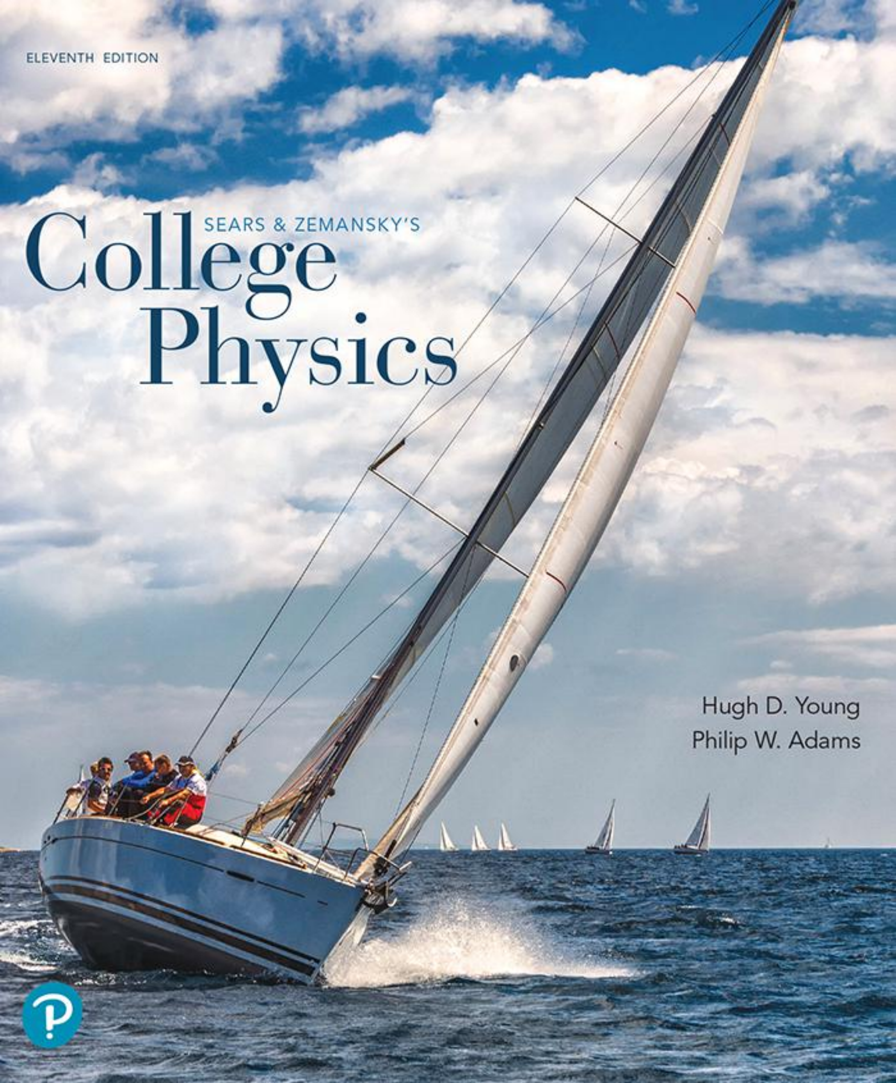
ELEVENTH EDITION

SEARS & ZEMANSKY'S

College Physics

Hugh D. Young
Philip W. Adams



COLLEGE PHYSICS

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SEARS AND ZEMANSKY'S

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ELEVENTH EDITION

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BRIEF CONTENTS

ABOUT THE AUTHORS

Philip W. Adams is a Professor of Physics at Louisiana State University in Baton Rouge, Louisiana. He obtained his Ph.D. in Physics from Rutgers University in 1986 and then held a postdoctoral research position at AT&T Bell Laboratories in Murray Hill, NJ for two years. He joined the faculty of LSU 1988 and has since become an internationally recognized low temperature experimentalist and has published over 90 papers in peer-reviewed scientific journals. He is a Fellow of the American Physical Society and has given many invited presentations on his work at international workshops and conferences on superconductivity and other topics in low temperature condensed matter physics.

Dr. Adams has had a career-long interest in physics education. He has taught introductory physics for engineers and for non-engineers many times in his 30-year tenure at LSU and has been the recipient of numerous teaching awards.

IN MEMORIAM: HUGH YOUNG (1930–2013)

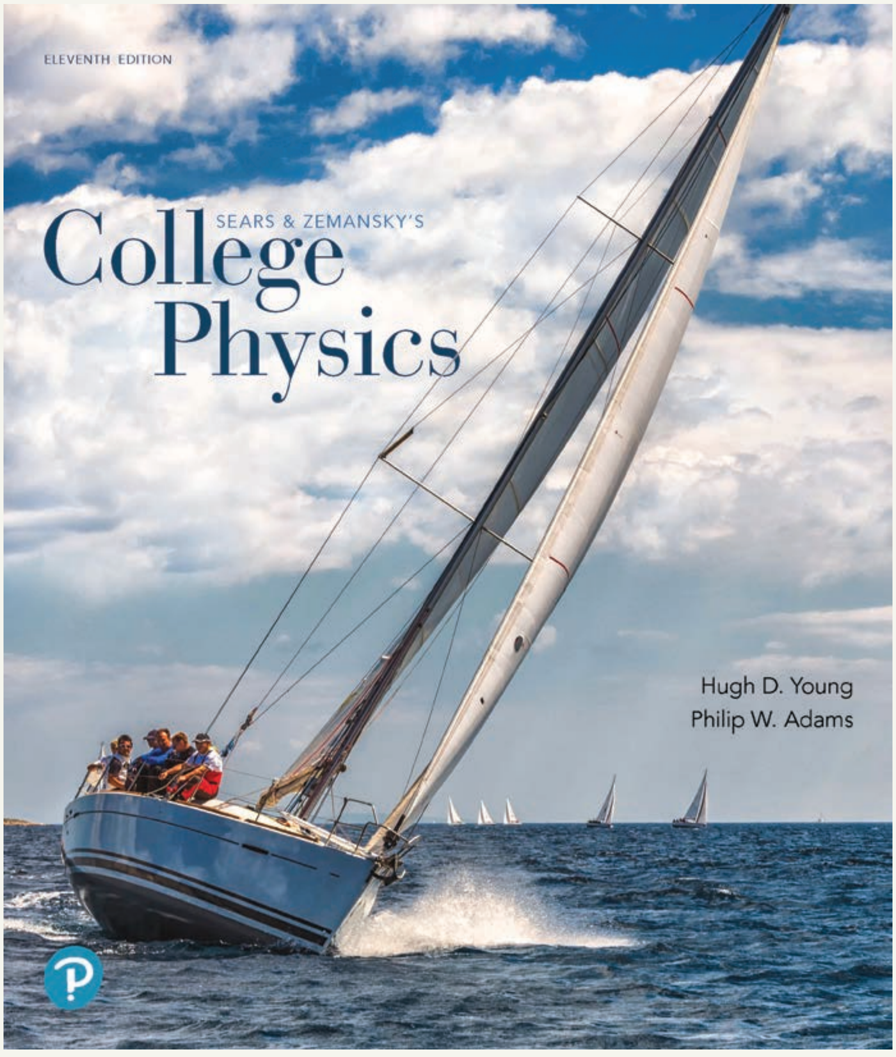
Hugh D. Young was Emeritus Professor of Physics at Carnegie Mellon University. He earned his Ph.D. from Carnegie Mellon in fundamental particle theory under the direction of the late Richard Cutkosky. He also had two visiting professorships at the University of California, Berkeley.

Dr. Young's career was centered entirely on undergraduate education. He wrote several undergraduate-level textbooks, and in 1973 he became a coauthor with Francis Sears and Mark Zemansky of their well-known introductory texts *University Physics* and *College Physics*.

Dr. Young earned a bachelor's degree in organ performance from Carnegie Mellon in 1972 and spent several years as Associate Organist at St. Paul's Cathedral in Pittsburgh. We at Pearson appreciated his professionalism, good nature, and collaboration. He will be missed.

Help students see the connections between problem types and understand how to solve them

The new **11th Edition of *College Physics*** incorporates data from thousands of surveyed students detailing their use and reliance on worked examples, video tutorials, and just-in-time remediation when working homework problems and preparing for exams. Driven by how students actually use the text and media, this edition offers multiple resources to help students see patterns and make connections between problem types, helping them to develop an understanding of problem-solving approaches, rather than simply plugging in an equation. **Mastering Physics** gives students additional problem sets with wrong answer specific feedback, hints, and links end-of-chapter problems directly to the Pearson eText for additional guidance.



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Help develop a greater understanding of . . .

Chapter 7 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

I, II, III: Difficulty levels. DATA: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. BIO: Biosciences Problems. SYM: Symbolic Problems. EST: Estimation Problems.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLE 7.8 before attempting these problems.

VP7.8.1 I A glider with mass $m = 0.2$ kg sits on a frictionless air track. It is connected to a massless spring with force constant $k = 20$ N/m. The glider is initially at $x = 0$, and the spring is relaxed. You then hit the glider with a hammer, which gives it an initial velocity of $v_0 = 5$ m/s in the positive x direction. At what x positions will the speed of the glider be zero?

VP7.8.2 II SYM A mass m is tied to an ideal spring with force constant k and rests on a frictionless surface. The mass moves along the x axis. Assume that $x = 0$ corresponds to the relaxed position of the spring. The mass is pulled out to a position x_m and released. Derive an expression for the positions at which the kinetic energy of the mass is equal to the elastic potential energy of the spring.

VP7.8.3 II A glider has a mass $m = 0.4$ kg. It is attached to a spring with force constant k. The relaxed position of the spring corresponds to $x = 0$. The glider is pulled out to $x = 0.2$ m and then released. If its speed at $x = 0.1$ m is 3 m/s, determine k.

Be sure to review EXAMPLE 7.13 before attempting these problems.

VP7.13.1 I A small box is released from rest at the top of a frictionless inclined plane as shown in Figure 7.36. The horizontal surface at the base of the plane is rough and has a coefficient of kinetic friction $\mu_k = 0.4$. If $H = 10$ m, how far does the box slide on the rough surface before coming to rest, d?

Figure 7.36

VP7.13.2 II SYM A box is released from rest at the top of an inclined plane that makes an angle θ with respect to the horizontal. The length of the plane is L (as measured along its surface), and the coefficient of kinetic friction between the box and the inclined surface is $\mu_k = \frac{1}{2}\tan\theta$. Use conservation of energy to derive an expression for the speed of the box when it reaches the bottom of the inclined plane.

VP7.13.3 II A glider has a mass $m = 0.4$ kg and is resting on a track that has a small but finite coefficient of kinetic friction. It is attached to a spring with force constant $k = 20$ N/m. The relaxed position of the spring corresponds to $x = 0$. The glider is pulled out to $x = 0.2$ m and then released. If its speed is 1.0 m/s when it first reaches $x = 0.1$ m, determine the work done by friction during that segment of the motion.

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NEW! Example Variation Problems build in difficulty by adjusting scenarios, changing the knowns vs. unknowns, and adding complexity and a step of reasoning to provide the most helpful range of related problems that use the same fundamental approach to solve. These scaffolded problem sets help students see patterns and make connections between problems types that can be solved applying the same fundamental principles so that they are less surprised by variations on problems when exam time comes. They are assignable in Mastering Physics.

UPDATED! Worked Examples follow a consistent and explicit global problem-solving strategy drawn from educational research that shows students find access to worked examples at the point of need most helpful. This 3-step approach emphasizes setting up a problem effectively before any attempts to solve it as well as the importance of reflecting on whether the answer is sensible. This focus helps students understand how to solve problems rather than hunting for an equation they can plug in.

EXAMPLE 5.3 Car on a ramp

In this example, we will see how to handle the case of an object resting on an inclined plane. A car with a weight of 1.76×10^4 N rests on the ramp of a trailer (Figure 5.3a). The car's brakes and transmission lock are released; only the cable prevents the car from rolling backward off the trailer. The ramp makes an angle of 26.0° with the horizontal. Find the tension in the cable and the force with which the ramp pushes on the car's tires.

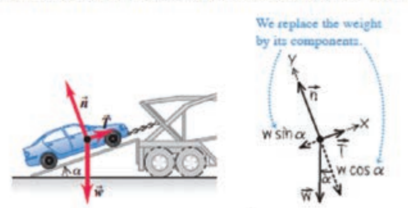

Figure 5.3 (a) Car on ramp (b) Free-body diagram for car

SOLUTION

SET UP Figure 5.3b shows our free-body diagram for the car. The three forces exerted on the car are its weight (magnitude w), the tension in the cable (magnitude T), and the normal force with magnitude n exerted by the ramp. (Because we treat the car as a particle, we can lump the normal forces on the four wheels together as a single force.) We orient our coordinate axes parallel and perpendicular to the ramp, and we replace the weight force by its components.

SOLVE The car is in equilibrium, so we first find the components of each force in our axis system and then apply Newton's first law. To find the components of the weight, we note that the angle α between the ramp and the horizontal is equal to the angle α between the weight vector and the normal to the ramp, as shown. The angle α is *not* measured in the usual way, counterclockwise from the $+x$ axis. To find the components of the weight (w_x and w_y), we use the right triangles in Figure 5.3b. We find that $w_x = -w\sin\alpha$ and $w_y = -w\cos\alpha$. The equilibrium conditions then give us

$$\Sigma F_x = 0, \quad T + (-w\sin\alpha) = 0,$$
$$\Sigma F_y = 0, \quad n + (-w\cos\alpha) = 0.$$

Be sure you understand how the signs are related to our choice of coordinate axis directions. Remember that, by definition, T, w, and n are *magnitudes* of vectors and are therefore positive.

Solving these equations for T and n, we find

$$T = w\sin\alpha,$$
$$n = w\cos\alpha.$$

Finally, inserting the numerical values $w = 1.76 \times 10^4$ N and $\alpha = 26°$, we obtain

$$T = (1.76 \times 10^4\ \text{N})(\sin 26°) = 7.72 \times 10^3\ \text{N},$$
$$n = (1.76 \times 10^4\ \text{N})(\cos 26°) = 1.58 \times 10^4\ \text{N}.$$

REFLECT To check some special cases, note that if the angle α is zero, then $\sin\alpha = 0$ and $\cos\alpha = 1$. In this case, the ramp is horizontal; no cable tension T is needed to hold the car, and the magnitude of the total normal force n is equal to the car's weight. If the angle is 90° (the ramp is vertical), then $\sin\alpha = 1$ and $\cos\alpha = 0$. In that case, the cable tension T equals the weight w and the normal force n is zero.

We also note that our results would still be correct if the car were on a ramp on a car transport trailer traveling down a straight highway at a constant speed of 65 mi/h. Do you see why?

Practice Problem: What ramp angle would be needed in order for the cable tension to equal one-half of the car's weight? *Answer:* 30°.

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Problem-Solving skills

BRIDGING PROBLEM

A large pressurized tank is filled with water. The air pressure above the water surface is $p_0 > p_{atm}$. The water has a depth h and density ρ_w. A cube of aluminum with sides of length L and density ρ_{Al} sits on the bottom of the tank (Figure 13.37a). (a) Derive an expression for the pressure at the bottom of the tank. (b) Derive an expression for the normal force between the cube and the bottom of the tank. (c) Should your answer for part (b) depend on p_0? Explain why or why not. (d) A small hole is punched into the wall of the tank at a depth of $h/2$, as shown in Figure 13.37b. What is the speed of the water flowing out of the hole? (Assume that the tank is large enough so that the water velocity inside the tank is near zero.)

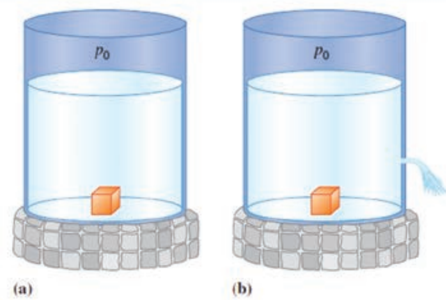

Figure 13.37

Set Up

- Write down the equation for pressure as a function of depth in a liquid.
- Draw a free-body diagram for the aluminum cube and identify all of the forces acting on it.
- Consider what role Pascal's law plays in this problem.
- Write down Bernoulli's equation and identify the pressures and velocities that are appropriate for part (d) of the problem.

Solve

- Solve for the pressure at the bottom of the tank.
- Apply Newton's second law to the cube and solve for the perpendicular force.
- Evaluate the effect of p on the buoyancy force.
- Apply Bernoulli's equation to the flow out of the hole and solve for the speed.

Evaluate

- What effect does the pressure at the top of the tank have on the pressure at the bottom of the tank?
- Would the perpendicular force in part (b) be larger or smaller if the cube were made of lead instead of aluminum? Explain.
- Would the perpendicular force in part (b) be larger or smaller if the liquid in the tank were ethanol instead of water? Explain.
- How would the speed of the escaping water change if the pressure in the tank were lowered so that $p = p_{atm}$?

