SULLIVAN

TRIGONOMETRY

A UNIT CIRCLE APPROACH

ELEVENTH EDITION

To the Student

As you begin, you may feel anxious about the number of theorems, definitions, procedures, and equations. You may wonder if you can learn it all in time. Don't worry–your concerns are normal. This textbook was written with you in mind. If you attend class, work hard, and read and study this text, you will build the knowledge and skills you need to be successful. Here's how you can use the text to your benefit.

Read Carefully

When you get busy, it's easy to skip reading and go right to the problems. Don't … the text has a large number of examples and clear explanations to help you break down the mathematics into easy-to-understand steps. Reading will provide you with a clearer understanding, beyond simple memorization. Read before class (not after) so you can ask questions about anything you didn't understand. You'll be amazed at how much more you'll get out of class if you do this.

Use the Features

I use many different methods in the classroom to communicate. Those methods, when incorporated into the text, are called "features." The features serve many purposes, from providing timely review of material you learned before (just when you need it) to providing organized review sessions to help you prepare for quizzes and tests. Take advantage of the features and you will master the material.

To make this easier, we've provided a brief guide to getting the most from this text. Refer to "Prepare for Class," "Practice," and "Review" at the front of the text. Spend fifteen minutes reviewing the guide and familiarizing yourself with the features by flipping to the page numbers provided. Then, as you read, use them. This is the best way to make the most of your text.

Please do not hesitate to contact me through Pearson Education, with any questions, comments, or suggestions for improving this text. I look forward to hearing from you, and good luck with all of your studies.

Best Wishes!

Michael Sullivan

Prepare for Class "Read the Book"

Feature	Description	Benefit	Page
Every Chapter Opener begins with . . .			
Chapter-Opening Topic & Project	Each chapter begins with a discussion of a topic of current interest and ends with a related project.	The project lets you apply what you learned to solve a problem related to the topic.	101
Internet-Based Projects	The projects allow for the integration of spreadsheet technology that you will need to be a productive member of the workforce.	The projects give you an opportunity to collaborate and use mathematics to deal with issues of current interest.	188
Every Section begins with . . .			
LEARNING OBJECTIVES 2	Each section begins with a list of objectives. Objectives also appear in the text where the objective is covered.	These focus your study by emphasizing what's most important and where to find it.	102
Sections contain . . .			
PREPARING FOR THIS SECTION	Most sections begin with a list of key concepts to review with page numbers.	Ever forget what you've learned? This feature highlights previously learned material to be used in this section. Review it, and you'll always be prepared to move forward.	115
Now Work the **'Are You Prepared?' Problems**	Problems that assess whether you have the prerequisite knowledge for the upcoming section.	Not sure you need the Preparing for This Section review? Work the 'Are You Prepared?' problems. If you get one wrong, you'll know exactly what you need to review and where to review it!	115, 127
Now Work PROBLEMS	These follow most examples and direct you to a related exercise.	We learn best by doing. You'll solidify your understanding of examples if you try a similar problem right away, to be sure you understand what you've just read.	117, 127
WARNING	Warnings are provided in the text.	These point out common mistakes and help you to avoid them.	117
Exploration and **Seeing the Concept**	These graphing utility activities foreshadow a concept or solidify a concept just presented.	You will obtain a deeper and more intuitive understanding of theorems and definitions.	68, 151
In Words	These provide alternative descriptions of select definitions and theorems.	Does math ever look foreign to you? This feature translates math into plain English.	116
Calculus	These appear next to information essential for the study of calculus.	Pay attention–if you spend extra time now, you'll do better later!	104, 129
SHOWCASE EXAMPLES	These examples provide "how-to" instruction by offering a guided, step-by-step approach to solving a problem.	With each step presented on the left and the mathematics displayed on the right, you can immediately see how each step is used.	153
Model It! Examples and Problems	These examples and problems require you to build a mathematical model from either a verbal description or data. The homework Model It! problems are marked by purple headings.	It is rare for a problem to come in the form "*Solve the following equation.*" Rather, the equation must be developed based on an explanation of the problem. These problems require you to develop models to find a solution to the problem.	108, 112
NEW! **Need to Review?**	These margin notes provide a just-in-time reminder of a concept needed now, but covered in an earlier section of the book. Each note is back-referenced to the chapter, section and page where the concept was originally discussed.	Sometimes as you read, you encounter a word or concept you know you've seen before, but don't remember exactly what it means. This feature will point you to where you first learned the word or concept. A quick review now will help you see the connection to what you are learning for the first time and make remembering easier the next time.	102

Practice “Work the Problems”

Feature	Description	Benefit	Page
‘Are You Prepared?’ Problems	These assess your retention of the prerequisite material you’ll need. Answers are given at the end of the section exercises. This feature is related to the Preparing for This Section feature.	Do you always remember what you’ve learned? Working these problems is the best way to find out. If you get one wrong, you’ll know exactly what you need to review and where to review it!	147, 156
Concepts and Vocabulary	These short-answer questions, mainly Fill-in-the-Blank, Multiple-Choice and True/False items, assess your understanding of key definitions and concepts in the current section.	It is difficult to learn math without knowing the language of mathematics. These problems test your understanding of the formulas and vocabulary.	157
Skill Building	Correlated with section examples, these problems provide straightforward practice.	It’s important to dig in and develop your skills. These problems provide you with ample opportunity to do so.	157–159
Applications and Extensions	These problems allow you to apply your skills to real-world problems. They also allow you to extend concepts learned in the section.	You will see that the material learned within the section has many uses in everyday life.	159–161
NEW! Challenge Problems	These problems have been added in most sections and appear at the end of the Applications and Extensions exercises. They are intended to be thought-provoking, requiring some ingenuity to solve.	Are you a student who likes being challenged? Then the Challenge Problems are for you! Your professor might also choose to assign a challenge problem as a group project. The ability to work with a team is a highly regarded skill in the working world.	161
Explaining Concepts: Discussion and Writing	“Discussion and Writing” problems are colored red. They support class discussion, verbalization of mathematical ideas, and writing and research projects.	To verbalize an idea, or to describe it clearly in writing, shows real understanding. These problems nurture that understanding. Many are challenging, but you’ll get out what you put in.	161
Retain Your Knowledge	These problems allow you to practice content learned earlier in the course.	Remembering how to solve all the different kinds of problems that you encounter throughout the course is difficult. This practice helps you remember.	161
Now Work PROBLEMS	Many examples refer you to a related homework problem. These related problems are marked by a pencil and orange numbers.	If you get stuck while working problems, look for the closest Now Work problem, and refer to the related example to see if it helps.	148, 149, 157, 158
Review Exercises	Every chapter concludes with a comprehensive list of exercises to practice. Use the list of objectives to determine the objective and examples that correspond to the problems.	Work these problems to ensure that you understand all the skills and concepts of the chapter. Think of it as a comprehensive review of the chapter.	183–186

Review “Study for Quizzes and Tests”

Feature	Description	Benefit	Page
The Chapter Review at the end of each chapter contains . . .			
Things to Know	A detailed list of important theorems, formulas, and definitions from the chapter.	Review these and you’ll know the most important material in the chapter!	181–183
You Should Be Able to . . .	Contains a complete list of objectives by section, examples that illustrate the objective, and practice exercises that test your understanding of the objective.	Do the recommended exercises and you’ll have mastered the key material. If you get something wrong, go back and work through the objective listed and try again.	183, 184
Review Exercises	These provide comprehensive review and practice of key skills, matched to the Learning Objectives for each section.	Practice makes perfect. These problems combine exercises from all sections, giving you a comprehensive review in one place.	184–186
Chapter Test	About 15–20 problems that can be taken as a Chapter Test. Be sure to take the Chapter Test under test conditions—no notes!	Be prepared. Take the sample practice test under test conditions. This will get you ready for your instructor’s test. If you get a problem wrong, you can watch the Chapter Test Prep Video.	186, 187
Cumulative Review	These problem sets appear at the end of each chapter, beginning with Chapter 2. They combine problems from previous chapters, providing an ongoing cumulative review. When you use them in conjunction with the Retain Your Knowledge problems, you will be ready for the final exam.	These problem sets are really important. Completing them will ensure that you are not forgetting anything as you go. This will go a long way toward keeping you primed for the final exam.	187
Chapter Projects	The Chapter Projects apply to what you’ve learned in the chapter. Additional projects are available on the Instructor’s Resource Center (IRC).	The Chapter Projects give you an opportunity to use what you’ve learned in the chapter to the opening topic. If your instructor allows, these make excellent opportunities to work in a group, which is often the best way to learn math.	188
Internet-Based Projects	In selected chapters, a Web-based project is given.	These projects give you an opportunity to collaborate and use mathematics to deal with issues of current interest by using the Internet to research and collect data.	188

To the Memory of Joe and Rita
and My Sister Maryrose

Trigonometry
A Unit Circle Approach

Eleventh Edition

Michael Sullivan
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Pearson

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About the Cover:
The image on this book's cover was inspired by a talk given by Michael Sullivan III: *Is Mathematical Talent Overrated?*

The answer is yes. In mathematics, innate talent plays a much smaller role than grit and motivation as you work toward your goal. If you put in the time and hard work, you can succeed in your math course—just as an athlete must work to medal in their sport.

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Three Distinct Series

Students have different goals, learning styles, and levels of preparation. Instructors have different teaching philosophies, styles, and techniques. Rather than write one series to fit all, the Sullivans have written three distinct series. All share the same goal—to develop a high level of mathematical understanding and an appreciation for the way mathematics can describe the world around us. The manner of reaching that goal, however, differs from series to series.

Flagship Series, Eleventh Edition

The Flagship Series is the most traditional in approach yet modern in its treatment of precalculus mathematics. In each text, needed review material is included, and is referenced when it is used. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor: *College Algebra, Algebra & Trigonometry, Trigonometry: A Unit Circle Approach, Precalculus.*

Enhanced with Graphing Utilities Series, Seventh Edition

This series provides a thorough integration of graphing utilities into topics, allowing students to explore mathematical concepts and encounter ideas usually studied in later courses. Many examples show solutions using algebra side-by-side with graphing techniques. Using technology, the approach to solving certain problems differs from the Flagship Series, while the emphasis on understanding concepts and building strong skills is maintained: *College Algebra, Algebra & Trigonometry, Precalculus*.

Concepts through Functions Series, Fourth Edition

This series differs from the others, utilizing a functions approach that serves as the organizing principle tying concepts together. Functions are introduced early in various formats. The approach supports the Rule of Four, which states that functions can be represented symbolically, numerically, graphically, and verbally. Each chapter introduces a new type of function and then develops all concepts pertaining to that particular function. The solutions of equations and inequalities, instead of being developed as stand-alone topics, are developed in the context of the underlying functions. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor: *College Algebra; Precalculus, with a Unit Circle Approach to Trigonometry; Precalculus, with a Right Triangle Approach to Trigonometry*.

The Flagship Series

College Algebra, Eleventh Edition

This text provides a contemporary approach to college algebra, with three chapters of review material preceding the chapters on functions. Graphing calculator usage is provided, but is optional. After completing this book, a student will be adequately prepared for trigonometry, finite mathematics, and business calculus.

Algebra & Trigonometry, Eleventh Edition

This text contains all the material in *College Algebra*, but also develops the trigonometric functions using a right triangle approach and shows how it relates to the unit circle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Vectors in the plane, sequences, induction, and the binomial theorem are also presented. Graphing calculator usage is provided, but is optional. After completing this book, a student will be adequately prepared for finite mathematics, business calculus, and engineering calculus.

Precalculus, Eleventh Edition

This text contains one review chapter before covering the traditional precalculus topics of polynomial, rational, exponential, and logarithmic functions and their graphs. The trigonometric functions are introduced using a unit circle approach and showing how it relates to the right triangle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Vectors in the plane and in space, including the dot and cross products, sequences, induction, and the binomial theorem are also presented. Graphing calculator usage is provided, but is optional. The final chapter provides an introduction to calculus, with a discussion of the limit, the derivative, and the integral of a function. After completing this book, a student will be adequately prepared for finite mathematics, business calculus, and engineering calculus.

Trigonometry: a Unit Circle Approach, Eleventh Edition

This text, designed for stand-alone courses in trigonometry, develops the trigonometric functions using a unit circle approach and shows how it relates to the right triangle approach. Vectors in the plane and in space, including the dot and cross products, are presented. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Graphing calculator usage is provided, but is optional. After completing this book, a student will be adequately prepared for finite mathematics, business calculus, and engineering calculus.

Preface to the Instructor

As a professor of mathematics at an urban public university for 35 years, I understand the varied needs of students taking trigonometry. Students range from being underprepared with little mathematical background and a fear of mathematics, to being highly prepared and motivated. For some, this is their final course in mathematics. For others, it is preparation for future mathematics courses. I have written this text with both groups in mind.

A tremendous benefit of authoring a successful series is the broad-based feedback I receive from instructors and students who have used previous editions. I am sincerely grateful for their support. Virtually every change to this edition is the result of their thoughtful comments and suggestions. I hope that I have been able to take their ideas and, building upon a successful foundation of the tenth edition, make this series an even better learning and teaching tool for students and instructors.

Features in the Eleventh Edition

A descriptive list of the many special features of *Trigonometry* can be found on the endpapers in the front of this text. This list places the features in their proper context, as building blocks of an overall learning system that has been carefully crafted over the years to help students get the most out of the time they put into studying. Please take the time to review it and to discuss it with your students at the beginning of your course. My experience has been that when students use these features, they are more successful in the course.

