



# Mathematics for the Trades

## A GUIDED APPROACH

SAUNDERS

eleventh edition

CARMAN



# **Mathematics for the Trades**

## **A Guided Approach**

**Eleventh Edition**

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*Dedicated to my mother Bernice who, at 95,  
is still providing me with love and encouragement.  
I could not have done it without you!*

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# Preface

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*Mathematics for the Trades: A Guided Approach* provides the practical mathematics skills needed in a wide variety of trade, technical, and other occupational areas, including plumbing, automotive, electrical and construction trades, machine technology, landscaping, HVAC, allied health, and many more. It is especially intended for students who find math challenging and for adults who have been out of school for a time. This text assists students by providing a direct, practical approach that emphasizes careful, complete explanations and actual on-the-job applications. It is intended to provide practical help with real math, beginning at each student's individual level of ability. Careful attention has been given to readability, and reading specialists have helped plan both the written text and the visual organization.

Several special features are designed with the math-challenged student in mind. Each chapter begins with a preview quiz keyed to the textbook, which shows concepts that will be covered in that chapter. A summary is provided for each chapter, and each chapter ends with a set of problems to check the student's progress. The book can be used for a traditional course, a course of self-study, or a tutor-assisted study program.

The format is clear and easy to follow. It respects the individual needs of each reader, providing immediate feedback at each step to ensure understanding and continued attention. The emphasis is on *explaining* concepts rather than simply *presenting* them. This is a practical presentation rather than a theoretical one.

A calculator is a necessary tool for workers in trade and technical areas. We have integrated calculators extensively into the text—in finding numerical solutions to problems (including specific keystroke sequences) and in determining the values of transcendental functions. We have taken care to first explain all concepts and problem solving without the use of the calculator and describe how to estimate and check answers. Many realistic problems in the exercise sets involve large numbers, repeated calculations, and large quantities of information and are thus ideally suited to calculator use. They are representative of actual job situations in which a calculator is needed. Detailed instruction on the use of calculators is included in special sections at the end of Chapters 1, 2, and 3 as well as being integrated into the text.

Special attention has been given to on-the-job math skills by using a wide variety of real problems and situations. Many problems parallel those that appear on professional and apprenticeship exams. The answers to the odd-numbered exercises are given in the back of the book.

## New to This Edition

- We have added a new “Case Study” feature, which appears at the end of every chapter. Each Case Study is a multi-part problem that delves deeply into a specific real-world application and uses many of the mathematical skills covered in the chapter. Each of the chapter opening photos has been chosen to illustrate the subject matter of the Case Study, and the chapter introduction includes a brief preview of the Case Study.
- At the beginning of each section in a chapter there are two “Learning Catalytics” problems in the margin. These are designed to be quick and simple problems

reviewing skills that are needed to learn the mathematics in the upcoming section. Instructors can use this feature to get immediate feedback on the readiness of their class to take on the new material.

- In Chapters 2 and 3, special attention has now been given to the order of operations with fractions and decimals.
- In Chapter 3, more examples and problems have been added that contain fractions and decimals mixed together. Also in this chapter, the table of decimal-fraction equivalents on page 168 is now used to find the closest fractional equivalent to a given decimal number.
- The old Section 5-3 has been split into two smaller sections. The new Section 5-3 focuses strictly on the metric system itself, while the new Section 5-4 covers conversions between the metric and the U.S. customary systems. The concept of “greatest possible error” has also been added to Chapter 5.
- At the request of many instructors, the use of significant digits has been de-emphasized in rounding instructions. However, this topic is still covered in Chapter 5 and it is still used in Chapter 10 for rounding with trig calculations.
- We have added a new section to Chapter 12 covering measures of dispersion, which includes the concepts of range and standard deviation, as well as a basic introduction to normal distribution. This section had appeared in some earlier editions of the textbook, and it has been extensively revised and updated for this edition.
- As with every new edition, hundreds of applied problems have either been added or revised in both the exercise sets and the worked examples so that they continue to be authentic, topical, up to date, and representative of a wide variety of occupational areas.
- Many of the generic sections of the exercise sets have been reorganized to achieve a better progression from easier to more difficult problems as well as to create a better odd–even balance.
- In addition to all of these specific changes and added features, thousands of smaller changes have been made to improve the clarity of the explanations and the consistency of the presentations.



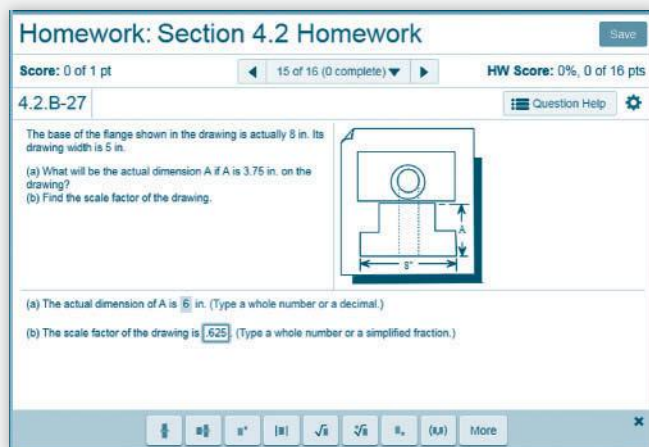
# Resources for Success

## Get the Most Out of MyLab Math

for *Mathematics for the Trades*, Eleventh Edition by Hal Saunders and Robert Carman

*Mathematics for Trades: A Guided Approach*, Eleventh edition offers market-leading content written by author-educators, tightly integrated with the #1 choice in digital learning—MyLab Math. MyLab Math courses can be tailored to the needs of instructors and students, while weaving the author team's voice and unique approach into all elements of the course. Teaching students the mathematical skills needed in realistic trade applications in both the text and MyLab Math course enables students to succeed not just in this course, but in their future trade.

Take advantage of the following resources, many of which are new or improved, to get the most out of your MyLab Math course.



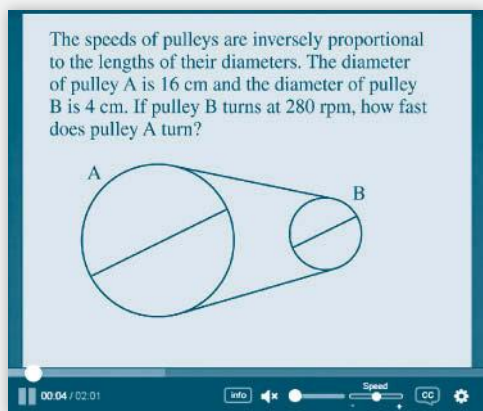
## Motivate Students through Math in Context

**NEW! A trade application question library** provides a wide range of exercises available to assign to work for any instructor's class dynamics. Available in the MyLab Math Assignment Manager, this library of exercises now allows instructors to pull in additional application questions from particular trades or industries.

**NEW! Case Study videos** complement the Case Studies that have been added to the text in this revision. These videos demonstrate a real-world trade application using math concepts to bring the math to life for students. Videos can be assigned in MyLab as homework, or could be used in the classroom to kick off a lecture.

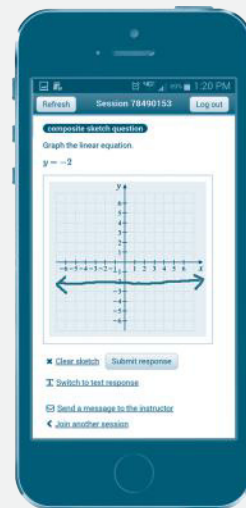
# Resources for Success

## Motivate and Support Students with Updated Resources



**Updated video program** provides section lectures for each section of the textbook. Lectures highlight key examples and exercises for every section of the textbook within a mobile-ready player that allows videos to be played on any device. Support students no matter where they are.

**NEW! Learning Catalytics** is an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking. Available through the MyLab Math course, pre-made questions specific to *Mathematics for the Trades* are available to use with students. Margin references in the text indicate where an existing Learning Catalytics question would be applicable to use. To find these questions in Learning Catalytics, search for **Saunders#**, where **#** is the chapter number.



### Resources for Instructors

The following resources can be downloaded from the *Instructor's Resource Center* on [www.pearson.com/us/higher-education](http://www.pearson.com/us/higher-education), or in the MyLab Math course.

#### NEW! PowerPoints

For the first time, PowerPoints for every section of the text are available. Fully editable, these can be used as a starting point for instructors to use in lecture, or for students to use as a study tool.

### Instructor's Solutions Manual

This manual contains detailed, worked-out solutions to all exercises in the text.

#### UPDATED! TestGen

TestGen® ([www.pearsoned.com/testgen](http://www.pearsoned.com/testgen)) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. Updated for the Eleventh edition, TestGen is now algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions.

## Acknowledgements

I would like to acknowledge the many people who have contributed to this Eleventh edition of *Mathematics for the Trades*. First and foremost, I am especially grateful to my editor, Matt Summers, who not only provided all the resources I needed, but who also gave me invaluable advice and encouragement from the first of the questionnaires to the last of the page proofs. Many other staff members at Pearson have played key roles in the revision process, including project manager Lauren Morse, editor-in-chief Michael Hirsch, producer Erin Carreiro, and product marketing manager Alicia Frankel, and I thank all of you for your contributions.

Special thanks go to Rachel Johnson from Northeast Wisconsin Technical College and to Cheryl Anderson from City College Montana State University for their creativity and expertise in providing most of the Case Studies. It was a pleasure working with both of you on this exciting new feature.

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A successful revision must be as error-free as possible, and there were two people who helped assure this outcome. My good friend Sally Pittman-Rabbin spent endless hours working all the new and revised problems so I could double-check my answers and correct any mistakes. And Deana Richmond not only triple-checked the mathematical accuracy, but proofread the entire revised text and provided many great suggestions for refining the wording of the explanations and improving consistency of the presentations. I offer my sincere thanks to both of you.

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This book has benefited greatly from their excellence as teachers.

Hal M. Saunders  
*Santa Barbara, California*

# How to Use This Book

---

In this book you will find many questions—not only at the end of each chapter or section, but on every page. This textbook is designed for those who need to learn the practical math used in the trades, and who want it explained carefully and completely at each step. The questions and explanations are designed so that you can:

- Start either at the beginning or where you need to start.
- Work on only what you need to know.
- Move as fast or as slowly as you wish.
- Receive the guidance and explanation you need.
- Skip material that you already understand.
- Do as many practice problems as you need.
- Test yourself often to measure your progress.

In other words, if you find mathematics difficult and you want to be guided carefully through it, this book is designed for you.

This is no ordinary book. You cannot browse through it; you don't read it. You *work* your way through it.

This textbook has been designed for students who will work through it to achieve understanding and practical skills. The alert student will look for and use the helpful features described below.

You will learn to do mathematics problems because you will follow our examples. The signal that a worked example is coming up looks like this:

## EXAMPLE 1

You should respond by following each step and questioning each step. If necessary, seek specific help from your instructor or tutor. ●

When you are confident that you understand the process, move on to the section labeled

---

### Your Turn

This is your chance to show that you are ready to try a similar problem or two on your own. Use the step-by-step procedure you learned in the *Example* to solve the problems.

### Solution

The *Your Turn* section is followed by the answers or worked solutions to the problems in the *Your Turn*.

---

Further drill problems are often provided in another set, which is labeled as follows:

### More Practice Answers

---

These problems may be followed by worked solutions or by a list of answers.

---



Keep on the lookout for the following helpers:



### Note

Every experienced teacher knows that certain mathematical concepts and procedures will present special difficulties for students. To help you with these, special notes are included in the text. A pencil symbol and a warning word appear in the left margin to indicate the start of the comment, and a bullet ● shows when it is completed. The word **Note**, as used at the start of this paragraph, calls your attention to conclusions or consequences that might be overlooked, common mistakes students make, or alternative explanations. ●



### Careful

A **Careful** comment points out a common mistake that you might make and shows you how to avoid it. ●



### Learning Help

A **Learning Help** gives you an alternative explanation or slightly different way of thinking about and working with the concepts being presented. ●



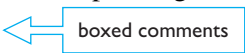
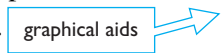
### A Closer Look

This phrase signals a follow-up to an answer or a worked solution. It may provide a more detailed or an alternative look at the whole process. ●

Examples often include step-by-step explanations that look like this:

**Step 1** The solution of each worked example is usually organized in a step-by-step format, similar to this paragraph.

**Step 2** In each worked example, explanations for each step are provided alongside the corresponding mathematical operations.

**Step 3** Color, , and other  are used to highlight the important or tricky aspects of a solution, if needed.



A **check icon** appears in a problem solution to remind you to check your work.



The calculator is an important tool for the modern trades worker or technician. We assume in this textbook that once you have learned the basic operations of arithmetic you will use a calculator. Problems in the exercise sets or examples in the text that involve the use of a scientific calculator are preceded by the calculator symbol shown here.

Solutions often include a display of the proper calculator key sequences. For example, the calculation

$$\frac{85.7 + (12.9)^2}{71.6}$$

would be shown as

$$85.7 \boxed{+} 12.9 \boxed{x^2} \boxed{=} \boxed{\div} 71.6 \boxed{=} \rightarrow 3.521089385$$

Not every student owns the same brand or model of calculator, and there are often variations in the way different calculators operate. The calculator key sequences shown in this textbook are based on the way that the newer generation of scientific calculators operate, and they will also be appropriate for graphing calculators. On these calculators, you enter calculations exactly as they would be correctly written or spoken aloud. Furthermore, their windows can display at least two lines—one to show the sequence of keys that you pressed (except for the equals sign), and the other to show the final answer.

