To Sophie, Alex, and Claire

PREFACE

Why Fundamentals of Engineering Economics?

Engineering economics is one of the most practical subject matters in the engineering curriculum, but it is an always challenging, ever-changing discipline. *Contemporary Engineering Economics* (*CEE*), now in its sixth edition, was first published in 1993, and since then, we have tried to reflect changes in the business world in each new edition along with the latest innovations in education and publishing. These changes have resulted in a better, more complete textbook, but one that is much longer than it was originally intended. This may present a problem: Today, covering the textbook in a single term is increasingly difficult. Therefore, we decided to create *Fundamentals of Engineering Economics* (*FEE*) for those who like *contemporary* but think a smaller, more concise textbook would better serve their needs.

Goals of the Text

This text aims not only to provide sound and comprehensive coverage of the concepts of engineering economics but also to address the practical concerns of engineering economics. More specifically, this text has the following goals:

1. To build a thorough understanding of the theoretical and conceptual basis upon which the practice of financial project analysis is built.
2. To satisfy the very practical needs of the engineer toward making informed financial decisions when acting as a team member or project manager for an engineering project.
3. To incorporate all critical decision-making tools—including the most contemporary, computer-oriented ones that engineers bring to the task of making informed financial decisions.
4. To appeal to the full range of engineering disciplines for which this course is often required: industrial, civil, mechanical, electrical, computer, aerospace, chemical, and manufacturing engineering as well as engineering technology.

Intended Market and Use

This text is intended for use in introductory engineering economics courses. Unlike the larger textbook (*CEE*), it is possible to cover *FEE* in a single term and perhaps even to supplement it with a few outside readings or case studies. Although the chapters in *FEE* are arranged logically, they are written in a flexible, modular format, allowing instructors to cover the material in a different sequence.

New to This Edition

Much of the content has been streamlined to provide materials in depth and to reflect the challenges in contemporary engineering economics. Some of the highlighted changes are as follows:

- All chapter opening vignettes—a trademark of *Fundamentals of Engineering Economics*—have been completely replaced with more current and thought-provoking examples from both service and manufacturing sectors.

Chapters	Chapter opening vignettes	Company	Sector	Industry
1	• A car for hire	Uber Technologies	Communications	Media, Internet-based services
2	• Powerball lottery	Personal	Consumer discretionary	Gaming
3	• College loans	Personal	Financials	Banking
4	• Baseball tickets	Boston Red Sox	Consumer discretionary	Recreational facilities
5	• Commercial building	The Endeavor Real Estate	Housing	Real estate
6	• Owning a dump truck	The City of Flagstaff	Government	Public works
7	• Value of a college degree	Personal	Consumer discretionary	Education
8	• Robot cargo handling	The Port of Los Angeles	Government	Transportation
9	• 3D Printing	Alcoa Aluminum	Materials	Metals & Mining
10	• Solar power plants	NRG Energy Co.	Utilities	Energy
11	• Pumped storage	Eagle Crest Energy	Utilities	Utility network
12	• Bio-solids fertilizer	Milorganite Factory	Private	Manufacturing
13	• Acquiring Brocade	Broadcom Company	Technology	Software

- **Self-Test Questions** have been expanded at the end of each chapter (184 problems in total), and worked-out solutions to the questions are provided in Appendix A. These questions are formatted in a style suitable for Fundamentals Engineering Exam review and were created to help students prepare for a typical class exam common to introductory engineering economic courses.

- Most of the end-of-chapter problems are revised to reflect the changes in the main text. There are 720 problems, including 184 self-test questions, 75% of which are new or updated.
- Various Excel® spreadsheet modeling techniques are introduced throughout the chapters, and the original Excel files are provided online at the Companion Website.
- Some other specific content changes made in the fourth edition are as follows:
  - In Chapter 1, updated a buy-lease decision problem, and introduced the Tesla's Gigafactory project to illustrate the scope of a large-scale engineering project.
  - In Chapter 2, updated the tuition prepayment plan and lottery examples.
  - In Chapter 3, introduced a new example to compare two different financial products.
  - In Chapter 4, updated all consumer price index (CPI) and inflation related data, restructured many examples to facilitate the understanding of equivalence calculation under inflation.
  - In Chapter 6, expanded an example of life-cycle cost analysis for an electric motor selection problem.
  - In Chapter 7, added a new section on modified internal rate of return.
  - In Chapter 8, added a new benefit–cost analysis example of comparison of mutually exclusive public projects.
  - In Chapter 9, updated any tax law changes from the 2017 Tax Cuts and Jobs Act.
  - In Chapter 10, revised a section on the tax rate to use in project analysis.
  - In Chapter 11, added a new section on the concept of value at risk (VaR) as a risk measure.
  - In Chapter 13, replaced all financial statements for Lam Research Corporation with those of J&M Corporation, and provided a new set of financial ratio analysis.
  - In Appendix A, updated all solutions to be consistent with new set of self-test questions.

Features of the Book

FEE is significantly different from *CEE*, but most of the chapters will be familiar to users of *CEE*. Although we pruned some material and clarified, updated, and otherwise improved all of the chapters, *FEE* should still be considered an alternative and streamlined version of *CEE*.

We did retain all of the pedagogical elements and supporting materials that helped make *CEE* so successful. For example:

- Each chapter opens with a real economic vignette describing how an individual decision maker or actual corporation has wrestled with the issues discussed in the chapter. These opening cases heighten students' interest by pointing out the real-world relevance and applicability of what might otherwise seem to be dry technical material.
- In working out each individual chapters example problems, students are encouraged to highlight the critical data provided by each question, isolate the question being asked, and outline the correct approach in the solution under the headings **Given, Find, Approach**, and **Comments**, respectively. This convention is employed throughout the text. This guidance is intended to stimulate student curiosity to look beyond the mechanics of problem solving to explore "what-if" issues, alternative solution methods, and the interpretation of the solutions.

- There are a large number of end-of-chapter problems and exam-type questions varying in level of difficulty; these problems thoroughly cover the book's various topics.
- Most chapters contain a section titled "Short Case Studies with Excel," enabling students to use Excel to answer a set of questions. These problems reinforce the concepts covered in the chapter and provide students an opportunity to become more proficient with the use of an electronic spreadsheet.
- Many of Excel spreadsheets now contain easy-to-follow call-out formulas. The integration of Excel is another important feature of FEE. Students have increased access to and familiarity with Excel, and instructors have more inclination either to treat these topics explicitly in the course or to encourage students to experiment independently. One could argue that the use of Excel will undermine true understanding of course concepts. This text does not promote the trivial or mindless use of Excel as a replacement for genuine understanding of and skill in applying traditional solution methods. Rather, it focuses on Excel's productivity-enhancing benefits for complex project cash flow development and analysis.

To Student: How to Prepare for the Fundamentals of Engineering (FE) Exam

The set of self-study questions at the end of each chapter is designed primarily to help you develop a working knowledge of the concepts and principles of engineering economics. However, the questions are also perfect resource to help you prepare the Fundamentals of Engineering (FE) exam. All questions are structured in multiple-choice format because these types of exam questions are used in the FE exam and, increasingly, in introductory engineering economics courses.

The FE exam typically consists of 180 multiple-choice questions. During the morning session (120 questions), all examinees take a general exam common to all disciplines. During the afternoon session (60 questions), examinees can opt to take a general exam or a discipline-specific (Chemical, Civil, Electrical, Environmental, Industrial, or Mechanical) exam.

The general exam includes four questions related to engineering economics in the morning session and five in the afternoon session. The specific engineering economics topics covered in the FE exam are

- Discounted cash flow (e.g., equivalence, PW, equivalent annual, FW, and rate of return)
- Cost (e.g., incremental, average, sunk, estimating)
- Analyses (e.g., breakeven, benefit–cost)
- Uncertainty (e.g., expected value and risk)
- Valuation and depreciation

Some sample questions are also provided by the National Council of Examiners for Engineering and Surveying (www.ncees.org/exams).

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- **Learning Catalytics™:** Learning Catalytics is an interactive student response tool that encourages team-based learning by using students' smartphones, tablets, or laptops to engage them in interactive tasks and thinking.

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Chan S. Park
Sedona, Arizona

CONTENTS

PART 1

Understanding Money and Its Management

CHAPTER ONE

Engineering Economic Decisions

Uber: The Transportation Service Has Become a Global Brand[1] Travis Kalanick, the founder of Uber, was born in Los Angeles, California, in 1976; he learned to code at an early age and went on to study computer engineering at UCLA but left with a few months to go before graduation. After a couple of startups, he had the financial means and time to create Uber. The story goes that on a snowy night in Paris, Kalanick and his cofounder (Garret Camp) struggled to find a taxi and the idea for a "car-on-demand" app was born out of their frustration.[2] Uber Garage was set up in April 2012 and was described as "a workshop where the company will experiment with new ideas for urban transportation."

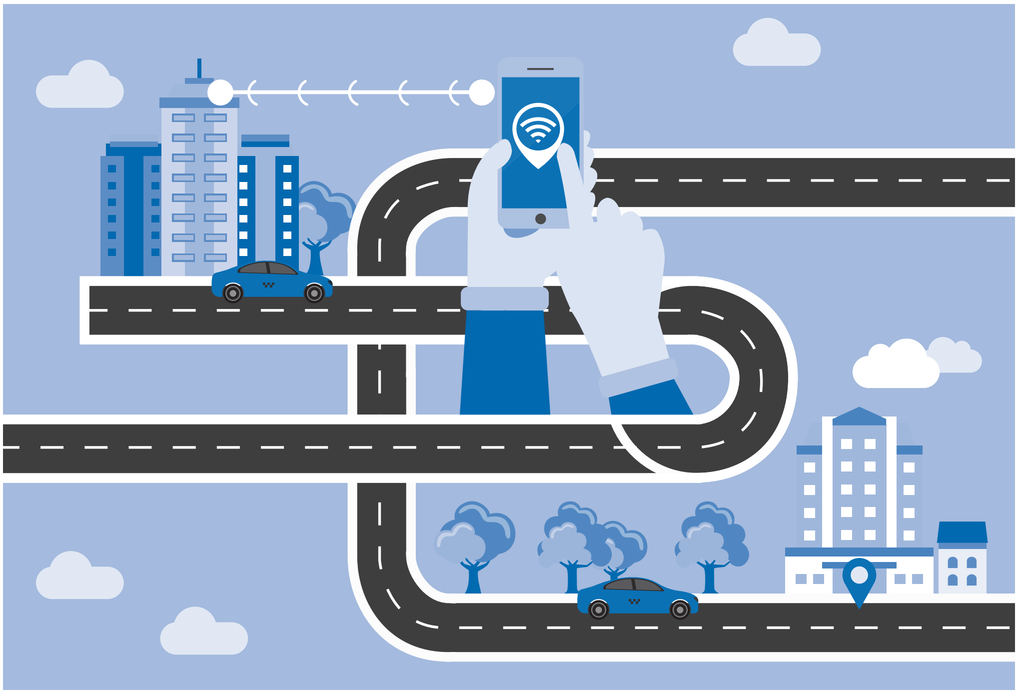
On launching, Uber entered into direct competition with the traditional taxi industry, which was highly fragmented globally. Licenses to operate taxis were generally tightly controlled by local authorities and regulatory bodies; new or additional licenses were not readily granted and no consideration was given to evolving population figures. Existing operators were therefore heavily protected, allowing fares to climb in the absence of free competition. It was ripe for Uber's entry because everybody hates it.[3]

Uber's overriding service aim was to get cars to customers as fast as possible and at the lowest possible price. To do so, it developed a proprietary system and mechanism for managing the volume and flow of vehicles. Essentially, the algorithm tries to predict urban traffic flows based on the existing data and therefore be as accurate as possible in determining where and when customers would need a car. Uber itself extracted a flat 20% commission before paying the driver.

[1] Max Chafkin, "What Makes Uber Run," Fast Company, September 08, 2015. https://www.fastcompany.com/3050250/what-makes-uber-run.

[2] Uber.com/our-story.

[3] IMD, "Uber: An Empire in the Making?," International Institute for Management Development, Lausanne, Switzerland (www.imd.org), Copyright © 2015.



By December 2014, Uber had operations in over 250 cities across the U.S., Central America, Africa, Europe, the Middle East, and Asia Pacific. By the end of 2016, Uber expected to be operating at an annual net booking revenue rate of close to $26 billion.[4]

Another innovation is the introduction of UberPool; this service allows riders heading the same way to share an Uber and save on cost. Kalanick believes that UberPool has the potential to be as affordable as taking a subway, or a bus, or other means of transportation. At a $68 billion valuation on Wall Street, Uber will be bigger than GM, Ford, and Honda. It took Uber only five and a half years to surpass the valuation of 107-year-old General Motors. Does Uber really deserve a higher valuation than the companies that manufacture and sell the bulk of cars around the worlds? That remains to be seen.

