COLLEGE ALGEBRA

CONCEPTS THROUGH FUNCTIONS

Fourth Edition

Sullivan
Sullivan

Pearson

Get the Most Out of MyLab Math

MyLab™ Math, Pearson's online homework, tutorial, and assessment program, creates personalized experiences for students and provides powerful tools for instructors. With a wealth of tested and proven resources, each course can be tailored to fit your specific needs.

Learning In Any Environment

- With an updated and streamlined design, students and instructors can access MyLab Math from most **mobile devices**.
- Because classroom formats and student needs continually change and evolve, MyLab Math has built-in flexibility to accommodate various course designs and formats.

Personalized Learning

Not every student learns the same way or at the same rate. Thanks to our advances in adaptive learning technology capabilities, you no longer have to teach as if they do.

- MyLab Math's **adaptive Study Plan** acts as a personal tutor, updating in real-time based on student performance throughout the course to provide personalized recommendations for practice. You can now assign the Study Plan as a prerequisite to a test or a quiz with **Companion Study Plan Assignments**.
- MyLab Math can **personalize homework assignments** for students based on their performance on a test or quiz. This way, students can focus on just the topics they have not yet mastered.
- New - **Skill Builder** exercises offer just-in-time adaptive practice from within homework assignments to help students build the skills needed to successfully complete their work.

Visit pearson.com/mylab/math and click Get Trained to make sure you're getting the most out of MyLab Math.

MyLab Math Online Course for College Algebra: Concepts Through Functions by Sullivan and Sullivan

(access code required)

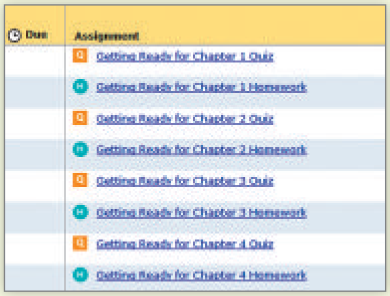
Pearson
MyLab

Achieve Your Potential

Success in math can make a difference in life. MyLab Math is a learning program with resources to help you achieve your potential in this course and beyond. MyLab Math helps you get up to speed on course material and understand how math will play a role in your future career.

Preparedness

One of the biggest challenges in College Algebra, Trigonometry, and Precalculus is being adequately prepared for the course with prerequisite knowledge. MyLab Math's learning resources help you refresh your knowledge of topics you previously learned. Brushing up on these essential algebra skills at the start of a course can dramatically help increase success.



Getting Ready

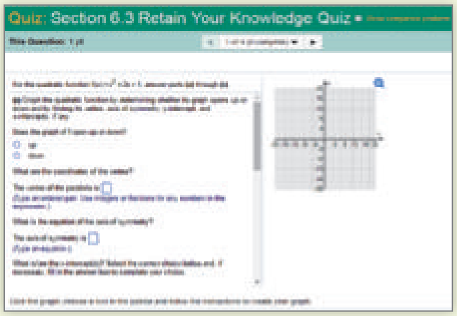
MyLab Math allows you to refresh your understanding of prerequisite topics through skill review quizzes and personalized homework. With Getting Ready content in MyLab Math, you'll get just the help you need to be prepared to learn the new material.

Maintaining Skills

The Sullivans are committed to students, helping them retain essential information and maintain skills needed for success in current and future math courses.

Retain Your Knowledge Exercises

Updated! Retain Your Knowledge Exercises support ongoing review at the course level and help students maintain essential skills. These are excellent cumulative review problems and are perfect for studying for final exams. Retain Your Knowledge Exercises are assignable in MyLab Math and available within each chapter.



Guided Lecture Notes

Get help focusing on important concepts with the use of this structured organized note-taking tool. The Guided Lecture Notes ask students to complete definitions, procedures, and examples based on the content of the Author-in-Action videos and textbook. By directing students into essential material, students can focus and retain the most important concepts.

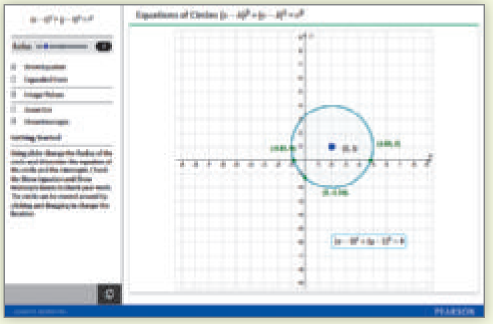
Connect the Concepts and Relate the Math

Visualization and Conceptual Understanding

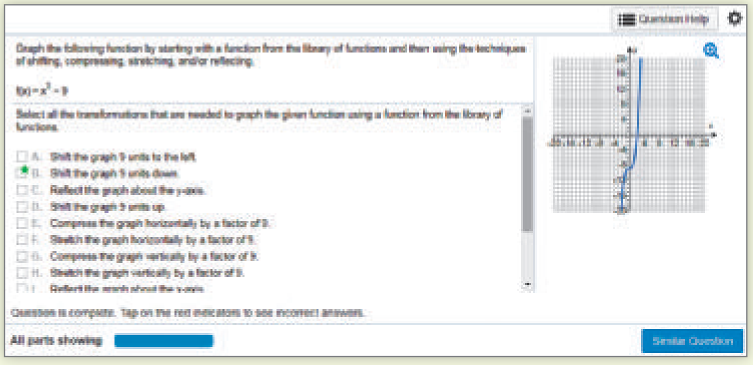
These MyLab Math resources will help students connect the concepts and think visually.

Guided Visualizations

Engaging interactive figures bring mathematical concepts to life, helping you visualize the concepts through directed explorations and purposeful manipulation. Guided Visualizations, assignable in MyLab Math, encourage active learning, critical thinking, and conceptual understanding.



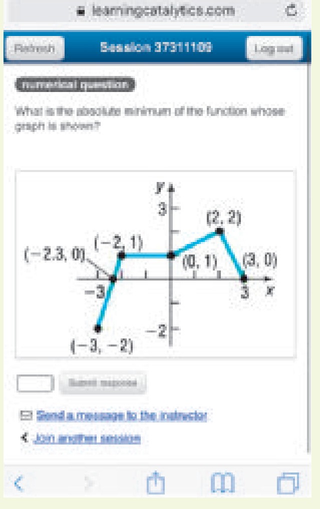
Setup & Solve Exercises



Stepped-out exercises ask students to first describe how they will set up and approach the problem. This reinforces conceptual understanding of the process applied in solving the problem and promotes long term retention of the skill. Access to the eText is available for additional support.

Learning Catalytics

Learning Catalytics helps you generate class discussion, customize your lecture, and promote peer-to-peer learning with real-time analytics. As a student response tool, Learning Catalytics uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking. Michael Sullivan III uses Learning Catalytics in his own classroom and has created a library of questions for instructors to use in their classes.



pearson.com/mylab/math

Prepare for Class "Read the Book"

Feature	Description	Benefit	Page
Every Chapter Opener begins with...			
Chapter Opening Article & Project	Each chapter begins with a current article and ends with a related project. The article describes a real situation.	The Article describes a real situation. The Project lets you apply what you learned to solve a related problem.	280, 385
Internet Based Projects	The projects allow for the integration of spreadsheet technology that students will need to be a productive member of the workforce.	The projects allow the opportunity for students to collaborate and use mathematics to deal with issues that come up in their lives.	280, 385
Every Section begins with...			
Learning Objectives 2	Each section begins with a list of objectives. Objectives also appear in the text where the objective is covered.	These focus your studying by emphasizing what's most important and where to find it.	302
Sections contain...			
Preparing for this Section	Most sections begin with a list of key concepts to review with page numbers.	Ever forget what you've learned? This feature highlights previously learned material to be used in this section. Review it, and you'll always be prepared to move forward.	302
Now Work the 'Are You Prepared?' Problems	Problems that assess whether you have the prerequisite knowledge for the upcoming section.	Not sure you need the Preparing for This Section review? Work the 'Are You Prepared?' problems. If you get one wrong, you'll know exactly what you need to review and where to review it!	302, 313
Now Work PROBLEMS	These follow most examples and direct you to a related exercise.	We learn best by doing. You'll solidify your understanding of examples if you try a similar problem right away, to be sure you understand what you've just read.	309, 314
WARNING	Warnings are provided in the text.	These point out common mistakes and help you to avoid them.	335
Exploration and Seeing the Concept	These represent graphing utility activities to foreshadow a concept or solidify a concept just presented.	You will obtain a deeper and more intuitive understanding of theorems and definitions.	206, 322
In Words	These provide alternative descriptions of select definitions and theorems.	Does math ever look foreign to you? This feature translates math into plain English.	319
CALCULUS	These appear next to information essential for the study of calculus.	Pay attention–if you spend extra time now, you'll do better later!	74, 309
SHOWCASE EXAMPLES	These examples provide "how-to" instruction by offering a guided, step-by-step approach to solving a problem.	With each step presented on the left and the mathematics displayed on the right, students can immediately see how each step is employed.	210–211
Model It! Examples and Problems	These are examples and problems that require you to build a mathematical model from either a verbal description or data. The homework Model It! problems are marked by purple headings.	It is rare for a problem to come in the form, *"Solve the following equation"*. Rather, the equation must be developed based on an explanation of the problem. These problems require you to develop models that will allow you to describe the problem mathematically and suggest a solution to the problem.	325, 356

Practice "Work the Problems"

Feature	Description	Benefit	Page
'Are You Prepared?' Problems	These assess your retention of the prerequisite material you'll need. Answers are given at the end of the section exercises. This feature is related to the Preparing for This Section feature.	Do you always remember what you've learned? Working these problems is the best way to find out. If you get one wrong, you'll know exactly what you need to review and where to review it!	302, 313
Concepts and Vocabulary	These short-answer questions, mainly Fill-in-the-Blank and True/False items, assess your understanding of key definitions and concepts in the current section.	It is difficult to learn math without knowing the language of mathematics. These problems test your understanding of the formulas and vocabulary.	313
Skill Building	Correlated to section examples, these problems provide straightforward practice.	It's important to dig in and develop your skills. These problems provide you with ample practice to do so.	313–315
Mixed Practice	These problems offer comprehensive assessment of the skills learned in the section by asking problems that relate to more than one concept or objective. These problems may also require you to utilize skills learned in previous sections.	Learning mathematics is a building process. Many concepts are interrelated. These problems help you see how mathematics builds on itself and also see how the concepts tie together.	315
Applications and Extensions	These problems allow you to apply your skills to real-world problems. They also allow you to extend concepts leamed in the section.	You will see that the material learned within the section has many uses in everyday life.	315–318
Explaining Concepts: Discussion and Writing	"Discussion and Writing" problems are colored red. These support class discussion, verbalization of mathematical ideas, and writing and research projects.	To verbalize an idea, or to describe it clearly in writing, shows real understanding. These problems nurture that understanding. Many are challenging but you'll get out what you put in.	318
Retain Your Knowledge	These problems allow you to practice content learned earlier in the course.	The ability to remember how to solve all the different problems learned throughout the course is difficult. These help you remember.	318
Now Work PROBLEMS	Many examples refer you to a related homework problem. These related problems are marked by a pencil and orange numbers.	If you get stuck while working problems, look for the closest Now Work problem and refer back to the related example to see if it helps.	311, 314
Chapter Review Problems	Every chapter concludes with a comprehensive list of exercises to practice. Use the list of objectives to determine the objective and examples that correspond to the problems.	Work these problems to verify you understand all the skills and concepts of the chapter. Think of it as a comprehensive review of the chapter.	380–383

Review "Study for Quizzes and Tests"

Feature	Description	Benefit	Page
Chapter Review at the end of each chapter contains…			
Things to Know	A detailed list of important theorems, formulas, and definitions from the chapter.	Review these and you'll know the most important material in the chapter!	378–379
You Should Be able to…	Contains a complete list of objectives by section, examples that illustrate the objective, and practice exercises that test your understanding of the objective.	Do the recommended exercises and you'll have mastery over the key material. If you get something wrong, review the suggested page numbers and try again.	379–380
Review Exercises	These provide comprehensive review and practice of key skills, matched to the Learning Objectives for each section.	Practice makes perfect. These problems combine exercises from all sections, giving you a comprehensive review in one place.	380–383
Chapter Test	About 15–20 problems that can be taken as a Chapter Test. Be sure to take the Chapter Test under test conditions—no notes!	Be prepared. Take the sample practice test under test conditions. This will get you ready for your instructor's test. If you get a problem wrong, you can watch the Chapter Test Prep Video.	383
Cumulative Review	These problem sets appear at the end of each chapter, beginning with Chapter 2. They combine problems from previous chapters, providing an ongoing cumulative review.	These are really important. They will ensure that you are not forgetting anything as you go. These will go a long way toward keeping you primed for the final exam.	384
Chapter Project	The Chapter Project applies to what you've learned in the chapter. Additional projects are available on the Instructor's Resource Center (IRC).	The Project gives you an opportunity to apply what you've learned in the chapter to the opening article. If your instructor allows, these make excellent opportunities to work in a group, which is often the best way of learning math.	385
Internet Based Projects	In selected chapters, a web-based project is given.	The projects allow the opportunity for students to collaborate and use mathematics to deal with issues that come up in their lives.	385