If you are wondering whether or not you own one of the newer-generation calculators, here is a simple test: Try to take the square root of 9 by pressing the square-root key first, then the 9, and then = or **ENTER** ( $\sqrt{\phantom{x}}$  9  $=$ ). If you are able to do this and obtain the correct answer of 3, then you own the type of calculator that will work for the sequences shown in the text. If, on the other hand, you need to enter 9 first and then the square-root key to obtain 3, you own one of the older-generation calculators, and you will need to consult your owner's manual to learn the proper key sequences (or buy a new calculator!). Another clue is that the older style machines normally have only a one-line display window.

**Exercises** At the conclusion of each section of each chapter you will find a set of problems covering the work of that section. These will include a number of routine or drill problems as well as applications or word problems. Each applied problem begins with an indication of the occupational area from which it has been taken.

**Chapter Summary** A chapter summary is included at the end of each chapter. It contains a list of objectives with corresponding reviews and worked examples.

**Problem Set** Following each chapter summary is a set of problems reviewing all of the material covered in the chapter.

Important rules, definitions, equations, or helpful hints are often placed in a box like this so they will be easy to find.

If your approach to learning mathematics is to skim the text lightly on the way to puzzling through a homework assignment, you will have difficulty with this or any other textbook. If you are motivated to study mathematics so that you understand it and can use it correctly, this textbook is designed for you.

According to an old Spanish proverb, the world is an ocean and one who cannot swim will sink to the bottom. If the modern world of work is an ocean, the skill needed to keep afloat or even swim to the top is clearly mathematics. It is the purpose of this book to help you learn these basic skills.

Now, turn to page 1 and let's begin.

H. M. S.

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# Arithmetic of Whole Numbers

Objective	Sample problems	For help, go to page
<b>When you finish this chapter you will be able to:</b>		
<b>1.</b> (a) Write whole numbers in words. (b) Write, in numerical form, whole numbers that are spoken or written in words.	(a) Write 250,374 in words _____	5
	(b) Write, in numerical form: “one million, sixty-five thousand, eight” _____	6
<b>2.</b> Round whole numbers.	Round 214,659	
	(a) to the nearest ten thousand _____	6
	(b) to the nearest hundred _____	
<b>3.</b> Add whole numbers.	(a) $67 + 58$ _____	7
	(b) $7009 + 1598$ _____	
<b>4.</b> Subtract whole numbers.	(a) $82 - 45$ _____	21
	(b) $4035 - 1967$ _____	
	(c) $14 + 31 + 59 - 67$ $+ 22 + 37 - 19$ _____	
<b>5.</b> Multiply whole numbers.	(a) $64 \times 37$ _____	30
	(b) $305 \times 243$ _____	
	(c) $908 \times 705$ _____	

Name \_\_\_\_\_  
 Date \_\_\_\_\_  
 Course/Section \_\_\_\_\_

Objective	Sample problems	For help, go to page
6. Divide whole numbers.	(a) $2006 \div 6$ _____	41
	(b) $7511 \div 37$ _____	
7. Determine factors and prime factors.	(a) List all the factors of 12. _____	46
	(b) Write 12 as a product of its prime factors. _____	47
8. Use the correct order of operations with addition, subtraction, multiplication, and division.	(a) $6 + 9 \times 3$ _____	53
	(b) $35 - 14 \div 7$ _____	
	(c) $56 \div 4 \times 2 + 9 - 4$ _____	
	(d) $(23 - 7) \times 24 \div (12 - 4)$ _____	
9. Solve practical applications involving whole numbers.	<b>Machine Trades</b> A metal casting weighs 680 lb. 235 lb of metal is removed during shaping. What is its finished weight? _____	

(Answers to these preview problems are given in the Appendix. Also, worked solutions to many of these problems appear in the chapter Summary. Don't peek.)

If you are certain that you can work *all* these problems correctly, turn to page 62 for a set of practice problems. If you cannot work one or more of the preview problems, turn to the page indicated to the right of the problem. For those who wish to master this material with the greatest success, turn to Section 1-1 and begin to work there.

# Arithmetic of Whole Numbers



A century ago, the average person used numbers to tell time, count, and keep track of money. Today, most people need to develop technical skills based on their ability to read, write, and work with numbers in order to earn a living. Although we live in an age of computers and calculators, much of the simple arithmetic used in industry, business, and the skilled trades is still done mentally or by hand. In fact, many trade and technical areas require you to *prove* that you can do the calculations by hand before you can get a job.

In this opening chapter, we take a practical, how-to-do-it look at the four basic operations of whole numbers and the order of operations. Once we get past the basics, we will show you how to use a calculator to do such calculations.

In addition to learning the basic mathematical skills, the other main purpose of this text is to teach you how to apply these skills in practical, on-the-job situations. Every section features worked examples of practical applications, and the exercise sets contain problems from a variety of trade, technical, and other occupations.

## CASE STUDY: Robin's New Car

At the end of each chapter you will be guided through a **Case Study**, a multi-step application that uses some of the math skills you learned in the chapter. The Case Study for Chapter 1 (p. 67) teaches you the difference between financing and leasing a new car and how to use the mathematics of whole numbers to help you choose the best option.

1-1	Reading, Writing, Rounding, and Adding Whole Numbers	p. 4
1-2	Subtraction of Whole Numbers	p. 21
1-3	Multiplication of Whole Numbers	p. 30
1-4	Division of Whole Numbers	p. 41
1-5	Order of Operations	p. 53

## 1-1

## Reading, Writing, Rounding, and Adding Whole Numbers

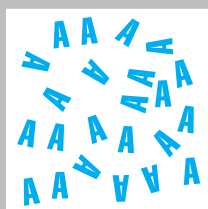
### learning|catalytics™

- Write in numerical form: two hundred six thousand, seven hundred thirty
- Round the number in problem 1 to the nearest thousand.

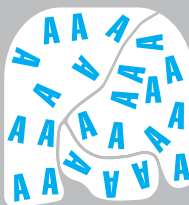
#### EXAMPLE 1

How many letters are in the collection shown in the margin?

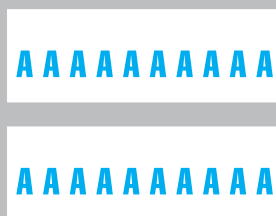
We counted 23. Notice that we can count the letters by grouping them into sets of ten:



becomes



or



2 tens + 3 ones

20 + 3 or 23

**Expanded Form** Mathematicians call this the **expanded form** of a number. For example,

$$46 = 40 + 6 = 4 \text{ tens} + 6 \text{ ones}$$

$$874 = 800 + 70 + 4 = 8 \text{ hundreds} + 7 \text{ tens} + 4 \text{ ones}$$

$$305 = 300 + 5 = 3 \text{ hundreds} + 0 \text{ tens} + 5 \text{ ones}$$

Only ten numerals or number symbols—0, 1, 2, 3, 4, 5, 6, 7, 8, and 9—are needed to write any number. These ten basic numerals are called the **digits** of the number. The digits 4 and 6 are used to write 46, the number 274 is a three-digit number, and so on.

#### Your Turn

Write out the following three-digit numbers in expanded form:

(a)  $762 = \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad} \text{ hundreds} + \underline{\quad} \text{ tens} + \underline{\quad} \text{ ones}$

(b)  $425 = \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad} \text{ hundreds} + \underline{\quad} \text{ tens} + \underline{\quad} \text{ ones}$

(c)  $208 = \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad} \text{ hundreds} + \underline{\quad} \text{ tens} + \underline{\quad} \text{ ones}$

#### Solutions

(a)  $762 = 700 + 60 + 2 = 7 \text{ hundreds} + 6 \text{ tens} + 2 \text{ ones}$

(b)  $425 = 400 + 20 + 5 = 4 \text{ hundreds} + 2 \text{ tens} + 5 \text{ ones}$

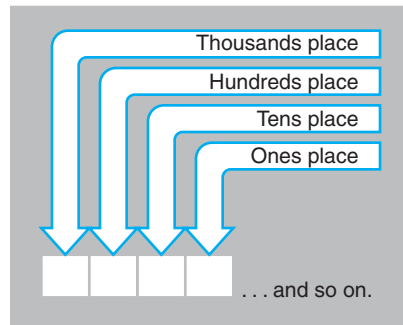
(c)  $208 = 200 + 0 + 8 = 2 \text{ hundreds} + 0 \text{ tens} + 8 \text{ ones}$



#### A Closer Look

Notice that the 2 in 762 means something different from the 2 in 425 or 208. In 762, the 2 signifies two ones. In 425, the 2 signifies two tens. In 208, the 2 signifies two

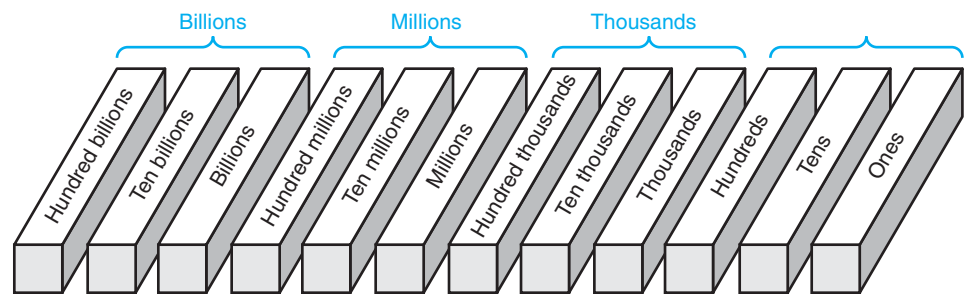
hundreds. Ours is a *place-value* system of naming numbers: the value of any digit depends on the place where it is located.



Being able to write a number in expanded form will help you to understand and remember the basic operations of arithmetic—even though you’ll never find it on a blueprint or in a technical handbook.

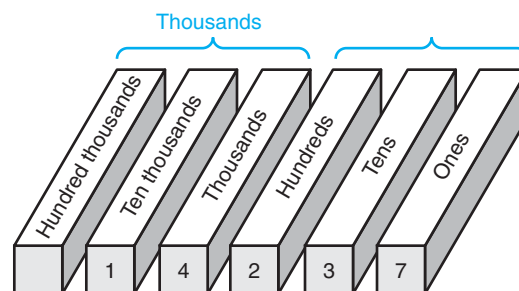
### Reading and Writing Whole Numbers

This expanded-form idea is useful especially in naming very large numbers. Any large number given in numerical form may be translated to words by using the following diagram:



#### EXAMPLE 2

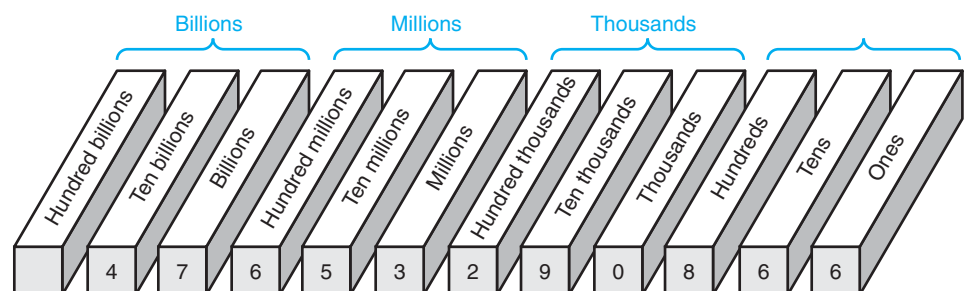
The number 14,237 can be placed in the diagram like this:



and read “fourteen thousand, two hundred thirty-seven.”

#### EXAMPLE 3

The number 47,653,290,866 becomes





and is read “forty-seven billion, six hundred fifty-three million, two hundred ninety thousand, eight hundred sixty-six.”

In each block of three digits read the digits in the normal way (“forty-seven,” “six hundred fifty-three”) and add the name of the block (“billion,” “million”). Notice that the word “and” is not used in naming these numbers. ●

**Your Turn**

Use the diagram to name the following numbers.

- |               |                    |
|---------------|--------------------|
| (a) 4072      | (b) 1,360,105      |
| (c) 3,000,210 | (d) 21,862,031,001 |

**Answers**

- (a) Four thousand, seventy-two  
 (b) One million, three hundred sixty thousand, one hundred five  
 (c) Three million, two hundred ten  
 (d) Twenty-one billion, eight hundred sixty-two million, thirty-one thousand, one

**Note**

Notice in part (a), we did not insert a comma between the 4 and the 0. In a four-digit number such as this, the comma is optional, and we will usually omit it. ●

It is also important to be able to write numbers correctly when you hear them spoken or when they are written in words.

**More Practice**

Read each of the following aloud and then write them in correct numerical form.

- (a) Fifty-eight thousand, four hundred six  
 (b) Two hundred seventy-three million, five hundred forty thousand  
 (c) Seven thousand, sixty  
 (d) Nine billion, six million, two hundred twenty-three thousand, fifty-eight

**Answers**

- |            |                   |
|------------|-------------------|
| (a) 58,406 | (b) 273,540,000   |
| (c) 7060   | (d) 9,006,223,058 |

**Rounding Whole Numbers**

In many situations a simplified approximation of a number is more useful than its exact value. For example, the accountant for a business may calculate its total monthly revenue as \$247,563, but the owner of the business may find it easier to talk about the revenue as “about \$250,000.” The process of approximating a number is called *rounding*. Rounding numbers comes in handy when we need to make estimates or do “mental mathematics.”

