically or numerically, but only after they have solved them with pencil and paper.

There are over 6000 exercises, including 720 Quick Review Exercises. The section exercises have been carefully graded from routine to challenging. The following types of skills are tested in each exercise set:

- Algebraic understanding and procedures
- Applications of mathematics
- Connecting algebra to geometry
- Interpretation of graphs
- Graphical and numerical representations of functions
- Data analysis

The exercise sets include distinctive kinds of thought-provoking exercises:

- **Standardized Test Questions** include two true-false problems with justifications and four multiple-choice questions.
- **Explorations** are opportunities for students to discover mathematics on their own or in groups. These exercises often require the use of critical thinking to explore the ideas involved.
- **Writing to Learn** exercises give students practice at communicating about mathematics and opportunities to demonstrate their understanding of important ideas.
- **Group Activity** exercises ask students to work collaboratively to solve problems while interacting with a few of their classmates.
- **Extending the Ideas** exercises go beyond what is presented in the text. These exercises are challenging extensions of the material in the text.

This variety of exercises provides sufficient flexibility to emphasize the skills and concepts most needed for each student or class.

Technology Resources

The following supplements are available for purchase:

MathXL® for School (optional, for purchase only)—access code required www.mathxlforschool.com

MathXL for School is a powerful online homework, tutorial, and assessment supplement aligned with Pearson Education's texts in mathematics or statistics. MathXL for School is the homework and assessment engine that runs MyMathLab for School.

With MathXL for School, teachers can do the following:

- Create, edit, and assign auto-graded online homework and tests correlated at the objective level to the text.
- Use automatic grading to rapidly assess students' understanding.
- Track both student and group performance in an online gradebook.
- Prepare students for high-stakes testing, including aligning assignments with associated standards and assessments.
- Deliver quality, effective instruction regardless of experience level.

With MathXL for School, students can do the following:

- Complete their homework and receive immediate feedback.
- Get self-paced assistance on problems in a variety of ways (guided solutions, step-by-step examples, video clips, and animations).
- Choose from a large number of practice problems, helping them master a topic.
- Receive personalized study plans and homework based on test results.

For more information and to purchase student access codes after the first year, visit our Web site at www.mathxlforschool.com or contact your Pearson School Sales Representative.

MyMathLab® for School Online Course (optional, for purchase only)—access code required

MyMathLab for School delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from a **trusted partner** with educational expertise and an eye on the future. To learn more about how MyMathLab® combines proven learning applications with powerful assessment, visit **www.mymathlabforschool.com** or contact your Pearson School Sales Representative. In this **MyMathLab® for School** course, you have access to the most cutting-edge, innovative study solutions proven to increase students' success.

Additional Teacher Resources

Most of the teacher supplements and resources available for this text are available electronically for download at the Instructor Resource Center (IRC). Please go to **www.PearsonSchool.com/Access_Request** and select "we need IRC (Instructor Resource Access)." You will be required to complete a one-time registration subject to verification before being emailed access information for download materials. Once logged into the IRC, enter the Student Edition ISBN in the **Search our Catalog** box to locate your resources.

The following supplements are available to qualified adopters:

Annotated Teacher's Edition

- Provides answers for all of the explorations, exercises, and projects in the Complete Answers section in the back of the text.
- Where space permits, each answer also appears adjacent to the corresponding exploration or exercise.
- Various examples marked with the PPT icon indicate that alternative examples are provided in the PowerPoint Slides.
- Provides notes written specifically for the teacher. These notes include chapter and section objectives, suggested assignments, lesson guides, and teaching tips.

Online Solutions Manual (Download Only)

Provides complete solutions to all exercises, including Explorations, Quick Reviews, Exercises, Review Exercises, and Modeling Projects.

Online Resource Manual (Download Only)

Provides Major Concepts Review, Group Activity Worksheets, Sample Chapter Tests, Standardized Test Preparation Questions, and Contest Problems.

Online Tests and Quizzes (Download Only)

Provides two parallel tests per chapter, two quizzes for every three to four sections, two parallel midterm tests covering Chapters P–5, and two parallel end-of-year tests covering Chapters 6–11.

TestGen® (Download Only)

TestGen enables teachers to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing teachers to create multiple but equivalent versions of the same question or test with the click of a button. Teachers can also modify test bank questions or add new questions. Tests can be printed or administered online.

PowerPoint Slides (Download Only)

Features presentations written and designed specifically for this text, including figures, alternative examples, definitions, and key concepts.

Common Core: Student Practice and Review Guide (Download Only)

This resource provides complete daily support for every lesson. The following resources are included: Problem Solving, Practice, and Standardized Test Prep.

Common Core: Implementation Guide (Download Only)

All the support a teacher needs to make the transition to a Common Core curriculum. Includes:

- Overview of the Common Core State Standards
- Standards for Mathematical Practice Observational Protocol
- Common Core Correlations
- Common Core assessment resources

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—F. D. D.
—G. D. F.
—D. K.
—D. E. B.

CHAPTER P

Prerequisites

Large distances are measured in *light years*, the distance that light travels in one year. Scientists use the speed of light, which is roughly 299,800 km/sec, to approximate distances within the solar system. For examples, see page 35.

Chapter P Overview

Historically, algebra was used to represent problems with symbols (algebraic models) and solve them by reducing the solution to algebraic manipulation of symbols. This technique is still important today. In addition, graphing calculators are now used to represent problems with graphs (graphical models) and solve them with the numerical and graphical techniques of technology.

We begin with basic properties of real numbers and introduce absolute value, distance formulas, midpoint formulas, and equations of circles. We use the slope of a line to write equations for the line, and we use these equations to solve practical problems. We then explore the basic ideas of complex numbers. We close the chapter by solving equations and inequalities using both algebraic and graphical techniques.

P.1 Real Numbers

What you'll learn about

- Representing Real Numbers
- Order and Interval Notation
- Basic Properties of Algebra
- Integer Exponents
- Scientific Notation

... and why

These topics are fundamental in the study of mathematics and science.

Representing Real Numbers

A **real number** is any number that can be written as a decimal. Real numbers are represented by symbols such as -8, 0, 1.75, $2.333...$, $0.\overline{36}$, $8/5$, $\sqrt{3}$, $\sqrt[3]{16}$, e, and π.

The set of real numbers contains several important subsets:

The **natural (or counting) numbers**:	$\{1, 2, 3, \ldots\}$
The **whole numbers**:	$\{0, 1, 2, 3, \ldots\}$
The **integers**:	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

We use braces $\{\ \}$ to enclose the **elements**, or **objects**, of a set. The rational numbers are another important subset of the real numbers. A **rational number** is any number that can be written as a quotient a/b of two integers, where $b \neq 0$. We can use **set-builder notation** to define the rational numbers:

$$\left\{ \frac{a}{b} \,\middle|\, a, b \text{ are integers, and } b \neq 0 \right\}$$

These symbols are read as "the set of all a over b such that a and b are integers, and b is not equal to zero."

The decimal form of a rational number either **terminates** like $7/4 = 1.75$, or is **infinitely repeating** like $4/11 = 0.363636... = 0.\overline{36}$. The bar over the 36 indicates the block of digits that repeats. A real number is **irrational** if it is *not* rational. The decimal form of an irrational number is infinitely nonrepeating. For example,

$$\sqrt{3} = 1.7320508... \text{ and } \pi = 3.14159265....$$

A real number can be approximated by giving a few of its digits. Sometimes we can find the decimal form of rational numbers with calculators, but not very often.

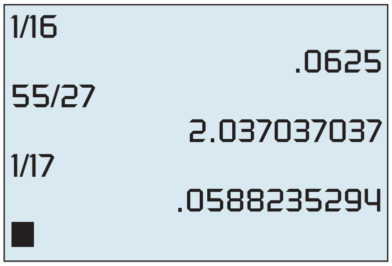


Figure P.1 Calculator decimal representations of $1/16$, $55/27$, and $1/17$ with the calculator set in Floating decimal mode. (Example 1)

EXAMPLE 1 Examining Decimal Forms of Rational Numbers

Determine the decimal form of $1/16$, $55/27$, and $1/17$.

SOLUTION Figure P.1 suggests that the decimal form of $1/16$ terminates and that of $55/27$ repeats in blocks of 037.

$$\frac{1}{16} = 0.0625 \qquad \text{and} \qquad \frac{55}{27} = 2.\overline{037}$$

We cannot predict the *exact* decimal form of $1/17$ from Figure P.1; however, we can say that $1/17 \approx 0.0588235294$. The symbol $\approx$ is read "*is approximately equal to*." We can use long division (see Exercise 66) to prove that

$$\frac{1}{17} = 0.\overline{0588235294117647}.$$

Now try Exercise 3.

The real numbers and the points of a line can be matched *one-to-one* to form a **real number line**. We start with a horizontal line and match the real number zero with a point O, the **origin**. **Positive numbers** are assigned to the right of the origin, and **negative numbers** to the left, as shown in Figure P.2.

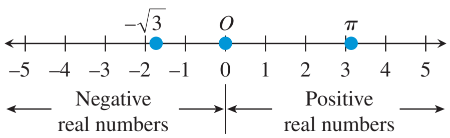


Figure P.2 The real number line.

Every real number corresponds to one and only one point on the real number line, and every point on the real number line corresponds to one and only one real number. Between every pair of real numbers on the number line there are infinitely many more real numbers.

The number associated with a point is **the coordinate of the point**. As long as the context is clear, we will follow the standard convention of using the real number for both the name of the point and its coordinate.

Order and Interval Notation

The set of real numbers is **ordered**. This means that we can use inequalities to compare any two real numbers that are not equal and say that one is "less than" or "greater than" the other.

Unordered Systems

Not all number systems are ordered. For example, the complex number system, introduced in Section P.6, has no natural ordering.

Order of Real Numbers

Let a and b be any real numbers.

Symbol	Definition	Read
$a > b$	$a - b$ is positive	a is greater than b
$a < b$	$a - b$ is negative	a is less than b
$a \geq b$	$a - b$ is positive or zero	a is greater than or equal to b
$a \leq b$	$a - b$ is negative or zero	a is less than or equal to b

The symbols $>$, $<$, $\geq$, and $\leq$ are **inequality symbols**.

Opposites and Number Line

$a < 0 \Leftrightarrow -a > 0$

If $a < 0$, then a is to the left of 0 on the real number line, and its opposite, $-a$, is to the right of 0. Thus, $-a > 0$. This logic can be reversed: If $-a > 0$, then $a < 0$.

Geometrically, $a > b$ means that a is to the right of b (equivalently, b is to the left of a) on the real number line. For example, $6 > 3$ implies that 6 is to the right of 3 on the real number line. Note also that $a > 0$ means that $a - 0$, or simply a, is positive, and $a < 0$ means that a is negative.

We are able to compare any two real numbers because of the following important property of the real numbers.

Trichotomy Property

Let a and b be any real numbers. Exactly one of the following is true:

$$a < b, \quad a = b, \quad \text{or} \quad a > b$$

Inequalities can be used to specify **intervals** of real numbers, as illustrated in Example 2.

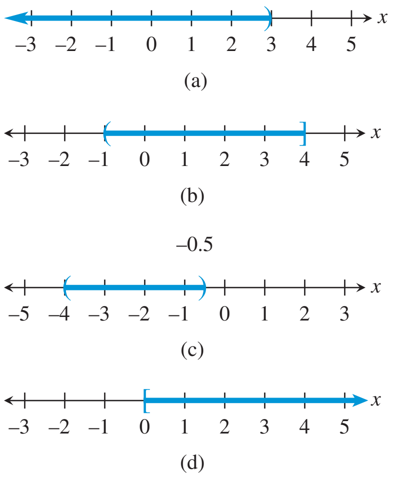


Figure P.3 In graphs of inequalities, parentheses correspond to $<$ and $>$, and brackets correspond to $\leq$ and $\geq$. (Examples 2 and 3)

EXAMPLE 2 Interpreting Inequalities

Interpret the meaning of, and graph, the interval of real numbers for the inequality.

(a) $x < 3$ **(b)** $-1 < x \leq 4$

SOLUTION

(a) The inequality $x < 3$ describes all real numbers less than 3 (Figure P.3a).

(b) The *double inequality* $-1 < x \leq 4$ represents all real numbers between -1 and 4, excluding -1 and including 4 (Figure P.3b). **Now try Exercise 5.**

EXAMPLE 3 Writing Inequalities

Write an inequality based on the description, and draw its graph.

(a) The real numbers between -4 and -0.5

(b) The real numbers greater than or equal to zero

SOLUTION

(a) $-4 < x < -0.5$ (Figure P.3c)

(b) $x \geq 0$ (Figure P.3d) **Now try Exercise 15.**

As shown in Example 2, inequalities define *intervals* on the real number line. We often use $[2, 5]$ to describe the *bounded interval* determined by $2 \leq x \leq 5$. This interval is **closed** because it contains its *endpoints* 2 and 5. There are four types of **bounded intervals**.

