Mathematics with Applications

in the Management, Natural, and Social Sciences

Twelfth Edition

Lial | Hungerford | Holcomb | Mullins

Features applications of college algebra, finite mathematics, and calculus

Pearson
MyLab

MyLab Math for *Mathematics with Applications* 12e and *Finite Mathematics with Applications* 12e **(access code required)**

Used by over 3 million students a year, MyLab™ Math is the world's leading online program for teaching and learning mathematics. MyLab Math delivers assessment, tutorials, and multimedia resources that provide engaging and personalized experiences for each student, so learning can happen in any environment.

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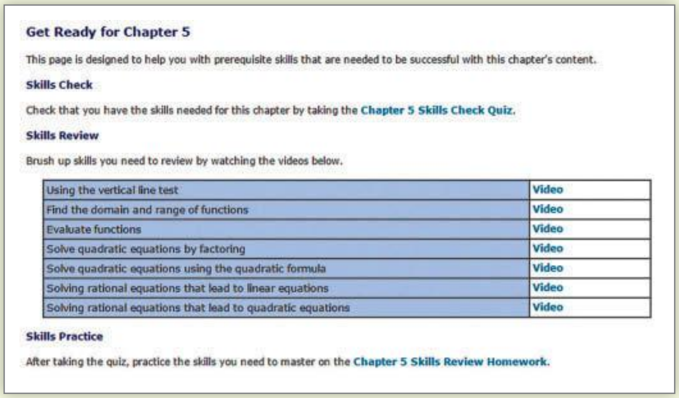
An Integrated Review version of the MyLab Math course contains premade, assignable quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified. Each student, therefore, receives just the help that he or she needs—no more, no less.


Get Ready for Chapter 5

This page is designed to help you with prerequisite skills that are needed to be successful with this chapter's content.

Skills Check

Check that you have the skills needed for this chapter by taking the **Chapter 5 Skills Check Quiz**.

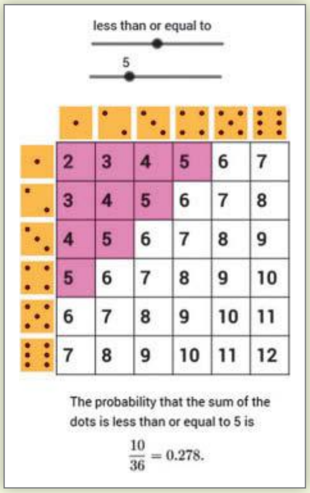
Skills Review

Brush up skills you need to review by watching the videos below.

Using the vertical line test	Video
Find the domain and range of functions	Video
Evaluate functions	Video
Solve quadratic equations by factoring	Video
Solve quadratic equations using the quadratic formula	Video
Solving rational equations that lead to linear equations	Video
Solving rational equations that lead to quadratic equations	Video

Skills Practice

After taking the quiz, practice the skills you need to master on the **Chapter 5 Skills Review Homework**.

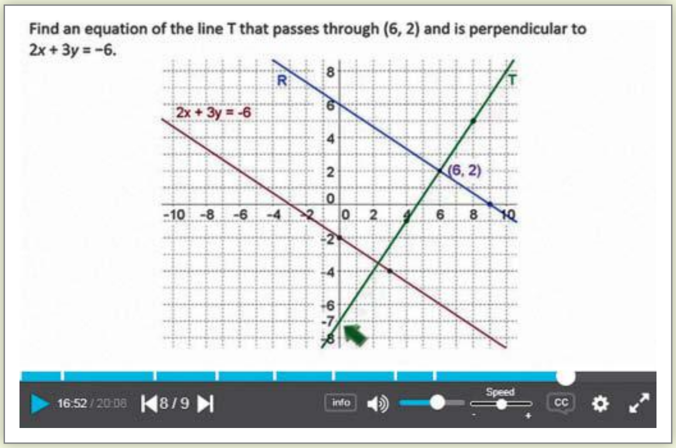


Interactive Figures

A full suite of interactive figures has been added to support teaching and learning. The figures illustrate key concepts and allow manipulation. They have been designed for use during lectures as well as by students working independently.

Instructional Videos

The instructional videos for this course are *all new* and include a full lecture video for each section of the text. These videos are segmented into parts (Introduction, Example, and Summary), making them easy for students and instructors to navigate. In addition, MathTalk and StatTalk videos are available to connect the content of the course to business and management applications.



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Algebra Review

Rules of Exponents

- **Multiplication of Exponents** To multiply a^m by a^n, *add* the exponents: $a^m a^n = a^{m+n}$.

- **Division of Exponents** To divide a^m by a^n, *subtract* the exponents: $\frac{a^m}{a^n} = a^{m-n}$.

- **Power of a Power** To find a power of a power, multiply the exponents: $(a^m)^n = a^{mn}$.

- **Product to a Power** To find $(ab)^n$, apply the exponent to *every* term inside the parentheses: $(ab)^n = a^n b^n$.

- **Quotient to a Power** To find $\left(\frac{a}{b}\right)^n$, apply the exponent to *both* the numerator and the denominator: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

- **Zero Exponent** If a is any nonzero real number, then $a^0 = 1$.

- **Negative Exponent** If n is a natural number, and if $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$.

- **Inversion Property** $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

Radicals

- ***n*th root of *a*** If n is an even natural number and $a \geq 0$, or if n is an odd natural number, $\sqrt[n]{a} = a^{1/n}$.

- For all rational numbers m/n and all real numbers a for which $\sqrt[n]{a}$ exists, $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$.

- For any real number a and any natural number n, $\sqrt[n]{a^n} = |a|$ if n is even and $\sqrt[n]{a^n} = a$ if n is odd.

- For all real numbers a and b, and for positive integers n for which all indicated roots exists, $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ and $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \quad (b \neq 0)$.

Factoring

- **Difference of Squares** $x^2 - y^2 = (x + y)(x - y)$.

- **Perfect Squares** $x^2 + 2xy + y^2 = (x + y)^2$ and $x^2 - 2xy + y^2 = (x - y)^2$.

- **Difference of Cubes** $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

- **Sum of Cubes** $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$.

Quadratic Equation and Formulas

- **Square Root Property** If $b > 0$, then the solutions of $x^2 = b$ are $x = \sqrt{b}$ and $x = -\sqrt{b}$.
- **Quadratic Formula** The solutions of the quadratic equation $ax^2 + bx - c$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- **Graph of a Quadratic Function** The graph of $f(x) = a(x - h)^2 + k$ is a parabola with vertex (h, k). It opens upward when $a > 0$ and downward when $a < 0$. The graph of $f(x) = ax^2 + bx + c$ is a parabola with vertex (h, k), where $h = \dfrac{-b}{2a}$ and $k = f(h)$.

Equations of Lines

- **Slope** The slope of the line m through the two points (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$ is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- The slope of every **horizontal** line is 0, and the slope of every **vertical** line is undefined.
- **Slope–Intercept Form** If a line has slope m and y-intercept b, then it is the graph of the equation $y = mx + b$.
- **Parallel Lines** Two nonvertical lines are parallel whenever they have the same slope.
- **Perpendicular Lines** Two nonvertical lines are perpendicular whenever the product of their slopes is -1 (in other words, they are negative reciprocals of each other).
- **Point–Slope Form** If a line has slope m and passes through the point (x_1, y_1), then $y - y_1 = m(x - x_1)$.

Properties of Logarithms

- **Definition of Logarithms to the Base *a*** $y = \log_a x$ means $a^y = x$.
- Let x and a be any positive real numbers, with $a \neq 1$, and r be any real number. Then $\log_a 1 = 0$; $\log_a a = 1$; $\log_a a^r = r$; $a^{\log_a x} = x$.
- **Product Property** $\log_a xy = \log_a x + \log_a y$
- **Quotient Property** $\log_a \dfrac{x}{y} = \log_a x - \log_a y$
- **Power Property** $\log_a x^r = r \log_a x$

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Mathematics with Applications

IN THE MANAGEMENT, NATURAL, AND SOCIAL SCIENCES

TWELFTH EDITION

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Pearson

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Pearson

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We would like to dedicate this twelfth edition to Thomas W. Hungerford, who passed away from a sudden illness in 2014. Tom had a major impact on both our lives. For Bernadette, it was through his authorship of his classic graduate text, Algebra*; for John it was through working closely with Tom on the ninth and tenth editions of this text. Tom was exact and thorough, and he was a deep thinker of how students and teachers would read and use the material in his books. He is greatly missed.*

We would also like to thank Greg Webster and Stan Palla for their enthusiastic support of our work on this project. Without their understanding, we never would have been able to make enough time to complete the revision. We thank you from the bottoms of our hearts.

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Contents

Preface

Mathematics with Applications is an applications-centered text for students in business, management, and natural and social sciences. It covers the basics of college algebra, followed by topics in finite mathematics and concluding with a treatment of applied calculus. The text can be used for a variety of different courses, and the only prerequisite is a basic course in algebra. Chapter 1 provides a thorough review of basic algebra for those students who need it.

It has been our primary goal to present sound mathematics in an understandable manner, proceeding from the familiar to new material, and from concrete examples to general rules and formulas. There is an ongoing focus on real-world problem solving, and almost every section includes relevant, contemporary applications.

New to This Edition

We have revised and added content, updated and added new applications, fine-tuned pedagogical devices, and evaluated and enhanced the exercise sets. In addition, both the functionality of MyLab™ Math and the resources within it have been greatly improved and expanded for this edition. These improvements were incorporated after careful consideration and much feedback from those who teach this course regularly. Following is a list of some of the more substantive revisions made to this edition.

- We updated or added real-world data for hundreds of examples and exercises. We have tried to make this text the most relevant and interesting of its kind, and the main way we do this is by immersing ourselves in the kinds of applications that we know from experience will motivate students. We realize that motivating reluctant learners is a major part of this course; the applications in this text are designed to give instructors a big advantage in facing this challenge.
- We analyzed aggregated student usage and performance data from MyLab Math for the previous edition of this text. The results of this analysis helped improve the quality and quantity of exercises that matter the most to instructors and students. Also, within exercise sets, we improved even-odd pairing of exercises and better gradated them by level.
- In Chapter 5, we changed the notation for financial formulas to match the notation used in the TVM Solver from the TI-84 calculator.
- We added weighted averages to Section 10.2 so that students can understand applications such as calculating their own final grade when different components of a course are weighted at different percentages.
- We moved the discussion about boxplots from Section 10.4 to Section 10.3 because boxplots are tools for visualizing variation, and variation is discussed in Section 10.3.
- Material in the previous edition on using the normal distribution to approximate the binomial distribution in Section 10.5 was consolidated and moved to Section 10.4. The normal approximation to the binomial is not as important a topic today as it once was because technology makes calculating exact binomial probabilities easy. However, the conceptual understanding of the ideas can be important for students to learn, so we condensed the material and put it in the section on the normal distribution.
- We added Section 13.3, an entirely new section on the topic of integration by parts. This was done in response to requests from users of the text. The section features six new examples, including antiderivatives that require one or two applications of integration by parts, the antiderivatives of the logarithmic function, and applications. It also features fifty new exercises, including applied problems involving energy consumption and

pharmaceutical sales. The previous Section 13.6, Tables of Integrals, was deleted because users indicated they did not cover this section.

- Labels for the applications in the previous edition were very general (e.g., Business, Life Sciences). In this edition, we made them more specific (e.g., Google Profits) to pique student interest and to allow students to find applications that relate to their specific major and areas of interest.

The copy below is an example of the efforts put forth to update each chapter. **To see this type of detailed information for *all chapters* in the text, please see the "Features" portion of the Pearson online catalog for this text.**

New to Chapter 4

- In Section 4.1, updated application Examples 5 and 6 and Checkpoint 7 with data on wine consumption and assets of AIG. Replaced or updated ten of the exercises with current data on the assets of Prudential Financial, Inc.; Netflix costs; GDP for China and the U.S.; asset management; imports from Vietnam; and subprime mortgages.
- Replaced Examples 1 and 2 in Section 4.2 with current data on debt in the U.S. and on sales of single-family homes. Updated Example 4 with more recent data on infant mortality rates. Replaced Example 6 and Checkpoint 4 with a current example on the price of scrap steel. Replaced or updated eleven of the application exercises with data on wind power, oil production, office rent, personal consumption, Medicare expenditures, Chinese assets in banks, Internet access in China, seat-belt use, death rates, food assistance, and labor force participation.
- In Section 4.3, added a graph of several logarithmic functions of different bases to help students visualize logarithmic functions better. Replaced an application example with a current logarithmic model function on wind energy generated in the U.S. Updated or replaced eight of the application exercises with data on health insurance costs, dairy expenditures, credit union assets, border patrol budgets, opioid deaths, iPhone sales, and vehicle miles traveled.
- In Section 4.4, utilized color to indicate nonpossible solutions to logarithmic equations more clearly. Replaced Example 7 and Checkpoint 7 with a new example on new jobs added to the U.S. economy. Also added a new example and checkpoint (using data on the digital grocery market) to illustrate solving for x with logarithmic and exponential equations. Updated or replaced twelve of the application exercises with data on foreign earnings, nursing degrees, veterans' benefits, Snapchat users, wind energy, Japanese messaging, CVS Health earnings and revenue, the number of teachers in the U.S., Best Buy revenue, Twitter stock price, and outstanding loans in U.S. banks.
- In the Review Exercises, updated or replaced ten of the application exercises with current data on exports to Mexico, Royal Caribbean share price, the number of murders in Chicago, crude oil and coal futures, recent earthquakes, FedEx profits, Starbucks and Dunkin' Donuts App users, and bank capital.
- Updated Case Study examples and exercises with more recent data and graphs from gapminder.org.

New to MyLab Math

Many improvements have been made to the overall functionality of MyLab Math since the previous edition. Beyond that, however, we have also increased and improved the content specific to this text.

- Instructors now have **more exercises** than ever to choose from when assigning homework. Most new questions are application-oriented. There are approximately 5900 assignable exercises in MyLab Math for this text. New exercise types include:
    - Additional Conceptual Questions provide support for assessing concepts and vocabulary. Many of these questions are application-oriented.

- Setup & Solve exercises require students to show how they set up a problem as well as the solution, which approximates more closely what is required of students on tests.

- The **videos are *all new***, and they feature veteran instructors Thomas Hartfield (University of North Georgia), Mike Rosenthal (Florida International University), and Kate Haynes (Delaware Technical Community College).
  - Each section of the text now has an accompanying full lecture video. To make it easier for students to navigate to the content they need, each lecture video is segmented into shorter clips (labeled Introduction, Example, or Summary).
  - Both the video lectures and video segments are assignable within MyLab Math. We have included a Guide to Video-Based Assignments within the Instructor Resources section of MyLab Math that allows you to assign exercises for each video.
  - MathTalk and StatTalk videos highlight applications of the content of the course to business. The videos are supported by assignable exercises.
- A full suite of **Interactive Figures** has been added to support teaching and learning. The figures illustrate key concepts and allow manipulation. They have been designed to be used during lectures as well as by students working independently.
- An **Integrated Review** version of the MyLab Math course contains premade quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified.
- **Study Skills Modules** help students with the life skills that can make the difference between passing and failing.
- The **Graphing Calculator Manual** and **Excel Spreadsheet Manual**, both specific to this course, have been updated to support the TI-84 CE (color edition) and Excel 2016, respectively. Both manuals also contain additional topics to support the course.
- We heard from users that the Annotated Instructor Edition for the previous edition required too much flipping of pages to find answers, so MyLab Math now contains a downloadable **Instructor Answers document**—with *all answers in one place*. (This augments the downloadable Instructor Solutions Manual, which contains all solutions.)