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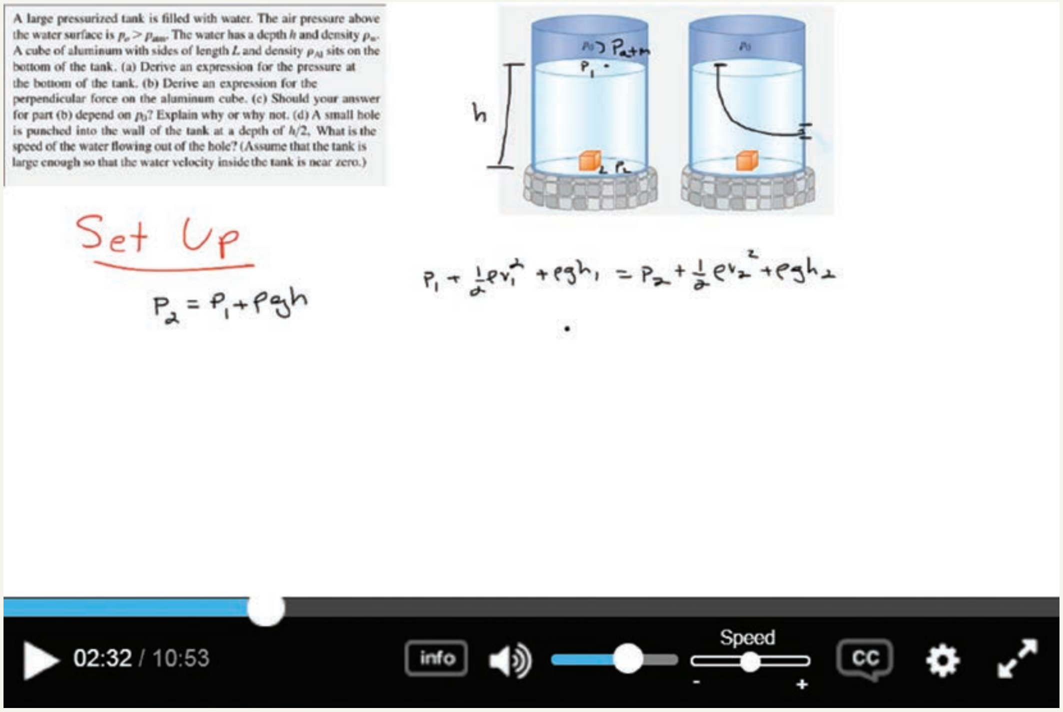
Bridging Problems at the end of each chapter help students move from single concept worked examples to multi-concept problems. New! Assignable as Tutorials in Mastering Physics.

Video Tutor Solutions (VTSs) for every Example and Bridging Problem in the book walk students through the problem-solving process, providing a virtual teaching assistant on a round-the-clock basis. New VTSs correspond to new and revised worked examples.



Tools that build conceptual understanding . . .

Test Your Understanding of SECTION 10.1

Torques on a rod

Three forces with equal magnitudes act on the ends of a long, thin rod of length L, as shown in Figure 10.5. The rod is pinned so that it can rotate about its center of mass. Which choice correctly ranks the magnitudes of the torques produced by all three forces, from largest to smallest?

A. $\tau_1 = \tau_2 = \tau_3$
B. $\tau_1 > \tau_3 > \tau_2$
C. $\tau_2 > \tau_3 > \tau_1$

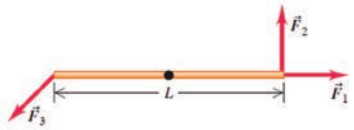


Figure 10.5

SOLUTION Because all three forces have the same magnitude, the force that acts over the longest moment arm produces the greatest torque. Because the line of action of $\vec{F}_1$ runs through the pivot, the first force produces no torque and $\tau_1 = 0$. For $\vec{F}_2$, the line of action is perpendicular to the rod, so the moment arm is $L/2$. Because $\vec{F}_3$ is not perpendicular to the rod, the moment arm must be shorter than $L/2$. Therefore, the correct answer is C.

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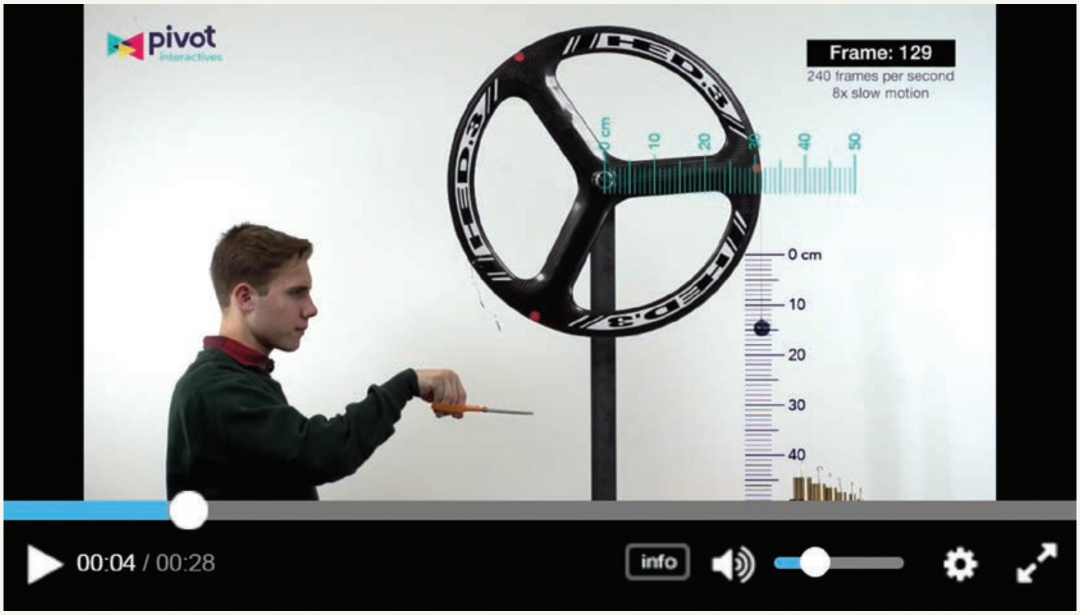
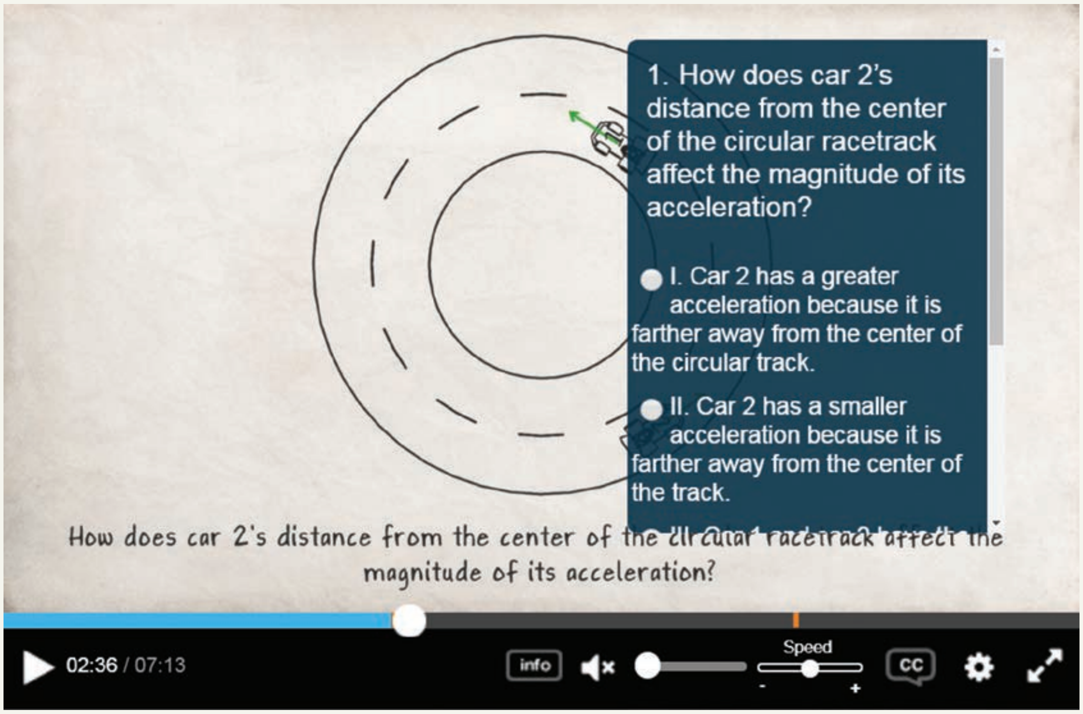
NEW! Test Your Understanding problems are added strategically throughout the chapters, helping students complete an important step of ensuring that their answer makes sense in the real world. Test Your Understanding problems in the eText provide the full solution when students mouseover a problem.



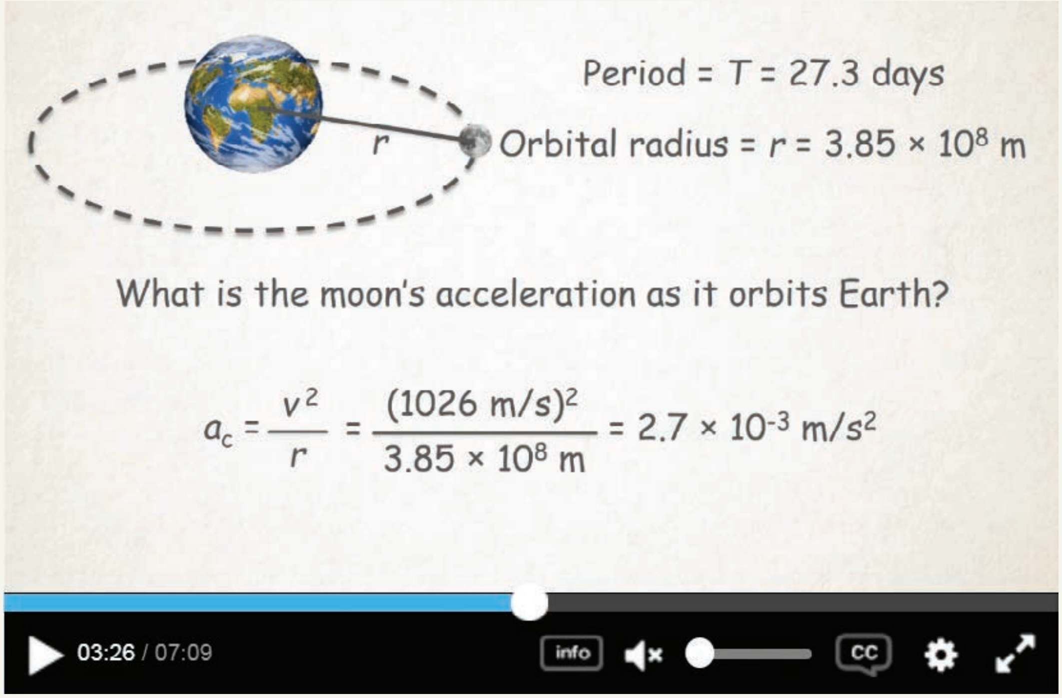
NEW! Direct Measurement Videos show real situations of physical phenomena. Grids, rulers, and frame counters appear as overlays, helping students to make precise measurements of quantities such as position and time. Students then apply these quantities along with physics concepts to solve problems and answer questions about the motion of the objects in the video. The problems are assignable in Mastering and can be used to replace or supplement traditional word problems, or as open-ended questions to help develop problem-solving skills.

even before students come to class



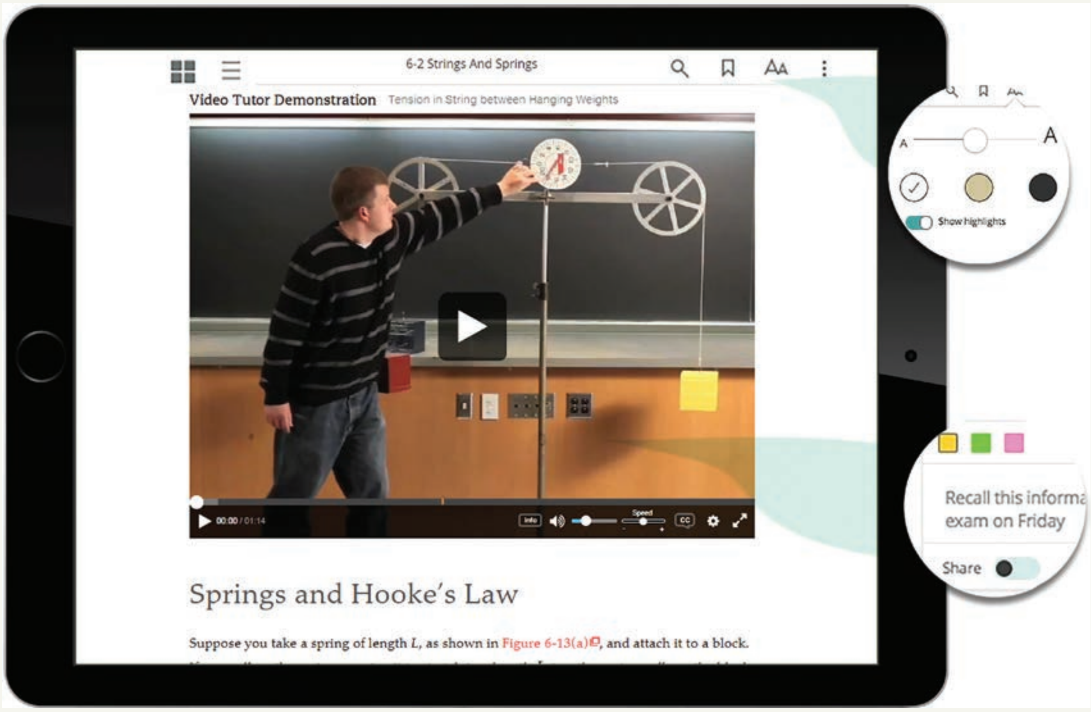
Interactive Pre-lecture Videos provide an introduction to topics with embedded assessment to help students prepare before lecture and to help professors identify student misconceptions.

NEW! 30 Quantitative Pre-lecture Videos now complement the conceptual Interactive Pre-lecture Videos. These videos are designed to expose students to concepts before class and help them learn how problems for a specific concept are worked.

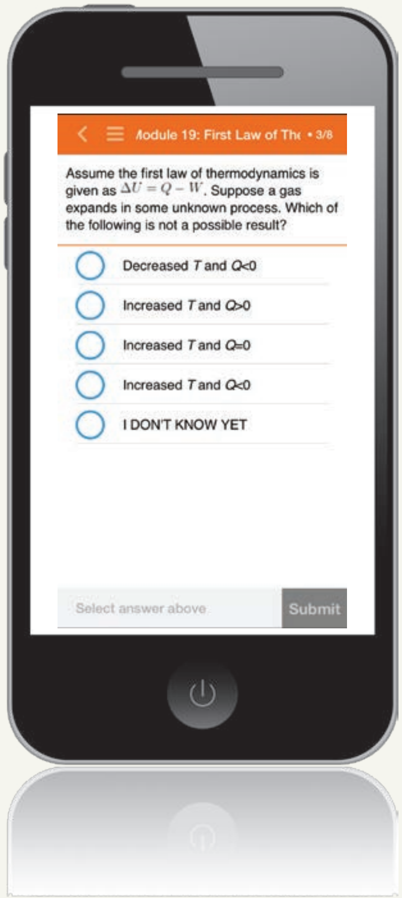


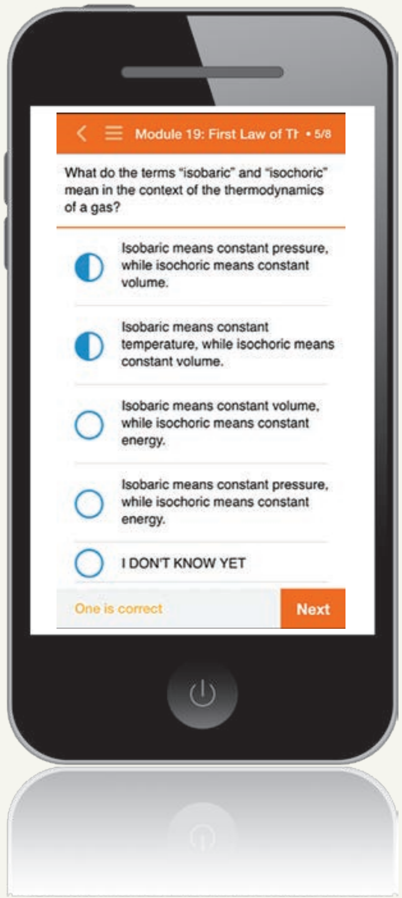
Reach every student . . .