This page intentionally left blank

College Algebra

CONCEPTS THROUGH FUNCTIONS

This page intentionally left blank

FOURTH EDITION

College Algebra

CONCEPTS THROUGH FUNCTIONS

Michael Sullivan

Chicago State University

Michael Sullivan, III

Joliet Junior College

Pearson

Director, Portfolio Management, Collegiate Math: *Anne Kelly*
Senior Editor: *Dawn Murrin*
Editorial Administrator: *Joseph Colella*
Manager, Courseware QA: *Mary Durnwald*
Director, Production & Digital Studio, Mathematics & Statistics: *Ruth Berry*
Producer, Production & Digital Studio, Mathematics & Statistics: *Marielle Guiney*
Manager, Content Development: *Kristina Evans*
Senior Content Developer: *Megan M. Burns*
Managing Producer: *Scott Disanno*
Content Producer: *Peggy McMahon*
Product Marketing Director: *Erin Kelly*
Product Marketer for Precalculus: *Stacey Sveum*
Product Marketing Assistant: *Shannon McCormack*
Field Marketing Manager: *Peggy Lucas*
Field Marketing Assistant: *Adrianna Valencia*
Senior Author Support/Technology Specialist: *Joe Vetere*
Manager, Rights and Permissions: *Gina Cheselka*
Manufacturing Buyer: *Carol Melville, LSC Communications*
Cover Design, Text Design, Production Coordination, Composition, and Illustrations: *Cenveo® Publisher Services*
Cover Image: © *Shutterstock, Inc.*

Cataloging-in-Publication Data on File

1—17

Pearson

ISBN 10: 0-13-4-68696-9
ISBN 13: 978-0-13-4-68696-7

In Memory of Mary...

Wife and Mother

Contents

This page intentionally left blank

To the Student

As you begin, you may feel anxious about the number of theorems, definitions, procedures, and equations. You may wonder if you can learn it all in time. Don't worry, your concerns are normal. This textbook was written with you in mind. If you attend class, work hard, and read and study this book, you will build the knowledge and skills you need to be successful. Here's how you can use the book to your benefit.

Read Carefully

When you get busy, it's easy to skip reading and go right to the problems. Don't . . . the book has a large number of examples and clear explanations to help you break down the mathematics into easy-to-understand steps. Reading will provide you with a clearer understanding, beyond simple memorization. Read before class (not after) so you can ask questions about anything you didn't understand. You'll be amazed at how much more you'll get out of class if you do this.

Use the Features

We use many different methods in the classroom to communicate. Those methods, when incorporated into the book, are called "features." The features serve many purposes, from providing timely review of material you learned before (just when you need it), to providing organized review sessions to help you prepare for quizzes and tests. Take advantage of the features and you will master the material.

To make this easier, we've provided a brief guide to getting the most from this book. Refer to the "Prepare for Class," "Practice," and "Review" on pages i–iii. Spend fifteen minutes reviewing the guide and familiarizing yourself with the features by flipping to the page numbers provided. Then, as you read, use them. This is the best way to make the most of your textbook.

Please do not hesitate to contact us, through Pearson Education, with any questions, suggestions, or comments that would improve this text. We look forward to hearing from you, and good luck with all of your studies.

Best Wishes!

Michael Sullivan
Michael Sullivan, III

Three Distinct Series

Students have different goals, learning styles, and levels of preparation. Instructors have different teaching philosophies, styles, and techniques. Rather than write one series to fit all, the Sullivans have written three distinct series. All share the same goal—to develop a high level of mathematical understanding and an appreciation for the way mathematics can describe the world around us. The manner of reaching that goal, however, differs from series to series.

Concepts through Functions Series, Fourth Edition

This series differs from the others, utilizing a functions approach that serves as the organizing principle tying concepts together. Functions are introduced early in various formats. This approach supports the Rule of Four, which states that functions are represented symbolically, numerically, graphically, and verbally. Each chapter introduces a new type of function and then develops all concepts pertaining to that particular function. The solutions of equations and inequalities, instead of being developed as stand-alone topics, are developed in the context of the underlying functions. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor: *College Algebra; Precalculus, with a Unit Circle Approach to Trigonometry; Precalculus, with a Right Triangle Approach to Trigonometry*.

Contemporary Series, Tenth Edition

The Contemporary Series is the most traditional in approach yet modern in its treatment of precalculus mathematics. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor: *College Algebra, Algebra & Trigonometry, Trigonometry, Precalculus.*

Enhanced with Graphing Utilities Series, Seventh Edition

This series provides a thorough integration of graphing utilities into topics, allowing students to explore mathematical concepts and foreshadow ideas usually studied in later courses. Using technology, the approach to solving certain problems differs from the Concepts or Contemporary Series, while the emphasis on understanding concepts and building strong skills does not: *College Algebra, Algebra & Trigonometry, Precalculus*.

The Concepts through Functions Series

College Algebra

This text provides an approach to college algebra that introduces functions early (Chapter 1). All material is presented through the eyes of a function. So, rather than have a chapter dedicated to review, the material is presented from a function point of view. For example, rather than reviewing the various approaches to solving quadratic equations, students are asked to find the zeros of a quadratic function or the x-intercepts of a quadratic function. This allows for review of the concepts, but also requires students to solve equations in the form $f(x) = 0$, which foreshadows solving $f'(x) = 0$ in calculus. Graphing calculator and Desmos usage is provided, but optional. Examples that utilize graphing technology are clearly marked with an icon. Exercises that require graphing technology are indicated with an icon and the problem number is green. After completing the material in this text, a student will be adequately prepared for trigonometry, finite mathematics, and business calculus.

Precalculus: A Right Triangle Approach to Trigonometry

This text contains all the material in *College Algebra*, but also develops the trigonometric functions using a right triangle approach and showing how it relates to the unit circle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Graphing calculator and Desmos usage is provided, but is optional. The final chapter provides an introduction to calculus, with a discussion of the limit, the derivative, and the integral of a function. After completing this text, a student will be adequately prepared for engineering calculus, business calculus, and finite mathematics.

Precalculus: A Unit Circle Approach to Trigonometry

This text contains all the material in *College Algebra*, but also develops the trigonometric functions using a unit circle approach and showing how it relates to the right triangle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Graphing calculator and Desmos usage is provided, but is optional. The final chapter provides an introduction to calculus, with a discussion of the limit, the derivative, and the integral of a function. After completing this text, a student will be adequately prepared for engineering calculus, business calculus, and finite mathematics.

This page intentionally left blank

Preface to the Instructor

As professors at both an urban university and a community college, Michael Sullivan and Michael Sullivan, III, are aware of the varied needs of College Algebra students, ranging from those who have little mathematical background and a fear of mathematics courses, to those having a strong mathematical education and a high level of motivation. For some of your students, this will be their last course in mathematics, whereas others will further their mathematical education. This text is written for both groups.

As a teacher, and as an author of precalculus, engineering calculus, finite mathematics, and business calculus texts, Michael Sullivan understands what students must know if they are to be focused and successful in upper-level math courses. However, as a father of four, he also understands the realities of college life. As an author of a developmental mathematics series, Michael's co-author and son, Michael Sullivan, III, understands the trepidations and skills students bring to the College Algebra course. Michael, III also believes in the value of technology as a tool for learning that enhances understanding without sacrificing math skills. Together, both authors have taken great pains to ensure that the text contains solid, student-friendly examples and problems, as well as a clear and seamless writing style.

A tremendous benefit of authoring a successful series is the broad-based feedback we receive from teachers and students. We are sincerely grateful for their support. Virtually every change in this edition is the result of their thoughtful comments and suggestions. We are sincerely grateful for this support and hope that we have been able to take these ideas and, building upon a successful first edition, make this series an even better tool for learning and teaching. We continue to encourage you to share with us your experiences teaching from this text.

About This Book

This book utilizes a functions approach to College Algebra. Functions are introduced early (Chapter 1) in various formats: maps, tables, sets of ordered pairs, equations, and graphs. Our approach to functions illustrates the symbolic, numeric, graphic, and verbal representations of functions. This allows students to make connections between the visual representation of a function and its algebraic representation.

It is our belief that students need to "hit the ground running" so that they do not become complacent in their studies. After all, it is highly likely that students have been exposed to solving equations and inequalities prior to entering this class. By spending precious time reviewing these concepts, students are likely to think of the course as a rehash of material learned in other courses and say to themselves, "I know this material, so I don't have to study." This may result in the students developing poor study habits for this course. By introducing functions early in the course, students are less likely to develop bad habits.

Another advantage of the early introduction of functions is that the discussion of equations and inequalities can focus around the concept of a function. For example, rather than asking students to solve an equation such as $2x^2 + 5x + 2 = 0$, we ask students to find the zeros of $f(x) = 2x^2 + 5x + 2$ or solve $f(x) = 0$ when $f(x) = 2x^2 + 5x + 2$. While the technique used to solve this type of problem is the same, the fact that the problem looks different to the student means the student is less apt to say, "Oh, I already have seen this problem before, and I know how to solve it." In addition, in Calculus students are going to be asked to solve equations such as $f'(x) = 0$, so solving $f(x) = 0$ is a logical prerequisite skill to practice in Precalculus. Another advantage to solving equations through the eyes of a function is that the properties of functions can be included in the solution. For example, the linear function $f(x) = 2x - 3$ has one real zero because the function f is increasing on its domain.

Features in the Fourth Edition

Rather than provide a list of new features here, that information can be found on pages i–iii.

This places the features in their proper context, as building blocks of an overall learning system that has been carefully crafted over the years to help students get the most out of the time they put into studying. Please take the time to review the features listed on pages i–iii and to discuss them with your students at the beginning of your course. Our experience has been that when students utilize these features, they are more successful in the course.

Changes in the Fourth Edition

Content

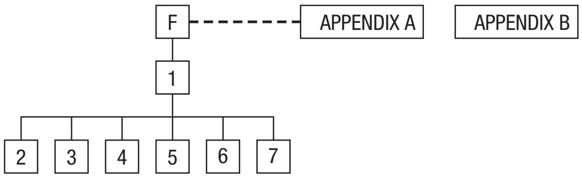
- Desmos screen captures have been added throughout the text. This is done to recognize that graphing technology expands beyond graphing calculators.
- Definitions have been reviewed, and in a few cases, revised to be consistent with those presented in Calculus. For example, in the definitions for increasing/decreasing functions, we deleted the word "open", allowing for functions to increase/decrease on any type of interval.

Organization

- **Chapter F, Section 3** We moved the objective "Find the Equation of a Line Given Two Points" after the objective "Identify the Slope and *y*-Intercept of a Line from Its Equation". This allows us to express lines in slope-intercept form.

Using this Book Effectively and Efficiently with Your Syllabus

To meet the varied needs of diverse syllabi, this book contains more content than is likely to be covered in a typical College Algebra course. As the chart illustrates, this book has been organized with flexibility of use in mind. Even within a given chapter, certain sections are optional and can be omitted without loss of continuity. See the detail following the flow chart.



Foundations A Prelude to Functions

Quick coverage of this chapter, which is mainly review material, will enable you to get to Chapter 1, *Functions and Their Graphs*, earlier.

Chapter 1 Functions and Their Graphs

Perhaps the most important chapter. Sections 1.6 and 1.7 are optional.

Chapter 2 Linear and Quadratic Functions

Topic selection depends on your syllabus. Sections 2.2, 2.6, and 2.7 may be omitted without a loss of continuity.

Chapter 3 Polynomial and Rational Functions

Topic selection depends on your syllabus. Section 3.6 is optional.

Chapter 4 Exponential and Logarithmic Functions

Sections 4.1–4.6 follow in sequence. Sections 4.7–4.9 are optional.