Continued Pedagogical Support

- **Real-Data Examples and Explanations:** Real-data exercises have long been a popular and integral aspect of this text. A significant number of new real-data examples and exercises have also been introduced into the text. Applications are noted with a **green header** to indicate the subject of the problem so instructors or students can focus on applications that are in line with students' majors.
- **Balanced Approach:** Multiple representations of a topic (symbolic, numerical, graphical, verbal) are given when appropriate. However, we do not believe that all representations are useful for all topics, so effective alternatives are discussed only when they are likely to increase student understanding.
- **Strong Algebra Foundation:** The text begins with four thorough chapters of college algebra that can be used in a variety of ways based on the needs of the students and the goals of the course. Take advantage of the content in these chapters as needed so students will be more successful with later topics and future courses.
- **Help for Skill Gaps:** The Prerequisite Skills Test (for Chapters 1–4) and Calculus Readiness Test (for Chapters 11–14) at the front of the text can help students determine where remediation is needed. The text contains solutions to the test exercises to help students remediate any gaps in basic skills.
- **Checkpoint** exercises are marked with icons such as ✓1 and provide an opportunity for students to stop, check their understanding of the specific concept at hand, and move forward with confidence. Answers to Checkpoint exercises are located at the end of the section to encourage students to work the problems before looking at the answers. (See pages 185 and 186.)

- **Caution** notes highlight common student difficulties or warn against frequently made mistakes. (See page 209.)
- **Exercises:** In addition to skill-based practice, conceptual, and application-based exercises, the text includes some specially marked exercises:
  - Writing Exercises (See page 188.)
  - Connection Exercises relate current topics to earlier sections (See page 213.)
  - Graphing Calculator Exercises (See page 205.)
  - Spreadsheet Exercises (See page 260.)
- **Example/Exercise Connection:** Selected exercises include a reference to related example(s) within the section (e.g., "See Examples 6 and 7") to facilitate what students do naturally when they use a book—that is, look for specific examples when they get stuck on a problem. Later exercises leave this information out and provide opportunities for mixed skill practice.
- **Graphing Calculators and Spreadsheets:** It is assumed that all students have a calculator that will handle exponential and logarithmic functions. Graphing calculator and spreadsheet references are highlighted in the text so that those who use the technology can easily incorporate it and those who do not can easily omit it. Examples and exercises that require a graphing calculator are marked with and those that require a spreadsheet are marked with , making it obvious where technology is being included.
- **Technology Tips:** These tips are placed at appropriate points in the text to inform students of various features of their graphing calculator, spreadsheet, or other computer programs. Note that Technology Tips designed for TI-84 CE also apply to the TI-84 Plus, TI-83, and TI-Nspire.
- **End-of-chapter materials:** are designed to help students prepare for exams. These materials include a List of Key Terms and Symbols and Summary of Key Concepts, as well as a thorough set of Chapter Review Exercises.
- **Case Studies:** appear at the end of each chapter and offer contemporary, real-world applications of some of the mathematics presented in the chapter. Not only do these provide an opportunity for students to see the mathematics they are learning in action, but they also provide at least a partial answer to the question, "What is this stuff good for?" These have been expanded to include options for longer-term projects if the instructor should choose to use them.

Course Flexibility

The content of the text is divided into three parts:

1. College Algebra (Chapters 1–4)
2. Finite Mathematics (Chapters 5–10)
3. Applied Calculus (Chapters 11–14)

This coverage of the material offers flexibility, making the text appropriate for a variety of courses, including:

- **Finite Mathematics and Applied Calculus** (one year or less). Use the entire book; cover topics from Chapters 1–4 as needed before proceeding to further topics.
- **Finite Mathematics** (one semester or two quarters). Use as much of Chapters 1–4 as needed, and then go into Chapters 5–10 as time permits and local needs require.
- **Applied Calculus** (one semester or quarter). Cover the precalculus topics in Chapters 1–4 as necessary, and then use Chapters 11–14.
- **College Algebra with Applications** (one semester or quarter). Use Chapters 1–8, with Chapters 7 and 8 being optional.

Pearson regularly produces custom versions of this text (and its accompanying MyLab Math course) to address the needs of specific course sequences. Custom versions can be produced for even smaller-enrollment courses due to advances in digital printing. Please contact your local Pearson representative for more details.

Chapter interdependence is as follows:

	Chapter	Prerequisite
1	Algebra and Equations	None
2	Graphs, Lines, and Inequalities	Chapter 1
3	Functions and Graphs	Chapters 1 and 2
4	Exponential and Logarithmic Functions	Chapter 3
5	Mathematics of Finance	Chapter 4
6	Systems of Linear Equations and Matrices	Chapters 1 and 2
7	Linear Programming	Chapters 3 and 6
8	Sets and Probability	None
9	Counting, Probability Distributions, and Further Topics in Probability	Chapter 8
10	Introduction to Statistics	Chapter 8
11	Differential Calculus	Chapters 1–4
12	Applications of the Derivative	Chapter 11
13	Integral Calculus	Chapters 11 and 12
14	Multivariate Calculus	Chapters 11–13

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John P. Holcomb, Jr.
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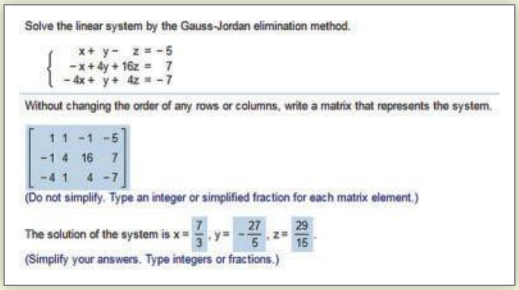

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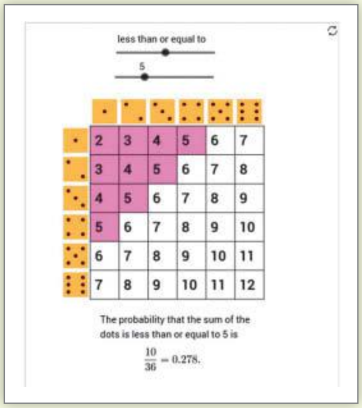

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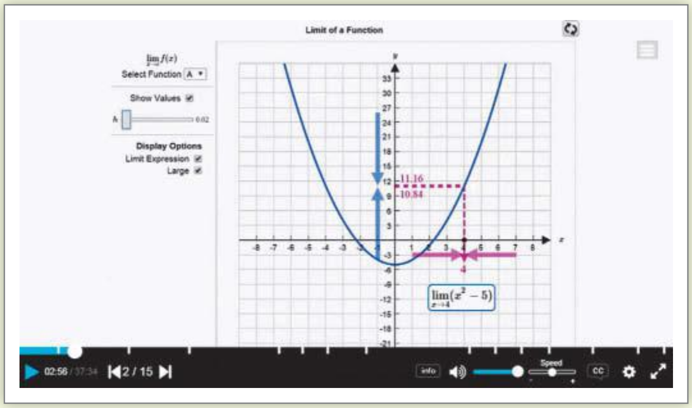

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Technology Manuals and Projects (downloadable)

- *Excel Spreadsheet Manual and Projects* by Stela Pudar-Hozo, Indiana University–Northwest
- *Graphing Calculator Manual and Projects* by Chris True, University of Nebraska

These manuals, both specific to this course, have been updated to support the TI-84 CE (color edition) and Excel 2016, respectively. Instructions are ordered by mathematical topic. The files can be downloaded from within MyLab Math.

Student's Solutions Manual (softcover and downloadable)

ISBN: 0-13-477267-9 | 978-0-13-477267-7

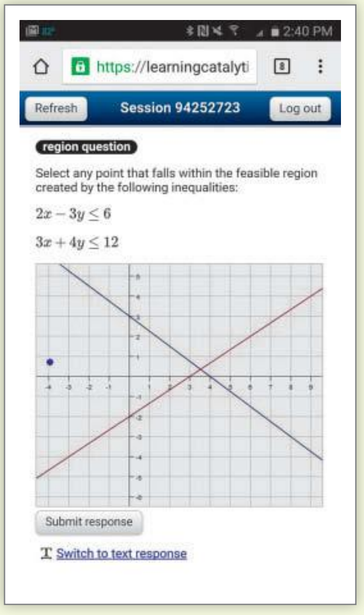
Written by Salvatore Sciandra from Niagara County Community College, the Student's Solutions Manual contains worked-out solutions to all the odd-numbered exercises and all Chapter Review and Case Studies. This manual is available in print and can be download from within MyLab Math.

A Complete eText Students get unlimited access to the eText within any MyLab Math course using that edition of the textbook. The Pearson eText app allows existing subscribers to access their titles on an iPad or Android tablet for either online or offline viewing.

SUPPORTING INSTRUCTION

MyLab Math comes from an experienced partner with educational expertise and an eye on the future. It provides resources to help you assess and improve students' results at every turn, and unparalleled flexibility to create a course tailored to you and your students.

Learning Catalytics™ Now included in all MyLab Math courses, this student response tool uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking during lecture. Learning Catalytics fosters student engagement and peer-to-peer learning with real-time analytics. Access prebuilt exercises created specifically for this course.



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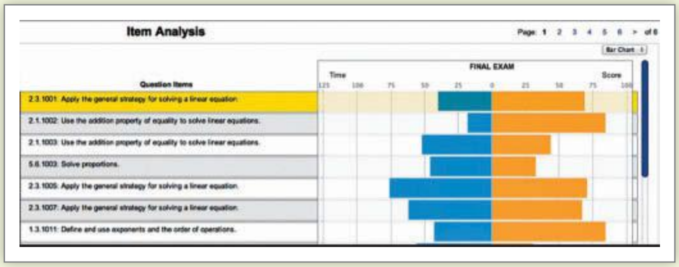

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To the Student

The key to succeeding in this course is to remember that *mathematics is not a spectator sport.* You can't expect to learn mathematics without *doing* mathematics any more than you could learn to swim without getting wet. You must take an active role and use all the resources at your disposal: your instructor, your fellow students, this book, and the supplements that accompany this book. Following are some general tips on how to be successful in the course and some specific tips on how to get the most out of this text and supplementary resources.

Ask Questions! Remember the words of the great Hillel: "The bashful do not learn." There is no such thing as a "dumb question" (assuming, of course, that you have read the book and your class notes and attempted the homework). Your instructor will welcome questions that arise from a serious effort on your part. So get your money's worth: Ask questions!

Read the Book Interactively! There is more to a math textbook than just the exercise sets. Each section introduces topics carefully with many examples—both mathematical and contextual. Take note of the "Caution" and "Note" comments, and bookmark pages with key definitions or formulas. After reading the example, try the Checkpoint exercise that appears next to it in the margin to check your understanding of the concept. This will help you solidify your understanding or diagnose if you do not fully understand the concept. The answers to the Checkpoint exercises are right after the homework exercises in each section. Resist the temptation to flip to the answer until you've worked the problem completely!

Take Advantage of the Supplementary Material! Many resources are at your disposal within and outside the text. Take the time to interact with them and determine which resources suit your learning style best.

- If your instructor allows the use of graphing calculators and/or spreadsheets, work through the examples and exercises marked with ▦ or ▤. Some instructors may make this material part of the course, whereas others will not require you to use technology. If your instructor asks that you use technology, a Graphing Calculator Manual and an Excel Spreadsheet Manual are available in MyLab Math. In addition, there are ***Technology Tips*** throughout the text that describe the proper menu or keys to use for various procedures on a graphing calculator. Note that Technology Tips for the TI-84+ CE also apply to TI-83+, TI-Nspire, and usually TI-83.
- MyLab Math has a variety of types of resources to help you learn, including videos for every section of the text; Interactive Figures to help visualize difficult concepts; unlimited practice and assessment on newly learned or prerequisite skills; and access to the Student Solutions Manual, Graphing Calculator Manual, Excel Spreadsheet Manual, and a variety of helpful reference cards.

Do Your Homework! Whether the homework is paper and pencil or assigned online, you must practice what you have learned. This is your opportunity to practice those essential skills needed for passing this course and those skills needed for application in future courses or your career.

We wish you the best in your efforts throughout this course, in future courses, and beyond school.

John and Bernadette

Prerequisite Skills Test*

The following test is unlike your typical math test. Rather than testing your skills after you have worked on them, this test assesses skills that you should know from previous coursework and will use in this class. It is intended to diagnose any areas that you may need to remediate. Take advantage of the results of this test by checking your answers in Appendix B. The full solutions are included to remind you of the steps to answer the problem.

Find the most simplified answer for the given problems involving fractions:

1. $\frac{5}{2} - 6 =$

2. $\frac{1}{2} \div \frac{2}{5} =$

3. $\frac{1}{3} \div 3 =$

Simplify the given expression, keeping in mind the appropriate order of operations:

4. $7 + 2 - 3(2 \div 6) =$

5. $\frac{2 \times 3 + 12}{1 + 5} - 1 =$

Indicate whether each of the statements is true or false:

6. $\frac{4 + 3}{3} = 5$

7. $\frac{5}{7} + \frac{7}{5} = 1$

8. $\frac{3}{5} + 1 = \frac{6}{5}$

Translate each of the following written expressions into a corresponding mathematical statement. If possible, solve for the unknown value or values.

9. Alicia has n pairs of shoes. Manuel has two more pairs of shoes than Alicia. How many pairs of shoes does Manuel have?

10. David's age and Selina's age, when added together, equals 42. Selena is 6 years older than David. What are David's and Selina's ages?

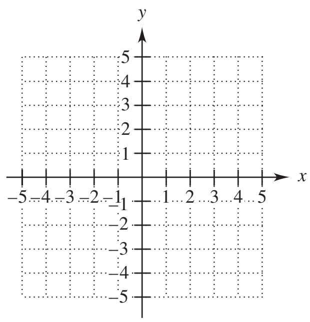
Solve the following problem.

11. The price of a sweater, originally sold for \$72, is reduced by 20%. What is the new sale price of the sweater?

Given the following rectangular coordinate system, graph and label the following points:

12. A: $(3, -2)$ B: $(4, 0)$ C: $(-2, -3)$

13. D: $(3, 5)$ E: $(-1, 4)$ F: $(-4, -5)$



Round the following values as indicated:

14. **(a)** 4.27659 to the nearest tenth

(b) 245.984 to the nearest unit (whole number)

15. **(a)** 16.38572 to the nearest hundredth

(b) 1,763,304.42 to the nearest thousand

Write the number that corresponds with the given numerical statement:

16. **(a)** The Company's liabilities totaled 34 million dollars.

(b) The total of investments was 2.2 thousand dollars.

17. **(a)** The population of a country is 17 hundred thousand.

(b) The cost of the new airport could run as high as three and a quarter billion dollars.

Answer the following. If there is no solution, indicate that there is no solution and provide a reason.

18. $\frac{5}{0} =$

19. A car is traveling 60 miles per hour. Is the car traveling at 1 mile per minute?

20. Which number is greater, -9 or -900?

*Full Solutions to this test are provided in the Appendix B.