NEW! College Physics is now available in Pearson eText. Pearson eText is a simple-to-use, mobile-optimized, personalized reading experience available within Mastering. It allows students to easily highlight, take notes, and review key vocabulary all in one place—even when offline. Seamlessly integrated videos, rich media, and interactive self-assessment questions engage students and give them access to the help they need, when they need it. Pearson eText is available within Mastering when packaged with a new book; students can also purchase Mastering with Pearson eText online.



with Mastering Physics

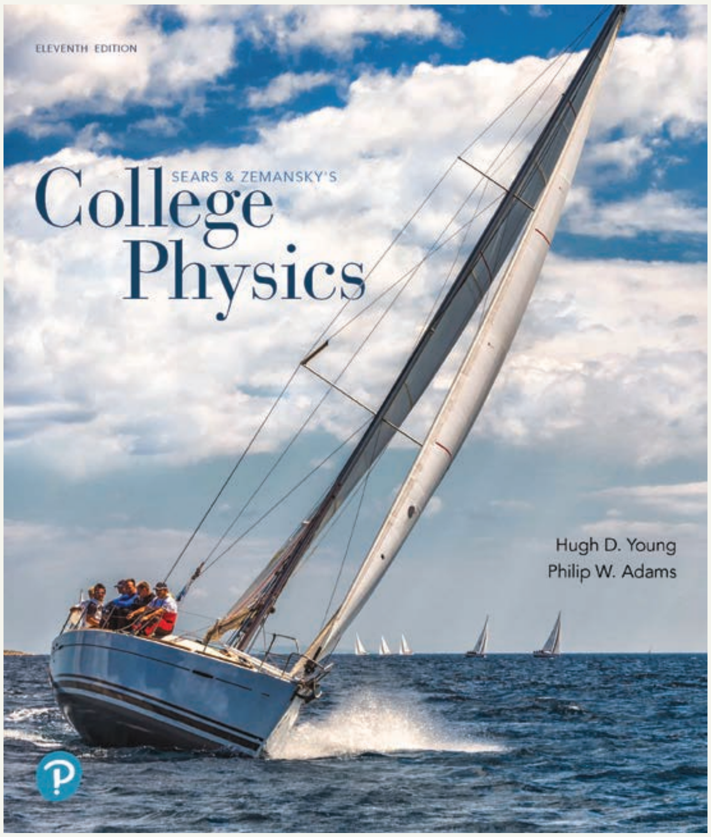




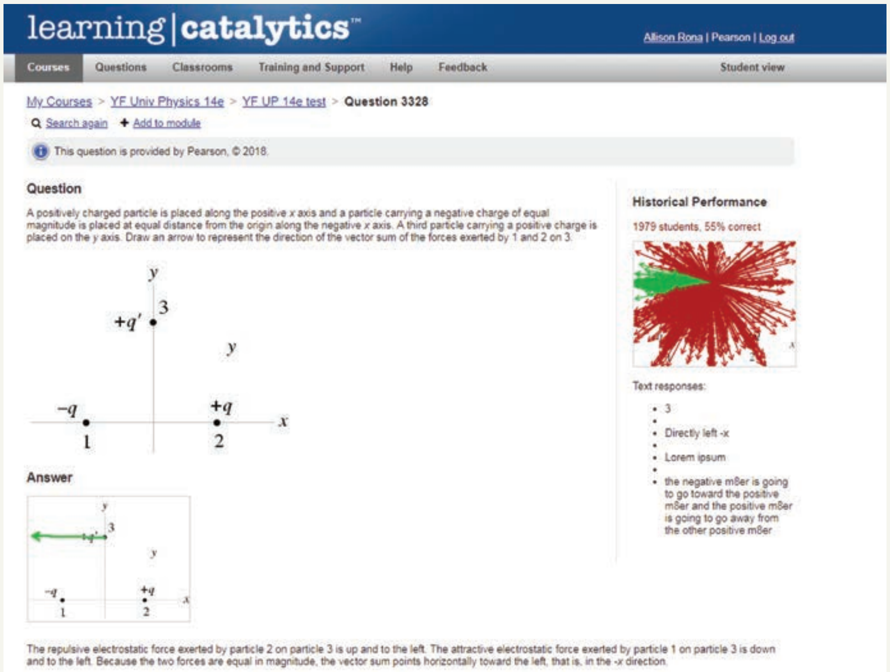
Dynamic Study Modules in Mastering Physics help students study effectively—and at their own pace—by keeping them motivated and engaged. The assignable modules rely on the latest research in cognitive science, using methods—such as adaptivity, gamification, and intermittent rewards—to stimulate learning and improve retention. DSM are available to use on any mobile device.

The Physics Primer relies on videos, hints, and feedback to refresh students' math skills in the context of physics and prepares them for success in the course. These tutorials can be assigned before the course begins or throughout the course as just-in-time remediation. They ensure students practice and maintain their math skills, while tying together mathematical operations and physics analysis.

Instructor support you can rely on



College Physics includes a full suite of instructor support materials in the Instructor Resources area in Mastering Physics. Resources include PowerPoint lecture outlines; all chapter summaries, key equations, and problem-solving strategies from the text; all figures and images from the text; plus a solutions manual and test bank.



Instructors also have access to **Learning Catalytics**. With Learning Catalytics, you'll hear from every student when it matters most. You pose a variety of questions that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops. You can monitor responses with real-time analytics and find out what your students do — and don't — understand. Then, you can adjust your teaching accordingly and even facilitate peer-to-peer learning, helping students stay motivated and engaged.

REAL-WORLD APPLICATIONS

BIO indicates bioscience applications.

TO THE STUDENT

HOW TO SUCCEED IN PHYSICS

"Is physics hard? Is it too hard for me?" Many students are apprehensive about their physics course. However, while the course can be challenging, almost certainly it is *not* too hard for you. If you devote time to the course and use that time wisely, you can succeed.

Here's how to succeed in physics.

1. **Spend time studying.** The rule of thumb for college courses is that you should expect to study about 2 to 3 hours per week for each unit of credit, *in addition* to the time you spend in class. And budget your time: 3 hours every other day is much more effective than 33 hours right before the exam.

   The good news is that physics is consistent. Once you've learned how to tackle one topic, you'll use the same study techniques to tackle the rest of the course. So if you find you need to give the course extra time at first, do so and don't worry—it'll pay dividends as the course progresses.

2. **Don't miss class.** Yes, you could borrow a friend's notes, but listening and participating in class are far more effective. Of course, *participating* means paying active attention, and interacting when you have the chance!

3. **Make this book work for you.** This text is packed with decades of teaching experience—but to make it work for you, you must read and use it *actively*. *Think* about what the text is saying. *Use* the illustrations. Try to *solve* the Test Your Understanding problems on your own, before reading the solutions. If you *underline,* do so thoughtfully and not mechanically.

   Use the Variation Problems to hone your problem solving skills. These problems represent progressive variations on the Key Examples. They are designed to help you recognize and exploit the underlying mathematical similarities of two closely related, yet seemingly completely different, physics problems. This is an important exam skill and it can serve you well!

   A good practice is to skim the chapter before going to class to get a sense for the topic, and then read it carefully and work the examples after class.

4. **Approach physics problems systematically.** While it's important to attend class and use the book, your *real* learning will happen mostly as you work problems—*if* you approach them correctly. Physics problems aren't math problems. You need to approach them in a different way. (If you're "not good at math," this may be good news for you!) What you do before and after solving an equation is more important than the math itself. The worked examples in this book help you develop good habits by consistently following three steps—*Set Up, Solve,* and *Reflect.* (In fact, this global approach will help you with problem solving in all disciplines—chemistry, medicine, business, etc.)

5. **Use campus resources.** If you get stuck, get help. Your professor probably has office hours and email; use them. Use your TA or campus tutoring center if you have one. Partner with a friend to study together. But also try to get unstuck on your own *before* you go for help. That way, you'll benefit more from the help you get.

6. **Honestly assess your level of understanding.** It is crucially important that you honestly acknowledge those concepts and problems that you don't really understand. Too often, students simply tell themselves that they understand a point made in the lecture, or a homework problem, or a concept when, in fact, they don't. Or, even worse, they simply hope that certain questions will not appear on the next exam.

So remember, you *can* succeed in physics. Just devote time to the job, work lots of problems, and get help when you need it. Your book is here to help. Have fun!

SET UP

Think about the physics involved in the situation the problem describes. What information are you given and what do you need to find out? Which physics principles do you need to apply? Almost always you should *draw a sketch* and label it with the relevant known and unknown information. (Many of the worked examples in this book include hand-drawn sketches to coach you on what to draw.)

SOLVE

Based on what you did in Set Up, identify the physics and appropriate equation or equations and do the algebra. Because you started by *thinking about the physics* (and *drawing a diagram*), you'll know which physics equations apply to the situation—you'll avoid the "plug and pray" trap of picking any equation that seems to have the right variables.

REFLECT

Once you have an answer, ask yourself whether it is plausible. If you calculated your weight on the Moon to be 10,423 kg—you must have made a mistake somewhere! Next, check that your answer has the right units. Finally, think about what *you* learned from the problem that will help you later.

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PREFACE

College Physics places equal emphasis on conceptual, qualitative, and quantitative understanding. This classic text gives students a solid understanding of the fundamentals, helps them develop critical thinking, quantitative reasoning, and problem-solving skills, and sparks interest in physics with real-world applications. Informed by physics education research and data of thousands of student users of Mastering Physics, this edition emphasizes learning to solve physics problems in a variety of contexts, and applying physics to the real world.

This text provides a comprehensive introduction to physics. It is intended for students whose mathematics preparation includes high-school algebra and trigonometry but not calculus. The complete text may be taught in a two-semester or three-quarter course, and the book is also adaptable to a wide variety of shorter courses.

NEW TO THIS EDITION

- **83 new Test Your Understanding questions were added to the text.** Now there is a Test Your Understanding box for every quantitative section of the book. These are intended to help students complete the important step of ensuring that their answer makes sense in the real world.
- **New! 180 Example Variation Problems build in difficulty** by adjusting scenarios, changing the knowns vs. unknowns, and adding complexity and a step of reasoning to provide the most helpful range of related problems that use the same fundamental approach to solve. These scaffolded problem sets help students see patterns and make connections between problems types that can be solved by applying the same fundamental principles so that they are less surprised by variations on problems when exam time comes.
- **Proportional Reasoning questions have been further developed.** These are designed to help the students recognize and exploit algebraic relationships between relevant physical quantities. These types of questions commonly appear on physics exams.
- **Streamlined and improved design.** The eleventh edition is more concise than previous editions and now features an open, inviting presentation.
- **Over 70 PhET simulations** are provided in the study area of the Mastering Physics website. These powerful simulations allow students to interact productively with the physics concepts they are learning.
- **Video Tutors bring key content to life throughout the text:**
  - **Over 50 Video Tutor Demonstrations feature interactive "pause-and-predict" demonstrations of key concepts.** The videos actively engage students and help uncover misconceptions.
  - **Video Tutor Solutions accompany every Worked Example and Bridging Problem in the book.** These narrated videos walk students through the problem-solving process, acting as a virtual teaching assistant on a round-the-clock basis. Students can access the Video Tutor Solutions using QR codes conveniently placed in the text, through links in the eText, or through the study area within Mastering Physics.
- **Assignable Mastering Physics activities are based on the Pause and Predict Video Tutors and PhET simulations.**
  - **Video Tutor Demonstrations with assessment allow the student** to extend their understanding by answering a follow-up question.
  - **PhET tutorials prompt students to explore the PhET simulations** and use them to answer questions and solve problems, helping them to make connections between real life phenomena and the underlying physics that explains such phenomena.

Complete and Two-Volume Editions

With Mastering Physics:

- **Complete Edition:** Chapters 0–30 (ISBN 978-0-134-87947-5)

Without Mastering Physics:

- **Complete Edition:** Chapters 0–30 (ISBN 978-0-134-87698-6)
- **Volume 1:** Chapters 0 –16 (ISBN 978-0-134-98732-2)
- **Volume 2:** Chapters 17–30 (ISBN 978-0-134-98731-6)

KEY FEATURES OF *COLLEGE PHYSICS*

- **A systematic approach to problem solving.** To solve problems with confidence, students must learn to approach problems effectively at a global level, must understand the physics in question, and must acquire the specific skills needed for particular types of problems. The Tenth Edition provides research-proven tools for students to tackle each goal.
  - Expanded Bridging Problems, now available in Mastering Physics, and additional **Practice Problems** provide extra support for students as they learn to solve problems in physics.
  - Each **worked example** follows a consistent and explicit **global problem-solving strategy** drawn from educational research. This three-step approach puts special emphasis on how to **set up** the problem before trying to **solve** it, and the importance of how to **reflect** on whether the answer is sensible.
  - **New - Example Variation Problems** build in difficulty by adjusting scenarios, changing the knowns vs. unknowns, and adding complexity and a step of reasoning to provide the most helpful range of related problems that use the same fundamental approach to solve. These scaffolded problem sets help students see patterns and make connections between problems types that can be solved, applying the same fundamental principles so that they are less surprised by variations on problems when exam time comes. Assignable in Mastering Physics.
  - Worked example solutions model the steps and decisions students should use but often skip. Worked examples include new **pencil diagrams:** hand-drawn diagrams that show exactly what a student should draw in the **set up** step of solving the problem. Also included are practice problems for the worked examples. These practice problems are now assignable in Mastering Physics.
- **Test Your Understanding** problems help students practice their qualitative and quantitative understanding of the physics. These featured problems focus on skills of quantitative and proportional reasoning—skills that are key to success on the MCATs. The TYU's use a multiple-choice format to elicit specific common misconceptions.
- **Problem-solving strategies** sections walk students through tactics for tackling particular types of problems—such as problems requiring Newton's second law or energy conservation—and follow the same 3-step global approach (set up, solve, and reflect).
- **Highly effective figures incorporate the latest ideas from educational research.** Color is used only for strict pedagogical purposes—for instance, in mechanics, **color is used to identify the object of interest,** while all other objects are gray. **Blue annotated comments** guide students in "reading" graphs and figures.
- **Visual chapter summaries** show each concept in words, math, and figures to reinforce how to "translate" between different representations and address different student learning styles.
- **Rich and diverse end-of-chapter problem sets.** *College Physics* features the renowned Sears/Zemansky problems, refined over five decades. We've used data from Mastering Physics to identify the strongest and most successful problems to retain for the tenth edition and we've added new problems. Multiple estimation questions were added to the Conceptual and Estimation Questions section of each chapter.
- Each chapter includes a set of **multiple-choice problems** that test the skills developed by the Test Your Understanding problems in the chapter text. The multiple-choice format elicits specific common misconceptions, enabling students to pinpoint their misunderstandings.
- The General Problems contain many **context-rich problems** that require students to simplify and model more complex real-world situations. Many problems relate to the fields of biology and medicine; these are all labeled BIO.
- **MCAT-style Passage Problems** appear in each chapter and follow the format used in the MCAT exam. These problems require students to investigate multiple aspects of a real-life physical situation, typically biological in nature, as described in a reading passage.

- **Connections of physics to the student's world.** Even more in-margin applications provide diverse, interesting, and self-contained examples of physics at work in the world. Many of these real-world applications are related to the fields of biology and medicine and are labeled BIO.
- **Writing that is easy to follow and rigorous.** The writing is friendly yet focused; it conveys an exact, careful, straightforward understanding of the physics, with an emphasis on the connections between concepts.

INSTRUCTOR SUPPLEMENTS

Note: *For convenience, all of the following instructor supplements can be accessed via Mastering Physics (www.masteringphysics.com).*

Instructor Solutions, prepared by A. Lewis Ford (Texas A&M University) and Brett Kraabel contain complete and detailed solutions to all end-of-chapter problems. All solutions follow consistently the same Set Up/Solve/Reflect problem-solving framework used in the textbook. Download only from the Mastering Physics Instructor Area or from the Instructor Resource Center (www.pearsonhighered.com/irc).

The **Instructor Resource** Collection, available on Mastering Physics, provides all line figures from the textbook in JPEG format. In addition, all the key equations, problem-solving strategies, tables, and chapter summaries are provided in editable Word format. Lecture outlines in PowerPoint are also included, along with over 70 PhET simulations as well as Video Tutor Demonstrations and Video Tutor Solutions.

Mastering Physics® (www.masteringphysics.com) is the most advanced, educationally effective, and widely used physics homework and tutorial system in the world. Eight years in development, it provides instructors with a library of extensively pre-tested end-of-chapter problems and rich, multipart, multistep tutorials that incorporate a wide variety of answer types, wrong answer feedback, individualized help (comprising hints or simpler sub-problems upon request), all driven by the largest metadatabase of student problem-solving in the world. NSF-sponsored published research (and subsequent studies) show that Mastering Physics has dramatic educational results. Mastering Physics allows instructors to build wide-ranging homework assignments of just the right difficulty and length and provides them with efficient tools to analyze both class trends and the work of any student in unprecedented detail.

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- **New–Enhanced End-of-Chapter Questions** provide instructional support when and where students need it including links to the eText, Video Tutor Solutions, math remediation and wrong-answer feedback for homework assignments.

