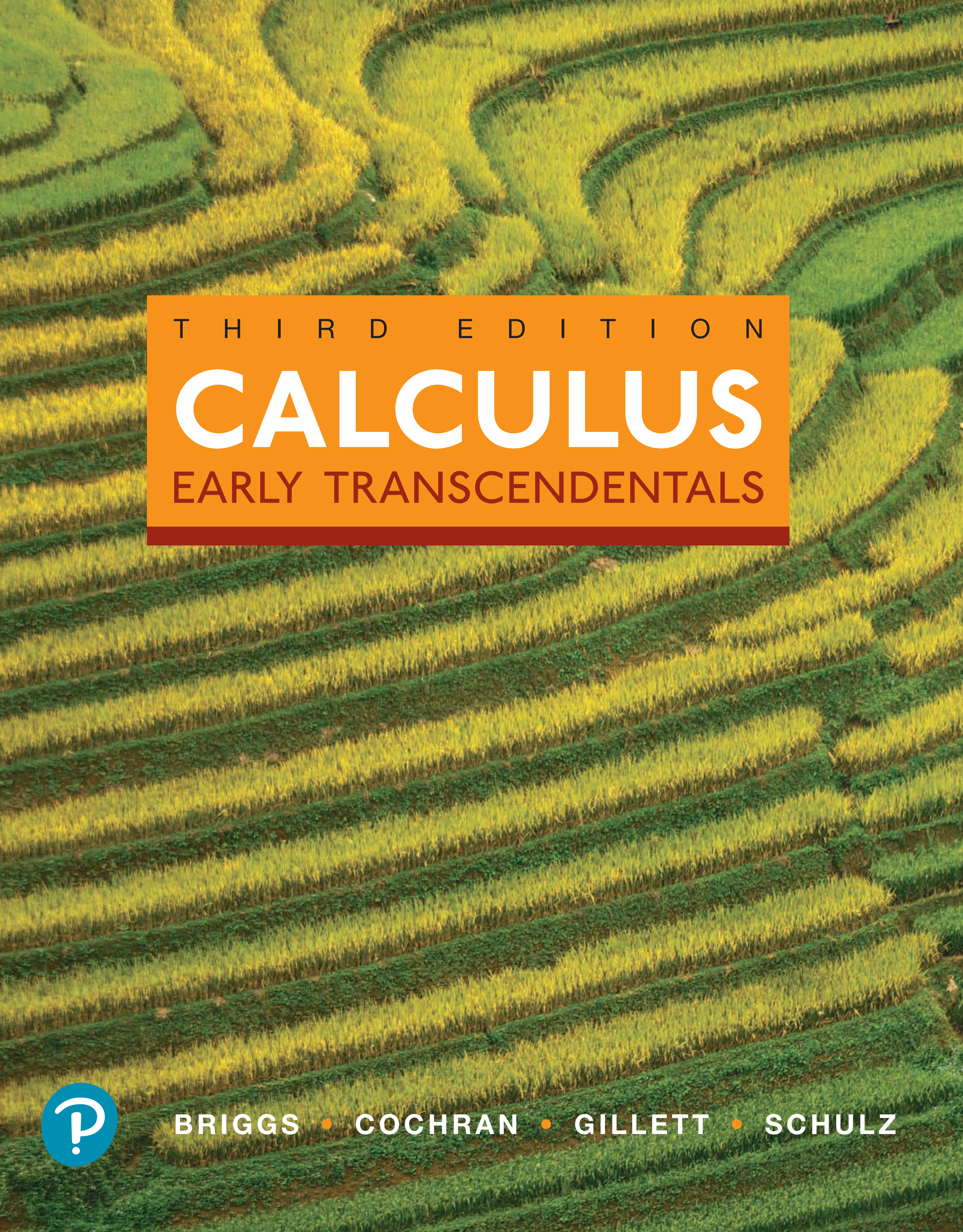


THIRD EDITION

CALCULUS

EARLY TRANSCENDENTALS

BRIGGS • COCHRAN • GILLETT • SCHULZ

Pearson MyLab

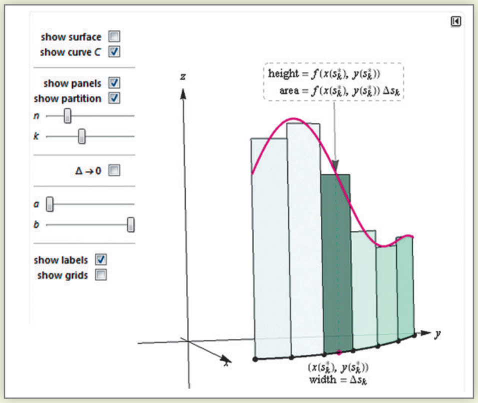
MyLab Math for *Calculus: Early Transcendentals*, 3e

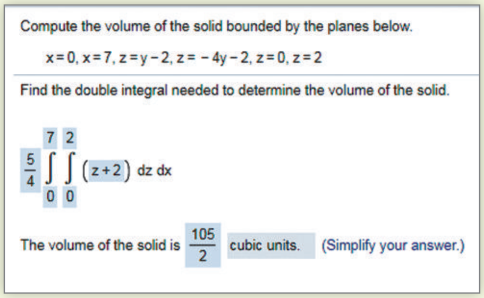
(access code required)

Used by over 3 million students a year, MyLab™ Math is the world's leading online program for teaching and learning mathematics. MyLab Math for *Calculus: Early Transcendentals,* 3e delivers text-specific assessment, tutorials, and multimedia resources that provide engaging and personalized experiences, so learning can happen in any environment and course format.

eBook with Interactive Figures

This groundbreaking eBook includes approximately 700 figures that can be manipulated to provide a deeper geometric understanding of key concepts. Plus, all-new **Enhanced Interactive Figures** incorporate functionality from several standard Interactive Figures, making each one mathematically richer and ideal for in-class demonstrations.




Compute the volume of the solid bounded by the planes below.

$x=0, x=7, z=y-2, z=-4y-2, z=0, z=2$

Find the double integral needed to determine the volume of the solid.

$$\frac{5}{4}\int_0^7\int_0^2 (z+2)\,dz\,dx$$

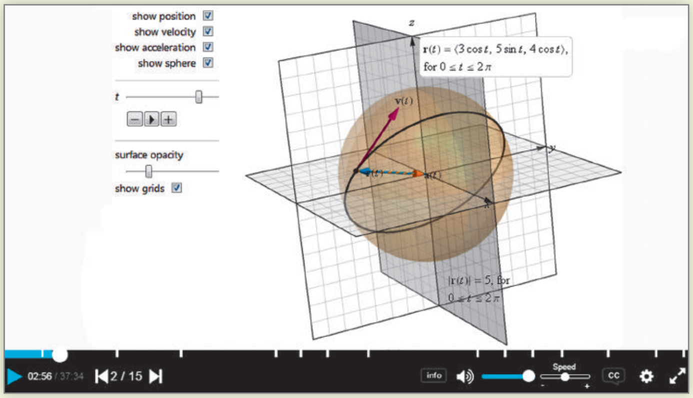
The volume of the solid is $\frac{105}{2}$ cubic units. (Simplify your answer.)

Questions that Deepen Understanding

MyLab Math includes a variety of question types designed to help students succeed in the course. In **Setup & Solve** questions, students show how they set up a problem as well as the solution, better mirroring what is required on tests. **Additional Conceptual Questions** were written by faculty at Cornell University to support deeper, theoretical understanding of the key concepts in calculus.

ALL NEW! Instructional Videos

For each section of the text, there is a newly recorded full-lecture video. Many videos make use of Interactive Figures to enhance understanding of concepts. To make them easier to navigate, each lecture video is segmented into parts (Introduction, Example, or Summary). The videos are assignable within MyLab Math, and a Guide to Video-Based Assignments shows which MyLab Math exercises correspond to each video.



pearson.com/mylab/math

ALGEBRA

Exponents and Radicals

$$x^a x^b = x^{a+b} \qquad \frac{x^a}{x^b} = x^{a-b} \qquad x^{-a} = \frac{1}{x^a} \qquad (x^a)^b = x^{ab} \qquad \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

$$x^{1/n} = \sqrt[n]{x} \qquad x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m \qquad \sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y} \qquad \sqrt[n]{x/y} = \sqrt[n]{x}/\sqrt[n]{y}$$

Factoring Formulas

$a^2 - b^2 = (a - b)(a + b)$ $\qquad$ $a^2 + b^2$ does not factor over real numbers.

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ $\qquad$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1})$

Binomials

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + b^n,$$

where $\displaystyle\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k(k-1)(k-2)\cdots 3\cdot 2\cdot 1} = \frac{n!}{k!(n-k)!}$

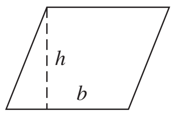
Quadratic Formula

The solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

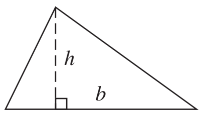
GEOMETRY

Parallelogram



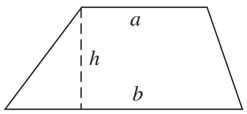
$A = bh$

Triangle



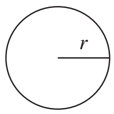
$A = \frac{1}{2}bh$

Trapezoid



$A = \frac{1}{2}(a + b)h$

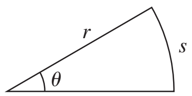
Circle



$A = \pi r^2$

$C = 2\pi r$

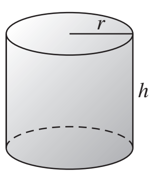
Sector



$A = \frac{1}{2}r^2\theta$

$s = r\theta$ (θ in radians)

Cylinder

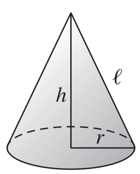


$V = \pi r^2 h$

$S = 2\pi rh$

(lateral surface area)

Cone

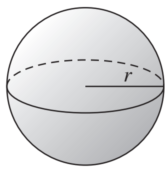


$V = \frac{1}{3}\pi r^2 h$

$S = \pi r\ell$

(lateral surface area)

Sphere

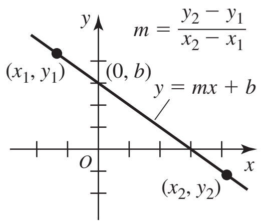


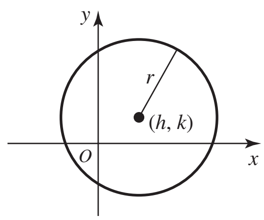
$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Equations of Lines and Circles

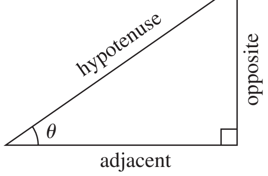
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	slope of line through (x_1, y_1) and (x_2, y_2)
$y - y_1 = m(x - x_1)$	point–slope form of line through (x_1, y_1) with slope m
$y = mx + b$	slope–intercept form of line with slope m and y-intercept $(0, b)$
$(x - h)^2 + (y - k)^2 = r^2$	circle of radius r with center (h, k)





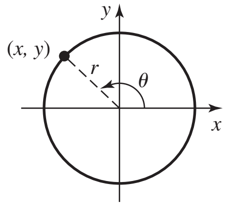
$(x - h)^2 + (y - k)^2 = r^2$

TRIGONOMETRY



$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

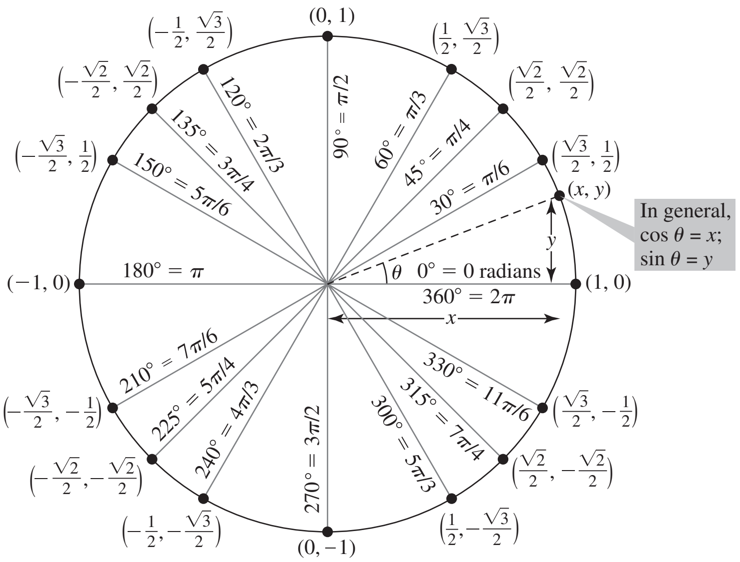


$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

(*Continued*)



Reciprocal Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \csc\theta = \frac{1}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \quad \tan^2\theta + 1 = \sec^2\theta \quad 1 + \cot^2\theta = \csc^2\theta$$

Sign Identities

$$\sin(-\theta) = -\sin\theta \quad \cos(-\theta) = \cos\theta \quad \tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta \quad \sec(-\theta) = \sec\theta \quad \cot(-\theta) = -\cot\theta$$

Double-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Identities

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2} \qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

Addition Formulas

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

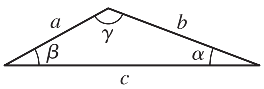
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Law of Sines

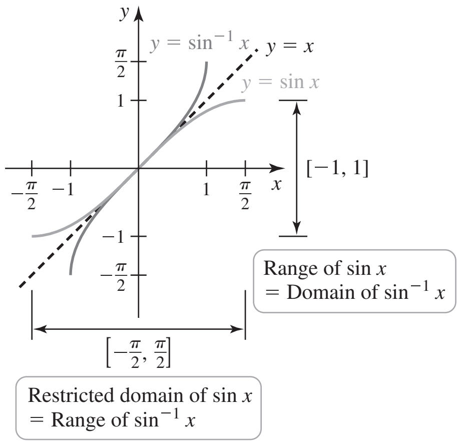
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

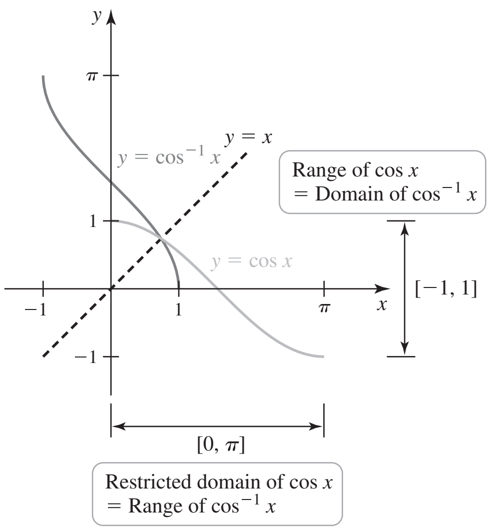


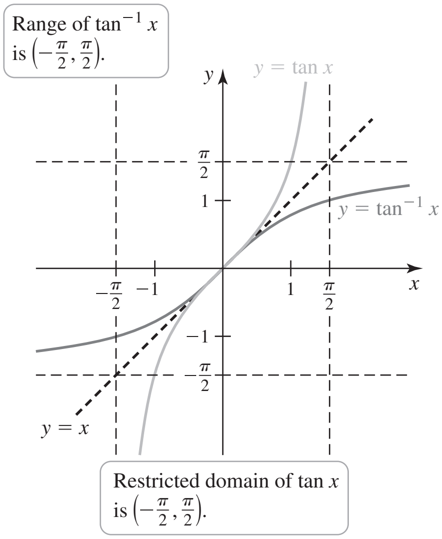
Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Graphs of Trigonometric Functions and Their Inverses







Calculus

EARLY TRANSCENDENTALS

Third Edition

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For Julie, Susan, Sally, Sue,
Katie, Jeremy, Elise, Mary, Claire, Katie, Chris, and Annie,
whose support, patience, and encouragement made this book possible.

Contents

Preface

The third edition of *Calculus: Early Transcendentals* supports a three-semester or four-quarter calculus sequence typically taken by students studying mathematics, engineering, the natural sciences, or economics. The third edition has the same goals as the first edition:

- to motivate the essential ideas of calculus with a lively narrative, demonstrating the utility of calculus with applications in diverse fields;
- to introduce new topics through concrete examples, applications, and analogies, appealing to students' intuition and geometric instincts to make calculus natural and believable; and
- once this intuitive foundation is established, to present generalizations and abstractions and to treat theoretical matters in a rigorous way.

The third edition both builds on the success of the previous two editions and addresses the feedback we have received. We have listened to and learned from the instructors who used the text. They have given us wise guidance about how to make the third edition an even more effective learning tool for students and a more powerful resource for instructors. Users of the text continue to tell us that it mirrors the course they teach—and, more important, that students actually read it! Of course, the third edition also benefits from our own experiences using the text, as well as from our experiences teaching mathematics at diverse institutions over the past 30 years.

New to the Third Edition

Exercises

The exercise sets are a major focus of the revision. In response to reviewer and instructor feedback, we've made some significant changes to the exercise sets by rearranging and relabeling exercises, modifying some exercises, and adding many new ones. Of the approximately 10,400 exercises appearing in this edition, 18% are new, and many of the exercises from the second edition were revised for this edition. We analyzed aggregated student usage and performance data from MyLab™ Math for the previous edition of this text. The results of this analysis helped us improve the quality and quantity of exercises that matter the most to instructors and students. We have also simplified the structure of the exercises sets from five parts to the following three:

1. **Getting Started** contains some of the former Review Questions but goes beyond those to include more conceptual exercises, along with new basic skills and short-answer exercises. Our goal in this section is to provide an excellent overall assessment of understanding of the key ideas of a section.
2. **Practice Exercises** consist primarily of exercises from the former Basic Skills, but they also include intermediate-level exercises from the former Further Explorations and Application sections. Unlike previous editions, these exercises are not necessarily organized into groups corresponding to specific examples. For instance, instead of separating out Product Rule exercises from Quotient Rule exercises in Section 3.4, we

have merged these problems into one larger group of exercises. Consequently, specific instructions such as "Use the Product Rule to find the derivative of the following functions" and "Use the Quotient Rule to find the derivative of the given functions" have been replaced with the general instruction "Find the derivative of the following functions." With Product Rule and Quotient Rule exercises mixed together, students must first choose the correct method for evaluating derivatives before solving the problems.

3. **Explorations and Challenges** consist of more challenging problems and those that extend the content of the section.

We no longer have a section of the exercises called "Applications," but (somewhat ironically) in eliminating this section, we feel we are providing better coverage of applications because these exercises have been placed strategically *throughout the exercise sets.* Some are in Getting Started, most are in Practice Exercises, and some are in Explorations and Challenges. The applications nearly always have a boldface heading so that the topic of the application is readily apparent.

Regarding the boldface heads that precede exercises: These heads provide instructors with a quick way to discern the topic of a problem when creating assignments. We heard from users of earlier editions, however, that some of these heads provided too much guidance in how to solve a given problem. In this edition, therefore, we eliminated or reworded run-in heads that provided too much information about the solution method for a problem.

Finally, the **Chapter Review exercises** received a major revamp to provide more exercises (particularly intermediate-level problems) and more opportunities for students to choose a strategy of solution. More than 26% of the Chapter Review exercises are new.

Content Changes

Below are noteworthy changes from the previous edition of the text. Many other detailed changes, not noted here, were made to improve the quality of the narrative and exercises. Bullet points with a icon represent major content changes from the previous edition.

Chapter 1 Functions

- Example 2 in Section 1.1 was modified with more emphasis on using algebraic techniques to determine the domain and range of a function. To better illustrate a common feature of limits, we replaced part (c) with a rational function that has a common factor in the numerator and denominator.
- Examples 7 and 8 in Section 1.1 from the second edition (2e) were moved forward in the narrative so that students get an intuitive feel for the composition of two functions using graphs and tables; compositions of functions using algebraic techniques follow.
- Example 10 in Section 1.1, illustrating the importance of secant lines, was made more relevant to students by using real data from a GPS watch during a hike. Corresponding exercises were also added.
- Exercises were added to Section 1.3 to give students practice at finding inverses of functions using the properties of exponential and logarithmic functions.
- New application exercises (investment problems and a biology problem) were added to Section 1.3 to further illustrate the usefulness of logarithmic and exponential functions.

Chapter 2 Limits

- Example 4 in Section 2.2 was revised, emphasizing an algebraic approach to a function with a jump discontinuity, rather than a graphical approach.
- Theorems 2.3 and 2.13 were modified, simplifying the notation to better connect with upcoming material.
- Example 7 in Section 2.3 was added to solidify the notions of left-, right-, and two-sided limits.
- The material explaining the end behavior of exponential and logarithmic functions was reworked, and Example 6 in Section 2.5 was added to show how substitution is used in evaluating limits.
- Exercises were added to Section 2.5 to illustrate the similarities and differences between limits at infinity and infinite limits. We also included some easier exercises in Section 2.5 involving limits at infinity of functions containing square roots.
- Example 5 in Section 2.7 was added to demonstrate an epsilon-delta proof of a limit of a quadratic function.
- We added 17 epsilon-delta exercises to Section 2.7 to provide a greater variety of problems involving limits of quadratic, cubic, trigonometric, and absolute value functions.

Chapter 3 Derivatives

- Chapter 3 now begins with a look back at average and instantaneous velocity, first encountered in Section 2.1, with a corresponding revised example in Section 3.1.
- The derivative at a point and the derivative as a function are now treated separately in Sections 3.1 and 3.2.
- After defining the derivative at a point in Section 3.1 with a supporting example, we added a new subsection: Interpreting the Derivative (with two supporting examples).
- Several exercises were added to Section 3.3 that require students to use the Sum and Constant Rules, together with geometry, to evaluate derivatives.
- The Power Rule for derivatives in Section 3.4 is stated for all real powers (later proved in Section 3.9). Example 4

in Section 3.4 includes two additional parts to highlight this change, and subsequent examples in upcoming sections rely on the more robust version of the Power Rule. The Power Rule for Rational Exponents in Section 3.8 was deleted because of this change.