Calculus Readiness Test*

1. Subtract the polynomials: $(7x^4 - 8x^3 + 2x^2 - 3) - (9x^4 - 2x^2 + 5x + 6)$.
2. Add the rational functions: $\frac{-6}{3x + 4} + \frac{2}{x}$.
3. Find the product: $(3x + 7)(5x - 8)$.
4. Find the product: $(7x + 3)(6x^2 + x - 8)$.
5. Simplify: $\frac{(x^5y^{-5})^3}{(x^2\sqrt{y})^6}$.
6. Multiply out and simplify: $(8x - 9)^2$.
7. Factor: $9x^2 - 49$.
8. Solve for x: $9x - 4 = 8 + 7(x - 5)$.
9. Solve for x: $3x^2 + 2x + 8 = 20x - 7$.
10. Solve the inequality for x: $x^2 - x - 6 < 0$.
11. Solve for x: $5e^{x-3} - 1 = 9$.
12. Solve for x: $\ln(5x + 1) = 2$.
13. Given that $f(x) = 5x^3 - 3x^2 + 9x + 60$, find $f(-2)$.
14. Given that $f(x) = 6x^2 - 7x - 20$, solve $f(x) = 0$.
15. Find an equation of a line through the point $(8, -3)$ with slope -5.

*Full Solutions to this test are provided in the Appendix B.

Algebra and Equations

1

CHAPTER

CHAPTER OUTLINE

Mathematics is widely used in business, finance, and the biological, social, and physical sciences, from developing efficient production schedules for a factory to mapping the human genome. Mathematics also plays a role in determining interest on a loan from a bank, the growth of traffic on websites, and the study of falling objects.

Algebra and equations are the basic mathematical tools for handling many applications. Your success in this course will depend on your having the algebraic skills presented in this chapter.

1.1 The Real Numbers

Only real numbers will be used in this book.* The names of the most common types of real numbers are as follows.

The Real Numbers

Natural (counting) numbers	1, 2, 3, 4, . . .
Whole numbers	0, 1, 2, 3, 4, . . .

* Not all numbers are real numbers. For example, $\sqrt{-1}$ is a number that is *not* a real number.

Integers	$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
Rational numbers	All numbers that can be written in the form p/q, where p and q are integers and $q \neq 0$
Irrational numbers	Real numbers that are not rational

As you can see, every natural number is a whole number, and every whole number is an integer. Furthermore, every integer is a rational number. For instance, the integer 7 can be written as the fraction $\frac{7}{1}$ and is therefore a rational number.

One example of an irrational number is π, the ratio of the circumference of a circle to its diameter. The number π can be approximated as $\pi \approx 3.14159$ ($\approx$ means "is approximately equal to"), but there is no rational number that is exactly equal to π.

EXAMPLE 1 What kind of number is each of the following?

(a) 6

Solution The number 6 is a natural number, a whole number, an integer, a rational number, and a real number.

(b) $\frac{3}{4}$

Solution This number is rational and real.

(c) 3π

Solution Because π is not a rational number, 3π is irrational and real. ✓1*

✓Checkpoint 1

Name all the types of numbers that apply to the following.

(a) -2

(b) $-5/8$

(c) $\pi/5$

Answers to Checkpoint exercises are found at the end of the section.

All real numbers can be written in decimal form. A rational number, when written in decimal form, is either a terminating decimal, such as .5 or .128, or a repeating decimal, in which some block of digits eventually repeats forever, such as 1.3333 . . . or 4.7234234234 . . . .† Irrational numbers are decimals that neither terminate nor repeat.

When a calculator is used for computations, the answers it produces are often decimal *approximations* of the actual answers; they are accurate enough for most applications. To ensure that your final answer is as accurate as possible,

you should not round off any numbers during long calculator computations.

It is usually OK to round off the final answer to a reasonable number of decimal places once the computation is finished.

The important basic properties of the real numbers are as follows.

Properties of the Real Numbers

For all real numbers, a, b, and c, the following properties hold true:

Commutative properties	$a + b = b + a$ and $ab = ba$.
Associative properties	$(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

*The use of Checkpoint exercises is explained in the "To the Student" section preceding this chapter.

†Some graphing calculators have a FRAC key that automatically converts some repeating decimals to fraction form.

Identity properties	There exists a unique real number 0, called the **additive identity,** such that $\mathbf{a + 0 = a}$ and $\mathbf{0 + a = a.}$ There exists a unique real number 1, called the **multiplicative identity,** such that $\mathbf{a \cdot 1 = a}$ and $\mathbf{1 \cdot a = a.}$
Inverse properties	For each real number a, there exists a unique real number $-a$, called the **additive inverse** of a, such that $\mathbf{a + (-a) = 0}$ and $\mathbf{(-a) + a = 0.}$ If $a \neq 0$, there exists a unique real number $1/a$, called the **multiplicative inverse** of a, such that $\mathbf{a \cdot \frac{1}{a} = 1}$ and $\mathbf{\frac{1}{a} \cdot a = 1.}$
Distributive property	$\mathbf{a(b + c) = ab + ac}$ and $\mathbf{(b + c)a = ba + ca}$.

The next five examples illustrate the properties listed in the preceding box.

EXAMPLE 2 The commutative property says that the order in which you add or multiply two quantities doesn't matter.

(a) $(6 + x) + 9 = 9 + (6 + x) = 9 + (x + 6)$ **(b)** $5 \cdot (9 \cdot 8) = (9 \cdot 8) \cdot 5$

EXAMPLE 3 When the associative property is used, the order of the numbers does not change, but the placement of parentheses does.

(a) $4 + (9 + 8) = (4 + 9) + 8$ **(b)** $3(9x) = (3 \cdot 9)x$ ✓2

✓ Checkpoint 2

Name the property illustrated in each of the following examples.

(a) $(2 + 3) + 9 = (3 + 2) + 9$

(b) $(2 + 3) + 9 = 2 + (3 + 9)$

(c) $(2 + 3) + 9 = 9 + (2 + 3)$

(d) $(4 \cdot 6)p = (6 \cdot 4)p$

(e) $4(6p) = (4 \cdot 6)p$

EXAMPLE 4 By the identity properties,

(a) $-8 + 0 = -8$ **(b)** $(-9) \cdot 1 = -9.$

TECHNOLOGY TIP To enter −8 on a calculator, use the negation key (labeled (−) or +/−), *not* the subtraction key. On most one-line scientific calculators, key in 8 +/−. On graphing calculators or two-line scientific calculators, key in either (−) 8 or +/− 8.

✓ Checkpoint 3

Name the property illustrated in each of the following examples.

(a) $2 + 0 = 2$

(b) $-\frac{1}{4} \cdot (-4) = 1$

(c) $-\frac{1}{4} + \frac{1}{4} = 0$

(d) $1 \cdot \frac{2}{3} = \frac{2}{3}$

EXAMPLE 5 By the inverse properties, the statements in parts (a) through (d) are true.

(a) $9 + (-9) = 0$ **(b)** $-15 + 15 = 0$

(c) $-8 \cdot \left(\frac{1}{-8}\right) = 1$ **(d)** $\frac{1}{\sqrt{5}} \cdot \sqrt{5} = 1$

NOTE There is no real number x such that $0 \cdot x = 1$, so 0 has no multiplicative inverse. ✓3

EXAMPLE 6 By the distributive property,

(a) $9(6 + 4) = 9 \cdot 6 + 9 \cdot 4$

(b) $3(x + y) = 3x + 3y$

(c) $-8(m + 2) = (-8)(m) + (-8)(2) = -8m - 16$

(d) $(5 + x)y = 5y + xy$. ✓4

✓Checkpoint 4

Use the distributive property to rewrite each of the following.

(a) $4(-2 + 5)$

(b) $2(a + b)$

(c) $-3(p + 1)$

(d) $(8 - k)m$

(e) $5x + 3x$

Order of Operations

Some complicated expressions may contain many sets of parentheses. To avoid ambiguity, the following procedure should be used.

Parentheses

Work separately above and below any fraction bar. Within each set of parentheses or square brackets, start with the innermost set and work outward.

EXAMPLE 7 Simplify: $[(3 + 2) - 7]5 + 2([6 \cdot 3] - 13)$.

Solution On each segment, work from the inside out:

$$\begin{aligned} &[(3 + 2) - 7]5 + 2([6 \cdot 3] - 13) \\ &= [5 - 7]5 + 2(18 - 13) \\ &= [-2]5 + 2(5) \\ &= -10 + 10 = 0. \end{aligned}$$

Does the expression $2 + 4 \times 3$ mean

$$(2 + 4) \times 3 = 6 \times 3 = 18?$$

Or does it mean

$$2 + (4 \times 3) = 2 + 12 = 14?$$

To avoid this ambiguity, mathematicians have adopted the following rules (which are also followed by almost all scientific and graphing calculators).

Order of Operations

1. Find all powers and roots, working from left to right.
2. Do any multiplications or divisions in the order in which they occur, working from left to right.
3. Finally, do any additions or subtractions in the order in which they occur, working from left to right.

If sets of parentheses or square brackets are present, use the rules in the preceding box within each set, working from the innermost set outward.

According to these rules, multiplication is done *before* addition, so $2 + 4 \times 3 = 2 + 12 = 14$. Here are some additional examples.

EXAMPLE 8 Use the order of operations to evaluate each expression if $x = -2$, $y = 5$, and $z = -3$.

(a) $-4x^2 - 7y + 4z$

Solution Use parentheses when replacing letters with numbers:

$$\begin{aligned} -4x^2 - 7y + 4z &= -4(-2)^2 - 7(5) + 4(-3) \\ &= -4(4) - 7(5) + 4(-3) = -16 - 35 - 12 = -63. \end{aligned}$$

(b) $$\begin{aligned} \frac{2(x-5)^2 + 4y}{z+4} &= \frac{2(-2-5)^2 + 4(5)}{-3+4} \\ &= \frac{2(-7)^2 + 20}{1} \\ &= 2(49) + 20 = 118. \end{aligned}$$

✓5

✓Checkpoint 5

Evaluate the following if $m = -5$ and $n = 8$.

(a) $-2mn - 2m^2$

(b) $\dfrac{4(n-5)^2 - m}{m+n}$

EXAMPLE 9 Use a calculator to evaluate

$$\frac{-9(-3) + (-5)}{3(-4) - 5(2)}.$$

Solution Use extra parentheses (shown here in blue) around the numerator and denominator when you enter the number in your calculator, and be careful to distinguish the negation key from the subtraction key.

numerator, denominator

$$(-9\,(-3) + (-5))/(3\,(-4) - 5(2))$$

Negation key, Subtraction key

If you don't get -1 as the answer, then you are entering something incorrectly. ✓6

✓Checkpoint 6

Use a calculator to evaluate the following.

(a) $4^2 \div 8 + 3^2 \div 3$

(b) $[-7 + (-9)]\cdot(-4) - 8(3)$

(c) $\dfrac{-11 - (-12) - 4\cdot 5}{4(-2) - (-6)(-5)}$

(d) $\dfrac{36 \div 4\cdot 3 \div 9 + 1}{9 \div (-6)\cdot 8 - 4}$

Square Roots

There are two numbers whose square is 16, namely, 4 and -4. The positive one, 4, is called the **square root** of 16. Similarly, the square root of a nonnegative number d is defined to be the *nonnegative* number whose square is d; this number is denoted $\sqrt{d}$. For instance,

$$\sqrt{36} = 6 \text{ because } 6^2 = 36, \quad \sqrt{0} = 0 \text{ because } 0^2 = 0, \text{ and}$$
$$\sqrt{1.44} = 1.2 \text{ because } (1.2)^2 = 1.44.$$

No negative number has a square root that is a real number. For instance, there is no real number whose square is -4, so -4 has no square root.

Every nonnegative real number has a square root. Unless an integer is a perfect square (such as $64 = 8^2$), its square root is an irrational number. A calculator can be used to obtain a rational approximation of these square roots.

TECHNOLOGY TIP

On one-line scientific calculators, $\sqrt{40}$ is entered as 40 $\sqrt{\ }$. On graphing calculators and two-line scientific calculators, key in $\sqrt{\ }$ 40 ENTER (or EXE).

EXAMPLE 10 Estimate each of the given quantities. Verify your estimates with a calculator.

(a) $\sqrt{40}$

Solution Since $6^2 = 36$ and $7^2 = 49$, $\sqrt{40}$ must be a number between 6 and 7. A typical calculator shows that $\sqrt{40} \approx 6.32455532$.

(b) $5\sqrt{7}$

Solution $\sqrt{7}$ is between 2 and 3 because $2^2 = 4$ and $3^2 = 9$, so $5\sqrt{7}$ must be a number between $5 \cdot 2 = 10$ and $5 \cdot 3 = 15$. A calculator shows that $5\sqrt{7} \approx 13.22875656$. ✓7

✓Checkpoint 7

Estimate each of the following.

(a) $\sqrt{73}$

(b) $\sqrt{22} + 3$

(c) Confirm your estimates in parts (a) and (b) with a calculator.

CAUTION If c and d are positive real numbers, then $\sqrt{c + d}$ is *not* equal to $\sqrt{c} + \sqrt{d}$. For example, $\sqrt{9 + 16} = \sqrt{25} = 5$, but $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$.

The Number Line

The real numbers can be illustrated geometrically with a diagram called a **number line.** Each real number corresponds to exactly one point on the line and vice versa. A number line with several sample numbers located (or **graphed**) on it is shown in Figure 1.1. ✓8

✓Checkpoint 8

Draw a number line, and graph the numbers −4, −1, 0, 1, 2.5, and 13/4 on it.

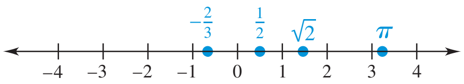


Figure 1.1

When comparing the sizes of two real numbers, the following symbols are used.

Symbol	Read	Meaning
$a < b$	a is less than b.	a lies to the *left* of b on the number line.
$b > a$	b is greater than a.	b lies to the *right* of a on the number line.

Note that $a < b$ means the same thing as $b > a$. The inequality symbols are sometimes joined with the equals sign, as follows.

Symbol	Read	Meaning
$a \le b$	a is less than or equal to b.	either $a < b$ or $a = b$
$b \ge a$	b is greater than or equal to a.	either $b > a$ or $b = a$

TECHNOLOGY TIP

If your graphing calculator has inequality symbols (usually located on the TEST menu), you can key in statements such as "5 < 12" or "−2 ≥ 3." When you press ENTER, the calculator will display 1 if the statement is true and 0 if it is false.

Only one part of an "either . . . or" statement needs to be true for the entire statement to be considered true. So the statement $3 \le 7$ is true because $3 < 7$, and the statement $3 \le 3$ is true because $3 = 3$.

EXAMPLE 11 Write *true* or *false* for each of the following.

(a) $8 < 12$

Solution This statement says that 8 is less than 12, which is true.

(b) $-6 > -3$

Solution The graph in Figure 1.2 shows that −6 is to the *left* of −3. Thus, $-6 < -3$, and the given statement is false.

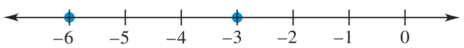


Figure 1.2

(c) $-2 \le -2$

Solution Because $-2 = -2$, this statement is true. ✓9

✓Checkpoint 9

Write *true* or *false* for the following.

(a) $-9 \le -2$

(b) $8 > -3$

(c) $-14 \le -20$

A number line can be used to draw the graph of a set of numbers, as shown in the next few examples.

EXAMPLE 12 Graph all real numbers x such that $1 < x < 5$.