Bounded Intervals of Real Numbers

Let a and b be real numbers with $a < b$.

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, b]$	Closed	$a \leq x \leq b$	
(a, b)	Open	$a < x < b$	
$[a, b)$	Half-open	$a \leq x < b$	
$(a, b]$	Half-open	$a < x \leq b$	

The numbers a and b are the **endpoints** of each interval.

We use the interval notation $(-\infty, \infty)$ to represent the entire set of real numbers. The symbols $-\infty$ (*negative infinity*) and ∞ (*positive infinity*) allow us to use interval notation for unbounded intervals and are *not* real numbers.

Interval Notation Using $\pm\infty$

Because $-\infty$ is *not* a real number, we use $(-\infty, 2)$ instead of $[-\infty, 2)$ to describe $x < 2$. Similarly, we use $[-1, \infty)$ instead of $[-1, \infty]$ to describe $x \geq -1$.

The interval of real numbers determined by the inequality $x < 2$ can be described by the *unbounded interval* $(-\infty, 2)$. This interval is **open** because it does *not* contain its endpoint 2. In addition to $(-\infty, \infty)$, there are four types of **unbounded intervals**.

Unbounded Intervals of Real Numbers

Let a and b be real numbers.

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, \infty)$	Closed	$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$	Closed	$x \leq b$	
$(-\infty, b)$	Open	$x < b$	

Each of these intervals has exactly one endpoint, namely a or b.

EXAMPLE 4 Converting Between Intervals and Inequalities

Convert interval notation to inequality notation, or vice versa. State whether the interval is bounded or unbounded, and open or closed. Graph the interval and identify its endpoints.

(a) $[-6, 3)$ **(b)** $(-\infty, -1)$ **(c)** $-2 \leq x \leq 3$

SOLUTION

(a) The interval $[-6, 3)$ corresponds to $-6 \leq x < 3$ and is bounded and half-open (Figure P.4a). The endpoints are -6 and 3.

(b) The interval $(-\infty, -1)$ corresponds to $x < -1$ and is unbounded and open (Figure P.4b). The only endpoint is -1.

(c) The inequality $-2 \leq x \leq 3$ corresponds to the closed, bounded interval $[-2, 3]$ (Figure P.4c). The endpoints are -2 and 3. **Now try Exercise 29.**

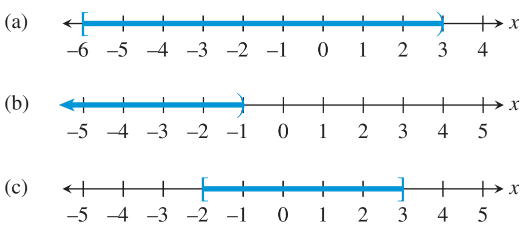


Figure P.4 Graphs of the intervals of real numbers in Example 4.

Basic Properties of Algebra

Algebra involves the use of letters and other symbols to represent real numbers. A **variable** is a letter or symbol (for example, x, y, t, θ) that represents an unspecified real number. A **constant** is a letter or symbol (for example, $-2, 0, \sqrt{3}, \pi$) that represents a specific real number. An **algebraic expression** is a combination of variables and constants involving addition, subtraction, multiplication, division, powers, and roots.

We state some of the properties of the arithmetic operations of addition, subtraction, multiplication, and division, represented by the symbols $+, -, \times$ (or $\cdot$) and $\div$ (or $/$), respectively. Addition and multiplication are the primary operations. Subtraction and division are defined in terms of addition and multiplication.

Subtraction: $a - b = a + (-b)$

Division: $\dfrac{a}{b} = a\left(\dfrac{1}{b}\right), b \neq 0$

In the above definitions, $-b$ is the **additive inverse** or **opposite** of b, and $1/b$ is the **multiplicative inverse** or **reciprocal** of b. Perhaps surprisingly, additive inverses are not always negative numbers. The additive inverse of 5 is the negative number -5. However, the additive inverse of -3 is the positive number 3.

Subtraction vs. Negative Numbers

On many calculators, there are two "–" keys, one for subtraction and one for negative numbers or opposites. Be sure you know how to use both keys correctly. Misuse can lead to incorrect results.

The following properties hold for real numbers, variables, and algebraic expressions.

Properties of Algebra

Let u, v, and w be real numbers, variables, or algebraic expressions.

1. **Commutative properties**
   Addition: $u + v = v + u$
   Multiplication: $uv = vu$
2. **Associative properties**
   Addition:
   $(u + v) + w = u + (v + w)$
   Multiplication: $(uv)w = u(vw)$
3. **Identity properties**
   Addition: $u + 0 = u$
   Multiplication: $u \cdot 1 = u$
4. **Inverse properties**
   Addition: $u + (-u) = 0$
   Multiplication: $u \cdot \dfrac{1}{u} = 1, u \neq 0$
5. **Distributive properties**
   Multiplication over addition:
   $u(v + w) = uv + uw$
   $(u + v)w = uw + vw$
   Multiplication over subtraction:
   $u(v - w) = uv - uw$
   $(u - v)w = uw - vw$

For each distributive property, the left-hand side of the equation shows the **factored form** of the algebraic expression, and the right-hand side shows the **expanded form**.

EXAMPLE 5 Using the Distributive Property

(a) Write the expanded form of $(a + 2)x$.

(b) Write the factored form of $3y - by$.

SOLUTION

(a) $(a + 2)x = ax + 2x$

(b) $3y - by = (3 - b)y$

Now try Exercise 37.

Here are some properties of the additive inverse, together with examples that help illustrate their meanings.

Properties of the Additive Inverse

Let u and v be real numbers, variables, or algebraic expressions.

Property	Example
1. $-(-u) = u$	$-(-3) = 3$
2. $(-u)v = u(-v) = -(uv)$	$(-4)3 = 4(-3) = -(4 \cdot 3) = -12$
3. $(-u)(-v) = uv$	$(-6)(-7) = 6 \cdot 7 = 42$
4. $(-1)u = -u$	$(-1)5 = -5$
5. $-(u + v) = (-u) + (-v)$	$-(7 + 9) = (-7) + (-9) = -16$

Integer Exponents

Exponential notation is used to shorten products of factors that repeat. For example,

$$(-3)(-3)(-3)(-3) = (-3)^4 \quad \text{and} \quad (2x + 1)(2x + 1) = (2x + 1)^2.$$

Exponential Notation

Let a be a real number, variable, or algebraic expression and n be a positive integer. Then

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}},$$

where n is the **exponent**, a is the **base**, and a^n is the ***n*th power of *a***, read as "a to the nth power."

The two exponential expressions in Example 6 have the same value but have different bases. Be sure you understand the distinction.

Understanding Notation

$(-3)^2 = 9$

$-3^2 = -9$

Be careful!

EXAMPLE 6 Identifying the Base

(a) In $(-3)^5$, the base is -3.

(b) In -3^5, the base is 3.

Now try Exercise 43.

Here are the basic properties of exponents, together with examples to illustrate their meanings.

Properties of Exponents

Let u and v be real numbers, variables, or algebraic expressions and m and n be integers. All bases are assumed to be nonzero.

Property	Example
1. $u^m u^n = u^{m+n}$	$5^3 \cdot 5^4 = 5^{3+4} = 5^7$
2. $\dfrac{u^m}{u^n} = u^{m-n}$	$\dfrac{x^9}{x^4} = x^{9-4} = x^5$
3. $u^0 = 1$	$8^0 = 1$
4. $u^{-n} = \dfrac{1}{u^n}$	$y^{-3} = \dfrac{1}{y^3}$
5. $(uv)^m = u^m v^m$	$(2z)^5 = 2^5 z^5 = 32z^5$
6. $(u^m)^n = u^{mn}$	$(x^2)^3 = x^{2 \cdot 3} = x^6$
7. $\left(\dfrac{u}{v}\right)^m = \dfrac{u^m}{v^m}$	$\left(\dfrac{a}{b}\right)^7 = \dfrac{a^7}{b^7}$

To simplify an expression involving powers means to rewrite it so that each factor appears only once, all exponents are positive, and exponents and constants are combined as much as possible.

Moving Factors

Be sure you understand how exponent property 4 permits us to move factors from the numerator to the denominator, and vice versa:

$$\frac{v^{-m}}{u^{-n}} = \frac{u^n}{v^m}$$

EXAMPLE 7 Simplifying Expressions Involving Powers

(a) $(2ab^3)(5a^2b^5) = 10(aa^2)(b^3b^5) = 10a^3b^8$

(b) $\dfrac{u^2v^{-2}}{u^{-1}v^3} = \dfrac{u^2u^1}{v^2v^3} = \dfrac{u^3}{v^5}$

(c) $\left(\dfrac{x^2}{2}\right)^{-3} = \left(\dfrac{2}{x^2}\right)^3 = \dfrac{2^3}{(x^2)^3} = \dfrac{8}{x^6}$

Now try Exercise 47.

Scientific Notation

Any positive number can be written in **scientific notation**,

$$c \times 10^m, \text{ where } 1 \le c < 10 \text{ and } m \text{ is an integer.}$$

This notation provides a way to work with very large and very small numbers. For example, the distance between Earth and the Sun is about 93,000,000 miles. In scientific notation,

$$93{,}000{,}000 \text{ mi} = 9.3 \times 10^7 \text{ mi.}$$

The *positive exponent* 7 indicates that moving the decimal point in 9.3 to the right 7 places produces the decimal form of the number.

The mass of an oxygen molecule is about

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 054 \text{ g.}$$

In scientific notation,

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 054 \text{ g} = 5.4 \times 10^{-23} \text{ g.}$$

The *negative exponent* −23 indicates that moving the decimal point in 5.4 to the left 23 places produces the decimal form of the number.

EXAMPLE 8 Converting to and from Scientific Notation

(a) $2.375 \times 10^8 = 237{,}500{,}000$

(b) $0.000000349 = 3.49 \times 10^{-7}$

Now try Exercises 57 and 59.

EXAMPLE 9 Using Scientific Notation

Simplify $\dfrac{(360{,}000)(4{,}500{,}000{,}000)}{18{,}000}$.

SOLUTION

$$\begin{aligned}\frac{(360{,}000)(4{,}500{,}000{,}000)}{18{,}000} &= \frac{(3.6 \times 10^5)(4.5 \times 10^9)}{1.8 \times 10^4} \\ &= \frac{(3.6)(4.5)}{1.8} \times 10^{5+9-4} \\ &= 9 \times 10^{10} \\ &= 90{,}000{,}000{,}000\end{aligned}$$

Now try Exercise 63.

Using a Calculator Figure P.5 shows two ways to perform the computation. In the first, the numbers are entered in decimal form. In the second, the numbers are entered in scientific notation. The calculator uses "9E10" to stand for 9×10^{10}.

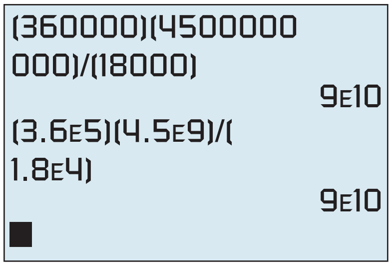


Figure P.5 Be sure you understand how your calculator displays scientific notation. (Example 9)

QUICK REVIEW P.1

1. List the positive integers between -3 and 7.
2. List the integers between -3 and 7.
3. List all negative integers greater than -4.
4. List all positive integers less than 5.

In Exercises 5 and 6, use a calculator to evaluate the expression. Round the value to two decimal places.

5. (a) $4(-3.1)^3 - (-4.2)^5$ (b) $\dfrac{2(-5.5) - 6}{7.4 - 3.8}$
6. (a) $5[3(-1.1)^2 - 4(-0.5)^3]$ (b) $5^{-2} + 2^{-4}$

In Exercises 7 and 8, evaluate the algebraic expression for the given values of the variables.

7. $x^3 - 2x + 1, x = -2, 1.5$
8. $a^2 + ab + b^2, a = -3, b = 2$

In Exercises 9 and 10, list the possible remainders.

9. When the positive integer n is divided by 7
10. When the positive integer n is divided by 13

SECTION P.1 Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

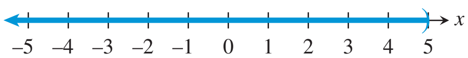
In Exercises 1–4, find the decimal form for the rational number. State whether it repeats or terminates.

1. $-37/8$
2. $15/99$
3. $-13/6$
4. $5/37$

In Exercises 5–10, interpret the meaning of, and graph, the interval of real numbers.

5. $x \le 2$
6. $-2 \le x < 5$
7. $(-\infty, 7)$
8. $[-3, 3]$
9. x is negative.
10. x is greater than or equal to 2 and less than or equal to 6.