- We combined the intermediate-level exercises in Section 3.4 involving the Product Rule and Quotient Rule together under one unified set of directions.
- The derivative of e^x still appears early in the chapter, but the derivative of e^{kx} is delayed; it appears only after the Chain Rule is introduced in Section 3.7.
- In Section 3.7, we deleted references to Version 1 and Version 2 of the Chain Rule. Additionally, Chain Rule exercises involving repeated use of the rule were merged with the standard exercises.
- In Section 3.8, we added emphasis on simplifying derivative formulas for implicitly defined functions; see Examples 4 and 5.
- Example 3 in Section 3.11 was replaced; the new version shows how similar triangles are used in solving a related-rates problem.

Chapter 4 Applications of the Derivative

- The Mean Value Theorem (MVT) was moved from Section 4.6 to 4.2 so that the proof of Theorem 4.7 is not delayed. We added exercises to Section 4.2 that help students better understand the MVT geometrically, and we included exercises where the MVT is used to prove some well-known identities and inequalities.
- Example 5 in Section 4.5 was added to give guidance on a certain class of optimization problems.
- Example 3b in Section 4.7 was replaced to better drive home the need to simplify after applying l'Hôpital's Rule.
- Most of the intermediate exercises in Section 4.7 are no longer separated out by the type of indeterminate form, and we added some problems in which l'Hôpital's Rule does not apply.
- Indefinite integrals of trigonometric functions with argument ax (Table 4.9) were relocated to Section 5.5, where they are derived with the Substitution Rule. A similar change was made to Table 4.10.
- Example 7b in Section 4.9 was added to foreshadow a more complete treatment of the domain of an initial value problem found in Chapter 9.
- We added to Section 4.9 a significant number of intermediate antiderivative exercises that require some preliminary work (e.g., factoring, cancellation, expansion) before the antiderivatives can be determined.

Chapter 5 Integration

- Examples 2 and 3 in Section 5.1 on approximating areas were replaced with a friendlier function where the grid points are more transparent; we return to these approximations in Section 5.3, where an exact result is given (Example 3b).
- Three properties of integrals (bounds on definite integrals) were added in Section 5.2 (Table 5.5); the last of these properties is used in the proof of the Fundamental Theorem (Section 5.3).
- Exercises were added to Sections 5.1 and 5.2 where students are required to evaluate Riemann sums using graphs or tables instead of formulas. These exercises will help students better understand the geometric meaning of Riemann sums.
- We added to Section 5.3 more exercises in which the integrand must be simplified before the integrals can be evaluated.
- A proof of Theorem 5.7 is now offered in Section 5.5.
- Table 5.6 lists the general integration formulas that were relocated from Section 4.9 to Section 5.5; Example 4 in Section 5.5 derives these formulas.

Chapter 6 Applications of Integration
Chapter 7 Logarithmic, Exponential, and Hyperbolic Functions

- Chapter 6 from the 2e was split into two chapters in order to match the number of chapters in *Calculus* (Late Transcendentals). The result is a compact Chapter 7.
- Exercises requiring students to evaluate net change using graphs were added to Section 6.1.
- Exercises in Section 6.2 involving area calculations with respect to x and y are now combined under one unified set of directions (so that students must first determine the appropriate variable of integration).
- We increased the number of exercises in Sections 6.3 and 6.4 in which curves are revolved about lines other than the x- and y-axes. We also added introductory exercises that guide students, step by step, through the processes used to find volumes.
- A more gentle introduction to lifting problems (specifically, lifting a chain) was added in Section 6.7 and illustrated in Example 3, accompanied by additional exercises.
- The introduction to exponential growth (Section 7.2) was rewritten to make a clear distinction between the relative growth rate (or percent change) of a quantity and the rate constant k. We revised the narrative so that the equation $y = y_0e^{kt}$ applies to both growth and decay models. This revision resulted in a small change to the half-life formula.
- The variety of applied exercises in Section 7.2 was increased to further illustrate the utility of calculus in the study of exponential growth and decay.

Chapter 8 Integration Techniques

- Table 8.1 now includes four standard trigonometric integrals that previously appeared in the section Trigonometric Integrals (8.3); these integrals are derived in Examples 1 and 2 in Section 8.1.
- A new section (8.6) was added so that students can master integration techniques (that is, choose a strategy) apart from the context given in the previous five sections.
- In Section 8.5 we increased the number and variety of exercises where students must set up the appropriate form of the partial fraction decomposition of a rational function, including more with irreducible quadratic factors.
- A full derivation of Simpson's Rule was added to Section 8.8, accompanied by Example 7, additional figures, and an expanded exercise set.

- The Comparison Test for improper integrals was added to Section 8.9, accompanied by Example 7, a two-part example. New exercises in Section 8.9 include some covering doubly infinite improper integrals over infinite intervals.

Chapter 9 Differential Equations

- The chapter on differential equations that was available only online in the 2e was converted to a chapter of the text, replacing the single-section coverage found in the 2e.
- More attention was given to the domain of an initial value problem, resulting in the addition and revision of several examples and exercises throughout the chapter.

Chapter 10 Sequences and Infinite Series

- The second half of Chapter 10 was reordered: Comparison Tests (Section 10.5), Alternating Series (Section 10.6, which includes the topic of absolute convergence), The Ratio and Root Tests (Section 10.7), and Choosing a Convergence Test (Section 10.8; new section). We split the 2e section that covered the comparison, ratio, and root tests to avoid overwhelming students with too many tests at one time. Section 10.5 focuses entirely on the comparison tests; 39% of the exercises are new. The topic of alternating series now appears before the Ratio and Root Tests so that the latter tests may be stated in their more general form (they now apply to any series rather than only to series with positive terms). The final section (10.8) gives students an opportunity to master convergence tests after encountering each of them separately.
- The terminology associated with sequences (10.2) now includes *bounded above, bounded below,* and *bounded* (rather than only *bounded,* as found in earlier editions).
- Theorem 10.3 (Geometric Sequences) is now developed in the narrative rather than within an example, and an additional example (10.2.3) was added to reinforce the theorem and limit laws from Theorem 10.2.
- Example 5c in Section 10.2 uses mathematical induction to find the limit of a sequence defined recursively; this technique is reinforced in the exercise set.
- Example 3 in Section 10.3 was replaced with telescoping series that are not geometric and that require re-indexing.
- We increased the number and variety of exercises where the student must determine the appropriate series test necessary to determine convergence of a given series.
- We added some easier intermediate-level exercises to Section 10.6, where series are estimated using nth partial sums for a given value of n.
- Properties of Convergent Series (Theorem 10.8) was expanded (two more properties) and moved to Section 10.3 to better balance the material presented in Sections 10.3 and 10.4. Example 4 in Section 10.3 now has two parts to give students more exposure to the theorem.

Chapter 11 Power Series

- Chapter 11 was revised to mesh with the changes made in Chapter 10.
- We included in Section 11.2 more exercises where the student must find the radius and interval of convergence.
- Example 2 in Section 11.3 was added to illustrate how to choose a different center for a series representation of a function when the original series for the function converges to the function on only part of its domain.
- We addressed an issue with the exercises in Section 11.2 of the previous edition by adding more exercises where the intervals of convergence either are closed or contain one, but not both, endpoints.
- We addressed an issue with exercises in the previous edition by adding many exercises that involve power series centered at locations other than 0.

Chapter 12 Parametric and Polar Curves

- The arc length of a two-dimensional curve described by parametric equations was added to Section 12.1, supported by two examples and additional exercises. Area and surfaces of revolution associated with parametric curves were also added to the exercises.
- In Example 3 in Section 12.2, we derive more general polar coordinate equations for circles.
- The arc length of a curve described in polar coordinates is given in Section 12.3.

Chapter 13 Vectors and the Geometry of Space

- The material from the 2e chapter Vectors and Vector-Valued Functions is now covered in this chapter and the following chapter.
- Example 5c in Section 13.1 was added to illustrate how to express a vector as a product of its magnitude and its direction.
- We increased the number of applied vector exercises in Section 13.1, starting with some easier exercises, resulting in a wider gradation of exercises.
- We adopted a more traditional approach to lines and planes; these topics are now covered together in Section 13.5, followed by cylinders and quadric surfaces in Section 13.6. This arrangement gives students early exposure to all the basic three-dimensional objects that they will encounter throughout the remainder of the text.
- A discussion of the distance from a point to a line was moved from the exercises into the narrative, supported with Example 3 in Section 13.5. Example 4 finds the point of intersection of two lines. Several related exercises were added to this section.
- In Section 13.6 there is a larger selection of exercises where the student must identify the quadric surface associated with a given equation. Exercises are also included where students design shapes using quadric surfaces.

Chapter 14 Vector-Valued Functions

- More emphasis was placed on the surface(s) on which a space curve lies in Sections 14.1 and 14.3.

- We added exercises in Section 14.1 where students are asked to find the curve of intersection of two surfaces and where students must verify that a curve lies on a given surface.
- Example 3c in Section 14.3 was added to illustrate how a space curve can be mapped onto a sphere.
- Because the arc length of plane curves (described parametrically in Section 12.1 and with polar coordinates in Section 12.3) was moved to an earlier location in the text, Section 14.4 is now a shorter section.

Chapter 15 Functions of Several Variables

- Equations of planes and quadric surfaces were removed from this chapter and now appear in Chapter 13.
- The notation in Theorem 15.2 was simplified to match changes made to Theorem 2.3.
- Example 7 in Section 15.4 was added to illustrate how the Chain Rule is used to compute second partial derivatives.
- We added more challenging partial derivative exercises to Section 15.3 and more challenging Chain Rule exercises to Section 15.4.
- Example 7 in Section 15.5 was expanded to give students more practice finding equations of curves that lie on surfaces.
- Theorem 15.13 was added in Section 15.5; it's a three-dimensional version of Theorem 15.11.
- Example 7 in Section 15.7 was replaced with a more interesting example; the accompanying figure helps tell the story of maximum/minimum problems and can be used to preview Lagrange multipliers.
- We added to Section 15.7 some basic exercises that help students better understand the second derivative test for functions of two variables.
- Example 1 in Section 15.8 was modified so that using Lagrange multipliers is the clear path to a solution, rather than eliminating one of the variables and using standard techniques. We also make it clear that care must be taken when using the method of Lagrange multipliers on sets that are not closed and bounded (absolute maximum and minimum values may not exist).

Chapter 16 Multiple Integration

- Example 2 in Section 16.3 was modified because it was too similar to Example 1.
- More care was given to the notation used with polar, cylindrical, and spherical coordinates (see, for example, Theorem 16.3 and the development of integration in different coordinate systems).
- Example 3 in Section 16.4 was modified to make the integration a little more transparent and to show that changing variables to polar coordinates is permissible in more than just the xy-plane.
- More multiple integral exercises were added to Sections 16.1, 16.2, and 16.4, where integration by substitution or integration by parts is needed to evaluate the integrals.
- In Section 16.4 we added more exercises in which the integrals must first be evaluated with respect to x or y instead of z. We also included more exercises that require triple integrals to be expressed in several orderings.

Chapter 17 Vector Calculus

- Our approach to scalar line integrals was streamlined; Example 1 in Section 17.2 was modified to reflect this fact.
- We added basic exercises in Section 17.2 emphasizing the geometric meaning of line integrals in a vector field. A subset of exercises was added where line integrals are grouped so that the student must determine the type of line integral before evaluating the integral.
- Theorem 17.5 was added to Section 17.3; it addresses the converse of Theorem 17.4. We also promoted the area of a plane region by a line integral to theorem status (Theorem 17.8 in Section 17.4).
- Example 3 in Section 17.7 was replaced to give an example of a surface whose bounding curve is not a plane curve and to provide an example that buttresses the claims made at the end of the section (that is, Two Final Notes on Stokes' Theorem).
- More line integral exercises were added to Section 17.3 where the student must first find the potential function before evaluating the line integral over a conservative vector field using the Fundamental Theorem of Line Integrals.
- We added to Section 17.7 more challenging surface integrals that are evaluated using Stokes' Theorem.

New to MyLab Math

- **Assignable Exercises** To better support students and instructors, we made the following changes to the assignable exercises:
  - Updated the solution processes in Help Me Solve This and View an Example to better match the techniques used in the text.
  - Added more Setup & Solve exercises to better mirror the types of responses that students are expected to provide on tests. We also added a parallel "standard" version of each Setup & Solve exercise, to allow the instructor to determine which version to assign.
  - Added exercises corresponding to new exercises in the text.

- Added exercises where MyLab Math users had identified gaps in coverage in the 2e.
- Added extra practice exercises to each section (clearly labeled EXTRA). These "beyond the text" exercises are perfect for chapter reviews, quizzes, and tests.
- Analyzed aggregated student usage and performance data from MyLab Math for the previous edition of this text. The results of this analysis helped improve the quality and quantity of exercises that matter the most to instructors and students.

- **Instructional Videos** For each section of the text, there is now a new full-lecture video. Many of these videos make use of Interactive Figures to enhance student understanding of concepts. To make it easier for students to navigate to the specific content they need, each lecture video is segmented into shorter clips (labeled Introduction, Example, or Summary). Both the full lectures and the video segments are assignable within MyLab Math. The videos were created by the following team: Matt Hudelson (Washington State University), Deb Carney and Rebecca Swanson (Colorado School of Mines), Greg Wisloski and Dan Radelet (Indiana University of Pennsylvania), and Nick Ormes (University of Denver).
- **Enhanced Interactive Figures** Incorporating functionality from several standard Interactive Figures makes Enhanced Interactive Figures mathematically richer and ideal for in-class demonstrations. Using a single figure, instructors can illustrate concepts that are difficult for students to visualize and can make important connections to key themes of calculus.
- **Enhanced Sample Assignments** These section-level assignments address gaps in precalculus skills with a personalized review of prerequisites, help keep skills fresh with spaced practice using key calculus concepts, and provide opportunities to work exercises without learning aids so students can check their understanding. They are assignable and editable.
- **Quick Quizzes** have been added to Learning Catalytics™ (an in-class assessment system) for every section of the text.
- **Maple™, Mathematica®, and Texas Instruments® Manuals and Projects** have all been updated to align with the latest software and hardware.

Noteworthy Features

Figures

Given the power of graphics software and the ease with which many students assimilate visual images, we devoted considerable time and deliberation to the figures in this text. Whenever possible, we let the figures communicate essential ideas using annotations reminiscent of an instructor's voice at the board. Readers will quickly find that the figures facilitate learning in new ways.

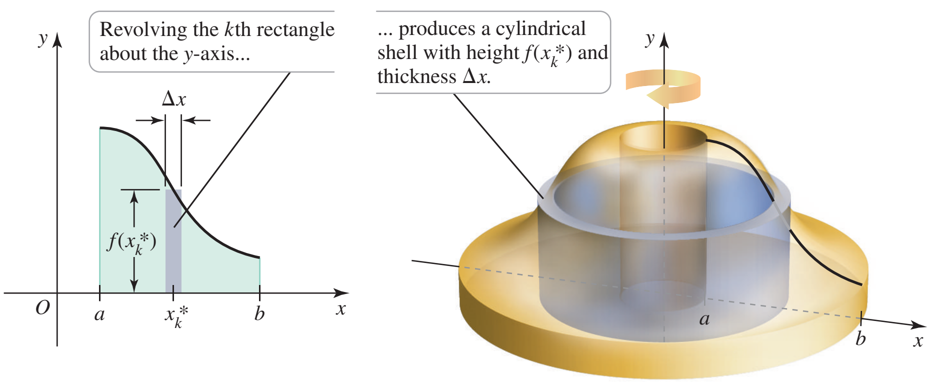

Figure 6.40

Annotated Examples

Worked-out examples feature annotations in blue to guide students through the process of solving the example and to emphasize that each step in a mathematical argument must be rigorously justified. These annotations are designed to echo how instructors "talk through" examples in lecture. They also provide help for students who may struggle with the algebra and trigonometry steps within the solution process.

Quick Checks

The narrative is interspersed with Quick Check questions that encourage students to do the calculus as they are reading about it. These questions resemble the kinds of questions instructors pose in class. Answers to the Quick Check questions are found at the end of the section in which they occur.

Guided Projects

MyLab Math contains 78 Guided Projects that allow students to work in a directed, step-by-step fashion, with various objectives: to carry out extended calculations, to derive physical models, to explore related theoretical topics, or to investigate new applications of calculus. The Guided Projects vividly demonstrate the breadth of calculus and provide a wealth of mathematical excursions that go beyond the typical classroom experience. A list of related Guided Projects is included at the end of each chapter.

Incorporating Technology

We believe that a calculus text should help students strengthen their analytical skills and demonstrate how technology can extend (not replace) those skills. Calculators and graphing utilities are additional tools in the kit, and students must learn when and when not to use them. Our goal is to accommodate the different policies regarding technology adopted by various instructors.

Throughout the text, exercises marked with **T** indicate that the use of technology—ranging from plotting a function with a graphing calculator to carrying out a calculation using a computer algebra system—may be needed. See page xx for information regarding our technology resource manuals covering Maple, Mathematica, and Texas Instruments graphing calculators.

Text Versions

- **eBook with Interactive Figures** The text is supported by a groundbreaking and award-winning electronic book created by Eric Schulz of Walla Walla Community College. This "live book" runs in Wolfram CDF Player (the free version of Mathematica) and contains the complete text of the print book plus interactive versions of approximately 700 figures. Instructors can use these interactive figures in the classroom to illustrate the important ideas of calculus, and students can explore them while they are reading the text. Our experience confirms that the interactive figures help build students' geometric intuition of calculus. The authors have written Interactive Figure Exercises that can be assigned via MyLab Math so that students can engage with the figures outside of class in a directed way. Available only within MyLab Math, the eBook provides instructors with powerful new teaching tools that expand and enrich the learning experience for students.
- **Other eBook Formats** The text is also available in various stand-alone eBook formats. These are listed in the Pearson online catalog: **www.pearson.com**. MyLab Math also contains an HTML eBook that is screen-reader accessible.
- **Other Print Formats** The text is also available in split editions (Single Variable [Chapters 1–12] and Multivariable [Chapters 10–17]) and in unbound (3-hole punched) formats. Again, see the Pearson online catalog for details: **www.pearson.com.**

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Yan Tang, *Embry-Riddle Aeronautical University*

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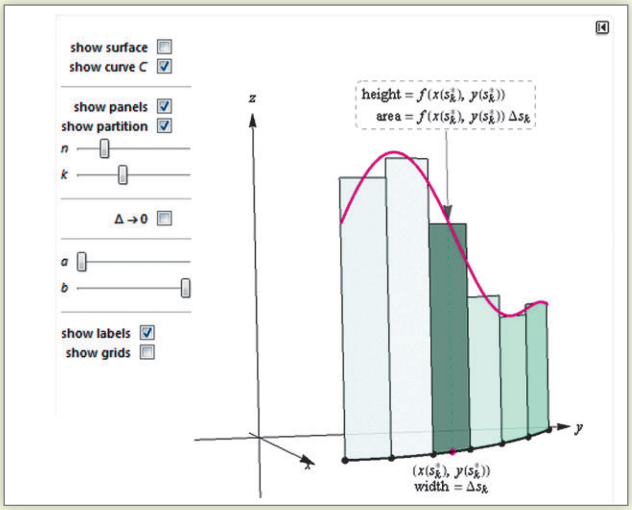
The eBook includes approximately 700 figures that can be manipulated by students to provide a deeper geometric understanding of key concepts and examples as they read and learn new material. Students get unlimited access to the eBook within any MyLab Math course using that edition of the text. The authors have written Interactive Figure Exercises that can be assigned for homework so that students can engage with the figures outside of the classroom.



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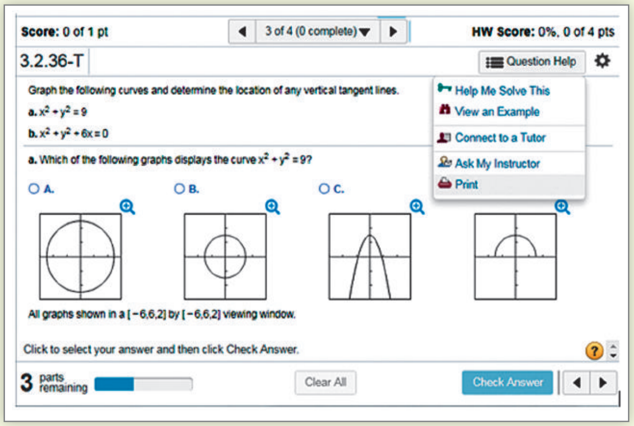


Exercises with Immediate Feedback

The over 8000 homework and practice exercises for this text regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Math provides helpful feedback when students enter incorrect answers and includes the optional learning aids Help Me Solve This, View an Example, videos, and/or the eBook.

NEW! Enhanced Sample Assignments

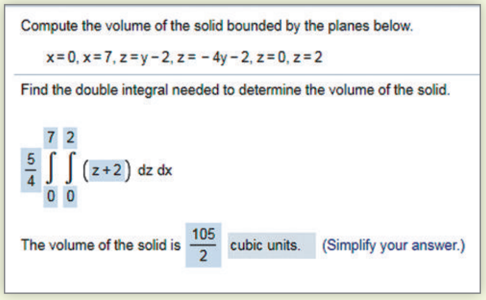
These section-level assignments include just-in-time review of prerequisites, help keep skills fresh with spaced practice of key concepts, and provide opportunities to work exercises without learning aids so students can check their understanding. They are assignable and editable within MyLab Math.

Additional Conceptual Questions

Additional Conceptual Questions focus on deeper, theoretical understanding of the key concepts in calculus. These questions were written by faculty at Cornell University under an NSF grant and are also assignable through Learning Catalytics™.

Setup & Solve Exercises

These exercises require students to show how they set up a problem, as well as the solution, thus better mirroring what is required on tests. This new type of exercise was widely praised by users of the second edition, so more were added to the third edition.