A number can be rounded to any desired place. For example, \$247,563 is approximately

\$247,560 rounded to the nearest ten,  
 \$247,600 rounded to the nearest hundred,  
 \$248,000 rounded to the nearest thousand,  
 \$250,000 rounded to the nearest ten thousand, and  
 \$200,000 rounded to the nearest hundred thousand.

To round a whole number, follow this step-by-step process:

**EXAMPLE 4**

Round 247,563

to the nearest  
hundred thousand

to the nearest  
ten thousand

**Step 1** Determine the place to which the number is to be rounded. Mark it on the right with a  $\wedge$ .

\$2  $\wedge$  47,563

\$24  $\wedge$  7,563

**Step 2** If the digit to the right of the mark is less than 5, replace all digits to the right of the mark with zeros.

\$200,000

**Step 3** If the digit to the right of the mark is equal to or larger than 5, increase the digit to the left by 1 and replace all digits to the right with zeros.

\$250,000

**Your Turn**

Try these for practice. Round

- (a) 73,856 to the nearest thousand
- (b) 64 to the nearest ten
- (c) 4852 to the nearest hundred
- (d) 350,000 to the nearest hundred thousand
- (e) 726 to the nearest hundred

**Solutions**

(a) **Step 1** Place a mark to the right of the thousands place. The digit 3 is in the thousands place.

73  $\wedge$  856

**Step 2** Does not apply.

**Step 3** The digit to the right of the mark, 8, is larger than 5. Increase the 3 to a 4 and replace all digits to the right with zeros.

74,000

(b) **Step 1** Place a mark to the right of the tens place. The digit 6 is in the tens place.

6  $\wedge$  4

**Step 2** The digit to the right of the mark, 4, is less than 5. Replace it with a zero.

60

(c) 4900

(d) 400,000

(e) 700

**Addition of Whole Numbers**

Adding whole numbers is fairly easy provided that you have stored in your memory a few simple addition facts. It is most important that you be able to add simple one-digit numbers mentally.

The following sets of problems in one-digit addition are designed to give you some practice. Work quickly. You should be able to answer all problems in a set in the time shown.

**PROBLEMS** One-Digit Addition

A. Add.

7	5	2	5	8	2	3	8	9	7
<u>3</u>	<u>6</u>	<u>9</u>	<u>7</u>	<u>8</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>3</u>	<u>6</u>
6	8	9	3	7	2	9	9	7	4
<u>4</u>	<u>5</u>	<u>6</u>	<u>5</u>	<u>7</u>	<u>7</u>	<u>4</u>	<u>9</u>	<u>2</u>	<u>7</u>
9	2	5	8	4	9	6	4	8	8
<u>7</u>	<u>6</u>	<u>5</u>	<u>9</u>	<u>5</u>	<u>5</u>	<u>6</u>	<u>3</u>	<u>2</u>	<u>3</u>
5	6	7	7	5	6	2	3	6	9
<u>8</u>	<u>7</u>	<u>5</u>	<u>9</u>	<u>4</u>	<u>5</u>	<u>8</u>	<u>7</u>	<u>8</u>	<u>8</u>
7	5	9	4	3	8	4	8	5	7
<u>4</u>	<u>9</u>	<u>2</u>	<u>6</u>	<u>8</u>	<u>6</u>	<u>9</u>	<u>4</u>	<u>8</u>	<u>8</u>

Mastery time = 80 seconds

B. Add. Try to do all addition mentally.

2	7	3	4	2	6	3	5	9	5
<u>5</u>	<u>3</u>	<u>6</u>	<u>5</u>	<u>7</u>	<u>7</u>	<u>4</u>	<u>7</u>	<u>6</u>	<u>2</u>
4	2	5	8	9	8	4	8	3	8
<u>4</u>	<u>2</u>	<u>5</u>	<u>8</u>	<u>9</u>	<u>8</u>	<u>4</u>	<u>8</u>	<u>3</u>	<u>8</u>
6	5	4	8	6	9	7	4	8	1
<u>2</u>	<u>4</u>	<u>2</u>	<u>1</u>	<u>8</u>	<u>3</u>	<u>1</u>	<u>9</u>	<u>4</u>	<u>8</u>
7	5	9	9	8	5	6	1	6	7
<u>7</u>	<u>5</u>	<u>9</u>	<u>9</u>	<u>8</u>	<u>5</u>	<u>6</u>	<u>1</u>	<u>6</u>	<u>7</u>
1	9	3	1	7	2	9	9	8	5
<u>9</u>	<u>9</u>	<u>1</u>	<u>6</u>	<u>9</u>	<u>9</u>	<u>8</u>	<u>5</u>	<u>3</u>	<u>4</u>
2	1	4	3	6	1	2	1	3	7
<u>2</u>	<u>1</u>	<u>4</u>	<u>3</u>	<u>6</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>3</u>	<u>7</u>

Mastery time = 90 seconds

The answers are given in the Appendix.

**Note**The answer to an addition problem is called the **sum**. ●

Adding one-digit numbers is very important. It is the key to many mathematical computations—even if you do the work on a calculator. But you also must be able to add numbers with two or more digits. Suppose that, as a contractor, you need to find the total time spent on a job by two workers who have put in 31 hours and 48 hours, respectively. You will need to find the sum

31 hours + 48 hours = \_\_\_\_\_

**Estimating**

What is the first step? Start adding digits? Punch in some numbers on your trusty calculator? Rattle your abacus? None of these. The first step is to *estimate* your answer. The most important rule in any mathematical calculation is:

Know the approximate answer to any calculation before you calculate it.

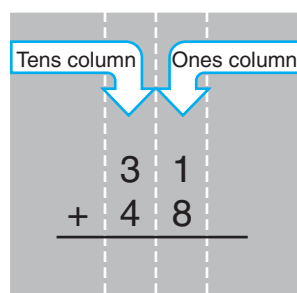
Never do an arithmetic calculation until you know roughly what the answer is going to be. Always know where you are going.

Rounding to the nearest 10 hours, the preceding sum can be estimated as 31 hours + 48 hours or approximately 30 hours + 50 hours or 80 hours, not 8 or 8000 or 800 hours. Having the estimate will keep you from making any major (and embarrassing) mistakes. Once you have a rough estimate of the answer, you are ready to do the arithmetic work.

Calculate  $31 + 48 = \underline{\hspace{2cm}}$

You don't really need an air-conditioned, solar-powered, talking calculator for that, do you?

You should set it up like this:



1. The numbers to be added are arranged vertically (up and down) in columns.
2. The right end or ones digits are placed in the ones column, the tens digits are placed in the tens column, and so on.

Avoid the confusion of  $\begin{array}{r} 31 \\ + 48 \end{array}$  or  $\begin{array}{r} 31 \\ + 48 \end{array}$



**Careful**

Most often the cause of errors in arithmetic is carelessness, especially in simple tasks such as lining up the digits correctly. ●

Once the digits are lined up the problem is easy.

$$\begin{array}{r} 31 \\ + 48 \\ \hline 79 \end{array}$$

**First**, find the sum of the ones column:  $1 + 8 = 9$   
**Then**, find the sum of the tens column:  $3 + 4 = 7$

Does the answer agree with your original estimate? Yes. The estimate, 80, is roughly equal to the actual sum, 79.

What we have just shown you is called the *guess and check method* of doing mathematics.

1. **Estimate** the answer using rounded numbers.
2. **Work** the problem carefully.
3. **Check** your answer against the estimate. If they disagree, repeat both Steps 1 and 2. The check icon reminds you to check your work.



Most students worry about estimating, either because they won't take the time to do it or because they are afraid they might do it incorrectly. Relax. You are the only one who will know your estimate. Do it in your head, do it quickly, and make it reasonably accurate. Step 3 helps you to find incorrect answers before you finish the problem. The *guess and check method* means that you never work in the dark; you always know where you are going.



**Note**

Estimating is especially important in practical math, where a wrong answer is not just a mark on a piece of paper. An error may mean time and money lost. ●

#### EXAMPLE 5

Here is a slightly more difficult problem:

$$27 \text{ lb} + 58 \text{ lb} = \underline{\hspace{2cm}}$$

**First**, estimate the answer.  $27 + 58$  is roughly  
 $30 + 60$  or about 90.  
 The answer is about 90 lb.

**Second**, line up the digits in columns.  $27$   
 $+ 58$

(The numbers to be added, 27 and 58 in this case, are called *addends*.)

**Third**, add carefully.  $\overset{1}{27}$   
 $+ 58$   
 $\hline 85$



**Finally**, check your answer by comparing it with the estimate. The estimate 90 lb is roughly equal to the answer 85 lb—at least you have an answer in the right ballpark. ●



### A Closer Look

What does that little 1 above the tens digit mean? What really happens when you “carry” a digit? Let’s look at it in detail. In expanded notation,

$$\begin{aligned}
 27 &\rightarrow 2 \text{ tens} + 7 \text{ ones} \\
 + 58 &\rightarrow 5 \text{ tens} + 8 \text{ ones} \\
 &= 7 \text{ tens} + 15 \text{ ones} \\
 &= 7 \text{ tens} + 1 \text{ ten} + 5 \text{ ones} \\
 &= 8 \text{ tens} + 5 \text{ ones} \\
 &= 85
 \end{aligned}$$

The 1 that is carried over to the tens column is really a ten. ●



### Learning Help

Trades people often must calculate exact answers mentally. To do a problem such as  $27 + 58$ , a trick called “balancing” works nicely. Simply add 2 to the 58 to get a “round” number, 60, and subtract 2 from 27 to balance, keeping the total the same. Therefore,  $27 + 58$  is the same as  $25 + 60$ , which is easy to add mentally to get 85. ●

### Your Turn

Try this one:  $456 + 87 = \underline{\hspace{2cm}}$

### Solution

**First**, estimate the answer.  $456 + 87$  is roughly  $460 + 90$  or about 550.

**Then**, line up the digits in columns and add.  $\overset{1}{\overset{1}{456}}$   
 $+ 87$   
 $\hline 543$

The answer is very close to the estimate.

We use the same procedure to add three or more numbers. Estimating and checking become even more important when the problem gets more complicated.

### EXAMPLE 6

To add  $536 + 1473 + 875 + 88$

**Estimate:** Rounding each number to the nearest hundred,

$$500 + 1500 + 900 + 100 = 3000$$

**Step 1** To help avoid careless errors, put the number with the most digits, 1473, on top. Put the number with the fewest digits, 88, on the bottom.

$$\begin{array}{r} 1473 \\ 536 \\ 875 \\ + 88 \\ \hline \end{array}$$

**Step 2**

$$\begin{array}{r} 2 \\ 1473 \\ 536 \\ 875 \\ + 88 \\ \hline 2 \end{array} \quad 3 + 6 + 5 + 8 = 22 \quad \text{Write 2; carry 2 tens}$$

**Step 3**

$$\begin{array}{r} 22 \\ 1473 \\ 536 \\ 875 \\ + 88 \\ \hline 72 \end{array} \quad 2 + 7 + 3 + 7 + 8 = 27 \quad \text{Write 7; carry 2 hundreds}$$

**Step 4**

$$\begin{array}{r} 122 \\ 1473 \\ 536 \\ 875 \\ + 88 \\ \hline 972 \end{array} \quad 2 + 4 + 5 + 8 = 19 \quad \text{Write 9; carry 1 thousand}$$

**Step 5**

$$\begin{array}{r} 122 \\ 1473 \\ 536 \\ 875 \\ + 88 \\ \hline 2972 \end{array} \quad 1 + 1 = 2 \quad \text{Write 2.}$$


**Check:** The estimate 3000 and the answer 2972 are very close.

### Your Turn

The following is a short set of problems. Add, and be sure to estimate your answers first. Check your answers with your estimates.

- (a)  $429 + 738 = \underline{\hspace{2cm}}$       (b)  $1446 + 867 = \underline{\hspace{2cm}}$   
 (c)  $82 + 2368 + 744 = \underline{\hspace{2cm}}$       (d)  $409 + 2572 + 3685 + 94 = \underline{\hspace{2cm}}$

### Solutions

- (a) **Estimate:** Rounding each number to the nearest hundred,  $400 + 700 = 1100$   
 Line up the digits:

$$\begin{array}{r} 429 \\ + 738 \\ \hline \end{array}$$

**Calculate:**

**Step 1**

$$\begin{array}{r} 1 \\ 429 \\ + 738 \\ \hline 7 \end{array} \quad 9 + 8 = 17 \quad \text{Write 7; carry 1 ten.}$$

**Step 2**

$$\begin{array}{r} 1 \\ 429 \\ + 738 \\ \hline 67 \end{array} \quad 1 + 2 + 3 = 6 \quad \text{Write 6.}$$

**Step 3**

$$\begin{array}{r} 429 \\ + 738 \\ \hline 1167 \end{array}$$

$4 + 7 = 11$  Write 11.



**Check:** The estimate 1100 and the answer 1167 are roughly equal.

(b) **Estimate:**  $1400 + 900 = 2300$

**Calculate:**

**Step 1**

$$\begin{array}{r} 1446 \\ + 867 \\ \hline 3 \end{array}$$

$6 + 7 = 13$  Write 3; carry 1 ten.