- **Updated! Retain Your Knowledge Problems** These problems, which were new to the previous edition, are based on the article *"To Retain New Learning, Do the Math,"* published in the *Edurati Review.* In this article, Kevin Washburn suggests that "the more students are required to recall new content or skills, the better their memory will be." The Retain Your Knowledge problems were so well received that they have been expanded in this edition. Moreover, while the focus remains to help students maintain their skills, in most sections, problems were chosen that preview skills required to succeed in subsequent sections or in calculus. These are easily identified by the calculus icon (△). All answers to Retain Your Knowledge problems are given in the back of the text and all are assignable in MyLab Math.
- **Guided Lecture Notes** Ideal for online, emporium/redesign courses, inverted classrooms, or traditional lecture classrooms. These lecture notes help students take thorough, organized, and understandable notes as they watch the Author in Action videos. They ask students to complete definitions, procedures, and examples based on the content of the videos and text. In addition, experience suggests that students learn by doing and understanding the why/how of the concept or property. Therefore, many sections will have an exploration activity to motivate student learning. These explorations introduce the topic and/or connect it to either a real-world application or a previous section. For example, when the vertical-line test is discussed in Section 1.3, after the theorem statement, the notes ask the students to explain why the vertical-line test works by using the definition of a function. This challenge helps students process the information at a higher level of understanding.
- **Illustrations** Many of the figures have captions to help connect the illustrations to the explanations in the body of the text.
- **Graphing Utility Screen Captures** In several instances we have added Desmos screen captures along with the TI-84 Plus C screen captures. These updated screen captures provide alternate ways of visualizing concepts and making connections between equations, data and graphs in full color.
- **Chapter Projects**, which apply the concepts of each chapter to a real-world situation, have been enhanced to give students an up-to-the-minute experience. Many of these projects are new requiring the student to research information online in order to solve problems.
- **Exercise Sets** The exercises in the text have been reviewed and analyzed, some have been removed, and new ones have been added. All time-sensitive problems have been updated to the most recent information available. The problem sets remain classified according to purpose.

  The ***'Are You Prepared?'*** problems have been improved to better serve their purpose as a just-in-time review of concepts that the student will need to apply in the upcoming section.

  The ***Concepts and Vocabulary*** problems have been expanded to cover each objective of the section. These multiple-choice, fill-in-the-blank, and True/False exercises have been written to also serve as reading quizzes.

  Skill Building problems develop the student's computational skills with a large selection of exercises that are directly related to the objectives of the section. Within these exercises, ***Mixed Practice*** problems offer a comprehensive assessment of skills that relate to more than one objective. Often these require skills learned earlier in the course.

  Applications and Extensions problems have been updated. Further, many new application-type exercises have been added, especially ones involving information and data drawn from sources the student will recognize, to improve relevance and timeliness.

  At the end of Applications and Extensions, we have a collection of one or more ***Challenge Problems***. These problems, as the title suggests, are intended to be thought-provoking, requiring some ingenuity to solve. They can be used for group work or to challenge students. At the end of the Annotated Instructor's

Edition and in the online Instructor's Solutions Manual, we have provided solutions to all these problems.

The ***Explaining Concepts: Discussion and Writing*** exercises provide opportunity for classroom discussion and group projects.

Updated! *Retain Your Knowledge* has been improved and expanded. The problems are based on material learned earlier in the course, especially calculus-related material. They serve to keep information that has already been learned "fresh" in the mind of the student. Answers to all these problems appear in the Student Edition.

NEW Need to Review? These margin notes provide a just-in-time reminder of a concept needed now, but covered in an earlier section of the book. Each note includes a reference to the chapter, section, and page where the concept was originally discussed.

Content Changes to the 11th edition

- ***Challenge Problems*** have been added in most sections at the end of the Applications and Extensions exercises. Challenge Problems are intended to be thought-provoking problems that require some ingenuity to solve. They can be used to challenge students or for group work.
- **Need to Review?** These margin notes provide a just-in-time review for a concept needed now, but covered earlier. Each note is back-referenced to the chapter, section, and page where the concept was originally discussed.
- Additional **Retain Your Knowledge** exercises, whose purpose is to keep learned material fresh in a student's mind, have been added to each section. Many of these new problems preview skills required for calculus or for concepts needed in subsequent sections.
- **Desmos** screen captures have been added throughout the text. This was done to recognize that graphing technology expands beyond graphing calculators.
- Examples and exercises throughout the text have been augmented to reflect a broader selection of STEM applications.
- Concepts and Vocabulary exercises have been expanded to cover each objective of a section.
- Skill building exercises have been expanded to assess a wider range of difficulty.
- Applied problems and those based on real data have been updated where appropriate.

Appendix A

- Section A.7 Objective 3 now includes rationalizing the numerator
  - NEW Example 6 Rationalizing Numerators
  - Problems 69–76 provide practice.
- Section A.7 Exercises now include more practice in simplifying radicals
- Section A.8 has been reorganized to treat the slope-intercept form of the equation of a line before finding an equation of a line using two points.

Chapter 1

- NEW Section 1.2 Example 9 Testing an Equation for Symmetry
- NEW Section 1.3 Objective 1 Describe a Relation
- NEW Section 1.3 Example 1 Describing a Relation demonstrates using the Rule of Four to express a relation numerically, as a mapping, and graphically given a verbal description.
- NEW Section 1.5 Example 4 Analyzing a Piecewise-defined Function
- Section 1.7 now finds and verifies inverse functions analytically and graphically.

Chapter 2

- NEW Section 2.1 Example 6 Field Width of a Digital Lens Reflex Camera Lens
- Sections 2.4 and 2.5 were reorganized for increased clarity.

Chapter 3

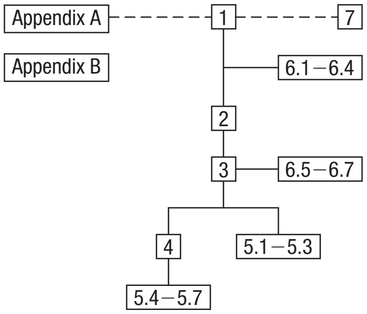
- Sections 3.1 and 3.2 were reorganized for increased clarity.

Chapter 5

- Section 5.3 The complex plane; DeMoivre's Theorem, was rewritten to support the exponential form of a complex number.
  - Euler's Formula is introduced to express a complex number in exponential form.
  - The exponential form of a complex number is used to compute products and quotients.
  - DeMoivre's Theorem is expressed using the exponential form of a complex number.
  - The exponential form is used to find complex roots.

Using the Eleventh Edition Effectively with Your Syllabus

To meet the varied needs of diverse syllabi, this text contains more content than is likely to be covered in a *Trigonometry* course. As the chart illustrates, this text has been organized with flexibility of use in mind. Within a given chapter, certain sections are optional (see the details that follow the figure below) and can be omitted without loss of continuity.



Appendix A Review

This chapter consists of review material. It may be used as the first part of the course or later as a just-in-time review when the content is required. Specific references to this chapter occur throughout the text to assist in the review process.

Chapter 1 Graphs and Functions

This chapter lays the foundation for Chapters 2, 6, and 7.

Chapter 2 Trigonometric Functions

Section 2.6 may be omitted in a brief course.

Chapter 3 Analytic Trigonometry

Section 3.7 may be omitted in a brief course.

Chapter 4 Applications of Trigonometric Functions

Sections 4.4 and 4.5 may be omitted in a brief course.

Chapter 5 Polar Coordinates; Vectors

Sections 5.1–5.3 and Sections 5.4–5.7 are independent and may be covered separately.

Chapter 6 Analytic Geometry

Sections 6.1–6.4 follow in sequence. Sections 6.5, 6.6, and 6.7 are independent of each other, but each requires Sections 6.1–6.4.

Chapter 7 Exponential and Logarithmic Functions

Sections 7.1–7.4 follow in sequence. Sections 7.5, 7.6, and 7.7 are optional.

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Textbooks are written by authors, but evolve from an idea to final form through the efforts of many people. It was Don Dellen who first suggested this text and series to me. Don is remembered for his extensive contributions to publishing and mathematics.

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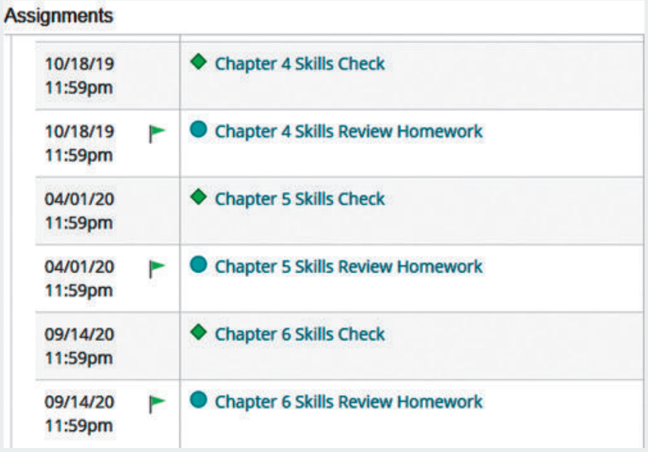
One of the biggest challenges in College Algebra, Trigonometry, and Precalculus is making sure students are adequately prepared with prerequisite knowledge. For a student, having the essential algebra skills upfront in this course can dramatically increase success.

- **MyLab Math with Integrated Review** can be used in corequisite courses, or simply to help students who enter without a full understanding of prerequisite skills and concepts. **Integrated Review** provides videos on review topics with a corresponding worksheet, along with premade, assignable skills-check quizzes and personalized review homework assignments. **Integrated Review** is now available within all Sullivan 11th Edition MyLab Math courses.



Assignments

10/18/19 11:59pm		◆ Chapter 4 Skills Check
10/18/19 11:59pm	⚑	● Chapter 4 Skills Review Homework
04/01/20 11:59pm		◆ Chapter 5 Skills Check
04/01/20 11:59pm	⚑	● Chapter 5 Skills Review Homework
09/14/20 11:59pm		◆ Chapter 6 Skills Check
09/14/20 11:59pm	⚑	● Chapter 6 Skills Review Homework

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Resources for Success

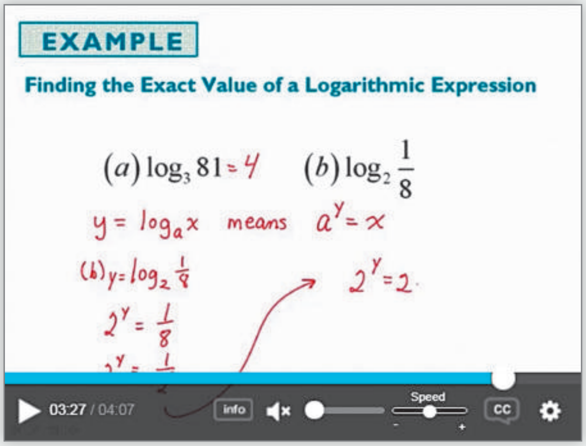
MyLab Math Online Course for Trigonometry, 11th Edition by Michael Sullivan (access code required)

MyLab™ Math is tightly integrated with each author's style, offering a range of author-created multimedia resources, so your students have a consistent experience.

Video Program and Resources

Author in Action Videos are actual classroom lectures by Michael Sullivan III with fully worked-out examples.

- **Video assessment** questions are available to assign in MyLab Math for key videos.
- **Updated!** The corresponding **Guided Lecture Notes** assist students in taking thorough, organized, and understandable notes while watching Author in Action videos.



EXAMPLE

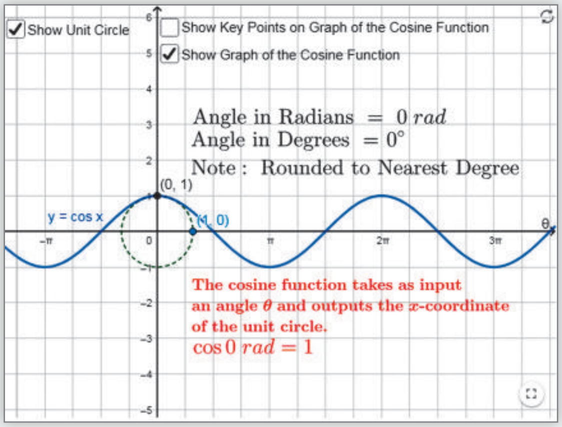
Finding the Exact Value of a Logarithmic Expression

$(a)\ \log_3 81 = 4 \qquad (b)\ \log_2 \frac{1}{8}$

$y = \log_a x$ means $a^y = x$

$(b)\ y = \log_2 \frac{1}{8} \qquad 2^y = 2$

$2^y = \frac{1}{8}$



☑ Show Unit Circle

☐ Show Key Points on Graph of the Cosine Function

☑ Show Graph of the Cosine Function

Angle in Radians = 0 *rad*
Angle in Degrees = 0°
Note : Rounded to Nearest Degree

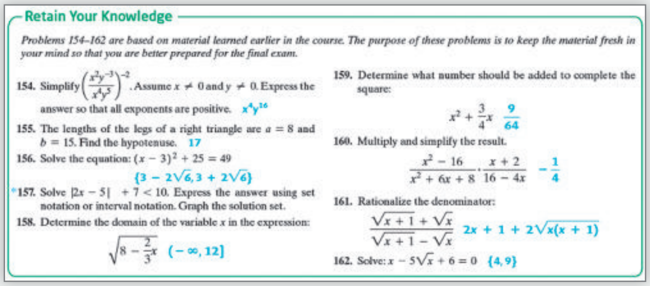
The cosine function takes as input an angle θ and outputs the x-coordinate of the unit circle.
$\cos 0\ rad = 1$

Guided Visualizations

New! **Guided Visualizations**, created in GeoGebra by Michael Sullivan III, bring mathematical concepts to life, helping students visualize the concept through directed exploration and purposeful manipulation. Assignable in MyLab Math with assessment questions to check students' conceptual understanding.

Retain Your Knowledge Exercises

Updated! **Retain Your Knowledge Exercises,** assignable in MyLab Math, improve students' recall of concepts learned earlier in the course. New for the 11th Edition, additional exercises will be included that will have an emphasis on content that students will build upon in the immediate upcoming section.