ALL NEW! Instructional Videos

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UPDATED! Technology Manuals (downloadable)

- Maple™ Manual and Projects by Kevin Reeves, East Texas Baptist University
- Mathematica® Manual and Projects by Todd Lee, Elon University
- TI-Graphing Calculator Manual by Elaine McDonald-Newman, Sonoma State University

These manuals cover *Maple* 2017, *Mathematica 11,* and the TI-84 Plus and TI-89, respectively. Each manual provides detailed guidance for integrating the software package or graphing calculator throughout the course, including syntax and commands. The projects include instructions and ready-made application files for Maple and Mathematica. The files can be downloaded from within MyLab Math.

Student's Solutions Manuals (softcover and downloadable)

Single Variable Calculus: Early Transcendentals (Chapters 1–12)
ISBN: 0-13-477048-X | 978-0-13-477048-2
Multivariable Calculus (Chapters 10–17)
ISBN: 0-13-476682-2 | 978-0-13-476682-9
Written by Mark Woodard (Furman University), the Student's Solutions Manual contains worked-out solutions to all the odd-numbered exercises. This manual is available in print and can be downloaded from within MyLab Math.

SUPPORTING INSTRUCTION

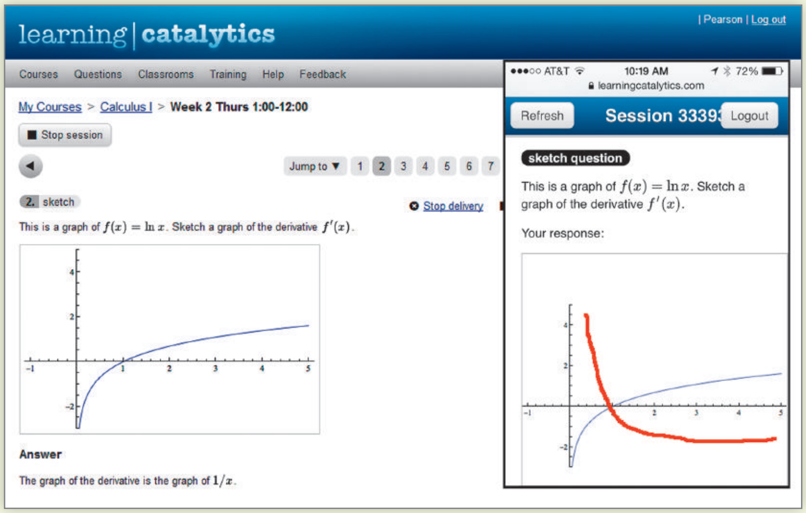
MyLab Math comes from an experienced partner with educational expertise and an eye on the future. It provides resources to help you assess and improve student results at every turn and unparalleled flexibility to create a course tailored to you and your students.

NEW! Enhanced Interactive Figures

Incorporating functionality from several standard Interactive Figures makes Enhanced Interactive Figures mathematically richer and ideal for in-class demonstrations. Using a single enhanced figure, instructors can illustrate concepts that are difficult for students to visualize and can make important connections to key themes of calculus.

Learning Catalytics

Now included in all MyLab Math courses, this student response tool uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking during lecture. Learning Catalytics™ fosters student engagement and peer-to-peer learning with real-time analytics. Access pre-built exercises created specifically for calculus, including Quick Quiz exercises for each section of the text.



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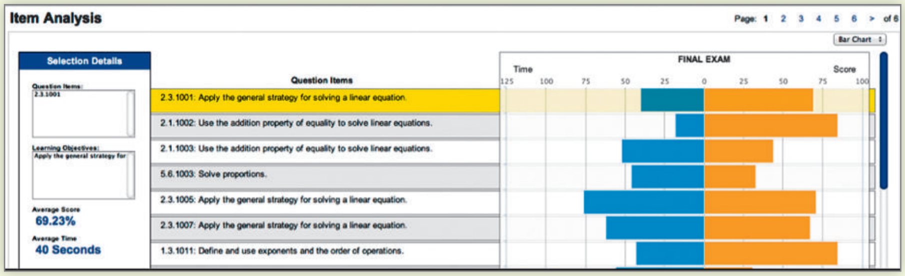
PowerPoint Lecture Resources (downloadable)

Slides contain presentation resources such as key concepts, examples, definitions, figures, and tables from this text. They can be downloaded from within MyLab Math or from Pearson's online catalog at **www.pearson.com**.

Comprehensive Gradebook

The gradebook includes enhanced reporting functionality, such as item analysis and a reporting dashboard to enable you to efficiently manage your course. Student performance data are presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.



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TestGen® (**www.pearson.com/testgen**) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple, but equivalent, versions of the same question or test with the click of a button. Instructors can also modify test bank questions and/or add new questions. The software and test bank are available for download from Pearson's online catalog, **www.pearson.com**. The questions are also assignable in MyLab Math.

Instructor's Solutions Manual (downloadable)

Written by Mark Woodard (Furman University), the Instructor's Solutions Manual contains complete solutions to all the exercises in the text. It can be downloaded from within MyLab Math or from Pearson's online catalog, **www.pearson.com**.

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Credits

Chapter opener art: Andrey Pavlov/123RF

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1

Functions

Chapter Preview Mathematics is a language with an alphabet, a vocabulary, and many rules. Before beginning your calculus journey, you should be familiar with the elements of this language. Among these elements are algebra skills; the notation and terminology for various sets of real numbers; and the descriptions of lines, circles, and other basic sets in the coordinate plane. A review of this material is found in Appendix B, online at bit.ly/2y3Nck3. This chapter begins with the fundamental concept of a function and then presents the entire cast of functions needed for calculus: polynomials, rational functions, algebraic functions, exponential and logarithmic functions, and the trigonometric functions, along with their inverses. Before you begin studying calculus, it is important that you master the ideas in this chapter.

1.1 Review of Functions

Everywhere around us we see relationships among quantities, or **variables**. For example, the consumer price index changes in time and the temperature of the ocean varies with latitude. These relationships can often be expressed by mathematical objects called *functions*. Calculus is the study of functions, and because we use functions to describe the world around us, calculus is a universal language for human inquiry.

DEFINITION Function

A **function** f is a rule that assigns to each value x in a set D a *unique* value denoted $f(x)$. The set D is the **domain** of the function. The **range** is the set of all values of $f(x)$ produced as x varies over the entire domain (Figure 1.1).

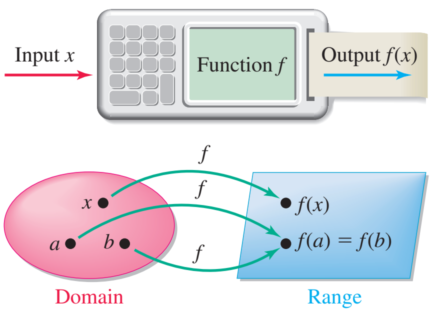


Figure 1.1

The **independent variable** is the variable associated with the domain; the **dependent variable** belongs to the range. The **graph** of a function f is the set of all points (x, y) in the xy-plane that satisfy the equation $y = f(x)$. The **argument** of a function is the expression on which the function works. For example, x is the argument when we write $f(x)$. Similarly, 2 is the argument in $f(2)$ and $x^2 + 4$ is the argument in $f(x^2 + 4)$.

QUICK CHECK 1 If $f(x) = x^2 - 2x$, find $f(-1)$, $f(x^2)$, $f(t)$, and $f(p - 1)$. ◂

The requirement that a function assigns a *unique* value of the dependent variable to each value in the domain is expressed in the vertical line test (Figure 1.2a). For example, the outside temperature as it varies over the course of a day is a function of time (Figure 1.2b).

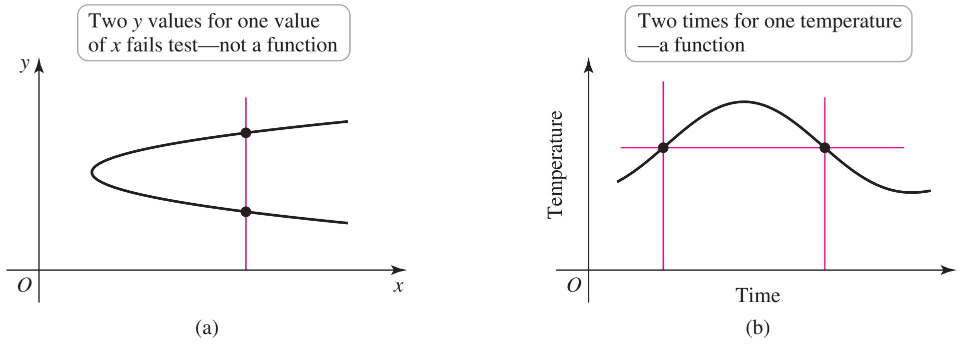


Figure 1.2

➤ If the domain is not specified, we take it to be the set of all values of x for which f is defined. We will see shortly that the domain and range of a function may be restricted by the context of the problem.

➤ A set of points or a graph that does *not* correspond to a function represents a **relation** between the variables. All functions are relations, but not all relations are functions.

> **Vertical Line Test**
>
> A graph represents a function if and only if it passes the **vertical line test**: Every vertical line intersects the graph at most once. A graph that fails this test does not represent a function.

EXAMPLE 1 **Identifying functions** State whether each graph in Figure 1.3 represents a function.

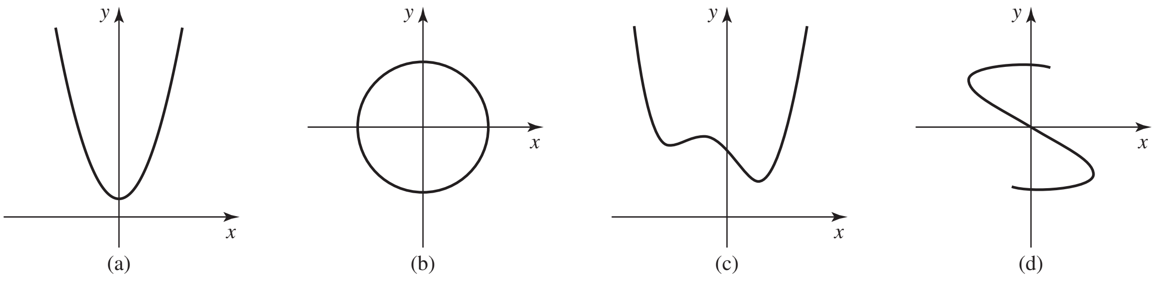


Figure 1.3

SOLUTION The vertical line test indicates that only graphs (a) and (c) represent functions. In graphs (b) and (d), there are vertical lines that intersect the graph more than once. Equivalently, there are values of x that correspond to more than one value of y. Therefore, graphs (b) and (d) do not pass the vertical line test and do not represent functions.

Related Exercise 3 ◂

EXAMPLE 2 **Domain and range** Determine the domain and range of each function.

a. $f(x) = x^2 + 1$ **b.** $g(x) = \sqrt{4 - x^2}$ **c.** $h(x) = \dfrac{x^2 - 3x + 2}{x - 1}$

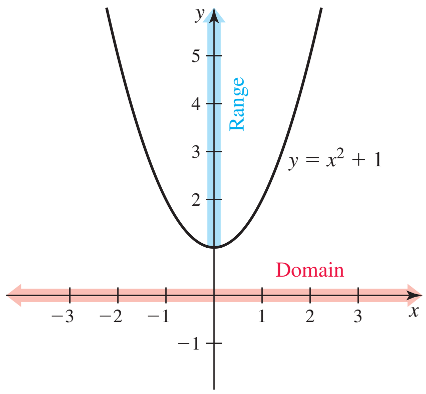


Figure 1.4

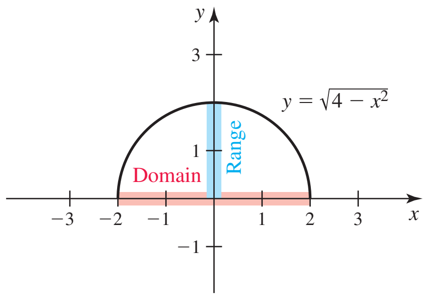


Figure 1.5

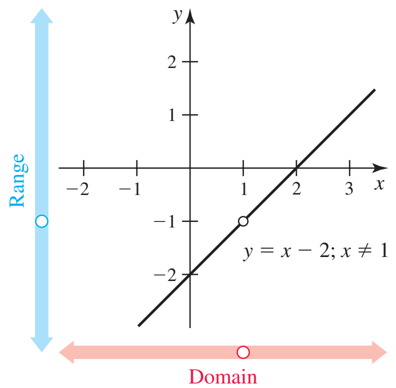


Figure 1.6

SOLUTION

a. Note that f is defined for all values of x; therefore, its domain is the set of all real numbers, written $(-\infty, \infty)$ or $\mathbb{R}$. Because $x^2 \geq 0$ for all x, it follows that $x^2 + 1 \geq 1$, which implies that the range of f is $[1, \infty)$. **Figure 1.4** shows the graph of f along with its domain and range.

b. Functions involving square roots are defined provided the quantity under the root is nonnegative (additional restrictions may also apply). In this case, the function g is defined provided $4 - x^2 \geq 0$, which means $x^2 \leq 4$, or $-2 \leq x \leq 2$. Therefore, the domain of g is $[-2, 2]$. The graph of $g(x) = \sqrt{4 - x^2}$ is the upper half of a circle centered at the origin with radius 2 (**Figure 1.5**; see Appendix B, online at bit.ly/2y3Nck3). From the graph we see that the range of g is $[0, 2]$.

c. The function h is defined for all values of $x \neq 1$, so its domain is $\{x: x \neq 1\}$. Factoring the numerator, we find that

$$h(x) = \frac{x^2 - 3x + 2}{x - 1} = \frac{(x - 1)(x - 2)}{x - 1} = x - 2, \text{ provided } x \neq 1.$$

The graph of $y = h(x)$, shown in **Figure 1.6**, is identical to the graph of the line $y = x - 2$ except that it has a hole at $(1, -1)$ because h is undefined at $x = 1$. Therefore, the range of h is $\{y: y \neq -1\}$. *Related Exercises 23, 25*

EXAMPLE 3 Domain and range in context At time $t = 0$, a stone is thrown vertically upward from the ground at a speed of 30 m/s. Its height h above the ground in meters (neglecting air resistance) is approximated by the function $f(t) = 30t - 5t^2$, where t is measured in seconds. Find the domain and range of f in the context of this particular problem.

SOLUTION Although f is defined for all values of t, the only relevant times are between the time the stone is thrown $(t = 0)$ and the time it strikes the ground, when $h = 0$. Solving the equation $h = 30t - 5t^2 = 0$, we find that

$$30t - 5t^2 = 0$$
$$5t(6 - t) = 0 \quad \text{Factor.}$$
$$5t = 0 \text{ or } 6 - t = 0 \quad \text{Set each factor equal to 0.}$$
$$t = 0 \text{ or } t = 6. \quad \text{Solve.}$$

Therefore, the stone leaves the ground at $t = 0$ and returns to the ground at $t = 6$. An appropriate domain that fits the context of this problem is $\{t: 0 \leq t \leq 6\}$. The range consists of all values of $h = 30t - 5t^2$ as t varies over $[0, 6]$. The largest value of h occurs when the stone reaches its highest point at $t = 3$ (halfway through its flight), which is $h = f(3) = 45$. Therefore, the range is $[0, 45]$. These observations are confirmed by the graph of the height function (**Figure 1.7**). Note that this graph is *not* the trajectory of the stone; the stone moves vertically.

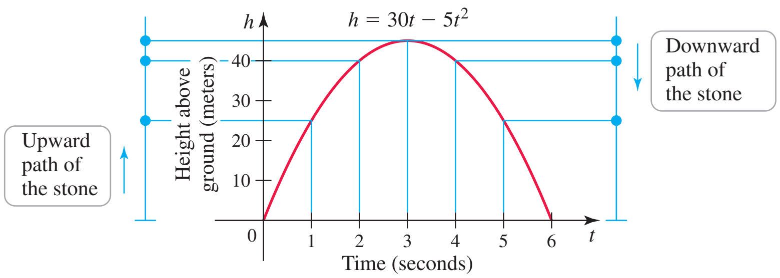


Figure 1.7

Related Exercises 8–9

QUICK CHECK 2 State the domain and range of $f(x) = (x^2 + 1)^{-1}$.

Composite Functions

Functions may be combined using sums $(f + g)$, differences $(f - g)$, products (fg), or quotients (f/g). The process called *composition* also produces new functions.

➤ In the composition $y = f(g(x))$, f is the *outer function* and g is the *inner function*.

DEFINITION Composite Functions

Given two functions f and g, the composite function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$. It is evaluated in two steps: $y = f(u)$, where $u = g(x)$. The domain of $f \circ g$ consists of all x in the domain of g such that $u = g(x)$ is in the domain of f (**Figure 1.8**).

➤ Three different notations for intervals on the real number line will be used throughout the text:
- $[-2, 3)$ is an example of interval notation,
- $-2 \le x < 3$ is inequality notation, and
- $\{x: -2 \le x < 3\}$ is set notation.

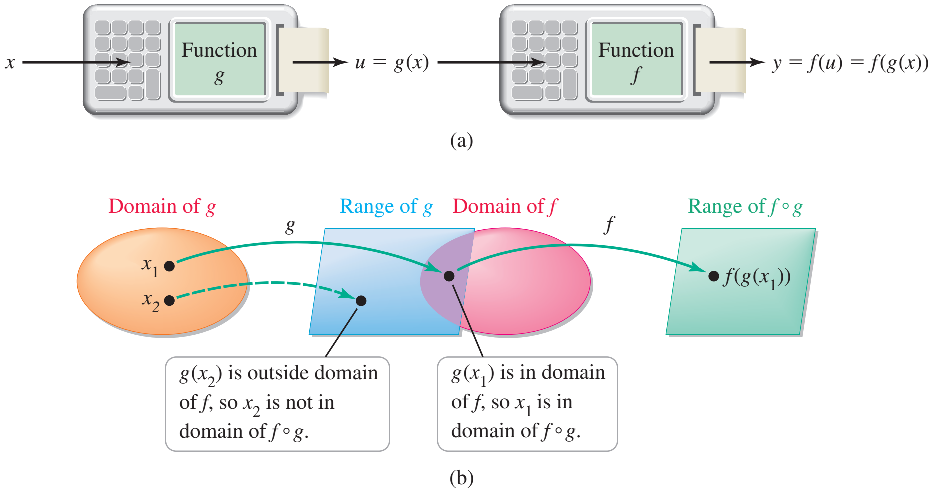


Figure 1.8

EXAMPLE 4 Using graphs to evaluate composite functions Use the graphs of f and g in **Figure 1.9** to find the following values.

a. $f(g(3))$ **b.** $g(f(3))$ **c.** $f(f(4))$ **d.** $f(g(f(8)))$

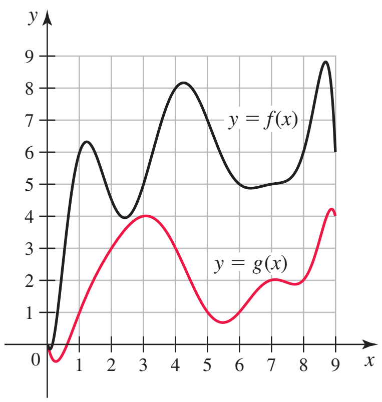


Figure 1.9

SOLUTION

a. The graphs indicate that $g(3) = 4$ and $f(4) = 8$, so $f(g(3)) = f(4) = 8$.

b. We see that $g(f(3)) = g(5) = 1$. Observe that $f(g(3)) \neq g(f(3))$.

c. In this case, $f(\underbrace{f(4)}_{8}) = f(8) = 6$.

d. Starting on the inside,

$$f(g(\underbrace{f(8)}_{6})) = f(\underbrace{g(6)}_{1}) = f(1) = 6.$$

Related Exercise 15 ◄

EXAMPLE 5 Using a table to evaluate composite functions Use the function values in the table to evaluate the following composite functions.

a. $(f \circ g)(0)$ **b.** $g(f(-1))$ **c.** $f(g(g(-1)))$

x	-2	-1	0	1	2
$f(x)$	0	1	3	4	2
$g(x)$	-1	0	-2	-3	-4

SOLUTION

a. Using the table, we see that $g(0) = -2$ and $f(-2) = 0$. Therefore, $(f \circ g)(0) = 0$.

b. Because $f(-1) = 1$ and $g(1) = -3$, it follows that $g(f(-1)) = -3$.

c. Starting with the inner function,

$$f(g(\underbrace{g(-1)}_{0})) = f(\underbrace{g(0)}_{-2}) = f(-2) = 0.$$

Related Exercise 16 ◄

EXAMPLE 6 **Composite functions and notation** Let $f(x) = 3x^2 - x$ and $g(x) = 1/x$. Simplify the following expressions.

a. $f(5p + 1)$ **b.** $g(1/x)$ **c.** $f(g(x))$ **d.** $g(f(x))$

SOLUTION In each case, the functions work on their arguments.

a. The argument of f is $5p + 1$, so

$$f(5p + 1) = 3(5p + 1)^2 - (5p + 1) = 75p^2 + 25p + 2.$$

b. Because g requires taking the reciprocal of the argument, we take the reciprocal of $1/x$ and find that $g(1/x) = 1/(1/x) = x$.

c. The argument of f is $g(x)$, so

$$f(g(x)) = f\left(\frac{1}{x}\right) = 3\left(\frac{1}{x}\right)^2 - \left(\frac{1}{x}\right) = \frac{3}{x^2} - \frac{1}{x} = \frac{3 - x}{x^2}.$$

➤ Examples 6c and 6d demonstrate that, in general,

$$f(g(x)) \neq g(f(x)).$$

d. The argument of g is $f(x)$, so

$$g(f(x)) = g(3x^2 - x) = \frac{1}{3x^2 - x}.$$

Related Exercises 33–37 ◄

EXAMPLE 7 **Working with composite functions** Identify possible choices for the inner and outer functions in the following composite functions. Give the domain of the composite function.

a. $h(x) = \sqrt{9x - x^2}$ **b.** $h(x) = \dfrac{2}{(x^2 - 1)^3}$

SOLUTION

a. An obvious outer function is $f(x) = \sqrt{x}$, which works on the inner function $g(x) = 9x - x^2$. Therefore, h can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of x such that $9x - x^2 \geq 0$. Solving this inequality gives $\{x: 0 \leq x \leq 9\}$ as the domain of $f \circ g$.