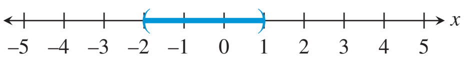
In Exercises 11–16, write an inequality for the interval of real numbers.

11. $[-1, 1)$
12. $(-\infty, 4]$
13. 

14. 

15. x is between -1 and 2.
16. x is greater than or equal to 5.

In Exercises 17–22, use interval notation to describe the interval of real numbers.

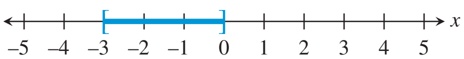
17. $x > -3$
18. $-7 < x < -2$
19. 

20. 

21. x is greater than -3 and less than or equal to 4.
22. x is positive.

In Exercises 23–28, use words to describe the interval of real numbers.

23. $4 < x \le 9$
24. $x \ge -1$
25. $[-3, \infty)$
26. $(-5, 7)$
27. 

28. 

In Exercises 29–32, convert to inequality notation. State whether the interval is bounded or unbounded and whether it is open or closed. Identify the endpoints.

29. $(-3, 4]$ **30.** $(-3, -1)$

31. $(-\infty, 5)$ **32.** $[-6, \infty)$

In Exercises 33–36, use both inequality and interval notation to describe the set of numbers. State the meaning of any variables you use.

33. Writing to Learn Bill is at least 29 years old.

34. Writing to Learn No item at Sarah's Variety Store costs more than \$2.00.

35. Writing to Learn The price of a gallon of gasoline varies from \$3.099 to \$4.399.

36. Writing to Learn Salary raises at California State University at Chico will be between 2% and 6.5% this year.

In Exercises 37–40, use the distributive property to write the factored form or the expanded form of the given expression.

37. $a(x^2 + b)$ **38.** $(y - z^3)c$

39. $ax^2 + dx^2$ **40.** $a^3z + a^3w$

In Exercises 41 and 42, find the additive inverse of the number.

41. $6 - \pi$ **42.** -7

In Exercises 43 and 44, identify the base of the exponential expression.

43. -5^2 **44.** $(-2)^7$

45. Group Activity Discuss which algebraic property or properties are illustrated by the equation. Try to reach a consensus.

(a) $(3x)y = 3(xy)$ (b) $a^2b = ba^2$

(c) $a^2b + (-a^2b) = 0$ (d) $(x + 3)^2 + 0 = (x + 3)^2$

(e) $a(x + y) = ax + ay$

46. Group Activity Discuss which algebraic property or properties are illustrated by the equation. Try to reach a consensus.

(a) $(x + 2)\dfrac{1}{x + 2} = 1$ (b) $1 \cdot (x + y) = x + y$

(c) $2(x - y) = 2x - 2y$

(d) $2x + (y - z) = 2x + (y + (-z))$
$= (2x + y) + (-z) =$
$(2x + y) - z$

(e) $\dfrac{1}{a}(ab) = \left(\dfrac{1}{a}a\right)b = 1 \cdot b = b$

In Exercises 47–52, simplify the expression. Assume that the variables in the denominators are nonzero.

47. $\dfrac{x^4y^3}{x^2y^5}$ **48.** $\dfrac{(3x^2)^2y^4}{3y^2}$

49. $\left(\dfrac{4}{x^2}\right)^2$ **50.** $\left(\dfrac{2}{xy}\right)^{-3}$

51. $\dfrac{(x^{-3}y^2)^{-4}}{(y^6x^{-4})^{-2}}$ **52.** $\left(\dfrac{4a^3b}{a^2b^3}\right)\left(\dfrac{3b^2}{2a^2b^4}\right)$

The data in Table P.1 give the expenditures in millions of dollars for U.S. public schools for the 2013–2014 school year.

Table P.1 U.S. Public School Expenditures

Category	Amount (millions of \$)
Current expenditures	535,665
Capital outlay	45,474
Interest on school debt	17,247
Total	606,490

Source: National Center for Education Statistics, U.S. Department of Education, as reported in The World Almanac and Book of Facts 2017.

In Exercises 53–56, write the amount of expenditures in dollars obtained from the category in scientific notation.

53. Current expenditures

54. Capital outlay

55. Interest on school debt

56. Total

In Exercises 57 and 58, write the number in scientific notation.

57. The mean distance from Jupiter to the Sun is about 483,900,000 miles.

58. The electric charge, in coulombs, of an electron is about −0.000 000 000 000 000 000 16.

In Exercises 59–62, write the number in decimal form.

59. 3.33×10^{-8}

60. 6.73×10^{11}

61. The distance that light travels in 1 year (*one light year*) is about 5.87×10^{12} mi.

62. The mass of a neutron is about 1.6747×10^{-24} g.

In Exercises 63 and 64, use scientific notation to simplify.

63. $\dfrac{(1.3 \times 10^{-7})(2.4 \times 10^{8})}{1.3 \times 10^{9}}$ without using a calculator

64. $\dfrac{(3.7 \times 10^{-7})(4.3 \times 10^{6})}{2.5 \times 10^{7}}$

Explorations

65. Investigating Exponents For positive integers m and n, we can use the definition to show that $a^m a^n = a^{m+n}$.

(a) Examine the equation $a^m a^n = a^{m+n}$ for $n = 0$ and explain why it is reasonable to define $a^0 = 1$ for $a \neq 0$.

(b) Examine the equation $a^m a^n = a^{m+n}$ for $n = -m$ and explain why it is reasonable to define $a^{-m} = 1/a^m$ for $a \neq 0$.

66. **Decimal Forms of Rational Numbers** Here is the third step when we divide 1 by 17. (The first two steps are not shown because the quotient is 0 in each case.)

$$\begin{array}{r} 0.05 \\ 17\overline{)1.00} \\ \underline{85} \\ 15 \end{array}$$

By convention we say that 1 is the first remainder in the long division process, 10 is the second, and 15 is the third remainder.

(a) Continue this long division process until a remainder is repeated, and complete the following table:

Step	Quotient	Remainder
1	0	1
2	0	10
3	5	15
⋮	⋮	⋮

(b) Explain why the digits that occur in the quotient between the pair of repeating remainders determine the infinitely repeating portion of the decimal representation. In this case

$$\frac{1}{17} = 0.\overline{0588235294117647}.$$

(c) Explain why this procedure will always determine the infinitely repeating portion of a rational number whose decimal representation does not terminate.

Standardized Test Questions

67. **True or False** The additive inverse of a real number must be negative. Justify your answer.

68. **True or False** The reciprocal of a positive real number must be less than 1. Justify your answer.

In Exercises 69–72, solve these problems without using a calculator.

69. **Multiple Choice** Which of the following inequalities corresponds to the interval $[-2, 1)$?

(A) $x \leq -2$ (B) $-2 \leq x \leq 1$

(C) $-2 < x < 1$ (D) $-2 < x \leq 1$

(E) $-2 \leq x < 1$

70. **Multiple Choice** What is the value of $(-2)^4$?

(A) 16 (B) 8

(C) 6 (D) −8

(E) −16

71. **Multiple Choice** What is the base of the exponential expression -7^2?

(A) −7 (B) 7

(C) −2 (D) 2

(E) 1

72. **Multiple Choice** Which of the following is the simplified form of $\frac{x^6}{x^2}$, $x \neq 0$?

(A) x^{-4} (B) x^2

(C) x^3 (D) x^4

(E) x^8

Extending the Ideas

The **magnitude** of a real number is its distance from the origin.

73. List the whole numbers whose magnitudes are less than 7.

74. List the natural numbers whose magnitudes are less than 7.

75. List the integers whose magnitudes are less than 7.

76. **Writing to Learn** **Combining Rational and Irrational Numbers** In each case, write an explanation to justify your answer.

(a) When two rational numbers are added, is the sum a rational number?

(b) When two rational numbers are multiplied, is the product a rational number?

(c) When a rational number and an irrational number are added, is the sum a rational number?

(d) When a *nonzero* rational number and an irrational number are multiplied, is the product a rational number?

P.2 Cartesian Coordinate System

What you'll learn about

- Cartesian Plane
- Absolute Value of a Real Number
- Distance Formulas
- Midpoint Formulas
- Equations of Circles
- Applications

... and why

These topics provide the foundation for the material that will be addressed in this text.

Cartesian Plane

The points in a plane correspond to ordered pairs of real numbers, just as the points on a line are associated with individual real numbers. This correspondence creates the **Cartesian plane**, or the **rectangular coordinate system** in the plane.

To construct a rectangular coordinate system (Cartesian plane), draw a pair of perpendicular real number lines, one horizontal and the other vertical, with the lines intersecting at their respective origins (Figure P.6). Their point of intersection, O, is the **origin** of the Cartesian plane. The horizontal line is usually the ***x*-axis**, and the vertical line is usually the ***y*-axis**. The positive direction on the x-axis is to the right, and the positive direction on the y-axis is up. The two axes divide the Cartesian plane into four **quadrants**, as shown in Figure P.7.

Each point P of the plane is associated with an **ordered pair (x, y)** of real numbers, the **(Cartesian) coordinates of the point**. The ***x*-coordinate** is the coordinate of the point on the x-axis that intersects with the vertical line from P. The ***y*-coordinate** is the coordinate of the point on the y-axis that intersects with the horizontal line from P (Figure P.7). Figure P.6 shows the points P and Q with coordinates $(4, 2)$ and $(-6, -4)$, respectively. As long as the context is clear, we use ordered pairs of real numbers to name points, not just their coordinates. For example, we can use $(-6, -4)$ to name point Q.

Not always *x* and *y*

In applications, the horizontal axis often represents time, typically denoted by the variable t. The vertical axis can represent any attribute of interest. For example, if the vertical axis represents force, we may use F as the variable.

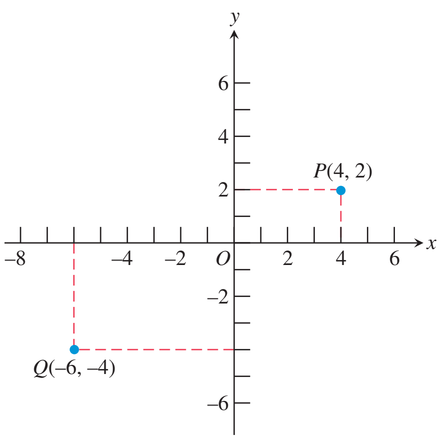


Figure P.6 The Cartesian coordinate plane.

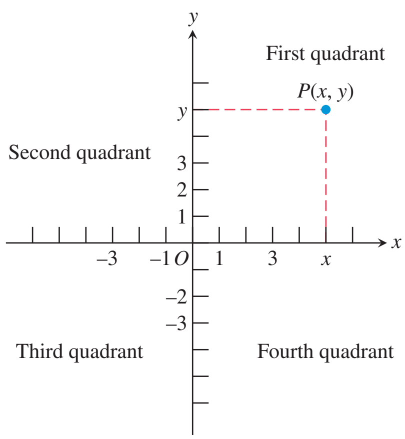


Figure P.7 The four quadrants. Points on the x- or y-axis are not in any quadrant.

Table P.2 U.S. Exports to Mexico

Time (years)	U.S. Exports (billions of $)
2005	120.2
2010	163.7
2012	215.9
2013	226.0
2014	240.3
2015	235.7

Source: U.S. Census Bureau, The World Almanac and Book of Facts 2017.

EXAMPLE 1 Plotting Data on U.S. Exports to Mexico

The values in billions of dollars of U.S. exports to Mexico for selected years from 2005 through 2015 are given in Table P.2. Plot the (time, export value) ordered pairs on a rectangular coordinate system.

SOLUTION The points are plotted in Figure P.8 on page 13. **Now try Exercise 31.**

A **scatter plot** is a graph of (x, y) data pairs on a Cartesian plane. Figure P.8 is a scatter plot of the data from Table P.2.

Absolute Value of a Real Number

The *absolute value of a real number* is its **magnitude** (size). For example, the absolute value of 3 is 3, and the absolute value of -5 is 5.

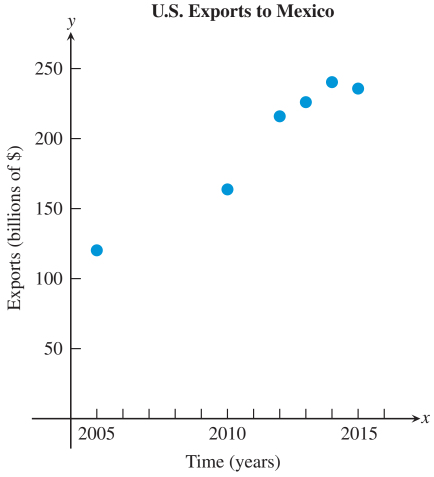


Figure P.8 The graph for Example 1.