Retain Your Knowledge

Problems 154–162 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

154. Simplify $\left(\frac{x^2y^{-3}}{x^4y^5}\right)^{-2}$. Assume $x \neq 0$ and $y \neq 0$. Express the answer so that all exponents are positive. x^4y^{16}

155. The lengths of the legs of a right triangle are $a = 8$ and $b = 15$. Find the hypotenuse. 17

156. Solve the equation: $(x-3)^2 + 25 = 49$ $\{3 - 2\sqrt{6}, 3 + 2\sqrt{6}\}$

157. Solve $|2x - 5| + 7 < 10$. Express the answer using set notation or interval notation. Graph the solution set.

158. Determine the domain of the variable x in the expression: $\sqrt{8 - \frac{2}{3}x}$ $(-\infty, 12]$

159. Determine what number should be added to complete the square: $x^2 + \frac{3}{4}x$ $\frac{9}{64}$

160. Multiply and simplify the result. $\frac{x^2-16}{x^2+6x+8} \cdot \frac{x+2}{16-4x}$ $-\frac{1}{4}$

161. Rationalize the denominator: $\frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}$ $2x + 1 + 2\sqrt{x(x+1)}$

162. Solve: $x - 5\sqrt{x} + 6 = 0$ $\{4, 9\}$

Resources for Success

Instructor Resources

Online resources can be downloaded from www.pearson.com, or hardcopy resources can be ordered from your sales representative.

Annotated Instructor's Edition
Trigonometry, 11th Edition
ISBN – 0135229065 / 9780135229064
Shorter answers are on the page beside the exercises. Longer answers are in the back of the text.

Instructor's Solutions Manual

ISBN – 0135228999 / 9780135228999
Includes fully worked solutions to all exercises in the text.

Learning Catalytics Question Library

Questions written by Michael Sullivan III are available within MyLab Math to deliver through Learning Catalytics to engage students in your course.

Powerpoint® Lecture Slides

Fully editable slides correlate to the textbook.

Mini Lecture Notes

Includes additional examples and helpful teaching tips, by section.

Testgen®

TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

Online Chapter Projects

Additional projects that give students an opportunity to apply what they learned in the chapter.

Student Resources

Additional resources to enhance student success.

Lecture Video

Author in Action videos are actual classroom lectures with fully worked out examples presented by Michael Sullivan, III. All video is assignable within MyLab Math.

Chapter Test Prep Videos

Students can watch instructors work through step-by-step solutions to all chapter test exercises from the text. These are available in MyLab Math and on YouTube.

Student's Solutions Manual

ISBN - 0134999312 / 9780134999319
Provides detailed worked-out solutions to odd-numbered exercises.

Guided Lecture Notes

ISBN – 0135244749 / 9780135244746
These lecture notes assist students in taking thorough, organized, and understandable notes while watching Author in Action videos. Students actively participate in learning the how/why of important concepts through explorations and activities. The Guided Lecture Notes are available as PDF's and customizable Word files in MyLab Math. They can also be packaged with the text and the MyLab Math access code.

Algebra Review

ISBN: 0131480065 / 9780131480063
Four printed chapters of Intermediate Algebra review available. Perfect for a corequisite course or for individual review.

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Applications Index

Graphs and Functions

1

Choosing a Data Plan

When selecting a data plan for a device, most consumers choose a service provider first and then select an appropriate data plan from that provider. The choice as to the type of plan selected depends on your use of the device. For example, is online gaming important? Do you want to stream audio or video? The mathematics learned in this chapter can help you decide what plan is best suited to your particular needs.

—See the Internet-based Chapter Project—

A Look Back

Appendix A reviews skills from intermediate algebra.

A Look Ahead

Here we connect algebra and geometry using the rectangular coordinate system to graph equations in two variables. Then we look at a special type of equation involving two variables called a *function*. This chapter deals with what a function is, how to graph functions, properties of functions, and how functions are used in applications. The word *function* apparently was introduced by René Descartes in 1637. For him, a function was simply any positive integral power of a variable *x*. Gottfried Wilhelm Leibniz (1646–1716), who always emphasized the geometric side of mathematics, used the word *function* to denote any quantity associated with a curve, such as the coordinates of a point on the curve. Leonhard Euler (1707–1783) used the word to mean any equation or formula involving variables and constants. His idea of a function is similar to the one most often seen in courses that precede calculus. Later, the use of functions in investigating heat flow equations led to a very broad definition that originated with Lejeune Dirichlet (1805–1859), which describes a function as a correspondence between two sets. That is the definition used in this text.

Outline

- **1.1** The Distance and Midpoint Formulas
- **1.2** Graphs of Equations in Two Variables; Circles
- **1.3** Functions and Their Graphs
- **1.4** Properties of Functions
- **1.5** Library of Functions; Piecewise-defined Functions
- **1.6** Graphing Techniques: Transformations
- **1.7** One-to-one Functions; Inverse Functions
- Chapter Review
- Chapter Test
- Chapter Projects

1.1 The Distance and Midpoint Formulas

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Algebra Essentials (Section A.1, pp. A1–A10)
- Geometry Essentials (Section A.2, pp. A14–A19)

Now Work the **'Are You Prepared?'** problems on page 5.

OBJECTIVES
1 Use the Distance Formula (p. 3)
2 Use the Midpoint Formula (p. 5)

Rectangular Coordinates

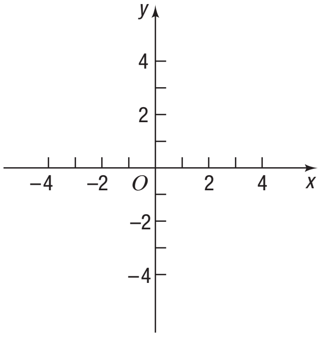


Figure 1 *xy*-Plane

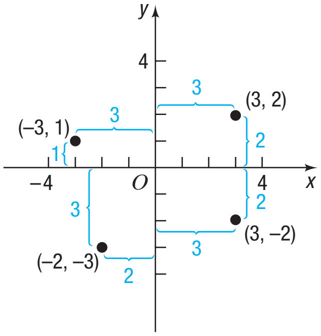


Figure 2

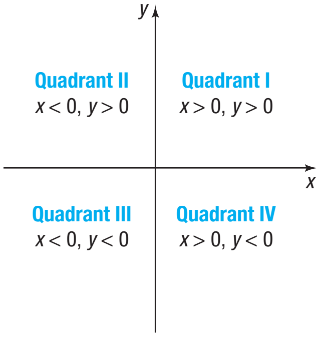


Figure 3

We locate a point on the real number line by assigning it a single real number, called the *coordinate of the point*. For work in a two-dimensional plane, we locate points by using two numbers.

Begin with two real number lines located in the same plane: one horizontal and the other vertical. The horizontal line is called the **x-axis**, the vertical line the **y-axis**, and the point of intersection the **origin O**. See Figure 1. Assign coordinates to every point on these number lines using a convenient scale. In mathematics, we usually use the same scale on each axis, but in applications, different scales appropriate to the application may be used.

The origin O has a value of 0 on both the x-axis and the y-axis. Points on the x-axis to the right of O are associated with positive real numbers, and those to the left of O are associated with negative real numbers. Points on the y-axis above O are associated with positive real numbers, and those below O are associated with negative real numbers. In Figure 1, the x-axis and y-axis are labeled as x and y, respectively, and an arrow at the end of each axis is used to denote the positive direction.

The coordinate system described here is called a **rectangular** or **Cartesian*** **coordinate system**. The x-axis and y-axis lie in a *plane* called the **xy-plane**, and the x-axis and y-axis are referred to as the **coordinate axes**.

Any point P in the xy-plane can be located by using an **ordered pair** (x, y) of real numbers. Let x denote the signed distance of P from the y-axis (*signed* means that if P is to the right of the y-axis, then $x > 0$, and if P is to the left of the y-axis, then $x < 0$); and let y denote the signed distance of P from the x-axis. The ordered pair (x, y), also called the **coordinates** of P, gives us enough information to locate the point P in the plane.

For example, to locate the point whose coordinates are $(-3, 1)$, go 3 units along the x-axis to the left of O and then go straight up 1 unit. We **plot** this point by placing a dot at this location. See Figure 2, in which the points with coordinates $(-3, 1)$, $(-2, -3)$, $(3, -2)$, and $(3, 2)$ are plotted.

The origin has coordinates $(0, 0)$. Any point on the x-axis has coordinates of the form $(x, 0)$, and any point on the y-axis has coordinates of the form $(0, y)$.

If (x, y) are the coordinates of a point P, then x is called the **x-coordinate**, or **abscissa**, of P, and y is the **y-coordinate**, or **ordinate**, of P. We identify the point P by its coordinates (x, y) by writing $P = (x, y)$. Usually, we will simply say "the point (x, y)" rather than "the point whose coordinates are (x, y)."

The coordinate axes partition the xy-plane into four sections called **quadrants**, as shown in Figure 3. In quadrant I, both the x-coordinate and the y-coordinate of all points are positive; in quadrant II, x is negative and y is positive; in quadrant III, both x and y are negative; and in quadrant IV, x is positive and y is negative. Points on the coordinate axes belong to no quadrant.

Now Work PROBLEM **15**

---

*Named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian.

COMMENT On a graphing calculator, you can set the scale on each axis. Once this has been done, you obtain the **viewing rectangle**. See Figure 4 for a typical viewing rectangle. You should now read Section B.1, *The Viewing Rectangle*.

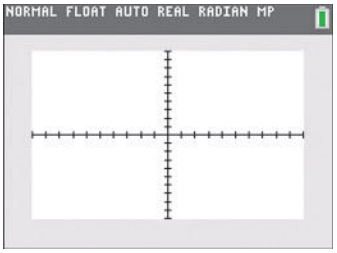


Figure 4 TI-84 Plus C Standard Viewing Rectangle ■

1 Use the Distance Formula

If the same units of measurement (such as inches, centimeters, and so on) are used for both the *x*-axis and *y*-axis, then all distances in the *xy*-plane can be measured using this unit of measurement.

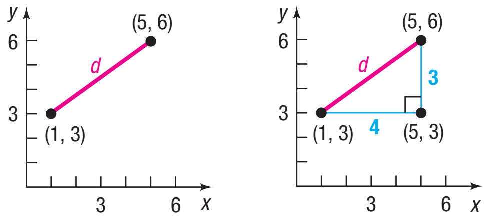
EXAMPLE 1 Finding the Distance between Two Points

Find the distance d between the points $(1, 3)$ and $(5, 6)$.

Solution First plot the points $(1, 3)$ and $(5, 6)$ and connect them with a line segment. See Figure 5(a). To find the length d, begin by drawing a horizontal line segment from $(1,3)$ to $(5,3)$ and a vertical line segment from $(5,3)$ to $(5,6)$, forming a right triangle, as shown in Figure 5(b). One leg of the triangle is of length 4 (since $|5 - 1| = 4$), and the other is of length 3 (since $|6 - 3| = 3$). By the Pythagorean Theorem, the square of the distance d that we seek is

Need to Review?
The Pythagorean Theorem and its converse are discussed in Section A.2, pp. A14–A15.

$$d^2 = 4^2 + 3^2 = 16 + 9 = 25$$
$$d = \sqrt{25} = 5$$



(a) (b)

Figure 5

The **distance formula** provides a straightforward method for computing the distance between two points.

In Words
To compute the distance between two points, find the difference of the x-coordinates, square it, and add this to the square of the difference of the y-coordinates. The square root of this sum is the distance.

THEOREM Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (1)$$

EXAMPLE 2 Using the Distance Formula

Find the distance d between the points $(-4, 5)$ and $(3, 2)$.

Solution Using the distance formula, equation (1), reveals that the distance d is

$$\begin{aligned} d &= \sqrt{[3-(-4)]^2 + (2-5)^2} = \sqrt{7^2 + (-3)^2} \\ &= \sqrt{49+9} = \sqrt{58} \approx 7.62 \end{aligned}$$

Now Work PROBLEMS 19 AND 23

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is never a negative number. Also, the distance between two points is 0 only when the points are identical—that is, when $x_1 = x_2$ and $y_1 = y_2$. And, because $(x_2 - x_1)^2 = (x_1 - x_2)^2$ and $(y_2 - y_1)^2 = (y_1 - y_2)^2$, it makes no difference whether the distance is computed from P_1 to P_2 or from P_2 to P_1; that is, $d(P_1, P_2) = d(P_2, P_1)$.

The introduction to this chapter mentioned that algebra and geometry are connected by the rectangular coordinate system. The next example shows how algebra (the distance formula) can be used to solve geometry problems.

EXAMPLE 3 Using Algebra to Solve a Geometry Problem

Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

(a) Plot each point and form the triangle ABC.
(b) Find the length of each side of the triangle.
(c) Show that the triangle is a right triangle.
(d) Find the area of the triangle.

Solution

(a) Figure 6 shows the points A, B, C and the triangle ABC.

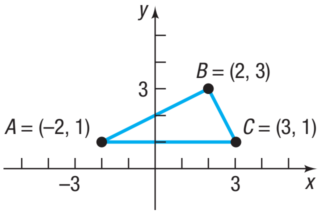


Figure 6

(b) To find the length of each side of the triangle, use the distance formula, equation (1).

$$\begin{aligned} d(A,B) &= \sqrt{[2-(-2)]^2 + (3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \\ d(B,C) &= \sqrt{(3-2)^2 + (1-3)^2} = \sqrt{1+4} = \sqrt{5} \\ d(A,C) &= \sqrt{[3-(-2)]^2 + (1-1)^2} = \sqrt{25+0} = 5 \end{aligned}$$

(c) If the sum of the squares of the lengths of two of the sides equals the square of the length of the third side, then the triangle is a right triangle. Looking at Figure 6, it seems reasonable to conjecture that the angle at vertex B might be a right angle. We shall check to see whether

$$[d(A,B)]^2 + [d(B,C)]^2 = [d(A,C)]^2$$

Using the results in part (b) yields

$$\begin{aligned} [d(A,B)]^2 + [d(B,C)]^2 &= \left(2\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2 \\ &= 20 + 5 = 25 = [d(A,C)]^2 \end{aligned}$$

It follows from the converse of the Pythagorean Theorem that triangle ABC is a right triangle.