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DETAILED CONTENTS


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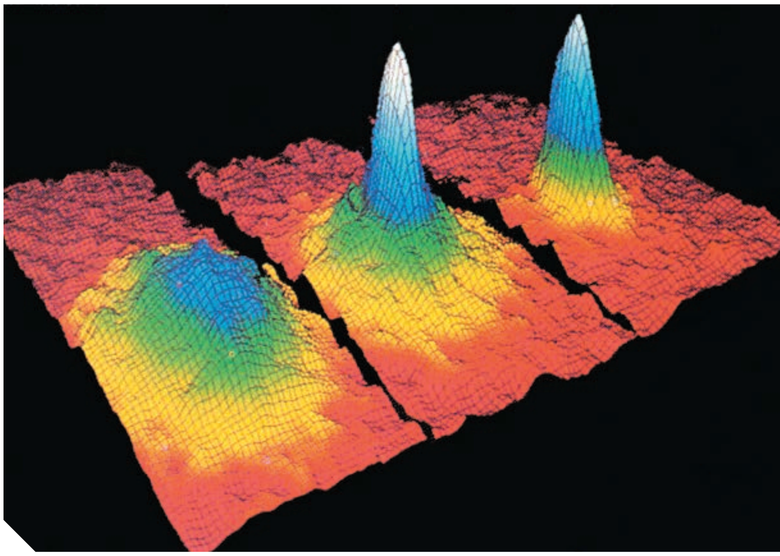


APPENDICES

The spiral arrangement of buds on this Romanesco broccoli plant is a classic example of how natural processes can give rise to geometrical patterns that can be expressed by means of mathematics. In this chapter, we will review the most important mathematical concepts used in this course.

0 Mathematics Review

Much of the natural world is arranged in patterns that can be described by means of fairly simple mathematics. Like all sciences that seek to explain the natural world, physics relies on a certain amount of mathematics to express its concepts in precise ways. In studying physics, then, you will need some basic math skills in order to understand lectures, read this textbook, and succeed with your homework and on exams. We strongly recommend that you review the material in this chapter and practice with the end-of-chapter problems before you read further. The beauty of physics cannot be fully appreciated if you do not have adequate mastery of basic mathematical skills.

0.1 EXPONENTS

Exponents are used frequently in physics—for example, when describing areas or three-dimensional space. When we write 3^4, the superscript 4 is called an **exponent** and the **base number** 3 is said to be raised to the fourth power. The quantity 3^4 is equal to $3 \times 3 \times 3 \times 3 = 81$. Algebraic symbols can also be raised to a power—for example, x^4. There are special names for the operation when the exponent is 2 or 3. When the exponent is 2, we say that the quantity is **squared;** thus, x^2 means x is squared. When the exponent is 3, the quantity is **cubed;** x^3 means x is cubed.

Note that $x^1 = x$ and that the exponent 1 is typically not written. Any quantity raised to the zero power is defined to be unity (that is, 1). Negative exponents are used for reciprocals: $x^{-4} = 1/x^4$.

An exponent can also be a fraction, as in $x^{1/4}$. The exponent $\frac{1}{2}$ is called a **square root,** and the exponent $\frac{1}{3}$ is called a **cube root.** For example, $\sqrt{6}$ can also be written as $6^{1/2}$.

Most calculators have special keys for calculating numbers raised to a power—for instance, a key labeled y^x or one labeled x^2.

Exponents obey several simple rules that follow directly from the meaning of raising a quantity to a power:

1. The product rule: $(x^m)(x^n) = x^{m+n}$.

   For example, $(3^3)(3^2) = 3^5 = 243$. To verify this result, note that $3^3 = 27$, $3^2 = 9$, and $(27)(9) = 243$.

LEARNING OUTCOMES

By the end of this chapter, you will be able to:

1. Use the rules for exponents to simplify algebraic expressions.
2. Express numbers in scientific notation and combine numbers in scientific notation by using addition, subtraction, multiplication, or division.
3. Use the quadratic formula to find both roots for a quadratic equation.
4. Solve a system of two equations with two unknown quantities.
5. Recognize direct, inverse, and inverse-square relationships either algebraically or graphically and solve such a relationship for an unknown quantity.
6. Use tables of data to create linear graphs, which can be used to solve for an unknown quantity.
7. Solve both base-10 logarithm and natural logarithm equations for an unknown quantity.
8. Use geometric expressions to solve for angles, lengths, areas, and volumes in a particular problem.
9. Use trigonometric identities to relate the angles and sides of a right triangle and the law of cosines and the law of sines to relate the angles and sides of any triangle.

2. The quotient rule: $\frac{x^m}{x^n} = x^{m-n}$.

   For example, $\frac{3^3}{3^2} = 3^{3-2} = 3^1 = 3$. To verify this result, note that $\frac{3^3}{3^2} = \frac{27}{9} = 3$.

   A special case of this rule is $\frac{x^m}{x^m} = x^{m-m} = x^0 = 1$.

3. The first power rule: $(x^m)^n = x^{mn}$.

   For example, $(2^2)^3 = 2^6 = 64$. To verify this result, note that $2^2 = 4$, so $(2^2)^3 = (4)^3 = 64$.

4. Other power rules:

$$(xy)^m = (x^m)(y^m) \quad \text{and} \quad \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}.$$

   For example, $(3 \times 2)^4 = 6^4 = 1296$. To verify the first result, note that $3^4 = 81$, $2^4 = 16$, and $(81)(16) = 1296$.

If the base number is negative, it is helpful to know that $(-x)^n = (-1)^n x^n$, and $(-1)^n$ is $+1$ if n is even and -1 if n is odd. You can verify easily the other power rules for any x and y.

EXAMPLE 0.1 Simplifying an exponential expression

Let's start by simplifying the expression $\frac{x^3y^{-3}xy^{4/3}}{x^{-4}y^{1/3}(x^2)^3}$ and calculating its numerical value when $x = 6$ and $y = 3$.

SOLUTION

SET UP AND SOLVE We simplify the expression as follows:

$$\frac{x^3x}{x^{-4}(x^2)^3} = x^3x^1x^4x^{-6} = x^{3+1+4-6} = x^2;$$

$$\frac{y^{-3}y^{4/3}}{y^{1/3}} = y^{-3+\frac{4}{3}-\frac{1}{3}} = y^{-2}.$$

Therefore,

$$\frac{x^3y^{-3}xy^{4/3}}{x^{-4}y^{1/3}(x^2)^3} = x^2y^{-2} = x^2\left(\frac{1}{y}\right)^2 = \left(\frac{x}{y}\right)^2.$$

For $x = 6$ and $y = 3$, $\left(\frac{x}{y}\right)^2 = \left(\frac{6}{3}\right)^2 = 4$.

If we evaluate the original expression directly, we obtain

$$\frac{x^3y^{-3}xy^{4/3}}{x^{-4}y^{1/3}(x^2)^3} = \frac{(6^3)(3^{-3})(6)(3^{4/3})}{(6^{-4})(3^{1/3})([6^2]^3)}$$

$$= \frac{(216)(1/27)(6)(4.33)}{(1/1296)(1.44)(46{,}656)} = 4.00,$$

which checks.

REFLECT This example demonstrates the usefulness of the rules for manipulating exponents. Often an intimidating-looking algebraic expression turns out to be quite simple once you have rearranged it.

EXAMPLE 0.2 Solving an exponential expression for the base number

If $x^4 = 81$, what is x?

SOLUTION

SET UP AND SOLVE We raise each side of the equation to the $\frac{1}{4}$ power: $(x^4)^{1/4} = (81)^{1/4}$. Then $(x^4)^{1/4} = x^1 = x$, so $x = (81)^{1/4}$ and $x = +3$ or $x = -3$. Either of these values of x gives $x^4 = 81$.

REFLECT Notice that we raised *both sides* of the equation to the $\frac{1}{4}$ power. As explained later in this chapter, an operation performed on both sides of an equation does not affect the equation's validity.

0.2 SCIENTIFIC NOTATION AND POWERS OF 10

In physics, we frequently encounter very large and very small numbers, and it is important to use the proper number of significant figures when expressing a physical quantity. Both of these issues are addressed by using **scientific notation,** in which a quantity is expressed as a decimal number with one digit to the left of the decimal point, multiplied by the appropriate power of 10. If the power of 10 is positive, it is the number of places the decimal point is moved to the right to obtain the fully written-out number—for example, $6.3 \times 10^4 = 63{,}000$. If the power of 10 is negative, it is the number of places the decimal point is moved to the left to obtain the fully written-out number—for example, $6.56 \times 10^{-3} = 0.00656$. In going from 6.56 to 0.00656, we move the decimal point three places to the left, so 10^{-3} is the correct power of 10 to use when the number is written in scientific notation. Most calculators have keys for expressing a number in either decimal (floating-point) or scientific notation.

When two numbers written in scientific notation are multiplied (or divided), multiply (or divide) the decimal parts to get the decimal part of the result, and multiply (or divide) the powers of 10 to get the power-of-10 portion of the result. You may have to adjust the location of the decimal point in the answer to express it in scientific notation. For example,

$$\begin{aligned}(8.43 \times 10^8)(2.21 \times 10^{-5}) &= (8.43 \times 2.21)(10^8 \times 10^{-5})\\ &= (18.6) \times (10^{8-5}) = 18.6 \times 10^3\\ &= 1.86 \times 10^4.\end{aligned}$$

Similarly,

$$\frac{5.6 \times 10^{-3}}{2.8 \times 10^{-6}} = \left(\frac{5.6}{2.8}\right) \times \left(\frac{10^{-3}}{10^{-6}}\right) = 2.0 \times 10^{-3-(-6)} = 2.0 \times 10^3.$$

Your calculator can handle these operations for you automatically, but it is important for you to develop good "number sense" for scientific notation manipulations.

When you are adding, subtracting, multiplying, or dividing numbers, keeping the proper number of significant figures is important. See Section 1.5 to review how to keep the proper number of significant figures in these cases.

0.3 ALGEBRA

Solving equations

Throughout your study of physics, you will encounter equations written with symbols that represent quantities. An **equation** consists of an equal sign and quantities to its left and to its right. Every equation tells us that the combination of quantities on the left of the equal sign has the same value as (that is, equals) the combination on the right of the equal sign. For example, the equation $y + 4 = x^2 + 8$ tells us that $y + 4$ has the same value as $x^2 + 8$. If $x = 3$, then the equation $y + 4 = x^2 + 8$ says that $y = 13$.

Often, one of the symbols in an equation is considered to be the *unknown,* and we wish to solve for the unknown in terms of the other quantities. For example, we might wish to solve the equation $2x^2 + 4 = 22$ for the value of x. Or we might wish to solve the equation $x = v_0 t + \frac{1}{2}at^2$ for the unknown a in terms of x, t, and v_0. Use the following rule to solve an equation: **An equation remains true if any valid operation performed on one side of the equation is also performed on the other side.** The operation could be adding or subtracting a number or symbol, multiplying or dividing by a number or symbol, or raising each side of the equation to the same power.

EXAMPLE 0.3 Solving a numerical equation

Here we will solve the simple equation $2x^2 + 4 = 22$ for x.

SOLUTION

SET UP AND SOLVE First we subtract 4 from both sides. This gives $2x^2 = 18$. Then we divide both sides by 2 to get $x^2 = 9$. Finally, we raise both sides of the equation to the $\frac{1}{2}$ power. (In other words, we take the square root of both sides of the equation.) This gives $x = \pm\sqrt{9} = \pm 3$; that is, $x = +3$ or $x = -3$. We can verify our answers by substituting our result back into the original equation: $2x^2 + 4 = 2(\pm 3)^2 + 4 = 2(9) + 4 = 18 + 4 = 22$, so $x = \pm 3$ does satisfy the equation.

REFLECT Notice that a square root always has *two* possible values, one positive and one negative. For instance, $\sqrt{4} = \pm 2$ because $(2)(2) = 4$ and $(-2)(-2) = 4$. A calculator will give you only a positive root; it's up to you to remember that there are actually two. Both roots are correct mathematically, but in a physics problem only one may represent the answer. For instance, if you can get dressed in $\sqrt{4}$ minutes, the only physically meaningful root is 2 minutes!

EXAMPLE 0.4 Solving a symbolic equation

Now let's solve for a parameter in a symbolic equation. Solve the equation $x = v_0 t + \frac{1}{2}at^2$ for a.

SOLUTION

SET UP AND SOLVE We subtract $v_0 t$ from both sides. This gives $x - v_0 t = \frac{1}{2}at^2$. Now we multiply both sides by 2 and divide both sides by t^2, giving

$$a = \frac{2(x - v_0 t)}{t^2}.$$

REFLECT As we've indicated, it makes no difference whether the quantities in an equation are represented by variables (such as x, v, and t) or by numerical values.

The quadratic formula

Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$ is called a **quadratic equation.** Each quadratic equation has two solutions. For certain types of quadratic equations, we can use the rules of algebra we have already discussed to find the two solutions. If $b \neq 0$ in a quadratic equation (meaning that the equation has no linear term), we can easily solve the equation $ax^2 + c = 0$ for x:

$$x = \pm\sqrt{\frac{-c}{a}}.$$

For example, if $a = 2$ and $c = -8$, the equation is $2x^2 - 8 = 0$ and the solution is

$$x = \pm\sqrt{\frac{-(-8)}{2}} = \pm\sqrt{4} = \pm 2.$$

The equation $ax^2 + bx = 0$ is also easily solved by factoring out an x on the left side of the equation, which gives $x(ax + b) = 0$. (To *factor out* a quantity means to isolate it so that the rest of the expression is either multiplied or divided by that quantity.) The equation $x(ax + b) = 0$ is true (that is, the left side equals zero) if either $x = 0$ or $x = -\frac{b}{a}$. These are the two solutions of the equation. For example, if $a = 2$ and $b = 8$, the equation is $2x^2 + 8x = 0$ and the solutions are $x = 0$ and $x = -\frac{8}{2} = -4$.

But if the equation is in the form $ax^2 + bx + c = 0$, with a, b, and c all nonzero, we cannot use our simple methods to solve for x. In this case, the easiest way to find the two solutions is to use the **quadratic formula:**

QUADRATIC FORMULA

For a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, the solutions are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Notes:

- In general, a quadratic equation has two roots (solutions), which may be real or complex numbers.
- If $b^2 = 4ac$, then the two roots are equal and real numbers.
- If $b^2 > 4ac$, that is, $b^2 - 4ac$ is positive, then the two roots are unequal and real numbers.
- If $b^2 < 4ac$, that is, $b^2 - 4ac$ is negative, then the roots are unequal complex numbers and cannot represent physical quantities. In that case, the quadratic equation has mathematical solutions but no physical solutions.

Throughout this text, we will make extensive use of the quadratic formula in analyzing the motion of freely falling objects.

EXAMPLE 0.5 Solving a quadratic equation

Let's apply the quadratic formula to a specific case. Find the values of x that satisfy the equation $2x^2 - 2x = 24$.

SOLUTION

SET UP AND SOLVE First we write the equation in the standard form $ax^2 + bx + c = 0$: $2x^2 - 2x - 24 = 0$. Then $a = 2$, $b = -2$, and $c = -24$. Next, the quadratic formula gives the two roots as

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-24)}}{(2)(2)}$$

$$= \frac{+2 \pm \sqrt{4 + 192}}{4} = \frac{2 \pm 14}{4},$$

so $x = 4$ or $x = -3$. If x represents a physical quantity that takes only nonnegative values, then the negative root $x = -3$ is nonphysical and is discarded.

REFLECT As we've mentioned, when an equation has more than one mathematical solution or root, it's up to *you* to decide whether one or the other or both represent the true physical answer. (If neither solution seems physically plausible, you should review your work.)

Simultaneous equations

If a problem has two unknowns—for example, x and y—then we need two independent equations in x and y (that is, two equations for x and y, where one equation is not simply a multiple of the other) to determine their values uniquely. Such equations are called **simultaneous equations** because we solve them together. A typical procedure is to solve one equation for x in terms of y and then substitute the result into the second equation to obtain an equation in which y is the only unknown. We then solve this equation for y and use the value of y in either of the original equations in order to solve for x. A pair of equations in which all quantities are symbols can be combined to eliminate one of the common unknowns. In general, to solve for n unknowns, we must have n independent equations. Simultaneous equations can also be solved **graphically** by plotting both equations using the same scale on the same graph paper. The solutions are the coordinates of the points of intersection of the graphs.

EXAMPLE 0.6 Solving two equations in two unknowns

Solve this pair of equations for x and y:

$$x + 4y = 14$$

$$3x - 5y = -9.$$

SOLUTION

SET UP AND SOLVE The first equation, once rearranged, gives $x = 14 - 4y$. Substituting this expression for x in the second equation yields, successively, $3(14 - 4y) - 5y = -9$, $42 - 12y - 5y = -9$, and $-17y = -51$. Thus, $y = \frac{-51}{-17} = 3$. Then $x = 14 - 4y = 14 - 12 = 2$. We can verify that $x = 2$, $y = 3$ satisfies both equations.

An alternative approach is to multiply the first equation by -3, which gives us $-3x - 12y = -42$. Adding this to the second equation gives, successively, $3x - 5y + (-3x) + (-12y) = -9 + (-42)$, $-17y = -51$, and $y = 3$, which agrees with our earlier result.

REFLECT As shown by the alternative approach, simultaneous equations can be solved in more than one way. The basic methods we describe are easy to keep straight; other methods may be quicker, but they may require more insight or forethought. Use the method you're comfortable with.

