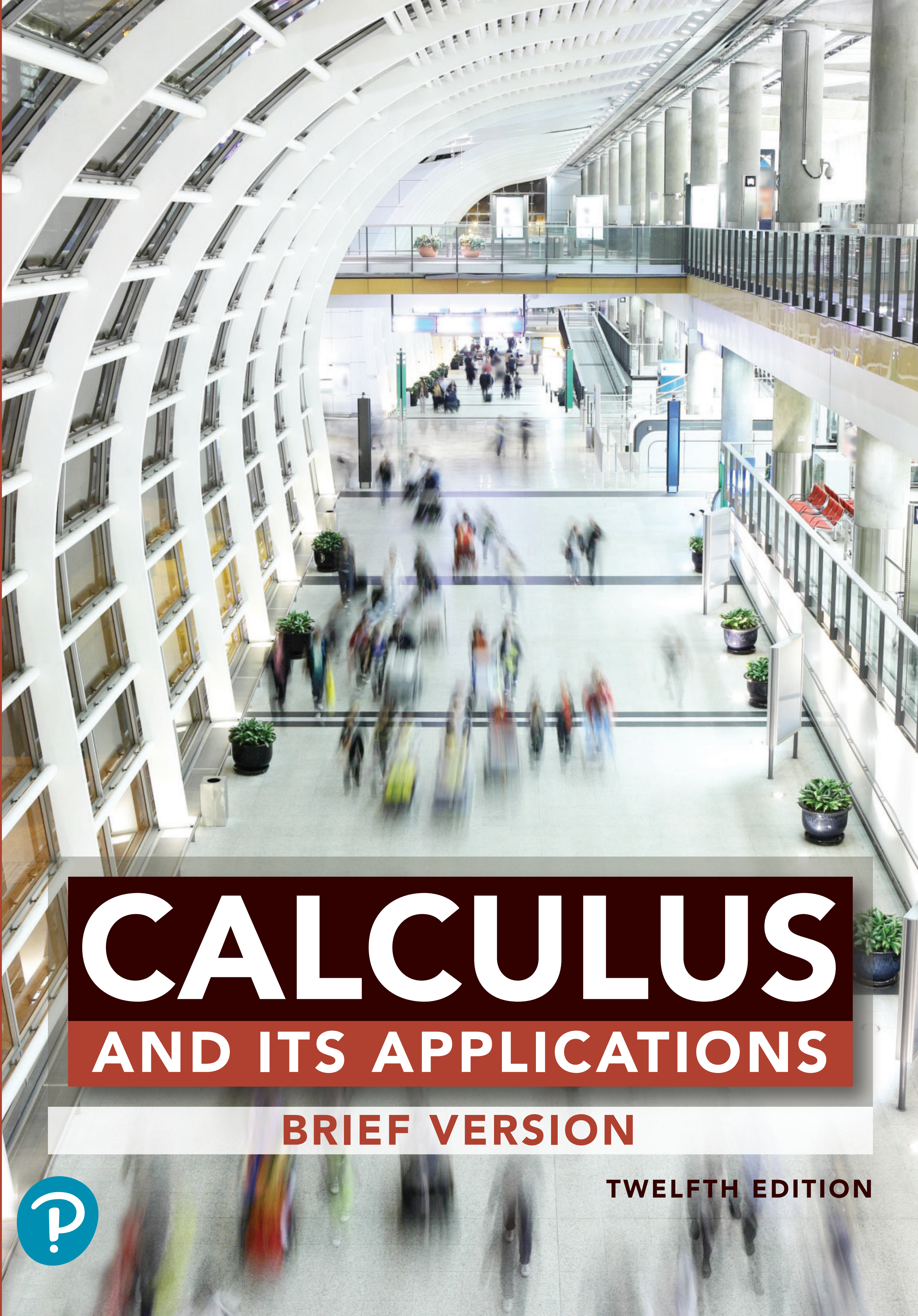

CALCULUS AND ITS APPLICATIONS

BRIEF VERSION

TWELFTH EDITION

BITTINGER | ELLENBOGEN | SURGENT

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Calculus

AND ITS APPLICATIONS

BRIEF VERSION

TWELFTH EDITION

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To Elaine, Kit, Beth, and Julie

Contents

The following chapters are included in the longer version of this text (entitled *Calculus and Its Applications*):

Preface

Calculus and Its Applications, Brief Version, is designed for a one-semester course in calculus for students majoring in business, economics, social sciences, or health/life sciences. We have strived to produce the most student-oriented applied calculus text on the market. We believe that appealing to students' intuition and writing in a direct, down-to-earth manner make this text accessible to any student possessing the prerequisite math skills. By presenting more topics in a conceptual and often visual manner and adding student self-assessment and teaching aids to this edition of the text, we address students' needs better than ever before. Tapping into areas of student interest, we provide motivation through an abundant supply of examples and exercises rich in real-world data from business, economics, environmental studies, health care, and the life sciences.

New examples cover a variety of applications designed to appeal to students taking the course. Found in every chapter, realistic applications draw students into the discipline and help them to generalize the material and apply it to new situations. To further spark student interest, hundreds of meticulously rendered graphs and illustrations appear throughout the text, making it a favorite among students who are visual learners.

A course in intermediate algebra is a prerequisite to this applied calculus text, although "Appendix A: Review of Basic Algebra" and "Chapter R: Functions, Graphs, and Models" provide a sufficient foundation to unify the diverse backgrounds of most students. Note the availability of two different versions of this text by the same authors:

- The **brief version** contains Chapters R–6 and is generally used for a one-semester course.
- The **complete** *Calculus and Its Applications* contains Chapters R–12 and is generally used for a two-semester course.

New to This Edition

We welcome to this edition co-author Gene Kramer of University of Cincinnati Blue Ash. Gene's primary focus was updating the contents of the MyLab Math course for the text.

This is a substantial revision of the text and its associated technology. The revisions were informed by feedback from users of the text and the accompanying MyLab Math course as well as our own classroom experiences.

New to MyLab Math

Many improvements have been made to the overall functionality of MyLab Math since the previous edition. However, beyond that, we have also increased and improved the content specific to this text.

- All MyLab Math **exercises have been reviewed and edited** where necessary by author Gene Kramer for improved quality and fidelity to the text.
- Instructors now have **more exercises** than ever to choose from when assigning homework. Most new questions are application-oriented. There are approximately 3270 assignable exercises in MyLab Math for this text. New exercise types include:
  - Setup and Solve exercises require students to show how they set up a problem as well as giving the solution, better mirroring what is required of students on tests.
  - Additional Conceptual Questions (labeled ACQ) provide support for assessing concepts and vocabulary. Many of these questions are application-oriented.

- About **90% of the instructional videos are brand-new**. The new videos were made using the latest technology and feature authors Gene Kramer and Scott Surgent along with instructors Mary Ann Barber (University of North Texas) and Thomas Hartfield (University of North Georgia).
  - The Guide to Video-Based Assignments shows which MyLab Math exercises can be assigned for each video. This resource is especially useful for online or flipped classes.
- The **suite of interactive figures has been expanded** to support teaching and learning. These figures (created in GeoGebra) illustrate key concepts and can be manipulated by users. They have been designed to be used in lectures as well as by students independently.
- **Enhanced Sample Assignments** are section-level assignments that (1) address gaps in precalculus skills with personalized prerequisite review, (2) help keep skills fresh with spaced practice of key calculus concepts, and (3) provide opportunities to work exercises without learning aids so that students can check their understanding. They are assignable and editable.
- **Study skills modules** help students with the life skills that can make the difference between passing and failing.
- The **Graphing Calculator Manual** and **Excel Spreadsheet Manual**, both specific to this course, have been updated to support the TI-84 Plus CE (color display) and Excel 2016, respectively. Both manuals also contain additional topics to support the course.
- We heard from users that the Annotated Instructor's Edition for the previous edition required too much flipping of pages to find answers, so MyLab Math now contains a **downloadable Instructor's Answers** document—*with all the answers in one place*. (This augments the downloadable Instructor Solutions Manual, which contains all *solutions*.)

Content Updates

Our overall goals in revising the text were as follows:

- Tighten language and consolidate examples and exercises whenever possible.
- Help students master exponential and logarithmic functions and realize their utility by introducing them earlier and using them more often.
- Bolster themes that endure throughout the book—for example, by adding material in Section R.3 on disjoint intervals, which is something students apply later when considering appropriate intervals of domain.
- Incorporate spreadsheets as a means of explaining concepts, when appropriate.
- Remove unnecessary or out-of-date Technology Connection features.
- Add new examples or subsections to pre-existing examples or sections and reference earlier examples or themes wherever appropriate.

Detailed changes to each chapter are as follows:

Chapter R

- In Section R.3, material on disjoint intervals, which are used elsewhere in the book, has been added.
- In Section R.4, the presentation of lines has been reordered, building from the most basic to the most general form.
- In Section R.5, all supply-demand curves have been revised so that price now sits on the vertical axis. This matches what students see in business and economics courses.
- New Section R.6 provides a deeper treatment of logarithms and exponentials. Some material from the previous edition's Section R.5 is used here but a significant portion is brand-new.

Chapter 1

- In Section 1.1, the introduction to limits along the number line has been rewritten to explain the idea of left- and right-hand approaches, thus setting up the general limit approach more intuitively.
- Section 1.2 contains added material on limits at infinity, to better set the stage for the introduction of asymptotic behavior in Chapter 3.

- In Section 1.7, composition of functions is discussed earlier, and the Extended Power Rule is treated as a special case of the general case. (Previously, we treated it as its own theorem.)

Chapters 2 and 3

Based on the feedback we received on the importance of exponential and logarithmic functions to the students in this class (particularly business students), we now introduce exponential and logarithmic functions (and their derivatives) in Chapter 2, then cover applications of the derivative in Chapter 3 (including applications involving exponential and logarithmic functions). This change helps students better learn exponential and logarithmic functions by introducing them earlier and providing increased opportunities to practice and apply them in both chapters.

- New Section 2.1 has increased coverage of how the natural base *e* is derived, including new examples and significant expansion of the exercise set.
- Former Section 3.2 is split into two new sections (2.2 and 2.3), which discuss the derivatives of the natural base and natural logarithmic functions, respectively. This allowed us to add more examples and give greater depth to the discussion of these topics.
- In Section 2.6 (formerly 3.5), discussion of models where base 10 or base *e* are not used has been expanded. We use general bases and show how one can go back and forth, the advantages of each, and so on.
- In new Sections 3.1–3.4, exponential and logarithmic functions have been added to the curve-sketching discussions. This is significant because we now have a richer set of examples and can go deeper into these topics.
- Optimization questions involving exponentials and logarithms have been added to Section 3.5. We include some discussion of the use of spreadsheets to show how a student could "solve" a maximum–minimum problem in this manner.
- New material on linearization has been included in Section 3.6.
- Exponential models are now covered in the discussion of elasticity of demand in Section 3.7, which includes three examples (rather than just one, as in the previous edition).
- Former Section 2.8 has been split into two sections (3.8 and 3.9) to lighten the content load. Section 3.8 covers implicit differentiation (including a subsection on logarithmic differentiation), and Section 3.9 covers related rates and includes more examples than in the previous edition.

Chapters 4 and 5

- Section 4.1 has increased focus on antidifferentiation as accumulation, using easy intuitive examples to appeal to students' familiarity with real life.
- Section 4.4 has a new subsection on moving averages.
- In Section 4.6, a new example involving exponential decay provides a life sciences application.
- New Section 4.7 covers numerical methods of integration.
- The previous edition's little-used section on integral tables is now Appendix E.
- Section 5.1 has been expanded to include material on price ceilings, price floors, and deadweight loss. This material is a natural extension of the topic of consumer and producer surplus, and the level of mathematics is a perfect fit.
- Problems in Section 5.2 have been updated with more realistic interest rates and current real-world data.

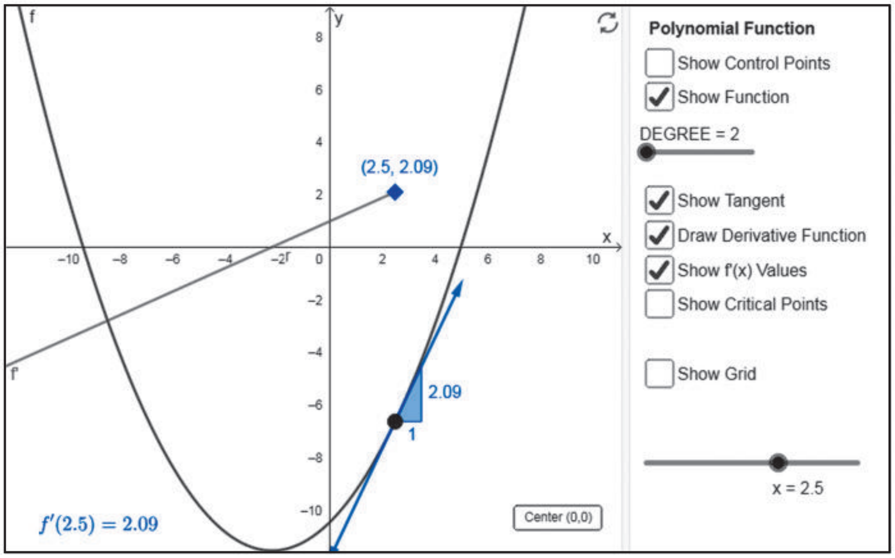
Chapter 6

- New material on differentials and tolerances is included in Section 6.2. This expands on ideas developed in Section 3.6.
- Section 6.3 includes new content on constrained optimization in three variables, showing how a constraint reduces a problem to a two-variable maximum–minimum problem. This sets the stage for Section 6.5.
- In Section 6.5, a new example (and exercises) shows how Lagrange multipliers can be used to solve constrained maximum–minimum problems in three variables.
- Examples in Section 6.6 have been revised to eliminate unnecessary fractions.

Our Approach

Intuitive Presentation

Although the word *intuitive* has many meanings and interpretations, we use it to mean "experience based, without proof." Throughout the text, when a concept is discussed, its presentation is designed so that the students' learning process is based on their earlier mathematical experience. This is illustrated by the following situations.



- Within MyLab Math, students and instructors have access to interactive figures that illustrate concepts and allow manipulation so that students can better predict and understand the underlying concepts.
- Before the formal definition of *continuity* is presented, an informal explanation is given, complete with graphs that take advantage of student intuition about ways in which a function could be discontinuous (see pp. 120–121).
- The definition of *derivative*, in Chapter 1, is presented in the context of a discussion of average rates of change (see p. 139). This presentation is more accessible and realistic than the strictly geometric idea of slope.
- When maximization problems involving volume are introduced (see pp. 318–319), a function that is to be maximized is derived. Instead of forging ahead with the standard calculus solution, the text first asks students to make a table of function values, graph the function, and then estimate the maximum value. This experience provides students with more insight into the problem. They recognize not only that different dimensions yield different volumes, but also that the dimensions yielding the maximum volume may be conjectured or estimated as a result of the calculations.

Timely Help for Gaps in Algebra Skills

One of the most critical factors underlying success in this course is a strong foundation in algebra skills. We recognize that students start this course with varying degrees of skills, so we have included multiple opportunities in both the text and MyLab Math to help students target their weak areas and remediate or refresh the needed skills.

In the Text

- **Prerequisite Skills Diagnostic Test (Part A).** This portion of the diagnostic test assesses skills refreshed in "Appendix A: Review of Basic Algebra." Answers to the questions (provided at the back of the book) reference specific examples within the appendix.
- **Appendix A: Review of Basic Algebra.** This 13-page appendix provides examples on topics such as exponents, equations, and inequalities and applied problems. It ends with an exercise set, for which answers are provided at the back of the book so students can check their understanding.
- **Prerequisite Skills Diagnostic Test (Part B).** This portion of the diagnostic test assesses skills that are reviewed in "Chapter R: Functions, Graphs, and Models," and the answers (provided at the back of the book) reference specific examples in that chapter. Some instructors may choose to cover these topics thoroughly in class, making this assessment less critical. Other instructors may use some or all of the questions in this test to determine whether there is a need to spend time remediating before moving on with Chapter 1.

- **Chapter R: Functions, Graphs, and Models.** This chapter covers basic concepts related to functions, graphing, and modeling. It can be an optional chapter, depending on students' prerequisite skills.

In MyLab Math

You can diagnose weak prerequisite skills through built-in diagnostic quizzes. By coupling these quizzes with personalized homework, MyLab Math provides remediation for just those skills a student lacks. Even if you choose not to assign the diagnostic quizzes with personalized homework, students can self-remediate through videos and practice exercises provided at the objective level. MyLab Math provides the just-in-time help that students need, so you can focus on the course content.

Exercises and Applications

There are 4114 assignable homework exercises in this edition. A large percentage of these exercises are rendered algorithmically in MyLab Math. All exercise sets are enhanced by the inclusion of real-world applications, detailed figures, and illustrative graphs. There are a variety of types of exercises, too, so different levels of understanding and varying approaches to problems can be assessed. In addition to applications, the exercise sets include Thinking and Writing, Synthesis, Technology Connection, and Concept Reinforcement exercises. The exercises in MyLab Math reflect the depth and variety of those in the printed text.

The authors also provide Quick Check exercises following selected examples to give students the opportunity to check their understanding of new concepts or skills as soon as they learn them and one skill at a time. Instructors may include these as part of a lecture as a means of gauging skills and gaining immediate student feedback. Answers to the Quick Check exercises are provided following the exercise set at the end of each section.

Relevant and factual applications drawn from a broad spectrum of fields are integrated throughout the text as applied examples and exercises, and are also featured in separate application subsections. Applications have been updated and expanded in this edition to include even more real-world data. In addition, each chapter opener features an application that serves as a preview of what students will learn in the chapter. The Index of Applications at the back of the book provides students and instructors with a comprehensive list of the many different fields featured in examples and exercises throughout the text.

The applications in the exercise sets in the text and within MyLab Math are grouped under headings that identify them as reflecting real-world situations: Business and Economics, Life and Physical Sciences, Social Sciences, and General Interest (abbreviated as BE, LS, PS, SS, and GI within MyLab Math). This organization allows the instructor to gear the assigned exercises to specific students and also allows each student to know whether a particular exercise applies to his or her major.

Opportunities to Incorporate Technology

This edition continues to emphasize mathematical modeling, utilizing the advantages of technology as appropriate. The use of Excel as a tool for solving problems has been expanded in this edition. Though the use of technology is optional with this text, its use meshes well with the text's more intuitive approach to applied calculus.

Technology Connections

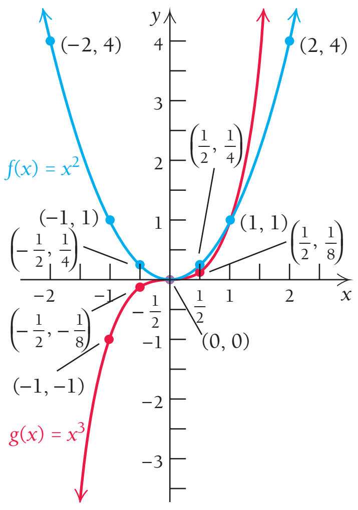
Technology Connections are included throughout the text to illustrate the use of technology, including graphing calculators and Excel spreadsheets. Whenever appropriate, figures that simulate graphs or tables generated by a graphing calculator are included. The goal is to take advantage of technology to which many students have access, wherever it makes sense, given the mathematical situation.