DEFINITION Absolute Value of a Real Number

The **absolute value of a real number *a*** is

$$|a| = \begin{cases} a, \text{ if } a > 0 \\ 0, \text{ if } a = 0 \\ -a, \text{ if } a < 0. \end{cases}$$

EXAMPLE 2 Using the Definition of Absolute Value

Evaluate:

(a) $|-4|$

(b) $|\pi - 6|$

SOLUTION

(a) Because $-4 < 0$, $|-4| = -(-4) = 4$.

(b) Because $\pi \approx 3.142$, $\pi - 6$ is negative, so $\pi - 6 < 0$. Thus, $|\pi - 6| = -(\pi - 6) = 6 - \pi \approx 2.858$.

Now try Exercise 9.

Here is a summary of some important properties of absolute value.

Properties of Absolute Value

Let a and b be real numbers.

1. $|a| \geq 0$

2. $|-a| = |a|$

3. $|ab| = |a||b|$

4. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, b \neq 0$

Distance Formulas

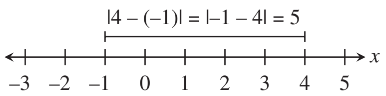


Figure P.9 Finding the distance between -1 and 4.

The *distance* between -1 and 4 on the number line is 5 (Figure P.9). This distance may be found by subtracting the smaller number from the larger: $4 - (-1) = 5$.
If we use absolute value, the order of subtraction does not matter:

$$|4 - (-1)| = |-1 - 4| = 5$$

Absolute Value and Distance

If we let $b = 0$ in the distance formula, we see that the distance between a and 0 is $|a|$. Thus, the absolute value of a number is its distance from zero.

Distance Formula (Number Line)

Let a and b be real numbers. The **distance between *a* and *b*** is

$$|a - b|.$$

Note that $|a - b| = |b - a|$.

To find the *distance* between two points that lie on the same horizontal or vertical line in the Cartesian plane, we use the distance formula for points on a number line. For example, the distance between points x_1 and x_2 on the x-axis is $|x_1 - x_2| = |x_2 - x_1|$ and the distance between points y_1 and y_2 on the y-axis is $|y_1 - y_2| = |y_2 - y_1|$.

To find the distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ that do not lie on the same horizontal or vertical line, we form the right triangle determined by P, Q, and $R(x_2, y_1)$ (Figure P.10).

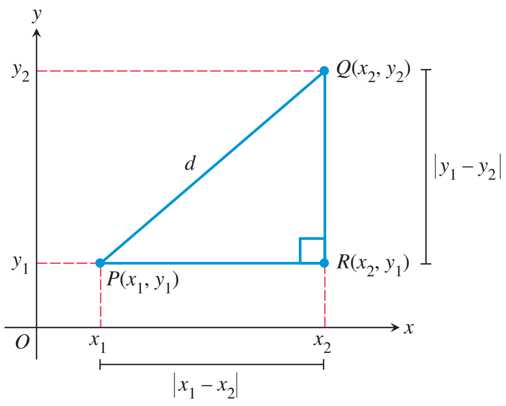


Figure P.10 Forming a right triangle with hypotenuse $\overline{PQ}$.

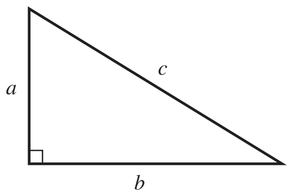


Figure P.11 The Pythagorean Theorem: In a right triangle, $c^2 = a^2 + b^2$.

The distance from P to R is $|x_1 - x_2|$, and the distance from R to Q is $|y_1 - y_2|$. By the **Pythagorean Theorem** (Figure P.11), the distance d between P and Q is

$$d = \sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2}.$$

Because $|x_1 - x_2|^2 = (x_1 - x_2)^2$ and $|y_1 - y_2|^2 = (y_1 - y_2)^2$, we obtain the following formula.

Distance Formula (Cartesian Plane)

The **distance d between points $P(x_1, y_1)$ and $Q(x_2, y_2)$** in a Cartesian plane is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

EXAMPLE 3 Finding the Distance Between Two Points

Find the distance d between the points $(1, 5)$ and $(6, 2)$.

SOLUTION

$$\begin{aligned} d &= \sqrt{(1 - 6)^2 + (5 - 2)^2} && \text{The distance formula} \\ &= \sqrt{(-5)^2 + 3^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \approx 5.831 && \text{Using a calculator} \end{aligned}$$

Now try Exercise 13.

Midpoint Formulas

When the endpoints of a segment on a number line are known, we take the average of their coordinates to find the midpoint of the segment.

Midpoint Formula (Number Line)

The **midpoint of the line segment with endpoints a and b** is

$$\frac{a + b}{2}.$$

EXAMPLE 4 Finding the Midpoint of a Line Segment

The midpoint of the line segment with endpoints -9 and 3 on a number line is

$$\frac{(-9) + 3}{2} = \frac{-6}{2} = -3.$$

See Figure P.12. **Now try Exercise 23.**

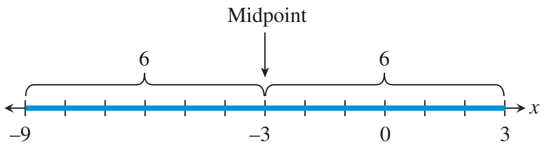


Figure P.12 Notice that the distance from the midpoint, -3, to 3 or to -9 is 6. (Example 4)

Just as with number lines, the midpoint of a line segment in the Cartesian plane involves averaging. Each coordinate of the midpoint is the average of the corresponding coordinates of its endpoints.

Midpoint Formula (Cartesian Plane)

The **midpoint of the line segment with endpoints (a, b) and (c, d)** is

$$\left(\frac{a + c}{2}, \frac{b + d}{2}\right).$$

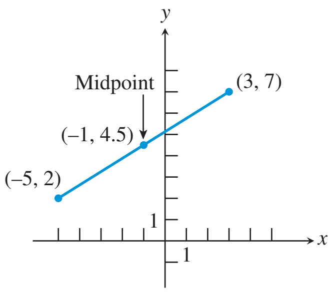


Figure P.13 Midpoint of a line segment. (Example 5)

EXAMPLE 5 Finding the Midpoint of a Line Segment

The midpoint of the line segment with endpoints $(-5, 2)$ and $(3, 7)$ is

$$(x, y) = \left(\frac{-5 + 3}{2}, \frac{2 + 7}{2}\right) = (-1, 4.5).$$

See Figure P.13. **Now try Exercise 25.**

Equations of Circles

A **circle** is the set of points in a plane at a fixed distance (**radius**) from a fixed point (**center**) in the plane. Figure P.14 shows the circle with center (h, k) and radius r. If (x, y) is any point on the circle, the distance formula gives

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$

Squaring both sides, we obtain the following equation for a circle.

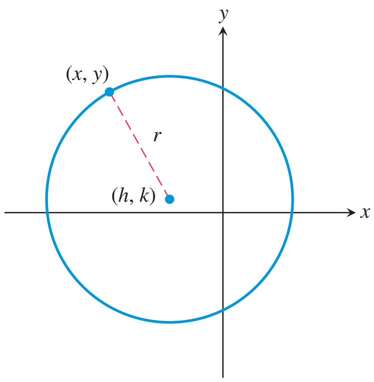


Figure P.14 The circle with center (h, k) and radius r.

DEFINITION Standard Form Equation of a Circle

The **standard form equation of a circle** with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

EXAMPLE 6 Finding Standard Form Equations of Circles

Find the standard form equation of the circle.

(a) Center $(-4, 1)$, radius 8 **(b)** Center $(0, 0)$, radius 5

SOLUTION

(a) $(x - h)^2 + (y - k)^2 = r^2$ Standard form equation

$(x - (-4))^2 + (y - 1)^2 = 8^2$ Substitute $h = -4$, $k = 1$, $r = 8$.

$(x + 4)^2 + (y - 1)^2 = 64$

(b) $(x - h)^2 + (y - k)^2 = r^2$ Standard form equation

$(x - 0)^2 + (y - 0)^2 = 5^2$ Substitute $h = 0$, $k = 0$, $r = 5$.

$x^2 + y^2 = 25$

Now try Exercise 41.

Applications

EXAMPLE 7 Using an Inequality to Express Distance

We can state that "the distance between x and -3 is less than 9" using the inequality

$$|x - (-3)| < 9 \quad \text{or} \quad |x + 3| < 9.$$

Now try Exercise 51.

The converse of the Pythagorean Theorem is true. That is, if the sum of squares of the lengths of the two sides of a triangle equals the square of the length of the third side, then the triangle is a right triangle.

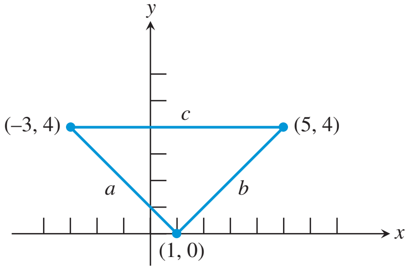


Figure P.15 The triangle in Example 8.

EXAMPLE 8 Verifying Right Triangles

Use the converse of the Pythagorean Theorem and the distance formula to prove that the points $(-3, 4)$, $(1, 0)$, and $(5, 4)$ determine a right triangle.

SOLUTION The three points are plotted in Figure P.15. We need to show that the lengths of the sides of the triangle satisfy the Pythagorean relationship $a^2 + b^2 = c^2$. Applying the distance formula, we find that

$$a = \sqrt{(-3 - 1)^2 + (4 - 0)^2} = \sqrt{32}$$
$$b = \sqrt{(1 - 5)^2 + (0 - 4)^2} = \sqrt{32}$$
$$c = \sqrt{(-3 - 5)^2 + (4 - 4)^2} = \sqrt{64}$$

The triangle is a right triangle because

$$a^2 + b^2 = \left(\sqrt{32}\right)^2 + \left(\sqrt{32}\right)^2 = 32 + 32 = 64 = c^2.$$

Now try Exercise 39.

Properties of geometric figures can sometimes be confirmed using analytic methods such as the midpoint formulas.

EXAMPLE 9 Using the Midpoint Formula

It is a fact from geometry that the diagonals of a parallelogram bisect each other. Prove this with a midpoint formula.

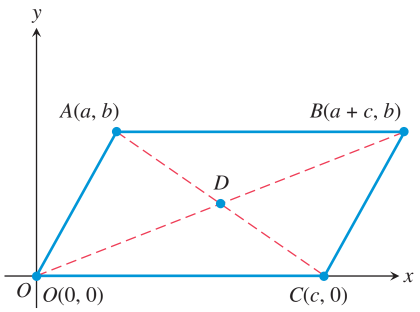


Figure P.16 The coordinates of B must be $(a + c, b)$ in order for CB to be parallel to OA. (Example 9)

SOLUTION We can position a parallelogram in the rectangular coordinate plane as shown in Figure P.16. Applying the midpoint formula for the Cartesian plane to segments OB and AC, we find that

$$\text{midpoint of segment } OB = \left(\frac{0 + a + c}{2}, \frac{0 + b}{2}\right) = \left(\frac{a + c}{2}, \frac{b}{2}\right)$$

$$\text{midpoint of segment } AC = \left(\frac{a + c}{2}, \frac{b + 0}{2}\right) = \left(\frac{a + c}{2}, \frac{b}{2}\right)$$

The midpoints of segments OA and AC are the same, so the diagonals of the parallelogram $OABC$ meet at their midpoints and thus bisect each other.

Now try Exercise 37.

QUICK REVIEW P.2

In Exercises 1 and 2, plot the two numbers on a number line. Then find the distance between them.

1. $\sqrt{7}, \sqrt{2}$ **2.** $-\frac{5}{3}, -\frac{9}{5}$

In Exercises 3 and 4, plot the real numbers on a number line.

3. $-3, 4, 2.5, 0, -1.5$ **4.** $-\frac{5}{2}, -\frac{1}{2}, \frac{2}{3}, 0, -1$

In Exercises 5 and 6, plot the points.

5. $A(3, 5), B(-2, 4), C(3, 0), D(0, -3)$

6. $A(-3, -5), B(2, -4), C(0, 5), D(-4, 0)$

In Exercises 7–10, use a calculator to evaluate the expression. Round your answer to two decimal places.

7. $\dfrac{-17 + 28}{2}$ **8.** $\sqrt{13^2 + 17^2}$

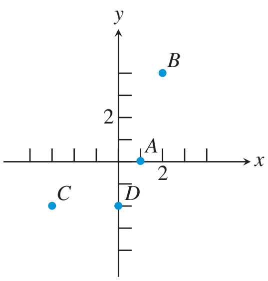
9. $\sqrt{6^2 + 8^2}$ **10.** $\sqrt{(17 - 3)^2 + (-4 - 8)^2}$

SECTION P.2 Exercises

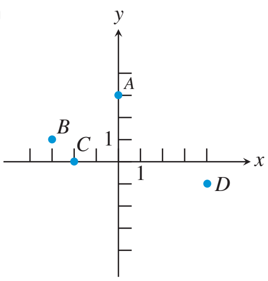
Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1 and 2, estimate the coordinates of the points.