Solution This graph includes all the real numbers between 1 and 5, not just the integers. Graph these numbers by drawing a heavy line from 1 to 5 on the number line, as in Figure 1.3. Parentheses at 1 and 5 show that neither of these points belongs to the graph. ✓10

Figure 1.3

✓Checkpoint 10

Graph all real numbers x such that

(a) $-5 < x < 1$

(b) $4 < x < 7$.

A set that consists of all the real numbers between two points, such as $1 < x < 5$ in Example 12, is called an **interval.** A special notation called **interval notation** is used to indicate an interval on the number line. For example, the interval including all numbers x such that $-2 < x < 3$ is written as $(-2, 3)$. The parentheses indicate that the numbers -2 and 3 are *not* included. If -2 and 3 are to be included in the interval, square brackets are used, as in $[-2, 3]$. The following chart shows several typical intervals, where $a < b$.

Intervals

Inequality	Interval Notation	Explanation
$a \le x \le b$	$[a, b]$	Both a and b are included.
$a \le x < b$	$[a, b)$	a is included; b is not.
$a < x \le b$	$(a, b]$	b is included; a is not.
$a < x < b$	(a, b)	Neither a nor b is included.

Interval notation is also used to describe sets such as the set of all numbers x such that $x \ge -2$. This interval is written $[-2, \infty)$. The set of all real numbers is written $(-\infty, \infty)$ in interval notation.

EXAMPLE 13 Graph the interval $[-2, \infty)$.

Solution Start at -2 and draw a heavy line to the right, as in Figure 1.4. Use a square bracket at -2 to show that -2 itself is part of the graph. The symbol ∞, read "infinity," *does not* represent a number. This notation simply indicates that *all* numbers greater than -2 are in the interval. Similarly, the notation $(-\infty, 2)$ indicates the set of all numbers x such that $x < 2$. ✓11

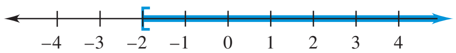


Figure 1.4

✓Checkpoint 11

Graph all real numbers x in the given interval.

(a) $(-\infty, 4]$

(b) $[-2, 1]$

Absolute Value

The **absolute value** of a real number a is the distance from a to 0 on the number line and is written $|a|$. For example, Figure 1.5 shows that the distance from 9 to 0 on the number line is 9, so we have $|9| = 9$. The figure also shows that $|-9| = 9$, because the distance from -9 to 0 is also 9.

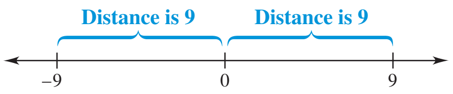


Figure 1.5

The facts that $|9| = 9$ and $|-9| = 9 = -(-9)$ suggest the following algebraic definition of absolute value.

Absolute Value

For any real number a,

$$|a| = a \quad \text{if } a \geq 0$$
$$|a| = -a \quad \text{if } a < 0.$$

The first part of the definition shows that $|0| = 0$ (because $0 \geq 0$). It also shows that the absolute value of any positive number a is the number itself, so $|a|$ is positive in such cases. The second part of the definition says that the absolute value of a negative number a is the *negative* of a. For instance, if $a = -5$, then $|-5| = -(-5) = 5$. So $|-5|$ is positive. The same thing works for any negative number—that is, its absolute value (the negative of a negative number) is positive. Thus, we can state the following:

For every nonzero real number a, the number $|a|$ is positive.

✓Checkpoint 12

Find the following.

(a) $|-6|$

(b) $-|7|$

(c) $-|-2|$

(d) $|-3 -4|$

(e) $|2 - 7|$

EXAMPLE 14 Evaluate $|8 - 9|$.

Solution First, simplify the expression within the absolute-value bars:

$$|8 - 9| = |-1| = 1. \quad ✓12$$

1.1 Exercises

In Exercises 1 and 2, label the statement true or false. (See Example 1.)

1. Every integer is a rational number.
2. Every real number is an irrational number.
3. The decimal expansion of the irrational number π begins 3.141592653589793 . . . . Use your calculator to determine which of the following rational numbers is the best approximation for the irrational number π:

$$\frac{22}{7}, \quad \frac{355}{113}, \quad \frac{103{,}993}{33{,}102}, \quad \frac{2{,}508{,}429{,}787}{798{,}458{,}000}.$$

Your calculator may tell you that some of these numbers are equal to π, but that just indicates that the number agrees with π for as many decimal places as your calculator can handle (usually 10–14). No rational number is exactly equal to π.

Identify the properties that are illustrated in each of the following. (See Examples 2–6.)

4. $0 + (-7) = -7 + 0$
5. $6(t + 4) = 6t + 6 \cdot 4$
6. $3 + (-3) = (-3) + 3$
7. $-5 + 0 = -5$
8. $(-4) \cdot \left(\frac{-1}{4}\right) = 1$
9. $8 + (12 + 6) = (8 + 12) + 6$
10. $1 \cdot (-20) = -20$
11. How is the additive inverse property related to the additive identity property? the multiplicative inverse property to the multiplicative identity property?
12. Explain the distinction between the commutative and associative properties.

Evaluate each of the following if $p = -2$, $q = 3$, and $r = -5$. (See Examples 7–9.)

13. $-3(p + 5q)$
14. $2(q - r)$
15. $\dfrac{q + r}{q + p}$
16. $\dfrac{3q}{3p - 2r}$

Interest Rates *The nominal annual percentage rate (APR) reported by lenders has the formula APR = 12r, where r is the monthly interest rate. Find the APR when*

17. $r = 3.8$ **18.** $r = 0.8$

Find the monthly interest rate r when

19. $APR = 11$ **20.** $APR = 13.2$

Evaluate each expression, using the order of operations given in the text. (See Examples 7–9.)

21. $3 - 4 \cdot 5 + 5$ **22.** $8 - (-4)^2 - (-12)$

23. $(4 - 5) \cdot 6 + 6$ **24.** $\dfrac{2(3 - 7) + 4(8)}{4(-3) + (-3)(-2)}$

25. $8 - 4^2 - (-12)$

26. $-(3 - 5) - [2 - (3^2 - 13)]$

27. $\dfrac{2(-3) + 3/(-2) - 2/(-\sqrt{16})}{\sqrt{64} - 1}$

28. $\dfrac{6^2 - 3\sqrt{25}}{\sqrt{6^2 + 13}}$

Use a calculator to help you list the given numbers in order from smallest to largest. (See Example 10.)

29. $\dfrac{189}{37}$, $\dfrac{4587}{691}$, $\sqrt{47}$, 6.735, $\sqrt{27}$, $\dfrac{2040}{523}$

30. $\dfrac{385}{117}$, $\sqrt{10}$, $\dfrac{187}{63}$, π, $\sqrt{\sqrt{85}}$, 2.9884

Express each of the following statements in symbols, using $<$, $>$, $\le$, *or* $\ge$.

31. 12 is less than 18.5.

32. -2 is greater than -20.

33. x is greater than or equal to 5.7.

34. y is less than or equal to -5.

35. z is at most 7.5.

36. w is negative.

Fill in the blank with $<$, $=$, *or* $>$ *so that the resulting statement is true.*

37. -6 _____ -2 **38.** 3.14 _____ π

39. $3/4$ _____ $.75$ **40.** $1/3$ _____ $.33$

Fill in the blank so as to produce two equivalent statements. For example, the arithmetic statement "a is negative" is equivalent to the geometric statement "the point a lies to the left of the point 0."

	Arithmetic Statement	*Geometric Statement*
41.	$a \ge b$	__________
42.	__________	a lies c units to the right of b
43.	__________	a lies between b and c, and to the right of c
44.	a is positive	__________

Graph the given intervals on a number line. (See Examples 12 and 13.)

45. $(-8, -1)$ **46.** $[-1, 10]$

47. $(-2, 3]$ **48.** $[-2, 2)$

49. $(-2, \infty)$ **50.** $(-\infty, -2]$

Evaluate each of the following expressions (see Example 14).

51. $|-9| - |-12|$

52. $|8| - |-4|$

53. $-|-4| - |-1 - 14|$

54. $-|6| - |-12 - 4|$

In each of the following problems, fill in the blank with either $=$, $<$, *or* $>$, *so that the resulting statement is true.*

55. $|5|$ _____ $|-5|$

56. $-|-4|$ _____ $|4|$

57. $|10 - 3|$ _____ $|3 - 10|$

58. $|6 - (-4)|$ _____ $|-4 - 6|$

59. $|-2 + 8|$ _____ $|2 - 8|$

60. $|3| \cdot |-5|$ _____ $|3(-5)|$

61. $|3 - 5|$ _____ $|3| - |5|$

62. $|-5 + 1|$ _____ $|-5| + |1|$

Write the expression without using absolute-value notation.

63. $|a - 7|$ if $a < 7$

64. $|b - c|$ if $b \ge c$

65. If a and b are any real numbers, is it always true that $|a + b| = |a| + |b|$? Explain your answer.

66. If a and b are any two real numbers, is it always true that $|a - b| = |b - a|$? Explain your answer.

67. For which real numbers b does $|2 - b| = |2 + b|$? Explain your answer.

68. **Health** Data from the National Health and Nutrition Examination Study estimates that 95% of adult heights (inches) are in the following ranges for females and males. (Data from: www.cdc.gov/nchs/nhanes.htm.)

Females	63.5 ± 8.4
Males	68.9 ± 9.3

Express the ranges as an absolute-value inequality in which x is the height of the person.

Consumer Price Index *The Consumer Price Index (CPI) tracks the cost of a typical sample of a consumer goods. The following table shows the percentage increase in the CPI for each year in a 10-year period.*

Year	2006	2007	2008	2009	2010
% Increase in CPI	3.2	4.1	.1	2.7	1.5

Year	2011	2012	2013	2014	2015
% Increase in CPI	3.0	1.7	1.5	1.6	.1

Let r denote the yearly percentage increase in the CPI. For each of the following inequalities, find the number of years during the given period that r satisfied the inequality. (Data from: U.S. Bureau of Labor Statistics.)

69. $r > 3.2$

70. $r < 3.2$

71. $r \leq 3.2$

72. $r \leq 1.5$

73. $r \geq 2.1$

74. $r \geq 1.3$

Stocks *The table presents the 2016 percentage change in the stock price for six well-known companies. (Data from: finance.yahoo.com.)*

Company	Percentage Change
American Express	+10.6%
Coca-Cola	−1.0%
ExxonMobil	+14.9%
Hewlett Packard	+63.1%
Ford	−5.7%
Red Robin Gourmet Burgers	−8.0%

Suppose that we wish to determine the difference in percentage change between two of the companies in the table, and suppose that we are interested only in the magnitude, or absolute value, of this difference. Then we subtract the two entries and find the absolute value. For example, the difference in percentage change of stock price for American Express and Ford is $|10.6\% - (-5.7\%)| = 16.3\%$.

Find the absolute value of the difference between the two indicated changes in stock price.

75. American Express and ExxonMobil

76. Hewlett Packard and Red Robin

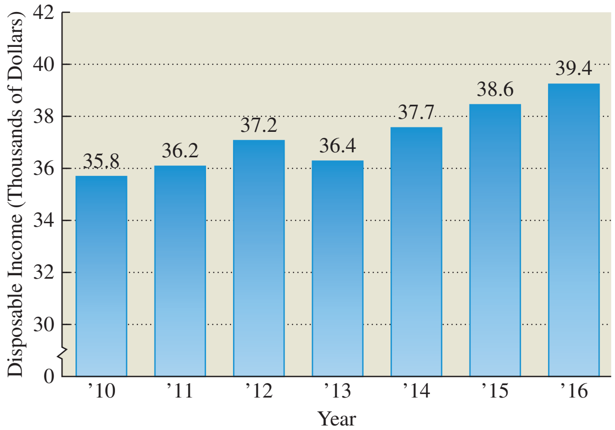
77. Coca-Cola and Hewlett Packard

78. American Express and Ford

79. Ford and Red Robin

80. Coca-Cola and Ford

Disposable Income *The following graph shows the real per capita amount of disposable income (in thousands of dollars) in the United States. (Data from: Federal Reserve Bank of St. Louis.)*



For each of the following, determine the years for which the expression is true, where x is the per capita disposable income.

81. $|x - 37{,}000| > 1000$

82. $|x - 32{,}000| > 3500$

83. $|x - 36{,}500| \leq 2200$

84. $|x - 38{,}500| \leq 1000$

Checkpoint Answers

1. **(a)** Integer, rational, real
(b) Rational, real
(c) Irrational, real

2. **(a)** Commutative property
(b) Associative property
(c) Commutative property
(d) Commutative property
(e) Associative property

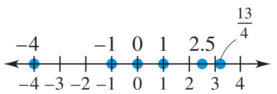
3. **(a)** Additive identity property
(b) Multiplicative inverse property
(c) Additive inverse property
(d) Multiplicative identity property

4. **(a)** $4(-2) + 4(5) = 12$
(b) $2a + 2b$
(c) $-3p - 3$
(d) $8m - km$
(e) $(5 + 3)x = 8x$

5. **(a)** 30 **(b)** $\frac{41}{3}$

6. **(a)** 5 **(b)** 40
(c) $\frac{19}{38} = \frac{1}{2} = .5$ **(d)** $-\frac{1}{4} = -.25$

7. **(a)** Between 8 and 9
(b) Between 7 and 8
(c) 8.5440; 7.6904

8.


9. **(a)** True **(b)** True **(c)** False

10. **(a)**


(b)


11. **(a)**


(b)


12. **(a)** 6 **(b)** −7
(c) −2 **(d)** 7
(e) 5

1.2 Polynomials

Polynomials are the fundamental tools of algebra and will play a central role in this course. In order to do polynomial arithmetic, you must first understand exponents. So we begin with them. You are familiar with the usual notation for squares and cubes, such as:

$$5^2 = 5 \cdot 5 \quad \text{and} \quad 6^3 = 6 \cdot 6 \cdot 6.$$

We now extend this convenient notation to other cases.

If n is a natural number and a is any real number, then

$$\boldsymbol{a^n} \quad \text{denotes the product} \quad \boldsymbol{a \cdot a \cdot a \cdots a}\ (n \text{ factors}).$$

The number a is the **base,** and the number n is the **exponent.**

EXAMPLE 1 4^6, which is read "four to the sixth," or "four to the sixth power," is the number

$$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4096.$$

Similarly, $(-5)^3 = (-5)(-5)(-5) = -125$, and

$$\left(\frac{3}{2}\right)^4 = \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} = \frac{81}{16}.$$

EXAMPLE 2 Use a calculator to approximate the given expressions.

(a) $(1.2)^8$

Solution Key in 1.2 and then use the x^y key (labeled ∧ on some calculators); finally, key in the exponent 8. The calculator displays the (exact) answer 4.29981696.

(b) $\left(\frac{12}{7}\right)^{23}$

Solution Don't compute 12/7 separately. Use parentheses and key in (12/7), followed by the x^y key and the exponent 23 to obtain the approximate answer 242,054.822. ✓1

✓Checkpoint 1

Evaluate the following.

(a) 6^3

(b) 5^{12}

(c) 1^9

(d) $\left(\frac{7}{5}\right)^8$

CAUTION A common error in using exponents occurs with expressions such as $4 \cdot 3^2$. The exponent of 2 applies only to the base 3, so that

$$4 \cdot 3^2 = 4 \cdot 3 \cdot 3 = 36.$$

On the other hand,

$$(4 \cdot 3)^2 = (4 \cdot 3)(4 \cdot 3) = 12 \cdot 12 = 144,$$

so

$$4 \cdot 3^2 \neq (4 \cdot 3)^2.$$

Be careful to distinguish between expressions like -2^4 and $(-2)^4$:

$$-2^4 = -(2^4) = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$$
$$(-2)^4 = (-2)(-2)(-2)(-2) = 16,$$

so

$$-2^4 \neq (-2)^4. \text{ ✓2}$$

✓Checkpoint 2

Evaluate the following.