Four types of Technology Connections allow students and instructors to explore key ideas:

1. **Lesson/Teaching—**provide students with an example, followed by exercises to work within the lesson.
2. **Checking—**tell the students how to verify a solution within an example by using a graphing calculator.

3. **Exploratory/Investigation**—provide questions to guide students through an investigation.
4. **Technology Connection Exercises**—found in most section exercise sets, Chapter Review Exercises, and Chapter Tests.

Extended Technology Applications at the ends of all chapters use real applications and real data. They require a step-by-step analysis that encourages group work. More challenging in nature, the exercises in these features often involve the use of regression to create models on a graphing calculator.



Use of Art and Color

One of the hallmarks of this text is the pervasive use of color as a pedagogical tool. Color is used in a methodical and precise manner that enhances the readability of the text for students and instructors.

- When two curves are graphed using the same set of axes, one is usually red and the other blue, with the red graph being the curve of major importance. As in the graph to the left, equation labels are the same color as the curve for clarity.
- When the instructions say "Graph," the dots match the color of the curve. When dots are used for emphasis other than just for plotting, they are black.
- Throughout the text, blue is used for secant lines and red for tangent lines (see p. 138).
- Red denotes substitution in equations while blue highlights the corresponding outputs, and the specific use of color is carried out in related figures (see pp. 282–284).
- Beginning with the discussion of integration, an amber color is used to highlight areas in graphs (see p. 408).

Opportunity for Review and Synthesis

Recognizing that it is often while preparing for exams that concepts gel for students, this text offers abundant opportunities for students to review, analyze, and synthesize recently learned concepts and skills.

- A **Section Summary** precedes every exercise set to assist students in identifying the key topics for each section and to serve as a mini-review.
- A **Chapter Summary** at the end of every chapter includes a section-by-section list of key definitions, concepts, and theorems, with examples for further clarification.
- **Chapter Review Exercises**, which include bracketed references to the section in which the related concept is first introduced, provide comprehensive coverage and appropriate referencing of each chapter's material.
- A **Chapter Test** at the end of every chapter and a **Cumulative Review** at the end of the text give students an authentic exam-like environment for testing their mastery. Answers to the chapter tests and the Cumulative Review are at the back of the text and include section references so students can diagnose their mistakes while preparing for exams.

Acknowledgments

As authors, we have taken many steps to ensure the accuracy of this text. Many devoted individuals comprised the team that was responsible for monitoring the revision and production process in a manner that makes this a work of which we can all be proud. We are thankful for our publishing team at Pearson, as well as all of the Pearson representatives who share our book with educators across the country. We would like to thank Jane Hoover for her many helpful suggestions, proofreading, and checking of art. Geri Davis deserves credit for both the attractive design of the text and the coordination of the many illustrations, photos, and graphs. Many thanks also to John Samons and Laurie Hurley for their careful checking of the manuscript and typeset pages and to Sal Sciandra for his work on the Solutions Manuals. Special thanks go to Cara McDaniel of Arizona State University for her thoughtful insights on the new material in the text. We are grateful to all those who have contributed to the improvement of this text over the years, including those who were instrumental in helping us to shape this 12th edition.

The following individuals provided terrific insights and meaningful suggestions for improving this text. We thank them:

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James Martin, *Christopher Newport University*
Tamara Miller, *Ivy Tech Community College of Indiana*
Michael R. Payne, *Pueblo Community College*
Brendan Patrick Purdy, *Moorpark College*
Thomas C. Redd, *North Carolina A&T State University*
Shumaila Saeed, *Indiana University Purdue University Indianapolis*
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Robert Vilardi, *Troy University*
Kimberly W. Walters, *Mississippi State University*

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Cheryl Van Rhein, *University of Missouri–Kansas City*
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Pearson MyLab

MyLab Math Online Course for *Calculus and Its Applications, Brief Version,* 12th edition

(access code required)

MyLab™ Math is available to accompany Pearson's market-leading text offerings. To give students a consistent tone, voice, and teaching method, each text's flavor and approach are tightly integrated throughout the accompanying MyLab Math course, making learning the material as seamless as possible.

PREPAREDNESS

One of the biggest challenges in applied calculus courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyLab Math supports students with just-in-time remediation and key-concept review.

Integrated Review

The MyLab Math course contains pre-made, assignable quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified. Each student, therefore, receives just the help that he or she needs—no more, no less.

NEW! Study Skills Modules

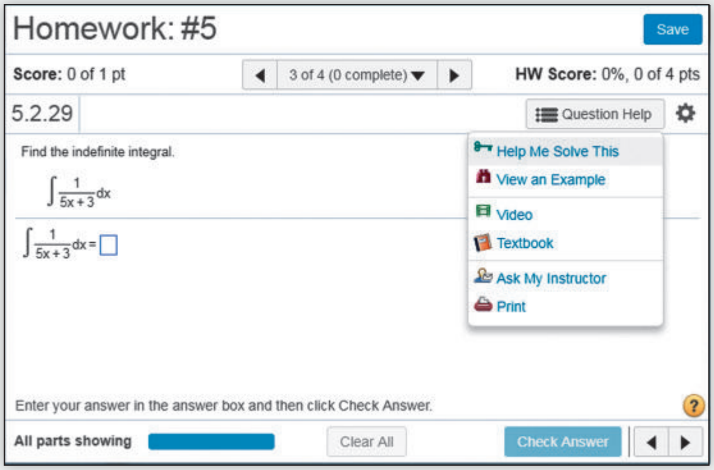
Study skills modules help students with the life skills that can make the difference between passing and failing.

DEVELOPING DEEPER UNDERSTANDING

MyLab Math provides content and tools that help students build a deeper understanding of course content than would otherwise be possible.

Exercises with Immediate Feedback

The approximately 3270 homework and practice exercises for this text regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Math provides helpful feedback when students enter incorrect answers and includes several optional learning aids: Help Me Solve This, View an Example, Video, and Textbook (links to the eText). All exercises have been reviewed and edited for this edition by author Gene Kramer.



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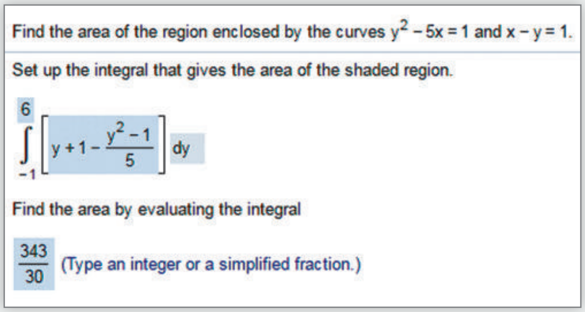
NEW! Setup and Solve Exercises

These exercises require students to show how they set up a problem as well as giving the solution, better mirroring what is required on tests.

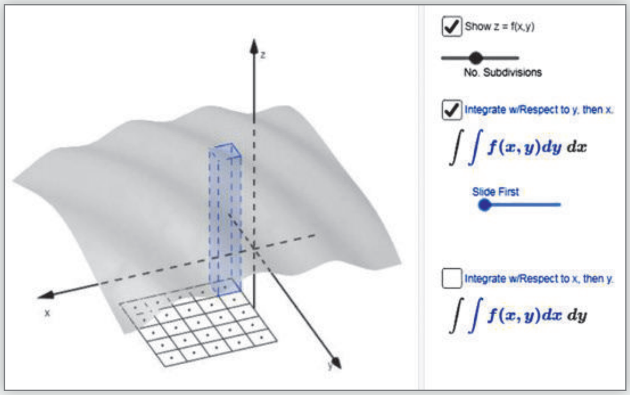


NEW! Additional Conceptual Questions

Additional Conceptual Questions provide support for assessing concepts and vocabulary. Many of these questions are application-oriented. They are clearly labeled ACQ in the Assignment Manager.

EXPANDED! Instructional Videos

About 90% of the instructional videos are brand-new. The new videos were made using the latest technology and feature authors Gene Kramer and Scott Surgent along with instructors Mary Ann Barber (University of North Texas) and Thomas Hartfield (University of North Georgia). In addition, there are MathTalk videos that highlight applications of the content of the course to business and are supported by assignable exercises.



EXPANDED! Interactive Figures

An expanded full suite of interactive figures illustrate key concepts and allow manipulation. They are designed to be used in lecture as well as by students independently.

NEW! Enhanced Sample Assignments

These assignments include just-in-time prerequisite review, help keep skills fresh with spaced practice of key concepts, and provide opportunities to work exercises without learning aids so that students can check their understanding. They are assignable and editable within MyLab Math.

UPDATED! Graphing Calculator and Excel Spreadsheet Manuals (downloadable)

Graphing Calculator Manual by Chris True, University of Nebraska
Excel Spreadsheet Manual by Stela Pudar-Hozo, Indiana University–Northwest
These manuals, both specific to this course, have been updated to support the TI-84 Plus CE (color display) and Excel 2016, respectively. Instructions are ordered by mathematical topic.

Student's Solutions Manual (softcover or downloadable)

ISBN: 0-13-516568-7 | 978-0-13-516568-3
The Student's Solutions Manual contains worked-out solutions to all the odd-numbered exercises and all Chapter Test exercises. This manual is available in print and can be downloaded from within MyLab Math.

A Complete eText

Students get unlimited access to the eText within any MyLab Math course using that edition of the textbook. The Pearson eText app allows existing subscribers to access their titles on an iPad or Android tablet for either online or offline viewing.

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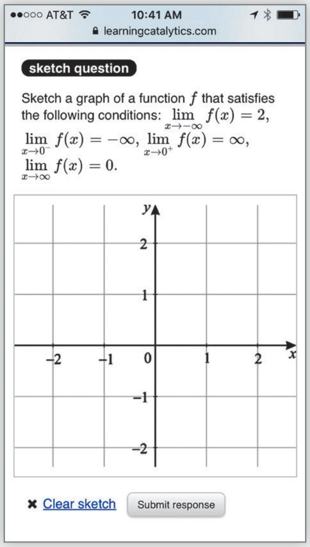
Pearson MyLab

SUPPORTING INSTRUCTION

MyLab Math comes from an experienced partner with educational expertise and an eye on the future. It provides resources to help you assess and improve students' results at every turn and unparalleled flexibility to create a course tailored to you and your students.



NEW! Learning Catalytics™

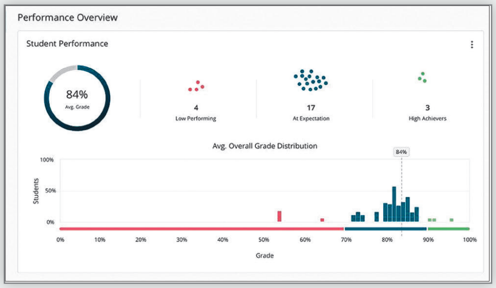
Now included in all MyLab Math courses, this student response tool uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking during lecture. Learning Catalytics fosters student engagement and peer-to-peer learning with real-time analytics. Access pre-built exercises created specifically for this course.

PowerPoint® Lectures (downloadable)

Slides contain presentation resources such as key concepts, examples, definitions, figures, and tables from this text. They can be downloaded from within MyLab Math or from Pearson's online catalog at **www.pearson.com**.

Comprehensive Gradebook

The gradebook includes enhanced reporting functionality such as item analysis and a reporting dashboard to allow you to efficiently manage your course. Student performance data is presented at the class, section, and program levels in an accessible, visual manner, providing the information you need to keep your students on track.



TestGen®

TestGen (**www.pearson.com/testgen**) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson's online catalog at **www.pearson.com**. The questions are also assignable in MyLab Math.

Instructor's Solutions Manual and NEW! Instructor's Answers (downloadable)

Solutions Manual ISBN: 0-13-518244-1 | 978-0-13-518244-4
Answers ISBN: 0-13-522497-7 | 978-0-13-522497-7

The Instructor's Solutions Manual contains worked-out solutions to all text exercises. The Instructor's Answers contain just the answers to exercises. Both of these can be downloaded *by instructors only* from within MyLab Math or from Pearson's online catalog at **www.pearson.com**.

Accessibility

Pearson works continuously to ensure our products are as accessible as possible to all students. We are working toward achieving WCAG 2.0 Level AA and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media, **www.pearson.com/mylab/math/accessibility**. Please see the "Accessibility" tab within the MyLab Math course for accessible student resources for this text.

pearson.com/mylab/math

Prerequisite Skills Diagnostic Test

To the Student and the Instructor

This diagnostic test can be used to assess student needs for this course. Part A covers algebra concepts discussed in Appendix A. Part B covers topics discussed in Chapter R, most of which come from a course in intermediate or college algebra. (This diagnostic test does not cover regression, though it is considered in Section R.7 and used throughout the text.) Students who have difficulty with the questions in part A should study Appendix A before moving to Chapter R. Those who have difficulty with the questions in part B should study Chapter R. Students who miss just a few questions might study the related topics in either Appendix A or Chapter R before continuing with the calculus chapters.

Part A: *Answers and locations of step-by-step solutions appear on p. A-1.*

Write an equivalent expression for each of the following without an exponent.

1. 4^3 **2.** $(-2)^5$ **3.** $\left(\frac{1}{2}\right)^3$ **4.** $(-2x)^1$ **5.** e^0

Write an equivalent expression for each of the following without a negative exponent.

6. x^{-5} **7.** $\left(\frac{1}{4}\right)^{-2}$ **8.** t^{-1}

Multiply.

9. $x^5 \cdot x^6$ **10.** $x^{-2} \cdot x^9$ **11.** $2x^{-3} \cdot 5x^{-4} \cdot 4x^{10}$

Divide.

12. $\dfrac{a^3}{a^2}$ **13.** $\dfrac{b^3}{b^{-5}}$

Simplify. Express each answer without a negative exponent.

14. $(x^{-2})^3$ **15.** $(2x^4y^{-5}z^3)^{-3}$

Multiply.

16. $3(x - 5)$ **17.** $(x - 5)(x + 3)$

18. $(a + b)(a + b)$ **19.** $(2x - t)^2$

20. $(3c + d)(3c - d)$

Factor.

21. $2xh + h^2$ **22.** $x^2 - 6xy + 9y^2$

23. $x^2 - 5x - 14$ **24.** $6x^2 + 7x - 5$

25. $x^3 - 7x^2 - 4x + 28$

Solve.

26. $-\frac{5}{6}x + 10 = \frac{1}{2}x + 2$ **27.** $3x(x - 2)(5x + 4) = 0$

28. $4x^3 = x$ **29.** $\dfrac{2x}{x-3} - \dfrac{6}{x} = \dfrac{18}{x^2 - 3x}$

30. $17 - 8x \geq 5x - 4$

31. After a 5% gain in weight, a grizzly bear weighs 693 lb. What was the bear's original weight?

32. Raggs, Ltd., a clothing firm, determines that its total revenue, R, in dollars, from the sale of x suits is given by $R = 350x + 500$. Find the number of suits that must be sold so that total revenue is more than \$70,050.

Part B: *Answers and locations of step-by-step solutions appear on p. A-1.*

Graph.

1. $y = 2x + 1$ **2.** $3x + 5y = 10$

3. $y = x^2 - 1$ **4.** $x = y^2$

5. A function g is given by $g(x) = 3x^2 - 2x + 8$. Find each of the following: $g(0)$, $g(-5)$, and $g(7a)$.

6. A function f is given by $f(x) = 3x - 12$. Find all x such that $f(x) = 0$.

7. Graph the function f:

$$f(x) = \begin{cases} 4, & \text{for } x \leq 0, \\ 3 - x^2, & \text{for } 0 < x \leq 2, \\ 2x - 6, & \text{for } x > 2. \end{cases}$$

8. Write interval notation for $\{x \mid -4 < x \leq 5\}$.

9. Find the domain of f: $f(x) = \dfrac{3}{2x - 5}$.

10. Find the slope and y-intercept of the graph of $2x - 4y - 7 = 0$.

11. Find an equation of the line with slope 3 containing the point $(-1, -5)$.

12. Find the slope of the line containing the points $(-2, 6)$ and $(-4, 9)$.

13. Solve for x: $\log_2 x = 32$. **14.** Solve for x: $7^x = \dfrac{1}{49}$.

Graph.

15. $f(x) = x^2 - 2x - 3$ **16.** $g(x) = x^3$

17. $f(x) = \dfrac{1}{x}$ **18.** $f(x) = |x|$

19. $f(x) = -\sqrt{x}$

20. Suppose that \$1000 is earning 5% interest, compounded annually. How much is the investment worth at the end of 2 yr?

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R Functions, Graphs, and Models

What You'll Learn

R.1 Graphs and Equations
R.2 Functions and Models
R.3 Finding Domain and Range
R.4 Slope and Linear Functions
R.5 Nonlinear Functions and Models
R.6 Exponential and Logarithmic Functions
R.7 Mathematical Modeling and Curve Fitting

Why It's Important

This chapter introduces functions and their notation, graphs, and applications. Also presented are many topics considered often throughout the text: supply and demand; total cost, revenue, and profit; the concept of a mathematical model; and curve fitting.

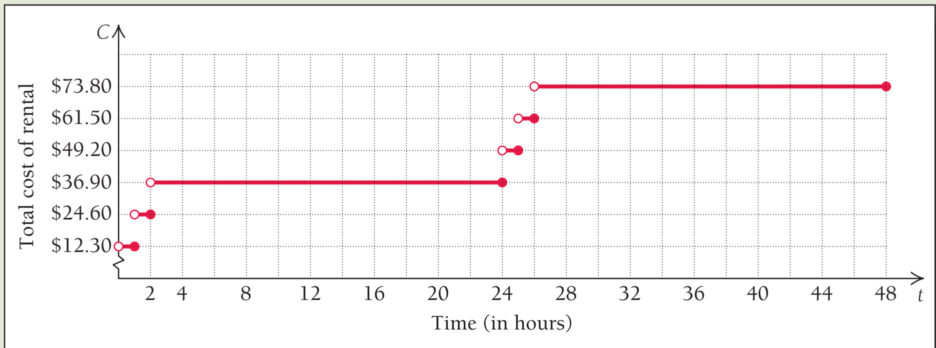
Skills in using a graphing calculator and other forms of technology are introduced and discussed in the Technology Connections. Details on keystrokes are given in the *Graphing Calculator Manual* (GCM).

Part A of the diagnostic test (p. xvii), on basic algebra concepts, allows students to determine whether they need to review Appendix A (p. 615) before studying this chapter. Part B, on college algebra topics, assesses the need to study this chapter before the calculus chapters.

Where It's Used

Rental Car Rates: What would be the total cost of renting a minivan in Florida for 36 hr?

(This problem appears as Example 8 in Section R.3.)



Graphs and Equations

R.1

- Graph equations.
- Use graphs as mathematical models to make predictions.
- Carry out calculations involving compound interest.

What Is Calculus?

What is calculus? How does calculus differ from algebra? These are common questions at the start of a course like this. Consider the following problems that one might encounter in an algebra course:

- A hotel manager charges \$120 per room per night. In one night, she had total revenue of \$5160. How many rooms did she rent that night?
- Juanita's car is moving at 40 mi/hr (58.67 ft/sec) when she applies the brakes. The car's velocity, t seconds after Juanita applies the brakes, is given by $v = -1.197t^2 + 58.67$, where v is in feet per second and $0 \leq t \leq 7$. How fast is the car traveling at $t = 3$ sec?
- A cylindrical soup can has a volume of 250 cm^3. If its height is twice the length of its radius, find the dimensions of the can.