EXAMPLE 0.7 Solving two symbolic equations in two unknowns

We can also use the substitution technique to solve symbolic equations that have no numbers in them. Use the equations $v = v_0 + at$ and $x = v_0 t + \frac{1}{2}at^2$ to obtain an equation for x that does not contain a.

SOLUTION

SET UP AND SOLVE We solve the first equation for a:

$$a = \frac{v - v_0}{t}.$$

We substitute this expression into the second equation:

$$x = v_0 t + \frac{1}{2}\left(\frac{v - v_0}{t}\right)t^2 = v_0 t + \frac{1}{2}vt - \frac{1}{2}v_0 t$$

$$= \frac{1}{2}v_0 t + \frac{1}{2}vt = \left(\frac{v_0 + v}{2}\right)t.$$

REFLECT When you solve a physics problem, it's often best to work with symbols for all but the final step of the problem. Once you've arrived at the final equation, you can plug in numerical values and solve for an answer.

0.4 ALGEBRAIC RELATIONSHIPS AND PROPORTIONAL REASONING

The essence of physics is to describe and verify the relationships among physical quantities. The relationships are often simple. For example, two quantities may be directly proportional to each other, they may be inversely proportional to each other, or one quantity may be inversely proportional to the square of the other quantity.

Direct relationship

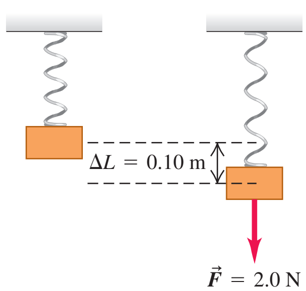


Figure 0.1

Two quantities are said to be **directly proportional** to each other if an increase (or decrease) of the first quantity causes an increase (or decrease) of the second quantity by the same factor. If y is directly proportional to x, the direct proportionality is written as $y \propto x$.The ratio y/x is a constant, say, k; that is, $\frac{y_1}{x_1} = \frac{y_2}{x_2} = k$. For example, the ratio of the circumference C to the diameter d of a circle is always π (pi), which we often approximate with the value 3.14. Therefore, the circumference of a circle is directly proportional to its diameter as $C = \pi d$, where π is the constant of proportionality. Another simple example of direct proportionality is the stretching or compression of an ordinary helical spring (discussed in Section 5.4). The spring has a certain length at rest, and that length increases, when a force F pulls on it, as shown in Figure 0.1. If the amount of stretch is not too great, the amount of force F, measured in newtons, on the spring and the amount of stretch ΔL are directly proportional to each other. (The newton, abbreviated N, is the SI unit for force. The Δ symbol means "change in," so in this case ΔL represents the change in length.) Thus, $F = k\Delta L$, where k is the constant of proportionality.

In general, in a direct proportion, $\frac{a}{b} = \frac{c}{d}$. Multiplying both sides by bd, we find

$$\cancel{b}d \cdot \frac{a}{\cancel{b}} = b\cancel{d} \cdot \frac{c}{\cancel{d}} \quad \text{or} \quad a \cdot d = b \cdot c.$$

Graph of direct proportionality relationship

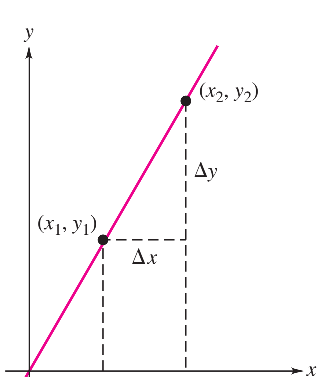


Figure 0.2

When y is directly proportional to x, $y = kx$, and the graph of y versus x is a straight line passing through the origin, as shown in Figure 0.2. In the graph, the change of the quantity x is labeled as Δx (which is often called "run"), and the corresponding change in y is labeled as Δy (which is often called "rise"). We have

$$\Delta x = x_2 - x_1$$
$$\Delta y = y_2 - y_1,$$

where (x_1, y_1) and (x_2, y_2) are coordinates of the two points on the line. The constant of proportionality between Δy and Δx is also k. Thus,

$$\Delta y = k\Delta x.$$

The steepness of the line is measured by the ratio $\Delta y/\Delta x$ and is called the *slope* of the line. Thus,

$$\text{Slope} = \frac{\Delta y}{\Delta x} = k.$$

The slope of a line can be *positive*, *negative*, *zero*, or *undefined*, as shown in Figure 0.3.

Note: Slope *positive* means *y increases as x increases*; slope *negative* means *y decreases as x increases*; slope *zero* means *y* does not change—that is, the line is *parallel to the x axis*; slope *undefined* means *x* does not change—that is, the line is *parallel to the y axis*.

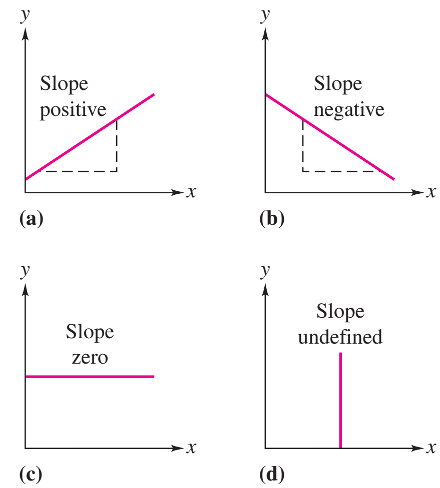


Figure **0.3**

EXAMPLE 0.8 Solving for a quantity in direct proportion

If y is directly proportional to x, and $x = 2$ when $y = 8$, what is y when $x = 10$?

SOLUTION

SET UP AND SOLVE Since y is directly proportional to x, we have $\frac{y_1}{x_1} = \frac{y_2}{x_2} = k$.

Substituting the values, we get $\frac{y}{10} = \frac{8}{2}$.

Multiplying both sides by 2 and 10 to get rid of the fractions gives $2y = 10 \cdot 8 = 80$.

Then we divide by 2 to isolate y: $y = \frac{80}{2} = 40$.

REFLECT This simple problem gives you the strategy for how to solve problems in direct proportion. Note that x has increased by a factor of 5, so y must also increase by the same factor.

EXAMPLE 0.9 Solving for the stiffness constant of a spring

Here we will use the concept of direct proportionality to analyze a spring system. Consider a spring that is suspended vertically from a fixed support. When a weight of 2.0 newtons is attached to the bottom of the spring, the spring stretches by 0.10 m. Determine the spring constant of the spring.

SOLUTION

SET UP AND SOLVE We have the force, $F = 2.0\text{ N}$, and the stretch, $\Delta L = 0.10\text{ m}$. The sketch of the problem is shown in Figure 0.1. The applied weight and the amount of stretch are related by a direct proportion expressed as $F = k\Delta L$. Using this equation, we can solve for the stiffness constant:

$$k = \frac{F}{\Delta L} = \frac{2.0\text{ N}}{0.10\text{ m}} = 20\text{ N/m}.$$

REFLECT The spring constant in this equation is the constant of proportionality. Its unit is the ratio of the units of F and ΔL.

Inverse proportion

When one quantity increases and a second quantity decreases in such a way that their product stays the same, they are said to be in **inverse proportion.** In inverse proportion, when one quantity approaches zero, the other quantity becomes extremely large, so that the product remains the same. For example, the product of the pressure and volume of an ideal gas remains constant if the temperature of the gas is maintained constant (as you will find in Section 15.2). Mathematically, if y is in inverse proportion to x, then $y \propto 1/x$. This gives $y = k/x$, or $xy = k$, where k is the constant of proportionality. That is, when x changes from x_1 to x_2, y changes from y_1 to y_2 so that $x_1y_1 = x_2y_2 = k$. This type of behavior is illustrated in Figure 0.4 (for an arbitrary choice of $k = 100$).

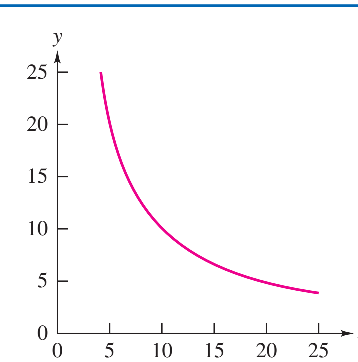


Figure **0.4**

EXAMPLE 0.10 Solving for a quantity (volume of an ideal gas) in inverse proportion

As we will learn (in Chapter 15), if the temperature of an ideal gas is kept constant, its pressure P is *inversely proportional* to its volume V. A cylindrical flask is fitted with an airtight piston and contains an ideal gas. Initially, the pressure of the inside gas is 11×10^4 pascals (Pa), and the volume of the gas is $8.0 \times 10^{-3}\ \text{m}^3$. Assuming that the system is always at the temperature of 330 kelvins (K), determine the volume of the gas when its pressure increases to 24×10^4 Pa.

SOLUTION

SET UP AND SOLVE Since the pressure P is inversely proportional to the volume V, the product PV remains constant; that is,

$$P_1V_1 = P_2V_2.$$

We divide by P_2 to solve for V_2: $V_2 = \dfrac{P_1V_1}{P_2}$.

In this problem, $P_1 = 11 \times 10^4$ Pa, $V_1 = 8.0 \times 10^{-3}\ \text{m}^3$, and $P_2 = 24 \times 10^4$ Pa. Substituting the values of P_1, V_1, and P_2, we solve for V_2:

$$V_2 = \frac{(11 \times 10^4\ \text{Pa}) \times (8.0 \times 10^{-3}\ \text{m}^3)}{24 \times 10^4\ \text{Pa}} = \frac{11 \times 8.0}{24} \times 10^{-3}\ \text{m}^3$$
$$= 3.7 \times 10^{-3}\ \text{m}^3.$$

REFLECT Since the pressure increased, the final volume has decreased, as we expect in an inverse proportion. Note that the pascal (Pa) and kelvin (K) are the SI units for pressure and temperature, respectively.

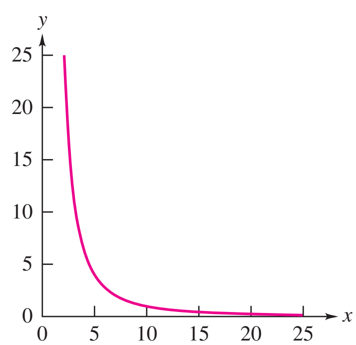


Figure 0.5

Inverse-square proportion

Inverse-square dependence is common in the laws of nature. For example, the force of gravity due to a particle decreases as the inverse square of the distance from the particle (as expressed by Newton's law of gravitation in Section 6.3). Similarly, the electrostatic force due to a point electric charge decreases as the square of the distance from the charge (as expressed by Coulomb's law in Section 17.4). The intensity of sound and of light also decreases as the inverse square of the distance from a point source (as you will find in Section 12.10). (Intensity in these cases is a measure of the power of the sound or light per unit area.)

Mathematically, if y varies inversely with the square of x, then

$$y \propto \frac{1}{x^2} \quad \text{or} \quad y = \frac{k}{x^2} \quad \text{or} \quad x^2y = k,$$

where k is the constant of proportionality. That is, when x changes from x_1 to x_2, y changes from y_1 to y_2 so that $x_1^2y_1 = x_2^2y_2 = k$. Or

$$\frac{y_1}{y_2} = \frac{x_2^2}{x_1^2}.$$

This relationship is illustrated in Figure 0.5 (for an arbitrary choice of $k = 100$).

EXAMPLE 0.11 Solving for a quantity (sound intensity) that varies as the inverse square

As we will discover (in Chapter 12), if a source of sound emits uniformly in all directions, then the intensity I of the emitted sound at a distance r from the source is given by the equation $I = k/r^2$, where k is a constant of proportionality. If the sound intensity is 0.05 watt/meter^2 (W/m^2) at a distance of 0.5 m from the source, find the sound intensity at a distance of 20 m from the source.

SOLUTION

SET UP AND SOLVE The intensity of sound is given by the equation $I = k/r^2$. We apply this equation to solve the problem. However, we do not need to know the value of the constant k. In this problem, we have the initial distance, $r_1 = 0.5$ m, sound intensity, $I_1 = 0.05\ \text{W/m}^2$, and final distance, $r_2 = 20$ m; the intensity I_2 is to be determined.

From the inverse-square relationship, we have (where the variables x and y have been replaced by r and I in the equation)

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}.$$

We take the reciprocal of both sides and multiply by I_1 to solve for I_2:

$$I_2 = I_1\frac{r_1^2}{r_2^2} = (0.05\ \text{W/m}^2)\frac{(0.5\ \text{m})^2}{(20\ \text{m})^2}$$
$$= 3.1 \times 10^{-5}\ \text{W/m}^2.$$

REFLECT As the distance increases, the intensity decreases. Note that because the intensity decreases as the square of the distance, the result for intensity is less than it would have been in the case of a simple inverse proportion.

0.5 DATA-DRIVEN PROBLEMS

Physics is an experimental science. In much of the current research in physics, sophisticated instruments are used to take precise measurements. But we can also learn a lot about the way the world works by conducting simple experiments. These experiments provide data that can be analyzed to determine how two variables relate to each other as well as to calculate some unknown physical quantity. Whether we measure the period of a pendulum with a stopwatch or the speed of a spinning disk as it slows to a stop with a tachometer, we can use the data we gather to verify the expressions relating two physical quantities that we will encounter throughout the text.

So, what can we do with our data once we have gathered them? First, we can organize data into a table. Often, this allows us to notice any obvious patterns, such as whether the values in one column increase by a constant amount when we increase the values in the other column in a systematic way. Second, we can plot our data, putting the values from one column in our table on the x axis and values from the other on the y axis. Using a graph is often the best way to identify the particular relationship between the two quantities we have measured. In the preceding sections, we have seen what the graphs look like for the most common mathematical relationships, such as when two variables are linearly related or inversely related. A graph will often show us how two variables are related to each other. We can then compare this relationship to the predicted relationship as given by an equation that we have derived in the text.

Often we can create a linear plot with our data from which we can determine some unknown value. For example, we will learn (in Chapter 14) that if we heat an object, its length increases. The specific increase in length, ΔL, is directly proportional to the increase in temperature ΔT. As a simple experiment, we can measure the length of the object at different temperatures. Table 0.1 shows the calculated temperature changes and the corresponding changes in the length of the object that we determine from our measurements. The data from the table are plotted in the ΔL-versus-ΔT graph in Figure 0.6. As we can see, because our two variables are directly proportional, the data in the plot lie along a straight line. The slope of this line has a physical meaning and is related to the coefficient of thermal expansion of the material.

If our variables are related in some other way, however, then we need to create a new plot so that our data still lie along a straight line. For example, we will learn (in Chapter 12) that the frequency f of a wave traveling through a medium is inversely proportional to the wavelength of the wave (which is denoted by the Greek letter lambda λ). The frequency and wavelength measurements listed in Table 0.2 are plotted in Figure 0.7a, where we have put the frequencies on the x axis and the wavelengths on the y axis. Notice that the shape of the plot is the same as the shape in Figure 0.4, where x and y are inversely proportional. To make a linear plot, we need to change our plot so that the inverse of the frequency (or $1/f$) is on the x axis. As we can see in Figure 0.7b, when we make a plot of λ versus $1/f$, the data lie along a straight line. This process is known as *linearizing* the data. Again, the slope of the line in Figure 0.7b has a physical meaning; in this case, it represents the speed of the wave.

Once we have linearized the data, we can find the best-fit line and use it to determine the slope and y-intercept. The slope of the best-fit line for the data can often be used to

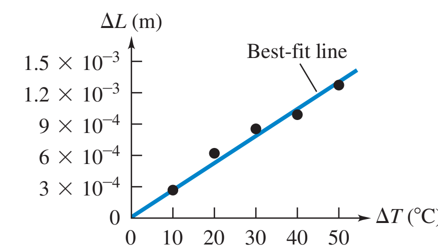
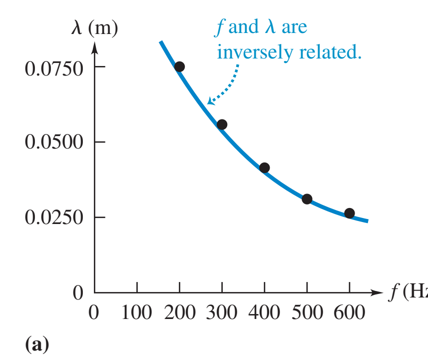


Figure **0.6**



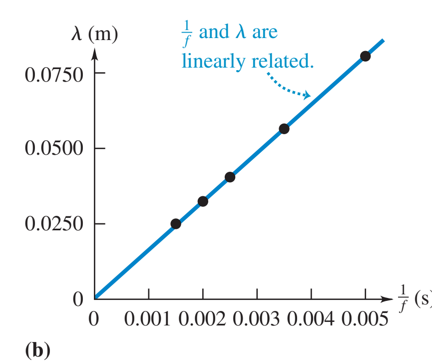


Figure **0.7**

TABLE 0.1 Data from temperature and length measurements

ΔT (°C)	ΔL (m)
10.0	2.57×10^{-4}
20.0	5.09×10^{-4}
30.0	7.68×10^{-4}
40.0	1.03×10^{-3}
50.0	1.28×10^{-3}

TABLE 0.2 Data from frequency and wavelength measurements

Frequency (Hz)	Wavelength (m)
200	0.0750
300	0.0500
400	0.0375
500	0.0300
600	0.0250

calculate other physical parameters, many of which might be difficult or even impossible to measure directly. In our first example, if we know the length of the solid object at our initial temperature, then we can use the slope of the best-fit line to calculate the coefficient of linear expansion for the object. Similarly, the slope of the linearized data in our second example tells us the speed of the wave as it travels through the medium. Fortunately, this process of using plots created from relatively simple measurements is powerful and can be used across the entire breadth of introductory physics. Let's look at two simple examples of an object in motion to see how this process works.