1.



2.



In Exercises 3 and 4, name the quadrants containing the points.

3. (a) $(2, 4)$ (b) $(0, 3)$ (c) $(-2, 3)$ (d) $(-1, -4)$

4. (a) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (b) $(-2, 0)$ (c) $(-1, -2)$ (d) $\left(-\frac{3}{2}, -\frac{7}{3}\right)$

In Exercises 5–8, evaluate the expression.

5. $3 + |-3|$ **6.** $2 - |-2|$

7. $|(-2)3|$ **8.** $\dfrac{-2}{|-2|}$

In Exercises 9 and 10, rewrite the expression without using absolute value symbols.

9. $|\pi - 4|$ **10.** $|\sqrt{5} - 5/2|$

In Exercises 11–18, find the distance between the points.

11. $-9.3, 10.6$ **12.** $-5, -17$

13. $(-4, -3), (1, 1)$ **14.** $(-3, -1), (5, -1)$

15. $(0, 0), (3, 4)$ **16.** $(-1, 2), (2, -3)$

17. $(-2, 0), (5, 0)$ **18.** $(0, -8), (0, -1)$

In Exercises 19–22, find the perimeter and area of the figure determined by the points.

19. $(-5, 3), (0, -1), (4, 4)$

20. $(-2, -2), (-2, 2), (2, 2), (2, -2)$

21. $(-3, -1), (-1, 3), (7, 3), (5, -1)$

22. $(-2, 1), (-2, 6), (4, 6), (4, 1)$

In Exercises 23–28, find the midpoint of the line segment with the given endpoints.

23. $-9.3, 10.6$

24. $-5, -17$

25. $(-1, 3), (5, 9)$

26. $(3, \sqrt{2}), (6, 2)$

27. $(-7/3, 3/4), (5/3, -9/4)$

28. $(5, -2), (-1, -4)$

In Exercises 29–34, draw a scatter plot of the data given in the table.

29. U.S. Motor Vehicle Production The total number of motor vehicles in millions (y) produced by the United States each year from 2009 through 2015 is given in the table. *(Source: Automotive News Data Center and R. L. Polk Marketing Systems as reported in The World Almanac and Book of Facts 2017.)*

x	2009	2010	2011	2012	2013	2014	2015
y	5.59	7.63	8.46	10.14	11.07	11.66	12.10

30. World Motor Vehicle Production The total number of motor vehicles in millions (y) produced in the world each year from 2009 through 2015 is given in the table. *(Source: Automotive News Data Center and R. L. Polk Marketing Systems as reported in The World Almanac and Book of Facts 2017.)*

x	2009	2010	2011	2012	2013	2014	2015
y	59.1	73.3	76.0	81.1	87.5	89.8	90.8

31. U.S. Imports from Mexico The total in billions of dollars of U.S. imports from Mexico for selected years is given in Table P.3.

Table P.3 U.S. Imports from Mexico

Time (years)	U.S. Imports (billions of $)
2005	170.1
2010	230.0
2012	277.6
2013	280.6
2014	295.7
2015	296.4

Source: U.S. Census Bureau, The World Almanac and Book of Facts 2017.

32. U.S. Agricultural Exports The total in billions of dollars of U.S. agricultural exports for selected years is given in Table P.4.

Table P.4 U.S. Agricultural Exports

Time (years)	U.S. Exports (billions of $)
2005	62.5
2010	108.5
2012	135.9
2013	141.1
2014	152.3
2015	139.7

Source: U.S. Department of Agriculture, The World Almanac and Book of Facts 2017.

33. U.S. Exports to China The total in billions of dollars of U.S. exports to China for selected years is given in Table P.5.

Table P.5 U.S. Exports to China

Time (years)	U.S. Exports (billions of $)
2005	41.2
2010	91.9
2012	110.5
2013	121.7
2014	123.6
2015	116.1

Source: U.S. Department of Agriculture, The World Almanac and Book of Facts 2017.

34. U.S. Exports to Canada The total in billions of dollars of U.S. exports to Canada for selected years is given in Table P.6.

Table P.6 U.S. Exports to Canada

Time (years)	U.S. Exports (billions of $)
2005	211.9
2010	249.3
2012	292.7
2013	300.8
2014	312.8
2015	280.6

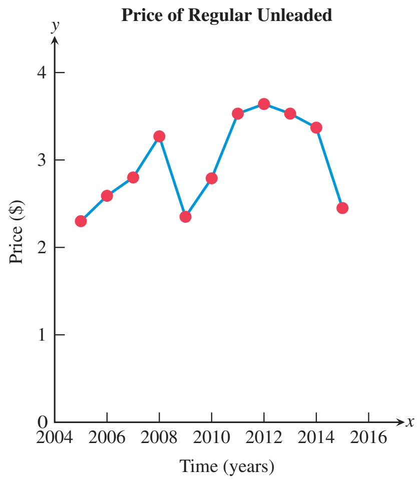
Source: U.S. Census Bureau, The World Almanac and Book of Facts 2017.

35. Reading from Graphs Using the graph below, estimate the price of gasoline (in dollars) for

(a) 2006 (b) 2010 (c) 2014

36. Percent Increase Using the graph below, estimate the percent increase (or decrease) in the price of gasoline from

(a) 2005 to 2010 (b) 2010 to 2015



Source: U.S. Census Bureau, The World Almanac and Book of Facts 2017.

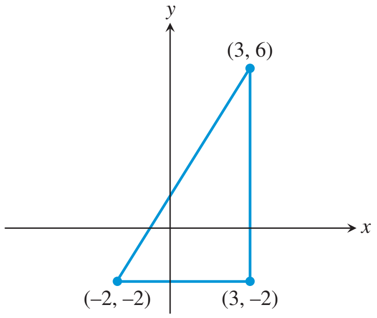
37. Prove that the figure determined by the points is an isosceles triangle: $(1, 3)$, $(4, 7)$, $(8, 4)$

38. **Group Activity** Prove that the diagonals of the figure determined by the points bisect each other.

(a) Square $(-7, -1)$, $(-2, 4)$, $(3, -1)$, $(-2, -6)$

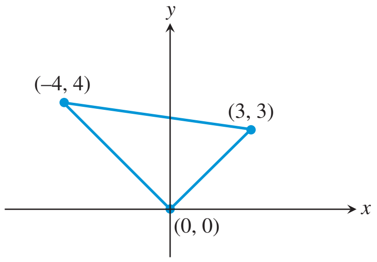
(b) Parallelogram $(-2, -3)$, $(0, 1)$, $(6, 7)$, $(4, 3)$

39. (a) Find the lengths of the sides of the triangle in the figure.



(b) **Writing to Learn** Prove that the triangle is a right triangle.

40. (a) Find the lengths of the sides of the triangle in the figure.



(b) **Writing to Learn** Prove that the triangle is a right triangle.

In Exercises 41–44, find the standard form equation for the circle.

41. Center $(1, 2)$, radius 5

42. Center $(-3, 2)$, radius 1

43. Center $(-1, -4)$, radius 3

44. Center $(0, 0)$, radius $\sqrt{3}$

In Exercises 45–48, find the center and radius of the circle.

45. $(x - 3)^2 + (y - 1)^2 = 36$

46. $(x + 4)^2 + (y - 2)^2 = 121$

47. $x^2 + y^2 = 5$

48. $(x - 2)^2 + (y + 6)^2 = 25$

In Exercises 49–52, write the statement using absolute value notation.

49. The distance between x and 4 is 3.

50. The distance between y and -2 is greater than or equal to 4.

51. The distance between x and c is less than d units.

52. y is more than d units from c.

53. Let $(4, 4)$ be the midpoint of the line segment determined by the points $(1, 2)$ and (a, b). Determine a and b.

54. **Writing to Learn** **Isosceles but Not Equilateral** Prove that the triangle determined by the points $(3, 0)$, $(-1, 2)$, and $(5, 4)$ is isosceles but not equilateral.

55. **Writing to Learn** **Equidistant Point** Prove that the midpoint of the hypotenuse of the right triangle with vertices $(0, 0)$, $(5, 0)$, and $(0, 7)$ is equidistant from the three vertices.

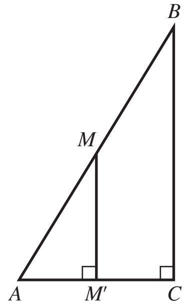
56. **Writing to Learn** Describe the set of real numbers that satisfy $|x - 2| < 3$.

57. **Writing to Learn** Describe the set of real numbers that satisfy $|x + 3| \geq 5$.

Standardized Test Questions

58. **True or False** If a is a real number, then $|a| \geq 0$. Justify your answer.

59. **True or False** Let ΔABC and $\Delta AMM'$ be right triangles as shown in the figure. If M is the midpoint of segment AB, then M' is the midpoint of segment AC. Justify your answer.



In Exercises 60–63, solve these problems without using a calculator.

60. **Multiple Choice** Which of the following is equal to $|1 - \sqrt{3}|$?

(A) $1 - \sqrt{3}$ (B) $\sqrt{3} - 1$

(C) $(1 - \sqrt{3})^2$ (D) $\sqrt{2}$

(E) $\sqrt{1/3}$

61. **Multiple Choice** Which of the following is the midpoint of the line segment with endpoints -3 and 2?

(A) $5/2$ (B) 1

(C) $-1/2$ (D) -1

(E) $-5/2$

62. **Multiple Choice** Which of the following is the center of the circle $(x - 3)^2 + (y + 4)^2 = 2$?

(A) $(3, -4)$ (B) $(-3, 4)$

(C) $(4, -3)$ (D) $(-4, 3)$

(E) $(3/2, -2)$

63. **Multiple Choice** Which of the following points is in the third quadrant?

(A) $(0, -3)$ (B) $(-1, 0)$

(C) $(2, -1)$ (D) $(-1, 2)$

(E) $(-2, -3)$

Explorations

64. Dividing a Line Segment into Thirds

(a) Find the coordinates of the points one-third and two-thirds of the way from $a = 2$ to $b = 8$ on a number line.

(b) Repeat (a) for $a = -3$ and $b = 7$.

(c) Find the coordinates of the points one-third and two-thirds of the way from a to b on a number line.

(d) Find the coordinates of the points one-third and two-thirds of the way from the point $(1, 2)$ to the point $(7, 11)$ in the Cartesian plane.

(e) Find the coordinates of the points one-third and two-thirds of the way from the point (a, b) to the point (c, d) in the Cartesian plane.

Extending the Ideas

65. Writing to Learn Equidistant Point from Vertices of a Right Triangle Prove that the midpoint of the hypotenuse of any right triangle is equidistant from the three vertices.

66. Comparing Areas Consider the four points $A(0, 0)$, $B(0, a)$, $C(a, a)$, and $D(a, 0)$. Let P be the midpoint of the line segment CD and Q the point one-fourth of the way from A to D on segment AD.

(a) Find the area of triangle BPQ.

(b) Compare the area of triangle BPQ with the area of square $ABCD$.

In Exercises 67–69, let $P(a, b)$ be a point in the first quadrant.

67. Find the coordinates of the point Q in the fourth quadrant so that the x-axis is the perpendicular bisector of PQ.

68. Find the coordinates of the point Q in the second quadrant so that the y-axis is the perpendicular bisector of PQ.

69. Find the coordinates of the point Q in the third quadrant so that the origin is the midpoint of the segment PQ.

70. Writing to Learn Prove that the distance formula for the number line is a special case of the distance formula for the Cartesian plane.

P.3 Linear Equations and Inequalities

What you'll learn about
- Equations
- Solving Equations
- Linear Equations in One Variable
- Linear Inequalities in One Variable

... and why
These topics provide the foundation for algebraic techniques needed throughout this text.

Equations

An **equation** is a statement of equality between two expressions. Here are some properties of equality that we use to solve equations algebraically.

Properties of Equality

Let u, v, w, and z be real numbers, variables, or algebraic expressions.

1. **Reflexive** $u = u$
2. **Symmetric** If $u = v$, then $v = u$.
3. **Transitive** If $u = v$ and $v = w$, then $u = w$.
4. **Additive** If $u = v$ and $w = z$, then $u + w = v + z$.
5. **Multiplicative** If $u = v$ and $w = z$, then $uw = vz$.

Solving Equations

A **solution of an equation in *x*** is a value of x for which the equation is true. To **solve an equation in *x*** means to find all values of x for which the equation is true, that is, to find all solutions of the equation.