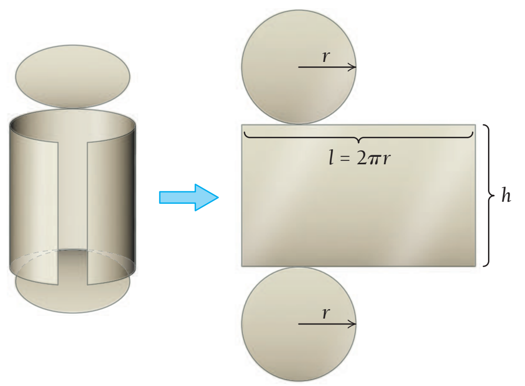
Now, consider similar problems one might encounter in a calculus course:

- At \$120 per room per night, a hotel manager typically rents 50 rooms each night. For every \$10 decrease in the price per room per night, she will rent 5 more rooms per night. Find the price per room that will maximize total nightly revenue.
- Juanita's car is moving at 40 mi/hr (58.67 ft/sec) when she applies the brakes. The car's velocity, t seconds after Juanita applies the brakes, is given by $v = -1.197t^2 + 58.67$, where v is in feet per second and $0 \leq t \leq 7$. How far does the car travel during the 7 sec it takes to come to a stop?
- A cylindrical soup can has a volume of 250 cm^3. If the cost of material for the top and bottom of the can is \$0.0008 per square centimeter, and the cost of material for the side is \$0.0015 per square centimeter, what dimensions minimize the cost of material for the can?

Problems about minimizing, maximizing, or finding a total accumulation of a quantity can be solved using calculus. A hotel manager might want to find the price that *maximizes* nightly revenue. A traffic engineer might want to determine a *total* distance traveled based on information about a vehicle's velocity. A manufacturer might want to find the dimensions that *minimize* the cost of producing a can.

Let's model this last problem using algebra. The combined area of the top and bottom of the soup can is $2\pi r^2$, and the area of the side is $2\pi rh$. Using the earlier information on material cost, the cost, C, of material for one can is

$$C = 0.0008(2\pi r^2) + 0.0015(2\pi rh),$$
$$= 0.0016\pi r^2 + 0.003\pi rh. \quad \text{Simplifying}$$



Each of the circular ends of the disassembled can has an area of πr^2, and the side has an area of $2\pi rh$.

The volume V of a circular cylinder with radius r and height h is given by $V = \pi r^2 h$. Using $V = 250\text{ cm}^3$, we have $h = \dfrac{250}{\pi r^2}$. Thus,

$$\begin{aligned} C &= 0.0016\pi r^2 + 0.003\pi r\left(\frac{250}{\pi r^2}\right) && \text{Replacing } h \text{ with } \frac{250}{\pi r^2} \\ &= 0.0016\pi r^2 + \frac{0.75}{r}. && \text{Simplifying} \end{aligned}$$

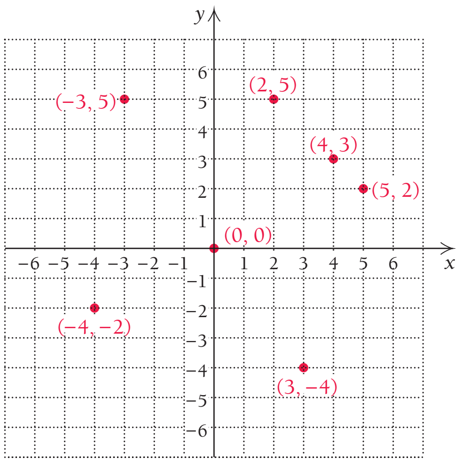
Using the above formula in a spreadsheet, we see how values of C, the cost of material for one can, vary with values of r, the radius, and from this, infer the value of r that results in the possible lowest cost C:

	A	B
1	radius	cost
2	r	C
33	3.5	0.275868914
34	3.6	0.273485845
35	3.7	0.271525071
36	3.8	0.269961189
37	3.9	0.268771404
38	4	0.267924772
39	4.1	0.267423105
40	4.2	0.26723974 ← Possible lowest cost
41	4.3	0.267359482
42	4.4	0.267768519
43	4.5	0.268454269
44	4.6	0.26941903
45	4.7	0.270625316
46	4.8	0.272076688
47	4.9	0.273764296
48	5	0.27568

When the radius of the can is 4.2 cm, the cost of material for the can is about \$0.26724. This appears to be the radius that results in the lowest cost. *But how can we be certain that no other dimensions will give a lower cost*? This is a question that algebra alone cannot answer. We need the tools of calculus to answer this. In Chapter 3, we study such maximum-minimum problems, including a complete solution to this one about the material cost for a can, which appears as Example 3 in Section 3.5.

Other topics we consider in calculus are the slope of a line touching a curve at a point, rates of change, areas under curves, accumulations of quantities, and statistical applications such as finding lines that best "fit" data (regression) and finding probabilities using certain distribution models.

Ordered Pairs and Graphs

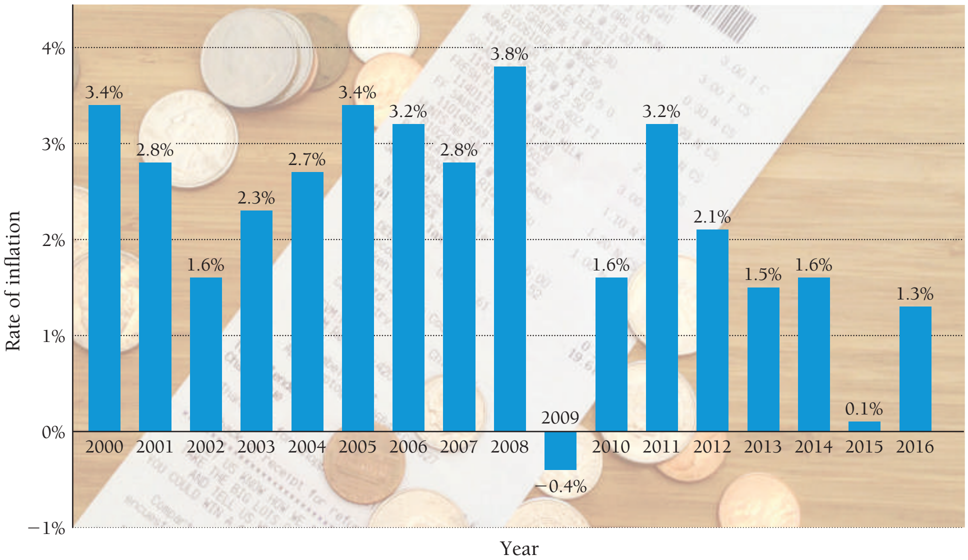


Each point in a plane corresponds to an ordered pair of numbers. Note in the figure at the right that the point corresponding to the pair (2, 5) is different from the point corresponding to the pair (5, 2). This is why we call a pair like (2, 5) an *ordered pair*. The first number is called the *first coordinate* of the point, and the second number is called the *second coordinate*. Together these are the *coordinates of the point*. The horizontal line is often labeled as the *x-axis*, and the vertical line is often labeled as the *y-axis*. The two axes intersect at the *origin*, (0, 0). The use of axes to display all points in the plane is often called the *xy-coordinate system*, the *xy-coordinate plane*, or simply the Cartesian plane.

Graphs

The study of graphs is an essential aspect of calculus. A graph allows us to visualize relationships between two variables. For instance, the graph below shows how the annual inflation rate in the United States changed from 2000 to 2016. A graph can be a powerful aid in understanding how change in one variable affects change in another.

ANNUAL INFLATION RATE IN THE UNITED STATES, 2000–2016



(*Source*: Bureau of Labor Statistics.)

Graphs of Equations

Equations such as $y = 2x - 1$ and $v = 125t^2 + 25$ are called *equations in two variables*. A *solution* of an equation in two variables is an ordered pair of numbers that, when substituted for the variables, forms a true statement. If not directed otherwise, we usually take the variables in *alphabetical* order. For example, $(-1, 2)$ is a solution of the equation $3x^2 + y = 5$, because when we substitute -1 for x and 2 for y, we get a true statement:

$$\begin{array}{r|l} 3x^2 + y & = 5 \\ \hline 3(-1)^2 + 2 & ?\ 5 \\ 3 + 2 & \ 5 \\ 5 & \ 5 \quad \text{TRUE} \end{array}$$

DEFINITION

The **graph** of an equation is a drawing that represents all ordered pairs that are solutions of the equation.

We obtain the graph of an equation by plotting enough ordered pairs (that are solutions) to see a pattern.

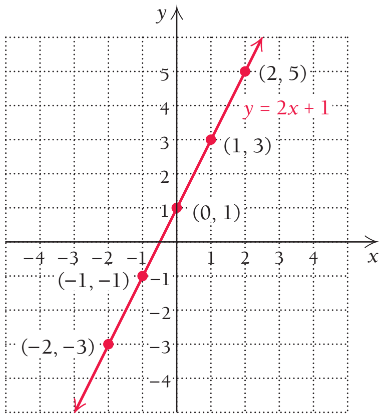
EXAMPLE 1 Graph: $y = 2x + 1$.

Solution We first find ordered pairs that are solutions and arrange them in a table. To find an ordered pair, we choose a number for x and then determine y.

For example, if we choose -2 for x and substitute that value in $y = 2x + 1$, we find that $y = 2(-2) + 1 = -4 + 1 = -3$. Thus, $(-2, -3)$ is a solution. We can select both negative numbers and positive numbers, as well as 0, for x.

x	y	(x, y)
-2	-3	$(-2, -3)$
-1	-1	$(-1, -1)$
0	1	$(0, 1)$
1	3	$(1, 3)$
2	5	$(2, 5)$

(1) Choose any x.
(2) Compute y.
(3) Form the pair (x, y).
(4) Plot the points.



Quick Check 1 ✓
Graph: $y = 3 - x$.

After we plot the points, we look for a pattern in the graph. In this case, the points suggest a line. We draw the line with a straightedge and label it $y = 2x + 1$. **1 ✓**

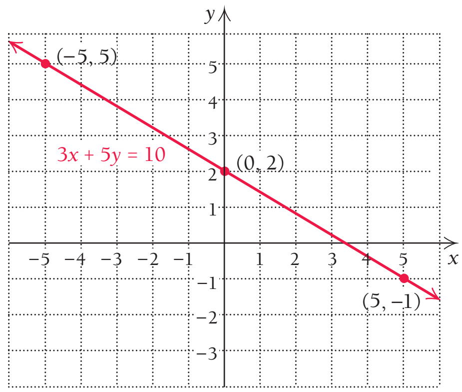
EXAMPLE 2 Graph: $3x + 5y = 10$.

Solution We could choose x-values, substitute, and determine corresponding y-values, but it is easier to first solve for y.*

$$3x + 5y = 10$$
$$3x + 5y - 3x = 10 - 3x \qquad \text{Subtracting } 3x \text{ from both sides}$$
$$5y = 10 - 3x \qquad \text{Simplifying}$$
$$\tfrac{1}{5} \cdot 5y = \tfrac{1}{5} \cdot (10 - 3x) \qquad \text{Multiplying both sides by } \tfrac{1}{5}$$
$$y = \tfrac{1}{5} \cdot (10) - \tfrac{1}{5} \cdot (3x) \qquad \text{Using the distributive law}$$
$$y = 2 - \tfrac{3}{5}x \qquad \text{Simplifying}$$
$$y = -\tfrac{3}{5}x + 2$$

Next we use $y = -\frac{3}{5}x + 2$ to find solutions that are ordered pairs. Choosing multiples of 5 for x avoids obtaining fractions as values of y.

x	y	(x, y)
0	2	$(0, 2)$
5	-1	$(5, -1)$
-5	5	$(-5, 5)$



Quick Check 2 ✓
Graph: $3x - 5y = 10$.

We plot the points, draw the line, and label the graph as shown. **2 ✓**

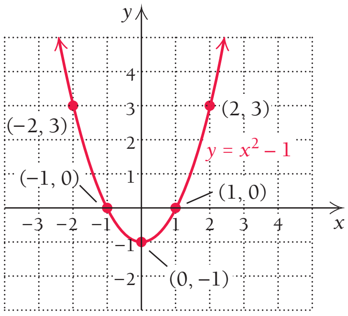
*Be sure to consult Appendix A, as needed, for a review of algebra.

Examples 1 and 2 show graphs of linear equations. Such graphs are considered in greater detail in Section R.4.

EXAMPLE 3 Graph: $y = x^2 - 1$.

Solution

x	y	(x, y)
-2	3	$(-2, 3)$
-1	0	$(-1, 0)$
0	-1	$(0, -1)$
1	0	$(1, 0)$
2	3	$(2, 3)$



Quick Check 3 ✓
Graph: $y = 2 - x^2$.

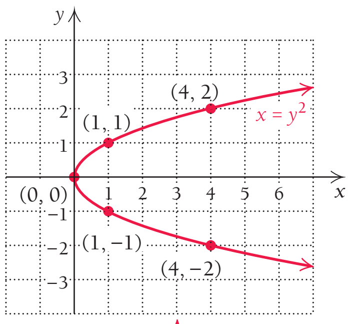
This time the pattern of the points is a curve called a *parabola*. We plot enough points to see a pattern and draw the graph. **3 ✓**

EXAMPLE 4 Graph: $x = y^2$.

Solution In this case, x is expressed in terms of the variable y, making it simpler to first choose numbers for y and then compute x.

x	y	(x, y)
4	-2	$(4, -2)$
1	-1	$(1, -1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
4	2	$(4, 2)$

(1) Choose any y.
(2) Compute x.
(3) Form the pair (x, y).
(4) Plot the points.



Quick Check 4 ✓
Graph: $x = 1 + y^2$.

We plot these points, keeping in mind that x is still the first coordinate and y the second. We look for a pattern and complete the graph by connecting the points. **4 ✓**

In calculus, we often study how the change in one variable affects the change in the other variable in the equation.

Mathematical Models

A real-world situation described using mathematics is called a **mathematical model**. For example, the speed at which a body falls due to gravity can be described using a mathematical model.

Mathematical models often allow us to predict what will happen in the real world. If the predictions are too inaccurate or if actual results do not conform to the model, the model must be changed or discarded.

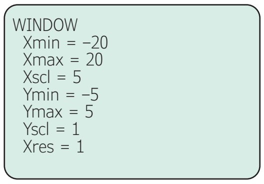
Technology Connection

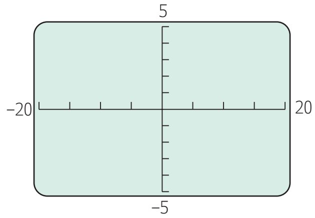
Introduction to the Graphing Calculator: Windows and Graphs

Viewing Windows

In this first of the optional Technology Connections, we begin to create graphs using a graphing calculator. Although some keystrokes are listed, exact keystrokes can be found in the owner's manual for your calculator or in the *Graphing Calculator Manual* (GCM) that accompanies this text.

The **viewing window** is the rectangular screen in which a graph appears. Windows are described by four numbers in the format [L, R, B, T], representing the Left and Right endpoints of the x-axis and the Bottom and Top endpoints of the y-axis. The WINDOW feature is used to set these dimensions. Below is a window setting of $[-20, 20, -5, 5]$ with axis scaling denoted as Xscl = 5 and Yscl = 1, which produces 5 units between tick marks from -20 to 20 on the x-axis and 1 unit between tick marks from -5 to 5 on the y-axis.

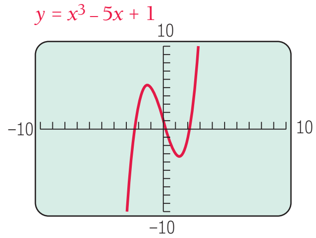




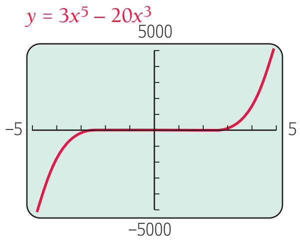
Scales should be chosen with care, since tick marks become indistinguishable when too many appear. On most graphing calculators, a window setting of $[-10, 10, -10, 10]$, Xscl = 1, Yscl = 1, Xres = 1 is considered **standard**.

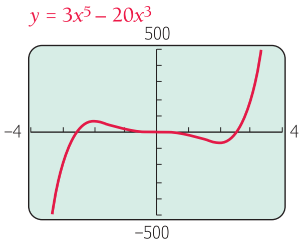
Graphs

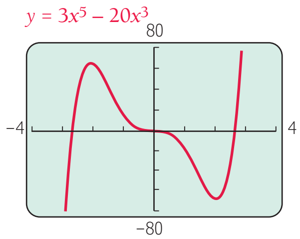
To graph the equation $y = x^3 - 5x + 1$, we press Y= and enter x^3−5x+1. We obtain the following graph in the standard viewing window.



It is often necessary to change viewing windows in order to get the best display of a graph. For example, each of the following is a graph of $y = 3x^5 - 20x^3$, but with a different viewing window. Which best displays the curvature of the graph?







Often, choosing a window that best reveals a graph's characteristics involves some trial and error and, in some cases, knowledge about the shape of the graph.

EXERCISE

Using your graphing calculator, reproduce the graphs shown in Examples 1, 2, and 3 of this section and those on this page. Adjust the viewing window in each case until you obtain a satisfactory graph.

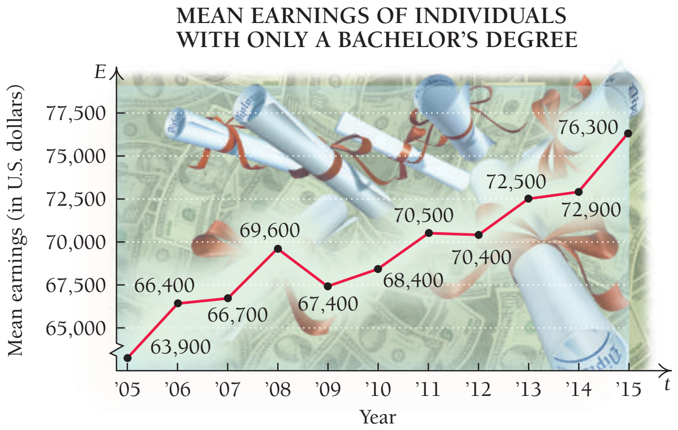
Creating a mathematical model is a process involving several steps.

CREATING A MATHEMATICAL MODEL

1. Recognize a real-world problem. → 2. Collect data. → 3. Analyze the data. → 4. Construct a model. → 5. Test and refine the model. → 6. Explain and predict.

Mathematical modeling is often an ongoing process. For example, finding a mathematical model that will accurately predict earnings is not simple. Most models will need to be reshaped as further information is acquired.

EXAMPLE 5 The following graph shows the mean (average) yearly earnings of individuals in the United States who have a bachelor's degree but no higher degree.



(*Source*: U.S. Census Bureau.)

Use the model $E = 1000t + 64{,}540$, where t is the number of years after 2005 and E is the yearly earnings, in dollars, to predict the yearly earnings in 2020 of an individual who holds a bachelor's degree but no higher degree.