➤ Techniques for solving inequalities are discussed in Appendix B, online at bit.ly/2y3Nck3.

b. A good choice for an outer function is $f(x) = 2/x^3 = 2x^{-3}$, which works on the inner function $g(x) = x^2 - 1$. Therefore, h can be expressed as $h = f \circ g$ or $h(x) = f(g(x))$. The domain of $f \circ g$ consists of all values of $g(x)$ such that $g(x) \neq 0$, which is $\{x: x \neq \pm 1\}$.

Related Exercises 44–45 ◄

EXAMPLE 8 **More composite functions** Given $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - x - 6$, find the following composite functions and their domains.

a. $g \circ f$ **b.** $g \circ g$

SOLUTION

a. We have

$$(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x}) = \underbrace{(\sqrt[3]{x})}_{f(x)}{}^2 - \underbrace{\sqrt[3]{x}}_{f(x)} - 6 = x^{2/3} - x^{1/3} - 6.$$

Because the domains of f and g are $(-\infty, \infty)$, the domain of $f \circ g$ is also $(-\infty, \infty)$.

b. In this case, we have the composition of two polynomials:

$$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= g(x^2 - x - 6) \\ &= \underbrace{(x^2 - x - 6)}_{g(x)}{}^2 - \underbrace{(x^2 - x - 6)}_{g(x)} - 6 \\ &= x^4 - 2x^3 - 12x^2 + 13x + 36.\end{aligned}$$

The domain of the composition of two polynomials is $(-\infty, \infty)$.

Related Exercises 47–48 ◄

QUICK CHECK 3 If $f(x) = x^2 + 1$ and $g(x) = x^2$, find $f \circ g$ and $g \circ f$. ◄

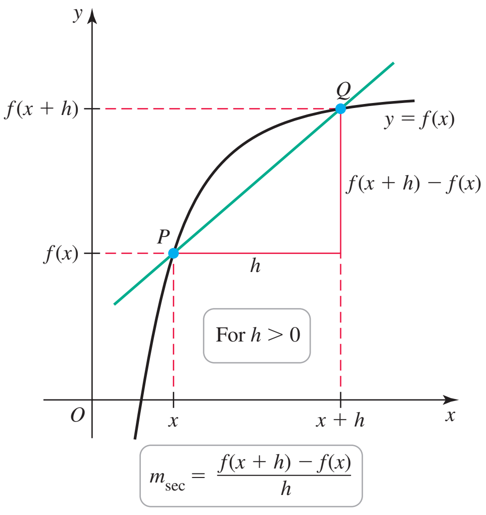


Figure 1.10

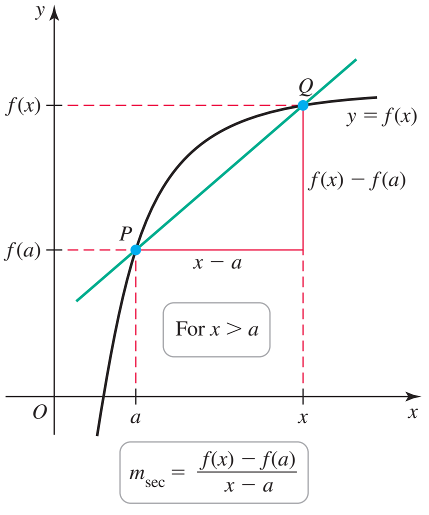


Figure 1.11

➤ Treat $f(x+h)$ like the composition $f(g(x))$, where $x+h$ plays the role of $g(x)$. It may help to establish a pattern in your mind before evaluating $f(x+h)$. For instance, using the function in Example 9a, we have

$$f(x) = 3x^2 - x;$$
$$f(12) = 3 \cdot 12^2 - 12;$$
$$f(b) = 3b^2 - b;$$
$$f(\text{math}) = 3 \cdot \text{math}^2 - \text{math};$$

therefore,

$$f(x+h) = 3(x+h)^2 - (x+h).$$

➤ See the front papers of this text for a review of factoring formulas.

Secant Lines and the Difference Quotient

As you will see shortly, slopes of lines and curves play a fundamental role in calculus. **Figure 1.10** shows two points $P(x, f(x))$ and $Q(x+h, f(x+h))$ on the graph of $y = f(x)$ in the case that $h > 0$. A line through any two points on a curve is called a **secant line**; its importance in the study of calculus is explained in Chapters 2 and 3. For now, we focus on the slope of the secant line through P and Q, which is denoted m_{sec} and is given by

$$m_{\text{sec}} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}.$$

The slope formula $\dfrac{f(x+h) - f(x)}{h}$ is also known as a **difference quotient**, and it can be expressed in several ways depending on how the coordinates of P and Q are labeled. For example, given the coordinates $P(a, f(a))$ and $Q(x, f(x))$ (**Figure 1.11**), the difference quotient is

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}.$$

We interpret the slope of the secant line in this form as the **average rate of change** of f over the interval $[a, x]$.

EXAMPLE 9 Working with difference quotients

a. Simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$, for $f(x) = 3x^2 - x$.

b. Simplify the difference quotient $\dfrac{f(x) - f(a)}{x - a}$, for $f(x) = x^3$.

SOLUTION

a. First note that $f(x+h) = 3(x+h)^2 - (x+h)$. We substitute this expression into the difference quotient and simplify:

$$\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{3(x+h)^2 - (x+h)}^{f(x+h)} - \overbrace{(3x^2 - x)}^{f(x)}}{h}$$

$$= \frac{3(x^2 + 2xh + h^2) - (x+h) - (3x^2 - x)}{h} \quad \text{Expand } (x+h)^2.$$

$$= \frac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h} \quad \text{Distribute.}$$

$$= \frac{6xh + 3h^2 - h}{h} \quad \text{Simplify.}$$

$$= \frac{h(6x + 3h - 1)}{h} = 6x + 3h - 1. \quad \text{Factor and simplify.}$$

b. The factoring formula for the difference of perfect cubes is needed:

$$\frac{f(x) - f(a)}{x - a} = \frac{x^3 - a^3}{x - a}$$

$$= \frac{(x-a)(x^2 + ax + a^2)}{x - a} \quad \text{Factoring formula}$$

$$= x^2 + ax + a^2. \quad \text{Simplify.}$$

Related Exercises 66, 72 ◄

EXAMPLE 10 Interpreting the slope of the secant line The position of a hiker on a trail at various times t is recorded by a GPS watch worn by the hiker. These data are then uploaded to a computer to produce the graph of the distance function $d = f(t)$ shown in Figure 1.12, where d measures the distance traveled on the trail in miles and t is the elapsed time in hours from the beginning of the hike.

a. Find the slope of the secant line that passes through the points on the graph corresponding to the trail segment between milepost 3 and milepost 5, and interpret the result.

b. Estimate the slope of the secant line that passes through points A and B in Figure 1.12, and compare it to the slope of the secant line found in part (a).

➤ Figure 1.12 contains actual GPS data collected in Rocky Mountain National Park. See Exercises 75–76 for another look at the data set.

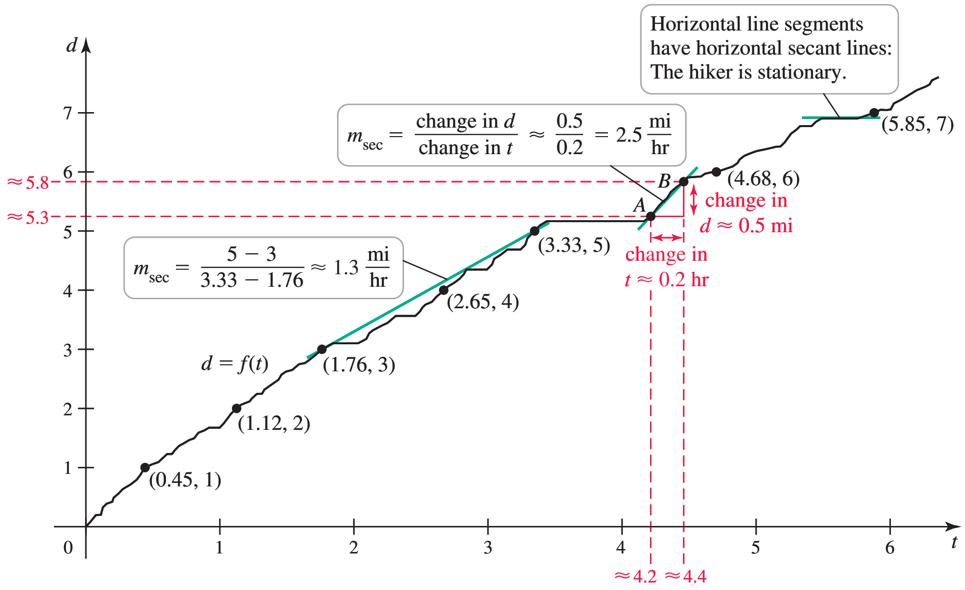


Figure 1.12

SOLUTION

a. We see from the graph of $d = f(t)$ that 1.76 hours (about 1 hour and 46 minutes) has elapsed when the hiker arrives at milepost 3, while milepost 5 is reached 3.33 hours into the hike. This information is also expressed as $f(1.76) = 3$ and $f(3.33) = 5$. To find the slope of the secant line through these points, we compute the change in distance divided by the change in time:

$$m_{sec} = \frac{f(3.33) - f(1.76)}{3.33 - 1.76} = \frac{5 - 3}{3.33 - 1.76} \approx 1.3 \frac{\text{mi}}{\text{hr}}.$$

The units provide a clue about the physical meaning of the slope: It measures the average rate at which the distance changes per hour, which is the average speed of the hiker. In this case, the hiker walks with an average speed of approximately 1.3 mi/hr between mileposts 3 and 5.

b. From the graph we see that the coordinates of points A and B are approximately $(4.2, 5.3)$ and $(4.4, 5.8)$, respectively, which implies the hiker walks $5.8 - 5.3 = 0.5$ mi in $4.4 - 4.2 = 0.2$ hr. The slope of the secant line through A and B is

$$m_{sec} = \frac{\text{change in } d}{\text{change in } t} \approx \frac{0.5}{0.2} = 2.5 \frac{\text{mi}}{\text{hr}}.$$

For this segment of the trail, the hiker walks at an average speed of about 2.5 mi/hr, nearly twice as fast as the average speed computed in part (a). Expressed another way, steep sections of the distance curve yield steep secant lines, which correspond to faster average hiking speeds. Conversely, any secant line with slope equal to 0 corresponds to an average speed of 0. Looking one last time at Figure 1.12, we can identify the time intervals during which the hiker was resting alongside the trail—whenever the distance curve is horizontal, the hiker is not moving.

Related Exercise 75 ➤

QUICK CHECK 4 Refer to Figure 1.12. Find the hiker's average speed during the first mile of the trail and then determine the hiker's average speed in the time interval from 3.9 to 4.1 hours. ➤

Symmetry

The word *symmetry* has many meanings in mathematics. Here we consider symmetries of graphs and the relations they represent. Taking advantage of symmetry often saves time and leads to insights.

DEFINITION Symmetry in Graphs

A graph is **symmetric with respect to the y-axis** if whenever the point (x, y) is on the graph, the point $(-x, y)$ is also on the graph. This property means that the graph is unchanged when reflected across the y-axis (**Figure 1.13a**).

A graph is **symmetric with respect to the x-axis** if whenever the point (x, y) is on the graph, the point $(x, -y)$ is also on the graph. This property means that the graph is unchanged when reflected across the x-axis (**Figure 1.13b**).

A graph is **symmetric with respect to the origin** if whenever the point (x, y) is on the graph, the point $(-x, -y)$ is also on the graph (**Figure 1.13c**). Symmetry about both the x- and y-axes implies symmetry about the origin, but not vice versa.

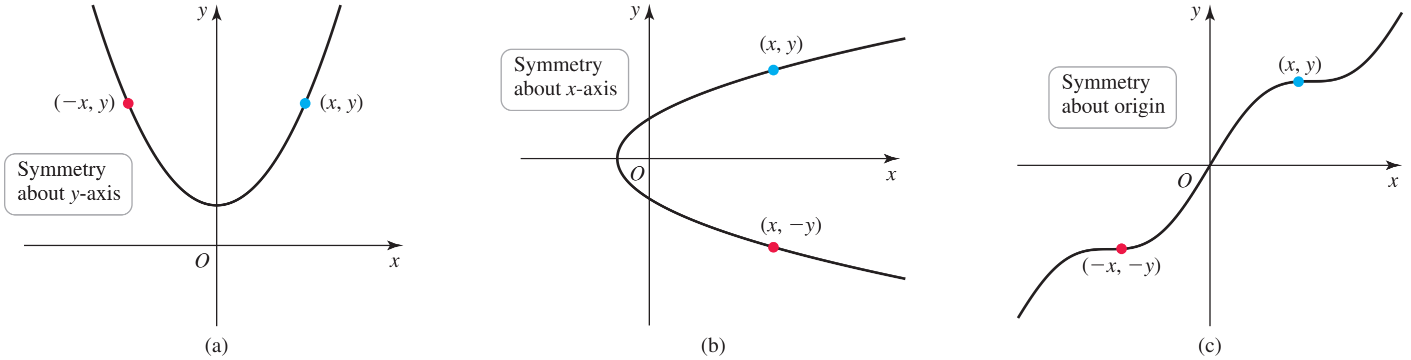


Figure 1.13

DEFINITION Symmetry in Functions

An **even function** f has the property that $f(-x) = f(x)$, for all x in the domain. The graph of an even function is symmetric about the y-axis.

An **odd function** f has the property that $f(-x) = -f(x)$, for all x in the domain. The graph of an odd function is symmetric about the origin.

Polynomials consisting of only even powers of the variable (of the form x^{2n}, where n is a nonnegative integer) are even functions. Polynomials consisting of only odd powers of the variable (of the form x^{2n+1}, where n is a nonnegative integer) are odd functions.

QUICK CHECK 5 Explain why the graph of a nonzero function is never symmetric with respect to the x-axis. ➤

EXAMPLE 11 **Identifying symmetry in functions** Identify the symmetry, if any, in the following functions.

a. $f(x) = x^4 - 2x^2 - 20$ **b.** $g(x) = x^3 - 3x + 1$ **c.** $h(x) = \dfrac{1}{x^3 - x}$

SOLUTION

a. The function f consists of only even powers of x (where $20 = 20 \cdot 1 = 20x^0$ and x^0 is considered an even power). Therefore, f is an even function (Figure 1.14). This fact is verified by showing that $f(-x) = f(x)$:

$$f(-x) = (-x)^4 - 2(-x)^2 - 20 = x^4 - 2x^2 - 20 = f(x).$$

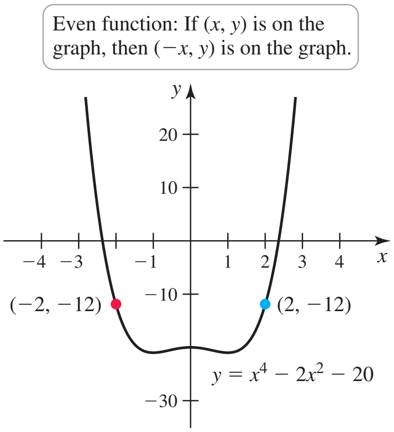


Figure 1.14

b. The function g consists of two odd powers and one even power (again, $1 = x^0$ is an even power). Therefore, we expect that g has no symmetry about the y-axis or the origin (Figure 1.15). Note that

$$g(-x) = (-x)^3 - 3(-x) + 1 = -x^3 + 3x + 1,$$

so $g(-x)$ equals neither $g(x)$ nor $-g(x)$; therefore, g has no symmetry.

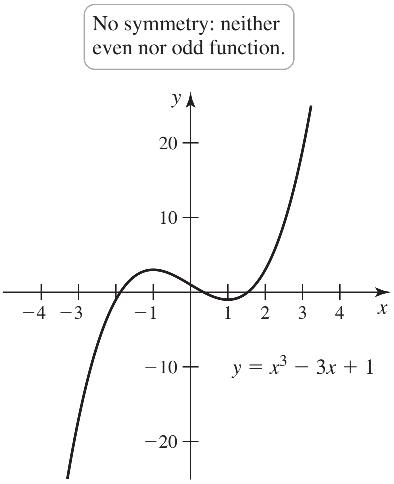


Figure 1.15

c. In this case, h is a composition of an odd function $f(x) = 1/x$ with an odd function $g(x) = x^3 - x$. Note that

$$h(-x) = \frac{1}{(-x)^3 - (-x)} = -\frac{1}{x^3 - x} = -h(x).$$

Because $h(-x) = -h(x)$, h is an odd function (Figure 1.16).

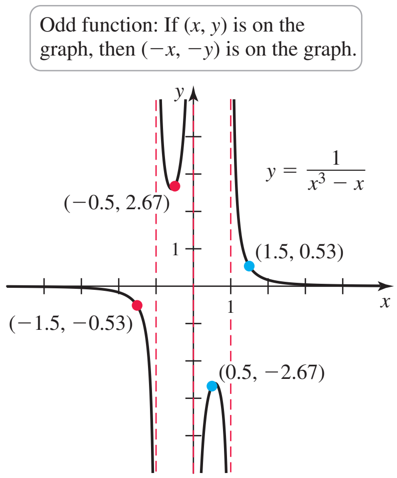


Figure 1.16

➤ The symmetry of compositions of even and odd functions is considered in Exercises 101–104.

Related Exercises 79–81 ◄

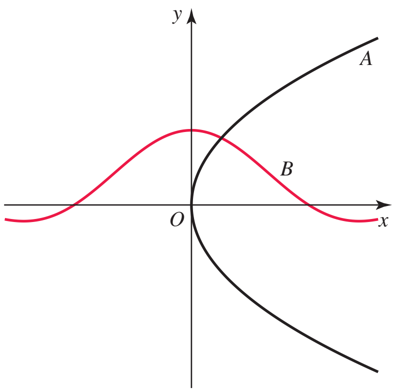
SECTION 1.1 EXERCISES

Getting Started

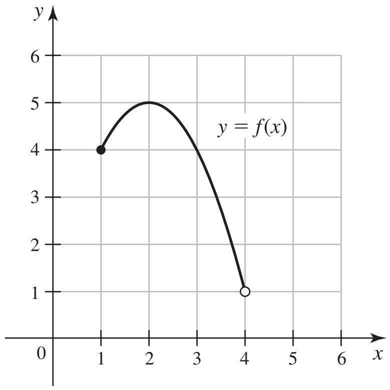
1. Use the terms *domain*, *range*, *independent variable*, and *dependent variable* to explain how a function relates one variable to another variable.

2. Is the independent variable of a function associated with the domain or range? Is the dependent variable associated with the domain or range?

3. Decide whether graph A, graph B, or both represent functions.



4. The entire graph of f is given. State the domain and range of f.



5. Which statement about a function is true? (i) For each value of x in the domain, there corresponds one unique value of y in the range; (ii) for each value of y in the range, there corresponds one unique value of x in the domain. Explain.

6. Determine the domain and range of $g(x) = \dfrac{x^2 - 1}{x - 1}$. Sketch a graph of g.

7. Determine the domain and range of $f(x) = 3x^2 - 10$.

8. **Throwing a stone** A stone is thrown vertically upward from the ground at a speed of 40 m/s at time $t = 0$. Its distance d (in meters) above the ground (neglecting air resistance) is approximated by the function $f(t) = 40t - 5t^2$. Determine an appropriate domain for this function. Identify the independent and dependent variables.

9. **Water tower** A cylindrical water tower with a radius of 10 m and a height of 50 m is filled to a height of h m. The volume V of water (in cubic meters) is given by the function $g(h) = 100\pi h$. Identify the independent and dependent variables for this function, and then determine an appropriate domain.

10. Let $f(x) = 1/(x^3 + 1)$. Compute $f(2)$ and $f(y^2)$.

11. Let $f(x) = 2x + 1$ and $g(x) = 1/(x - 1)$. Simplify the expressions $f(g(1/2))$, $g(f(4))$, and $g(f(x))$.

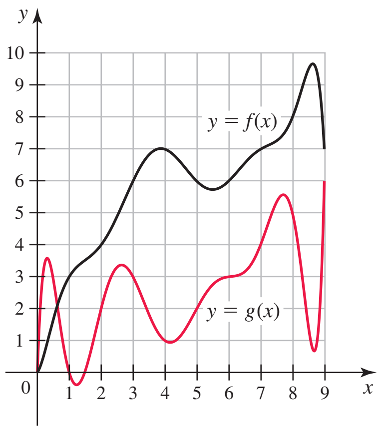
12. Find functions f and g such that $f(g(x)) = (x^2 + 1)^5$. Find a different pair of functions f and g that also satisfy $f(g(x)) = (x^2 + 1)^5$.

13. Explain how to find the domain of $f \circ g$ if you know the domain and range of f and g.

14. If $f(x) = \sqrt{x}$ and $g(x) = x^3 - 2$, simplify the expressions $(f \circ g)(3)$, $(f \circ f)(64)$, $(g \circ f)(x)$, and $(f \circ g)(x)$.