[4] Ibid.

The story of how an engineering student was motivated to invent a transportation service and eventually transform his invention into a multibillion-dollar business is not an uncommon phenomenon in today's market. Companies like Snap, Facebook, Google, Dell, and Microsoft produce computer-related products and have market values of ten to hundred billion dollars. These companies were all founded by highly motivated young college students just like Mr. Kalanick. Also common among these successful businesses is their capable and imaginative engineers who constantly generate sound ideas for capital investment, execute them well, and obtain good results. You might wonder what role these engineers play in making such business decisions: What specific tasks are assigned to these engineers, and what tools and techniques are available to them for making such capital-investment decisions? In this book, we will consider many investment situations, personal as well as business. The focus will be to evaluate engineering projects based on the merits of economic desirability and the respective firm's investment climate.

1.1 The Rational Decision-Making Process

We, as individuals or business-persons, constantly make decisions in our daily lives. Most are made automatically without realizing that we are actually following some sort of logical decision flowchart. Rational decision making is often a complex process that includes a number of essential elements. This chapter will provide examples of how two engineering students approached their financial and engineering design problems using flexible, rational decision making. By reviewing these examples, we will be able to identify some essential elements common to any rational decision-making process. The first example illustrates how a student named Maria Clark narrowed down her choice between two competing alternatives when financing an automobile. The second example illustrates how a typical engineering design class project idea evolves and how a student named Sonya approached the design problem by following a logical method of analysis.

1.1.1 How Do We Make Typical Personal Decisions?

For Maria Clark, a senior at the University of Washington, the future holds a new car. Her 2008 Kia Sportage has clocked almost 108,000 miles, and she wants to replace it soon. But how to do it—should she buy or lease? In either case, "Car payments would be difficult," said the engineering major, who works as a part-time cashier at a local supermarket. "I have never leased before, but I am leaning toward it this time to save on the down payment. I also don't want to worry about major repairs." For Maria, leasing would provide the warranty protection she wants, along with a new car every three years. On the other hand, she would be limited to driving only a specified number of miles, about 12,000 per year, after which she would have to pay 20 cents or more per mile. Maria is well aware that choosing the right vehicle and the best possible financing are important decisions. Yet, at this point, Maria is unsure of the implications of buying versus leasing.

Establishing the Goal or Objective

Maria decided to research the local papers and Internet for the latest lease programs, including factory-subsidized "sweetheart" deals and special incentive packages. Of the cars that were within her budget, the 2017 Chevy Sonic appeared to be attractive in terms of style, price, and options. Maria decided to visit the dealers' lots to see how the model looked and to take it for a test drive. After having very satisfactory driving experiences, Maria thought that it would be prudent to thoroughly examine the many technical and safety features of the vehicle. After her examination, she concluded that Sonic model would meet her expectation in terms of reliability, safety features, and quality.

Evaluation of Financing Alternatives

Maria estimated that her 2008 Kia Sportage could be traded in for around $5,500. This amount would be just enough to make any down payment required for buying or leasing the new automobile. Since Maria is also considering the option of buying the car, it is even more challenging to determine precisely whether she would be better off buying than leasing. To make a comparison of leasing versus buying, Maria could have considered what she likely would pay for the same vehicle under both scenarios.

- If she would own the car for as long as she would lease it, she could sell the car and use the proceeds to pay off any outstanding loan. If finances were her only consideration, her choice would depend on the specifics of the deal. But beyond finances, she would need to consider the positives and negatives of her personal preferences. By leasing, she would never experience the joy of the final payment—but she would have a new car every three years.
- Through her research, Maria learned that there are two types of leases: open-end and closed-end. The most popular by far was closed-end because open-end leases potentially expose the consumer to higher payments at the end of the lease if the car depreciates faster than expected. If Maria were to take a closed-end lease, she could just return the vehicle at the end of the lease and "walk away" to lease or buy another vehicle; however, she would still have to pay for extra mileage or excess wear or damage. She thought that since she would not be a "pedal-to-the-metal driver," closed-end charges would not be a problem for her.

To get the best financial deal, Maria obtained some financial facts from the dealer on their best offers. With each offer, she added up all the costs of each option due at signing. This sum does not reflect the total cost of either leasing or buying that vehicle over 39 months, as counting routine items such as oil changes and other maintenance are not considered. (See Table 1.1 for a comparison of the costs of both offers. Disposition fee is a paperwork charge for getting the vehicle ready for resale after the lease ends.) The monthly payment for buying option is based on 2.6% APR (annual percentage rate) over 72 months.

- Buy option:
  - Monthly payments: $\$298 \times 39 = \$11,622$
  - Cash due at signing: $3,572
  - Outstanding loan balance at end of 39 months: $9,475
  - Resale value at end of 39 months: $10,420
  - Total cost: $\$11,622 + \$3,572 + \$9,475 - \$10,420 = \$14,249$
- Lease option:
  - Monthly payments: $\$219 \times 38 = \$8,322$
  - Cash due at signing: $2,029
  - Disposition fee at lease end: $395
  - Total cost: $\$8,322 + \$2,029 + \$395 = \$10,746$

TABLE 1.1 **Financial Data for Auto Buying versus Leasing**

Item	Buy	Lease
1. Manufacturer's suggested retail price (MSRP)	$19,845	$19,845
2. Lease length (months)	39	39
3. Allowed mileage (miles)		32,500
4. Monthly payment	$298	$219
5. Mileage surcharge over 48,000 miles		$0.25
6. Disposition fee at lease end		$395
7. Purchase (resale) price of the vehicle at the end of lease	$10,420	$10,420
8. Outstanding loan balance at end of 39 months	$9,475	
9. Total due at signing:		
• First month's lease payment		$219
• Down payment	$3,572	$1,910
• Refundable security deposit		
Total	**$3,572**	**$2,029**

Maria was leaning toward taking the lease option as it appeared that by leasing, Maria could save about $3,503(=$14,249 − $10,746). However, if she were to drive any additional miles over the limit, her savings would be reduced by 25 cents for each additional mile driven. Maria would need to drive 14,012 extra miles over the limit in order to lose all the savings. Because she could not anticipate her exact driving needs after graduation and it was difficult to come up with $3,572 due at signing, she eventually decided to lease the vehicle.

Review of Maria's Decision-Making Process

Did Maria make the correct decision? When it comes to buying and leasing, there's no one-size-fits-all answer. We need to carefully consider all of the pro, cons, and costs involved and determine which best fits the individual situation. In no way are we saying what Maria did was a logical way to reach the sound economic decision. As you will see in Chapter 2, if Maria had considered the time value of money in her comparison, the amount of actual savings would be far less than $3,503. Even in many situations, the decision could be favoring the buy option.

Now let us revisit the decision-making process in a more structured way. The analysis can be thought of as including the six steps summarized in Figure 1.1. These six steps are known as the *rational decision-making process*. Certainly, we do not follow all six steps in every decision problem. Some decision problems may not require much time and effort. Quite often, we base our decisions solely on emotional reasons. However, for any complex economic decision problem, a structured framework proves to be worthwhile, which is no exception to Maria's buy versus lease decision.

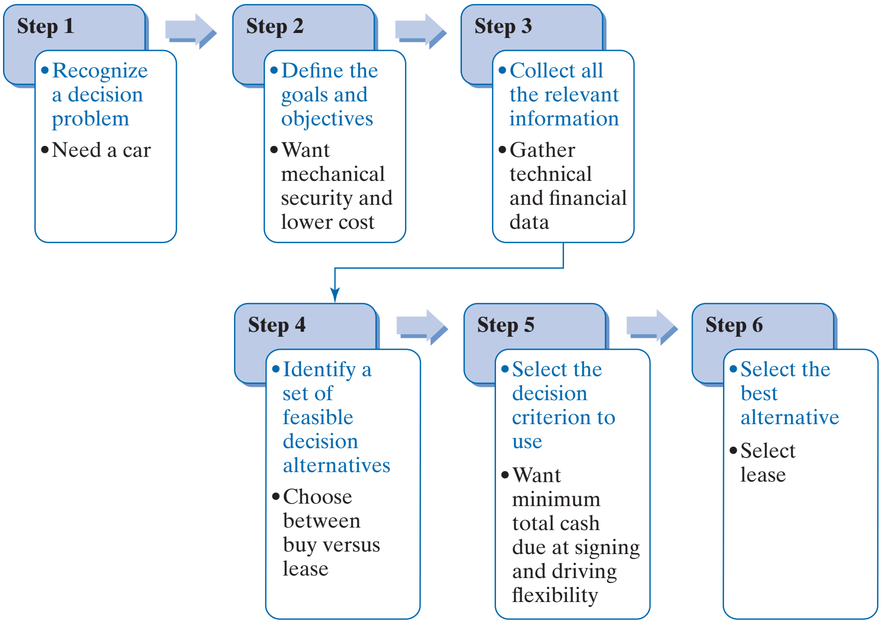


Figure 1.1 Logical steps to follow in a car-financing decision.

1.1.2 How Do We Approach an Engineering Design Problem?

The idea of design and development is what most distinguishes engineering from science, the latter being concerned principally with understanding the world as it is. Decisions made during the engineering design phase of a product's development determine the majority of the costs for manufacturing that product. As design and manufacturing processes become more complex, the engineer will increasingly be called upon to make decisions that involve cost. In this section, we provide an example of how engineers move from "concept" to "product." The story of how an electrical engineering student approached her design problem and exercised her judgment has much to teach us about some of the fundamental characteristics of the human endeavor known as *engineering decision making*.[5]

Getting an Idea: Necessity Is the Mother of Invention

Throughout history, necessity has proven to be the mother of invention. Most people abhor lukewarm beverages, especially during the hot days of summer. So, several years ago, Sonya Talton, an electrical engineering student at Johns Hopkins University, had a revolutionary idea—a self-chilling soda can!

Picture this: It is one of those sweltering, muggy August afternoons. Your friends have finally gotten their acts together for a picnic at the lake. Together, you pull out the items you brought with you: blankets, sunscreen, sandwiches, chips, and soda. You

[5] The original ideas for self-chilling soda can were introduced in *1991 Annual Report,* GWC Whiting School of Engineering, Johns Hopkins University. As of this writing, several versions of self-chilling beverage can appeared on the market.

wipe the sweat from your neck, reach for a soda, and realize that it is about the same temperature as the 90°F afternoon. Great start! And, of course, no one wants to go back to the store for more ice! Why does someone not come up with a soda container that can chill itself?

Setting Design Goals and Objectives

Sonya decided to devise a self chilling soda can as the term project in her engineering graphics and design course. The professor stressed innovative thinking and urged students to consider practical, but novel, concepts. The first thing Sonya needed to do was to establish some goals for the project:

- Get the soda as cold as possible in the shortest possible time.
- Keep the container design simple.
- Keep the size and weight of the newly designed container similar to that of the traditional soda can. (This factor would allow beverage companies to use existing vending machines and storage equipment.)
- Keep the production costs low.
- Make the product environmentally safe.

Evaluating Design Alternatives

With these goals in mind, Sonya had to think of a practical, yet innovative, way of chilling the can. Ice was the obvious choice—practical, but not innovative. Sonya had a great idea: What about a chemical ice pack? Sonya asked herself what would go inside such an ice pack. The answer she came up with was ammonium nitrate (NH_4NO_3) and a water pouch. When pressure is applied to the chemical ice pack, the water pouch breaks and mixes with the NH_4NO_3, creating an endothermic reaction (the absorption of heat). The NH_4NO_3 draws the heat out of the soda, causing it to chill (see Figure 1.2). How much water should go in the water pouch? After several trials involving different amounts of water, Sonya found that she could chill the soda can from 80°F to 48°F in a three-minute period using 115 mL of water. Next, she needed to determine how cold a refrigerated soda gets as a basis for comparison. She put a can in the fridge for two days and found that it chilled to 41°F. Sonya's idea was definitely feasible. But was it economically marketable?

Gauging Product Cost and Price

In Sonya's engineering graphics and design course, the professor emphasized the importance of marketing surveys and benefit–cost analyses as ways to gauge a product's potential and economic feasibility. To determine the marketability of her self-chilling soda can, Sonya surveyed approximately 80 people. She asked them only two questions: (1) How old were they? and (2) How much would they be willing to pay for a self-chilling can of soda? The under-21 group was willing to pay the most, 84 cents, on average. The 40-plus bunch wanted to pay only 68 cents, on average. Overall, members of the entire surveyed group would be willing to spend 75 cents for a self-chilling soda can. (This poll was hardly a scientific market survey, but it did give Sonya a feel for what would be a reasonable price for her product.)