Solution Since 2020 is 15 years after 2005, we substitute 15 for t:

$$E = 1000(15) + 64{,}540$$
$$= 15{,}000 + 64{,}540$$
$$= 79{,}540.$$

According to this model, the expected yearly earnings in 2020 of an individual with a bachelor's degree but no higher degree will be \$79,540. **5 ✓**

Quick Check 5 ✓

Using the model in Example 5, determine the year in which earnings of the individual will first exceed \$82,000.

As is the case with most models, the model in Example 5 is not perfect. For example, for $t = 10$, we get $E = \$74{,}540$, a number slightly different from the \$76,300 in the original data. But, for purposes of estimating, the model is adequate.

Compound Interest

One important model that is extremely precise involves **compound interest**. Suppose we invest P dollars at interest rate r, expressed as a decimal, and compounded annually. The amount A_1 in the account at the end of the first year is given by

$$A_1 = P + Pr = P(1 + r).$$

The original amount invested, P, is called the *principal*.

Going into the second year, we have $P(1 + r)$ dollars, so by the end of the second year, we will have the amount A_2 given by

$$A_2 = A_1 \cdot (1 + r) = [P(1 + r)](1 + r) = P(1 + r)^2.$$

Going into the third year, we have $P(1 + r)^2$ dollars, so by the end of the third year, we will have the amount A_3 given by

$$A_3 = A_2 \cdot (1 + r) = [P(1 + r)^2](1 + r) = P(1 + r)^3.$$

In general, we have the following theorem.

THEOREM 1

If an amount P is invested at interest rate r, expressed as a decimal, and compounded annually, then in t years it will grow to an amount A, where

$$A = P(1 + r)^t.$$

EXAMPLE 6 **Business: Compound Interest.** Suppose \$1000 is invested in Fibonacci Investment Fund at 5%, compounded annually. How much is in the account at the end of the second year?

Solution We substitute 1000 for P, 0.05 for r, and 2 for t in $A = P(1 + r)^t$ and get

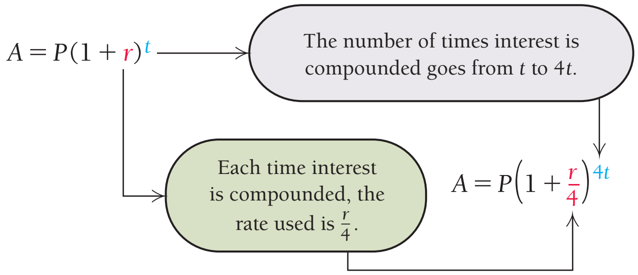
$$\begin{aligned} A &= 1000(1 + 0.05)^2 && \text{Substituting} \\ &= 1000(1.05)^2 && \text{Adding terms in parentheses} \\ &= 1000(1.1025) && \text{Squaring} \\ &= \$1102.50. && \text{Multiplying} \end{aligned}$$

There is \$1102.50 in the account after 2 yr. **6 ✓**

Quick Check 6 ✓

Business. Repeat Example 6 for an interest rate of 6%.

For interest compounded quarterly (four times per year), we can find a formula like the one above, as illustrated in the following diagram.



"Compounded quarterly" means that the given interest rate is divided by 4 and compounded four times per year. In general, the following theorem applies.

Compounding Frequency	n
Annually	1
Semiannually	2
Quarterly	4
Monthly	12
Daily	365
Hourly	8760

THEOREM 2

If a principal P is invested at interest rate r, expressed as a decimal, and compounded n times a year, in t years it grows to an amount A given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

EXAMPLE 7 **Business: Compound Interest.** Jan invests \$1000 in the Green View Fund at 2.75%, compounded quarterly. How much is in the account at the end of 3 yr?

Solution We use $A = P(1 + r/n)^{nt}$, substituting 1000 for P, 0.0275 for r, 4 for n (compounding quarterly), and 3 for t:

$$\begin{aligned} A &= 1000\left(1 + \frac{0.0275}{4}\right)^{4\cdot 3} && \text{Substituting} \\ &= 1000(1 + 0.006875)^{12} \\ &= 1000(1.006875)^{12} \\ &= 1000(1.085692139) && \text{Using a calculator to approximate } (1.006875)^{12} \\ &= 1085.692139 \\ &\approx \$1085.69. && \text{The symbol} \approx \text{means "approximately equal to."} \end{aligned}$$

There is \$1085.69 in the account after 3 yr. **7 ✓**

Quick Check 7 ✓

Business. Repeat Example 7 with a principal of \$25,000 and an interest rate of 3.25%, compounded monthly ($n = 12$), for 5 yr.

A calculator with a y^x or ^ key and a ten-digit display was used to find $(1.006875)^{12}$ in Example 7. The number of places on a calculator may affect the accuracy of the answer. Thus, you may occasionally find that your answers do not agree with those at the back of the text, which were found on a calculator with a ten-digit display. In general, when using a calculator, do all computations and round only at the end, as in Example 7. Usually, your answer will agree to at least four digits. It is a good idea to consult with your instructor about the accuracy required.

Section Summary

- Most graphs can be created by plotting points and looking for patterns. A graphing calculator creates graphs rapidly.
- Mathematical equations can serve as models in a wide variety of applications.
- An example of a mathematical model is the formula for compound interest. If P dollars are invested at interest rate r, compounded n times a year for t years, then the amount A in the account at the end of the t years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

R.1 Exercise Set

Exercises designated by the symbol are thinking and writing exercises. They should be answered using one or two English sentences. Because answers to many such exercises will vary, solutions are not given at the back of the book.

In Exercises 1–22, graph each equation. Use a graphing calculator only as a check.

1. $y = x - 1$

2. $y = x + 4$

3. $y = -\frac{1}{4}x$

4. $y = -3x$

5. $y = -\frac{5}{3}x + 3$

6. $y = \frac{2}{3}x - 4$

7. $x + y = 5$

8. $x - y = 4$

9. $6x + 3y = -9$

10. $8y - 2x = 4$

11. $2x + 5y = 10$

12. $5x - 6y = 12$

13. $y = x^2 - 5$

14. $y = x^2 - 3$

15. $x = y^2 + 2$

16. $x = 2 - y^2$

17. $y = 5$

18. $y = -2$

19. $y = 7 - x^2$

20. $y = 5 - x^2$

21. $y - 7 = x^3$

22. $y + 1 = x^3$

APPLICATIONS

23. Medicine. Ibuprofen is a medication used to relieve pain. The model given by

$$A = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t,\ 0 \le t \le 6,$$

can be used to estimate the number of milligrams, A, of ibuprofen in the bloodstream t hours after 400 mg of the medication has been swallowed. (*Source*: Based on data from Dr. P. Carey, Burlington, VT.) How many milligrams of ibuprofen are in the bloodstream 2 hr after 400 mg has been swallowed?

24. Running records. According to at least one study, the world record in any running race can be modeled by a linear equation. In particular, the world record R, in minutes, for the mile run in year x can be modeled by

$$R = -0.006x + 15.714.$$

Use this model to estimate the world records for the mile run in 1954, 2000, and 2025. Round your answers to the nearest hundredth of a minute.

25. Optimum solar panel angle. The optimum angle A, in degrees, to tilt a solar panel to capture the most sunlight is approximated by $A = -0.002x^2 + 0.924x - 0.152$, where x is the location of the panel in degrees latitude north of the equator. (*Source*: Based on data from solarpaneltilt.com.)

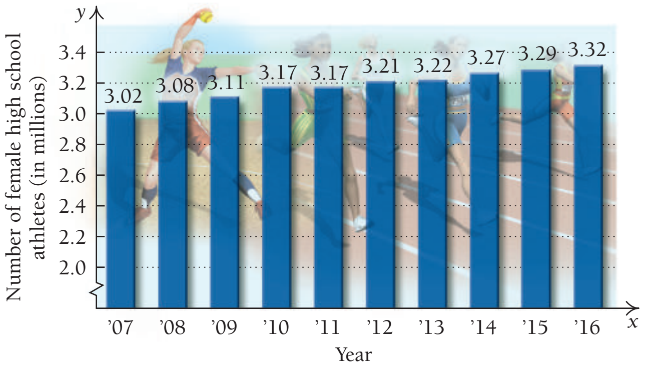
a) At what angle should a solar panel tilt in Honolulu (21.3° N)?
b) At what angle should a solar panel tilt in Kansas City (39.1° N)?
c) At what angle should a solar panel tilt in Edmonton (53.5° N)?

26. Rise in sea level. The rise in sea level t years after 1993 can be modeled by $S = -0.00173t^2 + 3.477t + 0.924$, where S is in millimeters. (*Source*: Based on data from climate.nasa.gov.)

a) How much had the sea level risen by 2003?
b) Estimate the rise in sea level in 2020.
c) When will sea level have risen 100 mm over the 1993 level?

27. The graph below shows participation by females in high school athletics from 2007 to 2016.

PARTICIPATION OF FEMALES IN HIGH SCHOOL ATHLETICS



(*Source*: National Federation of State High School Associations.)

a) Use the model $N = 0.0319t + 3.081$, where t is the number of years since 2007 and N is the number of participants, in millions, to predict the number of female high school athletes in 2020.

b) Use the model from part (a) to predict the year in which the number of female high school athletes exceeds 3.6 million.

28. Use the model $N = -0.0011t^2 + 0.0412t + 3.032$, where t is the number of years since 2007 and N is the number, in millions, of female high school athletes (see the graph in Exercise 27).

a) Use this model to predict the number of female high school athletes in 2020. Compare your answer to the number found in part (a) of Exercise 27.

b) Use this model to predict the number of female high school athletes in 2037.

c) Which of the two models better predicts the number of female high school athletes in the future? Why?

29. Snowboarding in the half-pipe. Shaun White, "The Flying Tomato," won a gold medal in the 2010 Winter Olympics for snowboarding in the half-pipe. He soared an unprecedented 25 ft above the edge of the half-pipe (which was still the world record in 2017). His speed $v(t)$, in miles per hour, upon reentering the pipe can be approximated by $v(t) = 10.9t$, where t is the number of seconds for which he was airborne. White was airborne for 2.5 sec. (*Source*: "White Rides to Repeat in Halfpipe, Lago Takes Bronze," Associated Press, 2/18/2010.) How fast was he going when he reentered the half-pipe?

30. Skateboard bomb drop. The distance $s(t)$, in feet, traveled by a body falling freely from rest in t seconds is approximated by

$$s(t) = 16t^2.$$

On April 6, 2006, pro skateboarder Danny Way smashed the world record for the "bomb drop" by free-falling 28 ft from the Fender Stratocaster guitar atop the Hard Rock Hotel & Casino in Las Vegas onto a ramp below (as of 2017 this was still the world record). (*Source*: www.skateboardingmagazine.com.) How long did it take until he hit the ramp?

31. Compound interest. Southside Investments purchases a \$100,000 certificate of deposit from Newton Bank, at 2.8%. How much is the investment worth (rounded to the nearest cent) at the end of 1 yr, if interest is compounded:

a) annually? **b)** semiannually?
c) quarterly? **d)** daily?
e) hourly?

32. Compound interest. Greenleaf Investments purchases a \$300,000 certificate of deposit from Descartes Bank, at 2.2%. How much is the investment worth (rounded to the nearest cent) at the end of 1 yr, if interest is compounded:

a) annually? **b)** semiannually?
c) quarterly? **d)** daily?
e) hourly?

33. Compound interest. Stateside Brokers deposit \$30,000 in Godel Municipal Bond Funds, at 4%. How much is the investment worth (rounded to the nearest cent) at the end of 3 yr, if interest is compounded:

a) annually? **b)** semiannually?
c) quarterly? **d)** daily?
e) hourly?

34. Compound interest. The Kims deposit \$1000 in Wiles Municipal Bond Funds, at 5%. How much is the investment worth (rounded to the nearest cent) at the end of 4 yr, if interest is compounded:

a) annually? **b)** semiannually?
c) quarterly? **d)** daily?
e) hourly?

Determining monthly loan payments. *If P dollars are borrowed at an annual interest rate r, expressed as a decimal, the payment M made each month for a total of n months is*

$$M = P\frac{\frac{r}{12}\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}.$$

35. Fermat's Last Bank makes a car loan of \$18,000, at 4.6% interest and with a loan period of 3 yr. What is the monthly payment?

36. At Haken Bank, Ken Appel takes out a \$100,000 mortgage at an interest rate of 2.4% for a loan period of 30 yr. What is the monthly payment?

Annuities. *If P dollars are invested annually in an annuity (investment fund), after n years, the annuity is worth*

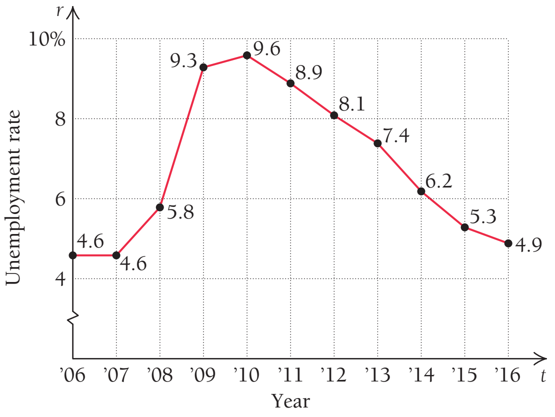
$$W = P\left[\frac{(1 + r)^n - 1}{r}\right],$$

where r is the interest rate, expressed as a decimal and compounded annually.

37. Kate invests \$3000 annually in an annuity from Mersenne Fund that earns 3.05% interest. How much is the investment worth after 18 yr? Round to the nearest cent.

38. Paulo establishes an annuity that earns $4\frac{1}{4}\%$ interest and wants it to be worth \$50,000 in 20 yr. How much will he need to invest annually to achieve this goal?

39. **Unemployment rate.** The unemployment rate in the United States from 2006 to 2016 is shown in the graph below.



(*Source*: Bureau of Labor Statistics.)

a) In what years was the unemployment rate at or above 5.8%?
b) In what years was the unemployment rate below 7%?
c) When was unemployment highest? What was the rate?
d) When was unemployment lowest? What was the rate?

40. Assuming that U.S. unemployment follows the trend shown between 2010 and 2016 (see Exercise 39), estimate the unemployment rate in 2019. Would you use this same trend to estimate the unemployment rate in 2025? Why or why not?

SYNTHESIS

Retirement account. *Sally makes deposits into a retirement account every year from age 30 until she retires at age 65.*

41. a) If Sally deposits \$1200 per year and the account earns interest at a rate of 4% per year, compounded annually, how much will she have in the account when she retires? (*Hint*: Use the annuity formula given for Exercises 37 and 38.)

b) How much of that total amount is from Sally's deposits? How much is interest?

42. a) Sally plans to take regular monthly distributions from her retirement account from the time she retires until she is 80 years old, when the account will have a value of \$0. How much should she take each month? Assume an interest rate of 4% per year, compounded monthly. (*Hint*: Use the formula given for Exercises 35 and 36 that calculates the monthly payments on a loan.)

b) What is the total of the payments Sally will receive? How much of the total will be her own money (see part b of Exercise 41), and how much will be interest?

Annual yield. *The annual interest rate r, when compounded more than once a year, results in a slightly higher yearly interest rate; this is called the* annual (or effective) yield *and denoted as Y. For example, \$1000 deposited at 5%, compounded monthly for 1 yr (12 months), will have a value given by* $A = 1000\left(1 + \frac{0.05}{12}\right)^{12} = \$1051.16$. *The interest earned is \$51.16/\$1000, or 0.05116, which is 5.116% of the original deposit. Thus, we say that this account has a yield of* $Y = 0.05116$, *or 5.116%. The formula for annual yield depends on the annual interest rate r and the compounding frequency n:*

$$Y = \left(1 + \frac{r}{n}\right)^n - 1.$$

For Exercises 43–46, find the annual yield as a percentage, to two decimal places, given the annual interest rate and the compounding frequency.

43. Annual interest rate of 5.3%, compounded monthly

44. Annual interest rate of 4.1%, compounded quarterly

45. Annual interest rate of 3.75%, compounded weekly

46. Annual interest rate of 4%, compounded daily

47. Lena is considering two savings accounts: Western Bank offers 2.5%, compounded annually, on savings accounts, while Commonwealth Savings offers 2.43%, compounded monthly.

a) Find the annual yield for both accounts.
b) Which account has the higher annual yield?

48. Chris is considering two savings accounts: Sierra Savings offers 3%, compounded annually, on savings accounts, while Foothill Bank offers 2.97%, compounded weekly.

a) Find the annual yield for both accounts.
b) Which account has the higher annual yield?

49. Stockman's Bank will pay 2.2%, compounded annually, on a savings account. A competitor, Mesalands Savings, offers monthly compounding on savings accounts. What is the minimum annual interest rate that Mesalands needs to pay to make its annual yield exceed that of Stockman's?

50. Belltown Bank offers a certificate of deposit at 3.75%, compounded annually. Shea Savings offers savings accounts with interest compounded quarterly. What is the minimum annual interest rate that Shea needs to pay to make its annual yield exceed that of Belltown?

Technology Connection

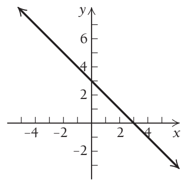
The Technology Connection heading indicates exercises that provide practice using a graphing calculator.

51. Graph $y = x^3 - x^2$ in the standard window, then in the window $[-2, 2, -2, 2]$. Which window shows better detail? Why?

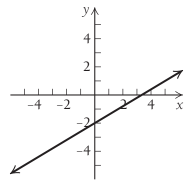
52. Graph $y = 2^x$ in the standard window. Does the graph actually touch the x-axis? Why or why not?

Answers to Quick Checks

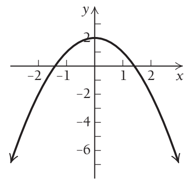
1. $y = 3 - x$



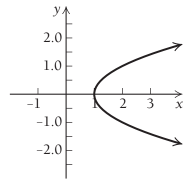
2. $3x - 5y = 10$



3. $y = 2 - x^2$



4. $x = 1 + y^2$



5. In 2022 **6.** There is \$1123.60 in the account after 2 yr. **7.** There is \$29,404.75 in the account after 5 yr.

R.2

- Determine whether a correspondence is a function.
- Find function values.
- Graph functions and determine whether a graph represents a function.
- Graph functions that are piecewise-defined.

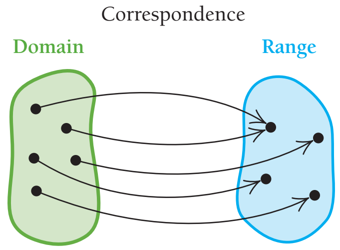
Functions and Models

Identifying Functions

A function is a special kind of correspondence between two sets that is a fundamental concept in mathematics. Consider the following situations:

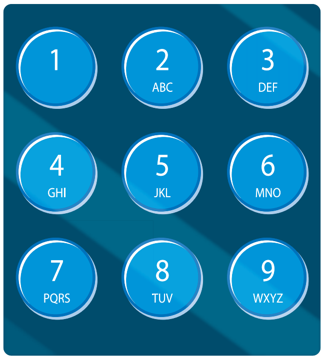
- For each weekday, there corresponds the closing value of the Dow Jones Industrial Average.
- For each student at a university, there corresponds the student's identification number.
- For each letter on a phone's keypad, there corresponds a number.
- For each real number x, there corresponds that number's square, x^2.