(a) $3 \cdot 6^2$

(b) $5 \cdot 4^3$

(c) -3^6

(d) $(-3)^6$

(e) $-2 \cdot (-3)^5$

By the definition of an exponent,

$$3^4 \cdot 3^2 = (3 \cdot 3 \cdot 3 \cdot 3)(3 \cdot 3) = 3^6.$$

Since $6 = 4 + 2$, we can write the preceding equation as $3^4 \cdot 3^2 = 3^{4+2}$. This result suggests the following fact, which applies to any real number a and natural numbers m and n.

Multiplication with Exponents

To multiply a^m by a^n, *add* the exponents:

$$a^m \cdot a^n = a^{m+n}.$$

EXAMPLE 3 Verify each of the following simplifications.

(a) $7^4 \cdot 7^6 = 7^{4+6} = 7^{10}$

(b) $(-2)^3 \cdot (-2)^5 = (-2)^{3+5} = (-2)^8$

(c) $(3k)^2 \cdot (3k)^3 = (3k)^5$

(d) $(m + n)^2 \cdot (m + n)^5 = (m + n)^7$ ✓3

Checkpoint 3

Simplify the following.

(a) $5^3 \cdot 5^6$

(b) $(-3)^4 \cdot (-3)^{10}$

(c) $(5p)^2 \cdot (5p)^8$

The multiplication property of exponents has a convenient consequence. By definition,

$$(5^2)^3 = 5^2 \cdot 5^2 \cdot 5^2 = 5^{2+2+2} = 5^6.$$

Note that $2 + 2 + 2$ is $3 \cdot 2 = 6$. This is an example of a more general fact about any real number a and natural numbers m and n.

Power of a Power

To find a power of a power, $(a^m)^n$, *multiply* the exponents:

$$(a^m)^n = a^{mn}.$$

EXAMPLE 4 Verify the following computations.

(a) $(x^3)^4 = x^{3\cdot4} = x^{12}$.

(b) $[(-3)^5]^3 = (-3)^{5\cdot3} = (-3)^{15}$.

(c) $[(6z)^4]^4 = (6z)^{4\cdot4} = (6z)^{16}$. ✓4

Checkpoint 4

Compute the following.

(a) $(6^3)^7$

(b) $[(4k)^5]^6$

It will be convenient to give a zero exponent a meaning. If the multiplication property of exponents is to remain valid, we should have, for example, $3^5 \cdot 3^0 = 3^{5+0} = 3^5$. But this will be true only when $3^0 = 1$. So we make the following definition.

Zero Exponent

If a is any nonzero real number, then

$$a^0 = 1.$$

Checkpoint 5

Evaluate the following.

(a) 17^0

(b) 30^0

(c) $(-10)^0$

(d) $-(12)^0$

For example, $6^0 = 1$ and $(-9)^0 = 1$. Note that 0^0 is *not* defined. ✓5

Polynomials

A **polynomial** is an algebraic expression such as

$$5x^4 + 2x^3 + 6x, \qquad 8m^3 + 9m^2 + \frac{3}{2}m + 3, \qquad -10p, \qquad \text{or} \qquad 8.$$

The letter used is called a **variable,** and a polynomial is a sum of **terms** of the form

(constant) × (nonnegative integer power of the variable).

We assume that $x^0 = 1$, $m^0 = 1$, etc., so terms such as 3 or 8 may be thought of as $3x^0$ and $8x^0$, respectively. The constants that appear in each term of a polynomial are called the **coefficients** of the polynomial. The coefficient of x^0 is called the **constant term.**

EXAMPLE 5 Identify the coefficients and the constant term of the given polynomials.

(a) $5x^2 - x + 12$

Solution The coefficients are 5, −1, and 12, and the constant term is 12.

(b) $7x^3 + 2x - 4$

Solution The coefficients are 7, 0, 2, and −4, because the polynomial can be written $7x^3 + 0x^2 + 2x - 4$. The constant term is −4.

A polynomial that consists only of a constant term, such as 15, is called a **constant polynomial.** The **zero polynomial** is the constant polynomial 0. The **degree** of a polynomial is the *exponent* of the highest power of x that appears with a *nonzero* coefficient, and the nonzero coefficient of this highest power of x is the **leading coefficient** of the polynomial. For example,

Polynomial	Degree	Leading Coefficient	Constant Term
$6x^7 + 4x^3 + 5x^2 - 7x + 10$	7	6	10
$-x^4 + 2x^3 + \frac{1}{2}$	4	−1	$\frac{1}{2}$
x^3	3	1	0
12	0	12	12

The degree of the zero polynomial is *not defined*, since no exponent of x occurs with a nonzero coefficient. First-degree polynomials are often called **linear polynomials.** Second- and third-degree polynomials are called **quadratics** and **cubics,** respectively. ✓6

✓Checkpoint 6

Find the degree of each polynomial.

(a) $x^4 - x^2 + x + 5$

(b) $7x^5 + 6x^3 - 3x^8 + 2$

(c) 17

(d) 0

Addition and Subtraction

Two terms having the same variable with the same exponent are called **like terms;** other terms are called **unlike terms.** Polynomials can be added or subtracted by using the distributive property to combine like terms. Only like terms can be combined. For example,

$$12y^4 + 6y^4 = (12 + 6)y^4 = 18y^4$$

and

$$-2m^2 + 8m^2 = (-2 + 8)m^2 = 6m^2.$$

The polynomial $8y^4 + 2y^5$ has unlike terms, so it cannot be further simplified.

In more complicated cases of addition, you may have to eliminate parentheses, use the commutative and associative laws to regroup like terms, and then combine them.

EXAMPLE 6 Add the following polynomials.

(a) $(8x^3 - 4x^2 + 6x) + (3x^3 + 5x^2 - 9x + 8)$

Solution
$$8x^3 - 4x^2 + 6x + 3x^3 + 5x^2 - 9x + 8 \qquad \text{Eliminate parentheses.}$$
$$= (8x^3 + 3x^3) + (-4x^2 + 5x^2) + (6x - 9x) + 8 \qquad \text{Group like terms.}$$
$$= 11x^3 + x^2 - 3x + 8 \qquad \text{Combine like terms.}$$

(b) $(-4x^4 + 6x^3 - 9x^2 - 12) + (-3x^3 + 8x^2 - 11x + 7)$

Solution
$$-4x^4 + 6x^3 - 9x^2 - 12 - 3x^3 + 8x^2 - 11x + 7 \qquad \text{Eliminate parentheses.}$$
$$= -4x^4 + (6x^3 - 3x^3) + (-9x^2 + 8x^2) - 11x + (-12 + 7) \qquad \text{Group like terms.}$$
$$= -4x^4 + 3x^3 - x^2 - 11x - 5 \qquad \text{Combine like terms.}$$

Care must be used when parentheses are preceded by a minus sign. For example, we know that

$$-(4 + 3) = -(7) = -7 = -4 - 3.$$

If you simply delete the parentheses in $-(4 + 3)$, you obtain $-4 + 3 = -1$, which is the wrong answer. This fact and the preceding examples illustrate the following rules.

Rules for Eliminating Parentheses

Parentheses preceded by a plus sign (or no sign) may be deleted.

Parentheses preceded by a minus sign may be deleted *provided that* the sign of every term within the parentheses is changed.

EXAMPLE 7 Subtract: $(2x^2 - 11x + 8) - (7x^2 - 6x + 2)$.

Solution
$$2x^2 - 11x + 8 - 7x^2 + 6x - 2 \qquad \text{Eliminate parentheses.}$$
$$= (2x^2 - 7x^2) + (-11x + 6x) + (8 - 2) \qquad \text{Group like terms.}$$
$$= -5x^2 - 5x + 6 \qquad \text{Combine like terms. ✓7}$$

Checkpoint 7

Add or subtract as indicated.

(a) $(-2x^2 + 7x + 9) + (3x^2 + 2x - 7)$

(b) $(4x + 6) - (13x - 9)$

(c) $(9x^3 - 8x^2 + 2x) - (9x^3 - 2x^2 - 10)$

Multiplication

The distributive property is also used to multiply polynomials. For example, the product of $8x$ and $6x - 4$ is found as follows:

$$8x(6x - 4) = 8x(6x) - 8x(4) \qquad \text{Distributive property}$$
$$= 48x^2 - 32x. \qquad x \cdot x = x^2$$

EXAMPLE 8 Use the distributive property to find each product.

(a) $2p^3(3p^2 - 2p + 5) = 2p^3(3p^2) + 2p^3(-2p) + 2p^3(5)$
$$= 6p^5 - 4p^4 + 10p^3$$

(b) $(3k - 2)(k^2 + 5k - 4) = 3k(k^2 + 5k - 4) - 2(k^2 + 5k - 4)$
$$= 3k^3 + 15k^2 - 12k - 2k^2 - 10k + 8$$
$$= 3k^3 + 13k^2 - 22k + 8 \quad ✓8$$

Checkpoint 8

Find the following products.

(a) $-6r(2r - 5)$

(b) $(8m + 3) \cdot (m^4 - 2m^2 + 6m)$

EXAMPLE 9 The product $(2x - 5)(3x + 4)$ can be found by using the distributive property twice:

$$\begin{aligned}(2x - 5)(3x + 4) &= 2x(3x + 4) - 5(3x + 4)\\ &= 2x \cdot 3x + 2x \cdot 4 + (-5) \cdot 3x + (-5) \cdot 4\\ &= 6x^2 + 8x - 15x - 20\\ &= 6x^2 - 7x - 20.\end{aligned}$$

Observe the pattern in the second line of Example 9 and its relationship to the terms being multiplied.

$$(2x - 5)(3x + 4) = 2x \cdot 3x + 2x \cdot 4 + (-5) \cdot 3x + (-5) \cdot 4$$

First terms: $(2x - 5)(3x + 4)$ → $2x \cdot 3x$

Outside terms: $(2x - 5)(3x + 4)$ → $2x \cdot 4$

Inside terms: $(2x - 5)(3x + 4)$ → $(-5) \cdot 3x$

Last terms: $(2x - 5)(3x + 4)$ → $(-5) \cdot 4$

This pattern is easy to remember by using the acronym **FOIL** (**F**irst, **O**utside, **I**nside, **L**ast). The FOIL method makes it easy to find products such as this one mentally, without the necessity of writing out the intermediate steps.

EXAMPLE 10 Use FOIL to find the product of the given polynomials.

(a) $(3x + 2)(x + 5) = \underset{\text{First}}{3x^2} + \underset{\text{Outside}}{15x} + \underset{\text{Inside}}{2x} + \underset{\text{Last}}{10} = 3x^2 + 17x + 10$

(b) $(x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$

(c) $(2x + 1)(2x - 1) = 4x^2 - 2x + 2x - 1 = 4x^2 - 1$ ✓9

Checkpoint 9

Use FOIL to find these products.

(a) $(5k - 1)(2k + 3)$

(b) $(7z - 3)(2z + 5)$

Applications

In business, the *revenue* from the sales of an item is given by

$$\textbf{Revenue} = \textbf{(price per item)} \times \textbf{(number of items sold).}$$

The *cost* to manufacture and sell these items is given by

$$\textbf{Cost} = \textbf{Fixed Costs} + \textbf{Variable Costs,}$$

where the fixed costs include such things as buildings and machinery (which do not depend on how many items are made) and variable costs include such things as labor and materials (which vary, depending on how many items are made). Then

$$\textbf{Profit} = \textbf{Revenue} - \textbf{Cost.}$$

EXAMPLE 11 **Profit** A manufacturer of scientific calculators sells calculators for \$12 each (wholesale) and can produce a maximum of 150,000. The variable cost of producing x thousand calculators is $6995x - 7.2x^2$ dollars, and the fixed costs for the manufacturing operation are \$230,000. If x thousand calculators are manufactured and sold, find expressions for the revenue, cost, and profit.

Solution If x thousand calculators are sold at \$12 each, then

$$\text{Revenue} = (\text{price per item}) \times (\text{number of items sold})$$
$$R = 12 \times 1000x = 12{,}000x,$$

Checkpoint 10

Suppose revenue is given by $7x^2 - 3x$, fixed costs are \$500, and variable costs are given by $3x^2 + 5x - 25$. Write an expression for

(a) Cost

(b) Profit

where $x \leq 150$ (because only 150,000 calculators can be made). The variable cost of making x thousand calculators is $6995x - 7.2x^2$, so that

$$\text{Cost} = \text{Fixed Costs} + \text{Variable Costs},$$
$$C = 230{,}000 + (6995x - 7.2x^2) \quad (x \leq 150).$$

Therefore, the profit is given by

$$\begin{aligned} P = R - C &= 12{,}000x - (230{,}000 + 6995x - 7.2x^2) \\ &= 12{,}000x - 230{,}000 - 6995x + 7.2x^2 \\ P &= 7.2x^2 + 5005x - 230{,}000 \quad (x \leq 150). \end{aligned}$$ ✓10

1.2 Exercises

Use a calculator to approximate these numbers. (See Examples 1 and 2.)

1. 11.2^6 **2.** $(-6.54)^{11}$

3. $(-18/7)^6$ **4.** $(5/9)^7$

5. Explain how the value of -3^2 differs from $(-3)^2$. Do -3^3 and $(-3)^3$ differ in the same way? Why or why not?

6. Describe the steps used to multiply 4^3 and 4^5. Is the product of 4^3 and 3^4 found in the same way? Explain.

Simplify each of the given expressions. Leave your answers in exponential notation. (See Examples 3 and 4.)

7. $4^2 \cdot 4^3$ **8.** $(-4)^4 \cdot (-4)^6$

9. $(-6)^2 \cdot (-6)^5$ **10.** $(2z)^5 \cdot (2z)^6$

11. $[(5u)^4]^7$ **12.** $(6y)^3 \cdot [(6y)^5]^4$

List the degree of the given polynomial, its coefficients, and its constant term. (See Example 5.)

13. $6.2x^4 - 5x^3 + 4x^2 - 3x + 3.7$

14. $6x^7 + 4x^6 - x^3 + x$

State the degree of the given polynomial.

15. $1 + x + 2x^2 + 3x^3$

16. $5x^4 - 4x^5 - 6x^3 + 7x^4 - 2x + 8$

Add or subtract as indicated. (See Examples 6 and 7.)

17. $(3x^3 + 2x^2 - 5x) + (-4x^3 - x^2 - 8x)$

18. $(-2p^3 - 5p + 7) + (-4p^2 + 8p + 2)$

19. $(-4y^2 - 3y + 8) - (2y^2 - 6y + 2)$

20. $(7b^2 + 2b - 5) - (3b^2 + 2b - 6)$

21. $(2x^3 + 2x^2 + 4x - 3) - (2x^3 + 8x^2 + 1)$

22. $(3y^3 + 9y^2 - 11y + 8) - (-4y^2 + 10y - 6)$

Find each of the given products. (See Examples 8–10.)