EXAMPLE 0.12 Going for a run

Suppose you step out your front door to go for a jog. You are wearing a stopwatch and an appropriately calibrated pedometer, so you can keep track of both how long and how far you run. You walk for the first minute and record the readings of the pedometer every 15 seconds. At the 1-minute mark, you start jogging and then record the pedometer readings every 15 seconds for the next minute as well. Table 0.3 lists both the time and distance measurements for the 2 minutes during which you collected data. What was your average speed during each minute?

TABLE 0.3 Data for Example 0.12

Elapsed time (s)	Distance from starting point (m)
15.0	22.5
30.0	45.0
45.0	67.5
60.0	90.0
75.0	127.5
90.0	165.0
105.0	202.5
120.0	240.0

SOLUTION

SET UP For an object moving at a constant speed, the distance the object travels is related to its speed by $d = vt$, where d is the distance from the starting point, t is the time elapsed, and v is the object's speed. We can compare the equation $d = vt$ to the equation of a line, $y = mx + b$. Then we recognize that if we put t on the x axis and d on the y axis of the graph, the slope of the best-fit line must be equal to your speed. Figure 0.8 shows the graph of our data, with time on the x axis and the distance traveled from the starting point on the y axis. As we can see from the graph, the data don't follow a single line over the entire 2-minute span, but rather are broken up into two line segments, one over the first minute and the other over the second.

SOLVE To calculate the speed you traveled during the first minute, we need to start by calculating the slope of the line segment between $t = 0$ s and $t = 60$ s. Because all of the data points lie along the best-fit line, we could use any two points during the first minute to compute the slope. If we use the data at $t = 0$ s and $t = 60$ s, we calculate the slope during the first minute as

$$m_1 = \frac{\Delta y}{\Delta x} = \frac{90\ \text{m} - 0\ \text{m}}{60\ \text{s} - 0\ \text{s}} = 1.5\ \text{m/s}.$$

Repeating the process for the second minute with the data at $t = 60$ s and $t = 120$ s, we see that the slope during the second minute is

$$m_2 = \frac{240\ \text{m} - 90\ \text{m}}{120\ \text{s} - 60\ \text{s}} = 2.5\ \text{m/s}.$$

Therefore, your speed was 1.5 m/s during the first minute and 2.5 m/s during the second minute.

REFLECT Looking at the plot of the data, we see that the slope of the line segment over the second minute is clearly steeper than the slope of the line segment over the first minute. This is the graphical way of representing the fact that you traveled with a greater speed when you were jogging than when you were walking.

Because it is such a common source of error for students, it is worth pointing out that *we always need to use two points that lie along a line to calculate its slope.* Simply dividing the y value of a single data point by the x value may seem to get you the correct answer for the first minute, but that is only because you hadn't traveled any distance yet when you started your watch at $t = 0$ s. Notice that just dividing a distance by a time will not give you the correct answer during the second minute.

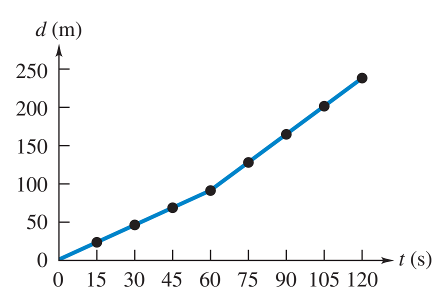


Figure 0.8

EXAMPLE 0.13 Rolling a marble down a track

You release a marble from the top of a long, straight track as shown in Figure 0.9. You have set up a digital camera that will take a picture every 0.50 second starting at the instant you release the marble. You can then use the pictures to measure how far down the track the marble has rolled in half-second increments. Your measurements for the time of each picture and the distances the marble had rolled are given in Table 0.4. What is the acceleration of the marble as it rolls down the track?

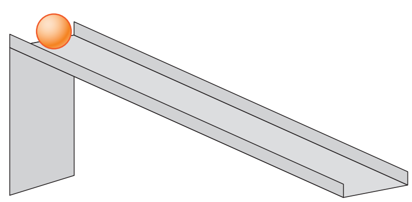


Figure **0.9**

TABLE 0.4 Data for Example 0.13

Time (s)	Distance from release point (m)
0.50	0.0409
1.00	0.154
1.50	0.334
2.00	0.597
2.50	0.935

SOLUTION

SET UP As we will discuss when we begin using Newton's laws of motion, the acceleration of the marble must be constant as it rolls down the track because the track is straight rather than curved. For an object that experiences a constant acceleration and starts from rest, the distance d it travels as a function of time t is given by $d = \frac{1}{2}at^2$, where a is the object's acceleration. Figure 0.10a shows a plot of distance versus time for the data in Table 0.4. As we can see, the graph is nonlinear. To linearize the data, we need to create a plot of distance versus time squared, as shown in Figure 0.10b. Then we can compare $d = \frac{1}{2}at^2$ to $y = mx + b$. We see that if we put t^2 on the x axis and d on the y axis, then the slope of the best-fit line is equal to $\frac{1}{2}a$.

SOLVE The slope of the best-fit line shown in Figure 0.10b is $m = 0.149 \text{ m/s}^2$. (Because the data do not lie exactly on the best-fit line, it is often convenient to use a plotting program to calculate the slope of the best-fit line.) Using $m = \frac{1}{2}a$, we calculate the acceleration of the marble to be $a = 2m = 0.298 \text{ m/s}^2$.

REFLECT Being able to compare a physics equation to $y = mx + b$ is a useful skill when dealing with data. The comparison helps us determine which graph we need to make to linearize the data in addition to which expression from the physics equation is equal to the value of the slope or y-intercept.

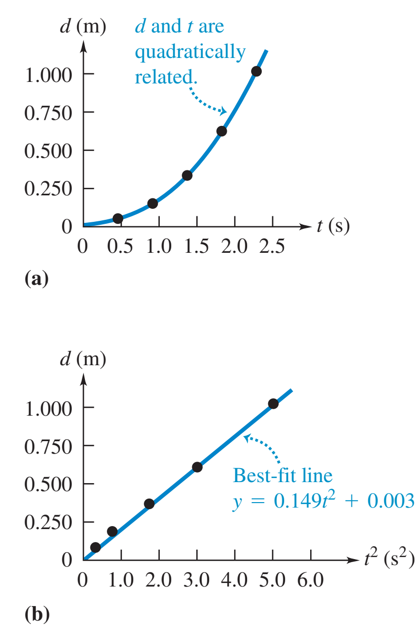


Figure **0.10**

0.6 LOGARITHMIC AND EXPONENTIAL FUNCTIONS

Exponential functions and logarithms are used for many natural phenomena (such as radioactive decay) where the rate of increase or decrease of a quantity is proportional to the current value of that quantity. We also work with exponential growth and decay in the study of electric circuits.

We will encounter two types of logarithms: the common logarithm and the natural logarithm. The base-10 logarithm, or **common logarithm** (log), of a number y is the power to which 10 must be raised to obtain y: $y = 10^{\log y}$. For example, $1000 = 10^3$, so $\log(1000) = 3$; we must raise 10 to the power 3 to obtain 1000. Most calculators have a key for calculating the log of a number.

Sometimes we are given the log of a number and asked to find the number. That is, if $\log y = x$ and x is given, what is y? To solve for y, we write an equation in which 10 is raised to the power equal to either side of the original equation: $10^{\log y} = 10^x$.

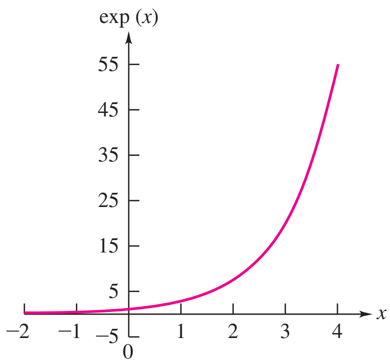


Figure **0.11**

But $10^{\log y} = y$, so $y = 10^x$. In this case, y is called the **antilog** of x. For example, if $\log y = -2.0$, then $y = 10^{-2.0} = 1.0 \times 10^{-2} = 0.010$.

The log of a number is positive if the number is greater than 1. The log of a number is negative if the number is less than 1 but greater than zero. The log of zero or of a negative number is not defined, and $\log 1 = 0$.

Another base that occurs frequently in physics is the quantity $e = 2.718\ldots$. The **natural logarithm** (ln) of a number y is the power to which e must be raised to obtain y: $y = e^{\ln y}$. If $x = \ln y$, then $y = e^x$, which is called an **exponential function**, also written as exp (x). Most calculators have keys for $\ln x$ and for e^x. For example, $\ln 10.0 = 2.30$, and if $\ln x = 3.00$, then $x = e^{3.00} = 20.1$. Note that $\ln 1 = 0$. A plot of the function $y = e^x$ is shown in Figure 0.11. If we were to plot $y = e^{-x}$, it would be a curve that is a reflection of Figure 0.11 about the y axis.

Logarithms with any choice of base, including base 10 or base e, obey several simple and useful rules:

1. $\log(ab) = \log a + \log b$. 2. $\log\left(\dfrac{a}{b}\right) = \log a - \log b$. 3. $\log(a^n) = n \log a$.

A particular example of the second rule is (because $\log 1 = 0$)

$$\log\left(\frac{1}{a}\right) = \log 1 - \log a = -\log a.$$

EXAMPLE 0.14 Solving a logarithmic equation

Now let's apply the logarithmic rules to the equation $\frac{1}{2} = e^{-\alpha T}$ and solve for T in terms of α.

SOLUTION

SET UP AND SOLVE We take the natural logarithm of both sides of the equation: $\ln(\frac{1}{2}) = -\ln 2$ and $\ln(e^{-\alpha T}) = -\alpha T$. The equation thus becomes $-\alpha T = -\ln 2$, and it follows that $T = \dfrac{\ln 2}{\alpha}$.

REFLECT The equation $y = e^{\alpha x}$ expresses y in terms of the exponential function $e^{\alpha x}$. The general rules for exponents in Section 0.1 apply when the base is e, so $e^x e^y = e^{x+y}$, $e^x e^{-x} = e^{x+(-x)} = e^0 = 1$, and $(e^x)^2 = e^{2x}$.

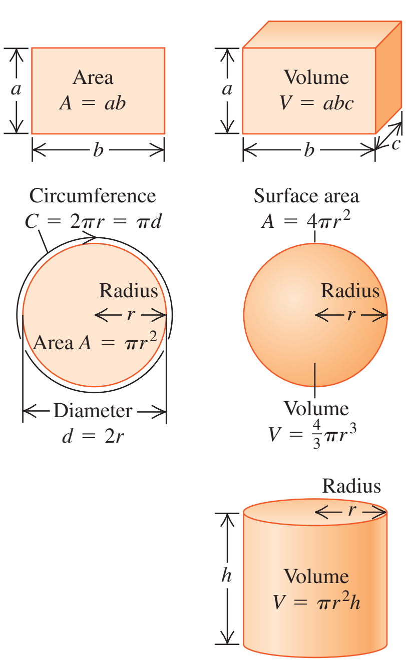


Figure **0.12**

0.7 AREAS AND VOLUMES

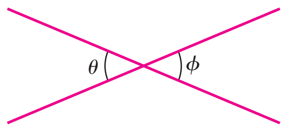
Figure 0.12 illustrates formulas for the areas and volumes of common geometric shapes:

- A rectangle with length a and width b has area $A = ab$.
- A rectangular solid (a box) with length a, width b, and height c has volume $V = abc$.
- A circle with radius r has diameter $d = 2r$, circumference $C = 2\pi r = \pi d$, and area $A = \pi r^2 = \pi d^2/4$.
- A sphere with radius r has surface area $A = 4\pi r^2$ and volume $V = \frac{4}{3}\pi r^3$.
- A cylinder with radius r and height h has volume $V = \pi r^2 h$.

0.8 PLANE GEOMETRY AND TRIGONOMETRY

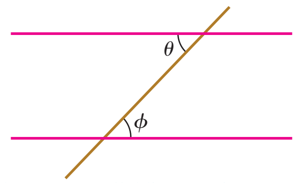
We present some useful results about angles:

1. Interior angles that are formed when two straight lines intersect are equal. For example, in Figure 0.13, the two angles θ and ϕ are equal.
2. When two parallel lines are intersected by a diagonal straight line, the alternate interior angles are equal. For example, in Figure 0.14, the two angles θ and ϕ are equal.
3. When the sides of one angle are each perpendicular to the corresponding sides of a second angle, the two angles are equal. For example, in Figure 0.15, the two angles θ and ϕ are equal.
4. The sum of the angles on one side of a straight line is 180°. In Figure 0.16, $\theta + \phi = 180°$.
5. The sum of the angles in any triangle is 180°.



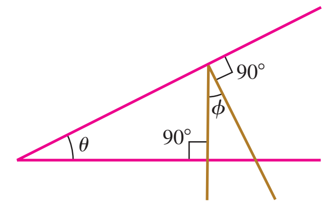
Interior angles formed when two straight lines intersect are equal: $\theta = \phi$

Figure **0.13**



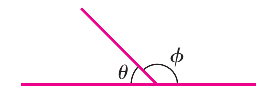
When two parallel lines are intersected by a diagonal straight line, the alternate interior angles are equal: $\theta = \phi$

Figure **0.14**



When the sides of one angle are each perpendicular to the corresponding sides of a second angle, the two angles are equal: $\theta = \phi$

Figure **0.15**



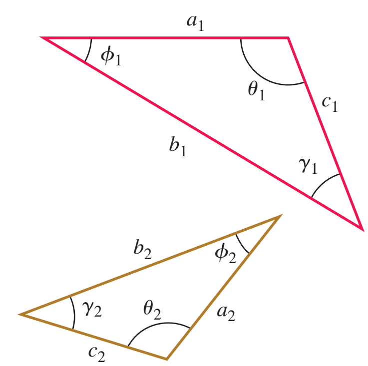
The sum of the angles on one side of a straight line is 180°: $\theta + \phi = 180°$

Figure **0.16**

Similar triangles

Triangles are **similar** if they have the same shape but different sizes or orientations. Similar triangles have equal corresponding angles and equal ratios of corresponding sides. If the two triangles in Figure 0.17 are similar, then $\theta_1 = \theta_2$, $\phi_1 = \phi_2$, $\gamma_1 = \gamma_2$, and $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$.

If two similar triangles are the same size, they are said to be **congruent.** If triangles are congruent, one can be flipped and rotated so that it can be placed precisely on top of the other.



Two similar triangles: Same shape but not necessarily the same size

Figure **0.17**

Right triangles and trig functions

In a **right triangle,** one angle is 90°. Therefore, the other two acute angles ("acute" means less than 90°) have a sum of 90°. In Figure 0.18, $\theta + \phi = 90°$. The side opposite the right angle is called the **hypotenuse** (side c in the figure). In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides. For the triangle in Figure 0.18, $c^2 = a^2 + b^2$. This formula is called the **Pythagorean theorem.**

If two right triangles have the same value for one acute angle, then the two triangles are similar and have the same ratio of corresponding sides. This statement allows us to define the functions **sine, cosine,** and **tangent,** which are ratios of a pair of sides. These functions, called **trigonometric functions** or **trig functions,** depend on only one of the angles in the right triangle. For an angle θ, these functions are written $\sin\theta$, $\cos\theta$, and $\tan\theta$.

In terms of the triangle in Figure 0.18, the sine, cosine, and tangent of the angle θ are:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c},$$

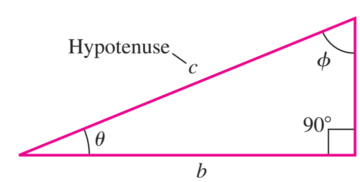
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}, \quad \text{and}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}.$$

Note that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. For angle ϕ, $\sin\phi = \dfrac{b}{c}$, $\cos\phi = \dfrac{a}{c}$, and $\tan\phi = \dfrac{b}{a}$.

In physics, angles are expressed in either degrees or radians, where π radians $= 180°$. (For more on radians, see Section 9.1.) Most calculators have a key for switching between degrees and radians. Always be sure that your calculator is set to the appropriate angular measure.

Inverse trig functions—denoted, for example, by $\sin^{-1}x$ (or arcsin x)—have a value equal to the angle that has the value x for the trig function. For example, $\sin 30° = 0.500$, so $\sin^{-1}(0.500) = \arcsin(0.500) = 30°$. Note that $\sin^{-1}x$ does *not* mean $\dfrac{1}{\sin x}$. Also, note that when you determine an angle using inverse trigonometric functions, the



For a right triangle:
$\theta + \phi = 90°$
$c^2 = a^2 + b^2$ (Pythagorean theorem)

Figure **0.18**

calculator always gives you the smallest correct angle, which may or may not be the right answer. Use the knowledge of which quadrant you are working in to determine the correct angle in the situation.