(d) Because the right angle is at vertex B, the sides AB and BC form the base and height of the triangle. Its area is

$$\text{Area} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height} = \frac{1}{2} \cdot 2\sqrt{5} \cdot \sqrt{5} = 5 \text{ square units}$$

Now Work PROBLEM 33

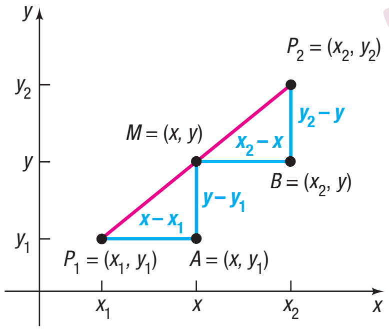


Figure 7

2 Use the Midpoint Formula

We now derive a formula for the coordinates of the **midpoint of a line segment**. Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be the endpoints of a line segment, and let $M = (x, y)$ be the point on the line segment that is the same distance from P_1 as it is from P_2. See Figure 7. The triangles P_1AM and MBP_2 are congruent. [Do you see why? $d(P_1, M) = d(M, P_2)$ is given; also, $\angle AP_1M = \angle BMP_2$* and $\angle P_1MA = \angle MP_2B$. So, we have angle–side–angle.] Because triangles P_1AM and MBP_2 are congruent, corresponding sides are equal in length. That is,

$$\begin{aligned} x - x_1 &= x_2 - x & \quad\text{and}\quad && y - y_1 &= y_2 - y \\ 2x &= x_1 + x_2 & && 2y &= y_1 + y_2 \\ x &= \frac{x_1 + x_2}{2} & && y &= \frac{y_1 + y_2}{2} \end{aligned}$$

THEOREM Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$\boxed{M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \textbf{(2)}}$$

In Words
To find the midpoint of a line segment, average the x-coordinates of the endpoints, and average the y-coordinates of the endpoints.

EXAMPLE 4 Finding the Midpoint of a Line Segment

Find the midpoint of the line segment from $P_1 = (-5, 5)$ to $P_2 = (3, 1)$. Plot the points P_1 and P_2 and their midpoint.

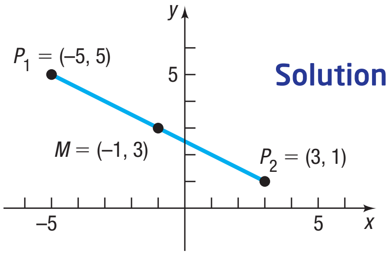


Figure 8

Solution Use the midpoint formula (2) with $x_1 = -5$, $y_1 = 5$, $x_2 = 3$, and $y_2 = 1$. The coordinates (x, y) of the midpoint M are

$$x = \frac{x_1 + x_2}{2} = \frac{-5 + 3}{2} = -1 \quad\text{and}\quad y = \frac{y_1 + y_2}{2} = \frac{5 + 1}{2} = 3$$

That is, $M = (-1, 3)$. See Figure 8.

Now Work PROBLEM 39

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*A postulate from geometry states that the transversal $\overline{P_1P_2}$ forms congruent corresponding angles with the parallel line segments $\overline{P_1A}$ and $\overline{MB}$.

1.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. On the real number line, the origin is assigned the number ________. (p. A4)
2. If −3 and 5 are the coordinates of two points on the real number line, the distance between these points is ________. (pp. A5–A6)
3. If 3 and 4 are the legs of a right triangle, the hypotenuse is ________. (p. A14)
4. Use the converse of the Pythagorean Theorem to show that a triangle whose sides are of lengths 11, 60, and 61 is a right triangle. (pp. A14–A15)
5. The area A of a triangle whose base is b and whose altitude is h is $A =$ __________. (p. A15)
6. ***True or False*** Two triangles are congruent if two angles and the included side of one equals two angles and the included side of the other. (pp. A16–A17)

Concepts and Vocabulary

7. If (x, y) are the coordinates of a point P in the xy-plane, then x is called the ____________________ of P, and y is the ____________________ of P.

8. The coordinate axes partition the xy-plane into four sections called __________.

9. If three distinct points P, Q, and R all lie on a line, and if $d(P, Q) = d(Q, R)$, then Q is called the __________ of the line segment from P to R.

10. ***True or False*** The distance between two points is sometimes a negative number.

11. ***True or False*** The point $(-1, 4)$ lies in quadrant IV of the Cartesian plane.

12. ***True or False*** The midpoint of a line segment is found by averaging the x-coordinates and averaging the y-coordinates of the endpoints.

13. ***Multiple Choice*** Which of the following statements is true for a point (x, y) that lies in quadrant III?

(a) Both x and y are positive.
(b) Both x and y are negative.
(c) x is positive, and y is negative.
(d) x is negative, and y is positive.

14. ***Multiple Choice*** Choose the expression that equals the distance between two points (x_1, y_1) and (x_2, y_2).

(a) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(b) $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$
(c) $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$
(d) $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$

Skill Building

In Problems 15 and 16, plot each point in the xy-plane. State which quadrant or on what coordinate axis each point lies.

15. **(a)** $A = (-3, 2)$ **(b)** $B = (6, 0)$ **(c)** $C = (-2, -2)$ **(d)** $D = (6, 5)$ **(e)** $E = (0, -3)$ **(f)** $F = (6, -3)$

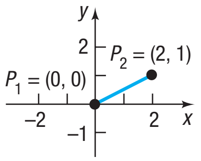
16. **(a)** $A = (1, 4)$ **(b)** $B = (-3, -4)$ **(c)** $C = (-3, 4)$ **(d)** $D = (4, 1)$ **(e)** $E = (0, 1)$ **(f)** $F = (-3, 0)$

17. Plot the points $(2, 0)$, $(2, -3)$, $(2, 4)$, $(2, 1)$, and $(2, -1)$. Describe the set of all points of the form $(2, y)$, where y is a real number.

18. Plot the points $(0, 3)$, $(1, 3)$, $(-2, 3)$, $(5, 3)$, and $(-4, 3)$. Describe the set of all points of the form $(x, 3)$, where x is a real number.

In Problems 19–32, find the distance d between the points P_1 and P_2.

19.



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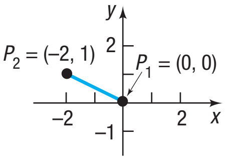

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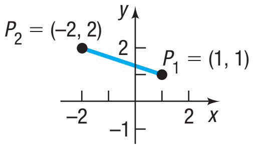
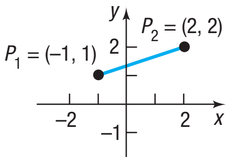


22.



23. $P_1 = (3, -4)$; $P_2 = (5, 4)$
24. $P_1 = (-1, 0)$; $P_2 = (2, 4)$
25. $P_1 = (-7, 3)$; $P_2 = (4, 0)$
26. $P_1 = (2, -3)$; $P_2 = (4, 2)$
27. $P_1 = (5, -2)$; $P_2 = (6, 1)$
28. $P_1 = (-4, -3)$; $P_2 = (6, 2)$
29. $P_1 = (-0.2, 0.3)$; $P_2 = (2.3, 1.1)$
30. $P_1 = (1.2, 2.3)$; $P_2 = (-0.3, 1.1)$
31. $P_1 = (a, b)$; $P_2 = (0, 0)$
32. $P_1 = (a, a)$; $P_2 = (0, 0)$

In Problems 33–38, plot each point and form the triangle ABC. Show that the triangle is a right triangle. Find its area.

33. $A = (-2, 5)$; $B = (1, 3)$; $C = (-1, 0)$
34. $A = (-2, 5)$; $B = (12, 3)$; $C = (10, -11)$
35. $A = (-5, 3)$; $B = (6, 0)$; $C = (5, 5)$
36. $A = (-6, 3)$; $B = (3, -5)$; $C = (-1, 5)$
37. $A = (4, -3)$; $B = (0, -3)$; $C = (4, 2)$
38. $A = (4, -3)$; $B = (4, 1)$; $C = (2, 1)$

In Problems 39–46, find the midpoint of the line segment joining the points P_1 and P_2.

39. $P_1 = (3, -4)$; $P_2 = (5, 4)$
40. $P_1 = (-2, 0)$; $P_2 = (2, 4)$
41. $P_1 = (-1, 4)$; $P_2 = (8, 0)$
42. $P_1 = (2, -3)$; $P_2 = (4, 2)$
43. $P_1 = (7, -5)$; $P_2 = (9, 1)$
44. $P_1 = (-4, -3)$; $P_2 = (2, 2)$
45. $P_1 = (a, b)$; $P_2 = (0, 0)$
46. $P_1 = (a, a)$; $P_2 = (0, 0)$

Applications and Extensions

47. If the point $(2, 5)$ is shifted 3 units to the right and 2 units down, what are its new coordinates?

48. If the point $(-1, 6)$ is shifted 2 units to the left and 4 units up, what are its new coordinates?

49. Find all points having an x-coordinate of 3 whose distance from the point $(-2, -1)$ is 13.

(a) By using the Pythagorean Theorem.

(b) By using the distance formula.

50. Find all points having a y-coordinate of -6 whose distance from the point $(1, 2)$ is 17.

(a) By using the Pythagorean Theorem.

(b) By using the distance formula.

51. Find all points on the x-axis that are 6 units from the point $(4, -3)$.

52. Find all points on the y-axis that are 6 units from the point $(4, -3)$.

53. Suppose that $A = (2, 5)$ are the coordinates of a point in the xy-plane.

(a) Find the coordinates of the point if A is shifted 3 units to the left and 4 units down.

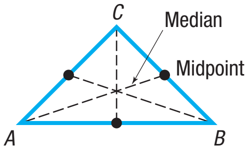
(b) Find the coordinates of the point if A is shifted 2 units to the left and 8 units up.

54. Plot the points $A = (-1, 8)$ and $M = (2, 3)$ in the xy-plane. If M is the midpoint of a line segment AB, find the coordinates of B.

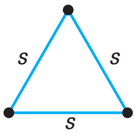
55. The midpoint of the line segment from P_1 to P_2 is $(-1, 4)$. If $P_1 = (-3, 6)$, what is P_2?

56. The midpoint of the line segment from P_1 to P_2 is $(5, -4)$. If $P_2 = (7, -2)$, what is P_1?

57. Geometry The **medians** of a triangle are the line segments from each vertex to the midpoint of the opposite side (see the figure). Find the lengths of the medians of the triangle with vertices at $A = (0, 0)$, $B = (6, 0)$, and $C = (4, 4)$.



58. Geometry An **equilateral triangle** has three sides of equal length. If two vertices of an equilateral triangle are $(0, 4)$ and $(0, 0)$ find the third vertex. How many of these triangles are possible?



*In Problems 59–62, find the length of each side of the triangle determined by the three points P_1, P_2, and P_3. State whether the triangle is an isosceles triangle, a right triangle, neither of these, or both. (An **isosceles triangle** is one in which at least two of the sides are of equal length.)*

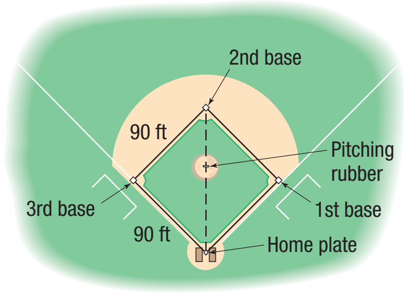
59. $P_1 = (2, 1);\quad P_2 = (-4, 1);\quad P_3 = (-4, -3)$

60. $P_1 = (-1, 4);\quad P_2 = (6, 2);\quad P_3 = (4, -5)$

61. $P_1 = (-2, -1);\quad P_2 = (0, 7);\quad P_3 = (3, 2)$

62. $P_1 = (7, 2);\quad P_2 = (-4, 0);\quad P_3 = (4, 6)$

63. Baseball A major league baseball "diamond" is actually a square 90 feet on a side (see the figure). What is the distance directly from home plate to second base (the diagonal of the square)?



64. Little League Baseball The layout of a Little League playing field is a square 60 feet on a side. How far is it directly from home plate to second base (the diagonal of the square)?

***Source**: 2018 Little League Baseball Official Regulations, Playing Rules, and Operating Policies*

65. Baseball Refer to Problem 63. Overlay a rectangular coordinate system on a major league baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.

(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.

(b) If the right fielder is located at $(310, 15)$, how far is it from the right fielder to second base?

(c) If the center fielder is located at $(300, 300)$, how far is it from the center fielder to third base?

66. Little League Baseball Refer to Problem 64. Overlay a rectangular coordinate system on a Little League baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.

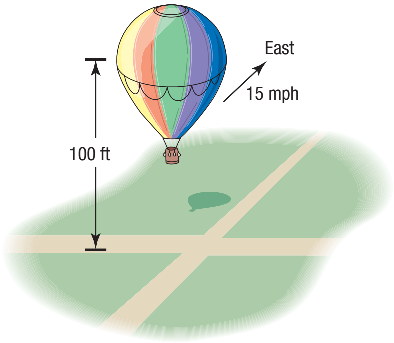
(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.

(b) If the right fielder is located at $(180, 20)$, how far is it from the right fielder to second base?

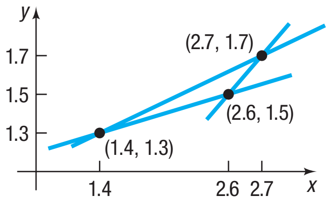
(c) If the center fielder is located at $(220, 220)$, how far is it from the center fielder to third base?

67. Distance between Moving Objects A Ford Focus and a Freightliner Cascadia truck leave an intersection at the same time. The Focus heads east at an average speed of 60 miles per hour, while the Cascadia heads south at an average speed of 45 miles per hour. Find an expression for their distance apart d (in miles) at the end of t hours.

68. **Distance of a Moving Object from a Fixed Point** A hot-air balloon, headed due east at an average speed of 15 miles per hour and at a constant altitude of 100 feet, passes over an intersection (see the figure). Find an expression for the distance d (measured in feet) from the balloon to the intersection t seconds later.



69. **Drafting Error** When a draftsman draws three lines that are to intersect at one point, the lines may not intersect as intended and subsequently will form an **error triangle**. If this error triangle is long and thin, one estimate for the location of the desired point is the midpoint of the shortest side. The figure shows one such error triangle.