Chapter 5 Analytic Geometry

Sections 5.1–5.4 follow in sequence.

Chapter 6 Systems of Equations and Inequalities

Sections 6.2–6.7 may be covered in any order. Section 6.8 requires Section 6.7.

Chapter 7 Sequences; Induction; the Binomial Theorem

There are three independent parts: Sections 7.1–7.3, Section 7.4, and Section 7.5.

Chapter 8 Counting and Probability

The sections follow in sequence.

Appendix A Review

This review material may be covered at the start of a course or used as a just-in-time review. Specific references to this material occur throughout the text to assist in the review process.

Appendix B Graphing Utilities

Reference is made to these sections at the appropriate place in the text.

Acknowledgments

Textbooks are written by authors, but evolve from an idea to final form through the efforts of many people. It was Don Dellen who first suggested this book and series. Don is remembered for his extensive contributions to publishing and mathematics.

Thanks are due to the following people for their assistance and encouragement to the preparation of this edition:

- From Pearson Education: Anne Kelly for her substantial contributions, ideas, and enthusiasm; Peggy Lucas, who is a huge fan and works tirelessly to get the word out; Dawn Murrin, for her unmatched talent at getting the details right; Peggy McMahon for her organizational skills and leadership in overseeing production; and the Pearson Math and Science Sales team, for their continued confidence and personal support of our books.
- Accuracy checkers: C. Brad Davis, who read the entire manuscript and accuracy checked answers. His attention to detail is amazing; Timothy Britt, for creating the Solutions Manuals and accuracy checking answers.
- Michael Sullivan, III would like to thank his colleagues at Joliet Junior College for their support and feedback.

Finally, we offer our grateful thanks to the dedicated users and reviewers of our books, whose collective insights form the backbone of each textbook revision.

Our list of indebtedness just grows and grows. And, if we've forgotten anyone, please accept our apology. Thank you all.

Ryan Adams — Northwest Florida State College
James Africh — College of DuPage
Steve Agronsky — Cal Poly State University
Gerardo Aladro — Florida International University
Grant Alexander — Joliet Junior College
Dave Anderson — South Suburban College
Richard Andrews — Florida A&M University
Joby Milo Anthony — University of Central Florida
Karen Appel — Mesa Community College
James E. Arnold — University of Wisconsin-Milwaukee
Adel Arshaghi — Center for Educational Merit
Carolyn Autray — University of West Georgia
Agnes Azzolino — Middlesex County College
Taoufik Bahadi — University of Tampa
Wilson P. Banks — Illinois State University
Scott Barnett — Henry Ford Community College
Sudeshna Basu — Howard University
Dale R. Bedgood — East Texas State University
Beth Beno — South Suburban College
Carolyn Bernath — Tallahassee Community College
Rebecca Berthiaume — Edison State College
William H. Beyer — University of Akron
John Bialas — Joliet Junior College
Annette Blackwelder — Florida State University
Richelle Blair — Lakeland Community College
Linda Blanco — Joliet Junior College
Kevin Bodden — Lewis and Clark College
Jeffrey Boerner — University of Wisconsin-Stout
Barry Booten — Florida Atlantic University
Larry Bouldin — Roane State Community College
Bob Bradshaw — Ohlone College
Trudy Bratten — Grossmont College
Martin Bredeck — Northern Virginia Community College (Annandale Campus)
Tim Bremer — Broome Community College

Tim Britt — Jackson State Community College
Michael Brook — University of Delaware
Joanne Brunner — Joliet Junior College
Warren Burch — Brevard Community College
Mary Butler — Lincoln Public Schools
Melanie Butler — West Virginia University
Jim Butterbach — Joliet Junior College
William J. Cable — University of Wisconsin-Stevens Point
Lois Calamia — Brookdale Community College
Jim Campbell — Lincoln Public Schools
Roger Carlsen — Moraine Valley Community College
Elena Catoiu — Joliet Junior College
Mathews Chakkanakuzhi — Palomar College
Tim Chappell — Penn Valley Community College
John Collado — South Suburban College
Alicia Collins — Mesa Community College
Nelson Collins — Joliet Junior College
Rebecca Connell — Troy University
Jim Cooper — Joliet Junior College
Denise Corbett — East Carolina University
Carlos C. Corona — San Antonio College
Theodore C. Coskey — South Seattle Community College
Rebecca Connell — Troy University
Donna Costello — Plano Senior High School
Paul Crittenden — University of Nebraska at Lincoln
John Davenport — East Texas State University
Faye Dang — Joliet Junior College
Antonio David — Del Mar College
Stephanie Deacon — Liberty University
Duane E. Deal — Ball State University
Jerry DeGroot — Purdue North Central
Timothy Deis — University of Wisconsin-Platteville
Joanna DelMonaco — Middlesex Community College
Vivian Dennis — Eastfield College
Deborah Dillon — R. L. Turner High School
Guesna Dohrman — Tallahassee Community College
Cheryl Doolittle — Iowa State University
Karen R. Dougan — University of Florida
Jerrett Dumouchel — Florida Community College at Jacksonville
Louise Dyson — Clark College
Paul D. East — Lexington Community College
Don Edmondson — University of Texas-Austin
Erica Egizio — Lewis University
Laura Egner — Joliet Junior College
Jason Eltrevoog — Joliet Junior College
Christopher Ennis — University of Minnesota
Kathy Eppler — Salt Lake Community College
Ralph Esparza Jr. — Richland College
Garret J. Etgen — University of Houston
Scott Fallstrom — Shoreline Community College
Pete Falzone — Pensacola Junior College
Arash Farahmand — Skyline College
W.A. Ferguson — University of Illinois-Urbana/Champaign
Iris B. Fetta — Clemson University
Mason Flake — student at Edison Community College
Timothy W. Flood — Pittsburg State University
Robert Frank — Westmoreland County Community College
Merle Friel — Humboldt State University
Richard A. Fritz — Moraine Valley Community College
Dewey Furness — Ricks College
Michael Furuto — University of Hawai'i - West O'ahu
Mary Jule Gabiou — North Idaho College
Randy Gallaher — Lewis and Clark College
Tina Garn — University of Arizona
Dawit Getachew — Chicago State University
Wayne Gibson — Rancho Santiago College
Loran W. Gierhart — University of Texas at San Antonio and Palo Alto College
Robert Gill — University of Minnesota Duluth
Nina Girard — University of Pittsburgh at Johnstown
Rebecca Goad — Joliet Junior College
Sudhir Kumar Goel — Valdosta State University
Adrienne Goldstein — Miami Dade College, Kendall Campus
Joan Goliday — Sante Fe Community College
Lourdes Gonzalez — Miami Dade College, Kendall Campus
Frederic Gooding — Goucher College
Donald Goral — Northern Virginia Community College
Sue Graupner — Lincoln Public Schools
Mary Beth Grayson — Liberty University
Jennifer L. Grimsley — University of Charleston
Ken Gurganus — University of North Carolina
James E. Hall — University of Wisconsin-Madison
Judy Hall — West Virginia University
Edward R. Hancock — DeVry Institute of Technology
Julia Hassett — DeVry Institute, Dupage
Christopher Hay-Jahans — University of South Dakota
Michah Heibel — Lincoln Public Schools
LaRae Helliwell — San Jose City College
Celeste Hernandez — Richland College
Gloria P. Hernandez — Louisiana State University at Eunice
Brother Herron — Brother Rice High School
Robert Hoburg — Western Connecticut State University
Lynda Hollingsworth — Northwest Missouri State University
Deltrye Holt — Augusta State University
Charla Holzbog — Denison High School
Lee Hruby — Naperville North High School
Miles Hubbard — St. Cloud State University
Kim Hughes — California State College-San Bernardino
Stanislav, Jabuka — University of Nevada, Reno
Ron Jamison — Brigham Young University
Lars Jensen — Truckee Meadows Community College
Richard A. Jensen — Manatee Community College
Glenn Johnson — Middlesex Community College
Sandra G. Johnson — St. Cloud State University
Tuesday Johnson — New Mexico State University
Susitha Karunaratne — Purdue University North Central
Moana H. Karsteter — Tallahassee Community College
Donna Katula — Joliet Junior College
Arthur Kaufman — College of Staten Island
Thomas Kearns — North Kentucky University
Jack Keating — Massasoit Community College
Shelia Kellenbarger — Lincoln Public Schools
Rachael Kenney — North Carolina State University
John B. Klassen — North Idaho College
Debra Kopcso — Louisiana State University
Lynne Kowski — Raritan Valley Community College
Yelena Kravchuk — University of Alabama at Birmingham
Ray S. Kuan — Skyline College
Keith Kuchar — Manatee Community College
Tor Kwembe — Chicago State University
Linda J. Kyle — Tarrant Country Jr. College
H.E. Lacey — Texas A & M University
Harriet Lamm — Coastal Bend College
James Lapp — Fort Lewis College
Matt Larson — Lincoln Public Schools
Christopher Lattin — Oakton Community College
Julia Ledet — Lousiana State University
Adele LeGere — Oakton Community College
Kevin Leith — University of Houston
JoAnn Lewin — Edison College
Jeff Lewis — Johnson County Community College
Heidi Lyne — Joliet Junior College
Janice C. Lyon — Tallahassee Community College
Jean McArthur — Joliet Junior College
Virginia McCarthy — Iowa State University
Karla McCavit — Albion College
Michael McClendon — University of Central Oklahoma
Tom McCollow — DeVry Institute of Technology
Marilyn McCollum — North Carolina State University
Jill McGowan — Howard University
Will McGowant — Howard University
Dave McGuire — Joliet Junior College
Angela McNulty — Joliet Junior College
Laurence Maher — North Texas State University
Jay A. Malmstrom — Oklahoma City Community College
Rebecca Mann — Apollo High School
Lynn Marecek — Santa Ana College
Ashley Martin — Jamestown Community College
Sherry Martina — Naperville North High School
Alec Matheson — Lamar University
Nancy Matthews — University of Oklahoma
James Maxwell — Oklahoma State University-Stillwater
Marsha May — Midwestern State University
James McLaughlin — West Chester University
Judy Meckley — Joliet Junior College
David Meel — Bowling Green State University
Carolyn Meitler — Concordia University
Samia Metwali — Erie Community College
Rich Meyers — Joliet Junior College
Matthew Michaelson — Glendale Community College
Eldon Miller — University of Mississippi
James Miller — West Virginia University
Michael Miller — Iowa State University
Kathleen Miranda — SUNY at Old Westbury
Chris Mirbaha — The Community College of Baltimore County
Karla Mitchell — North Idaho College
Val Mohanakumar — Hillsborough Community College
Thomas Monaghan — Naperville North High School
Miguel Montanez — Miami Dade College, Wolfson Campus
Maria Montoya — Our Lady of the Lake University
Susan Moosai — Florida Atlantic University
Craig Morse — Naperville North High School
Samad Mortabit — Metropolitan State University
Pat Mower — Washburn University
Tammy Muhs — University of Central Florida
A. Muhundan — Manatee Community College
Jane Murphy — Middlesex Community College
Richard Nadel — Florida International University