In the next two examples, we apply the trigonometric functions to problems involving right triangles. It is important that you feel comfortable with these examples because we use similar techniques throughout the text when dealing with vector quantities.

EXAMPLE 0.15 Using trigonometry I

A right triangle has one angle of 30° and one side with length 8.0 cm, as shown in Figure 0.19. What is the angle ϕ, and what are the lengths x and y of the other two sides of the triangle?

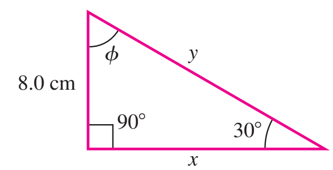


Figure **0.19**

SOLUTION

SET UP AND SOLVE We have $\phi + 30° = 90°$, so $\phi = 60°$. Then $\tan 30° = \dfrac{8.0\text{ cm}}{x}$, so $x = \dfrac{8.0\text{ cm}}{\tan 30^\circ} = 13.9\text{ cm}$. To find y, we use the Pythagorean theorem: $y^2 = (8.0\text{ cm})^2 + (13.9\text{ cm})^2$, so $y = 16.0\text{ cm}$.

Or we can say $\sin 30° = 8.0\text{ cm}/y$, so $y = 8.0\text{ cm}/\sin 30^\circ = 16\text{ cm}$, which agrees with the earlier result.

REFLECT Notice how we used the Pythagorean theorem in combination with a trig function. You will use these tools constantly in physics, so make sure that you can employ them with confidence.

EXAMPLE 0.16 Using trigonometry II

A right triangle has two sides with lengths as shown in Figure 0.20. What is the length x of the third side of the triangle, and what is the angle θ in degrees?

SOLUTION

SET UP AND SOLVE The Pythagorean theorem applied to this right triangle gives $(3.0\text{ m})^2 + x^2 = (5.0\text{ m})^2$, so $x = \sqrt{(5.0\text{ m})^2 - (3.0\text{ m})^2} = 4.0\text{ m}$. (Since x is a length, we take the positive root of the equation.) We also have

$$\cos\theta = \frac{3.0\text{ m}}{5.0\text{ m}} = 0.60, \text{ so } \theta = \cos^{-1}(0.60) = 53°.$$

REFLECT In this case, we knew the lengths of two sides but none of the acute angles, so we used the Pythagorean theorem first and then an appropriate trig function.

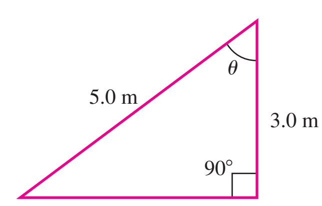


Figure **0.20**

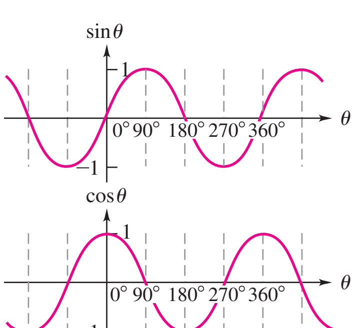


Figure **0.21**

In a right triangle, all angles are in the range from 0° to 90°, and the sine, cosine, and tangent of the angles are all positive. This must be the case, since the trig functions are ratios of lengths. But for other applications, such as finding the components of vectors, calculating the oscillatory motion of a mass on a spring, or describing wave motion, it is useful to define the sine, cosine, and tangent for angles outside that range. Graphs of $\sin\theta$ and $\cos\theta$ are given in Figure 0.21. The values of $\sin\theta$ and $\cos\theta$ vary between +1 and −1. Each function is periodic, with a period of 360°. Note the range of angles between 0° and 360° for which each function is positive and negative. The two functions $\sin\theta$ and $\cos\theta$ are shifted 90° relative to each other. When one is zero, the other has its maximum magnitude (i.e., its maximum or minimum value).

For any triangle (see Figure 0.22)—in other words, not necessarily a right triangle—these two relationships apply:

1. $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$ (law of sines).
2. $c^2 = a^2 + b^2 - 2ab\cos\gamma$ (law of cosines).

Some of the relationships among trig functions are called trig identities. The following list includes many of the most useful trig identities in introductory physics:

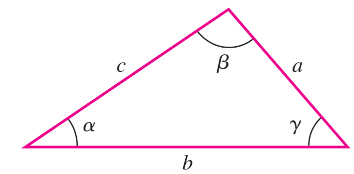


Figure 0.22

Useful trigonometric identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\sin(-\theta) = -\sin(\theta)$$
$$\cos(-\theta) = \cos(\theta)$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$
$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$
$$\sin(180° - \theta) = \sin\theta$$
$$\cos(180° - \theta) = -\cos\theta$$
$$\sin(90° - \theta) = \cos\theta$$
$$\cos(90° - \theta) = \sin\theta$$

For small angle θ (in radians),

$$\cos\theta \approx 1 - \frac{\theta^2}{2} \approx 1$$
$$\sin\theta \approx \theta$$

PROBLEMS

0.1 Exponents

Use the exponent rules to simplify the following expressions:

1. $(-3x^4y^2)^2$
2. $\frac{(2^3 4^4)^2}{(8)^4}$
3. $\left(\frac{8x^3y^2}{2y^5}\right)^2$
4. $\left(-\frac{x^{-4}y^{-4}}{x^2y^{-2}}\right)^5$

0.2 Scientific Notation and Powers of 10

Express the following expressions in scientific notation:

5. 475000
6. 0.00000472
7. 123×10^{-6}
8. $\frac{8.3 \times 10^5}{7.8 \times 10^2}$

0.3 Algebra

Solve the following equations using any method:

9. $4x + 6 = 9x - 14$
10. $E = mc^2$, solve for c in terms of E and m.
11. $4x^2 + 6 = 3x^2 + 18$
12. $-196 = -9.8t^2$
13. $x^2 - 5x + 6 = 0$
14. $(x - 5)(x + 3) = 0$
15. $4.9t^2 + 2t - 20 = 0$
16. $5x - 4y = 1, 6y = 10x - 4$
17. $2x + 3 = 5y, \frac{2}{3}y - 1 = -4x$

0.4 Algebraic Relationships and Proportional Reasoning

18. If x is proportional to y and $x = 2$ when $y = 10$, what is the value of x when $y = 8$?
19. The gravitational force F on an object is directly proportional to the object's mass m. If the force on an object with $m = 1$ kg is $F = 9.8$ N, what is the force on an object with $m = 2.8$ kg?
20. According to the ideal-gas law (Section 15.2), the volume of an ideal gas is directly proportional to its temperature in kelvins (K) if the pressure of the gas is constant. An ideal gas occupies a volume of 4.0 liters at 100 K. Determine its volume when it is heated to 300 K while held at a constant pressure.
21. For a sound coming from a point source, the amplitude of sound is inversely proportional to the distance. If the displacement amplitude of an air molecule in a sound wave is 4.8×10^{-6} m at a point 1.0 m from the source, what would be the displacement amplitude of the same sound when the distance increases to 4.0 m?
22. If an object is moving at a constant speed v, then the time the object takes to traverse a distance d is inversely proportional to its speed v. If it takes a car 1 h to travel a distance of 60 mi, how long will it take the car to travel the same distance if it slows down to one-third of its original speed?
23. The force of gravity on an object (which we experience as the object's weight) varies inversely as the square of the distance from the center of the earth. Determine the force of gravity on an astronaut when he is at a height of 6000 km from the surface of the earth if he weighs 700 newtons (N) when on the surface of the earth. The radius of the earth is 6.38×10^6 m. (If the astronaut is in orbit, he will float "weightlessly," but gravity still acts on him—he and his spaceship appear weightless because they are falling freely in their orbit around the earth.)

0.5 Data-Driven Problems

24. The data in Table 0.5 are expected to obey the relationship $y = kx^n$, where k is a positive constant and n is a positive integer. Make several plots of the y values from the table as a function of x^n, where x represents the corresponding x values and n is an integer of your choosing. Find the value of n that produces a linearized plot. From your linearized plot, determine the value of k.

TABLE 0.5

x value	y value
0.40	0.125
0.80	0.512
1.2	1.73
1.4	2.75
1.8	5.83

25. You are trying to determine whether a produce scale is properly calibrated. To do this, you put known weights on the scale and record the scale's reading for each. Your results are given in Table 0.6. Make a plot with the readings on the y axis and the weights on the x axis. Are the scale readings accurate? If not, describe in what way they are inaccurate.

TABLE 0.6

Weight (lb)	Reading (lb)
0.5	0.7
1.0	1.2
1.5	1.7
2.0	2.2

0.6 Logarithmic and Exponential Functions

26. Use the properties of logarithms and write each expression in terms of logarithms of x, y, and z.

a. $\log(x^4y^2z^8)$ b. $\log\sqrt{x^3y^7}$ c. $\log\sqrt{\frac{x^2y^6}{z^3}}$

27. Simplify the expression.

a. $4\log x + \log y - 3\log(x + y)$

b. $\log(xy + x^2) - \log(xz + yz) + 2\log z$

28. If $\beta = 10\log\left(\frac{I}{10^{-12}}\right)$, find β when $I = 10^{-4}$.

29. If $40 = 10\log\left(\frac{5}{r^2}\right)$, solve for r.

30. $N = N_0e^{-(0.210)t}$. If $N_0 = 2.00 \times 10^6$, solve for t when $N = 2.50 \times 10^4$.

0.7 Areas and Volumes

31. (a) Compute the circumference and area of a circle of radius 0.12 m. (b) Compute the surface area and volume of a sphere of radius 0.21 m. (c) Compute the total surface area and volume of a rectangular solid of length 0.18 m, width 0.15 m, and height 0.8 m. (d) Compute the total surface area and volume of a cylinder of radius 0.18 m and height 0.33 m.

0.8 Plane Geometry and Trigonometry

32. A right triangle has a hypotenuse of length 20 cm and another side of length 16 cm. Determine the third side of the triangle and the other two angles of the triangle.
33. In a stairway, each step is set back 30 cm from the next lower step. If the stairway rises at an angle of 36° with the horizontal, what is the height of each step?
34. A ladder is leaning against a building. The ladder has a length of 3 m, and the foot of the ladder is 0.5 m from the base of the building. (a) What angle does the ladder make with the horizontal? (b) How far above ground level is the top of the ladder?
35. A right triangle has a height of 1 m and a base of 2 m. Find the hypotenuse and all of the angles of the triangle.

In the game of pool, a player first strikes the cue ball, which then rolls to impact the other balls on the table. Although the collective motion of the pool balls can be quite complex, it is, in fact, governed by the laws of physics. It is the task of the player to adjust the speed and direction of the cue ball in order to obtain the desired result, which in this case is to sink a specific ball in one of the pockets.

1 Models, Measurements, and Vectors

The study of physics is an adventure, one that can be challenging and sometimes frustrating, but it is ultimately richly rewarding and satisfying. Studying physics will appeal to both your sense of beauty and your rational intelligence. Physics explains how the *aurora borealis* lights up the northern skies as well as how a marble rolls down an inclined plane. Both of these phenomena—one exotic and almost magical, the other mundane and commonplace—are governed by the laws of physics.

Physics attempts to describe the world around us in terms of rigorous mathematical models. The ultimate goal is not only to understand the physical world but also to develop the tools necessary to predict its behavior. Our present understanding of the physical world has been built on foundations laid by scientific giants such as Galileo, Newton, and Einstein. You can share some of the excitement of their historic discoveries when you learn to use physics to solve practical problems and to gain insight into everyday phenomena.

In this opening chapter, we go over some important preliminaries that we'll need throughout our study. We start with the philosophical framework in which we operate—the nature of physical theory and the role of idealized *models* in representing physical systems. We then discuss systems of *units* that are used to describe physical quantities, such as length and time, and we examine the *precision* of a number, often described by means of *significant figures.* We look at examples of problems for which we can't or don't want to make precise calculations, but for which rough estimates can be interesting and useful. Finally, we use *vectors* to describe and analyze many physical quantities, such as velocity and force, that have direction as well as magnitude.

LEARNING OUTCOMES

By the end of this chapter, you will be able to:

1. Define the fundamental units in SI for length, mass, and time.
2. Convert from one unit to another within the metric system and between metric and British units for length and mass.
3. Use SI base units to calculate derived units for a physical quantity and determine whether an expression is dimensionally consistent.
4. Determine the number of significant figures in a given number and use the rules for adding, subtracting, multiplying, and dividing numbers to determine the significant figures in a calculated value.
5. Describe a vector in terms of its magnitude and direction or in unit vector notation.
6. Determine the magnitude and direction of a vector given its components.
7. Determine the components of a vector given its magnitude and direction.
8. Multiply a vector by a scalar and determine changes in both the vector's magnitude and direction.
9. Add or subtract two or more vectors both graphically and analytically.

1.1 INTRODUCTION

Physics is an *experimental* science. Physicists observe the phenomena of nature and try to discover patterns and principles that relate these phenomena. These patterns are called *physical theories* or, when they are very broad and well established, *physical laws.* The development of physical theory requires creativity at every stage. The physicist has to learn to ask appropriate questions, design experiments to try to answer those questions, and draw appropriate conclusions from the results. Figure 1.1 shows two famous experimental

APPLICATION Where are the tunes coming from? These dancers may not realize it, but they are enjoying the application of fundamental principles of physics that make their music possible. In later chapters, we will learn about electric charge and current and how batteries are used to store electric potential energy. We will see how this energy can be harnessed to perform the mechanical work of producing sound waves that we hear as music.

facilities, one old and one new. The Leaning Tower of Pisa is where Galileo studied the physics of free fall. The Chandra X-ray Observatory is in orbit around the earth and is currently being used to unravel the early history of the universe.

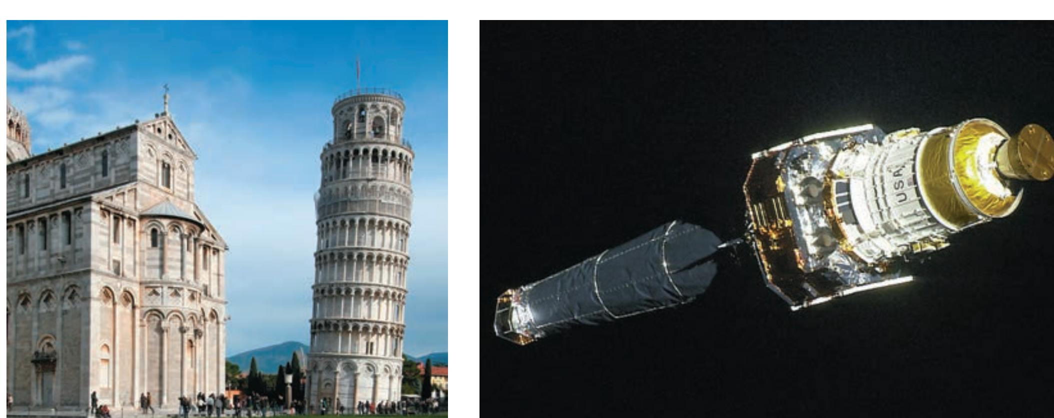
According to legend, Galileo Galilei (1564–1642) dropped light and heavy objects from the top of the Leaning Tower of Pisa to find out whether they would fall at the same rate or at different rates. Galileo recognized that only an *experimental* investigation could answer this question. From the results of his experiments, he had to make the inductive leap to the principle, or theory, that the rate at which an object falls is independent of its weight.

The development of physical theory is always a two-way process that starts and ends with observations or experiments. The study of physics does not merely involve the collection of facts and development of principles; it also involves the *process* by which we arrive at general principles that describe the behavior of the physical universe. And there is always the possibility that new observations will require revision of a theory. We can *disprove* a theory by finding a phenomenon that is inconsistent with it, but we can never *prove* that a theory is *always* correct.

Getting back to Galileo, suppose we drop a feather and a cannonball. They certainly *do not* fall at the same rate. This doesn't mean that our statement of Galileo's theory is wrong, however; it is simply incomplete. One complicating feature is air resistance. If we drop the feather and the cannonball in vacuum, they *do* fall at the same rate. Our statement of Galileo's theory is valid for bodies that are heavy enough that air resistance has almost no effect on them. A feather or a parachute clearly does not have this property.

Every physical theory has a range of validity and a boundary outside of which it is not applicable. The range of Galileo's work with falling bodies was greatly extended half a century later by Newton's laws of motion and his law of gravitation. Nearly all the principles in this book are so solidly established by experimental evidence that they have earned the title **physical law.** Yet there are some areas of physics in which present-day research is continuing to broaden our understanding of the physical world. We'll discuss some examples of these areas later (in the final four chapters of the book).