23. $-9m(2m^2 + 6m - 1)$

24. $2a(4a^2 - 6a + 8)$

25. $(3z + 5)(4z^2 - 2z + 1)$

26. $(2k + 3)(4k^3 - 3k^2 + k)$

27. $(6k - 1)(2k + 3)$

28. $(8r + 3)(r - 1)$

29. $(3y + 5)(2y + 1)$

30. $(5r - 3s)(5r - 4s)$

31. $(9k + q)(2k - q)$

32. $(.012x - .17)(.3x + .54)$

33. $(6.2m - 3.4)(.7m + 1.3)$

34. $2p - 3[4p - (8p + 1)]$

35. $5k - [k + (-3 + 5k)]$

36. $(3x - 1)(x + 2) - (2x + 5)^2$

Profit *Find expressions for the revenue, cost, and profit from selling x thousand items. (See Example 11.)*

	Item Price	*Fixed Costs*	*Variable Costs*
37.	\$5.00	\$200,000	$1800x$
38.	\$8.50	\$225,000	$4200x$

39. **Profit** Beauty Works sells its cologne wholesale for \$9.75 per bottle. The variable costs of producing x thousand bottles is $-3x^2 + 3480x - 325$ dollars, and the fixed costs of manufacturing are \$260,000. Find expressions for the revenue, cost, and profit from selling x thousand items.

40. **Profit** A self-help guru sells her book *Be Happy in 45 Easy Steps* for \$23.50 per copy. Her fixed costs are \$145,000 and the estimate of the variable cost of printing, binding, and distributing x thousand books is given by $-4.2x^2 + 3220x - 425$ dollars. Find expressions for the revenue, cost, and profit from selling x thousand copies of the book.

Work these problems.

Starbucks Earnings *The accompanying bar graph shows the net earnings (in millions of dollars) for the Starbucks Corporation. The polynomial*

$$\mathbf{4.79x^3 - 122.5x^2 + 1104x - 2863}$$

gives a good approximation of Starbuck's net earnings in year x, where $x = 7$ corresponds to 2007, and so on ($7 \le x \le 16$). For each of the given years,

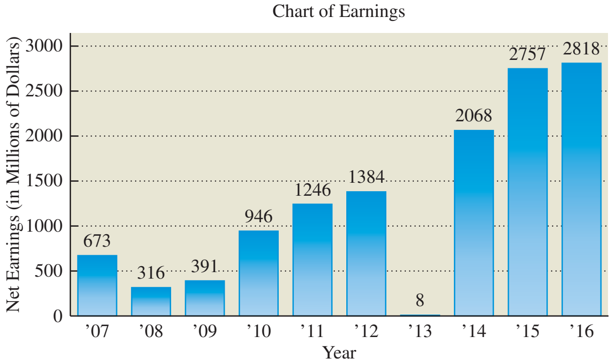
(a) *use the bar graph to determine the net earnings;*

(b) *use the polynomial to determine the net earnings.*

(Data from: www.morningstar.com.)

41. 2007 **42.** 2015

43. 2012 **44.** 2013



Assuming that the polynomial approximation in Exercises 41–44 remains accurate in later years, use it to estimate Starbucks's net earnings in each of the following years.

45. 2017 **46.** 2018 **47.** 2019

48. Do the estimates in Exercises 45–47 seem plausible? Explain.

Starbucks Costs *The costs (in millions of dollars) for the Starbucks Corporation can be approximated by the polynomial* $9.5x^3 - 401.6x^2 + 6122x - 25{,}598$, *where* $x = 7$ *corresponds to the year 2007. Determine whether each of the given statements is true or false. (Data from: www.morningstar.com.)*

49. The costs were higher than \$5000 million in 2010.

50. The costs were higher than \$7500 million in 2015.

51. The costs were higher in 2012 than in 2015.

52. The costs were lower in 2011 than in 2016.

PepsiCo Profits *The profits (in millions of dollars) for PepsiCo Inc. can be approximated by the polynomial* $-72.85x^3 + 2082x^2 - 16{,}532x + 59{,}357$, *where* $x = 7$ *corresponds to the year 2007. (Data from: www.morningstar.com.) Find the approximate profits for the following years.*

53. 2007 **54.** 2010 **55.** 2012 **56.** 2015

Determine whether each of the given statements is true or false.

57. Were profits higher in 2013 or 2009?

58. Were profits higher in 2012 or 2015?

Business *Use the table feature of a graphing calculator or use a spreadsheet to make a table of values for the profit expression in Example 11, with* $x = 0, 5, 10, \ldots, 150$. *Use the table to answer the following questions.*

59. What is the profit or loss (negative profit) when 25,000 calculators are sold? when 60,000 are sold? Explain these answers.

60. Approximately how many calculators must be sold in order for the company to make a profit?

61. What is the profit from selling 100,000 calculators?

62. What is the profit from selling 150,000 calculators?

Checkpoint Answers

1. **(a)** 216 **(b)** 244,140,625 **(c)** 1 **(d)** 14.75789056

2. **(a)** 108 **(b)** 320 **(c)** -729 **(d)** 729 **(e)** 486

3. **(a)** 5^9 **(b)** $(-3)^{14}$ **(c)** $(5p)^{10}$

4. **(a)** 6^{21} **(b)** $(4k)^{30}$

5. **(a)** 1 **(b)** 1 **(c)** 1 **(d)** -1

6. **(a)** 4 **(b)** 8 **(c)** 0 **(d)** Not defined

7. **(a)** $x^2 + 9x + 2$ **(b)** $-9x + 15$ **(c)** $-6x^2 + 2x + 10$

8. **(a)** $-12r^2 + 30r$ **(b)** $8m^5 + 3m^4 - 16m^3 + 42m^2 + 18m$

9. **(a)** $10k^2 + 13k - 3$ **(b)** $14z^2 + 29z - 15$

10. **(a)** $C = 3x^2 + 5x + 475$ **(b)** $P = 4x^2 - 8x - 475$

1.3 Factoring

The number 18 can be written as a product in several ways: $9 \cdot 2$, $(-3)(-6)$, $1 \cdot 18$, etc. The numbers in each product (9, 2, -3, etc.) are called **factors,** and the process of writing 18 as a product of factors is called **factoring.** Thus, factoring is the reverse of multiplication.

Factoring of polynomials is a means of simplifying many expressions and of solving certain types of equations. As is the usual custom, factoring of polynomials in this text will be restricted to finding factors with *integer* coefficients (otherwise there may be an infinite number of possible factors).

Greatest Common Factor

The algebraic expression $15m + 45$ is made up of two terms: $15m$ and 45. Each of these terms has 15 as a factor. In fact, $15m = 15 \cdot m$ and $45 = 15 \cdot 3$. By the distributive property,

$$15m + 45 = 15 \cdot m + 15 \cdot 3 = 15(m + 3).$$

Both 15 and $m + 3$ are factors of $15m + 45$. Since 15 is a factor of all terms of $15m + 45$ and is the largest such number, it is called the **greatest common factor** for the polynomial $15m + 45$. The process of writing $15m + 45$ as $15(m + 3)$ is called **factoring out** the greatest common factor.

EXAMPLE 1 Factor out the greatest common factor.

(a) $12p - 18q$

Solution Both $12p$ and $18q$ are divisible by 6, and

$$\begin{aligned} 12p - 18q &= 6 \cdot 2p - 6 \cdot 3q \\ &= 6(2p - 3q). \end{aligned}$$

(b) $8x^3 - 9x^2 + 15x$

Solution Each of these terms is divisible by x:

$$\begin{aligned} 8x^3 - 9x^2 + 15x &= (8x^2) \cdot x - (9x) \cdot x + 15 \cdot x \\ &= x(8x^2 - 9x + 15). \end{aligned}$$

(c) $5(4x - 3)^3 + 2(4x - 3)^2$

Solution The quantity $(4x - 3)^2$ is a common factor. Factoring it out gives

$$\begin{aligned} 5(4x - 3)^3 + 2(4x - 3)^2 &= (4x - 3)^2[5(4x - 3) + 2] \\ &= (4x - 3)^2(20x - 15 + 2) \\ &= (4x - 3)^2(20x - 13). \end{aligned}$$

✓1

✓ Checkpoint 1

Factor out the greatest common factor.

(a) $12r + 9k$

(b) $75m^2 + 100n^2$

(c) $6m^4 - 9m^3 + 12m^2$

(d) $3(2k + 1)^3 + 4(2k + 1)^4$

Factoring Quadratics

If we multiply two first-degree polynomials, the result is a quadratic. For instance, using FOIL, we see that $(x + 1)(x - 2) = x^2 - x - 2$. Since factoring is the reverse of multiplication, factoring quadratics requires using FOIL backward.

EXAMPLE 2 Factor $x^2 + 9x + 18$.

Solution We must find integers b and d such that

$$\begin{aligned} x^2 + 9x + 18 &= (x + b)(x + d) \\ &= x^2 + dx + bx + bd \\ x^2 + 9x + 18 &= x^2 + (b + d)x + bd. \end{aligned}$$

Since the constant coefficients on each side of the equation must be equal, we must have $bd = 18$; that is, b and d are factors of 18. Similarly, the coefficients of x must be the same, so that $b + d = 9$. The possibilities are summarized in this table:

Factors b, d of 18	**Sum $b + d$**
$18 \cdot 1$	$18 + 1 = 19$
$9 \cdot 2$	$9 + 2 = 11$
$6 \cdot 3$	$6 + 3 = 9$

There is no need to list negative factors, such as $(-3)(-6)$, because their sum is negative. The table suggests that 6 and 3 will work. Verify that

$$(x + 6)(x + 3) = x^2 + 9x + 18. \quad ✓2$$

Checkpoint 2

Factor the following.

(a) $r^2 + 7r + 10$

(b) $x^2 + 4x + 3$

(c) $y^2 + 6y + 8$

EXAMPLE 3

Factor $x^2 + 3x - 10$.

Solution As in Example 2, we must find factors b and d whose product is -10 (the constant term) and whose sum is 3 (the coefficient of x). The following table shows the possibilities.

Factors b, d of -10	Sum $b + d$
$1(-10)$	$1 + (-10) = -9$
$(-1)10$	$-1 + 10 = 9$
$2(-5)$	$2 + (-5) = -3$
$(-2)5$	$-2 + 5 = 3$

The only factors with product -10 and sum 3 are -2 and 5. So the correct factorization is

$$x^2 + 3x - 10 = (x - 2)(x + 5),$$

as you can readily verify.

It is usually not necessary to construct tables as was done in Examples 2 and 3—you can just mentally check the various possibilities. The approach used in Examples 2 and 3 (with minor modifications) also works for factoring quadratic polynomials whose leading coefficient is not 1.

EXAMPLE 4

Factor $4y^2 - 11y + 6$.

Solution We must find integers a, b, c, and d such that

$$\begin{aligned} 4y^2 - 11y + 6 &= (ay + b)(cy + d) \\ &= acy^2 + ady + bcy + bd \\ 4y^2 - 11y + 6 &= acy^2 + (ad + bc)y + bd. \end{aligned}$$

Since the coefficients of y^2 must be the same on both sides, we see that $ac = 4$. Similarly, the constant terms show that $bd = 6$. The positive factors of 4 are 4 and 1 or 2 and 2. Since the middle term is negative, we consider only negative factors of 6. The possibilities are -2 and -3 or -1 and -6. Now we try various arrangements of these factors until we find one that gives the correct coefficient of y:

$$(2y - 1)(2y - 6) = 4y^2 - \mathbf{14y} + 6 \qquad \text{Incorrect}$$
$$(2y - 2)(2y - 3) = 4y^2 - \mathbf{10y} + 6 \qquad \text{Incorrect}$$
$$(y - 2)(4y - 3) = 4y^2 - \mathbf{11y} + 6. \qquad \text{Correct}$$

The last trial gives the correct factorization. ✓3

Checkpoint 3

Factor the following.

(a) $x^2 - 4x + 3$

(b) $2y^2 - 5y + 2$

(c) $6z^2 - 13z + 6$

EXAMPLE 5

Factor $6p^2 - 7pq - 5q^2$.

Solution Again, we try various possibilities. The positive factors of 6 could be 2 and 3 or 1 and 6. As factors of -5, we have only -1 and 5 or -5 and 1. Try different combinations of these factors until the correct one is found:

$$(2p - 5q)(3p + q) = 6p^2 - \mathbf{13pq} - 5q^2 \qquad \text{Incorrect}$$
$$(3p - 5q)(2p + q) = 6p^2 - \mathbf{7pq} - 5q^2. \qquad \text{Correct}$$

So $6p^2 - 7pq - 5q^2$ factors as $(3p - 5q)(2p + q)$. ✓4

Checkpoint 4

Factor the following.

(a) $r^2 - 5r - 14$

(b) $3m^2 + 5m - 2$

(c) $6p^2 + 13pq - 5q^2$

NOTE In Examples 2–4, we chose positive factors of the positive first term. Of course, we could have used two negative factors, but the work is easier if positive factors are used.

EXAMPLE 6 Factor $x^2 + x + 3$.

Solution There are only two ways to factor 3, namely, $3 = 1 \cdot 3$ and $3 = (-1)(-3)$. They lead to these products:

$$(x + 1)(x + 3) = x^2 + 4x + 3 \quad \text{Incorrect}$$
$$(x - 1)(x - 3) = x^2 - 4x + 3. \quad \text{Incorrect}$$

Therefore, this polynomial cannot be factored.

Factoring Patterns

In some cases, you can factor a polynomial with a minimum amount of guesswork by recognizing common patterns. The easiest pattern to recognize is the *difference of squares.*

Difference of Squares

$$x^2 - y^2 = (x + y)(x - y).$$

To verify the accuracy of the preceding equation, multiply out the right side.

EXAMPLE 7 Factor each of the following.

(a) $4m^2 - 9$

Solution Notice that $4m^2 - 9$ is the difference of two squares, since $4m^2 = (2m)^2$ and $9 = 3^2$. Use the pattern for the difference of two squares, letting $2m$ replace x and 3 replace y. Then the pattern $x^2 - y^2 = (x + y)(x - y)$ becomes

$$\begin{aligned} 4m^2 - 9 &= (\mathbf{2m})^2 - \mathbf{3}^2 \\ &= (2m + 3)(2m - 3). \end{aligned}$$

(b) $128p^2 - 98q^2$

Solution First factor out the common factor of 2:

$$\begin{aligned} 128p^2 - 98q^2 &= 2(64p^2 - 49q^2) \\ &= 2[(\mathbf{8p})^2 - (\mathbf{7q})^2] \\ &= 2(8p + 7q)(8p - 7q). \end{aligned}$$

(c) $x^2 + 36$

Solution The sum of two perfect squares cannot be factored with real numbers. To understand why this is true, let us say for the sake of argument that $x^2 + 36$ can be factored with real numbers. Then there would exist two real numbers a and b where we could write

$$x^2 + 36 = (x + a)(x + b).$$

The above expression is an equality. This means that for any real number that we substitute for x, the expressions on each side of the equals sign must evaluate to the same value. However, if we let $x = -a$, then we obtain:

$$\begin{aligned} (-a)^2 + 36 &= (\mathbf{-a} + a)(\mathbf{-a} + b) && \text{Substituting } x = -a. \\ (-a)^2 + 36 &= 0(-a + b) && \text{Additive inverse of } a \\ (-a)^2 + 36 &= 0. && \text{Multiplication by } 0 \end{aligned}$$

The above is problematic. The left-hand side of the equation has to be greater than 0 because $(-a)^2$ has to be 0 or positive, and then, when added to 36, it will remain greater than 0. As we see above, the right-hand side has to be 0. This is a contradiction. Thus, our assumption that $x^2 + 36$ can be factored with real numbers must be incorrect.