Gabriel Nagy — Kansas State University
Bill Naegele — South Suburban College
Karla Neal — Lousiana State University
Lawrence E. Newman — Holyoke Community College
Dwight Newsome — Pasco-Hernando Community College
Victoria Noddings — MiraCosta College
Denise Nunley — Maricopa Community Colleges
James Nymann — University of Texas-El Paso
Mark Omodt — Anoka-Ramsey Community College
Seth F. Oppenheimer — Mississippi State University
Leticia Oropesa — University of Miami
Linda Padilla — Joliet Junior College
Sanja Pantic — University of Illinois at Chicago
Donnabelle Pascual — Leeward Community College
E. James Peake — Iowa State University
Kelly Pearson — Murray State University
Dashamir Petrela — Florida Atlantic University
Philip Pina — Florida Atlantic University
Charlotte Pisors — Baylor University
Michael Prophet — University of Northern Iowa
Laura Pyzdrowski — West Virginia University
Carrie Quesnell — Weber State University
Neal C. Raber — University of Akron
Thomas Radin — San Joaquin Delta College
Aibeng Serene Radulovic — Florida Atlantic University
Ken A. Rager — Metropolitan State College
Angela Raynor — The State University of New York at Fredonia
Kenneth D. Reeves — San Antonio College
Elsi Reinhardt — Truckee Meadows Community College
Jose Remesar — Miami Dade College, Wolfson Campus
Jane Ringwald — Iowa State University
Douglas F. Robertson — University of Minnesota, MPLS
Stephen Rodi — Austin Community College
William Rogge — Lincoln Northeast High School
Howard L. Rolf — Baylor University
Mike Rosenthal — Florida International University
Phoebe Rouse — Lousiana State University
Edward Rozema — University of Tennessee at Chattanooga
David Ruffato — Joliet Junior College
Dennis C. Runde — Manatee Community College
Alan Saleski — Loyola University of Chicago
Susan Sandmeyer — Jamestown Community College
Brenda Santistevan — Salt Lake Community College
Linda Schmidt — Greenville Technical College
Ingrid Scott — Montgomery College
A.K. Shamma — University of West Florida
Zachery Sharon — University of Texas at San Antonio
Martin Sherry — Lower Columbia College
Carmen Shershin — Florida International University
Tatrana Shubin — San Jose State University
Anita Sikes — Delgado Community College
Timothy Sipka — Alma College
Charlotte Smedberg — University of Tampa
Lori Smellegar — Manatee Community College
Gayle Smith — Loyola Blakefield
Cindy Soderstrom — Salt Lake Community College
Leslie Soltis — Mercyhurst College
John Spellman — Southwest Texas State University
Karen Spike — University of North Carolina
Rajalakshmi Sriram — Okaloosa-Walton Community College
Katrina Staley — North Carolina Agricultural and Technical State University
Becky Stamper — Western Kentucky University
Judy Staver — Florida Community College-South
Robin Steinberg — Pima Community College
Neil Stephens — Hinsdale South High School
Sonya Stephens — Florida A&M Univeristy
Patrick Stevens — Joliet Junior College
Mary Stinnett — Umpqua Community College
John Sumner — University of Tampa
Matthew TenHuisen — University of North Carolina, Wilmington
Christopher Terry — Augusta State University
Diane Tesar — South Suburban College
Theresa Thompson — Tulsa Community College
Tommy Thompson — Brookhaven College
Martha K. Tietze — Shawnee Mission Northwest High School
Richard J. Tondra — Iowa State University
Florentina Tone — University of West Florida
Suzanne Topp — Salt Lake Community College
Marilyn Toscano — University of Wisconsin, Superior
Marvel Townsend — University of Florida
Jim Trudnowski — Carroll College
Robert Tuskey — Joliet Junior College
Mihaela Vajiac — Chapman University-Orange
Julia Varbalow — Thomas Nelson Community College-Leesville
Richard G. Vinson — University of South Alabama
Jorge Viola-Prioli — Florida Atlantic University
Mary Voxman — University of Idaho
Jennifer Walsh — Daytona Beach Community College
Donna Wandke — Naperville North High School
Timothy L. Warkentin — Cloud County Community College
Melissa J. Watts — Virginia State University
Hayat Weiss — Middlesex Community College
Kathryn Wetzel — Amarillo College
Darlene Whitkenack — Northern Illinois University
Suzanne Williams — Central Piedmont Community College
Larissa Williamson — University of Florida
Christine Wilson — West Virginia University
Brad Wind — Florida International University
Anna Wiodarczyk — Florida International University
Mary Wolyniak — Broome Community College
Canton Woods — Auburn University
Tamara S. Worner — Wayne State College
Terri Wright — New Hampshire Community Technical College, Manchester
Aletheia Zambesi — University of West Florida
George Zazi — Chicago State University
Steve Zuro — Joliet Junior College

Michael Sullivan
Chicago State University

Michael Sullivan, III
Joliet Junior College

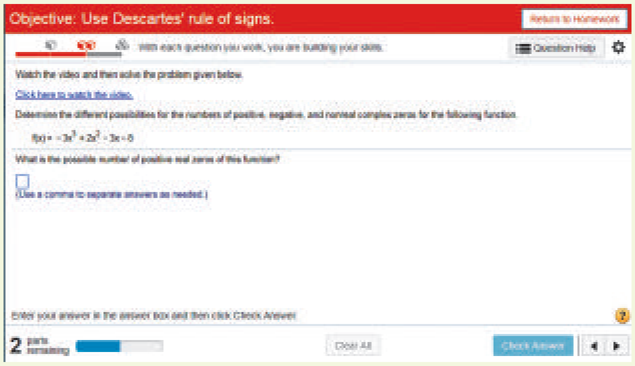
Get the Most Out of MyLab Math

Used by over 3 million students a year, MyLab™ Math is the world's leading online program for teaching and learning mathematics. MyLab Math delivers assessment, tutorials, and multimedia resources that provide engaging and personalized experiences for each student, so learning can happen in any environment. Each course is developed to accompany Pearson's best-selling content, authored by thought leaders across the math curriculum, and can be easily customized to fit any course format.

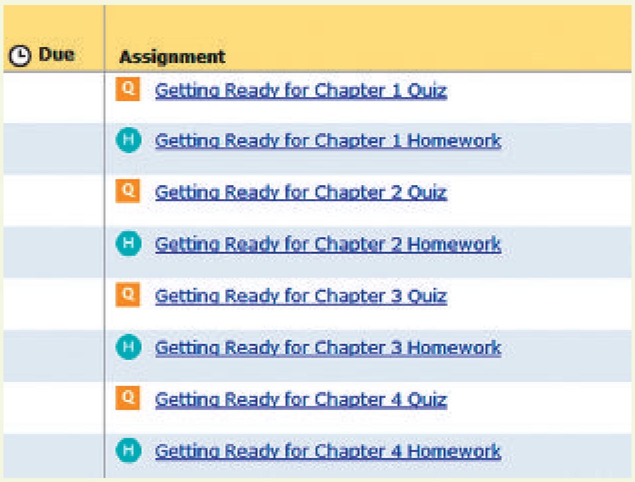
Preparedness

One of the biggest challenges in many mathematics courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyLab Math offers a variety of content and course options to support students with just-in-time remediation and key-concept review.

- **Skill Builder** offers adaptive practice that is designed to increase students' ability to complete their assignments. By monitoring student performance on their homework, Skill Builder adapts to each student's needs and provides just-in-time, in-assignment practice to help them improve their proficiency of key learning objectives.



- **Getting Ready** material provides just-in-time review, integrated throughout the course as needed to prepare students with prerequisite material to succeed. From a quick quiz, a personalized, just-in-time review assignment is generated for each student, allowing them to refresh forgotten concepts.

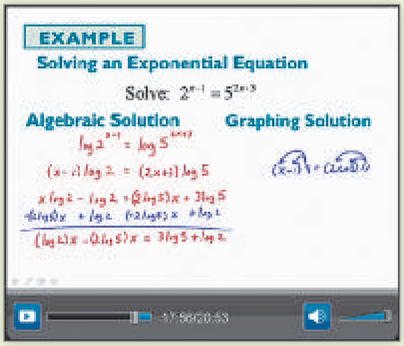


Resources for Success

MylLab Math Online Course **College Algebra: Concepts Through Functions**

by Sullivan and Sullivan (access code required)

MyLab Math is available to accompany Pearson's market leading text offerings. To give students a consistent tone, voice, and teaching method each text's flavor and approach is tightly integrated throughout the accompanying **MyLab Math** course, making learning the material as seamless as possible.



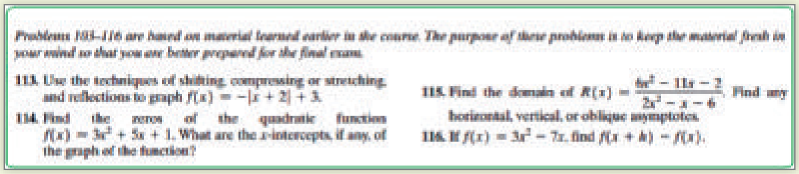
Video Program and Resources

Author in Action Videos are actual classroom lectures with fully worked out examples presented by Michael Sullivan III.

- **Video assessment** questions are available to assign in MyLab Math for key Author in Action videos.
- The corresponding **Guided Lecture Notes** assist students in taking thorough, organized, and understandable notes while watching Author in Action videos.

Retain Your Knowledge

Updated! Retain Your Knowledge Exercises support ongoing review at the course level and help students maintain essential skills. These are excellent cumulative review problems and are perfect for studying for final exams. Retain Your Knowledge Exercises are available to assign in **MyLab Math** and in the text.



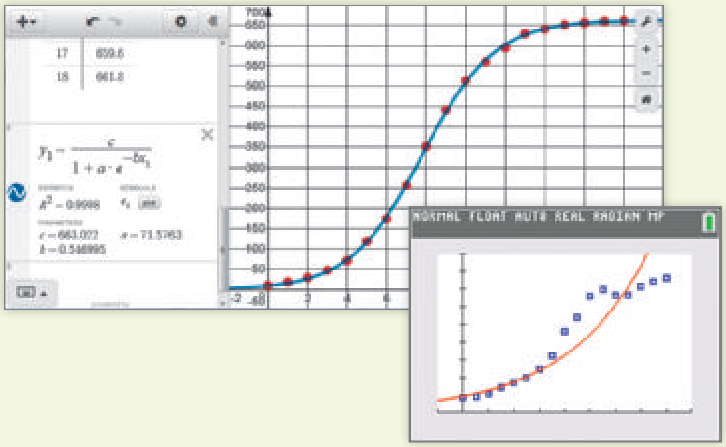
Problems 113–116 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

113. Use the techniques of shifting, compressing or stretching, and reflections to graph $f(x) = -|x+2| + 3$.

114. Find the zeros of the quadratic function $f(x) = 3x^2 + 5x + 1$. What are the x-intercepts, if any, of the graph of the function?

115. Find the domain of $R(x) = \dfrac{6x^2 - 11x - 2}{2x^2 - x - 6}$. Find any horizontal, vertical, or oblique asymptotes.

116. If $f(x) = 3x^2 - 7x$, find $f(x+h) - f(x)$.



Graphing Images

Updated! Throughout each chapter TI-84 Plus C and Desmos© screenshots appear. These images help students visualize concepts clearly and make stronger connections among equations, data and graphs in full color; using the graphing technology they are most familiar with.

Sample Assignments

Enhanced Sample Assignments make course set-up easier by giving instructors a starting point for each chapter. Each assignment, handpicked by the author to align with this text, includes a thoughtful mix of question types (e.g., conceptual, skills, etc.) specific to that topic.

pearson.com/mylab/math

Resources for Success

Pearson MyLab

Instructor Resources

Annotated Instructor's Edition

ISBN 10: 0134689801 ISBN 13: 9780134689807

The Annotated Instructor's Edition includes answers to the exercises sets. Shorter answers are on the page beside the exercises, and longer answers are in the back of the text.

The following resources can be downloaded from www.pearson.com or at www.pearson.com/mylab/math

Learning Catalytics Question Library

Questions written by Michael Sullivan III are available to deliver through Learning Catalytics to engage students in your course.

PowerPoint® Lecture Slides

Fully editable slides that correlate to the textbook.

Instructor Solutions Manual

Includes fully worked solutions to all textbook exercises.

TestGen®

TestGen® (www.pearson.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

Mini Lecture Notes

Includes additional examples and helpful teaching tips, by section.

Online Chapter Projects

Additional projects that let students apply what was learned in the chapter.

Student Resources

Additional resources to help student success:

Lecture Video

Author in Action videos are actual classroom lectures with fully worked-out examples presented by Michael Sullivan III. All videos are assignable with video assessment questions within MyLab Math.

Chapter Test Prep Videos

Students can watch instructors work through step-by-step solutions to all chapter test exercises from the textbook. These are available in MyLab Math and on YouTube.

Student Solutions Manual

Provides detailed worked-out solutions to odd-numbered exercises.

Guided Lecture Notes

These lecture notes assist students in taking thorough, organized, and understandable notes while watching Author in Action videos. Students actively participate in learning the how/why of important concepts through explorations and activities. The Guided Lecture Notes are available as PDF's and customizable Word files in MyLab Math. They can also be packaged with the textbook and MyLab Math access code.

Algebra Review

Four chapters of Intermediate Algebra review. Perfect for a slower-paced course or for individual review.

Skills for Success

Online module found in MyLab Math that supports students continued success in college. This module provides tutorial and guidance on a variety of topics, including transition to college, online learning, time management and professional development skills.

This page intentionally left blank

Applications Index

College Algebra

CONCEPTS THROUGH FUNCTIONS

Foundations: A Prelude to Functions

F

What Is My House Worth?