15. Use the graphs of f and g in the figure to determine the following function values.

a. $(f \circ g)(2)$ **b.** $g(f(2))$
c. $f(g(4))$ **d.** $g(f(5))$
e. $f(f(8))$ **f.** $g(f(g(5)))$



16. Use the table to evaluate the given compositions.

x	-1	0	1	2	3	4
$f(x)$	3	1	0	-1	-3	-1
$g(x)$	-1	0	2	3	4	5
$h(x)$	0	-1	0	3	0	4

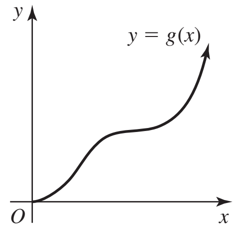
a. $h(g(0))$ **b.** $g(f(4))$
c. $h(h(0))$ **d.** $g(h(f(4)))$
e. $f(f(f(1)))$ **f.** $h(h(h(0)))$
g. $f(h(g(2)))$ **h.** $g(f(h(4)))$
i. $g(g(g(1)))$ **j.** $f(f(h(3)))$

17. **Rising radiosonde** The National Weather Service releases approximately 70,000 radiosondes every year to collect data from the atmosphere. Attached to a balloon, a radiosonde rises at about 1000 ft/min until the balloon bursts in the upper atmosphere. Suppose a radiosonde is released from a point 6 ft above the ground and that 5 seconds later, it is 83 ft above the ground. Let $f(t)$ represent the height (in feet) that the radiosonde is above the ground t seconds after it is released. Evaluate $\dfrac{f(5) - f(0)}{5 - 0}$ and interpret the meaning of this quotient.

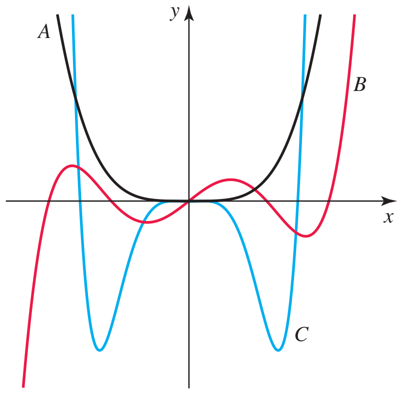
18. **World record free fall** On October 14, 2012, Felix Baumgartner stepped off a balloon capsule at an altitude of 127,852.4 feet and began his free fall. It is claimed that Felix reached the speed of sound 34 seconds into his fall at an altitude of 109,731 feet and that he continued to fall at supersonic speed for 30 seconds until he was at an altitude of 75,330.4 feet. Let $f(t)$ equal the distance that Felix had fallen t seconds after leaving his capsule. Calculate $f(0)$, $f(34)$, $f(64)$, and his average supersonic speed $\dfrac{f(64) - f(34)}{64 - 34}$ (in ft/s) over the time interval $[34, 64]$. (*Source:* http://www.redbullstratos.com)

19. Suppose f is an even function with $f(2) = 2$ and g is an odd function with $g(2) = -2$. Evaluate $f(-2)$, $g(-2)$, $f(g(2))$, and $g(f(-2))$.

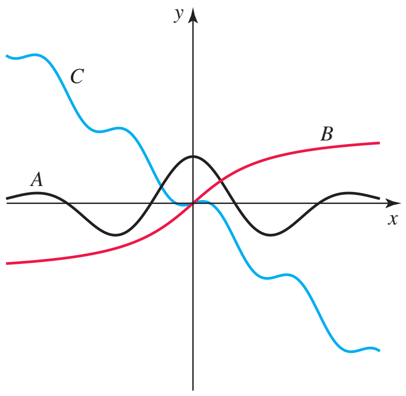
20. Complete the left half of the graph of g if g is an odd function.



21. State whether the functions represented by graphs A, B, and C in the figure are even, odd, or neither.



22. State whether the functions represented by graphs A, B, and C in the figure are even, odd, or neither.



Practice Exercises

23–26. Domain and range *State the domain and range of the function.*

23. $f(x) = \dfrac{x^2 - 5x + 6}{x - 2}$ **24.** $f(x) = \dfrac{x - 2}{2 - x}$

25. $f(x) = \sqrt{7 - x^2}$ **26.** $f(x) = -\sqrt{25 - x^2}$

27–30. Domain *State the domain of the function.*

27. $h(u) = \sqrt[3]{u - 1}$ **28.** $F(w) = \sqrt[4]{2 - w}$

29. $f(x) = (9 - x^2)^{3/2}$ **30.** $g(t) = \dfrac{1}{1 + t^2}$

T 31. Launching a rocket A small rocket is launched vertically upward from the edge of a cliff 80 ft above the ground at a speed of 96 ft/s. Its height (in feet) above the ground is given by $h(t) = -16t^2 + 96t + 80$, where t represents time measured in seconds.

a. Assuming the rocket is launched at $t = 0$, what is an appropriate domain for h?

b. Graph h and determine the time at which the rocket reaches its highest point. What is the height at that time?

32. Draining a tank (Torricelli's law) A cylindrical tank with a cross-sectional area of 10 m^2 is filled to a depth of 25 m with water. At $t = 0$ s, a drain in the bottom of the tank with an area of 1 m^2 is opened, allowing water to flow out of the tank. The depth of water in the tank (in meters) at time $t \geq 0$ is $d(t) = (5 - 0.22t)^2$.

a. Check that $d(0) = 25$, as specified.

b. At what time is the tank empty?

c. What is an appropriate domain for d?

33–42. Composite functions and notation *Let $f(x) = x^2 - 4$, $g(x) = x^3$, and $F(x) = 1/(x - 3)$. Simplify or evaluate the following expressions.*

33. $g(1/z)$ **34.** $F(y^4)$

35. $F(g(y))$ **36.** $f(g(w))$

37. $g(f(u))$ **38.** $\dfrac{f(2 + h) - f(2)}{h}$

39. $F(F(x))$ **40.** $g(F(f(x)))$

41. $f(\sqrt{x + 4})$ **42.** $F\left(\dfrac{3x + 1}{x}\right)$

43–46. Working with composite functions *Find possible choices for outer and inner functions f and g such that the given function h equals $f \circ g$.*

43. $h(x) = (x^3 - 5)^{10}$ **44.** $h(x) = \dfrac{2}{(x^6 + x^2 + 1)^2}$

45. $h(x) = \sqrt{x^4 + 2}$ **46.** $h(x) = \dfrac{1}{\sqrt{x^3 - 1}}$

47–54. More composite functions *Let $f(x) = |x|$, $g(x) = x^2 - 4$, $F(x) = \sqrt{x}$, and $G(x) = 1/(x - 2)$. Determine the following composite functions and give their domains.*

47. $f \circ g$ **48.** $g \circ f$

49. $f \circ G$ **50.** $f \circ g \circ G$

51. $G \circ g \circ f$ **52.** $g \circ F \circ F$

53. $g \circ g$ **54.** $G \circ G$

55–60. Missing piece *Let $g(x) = x^2 + 3$. Find a function f that produces the given composition.*

55. $(f \circ g)(x) = x^2$ **56.** $(f \circ g)(x) = \dfrac{1}{x^2 + 3}$

57. $(f \circ g)(x) = x^4 + 6x^2 + 9$ **58.** $(f \circ g)(x) = x^4 + 6x^2 + 20$

59. $(g \circ f)(x) = x^4 + 3$ **60.** $(g \circ f)(x) = x^{2/3} + 3$

61. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. The range of $f(x) = 2x - 38$ is all real numbers.
b. The relation $y = x^6 + 1$ is *not* a function because $y = 2$ for both $x = -1$ and $x = 1$.
c. If $f(x) = x^{-1}$, then $f(1/x) = 1/f(x)$.
d. In general, $f(f(x)) = (f(x))^2$.
e. In general, $f(g(x)) = g(f(x))$.
f. By definition, $f(g(x)) = (f \circ g)(x)$.
g. If $f(x)$ is an even function, then $cf(ax)$ is an even function, where a and c are nonzero real numbers.
h. If $f(x)$ is an odd function, then $f(x) + d$ is an odd function, where d is a nonzero real number.
i. If f is both even *and* odd, then $f(x) = 0$ for all x.

62–68. Working with difference quotients *Simplify the difference quotient* $\dfrac{f(x+h) - f(x)}{h}$ *for the following functions.*

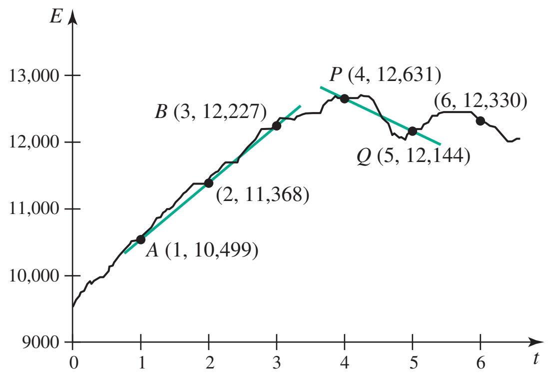
62. $f(x) = 10$
63. $f(x) = 3x$
64. $f(x) = 4x - 3$
65. $f(x) = x^2$
66. $f(x) = 2x^2 - 3x + 1$
67. $f(x) = \dfrac{2}{x}$
68. $f(x) = \dfrac{x}{x+1}$

69–74. Working with difference quotients *Simplify the difference quotient* $\dfrac{f(x) - f(a)}{x - a}$ *for the following functions.*

69. $f(x) = x^2 + x$
70. $f(x) = 4 - 4x - x^2$
71. $f(x) = x^3 - 2x$
72. $f(x) = x^4$
73. $f(x) = -\dfrac{4}{x^2}$
74. $f(x) = \dfrac{1}{x} - x^2$

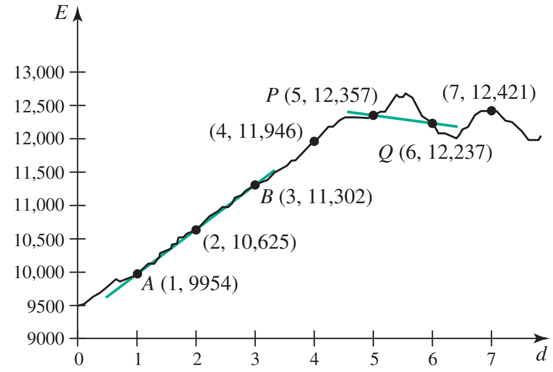
75. GPS data A GPS device tracks the elevation E (in feet) of a hiker walking in the mountains. The elevation t hours after beginning the hike is given in the figure.

a. Find the slope of the secant line that passes through points A and B. Interpret your answer as an average rate of change over the interval $1 \le t \le 3$.
b. Repeat the procedure outlined in part (a) for the secant line that passes through points P and Q.
c. Notice that the curve in the figure is horizontal for an interval of time near $t = 5.5$ hr. Give a plausible explanation for the horizontal line segment.



76. Elevation vs. Distance The following graph, obtained from GPS data, shows the elevation of a hiker as a function of the distance d from the starting point of the trail.

a. Find the slope of the secant line that passes through points A and B. Interpret your answer as an average rate of change over the interval $1 \le d \le 3$.
b. Repeat the procedure outlined in part (a) for the secant line that passes through points P and Q.
c. Notice that the elevation function is nearly constant over the segment of the trail from mile $d = 4.5$ to mile $d = 5$. Give a plausible explanation for the horizontal line segment.



77–78. Interpreting the slope of secant lines *In each exercise, a function and an interval of its independent variable are given. The endpoints of the interval are associated with points P and Q on the graph of the function.*

a. *Sketch a graph of the function and the secant line through P and Q.*
b. *Find the slope of the secant line in part (a), and interpret your answer in terms of an average rate of change over the interval. Include units in your answer.*

77. After t seconds, an object dropped from rest falls a distance $d = 16t^2$, where d is measured in feet and $2 \le t \le 5$.

78. The volume V of an ideal gas in cubic centimeters is given by $V = 2/p$, where p is the pressure in atmospheres and $0.5 \le p \le 2$.

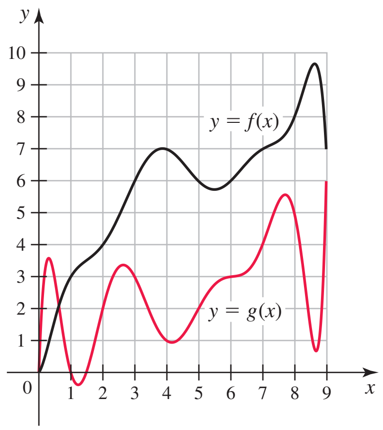
79–86. Symmetry *Determine whether the graphs of the following equations and functions are symmetric about the x-axis, the y-axis, or the origin. Check your work by graphing.*

79. $f(x) = x^4 + 5x^2 - 12$
80. $f(x) = 3x^5 + 2x^3 - x$
81. $f(x) = x^5 - x^3 - 2$
82. $f(x) = 2|x|$
83. $x^{2/3} + y^{2/3} = 1$
84. $x^3 - y^5 = 0$
85. $f(x) = x|x|$
86. $|x| + |y| = 1$

Explorations and Challenges

87. Composition of even and odd functions from graphs Assume f is an even function and g is an odd function. Use the (incomplete) graphs of f and g in the figure to determine the following function values.

a. $f(g(-2))$
b. $g(f(-2))$
c. $f(g(-4))$
d. $g(f(5) - 8)$
e. $g(g(-7))$
f. $f(1 - f(8))$



88. **Composition of even and odd functions from tables** Assume f is an even function, g is an odd function, and both are defined at 0. Use the (incomplete) table to evaluate the given compositions.

x	1	2	3	4
$f(x)$	2	−1	3	−4
$g(x)$	−3	−1	−4	−2

a. $f(g(-1))$ **b.** $g(f(-4))$
c. $f(g(-3))$ **d.** $f(g(-2))$
e. $g(g(-1))$ **f.** $f(g(0) - 1)$
g. $f(g(g(-2)))$ **h.** $g(f(f(-4)))$
i. $g(g(g(-1)))$

89. **Absolute value graphs** Use the definition of absolute value (see Appendix B, online at bit.ly/2y3Nck3) to graph the equation $|x| - |y| = 1$. Use a graphing utility to check your work.

90. **Graphing semicircles** Show that the graph of $f(x) = 10 + \sqrt{-x^2 + 10x - 9}$ is the upper half of a circle. Then determine the domain and range of the function.

91. **Graphing semicircles** Show that the graph of $g(x) = 2 - \sqrt{-x^2 + 6x + 16}$ is the lower half of a circle. Then determine the domain and range of the function.

92. **Even and odd at the origin**

a. If $f(0)$ is defined and f is an even function, is it necessarily true that $f(0) = 0$? Explain.

b. If $f(0)$ is defined and f is an odd function, is it necessarily true that $f(0) = 0$? Explain.

T **93–96. Polynomial calculations** *Find a polynomial f that satisfies the following properties. (Hint: Determine the degree of f; then substitute a polynomial of that degree and solve for its coefficients.)*

93. $f(f(x)) = 9x - 8$

94. $(f(x))^2 = 9x^2 - 12x + 4$

95. $f(f(x)) = x^4 - 12x^2 + 30$

96. $(f(x))^2 = x^4 - 12x^2 + 36$

97–100. Difference quotients *Simplify the difference quotients* $\frac{f(x+h) - f(x)}{h}$ *and* $\frac{f(x) - f(a)}{x - a}$ *by rationalizing the numerator.*

97. $f(x) = \sqrt{x}$

98. $f(x) = \sqrt{1 - 2x}$

99. $f(x) = -\frac{3}{\sqrt{x}}$

100. $f(x) = \sqrt{x^2 + 1}$

101–104. Combining even and odd functions *Let E be an even function and O be an odd function. Determine the symmetry, if any, of the following functions.*

101. $E + O$

102. $E \cdot O$

103. $O \circ E$

104. $E \circ O$

QUICK CHECK ANSWERS

1. $3, x^4 - 2x^2, t^2 - 2t, p^2 - 4p + 3$ **2.** Domain is all real numbers; range is $\{y: 0 < y \leq 1\}$. **3.** $(f \circ g)(x) = x^4 + 1$ and $(g \circ f)(x) = (x^2 + 1)^2$ **4.** Average speed ≈ 2.2 mi/hr for first mile; average speed $= 0$ on $3.9 \leq t \leq 4.1$. **5.** If the graph were symmetric with respect to the x-axis, it would not pass the vertical line test. ➤

1.2 Representing Functions

We consider four approaches to defining and representing functions: formulas, graphs, tables, and words.

Using Formulas

The following list is a brief catalog of the families of functions that are introduced in this chapter and studied systematically throughout this text; they are all defined by *formulas*.

➤ One version of the Fundamental Theorem of Algebra states that a nonzero polynomial of degree n has exactly n (possibly complex) roots, counting each root up to its multiplicity.

1. **Polynomials** are functions of the form

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where the **coefficients** $a_0, a_1, \ldots, a_n$ are real numbers with $a_n \neq 0$ and the nonnegative integer n is the **degree** of the polynomial. The domain of any polynomial is the set of all real numbers. An nth-degree polynomial can have as many as n real **zeros** or **roots**—values of x at which $p(x) = 0$; the zeros are points at which the graph of p intersects the x-axis.

2. **Rational functions** are ratios of the form $f(x) = p(x)/q(x)$, where p and q are polynomials. Because division by zero is prohibited, the domain of a rational function is the set of all real numbers except those for which the denominator is zero.
3. **Algebraic functions** are constructed using the operations of algebra: addition, subtraction, multiplication, division, and roots. Examples of algebraic functions are $f(x) = \sqrt{2x^3 + 4}$ and $g(x) = x^{1/4}(x^3 + 2)$. In general, if an even root (square root, fourth root, and so forth) appears, then the domain does not contain points at which the quantity under the root is negative (and perhaps other points).

➤ Exponential and logarithmic functions are introduced in Section 1.3.

4. **Exponential functions** have the form $f(x) = b^x$, where the base $b \neq 1$ is a positive real number. Closely associated with exponential functions are **logarithmic functions** of the form $f(x) = \log_b x$, where $b > 0$ and $b \neq 1$. Exponential functions have a domain consisting of all real numbers. Logarithmic functions are defined for positive real numbers.

   The **natural exponential function** is $f(x) = e^x$, with base $b = e$, where $e \approx 2.71828\ldots$ is one of the fundamental constants of mathematics. Associated with the natural exponential function is the **natural logarithm function** $f(x) = \ln x$, which also has the base $b = e$.

➤ Trigonometric functions and their inverses are introduced in Section 1.4.

5. The **trigonometric functions** are $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, and $\csc x$; they are fundamental to mathematics and many areas of application. Also important are their relatives, the **inverse trigonometric functions**.
6. Trigonometric, exponential, and logarithmic functions are a few examples of a large family called **transcendental functions**. Figure 1.17 shows the organization of these functions, which are explored in detail in upcoming chapters.

QUICK CHECK 1 Are all polynomials rational functions? Are all algebraic functions polynomials? ➤

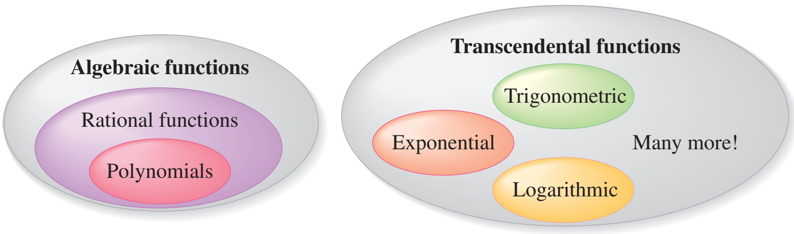


Figure 1.17

Using Graphs

Although formulas are the most compact way to represent many functions, graphs often provide the most illuminating representations. Two of countless examples of functions and their graphs are shown in Figure 1.18. Much of this text is devoted to creating and analyzing graphs of functions.

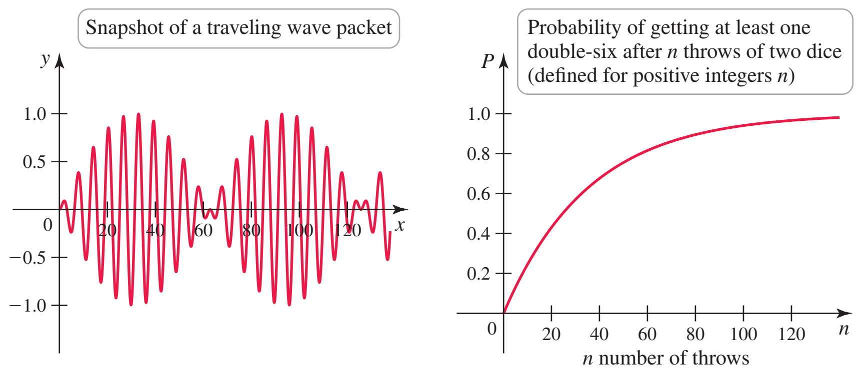


Figure 1.18

There are two approaches to graphing functions.

- Graphing calculators and graphing software are easy to use and powerful. Such **technology** easily produces graphs of most functions encountered in this text. We assume you know how to use a graphing utility.
- Graphing utilities, however, are not infallible. Therefore, you should also strive to master **analytical methods** (pencil-and-paper methods) in order to analyze functions and make accurate graphs by hand. Analytical methods rely heavily on calculus and are presented throughout this text.

The important message is this: Both technology and analytical methods are essential and must be used together in an integrated way to produce accurate graphs.

Linear Functions One form of the equation of a line (see Appendix B, online at bit.ly/2y3Nck3) is $y = mx + b$, where m and b are constants. Therefore, the function $f(x) = mx + b$ has a straight-line graph and is called a **linear function**.

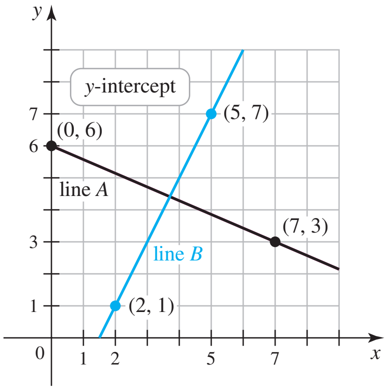


Figure 1.19

EXAMPLE 1 **Linear functions and their graphs** Determine the function represented by **(a)** line A in Figure 1.19 and **(b)** line B in Figure 1.19.