The next hurdle was to determine the existing production cost of one traditional can of soda. Also, how much more would it cost to produce the self-chiller? Would it be profitable? She went to the library, and there she found the bulk cost of the chemicals

Figure 1.2 Conceptual design for self-chilling soda can.

and materials she would need. Then she calculated how much money would be required for production of one unit of soda. She could not believe it! It would cost only 12 cents to manufacture (and transport) one can of soda. The self-chiller would cost 2 or 3 cents more. That was not bad, considering that the average consumer was willing to pay up to 25 cents more for the self-chilling can than for the traditional one that typically costs 50 cents.

Considering Green Engineering

The only two constraints left to consider were possible chemical contamination of the soda and recyclability. Theoretically, it should be possible to build a machine that would drain the solution from the can and recrystallize it. The ammonium nitrate could then be reused in future soda cans; in addition, the plastic outer can could be recycled. Chemical contamination of the soda, however, was a big concern. Unfortunately, there was absolutely no way to ensure that the chemical and the soda would never come in contact with one another inside the cans. To ease consumer fears, Sonya decided that a color or odor indicator could be added to alert the consumer to contamination if it occurred.

What Is the Next Step?

What is Sonya's conclusion? The self-chilling beverage container (can) would be a wonderful technological advancement. The product would be convenient for the beach, picnics, sporting events, and barbecues. Its design would incorporate consumer convenience while addressing environmental concerns. It would be innovative, yet inexpensive, and it would have an economic as well as a social impact on society. Sonya would explore the possibility of patent application of her idea.[6] In the meantime, she would shop for any business venture capitalist who would be interested in investing money to develop the product.

1.1.3 What Makes Economic Decisions Different from Other Design Decisions?

Economic decisions are fundamentally different from the types of decisions typically encountered in engineering design. In a design situation, the engineer uses known physical properties, the principles of chemistry and physics, engineering design correlations, and engineering judgment to arrive at a workable and optimal design. If the judgment is sound, the calculations are done correctly, and we ignore potential technological advances, the design is time invariant. In other words, if the engineering design to meet a particular need is done today, next year, or in five years' time, the final design will not need to change significantly.

In considering economic decisions, the measurement of investment attractiveness, which is the subject of this book, is relatively straightforward. However, information required in such evaluations always involves predicting, or forecasting, product sales, product selling price, and various costs over some future time frame—5 years, 10 years, even 25 years.

All such forecasts have two things in common. First, they are never completely accurate when compared with the actual values realized at future times. Second, a prediction or forecast made today is likely to be different than one made at some point in the future. It is this ever-changing view of the future that can make it necessary to revisit and even alter previous economic decisions. Thus, unlike engineering design outcomes, the conclusions reached through economic evaluation are not necessarily time invariant. Economic decisions have to be based on the best information available at the time of the decision and a thorough understanding of the uncertainties in the forecasted data.

1.2 The Engineer's Role in Business

What role do engineers play within a firm? What specific tasks are assigned to the engineering staff, and what tools and techniques are available to it to improve a firm's profits? Engineers are called upon to participate in a variety of decision-making processes ranging from manufacturing and marketing to finances. We will restrict our focus here to various economic decisions related to engineering projects. We refer to these decisions as **engineering economic decisions**.

1.2.1 Making Capital-Expenditure Decisions

In manufacturing, engineering is involved in every detail of producing goods, from conceptual design to shipping. In fact, engineering decisions account for the majority (some say 85%) of product costs. Engineers must consider the effective use of fixed capital assets such as buildings and machinery. One of the engineer's primary tasks is to plan for the acquisition of equipment (**capital expenditure**) that will enable the firm to design and manufacture products economically (see Figure 1.3).

[6]As of this printing, Sonya's invention has not been yet made to consumer market. However, a high-tech, $20 million plant that will produce self-chilling beverage cans and employ 250 people is earmarked for property on the East Side, Youngstown, Ohio (http://businessjournaldaily.com/20m-chill-can-tech-plant-coming-to-city/).

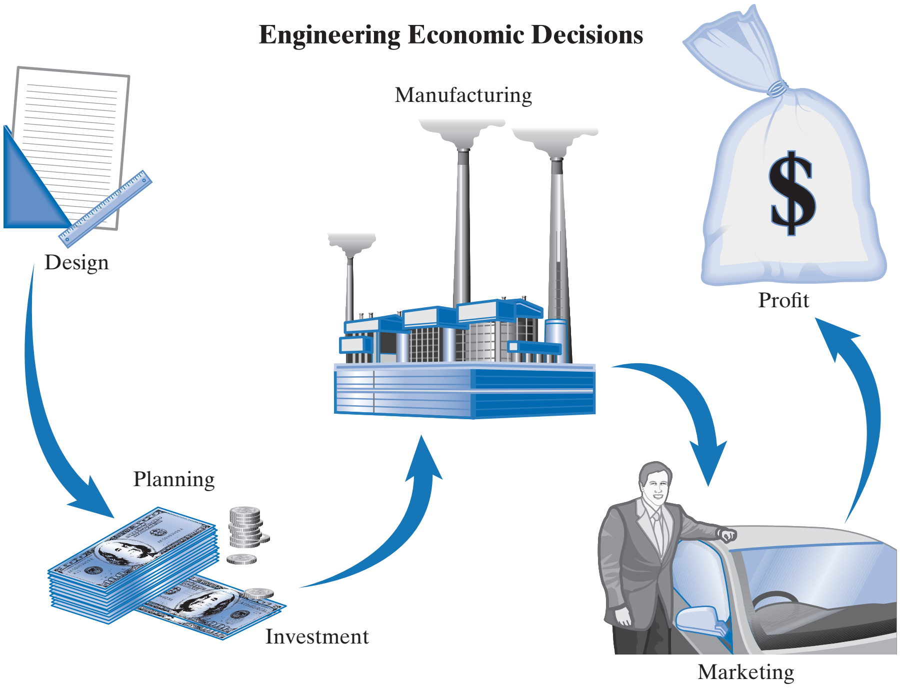


Figure 1.3 One of the primary functions of engineers: Making capital-budgeting decisions.

With the purchase of any fixed asset—equipment, for example—we need to estimate the profits (more precisely, the cash flows) that the asset will generate during its service period. In other words, we have to make capital-expenditure decisions based on predictions about the future. Suppose, for example, that you are considering the purchase of a deburring machine to meet the anticipated demand for hubs and sleeves used in the production of gear couplings. You expect the machine to last 10 years. This purchase decision thus involves an implicit 10-year sales forecast for the gear couplings, which means that a long waiting period will be required before you will know whether the purchase was justified.

An inaccurate estimate of asset needs can have serious consequences. If you invest too much in assets, you incur unnecessarily heavy expenses. Spending too little on fixed assets is also harmful, because your firm's equipment may be too obsolete to make products competitively; without an adequate capacity, you may lose a portion of your market share to rival firms. Regaining lost customers involves heavy marketing expenses and may even require price reductions or product improvements, both of which are costly.

1.2.2 Large-Scale Engineering Economic Decisions

The economic decisions that engineers make in business differ very little from those made by Sonya in designing the self-chilling soda can, except for the scale of the concern. In the development of any product, a company's engineers are called upon to translate an idea into reality. A firm's growth and development depend largely upon a constant flow of ideas for new products, and for the firm to remain competitive, it has to make existing products better or produce them at a lower cost. We will present an

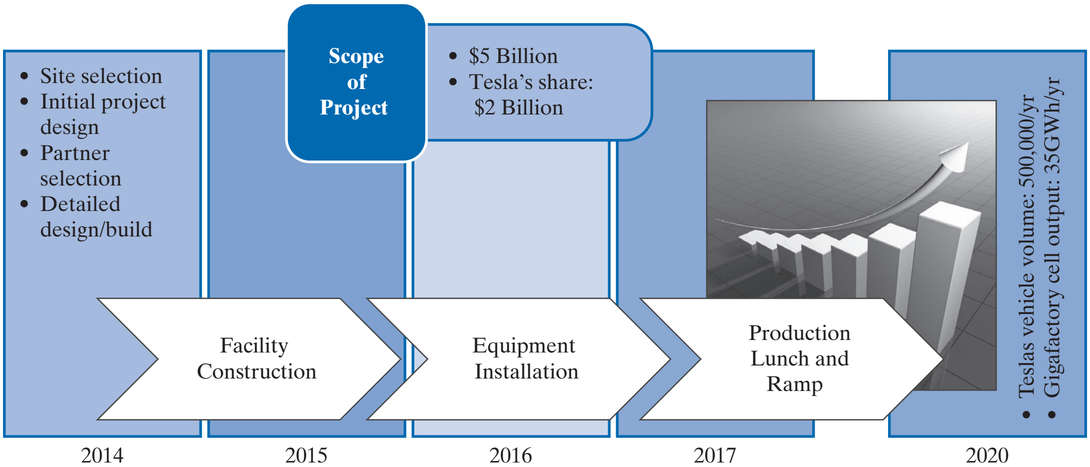


Figure 1.4 Projected timeline of Tesla's Gigafactory.

example of how a large-scale engineering project evolves and what types of financial decisions have to be considered in the process of executing such a project.

Are Tesla's Plans for a Gigafactory[7] Realistic?

Tesla Motors introduced the world's first luxury electrical vehicles whose engines cut air pollution to zero and boost operating efficiency to significant levels. Tesla, in short, wanted to launch and dominate a new "green" era for automobiles and plans to build one of the world's largest factories of any kind in the U.S. But it wouldn't build its electric cars there—it would make the batteries to power them. Tesla's mission is to accelerate the world's transition to sustainable energy. As shown in Figure 1.4, Tesla broke ground on the $5 billion Gigafactory in June 2014, outside Sparks, Nevada, and expects to begin battery cell production by the end of 2017. By 2018, the Gigafactory will reach full capacity and produce more lithium-ion batteries annually than what were produced worldwide in 2013. It says the scale will help drive the cost of batteries down, thereby helping them reach the mass manufacturing target.

How Economical Is Tesla's Plan?

Obviously, this level of an engineering decision is far more complex and more significant than a business decision about when to introduce a new product. Projects of this nature involve large sums of money over long periods of time, and it is difficult to estimate the magnitude of economic benefits in any precise manner. Even if we can justify the project on economic reasoning, how to finance the project is another issue. Any engineering economic decision pertaining to this type of a large-scale project will be extremely difficult to make.

How Much Would It Cost?

Tesla's Gigafactory is so big, in fact, that it will be the world's largest building by footprint. The biggest question remaining about the mass production of the electric vehicles

[7] Tesla Motors Corporation (https://www.tesla.com/gigafactory).

is battery production cost. Costs would need to come down for Tesla's electric cars to be competitive around the world, where gasoline prices are stable. Economies of scale would help as production volumes increase, but further advances in engineering also would be essential. With the initial engineering specification, Tesla has designed the powerpacks and their associated circuitry, each of them contains up to 7,000 standard lithium-ion cells of the sort found in laptops. The firm is said to buy more of these sorts of cells than all the world's computer makers combined. Tesla argues that its battery packs, including their power-management and cooling systems, currently cost less than $300 a kilowatt-hour (kWh) of storage capacity, about half the costs of its rivals.

The Gigafactory, which will eventually turn out batteries for 500,000 vehicles, should cut their costs by another 30%; two-thirds of that saving will come from scale alone, with the rest due to improved manufacturing technology. When costs drop below $200/kWh, battery-powered cars start to become competitive with conventional ones without government subsidies. The Gigafactory could bring Tesla close to that. The lowered cost of the batteries will enable the company to price its Model 3 at about $35,000.

What Is the Business Risk?

Although engineers at Tesla claim that they would be able to cut its current battery costs drastically, many financial analysts are skeptical as raw materials account for 70% of the price of a lithium battery. This would make the scope for savings limited, and even if the factory does turn out many cheap battery cells, it may not be enough. Technically, the key to increasing range and performance is to improve the efficiency, size, and price of the electronics that manage the power, along with overall vehicle weight. Tesla does not have the same advantages in these areas as it has with its batteries. Who is right? Nobody knows for sure at this point.

At a cost of $5 billion, which Tesla will share with Panasonic of Japan, its current battery supplier, and other partners, the Gigafactory is a big gamble. Also, if electric-car demand stalls, the question is what we do with the huge output of cheap batteries. There is a lot of cost that can be removed at larger scales of battery manufacturing, but it's all about the capacity utilization. A battery plant that is not running will cost Tesla a fortune.