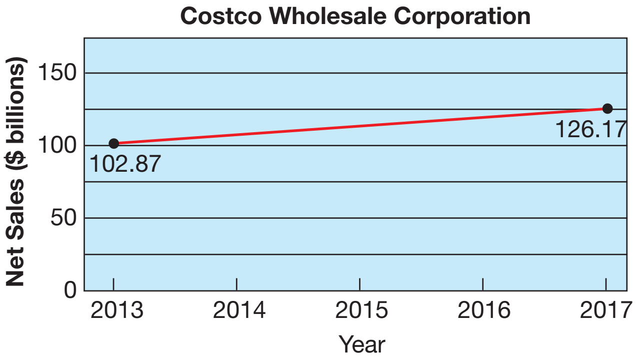


(a) Find an estimate for the desired intersection point.

(b) Find the distance from (1.4, 1.3) to the midpoint found in part (a).

70. **Net Sales** The figure at the top of the next column illustrates the net sales growth of Costco Wholesale Corporation from 2013 through 2017. Use the midpoint formula to estimate the net sales of Costco Wholesale Corporation in 2015. How does your result compare to the reported value of \$113.67 billion?

***Source**: Costco Wholesale Corporation 2017 Annual Report*



71. **Poverty Threshold** Poverty thresholds are determined by the U.S. Census Bureau. A poverty threshold represents the minimum annual household income for a family not to be considered poor. In 2009, the poverty threshold for a family of four with two children under the age of 18 years was \$21,756. In 2017, the poverty threshold for a family of four with two children under the age of 18 years was \$24,858. Assuming that poverty thresholds increase in a straight-line fashion, use the midpoint formula to estimate the poverty threshold for a family of four with two children under the age of 18 in 2013. How does your result compare to the actual poverty threshold in 2013 of \$23,624?

***Source**: U.S. Census Bureau*

72. ***Challenge Problem*** **Geometry** Verify that the points $(0,0)$, $(a,0)$, and $\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$ are the vertices of an equilateral triangle. Then show that the midpoints of the three sides are the vertices of a second equilateral triangle.

73. ***Challenge Problem*** **Geometry** Find the midpoint of each diagonal of a square with side of length s. Draw the conclusion that the diagonals of a square intersect at their midpoints.

[**Hint**: Use $(0, 0)$, $(0, s)$, $(s, 0)$, and (s, s) as the vertices of the square.]

74. ***Challenge Problem*** **Geometry** A point P is equidistant from $(-5, 1)$ and $(4, -4)$. Find the coordinates of P if its y-coordinate is twice its x-coordinate.

75. ***Challenge Problem*** **Geometry** For any parallelogram, prove that the sum of the squares of the lengths of the sides equals the sum of the squares of the lengths of the diagonals.

[**Hint**: Use $(0, 0)$, $(a, 0)$, $(a + b, c)$, and (b, c) as the vertices of the parallelogram. Assume a, b, and c are positive.]

Explaining Concepts: Discussion and Writing

76. Write a paragraph that describes a Cartesian plane. Then write a second paragraph that describes how to plot points in the Cartesian plane. Your paragraphs should include the terms "coordinate axes," "ordered pair," "coordinates," "plot," "x-coordinate," and "y-coordinate."

'Are You Prepared?' Answers

1. 0 **2.** 8 **3.** 5 **4.** $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$ **5.** $\frac{1}{2}bh$ **6.** True

1.2 Graphs of Equations in Two Variables; Circles

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Solving Equations (Section A.4, pp. A27–A34)
- Complete the Square (Section A.3, pp. A24–A25)

Now Work the **'Are You Prepared?'** problems on page 19.

OBJECTIVES
1 Graph Equations by Plotting Points (p. 9)
2 Find Intercepts from a Graph (p. 11)
3 Find Intercepts from an Equation (p. 11)
4 Test an Equation for Symmetry with Respect to the *x*-Axis, the *y*-Axis, and the Origin (p. 12)
5 Know How to Graph Key Equations (p. 14)
6 Write the Standard Form of the Equation of a Circle (p. 16)
7 Graph a Circle (p. 17)
8 Work with the General Form of the Equation of a Circle (p. 18)

1 Graph Equations by Plotting Points

An **equation in two variables**, say x and y, is a statement in which two expressions involving x and y are equal. The expressions are called the **sides** of the equation. Since an equation is a statement, it may be true or false, depending on the value of the variables. Any pair of values for x and y that result in a true statement are said to **satisfy** the equation.

For example, the following are all equations in two variables x and y:

$$x^2 + y^2 = 5 \qquad 2x - y = 6 \qquad y = 2x + 5 \qquad x^2 = y$$

The first of these, $x^2 + y^2 = 5$, is satisfied for $x = 1, y = 2$, since $1^2 + 2^2 = 5$. Other choices of x and y, such as $x = -1, y = -2$, also satisfy this equation. It is not satisfied for $x = 2$ and $y = 3$, since $2^2 + 3^2 = 4 + 9 = 13 \neq 5$.

The **graph of an equation in two variables** x and y consists of the set of points in the xy-plane whose coordinates (x, y) satisfy the equation.

EXAMPLE 1 Determining Whether a Point Is on the Graph of an Equation

Determine if the following points are on the graph of the equation $2x - y = 6$.

(a) $(2, 3)$ (b) $(2, -2)$

Solution

(a) For the point $(2, 3)$, check to see whether $x = 2, y = 3$ satisfies the equation $2x - y = 6$.

$$2x - y = 2 \cdot 2 - 3 = 4 - 3 = 1 \neq 6$$

The equation is not satisfied, so the point $(2, 3)$ is not on the graph of $2x - y = 6$.

(b) For the point $(2, -2)$,

$$2x - y = 2 \cdot 2 - (-2) = 4 + 2 = 6$$

The equation is satisfied, so the point $(2, -2)$ is on the graph of $2x - y = 6$.

Now Work PROBLEM 13

EXAMPLE 2 Graphing an Equation by Plotting Points

Graph the equation: $y = 2x + 5$

Solution

The graph consists of all points (x, y) that satisfy the equation. To locate some of these points (and get an idea of the pattern of the graph), assign some numbers to x, and find corresponding values for y.

If	Then	Point on Graph
$x = 0$	$y = 2 \cdot 0 + 5 = 5$	$(0, 5)$
$x = 1$	$y = 2 \cdot 1 + 5 = 7$	$(1, 7)$
$x = -5$	$y = 2 \cdot (-5) + 5 = -5$	$(-5, -5)$
$x = 10$	$y = 2 \cdot 10 + 5 = 25$	$(10, 25)$

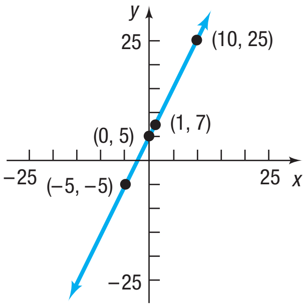


Figure 9 $y = 2x + 5$

By plotting these points and then connecting them, we obtain the graph (a *line*) of the equation $y = 2x + 5$, as shown in Figure 9.

EXAMPLE 3 Graphing an Equation by Plotting Points

Graph the equation: $y = x^2$

Solution

Table 1 provides several points on the graph of $y = x^2$. Plotting these points and connecting them with a smooth curve gives the graph (a *parabola*) shown in Figure 10.

Table 1

x	$y = x^2$	(x, y)
-4	16	$(-4, 16)$
-3	9	$(-3, 9)$
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$
3	9	$(3, 9)$
4	16	$(4, 16)$

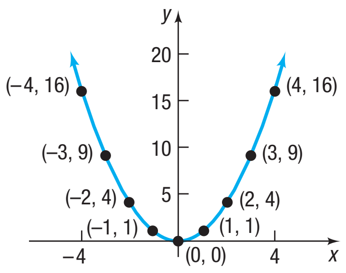


Figure 10 $y = x^2$

The graphs of the equations shown in Figures 9 and 10 do not show all points. For example, in Figure 9, the point $(20, 45)$ is a part of the graph of $y = 2x + 5$, but it is not shown. Since the graph of $y = 2x + 5$ can be extended out indefinitely, we use arrows to indicate that the pattern shown continues. It is important, when showing a graph, to present enough of the graph so that any viewer of the illustration will "see" the rest of it as an obvious continuation of what is actually there. This is referred to as a **complete graph**.

COMMENT Another way to obtain the graph of an equation is to use a graphing utility. Read Section B.2, *Using a Graphing Utility to Graph Equations.*

One way to obtain the complete graph of an equation is to plot enough points on the graph for a pattern to become evident. Then these points are connected with a smooth curve following the suggested pattern. But how many points are sufficient? Sometimes knowledge about the equation tells us. For example, we discuss in Section A.8 that if an equation is of the form $y = mx + b$, then its graph is a line. In this case, only two points are needed to obtain the complete graph.

One purpose of this text is to investigate the properties of equations in order to decide whether a graph is complete. Sometimes we shall graph equations by plotting points. Shortly, we shall investigate various techniques that will enable us to graph an equation without plotting so many points.

Two techniques that sometimes reduce the number of points required to graph an equation involve finding *intercepts* and checking for *symmetry*.

2 Find Intercepts from a Graph

The points, if any, at which a graph crosses or touches the coordinate axes are called the **intercepts** of the graph. See Figure 11. The x-coordinate of a point at which the graph crosses or touches the x-axis is an ***x*-intercept**, and the y-coordinate of a point at which the graph crosses or touches the y-axis is a ***y*-intercept**.

In Words
Intercepts are points (ordered pairs). An x-intercept or a y-intercept is a number. For example, the point (3, 0) is an intercept; the number 3 is an x-intercept.

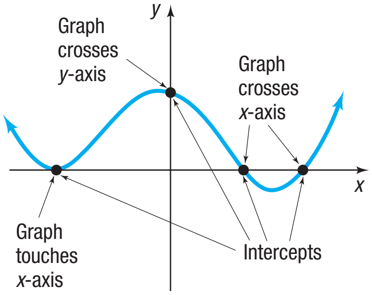


Figure 11

EXAMPLE 4 Finding Intercepts from a Graph

Find the intercepts of the graph in Figure 12. What are its x-intercepts? What are its y-intercepts?

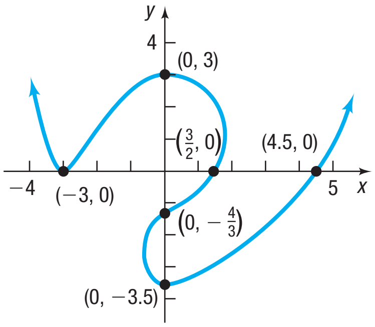


Figure 12

Solution The intercepts of the graph are the points

$$(-3, 0), \quad (0, 3), \quad \left(\frac{3}{2}, 0\right), \quad \left(0, -\frac{4}{3}\right), \quad (0, -3.5), \quad (4.5, 0)$$

The x-intercepts are -3, $\frac{3}{2}$, and 4.5; the y-intercepts are -3.5, $-\frac{4}{3}$, and 3.

In Example 4, notice that intercepts are listed as ordered pairs, and the x-intercepts and the y-intercepts are listed as numbers. We use this distinction throughout the text.

Now Work PROBLEM 41(a)

3 Find Intercepts from an Equation

The intercepts of a graph can be found from its equation by using the fact that points on the x-axis have y-coordinates equal to 0, and points on the y-axis have x-coordinates equal to 0.

COMMENT For many equations, finding intercepts may not be so easy. In such cases, a graphing utility can be used. Read the first part of Section B.3, *Using a Graphing Utility to Locate Intercepts and Check for Symmetry*, to find out how to locate intercepts using a graphing utility. ■

Procedure for Finding Intercepts

- To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x, where x is a real number.
- To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y, where y is a real number.

EXAMPLE 5 Finding Intercepts from an Equation

Find the x-intercept(s) and the y-intercept(s) of the graph of $y = x^2 - 4$. Then graph $y = x^2 - 4$ by plotting points.

Solution To find the x-intercept(s), let $y = 0$ and obtain the equation

$$\begin{aligned} x^2 - 4 &= 0 && y = x^2 - 4 \text{ with } y = 0 \\ (x+2)(x-2) &= 0 && \text{Factor.} \\ x + 2 = 0 \quad \text{or} \quad x - 2 &= 0 && \text{Use the Zero-Product Property.} \\ x = -2 \quad \text{or} \quad x &= 2 && \text{Solve.} \end{aligned}$$

The equation has two solutions, -2 and 2. The x-intercepts are -2 and 2.

To find the y-intercept(s), let $x = 0$ in the equation.

$$\begin{aligned} y &= x^2 - 4 \\ &= 0^2 - 4 = -4 \end{aligned}$$

The y-intercept is -4.

Since $x^2 \geq 0$ for all x, we deduce from the equation $y = x^2 - 4$ that $y \geq -4$ for all x. This information, the intercepts, and the points from Table 2 enable us to graph $y = x^2 - 4$. See Figure 13.

Table 2

x	$y = x^2 - 4$	(x, y)
-3	5	$(-3, 5)$
-1	-3	$(-1, -3)$
1	-3	$(1, -3)$
3	5	$(3, 5)$

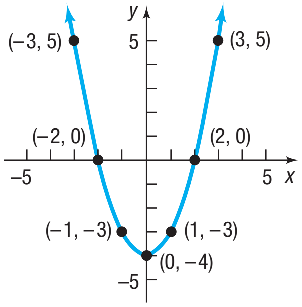


Figure 13 $y = x^2 - 4$

Now Work PROBLEM 23

4 Test an Equation for Symmetry with Respect to the x-Axis, the y-Axis, and the Origin

Another helpful tool for graphing equations by hand involves *symmetry*, particularly symmetry with respect to the x-axis, the y-axis, and the origin.

Symmetry often occurs in nature. Consider the picture of the butterfly. Do you see the symmetry?

DEFINITION Symmetry with Respect to the x-Axis

A graph is **symmetric with respect to the x-axis** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

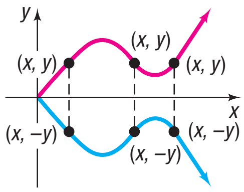


Figure 14 Symmetry with respect to the x-axis

Figure 14 illustrates the definition. Note that when a graph is symmetric with respect to the x-axis, the part of the graph above the x-axis is a reflection (or mirror image) of the part below it, and vice versa.