There are many factors that play a role in the value of a home. Everyone knows the golden rule of real estate—Location, Location, Location! Aside from where a property is located, one must consider the size of the home, number of bedrooms, number of bathrooms, status of updates within the home, and many, many other considerations.

Zillow (www.zillow.com) developed a model (an equation) that is used to approximate the value of a home. This approximate value is called a Zestimate. According to Zillow, the Zestimate is the estimated market value for an individual home. Zillow uses available information on the millions of homes that have sold around the country to arrive at its Zestimate. Mainly, Zillow uses the physical attributes of the home, tax assessments, and transaction data to arrive at its Zestimate. Homeowners are free to report updated home facts about their particular property in order to improve the Zestimate. The accuracy of the Zestimate is dependent upon the location of the home. For example, Zestimates in the Chicago area have some of the best Zestimates (60.9% of homes sold within 5% of the Zestimate), while Zestimates in Cleveland, OH are only fair (44.2% of the homes sold within 5% of the sale price).

—Michael Sullivan, III

Source: *https://www.zillow.com*

—See the Internet-based Chapter Project—

A Look Back

Appendix A reviews skills from Intermediate Algebra.

A Look Ahead

Here we connect algebra and geometry using the rectangular coordinate system. In the 1600s, algebra had developed to the point that René Descartes (1596–1650) and Pierre de Fermat (1601–1665) were able to use rectangular coordinates to translate geometry problems into algebra problems, and vice versa. This enabled both geometers and algebraists to gain new insights into their subjects, which had been thought to be separate but now were seen as connected.

Outline

F.1 The Distance and Midpoint Formulas

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Algebra Essentials (Appendix A, Section A.1, pp. A1–A10)
- Geometry Essentials (Appendix A, Section A.2, pp. A14–A19)

Now Work the **'Are You Prepared?'** problems on page 6.

OBJECTIVES
1 Use the Distance Formula (p. 3)
2 Use the Midpoint Formula (p. 5)

Rectangular Coordinates

A point on the real number line is located by a single real number called the *coordinate of the point*. For work in a two-dimensional plane, points are located by using two numbers.

Begin with two real number lines located in the same plane: one horizontal and the other vertical. The horizontal line is called the ***x*-axis,** the vertical line the ***y*-axis,** and the point of intersection the **origin *O*.** See Figure 1. Assign coordinates to every point on these number lines using a convenient scale. Recall that the scale of a number line is the distance between 0 and 1. In mathematics, we usually use the same scale on each axis, but in applications, a different scale is often used.

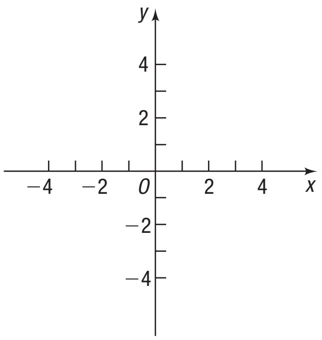


Figure 1 *xy*-Plane

The origin O has a value of 0 on both the x-axis and the y-axis. Points on the x-axis to the right of O are associated with positive real numbers, and those to the left of O are associated with negative real numbers. Points on the y-axis above O are associated with positive real numbers, and those below O are associated with negative real numbers. In Figure 1, the x-axis and y-axis are labeled as x and y, respectively, and an arrow at the end of each axis is used to denote the positive direction.

The coordinate system described here is called a **rectangular** or **Cartesian*** **coordinate system.** The plane formed by the x-axis and y-axis is sometimes called the ***xy*-plane,** and the x-axis and y-axis are referred to as the **coordinate axes.**

Any point P in the xy-plane can be located by using an **ordered pair** (x, y) of real numbers. Let x denote the signed distance of P from the y-axis (*signed* means that if P is to the right of the y-axis, then $x > 0$, and if P is to the left of the y-axis, then $x < 0$); and let y denote the signed distance of P from the x-axis. The ordered pair (x, y), also called the **coordinates** of P, then gives us enough information to locate the point P in the plane.

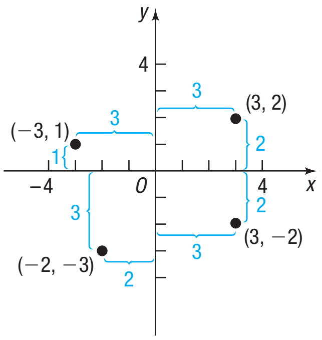


Figure 2

For example, to locate the point whose coordinates are $(-3, 1)$, go 3 units along the x-axis to the left of O and then go straight up 1 unit. We **plot** this point by placing a dot at this location. See Figure 2, in which the points with coordinates $(-3, 1)$, $(-2, -3)$, $(3, -2)$, and $(3, 2)$ are plotted.

The origin has coordinates $(0, 0)$. Any point on the x-axis has coordinates of the form $(x, 0)$, and any point on the y-axis has coordinates of the form $(0, y)$.

If (x, y) are the coordinates of a point P, then x is called the ***x*-coordinate,** or **abscissa,** of P, and y is the ***y*-coordinate,** or **ordinate,** of P. We identify the point P by its coordinates (x, y) by writing $P = (x, y)$. Usually, we will simply say, "the point (x, y)" rather than "the point whose coordinates are (x, y)."

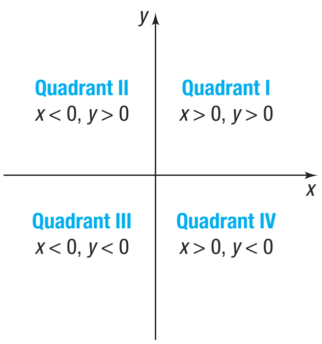


Figure 3

The coordinate axes divide the xy-plane into four sections called **quadrants,** as shown in Figure 3. In quadrant I, both the x-coordinate and the y-coordinate of all points are positive; in quadrant II, x is negative and y is positive; in quadrant III, both x and y are negative; and in quadrant IV, x is positive and y is negative. Points on the coordinate axes belong to no quadrant.

NOW WORK PROBLEM 15

*Named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian.

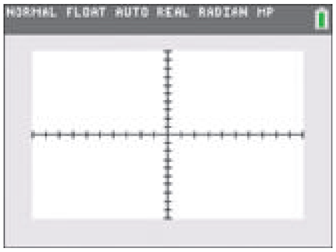


Figure 4 TI-84 Plus C standard viewing rectangle

COMMENT On a graphing calculator, you can set the scale on each axis. Once this has been done, you obtain the **viewing rectangle.** See Figure 4 for a typical viewing rectangle. You should now read Section B.1, *The Viewing Rectangle*, in Appendix B. ■

1 Use the Distance Formula

If the same units of measurement (such as inches, centimeters, and so on) are used for both the x-axis and y-axis, then all distances in the xy-plane can be measured using this unit of measurement.

EXAMPLE 1 Finding the Distance between Two Points

Find the distance d between the points $(1, 3)$ and $(5, 6)$.

Solution First plot the points $(1, 3)$ and $(5, 6)$ and connect them with a straight line. See Figure 5(a). To find the length d, begin by drawing a horizontal line from $(1, 3)$ to $(5, 3)$ and a vertical line from $(5, 3)$ to $(5, 6)$, forming a right triangle, as shown in Figure 5(b). One leg of the triangle is of length 4 (since $|5 - 1| = 4$), and the other is of length 3 (since $|6 - 3| = 3$). By the Pythagorean Theorem, the square of the distance d that we seek is

$$d^2 = 4^2 + 3^2 = 16 + 9 = 25$$
$$d = \sqrt{25} = 5$$

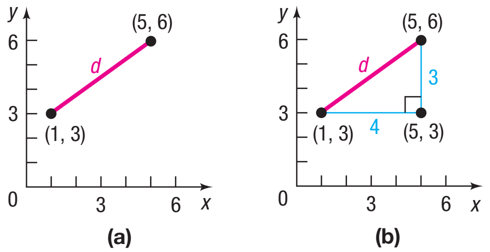


Figure 5

The **distance formula** provides a straightforward method for computing the distance between two points.

THEOREM

Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (1)$$

In Words
To compute the distance between two points, find the difference of the x-coordinates, square it, and add this to the square of the difference of the y-coordinates. The square root of this sum is the distance.

Proof of the Distance Formula Let (x_1, y_1) denote the coordinates of point P_1 and let (x_2, y_2) denote the coordinates of point P_2. Assume that the line joining P_1 and P_2 is neither horizontal nor vertical. Refer to Figure 6(a). The coordinates of P_3 are (x_2, y_1). The horizontal distance from P_1 to P_3 is the absolute value of the difference of the x-coordinates, $|x_2 - x_1|$. The vertical distance from P_3 to P_2 is the

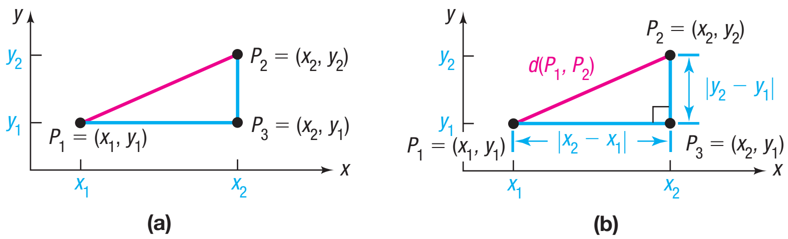


Figure 6

absolute value of the difference of the y-coordinates, $|y_2 - y_1|$. See Figure 6(b). The distance $d(P_1, P_2)$ that we seek is the length of the hypotenuse of the right triangle, so, by the Pythagorean Theorem, it follows that

$$[d(P_1,P_2)]^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$d(P_1,P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Now, if the line joining P_1 and P_2 is horizontal, then the y-coordinate of P_1 equals the y-coordinate of P_2; that is, $y_1 = y_2$. Refer to Figure 7(a). In this case, the distance formula (1) still works, because, for $y_1 = y_2$, it reduces to

$$d(P_1,P_2) = \sqrt{(x_2 - x_1)^2 + 0^2} = \sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$$

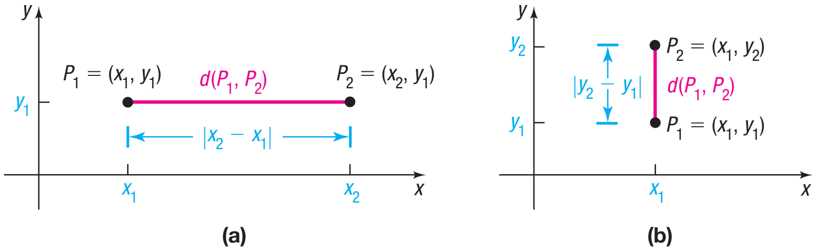


Figure 7

A similar argument holds if the line joining P_1 and P_2 is vertical. See Figure 7(b). ●

EXAMPLE 2 Using the Distance Formula

Find the distance d between the points $(-3, 5)$ and $(3, 2)$.

Solution Use the distance formula, equation (1), with $P_1 = (x_1, y_1) = (-3, 5)$ and $P_2 = (x_2, y_2) = (3, 2)$. Then

$$d = \sqrt{[3 - (-3)]^2 + (2 - 5)^2} = \sqrt{6^2 + (-3)^2}$$
$$= \sqrt{36 + 9}$$
$$= \sqrt{45}$$
$$= 3\sqrt{5} \approx 6.71$$ ●

NOW WORK PROBLEMS 19 AND 23

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is never a negative number. Furthermore, the distance between two points is 0 only when the points are identical—that is, when $x_1 = x_2$ and $y_1 = y_2$. Also, because $(x_2 - x_1)^2 = (x_1 - x_2)^2$ and $(y_2 - y_1)^2 = (y_1 - y_2)^2$, it makes no difference whether the distance is computed from P_1 to P_2 or from P_2 to P_1; that is, $d(P_1, P_2) = d(P_2, P_1)$.

The introduction to this chapter mentioned that rectangular coordinates enable us to translate geometry problems into algebra problems, and vice versa. The next example shows how algebra (the distance formula) can be used to solve geometry problems.

EXAMPLE 3 Using Algebra to Solve Geometry Problems

Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

(a) Plot each point and form the triangle ABC.
(b) Find the length of each side of the triangle.
(c) Verify that the triangle is a right triangle.
(d) Find the area of the triangle.

Solution

(a) Figure 8 shows the points A, B, C and the triangle ABC.