EXAMPLE 1 Confirming a Solution

Prove that $x = -2$ is a solution of the equation $x^3 - x + 6 = 0$.

SOLUTION Let $x = -2$. Then

$$\begin{aligned} x^3 - x + 6 &= (-2)^3 - (-2) + 6 \\ &= -8 + 2 + 6 \\ &= 0. \end{aligned}$$

Thus, by the transitive property of equality, -2 is a value of x for which the equation $x^3 - x + 6 = 0$ is true. Hence, $x = -2$ is a solution of the equation $x^3 - x + 6 = 0$. **Now try Exercise 1.**

Linear Equations in One Variable

The most basic equation in algebra is a *linear equation*.

DEFINITION Linear Equation in *x*

A **linear equation in *x*** is one that can be written in the form

$$ax + b = 0,$$

where a and b are real numbers and $a \neq 0$.

The equation $2z - 4 = 0$ is linear in the variable z. Because of the exponent 2, the equation $3u^2 - 12 = 0$ is *not* linear in the variable u. A linear equation in one variable has exactly one solution. We solve such an equation by transforming it into an *equivalent equation* whose solution is obvious. Two or more equations are **equivalent** if they have the same solutions. For example, the equations $2z - 4 = 0$, $2z = 4$, and $z = 2$ are all equivalent equations.

Operations for Equivalent Equations

An equivalent equation is obtained if one or more of the following operations are performed.

Operation	Given Equation	Equivalent Equation
1. Combine like terms, reduce fractions, and remove grouping symbols.	$2x + x = \frac{3}{9}$	$3x = \frac{1}{3}$
2. Perform the same operation on both sides.		
(a) Add (-3).	$x + 3 = 7$	$x = 4$
(b) Subtract $(2x)$.	$5x = 2x + 4$	$3x = 4$
(c) Multiply by a nonzero constant $(1/3)$.	$3x = 12$	$x = 4$
(d) Divide by a nonzero constant (3).	$3x = 12$	$x = 4$

The next two examples illustrate how to use equivalent equations to solve linear equations.

EXAMPLE 2 Solving a Linear Equation

Solve $2(2x - 3) + 3(x + 1) = 5x + 2$. Support the result with a calculator.

SOLUTION

$$\begin{aligned} 2(2x - 3) + 3(x + 1) &= 5x + 2 \\ 4x - 6 + 3x + 3 &= 5x + 2 && \text{Distributive properties} \\ 7x - 3 &= 5x + 2 && \text{Combine like terms.} \\ 2x &= 5 && \text{Add 3, and subtract } 5x. \\ x &= 2.5 && \text{Divide by 2.} \end{aligned}$$

To support our algebraic work we can evaluate the expressions in the original equation for $x = 2.5$. Figure P.17 shows that each side of the original equation is equal to 14.5 if $x = 2.5$. **Now try Exercise 23.**

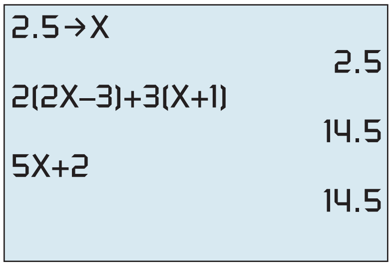


Figure P.17 The top line stores the number 2.5 into the variable x. (Example 2)

If an equation involves fractions, find the least common denominator (LCD) of the fractions and multiply both sides by the LCD. This is sometimes referred to as *clearing the equation of fractions*. Example 3 illustrates this method.

Integers and Fractions

Notice in Example 3 that $2 = \frac{2}{1}$.

EXAMPLE 3 Solving a Linear Equation Involving Fractions

Solve

$$\frac{5y-2}{8} = 2 + \frac{y}{4}.$$

SOLUTION The denominators are 8, 1, and 4. The LCD of the fractions is 8. (See Appendix A.3 if necessary.)

$$\frac{5y-2}{8} = 2 + \frac{y}{4}$$

$$8\left(\frac{5y-2}{8}\right) = 8\left(2 + \frac{y}{4}\right) \qquad \text{Multiply by the LCD 8.}$$

$$8 \cdot \frac{5y-2}{8} = 8 \cdot 2 + 8 \cdot \frac{y}{4} \qquad \text{Distributive property}$$

$$5y - 2 = 16 + 2y \qquad \text{Simplify.}$$

$$5y = 18 + 2y \qquad \text{Add 2.}$$

$$3y = 18 \qquad \text{Subtract } 2y.$$

$$y = 6 \qquad \text{Divide by 3.}$$

We leave it to you to check the solution using either paper and pencil or a calculator.

Now try Exercise 25.

Linear Inequalities in One Variable

We used inequalities to describe order on the number line in Section P.1. For example, if x is to the left of 2 on the number line, or if x is any real number less than 2, we write $x < 2$. The most basic inequality in algebra is a *linear inequality*.

DEFINITION Linear Inequality in *x*

A **linear inequality in *x*** is one that can be written in the form

$$ax + b < 0, \quad ax + b \le 0, \quad ax + b > 0, \quad \text{or} \quad ax + b \ge 0,$$

where a and b are real numbers and $a \neq 0$.

To **solve an inequality in *x*** means to find all values of x for which the inequality is true. A **solution of an inequality in *x*** is a value of x for which the inequality is true. The set of all solutions of an inequality is the **solution set** of the inequality. We **solve an inequality** by finding its solution set. Here is a list of properties we use to solve inequalities.

Direction of an Inequality

Multiplying (or dividing) an inequality by a positive number preserves the direction of the inequality. Multiplying (or dividing) an inequality by a negative number reverses the direction.

Properties of Inequalities

Let u, v, w, and z be real numbers, variables, or algebraic expressions, and c a real number.

1. **Transitive** If $u < v$ and $v < w$, then $u < w$.
2. **Additive** If $u < v$, then $u + w < v + w$.
   If $u < v$ and $w < z$, then $u + w < v + z$.
3. **Multiplicative** If $u < v$ and $c > 0$, then $uc < vc$.
   If $u < v$ and $c < 0$, then $uc > vc$.

There are similar properties for $\le$, $>$, and $\ge$.

The set of solutions of a linear inequality in one variable is an interval of real numbers. Just as with linear equations, we solve a linear inequality by transforming it into an *equivalent inequality* whose solutions are obvious. Two or more inequalities are **equivalent** if they have the same solution set. The properties of inequalities listed on the previous page describe operations that transform an inequality into an equivalent one.

EXAMPLE 4 Solving a Linear Inequality

Solve $3(x-1)+2 \leq 5x+6$.

SOLUTION

$$
\begin{aligned}
3(x-1)+2 &\leq 5x+6 \\
3x-3+2 &\leq 5x+6 && \text{Distributive property} \\
3x-1 &\leq 5x+6 && \text{Combine like terms.} \\
3x &\leq 5x+7 && \text{Add 1.} \\
-2x &\leq 7 && \text{Subtract } 5x. \\
\left(-\frac{1}{2}\right)\cdot -2x &\geq \left(-\frac{1}{2}\right)\cdot 7 && \text{Multiply by } -1/2. \text{ (The inequality reverses.)} \\
x &\geq -3.5
\end{aligned}
$$

The solution set of the inequality is the set of all real numbers greater than or equal to -3.5. In interval notation, the solution set is $[-3.5, \infty)$.

Now try Exercise 41.

Because the solution set of a linear inequality is an interval of real numbers, we can display the solution set with a number line graph as illustrated in Example 5.

EXAMPLE 5 Solving a Linear Inequality Involving Fractions

Solve the inequality, and graph its solution set.

$$\frac{x}{3}+\frac{1}{2} > \frac{x}{4}+\frac{1}{3}$$

SOLUTION The LCD of the fractions is 12.

$$
\begin{aligned}
\frac{x}{3}+\frac{1}{2} &> \frac{x}{4}+\frac{1}{3} \\
12\cdot\left(\frac{x}{3}+\frac{1}{2}\right) &> 12\cdot\left(\frac{x}{4}+\frac{1}{3}\right) && \text{Multiply by the LCD 12.} \\
4x+6 &> 3x+4 && \text{Simplify.} \\
x+6 &> 4 && \text{Subtract } 3x. \\
x &> -2 && \text{Subtract 6.}
\end{aligned}
$$

The solution set is the interval $(-2, \infty)$. Its graph is shown in Figure P.18.

Now try Exercise 37.

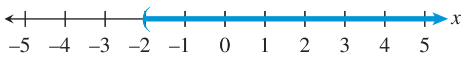


Figure P.18 The graph of the solution set of the inequality in Example 5.

Sometimes two inequalities are combined in a *double inequality,* which is solved by isolating x as the middle expression. Example 6 illustrates this.

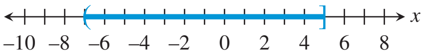


Figure P.19 The graph of the solution set of the double inequality in Example 6.

EXAMPLE 6 Solving a Double Inequality

Solve the inequality, and graph its solution set.

$$-3 < \frac{2x + 5}{3} \le 5$$

SOLUTION

$$-3 < \frac{2x + 5}{3} \le 5$$

$$-9 < 2x + 5 \le 15 \quad \text{Multiply by 3.}$$

$$-14 < 2x \le 10 \quad \text{Subtract 5.}$$

$$-7 < x \le 5 \quad \text{Divide by 2.}$$

The solution set is the set of all real numbers greater than -7 and less than or equal to 5. In interval notation, the solution set is $(-7, 5]$. Its graph is shown in Figure P.19.

Now try Exercise 47.

QUICK REVIEW P.3

In Exercises 1 and 2, simplify the expression by combining like terms.

1. $2x + 5x + 7 + y - 3x + 4y + 2$

2. $4 + 2x - 3z + 5y - x + 2y - z - 2$

In Exercises 3 and 4, use the distributive property to expand the products. Simplify the resulting expression by combining like terms.

3. $3(2x - y) + 4(y - x) + x + y$

4. $5(2x + y - 1) + 4(y - 3x + 2) + 1$

In Exercises 5–10, use the LCD to combine the fractions. Simplify the resulting fraction.

5. $\frac{2}{y} + \frac{3}{y}$ **6.** $\frac{1}{y - 1} + \frac{3}{y - 2}$

7. $2 + \frac{1}{x}$ **8.** $\frac{1}{x} + \frac{1}{y} - x$

9. $\frac{x + 4}{2} + \frac{3x - 1}{5}$ **10.** $\frac{x}{3} + \frac{x}{4}$

SECTION P.3 Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, which values of x are solutions of the equation?

1. $2x^2 + 5x = 3$

(a) $x = -3$ (b) $x = -\frac{1}{2}$ (c) $x = \frac{1}{2}$

2. $\frac{x}{2} + \frac{1}{6} = \frac{x}{3}$

(a) $x = -1$ (b) $x = 0$ (c) $x = 1$

3. $\sqrt{1 - x^2} + 2 = 3$

(a) $x = -2$ (b) $x = 0$ (c) $x = 2$

4. $(x - 2)^{1/3} = 2$

(a) $x = -6$ (b) $x = 8$ (c) $x = 10$

In Exercises 5–10, determine whether the equation is linear in x.

5. $5 - 3x = 0$ **6.** $5 = 10/2$

7. $x + 3 = x - 5$ **8.** $x - 3 = x^2$

9. $2\sqrt{x} + 5 = 10$ **10.** $x + \frac{1}{x} = 1$

In Exercises 11–24, solve the equation without using a calculator.

11. $3x = 24$ **12.** $4x = -16$

13. $3t - 4 = 8$ **14.** $2t - 9 = 3$

15. $2x - 3 = 4x - 5$ **16.** $4 - 2x = 3x - 6$

17. $4 - 3y = 2(y + 4)$ **18.** $4(y - 2) = 5y$

19. $\frac{1}{2}x = \frac{7}{8}$ **20.** $\frac{2}{3}x = \frac{4}{5}$

21. $\frac{1}{2}x + \frac{1}{3} = 1$

22. $\frac{1}{3}x + \frac{1}{4} = 1$

23. $2(3 - 4z) - 5(2z + 3) = z - 17$

24. $3(5z - 3) - 4(2z + 1) = 5z - 2$

In Exercises 25–28, solve the equation. Support your answer with a calculator.

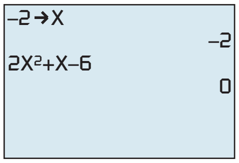
25. $\frac{2x - 3}{4} + 5 = 3x$

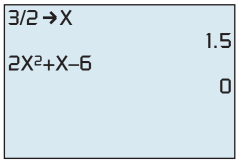
26. $2x - 4 = \frac{4x - 5}{3}$

27. $\frac{t + 5}{8} - \frac{t - 2}{2} = \frac{1}{3}$

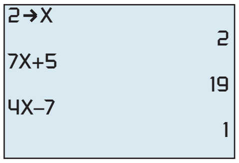
28. $\frac{t - 1}{3} + \frac{t + 5}{4} = \frac{1}{2}$

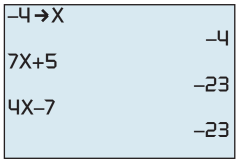
29. Writing to Learn Write a statement about a solution of an equation suggested by the computations in the figure.