Despite Tesla management's decision to build the giant battery factory, the financial analysts were still uncertain whether there would be enough demand. Furthermore, competitors, including U.S. automakers, just did not see how Tesla could achieve the economies of scale needed to produce electric cars at a profit. The primary advantage of the design, however, is that the electric vehicle could cut auto pollution to a zero level. This is a feature that could be very appealing at a time when government air-quality standards are becoming more rigorous and consumer interest in the environment is getting stronger. However, in the case of the Tesla products, if a significant reduction in production cost never materializes, demand might remain insufficient to justify the investment in the battery factory.

1.2.3 Impact of Engineering Projects on Financial Statements

Engineers must also understand the business environment in which a company's major business decisions are made. It is important for an engineering project to generate profits, but the project also must strengthen the firm's overall financial position. How do we measure Tesla's success in the battery project? Will enough batteries be produced, for example, to generate sufficient profits? While the Gigafactory will be of another level of engineering achievement, the bottom-line concern is its financial performance over the long run.

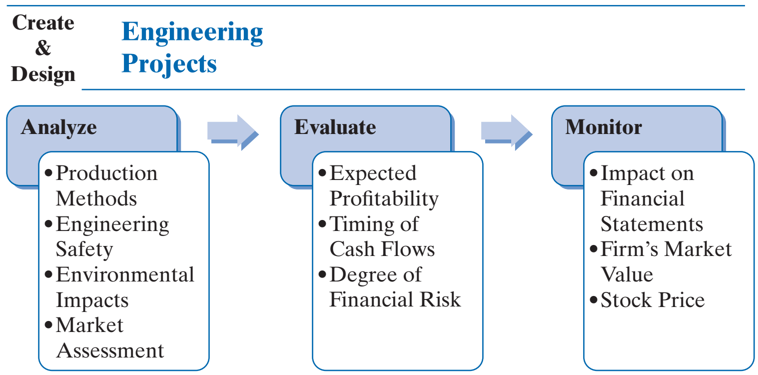


Figure 1.5 How a successful engineering project affects a firm's market value.

Regardless of the form of a business, each company has to produce basic financial statements at the end of each operating cycle (typically, a year). These financial statements provide the basis for future investment analysis. In practice, we seldom make investment decisions based solely on an estimate of a project's profitability because we must also consider the project's overall impact on the financial strength and position of the company. For example, some companies with low cash flow may be unable to bear the risk of a large project like Tesla's Gigafactory even if it might be profitable (see Figure 1.5).

Suppose that you are the president of Tesla. Also suppose that you hold some shares in the company, which makes you one of the company's many owners. What objectives would you set for the company? One of your objectives should be to increase the company's value to its owners (including yourself) as much as possible. While all firms operate to generate profit, what determines the market value of a company are not profits, per se, but rather, cash flows. It is, after all, the available cash that determines the future investments and growth of the firm. The market price of your company's stock to some extent represents the value of your company. Multiple factors affect your company's market value: present and expected future earnings, the timing and duration of these earnings, and the risks associated with the earnings. Certainly, any successful investment decision will increase a company's market value. Stock price can be a good indicator of your company's financial health and may also reflect the market's attitude about how well your company is managed for the benefit of its owners.

If investors like the battery project, the result will be an increased demand for the company's stock. This increased demand, in turn, will cause stock prices, and hence, shareholder wealth, to increase. Any successful investment decision on the battery's scale will tend to increase a firm's stock prices in the marketplace and promote long-term success. Thus, in making a large-scale engineering project decision, we must consider the project's possible effect on the firm's market value. (We will consider this important issue in Chapter 13.)

1.3 Types of Strategic Engineering Economic Decisions

A project idea such as constructing a battery plant can originate from many different levels in an organization. Since some ideas are good, while others are not, it is necessary to establish procedures for screening projects. Many large companies have a specialized project analysis division that actively searches for new ideas, projects, and ventures. Once project ideas are identified, they are typically classified as (1) new products or product expansion, (2) equipment and process selection, (3) cost reduction, (4) equipment replacement, or (5) service or quality improvement. This classification scheme allows management to address key questions such as the following: Can the existing plant be used to achieve the new production levels? Does the firm have the capital to undertake this new investment? Does the new proposal warrant the recruitment of new technical personnel? The answers to these questions help firms screen out proposals that are not feasible.

The Tesla's battery project represents a fairly complex engineering decision that required the approval of top executives and the board of directors. Virtually all big businesses at some time face investment decisions of this magnitude. In general, the larger the investment, the more detailed the analysis required to support the expenditure. For example, expenditures to increase the output of existing products or to manufacture a new product would invariably require a very detailed economic justification. Final decisions on new products and marketing decisions are generally made at a high level within the company. On the other hand, a decision to repair damaged equipment can be made at a lower level within a company. In this section, we will provide many real examples to illustrate each class of engineering economic decision. At this point, our intention is not to provide a solution for each example but to describe the nature of decision-making problems a typical engineer might face in the real world.

1.3.1 New Products or Product Expansion

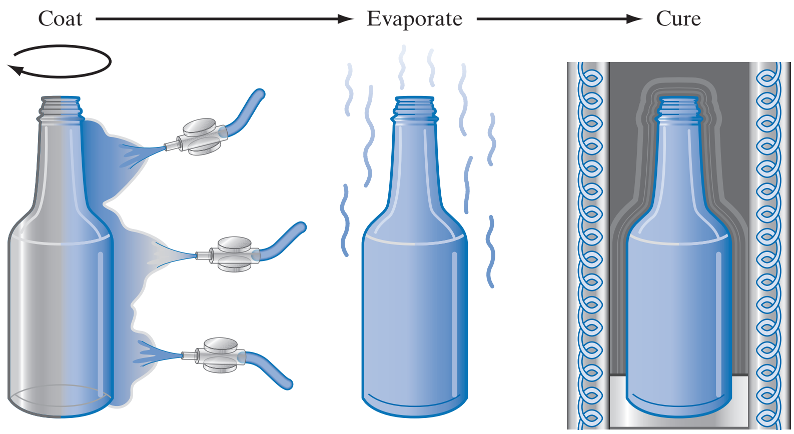
Investments in this category are those that increase the revenues of a company if output is increased. There are two common types of expansion decision problems. The first type includes decisions about expenditures to increase the output of existing production or distribution facilities. In these situations, we are basically asking, "Shall we build or otherwise acquire a new facility?" The expected future cash inflows in this investment category are the revenues from the goods and services produced in the new facility.

The second type of decision problem includes the consideration of expenditures necessary to produce a new product or to expand into a new geographic area. These projects normally require large sums of money over long periods. For example, Apple Computer's investment in iPad®'s A4 chip is estimated to be $1 billion. In 2017, Apple introduced an iPad with a whopping 12.9-inch display to bridge the gap between tablets and laptops. The new A9X chip (an advanced version of A4 chip) is about as powerful as a mid-range laptop, and the unit component cost is estimated to be around $40. The iPad Pro starts at $799 and can range all the way up to $1,129 for the 256 GB model with data connectivity. The cost for Apple to build the $799 base model iPad is estimated to be $366.50.

Clearly, the profit margin for the iPad varies with the different design features—the high-end product being more profitable. At the time of its introduction, the main question was, "Will there be enough demand for the iPad so that Apple could recoup the investment and be the market leader in the tablet PC market?"

1.3.2 Equipment and Process Selection

This class of engineering decision problem involves selecting the best course of action when there are several ways to meet a project's requirements. Which of several proposed items of equipment shall we purchase for a given purpose? The choice often hinges on which item is expected to generate the largest savings (or return on the investment). The choice of material will dictate the manufacturing process involved.



Spray coating of external PET bottles

	Five-Layer Bottle	Three-Layer with External Coating
• Capacity	20,000 bottles/hour	20,000 bottles/hour
• Capital investment	$10.8 million	$7.5 million
• Direct manufacturing cost	$59.35/1,000 bottles	$66.57/1,000 bottles

Figure 1.6 Making plastic beer bottles by two different manufacturing processes.

(See Figure 1.6 on making a 0.5 liter polyethylene terephthalate (PET) barrier beer bottle.) Many factors will affect the ultimate choice of the material, and engineers should consider all major cost elements, such as machinery and equipment, tooling, labor, and material. Other factors may include press and assembly, production and engineered scrap, the number of dies and tools, and the cycle times for various processes.

1.3.3 Cost Reduction

A cost-reduction project attempts to lower a firm's operating costs. Typically, we need to consider whether a company should buy equipment to perform an operation now done manually or in some other way spend money now in order to save more money later. The expected future cash inflows from this investment are savings resulting from lower operating costs. Or perhaps the firm needs to decide whether to produce a part for a product in-house or to buy it from a supplier to reduce the total production cost. This is commonly known as a *make-or-buy* (or *outsourcing*) *analysis*.

1.3.4 Equipment Replacement

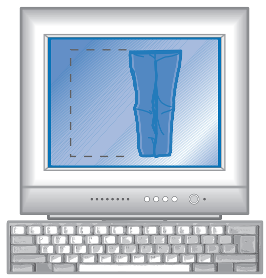
This category of investment decision involves considering the expenditure necessary to replace worn-out or obsolete equipment. For example, a company may purchase 10 large presses with the expectation that it will produce stamped metal parts for 10 years. After five years, however, it may become necessary to produce the parts in plastic, which would require retiring the presses early and purchasing plastic-molding machines. Similarly, a company may find that, for competitive reasons, larger and more accurate parts are required, which will make the purchased machines obsolete earlier than expected.

1.3.5 Service or Quality Improvement

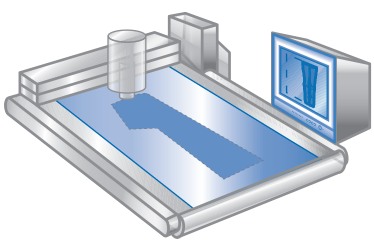
The service sector of the U.S. economy dominates both gross domestic product (GDP) and total employment. It is also the fastest growing part of the economy and the one offering the most fertile opportunities for productivity improvement. For example, service activities now approach 80% of U.S. employment, far outstripping sectors such as manufacturing (14%) and agriculture (2%). New service activities are continually emerging throughout the economy as forces such as globalization, e-commerce, and environmental reuse concerns create the need by businesses for ever more decentralization and outsourcing of operations and processes.

Investments in this category include any activities to support the improvement of productivity, quality, and customer satisfaction in the service sector, such as in the financial, healthcare, and retail industries. See Figure 1.7 for an example of a service improvement in retail where a blue jeans manufacturer is considering installing robotic tailors.

A sales clerk measures the customer, using instructions from a computer as an aid.



The clerk enters the measurements and adjusts the data, according to the customer's reaction to the samples.



The final measurements are relayed to a computerized fabric cutting machine at the factory.

Bar codes are attached to the clothing to track it as it is assembled, washed, and prepared for shipment.

Figure 1.7 To make customized blue jeans for women, a new computerized system is being installed at some retail stores, allowing women to place customized orders.

The manufacturer's main problem is to determine how much demand the women's line would generate: How many more jeans would the manufacturer need to sell to justify the cost of additional tailors? This analysis should involve a comparison of the cost of operating additional robotic tailors with additional revenues generated by increased jeans sales.

1.4 Fundamental Principles in Engineering Economics

This book focuses on the principles and procedures for making sound engineering economic decisions. To the first-time student of engineering economics, anything related to money matters may seem quite strange compared with other engineering subjects. However, the decision logic involved in problem solving is quite similar to any other engineering subject matter; there are basic fundamental principles to follow in any engineering economic decision. These principles unite to form the concepts and techniques presented in the text, thereby allowing us to focus on the logic underlying the practice of engineering economics.