When we see an expression of the form $x^2 + a^2$, we call this a sum of two squares. Such expressions cannot be factored with real numbers.

(d) $(x - 2)^2 - 49$

Solution Since $49 = 7^2$, this is a difference of two squares. So it factors as follows:

$$\begin{aligned}(x - 2)^2 - 49 &= (x - 2)^2 - 7^2\\ &= [(x - 2) + 7][(x - 2) - 7]\\ &= (x + 5)(x - 9).\end{aligned}$$ ✓5

✓Checkpoint 5

Factor the following.

(a) $9p^2 - 49$

(b) $y^2 + 100$

(c) $(x + 3)^2 - 64$

Another common pattern is the *perfect square.* Verify each of the following factorizations by multiplying out the right side.

Perfect Squares

$$x^2 + 2xy + y^2 = (x + y)^2$$
$$x^2 - 2xy + y^2 = (x - y)^2$$

Whenever you have a quadratic whose first and last terms are squares, it *may* factor as a perfect square. The key is to look at the middle term. To have a perfect square whose first and last terms are x^2 and y^2, the middle term must be $\pm 2xy$. To avoid errors, always check this.

EXAMPLE 8 Factor each polynomial, if possible.

(a) $16p^2 - 40pq + 25q^2$

Solution The first and last terms are squares, namely, $16p^2 = (4p)^2$ and $25q^2 = (5q)^2$. So the second perfect-square pattern, with $x = 4p$ and $y = 5q$, might work. To have a perfect square, the middle term $-40pq$ must equal $-2(4p)(5q)$, which it does. So the polynomial factors as

$$16p^2 - 40pq + 25q^2 = (4p - 5q)(4p - 5q),$$

as you can easily verify.

(b) $9u^2 + 5u + 1$

Solution Again, the first and last terms are squares: $9u^2 = (3u)^2$ and $1 = 1^2$. The middle term is positive, so the first perfect-square pattern might work, with $x = 3u$ and $y = 1$. To have a perfect square, however, the middle term would have to be $2(3u) \cdot 1 = 6u$, which is *not* the middle term of the given polynomial. So it is not a perfect square—in fact, it cannot be factored.

(c) $169x^2 + 104xy^2 + 16y^4$

Solution This polynomial may be factored as $(13x + 4y^2)^2$, since $169x^2 = (13x)^2$, $16y^4 = (4y^2)^2$, and $2(13x)(4y^2) = 104xy^2$. ✓6

✓Checkpoint 6

Factor.

(a) $4m^2 + 4m + 1$

(b) $25z^2 - 80zt + 64t^2$

(c) $9x^2 + 15x + 25$

EXAMPLE 9 Factor each of the given polynomials.

(a) $12x^2 - 26x - 10$

Solution Look first for a greatest common factor. Here, the greatest common factor is 2: $12x^2 - 26x - 10 = 2(6x^2 - 13x - 5)$. Now try to factor $6x^2 - 13x - 5$. Possible factors of 6 are 3 and 2 or 6 and 1. The only factors of -5 are -5 and 1 or 5 and -1. Try various combinations. You should find that the quadratic factors as $(3x + 1)(2x - 5)$. Thus,

$$12x^2 - 26x - 10 = 2(3x + 1)(2x - 5).$$

(b) $4z^2 + 12z + 9 - w^2$

Solution There is no common factor here, but notice that the first three terms can be factored as a perfect square:

$$4z^2 + 12z + 9 - w^2 = (2z + 3)^2 - w^2.$$

Written in this form, the expression is the difference of squares, which can be factored as follows:

$$\begin{aligned}(2z + 3)^2 - w^2 &= [(2z + 3) + w][(2z + 3) - w]\\ &= (2z + 3 + w)(2z + 3 - w).\end{aligned}$$

(c) $16a^2 - 100 - 48ac + 36c^2$

Solution Factor out the greatest common factor of 4 first:

$$\begin{aligned}16a^2 - 100 - 48ac + 36c^2 &= 4[4a^2 - 25 - 12ac + 9c^2]\\ &= 4[(4a^2 - 12ac + 9c^2) - 25] && \text{Rearrange terms and group.}\\ &= 4[(2a - 3c)^2 - 25] && \text{Factor.}\\ &= 4(2a - 3c + 5)(2a - 3c - 5). && \text{Factor the difference of squares.}\end{aligned}$$

✓7

Checkpoint 7

Factor the following.

(a) $6x^2 - 27x - 15$

(b) $9r^2 + 12r + 4 - t^2$

(c) $18 - 8xy - 2y^2 - 8x^2$

CAUTION Remember always to look first for a greatest common factor.

Higher Degree Polynomials

Polynomials of degree greater than 2 are often difficult to factor. However, factoring is relatively easy in two cases: *the difference and the sum of cubes.* By multiplying out the right side, you can readily verify each of the following factorizations.

Difference and Sum of Cubes

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$
$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

EXAMPLE 10 Factor each of the following polynomials.

(a) $k^3 - 8$

Solution Since $8 = 2^3$, use the pattern for the difference of two cubes to obtain

$$k^3 - 8 = k^3 - 2^3 = (k - 2)(k^2 + 2k + 4).$$

(b) $m^3 + 125$

Solution $m^3 + 125 = m^3 + 5^3 = (m + 5)(m^2 - 5m + 25)$

(c) $8k^3 - 27z^3$

Solution $8k^3 - 27z^3 = (2k)^3 - (3z)^3 = (2k - 3z)(4k^2 + 6kz + 9z^2)$ ✓8

✓Checkpoint 8

Factor the following.

(a) $a^3 + 1000$

(b) $z^3 - 64$

(c) $1000m^3 - 27z^3$

Substitution and appropriate factoring patterns can sometimes be used to factor higher degree expressions.

EXAMPLE 11 Factor the following polynomials.

(a) $x^8 + 4x^4 + 3$

Solution The idea is to make a substitution that reduces the polynomial to a quadratic or cubic that we can deal with. Note that $x^8 = (x^4)^2$. Let $u = x^4$. Then

$$\begin{aligned} x^8 + 4x^4 + 3 &= (x^4)^2 + 4x^4 + 3 && \text{Power of a power} \\ &= u^2 + 4u + 3 && \text{Substitute } x^4 = u. \\ &= (u + 3)(u + 1) && \text{Factor.} \\ &= (x^4 + 3)(x^4 + 1). && \text{Substitute } u = x^4. \end{aligned}$$

(b) $x^4 - y^4$

Solution Note that $x^4 = (x^2)^2$, and similarly for the y term. Let $u = x^2$ and $v = y^2$. Then

$$\begin{aligned} x^4 - y^4 &= (x^2)^2 - (y^2)^2 && \text{Power of a power} \\ &= u^2 - v^2 && \text{Substitute } x^2 = u \text{ and } y^2 = v. \\ &= (u + v)(u - v) && \text{Difference of squares} \\ &= (x^2 + y^2)(x^2 - y^2) && \text{Substitute } u = x^2 \text{ and } v = y^2. \\ &= (x^2 + y^2)(x + y)(x - y). && \text{Difference of squares} \end{aligned}$$ ✓9

✓Checkpoint 9

Factor each of the following.

(a) $2x^4 + 5x^2 + 2$

(b) $3x^4 - x^2 - 2$

Once you understand Example 11, you can often factor without making explicit substitutions.

EXAMPLE 12 Factor $256k^4 - 625m^4$.

Solution Use the difference of squares twice, as follows:

$$\begin{aligned} 256k^4 - 625m^4 &= (16k^2)^2 - (25m^2)^2 \\ &= (16k^2 + 25m^2)(16k^2 - 25m^2) \\ &= (16k^2 + 25m^2)(4k + 5m)(4k - 5m). \end{aligned}$$ ✓10

✓Checkpoint 10

Factor $81x^4 - 16y^4$.

1.3 Exercises

Factor out the greatest common factor in each of the given polynomials. (See Example 1.)

1. $12x^2 - 24x$

2. $5y - 65xy$

3. $r^3 - 5r^2 + r$

4. $t^3 + 3t^2 + 8t$

5. $6z^3 - 12z^2 + 18z$

6. $5x^3 + 55x^2 + 10x$

7. $3(2y - 1)^2 + 7(2y - 1)^3$

8. $(3x + 7)^5 - 4(3x + 7)^3$

9. $3(x + 5)^4 + (x + 5)^6$

10. $3(x + 6)^2 + 6(x + 6)^4$

Factor the polynomial. (See Examples 2 and 3.)

11. $x^2 + 5x + 4$

12. $u^2 + 7u + 6$

13. $x^2 + 7x + 12$

14. $y^2 + 8y + 12$

15. $x^2 + x - 6$

16. $x^2 + 4x - 5$

17. $x^2 + 2x - 3$

18. $y^2 + y - 12$

19. $x^2 - 3x - 4$

20. $u^2 - 2u - 8$

21. $z^2 - 9z + 14$ **22.** $w^2 - 6w - 16$

23. $z^2 + 10z + 24$ **24.** $r^2 + 16r + 60$

Factor the polynomial. (See Examples 4–6.)

25. $2x^2 - 9x + 4$ **26.** $3w^2 - 8w + 4$

27. $15p^2 - 23p + 4$ **28.** $8x^2 - 14x + 3$

29. $4z^2 - 16z + 15$ **30.** $12y^2 - 29y + 15$

31. $6x^2 - 5x - 4$ **32.** $12z^2 + z - 1$

33. $10y^2 + 21y - 10$ **34.** $15u^2 + 4u - 4$

35. $6x^2 + 5x - 4$ **36.** $12y^2 + 7y - 10$

Factor each polynomial completely. Factor out the greatest common factor as necessary. (See Examples 2–9.)

37. $3a^2 + 2a - 5$ **38.** $6a^2 - 48a - 120$

39. $x^2 - 81$ **40.** $x^2 + 17xy + 72y^2$

41. $9p^2 - 12p + 4$ **42.** $3r^2 - r - 2$

43. $r^2 + 3rt - 10t^2$ **44.** $2a^2 + ab - 6b^2$

45. $m^2 - 8mn + 16n^2$ **46.** $8k^2 - 16k - 10$

47. $4u^2 + 12u + 9$ **48.** $9p^2 - 16$

49. $25p^2 - 10p + 4$ **50.** $10x^2 - 17x + 3$

51. $4r^2 - 9v^2$ **52.** $x^2 + 3xy - 28y^2$

53. $x^2 + 4xy + 4y^2$ **54.** $16u^2 + 12u - 18$

55. $3a^2 - 13a - 30$ **56.** $3k^2 + 2k - 8$

57. $21m^2 + 13mn + 2n^2$ **58.** $81y^2 - 100$

59. $y^2 - 4yz - 21z^2$ **60.** $49a^2 + 9$

61. $121x^2 - 64$ **62.** $4z^2 + 56zy + 196y^2$

Factor each of these polynomials. (See Example 10.)

63. $a^3 - 64$ **64.** $b^3 + 216$

65. $8r^3 - 27s^3$ **66.** $1000p^3 + 27q^3$

67. $64m^3 + 125$ **68.** $216y^3 - 343$

69. $1000y^3 - z^3$ **70.** $125p^3 + 8q^3$

Factor each of these polynomials. (See Examples 11 and 12.)

71. $x^4 + 5x^2 + 6$ **72.** $y^4 + 7y^2 + 10$

73. $b^4 - b^2$ **74.** $z^4 - 3z^2 - 4$

75. $x^4 - x^2 - 12$ **76.** $4x^4 + 27x^2 - 81$

77. $16a^4 - 81b^4$ **78.** $x^6 - y^6$

79. $x^8 + 8x^2$ **80.** $x^9 - 64x^3$

81. When asked to factor $6x^4 - 3x^2 - 3$ completely, a student gave the following result:

$$6x^4 - 3x^2 - 3 = (2x^2 + 1)(3x^2 - 3).$$

Is this answer correct? Explain why.

82. When can the sum of two squares be factored? Give examples.

83. Explain why $(x + 2)^3$ is not the correct factorization of $x^3 + 8$, and give the correct factorization.

84. Describe how factoring and multiplication are related. Give examples.

✓ Checkpoint Answers

1. **(a)** $3(4r + 3k)$ **(b)** $25(3m^2 + 4n^2)$
   (c) $3m^2(2m^2 - 3m + 4)$ **(d)** $(2k + 1)^3(7 + 8k)$
2. **(a)** $(r + 2)(r + 5)$ **(b)** $(x + 3)(x + 1)$
   (c) $(y + 2)(y + 4)$
3. **(a)** $(x - 3)(x - 1)$ **(b)** $(2y - 1)(y - 2)$
   (c) $(3z - 2)(2z - 3)$
4. **(a)** $(r - 7)(r + 2)$ **(b)** $(3m - 1)(m + 2)$
   (c) $(2p + 5q)(3p - q)$
5. **(a)** $(3p + 7)(3p - 7)$ **(b)** Cannot be factored
   (c) $(x + 11)(x - 5)$
6. **(a)** $(2m + 1)^2$ **(b)** $(5z - 8t)^2$
   (c) Does not factor
7. **(a)** $3(2x + 1)(x - 5)$
   (b) $(3r + 2 + t)(3r + 2 - t)$
   (c) $2(3 - 2x - y)(3 + 2x + y)$
8. **(a)** $(a + 10)(a^2 - 10a + 100)$
   (b) $(z - 4)(z^2 + 4z + 16)$
   (c) $(10m - 3z)(100m^2 + 30mz + 9z^2)$
9. **(a)** $(2x^2 + 1)(x^2 + 2)$
   (b) $(3x^2 + 2)(x + 1)(x - 1)$
10. $(9x^2 + 4y^2)(3x + 2y)(3x - 2y)$

1.4 Rational Expressions

A **rational expression** is an expression that can be written as the quotient of two polynomials, such as

$$\frac{8}{x-1}, \qquad \frac{3x^2 + 4x}{5x - 6}, \qquad \text{and} \qquad \frac{2y + 1}{y^4 + 8}.$$

It is sometimes important to know the values of the variable that make the denominator 0 (in which case the quotient is not defined). For example, 1 cannot be used as a replacement for x in the first expression above and $6/5$ cannot be used in the second one, since these values make the respective denominators equal 0. *Throughout this section, we assume that all denominators are nonzero*, which means that some replacement values for the variables may have to be excluded. ✓1

✓Checkpoint 1

What value of the variable makes each denominator equal 0?

(a) $\frac{5}{x-3}$

(b) $\frac{2x-3}{4x-1}$

(c) $\frac{x+2}{x}$

(d) Why do we need to determine these values?

Simplifying Rational Expressions

A key tool for simplification is the following fact.

Cancellation Property

For all expressions P, Q, and S, with $Q \neq 0$ and $S \neq 0$,

$$\frac{PS}{QS} = \frac{P}{Q}.$$

EXAMPLE 1 Write each of the following rational expressions in lowest terms (so that the numerator and denominator have no common factor with integer coefficients except 1 or -1).

(a) $\frac{12m}{-18}$

Solution Both $12m$ and -18 are divisible by 6. By the cancellation property,

$$\frac{12m}{-18} = \frac{2m \cdot 6}{-3 \cdot 6} = \frac{2m}{-3} = -\frac{2m}{3}.$$

(b) $\frac{8x+16}{4}$

Solution Factor the numerator and cancel:

$$\frac{8x+16}{4} = \frac{8(x+2)}{4} = \frac{4 \cdot 2(x+2)}{4} = \frac{2(x+2)}{1} = 2(x+2).$$

The answer could also be written as $2x + 4$ if desired.