In each of these examples, the first set is called the *domain* and the second set is called the *range*. For any member of the domain, there is *exactly one* member of the range to which it corresponds (is matched). This type of correspondence is called a *function*.



DEFINITION

A **function** is a correspondence between a first set, called the **domain**, and a second set, called the **range**, such that each member of the domain corresponds to *exactly one* member of the range.



Each correspondence in the list on the preceding page is a function. On a phone's keypad, each of the 26 letters corresponds to exactly one number. For example, the letter H corresponds to the number 4. However, the reverse situation is not a function. The number 4 does not correspond to exactly one letter, since it corresponds to *three* letters: G, H, and I.

EXAMPLE 1 Determine whether or not each correspondence is a function.

a) *Number of iPhones sold yearly (in millions)*

Domain		Range
2014	→	169.2
2015	→	231.7
2016	→	211.9
2017	→	216.8

(*Source:* Apple Inc.)

b) *Squaring*

Domain		Range
3	→	9
4	→	16
5	→	25
−5	→	25

c) *Basketball teams*

Domain		Range
New York	→	Knicks
Los Angeles	→	Lakers
Los Angeles	→	Clippers
Atlanta	→	Hawks

Solution

a) The correspondence *is* a function because each member of the domain corresponds to only one member of the range.

b) The correspondence *is* a function because each member of the domain corresponds to only one member of the range, even though two members of the domain correspond to 25.

c) The correspondence *is not* a function because one member of the domain, Los Angeles, corresponds to two members of the range, Lakers and Clippers.

EXAMPLE 2 Determine whether or not each correspondence is a function.

	Domain	Correspondence	Range
a)	A group of people in an elevator	Each person's weight	A set of positive numbers
b)	The integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$	Each number's square	A set of nonnegative integers: $\{0, 1, 4, 9, 16, 25, \ldots\}$
c)	The set of all states	Each state's U.S. Senators	The set of all 100 U.S. Senators

Solution

a) The correspondence is a function because each person has *only one* weight.

b) The correspondence is a function because each integer has *only one* square.

c) The correspondence is *not* a function because each state has *two* U.S. Senators.

1 ✓

Quick Check 1 ✓

Determine whether or not each correspondence is a function.

a) The domain is the books in a library, the range is the set of positive integers, and the correspondence is the page count of each book.

b) The domain is the letters of the alphabet, the range is the set of all Americans, and the correspondence is the first letter of a person's last name.

Consistent with the definition on p. 13, we can regard a function as a set of ordered pairs, such that no two pairs have the same first coordinate paired with different second coordinates. When a function is written as a set of ordered pairs, the domain is the set of all first coordinates, and the range is the set of all second coordinates. Function names are usually represented by lowercase letters. Thus, if f represents the function in Example 2(b), we have

$$f = \{\ldots, (-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), \ldots\},$$

and

$$\text{domain of } f = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}; \quad \text{range of } f = \{0, 1, 4, 9, \ldots\}.$$

Finding Function Values

Functions can be described by equations such as $y = 2x + 3$ and $y = 4 - x^2$. To graph the function given by $y = 2x + 3$, we find ordered pairs by performing calculations for selected x values:

for $x = 4$, $y = 2 \cdot 4 + 3 = 11$; The graph includes (4, 11).

for $x = -5$, $y = 2 \cdot (-5) + 3 = -7$; The graph includes $(-5, -7)$.

for $x = 0$, $y = 2 \cdot 0 + 3 = 3$; and so on. The graph includes (0, 3).

The **inputs** (members of the domain) are the values of x substituted into the equation. The **outputs** (members of the range) are the resulting values of y. If we call the function f, we can use x to represent an arbitrary input and $f(x)$, read "f of x" or "f at x" or "the value of f at x," to represent the corresponding output. In this notation, the function given by $y = 2x + 3$ is written as $f(x) = 2x + 3$, and the calculations above can be written more concisely as

$$f(4) = 2 \cdot 4 + 3 = 11;$$

$$f(-5) = 2 \cdot (-5) + 3 = -7;$$

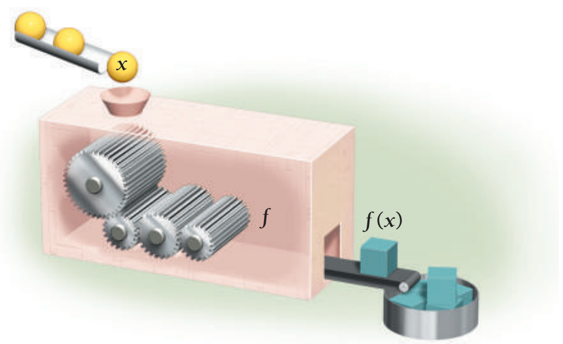
$$f(0) = 2 \cdot 0 + 3 = 3; \text{ and so on.}$$

Thus, instead of writing "when $x = 4$, the value of y is 11," we can simply write "$f(4) = 11$," which is most commonly read as "f of 4 is 11." Note that $f(4)$ *does not mean* "f times 4."

It helps to think of a function as a machine. Think of $f(4) = 11$ as the result of putting a member of the domain (an input), 4, into the machine. The machine knows the correspondence $f(x) = 2x + 3$, computes $2 \cdot 4 + 3$, and produces a member of the range (the output), 11.

Function: $f(x) = 2x + 3$

Input	Output
4	11
-5	-7
0	3
t	$2t + 3$
$a + h$	$2(a + h) + 3$



EXAMPLE 3 The squaring function f is given by $f(x) = x^2$. Find $f(-3)$, $f(1)$, $f(k)$, $f(\sqrt{k})$, and $f(x + h)$.

Solution We have

$$f(-3) = (-3)^2 = 9;$$
$$f(1) = 1^2 = 1;$$
$$f(k) = k^2 = k^2;$$
$$f(\sqrt{k}) = (\sqrt{k})^2 = k;$$
$$f(x + h) = (x + h)^2 = x^2 + 2xh + h^2.$$

For a review of algebra, see Appendix A .

To find $f(x + h)$, remember what the function does: It squares the input. Thus, $f(x + h) = (x + h)^2 = x^2 + 2xh + h^2$. This amounts to replacing x on both sides of $f(x) = x^2$ with $x + h$. **2 ✓**

Quick Check 2 ✓

A function f is given by $f(x) = 3x + 5$. Find $f(4)$, $f(-5)$, $f(0)$, and $f(x + h)$.

EXAMPLE 4 A function g is given by $g(x) = 3x^2 - 2x + 8$. Find $g(0)$, $g(-5)$, and $g(7a)$.

Solution One way to find function values when a formula is given is to think of the formula with blanks, or placeholders, as follows:

$$g(\square) = 3\cdot\square^2 - 2\cdot\square + 8.$$

To find an output for a given input, we think: "Whatever goes in the blank on the left goes in the blank(s) on the right."

$$g(0) = 3\cdot 0^2 - 2\cdot 0 + 8 = 8$$
$$g(-5) = 3(-5)^2 - 2\cdot(-5) + 8 = 3\cdot 25 + 10 + 8 = 75 + 10 + 8 = 93$$
$$g(7a) = 3(7a)^2 - 2(7a) + 8 = 3\cdot 49a^2 - 14a + 8 = 147a^2 - 14a + 8$$

3 ✓

Quick Check 3 ✓

A function h is given by $h(x) = 3x^2 + 2x - 7$. Find $h(4)$, $h(-5)$, $h(0)$, $h(a)$, and $h(5a)$.

Technology Connection

The TABLE Feature

The TABLE feature is one way to find ordered pairs of inputs and outputs of functions. Consider the function given by $f(x) = x^3 - 5x + 1$, entered as $y_1 = x^3 - 5x + 1$. To use the TABLE feature, we access the TABLE SETUP screen and enter the x-value at which the table will start and an increment for the x-value. For this equation, we let TblStart $= 0.3$ and ΔTbl $= 1$. This means that the table's x-values will start at 0.3 and increase by 1.

TABLE SETUP
TblStart = .3
ΔTbl = 1
Indpnt: Auto Ask
Depend: Auto Ask

We next set Indpnt and Depend to Auto and then press TABLE. The result is shown below.

X	Y1	
.3	−.473	
1.3	−3.303	
2.3	1.667	
3.3	20.437	
4.3	59.007	
5.3	123.38	
6.3	219.55	

X = .3

The arrow keys, ▲ and ▼, allow us to scroll up and down to view other values.

X	Y1	
12.3	1800.4	
13.3	2287.1	
14.3	2853.7	
15.3	3506.1	
16.3	4250.2	
17.3	5092.2	
18.3	6038	

X = 18.3

If we set Indpnt to Ask, leave Depend set to Auto, and press TABLE, we can enter any value for x.

EXERCISES

Use the function given by $f(x) = x^3 - 5x + 1$ for Exercises 1 and 2.

1. Use the TABLE feature to construct a table starting with $x = 10$ and ΔTbl $= 5$. Find the value of y when x is 10. Then find the value of y when x is 35.
2. Adjust the table settings to Indpnt: Ask. How does the table change? Enter a number of your choice and see what happens. Use this setting to find the value of y when x is 28.

EXAMPLE 5 A function f subtracts the square of an input from the input:

$$f(x) = x - x^2.$$

Find $f(4)$, $f(x+h)$, and $\dfrac{f(x+h)-f(x)}{h}$. Simplify your results.

Solution We have

$$f(4) = 4 - 4^2 = 4 - 16 = -12;$$

and

$$\begin{aligned} f(x+h) &= (x+h) - (x+h)^2 \\ &= x + h - (x^2 + 2xh + h^2) \qquad \text{Squaring the binomial} \\ &= x + h - x^2 - 2xh - h^2. \end{aligned}$$

For the expression $\dfrac{f(x+h)-f(x)}{h}$, we have

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{\overbrace{x + h - x^2 - 2xh - h^2}^{f(x+h),\text{ found above}} - \overbrace{(x - x^2)}^{f(x)}}{h} \\ &= \frac{h - 2xh - h^2}{h} \qquad \text{Simplifying} \\ &= \frac{h(1 - 2x - h)}{h} \qquad \text{Factoring} \\ &= 1 - 2x - h, \quad \text{for } h \neq 0. \end{aligned}$$

4 ✓

Quick Check 4 ✓

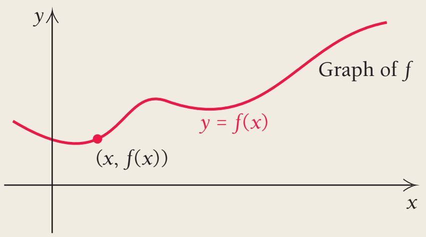
A function f is given by $f(x) = 2x - x^2$. Find $f(4)$, $f(x+h)$, and $\dfrac{f(x+h)-f(x)}{h}$. Simplify your results.

Graphs of Functions

Consider again the function given by $f(x) = x^2$. The input 3 is associated with the output 9. The input–output pair (3, 9) is one point on the *graph* of this function.

DEFINITION

The **graph** of a function f is a drawing that represents all the input–output pairs, $(x, f(x))$. When the function is given by an equation, the graph of the function is the graph of the equation, $y = f(x)$.



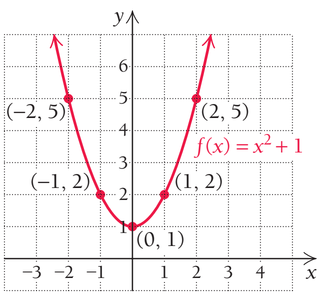
It is customary to locate input values (the domain) on the horizontal axis and output values (the range) on the vertical axis.

EXAMPLE 6 Graph: $f(x) = x^2 + 1$.

Solution

x	$f(x)$	$(x, f(x))$
−2	5	(−2, 5)
−1	2	(−1, 2)
0	1	(0, 1)
1	2	(1, 2)
2	5	(2, 5)

(1) Choose any x.
(2) Compute y.
(3) Form the pair (x, y).
(4) Plot the points.



Quick Check 5 ✓

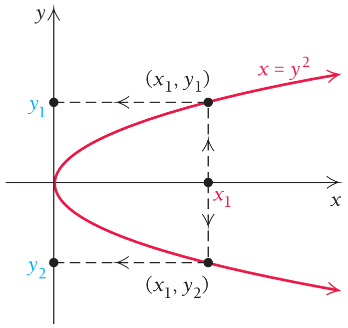
Graph: $f(x) = 2 - x^2$.

We plot the input–output pairs from the table and, in this case, draw a curve to complete the graph.

5 ✓

The Vertical-Line Test

Let's now determine how to look at a graph and decide whether it represents a function. In the graph at the right, note that x_1 has *two* outputs. Since a function must have exactly *one* output for every input, this graph cannot represent a function. The fact that a vertical line can intersect the graph in more than one place demonstrates that the graph does not represent a function.

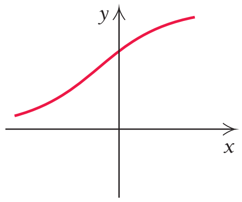


The Vertical-Line Test

A graph represents a function if it is impossible to draw a vertical line that intersects the graph more than once.

Equivalently, if a vertical line intersects a graph more than once, then the graph does not represent a function.

EXAMPLE 7 Determine whether each of the following is the graph of a function.

a)


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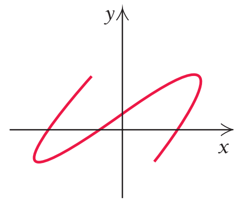
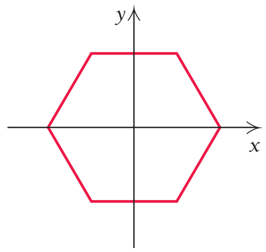
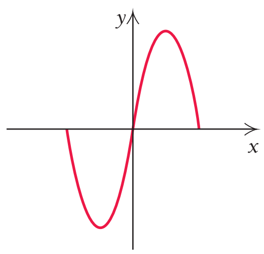
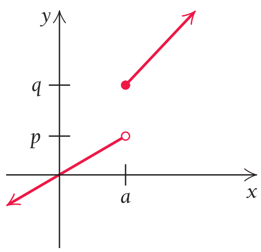
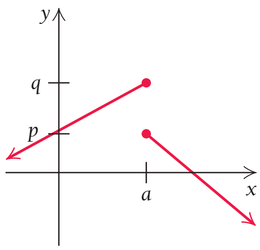


c)


d)


e)


f)


Solution

a) The graph is that of a function. Any vertical line that is drawn will intersect the graph no more than once.

b) The graph is not that of a function. Some vertical lines will intersect the graph more than once.

c) The graph is not that of a function.

d) The graph is that of a function.

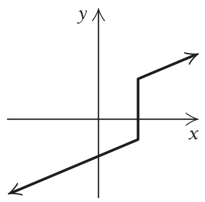
e) The graph is that of a function. Note that a vertical line at $x = a$ will intersect the graph once.

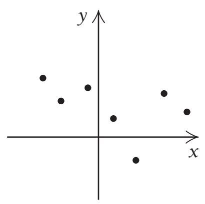
f) The graph is not that of a function. Note that a vertical line at $x = a$ will intersect the graph more than once.

In parts (e) and (f), an open dot represents an ordered pair that is not part of the graph, and a solid dot represents an ordered pair that is part of the graph. Thus, in part (e), the ordered pair (a, p) is not part of the graph, but the ordered pair (a, q) is. **6 ✓**

Quick Check 6 ✓

Determine whether each of the following is the graph of a function.

a)


b)


Functions Defined Piecewise

Some functions are defined *piecewise*, using different output formulas for different parts of the domain, as in parts (e) and (f) of Example 7. To graph a piecewise-defined function, we use the correspondence specified for each part of the domain.

EXAMPLE 8 Graph the function defined as follows:

$$f(x) = \begin{cases} 4, & \text{for } x \leq 0, \\ 3 - x^2, & \text{for } 0 < x \leq 2, \\ 2x - 6, & \text{for } x > 2. \end{cases}$$

This means that for any input x less than or equal to 0, the output is 4.

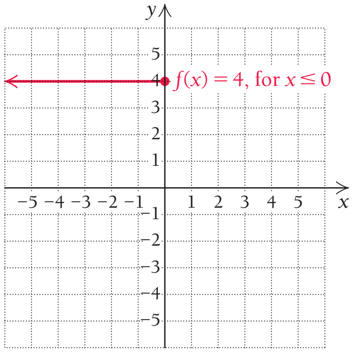
This means that for any input x greater than 0 and less than or equal to 2, the output is $3 - x^2$.

This means that for any input x greater than 2, the output is $2x - 6$.

Solution Working from left to right, note that for all x-values less than or equal to 0, the graph is the horizontal line $y = 4$. For example,

$$f(-2) = 4;$$
$$f(-1) = 4;$$
and $$f(0) = 4.$$

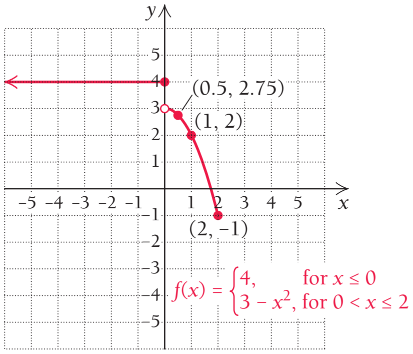
The solid dot at (0, 4) indicates that (0, 4) is part of the graph.



Next, observe that for x-values greater than 0 but not greater than 2, the graph is a portion of the parabola given by $y = 3 - x^2$. Note that for $f(x) = 3 - x^2$,

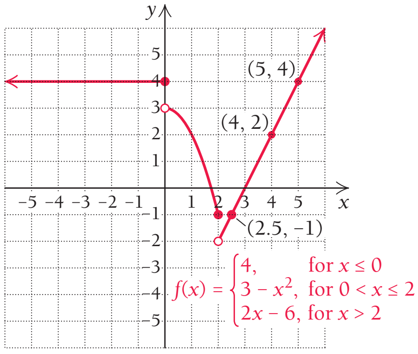
$$f(0.5) = 3 - 0.5^2 = 2.75;$$
$$f(1) = 2;$$
and $$f(2) = -1.$$

The open dot at (0, 3) indicates that this point is *not* part of the graph.



Finally, note that for x-values greater than 2, the graph is the line $y = 2x - 6$.

$$f(2.5) = 2 \cdot 2.5 - 6 = -1;$$
$$f(4) = 2;$$
and $$f(5) = 4.$$



7 ✓

Quick Check 7 ✓

Graph the function defined as follows:

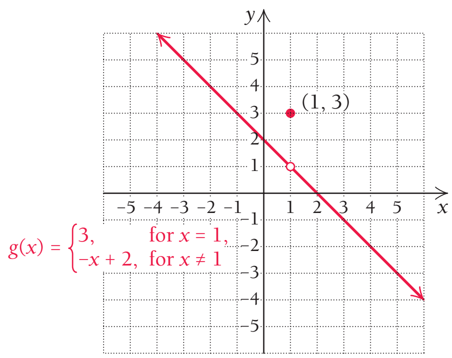
$$f(x) = \begin{cases} 4, & \text{for } x \leq 0, \\ 4 - x^2, & \text{for } 0 < x \leq 2, \\ 2x - 6, & \text{for } x > 2. \end{cases}$$

EXAMPLE 9 Graph the function defined as follows:

$$g(x) = \begin{cases} 3, & \text{for } x = 1, \\ -x + 2, & \text{for } x \neq 1. \end{cases}$$

Solution The function is defined such that $g(1) = 3$ and for all other x-values (that is, for $x \neq 1$), we have $g(x) = -x + 2$. Thus, to graph this function, we graph the line given by $y = -x + 2$, but with an open dot at the point corresponding to $x = 1$. To complete the graph, we plot the point $(1, 3)$ since $g(1) = 3$.

x	$g(x)$	$(x, g(x))$
-3	$-(-3) + 2$	$(-3, 5)$
0	$-0 + 2$	$(0, 2)$
1	3	$(1, 3)$
2	$-2 + 2$	$(2, 0)$
3	$-3 + 2$	$(3, -1)$



8 ✓

Quick Check 8 ✓

Graph the function defined as follows:

$$f(x) = \begin{cases} 1, & \text{for } x = -2, \\ 2 - x, & \text{for } x \neq -2. \end{cases}$$

Some Final Remarks

We sometimes use the terminology *y is a function of x*. This means that x is an input and y is an output. It also means that x is the **independent variable** because it represents inputs and y is the **dependent variable** because it represents outputs. We may refer to "a function $y = x^2$" without naming it using a letter f.