The four principles of engineering economics are as follows:

- **Principle 1: An earlier dollar is worth more than a later dollar.** A fundamental concept in engineering economics is that money has a time value associated with it. Because we can earn interest on money received today (compound interest), it is better to receive money earlier than later. This concept will be the basic foundation for all engineering project evaluation. As noted by Albert Einstein, the most powerful force in the universe is compound interest.
- **Principle 2: All that counts is the differences among alternatives.** An economic decision should be based on the differences among the alternatives considered. All that is common is irrelevant to the decision. Certainly, any economic decision is no better than any one of the alternatives being considered. Therefore, an economic decision should be based on the objective of making the best use of limited resources. Whenever a choice is made, something is given up. The *opportunity cost* of a choice is the value of the best alternative given up.
- **Principle 3: Marginal revenue must exceed marginal cost.** Any increased economic activity must be justified based on the following fundamental economic principle: Marginal revenue must exceed marginal cost. Here, the marginal revenue is the additional revenue made possible by increasing the activity by one unit (or a small unit). Similarly, marginal cost is the additional cost incurred by the same increase in activity. Productive resources, such as natural resources, human resources, and capital goods available to make goods and services, are limited. People cannot have all the goods and services they want; as a result, they must choose resources that produce with the best economy.
- **Principle 4: Additional risk is not taken without expected additional return.** For delaying consumption, investors demand a minimum return that must be greater than the anticipated rate of inflation or than any perceived risk. If they do not see themselves receiving enough to compensate for anticipated inflation and perceived investment risk, investors may purchase whatever goods they desire ahead of time or invest in assets that would provide a sufficient return to compensate for any loss from inflation or potential risk.

These four principles are as much statements of common sense as they are theoretical principles. They provide the logic behind what follows in this book. We build on the principles and attempt to draw out their implications for decision making. As we continue, try to keep in mind that while the topics being treated may change from chapter to chapter, the logic driving our treatment of them is constant and rooted in these four fundamental principles.

SUMMARY

- This chapter provides an overview of a variety of engineering economic problems that commonly are found in the business world. We examined the place of engineers in a firm, and we saw that engineers play an increasingly important role in companies, as evidenced in Tesla's development of giant battery factory. Commonly, engineers participate in a variety of strategic business decisions ranging from product design to marketing.
- The term *engineering economic decision* refers to all investment decisions relating to an engineering project. The most interesting facet of an economic decision, from an engineer's point of view, is the evaluation of costs and benefits associated with making a capital investment.
- The five main types of engineering economic decisions are (1) new products or product expansion, (2) equipment and process selection, (3) cost reduction, (4) equipment replacement, and (5) service or quality improvement,
- The factors of time and uncertainty are the defining aspects of any investment project.

SELF-TEST QUESTIONS

1s.1 Which of the following statements is incorrect?
(a) Economic decisions are time invariant.
(b) Time and risk are the most important factors in any investment evaluation.
(c) For a large-scale engineering project, engineers must consider the impact of the project on the company's financial statements.
(d) One of the primary roles of engineers is to make capital expenditure decisions.

1s.2 When evaluating a large-scale engineering project, which of the following items is important?
(a) Expected profitability
(b) Timing of cash flows
(c) Degree of financial risk
(d) All of the above

1s.3 Which of the following statements defines the discipline of engineering economics most closely?
(a) Economic decisions made by engineers.
(b) Economic decisions related to financial assets.
(c) Economic decisions primarily for real assets and services from engineering projects.
(d) Any economic decision related to the time value of money.

1s.4 Which of the following statements is not one of the four fundamental principles of engineering economics?

(a) Receiving a dollar today is worth more than a dollar received in the future.

(b) To expect a higher return on investment, you need to take a higher risk.

(c) Marginal revenue must exceed marginal cost to justify any production.

(d) When you are comparing different alternatives, you must not focus only on differences in alternatives.

PROBLEMS

1.1 Read the *Wall Street Journal* over a one-week period and identify the business investment news using one of the categories—(1) new products or product expansion, (2) equipment and process selection, (3) cost reduction, (4) equipment replacement, or (5) service or quality improvement.

1.2 By reading any business publication give examples that illustrate one of the four fundamental principles of engineering economics.

CHAPTER TWO

Time Value of Money

Biggest Lottery Jackpots in U.S. History – $1.58 Billion Powerball The world's biggest lottery jackpot of $1.58 billion ($1,586,400,000 to be exact) on January 13, 2016, was a three-way split of $528 million each. The odds of winning the $1.58 billion jackpot were 1 in 292.2 million, which are probabilistically impossible odds to hope for, but there are always a few lucky people to claim such a record-shattering jackpot. The three winners were from California, Florida, and Tennessee, and they opted to take the $327.8 million lump sum ($983,505,233 split three ways) rather than receiving the $528 million sum in 30 annual installments.

How much do they take home?[1] First, they have to pay federal and state income taxes. For state income taxes, none will have to pay income taxes to their states. Some states don't have an

[1] *Source:* Mike Tarson and Christine Romans, "How Much Will the Powerball Jackpot Winners Get?" @ CNNMoney, January 14, 2016. (http://money.cnn.com/2016/01/14/news/powerball-winnings/).

income tax like Florida and Tennessee, yet other states have longtime tax exemptions for lottery winners like California.

Second, after paying 39.6% in federal income taxes on their prizes, each will take home about $197 million. So what should they do with that money? A couple of easy options to think of are as follows:

- If they are looking for the safest investment, they might purchase a 30-year U.S. government bond. With a 2.85% yield, each winner would get about $5.64 million a year to live on for the next 30 years, without touching the principal amount.
- If they are willing to take some risk, they could invest their money in the stock market. For example, with that money, they could have bought 1.9 million shares of Apple (AAPL) stock around $104 a share. A year later they could have increased their wealth by 34% as Apple shares went up by 34%.

If you were the winner of the aforementioned jackpot, you might well wonder why the value of the single-sum payment—$197 million paid immediately—is so much lower than the total value of the annuity payments—$17.6 (=$528 million/30) million received in 30 installments over 29 years (the first installment is paid immediately). Isn't receiving the annuity of $17.6 million overall a lot better than receiving just $197 million now? The answer to your question involves the principles we will discuss in this chapter, namely, the operation of interest and the time value of money.

The question we just posed provides a good starting point for this chapter. If it is better to receive a dollar today than it is to receive a dollar in 10 years, how do we quantify the difference? Our lottery example is complex. Instead of a choice between two single payments, the lottery winners were faced with a decision between a single payment now and a series of future payments. First, most people familiar with investments would tell you that receiving $197 million today is likely to prove a better deal than taking $17.6 million a year for 30 annual installments with the first payment immediately. In fact, based on the principles you will learn in this chapter, the real present value of the 29-year payment series— the value that you could receive today in the financial marketplace for the promise of $17.6 million a year for the next 29 years—can be shown to be considerably less than $197 million. And that is even before we consider the effects of inflation! The reason for this surprising result is the **time value of money**; the earlier a sum of money is received, the more it is worth because over time money can earn more money via interest.

In engineering economic analysis, the principles discussed in this chapter are regarded as the underpinnings of nearly all project investment analysis. It is imperative to understand these principles because we always need to account for the effect of interest operating on sums of cash over time. Fortunately, we have interest formulas that allow us to place different cash flows received at different times in the same time frame to make comparisons possible. As will become apparent, almost our entire study of engineering economic analysis is built on the principles introduced in this chapter.

2.1 Interest: The Cost of Money

Most of us have a general appreciation of the concept of interest. We know that money left in a savings account earns interest so that the balance over time is higher than the sum of the deposits. We know that borrowing to buy a car means repaying an amount over time, including the interest, and thus the amount paid is more than the amount borrowed. However, what may be unfamiliar to us is that, in the financial world, money itself is a commodity, and like other goods that are bought and sold, money costs money.

The cost of money is established and measured by an **interest rate**, a percentage that is periodically applied and added to an amount (or to varying amounts) of money over a specified length of time. When money is borrowed, the interest paid is the charge to the borrower for the use of the lender's property. When money is loaned or invested, the interest earned is the lender's gain for providing a good to another person. **Interest**, then, may be defined as the cost of having money available for use. In this section, we examine how interest operates in a free-market economy and establish a basis for understanding the more complex interest relationships that are presented later in the chapter.

2.1.1 The Time Value of Money

The time value of money seems like a sophisticated concept, yet it is one that you encounter every day. Should you buy something today or buy it later? Here is a simple example of how your buying behavior can have varying results: Pretend you have $100 and you want to buy a $100 refrigerator for your dorm room. (Assume that you are currently sharing a large refrigerator with your roommates in a common area.)

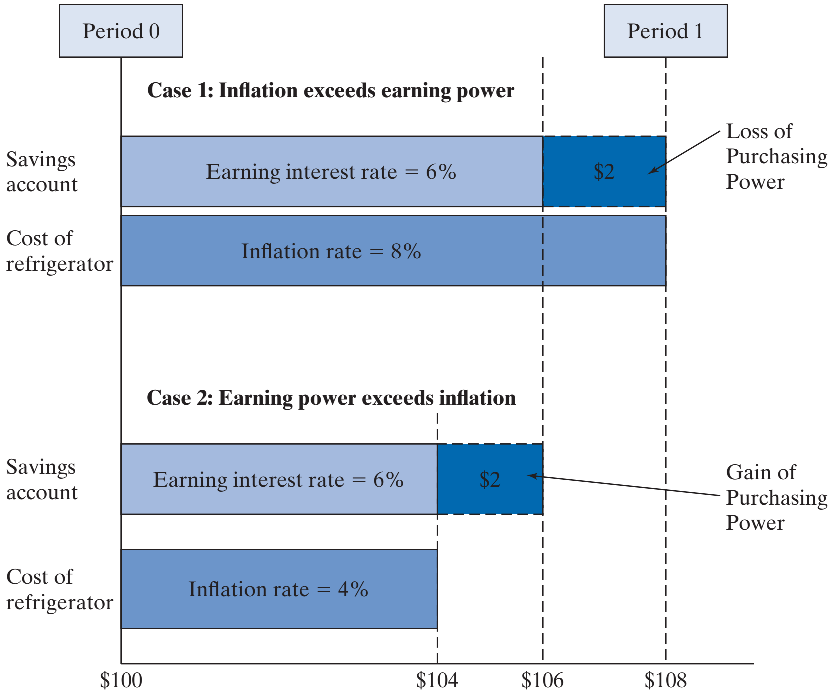


Figure 2.1 Gains achieved or losses incurred by delaying consumption.

- If you buy it now, you end up broke. But if you invest your money at 6% annual interest, then in a year you can still buy the refrigerator, and you will have $6 left over with no change in the price of the refrigerator. Clearly, you need to ask yourself whether the inconvenience of not having the refrigerator in your own room for a year can be compensated by the financial gain in the amount of $6.
- If the price of the refrigerator increases at an annual rate of 8% due to inflation, then you will not have enough money (you will be $2 short) to buy the refrigerator a year from now (Case 1 in Figure 2.1). In that case, you probably are better off buying the refrigerator now. If the inflation rate is running at only 4%, then you will have $2 left over if you buy the refrigerator a year from now (Case 2 in Figure 2.1).

Clearly, the rate at which you earn interest should be higher than the inflation rate in order to make any economic sense of the delayed purchase. In other words, in an inflationary economy, your purchasing power will continue to decrease as you further delay the purchase of the refrigerator. In order to make up for this future loss in purchasing power, the rate at which you earn interest must be sufficiently higher than the anticipated inflation rate. After all, time, like money, is a finite resource. There are only 24 hours in a day, so time has to be budgeted, too. What this example illustrates is that we must connect *earning power* and *purchasing power* to the concept of time.

The way interest operates reflects the fact that money has a time value. This is why amounts of interest depend on lengths of time; interest rates, for example, are typically given in terms of a percentage per year. We may define the principle of the time value of money as follows: The economic value of a sum depends on when the sum is received. Because money has both **earning power** and **purchasing power** over time (i.e., it can be put to work, earning more money for its owner), a dollar received today has a higher value than a dollar received at some future time. When we deal with large amounts of money, long periods of time, and high interest rates, a change in the value of a sum of money over time becomes extremely significant. For example, at a current annual interest rate of 10%, $1 million will earn $100,000 in interest in a year; thus, to wait a year to receive $1 million clearly involves a significant sacrifice. When deciding among alternative proposals, we must take into account the operation of interest and the time value of money in order to make valid comparisons of different amounts at various times.

When financial institutions quote lending or borrowing interest rates in the marketplace, those interest rates reflect the desired earning rate as well as any protection from loss in the future purchasing power of money because of inflation. Interest rates, adjusted for inflation, rise and fall to balance the amount saved with the amount borrowed, which affects the allocation of scarce resources between present and future uses.

Unless stated otherwise, we will assume that the interest rates used in this book reflect the **market interest rate**, which considers the earning power of money as well as the effect of inflation perceived in the marketplace. We will also assume that all cash flow transactions are given in terms of **actual dollars**, for which the effect of inflation, if any, is reflected in the amount.