An essential part of the interplay between theory and experiment is learning how to apply physical principles to practical problems. At various points in our study, we'll discuss systematic problem-solving procedures that will help you set up problems and solve them efficiently and accurately. Learning to solve problems is absolutely essential; you don't *know* physics unless you can *do* physics. This means learning not only the general principles but also how to apply them in a variety of specific situations.



(a) (b)

Figure 1.1 Two research laboratories. (a) The Leaning Tower of Pisa (Italy). According to legend, Galileo studied the motion of freely falling objects by dropping them from the tower. He is also said to have gained insights into pendulum motion by observing the swinging of the chandelier in the cathedral to the left of the tower. (b) The Chandra X-ray Observatory, the first orbital observatory to capture and probe X-ray radiation from hot, turbulent regions of the universe.

1.2 IDEALIZED MODELS

In everyday conversation, we often use the word **model** to mean either a small-scale replica, such as a model train or airplane, or a human figure that displays articles of clothing. **In physics, a *model* is a simplified version of a physical system that would be too complicated to analyze in full without the simplifications.**

Here's an example: We want to analyze the motion of a baseball thrown through the air. How complicated is this problem? The ball is neither perfectly spherical nor perfectly rigid; it flexes a little and spins as it moves through the air. Wind and air resistance influence its motion, the earth rotates beneath it, the ball's weight varies a little as its distance from the center of the earth changes, and so on. If we try to include all these features, the analysis gets pretty hopeless. Instead, we invent a simplified version of the problem. We ignore the size and shape of the ball and represent it as a *particle*—that is, as an object that has no size and is completely localized at a single point in space. We ignore air resistance, treating the ball as though it moves in vacuum; we forget about the earth's rotation, and we make the weight of the ball exactly constant. *Now* we have a problem that is simple enough to deal with: The ball is a particle moving along a simple parabolic path. (We'll analyze this model in detail in Chapter 3.)

The point is that we have to overlook quite a few minor effects in order to concentrate on the most important features of the motion. That's what we mean by making an idealized model of the system. Of course, we have to be careful not to overlook *too much.* If we ignore the effect of gravity completely, then when we throw the ball, it will travel in a straight line and disappear into space, never to be seen again! We need to use some judgment and creativity to construct a model that simplifies a system without throwing out its essential features.

When we analyze a system or predict its behavior on the basis of a model, we always need to remember that the validity of our predictions is limited by the validity of the model. If the model represents the real system quite precisely, then we expect predictions made from it to agree quite closely with the actual behavior of the system. Going back to Pisa with Galileo once more, we see that our statement of his theory of falling objects corresponds to an idealized model that does not include the effects of air resistance, the rotation of the earth, or the variation of weight with height. This model works well for a bullet and a cannonball, not so well for a feather.

The concept of an idealized model is of the utmost importance in all of physical science and technology. In applying physical principles to complex systems, we always make extensive use of idealized models, and it is important to be aware of the assumptions we are making. Indeed, the *principles* of physics themselves are stated in terms of idealized models; we speak about point masses, rigid bodies, ideal insulators, and so on. Idealized models will play a crucial role in our discussion of physical theories and their applications to specific problems throughout this book.

BIO APPLICATION How can I find lunch in the dark? Insect-eating bats utilize the laws of physics in their quest for prey in a process known as *echolocation.* To locate and capture an insect, these flying mammals first emit an extremely rapid series of 20 to 200 bursts of high-frequency sound. Without using a stopwatch or calculator, the bats are able to determine the insect's position by sensing the direction of the reflected sound waves and the time it takes for these waves to reach their ears. Luckily for us, the sounds bats emit are above the range of human hearing; they can be higher than 100 decibels, or about as loud as a typical smoke alarm.

1.3 STANDARDS AND UNITS

Any number that is used to describe an observation of a physical phenomenon quantitatively is called a **physical quantity.** Some physical quantities are so fundamental that we can define them only by describing a procedure for measuring them. Such a definition is called an **operational definition.** For instance, we might use a ruler to measure a length or a stopwatch to measure a time interval. In other cases, we define a physical quantity by describing a way to *calculate* the quantity from other quantities that we can measure. For example, we might define the average speed of a moving object as the distance traveled (measured with a ruler) divided by the time of travel (measured with a stopwatch).

DEFINITION OF UNITS

When we measure a physical quantity, we always compare it with some reference standard for that quantity. If a number is used to specify the value of that quantity, then it must carry with it a *unit* that reflects the quantity's physical manifestation. For example, when we say that a rope is 30 meters long, we mean that it is 30 times as long as a meterstick, which we *define* to be 1 meter long. Here *meter* is the *unit* associated with the physical property of length.

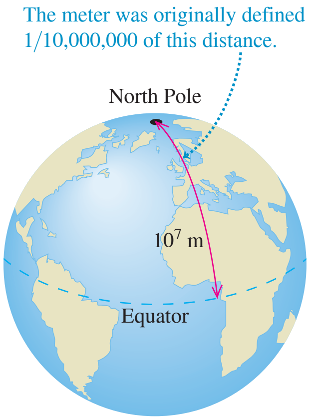


Figure 1.2 The original definition of the meter, from 1791.

To make precise measurements, we need definitions of units of measurement that do not change and that can be duplicated by observers in various locations. When the metric system was established in 1791 by the Paris Academy of Sciences, the **meter** was defined as one ten-millionth of the distance from the equator to the north pole (Figure 1.2), and the **second** was defined as the time it takes for a pendulum 1 meter long to swing from one side to the other.

These definitions were cumbersome and hard to duplicate precisely, and in more recent years they have been replaced by more refined definitions. Since 1889, the definitions of the basic units have been established by an international organization, the General Conference on Weights and Measures. The system of units defined by this organization is based on the metric system, and since 1960 it has been known officially as the **International System,** or **SI** (the abbreviation for the French equivalent, Système International).

Time

From 1889 until 1967, the unit of time was defined in terms of the length of the day, directly related to the time of the earth's rotation. The present standard, adopted in 1967, is much more precise. It is based on an atomic clock that uses the energy difference between the two lowest energy states of the cesium atom. Electromagnetic radiation (microwaves) of precisely the proper frequency causes transitions from one of these states to the other. One second is defined as the time required for 9,192,631,770 cycles of this radiation.

Length

In 1960, an atomic standard for the meter was established, using the orange-red light emitted by atoms of krypton in a glow discharge tube. In November 1983, the standard of length was changed again, in a more radical way. The new definition of the meter is the distance light travels (in vacuum) in 1/299,792,458 second. The adoption of this definition has the effect, in turn, of *defining* the speed of light to be precisely 299,792,458 m/s; we then define the meter to be consistent with this number and with the atomic-clock definition of the second. This approach provides a much more precise standard of length than the one based on a wavelength of light.

Mass

Until recently the unit of mass, the kilogram (abbreviated kg), was defined to be the mass of a metal cylinder kept at the International Bureau of Weights and Measures in France (Fig. 1.3). This was a very inconvenient standard to use. Since 2018 the value of the kilogram has been based on a fundamental constant of nature called *Planck's constant* (symbol h), whose defined value $h = 6.62607015 \times 10^{-34}\ \mathrm{kg \cdot m^2/s}$ is related to those of the kilogram, meter, and second. Given the values of the meter and the second, the masses of objects can be experimentally determined in terms of h. (We'll explain the meaning of h in Chapter 28.) The *gram* (which is not a fundamental unit) is 0.001 kilogram.

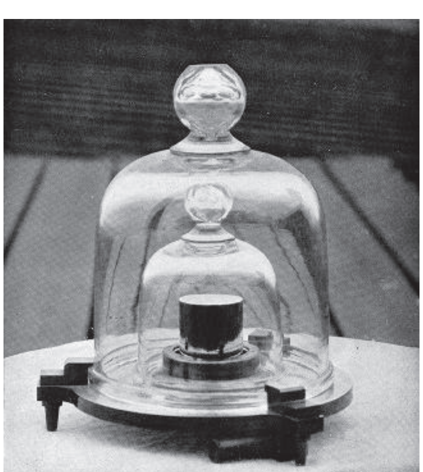


Figure 1.3 Until 2018 a metal cylinder was used to define the value of the kilogram. (The one shown here, a copy of the one in France, was maintained by the U.S. National Institute of Standards and Technology.) Today the kilogram is defined in terms of one of the fundamental constants of nature.

Prefixes

Once the fundamental units have been defined, it is easy to introduce larger and smaller units for the same physical quantities. One of the great strengths of the metric system is that these other units are related to the fundamental units by multiples of 10 or 1/10. (By comparison, the British system bristles with inconvenient factors such as 12, 16, and 36.) Thus, 1 kilometer (1 km) is 1000 meters, 1 centimeter (1 cm) is 1/100 meter, and so on. We usually express these multiplicative factors in exponential notation, typically called scientific notation or *powers-of-10* notation—for example, $1000 = 10^3$ and $1/100 = 10^{-2}$. With this notation,

$$1\ \mathrm{km} = 10^3\ \mathrm{m}, \quad 1\ \mathrm{cm} = 10^{-2}\ \mathrm{m}.$$

The names of additional units are derived by adding a **prefix** to the name of the fundamental unit. For example, the prefix "kilo-," abbreviated k, means a unit larger by a factor of 1000; thus,

$$1\ \text{kilometer} = 1\ \mathrm{km} = 10^3\ \text{meters} = 10^3\ \mathrm{m},$$
$$1\ \text{kilogram} = 1\ \mathrm{kg} = 10^3\ \text{grams} = 10^3\ \mathrm{g},$$
$$1\ \text{kilowatt} = 1\ \mathrm{kW} = 10^3\ \text{watts} = 10^3\ \mathrm{W}.$$

Table 1.1 lists the standard SI prefixes with their meanings and abbreviations. Note that most of these are multiples of 10^3.

When pronouncing unit names with prefixes, accent the *first* syllable: KIL-o-gram, KIL-o-meter, CEN-ti-meter, and MIC-ro-meter. (Kilometer is sometimes pronounced kil-OM-eter.)

Here are some examples of the use of multiples of 10 and their prefixes with the units of length, mass, and time. Additional time units are also included. Figure 1.4 gives you an idea of the range of length scales studied in physics.

TABLE 1.1 Prefixes for powers of 10

Power of 10	Prefix	Abbreviation
10^{-18}	atto-	a
10^{-15}	femto-	f
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	μ
10^{-3}	milli-	m
10^{-2}	centi-	c
10^{3}	kilo-	k
10^{6}	mega-	M
10^{9}	giga-	G
10^{12}	tera-	T
10^{15}	peta-	P
10^{18}	exa-	E

Length

1 nanometer = 1 nm = 10^{-9} m (a few times the size of an atom)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and cells)
1 millimeter = 1 mm = 10^{-3} m (size of the point of a ballpoint pen)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)
1 kilometer = 1 km = 10^{3} m (distance traveled in a 10-minute walk)

Mass

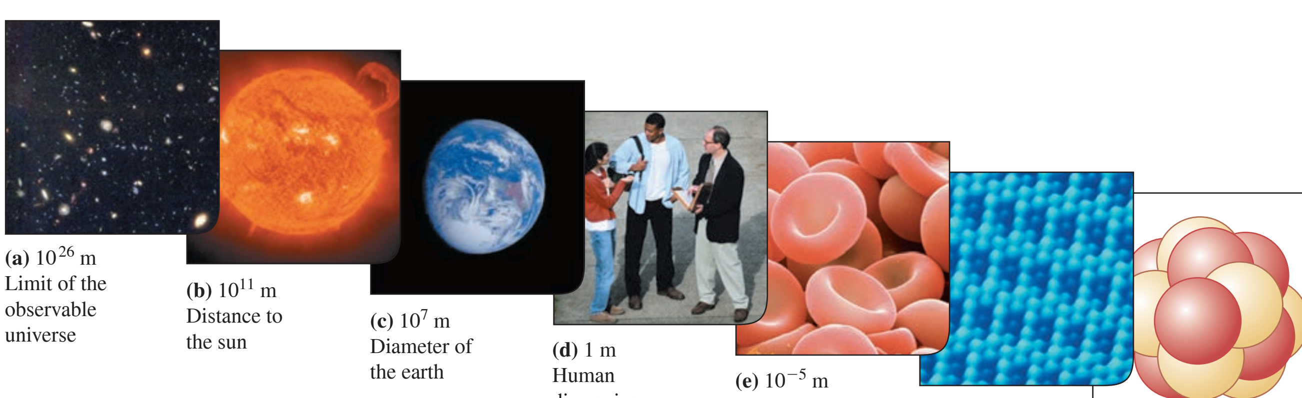
1 microgram = 1 μg = 10^{-9} kg (mass of a 1 mm length of hair)
1 milligram = 1 mg = 10^{-6} kg (mass of a grain of salt)
1 gram = 1 g = 10^{-3} kg (mass of a paper clip)
1 kilogram = 1 kg = 10^{3} g (mass of a 1 liter bottle of water)

Time

1 nanosecond = 1 ns = 10^{-9} s (time required for a personal computer to add two numbers)
1 microsecond = 1 μs = 10^{-6} s (time required for a 10-year-old personal computer to add two numbers)
1 millisecond = 1 ms = 10^{-3} s (time required for sound to travel 0.35 m)
1 minute = 1 min = 60 s
1 hour = 1 h = 3600 s
1 day = 1 day = 86,400 s

British units

Finally, we mention the British system of units. These units are used only in the United States and a few other countries, and in most of those countries they are being replaced by SI units. Even in the United States, we now use metric units in many everyday contexts;



(a) 10^{26} m Limit of the observable universe
(b) 10^{11} m Distance to the sun
(c) 10^{7} m Diameter of the earth
(d) 1 m Human dimensions
(e) 10^{-5} m Diameter of a red blood cell
(f) 10^{-10} m Radius of an atom
(g) 10^{-14} m Radius of an atomic nucleus

Figure 1.4 The laws of physics apply over an incredibly wide range of system sizes—from the cosmological scales of galaxies to the subnuclear realms of protons and neutrons.

we speak about 4 liter engines, 50 mm lenses, 10 km races, 2 liter soda bottles, and so on. British units are now officially *defined* in terms of SI units:

$$\text{Length: 1 inch} = 2.54\ \text{cm (exactly)}.$$

$$\text{Force: 1 pound} = 4.448221615260\ \text{newtons (exactly)}.$$

The fundamental British unit of time is the second, defined the same way as in SI. British units are used only in mechanics and thermodynamics; there is no British system of electrical units. In this book, we use SI units in almost all examples and problems, but in the early chapters we occasionally give approximate equivalents in British units.

Test Your Understanding of SECTION 1.3

SI measurement of your hand

With your hand flat and your fingers and thumb close together, the width of your hand is about

A. 50 cm.
B. 10 cm.
C. 10 mm.

SOLUTION If you're not familiar with metric units of length, you can use your body to develop intuition for them. One meter (100 cm) is slightly more than a yard, so the distance from elbow to fingertips is about 50 cm; this eliminates answer A. Ten millimeters (1 cm) is about the width of your little finger. Ten centimeters is about 4 inches and is about the width of your hand. The correct answer is B.

APPLICATION A $125 million unit consistency error. In 1998, the Mars Climate Orbiter was sent by NASA to orbit Mars at an altitude above the Martian atmosphere; instead, it entered the atmosphere in 1999 and burned up! The key error that led to this disaster was a miscommunication between the Jet Propulsion Laboratory (JPL) and the spacecraft engineers who built the orbiter. The engineers specified the amount of thrust required to steer the craft's trajectory in units of British pounds, while the JPL assumed the numbers were in SI units of newtons. Thus, each correction of the trajectory applied a force 4.45 times greater than needed. Although there were other compounding errors, this unit inconsistency was a major factor in the failure of the mission and the loss of the orbiter.

1.4 DIMENSIONAL CONSISTENCY AND UNIT CONVERSIONS

We use equations to express relationships among physical quantities that are represented by algebraic symbols. Most algebraic symbols for physical quantities denote both a number and a dimension. For example, if the symbol d represents distance, then it must have dimensions of length. If the symbol v represents speed, then it must have dimensions of length per time. The dimensions themselves can be expressed in a variety of units. For instance, the dimension of length can be expressed in meters, feet, inches, miles, and so on. However, independent of which units are used, an equation must always be **dimensionally consistent.** You can't add apples and pomegranates; two terms may be added or equated only if they have the same dimensions. For example, if a body moving with constant speed v travels a distance d in a time t, these quantities are related by the equation

$$d = vt\ (\text{distance} = \text{speed} \times \text{time}).$$

If d has dimensions of length and is measured in meters, then the product vt must also have dimensions of length and be expressed in meters. So, we may write

$$10\ \text{m} = (2\ \text{m/s})(5\ \text{s}).$$

Because the unit 1/s cancels the unit s in the last factor, the product vt does have units of meters, as it must. In calculations, with respect to multiplication and division, units are treated just like algebraic symbols.

DIMENSIONAL CONSISTENCY OF EQUATIONS

You cannot add, subtract, or equate quantities of different dimensions in an equation. However, you can multiply and divide quantities that have different dimensions.
Notes:

- The dimensional consistency rules hold true whether you are dealing with numbers or algebraic expressions.
- Always write numbers with the correct units and carry the units through the calculation.