Section Summary

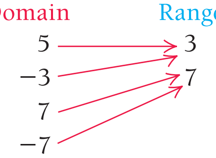
- A *function* is a correspondence between two sets such that for each member of the first set (the *domain*), there corresponds exactly one member of the second set (the *range*).
- A function's domain represents *inputs*, and its range represents *outputs*.
- A function given as an equation can be written using function notation: $y = f(x)$, where f is the name of the function. Ordered pairs are of the form $(x, f(x))$.

R.2 Exercise Set

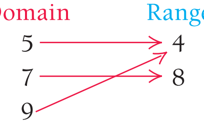
Note: A review of algebra can be found in Appendix A.

Determine whether each correspondence is a function.

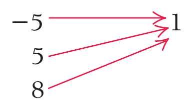
1.



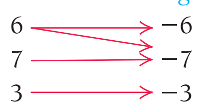
2.



3.



4.



5. Sandwich prices.

DOMAIN	RANGE
Hamburger	\$0.99
Cheeseburger	\$1.29
Filet-O-Fish®	\$3.49
Quarter Pounder® with cheese	\$3.69
Big Mac®	\$3.99
Crispy Chicken	\$4.29
Chicken McNuggets®	\$4.29
Double Quarter Pounder® with cheese	\$4.69

(*Source*: www.mcdonalds.com.)

6. **Sandwich calorie content.**

DOMAIN	RANGE
Hamburger →	250
Cheeseburger →	300
Quarter Pounder® →	410
Double Cheeseburger® →	440
Filet-O-Fish® →	380
Big Mac® →	540
Double Quarter Pounder® with cheese →	740

(*Source*: www.mcdonalds.com.)

Determine whether each of the following is a function.

	Domain	Correspondence	Range
7.	A set of numbers	A number doubled	A set of numbers
8.	A set of numbers	Two less than the number	A set of numbers
9.	A set of positive numbers	The square root of a number	A set of positive numbers
10.	A set of numbers	The cube root of a number	A set of numbers
11.	A set of positive numbers	Positive integers that are less than or equal to a number	A set of numbers
12.	A set of numbers	Odd integers that are less than or equal to a number	A set of numbers
13.	A set of books in a bookstore	A book's Library of Congress Catalog Number	A set of numeric codes
14.	A set of books in a bookstore	A book's ISBN	A set of numeric codes
15.	A set of people	A person's birthdate	A set of numerical codes of the form mm/dd/yyyy
16.	A set of people	A person's weight	A set of numbers
17.	A set of numerical codes of the form mm/dd/yyyy	A person's birthdate	A set of people
18.	A set of numbers	A person's weight	A set of people
19.	A set of rectangles	A rectangle's area	A set of numbers
20.	A set of rectangles	A rectangle's perimeter	A set of numbers
21.	A set of numbers	A rectangle with that area	A set of rectangles
22.	A set of numbers	A rectangle with that perimeter	A set of rectangles

23. A function f is given by

$$f(x) = 4x - 3.$$

This function takes a number x, multiplies it by 4, and subtracts 3.

a) Complete this table.

x	5.1	5.01	5.001	5
$f(x)$				

b) Find $f(4), f(3), f(-2), f(k)$, and $f(x + h)$.

24. A function f is given by

$$f(x) = 3x + 2.$$

This function takes a number x, multiplies it by 3, and adds 2.

a) Complete this table.

x	4.1	4.01	4.001	4
$f(x)$				

b) Find $f(5), f(-1), f(k)$, and $f(x + h)$.

25. A function g is given by

$$g(x) = x^2 - 3.$$

Find $g(-1), g(0), g(1), g(5), g(a + h)$, and

$$\frac{g(x + h) - g(x)}{h}.$$

26. A function g is given by

$$g(x) = x^2 + 4.$$

Find $g(-3), g(0), g(-1), g(7), g(a + h)$, and

$$\frac{g(x + h) - g(x)}{h}.$$

27. A function f is given by

$$f(x) = \frac{1}{(x + 3)^2}.$$

Find $f(4), f(0), f(a), f(x + h)$, and

$$\frac{f(x + h) - f(x)}{h}.$$

28. A function f is given by

$$f(x) = \frac{1}{(x - 5)^2}.$$

This function takes a number x, subtracts 5 from it, squares the result, and takes the reciprocal of the square. Find $f(3), f(-1), f(k)$, and $f(x + h)$.

29. A function f takes a number x, multiplies it by 4, and adds 2.

a) Write f as an equation. **b)** Graph f.

30. A function g takes a number x, multiplies it by -3, and subtracts 4.

a) Write g as an equation. **b)** Graph g.

31. A function h takes a number x, squares it, and adds x.

a) Write h as an equation.
b) Graph h.

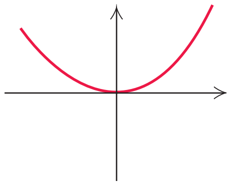
32. A function k takes a number x, squares it, and subtracts 3 times x.

a) Write k as an equation.
b) Graph k.

Graph each function.

33. $f(x) = 2x - 5$
34. $f(x) = 3x - 1$
35. $g(x) = -4x$
36. $g(x) = -2x$
37. $f(x) = x^2 - 2$
38. $f(x) = x^2 + 4$
39. $f(x) = 6 - x^2$
40. $g(x) = -x^2 + 1$
41. $g(x) = x^3$
42. $g(x) = \frac{1}{2}x^3$

Use the vertical-line test to determine whether each graph is that of a function. (In Exercises 51–55, the dashed lines are not part of the graphs.)

43.


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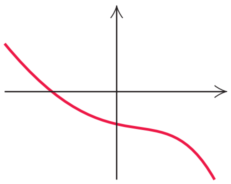

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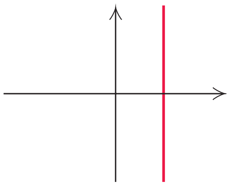

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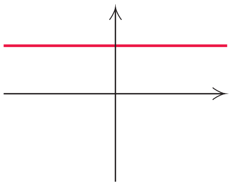

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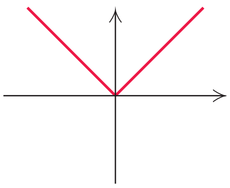

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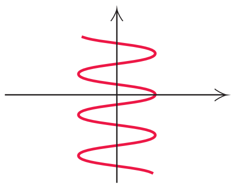

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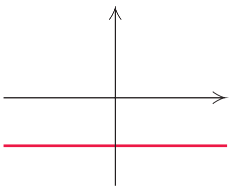

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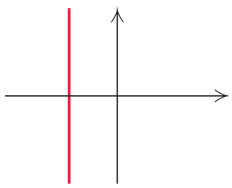
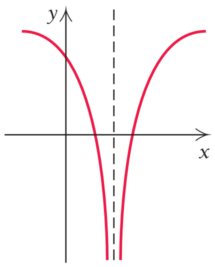
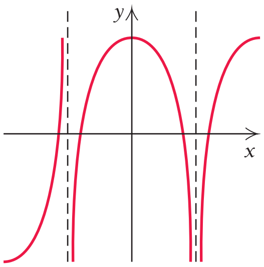


51.


52.


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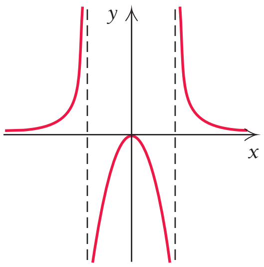

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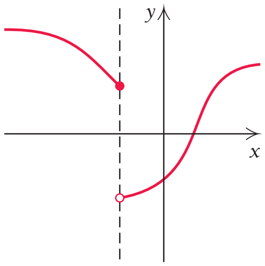
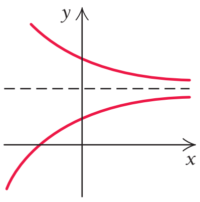


55.


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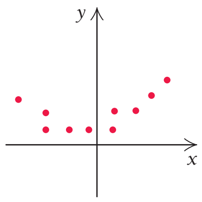


In Exercises 57 and 58, assume that x is the input and y is the output.

57. a) Graph $x = y^2 - 2$.
b) Is this a function? Why or why not?

58. a) Graph $x = y^2 - 3$.
b) Is this a function? Why or why not?

59. For $f(x) = x^2 - 3x$, find $\dfrac{f(x + h) - f(x)}{h}$.

60. For $f(x) = x^2 + 4x$, find $\dfrac{f(x + h) - f(x)}{h}$.

For Exercises 61–64, consider the function f given by

$$f(x) = \begin{cases} -2x + 1, & \text{for } x < 0, \\ 17, & \text{for } x = 0, \\ x^2 - 3, & \text{for } 0 < x < 4, \\ \frac{1}{2}x + 1, & \text{for } x \geq 4. \end{cases}$$

61. Find $f(-1)$ and $f(1)$.
62. Find $f(-3)$ and $f(3)$.
63. Find $f(0)$ and $f(10)$.
64. Find $f(-5)$ and $f(5)$.

Graph.

65. $f(x) = \begin{cases} 1, & \text{for } x < 0, \\ -1, & \text{for } x \geq 0 \end{cases}$

66. $f(x) = \begin{cases} 2, & \text{for } x \leq 3, \\ -2, & \text{for } x > 3 \end{cases}$

67. $f(x) = \begin{cases} 6, & \text{for } x = -2, \\ x^2, & \text{for } x \neq -2 \end{cases}$

68. $f(x) = \begin{cases} 5, & \text{for } x = 1, \\ x^3, & \text{for } x \neq 1 \end{cases}$

69. $f(x) = \begin{cases} -x, & \text{for } x < 0, \\ 4, & \text{for } x = 0, \\ x + 2, & \text{for } x > 0 \end{cases}$

70. $g(x) = \begin{cases} 2x - 3, & \text{for } x < 1, \\ 5, & \text{for } x = 1, \\ x - 2, & \text{for } x > 1 \end{cases}$

71. $g(x) = \begin{cases} \frac{1}{2}x - 1, & \text{for } x < 2, \\ -4, & \text{for } x = 2, \\ x - 3, & \text{for } x > 2 \end{cases}$

72. $g(x) = \begin{cases} x^2, & \text{for } x < 0, \\ -3, & \text{for } x = 0, \\ -2x + 3, & \text{for } x > 0 \end{cases}$

73. $f(x) = \begin{cases} -7, & \text{for } x = 2, \\ x^2 - 3, & \text{for } x \neq 2 \end{cases}$

74. $f(x) = \begin{cases} -6, & \text{for } x = -3, \\ -x^2 + 5, & \text{for } x \neq -3 \end{cases}$

Compound interest. *The amount of money, A(t), in a savings account that pays 3% interest, compounded quarterly for t years, with an initial investment of P dollars, is given by*

$$A(t) = P\left(1 + \frac{0.03}{4}\right)^{4t}.$$

75. If \$500 is invested at 3%, compounded quarterly, how much will the investment be worth after 2 yr?

76. If \$800 is invested at 3%, compounded quarterly, how much will the investment be worth after 3 yr?

Chemotherapy. *In computing the dosage for some chemotherapy patients, the measure of a patient's body surface area is needed. A good approximation of this area s, in square meters* (m^2), *is given by*

$$s = \sqrt{\frac{hw}{3600}},$$

where w is the patient's weight in kilograms (kg) and h is the patient's height in centimeters (cm). (Source: U.S. Oncology.) Use this information for Exercises 77 and 78. Round your answers to the nearest thousandth.

77. If a patient's height is 170 cm, approximate the patient's surface area assuming that:

a) The patient's weight is 70 kg.
b) The patient's weight is 100 kg.
c) The patient's weight is 50 kg.

78. If a patient's weight is 70 kg, approximate the patient's surface area assuming that:

a) The patient's height is 150 cm.
b) The patient's height is 180 cm.

79. **Business: monthly payments.** The table below shows the monthly payment for a \$20,000 auto loan at a rate of 2.5% for a term of 5, 6, or 7 yr.

Term (in years)	Interest Rate	Monthly Payment (in dollars)
5	0.025	354.95
6	0.025	299.42
7	0.025	259.78

a) If the inputs are the term and the outputs are the monthly payment, is this correspondence a function?
b) If the inputs are the monthly payment and the outputs are the term, is this correspondence a function?
c) If the input is the interest rate and the outputs are the monthly payments, is this correspondence a function?
d) If the inputs are the monthly payments and the output is the rate, is this correspondence a function?

80. **Scaling stress factors.** In psychology, a process called *scaling* is used to attach numerical ratings to a group of life experiences. In the following table, various events have been rated on a scale from 1 to 100 according to their stress levels.

Event	Scale of Impact
Death of spouse	100
Divorce	73
Jail term	63
Marriage	50
Lost job	47
Pregnancy	40
Death of close friend	37
Loan over \$10,000	31
Child leaving home	29
Change in schools	20
Loan less than \$10,000	17
Christmas	12

(*Source:* Thomas H. Holmes, University of Washington School of Medicine.)

a) Does the table represent a function? Why or why not?
b) What are the inputs? What are the outputs?

SYNTHESIS

Solve for y in terms of x, and determine if the resulting equation represents a function.

81. $2x + y - 16 = 4 - 3y + 2x$

82. $2y^2 + 3x = 4x + 5$

83. $(4y^{2/3})^3 = 64x$

84. $(3y^{3/2})^2 = 72x$

85. Explain why the vertical-line test works.

86. A function f is given by

$$f(x) = |x - 2| + |x + 1| - 5.$$

Find $f(-3)$, $f(-2)$, $f(0)$, and $f(4)$.

Technology Connection

In Exercises 87 and 88, use the TABLE *feature to construct a table for the function under the given conditions.*

87. $f(x) = x^3 + 2x^2 - 4x - 13$; TblStart $= -3$; ΔTbl $= 2$

88. $f(x) = \dfrac{3}{x^2 - 4}$; TblStart $= -3$; ΔTbl $= 1$

89. Graph the function in each of Exercises 87 and 88.

90. Use the TRACE feature to find several ordered-pair solutions of the function $f(x) = \sqrt{10 - x^2}$.

91. A function f takes a number x, adds 2, and then multiplies the result by 5, while a function g takes a number x, multiplies it by 5, and then adds 2.

- **a)** Express f and g as equations.
- **b)** Graph f and g on the same axes.
- **c)** Are f and g the same function?

92. A function f takes a number x, subtracts 4, and then squares the result, while a function g takes a number x, squares it, and then subtracts 4.

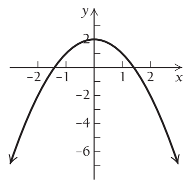
- **a)** Express f and g as equations.
- **b)** Graph f and g on the same axes.
- **c)** Are f and g the same function?

93. A function f takes a number x, multiplies it by 3, and then adds 6, while a function g takes a number x, adds h to it, and then multiplies the result by 3. Find h if f and g are the same function.

94. A function f takes a number x, adds 3, and then squares the result, while a function g takes a number x, squares it, adds 6 times x, and then adds h to the result. Find h if f and g are the same function.

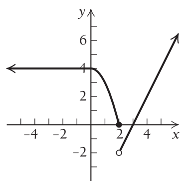
Answers to Quick Checks

1. (a) The correspondence is a function since each book has one page count associated with it.
(b) The correspondence is not a function since more than one person will have a last name that starts with any particular letter. That is, each letter has more than one last name associated with it.
2. $17, -10, 5, 3x + 3h + 5$
3. $49, 58, -7, 3a^2 + 2a - 7, 75a^2 + 10a - 7$
4. $-8, 2x + 2h - x^2 - 2xh - h^2, 2 - 2x - h$
5. $f(x) = 2 - x^2$

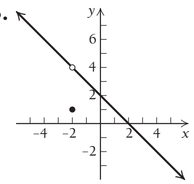


6. (a) The graph is not that of a function. **(b)** The graph is that of a function.

7.



8.



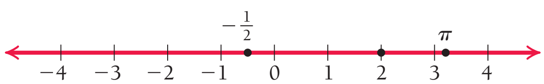
R.3 Finding Domain and Range

- Write interval notation for a set of points.
- Find the domain and the range of a function.

Set Notation

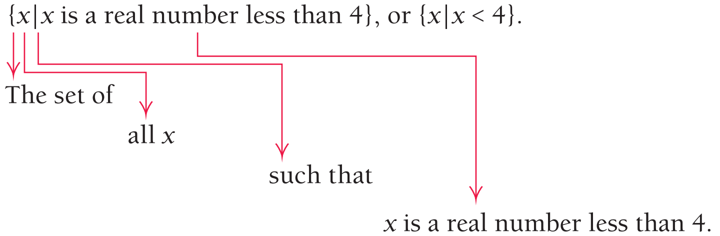
A **set** is any collection of objects, such as numbers. The set of numbers we consider most often in calculus is the set of **real numbers**, denoted $\mathbb{R}$. The real numbers can be represented by a line, called the **real number line**. Every point on the line represents a real number.

For example, the set consisting of 2, $-\frac{1}{2}$, and π can be written $\{2, -\frac{1}{2}, \pi\}$. On the real number line, a dot is placed at each number's location:



The method of describing a set by listing its members within braces $\{\ \}$ is known as the **roster method**. Larger sets can be described using **set-builder notation**, which specifies conditions for which an object is a member of the set. For example, the set of all real numbers less than 4 can be described in set-builder notation as follows:

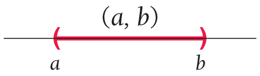
$\{x | x \text{ is a real number less than } 4\}$, or $\{x | x < 4\}$.



Interval Notation

We can also describe certain sets using **interval notation**. If a and b are real numbers, with $a < b$, we define the interval (a, b) as the set of all numbers between but not including a and b, that is, the set of all x for which $a < x < b$. Thus,

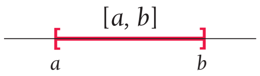
the set $(a, b) = \{x | a < x < b\}$
is displayed on the number line as



The points a and b are the **endpoints** of the interval. The parentheses indicate that the endpoints are *not* included in the interval.

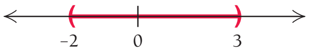
The interval $[a, b]$ is defined as the set of all x for which $a \leq x \leq b$. Thus,

the set $[a, b] = \{x | a \leq x \leq b\}$
is displayed on the number line as

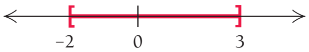


The brackets indicate that the endpoints *are* included in the interval.*

Do not confuse the *interval* (a, b) with the *ordered pair* (a, b) used to represent a point in the plane, as in Section R.1. The context in which the notation appears makes the meaning clear.