2.1.2 Elements of Transactions Involving Interest

Many types of transactions involve interest (e.g., borrowing money, investing money, or purchasing machinery on credit), and certain elements are common to all of these types of transactions. These elements are:

1. The initial amount of money invested or borrowed in a transaction is called the **principal** (P).
2. The **interest rate** (i) measures the cost or price of money and is expressed as a percentage per period of time.
3. A period of time called the **interest period** (n) determines how frequently interest is calculated. (Note that, even though the length of time of an interest period can vary, interest rates are frequently quoted in terms of an annual percentage rate. We will discuss this potentially confusing aspect of interest in Chapter 3.)
4. A specified length of time marks the duration of the transaction and thereby establishes a certain **number of interest periods** (N).
5. **A plan for receipts or disbursements** (A_n) yields a particular cash flow pattern over a specified length of time. (For example, we might have a series of equal monthly payments that repay a loan.)
6. A **future amount of money** (F) results from the cumulative effects of the interest rate over a number of interest periods.

Example of an Interest Transaction

As an example of how the elements we have just defined are used in a particular situation, let us suppose that you apply for an education loan in the amount of $30,000 from a bank at a 9% annual interest rate. In addition, you pay a $300 loan origination fee[2] when the

[2] The loan origination fee covers the administrative costs of processing the loan. It is often expressed in points. One point is 1% of the loan amount. In our example, the $30,000 loan with a loan origination fee of one point would mean the borrower pays a $300 fee. This is equivalent to financing $29,700, but the payments are based on a $30,000 loan. Both payment plans are based on a rate of 9% interest.

TABLE 2.1 Repayment Plans Offered by the Lender

End of Year	Receipts	Payments	
		Plan 1	Plan 2
Year 0	$30,000	$300.00	$300.00
Year 1		$7,712.77	0
Year 2		$7,712.77	0
Year 3		$7,712.77	0
Year 4		$7,712.77	0
Year 5		$7,712.77	$46,158.72

loan commences. The bank offers two repayment plans, one with equal payments made at the end of every year for the next five years (installment plan) and the other with a single payment made after the loan period of five years (deferment plan). These payment plans are summarized in Table 2.1.

- In Plan 1, the principal amount, P, is $30,000 and the interest rate, i, is 9%. The interest period, n, is one year, and the duration of the transaction is five years, which means that there are five interest periods $(N = 5)$. It bears repeating that while one year is a common interest period, interest is frequently calculated at other intervals as well— monthly, quarterly, or semiannually, for instance. For this reason, we used the term **period** rather than **year** when we defined the preceding list of variables. The receipts and disbursements planned over the duration of this transaction yield a cash flow pattern of five equal payments, A, of $7,712.77 each, paid at year-end during years 1 through 5.
- Plan 2 has most of the elements of Plan 1 except that instead of five equal repayments, we have a grace period followed by a single future repayment (lump sum), F, of $46,158.72.

Cash Flow Diagrams

Problems involving the time value of money can be conveniently represented in graphic form with a **cash flow diagram** (Figure 2.2). Cash flow diagrams represent time by a horizontal line marked off with the number of interest periods specified. Arrows represent the cash flows over time at relevant periods. Upward arrows represent positive flows (receipts), and downward arrows represent negative flows (expenditures). Note, too, that the arrows actually represent **net cash flows**; two or more receipts or disbursements made at the same time are summed and shown as a net single arrow. For example, $30,000 received during the same period as a $300 payment is being made would be recorded as an upward arrow of $29,700. The lengths of the arrows can also suggest the relative values of particular cash flows.

Cash flow diagrams function in a manner similar to free-body diagrams or circuit diagrams, which most engineers frequently use. Cash flow diagrams give a convenient summary of all the important elements of a problem and serve as a reference point for determining whether the elements of a problem have been converted into their appropriate parameters. This book frequently employs this graphic tool, and you are strongly encouraged to develop the habit of using well-labeled cash flow diagrams as a means

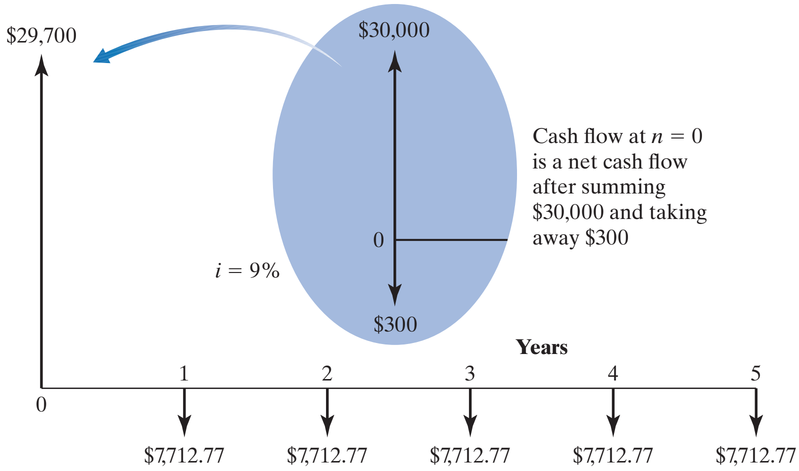


Figure 2.2 A cash flow diagram for Plan 1 of the loan repayment example.

to identify and summarize pertinent information in a cash flow problem. Similarly, a table such as Table 2.1 can help you organize information in another summary format.

End-of-Period Convention

In practice, cash flows can occur at the *beginning* or in the *middle* of an interest period or at practically any point in time. One of the simplifying assumptions we make in engineering economic analysis is the **end-of-period convention**, which is the practice of placing all cash flow transactions at the *end* of an interest period. This assumption relieves us of the responsibility of dealing with the effects of interest within an interest period, which would greatly complicate our calculations. Like many of the simplifying assumptions and estimates we make in modeling engineering economic problems, the end-of-period convention inevitably leads to some discrepancies between our model and real-world results.

Suppose, for example, that $100,000 is deposited during the first month of the year in an account with an interest period of one year and an interest rate of 10% per year. In such a case, if the deposit is withdrawn one month before the end of the year, the investor would experience a loss of $10,000— all of it interest! This results because, under the end-of-period convention, the $100,000 deposit made during the interest period is viewed as if it were made at the end of the year, as opposed to 11 months earlier. This example gives you a sense of why financial institutions choose interest periods that are less than one year, even though they usually quote their rate in terms of *annual percentage*. Armed with an understanding of the basic elements involved in interest problems, we can now begin to look at the details of calculating interest.

2.1.3 Methods of Calculating Interest

Money can be loaned and repaid in many ways, and similarly, money can earn interest in many different ways. Usually, however, at the end of each interest period, the interest earned on the principal amount is calculated according to a specified interest rate.

There are two computational schemes for calculating this earned interest yield: **simple interest** and **compound interest**.

Simple Interest

The first scheme considers interest earned on only the principal amount during each interest period. In other words, the interest earned during each interest period does not earn additional interest in the remaining periods, *even if you do not withdraw the earned interest*.

In general, for a deposit of P dollars at a simple interest rate of i for N periods, the total earned interest I would be

$$I = (iP)N. \tag{2.1}$$

The total amount available at the end of N periods, F, thus would be

$$F = P + I = P(1 + iN). \tag{2.2}$$

Simple interest is commonly used with add-on loans or bonds.

Compound Interest

Under a compound interest scheme, the interest earned in each period is calculated based on the total amount at the end of the previous period. This total amount includes the original principal plus the accumulated interest that has been left in the account. In this case, you are, in effect, increasing the deposit amount by the amount of interest earned. In general, if you deposited (invested) P dollars at an interest rate i, you would have $P + iP = P(1 + i)$ dollars at the end of one interest period. If the entire amount (principal and interest) were reinvested at the same rate i for next period, you would have at the end of the second period

$$\begin{aligned} P(1 + i) + i[P(1 + i)] &= P(1 + i)(1 + i) \\ &= P(1 + i)^2. \end{aligned}$$

Continuing, we see that the balance after period three is

$$P(1 + i)^2 + i[P(1 + i)^2] = P(1 + i)^3.$$

This interest-earning process repeats, and after N periods, the total accumulated value (balance) F will grow to

$$F = P(1 + i)^N. \tag{2.3}$$

Engineering economic analysis uses the compound interest scheme exclusively, as it is most frequently practiced in the real world.

EXAMPLE 2.1 Simple versus Compound Interest

Suppose you deposit $1,000 in a bank savings account that pays interest at a rate of 8% per year. Assume that you do not withdraw the interest earned at the end of each period (year) but instead let it accumulate. (1) How much would you have at the end of year 3 with simple interest? (2) How much would you have at the end of year 3 with compound interest?

DISSECTING THE PROBLEM

Given: $P = \$1{,}000$, $N = 3$ years, and $i = 8\%$ per year.
Find: F.

METHODOLOGY

Use Eqs. (2.2) and (2.3) to calculate the total amount accumulated under each computational scheme.

SOLUTION

(a) Simple interest: Using Eq. (2.2) we calculate F as

$$F = \$1{,}000\,[1 + (0.08)^3] = \$1{,}240.$$

Year by year, the interest accrues as shown:

End of Year	Beginning Balance	Interest Earned	Ending Balance
1	\$1,000	\$80	\$1,080
2	\$1,080	\$80	\$1,160
3	\$1,160	\$80	\$1,240

(b) Compound interest: Applying Eq. (2.3) to our three-year, 8% case, we obtain

$$F = \$1{,}000\,(1 + 0.08)^3 = \$1{,}259.71.$$

The total interest earned is \$259.71, which is \$19.71 more than accumulated under the simple-interest method. We can keep track of the interest-accruing process more precisely as follows:

End of Year	Beginning Balance	Interest Earned	Ending Balance
1	\$1,000.00	\$80.00	\$1,080.00
2	\$1,080.00	\$86.40	\$1,166.40
3	\$1,166.40	\$93.31	\$1,259.71

COMMENTS: At the end of the first year, you would have a total of \$1,080 which consists of \$1,000 in principal plus \$80 in interest. In effect, at the beginning of the second year, you would be depositing \$1,080, rather than \$1,000. Thus, at the end of the second year, the interest earned would be $0.08(\$1{,}080) = \$86.40$ and the balance would be $\$1{,}080 + \$86.40 = \$1{,}166.40$. This is the equivalent amount you would be depositing at the beginning of the third year, and the interest earned for that period would be $0.08(\$1{,}166.40) = \$93.31$. With a beginning principal amount of \$1,166.40 plus the \$93.31 interest, the total balance would be \$1,259.71 at the end of year 3.

2.2 Economic Equivalence

The observation that money has a time value leads us to an important question: If receiving \$100 today is not the same as receiving \$100 at any future point, how do we measure and compare various cash flows? How do we know, for example,

- Economic equivalence exists between cash flows that have the *same economic effect* and could therefore be traded for one another.
- Even though the amounts and timing of the cash flows may differ, the appropriate interest rate makes them equal.

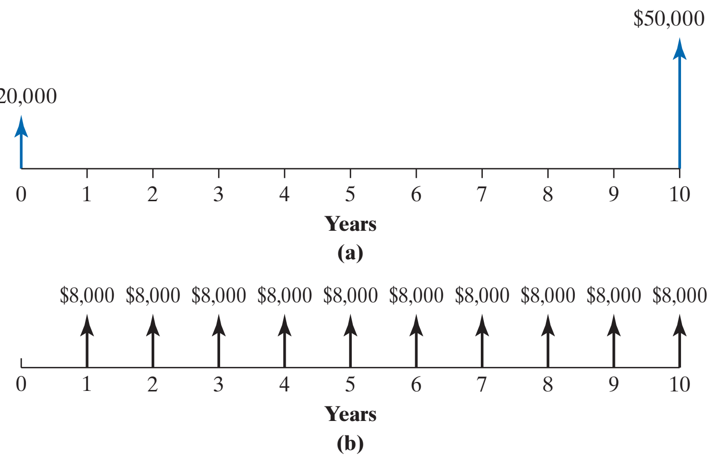


Figure 2.3 Which option would you prefer? (a) Two payments ($20,000 now and $50,000 at the end of 10 years) or (b) 10 equal annual receipts in the amount of $8,000 each.

whether we should prefer to have two payments, $20,000 today and $50,000 in 10 years from now, or $8,000 each year for the next 10 years? (See Figure 2.3.) In this section, we will describe the basic analytical techniques for making these comparisons. Then in Section 2.3, we will use these techniques to develop a series of formulas that can greatly simplify our calculations.

2.2.1 Definition and Simple Calculations

The central factor in deciding among alternative cash flows involves comparing their economic worth. This would be a simple matter if, in the comparison, we did not need to consider the time value of money. We could simply add up the individual payments within a cash flow, treating receipts as positive cash flows and payments (disbursements) as negative cash flows. Calculations for determining the economic effects of one or more cash flows are based on the concept of economic equivalence.