**Step 2**

$$\begin{array}{r} 1446 \\ + 867 \\ \hline 13 \end{array}$$

$1 + 4 + 6 = 11$  Write 1; carry 1 hundred.

**Step 3**

$$\begin{array}{r} 1446 \\ + 867 \\ \hline 313 \end{array}$$

$1 + 4 + 8 = 13$  Write 3; carry 1 thousand.

**Step 4**

$$\begin{array}{r} 1446 \\ + 867 \\ \hline 2313 \end{array}$$

$1 + 1 = 2$  Write 2.



**Check:** The estimate 2300 and the answer 2313 are roughly equal.

(c) **Estimate:** Rounding each number to the nearest hundred,  
 $100 + 2400 + 700 = 3200$

**Calculate:**

$$\begin{array}{r} 2368 \\ 744 \\ + 82 \\ \hline 3194 \end{array}$$



**Check:** The estimate 3200 and the answer 3194 are roughly equal.

(d) **Estimate:**  $400 + 2600 + 3700 + 100 = 6800$

**Calculate:**

$$\begin{array}{r} 2572 \\ 3685 \\ 409 \\ + 94 \\ \hline 6760 \end{array}$$



**Check:** The estimate 6800 and the answer 6760 are roughly equal.

Estimating answers is a very important part of any mathematics calculation, especially for the practical mathematics used in engineering, technology, and the trades. A successful builder, painter, or repairperson must make accurate estimates of job costs—business success depends on it. If you work in a technical trade, getting and keeping your job may depend on your ability to get the correct answer *every* time.

If you use a calculator to do the actual arithmetic, it is even more important to get a careful estimate of the answer first. If you plug a wrong number into the calculator, accidentally hit a wrong key, or unknowingly use a failing battery, the calculator may give you a wrong answer—lightning fast, but wrong. The estimate is your best insurance that a wrong answer will be caught immediately. Convinced?

### Practical Applications

Once you learn a basic math skill, you must also learn how to apply that skill to a real-world situation in your job or in your daily life. To help you practice applying the math, practical applications, also known to students as word problems, will be provided throughout this text in the examples and in the exercises.

Word problems that call for addition often contain key questions such as “how many . . .?” or “what is the total number of . . .?” Look for these clues that indicate you must add to find the answer.

#### EXAMPLE 7

**Transportation** In order to accommodate the weight of their equipment, the California Highway Patrol is now using an SUV instead of a sedan as their primary vehicle. The additional equipment they carry includes a push bumper (54 lb), a computer (12 lb), a gun tub with rifles (32 lb), a light bar (30 lb), a prisoner barrier (56 lb), radio and video equipment (270 lb), and other miscellaneous supplies such as flares, first aid supplies, and traffic cones (50 lb). What is the total weight of the additional equipment listed?

The phrase “What is the *total* weight . . .” indicates that addition is required to solve the problem.

**Estimate:**  $50 + 10 + 30 + 30 + 60 + 270 + 50 = 500$

**Calculate:** To avoid confusion, place the only 3-digit number, 270, on top.

$$\begin{array}{r} 270 \\ 54 \\ 12 \\ 32 \\ 30 \\ 56 \\ + 50 \\ \hline 504 \end{array}$$

The new SUVs will need to carry 504 lb of additional equipment. Notice that the estimate is very close to the final answer. ●

#### Your Turn

**Trades Management** During the 3-year construction of the Wilshire Grand Center in Los Angeles, a large number of workers from 9 different trades were needed. First estimates were as follows: 2421 ironworkers, 1853 carpenters, 1466 concrete contractors, 1198 electricians, 908 mechanical contractors, 747 stone and tile contractors, 587 elevator contractors, 462 plumbers, and 121 glaziers. What was the total number of estimated trades workers needed for the project? (Source: Turner Construction.)

#### Solution

Again, the phrase “What was the total number . . .” indicates that addition is needed to solve the problem.

**Estimate:**  $2400 + 1900 + 1500 + 1200 + 900 + 700 + 600 + 500 + 100 = 9800$



Calculate:

444

2421

1853

1466

1198

908

747

587

462

+ 121

9763

A total of 9763 workers from these 9 trades were needed for the project. Notice that the original estimate is very close to the final answer.

Units of Measure

Units of measure are important in every trade. Just as you want to make sure that a customer is not thinking in centimeters while you are thinking yards, you will want to make sure that the measurements required for a project have consistent units.

An in-depth study of measurement units is presented in Chapter 5, and we require only a few basic conversions between units until then. However, because it is nearly impossible to discuss numbers without discussing their units, many of the practical word problems in Chapters 1 through 4 contain units of measure. Therefore, we are providing you with the following table listing the units of measure used in Chapters 1 through 4, along with their most common abbreviations.

Type of Measurement	U.S. Customary Units	Metric Units
Length or distance	inch (in.* or ") foot (ft or ') yard (yd) mile (mi)	millimeter (mm) centimeter (cm) meter (m) kilometer (km)
Weight	ounce (oz) pound (lb) ton (t)	microgram (µg) milligram (mg) gram (g) kilogram (kg)
Area	square inch (sq in.) square foot (sq ft) square yard (sq yd) acre (a)	square centimeter (sq cm) square meter (sq m) square kilometer (sq km) hectare (ha)
Capacity or volume	fluid ounces (fl oz) pint (pt) quart (qt) gallon (gal) bushel (bu) cubic inch (cu in.) cubic foot (cu ft) cubic yard (cu yd)	cubic centimeter (cu cm, cc) milliliter (mL) liter (L) cubic meter (cu m)

(continued)

Type of Measurement	U.S. Customary Units	Metric Units
Velocity or speed	miles per hour (mph or mi/hr) beats per minute (bpm) cycles per second (hertz) revolutions per minute (rpm or rev/min)	meters per second (m/sec) kilometers per hour (km/hr)
Temperature	degrees Fahrenheit (°F)	degrees Celsius (°C)
Power, energy, and heat	ohm ( $\Omega$ ) watt (W) volt (V) ampere (A) horsepower (hp) British thermal unit (Btu)	cubic foot per meter (cfm) kilohertz (kHz) picofarad (pF) kilowatt (kW)
Pressure	pounds per square inch (psi or lb/in. <sup>2</sup> )	pascal (Pa)
Amount of lumber	board feet (bf or fbm)	

\*For abbreviations that might be mistaken for a word (e.g., “in” for inches), a period is included at the end of the abbreviation. For abbreviations that would not be mistaken for a word (e.g., “ft”), no period is added.

Now, try the following problems for practice in working with whole numbers.

### Exercises 1-1 Reading, Writing, Rounding, and Adding Whole Numbers

#### A. Reading, Writing, and Rounding Whole Numbers

Write in words.

1. 357      2. 2304      3. 17,092      4. 207,630      5. 2,000,034
6. 10,007      7. 740,106      8. 5,055,550      9. 118,180,018
10. 6,709,210,046

Write as numbers.

11. Three thousand, six      12. Seventeen thousand, twenty-four
13. Eleven thousand, one hundred      14. Three million, two thousand, seventeen
15. Four million, forty thousand, six      16. Five billion, seven hundred twenty million, ten

Round as indicated.

17. 357 to the nearest ten      18. 4386 to the nearest hundred
19. 4386 to the nearest thousand      20. 15,472 to the nearest thousand
21. 225,799 to the nearest ten thousand      22. 32,408,792 to the nearest hundred thousand

#### B. Add.

1.  $\begin{array}{r} 47 \\ 22 \\ \hline \end{array}$       2.  $\begin{array}{r} 27 \\ 31 \\ \hline \end{array}$       3.  $\begin{array}{r} 45 \\ 35 \\ \hline \end{array}$       4.  $\begin{array}{r} 38 \\ 55 \\ \hline \end{array}$       5.  $\begin{array}{r} 75 \\ 48 \\ \hline \end{array}$
6.  $\begin{array}{r} 26 \\ 98 \\ \hline \end{array}$       7.  $\begin{array}{r} 48 \\ 84 \\ \hline \end{array}$       8.  $\begin{array}{r} 67 \\ 69 \\ \hline \end{array}$       9.  $\begin{array}{r} 747 \\ 59 \\ \hline \end{array}$       10.  $\begin{array}{r} 508 \\ 95 \\ \hline \end{array}$

11. $\begin{array}{r} 684 \\ 706 \end{array}$	12. $\begin{array}{r} 432 \\ 399 \end{array}$	13. $\begin{array}{r} 621 \\ 388 \end{array}$	14. $\begin{array}{r} 189 \\ 204 \end{array}$	15. $\begin{array}{r} 375 \\ 486 \end{array}$
16. $\begin{array}{r} 4237 \\ 288 \end{array}$	17. $\begin{array}{r} 5076 \\ 385 \end{array}$	18. $\begin{array}{r} 7907 \\ 1395 \end{array}$	19. $\begin{array}{r} 3785 \\ 7643 \end{array}$	20. $\begin{array}{r} 6709 \\ 9006 \end{array}$
21. $\begin{array}{r} 10674 \\ 397 \end{array}$	22. $\begin{array}{r} 40026 \\ 7085 \end{array}$	23. $\begin{array}{r} 18745 \\ 6972 \end{array}$	24. $\begin{array}{r} 19876 \\ 4835 \end{array}$	25. $\begin{array}{r} 78044 \\ 97684 \end{array}$
26. $\begin{array}{r} 4728 \\ 683 \\ 29 \end{array}$	27. $\begin{array}{r} 5818 \\ 244 \\ 33 \end{array}$	28. $\begin{array}{r} 83754 \\ 66283 \\ 5984 \end{array}$	29. $\begin{array}{r} 498321 \\ 65466 \\ 95873 \\ 3604 \end{array}$	30. $\begin{array}{r} 843592 \\ 710662 \\ 497381 \\ 25738 \end{array}$

C. Arrange vertically and find the sum.

- $487 + 29 + 526 = \underline{\hspace{2cm}}$
- $65 + 796 + 87 = \underline{\hspace{2cm}}$
- $322 + 46 + 5984 = \underline{\hspace{2cm}}$
- $7268 + 209 + 178 = \underline{\hspace{2cm}}$
- $5016 + 423 + 1075 = \underline{\hspace{2cm}}$
- $715 + 4293 + 184 + 19 = \underline{\hspace{2cm}}$
- $1706 + 387 + 42 + 307 = \underline{\hspace{2cm}}$
- $456 + 978 + 1423 + 3584 = \underline{\hspace{2cm}}$
- $6284 + 28 + 674 + 97 = \underline{\hspace{2cm}}$
- $6842 + 9008 + 57 + 368 = \underline{\hspace{2cm}}$
- $268 + 1593 + 88 + 2165 = \underline{\hspace{2cm}}$
- $8764 + 85 + 983 + 19 = \underline{\hspace{2cm}}$
- $4 + 6 + 11 + 7 + 14 + 3 + 9 + 6 + 4 = \underline{\hspace{2cm}}$
- $12 + 7 + 15 + 16 + 21 + 8 + 10 + 5 + 30 + 17 = \underline{\hspace{2cm}}$
- $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = \underline{\hspace{2cm}}$
- $22 + 31 + 43 + 11 + 9 + 1 + 19 + 12 = \underline{\hspace{2cm}}$
- $75 + 4 + 81 + 12 + 14 + 65 + 47 + 22 + 37 = \underline{\hspace{2cm}}$
- $89,652 + 57,388 + 6506 = \underline{\hspace{2cm}}$
- $443,700 + 629,735 + 85,962 + 6643 = \underline{\hspace{2cm}}$
- $784,396 + 858,390 + 662,043 + 965,831 + 62,654 = \underline{\hspace{2cm}}$

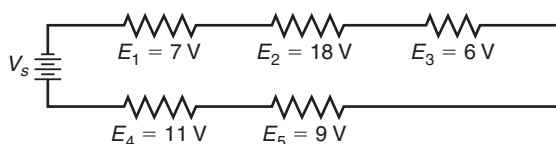
D. Practical Applications.

- Electrical Trades** In setting up his latest wiring job, an electrician cut the following lengths of wire: 387, 913, 76, 2640, and 845 ft. Find the total length of wire used.
- Flooring and Carpeting** The Acme Lumber Co. made four deliveries of 1-in. by 6-in. flooring: 3280, 2650, 2465, and 2970 fbm. What was the total number of board feet of flooring delivered?
- Machine Trades** The stockroom has eight boxes of No. 10 hexhead cap screws. How many screws of this type are in stock if the boxes contain 346, 275, 84, 128, 325, 98, 260, and 120 screws, respectively?