SOLUTION

a. From the graph, we see that the y-intercept of line A is $(0, 6)$. Using the points $(0, 6)$ and $(7, 3)$, we find that the slope of the line is

$$m = \frac{3 - 6}{7 - 0} = -\frac{3}{7}.$$

Therefore, the line is described by the function $f(x) = -\frac{3}{7}x + 6$.

b. Line B passes through the points $(2, 1)$ and $(5, 7)$, so the slope of this line is

$$m = \frac{7 - 1}{5 - 2} = 2,$$

which implies that the line is described by a function of the form $g(x) = 2x + b$. To determine the value of b, note that $g(2) = 1$:

$$g(2) = 2 \cdot 2 + b = 1.$$

Solving for b, we find that $b = 1 - 4 = -3$. Therefore, the line is described by the function $g(x) = 2x - 3$.

Related Exercises 3, 15–16 ◄

➤ In the solution to Example 1b, we used the point $(2, 1)$ to determine the value of b. Using the point $(5, 7)$—or, equivalently, the fact that $g(5) = 7$—leads to the same result.

EXAMPLE 2 **Demand function for pizzas** After studying sales for several months, the owner of a pizza chain knows that the number of two-topping pizzas sold in a week (called the *demand*) decreases as the price increases. Specifically, her data indicate that at a price of \$14 per pizza, an average of 400 pizzas are sold per week, while at a price of \$17 per pizza, an average of 250 pizzas are sold per week. Assume the demand d is a *linear* function of the price p.

a. Find the constants m and b in the demand function $d = f(p) = mp + b$. Then graph f.

b. According to this model, how many pizzas (on average) are sold per week at a price of \$20?

SOLUTION

a. Two points on the graph of the demand function are given: $(p, d) = (14, 400)$ and $(17, 250)$. Therefore, the slope of the demand line is

$$m = \frac{400 - 250}{14 - 17} = -50 \text{ pizzas per dollar.}$$

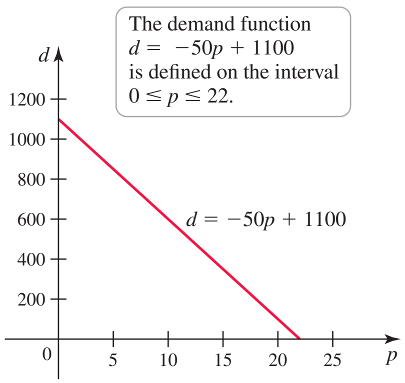


Figure 1.20

➤ The units of the slope have meaning: For every dollar the price is reduced, an average of 50 more pizzas can be sold.

It follows that the equation of the linear demand function is

$$d - 250 = -50(p - 17).$$

Expressing d as a function of p, we have $d = f(p) = -50p + 1100$ (Figure 1.20).

b. Using the demand function with a price of \$20, the average number of pizzas that could be sold per week is $f(20) = 100$.

Related Exercise 21 ◄

Piecewise Functions A function may have different definitions on different parts of its domain. For example, income tax is levied in tax brackets that have different tax rates. Functions that have different definitions on different parts of their domain are called **piecewise functions**. If all the pieces are linear, the function is **piecewise linear**. Here are some examples.

EXAMPLE 3 **Defining a piecewise function** The graph of a piecewise linear function g is shown in Figure 1.21. Find a formula for the function.

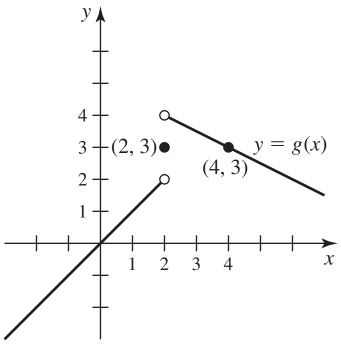


Figure 1.21

SOLUTION For $x < 2$, the graph is linear with a slope of 1 and a y-intercept of $(0, 0)$; its equation is $y = x$. For $x > 2$, the slope of the line is $-\frac{1}{2}$ and it passes through $(4, 3)$; an equation of this piece of the function is

$$y - 3 = -\frac{1}{2}(x - 4) \quad \text{or} \quad y = -\frac{1}{2}x + 5.$$

For $x = 2$, we have $g(2) = 3$. Therefore,

$$g(x) = \begin{cases} x & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ -\frac{1}{2}x + 5 & \text{if } x > 2. \end{cases}$$

Related Exercises 25–26 ◄

EXAMPLE 4 **Graphing piecewise functions** Graph the following functions.

a. $f(x) = \begin{cases} \dfrac{x^2 - 5x + 6}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

b. $f(x) = |x|$, the **absolute value** function

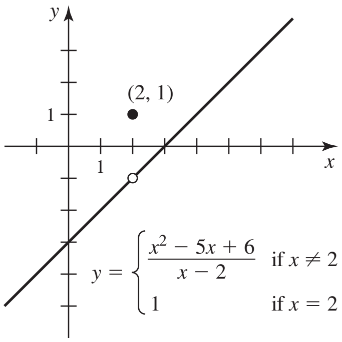


Figure 1.22

SOLUTION

a. The function f is simplified by factoring and then canceling $x - 2$, assuming $x \neq 2$:

$$\frac{x^2 - 5x + 6}{x - 2} = \frac{(x - 2)(x - 3)}{x - 2} = x - 3.$$

Therefore, the graph of f is identical to the graph of the line $y = x - 3$ when $x \neq 2$. We are given that $f(2) = 1$ (Figure 1.22).

b. The absolute value of a real number is defined as

$$f(x) = |x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0. \end{cases}$$

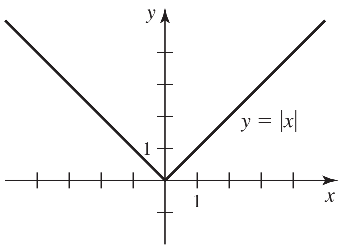


Figure 1.23

Graphing $y = -x$, for $x < 0$, and $y = x$, for $x \ge 0$, produces the graph in Figure 1.23.

Related Exercises 29–30 ◄

Power Functions Power functions are a special case of polynomials; they have the form $f(x) = x^n$, where n is a positive integer. When n is an even integer, the function values are nonnegative and the graph passes through the origin, opening upward (**Figure 1.24**). For odd integers, the power function $f(x) = x^n$ has values that are positive when x is positive and negative when x is negative (**Figure 1.25**).

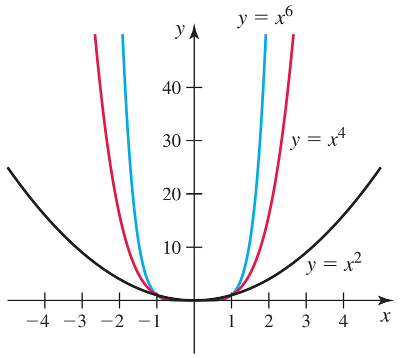


Figure 1.24

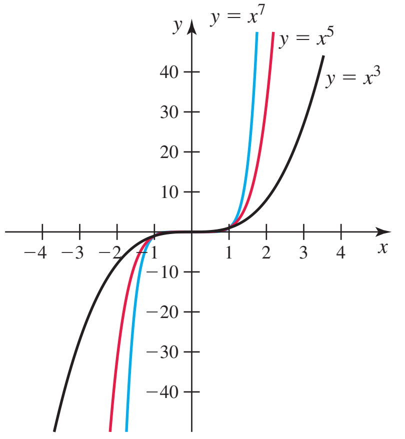


Figure 1.25

QUICK CHECK 2 What is the range of $f(x) = x^7$? What is the range of $f(x) = x^8$? ◄

Root Functions Root functions are a special case of algebraic functions; they have the form $f(x) = x^{1/n}$, where $n > 1$ is a positive integer. Notice that when n is even (square roots, fourth roots, and so forth), the domain and range consist of nonnegative numbers. Their graphs begin steeply at the origin and flatten out as x increases (**Figure 1.26**).

By contrast, the odd root functions (cube roots, fifth roots, and so forth) are defined for all real values of x, and their range is all real numbers. Their graphs pass through the origin, open upward for $x < 0$ and downward for $x > 0$, and flatten out as x increases in magnitude (**Figure 1.27**).

➤ Recall that if n is a positive integer, then $x^{1/n}$ is the nth root of x; that is, $f(x) = x^{1/n} = \sqrt[n]{x}$.

QUICK CHECK 3 What are the domain and range of $f(x) = x^{1/7}$? What are the domain and range of $f(x) = x^{1/10}$? ◄

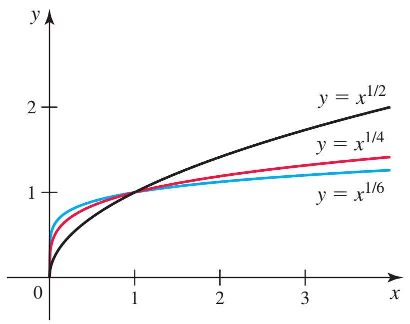


Figure 1.26

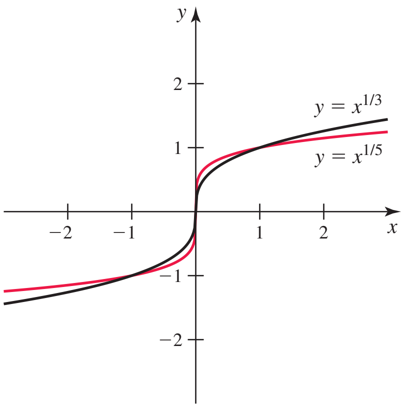


Figure 1.27

Rational Functions Rational functions appear frequently in this text, and much is said later about graphing rational functions. The following example illustrates how analysis and technology work together.

EXAMPLE 5 **Technology and analysis** Consider the rational function

$$f(x) = \frac{3x^3 - x - 1}{x^3 + 2x^2 - 6}.$$

a. What is the domain of f?

b. Find the roots (zeros) of f.

c. Graph the function using a graphing utility.

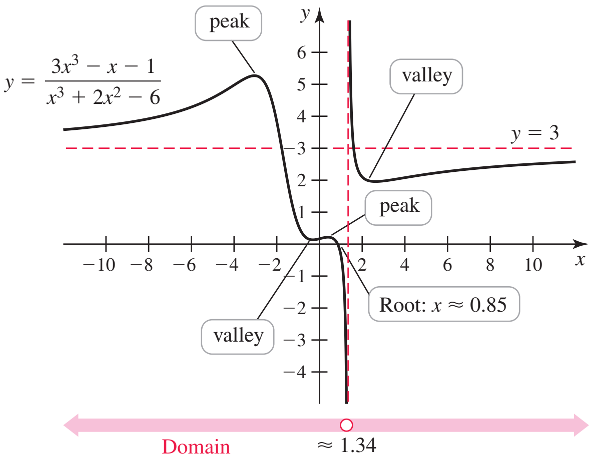


Figure 1.28

d. At what points does the function have peaks and valleys?

e. How does f behave as x grows large in magnitude?

SOLUTION

a. The domain consists of all real numbers except those at which the denominator is zero. A graphing utility shows that the denominator has one real zero at $x \approx 1.34$ and therefore, the domain of f is $\{x: x \neq 1.34\}$.

b. The roots of a rational function are the roots of the numerator, provided they are not also roots of the denominator. Using a graphing utility, the only real root of the numerator is $x \approx 0.85$.

c. After experimenting with the graphing window, a reasonable graph of f is obtained (Figure 1.28). Because the denominator is zero at $x \approx 1.34$, the function becomes large in magnitude at nearby points, and f has a *vertical asymptote*." Watch page break. Maintain current page break.

d. The function has two peaks (soon to be called *local maxima*), one near $x = -3.0$ and one near $x = 0.4$. The function also has two valleys (soon to be called *local minima*), one near $x = -0.3$ and one near $x = 2.6$.

e. By zooming out, it appears that as x increases in the positive direction, the graph approaches the *horizontal asymptote* $y = 3$ from below, and as x becomes large and negative, the graph approaches $y = 3$ from above.

Related Exercises 35–36 ◄

➤ In Chapter 4, we show how calculus is used to locate the local maximum and local minimum values of a function.

Using Tables

Sometimes functions do not originate as formulas or graphs; they may start as numbers or data. For example, suppose you do an experiment in which a marble is dropped into a cylinder filled with heavy oil and is allowed to fall freely. You measure the total distance d, in centimeters, that the marble falls at times $t = 0, 1, 2, 3, 4, 5, 6,$ and 7 seconds after it is dropped (Table 1.1). The first step might be to plot the data points (Figure 1.29).

Table 1.1

t (s)	d (cm)
0	0
1	2
2	6
3	14
4	24
5	34
6	44
7	54

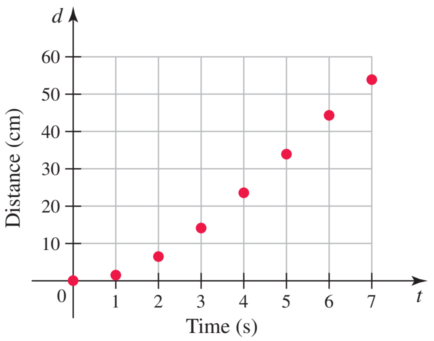


Figure 1.29

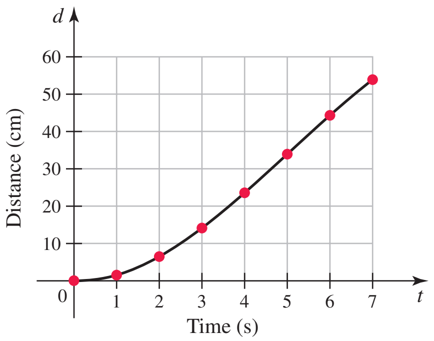


Figure 1.30

The data points suggest that there is a function $d = f(t)$ that gives the distance that the marble falls at *all* times of interest. Because the marble falls through the oil without abrupt changes, a smooth graph passing through the data points (Figure 1.30) is reasonable. Finding the function that best fits the data is a more difficult problem, which we discuss later in the text.

Using Words

Using words may be the least mathematical way to define functions, but it is often the way in which functions originate. Once a function is defined in words, it can often be tabulated, graphed, or expressed as a formula.

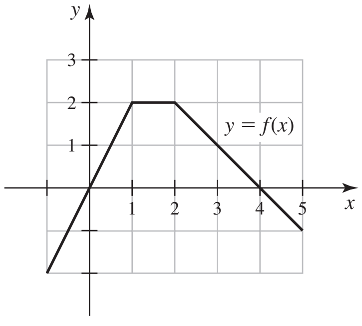


Figure 1.31

EXAMPLE 6 **A slope function** Let S be the **slope function** for a given function f. In words, this means that $S(x)$ is the slope of the curve $y = f(x)$ at the point $(x, f(x))$. Find and graph the slope function for the function f in Figure 1.31.

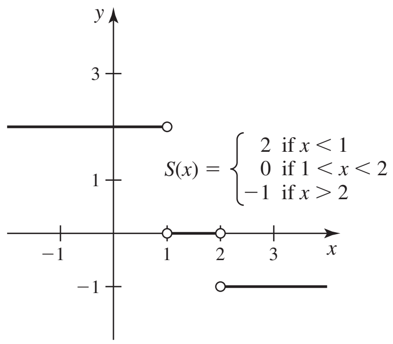


Figure 1.32

SOLUTION For $x < 1$, the slope of $y = f(x)$ is 2. The slope is 0 for $1 < x < 2$, and the slope is -1 for $x > 2$. At $x = 1$ and $x = 2$, the graph of f has a corner, so the slope is undefined at these points. Therefore, the domain of S is the set of all real numbers except $x = 1$ and $x = 2$, and the slope function (**Figure 1.32**) is defined by the piecewise function

$$S(x) = \begin{cases} 2 & \text{if } x < 1 \\ 0 & \text{if } 1 < x < 2 \\ -1 & \text{if } x > 2. \end{cases}$$

Related Exercises 47–48 ◄

EXAMPLE 7 **An area function** Let A be an **area function** for a positive function f. In words, this means that $A(x)$ is the area of the region bounded by the graph of f and the t-axis from $t = 0$ to $t = x$. Consider the function (**Figure 1.33**)

$$f(t) = \begin{cases} 2t & \text{if } 0 \le t \le 3 \\ 6 & \text{if } t > 3. \end{cases}$$

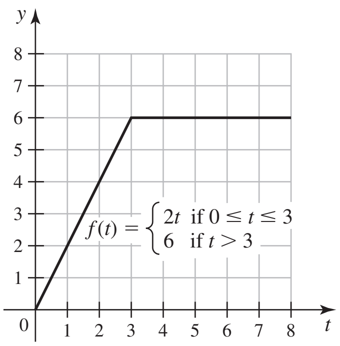


Figure 1.33

a. Find $A(2)$ and $A(5)$.

b. Find a piecewise formula for the area function for f.

SOLUTION

a. The value of $A(2)$ is the area of the shaded region between the graph of f and the t-axis from $t = 0$ to $t = 2$ (**Figure 1.34a**). Using the formula for the area of a triangle,

$$A(2) = \frac{1}{2}(2)(4) = 4.$$

➤ Slope functions and area functions reappear in upcoming chapters and play an essential part in calculus.

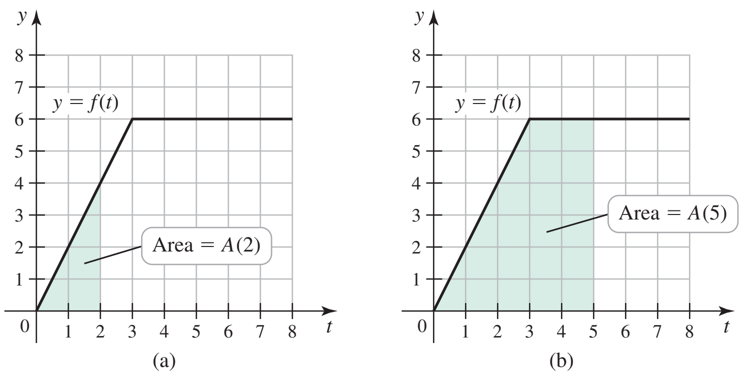


Figure 1.34

The value of $A(5)$ is the area of the shaded region between the graph of f and the t-axis on the interval $[0, 5]$ (**Figure 1.34b**). This area equals the area of the triangle whose base is the interval $[0, 3]$ plus the area of the rectangle whose base is the interval $[3, 5]$:

$$A(5) = \underbrace{\frac{1}{2}(3)(6)}_{\text{area of the triangle}} + \underbrace{(2)(6)}_{\text{area of the rectangle}} = 21.$$

b. For $0 \le x \le 3$ (**Figure 1.35a**), $A(x)$ is the area of the triangle whose base is the interval $[0, x]$. Because the height of the triangle at $t = x$ is $f(x)$,

$$A(x) = \frac{1}{2}x\,f(x) = \frac{1}{2}x\underbrace{(2x)}_{f(x)} = x^2.$$

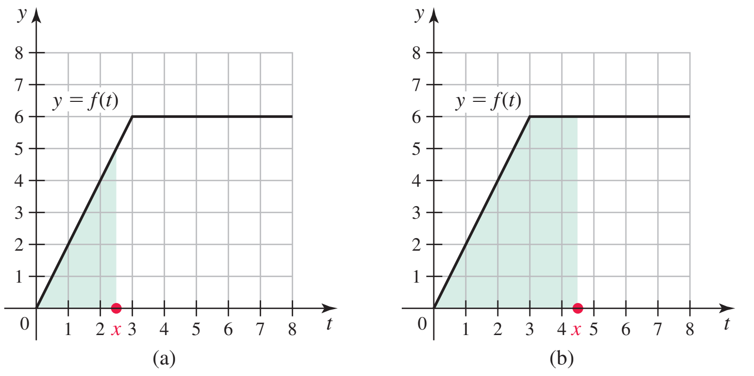


Figure 1.35

For $x > 3$ (**Figure 1.35b**), $A(x)$ is the area of the triangle on the interval $[0, 3]$ plus the area of the rectangle on the interval $[3, x]$:

$$A(x) = \underbrace{\frac{1}{2}(3)(6)}_{\text{area of the triangle}} + \underbrace{(x-3)(6)}_{\text{area of the rectangle}} = 6x - 9.$$

Therefore, the area function A (**Figure 1.36**) has the piecewise definition

$$A(x) = \begin{cases} x^2 & \text{if } 0 \le x \le 3 \\ 6x - 9 & \text{if } x > 3. \end{cases}$$

Related Exercises 51–52 ◄

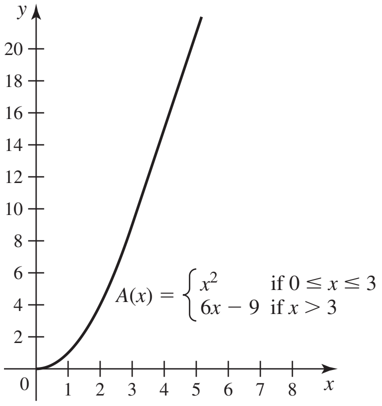


Figure 1.36

Transformations of Functions and Graphs

There are several ways to transform the graph of a function to produce graphs of new functions. Four transformations are common: *shifts* in the x- and y-directions and *scalings* in the x- and y-directions. These transformations, summarized in **Figures 1.37–1.42**, can save time in graphing and visualizing functions.

The graph of $y = f(x) + d$ is the graph of $y = f(x)$ shifted vertically by d units: up if $d > 0$ and down if $d < 0$.