Economic equivalence exists between cash flows that have the same economic effect and could therefore be traded for one another in the financial marketplace (which we assume to exist). Economic equivalence refers to the fact that any cash flow—whether a single payment or a series of payments—can be converted to an *equivalent* cash flow at any point in time. The critical thinking on the present value of future cash flows is that the present sum is equivalent in value to the future cash flows because, if you had the present value today, you could transform it into the future cash flows simply by investing it at the market interest rate, also referred to as the **discount rate**. This process is shown in Figure 2.4.

The strict concept of equivalence may be extended to include the comparison of alternatives. For instance, we could compare the values of two proposals by finding the equivalent values of each at any common point in time. If financial proposals that appear to be quite different could turn out to have the same monetary value, then we can be *economically indifferent* in choosing between them. Likewise, in terms of economic effect, one would be an even exchange for the other, so there is no reason to prefer one over the other.

- If you deposit P dollars today for N periods at i, you will have F dollars at the end of period N.
- F dollars at the end of period N is equal to a single sum of P dollars now if your earning power is measured in terms of the interest rate i.

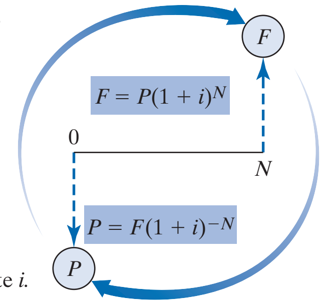


Figure 2.4 Using compound interest to establish economic equivalence.

A way to see the concepts of equivalence and economic indifference at work in the real world is to note the variety of payment plans offered by lending institutions for consumer loans. Recall Table 2.1, where we showed two different repayment plans for a loan of $30,000 for five years at an annual interest rate of 9%. You will notice that the two plans require significantly different repayment patterns and different total amounts of repayment. However, because of the time value of money, these plans are equivalent—economically, the bank is indifferent to the consumer's choice of plan. We will now discuss how such equivalence relationships are established.

Equivalence Calculations: A Simple Example

Equivalence calculations can be viewed as an application of the compound-interest relationships we learned in Section 2.1. Suppose, for example, that we invest $1,000 at 12% annual interest for five years. The formula developed for calculating compound interest, $F = P(1 + i)^N$ [Eq. (2.3)], expresses the equivalence between some present amount P and a future amount F for a given interest rate i and a number of interest periods, N. Therefore, at the end of the investment period, our sums grow to

$$\$1{,}000(1 + 0.12)^5 = \$1{,}762.34.$$

Thus, we can say that at 12% interest, $1,000 received now is equivalent to $1,762.34 received in five years, and we could trade $1,000 now for the promise of receiving $1,762.34 in five years. Example 2.2 further demonstrates the application of this basic technique.

EXAMPLE 2.2 Equivalence

Suppose you are offered the alternative of receiving either $2,007 at the end of five years or $1,500 today. There is no question that the $2,007 will be paid in full (i.e., there's no risk of nonreceipt). Assuming that the money will not be needed in the next five years, you would deposit the $1,500 in an account that pays i% interest. What value of i would make you indifferent to your choice between $1,500 today and the promise of $2,007 at the end of five years?

DISSECTING THE PROBLEM

Our job is to determine the present amount that is economically equivalent to $2,007 in five years, given the investment potential of i% per year. Note that the statement of the problem assumes that you would exercise the option of using the earning power of your money by depositing it. The "indifference" ascribed to you refers to economic indifference; that is, within a marketplace where i% is the applicable interest rate, you could trade one cash flow for the other.

Given: $F = \$2{,}007$, $N = 5$ years, $P = \$1{,}500$. See Figure 2.5a.
Find: i.

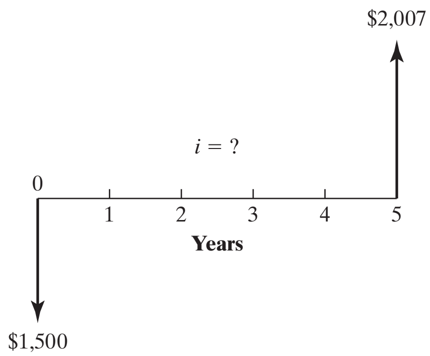


Figure 2.5a Cash flow diagram.

METHODOLOGY

Use Eq. (2.3), $F = P(1 + i)^N$ and solve for i.

SOLUTION

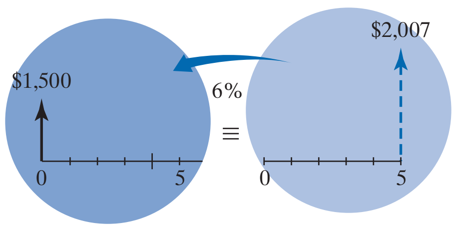
Using the expression of Eq. (2.3) we obtain

$$\$2{,}007 = \$1{,}500(1 + i)^5.$$

Solving for i yields:

$$i = \left(\frac{F}{P}\right)^{1/N} - 1 = \left(\frac{2{,}007}{1{,}500}\right)^{1/5} - 1$$
$$= 0.06 \text{ (or 6\%)}.$$

We can summarize the problem graphically as in Figure 2.5b.



- Step 1: Determine the base period, say, year 0.
- Step 2: Identify the interest rate to use.
- Step 3: Calculate the equivalent value at the base period.

$i = 3\%, P = \$2{,}007(1 + 0.03)^{-5} = \$1{,}731$
$i = 6\%, P = \$2{,}007(1 + 0.06)^{-5} = \$1{,}500$
$i = 9\%, P = \$2{,}007(1 + 0.09)^{-5} = \$1{,}304$

Figure 2.5b Equivalence calculations at varying interest rate.

COMMENTS: In this example, it is clear that if i is anything less than 6%, you would prefer the promise of \$2,007 in five years to \$1,500 today; if i is more than 6%, you would prefer \$1,500 now. As you may have already guessed, at a lower interest rate, P must be higher in order to be equivalent to the future amount. For example, at $i = 4\%$, $P = \$1{,}650$.

2.2.2 Equivalence Calculations Require a Common Time Basis for Comparison

Referring to Figure 2.3, how can we compare these two different cash flow series? Since we know how to calculate the equivalent value of a single cash flow, we may be able to convert each cash flow in the series to its equivalent value at a common base period. One aspect of this basis is the choice of a single point in time at which to make our calculations. In Example 2.2, if we had been given the magnitude of each cash flow and had been asked to determine whether the two were equivalent, we could have chosen any reference point and used the compound-interest formula to find the value of each cash flow at that point. As you can readily see, the choice of $n = 0$ or $n = 5$ would make our problem simpler, because we would need to make only one set of calculations: At 6% interest, either convert \$1,500 at time 0 to its equivalent value at time 5, or convert \$2,007 at time 5 to its equivalent value at time 0.

When selecting a point in time at which to compare the values of alternative cash flows, we commonly use either the present time, which yields what is called the **present worth** of the cash flows, or some point in the future, which yields their **future worth**. The choice of the point in time to use often depends on the circumstances surrounding a particular decision, or the choice may be made for convenience. For instance, if the present worth is known for the first two of three alternatives, simply calculating the present worth of the third will allow us to compare all three. For an illustration, consider Example 2.3.

EXAMPLE 2.3 Equivalence Calculations

Consider the cash flow series given in Figure 2.6. Compute the equivalent lump-sum amount at $n = 3$ at 10% annual interest.

DISSECTING THE PROBLEM	**Given:** The cash flows given in Figure 2.6, and $i = 10\%$ per year. **Find:** V_3 (or equivalent worth at $n = 3$).

METHODOLOGY

We find the equivalent worth at $n = 3$ in two steps. First, we find the future worth of each cash flow at $n = 3$ for all cash flows that occur before $n = 3$. Second, we find the present worth of each cash flow at $n = 3$ for all cash flows that occur after $n = 3$.

SOLUTION

- **Step 1:** Find the equivalent lump-sum payment of the first four payments at $n = 3$:

$$\$100(1 + 0.10)^3 + \$80(1 + 0.10)^2 + \$120(1 + 0.10)^1 + \$150 = \$511.90.$$

- **Step 2:** Find the equivalent lump-sum payment of the remaining two payments at $n = 3$:

$$\$200(1 + 0.10)^{-1} + \$100(1 + 0.10)^{-2} = \$264.46.$$

- **Step 3:** Find V_3, the total equivalent value:

$$V_3 = \$511.90 + \$264.46 = \$776.36.$$

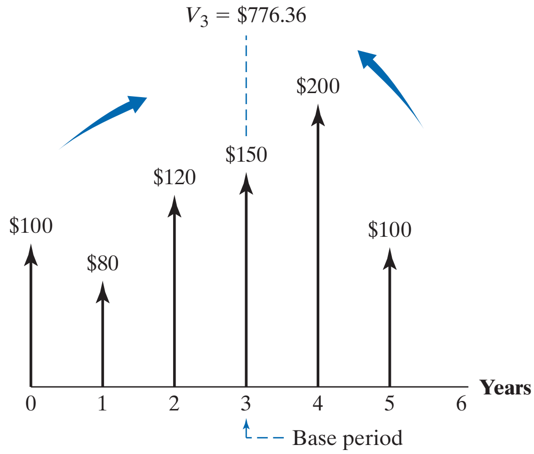


Figure 2.6 Equivalent worth-calculation at $n = 3$.

2.3 Interest Formulas for Single Cash Flows

We begin our coverage of interest formulas by considering the simplest of cash flows: single cash flows.

2.3.1 Compound-Amount Factor

Given a present sum P invested for N interest periods at interest rate i, what sum will have accumulated at the end of the N periods? You probably noticed right away that this description matches the case we first encountered in describing compound interest. To solve for F (the future sum), we use Eq. (2.3):

$$F = P(1 + i)^N.$$

Because of its origin in the compound-interest calculation, the factor $(1 + i)^N$ is known as the **compound-amount factor**. Like the concept of equivalence, this factor is one of the foundations of engineering economic analysis. Given this factor, all other important interest formulas can be derived.

The process of finding F is often called the **compounding process**. The cash flow transaction is illustrated in Figure 2.7. (Note the time-scale convention: The first period begins at $n = 0$ and ends at $n = 1$.) If a calculator is handy, it is easy enough to calculate $(1 + i)^N$ directly.

Interest Tables

Interest formulas such as the one developed in Eq. (2.3), $F = P(1 + i)^N$, allow us to substitute known values from a particular situation into the equation and solve for the unknown. Before the calculator was developed, solving these equations was very tedious. Imagine needing to solve by hand an equation with a large value of N, such as $F = \$20{,}000(1 + 0.12)^{15}$. More complex formulas required even more involved calculations. To simplify the process, tables of compound-interest factors were developed. These tables allow us to find the appropriate factor for a given interest rate and the number of interest periods. Even though many online financial calculators are now

- *Single-payment compound-amount factor* (growth factor)
- Given:
  $i = 10\%$
  $N = 8$ years
  $P = \$2,000$
- Find:
  $F = \$2,000(1 + 0.10)^8$
  $= \$2,000(F/P, 10\%, 8)$
  $= \$4,287.18$

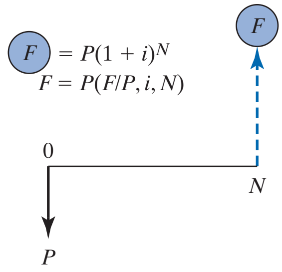


Figure 2.7 Compounding process: Find F, given P, i, and N.

readily available, it is still often convenient to use these tables, which are included in this text in Appendix B. Take some time now to become familiar with their arrangement. If you can, locate the compound-interest factor for the example just presented, in which we know P and, to find F, we need to know the factor by which to multiply \$20,000 when the interest rate i is 12% and the number of periods is 15:

$$F = \$20,000\underbrace{(1 + 0.12)^{15}}_{5.4736} = \$109,472.$$

Factor Notation

As we continue to develop interest formulas in the rest of this chapter, we will express the resulting compound-interest factors in a conventional notation that can be substituted into a formula to indicate precisely which table factor to use in solving an equation. In the preceding example, for instance, the formula derived as Eq. (2.3) is $F = P(1 + i)^N$. To specify how the interest tables are to be used, we may also express that factor in a functional notation as $(F/P, i, N)$, which is read as "Find F, given P, i, and N." This factor is known as the **single-payment compound-amount factor**. When we incorporate the table factor into the formula, the formula is expressed as follows:

$$F = P(1 + i)^N = P(F/P, i, N).$$

Thus, in the preceding example, where we had $F = \$20,000(1.12)^{15}$, we can now write $F = \$20,000(F/P, 12\%, 15)$. The table factor tells us to use the 12% interest table and find the factor in the F/P column for $N = 15$. Because using the interest tables is often the easiest way to solve an equation, this factor notation is included for each of the formulas derived in the upcoming sections.