4. **Trades Management** In calculating her weekly expenses, a contractor found that she had spent the following amounts: materials, \$53,860; labor, \$7854; salaried help, \$1542; overhead expense, \$1832. What was her total expense for the week?
5. **Trades Management** The head machinist at Tiger Tool Co. is responsible for totaling time cards to determine job costs. She found that five different jobs this week took 78, 428, 143, 96, and 384 minutes each. What was the total time in minutes for the five jobs?
6. **Roofing** On a home construction job, a roofer laid 1480 wood shingles the first day, 1240 the second, 1560 the third, 1320 the fourth, and 1070 the fifth day. How many shingles did he lay in five days?
7. **Industrial Technology** Eight individually powered machines in a small production shop have motors using 420, 260, 875, 340, 558, 564, 280, and 310 watts each. What is the total wattage used when (a) the total shop is in operation? (b) the three largest motors are running? (c) the three smallest motors are running?
8. **Automotive Trades** A mechanic is taking inventory of oil in stock. He has 24 quarts of 10W-30, 8 quarts of 30W, 42 quarts of 20W-50, 16 quarts of 10W-40, and 21 quarts of 20W-40. How many total quarts of oil does he have in stock?
9. **Construction** The Happy Helper building materials supplier has four piles of bricks containing 1250, 865, 742, and 257 bricks. What is the total number of bricks it has on hand?
10. **Machine Trades** A machinist needs the following lengths of 1-in. diameter rod: 8 in., 14 in., 6 in., 27 in., and 42 in. How long a rod is required to supply all five pieces? (Ignore cutting waste.)
11. **Landscaping** A new landscape maintenance business requires the following equipment:  
 1 rototiller for \$599  
 1 gas trimmer for \$309  
 1 mower for \$369  
 1 hedge trimmer for \$280  
 What is the total cost of this equipment?
12. **Electrical Trades** The Radius Electronics Company orders 325 resistors, 162 capacitors, 25 integrated circuit boards, and 68 transistors. Calculate the total number of parts ordered.
13. **Electronics** When resistors are connected in series, the total resistance is the sum of the individual resistors. If resistances of 520, 1160, 49, and 1200 ohms are connected in series, calculate the total resistance.
14. **Electronics** Kirchhoff's law states that the sum of the voltage drops around a closed circuit is equal to the source voltage; that is,

$$V_s = E_1 + E_2 + E_3 + E_4 + E_5.$$

Calculate the source voltage  $V_s$  for the circuit shown.



15. **Automotive Trades** To balance an engine, a mechanic must know the total weight of a piston assembly, also known as the reciprocating weight of the assembly. A certain piston assembly consisted of a 485-gram piston, a 74-gram wrist-pin, 51 grams of compression and oil rings, and the small end of the connecting rod weighing 146 grams. What is the reciprocating weight of this assembly?
16. **Allied Health** The following standard amounts of food were recorded in one patient's Intake/Output record: one carton of milk, 8 fl oz (fluid ounces); one carton of juice, 8 fl oz; one bowl of soup, 15 fl oz; one cup of coffee, 6 fl oz; one serving of sherbet, 12 fl oz; one glass of water, 8 fl oz. Determine the total fluid intake for this patient.
17. **Construction** For working on remote construction sites, GVM Construction needs a generator that can supply a 1400-W (watt) circular saw, an 1800-W table saw, a 600-W hand drill, and a 100-W radio simultaneously. What total wattage does the generator need to supply?
18. **Automotive Trades** The base price for a 2016 BMW 320i, including destination charge, is \$34,145. The following is a list of optional packages and extras:

Item	Cost	Item	Cost
Track Handling Package	\$2300	Driver Assistance Plus	\$ 950
Premium Package	\$3100	Navigation System	\$1950
Sport Package	\$1300	Lighting Package	\$ 700
USB/Smartphone Integration	\$ 350	Cold Weather Package	\$ 800
Fold-Down Rear Seat	\$ 475	Interior Color Upgrade	\$1450

What would be the total pretax cost of the car if a customer added the following features:

- (a) Track Handling Package, Navigation System, Cold Weather Package, and Fold-Down Rear Seat?
- (b) Driver Assistance Plus, Sport Package, USB/Smartphone Integration, and Interior Color Upgrade?
- (c) Premium Package, Lighting Package, Fold-Down Rear Seat, USB/Smartphone Integration, and Interior Color Upgrade?
19. **Sports and Leisure** In the 1988 Olympics in Seoul, South Korea, Jackie Joyner-Kersey (USA) won the gold medal in the heptathlon with a world-record performance. The heptathlon consists of seven separate events performed over two days, and the points for each event are added to determine the total score. Jackie's points (and performances) for each event were as follows:

Day	Event	Performance	Points Earned
Day 1	100-meter hurdles	12.69 sec	1172
	High jump	1.86 m	1054
	Shot put	15.80 m	915
Day 2	200-meter dash	22.56 sec	1123
	Long jump	7.27 m	1264
	Javelin	45.66 m	776
	800-meter run	2 min 8.51 sec	987

- (a) How many points did she earn on Day 1?
- (b) How many points did she earn on Day 2?
- (c) What was Jackie's world-record-setting point total?

#### E. Calculator Problems

You probably own a calculator and, of course, you are eager to put it to work doing practical math calculations. In this text we include problem sets for calculator users. These problems are taken from real-life situations and, unlike most textbook problems, involve big numbers and lots of calculations. If you think that having an electronic brain-in-a-box means that you do not need to know basic arithmetic, you will be disappointed. The calculator helps you to work faster, but it will not tell you *what* to do or *how* to do it.

Detailed instructions on using a calculator with whole numbers appears on page 68.

Here are a few helpful hints for calculator users:



1. Always *estimate* your answer before doing a calculation.
2. *Check* your answer by comparing it with the estimate or by the other methods shown in this text. Be certain that your answer makes sense.
3. If you doubt the calculator (they do break down, you know), put a problem in it whose answer you know, preferably a problem like the one you are solving.
1. **Electronics** An electronics mixing circuit adds two given input frequencies to produce an output signal. If the input frequencies are 35,244 kHz and 61,757 kHz, calculate the frequency of the output signal.
2. **Manufacturing** The following table lists the number of widget fasteners made by each of the five machines at the Ace Widget Co. during the last ten working days.

Day	Machine					Daily Totals
	A	B	C	D	E	
1	347	402	406	527	237	
2	451	483	312	563	316	
3	406	511	171	581	289	
4	378	413	0	512	291	
5	399	395	452	604	342	
6	421	367	322	535	308	
7	467	409	256	578	264	
8	512	514	117	588	257	
9	302	478	37	581	269	
10	391	490	112	596	310	
Machine Totals						

- (a) Complete the table by finding the number of fasteners produced each day. Enter these totals under the column "Daily Totals" on the right.
- (b) Find the number of fasteners produced by each machine during the ten-day period and enter these totals along the bottom row marked "Machine Totals."
- (c) Does the sum of the daily totals equal the sum of the machine totals?

3. Add the following as shown.

$$\begin{array}{r} (a) \ \$137427 \\ 67429 \\ 91006 \\ 6070 \\ 4894 \\ 399 \\ \hline \end{array}$$

$$\begin{array}{r} (b) \ \$216847 \\ 386738 \\ 86492 \\ 28104 \\ 9757 \\ 4875 \\ \hline \end{array}$$

$$\begin{array}{r} (c) \ \$693884 \\ 675489 \\ 560487 \\ 276921 \\ 44682 \\ 47039 \\ \hline \end{array}$$

$$(d) \ \$4299 + \$137 + \$20 + \$177 + \$63 + \$781 + \$1008 + \$671 = ?$$

4. **Trades Management** Joe's Air Conditioning Installation Co. has not been successful, and he is wondering if he should sell it and move to a better location. During the first three months of the year, his expenses were as follows:

Rent \$4260

Supplies \$2540

Part-time helper \$2100

Transportation \$948

Utilities \$815

Advertising \$750

Miscellaneous \$187

His monthly income was:

January \$1760

February \$2650

March \$3325

(a) What was his total expense for the three-month period?

(b) What was his total income for the three-month period?

(c) Now turn your calculator around to learn what Joe should do about this unhappy situation.

5. **Electrical Trades** A mapper is a person employed by an electrical utility company who has the job of reading diagrams of utility installations and listing the materials to be installed or removed by engineers. Part of a typical job list might look like this:

**INSTALLATION** (in feet of conductor)

Location Code	No. 12 BHD (bare, hard-drawn copper wire)	#TX (triplex)	410 AAC (all- aluminum conductor)	110 ACSR (aluminum- core steel- reinforced conductor)	6B (No. 6, bare conductor)
A3	1740	40	1400		350
A4	1132		5090		2190
B1	500			3794	
B5		87	3995		1400
B6	4132	96	845		
C4		35		3258	2780
C5	3949		1385	1740	705

(a) How many total feet of each kind of conductor must the installer have to complete the job?

(b) How many feet of conductor are to be installed at each of the seven locations?

When you have completed these exercises, check your answers to the odd-numbered problems in the Appendix and then continue with Section 1-2.

## 1-2

## Subtraction of Whole Numbers

## learning|catalytics™

Do the following without using a calculator:

1. Add:  $2649 + 857$
2. Subtract:  $287 - 52$

Subtraction is the reverse of addition.

Addition:  $3 + 4 = \square$

Subtraction:  $3 + \square = 7$

Written this way, a subtraction problem asks the question: How much must be added to a given number to produce a required amount?

Most often, however, the numbers in a subtraction problem are written using a minus sign ( $-$ ).

To fill in the box in  $3 + \square = 7$ , we must perform the subtraction  $7 - 3 = \square$ . If we remember that  $3 + 4 = 7$ , then we will know that  $7 - 3 = 4$ .



## Note

The **difference** is the name given to the answer in a subtraction problem. ●

Solving simple subtraction problems depends on your knowledge of the addition of one-digit numbers.

For example, to solve the problem

$$9 - 4 = \underline{\quad}$$

you probably go through a chain of thoughts something like this:

Nine minus four. Four added to what number gives nine? Five? Try it: four plus five equals nine. Right.

Subtraction problems with small whole numbers will be easy for you if you know your addition tables. See how quickly and accurately you can mentally compute the following subtraction facts.

## PROBLEMS Subtraction Facts

5	9	15	10	7	12	4	11	13	9	12
<u>-3</u>	<u>-1</u>	<u>-8</u>	<u>-5</u>	<u>-6</u>	<u>-9</u>	<u>-2</u>	<u>-6</u>	<u>-9</u>	<u>-2</u>	<u>-3</u>
18	10	14	11	9	16	8	6	11	10	7
<u>-9</u>	<u>-3</u>	<u>-7</u>	<u>-4</u>	<u>-7</u>	<u>-8</u>	<u>-6</u>	<u>-1</u>	<u>-7</u>	<u>-1</u>	<u>-4</u>
11	9	12	10	15	4	8	13	11	6	10
<u>-3</u>	<u>-4</u>	<u>-6</u>	<u>-2</u>	<u>-9</u>	<u>-1</u>	<u>-5</u>	<u>-6</u>	<u>-2</u>	<u>-3</u>	<u>-7</u>
14	11	8	17	9	12	7	6	8	5	3
<u>-8</u>	<u>-9</u>	<u>-7</u>	<u>-9</u>	<u>-6</u>	<u>-7</u>	<u>-1</u>	<u>-4</u>	<u>-1</u>	<u>-1</u>	<u>-2</u>
2	15	8	11	6	16	10	13	13	8	17
<u>-1</u>	<u>-7</u>	<u>-4</u>	<u>-5</u>	<u>-2</u>	<u>-7</u>	<u>-6</u>	<u>-4</u>	<u>-7</u>	<u>-3</u>	<u>-8</u>
8	12	9	15	4	14	7	13	14	5	12
<u>-2</u>	<u>-5</u>	<u>-3</u>	<u>-6</u>	<u>-3</u>	<u>-6</u>	<u>-2</u>	<u>-8</u>	<u>-9</u>	<u>-2</u>	<u>-5</u>
14	3	13	11	12	10	9	5	6	12	7
<u>-5</u>	<u>-1</u>	<u>-5</u>	<u>-8</u>	<u>-4</u>	<u>-8</u>	<u>-5</u>	<u>-4</u>	<u>-5</u>	<u>-8</u>	<u>-3</u>

Mastery time: 3 minutes

The answers are given in the Appendix.



**EXAMPLE 1** Here is a more difficult subtraction problem:

$$47 - 23 = \underline{\hspace{2cm}}$$

The **first** step is to estimate the answer—remember?

$$47 - 23 \quad \text{is roughly} \quad 50 - 20 \quad \text{or} \quad 30.$$

The difference, your answer, will be about 30—not 3 or 10 or 300.

The **second** step is to write the numbers vertically, as you did with addition. Be careful to keep the ones digits in line in one column, the tens digits in a second column, and so on.

$$\begin{array}{r} 47 \\ -23 \\ \hline \end{array}$$

Notice that the larger number is written above the smaller number.

Once the numbers have been arranged in this way, the difference may be written by performing the following two steps:

**Step 1**

$$\begin{array}{r} 47 \\ -23 \\ \hline 4 \end{array} \quad \text{ones digits: } 7 - 3 = 4$$

**Step 2**

$$\begin{array}{r} 47 \\ -23 \\ \hline 24 \end{array} \quad \text{tens digits: } 4 - 2 = 2$$



The difference is 24, which agrees roughly with our estimate.

**EXAMPLE 2** With some problems, it is necessary to rewrite the larger number before the problem can be solved. Let's try this one:

$$64 - 37 = \underline{\hspace{2cm}}$$

**First**, estimate the answer. Rounding to the nearest ten,  $64 - 37$  is roughly  $60 - 40$  or 20.

**Second**, arrange the numbers vertically in columns.

$$\begin{array}{r} 64 \\ -37 \\ \hline \end{array}$$

Because 7 is larger than 4, we must “borrow” one ten from the 6 tens in 64. We are actually rewriting 64 (6 tens + 4 ones) as 5 tens + 14 ones. In actual practice, our work would look like this:

**Step 1**

$$\begin{array}{r} 64 \\ -37 \\ \hline \end{array}$$

**Step 2**

$$\begin{array}{r} 5 \quad 14 \\ \cancel{6} \quad \cancel{4} \\ -3 \quad 7 \\ \hline \end{array}$$

Borrow one ten, change the 6 in the tens place to 5, change 4 to 14, subtract  $14 - 7 = 7$ .