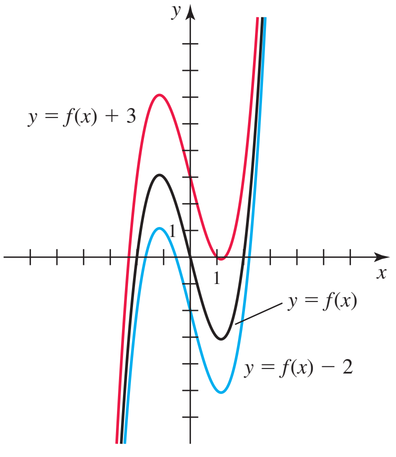


Figure 1.37

The graph of $y = f(x - b)$ is the graph of $y = f(x)$ shifted horizontally by b units: right if $b > 0$ and left if $b < 0$.

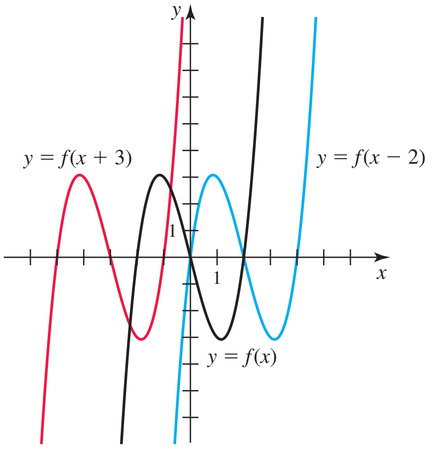


Figure 1.38

For $c > 0$ and $c \neq 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ scaled vertically by a factor of c: compressed if $0 < c < 1$ and stretched if $c > 1$.

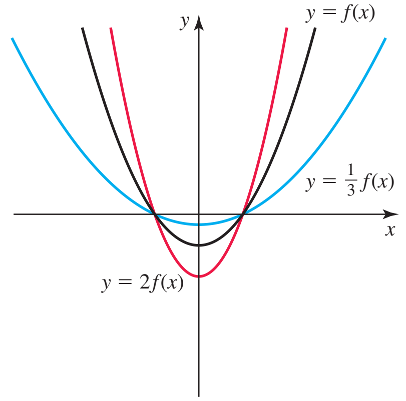


Figure 1.39

For $c < 0$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ reflected across the x-axis and scaled vertically (if $c \neq -1$) by a factor of $|c|$: compressed if $0 < |c| < 1$ and stretched if $|c| > 1$.

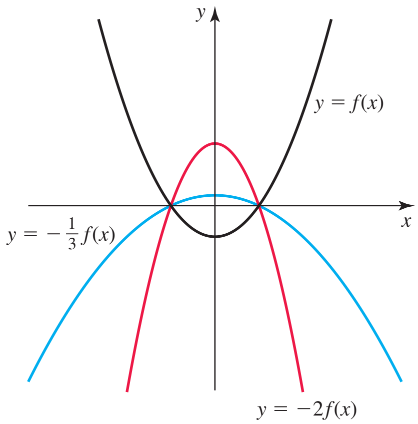


Figure 1.40

For $a > 0$ and $a \neq 1$, the graph of $y = f(ax)$ is the graph $y = f(x)$ scaled horizontally by a factor of $1/a$: compressed if $0 < 1/a < 1$ and stretched if $1/a > 1$.

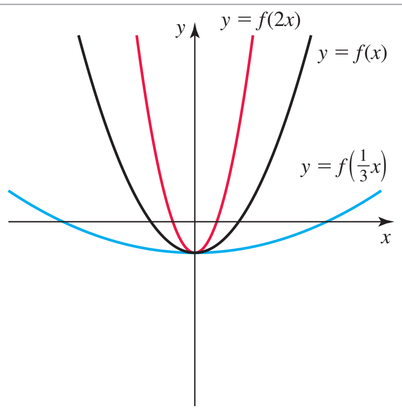


Figure 1.41

For $a < 0$, the graph of $y = f(ax)$ is the graph of $y = f(x)$ reflected across the y-axis and scaled horizontally (if $a \neq -1$) by a factor of $1/|a|$: compressed if $0 < 1/|a| < 1$ and stretched if $1/|a| > 1$.

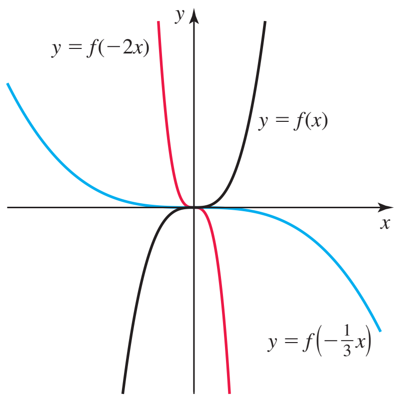


Figure 1.42

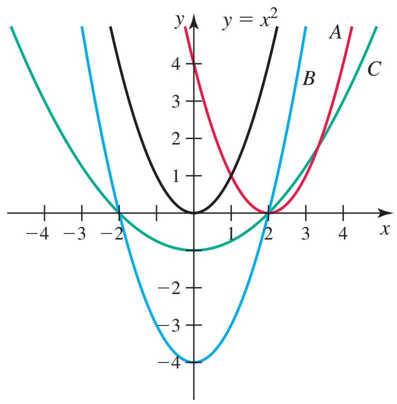


Figure 1.43

➤ You should verify that graph C also corresponds to a horizontal stretch and a vertical shift. It has the equation $y = f(ax) - 1$, where $a = \frac{1}{2}$.

EXAMPLE 8 **Transforming parabolas** The graphs A, B, and C in **Figure 1.43** are obtained from the graph of $f(x) = x^2$ using shifts and scalings. Find the function that describes each graph.

SOLUTION

a. Graph A is the graph of f shifted to the right by 2 units. It represents the function

$$f(x - 2) = (x - 2)^2 = x^2 - 4x + 4.$$

b. Graph B is the graph of f shifted down by 4 units. It represents the function

$$f(x) - 4 = x^2 - 4.$$

c. Graph C is a vertical scaling of the graph of f shifted down by 1 unit. Therefore, it represents $cf(x) - 1 = cx^2 - 1$, for some value of c, with $0 < c < 1$ (because the graph is vertically compressed). Using the fact that graph C passes through the points $(\pm 2, 0)$, we find that $c = \frac{1}{4}$. Therefore, the graph represents

$$y = \frac{1}{4}f(x) - 1 = \frac{1}{4}x^2 - 1.$$

Related Exercises 55, 58 ◀

QUICK CHECK 4 How do you modify the graph of $f(x) = 1/x$ to produce the graph of $g(x) = 1/(x + 4)$? ◄

➤ Note that we can also write $g(x) = 2\left|x + \frac{1}{2}\right|$, which means the graph of g may also be obtained by vertically stretching the graph of f by a factor of 2, followed by a horizontal shift.

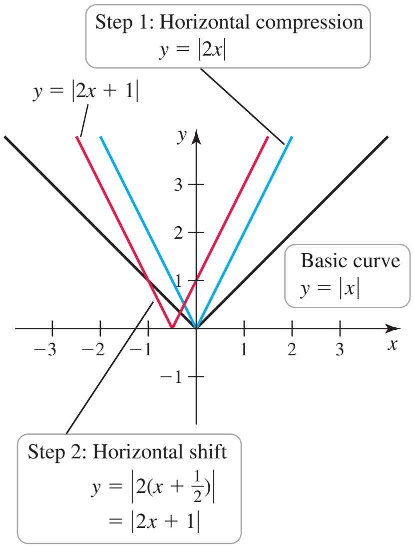


Figure 1.44

EXAMPLE 9 **Scaling and shifting** Graph $g(x) = |2x + 1|$.

SOLUTION We write the function as $g(x) = \left|2\left(x + \frac{1}{2}\right)\right|$. Letting $f(x) = |x|$, we have $g(x) = f\left(2\left(x + \frac{1}{2}\right)\right)$. Therefore, the graph of g is obtained by horizontally compressing the graph of f by a factor of $\frac{1}{2}$ and shifting it $\frac{1}{2}$ unit to the left (**Figure 1.44**).

Related Exercise 64 ◄

SUMMARY **Transformations**

Given real numbers a, b, c, and d and a function f, the graph of $y = cf(a(x - b)) + d$ can be obtained from the graph of $y = f(x)$ in the following steps.

$y = f(x)$ —— *horizontal scaling by a factor of* $1/|a|$ (and reflection across the y-axis if $a < 0$) ⟶ $y = f(ax)$

—— *horizontal shift by b units* ⟶ $y = f(a(x - b))$

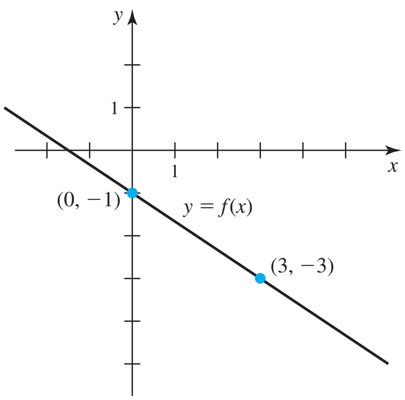
—— *vertical scaling by a factor of* $|c|$ (and reflection across the x-axis if $c < 0$) ⟶ $y = cf(a(x - b))$

—— *vertical shift by d units* ⟶ $y = cf(a(x - b)) + d$

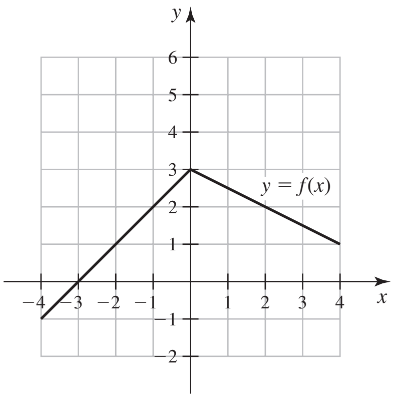
SECTION 1.2 EXERCISES

Getting Started

1. Give four ways in which functions may be defined and represented.
2. What is the domain of a polynomial?
3. Determine the function f represented by the graph of the line $y = f(x)$ in the figure.



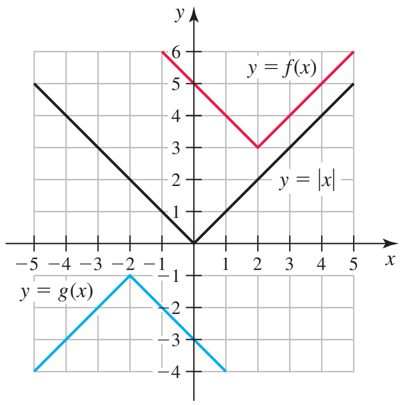
4. Determine the linear function g whose graph is parallel to the line $y = 2x + 1$ and passes through the point $(5, 0)$.
5. What is the domain of a rational function?
6. What is a piecewise linear function?
7. Write a definition of the piecewise linear function $y = f(x)$ that is given in the graph.



8. The graph of $y = \sqrt{x}$ is shifted 2 units to the right and 3 units up. Write an equation for this transformed graph.
9. How do you obtain the graph of $y = f(x + 2)$ from the graph of $y = f(x)$?
10. How do you obtain the graph of $y = -3f(x)$ from the graph of $y = f(x)$?
11. How do you obtain the graph of $y = f(3x)$ from the graph of $y = f(x)$?

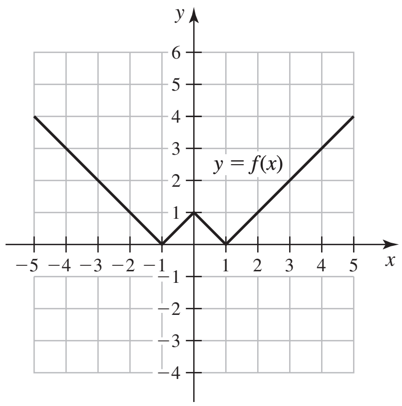
12. How do you obtain the graph of $y = 4(x + 3)^2 + 6$ from the graph of $y = x^2$?

13. The graphs of the functions f and g in the figure are obtained by vertical and horizontal shifts and scalings of $y = |x|$. Find formulas for f and g. Verify your answers with a graphing utility.



14. Transformations Use the graph of f in the figure to plot the following functions.

a. $y = -f(x)$ **b.** $y = f(x + 2)$
c. $y = f(x - 2)$ **d.** $y = f(2x)$
e. $y = f(x - 1) + 2$ **f.** $y = 2f(x)$



Practice Exercises

15. Graph of a linear function Find and graph the linear function that passes through the points $(1, 3)$ and $(2, 5)$.

16. Graph of a linear function Find and graph the linear function that passes through the points $(2, -3)$ and $(5, 0)$.

17. Linear function Find the linear function whose graph passes though the point $(3, 2)$ and is parallel to the line $y = 3x + 8$.

18. Linear function Find the linear function whose graph passes though the point $(-1, 4)$ and is perpendicular to the line $y = \frac{1}{4}x - 7$.

19–20. Yeast growth *Consider a colony of yeast cells that has the shape of a cylinder. As the number of yeast cells increases, the cross-sectional area A (in mm^2) of the colony increases but the height of the colony remains constant. If the colony starts from a single cell, the number of yeast cells (in millions) is approximated by the linear function $N(A) = C_sA$, where the constant C_s is known as the cell-surface coefficient. Use the given information to determine the cell-surface coefficient for each of the following colonies of yeast cells, and find the number of yeast cells in the colony when the cross-sectional area A reaches 150 mm^2.* (*Source: Letters in Applied Microbiology*, 594, 59, 2014)

19. The scientific name of baker's or brewer's yeast (used in making bread, wine, and beer) is *Saccharomyces cerevisiae.* When the cross-sectional area of a colony of this yeast reaches 100 mm^2, there are 571 million yeast cells.

20. The yeast *Rhodotorula glutinis* is a laboratory contaminant. When the cross-sectional area of a colony reaches 100 mm^2, there are 226 million yeast cells.

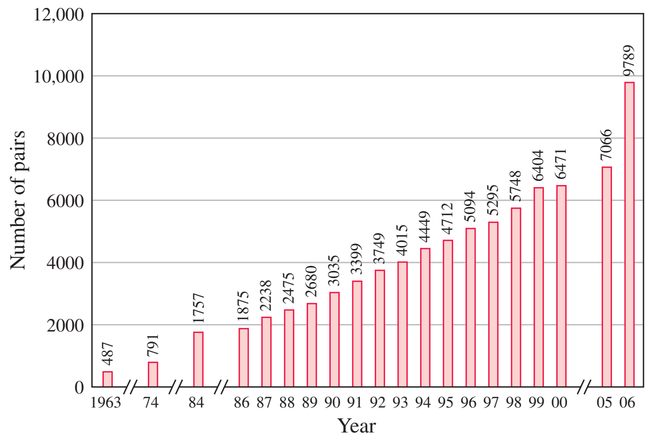
21. Demand function Sales records indicate that if Blu-ray players are priced at \$250, then a large store sells an average of 12 units per day. If they are priced at \$200, then the store sells an average of 15 units per day. Find and graph the linear demand function for Blu-ray sales. For what prices is the demand function defined?

22. Fundraiser The Biology Club plans to have a fundraiser for which \$8 tickets will be sold. The cost of room rental and refreshments is \$175. Find and graph the function $p = f(n)$ that gives the profit from the fundraiser when n tickets are sold. Notice that $f(0) = -\$175$; that is, the cost of room rental and refreshments must be paid regardless of how many tickets are sold. How many tickets must be sold for the fundraiser to break even (zero profit)?

23. Bald eagle population After DDT was banned and the Endangered Species Act was passed in 1973, the number of bald eagles in the United States increased dramatically. In the lower 48 states, the number of breeding pairs of bald eagles increased at a nearly linear rate from 1875 pairs in 1986 to 6471 pairs in 2000.

a. Use the data points for 1986 and 2000 to find a linear function p that models the number of breeding pairs from 1986 to 2000 $(0 \le t \le 14)$.

b. Using the function in part (a), approximately how many breeding pairs were in the lower 48 states in 1995?

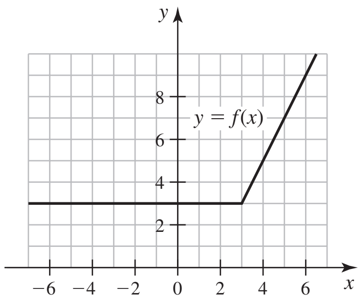


(*Source:* U.S. Fish and Wildlife Service)

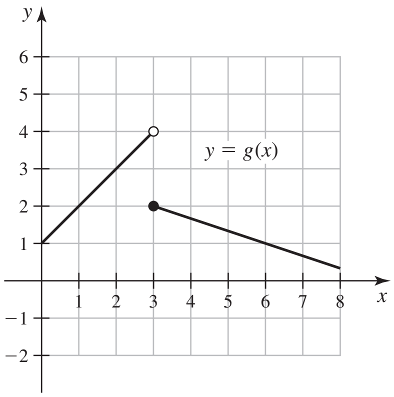
24. Taxicab fees A taxicab ride costs \$3.50 plus \$2.50 per mile. Let m be the distance (in miles) from the airport to a hotel. Find and graph the function $c(m)$ that represents the cost of taking a taxi from the airport to the hotel. Also determine how much it will cost if the hotel is 9 miles from the airport.

25–26. Defining piecewise functions *Write a definition of the function whose graph is given.*

25.



26.



27. **Parking fees** Suppose that it costs 5¢ per minute to park at the airport, with the rate dropping to 3¢ per minute after 9 P.M. Find and graph the cost function $c(t)$ for values of t satisfying $0 \leq t \leq 120$. Assume that t is the number of minutes after 8 P.M.

28. **Taxicab fees** A taxicab ride costs \$3.50 plus \$2.50 per mile for the first 5 miles, with the rate dropping to \$1.50 per mile after the fifth mile. Let m be the distance (in miles) from the airport to a hotel. Find and graph the piecewise linear function $c(m)$ that represents the cost of taking a taxi from the airport to a hotel m miles away.

29–34. Piecewise linear functions *Graph the following functions.*

29. $f(x) = \begin{cases} \dfrac{x^2 - x}{x - 1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$

30. $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$

31. $f(x) = \begin{cases} 3x - 1 & \text{if } x \leq 0 \\ -2x + 1 & \text{if } x > 0 \end{cases}$

32. $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$

33. $f(x) = \begin{cases} -2x - 1 & \text{if } x < -1 \\ 1 & \text{if } -1 \leq x \leq 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$

34. $f(x) = \begin{cases} 2x + 2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \leq x \leq 2 \\ 3 - \dfrac{x}{2} & \text{if } x > 2 \end{cases}$

T 35–40. Graphs of functions

a. *Use a graphing utility to produce a graph of the given function. Experiment with different windows to see how the graph changes on different scales. Sketch an accurate graph by hand after using the graphing utility.*

b. *Give the domain of the function.*

c. *Discuss interesting features of the function, such as peaks, valleys, and intercepts (as in Example 5).*

35. $f(x) = x^3 - 2x^2 + 6$

36. $f(x) = \sqrt[3]{2x^2 - 8}$

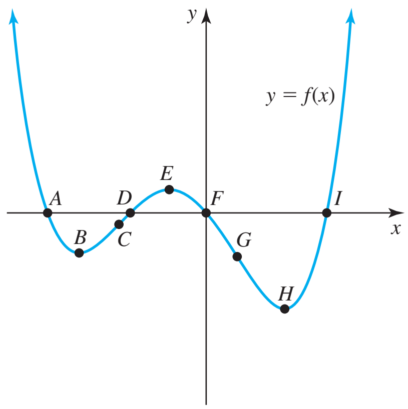
37. $g(x) = \left| \dfrac{x^2 - 4}{x + 3} \right|$

38. $f(x) = \dfrac{\sqrt{3x^2 - 12}}{x + 1}$

39. $f(x) = 3 - |2x - 1|$

40. $f(x) = \begin{cases} \dfrac{|x - 1|}{x - 1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

41. **Features of a graph** Consider the graph of the function f shown in the figure. Answer the following questions by referring to the points A–I.



a. Which points correspond to the roots (zeros) of f?

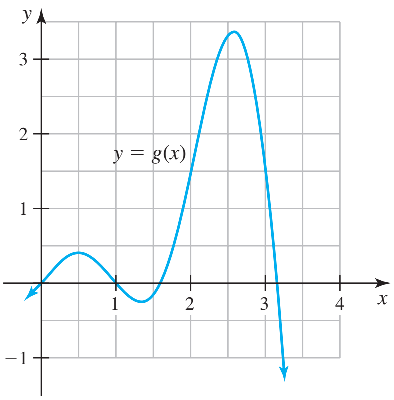
b. Which points on the graph correspond to high points or peaks (soon to be called *local maximum* values of f)?

c. Which points on the graph correspond to low points or valleys (soon to be called *local minimum* values of f)?

d. As you move along the curve in the positive x-direction, at which point is the graph rising most rapidly?

e. As you move along the curve in the positive x-direction, at which point is the graph falling most rapidly?

42. **Features of a graph** Consider the graph of the function g shown in the figure.



a. Give the approximate roots (zeros) of g.

b. Give the approximate coordinates of the high points or peaks (soon to be called *local maximum* values of g).

c. Give the approximate coordinates of the low points or valleys (soon to be called *local minimum* values of g).

d. Imagine moving along the curve in the positive x-direction on the interval $[0, 3]$. Give the approximate coordinates of the point at which the graph is rising most rapidly.

e. Imagine moving along the curve in the positive x-direction on the interval $[0, 3]$. Give the approximate coordinates of the point at which the graph is falling most rapidly.