An open interval

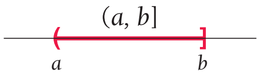


A closed interval

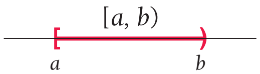
Intervals like $(-2, 3)$, in which neither endpoint is included, are called **open intervals**; intervals like $[-2, 3]$, which include both endpoints, are called **closed intervals**. Thus, (a, b) is read "the open interval a, b," and $[a, b]$ is read "the closed interval a, b."

Intervals that are **half-open** include one endpoint but not the other:

$(a, b] = \{x | a < x \leq b\}$. The graph excludes a and includes b.

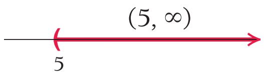


$[a, b) = \{x | a \leq x < b\}$. The graph includes a and excludes b.



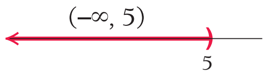
Intervals may extend without bound in one or both directions. We use the symbols ∞, read "infinity," and $-\infty$, read "negative infinity," to describe these intervals. The notation $(5, \infty)$ represents the set of all real numbers greater than 5. That is,

$(5, \infty) = \{x | x > 5\}$.



Similarly, the notation $(-\infty, 5)$ represents the set of all real numbers less than 5. That is,

$(-\infty, 5) = \{x | x < 5\}$.



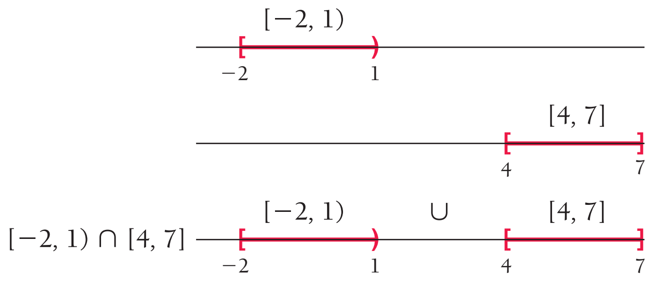
The notations $[5, \infty)$ and $(-\infty, 5]$ are used when we want to include an endpoint. The interval $(-\infty, \infty)$ describes the set of all real numbers.

$(-\infty, \infty) = \{x | x \text{ is a real number}\}$

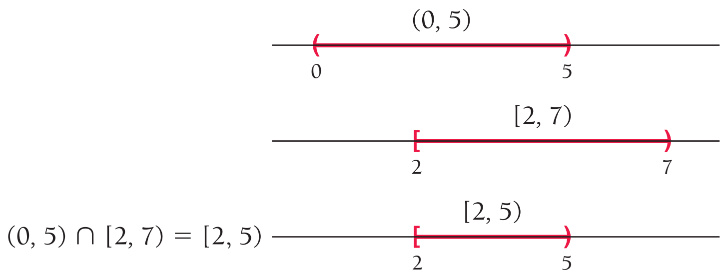
*The representations  and  are sometimes used instead of, respectively,  and .

Union and Intersection of Intervals

The *union* of two (or more) intervals consists of all real numbers x that are contained in one interval *or* the other *or* both. We use the symbol $\cup$ to represent the union of intervals. For example, $[-2, 1) \cup [4, 7]$ represents all real numbers x such that $-2 \le x < 1$ *or* $4 \le x \le 7$. That is,



The *intersection* of two (or more) intervals consists of all real numbers x that are contained in both (or all) intervals simultaneously. We use the symbol $\cap$ to represent the intersection of intervals. For example, $(0, 5) \cap [2, 7)$ represents all real numbers x such that $0 < x < 5$ *and* $2 \le x < 7$, and simplifies as the interval $[2, 5)$.



When two intervals do not contain any common elements, then their intersection is empty, denoted by $\varnothing$. For example, the intervals $[-2, 1)$ and $[4, 7]$ shown above do not have any real numbers in common (numbers that are in both intervals simultaneously), so $[-2, 1) \cap [4, 7] = \varnothing$. Such intervals are called *disjoint*.

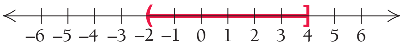
Interval notation is summarized in the table on the next page.

EXAMPLE 1 Write interval notation for each set or graph:

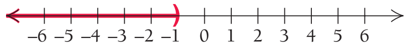
a) $\{x \mid -4 < x < 5\}$

b) $\{x \mid x \ge -2\}$

c)



d)



e) $\{x \mid -3 < x < 1 \text{ or } x \ge 2\}$

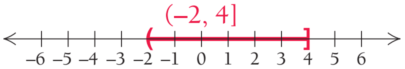
f) $\{x \mid x \le 2 \text{ and } x > 0\}$

Solution

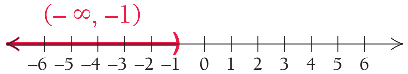
a) $\{x \mid -4 < x < 5\} = (-4, 5)$

b) $\{x \mid x \ge -2\} = [-2, \infty)$

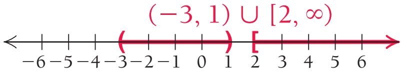
c) $(-2, 4]$



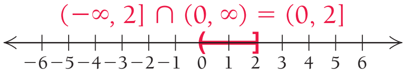
d) $(-\infty, -1)$



e) $(-3, 1) \cup [2, \infty)$



f) $(-\infty, 2] \cap (0, \infty) = (0, 2]$



1 ✓

Quick Check 1 ✓

Write interval notation for each set, and show it on a graph.

a) $\{x \mid -2 \le x \le 5\}$

b) $\{x \mid -2 \le x < 5\}$

c) $\{x \mid -2 < x \le 5\}$

d) $\{x \mid -2 < x < 5\}$

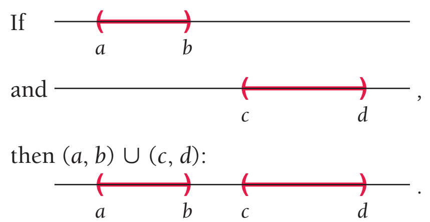
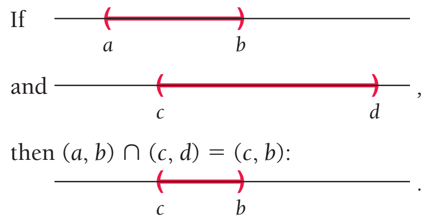
e) $\{x \mid x < 0 \text{ or } 2 \le x < 5\}$

f) $\{x \mid 2 < x < 5 \text{ and } 4 \le x\}$

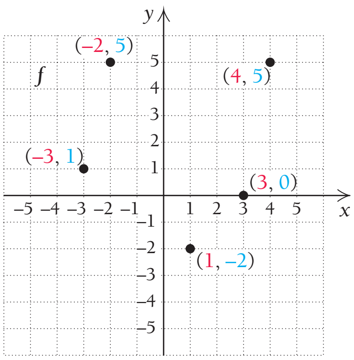
Finding Domain and Range

Recall that a set of ordered pairs in which no two different pairs share a common first coordinate is a function. The **domain** is the set of all first coordinates, and the **range** is the set of all second coordinates.

Intervals: Notation and Graphs

Interval Notation	Set Notation	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \ge a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\{x \mid x \text{ is a real number}\}$	
$(a, b) \cup (c, d)$	$\{x \mid a < x < b \text{ or } c < x < d\}$	
$(a, b) \cap (c, d)$	$\{x \mid a < x < b \text{ and } c < x < d\}$ If (a, b) and (c, d) have no common elements, then $(a, b) \cap (c, d) = \varnothing$.	

EXAMPLE 2 For the function f shown in the graph to the right, determine the domain and the range.



Solution This function consists of just five ordered pairs and can be written as

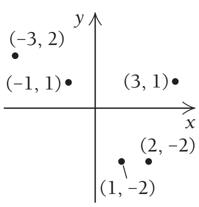
$$f = \{(-3, 1), (-2, 5), (1, -2), (3, 0), (4, 5)\}.$$

To determine the domain and the range, we read the x- and the y-values directly from the graph.

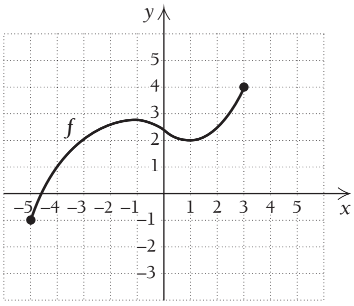
The domain is the set of all first coordinates, $\{-3, -2, 1, 3, 4\}$. The range is the set of all second coordinates, $\{-2, 0, 1, 5\}$. The ordering of the numbers within the set is not important, and repeated numbers are not listed. Although 5 appears twice as a second coordinate, we list it only once. **2 ✓**

Quick Check 2 ✓

For the function shown in the graph below, determine the domain and the range.



EXAMPLE 3 For the function f shown in the graph to the right, determine each of the following.



a) The number in the range that is paired with the input 1. That is, find $f(1)$.

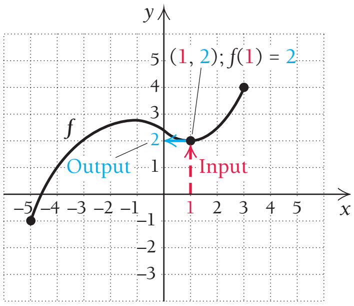
b) The domain of f

c) The number(s) in the domain that is (are) paired with the output 1. That is, find all x-values for which $f(x) = 1$.

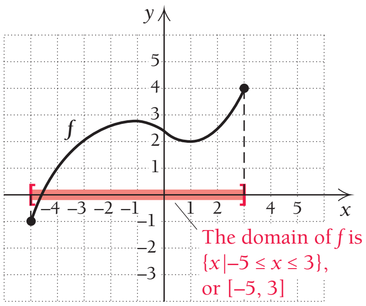
d) The range of f

Solution

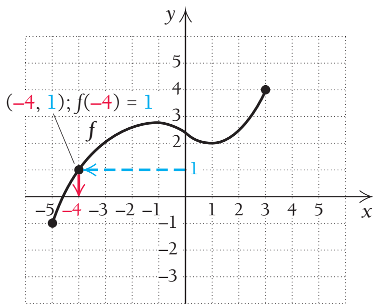
a) To determine which number is paired with the input 1, we locate 1 on the horizontal axis. Next, we identify the point on the graph of f for which 1 is the first coordinate. From that point, we look to the vertical axis to find the corresponding y-coordinate, 2. The input 1 has the output 2; that is, $f(1) = 2$.



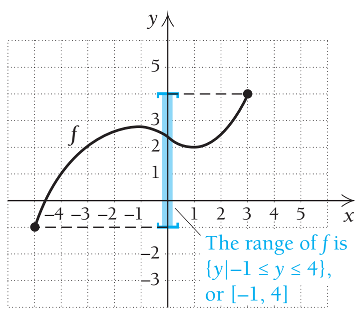
b) The domain of the function is the set of all x-values, or inputs, of the points on the graph. These extend from -5 to 3 and can be viewed as the curve's shadow on the x-axis. Thus, the domain is the set $\{x \mid -5 \le x \le 3\}$, or, in interval notation, $[-5, 3]$.



c) To determine which number(s) in the domain is (are) paired with the output 1, we locate 1 on the vertical axis. From there, we look left and right to the graph of f to identify any points for which 1 is the second coordinate. One such point exists: $(-4, 1)$. For this function, we note that $x = -4$ is the only member of the domain paired with the range value 1. There may be more than one member of the domain paired with a member of the range. For example, the output 2 appears to be paired with more than one input.



d) The range of the function is the set of all y-values, or outputs, of the points on the graph. These extend from -1 to 4 and can be viewed as the curve's shadow on the y-axis. Thus, the range is the set $\{y \mid -1 \le y \le 4\}$, or, in interval notation, $[-1, 4]$.



3 ✓

Quick Check 3 ✓

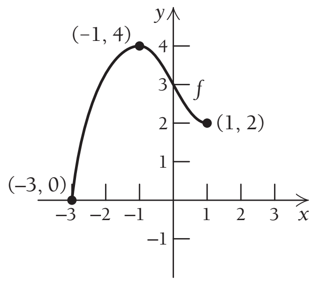
For the function f, shown in the graph below, determine each of the following:

a) $f(-1)$

b) $f(1)$

c) the domain

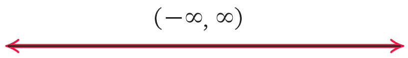
d) the range



When a function is given by an equation or formula, the domain is understood to be the largest set of real numbers (inputs) for which function values (outputs) can be calculated. That is, the domain is the set of all allowable inputs into the formula. To find the domain, we think, "For what input values does the function have an output?"

EXAMPLE 4 Find and graph the domain: $f(x) = 3x + 2$.

Solution Is there any number x for which we cannot calculate $3x + 2$? The answer is no. Thus, the domain of f is the set of all real numbers: $(-\infty, \infty)$. The graph of the domain of f is shown below.



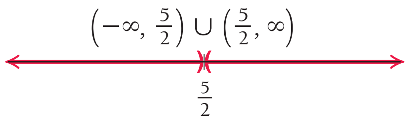
EXAMPLE 5 Find and graph the domain: $f(x) = \dfrac{3}{2x - 5}$.

Solution We recall that a denominator cannot equal zero. Since $3/(2x - 5)$ cannot be calculated when the denominator, $2x - 5$, is 0, we solve $2x - 5 = 0$ to find those real numbers that must be excluded from the domain of f:

$$2x - 5 = 0 \quad \text{Setting the denominator equal to 0}$$
$$2x = 5 \quad \text{Adding 5 to both sides}$$
$$x = \tfrac{5}{2}. \quad \text{Dividing both sides by 2}$$

Thus, $\frac{5}{2}$ is not in the domain, whereas all other real numbers are. We say that f *is not defined at* $\frac{5}{2}$, or $f(\frac{5}{2})$ *does not exist*.

The domain of f is $\{x \mid x \text{ is a real number } \textit{and } x \neq \frac{5}{2}\}$, or, in interval notation, $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$. The graph of the domain is shown below; only $\frac{5}{2}$ is not part of the domain.

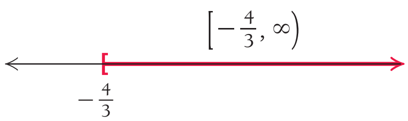


EXAMPLE 6 Find and graph the domain: $g(x) = \sqrt{4 + 3x}$.

Solution Since radicands in even roots cannot be negative, $\sqrt{4 + 3x}$ is not a real number when $4 + 3x$ is negative. The domain is all real numbers for which $4 + 3x \geq 0$. We find the domain by solving the inequality. (See Appendix A for a review of solving inequalities.)

$$4 + 3x \geq 0$$
$$3x \geq -4 \quad \text{Adding } -4 \text{ to both sides}$$
$$x \geq -\tfrac{4}{3} \quad \text{Dividing both sides by 3}$$

The domain is $\{x \mid -\frac{4}{3} \leq x < \infty\}$, or, in interval notation, $[-\frac{4}{3}, \infty)$. Its graph is shown below.



4 ✓

Quick Check 4 ✓

Find and graph the domain of each function.

a) $f(x) = \dfrac{5}{x - 8}$

b) $f(x) = \sqrt{2x - 8}$

EXAMPLE 7 Find and graph the domain: $h(x) = \sqrt{x} + \sqrt{3 - x}$.

Solution The radicands of both square root terms must be nonnegative. From the term $\sqrt{x}$, we see that x must be in the interval $[0, \infty)$, and from the term $\sqrt{3 - x}$, we see that x must be in the interval $(-\infty, 3]$. The domain of h is the set of real numbers x that are included in both intervals; that is, the domain is the intersection of the intervals $[0, \infty)$ and $(-\infty, 3]$. Thus, the domain of h is

$$(-\infty, 3] \cap [0, \infty) = [0, 3].$$

The graph of the domain is shown below.

The domain of $\sqrt{x}$ is $x \geq 0$, or the interval $[0, \infty)$.

The domain of $\sqrt{3 - x}$ is $x \leq 3$, or the interval $(-\infty, 3]$.

The domain for $h(x) = \sqrt{x} + \sqrt{3 - x}$ is the intersection of $[0, \infty)$ and $(-\infty, 3]$.

5 ✓

Quick Check 5 ✓

Find and graph the domain: $m(x) = \sqrt{x + 2} + \sqrt{4 - x}$.

Domains and Ranges in Applications

The domain and the range of a function given by a formula are sometimes affected by the context of an application. In such cases, we consider the function's practical domain and range. For example, for the function $y = 15x$, the domain is $\{x \mid x \text{ is any real number}\}$ and the range is $\{y \mid y \text{ is any real number}\}$. However, if this function represents the pay, y, for someone who works x hours at \$15 per hour, then negative values of x and y do not make sense. In this case, the function's practical domain is $\{x \mid x \geq 0\}$ and its practical range is $\{y \mid y \geq 0\}$.

EXAMPLE 8 **Rental Car Rates.** The hourly rate to rent a minivan in Florida is \$12.30 per hour or any part of an hour. For any rental period of more than 2 hr, the daily rate of \$36.90 applies, up to and including a maximum of 24 hr. This pricing also applies to subsequent 24-hr periods. Let $C(t)$ be the cost to rent a minivan for t hours. Disregard any extra taxes or surcharges. (*Sources*: avis.com; dms.myflorida.com.)

a) Find $C(18)$, and explain what this number represents.

b) Find $C(25.5)$, and explain what this number represents.

c) Find $C(30)$, and explain what this number represents.

d) Sketch the graph of C for $0 < t \leq 48$, and state the practical range of C.

Solution

a) Since 18 hr exceeds 2 hr and is less than 24 hr, we have $C(18) = \$36.90$, meaning that the charge to rent a minivan for 18 hr is the daily rate of \$36.90.

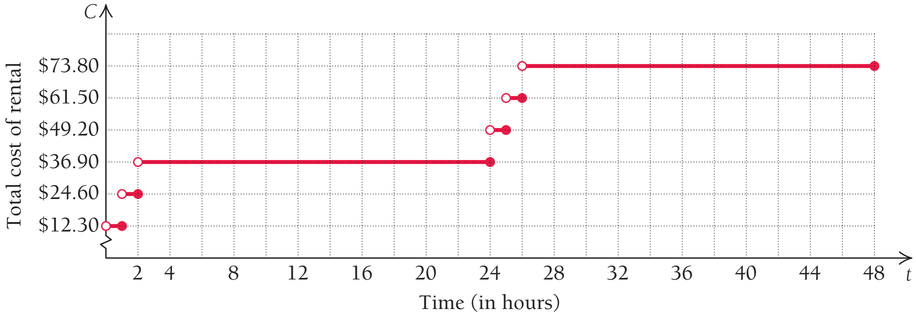
b) A rental period of 25.5 hr consists of a 24-hr period for which the daily rate is charged plus two 1-hr periods, since the extra 0.5 hr is considered part of a full second hour. Thus,

$$C(25.5) = \$36.90 + \$12.30 + \$12.30 = \$61.50.$$

c) A rental period of 30 hr consists of a 24-hr period for which the daily rate is charged plus an extra 6 hr for which the daily rate is charged again. Thus,

$$C(30) = \$36.90 + \$36.90 = \$73.80.$$

d) The graph of C, for $0 < t \leq 48$, is shown below.