**Step 3**

$$\begin{array}{r} 5 \quad 14 \\ \cancel{6} \quad \cancel{4} \\ -3 \quad 7 \\ \hline 2 \quad 7 \end{array}$$

$5 - 3 = 2$        $14 - 7 = 7$



Double-check subtraction problems by adding the answer (the *difference*) and the smaller number (the *subtrahend*); their sum should equal the larger number.

**Step 4 Check:**

$$\begin{array}{r} 37 \\ +27 \\ \hline 64 \end{array}$$

**Learning Help**

If you need to get an exact answer to a problem such as  $64 - 37$  mentally, add or subtract to make the smaller number, 37, a “round” number. In this case, add 3 to make it 40. Because we’re subtracting, we want the *difference*, not the *total*, to be the same or balance. Therefore, we also add 3 to the 64 to get 67. The problem becomes  $64 - 37 = (64 + 3) - (37 + 3) = 67 - 40$ . Subtracting a round number is easy mentally:  $67 - 40 = 27$ . ●

When a subtraction problem involves three-digit numbers, the procedure is exactly the same. There is simply one additional step.

**EXAMPLE 3**

To subtract  $426 - 128$ , follow these steps:

**Estimate:**  $400 - 100 = 300$

**Step 1**

$$\begin{array}{r} 426 \\ -128 \\ \hline \end{array}$$

**Step 2**

$$\begin{array}{r} 1 \quad 16 \\ 4 \quad 2 \quad 6 \\ -1 \quad 2 \quad 8 \\ \hline 8 \end{array}$$

**Steps 3 and 4**

$$\begin{array}{r} 3 \quad 11 \quad 16 \\ 4 \quad 2 \quad 6 \\ -1 \quad 2 \quad 8 \\ \hline 2 \quad 9 \quad 8 \end{array}$$

In this case we borrow twice. Borrow one ten from the 20 in 426 and make 16. Then borrow one hundred from the 400 in 426 to make 11 in the tens place.

$$16 - 8 = 8 \text{ Write 8}$$

$$11 - 2 = 9 \text{ Write 9}$$

$$3 - 1 = 2 \text{ Write 2}$$



The answer 298 is approximately equal to the estimate 300.

**Double-check:**

$$\begin{array}{r} 1 \quad 1 \\ 1 \quad 2 \quad 8 \\ + 2 \quad 9 \quad 8 \\ \hline 4 \quad 2 \quad 6 \end{array}$$

**Your Turn**

Subtract as indicated:

(a)  $\begin{array}{r} 59 \\ -24 \\ \hline \end{array}$  (b)  $\begin{array}{r} 71 \\ -39 \\ \hline \end{array}$  (c)  $\begin{array}{r} 687 \\ -194 \\ \hline \end{array}$  (d)  $\begin{array}{r} 902 \\ -65 \\ \hline \end{array}$

**Solutions**

(a) **Estimate:**  $60 - 20 = 40$

**Step 1**

$$\begin{array}{r} 5 \quad 9 \\ -2 \quad 4 \\ \hline 5 \end{array}$$

**Step 2**

$$\begin{array}{r} 5 \quad 9 \\ -2 \quad 4 \\ \hline 3 \quad 5 \end{array}$$



The difference is 35, which roughly agrees with our estimate.

(b) **Estimate:**  $70 - 40 = 30$

**Step 1**

$$\begin{array}{r} 6 \quad 11 \\ 7 \quad 1 \\ -3 \quad 9 \\ \hline \end{array}$$

Borrow one ten from 70, change the 7 in the tens place to 6, change the 1 in the ones place to 11.

**Steps 2 and 3**

$$\begin{array}{r} 6 \quad 11 \\ 7 \quad 1 \\ -3 \quad 9 \\ \hline 3 \quad 2 \end{array}$$

$$11 - 9 = 2 \text{ Write 2}$$

$$6 - 3 = 3 \text{ Write 3}$$



The answer 32 is approximately equal to the estimate 30. As a shortcut, mentally add 1 to each number,

$$71 - 39 = 72 - 40$$

then subtract.

$$72 - 40 = 32$$

(c) **Estimate:**  $700 - 200 = 500$

**Step 1**

$$\begin{array}{r} 687 \\ -194 \\ \hline 3 \end{array}$$

**Step 2**

$$\begin{array}{r} \overset{5}{6} \overset{18}{8} 7 \\ -194 \\ \hline 93 \end{array}$$

**Step 3**

$$\begin{array}{r} \overset{5}{6} \overset{18}{8} 7 \\ -194 \\ \hline 493 \end{array}$$



The answer is very close to the original estimate.

(d) **Estimate:**  $900 - 50 = 850$

**Step 1**

$$\begin{array}{r} 902 \\ -65 \\ \hline \end{array}$$

**Step 2**

$$\begin{array}{r} \overset{8}{9} \overset{10}{0} 2 \\ -65 \\ \hline \end{array}$$

**Step 3**

$$\begin{array}{r} \overset{8}{9} \overset{9}{0} \overset{12}{2} \\ -65 \\ \hline 837 \end{array}$$

12 - 5 = 7 Write 7

9 - 6 = 3 Write 3

8 - 0 = 8 Write 8



The answer 837 is roughly equal to the estimate 850.

In problem (d) we first borrow one hundred from 900 to get a 10 in the tens place. Then we borrow one 10 from the tens place to get a 12 in the ones place.

### Problems with Zero Digits

Let's work through a few examples of subtraction problems involving zero digits. These can be troublesome for some students.

#### EXAMPLE 4

(a)  $400 - 167 = ?$

**Step 1**

$$\begin{array}{r} 400 \\ -167 \\ \hline \end{array}$$

**Step 2**

$$\begin{array}{r} \overset{3}{4} \overset{10}{0} 0 \\ -167 \\ \hline \end{array}$$

**Step 3**

$$\begin{array}{r} \overset{3}{4} \overset{9}{0} \overset{10}{0} \\ -167 \\ \hline 233 \end{array}$$

**Check**

$$\begin{array}{r} 167 \\ +233 \\ \hline 400 \end{array}$$

Do you see in Steps 2 and 3 that we have rewritten 400 as  $300 + 90 + 10$ ?

(b)  $5006 - 2487 = ?$

**Step 1**

$$\begin{array}{r} 5006 \\ -2487 \\ \hline \end{array}$$

**Step 2**

$$\begin{array}{r} \overset{4}{5} \overset{10}{0} 0 6 \\ -2487 \\ \hline \end{array}$$

**Step 3**

$$\begin{array}{r} \overset{4}{5} \overset{9}{0} \overset{10}{0} 6 \\ -2487 \\ \hline \end{array}$$

**Step 4**

$$\begin{array}{r} \overset{4}{5} \overset{9}{0} \overset{9}{0} \overset{16}{6} \\ -2487 \\ \hline 2519 \end{array}$$

**Check**

$$\begin{array}{r} 2487 \\ +2519 \\ \hline 5006 \end{array}$$

## More Practice

Subtract.

$$\begin{array}{r} \text{(a) } 500 \\ - 234 \\ \hline \end{array} \quad \begin{array}{r} \text{(b) } 7008 \\ - 3605 \\ \hline \end{array} \quad \begin{array}{r} \text{(c) } 30206 \\ - 4738 \\ \hline \end{array}$$

## Answers

$$\begin{array}{l} \text{(a) } 266 \\ \text{(b) } 3403 \\ \text{(c) } 25,468 \end{array}$$

## Practical Applications

When solving word problems, you must look for key words or phrases to help you decide what operation to use. Here are some typical questions that call for subtraction:

“What is the *difference* between . . . ?”

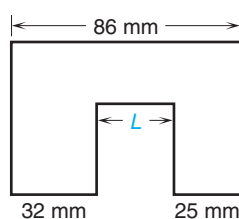
“How much *remains* . . . ?” or “How much *is left* . . . ?”

“By how much is A *greater than* B?” or “What was the *increase* from A to B?”

“How much do you *save*?”

As shown in the next example, finding a missing dimension in a drawing might also call for subtraction.

## EXAMPLE 5



**Machine Trades** Suppose we need to find the missing dimension  $L$  in the drawing in the margin. Along the top of the drawing, we have the total horizontal dimension of 86 mm. We can also see that the three lower horizontal dimensions must add up to this total. To find the missing dimension  $L$ , we must subtract the other two given dimensions from the total. Therefore, the missing dimension is

$$L = 86 \text{ mm} - 32 \text{ mm} - 25 \text{ mm}$$

**Estimate:**  $90 - 30 - 30 = 30$

## Step 1

$$\begin{array}{r} 86 \\ - 32 \\ \hline 54 \end{array}$$

## Step 2

$$\begin{array}{r} 54 \\ - 25 \\ \hline 29 \end{array}$$

The missing dimension  $L$  is 29 mm. This is very close to our estimate of 30 mm. ●

## Your Turn

**Welding** Three pieces measuring 14 in., 22 in., and 26 in. are cut from a steel bar 72 in. long. Allowing for a total of 1 in. for waste in cutting, what is the length of the piece remaining?

## Solution

The key words that indicate subtraction are *cut*, *waste*, and *remaining*. To find how much remains, we must subtract the amounts cut and the amount wasted from the original length. The remaining length in inches will be

$$72 - 14 - 22 - 26 - 1$$

But instead of performing four subtraction problems, it might be easier to add all the pieces cut and wasted, and then subtract this total from the original length.

## Step 1

$$\begin{array}{r} 14 \\ 22 \\ 26 \\ + 1 \\ \hline 63 \text{ in. cut and wasted} \end{array}$$

## Step 2

$$\begin{array}{r} 72 \leftarrow \text{Original length} \\ - 63 \\ \hline 9 \leftarrow \text{The remaining length is 9 inches.} \end{array}$$

Practical problems are not always simple and straightforward. They will often involve several steps and more than one operation, as shown in the next example. ●

**EXAMPLE 6**

**Life Skills** When consumers purchase new automobiles these days, they are often faced with the decision of whether to buy the hybrid version of a particular car. The hybrid version usually costs more but saves money over time because of lower fuel costs.

Suppose that a potential car buyer is deciding between a Toyota Camry and a Camry Hybrid. He estimates that he will keep the car at least 5 years and then sell it. He wants to know if the hybrid model will save him money and, if so, how much. The dealer provides him with the following information:

	Toyota Camry LE	Camry Hybrid LE
Sales Price (MSRP)	\$23,935	\$27,655
5-Year Fuel Cost*	\$ 5550	\$ 3570
Insurance and Fees	\$ 8200	\$ 8350
Maintenance and Repairs	\$ 4700	\$ 4400
5-Year Resale Value	\$ 8100	\$ 8000

\* Note: These costs are based on 12,000 annual miles of driving, mostly in the city, and gas costing \$2.50 per gallon.  
(Source: KBB.com)

We can use these figures to get a rough estimate of the total **5-year cost** of each model.

**First**, find the sum of the expenditures: The sales price, fuel cost, insurance and fees, and maintenance and repairs.

Toyota Camry LE	Camry Hybrid LE
\$23935	\$27655
5550	3570
8200	8350
+ 4700	+ 4400
<u>\$42385</u>	<u>\$43975</u>

**Then**, subtract the 5-year resale values from these results.

Toyota Camry LE	Camry Hybrid LE
\$42385	\$43975
– 8100	– 8000
<u>\$34285</u>	<u>\$35975</u>

These figures represent rough estimates of the total 5-year cost of each vehicle. They show that if the buyer keeps the car he chooses for at least 5 years and then sells it, choosing the non-hybrid Camry LE will save him money.

**Finally**, find the difference between these two costs to find the total amount of savings.

$$\begin{array}{r}
 \text{Camry Hybrid LE:} \quad \$35975 \\
 \text{Toyota Camry LE:} \quad - \quad 34285 \\
 \hline
 \$1690
 \end{array}$$

Overall, the non-hybrid Camry LE will save the car buyer \$1690 over 5 years based on initial price, fuel costs, insurance and fees, maintenance and repairs, and resale values. It should be noted that if the buyer decides to keep the car longer, the continued fuel savings of the hybrid might end up making that model a better buy.

Notice that this problem required two different operations and several steps to solve. Both addition and subtraction were used to calculate the 5-year cost of each model, and then subtraction was used to determine how much money was saved. ●

Now check your progress on subtraction in Exercises 1-2.

## Exercises 1-2 Subtraction of Whole Numbers

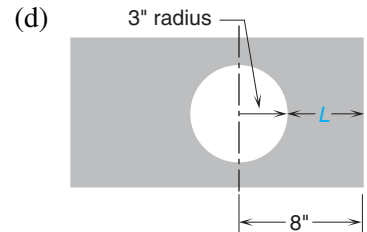
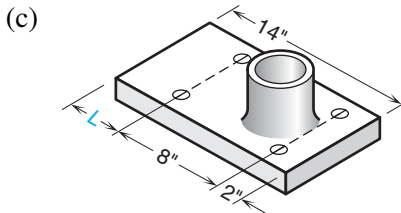
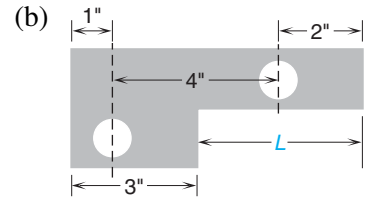
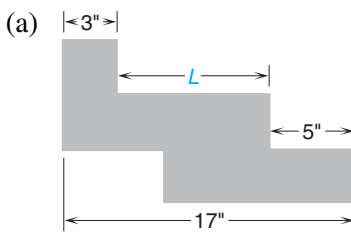
### A. Subtract.