(c) $\frac{k^2+7k+12}{k^2+2k-3}$

Solution Factor the numerator and denominator and cancel:

$$\frac{k^2+7k+12}{k^2+2k-3} = \frac{(k+4)(k+3)}{(k-1)(k+3)} = \frac{k+4}{k-1}.$$ ✓2

✓Checkpoint 2

Write each of the following in lowest terms.

(a) $\frac{12k+36}{18}$

(b) $\frac{15m+30m^2}{5m}$

(c) $\frac{2p^2+3p+1}{p^2+3p+2}$

Multiplication and Division

The rules for multiplying and dividing rational expressions are the same fraction rules you learned in arithmetic.

Multiplication and Division Rules

For all expressions P, Q, R, and S, with $Q \neq 0$ and $S \neq 0$,

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$$

and

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} \qquad (R \neq 0).$$

EXAMPLE 2

(a) Multiply $\frac{2}{3} \cdot \frac{y}{5}$.

Solution Use the multiplication rule. Multiply the numerators and then the denominators:

$$\frac{2}{3} \cdot \frac{y}{5} = \frac{2 \cdot y}{3 \cdot 5} = \frac{2y}{15}.$$

The result, $2y/15$, is in lowest terms.

(b) Multiply $\frac{3y + 9}{6} \cdot \frac{18}{5y + 15}$.

Solution Factor where possible:

$$\frac{3y + 9}{6} \cdot \frac{18}{5y + 15} = \frac{3(y + 3)}{6} \cdot \frac{18}{5(y + 3)}$$

$$= \frac{3 \cdot 18(y + 3)}{6 \cdot 5(y + 3)} \qquad \text{Multiply numerators and denominators.}$$

$$= \frac{3 \cdot 6 \cdot 3(y + 3)}{6 \cdot 5(y + 3)} \qquad 18 = 6 \cdot 3$$

$$= \frac{3 \cdot 3}{5} \qquad \text{Write in lowest terms.}$$

$$= \frac{9}{5}.$$

(c) Multiply $\frac{m^2 + 5m + 6}{m + 3} \cdot \frac{m^2 + m - 6}{m^2 + 3m + 2}$.

Solution Factor numerators and denominators:

$$\frac{(m + 2)(m + 3)}{m + 3} \cdot \frac{(m - 2)(m + 3)}{(m + 2)(m + 1)} \qquad \text{Factor.}$$

$$= \frac{(m + 2)(m + 3)(m - 2)(m + 3)}{(m + 3)(m + 2)(m + 1)} \qquad \text{Multiply.}$$

$$= \frac{(m - 2)(m + 3)}{m + 1} \qquad \text{Lowest terms}$$

$$= \frac{m^2 + m - 6}{m + 1}.$$ ✓3

✓Checkpoint 3

Multiply.

(a) $\frac{3r^2}{5} \cdot \frac{20}{9r}$

(b) $\frac{y - 4}{y^2 - 2y - 8} \cdot \frac{y^2 - 4}{3y}$

EXAMPLE 3

(a) Divide $\frac{8x}{5} \div \frac{11x^2}{20}$.

Solution Invert the second expression and multiply (division rule):

$$\frac{8x}{5} \div \frac{11x^2}{20} = \frac{8x}{5} \cdot \frac{20}{11x^2} \quad \text{Invert and multiply.}$$

$$= \frac{8x \cdot 20}{5 \cdot 11x^2} \quad \text{Multiply.}$$

$$= \frac{32}{11x}. \quad \text{Lowest terms}$$

(b) Divide $\frac{9p - 36}{12} \div \frac{5(p - 4)}{18}$.

Solution We have

$$\frac{9p - 36}{12} \cdot \frac{18}{5(p - 4)} \quad \text{Invert and multiply.}$$

$$= \frac{9(p - 4)}{12} \cdot \frac{18}{5(p - 4)} \quad \text{Factor.}$$

$$= \frac{27}{10}. \quad \text{Cancel, multiply, and write in lowest terms.}$$

✓ 4

Checkpoint 4

Divide.

(a) $\frac{5m}{16} \div \frac{m^2}{10}$

(b) $\frac{2y - 8}{6} \div \frac{5y - 20}{3}$

(c) $\frac{m^2 - 2m - 3}{m(m + 1)} \div \frac{m + 4}{5m}$

Addition and Subtraction

As you know, when two numerical fractions have the same denominator, they can be added or subtracted. The same rules apply to rational expressions.

Addition and Subtraction Rules

For all expressions P, Q, R, with $Q \neq 0$,

$$\frac{P}{Q} + \frac{R}{Q} = \frac{P + R}{Q}$$

and

$$\frac{P}{Q} - \frac{R}{Q} = \frac{P - R}{Q}.$$

EXAMPLE 4

Add or subtract as indicated.

(a) $\frac{4}{5k} + \frac{11}{5k}$

Solution Since the denominators are the same, we add the numerators:

$$\frac{4}{5k} + \frac{11}{5k} = \frac{4 + 11}{5k} = \frac{15}{5k} \quad \text{Addition rule}$$

$$= \frac{3}{k}. \quad \text{Lowest terms}$$

(b) $\dfrac{2x^2 + 3x + 1}{x^5 + 1} - \dfrac{x^2 - 7x}{x^5 + 1}$

Solution The denominators are the same, so we subtract numerators, paying careful attention to parentheses:

$$\frac{2x^2 + 3x + 1}{x^5 + 1} - \frac{x^2 - 7x}{x^5 + 1} = \frac{(2x^2 + 3x + 1) - (x^2 - 7x)}{x^5 + 1} \quad \text{Subtraction rule}$$

$$= \frac{2x^2 + 3x + 1 - x^2 - (-7x)}{x^5 + 1} \quad \text{Subtract numerators.}$$

$$= \frac{2x^2 + 3x + 1 - x^2 + 7x}{x^5 + 1}$$

$$= \frac{x^2 + 10x + 1}{x^5 + 1}. \quad \text{Simplify the numerator.}$$

When fractions do not have the same denominator, you must first find a common denominator before you can add or subtract. A common denominator is a denominator that has each fraction's denominator as a factor.

EXAMPLE 5 Add or subtract as indicated.

(a) $\dfrac{7}{p^2} + \dfrac{9}{2p} + \dfrac{1}{3p^2}$

Solution These three denominators are different, so we must find a common denominator that has each of p^2, $2p$, and $3p^2$ as factors. Observe that $6p^2$ satisfies these requirements. Use the cancellation property to rewrite each fraction as one that has $6p^2$ as its denominator and then add them:

$$\frac{7}{p^2} + \frac{9}{2p} + \frac{1}{3p^2} = \frac{6 \cdot 7}{6 \cdot p^2} + \frac{3p \cdot 9}{3p \cdot 2p} + \frac{2 \cdot 1}{2 \cdot 3p^2} \quad \text{Cancellation property}$$

$$= \frac{42}{6p^2} + \frac{27p}{6p^2} + \frac{2}{6p^2}$$

$$= \frac{42 + 27p + 2}{6p^2} \quad \text{Addition rule}$$

$$= \frac{27p + 44}{6p^2}. \quad \text{Simplify.}$$

(b) $\dfrac{k^2}{k^2 - 1} - \dfrac{2k^2 - k - 3}{k^2 + 3k + 2}$

Solution Factor the denominators to find a common denominator:

$$\frac{k^2}{k^2 - 1} - \frac{2k^2 - k - 3}{k^2 + 3k + 2} = \frac{k^2}{(k + 1)(k - 1)} - \frac{2k^2 - k - 3}{(k + 1)(k + 2)}.$$

A common denominator here is $(k + 1)(k - 1)(k + 2)$, because each of the preceding denominators is a factor of this common denominator. Write each fraction with the common denominator:

$$\frac{k^2}{(k+1)(k-1)} - \frac{2k^2-k-3}{(k+1)(k+2)}$$

$$= \frac{k^2(k+2)}{(k+1)(k-1)(k+2)} - \frac{(2k^2-k-3)(k-1)}{(k+1)(k-1)(k+2)}$$

$$= \frac{k^3+2k^2-(2k^2-k-3)(k-1)}{(k+1)(k-1)(k+2)}$$ Subtract fractions.

$$= \frac{k^3+2k^2-(2k^3-3k^2-2k+3)}{(k+1)(k-1)(k+2)}$$ Multiply $(2k^2\text{-}k\text{-}3)(k\text{-}1)$.

$$= \frac{k^3+2k^2-2k^3+3k^2+2k-3}{(k+1)(k-1)(k+2)}$$ Polynomial subtraction

$$= \frac{-k^3+5k^2+2k-3}{(k+1)(k-1)(k+2)}.$$ Combine terms. ✓5

✓Checkpoint 5

Add or subtract.

(a) $\frac{3}{4r} + \frac{8}{3r}$

(b) $\frac{1}{m-2} - \frac{3}{2(m-2)}$

(c) $\frac{p+1}{p^2-p} - \frac{p^2-1}{p^2+p-2}$

Complex Fractions

Any quotient of rational expressions is called a **complex fraction.** Complex fractions are simplified as demonstrated in Example 6.

EXAMPLE 6 Simplify the complex fraction

$$\frac{6-\frac{5}{k}}{1+\frac{5}{k}}.$$

Solution Multiply both numerator and denominator by the common denominator k:

$$\frac{6-\frac{5}{k}}{1+\frac{5}{k}} = \frac{k\left(6-\frac{5}{k}\right)}{k\left(1+\frac{5}{k}\right)}$$ Multiply by $\frac{k}{k}$.

$$= \frac{6k-k\left(\frac{5}{k}\right)}{k+k\left(\frac{5}{k}\right)}$$ Distributive property

$$= \frac{6k-5}{k+5}.$$ Simplify.

1.4 Exercises

Write each of the given expressions in lowest terms. Factor as necessary. (See Example 1.)

1. $\frac{8x^2}{56x}$

2. $\frac{27m}{81m^3}$

3. $\frac{25p^2}{35p^3}$

4. $\frac{18y^4}{24y^2}$

5. $\frac{5m+15}{4m+12}$

6. $\frac{10z+5}{20z+10}$

7. $\frac{4(w-3)}{(w-3)(w+6)}$

8. $\frac{-6(x+2)}{(x+4)(x+2)}$

9. $\frac{3y^2-12y}{9y^3}$

10. $\frac{15k^2+45k}{9k^2}$

11. $\frac{m^2-4m+4}{m^2+m-6}$

12. $\frac{r^2-r-6}{r^2+r-12}$

13. $\frac{x^2+2x-3}{x^2-1}$

14. $\frac{z^2+4z+4}{z^2-4}$

Multiply or divide as indicated in each of the exercises. Write all answers in lowest terms. (See Examples 2 and 3.)

15. $\frac{3a^2}{64} \cdot \frac{8}{2a^3}$

16. $\frac{2u^2}{8u^4} \cdot \frac{10u^3}{9u}$

17. $\frac{7x}{11} \div \frac{14x^3}{66y}$

18. $\frac{6x^2y}{2x} \div \frac{21xy}{y}$

19. $\frac{2a+b}{3c} \cdot \frac{15}{4(2a+b)}$

20. $\frac{4(x+2)}{w} \cdot \frac{3w^2}{8(x+2)}$

21. $\frac{15p-3}{6} \div \frac{10p-2}{3}$

22. $\frac{2k+8}{6} \div \frac{3k+12}{3}$

23. $\frac{9y-18}{6y+12} \cdot \frac{3y+6}{15y-30}$

24. $\frac{12r+24}{36r-36} \div \frac{6r+12}{8r-8}$

25. $\frac{4a+12}{2a-10} \div \frac{a^2-9}{a^2-a-20}$

26. $\frac{6r-18}{9r^2+6r-24} \cdot \frac{12r-16}{4r-12}$

27. $\frac{k^2-k-6}{k^2+k-12} \cdot \frac{k^2+3k-4}{k^2+2k-3}$

28. $\frac{n^2-n-6}{n^2-2n-8} \div \frac{n^2-9}{n^2+7n+12}$

29. In your own words, explain how to find the least common denominator of two fractions.

30. Describe the steps required to add three rational expressions. You may use an example to illustrate.

Add or subtract as indicated in each of the following. Write all answers in lowest terms. (See Example 4.)

31. $\frac{2}{7z} - \frac{1}{5z}$

32. $\frac{4}{3z} - \frac{5}{4z}$

33. $\frac{r+2}{3} - \frac{r-2}{3}$

34. $\frac{3y-1}{8} - \frac{3y+1}{8}$

35. $\frac{4}{x} + \frac{1}{5}$

36. $\frac{6}{r} - \frac{3}{4}$

37. $\frac{1}{m-1} + \frac{2}{m}$

38. $\frac{8}{y+2} - \frac{3}{y}$

39. $\frac{7}{b+2} + \frac{2}{5(b+2)}$

40. $\frac{4}{3(k+1)} + \frac{3}{k+1}$

41. $\frac{2}{5(k-2)} + \frac{5}{4(k-2)}$

42. $\frac{11}{3(p+4)} - \frac{5}{6(p+4)}$

43. $\frac{2}{x^2-4x+3} + \frac{5}{x^2-x-6}$

44. $\frac{3}{m^2-3m-10} + \frac{7}{m^2-m-20}$

45. $\frac{2y}{y^2+7y+12} - \frac{y}{y^2+5y+6}$

46. $\frac{-r}{r^2-10r+16} - \frac{3r}{r^2+2r-8}$

In each of the exercises in the next set, simplify the complex fraction. (See Example 6.)

47. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}$

48. $\dfrac{2-\frac{2}{y}}{2+\frac{2}{y}}$

49. $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

50. $\dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

Work these problems.

Natural Science *Each figure in the following exercises is a dartboard. The probability that a dart which hits the board lands in the shaded area is the fraction*

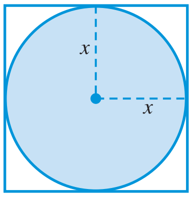
$$\frac{\text{area of the shaded region}}{\text{area of the dartboard}}.$$

Note: The area of a circle is πr^2, the area of a square is bh, and the area of a triangle is $\frac{1}{2}bh$.

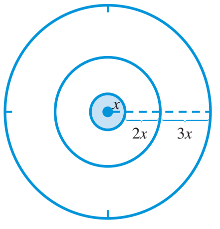
(a) *Express the probability as a rational expression in x.*

(b) *Then reduce the expression to lowest terms.*

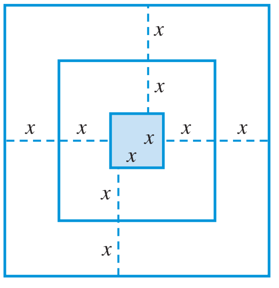
51.



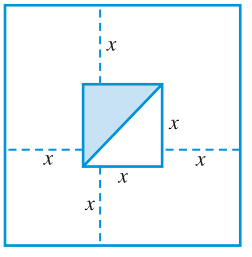
52.



53.



54.



Costs *In Example 11 of Section 1.2, we saw that the cost C of producing x thousand calculators is given by*

$$C = -7.2x^2 + 6995x + 230{,}000 \quad (x \le 150).$$

55. The average cost per calculator is the total cost C divided by the number of calculators produced. Write a rational expression that gives the average cost per calculator when x thousand are produced.

56. Find the average cost per calculator for each of these production levels: 20,000, 50,000, and 125,000.