EXAMPLE 2.4 Single Amounts: Find *F*, Given *P*, *i*, and *N*

If you had \$1,000 now and invested it at 7% interest compounded annually, how much would it be worth in eight years (Figure 2.8)?

DISSECTING THE PROBLEM	
	Given: $P = \$1,000$, $i = 7\%$ per year, and $N = 8$ years. **Find:** F.

METHODOLOGY	SOLUTION
Method 1: Using a Calculator You can simply use a calculator to evaluate the $(1 + i)^N$ term (financial calculators are preprogrammed to solve most future-value problems).	$F = \$1{,}000(1 + 0.07)^8$ $= \$1{,}718.19.$
Method 2: Using Compound-Interest Tables The interest tables can be used to locate the compound-amount factor for $i = 7\%$ and $N = 8$. The number you get can be substituted into the equation. Compound-interest tables are included in Appendix B of this book.	Using this method, we obtain $F = \$1{,}000(F/P, 7\%, 8) = \$1{,}000(1.7182) = \$1{,}718.20.$ This amount is essentially identical to the value obtained by direct evaluation of the single cash flow compound-amount factor. The slight deviation is due to rounding differences.
Method 3: Using a Computer Many financial software programs for solving compound-interest problems are available for use with personal computers. As summarized in the back cover, many spreadsheet programs such as Excel also provide financial functions to evaluate various interest formulas.	With Excel, the future-worth calculation looks like the following: = FV(7%,8,0,–1000,0) = $1,718.20.

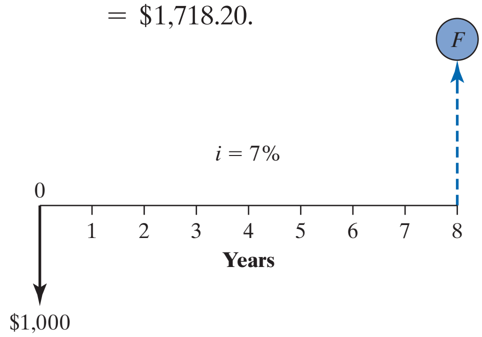


Figure 2.8 Cash flow diagram.

COMMENTS: A better way to take advantage of the powerful features of Excel is to develop a worksheet as shown in Table 2.2. Here, we are not calculating just the future worth of the single payment today. Instead, we can show in both tabular and graphical form how the deposit balances change over time. For example, the $1,000 deposit today will grow to $1,500.73 in six years. If you want to see how the cash balances change over time at a higher interest rate, say, 10%, you just change the interest rate in cell C6 and press the "ENTER" button.

2.3.2 Present-Worth Factor

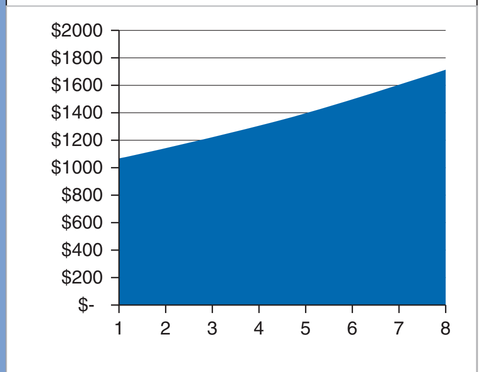
Finding the present worth of a future sum is simply the reverse of compounding and is known as the **discounting process**. (See Figure 2.9.) In Eq. (2.3), we can see that if we need to find a present sum P, given a future sum F, we simply solve for P:

$$P = F\left[\frac{1}{(1 + i)^N}\right] = F(P/F, i, N). \quad (2.4)$$

TABLE 2.2 **An Excel Worksheet to Illustrate How the Cash Balances Change over Time**

	A	B	C	D	E	F	G
1	Single Cash Flows						
2							
3	Inputs				Output		
4							
5	(*P*) Present Worth ($)		1000		(*F*) Future Worth ($)		1718.19
6	(*i*) Interest Rate (%)		7				
7	(*N*) Interest Periods		8		Cash Balance Over Time		
8							
9	Period (*n*)	Deposit (*P*)	Cash Balance				
10							
11	0	$ 1000	$ 1000.00				
12	1		$ 1070.00				
13	2		$ 1144.90				
14	3		$ 1225.04				
15	4		$ 1310.80				
16	5		$ 1402.55				
17	6		$ 1500.73				
18	7		$ 1605.78				
19	8		$ 1718.19				
20							

= C18*(1+C6%)+B19



The factor $1/(1+i)^N$ is known as the **single-payment present-worth factor** and is designated $(P/F, i, N)$. Tables have been constructed for *P/F* factors and for various values of *i* and *N*. The interest rate *i* and the *P/F* factor are also referred to as the **discount rate** and the **discounting factor**, respectively.

- *Single-payment present-worth factor (discount factor)*
- Given:
  $i = 12\%$
  $N = 5$ years
  $F = \$1{,}000$
- Find:
  $P = \$1{,}000(1+0.12)^{-5}$
  $= \$1{,}000(P/F, 12\%, 5)$
  $= \$567.40$

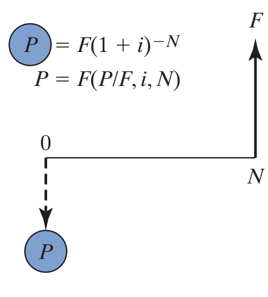


Figure 2.9 Discounting process: Find *P*, given *F*, *i*, and *N*.

EXAMPLE 2.5 Single Amounts: Find *P*, Given *F*, *i*, and *N*

A zero-coupon bond[3] is a popular variation on the bond theme for some investors. What should be the price of an eight-year zero-coupon bond with a face value of $1,000 if similar, nonzero-coupon bonds are yielding 6% annual interest?

DISSECTING THE PROBLEM

As an investor of a zero-coupon bond, you do not receive any interest payments until the bond reaches maturity. When the bond matures, you will receive $1,000 (the face value). In lieu of getting interest payments, you can buy the bond at a discount. The question is, "What should the price of the bond be in order to realize a 6% return on your investment?" (See Figure 2.10.)

Given: $F = \$1{,}000$, $i = 6\%$ per year, and $N = 8$ years.
Find: P.

METHODOLOGY

Using a calculator may be the best way to make this simple calculation. It is equivalent to finding the present value of the $1,000 face value at 6% interest.

SOLUTION

Using a calculator, we obtain

$$P = \$1{,}000(1 + 0.06)^{-8} = \$1{,}000(0.6274) = \$627.40.$$

We can also use the interest tables to find that

$$P = \$1{,}000\overbrace{(P/F, 6\%, 8)}^{(0.6274)} = \$627.40.$$

Again, you could also use a financial calculator or computer to find the present worth. With Excel, the present-value calculation looks like the following:

$$= \text{PV}(6\%,8,0,1000,0)$$
$$= -\$\ 627.40.$$

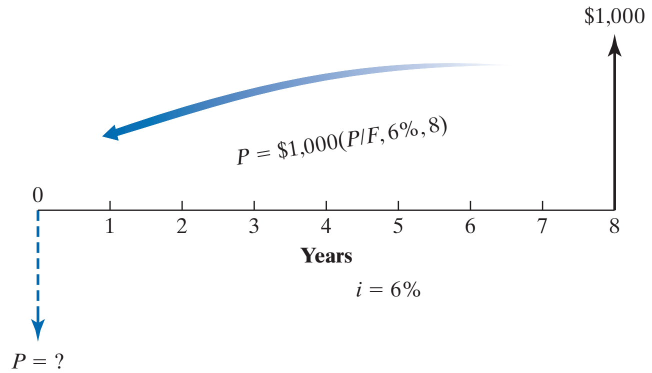


Figure 2.10 Cash flow diagram.

[3] Bonds are loans that investors make to corporations and governments. In Example 2.5, the $1,000 of principal is the **face value** of the bond, the yearly interest payment is its **coupon**, and the length of the loan is the bond's **maturity**.

2.3.3 Solving for Time and Interest Rates

At this point, you should realize that the compounding and discounting processes are reciprocals of one another and that we have been dealing with one equation in two forms:

$$\text{Future-value form: } F = P(1 + i)^N$$

and

$$\text{Present-value form: } P = F(1 + i)^{-N}.$$

There are four variables in these equations: P, F, N, and i. If you know the values of any three, you can find the value of the fourth. Thus far, we have always given you the interest rate (i) and the number of years (N), plus either P or F. In many situations, though, you will need to solve for i or N, as we discuss next.

EXAMPLE 2.6 Solving for *i*

Suppose you buy a share of stock for \$10 and sell it for \$20; your profit is thus \$10. If that happens within a year, your rate of return is an impressive 100% $(\$10/\$10 = 1)$. If it takes five years, what would be the annual rate of return on your investment? (See Figure 2.11.)

DISSECTING THE PROBLEM

Here, we know P, F, and N, but we do not know i, the interest rate you will earn on your investment. This type of rate of return calculation is straightforward, since you make only a one-time lump-sum investment.

Given: $P = \$10$, $F = \$20$, and $N = 5$ years.
Find: i.

METHODOLOGY

We start with the following relationship:

$$F = P(1 + i)^N.$$

We then substitute in the given values:

$$\$20 = \$10(1 + i)^5.$$

Next, we solve for i by one of two methods.

Method 1: Trial and Error
Go through a trial-and-error process in which you insert different values of i into the equation until you find a value that "works," in the sense that the right-hand side of the equation equals \$20.

SOLUTION

The solution value is $i = 14.87\%$. The trial-and-error procedure is extremely tedious and inefficient for most problems, so it is not widely practiced in the real world.

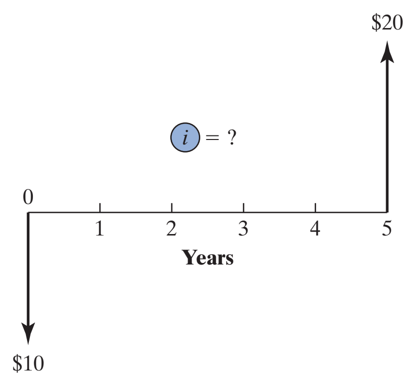


Figure 2.11 Cash flow diagram.

METHODOLOGY	SOLUTION
Method 2: Interest Tables You can solve the problem by using the interest tables in Appendix B.	Start with the equation $$\$20 = \$10(1 + i)^5,$$ which is equivalent to $$2 = (1 + i)^5 = (F/P, i, 5).$$ Now look across the $N = 5$ row under the $(F/P, i, 5)$ column until you can locate the value of 2. This value is approximated in the 15% interest table at $(F/P, 15\%, 5) = 2.0114$, so the interest rate at which \$10 grows to \$20 over five years is very close to 15%. This procedure will be very tedious for fractional interest rates or when N is not a whole number, as you may have to approximate the solution by linear interpolation.
Method 3: Practical Approach The most practical approach is to use either a financial calculator or an electronic spreadsheet such as Excel. A financial function such as RATE(N, 0, P, F) allows us to calculate an unknown interest rate.	The precise command statement would be as follows: $$= \text{RATE}(5{,}0{,}-10{,}20) = 14.87\%.$$ Note that we always enter the present value (P) as a negative number in order to indicate a cash outflow in Excel.

EXAMPLE 2.7 Single Amounts: Find *N*, Given *P*, *F*, and *i*

You have just purchased 200 shares of a biotechnology stock at \$15 per share. You will sell the stock when its market price doubles. If you expect the stock price to increase 12% per year, how long do you expect to wait before selling the stock? (See Figure 2.12.)

DISSECTING THE PROBLEM	
	Given: $P = \$3{,}000$, $F = \$6{,}000$, and $i = 12\%$ per year. **Find:** N (years).

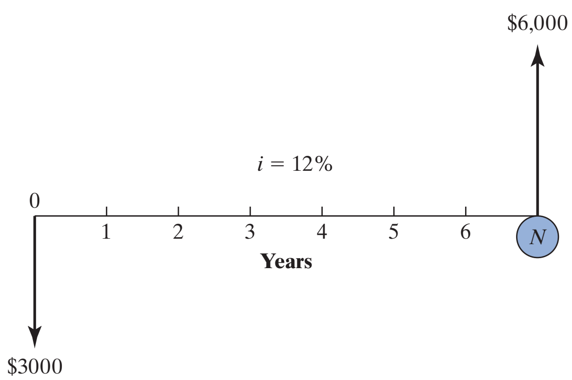


Figure 2.12 Cash flow diagram.