- |   |   |   |  |   |   |
|---|---|---|--|---|---|
| 1. $\begin{array}{r} 42 \\ 7 \\ \hline \end{array}$       | 2. $\begin{array}{r} 25 \\ 8 \\ \hline \end{array}$       | 3. $\begin{array}{r} 34 \\ 9 \\ \hline \end{array}$       | 4. $\begin{array}{r} 76 \\ 7 \\ \hline \end{array}$        | 5. $\begin{array}{r} 64 \\ 31 \\ \hline \end{array}$    | 6. $\begin{array}{r} 68 \\ 10 \\ \hline \end{array}$    |
| 7. $\begin{array}{r} 75 \\ 13 \\ \hline \end{array}$      | 8. $\begin{array}{r} 96 \\ 22 \\ \hline \end{array}$      | 9. $\begin{array}{r} 40 \\ 27 \\ \hline \end{array}$      | 10. $\begin{array}{r} 78 \\ 49 \\ \hline \end{array}$      | 11. $\begin{array}{r} 51 \\ 39 \\ \hline \end{array}$   | 12. $\begin{array}{r} 36 \\ 17 \\ \hline \end{array}$   |
| 13. $\begin{array}{r} 42 \\ 27 \\ \hline \end{array}$     | 14. $\begin{array}{r} 52 \\ 16 \\ \hline \end{array}$     | 15. $\begin{array}{r} 65 \\ 27 \\ \hline \end{array}$     | 16. $\begin{array}{r} 46 \\ 17 \\ \hline \end{array}$      | 17. $\begin{array}{r} 84 \\ 38 \\ \hline \end{array}$   | 18. $\begin{array}{r} 70 \\ 48 \\ \hline \end{array}$   |
| 19. $\begin{array}{r} 34 \\ 9 \\ \hline \end{array}$      | 20. $\begin{array}{r} 56 \\ 18 \\ \hline \end{array}$     | 21. $\begin{array}{r} 546 \\ 357 \\ \hline \end{array}$   | 22. $\begin{array}{r} 409 \\ 324 \\ \hline \end{array}$    | 23. $\begin{array}{r} 476 \\ 195 \\ \hline \end{array}$ | 24. $\begin{array}{r} 330 \\ 76 \\ \hline \end{array}$  |
| 25. $\begin{array}{r} 504 \\ 96 \\ \hline \end{array}$    | 26. $\begin{array}{r} 747 \\ 593 \\ \hline \end{array}$   | 27. $\begin{array}{r} 400 \\ 127 \\ \hline \end{array}$   | 28. $\begin{array}{r} 803 \\ 88 \\ \hline \end{array}$     | 29. $\begin{array}{r} 632 \\ 58 \\ \hline \end{array}$  | 30. $\begin{array}{r} 438 \\ 409 \\ \hline \end{array}$ |
| 31. $\begin{array}{r} 6218 \\ 3409 \\ \hline \end{array}$ | 32. $\begin{array}{r} 6084 \\ 386 \\ \hline \end{array}$  | 33. $\begin{array}{r} 13042 \\ 524 \\ \hline \end{array}$ | 34. $\begin{array}{r} 57022 \\ 980 \\ \hline \end{array}$  |   |   |
| 35. $\begin{array}{r} 5007 \\ 266 \\ \hline \end{array}$  | 36. $\begin{array}{r} 10000 \\ 386 \\ \hline \end{array}$ | 37. $\begin{array}{r} 48093 \\ 500 \\ \hline \end{array}$ | 38. $\begin{array}{r} 27004 \\ 4582 \\ \hline \end{array}$ |   |   |

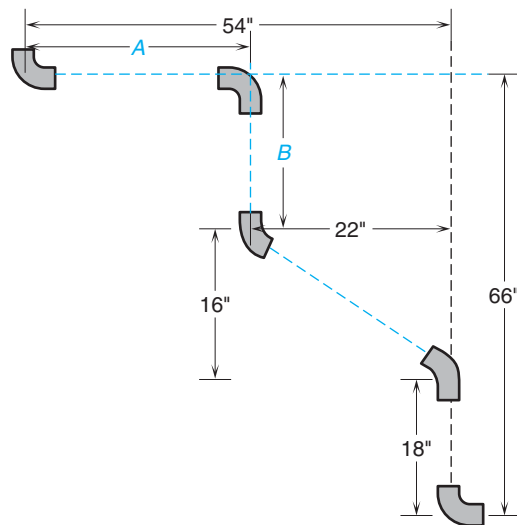
### B. Practical Applications

- Painting** In planning for a particular job, a painter buys \$486 worth of materials. When the job is completed, she returns some unused rollers and brushes for a credit of \$27. What was the net amount of her bill?
- Construction** How many square feet (sq ft) of plywood remain from an original supply of 8000 sq ft after 5647 sq ft is used?
- Welding** A storage rack at the Tiger Tool Company contains 3540 ft of 1-in. stock. On a certain job 1782 ft is used. How much is left?
- Welding** Five pieces measuring 26, 47, 38, 27, and 32 cm are cut from a steel bar that was 200 cm long. Allowing for a total of 1 cm for waste in cutting, what is the length of the piece remaining?
- Trades Management** Taxes on a group of factory buildings owned by the Ace Manufacturing Company amounted to \$875,977 eight years ago. Taxes on the same buildings last year amounted to \$1,206,512. Find the increase in taxes.
- Trades Management** To pay their bills, the owners of Edwards Plumbing Company made the following withdrawals from their bank account: \$72, \$375, \$84, \$617, and \$18. If the original balance was \$5820, what was the amount of the new balance?
- Manufacturing** Which total volume is greater, four drums containing 72, 45, 39, and 86 liters, or three drums containing 97, 115, and 74 liters? By how much is it greater?

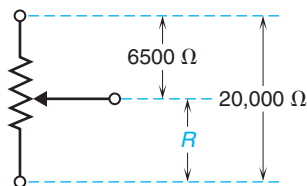
8. **Machine Trades** Determine the missing dimension ( $L$ ) in the following drawings. In the figures, feet are abbreviated with the (') symbol, and inches are abbreviated with the (") symbol.



9. **Automotive Trades** A service department began the day with 238 gallons of coolant. During the day 64 gallons were used. How many gallons remained at the end of the day?
10. **Construction** A truck loaded with rocks weighs 14,260 lb. If the truck weighs 8420 lb, how much do the rocks weigh?
11. **Printing** A press operator has a total of 22,000 impressions to run for a job. If the operator runs 14,250 the first day, how many are left to run?
12. **Plumbing** In the following plumbing diagram, find pipe lengths  $A$  and  $B$ .



Problem 12



Problem 13

13. **Electronics** A *potentiometer* is a device that acts as a variable resistor, allowing a resistance to change from 0 ohms to some maximum value. If a 20,000-ohm potentiometer is set to 6500 ohms ( $\Omega$ ) as shown in the figure, calculate the resistance  $R$ . (Hint:  $R + 6500 = 20,000$ .)
14. **Allied Health** The white blood cell (WBC) count is an important indicator of health. Before surgery, the WBC count of a patient was 9472. After surgery, his WBC count had dropped to 5786. Calculate the difference in WBC count for this patient.

15. **Electronics** An electronic mixer can produce a signal with frequency equal to the difference between two input signals. If the input signals have frequencies of 1,350,000 and 850,000 hertz, calculate the frequency difference.
16. **Automotive Trades** A set of tires was rated at 40,000 miles. A car's odometer read 53,216 when the tires were installed and 91,625 when they needed replacing.
  - (a) How long did they actually last?
  - (b) By how many miles did they miss the advertised rating?
17. **Office Services** A gas meter read 8701 at the beginning of the month and 8823 at the end of the month. Find the difference between these readings to calculate the number of HCF (hundred cubic feet) used.
18. **General Interest** The tallest building in the United States is One World Trade Center in New York City at 1776 feet. The tallest completed building in the world is the Burj Khalifa in Dubai at 2722 feet. How much taller is the Burj Khalifa than One World Trade Center?
19. **Life Skills** A potential new-car buyer is trying to decide between a Hyundai Sonata and a Sonata Hybrid. Use the following figures for the Limited models to calculate her total 5-year cost for each model. Then determine which model will cost her less and by how much. (See Example 6 on page 26.)

	Sonata	Sonata Hybrid
Sales Price (MSRP)	\$28,185	\$30,935
5-Year Fuel Cost	\$ 5444	\$ 3953
Insurance and Fees	\$ 8166	\$ 9101
Maintenance and Repairs	\$ 4716	\$ 4168
5-Year Resale Value	\$ 9419	\$ 9107

(Source: KBB.com)

### C. Calculator Problems

1. **Plumbing** The Karroll Plumbing Co. has 10 trucks and, for the month of April, the following mileage was recorded on each:

Truck No.	Mileage at Start	Mileage at End
1	58352	60027
2	42135	43302
3	76270	78007
4	40006	41322
5	08642	10002
6	35401	35700
7	79002	80101
8	39987	40122
9	10210	11671
10	71040	73121

Find the mileage traveled by each truck during the month of April and the total mileage of all vehicles.



2. Which sum is greater?

987654321		123456789
87654321		123456780
7654321		123456700
654321		123456000
54321	or	123450000
4321		123400000
321		123000000
21		120000000
<u>1</u>		<u>100000000</u>

3. **Trades Management** If an electrician's helper earns \$28,245 per year and she pays \$3814 in withholding taxes, what is her take-home pay?
4. **Trades Management** The revenue of the Smith Construction Company for the year is \$3,837,672, and the total expenses are \$3,420,867. Find the difference, Smith's profit, for that year.
5. **Life Skills** Balance the following checking account record:

Date	Deposits	Withdrawals	Balance
7/1			\$6375
7/3		\$ 379	
7/4	\$1683		
7/7	\$ 474		
7/10	\$ 487		
7/11		\$2373	
7/15		\$1990	
7/18		\$ 308	
7/22		\$1090	
7/26		\$ 814	
8/1			A

- (a) Find the new balance A.
- (b) Keep a running balance by filling each blank in the balance column.
6. **Construction** A water meter installed by the BetterBilt Construction Company at a work site read 9357 cubic feet on June 1 and 17,824 cubic feet on July 1. How much water was used at the site during the month of June?

When you have completed these exercises, check your answers to the odd-numbered problems in the Appendix, then turn to Section 1-3 to study the multiplication of whole numbers.

### 1-3

## Multiplication of Whole Numbers

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1. Subtract:  $572 - 286$   
 2. Multiply:  $6 \times 9$

In a certain football game, the West Newton Waterbugs scored five touchdowns at six points each. How many total points did they score through touchdowns? We can answer the question several ways:

1. Count points, .....

2. Add touchdowns,  $6 + 6 + 6 + 6 + 6 = ?$

or

3. Multiply  $5 \times 6 = ?$

We're not sure about the mathematical ability of the West Newton scorekeeper, but most people would multiply. Multiplication is a shortcut method of performing repeated addition.

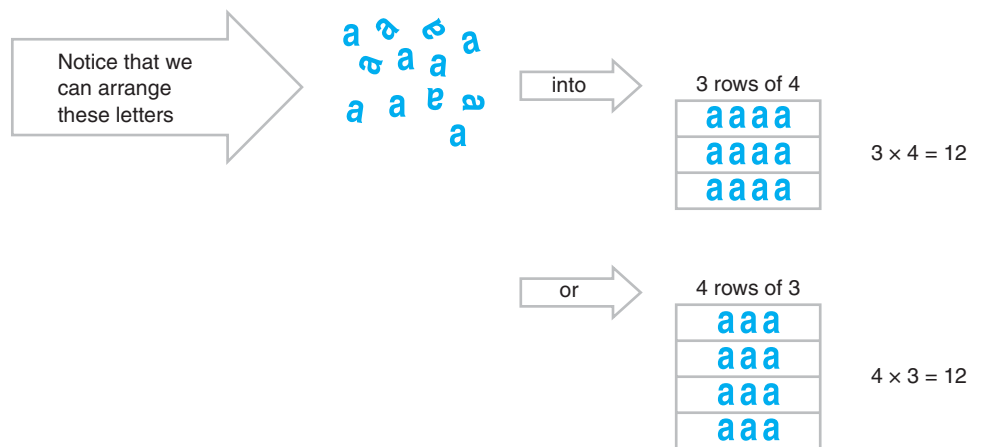
How many points did they score?

$$5 \times 6 = 30$$



**Note**

In a multiplication problem the **product** is the name given to the result of the multiplication. The numbers being multiplied are the **factors** of the product. ●



Changing the order of the factors does not change their product. This is called the **commutative property** of multiplication.

To become skillful at multiplication, you must know the basic one-digit multiplication facts from memory. Even if you use a calculator for your work, you need to know these products to make estimates and to check your work.

## PROBLEMS One-Digit Multiplication

Multiply as shown. Work quickly; you should be able to answer all problems in a set correctly in the time indicated.