(a) 

(b) 

30. Writing to Learn Write a statement about a solution of an equation suggested by the computations in the figure.

(a) 

(b) 

In Exercises 31–34, which values of x are solutions of the inequality?

31. $2x - 3 < 7$

(a) $x = 0$ (b) $x = 5$ (c) $x = 6$

32. $3x - 4 \geq 5$

(a) $x = 0$ (b) $x = 3$ (c) $x = 4$

33. $-1 < 4x - 1 \leq 11$

(a) $x = 0$ (b) $x = 2$ (c) $x = 3$

34. $-3 \leq 1 - 2x \leq 3$

(a) $x = -1$ (b) $x = 0$ (c) $x = 2$

In Exercises 35–42, solve the inequality, and draw a number line graph of the solution set.

35. $x - 4 < 2$

36. $x + 3 > 5$

37. $2x - 1 \leq 4x + 3$

38. $3x - 1 \geq 6x + 8$

39. $2 \leq x + 6 < 9$

40. $-1 \leq 3x - 2 < 7$

41. $2(5 - 3x) + 3(2x - 1) \leq 2x + 1$

42. $4(1 - x) + 5(1 + x) > 3x - 1$

In Exercises 43–54, solve the inequality.

43. $\frac{5x + 7}{4} \leq -3$

44. $\frac{3x - 2}{5} > -1$

45. $4 \geq \frac{2y - 5}{3} \geq -2$

46. $1 > \frac{3y - 1}{4} > -1$

47. $0 \leq 2z + 5 < 8$

48. $-6 < 5t - 1 < 0$

49. $\frac{x - 5}{4} + \frac{3 - 2x}{3} < -2$

50. $\frac{3 - x}{2} + \frac{5x - 2}{3} < -1$

51. $\frac{2y - 3}{2} + \frac{3y - 1}{5} < y - 1$

52. $\frac{3 - 4y}{6} - \frac{2y - 3}{8} \geq 2 - y$

53. $\frac{1}{2}(x - 4) - 2x \leq 5(3 - x)$

54. $\frac{1}{2}(x + 3) + 2(x - 4) < \frac{1}{3}(x - 3)$

In Exercises 55–58, find the solutions of the equation or inequality that are displayed in Figure P.20.

55. $x^2 - 2x < 0$

56. $x^2 - 2x = 0$

57. $x^2 - 2x > 0$

58. $x^2 - 2x \leq 0$

X	Y1	
0	0	
1	–1	
2	0	
3	3	
4	8	
5	15	
6	24	
Y1 = X²–2X		

Figure P.20 The second column gives values of $y_1 = x^2 - 2x$ for $x = 0, 1, 2, 3, 4, 5,$ and 6.

59. Writing to Learn Explain how the second equation was obtained from the first.

$$x - 3 = 2x + 3, \quad 2x - 6 = 4x + 6$$

60. Writing to Learn Explain how the second equation was obtained from the first.

$$2x - 1 = 2x - 4, \quad x - \frac{1}{2} = x - 2$$

61. Group Activity Determine whether the two equations are equivalent.

(a) $3x = 6x + 9, \quad x = 2x + 9$

(b) $6x + 2 = 4x + 10, \quad 3x + 1 = 2x + 5$

62. Group Activity Determine whether the two equations are equivalent.

(a) $3x + 2 = 5x - 7, \quad -2x + 2 = -7$

(b) $2x + 5 = x - 7, \quad 2x = x - 7$

Standardized Test Questions

63. True or False $-6 > -2$. Justify your answer.

64. True or False $2 \le \frac{6}{3}$. Justify your answer.

In Exercises 65–68, you may use a graphing calculator to solve these problems.

65. Multiple Choice Which of the following equations is equivalent to the equation $3x + 5 = 2x + 1$?

(A) $3x = 2x$ (B) $3x = 2x + 4$

(C) $\frac{3}{2}x + \frac{5}{2} = x + 1$ (D) $3x + 6 = 2x$

(E) $3x = 2x - 4$

66. Multiple Choice Which of the following inequalities is equivalent to the inequality $-3x < 6$?

(A) $3x < -6$ (B) $x < 10$

(C) $x > -2$ (D) $x > 2$

(E) $x > 3$

67. Multiple Choice Which of the following is the solution to the equation $x(x + 1) = 0$?

(A) $x = 0$ or $x = -1$ (B) $x = 0$ or $x = 1$

(C) Only $x = -1$ (D) Only $x = 0$

(E) Only $x = 1$

68. Multiple Choice Which of the following represents an equation equivalent to the equation

$$\frac{2x}{3} + \frac{1}{2} = \frac{x}{4} - \frac{1}{3}$$

that is cleared of fractions?

(A) $2x + 1 = x - 1$ (B) $8x + 6 = 3x - 4$

(C) $4x + 3 = \frac{3}{2}x - 2$ (D) $4x + 3 = 3x - 4$

(E) $4x + 6 = 3x - 4$

Explorations

69. Testing Inequalities on a Calculator

(a) The calculator we use indicates that the statement $2 < 3$ is true by returning the value 1 (for true) when $2 < 3$ is entered. Try it with your calculator.

(b) The calculator we use indicates that the statement $2 < 1$ is false by returning the value 0 (for false) when $2 < 1$ is entered. Try it with your calculator.

(c) Use your calculator to test which of these two numbers is larger: 799/800, 800/801.

(d) Use your calculator to test which of these two numbers is larger: −102/101, −103/102.

(e) If your calculator returns 0 when you enter $2x + 1 < 4$, what can you conclude about the value stored in x?

Extending the Ideas

70. Perimeter of a Rectangle The formula for the perimeter P of a rectangle is

$$P = 2(L + W).$$

Solve this equation for W.

71. Area of a Trapezoid The formula for the area A of a trapezoid is

$$A = \frac{1}{2}h(b_1 + b_2).$$

Solve this equation for b_1.

72. Volume of a Sphere

The formula for the volume V of a sphere is

$$V = \frac{4}{3}\pi r^3.$$

Solve this equation for r.

73. Celsius and Fahrenheit The formula for Celsius temperature in terms of Fahrenheit temperature is

$$C = \frac{5}{9}(F - 32).$$

Solve the equation for F.

P.4 Lines in the Plane

What you'll learn about

- Slope of a Line
- Point-Slope Form Equation of a Line
- Slope-Intercept Form Equation of a Line
- Graphing Linear Equations in Two Variables
- Parallel and Perpendicular Lines
- Applying Linear Equations in Two Variables

... and why

Linear equations are used extensively in applications involving business and behavioral science.

Slope of a Line

The slope of a nonvertical line is the vertical change divided by the horizontal change between any two points on the line. For the points (x_1, y_1) and (x_2, y_2), the vertical change is $\Delta y = y_2 - y_1$, and the horizontal change is $\Delta x = x_2 - x_1$. (Δy is read "delta" y.) See Figure P.21.

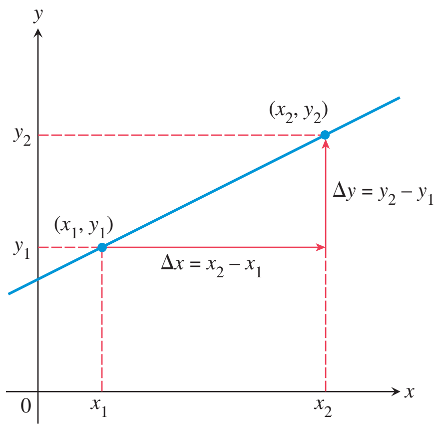


Figure P.21 The slope of a nonvertical line can be found from the coordinates of any two points on the line.

DEFINITION Slope of a Line

The **slope** of a nonvertical line through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

If the line is vertical, then $x_1 = x_2$ and the slope is undefined.

EXAMPLE 1 Finding the Slope of a Line

Find the slope of the line through the two points. Sketch a graph of the line.

(a) $(-1, 2)$ and $(4, -2)$

(b) $(1, 1)$ and $(3, 4)$

SOLUTION

(a) The two points are $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (4, -2)$. Thus,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - 2}{4 - (-1)} = -\frac{4}{5} = -0.8.$$

(b) The two points are $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (3, 4)$. Thus,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{3 - 1} = \frac{3}{2} = 1.5.$$

The graphs of these two lines are shown in Figure P.22.

Now try Exercise 3.

Slope Formula

The slope does not depend on the order of the points. We could use $(x_1, y_1) = (4, -2)$ and $(x_2, y_2) = (-1, 2)$ in Example 1a. Check it out.

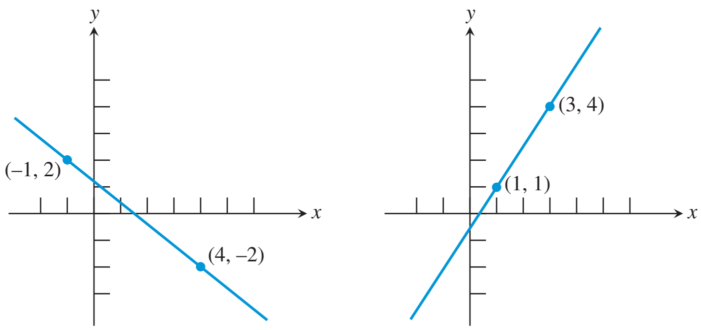


Figure P.22 The graphs of the two lines in Example 1.

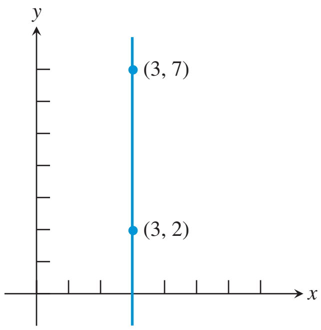


Figure P.23 Applying the slope formula to this vertical line gives $m = 5/0$, which is not defined. Thus, the slope of a vertical line is undefined.

Figure P.23 shows a vertical line through the points $(3, 2)$ and $(3, 7)$. If we try to calculate its slope using the slope formula $(y_2 - y_1)/(x_2 - x_1)$, we get zero in the denominator. So, it makes sense to say that a vertical line does not have a slope, or that its slope is undefined.

Point-Slope Form Equation of a Line

If we know the coordinates of one point on a line and the slope of the line, then we can find an equation for that line. For example, the line in Figure P.24 passes through the point (x_1, y_1) and has slope m. If (x, y) is any other point on this line, the definition of the slope yields the equation

$$m = \frac{y - y_1}{x - x_1} \quad \text{or} \quad y - y_1 = m(x - x_1).$$

An equation written this way is in the *point-slope form*.

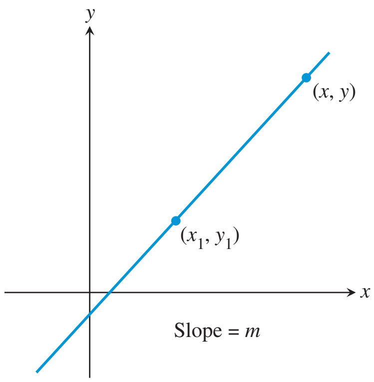


Figure P.24 The line through (x_1, y_1) with slope m.

DEFINITION Point-Slope Form of an Equation of a Line

The **point-slope form** of an equation of a line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1).$$

EXAMPLE 2 Using the Point-Slope Form

Use the point-slope form to find an equation of the line that passes through the point $(-3, -4)$ and has slope 2.

SOLUTION We substitute $x_1 = -3$, $y_1 = -4$, and $m = 2$ into the point-slope form, and simplify the resulting equation.

$$
\begin{aligned}
y - y_1 &= m(x - x_1) && \text{Point-slope form} \\
y - (-4) &= 2(x - (-3)) && x_1 = -3,\ y_1 = -4,\ m = 2 \\
y + 4 &= 2(x + 3) && \text{Simplify.}
\end{aligned}
$$

For graphing purposes, this equation can be written as $y = 2(x + 3) - 4$ or as $y = 2x + 2$.

Now try Exercise 11.

y-Intercept

The b in $y = mx + b$ is often referred to as "the y-intercept" instead of "the y-coordinate of the y-intercept."

Slope-Intercept Form Equation of a Line

The **y-intercept** of a nonvertical line is the point where the line intersects the y-axis. If we know the y-intercept and the slope of the line, we can apply the point-slope form to find an equation of the line.