(b) To find the length of each side of the triangle, use the distance formula, equation (1).

$$d(A, B) = \sqrt{[2 - (-2)]^2 + (3 - 1)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$
$$d(B, C) = \sqrt{(3 - 2)^2 + (1 - 3)^2} = \sqrt{1 + 4} = \sqrt{5}$$
$$d(A, C) = \sqrt{[3 - (-2)]^2 + (1 - 1)^2} = \sqrt{25 + 0} = 5$$

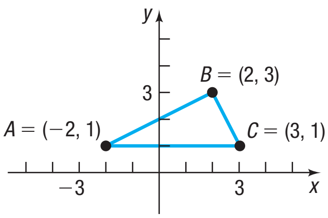


Figure 8

(c) If the sum of the squares of the lengths of two of the sides equals the square of the length of the third side, then the triangle is a right triangle. Looking at Figure 8, it seems reasonable to conjecture that the right angle is at vertex B. We shall check to see whether

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

Using the results from part (b) yields

$$[d(A, B)]^2 + [d(B, C)]^2 = (2\sqrt{5})^2 + (\sqrt{5})^2$$
$$= 20 + 5 = 25 = [d(A, C)]^2$$

It follows from the converse of the Pythagorean Theorem that triangle ABC is a right triangle.

(d) Because the right angle is at vertex B, the sides AB and BC form the base and height of the triangle. Its area is

$$\text{Area} = \frac{1}{2}(\text{Base})(\text{Height}) = \frac{1}{2}(2\sqrt{5})(\sqrt{5}) = 5 \text{ square units}$$

NOW WORK PROBLEM 33

2 Use the Midpoint Formula

We now derive a formula for the coordinates of the **midpoint of a line segment.** Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be the endpoints of a line segment, and let $M = (x, y)$ be the point on the line segment that is the same distance from P_1 as it is from P_2. See Figure 9. The triangles P_1AM and MBP_2 are congruent. [Do you see why? Angle AP_1M = angle BMP_2,* angle P_1MA = angle MP_2B, and $d(P_1, M) = d(M, P_2)$ is given. Thus we have angle–side–angle.] Hence, corresponding sides are equal in length. That is,

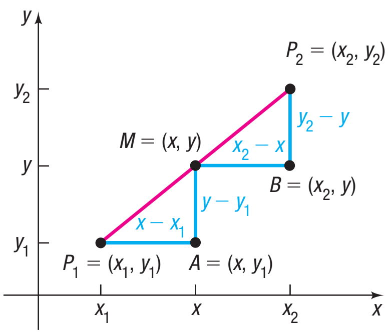


Figure 9 Illustration of midpoint

$$x - x_1 = x_2 - x \qquad \text{and} \qquad y - y_1 = y_2 - y$$
$$2x = x_1 + x_2 \qquad\qquad 2y = y_1 + y_2$$
$$x = \frac{x_1 + x_2}{2} \qquad\qquad y = \frac{y_1 + y_2}{2}$$

THEOREM

Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \textbf{(2)}$$

In Words

To find the midpoint of a line segment, average the x-coordinates of the endpoints, and average the y-coordinates of the endpoints.

*A postulate from geometry states that the transversal $\overline{P_1P_2}$ forms congruent corresponding angles with the parallel line segments $\overline{P_1A}$ and $\overline{MB}$.

EXAMPLE 4 **Finding the Midpoint of a Line Segment**

Find the midpoint of the line segment from $P_1 = (-5, 5)$ to $P_2 = (3, 1)$. Plot the points P_1 and P_2 and the midpoint.

Solution Apply the midpoint formula (2) using $x_1 = -5, y_1 = 5, x_2 = 3$, and $y_2 = 1$. Then the coordinates (x, y) of the midpoint M are

$$x = \frac{x_1 + x_2}{2} = \frac{-5 + 3}{2} = -1 \quad \text{and} \quad y = \frac{y_1 + y_2}{2} = \frac{5 + 1}{2} = 3$$

That is, $M = (-1, 3)$. See Figure 10.

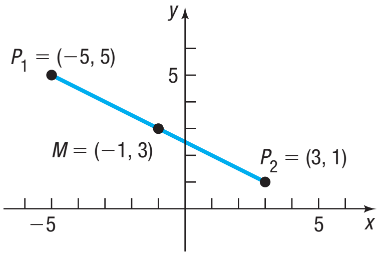


Figure 10

NOW WORK PROBLEM 39

F.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. On the real number line the origin is assigned the number __________. (p. A4)
2. If −3 and 5 are the coordinates of two points on the real number line, the distance between these points is __________. (p. A6)
3. If 3 and 4 are the legs of a right triangle, the hypotenuse is __________. (pp. A14–A15)
4. Use the converse of the Pythagorean Theorem to show that a triangle whose sides are of lengths 11, 60, and 61 is a right triangle. (pp. A14–A15)
5. The area A of a triangle whose base is b and whose altitude is h is $A =$ ______. (p. A15)
6. ***True or False*** Two triangles are congruent if two angles and the included side of one equals two angles and the included side of the other. (pp. A16–A17)

Concepts and Vocabulary

7. If (x, y) are the coordinates of a point P in the xy-plane, then x is called the ________________ of P, and y is the ________________ of P.
8. The coordinate axes divide the xy-plane into four sections called ____________.
9. If three distinct points P, Q, and R all lie on a line, and if $d(P, Q) = d(Q, R)$, then Q is called the __________ of the line segment from P to R.
10. ***True or False*** The distance between two points is sometimes a negative number.
11. ***True or False*** The point $(-1, 4)$ lies in quadrant IV of the Cartesian plane.
12. ***True or False*** The midpoint of a line segment is found by averaging the x-coordinates and averaging the y-coordinates of the endpoints.
13. Which of the following statements is true for a point (x, y) that lies in quadrant III?
    (a) Both x and y are positive.
    (b) Both x and y are negative.
    (c) x is positive, and y is negative.
    (d) x is negative, and y is positive.
14. Choose the formula that gives the distance between two points (x_1, y_1) and (x_2, y_2).
    (a) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
    (b) $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$
    (c) $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$
    (d) $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$

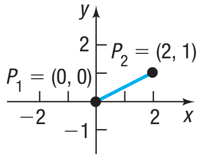
Skill Building

In Problems 15 and 16, plot each point in the xy-plane. Tell in which quadrant or on what coordinate axis each point lies.

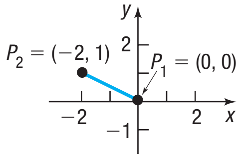
15. (a) $A = (-3, 2)$ (b) $B = (6, 0)$ (c) $C = (-2, -2)$ (d) $D = (6, 5)$ (e) $E = (0, -3)$ (f) $F = (6, -3)$
16. (a) $A = (1, 4)$ (b) $B = (-3, -4)$ (c) $C = (-3, 4)$ (d) $D = (4, 1)$ (e) $E = (0, 1)$ (f) $F = (-3, 0)$
17. Plot the points $(2, 0)$, $(2, -3)$, $(2, 4)$, $(2, 1)$, and $(2, -1)$. Describe the set of all points of the form $(2, y)$, where y is a real number.
18. Plot the points $(0, 3)$, $(1, 3)$, $(-2, 3)$, $(5, 3)$, and $(-4, 3)$. Describe the set of all points of the form $(x, 3)$, where x is a real number.

In Problems 19–32, find the distance $d(P_1, P_2)$ between the points P_1 and P_2.

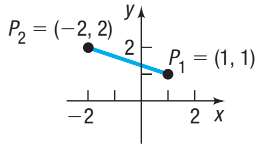
19.



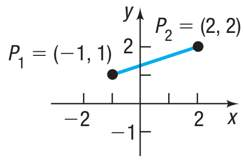
20.



21.



22.



23. $P_1 = (3, -4)$; $P_2 = (5, 4)$

24. $P_1 = (-1, 0)$; $P_2 = (2, 4)$

25. $P_1 = (-3, 2)$; $P_2 = (6, 0)$

26. $P_1 = (2, -3)$; $P_2 = (4, 2)$

27. $P_1 = (4, -2)$; $P_2 = (-2, -5)$

28. $P_1 = (-4, -3)$; $P_2 = (6, 2)$

29. $P_1 = (-0.2, 0.3)$; $P_2 = (2.3, 1.1)$

30. $P_1 = (1.2, 2.3)$; $P_2 = (-0.3, 1.1)$

31. $P_1 = (a, b)$; $P_2 = (0, 0)$

32. $P_1 = (a, a)$; $P_2 = (0, 0)$

In Problems 33–38, plot each point and form the triangle ABC. Verify that the triangle is a right triangle. Find its area.

33. $A = (-2, 5)$; $B = (1, 3)$; $C = (-1, 0)$

34. $A = (-2, 5)$; $B = (12, 3)$; $C = (10, -11)$

35. $A = (-5, 3)$; $B = (6, 0)$; $C = (5, 5)$

36. $A = (-6, 3)$; $B = (3, -5)$; $C = (-1, 5)$

37. $A = (4, -3)$; $B = (0, -3)$; $C = (4, 2)$

38. $A = (4, -3)$; $B = (4, 1)$; $C = (2, 1)$

In Problems 39–48, find the midpoint of the line segment joining the points P_1 and P_2.

39. $P_1 = (3, -4)$; $P_2 = (5, 4)$

40. $P_1 = (-2, 0)$; $P_2 = (2, 4)$

41. $P_1 = (-3, 2)$; $P_2 = (6, 0)$

42. $P_1 = (2, -3)$; $P_2 = (4, 2)$

43. $P_1 = (4, -2)$; $P_2 = (-2, -5)$

44. $P_1 = (-4, -3)$; $P_2 = (2, 2)$

45. $P_1 = (-0.2, 0.3)$; $P_2 = (2.3, 1.1)$

46. $P_1 = (1.2, 2.3)$; $P_2 = (-0.3, 1.1)$

47. $P_1 = (a, b)$; $P_2 = (0, 0)$

48. $P_1 = (a, a)$; $P_2 = (0, 0)$

Applications and Extensions

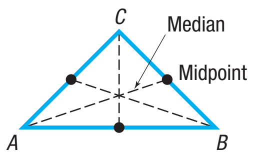
49. Find all points having an x-coordinate of 2 whose distance from the point $(-2, -1)$ is 5.

50. Find all points having a y-coordinate of -3 whose distance from the point $(1, 2)$ is 13.

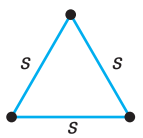
51. Find all points on the x-axis that are 5 units from the point $(4, -3)$.

52. Find all points on the y-axis that are 5 units from the point $(4, 4)$.

53. Geometry The **medians** of a triangle are the line segments from each vertex to the midpoint of the opposite side (see the figure). Find the lengths of the medians of the triangle with vertices at $A = (0, 0)$, $B = (6, 0)$, and $C = (4, 4)$.



54. Geometry An **equilateral triangle** is one in which all three sides are of equal length. If two vertices of an equilateral triangle are $(0, 4)$ and $(0, 0)$, find the third vertex. How many of these triangles are possible?



55. Geometry Find the midpoint of each diagonal of a square with side of length s. Draw the conclusion that the diagonals of a square intersect at their midpoints.
[**Hint:** Use $(0, 0)$, $(0, s)$, $(s, 0)$, and (s, s) as the vertices of the square.]

56. Geometry Verify that the points $(0, 0)$, $(a, 0)$, and $\left(\frac{a}{2}, \frac{\sqrt{3}\,a}{2}\right)$ are the vertices of an equilateral triangle. Then show that the midpoints of the three sides are the vertices of a second equilateral triangle (refer to Problem 54).

In Problems 57–60, find the length of each side of the triangle determined by the three points P_1, P_2, and P_3. State whether the triangle is an isosceles triangle, a right triangle, neither of these, or both. (An **isosceles triangle** *is one in which at least two of the sides are of equal length.)*

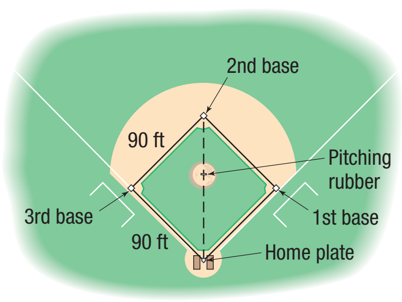
57. $P_1 = (2, 1)$; $P_2 = (-4, 1)$; $P_3 = (-4, -3)$

58. $P_1 = (-1, 4)$; $P_2 = (6, 2)$; $P_3 = (4, -5)$

59. $P_1 = (-2, -1)$; $P_2 = (0, 7)$; $P_3 = (3, 2)$

60. $P_1 = (7, 2)$; $P_2 = (-4, 0)$; $P_3 = (4, 6)$

61. Baseball A major league baseball "diamond" is actually a square 90 feet on a side (see the figure). What is the distance directly from home plate to second base (the diagonal of the square)?