Since the cost is in increments of \$12.30, we use the roster method to write the practical range of C, for $0 < t \leq 48$: $\{12.3, 24.6, 36.9, 49.2, 61.5, 73.8\}$.

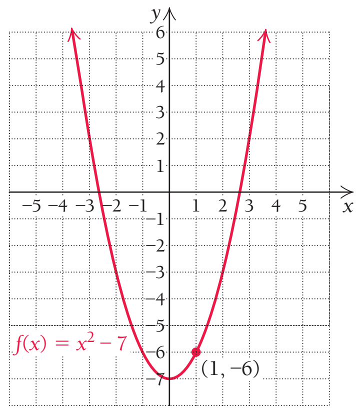
Section Summary

The following is a review of function terminology from Sections R.1–R.3.

Function Concepts

- Formula for f: $f(x) = x^2 - 7$
- For every input of f, there is exactly one output.
- For the input 1, the output is -6.
- For the output -3, the inputs are -2 and 2; $f(-2) = -3$ and $f(2) = -3$.
- The domain is the set of all inputs, or in this case, the set of all real numbers, $\mathbb{R}$.
- The range is the set of all outputs, or in this case, the interval $[-7, \infty)$.

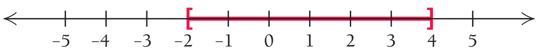
Graph



R.3 Exercise Set

In Exercises 1–10, write interval notation for each graph.

1.


2.


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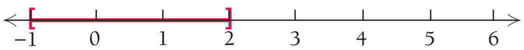

5.


6.


7.


8.


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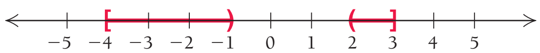
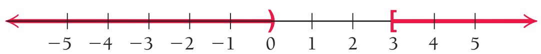


10.


Write interval notation for each of the following. Then graph the interval on a number line.

11. The set of all numbers x such that $-2 \leq x \leq 2$

12. The set of all numbers x such that $-5 < x < 5$

13. $\{x | 6 < x \leq 20\}$

14. $\{x | -4 \leq x < -1\}$

15. $\{x | x > -3\}$

16. $\{x | x \leq -2\}$

17. $\{x | -2 < x \leq 3\}$

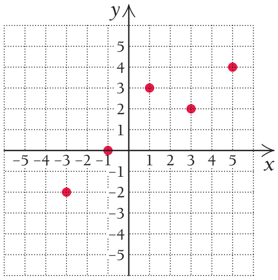
18. $\{x | -10 \leq x < 4\}$

19. $\{x | -4 \leq x < -3 \text{ or } 0 < x \leq 5\}$

20. $\{x | x < -2 \text{ or } 1 \leq x < 4\}$

In Exercises 21–32, each graph is that of a function, f. Determine (a) $f(1)$; (b) the domain; (c) all x-values such that $f(x) = 2$; and (d) the range.

21.



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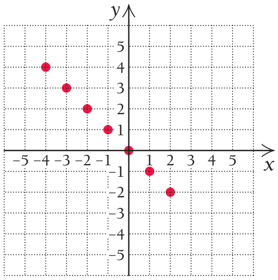

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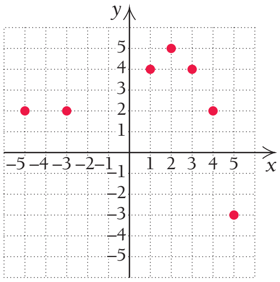

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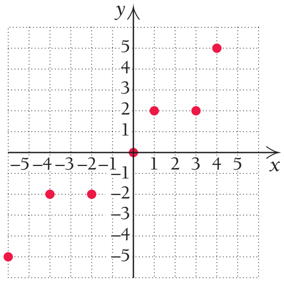

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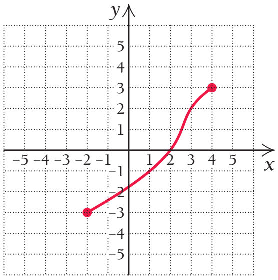

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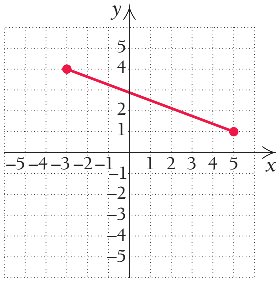

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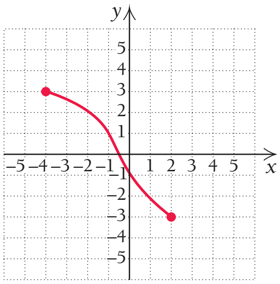

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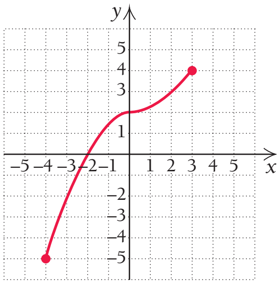

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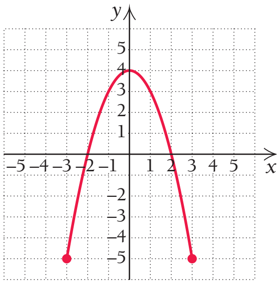

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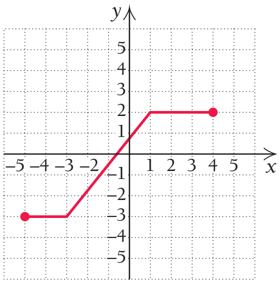
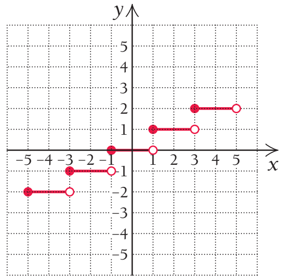
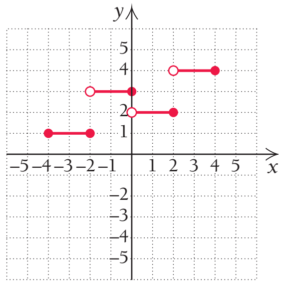


31.



32.



Write the domain of each function given below in interval notation; then graph the domain on a number line.

33. $f(x) = \dfrac{6}{2 - x}$

34. $f(x) = \dfrac{2}{x + 3}$

35. $f(x) = \sqrt{2x}$

36. $f(x) = \sqrt{x - 2}$

37. $f(x) = x^2 - 2x + 3$

38. $f(x) = x^2 + 3$

39. $f(x) = \dfrac{x - 2}{6x - 12}$

40. $f(x) = \dfrac{8}{3x - 6}$

41. $f(x) = x - 4$

42. $f(x) = 3x + 7$

43. $f(x) = \dfrac{3x - 1}{7 - 2x}$

44. $f(x) = \dfrac{2x - 1}{9 - 2x}$

45. $g(x) = \sqrt{4 + 5x}$

46. $g(x) = \sqrt{2 - 3x}$

47. $g(x) = x^2 - 2x + 1$

48. $g(x) = 4x^3 + 5x^2 - 2x$

49. $g(x) = \dfrac{2x}{x^2 - 25}$ (*Hint*: Factor the denominator.)

50. $g(x) = \dfrac{x - 1}{x^2 - 36}$ (*Hint*: Factor the denominator.)

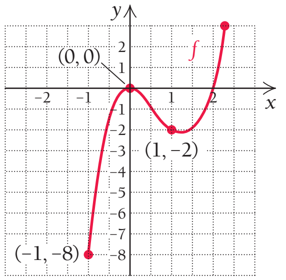
51. $g(x) = \dfrac{1}{x^2 + 9}$

52. $g(x) = \dfrac{x}{x^2 + 1}$

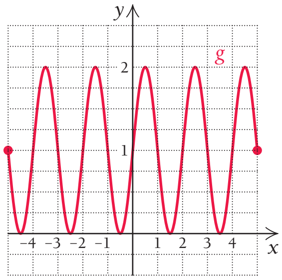
53. $f(x) = \sqrt{x + 1} + \sqrt{6 - 2x}$

54. $k(x) = \sqrt{2x + 3} - \sqrt{12 - 5x}$

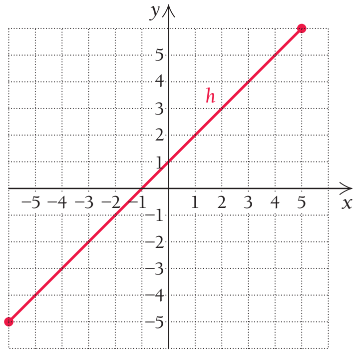
55. For the function f shown in the graph, find all x-values for which $f(x) \leq 0$.



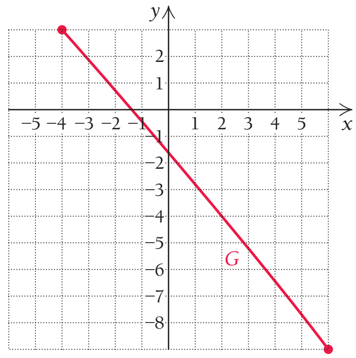
56. For the function g shown in the graph, find all x-values for which $g(x) = 1$.



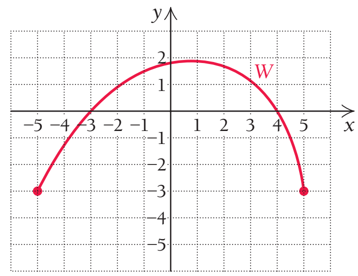
57. For the function h shown in the graph, find all x-values for which $h(x) = 2$.



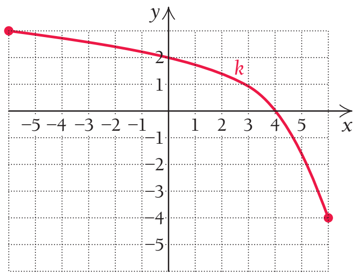
58. For the function G shown in the graph, find all x-values for which $G(x) > -4$.



59. For the function W shown in the graph, find all x-values for which $W(x) = 0$.



60. For the function k shown in the graph, find all x-values for which $k(x) \geq 0$.



APPLICATIONS

Business and Economics

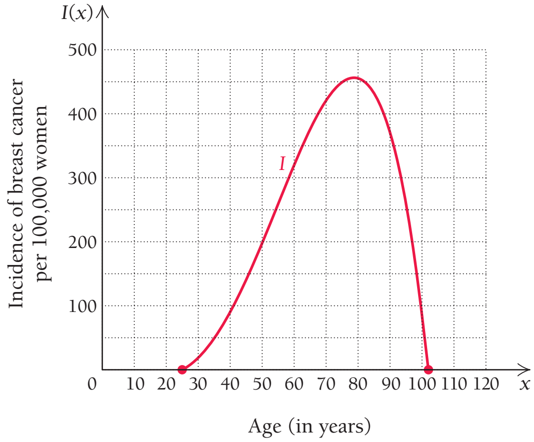
61. Hourly earnings. Karen works as a contractor, earning \$40 per hour. She will work at most 10 days, for at most 8 hours per day. Her total earnings P after working t hours are given by $P(t) = 40t$. Find the practical domain and range of this function.

62. Sales tax. Marcus plans to spend at most \$200 at the electronics store. The sales tax rate is 5% per dollar spent. The total tax T on x dollars spent is given by $T(x) = 0.05x$. Find the practical domain and range of this function.

Life and Physical Sciences

63. Incidence of breast cancer. The following graph approximates the incidence of breast cancer I, per 100,000 women, as a function of age x, in years. The equation for this graph is

$$I(x) = -0.0000554x^4 + 0.0067x^3 - 0.0997x^2 - 0.84x - 0.25.$$

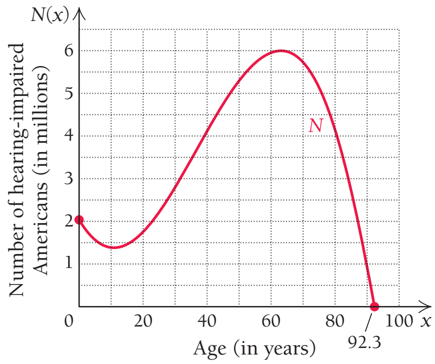


(*Source*: Based on data from the National Cancer Institute.)

a) Use the graph to estimate the domain of I.
b) Use the graph to estimate the range of I.
c) What 10-yr age interval sees the greatest increase in the incidence of breast cancer? Explain how you determined this.

64. Hearing-impaired Americans. The following graph approximates the number N, in millions, of hearing-impaired Americans who are x years old. The equation for this graph is

$$N(x) = -0.000065x^3 + 0.0072x^2 - 0.133x + 2.062.$$



(*Source*: Better Hearing Institute.)

a) Use the graph to determine the domain of N.
b) Use the graph to approximate the range of N.
c) If you were marketing a new type of hearing aid, at what age group (expressed as a 10-yr interval) would you target advertisements? Why?

65. Taxi fares. Goldenrod TaxiCab charges \$5 for the first mile and \$3.50 for each additional mile or part of a mile. Let $f(x)$ represent the fare, in dollars, for a trip of x miles.

a) Find $f(4)$, and explain what that value represents.
b) Find $f(4.25)$, and explain what that value represents.
c) Find the range of f for $0 < x \le 10$.

66. Shipping charges. Online Couture charges \$20 to ship orders for which the total amount spent is any amount up to and including \$100. For every extra \$20 spent, the shipping rate is reduced by \$5, with the possibility of free shipping if the total amount of the order is large enough. Let $S(x)$ be the shipping charge, in dollars, for an order totaling x dollars.

a) Find $S(95)$, and explain what that value represents.
b) Find $S(102)$, and explain what that value represents.
c) What is the minimum order total to receive free shipping?
d) Find the range of S when $0 < x \le 200$.

SYNTHESIS

67. Determine the domain of $f(x) = \dfrac{\sqrt{x}}{\sqrt{5-x}}$.

68. Determine the domain of $g(x) = \dfrac{\sqrt{3-x}}{x}$.

69. Determine the domain of $f(x) = \dfrac{1}{x(x^2-9)}$.

70. Determine the domain of $g(x) = \dfrac{4}{x(x^2-x-12)}$.

71. Write an equation for a function whose domain is all of the real numbers except $x = -2, 0, 2$. (*Hint*: See Exercise 69.)

72. Write an equation for a function whose domain is all of the real numbers except $x = -1, 0, 7$. (*Hint*: See Exercise 70.)

73. Explain why the domain of $f(x) = \sqrt{1-x} + \sqrt{x-3}$ is $\varnothing$.

Technology Connection

In Exercises 74 and 75, use a graphing calculator to find the domain of the function.

74. $f(x) = \sqrt{x^2 - 25}$

75. $g(x) = \sqrt{x^2 - 3x - 4}$

76. The function $P(t) = 2500(1.02)^t$ gives the population $P(t)$ of a city, t years after 2010 (where $t = 0$ represents 2010). Could the domain include negative values for t? If so, describe a situation in which a negative value of t makes sense within the context of the problem. How would the range be affected if negative values of t are included?

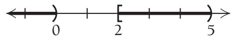
Answers to Quick Checks

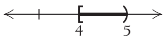
1. (a) $[-2, 5]$, 

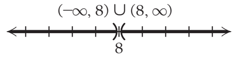
(b) $[-2, 5)$, 

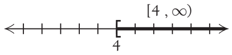
(c) $(-2, 5]$, 

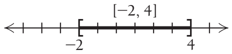
(d) $(-2, 5)$, 

(e) $(-\infty, 0) \cup [2, 5)$, 

(f) $[4, 5)$, 

2. Domain is $\{-3, -1, 1, 2, 3\}$, and range is $\{-2, 1, 2\}$.
3. (a) $f(-1) = 4$ **(b)** $f(1) = 2$ **(c)** domain is $[-3, 1]$ **(d)** range is $[0, 4]$
4. (a) $(-\infty, 8) \cup (8, \infty)$, 

(b) $[4, \infty)$, 

5. $[-2, 4]$, 

R.4 Slope and Linear Functions

- Find and interpret the slope of a line.
- Graph equations of the form $y = c$ and $x = a$.
- Graph linear functions.
- Find an equation of a line when given the slope and one point on the line and when given two points on the line.
- Solve applied problems involving slope and linear functions.

Given any two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, we can draw a line containing these points. A measure of the line's steepness is called the **slope**, denoted by the letter m.

DEFINITION

The **slope** of a line containing points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}.$$

EXAMPLE 1 Find the slope of the line containing the points $(-2, 6)$ and $(-4, 9)$.

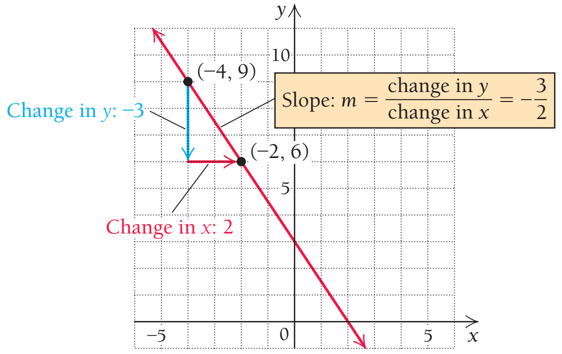
Solution We have

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 9}{-2 - (-4)} \quad \text{Regarding } (-2, 6) \text{ as } P_2 \text{ and } (-4, 9) \text{ as } P_1$$

$$= \frac{-3}{2} = -\frac{3}{2}.$$

It does not matter which point is regarded as P_1 or P_2, as long as we subtract the coordinates in the same order. Thus, we can also find m as follows:

$$m = \frac{9 - 6}{-4 - (-2)} = \frac{3}{-2} = -\frac{3}{2}. \quad \text{Here, } (-4, 9) \text{ serves as } P_2, \text{ and } (-2, 6) \text{ serves as } P_1.$$



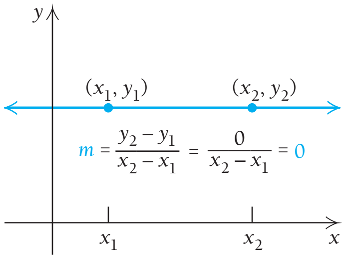
In the graph above, the points $(-2, 6)$ and $(-4, 9)$ are plotted, and a line is drawn through them. The slope $m = -\frac{3}{2}$ indicates that for each horizontal change of 2 units, the corresponding vertical change is -3 units. This line slopes down from left to right.

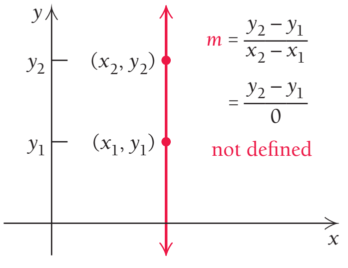
1 ✓

Quick Check 1 ✓

Find the slope of the line containing the points $(2, 3)$ and $(1, -4)$.

If a line is horizontal, the vertical change between any two points is 0. Thus, a horizontal line has slope 0. If a line is vertical, the horizontal change between any two points is 0. In this case, the slope is *not defined* because we cannot divide by 0. A vertical line has undefined slope. Thus, "0 slope" and "undefined slope" are two very different concepts.





Applications of Slope

Slope has many real-world applications. For example, numbers like 2%, 3%, and 6% are often used to represent the *grade*, or steepness, of a road. A 3% grade $(3\% = \frac{3}{100})$ means that for every horizontal distance of 100 ft, the road rises 3 ft. In solar arrays,