EXAMPLE 6 **Points Symmetric with Respect to the x-Axis**

If a graph is symmetric with respect to the x-axis, and the point $(3, 2)$ is on the graph, then the point $(3, -2)$ is also on the graph.

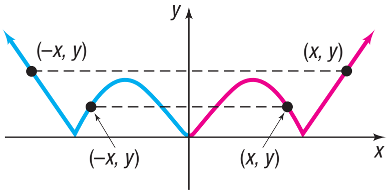


Figure 15 Symmetry with respect to the y-axis

DEFINITION Symmetry with Respect to the y-Axis

A graph is **symmetric with respect to the y-axis** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

Figure 15 illustrates the definition. When a graph is symmetric with respect to the y-axis, the part of the graph to the right of the y-axis is a reflection of the part to the left of it, and vice versa.

EXAMPLE 7

Points Symmetric with Respect to the y-Axis

If a graph is symmetric with respect to the y-axis and the point $(5, 8)$ is on the graph, then the point $(-5, 8)$ is also on the graph.

DEFINITION Symmetry with Respect to the Origin

A graph is **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.

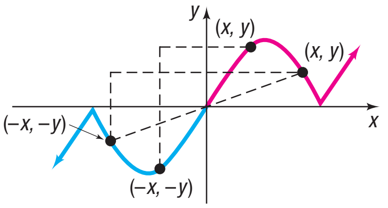


Figure 16 Symmetry with respect to the origin

Figure 16 illustrates the definition. Symmetry with respect to the origin may be viewed in three ways:

- As a reflection about the y-axis, followed by a reflection about the x-axis
- As a projection along a line through the origin so that the distances from the origin are equal
- As half of a complete revolution about the origin

EXAMPLE 8

Points Symmetric with Respect to the Origin

If a graph is symmetric with respect to the origin, and the point $(4, -2)$ is on the graph, then the point $(-4, 2)$ is also on the graph.

Now Work PROBLEMS 31 AND 41(b)

When the graph of an equation is symmetric with respect to a coordinate axis or the origin, the number of points that you need to plot in order to see the pattern is reduced. For example, if the graph of an equation is symmetric with respect to the y-axis, then once points to the right of the y-axis are plotted, an equal number of points on the graph can be obtained by reflecting them about the y-axis. Because of this, before we graph an equation, we should first determine whether it has any symmetry. The following tests are used for this purpose.

Tests for Symmetry

To test the graph of an equation for symmetry with respect to the

- *x-Axis* Replace y by $-y$ in the equation and simplify. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.
- *y-Axis* Replace x by $-x$ in the equation and simplify. If an equivalent equation results, the graph of the equation is symmetric with respect to the y-axis.
- *Origin* Replace x by $-x$ and y by $-y$ in the equation and simplify. If an equivalent equation results, the graph of the equation is symmetric with respect to the origin.

EXAMPLE 9 Testing an Equation for Symmetry

Test $4x^2 + 9y^2 = 36$ for symmetry.

Solution

x-Axis: To test for symmetry with respect to the x-axis, replace y by $-y$. Since $4x^2 + 9(-y)^2 = 36$ is equivalent to $4x^2 + 9y^2 = 36$, the graph of the equation is symmetric with respect to the x-axis.

y-Axis: To test for symmetry with respect to the y-axis, replace x by $-x$. Since $4(-x)^2 + 9y^2 = 36$ is equivalent to $4x^2 + 9y^2 = 36$, the graph of the equation is symmetric with respect to the y-axis.

Origin: To test for symmetry with respect to the origin, replace x by $-x$ and y by $-y$. Since $4(-x)^2 + 9(-y)^2 = 36$ is equivalent to $4x^2 + 9y^2 = 36$, the graph of the equation is symmetric with respect to the origin.

EXAMPLE 10 Testing an Equation for Symmetry

Test $y = \dfrac{4x^2}{x^2 + 1}$ for symmetry.

Solution

x-Axis: To test for symmetry with respect to the x-axis, replace y by $-y$. Since $-y = \dfrac{4x^2}{x^2 + 1}$ is not equivalent to $y = \dfrac{4x^2}{x^2 + 1}$, the graph of the equation is not symmetric with respect to the x-axis.

y-Axis: To test for symmetry with respect to the y-axis, replace x by $-x$. Since $y = \dfrac{4(-x)^2}{(-x)^2 + 1} = \dfrac{4x^2}{x^2 + 1}$ is equivalent to $y = \dfrac{4x^2}{x^2 + 1}$, the graph of the equation is symmetric with respect to the y-axis.

Origin: To test for symmetry with respect to the origin, replace x by $-x$ and y by $-y$.

$$-y = \frac{4(-x)^2}{(-x)^2 + 1} \qquad \text{Replace } x \text{ by } -x \text{ and } y \text{ by } -y.$$

$$-y = \frac{4x^2}{x^2 + 1} \qquad \text{Simplify.}$$

$$y = -\frac{4x^2}{x^2 + 1} \qquad \text{Multiply both sides by } -1.$$

Since the result is not equivalent to the original equation, the graph of the equation $y = \dfrac{4x^2}{x^2 + 1}$ is not symmetric with respect to the origin.

Seeing the Concept

Figure 17 shows the graph of $4x^2 + 9y^2 = 36$ using Desmos. Do you see the symmetry with respect to the x-axis, the y-axis, and the origin?

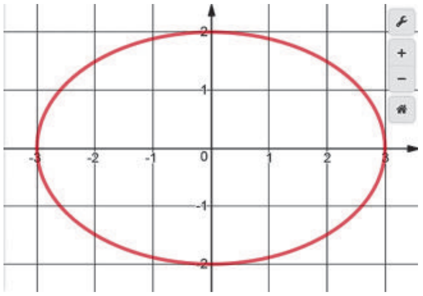


Figure 17 $4x^2 + 9y^2 = 36$

Seeing the Concept

Figure 18 shows the graph of $y = \dfrac{4x^2}{x^2 + 1}$ using a TI-84 Plus C graphing calculator. Do you see the symmetry with respect to the y-axis?

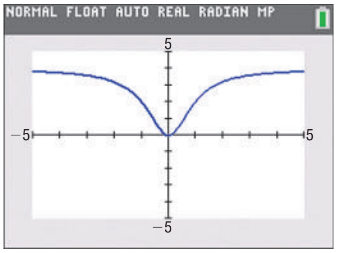


Figure 18 $y = \dfrac{4x^2}{x^2 + 1}$

Now Work PROBLEM 61

5 Know How to Graph Key Equations

The next three examples use intercepts, symmetry, and point plotting to obtain the graphs of key equations. It is important to know the graphs of these key equations because we use them later. The first of these is $y = x^3$.

EXAMPLE 11

Graphing the Equation $y = x^3$ by Finding Intercepts, Checking for Symmetry, and Plotting Points

Graph the equation $y = x^3$ by plotting points. Find any intercepts and check for symmetry first.

Solution First, find the intercepts. When $x = 0$, then $y = 0$; and when $y = 0$, then $x = 0$. The origin $(0, 0)$ is the only intercept. Now test for symmetry.

x-Axis: Replace y by $-y$. Since $-y = x^3$ is not equivalent to $y = x^3$, the graph is not symmetric with respect to the x-axis.

y-Axis: Replace x by $-x$. Since $y = (-x)^3 = -x^3$ is not equivalent to $y = x^3$, the graph is not symmetric with respect to the y-axis.

Origin: Replace x by $-x$ and y by $-y$. Since $-y = (-x)^3 = -x^3$ is equivalent to $y = x^3$ (multiply both sides by -1), the graph is symmetric with respect to the origin.

To graph $y = x^3$, use the equation to obtain several points on the graph. Because of the symmetry, we need to locate only points on the graph for which $x \geq 0$. See Table 3. Since $(1, 1)$ is on the graph, and the graph is symmetric with respect to the origin, the point $(-1, -1)$ is also on the graph. Plot the points from Table 3 and use the symmetry. Figure 19 shows the graph.

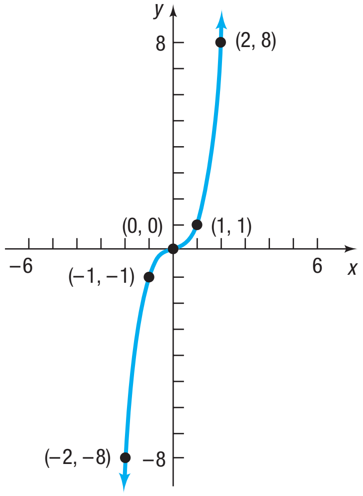


Figure 19 $y = x^3$

Table 3

x	$y = x^3$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	8	(2, 8)
3	27	(3, 27)

EXAMPLE 12

Graphing the Equation $x = y^2$

(a) Graph the equation $x = y^2$. Find any intercepts and check for symmetry first.

(b) Graph $x = y^2$, $y \geq 0$.

Solution (a) The lone intercept is $(0, 0)$. The graph is symmetric with respect to the x-axis. (Do you see why? Replace y by $-y$.) Figure 20(a) shows the graph.

(b) If we restrict y so that $y \geq 0$, the equation $x = y^2$, $y \geq 0$, may be written equivalently as $y = \sqrt{x}$. The portion of the graph of $x = y^2$ in quadrant I is therefore the graph of $y = \sqrt{x}$. See Figure 20(b).

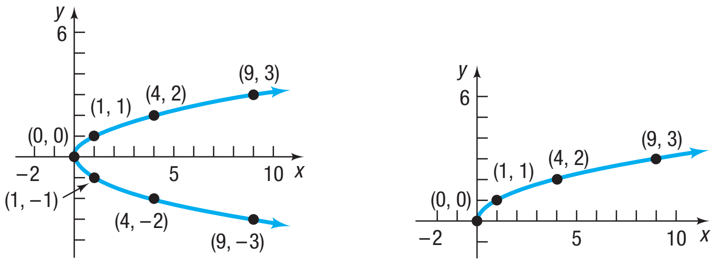


Figure 20 (a) $x = y^2$ (b) $y = \sqrt{x}$

COMMENT To see the graph of the equation $x = y^2$ on a graphing calculator, you will need to graph two equations: $Y_1 = \sqrt{x}$ and $Y_2 = -\sqrt{x}$. See Figure 21. ■

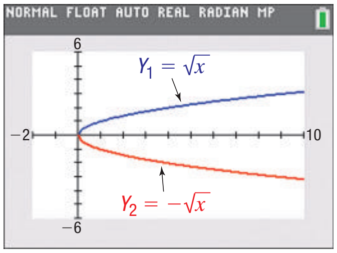


Figure 21

EXAMPLE 13 Graphing the Equation $y = \frac{1}{x}$

Graph the equation $y = \frac{1}{x}$. First, find any intercepts and check for symmetry.

Solution Check for intercepts first. If we let $x = 0$, we obtain 0 in the denominator, which makes y undefined. We conclude that there is no y-intercept. If we let $y = 0$, we get the equation $\frac{1}{x} = 0$, which has no solution. We conclude that there is no x-intercept. The graph of $y = \frac{1}{x}$ does not cross or touch the coordinate axes.

Next check for symmetry:

x-Axis: Replacing y by $-y$ yields $-y = \frac{1}{x}$, which is not equivalent to $y = \frac{1}{x}$.

y-Axis: Replacing x by $-x$ yields $y = \frac{1}{-x} = -\frac{1}{x}$, which is not equivalent to $y = \frac{1}{x}$.

Origin: Replacing x by $-x$ and y by $-y$ yields $-y = -\frac{1}{x}$, which is equivalent to $y = \frac{1}{x}$. The graph is symmetric with respect to the origin.

Table 4

x	$y = \frac{1}{x}$	(x, y)
$\frac{1}{10}$	10	$\left(\frac{1}{10}, 10\right)$
$\frac{1}{3}$	3	$\left(\frac{1}{3}, 3\right)$
$\frac{1}{2}$	2	$\left(\frac{1}{2}, 2\right)$
1	1	$(1, 1)$
2	$\frac{1}{2}$	$\left(2, \frac{1}{2}\right)$
3	$\frac{1}{3}$	$\left(3, \frac{1}{3}\right)$
10	$\frac{1}{10}$	$\left(10, \frac{1}{10}\right)$

Now set up Table 4, listing several points on the graph. Because of the symmetry with respect to the origin, we use only positive values of x. From Table 4 we infer that if x is a large and positive number, then $y = \frac{1}{x}$ is a positive number close to 0. We also infer that if x is a positive number close to 0, then $y = \frac{1}{x}$ is a large and positive number. Armed with this information, we can graph the equation.

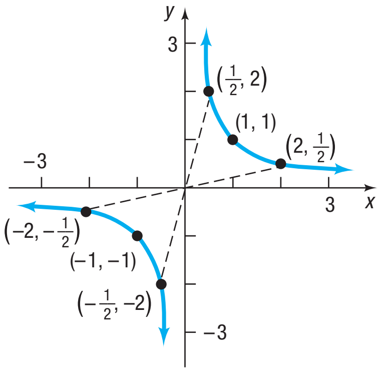


Figure 22 $y = \frac{1}{x}$

Figure 22 illustrates some of these points and the graph of $y = \frac{1}{x}$. Observe how the absence of intercepts and the existence of symmetry with respect to the origin were utilized.

COMMENT Refer to Example 2 in Section B.3, for the graph of $y = \frac{1}{x}$ using a graphing calculator. ■

6 Write the Standard Form of the Equation of a Circle

One advantage of a coordinate system is that it enables us to translate a geometric statement into an algebraic statement, and vice versa. Consider, for example, the following geometric statement that defines a circle.

DEFINITION Circle

A **circle** is a set of points in the xy-plane that are a fixed distance r from a fixed point (h, k). The fixed distance r is called the **radius**, and the fixed point (h, k) is called the **center** of the circle.