62. **Little League Baseball** The layout of a Little League playing field is a square 60 feet on a side. How far is it directly from home plate to second base (the diagonal of the square)?

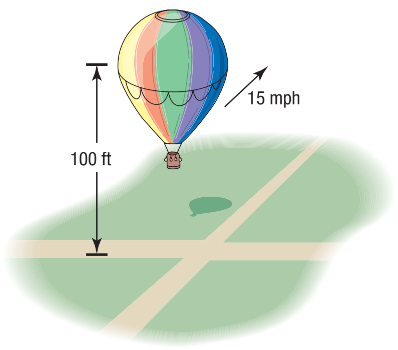
Source: *Little League Baseball, Official Regulations and Playing Rules, 2016*

63. **Baseball** Refer to Problem 61. Overlay a rectangular coordinate system on a major league baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.
   (a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.
   (b) If the right fielder is located at $(310, 15)$, how far is it from there to second base?
   (c) If the center fielder is located at $(300, 300)$, how far is it from there to third base?

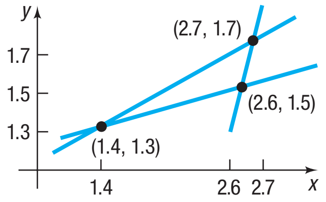
64. **Little League Baseball** Refer to Problem 62. Overlay a rectangular coordinate system on a Little League baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.
   (a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.
   (b) If the right fielder is located at $(180, 20)$, how far is it from there to second base?
   (c) If the center fielder is located at $(220, 220)$, how far is it from there to third base?

65. **Distance between Moving Objects** A Ford Focus and a Freightliner truck leave an intersection at the same time. The Focus heads east at an average speed of 30 miles per hour, while the truck heads south at an average speed of 40 miles per hour. Find an expression for their distance apart d (in miles) at the end of t hours.

66. **Distance of a Moving Object from a Fixed Point** A hot-air balloon, headed due east at an average speed of 15 miles per hour and at a constant altitude of 100 feet, passes over an intersection (see the figure). Find an expression for the distance d (measured in feet) from the balloon to the intersection t seconds later.



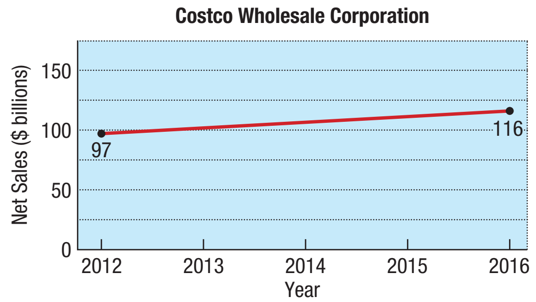
67. **Drafting Error** When a draftsperson draws three lines that are to intersect at one point, the lines may not intersect as intended and subsequently will form an **error triangle.** If this error triangle is long and thin, one estimate for the location of the desired point is the midpoint of the shortest side. The figure in the next column shows one such error triangle.



   (a) Find an estimate for the desired intersection point.
   (b) Find the length of the median for the midpoint found in part (a). See Problem 53.

68. **Net Sales** The figure illustrates how net sales of Costco Wholesale Corporation grew from 2012 through 2016. Use the midpoint formula to estimate the net sales of Costco Wholesale Corporation in 2014. How does your result compare to the reported value of $110 billion?

Source: *Costco Wholesale Corporation 2016 Annual Report*



69. **Poverty Threshold** Poverty thresholds are determined by the U.S. Census Bureau. A poverty threshold represents the minimum annual household income for a family not to be considered poor. In 2008, the poverty threshold for a family of four with two children under the age of 18 years was $21,834. In 2016, the poverty threshold for a family of four with two children under the age of 18 years was $24,339. Assuming poverty thresholds increase in a straight-line fashion, use the midpoint formula to estimate the poverty threshold of a family of four with two children under the age of 18 in 2012. How does your result compare to the actual poverty threshold in 2012 of $23,283?

Source: *U.S. Census Bureau*

70. **Horizontal and Vertical Shifts** Suppose that $A = (2, 5)$ are the coordinates of a point in the xy-plane.
   (a) Find the coordinates of the point if A is shifted 3 units to the right and 2 units down.
   (b) Find the coordinates of the point if A is shifted 2 units to the left and 8 units up.

71. **Completing a Line Segment** Plot the points $A = (-1, 8)$ and $M = (2, 3)$ in the xy-plane. If M is the midpoint of a line segment AB, find the coordinates of B.

Discussion and Writing

72. Write a paragraph that describes a Cartesian plane. Then write a second paragraph that describes how to plot points in the Cartesian plane. Your paragraphs should include the terms *coordinate axes*, *ordered pair*, *coordinates*, *plot*, *x-coordinate*, and *y-coordinate*.

'Are You Prepared?' Answers

1. 0 **2.** 8 **3.** 5 **4.** $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$ **5.** $\frac{1}{2}bh$ **6.** True

F.2 Graphs of Equations in Two Variables; Intercepts; Symmetry

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Solving Equations (Appendix A, Section A.8, pp. A66–A72)

Now Work the 'Are You Prepared?' problems on page 16.

OBJECTIVES

1 Graph Equations by Plotting Points

An **equation in two variables,** say x and y, is a statement in which two expressions involving x and y are equal. The expressions are called the **sides** of the equation. Since an equation is a statement, it may be true or false, depending on the value of the variables. Any values of x and y that result in a true statement are said to **satisfy** the equation.

For example, the following are all equations in two variables x and y:

$$x^2 + y^2 = 5 \qquad 2x - y = 6 \qquad y = 2x + 5 \qquad x^2 = y$$

The first of these, $x^2 + y^2 = 5$, is satisfied for $x = 1, y = 2$, since $1^2 + 2^2 = 1 + 4 = 5$. Other choices of x and y, such as $x = -1$, $y = -2$, also satisfy this equation. It is not satisfied for $x = 2$ and $y = 3$, since $2^2 + 3^2 = 4 + 9 = 13 \neq 5$.

The **graph of an equation in two variables** x and y consists of the set of points in the xy-plane whose coordinates (x, y) satisfy the equation.

EXAMPLE 1 **Determining Whether a Point Is on the Graph of an Equation**

Determine if the following points are on the graph of the equation $2x - y = 6$.

(a) $(2, 3)$ (b) $(2, -2)$

Solution (a) For the point $(2, 3)$, check to see whether $x = 2, y = 3$ satisfies the equation $2x - y = 6$.

$$2x - y = 2(2) - 3 = 4 - 3 = 1 \neq 6$$

The equation is not satisfied, so the point $(2, 3)$ is not on the graph.

(b) For the point $(2, -2)$,

$$2x - y = 2(2) - (-2) = 4 + 2 = 6$$

The equation is satisfied, so the point $(2, -2)$ is on the graph.

NOW WORK PROBLEM 11

EXAMPLE 2 How to Graph an Equation by Plotting Points

Graph the equation: $y = -2x + 3$

Step-by-Step Solution

Step 1 Find points (x, y) that satisfy the equation. To determine these points, choose values of x and use the equation to find the corresponding values for y. See Table 1.

Table 1

x	$y = -2x + 3$	(x, y)
-2	$-2(-2) + 3 = 7$	$(-2, 7)$
-1	$-2(-1) + 3 = 5$	$(-1, 5)$
0	$-2(0) + 3 = 3$	$(0, 3)$
1	$-2(1) + 3 = 1$	$(1, 1)$
2	$-2(2) + 3 = -1$	$(2, -1)$

Step 2 Plot the points found in the table as shown in Figure 11(a). Now connect the points to obtain the graph of the equation (a *line*), as shown in Figure 11(b).

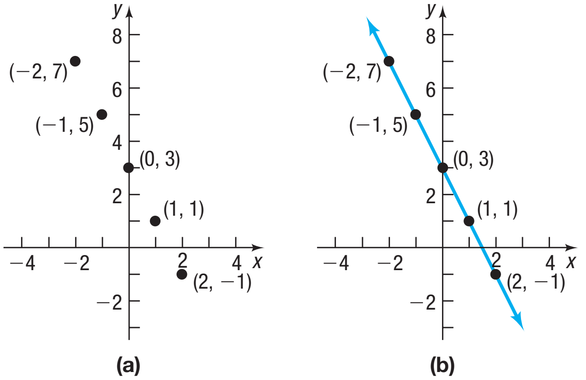


Figure 11 $y = -2x + 3$

EXAMPLE 3 Graphing an Equation by Plotting Points

Graph the equation: $y = x^2$

Solution

Table 2 provides several points on the graph. Plotting these points and connecting them with a smooth curve gives the graph (a *parabola*) shown in Figure 12.

Table 2

x	$y = x^2$	(x, y)
-4	16	$(-4, 16)$
-3	9	$(-3, 9)$
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$
3	9	$(3, 9)$
4	16	$(4, 16)$

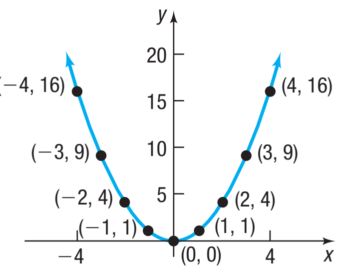


Figure 12 $y = x^2$

The graphs of the equations shown in Figures 11 and 12 do not show all the points that are on the graph. For example, in Figure 11 the point $(20, -37)$ is a part of the graph of $y = -2x + 3$, but it is not shown. Since the graph of $y = -2x + 3$ could be extended out indefinitely, we use arrows to indicate that the pattern shown continues. It is important when illustrating a graph to present enough of the graph so that any viewer of the illustration will "see" the rest of it as an obvious continuation of what is actually there. This is referred to as a **complete graph.**

One way to obtain a complete graph of an equation is to plot a sufficient number of points on the graph for a pattern to become evident. Then these points are connected with a smooth curve following the suggested pattern. But how many points are sufficient? Sometimes knowledge about the equation tells us. For example, we will learn in the next section that if an equation is of the form $y = mx + b$, then its graph is a line. In this case, only two points are needed to obtain the graph.

One purpose of this text is to investigate the properties of equations in order to decide whether a graph is complete. Sometimes we shall graph equations by plotting points. Shortly, we shall investigate various techniques that will enable us to graph an equation without plotting so many points.

COMMENT Another way to obtain the graph of an equation is to use a graphing utility. Read Section B.2, *Using a Graphing Utility to Graph Equations*, in Appendix B. ■

Two techniques that sometimes reduce the number of points required to graph an equation involve finding *intercepts* and checking for *symmetry*.

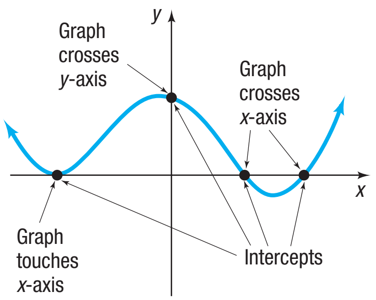


Figure 13

2 Find Intercepts from a Graph

The points, if any, at which a graph crosses or touches the coordinate axes are called the **intercepts.** See Figure 13. The x-coordinate of a point at which the graph crosses or touches the x-axis is an ***x*-intercept,** and the y-coordinate of a point at which the graph crosses or touches the y-axis is a ***y*-intercept.** For a graph to be complete, all its intercepts must be displayed.

EXAMPLE 4 Finding Intercepts from a Graph

Find the intercepts of the graph in Figure 14. What are its x-intercepts? What are its y-intercepts?