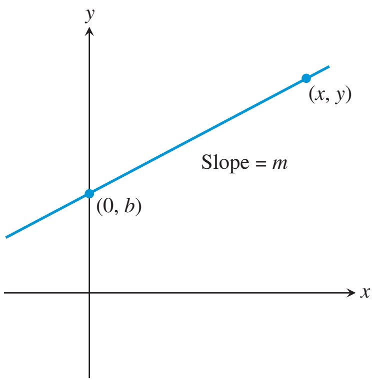


Figure P.25 The line with slope m and y-intercept b.

Alternative Solution

You could solve Example 3 using the point-slope form:

$$\begin{aligned} y - 6 &= 3(x - (-1)) \\ y &= 3(x + 1) + 6 \\ y &= 3x + 3 + 6 \\ y &= 3x + 9 \end{aligned}$$

Figure P.25 shows a line with slope m and y-intercept $(0, b)$, or b for short. A point-slope form equation for this line is $y - b = m(x - 0)$. By rewriting this equation, we obtain the form known as the *slope-intercept form*.

DEFINITION Slope-Intercept Form of an Equation of a Line

The **slope-intercept form** of an equation of a line with slope m and y-intercept $(0, b)$ is

$$y = mx + b.$$

EXAMPLE 3 Using the Slope-Intercept Form

Using the slope-intercept form, write an equation of the line with slope 3 that passes through the point $(-1, 6)$.

SOLUTION

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ y &= 3x + b && m = 3 \\ 6 &= 3(-1) + b && y = 6 \text{ when } x = -1 \\ b &= 9 \end{aligned}$$

The slope-intercept form of the equation is $y = 3x + 9$. **Now try Exercise 21.**

We should not use the phrase "*the* equation of a line" because each line has many equations. Every line has an equation that can be written in the form $Ax + By + C = 0$ where A and B are not both zero. This form is the **general form** for an equation of a line.

If $B \neq 0$, the general form can be changed to the slope-intercept form as follows:

$$\begin{aligned} Ax + By + C &= 0 \\ By &= -Ax - C \\ y &= \underbrace{-\frac{A}{B}}_{\text{slope}}x + \underbrace{\left(-\frac{C}{B}\right)}_{y\text{-intercept}} \end{aligned}$$

Forms of Equations of Lines

General form:	$Ax + By + C = 0$, A and B not both zero
Slope-intercept form:	$y = mx + b$
Point-slope form:	$y - y_1 = m(x - x_1)$
Vertical line:	$x = a$
Horizontal line:	$y = b$

Graphing Linear Equations in Two Variables

A **linear equation in x and y** is one that can be written in the form

$$Ax + By = C,$$

where A and B are not both zero. Rewriting the equation as $Ax + By - C = 0$, we see that it is closely related to the general form. If $B = 0$, the line is vertical, and if $A = 0$, the line is horizontal.

The **graph** of an equation in x and y consists of all pairs (x, y) that are solutions of the equation. For example, $(1, 2)$ is a **solution** of the equation $2x + 3y = 8$ because substituting $x = 1$ and $y = 2$ into the equation leads to the true statement $8 = 8$. The pairs $(-2, 4)$ and $(2, 4/3)$ are also solutions.

Because the graph of a linear equation in x and y is a straight line, to draw the graph we can find two solutions and then connect them with a straight line. If a line is neither horizontal nor vertical, then two easy points to find are its x-intercept and y-intercept. The **x-intercept** is the point $(a, 0)$ where the graph intersects the x-axis. Set $y = 0$ and solve for x to find the x-intercept. To find the y-intercept, set $x = 0$ and solve for y.

```
WINDOW
  Xmin=–10
  Xmax=10
  Xscl=1
  Ymin=–10
  Ymax=10
  Yscl=1
  Xres=1
```

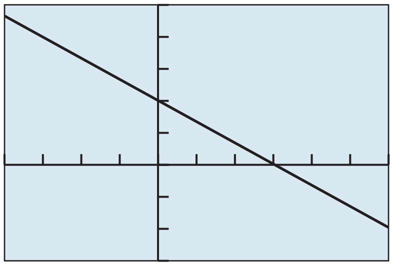
Figure P.26 The window dimensions for the *standard window*. The notation "$[-10, 10]$ by $[-10, 10]$" is used to represent window dimensions like these.

Graphing with a Graphing Utility

To draw a graph of an equation using a grapher:

1. Rewrite the equation in the form $y =$ (an expression in x).
2. Enter the expression into the grapher.
3. Select an appropriate *viewing window*. (See Figures P.26 and P.27, Example 4, and the margin note.)
4. Press the "graph" key.

A graphing utility, or *grapher*, computes y-values for a select set of x-values between Xmin and Xmax and plots the corresponding (x, y) points.



$[-4, 6]$ by $[-3, 5]$

Figure P.27 The graph of $2x + 3y = 6$. Notice that the points $(0, 2)$ (y-intercept) and $(3, 0)$ (x-intercept) lie on the graph and are solutions of the equation. (Example 4)

EXAMPLE 4 Using a Graphing Utility

Draw the graph of $2x + 3y = 6$.

SOLUTION First we solve for y.

$$2x + 3y = 6$$

$$3y = -2x + 6 \quad \text{Solve for } y.$$

$$y = -\frac{2}{3}x + 2 \quad \text{Divide by 3.}$$

Figure P.27 shows the graph of $y = -(2/3)x + 2$, or equivalently, the graph of the linear equation $2x + 3y = 6$ in the $[-4, 6]$ by $[-3, 5]$ viewing window.

Now try Exercise 27.

Viewing Window

The **viewing window** $[-4, 6]$ by $[-3, 5]$ in Example 4 and Figure P.27 means $-4 \le x \le 6$ and $-3 \le y \le 5$.

Parallel and Perpendicular Lines

EXPLORATION 1 Investigating Graphs of Linear Equations

1. What do the graphs of $y = mx + b$ and $y = mx + c$, $b \ne c$, have in common? How are they different?
2. Graph $y = 2x$ and $y = -(1/2)x$ in a *square viewing window*. (See margin note.) On the grapher we use, the "decimal window" is square. Estimate the angle between the two lines.
3. Repeat part 2 for $y = mx$ and $y = -(1/m)x$ with $m = 1, 3, 4,$ and 5.

Square Viewing Window

A **square viewing window** on a grapher is one in which angles appear to be true. For example, the line $y = x$ will appear to make a 45° angle with the positive x-axis. Furthermore, a distance of 1 on the x- and y-axes will appear to be the same.

Parallel lines and perpendicular lines were involved in Exploration 1. Using a grapher to decide whether lines are parallel or perpendicular is risky. Here is an algebraic test to determine whether two lines are parallel or perpendicular.

Parallel and Perpendicular Lines

1. Two nonvertical lines are parallel if and only if their slopes are equal. Any two distinct vertical lines are parallel.
2. Two nonvertical lines are perpendicular if and only if their slopes m_1 and m_2 are opposite reciprocals, that is, if and only if

$$m_1 = -\frac{1}{m_2}.$$

A vertical line is perpendicular to a horizontal line, and vice versa.

EXAMPLE 5 Finding an Equation of a Parallel Line

Find an equation of the line through $P(1, -2)$ that is parallel to the line l with equation $3x - 2y = 1$.

SOLUTION We find the slope of l by writing its equation in slope-intercept form.

$$3x - 2y = 1 \quad \text{Equation for } l$$
$$-2y = -3x + 1 \quad \text{Subtract } 3x.$$
$$y = \frac{3}{2}x - \frac{1}{2} \quad \text{Divide by } -2.$$

The slope of l is $3/2$.

The line whose equation we seek has slope $3/2$ and contains the point $(x_1, y_1) = (1, -2)$. Thus, the point-slope form equation for the line we seek is

$$y + 2 = \frac{3}{2}(x - 1),$$

which also can be written as

$$y = \frac{3}{2}x - \frac{7}{2} \quad \text{or} \quad 3x - 2y = 7.$$

Now try Exercise 41(a).

EXAMPLE 6 Finding an Equation of a Perpendicular Line

Find an equation of the line through $P(2, -3)$ that is perpendicular to the line l with equation $4x + y = 3$. Support the result with a grapher.

SOLUTION We find the slope of l by writing its equation in slope-intercept form.

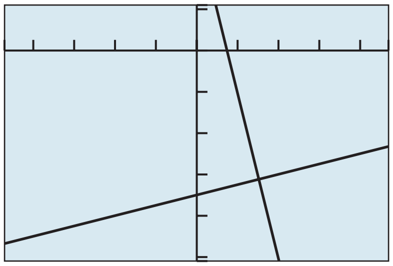
$$4x + y = 3 \quad \text{Equation for } l$$
$$y = -4x + 3 \quad \text{Subtract } 4x.$$

The slope of l is -4.

The line whose equation we seek has slope $-1/(-4) = 1/4$ and passes through the point $(x_1, y_1) = (2, -3)$. We use the point-slope form, then simplify the equation:

$$y - (-3) = \frac{1}{4}(x - 2)$$
$$y + 3 = \frac{1}{4}x - \frac{1}{2} \quad \text{Distributive property}$$
$$y = \frac{1}{4}x - \frac{7}{2}$$

Figure P.28 shows the graphs of the two equations in a square viewing window and suggests that the graphs are indeed perpendicular. **Now try Exercise 43(b).**



$[-4.7, 4.7]$ by $[-5.1, 1.1]$

Figure P.28 The graphs of $y = -4x + 3$ and $y = (1/4)x - 7/2$ in this square viewing window appear to intersect at a right angle. (Example 6)

Applying Linear Equations in Two Variables

Linear equations and their graphs occur frequently in applications. Algebraic solutions to these application problems often require finding an equation of a line and solving a linear equation in one variable. Grapher techniques complement algebraic ones.

EXAMPLE 7 Finding the Depreciation of Real Estate

Camelot Apartments purchased a \$50,000 building. For tax purposes, its value depreciates \$2000 per year over a 25-year period.

(a) Write a linear equation giving the value y of the building in terms of the years x after the purchase.

(b) In how many years will the value of the building be \$24,500?

SOLUTION

(a) We need to determine the value of m and b so that $y = mx + b$, where $0 \leq x \leq 25$. We know that $y = 50{,}000$ when $x = 0$, so the line has y-intercept $(0, 50{,}000)$ and $b = 50{,}000$. One year after purchase $(x = 1)$, the value of the building is $50{,}000 - 2000 = 48{,}000$. So when $x = 1$, $y = 48{,}000$. Therefore,

$$y = mx + b$$

$$48{,}000 = m \cdot 1 + 50{,}000 \qquad y = 48{,}000 \text{ when } x = 1.$$

$$-2000 = m \qquad \text{Subtract } 50{,}000.$$

The value y of the building x years after its purchase is

$$y = -2000x + 50{,}000.$$

(b) We need to find the value of x when $y = 24{,}500$. So, we substitute 24,500 for y in the equation $y = -2000x + 50{,}000$.

$$24{,}500 = -2000x + 50{,}000 \qquad \text{Set } y = 24{,}500.$$

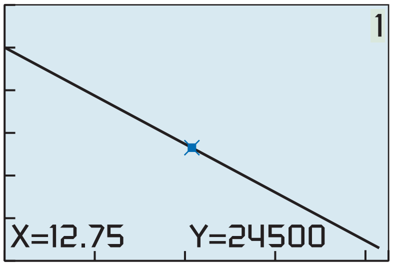
$$-25{,}500 = -2000x \qquad \text{Subtract } 50{,}000.$$

$$12.75 = x \qquad \text{Divide by } -2000.$$

The value of the building will be \$24,500 precisely 12.75 years, or 12 years 9 months, after the building was purchased by Camelot Apartments.

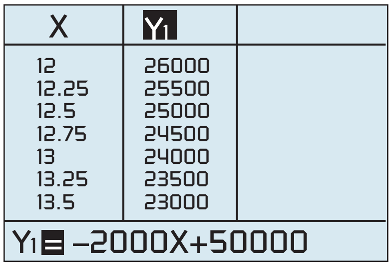
We can support our algebraic work both graphically and numerically. The trace coordinates in Figure P.29a show graphically that $(12.75, 24{,}500)$ is a solution of $y = -2000x + 50{,}000$. This means that $y = 24{,}500$ when $x = 12.75$. Figure P.29b is a table of values for $y = -2000x + 50{,}000$ for a few values of x. The fourth line of the table shows numerically that $y = 24{,}500$ when $x = 12.75$.

Now try Exercise 45.



[0, 23.5] by [0, 60000]

(a)



(b)

Figure P.29 A (a) graph and (b) table of values for $y = -2000x + 50{,}000$. (Example 7)

Figure P.30 on page 34 shows Americans' income from 2010 through 2015 in trillions of dollars and a corresponding scatter plot of these data. In Example 8, we model the data in Figure P.30 with a linear equation.