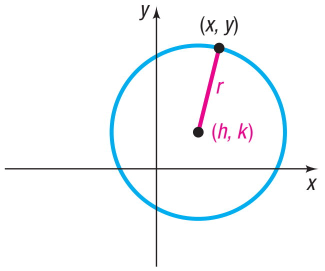


Figure 23 $(x - h)^2 + (y - k)^2 = r^2$

Need to Review?
The distance formula is discussed in Section 1.1, pp. 3–4.

Figure 23 shows the graph of a circle. To find the equation, let (x, y) represent the coordinates of any point on a circle with radius r and center (h, k). Then the distance between the points (x, y) and (h, k) must always equal r. That is, by the distance formula,

$$\sqrt{(x - h)^2 + (y - k)^2} = r$$

or, equivalently,

$$(x - h)^2 + (y - k)^2 = r^2$$

DEFINITION Standard Form of an Equation of a Circle

The **standard form of an equation of a circle** with radius r and center (h, k) is

$$(x-h)^2+(y-k)^2=r^2 \qquad \textbf{(1)}$$

THEOREM

The standard form of an equation of a circle of radius r with center at the origin $(0, 0)$ is

$$x^2+y^2=r^2$$

DEFINITION Unit Circle

If the radius $r = 1$, the circle whose center is at the origin is called the **unit circle** and has the equation

$$x^2+y^2=1$$

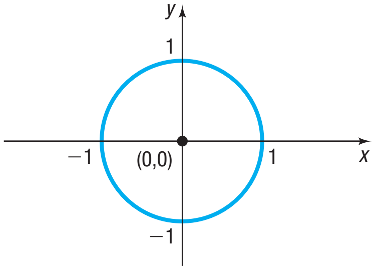


Figure 24
Unit circle $x^2 + y^2 = 1$

See Figure 24. Notice that the graph of the unit circle is symmetric with respect to the x-axis, the y-axis, and the origin.

EXAMPLE 14 Writing the Standard Form of the Equation of a Circle

Write the standard form of the equation of the circle with radius 5 and center $(-3, 6)$.

Solution Substitute the values $r = 5$, $h = -3$, and $k = 6$ into equation (1).

$$
\begin{aligned}
(x-h)^2+(y-k)^2&=r^2 \qquad \text{Equation (1)}\\
[x-(-3)]^2+(y-6)^2&=5^2\\
(x+3)^2+(y-6)^2&=25
\end{aligned}
$$

Now Work PROBLEM 79

7 Graph a Circle

EXAMPLE 15 Graphing a Circle

Graph the equation: $(x+3)^2+(y-2)^2=16$

Solution The given equation is in the standard form of an equation of a circle. To graph the circle, compare the equation to equation (1). The comparison gives information about the circle.

$$
\begin{aligned}
(x+3)^2+(y-2)^2&=16\\
(x-\underset{\uparrow}{(-3)})^2+(y-\underset{\uparrow}{2})^2&=\underset{\uparrow}{4^2}\\
(x-h)^2\;+(y-k)^2&=r^2
\end{aligned}
$$

We see that $h = -3$, $k = 2$, and $r = 4$. The circle has center $(-3, 2)$ and radius 4. To graph this circle, first plot the center $(-3, 2)$. Since the radius is 4, locate four points on the circle by plotting points 4 units to the left, to the right, up, and down from the center. These four points are then used as guides to obtain the graph. See Figure 25.

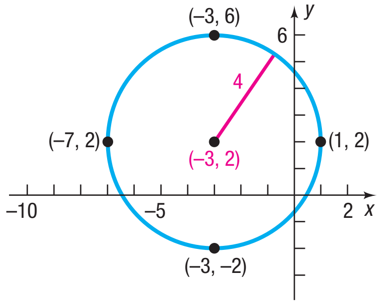


Figure 25 $(x+3)^2+(y-2)^2=16$

Now Work PROBLEMS 95(a) AND (b)

EXAMPLE 16 Finding the Intercepts of a Circle

Find the intercepts, if any, of the graph of the circle $(x+3)^2 + (y-2)^2 = 16$.

In Words

The symbol ± is read "plus or minus." It means to add and subtract the quantity following the ± symbol. For example, 5 ± 2 means "5 − 2 = 3 or 5 + 2 = 7."

Solution This is the equation graphed in Example 15. To find the x-intercepts, if any, let $y = 0$. Then

$$\begin{aligned}
(x+3)^2 + (y-2)^2 &= 16 \\
(x+3)^2 + (0-2)^2 &= 16 && y = 0 \\
(x+3)^2 + 4 &= 16 && \text{Simplify.} \\
(x+3)^2 &= 12 && \text{Simplify.} \\
x + 3 &= \pm\sqrt{12} && \text{Use the Square Root Method.} \\
x &= -3 \pm 2\sqrt{3} && \text{Solve for } x.
\end{aligned}$$

The x-intercepts are $-3 - 2\sqrt{3} \approx -6.46$ and $-3 + 2\sqrt{3} \approx 0.46$.

To find the y-intercepts, if any, let $x = 0$. Then

$$\begin{aligned}
(x+3)^2 + (y-2)^2 &= 16 \\
(0+3)^2 + (y-2)^2 &= 16 \\
9 + (y-2)^2 &= 16 \\
(y-2)^2 &= 7 \\
y - 2 &= \pm\sqrt{7} \\
y &= 2 \pm \sqrt{7}
\end{aligned}$$

The y-intercepts are $2 - \sqrt{7} \approx -0.65$ and $2 + \sqrt{7} \approx 4.65$.

Look back at Figure 25 to verify the approximate locations of the intercepts.

Now Work PROBLEM 95(c)

8 Work with the General Form of the Equation of a Circle

If we eliminate the parentheses from the standard form of the equation of the circle given in Example 15, we get

$$\begin{gathered}
(x+3)^2 + (y-2)^2 = 16 \\
x^2 + 6x + 9 + y^2 - 4y + 4 = 16
\end{gathered}$$

which simplifies to

$$x^2 + y^2 + 6x - 4y - 3 = 0$$

It can be shown that any equation of the form

$$x^2 + y^2 + ax + by + c = 0$$

has a graph that is a circle, is a point, or has no graph at all. For example, the graph of the equation $x^2 + y^2 = 0$ is the single point $(0, 0)$. The equation $x^2 + y^2 + 5 = 0$, or $x^2 + y^2 = -5$, has no graph, because sums of squares of real numbers are never negative.

DEFINITION General Form of the Equation of a Circle

When its graph is a circle, the equation

$$\boxed{x^2 + y^2 + ax + by + c = 0}$$

is the **general form of the equation of a circle**.

Now Work PROBLEM 85

Need to Review?
Completing the square is discussed in Section A.3, pp. A24–A25.

If an equation of a circle is in general form, we use the method of completing the square to put the equation in standard form so that we can identify its center and radius.

EXAMPLE 17 Graphing a Circle Whose Equation Is in General Form

Graph the equation: $x^2 + y^2 + 4x - 6y + 12 = 0$

Solution Group the terms involving x, group the terms involving y, and put the constant on the right side of the equation. The result is

$$(x^2 + 4x) + (y^2 - 6y) = -12$$

Next, complete the square of each expression in parentheses. Remember that any number added on the left side of the equation must also be added on the right.

$$(x^2 + 4x + \underbrace{4}_{\left(\frac{4}{2}\right)^2 = 4}) + (y^2 - 6y + \underbrace{9}_{\left(\frac{-6}{2}\right)^2 = 9}) = -12 + 4 + 9$$

$$(x+2)^2 + (y-3)^2 = 1 \qquad \text{Factor.}$$

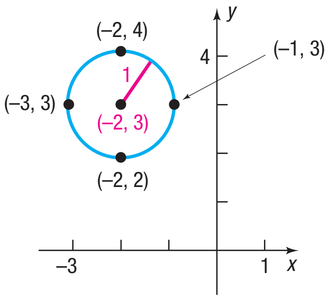


Figure 26 $(x+2)^2 + (y-3)^2 = 1$

This equation is the standard form of the equation of a circle with radius 1 and center $(-2, 3)$. To graph the equation, use the center $(-2, 3)$ and the radius 1. See Figure 26.

Now Work PROBLEM 99

EXAMPLE 18 Using a Graphing Utility to Graph a Circle

Graph the equation: $x^2 + y^2 = 4$

Solution This is the equation of a circle with center at the origin and radius 2. To graph this equation using a graphing utility, begin by solving for y.

$$\begin{aligned} x^2 + y^2 &= 4 \\ y^2 &= 4 - x^2 && \text{Subtract } x^2 \text{ from both sides.} \\ y &= \pm\sqrt{4 - x^2} && \text{Use the Square Root Method to solve for } y. \end{aligned}$$

There are two equations to graph: first graph $Y_1 = \sqrt{4 - x^2}$ and then graph $Y_2 = -\sqrt{4 - x^2}$ on the same square screen. (Your circle will appear oval if you do not use a square screen.*) See Figure 27.

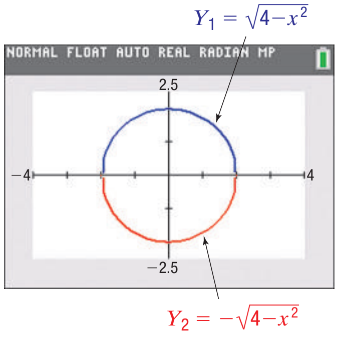


Figure 27 $x^2 + y^2 = 4$

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*The square screen ratio for the TI-84 Plus C graphing calculator is 8:5.

1.2 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. To complete the square of $x^2 + 10x$, you would _______ (*add*/*subtract*) the number ______. (pp. A24–A25)
2. Use the Square Root Method to solve the equation $(x-2)^2 = 9$. (p. A31)

Concepts and Vocabulary

3. The points, if any, at which a graph crosses or touches the coordinate axes are called __________.

4. If for every point (x, y) on the graph of an equation the point $(-x, y)$ is also on the graph, then the graph is symmetric with respect to the __________.

5. If the graph of an equation is symmetric with respect to the origin and $(3, -4)$ is a point on the graph, then __________ is also a point on the graph.

6. ***True or False*** To find the y-intercepts of the graph of an equation, let $x = 0$ and solve for y.

7. ***True or False*** If a graph is symmetric with respect to the x-axis, then it cannot be symmetric with respect to the y-axis.

8. For a circle, the __________ is the distance from the center to any point on the circle.

9. ***True or False*** The radius of the circle $x^2 + y^2 = 9$ is 3.

10. ***Multiple Choice*** Given that the intercepts of a graph are $(-4, 0)$ and $(0, 5)$, choose the statement that is true.
(a) The y-intercept is -4, and the x-intercept is 5.
(b) The y-intercepts are -4 and 5.
(c) The x-intercepts are -4 and 5.
(d) The x-intercept is -4, and the y-intercept is 5.

11. ***True or False*** The center of the circle

$$(x + 3)^2 + (y - 2)^2 = 13$$

is $(3, -2)$.

12. ***Multiple Choice*** The equation of a circle can be changed from general form to standard form by doing which of the following?
(a) completing the squares
(b) solving for x
(c) solving for y
(d) squaring both sides

Skill Building

In Problems 13–18, determine which of the given points are on the graph of the equation.

13. Equation: $y = x^4 - \sqrt{x}$
Points: $(0, 0)$; $(1, 1)$; $(2, 4)$

14. Equation: $y = x^3 - 2\sqrt{x}$
Points: $(0, 0)$; $(1, 1)$; $(1, -1)$

15. Equation: $y^2 = x^2 + 9$
Points: $(0, 3)$; $(3, 0)$; $(-3, 0)$

16. Equation: $y^3 = x + 1$
Points: $(1, 2)$; $(0, 1)$; $(-1, 0)$

17. Equation: $x^2 + y^2 = 4$
Points: $(0, 2)$; $(-2, 2)$; $\left(\sqrt{2}, \sqrt{2}\right)$

18. Equation: $x^2 + 4y^2 = 4$
Points: $(0, 1)$; $(2, 0)$; $\left(2, \frac{1}{2}\right)$

In Problems 19–30, find the intercepts and graph each equation by plotting points. Be sure to label the intercepts.

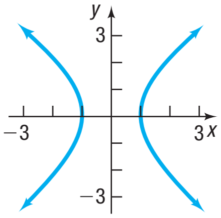
19. $y = x + 2$
20. $y = x - 6$
21. $y = 2x + 8$
22. $y = 3x - 9$
23. $y = x^2 - 1$
24. $y = x^2 - 9$
25. $y = -x^2 + 4$
26. $y = -x^2 + 1$
27. $2x + 3y = 6$
28. $5x + 2y = 10$
29. $9x^2 + 4y = 36$
30. $4x^2 + y = 4$

In Problems 31–40, plot each point. Then plot the point that is symmetric to it with respect to (a) the x-axis; (b) the y-axis; (c) the origin.

31. $(3, 4)$
32. $(5, 3)$
33. $(-2, 1)$
34. $(4, -2)$
35. $(5, -2)$
36. $(-1, -1)$
37. $(-3, -4)$
38. $(4, 0)$
39. $(0, -3)$
40. $(-3, 0)$

In Problems 41–52, the graph of an equation is given. (a) Find the intercepts. (b) Indicate whether the graph is symmetric with respect to the x-axis, the y-axis, or the origin.

41.