A. Multiply.

6	4	9	6	3	9	7	8	2
<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>
6	8	5	5	2	3	9	7	3
<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>
8	2	9	6	5	3	8	5	6
<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>
7	5	4	7	4	8	6	9	8
<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>
4	3	9	7	2	5	7	6	8
<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>
5	3	5	9	9	2	7	4	4
<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>
4	2	5	3	9	2	3	6	4
<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>	<u>  </u>

Mastery time = 90 sec

**B. Multiply.**

2	6	3	5	6	4	8	2	7
<u>8</u>	<u>5</u>	<u>3</u>	<u>7</u>	<u>3</u>	<u>7</u>	<u>6</u>	<u>6</u>	<u>9</u>
8	4	2	3	5	6	9	5	8
<u>4</u>	<u>5</u>	<u>9</u>	<u>8</u>	<u>5</u>	<u>4</u>	<u>5</u>	<u>2</u>	<u>9</u>
3	7	5	6	9	2	7	8	2
<u>5</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>4</u>	<u>4</u>	<u>6</u>	<u>8</u>	<u>2</u>
5	3	2	8	6	4	9	7	3
<u>5</u>	<u>9</u>	<u>3</u>	<u>7</u>	<u>6</u>	<u>3</u>	<u>9</u>	<u>2</u>	<u>7</u>

Mastery time = 90 sec

Check your answers in the Appendix.

If you are not able to perform these one-digit multiplications quickly from memory, you should practice until you can do so. A multiplication table is given in the Appendix on page 883. Use it if you need it.

**Multiplying by Zero and One**

We omitted multiplication by zero and one from the basic facts drill because these products are very simple. The product of any number and zero is zero. For example,

$$0 \times 2 = 0$$

$$0 \times 7 = 0$$

$$395 \times 0 = 0$$

The product of any number and 1 is that same number. For example,

$$1 \times 2 = 2$$

$$6 \times 1 = 6$$

or even

$$1 \times 753 = 753$$

**The Sexy Six**

Here are the six most often missed one-digit multiplications:

Inside  
digits

$$\begin{array}{l} 9 \times 8 = 72 \\ 9 \times 7 = 63 \\ 9 \times 6 = 54 \\ 8 \times 7 = 56 \\ 8 \times 6 = 48 \\ 7 \times 6 = 42 \end{array}$$

It may help you to notice that in these multiplications the “inside” digits, such as 8 and 7, are consecutive and the digits of the answer add to nine:  $7 + 2 = 9$ . This is true for *all* one-digit numbers multiplied by 9.

Be certain that you have these memorized.

(There is nothing very sexy about them, but we did get your attention, didn’t we?)

**Multiplying by Larger Numbers**

The multiplication of larger numbers is based on the one-digit number multiplication facts.

**EXAMPLE 1**

Consider the problem:

$$34 \times 2 = \underline{\hspace{2cm}}$$

**First**, estimate the answer:  $30 \times 2 = 60$ . The actual product of the multiplication will be about 60.

**Second**, arrange the factors to be multiplied vertically, with ones digits in a single column, tens digits in a second column, and so on.

$$\begin{array}{r} 34 \\ \times 2 \\ \hline \end{array}$$

**Finally**, to make the process clear, let's write it in expanded form.

$$\begin{array}{r} 34 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{l} 3 \text{ tens} + 4 \text{ ones} \\ \times 2 \\ \hline 6 \text{ tens} + 8 \text{ ones} = 60 + 8 = 68 \end{array}$$



The estimate 60 is roughly equal to the answer 68.

**EXAMPLE 2**

Here is a slightly more difficult problem. To find the product

$$\begin{array}{r} 28 \\ \times 3 \\ \hline \end{array}$$

**First**, estimate the answer:  $30 \times 3 = 90$  The answer is about 90.

**Then**, proceed as follows:

$$\begin{array}{r} 28 \\ \times 3 \\ \hline \end{array} \quad \begin{array}{l} 2 \text{ tens} + 8 \text{ ones} \\ \times 3 \\ \hline 6 \text{ tens} + 24 \text{ ones} \\ = 6 \text{ tens} + 2 \text{ tens} + 4 \text{ ones} \\ = 8 \text{ tens} + 4 \text{ ones} \\ = 80 + 4 \\ = 84 \end{array}$$



90 is roughly equal to 84.

Of course, we do not normally use the expanded form; instead we simplify the work like this:

$$\begin{array}{r} 28 \\ \times 3 \\ \hline 84 \end{array} \quad \begin{array}{l} 3 \times 8 = 24 \quad \text{Write 4 and carry 2 tens.} \\ 3 \times 2 \text{ tens} = 6 \text{ tens} \quad 6 \text{ tens} + 2 \text{ tens} = 8 \text{ tens} \\ \text{Write 8.} \end{array}$$

**Your Turn**

Now try these problems to be certain you understand the process. Multiply as shown.

$$\begin{array}{lll} \text{(a)} & \begin{array}{r} 43 \\ \times 5 \\ \hline \end{array} & \text{(b)} \quad \begin{array}{r} 29 \\ \times 6 \\ \hline \end{array} & \text{(c)} \quad \begin{array}{r} 258 \\ \times 7 \\ \hline \end{array} \end{array}$$

**Solutions** (a) **Estimate:**  $40 \times 5 = 200$  The answer is roughly 200.

$$\begin{array}{r} \phantom{0}^1 4 \phantom{0} 3 \\ \times \phantom{0} 5 \\ \hline 2 \phantom{0} 1 \phantom{0} 5 \end{array}$$

$5 \times 3 = 15$  Write 5; carry 1 ten.  
 $5 \times 4 \text{ tens} = 20 \text{ tens}$   
 $20 \text{ tens} + 1 \text{ ten} = 21 \text{ tens}$



The answer 215 is roughly equal to the estimate 200.

(b) **Estimate:**  $30 \times 6 = 180$

$$\begin{array}{r} \phantom{0}^5 2 \phantom{0} 9 \\ \times \phantom{0} 6 \\ \hline 1 \phantom{0} 7 \phantom{0} 4 \end{array}$$

$6 \times 9 = 54$  Write 4; carry 5 tens.  
 $6 \times 2 \text{ tens} = 12 \text{ tens}$   
 $12 \text{ tens} + 5 \text{ tens} = 17 \text{ tens}$



The answer 174 is roughly equal to the estimate 180.

(c) **Estimate:**  $300 \times 7 = 2100$

$$\begin{array}{r} \phantom{0}^4 \phantom{0}^5 2 \phantom{0} 5 \phantom{0} 8 \\ \times \phantom{0} 7 \\ \hline 1 \phantom{0} 8 \phantom{0} 0 \phantom{0} 6 \end{array}$$

$7 \times 8 = 56$  Write 6; carry 5 tens.  
 $7 \times 5 \text{ tens} = 35 \text{ tens}$   
 $35 \text{ tens} + 5 \text{ tens} = 40 \text{ tens}$  Write 0; carry 4 hundreds.  
 $7 \times 2 \text{ hundreds} = 14 \text{ hundreds}$   
 $14 \text{ hundreds} + 4 \text{ hundreds} = 18 \text{ hundreds}$



The answer and estimate are roughly equal.



### Learning Help

Multiplying a two- or three-digit number by a one-digit number can be done mentally. For example, think of  $43 \times 5$  as  $40 \times 5$  plus  $3 \times 5$ , or  $200 + 15 = 215$ . For a problem such as  $29 \times 6$ , think  $30 \times 6$  minus  $1 \times 6$  or  $180 - 6 = 174$ . Tricks like this are very useful on the job when neither paper and pencil nor a calculator is at hand. ●

### Two- and Three-Digit Multiplications

Calculations involving two- and three-digit multipliers are done in a similar way.

#### EXAMPLE 3

To multiply

$$\begin{array}{r} 89 \\ \times 24 \\ \hline \end{array}$$

**First**, estimate the answer:  $90 \times 20 = 1800$

**Second**, multiply by the ones digit 4.

$$\begin{array}{r} \phantom{0}^3 8 \phantom{0} 9 \\ \times 4 \\ \hline 3 \phantom{0} 5 \phantom{0} 6 \end{array}$$

$4 \times 9 = 36$  Write 6; carry 3 tens.  
 $4 \times 8 \text{ tens} = 32 \text{ tens}$   
 $32 \text{ tens} + 3 \text{ tens} = 35 \text{ tens}$

**Third**, multiply by the tens digit 2.

$$\begin{array}{r}
 \phantom{0}1 \\
 \phantom{0}3 \\
 89 \\
 \times 24 \\
 \hline
 356 \\
 178 \phantom{0} \\
 \hline
 \end{array}$$

$2 \times 9 = 18$       Write 8; carry 1.  
 $2 \times 8 = 16$   
 $16 + 1 = 17$   
 Leave a blank space here, because we are actually multiplying  $89 \times 20 = 1780$ .

**Fourth**, add the products obtained.

$$\begin{array}{r}
 356 \\
 178 \\
 \hline
 2136
 \end{array}$$



**Finally**, check it. The estimate and the answer are roughly the same, or at least in the same ballpark.

Notice that the product in the third step, 178, is written one digit space over from the product from the second step. When we multiplied  $2 \times 9$  to get 18 in Step 3, we were actually multiplying  $20 \times 9 = 180$ , but the zero in 180 is usually omitted to save time.

### Your Turn

Try these:

(a)  $\begin{array}{r} 64 \\ \times 37 \\ \hline \end{array}$       (b)  $\begin{array}{r} 327 \\ \times 45 \\ \hline \end{array}$

### Solutions

(a) **Estimate:**  $60 \times 40 = 2400$

$$\begin{array}{r}
 64 \\
 \times 37 \\
 \hline
 448 \\
 192 \phantom{0} \\
 \hline
 2368
 \end{array}$$

$7 \times 4 = 28$       Write 8; carry 2.  
 $7 \times 6 = 42$       Add carry 2 to get 44; write 44.  
 $3 \times 4 = 12$       Write 2; carry 1.  
 $3 \times 6 = 18$       Add carry 1 to get 19; write 19.  
 Add to obtain the answer.

(b) **Estimate:**  $300 \times 50 = 15,000$

$$\begin{array}{r}
 327 \\
 \times 45 \\
 \hline
 1635 \\
 1308 \phantom{0} \\
 \hline
 14715
 \end{array}$$

$5 \times 7 = 35$       Write 5; carry 3.  
 $5 \times 2 = 10$       Add carry 3 to get 13; write 3, carry 1.  
 $5 \times 3 = 15$       Add carry 1 to get 16; write 16.  
 $4 \times 7 = 28$       Write 8, carry 2.  
 $4 \times 2 = 8$       Add carry 2 to get 10; write 0, carry 1.  
 $4 \times 3 = 12$       Add carry 1 to get 13; write 13.

The product is 14,715.

Comparing Example 2 to Example 3, notice that adding a second digit to the bottom factor creates a second row in the solution. In the next example, we should see three rows in the solution in addition to the answer row.

### EXAMPLE 4

To multiply

$$\begin{array}{r}
 527 \\
 \times 231 \\
 \hline
 \end{array}$$

**First**, estimate the answer.  $500 \times 200 = 100,000$

**Next**, multiply 527 by each digit in 231, from right to left, creating a separate row for each product. Be sure to indent each row one space to the left of the row above it.

$$\begin{array}{r}
 527 \\
 \times 231 \\
 \hline
 527 \quad \leftarrow 527 \times 1 \\
 1581 \quad \leftarrow 527 \times 3 \\
 1054 \quad \leftarrow 527 \times 2 \\
 \hline
 121737
 \end{array}$$

The product is 121,737. This agrees roughly with our estimate. ●

**Your Turn** Multiply as indicated.

(a) $\begin{array}{r} 742 \\ \times 8 \\ \hline \end{array}$	(b) $\begin{array}{r} 56 \\ \times 17 \\ \hline \end{array}$	(c) $\begin{array}{r} 198 \\ \times 45 \\ \hline \end{array}$	(d) $\begin{array}{r} 691 \\ \times 382 \\ \hline \end{array}$	(e) $\begin{array}{r} 344 \\ \times 207 \\ \hline \end{array}$
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**Answers** (a) 5936 (b) 952 (c) 8910 (d) 263,962 (e) 71,208



### A Closer Look

Did you have trouble with the zero in problem (e)? The zero simply creates a row of zeros in your solution. Your work should look like this:

$$\begin{array}{r}
 344 \\
 \times 207 \\
 \hline
 2408 \\
 000 \quad \leftarrow 0 \times 4 = 0, 0 \times 4 = 0, \text{ and } 0 \times 3 = 0 \\
 688 \\
 \hline
 71208
 \end{array}$$

### Multiplication Shortcuts

There are hundreds of quick ways to multiply various numbers. Most of them are quick only if you are already a math whiz. If you are not, the shortcuts will confuse more than help you. Here are a few that are easy to do and easy to remember.

1. To multiply by 10, attach a zero to the right end of the first factor. For example,

$$34 \times 10 = 340$$


$$256 \times 10 = 2560$$

Multiplying by 100 or 1000 is similar.

$$34 \times 100 = 3400$$

$$256 \times 1000 = 256000$$

2. To multiply by a number ending in zeros, carry the zeros forward to the answer. For example,

$\begin{array}{r} 26 \\ \times 20 \\ \hline \end{array}$		$\begin{array}{r} 26 \\ \times 20 \\ \hline 520 \end{array}$	Multiply $26 \times 2$ and attach the zero on the right. The product is 520.
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(continued)