T 43. Relative acuity of the human eye The **fovea centralis** (or **fovea**) is responsible for the sharp central vision that humans use for reading and other detail-oriented eyesight. The relative acuity of a human eye, which measures the sharpness of vision, is modeled by the function

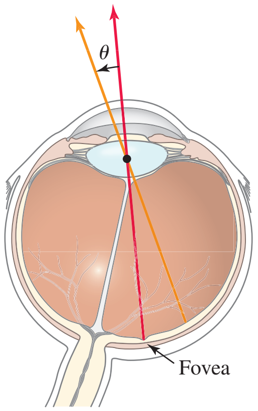
$$R(\theta) = \frac{0.568}{0.331|\theta| + 0.568},$$

where θ (in degrees) is the angular deviation of the line of sight from the center of the fovea (see figure).

a. Graph R, for $-15 \le \theta \le 15$.

b. For what value of θ is R maximized? What does this fact indicate about our eyesight?

c. For what values of θ do we maintain at least 90% of our maximum relative acuity? (*Source: The Journal of Experimental Biology*, 203, 24, Dec 2000)



44–48. Slope functions *Determine the slope function $S(x)$ for the following functions.*

44. $f(x) = 3$ **45.** $f(x) = 2x + 1$ **46.** $f(x) = |x|$

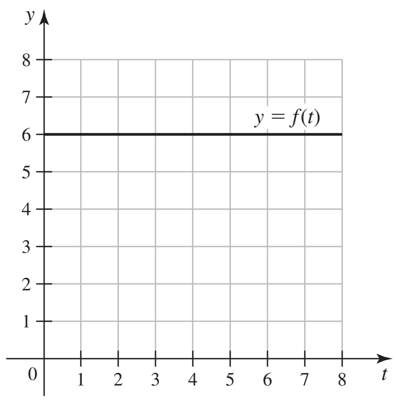
47. Use the figure for Exercise 7.

48. Use the figure for Exercise 26.

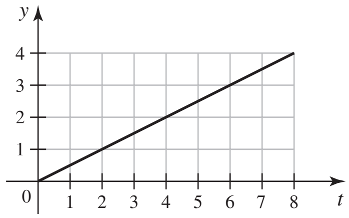
49–52. Area functions *Let $A(x)$ be the area of the region bounded by the t-axis and the graph of $y = f(t)$ from $t = 0$ to $t = x$. Consider the following functions and graphs.*

a. *Find $A(2)$.* ***b.*** *Find $A(6)$.* ***c.*** *Find a formula for $A(x)$.*

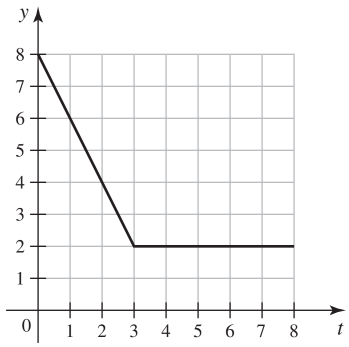
49. $f(t) = 6$



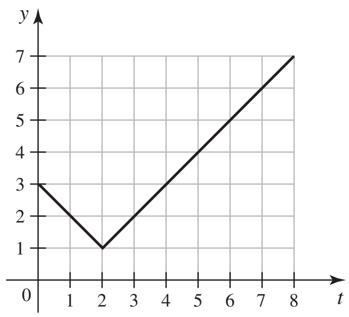
50. $f(t) = \frac{t}{2}$



51. $f(t) = \begin{cases} -2t + 8 & \text{if } t \le 3 \\ 2 & \text{if } t > 3 \end{cases}$



52. $f(t) = |t - 2| + 1$



53. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. All polynomials are rational functions, but not all rational functions are polynomials.

b. If f is a linear polynomial, then $f \circ f$ is a quadratic polynomial.

c. If f and g are polynomials, then the degrees of $f \circ g$ and $g \circ f$ are equal.

d. The graph of $g(x) = f(x + 2)$ is the graph of f shifted 2 units to the right.

54. Shifting a graph Use a shift to explain how the graph of $f(x) = \sqrt{-x^2 + 8x + 9}$ is obtained from the graph of $g(x) = \sqrt{25 - x^2}$. Sketch a graph of f.

55. Transformations of $f(x) = x^2$ Use shifts and scalings to transform the graph of $f(x) = x^2$ into the graph of g. Use a graphing utility to check your work.

a. $g(x) = f(x - 3)$

b. $g(x) = f(2x - 4)$

c. $g(x) = -3f(x - 2) + 4$

d. $g(x) = 6f\left(\frac{x - 2}{3}\right) + 1$

56. Transformations of $f(x) = \sqrt{x}$ Use shifts and scalings to transform the graph of $f(x) = \sqrt{x}$ into the graph of g. Use a graphing utility to check your work.

a. $g(x) = f(x + 4)$ **b.** $g(x) = 2f(2x - 1)$

c. $g(x) = \sqrt{x - 1}$ **d.** $g(x) = 3\sqrt{x - 1} - 5$

T 57–64. Shifting and scaling *Use shifts and scalings to graph the given functions. Then check your work with a graphing utility. Be sure to identify an original function on which the shifts and scalings are performed.*

57. $f(x) = (x - 2)^2 + 1$

58. $f(x) = x^2 - 2x + 3$ (*Hint:* Complete the square first.)

59. $g(x) = -3x^2$

60. $g(x) = 2x^3 - 1$

61. $g(x) = 2(x + 3)^2$

62. $p(x) = x^2 + 3x - 5$

63. $h(x) = -4x^2 - 4x + 12$

64. $h(x) = |3x - 6| + 1$

65–67. Intersection problems *Find the following points of intersection.*

65. The point(s) of intersection of the curves $y = 4\sqrt{2x}$ and $y = 2x^2$

66. The point(s) of intersection of the parabola $y = x^2 + 2$ and the line $y = x + 4$

67. The point(s) of intersection of the parabolas $y = x^2$ and $y = -x^2 + 8x$

Explorations and Challenges

68. Two semicircles The entire graph of f consists of the upper half of a circle of radius 2 centered at the origin and the lower half of a circle of radius 3 centered at $(5, 0)$. Find a piecewise function for f and plot a graph of f.

69. Piecewise function Plot a graph of the function

$$f(x) = \begin{cases} \frac{3}{2}x & \text{if } 0 \le x \le 2 \\ 3 + \sqrt{x - 2} & \text{if } 2 < x \le 6 \\ \sqrt{25 - (x - 6)^2} & \text{if } 6 < x \le 11. \end{cases}$$

70. Floor function The floor function, or greatest integer function, $f(x) = \lfloor x \rfloor$, gives the greatest integer less than or equal to x. Graph the floor function for $-3 \le x \le 3$.

71. Ceiling function The ceiling function, or smallest integer function, $f(x) = \lceil x \rceil$, gives the smallest integer greater than or equal to x. Graph the ceiling function for $-3 \le x \le 3$.

72. Sawtooth wave Graph the sawtooth wave defined by

$$f(x) = \begin{cases} \vdots \\ x + 1 & \text{if } -1 \le x < 0 \\ x & \text{if } 0 \le x < 1 \\ x - 1 & \text{if } 1 \le x < 2 \\ x - 2 & \text{if } 2 \le x < 3 \\ \vdots \end{cases}$$

73. Square wave Graph the square wave defined by

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } 0 \le x < 1 \\ 0 & \text{if } 1 \le x < 2 \\ 1 & \text{if } 2 \le x < 3 \\ \vdots \end{cases}$$

74–76. Roots and powers *Sketch a graph of the given pairs of functions. Be sure to draw the graphs accurately relative to each other.*

74. $y = x^4$ and $y = x^6$

75. $y = x^3$ and $y = x^7$

76. $y = x^{1/3}$ and $y = x^{1/5}$

77. Tennis probabilities Suppose the probability of a server winning any given point in a tennis match is a constant p, with $0 \le p \le 1$. Then the probability of the server winning a game when serving from deuce is

$$f(p) = \frac{p^2}{1 - 2p(1 - p)}.$$

a. Evaluate $f(0.75)$ and interpret the result.

b. Evaluate $f(0.25)$ and interpret the result.

(*Source: The College Mathematics Journal*, 38, 1, Jan 2007)

78. Temperature scales

a. Find the linear function $C = f(F)$ that gives the reading on the Celsius temperature scale corresponding to a reading on the Fahrenheit scale. Use the facts that $C = 0$ when $F = 32$ (freezing point) and $C = 100$ when $F = 212$ (boiling point).

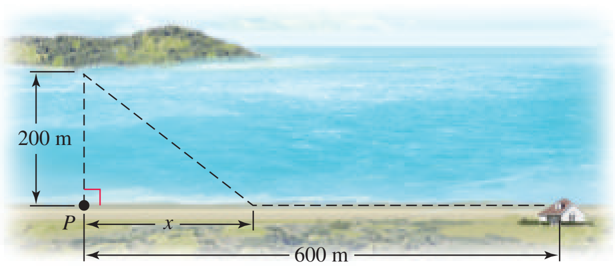
b. At what temperature are the Celsius and Fahrenheit readings equal?

79. Automobile lease vs. purchase A car dealer offers a purchase option and a lease option on all new cars. Suppose you are interested in a car that can be bought outright for \$25,000 or leased for a start-up fee of \$1200 plus monthly payments of \$350.

a. Find the linear function $y = f(m)$ that gives the total amount you have paid on the lease option after m months.

b. With the lease option, after a 48-month (4-year) term, the car has a residual value of \$10,000, which is the amount that you could pay to purchase the car. Assuming no other costs, should you lease or buy?

T 80. Walking and rowing Kelly has finished a picnic on an island that is 200 m off shore (see figure). She wants to return to a beach house that is 600 m from the point P on the shore closest to the island. She plans to row a boat to a point on shore x meters from P and then jog along the (straight) shore to the house.



a. Let $d(x)$ be the total length of her trip as a function of x. Find and graph this function.

b. Suppose that Kelly can row at 2 m/s and jog at 4 m/s. Let $T(x)$ be the total time for her trip as a function of x. Find and graph $y = T(x)$.

c. Based on your graph in part (b), estimate the point on the shore at which Kelly should land to minimize the total time of her trip. What is that minimum time?

T **81. Optimal boxes** Imagine a lidless box with height h and a square base whose sides have length x. The box must have a volume of 125 ft^3.

a. Find and graph the function $S(x)$ that gives the surface area of the box, for all values of $x > 0$.

b. Based on your graph in part (a), estimate the value of x that produces the box with a minimum surface area.

82. Composition of polynomials Let f be an nth-degree polynomial and let g be an mth-degree polynomial. What is the degree of the following polynomials?

a. $f \cdot f$ **b.** $f \circ f$ **c.** $f \cdot g$ **d.** $f \circ g$

83. Parabola vertex property Prove that if a parabola crosses the x-axis twice, the x-coordinate of the vertex of the parabola is halfway between the x-intercepts.

84. Parabola properties Consider the general quadratic function $f(x) = ax^2 + bx + c$, with $a \neq 0$.

a. Find the coordinates of the vertex of the graph of the parabola $y = f(x)$ in terms of a, b, and c.

b. Find the conditions on a, b, and c that guarantee that the graph of f crosses the x-axis twice.

T **85. Factorial function** The factorial function is defined for positive integers as $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$.

a. Make a table of the factorial function, for $n = 1, 2, 3, 4, 5$.

b. Graph these data points and then connect them with a smooth curve.

c. What is the least value of n for which $n! > 10^6$?

QUICK CHECK ANSWERS

1. Yes; no **2.** $(-\infty, \infty)$; $[0, \infty)$ **3.** Domain and range are $(-\infty, \infty)$. Domain and range are $[0, \infty)$. **4.** Shift the graph of f horizontally 4 units to the left. ◄

1.3 Inverse, Exponential, and Logarithmic Functions

Exponential functions are fundamental to all of mathematics. Many processes in the world around us are modeled by *exponential functions*—they appear in finance, medicine, ecology, biology, economics, anthropology, and physics (among other disciplines). Every exponential function has an inverse function, which is a member of the family of *logarithmic functions*, also discussed in this section.

➤ **Exponent Rules**
For any base $b > 0$ and real numbers x and y, the following relations hold:

E1. $b^x b^y = b^{x+y}$

E2. $\dfrac{b^x}{b^y} = b^{x-y}$ $\left(\text{which includes } \dfrac{1}{b^y} = b^{-y}\right)$

E3. $(b^x)^y = b^{xy}$

E4. $b^x > 0$, for all x

➤ $16^{3/4}$ can also be computed as $\sqrt[4]{16^3} = \sqrt[4]{4096} = 8$.

Exponential Functions

Exponential functions have the form $f(x) = b^x$, where the base $b \neq 1$ is a positive real number. An important question arises immediately: For what values of x can b^x be evaluated? We certainly know how to compute b^x when x is an integer. For example, $2^3 = 8$ and $2^{-4} = 1/2^4 = 1/16$. When x is rational, the numerator and denominator are interpreted as a power and root, respectively:

$$16^{3/4} = 16^{\overset{\text{power}}{3}/\underset{\text{root}}{4}} = \big(\underbrace{\sqrt[4]{16}}_{2}\big)^3 = 8.$$

But what happens when x is irrational? For example, how should 2^π be understood? Your calculator provides an approximation to 2^π, but where does the approximation come from? These questions will be answered eventually. For now, we assume that b^x can be defined for all real numbers x and that it can be approximated as closely as desired by using rational numbers as close to x as needed. In Section 7.1, we prove that the domain of an exponential function is all real numbers.

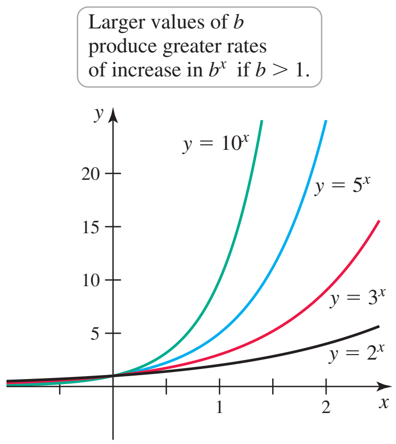


Figure 1.45

Properties of Exponential Functions $f(x) = b^x$

1. Because b^x is defined for all real numbers, the domain of f is $\{x: -\infty < x < \infty\}$. Because $b^x > 0$ for all values of x, the range of f is $\{y: 0 < y < \infty\}$.

2. For all $b > 0$, $b^0 = 1$, and therefore $f(0) = 1$.

3. If $b > 1$, then f is an increasing function of x (**Figure 1.45**). For example, if $b = 2$, then $2^x > 2^y$ whenever $x > y$.

QUICK CHECK 1 Is it possible to raise a positive number b to a power and obtain a negative number? Is it possible to obtain zero? ◄

QUICK CHECK 2 Explain why $f(x) = \left(\frac{1}{3}\right)^x$ is a decreasing function. ◄

4. If $0 < b < 1$, then f is a decreasing function of x. For example, if $b = \frac{1}{2}$, then

$$f(x) = \left(\frac{1}{2}\right)^x = \frac{1}{2^x} = 2^{-x},$$

and because 2^x increases with x, 2^{-x} decreases with x (**Figure 1.46**).

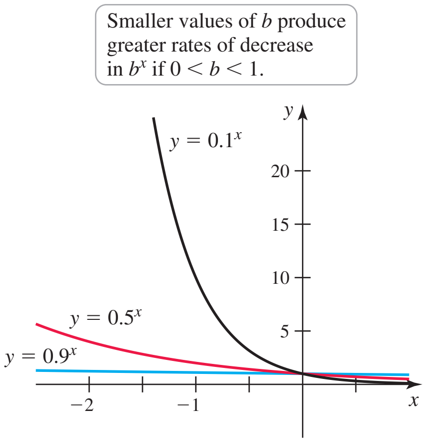


Figure 1.46

The Natural Exponential Function One of the bases used for exponential functions is special. For reasons that will become evident in upcoming chapters, the special base is e, one of the fundamental constants of mathematics. It is an irrational number with a value of $e = 2.718281828459\ldots$.

➤ The notation e was proposed by the Swiss mathematician Leonhard Euler (pronounced *oiler*) (1707–1783).

> **DEFINITION The Natural Exponential Function**
>
> The **natural exponential function** is $f(x) = e^x$, which has the base $e = 2.718281828459\ldots$.

The base e gives an exponential function that has a valuable property. As shown in **Figure 1.47a**, the graph of $y = e^x$ lies between the graphs of $y = 2^x$ and $y = 3^x$ (because $2 < e < 3$). At every point on the graph of $y = e^x$, it is possible to draw a *tangent line* (discussed in Chapters 2 and 3) that touches the graph only at that point. The natural exponential function is the only exponential function with the property that the slope of the tangent line at $x = 0$ is 1 (**Figure 1.47b**); therefore, e^x has both value and slope equal to 1 at $x = 0$. This property—minor as it may seem—leads to many simplifications when we do calculus with exponential functions.

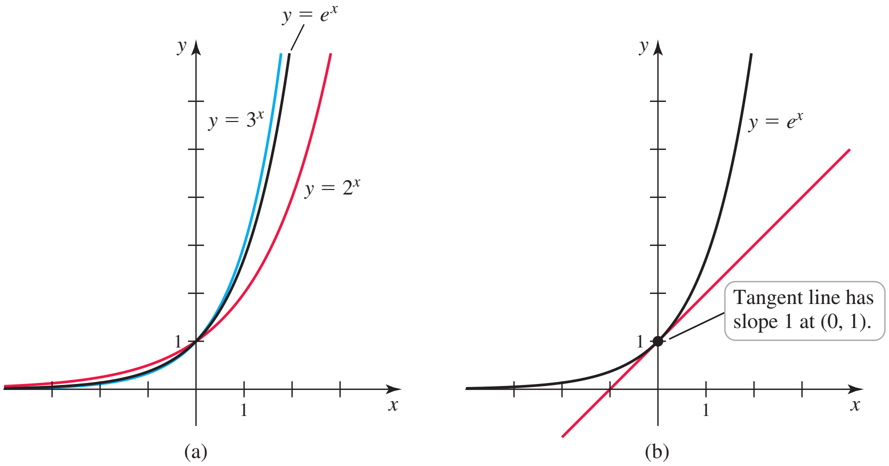


Figure 1.47

Inverse Functions

Consider the linear function $f(x) = 2x$, which takes any value of x and doubles it. The function that reverses this process by taking any value of $f(x) = 2x$ and mapping it back to x is called the *inverse function* of f, denoted f^{-1}. In this case, the inverse function is $f^{-1}(x) = x/2$. The effect of applying these two functions in succession looks like this:

$$x \xrightarrow{f} 2x \xrightarrow{f^{-1}} x.$$

We now generalize this idea.

QUICK CHECK 3 What is the inverse of $f(x) = \frac{1}{3}x$? What is the inverse of $f(x) = x - 7$? ◄

> **DEFINITION Inverse Function**
>
> Given a function f, its inverse (if it exists) is a function f^{-1} such that whenever $y = f(x)$, then $f^{-1}(y) = x$ (**Figure 1.48**).

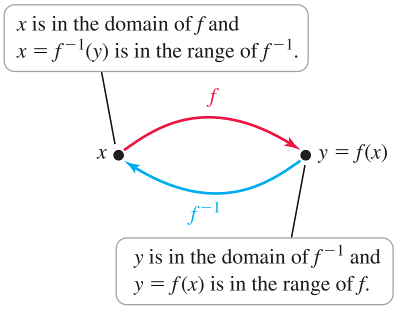
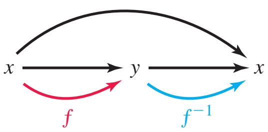


Figure 1.48

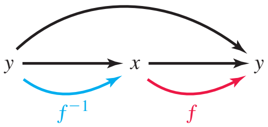
Because the inverse "undoes" the original function, if we start with a value of x, apply f to it, and then apply f^{-1} to the result, we recover the original value of x; that is,

$$f^{-1}(f(x)) = x.$$



Similarly, if we apply f^{-1} to a value of y and then apply f to the result, we recover the original value of y; that is,

$$f(f^{-1}(y)) = y.$$



➤ The notation f^{-1} for the inverse can be confusing. The inverse is not the reciprocal; that is, $f^{-1}(x)$ is not $1/f(x) = (f(x))^{-1}$. We adopt the common convention of using simply *inverse* to mean *inverse function*.

One-to-One Functions We have defined the inverse of a function, but said nothing about when it exists. To ensure that f has an inverse on a domain, f must be *one-to-one* on that domain. This property means that every output of the function f corresponds to exactly one input. The one-to-one property is checked graphically by using the *horizontal line test*.

> **DEFINITION One-to-One Functions and the Horizontal Line Test**
>
> A function f is **one-to-one** on a domain D if each value of $f(x)$ corresponds to exactly one value of x in D. More precisely, f is one-to-one on D if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$, for x_1 and x_2 in D. The **horizontal line test** says that every horizontal line intersects the graph of a one-to-one function at most once (**Figure 1.49**).

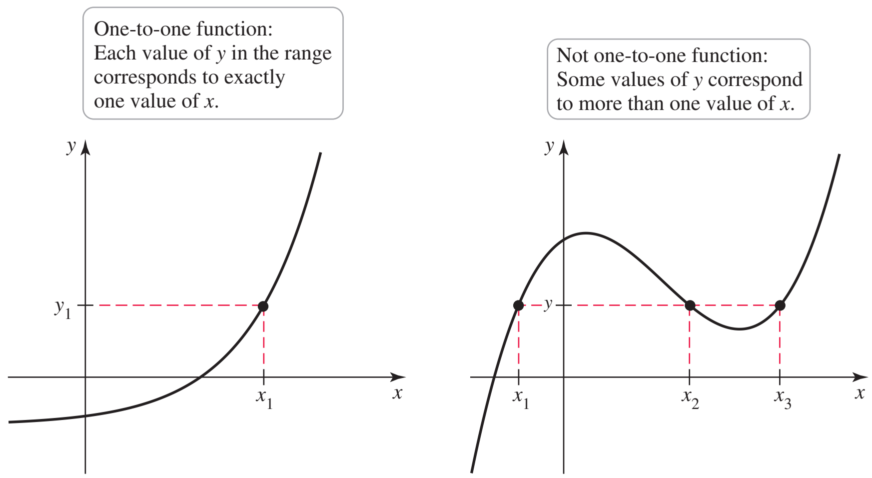


Figure 1.49