Solution The intercepts of the graph are the points

$$(-3, 0), \quad (0, 3), \quad \left(\frac{3}{2}, 0\right), \quad \left(0, -\frac{4}{3}\right), \quad (0, -3.5), \quad (4.5, 0)$$

The x-intercepts are $-3, \frac{3}{2}$, and 4.5; the y-intercepts are $-3.5, -\frac{4}{3}$, and 3. ●

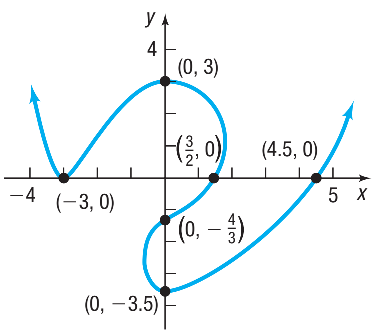


Figure 14

In Example 4, note the following usage: If the type of intercept (x- versus y-) is not specified, then report the intercept as an ordered pair. However, if the type of intercept is specified, then report only the coordinate of the specified intercept. For x-intercepts, report the x-coordinate of the intercept; for y-intercepts, report the y-coordinate of the intercept.

NOW WORK PROBLEM 39(A)

3 Find Intercepts from an Equation

The intercepts of a graph can be found from its equation by using the fact that points on the x-axis have y-coordinates equal to 0 and points on the y-axis have x-coordinates equal to 0.

Procedure for Finding Intercepts

1. To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x.
2. To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y.

Because the x-intercepts of the graph of an equation are those x-values for which $y = 0$, they are also called the **zeros** (or **roots**) of the equation.

EXAMPLE 5 Finding Intercepts from an Equation

Find the x-intercept(s) and the y-intercept(s) of the graph of $y = x^2 - 4$.

Solution To find the x-intercept(s), let $y = 0$ and obtain the equation

$$x^2 - 4 = 0$$

$$(x + 2)(x - 2) = 0 \quad \text{Factor.}$$

$$x + 2 = 0 \quad \text{or} \quad x - 2 = 0 \quad \text{Zero-Product Property}$$

$$x = -2 \quad \text{or} \quad x = 2 \quad \text{Solve.}$$

The equation has two solutions, -2 and 2. The x-intercepts (the zeros) are -2 and 2.

To find the y-intercept(s), let $x = 0$ in the equation.

$$\begin{aligned} y &= x^2 - 4 \\ &= 0^2 - 4 \\ &= -4 \end{aligned}$$

The y-intercept is -4.

NOW WORK PROBLEM 21

COMMENT For many equations, finding intercepts may not be so easy. In such cases, a graphing utility can be used. Read the first part of Section B.3, *Using a Graphing Utility to Locate Intercepts and Check for Symmetry*, in Appendix B to find out how a graphing utility locates intercepts.

4 Test an Equation for Symmetry

Another helpful tool for graphing equations by hand involves *symmetry*, particularly symmetry with respect to the x-axis, the y-axis, and the origin.

Symmetry often occurs in nature. Consider the picture of the butterfly. Do you see the symmetry?

DEFINITION

A graph is said to be **symmetric with respect to the x-axis** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

A graph is said to be **symmetric with respect to the y-axis** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

A graph is said to be **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.

Figure 15 illustrates the definition. Note that when a graph is symmetric with respect to the x-axis, the part of the graph above the x-axis is a reflection or mirror image of the part below it, and vice versa. When a graph is symmetric with respect to the y-axis, the part of the graph to the right of the y-axis is a reflection of the part to the left of it, and vice versa. Symmetry with respect to the origin may be viewed in two ways:

1. As a reflection about the y-axis, followed by a reflection about the x-axis
2. As a projection along a line through the origin so that the distances from the origin are equal

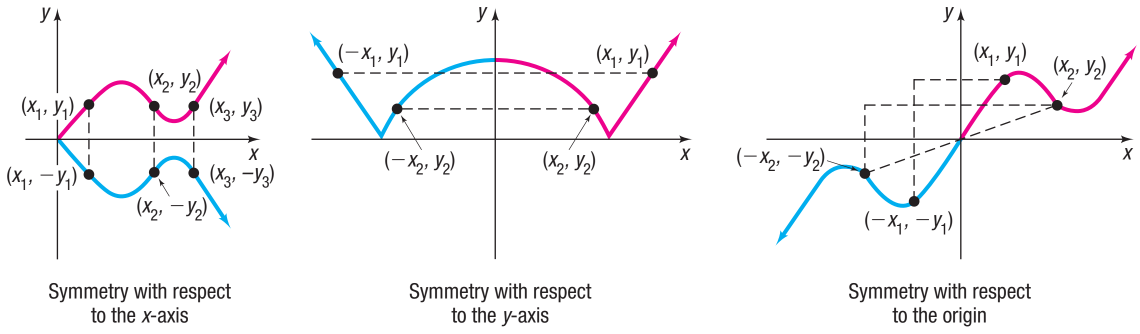


Figure 15

EXAMPLE 6 **Symmetric Points**

(a) If a graph is symmetric with respect to the x-axis, and the point $(4, 2)$ is on the graph, then the point $(4, -2)$ is also on the graph.

(b) If a graph is symmetric with respect to the y-axis, and the point $(4, 2)$ is on the graph, then the point $(-4, 2)$ is also on the graph.

(c) If a graph is symmetric with respect to the origin, and the point $(4, 2)$ is on the graph, then the point $(-4, -2)$ is also on the graph. ●

NOW WORK PROBLEM 29

When the graph of an equation is symmetric with respect to the x-axis, the y-axis, or the origin, the number of required points to plot in order to see the pattern is reduced. For example, if the graph of an equation is symmetric with respect to the y-axis, then once points to the right of the y-axis are plotted, an equal number of points on the graph can be obtained by reflecting them about the y-axis. Because of this, before graphing an equation, it is wise to determine whether any symmetry exists. The following tests are used for this purpose.

Tests for Symmetry

To test the graph of an equation for symmetry with respect to the

x-AXIS Replace y by $-y$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.

y-AXIS Replace x by $-x$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the y-axis.

ORIGIN Replace x by $-x$ and y by $-y$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the origin.

EXAMPLE 7 Testing an Equation for Symmetry

Test $y = \dfrac{4x^2}{x^2 + 1}$ for symmetry.

Solution

x-Axis: To test for symmetry with respect to the x-axis, replace y by $-y$. Since $-y = \dfrac{4x^2}{x^2 + 1}$ is not equivalent to $y = \dfrac{4x^2}{x^2 + 1}$, the graph of the equation is not symmetric with respect to the x-axis.

y-Axis: To test for symmetry with respect to the y-axis, replace x by $-x$. Since $y = \dfrac{4(-x)^2}{(-x)^2 + 1} = \dfrac{4x^2}{x^2 + 1}$ is equivalent to $y = \dfrac{4x^2}{x^2 + 1}$, the graph of the equation is symmetric with respect to the y-axis.

Origin: To test for symmetry with respect to the origin, replace x by $-x$ and y by $-y$.

$$-y = \frac{4(-x)^2}{(-x)^2 + 1} \quad \text{Replace } x \text{ by } -x \text{ and } y \text{ by } -y.$$

$$-y = \frac{4x^2}{x^2 + 1} \quad \text{Simplify.}$$

$$y = -\frac{4x^2}{x^2 + 1} \quad \text{Multiply both sides by } -1.$$

Since the result is not equivalent to the original equation, the graph of the equation $y = \dfrac{4x^2}{x^2 + 1}$ is not symmetric with respect to the origin. ●

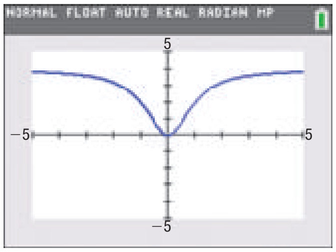


Figure 16 $y = \dfrac{4x^2}{x^2 + 1}$

Seeing the Concept

Figure 16 shows the graph of $y = \dfrac{4x^2}{x^2 + 1}$ using a TI-84 Plus C graphing calculator. Do you see the symmetry with respect to the y-axis?

NOW WORK PROBLEM 59

5 Know How to Graph Key Equations

The next three examples use intercepts, symmetry, and point plotting to obtain the graphs of key equations. It is important to know the graphs of these key equations because they will be used later. The first of these is $y = x^3$.

EXAMPLE 8 Graphing the Equation $y = x^3$ by Finding Intercepts and Checking for Symmetry

Graph the equation $y = x^3$ by plotting points. Find any intercepts and check for symmetry first.

Solution

First, find the intercepts. When $x = 0$, then $y = 0$; and when $y = 0$, then $x = 0$. The origin $(0, 0)$ is the only intercept. Now test for symmetry.

x-Axis: Replace y by $-y$. Since $-y = x^3$ is not equivalent to $y = x^3$, the graph is not symmetric with respect to the x-axis.

y-Axis: Replace x by $-x$. Since $y = (-x)^3 = -x^3$ is not equivalent to $y = x^3$, the graph is not symmetric with respect to the y-axis.

Origin: Replace x by $-x$ and y by $-y$. Since $-y = (-x)^3 = -x^3$ is equivalent to $y = x^3$ (multiply both sides by -1), the graph is symmetric with respect to the origin.

To graph $y = x^3$, use the equation to obtain several points on the graph. Because of the symmetry, we only need to locate points on the graph for which $x \geq 0$.

See Table 3. Since $(1, 1)$ is on the graph, and the graph is symmetric with respect to the origin, the point $(-1, -1)$ is also on the graph. Figure 17 shows the graph.

Table 3

x	$y = x^3$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	8	(2, 8)
3	27	(3, 27)

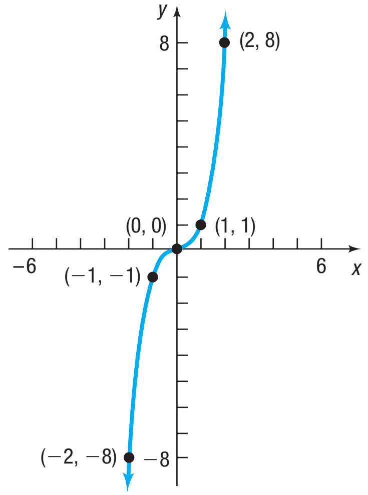


Figure 17 $y = x^3$

EXAMPLE 9 Graphing the Equation $x = y^2$

(a) Graph the equation $x = y^2$. Find any intercepts and check for symmetry first.
(b) Graph $x = y^2, y \geq 0$.

Solution (a) The lone intercept is $(0, 0)$. The graph is symmetric with respect to the x-axis since $x = (-y)^2$ is equivalent to $x = y^2$. The graph is not symmetric with respect to the y-axis or the origin.

To graph $x = y^2$, use the equation to obtain several points on the graph. Because the equation is solved for x, it is easier to assign values to y and use the equation to determine the corresponding values of x. Because of the symmetry, start by finding points whose y-coordinates are non-negative. Then use the symmetry to find additional points on the graph. See Table 4. For example, since $(1, 1)$ is on the graph, so is $(1, -1)$. Since $(4, 2)$ is on the graph, so is $(4, -2)$, and so on. Plot these points and connect them with a smooth curve to obtain Figure 18.

Table 4

y	$x = y^2$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	4	(4, 2)
3	9	(9, 3)

(b) If we restrict y so that $y \geq 0$, the equation $x = y^2$, $y \geq 0$, may be written equivalently as $y = \sqrt{x}$. The portion of the graph of $x = y^2$ in quadrant I is therefore the graph of $y = \sqrt{x}$. See Figure 19.

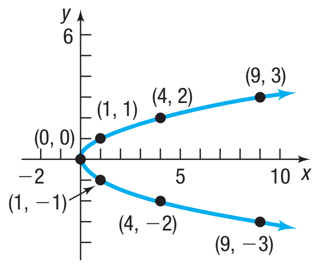


Figure 18 $x = y^2$

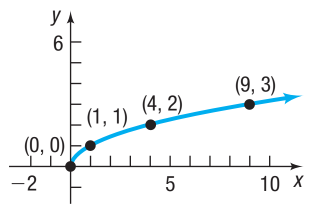


Figure 19 $y = \sqrt{x}$

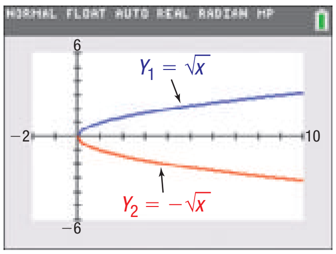


Figure 20 TI-84 Plus C

COMMENT To see the graph of the equation $x = y^2$ on a graphing calculator, graph two equations: $Y_1 = \sqrt{x}$ and $Y_2 = -\sqrt{x}$. See Figure 20. We discuss why in Chapter 1. ■

EXAMPLE 10 Graphing the Equation $y = \frac{1}{x}$

Graph the equation $y = \frac{1}{x}$. Find any intercepts and check for symmetry first.

(continued)