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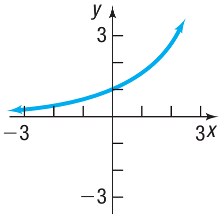

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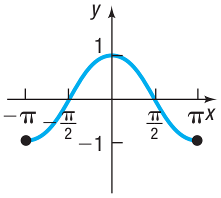

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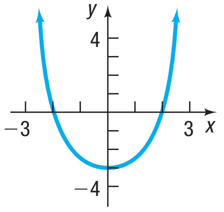

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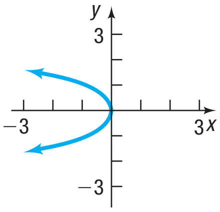

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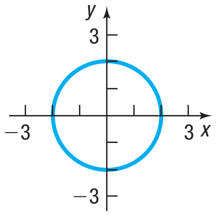

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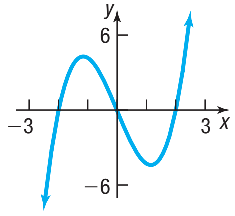

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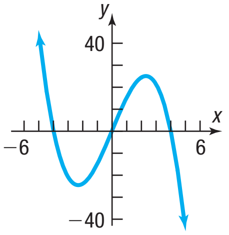

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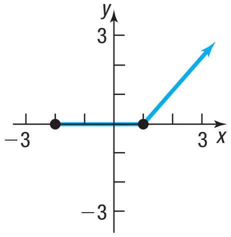

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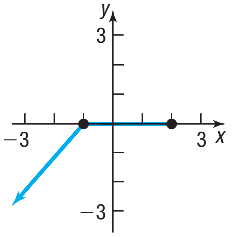

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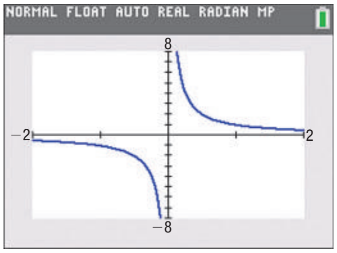

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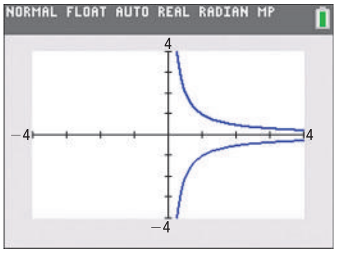
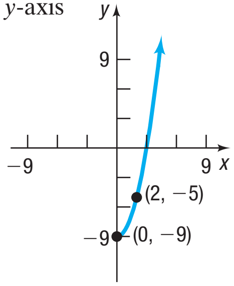
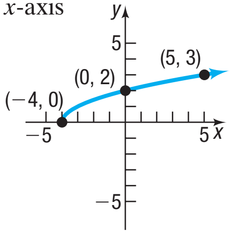


In Problems 53–56, draw a complete graph so that it has the type of symmetry indicated.

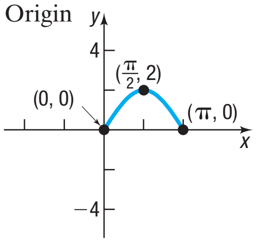
53. y-axis



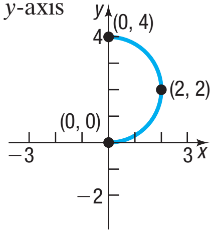
54. x-axis



55. Origin



56. y-axis



In Problems 57–72, list the intercepts and test for symmetry.

57. $y^2 = x + 4$

58. $y^2 = x + 9$

59. $y = \sqrt[3]{x}$

60. $y = \sqrt[5]{x}$

61. $x^2 + y - 9 = 0$

62. $x^2 - y - 4 = 0$

63. $9x^2 + 4y^2 = 36$

64. $4x^2 + y^2 = 4$

65. $y = x^3 - 27$

66. $y = x^4 - 1$

67. $y = x^2 - 3x - 4$

68. $y = x^2 + 4$

69. $y = \dfrac{3x}{x^2 + 9}$

70. $y = \dfrac{x^2 - 4}{2x}$

71. $y = \dfrac{-x^3}{x^2 - 9}$

72. $y = \dfrac{x^4 + 1}{2x^5}$

In Problems 73–76, draw a quick sketch of the graph of each equation.

73. $y = x^3$

74. $x = y^2$

75. $y = \sqrt{x}$

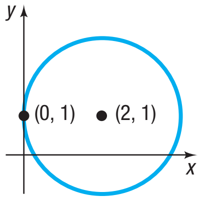
76. $y = \dfrac{1}{x}$

77. If $(a, 4)$ is a point on the graph of $y = x^2 + 3x$, what is a?

78. If $(a, -5)$ is a point on the graph of $y = x^2 + 6x$, what is a?

In Problems 79–82, find the center and radius of each circle. Write the standard form of the equation.

79.



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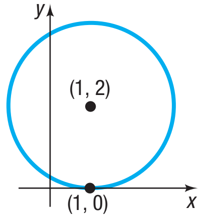

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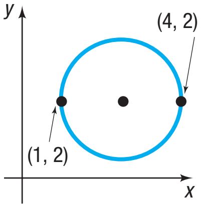

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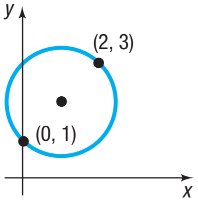


In Problems 83–92, write the standard form of the equation and the general form of the equation of each circle of radius r and center (h, k). Graph each circle.

83. $r = 2;\quad (h, k) = (0, 0)$

84. $r = 3;\quad (h, k) = (0, 0)$

85. $r = 2;\quad (h, k) = (0, 2)$

86. $r = 3;\quad (h, k) = (1, 0)$

87. $r = 5;\quad (h, k) = (4, -3)$

88. $r = 4;\quad (h, k) = (2, -3)$

89. $r = 4;\quad (h, k) = (-2, 1)$

90. $r = 7;\quad (h, k) = (-5, -2)$

91. $r = \frac{1}{2};\quad (h, k) = \left(\frac{1}{2}, 0\right)$

92. $r = \frac{1}{2};\quad (h, k) = \left(0, -\frac{1}{2}\right)$

In Problems 93–106, (a) find the center (h, k) and radius r of each circle; (b) graph each circle; (c) find the intercepts, if any.

93. $x^2 + y^2 = 4$

94. $x^2 + (y - 1)^2 = 1$

95. $2(x - 3)^2 + 2y^2 = 8$

96. $3(x + 1)^2 + 3(y - 1)^2 = 6$

97. $x^2 + y^2 - 2x - 4y - 4 = 0$

98. $x^2 + y^2 + 4x + 2y - 20 = 0$

99. $x^2 + y^2 + 4x - 4y - 1 = 0$

100. $x^2 + y^2 - 6x + 2y + 9 = 0$

101. $x^2 + y^2 - x + 2y + 1 = 0$

102. $x^2 + y^2 + x + y - \frac{1}{2} = 0$

103. $2x^2 + 2y^2 - 12x + 8y - 24 = 0$

104. $2x^2 + 2y^2 + 8x + 7 = 0$

105. $2x^2 + 8x + 2y^2 = 0$

106. $3x^2 + 3y^2 - 12y = 0$

In Problems 107–112, find the standard form of the equation of each circle.

107. Center at the origin and containing the point $(-2, 3)$

108. Center $(1, 0)$ and containing the point $(-3, 2)$

109. With endpoints of a diameter at $(1, 4)$ and $(-3, 2)$

110. With endpoints of a diameter at $(4, 3)$ and $(0, 1)$

111. Center $(2, -4)$ and circumference 16π

112. Center $(-5, 6)$ and area 49π

Applications and Extensions

113. Given that the point $(1, 2)$ is on the graph of an equation that is symmetric with respect to the origin, what other point is on the graph?

114. If the graph of an equation is symmetric with respect to the y-axis and 6 is an x-intercept of this graph, name another x-intercept.

115. If the graph of an equation is symmetric with respect to the origin and -4 is an x-intercept of this graph, name another x-intercept.

116. If the graph of an equation is symmetric with respect to the x-axis and 2 is a y-intercept, name another y-intercept.

117. Microphones In studios and on stages, cardioid microphones are often preferred for the richness they add to voices and for their ability to reduce the level of sound from the sides and rear of the microphone. Suppose one such cardioid pattern is given by the equation $(x^2 + y^2 - x)^2 = x^2 + y^2$.

(a) Find the intercepts of the graph of the equation.

(b) Test for symmetry with respect to the x-axis, the y-axis, and the origin.

Source: *www.notaviva.com*

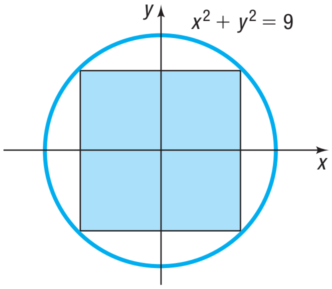
118. Solar Energy The solar electric generating systems at Kramer Junction, California, use parabolic troughs to heat a heat-transfer fluid to a high temperature. This fluid is used to generate steam that drives a power conversion system to produce electricity. For troughs 7.5 feet wide, an equation for the cross section is $16y^2 = 120x - 225$.

(a) Find the intercepts of the graph of the equation.

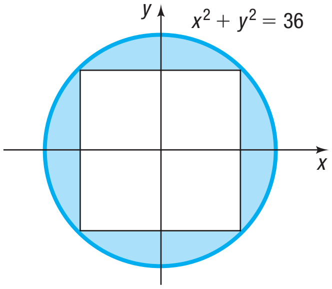
(b) Test for symmetry with respect to the x-axis, the y-axis, and the origin.

Source: *U.S. Department of Energy*

119. Find the area of the square in the figure.



120. Find the area of the blue shaded region in the figure, assuming the quadrilateral inside the circle is a square.



121. Ferris Wheel The original Ferris wheel was built in 1893 by Pittsburgh, Pennsylvania bridge builder George W. Ferris. The Ferris wheel was originally built for the 1893 World's Fair in Chicago, but it was also later reconstructed for the 1904 World's Fair in St. Louis. It had a maximum height of 264 feet and a wheel diameter of 250 feet. Find an equation for the wheel if the center of the wheel is on the y-axis.
Source: *guinnessworldrecords.com*

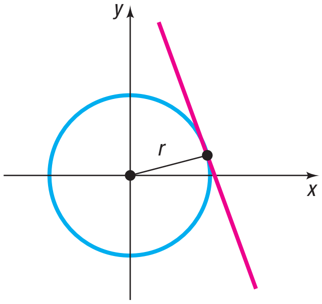
122. Ferris Wheel The High Roller observation wheel in Las Vegas has a maximum height of 550 feet and a diameter of 520 feet, with one full rotation taking approximately 30 minutes. Find an equation for the wheel if the center of the wheel is on the y-axis.
Source: *Las Vegas Review Journal*

123. Vertically Circular Building Located in Al Raha, Abu Dhabi, the headquarters of property developing company Aldar is a vertically circular building with a diameter of 121 meters. The tip of the building is 110 meters aboveground. Find an equation for the building's outline if the center of the building is on the y-axis.

124. Vertically Circular Building The Sunrise Kempinski Hotel in Beijing, China, is a vertically circular building whose outline is described by the equation $x^2 + y^2 - 78y - 1843 = 0$ if the center of the building is on the y-axis. If x and y are in meters, what is the height of the building?

Problems 125–130 require the following discussion.

The **tangent line** to a circle may be defined as the line that intersects the circle in a single point, called the **point of tangency**. See the figure.



In Problems 125–128, find the standard form of the equation of each circle. (Refer to the preceding discussion).

125. Center $(2, 3)$ and tangent to the x-axis

126. Center $(-3, 1)$ and tangent to the y-axis

127. Center $(-1, 3)$ and tangent to the line $y = 2$

128. Center $(4, -2)$ and tangent to the line $x = 1$

129. *Challenge Problem* If the equation of a circle is $x^2 + y^2 = r^2$ and the equation of a tangent line is $y = mx + b$, show that:

(a) $r^2(1 + m^2) = b^2$

[**Hint**: The quadratic equation $x^2 + (mx + b)^2 = r^2$ has exactly one solution.]

(b) The point of tangency is $\left(\frac{-r^2 m}{b}, \frac{r^2}{b}\right)$.

(c) The tangent line is perpendicular to the line containing the center of the circle and the point of tangency.

130. *Challenge Problem* Refer to Problem 129. The line $x - 2y + 4 = 0$ is tangent to a circle at $(0, 2)$. The line $y = 2x - 7$ is tangent to the same circle at $(3, -1)$. Find the center of the circle.

131. *Challenge Problem* If (x_1, y_1) and (x_2, y_2) are the endpoints of a diameter of a circle, show that an equation of the circle is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

132. *Challenge Problem* If $x^2 + y^2 + dx + ey + f = 0$ is the equation of a circle, show that

$$x_0x + y_0y + d\left(\frac{x + x_0}{2}\right) + e\left(\frac{y + y_0}{2}\right) + f = 0$$

is an equation of the tangent line to the circle at the point (x_0, y_0).

133. *Challenge Problem* **Lemniscate** For a nonzero constant a, find the intercepts of the graph of $(x^2 + y^2)^2 = a^2(x^2 - y^2)$. Then test for symmetry with respect to the x-axis, the y-axis, and the origin.

134. *Challenge Problem* **Limaçon** For nonzero constants a and b, find the intercepts of the graph of

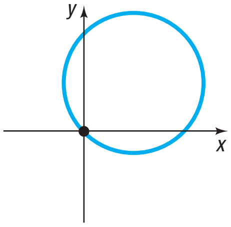
$$(x^2 + y^2 - ax)^2 = b^2(x^2 + y^2)$$

Then test for symmetry with respect to the x-axis, the y-axis, and the origin.

Explaining Concepts: Discussion and Writing

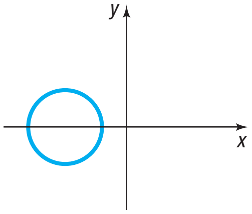
135. Which of the following equations might have the graph shown? (More than one answer is possible.)

(a) $(x - 2)^2 + (y + 3)^2 = 13$
(b) $(x - 2)^2 + (y - 2)^2 = 8$
(c) $(x - 2)^2 + (y - 3)^2 = 13$
(d) $(x + 2)^2 + (y - 2)^2 = 8$
(e) $x^2 + y^2 - 4x - 9y = 0$
(f) $x^2 + y^2 + 4x - 2y = 0$
(g) $x^2 + y^2 - 9x - 4y = 0$
(h) $x^2 + y^2 - 4x - 4y = 4$



136. Which of the following equations might have the graph shown? (More than one answer is possible.)

(a) $(x - 2)^2 + y^2 = 3$
(b) $(x + 2)^2 + y^2 = 3$
(c) $x^2 + (y - 2)^2 = 3$
(d) $(x + 2)^2 + y^2 = 4$
(e) $x^2 + y^2 + 10x + 16 = 0$
(f) $x^2 + y^2 + 10x - 2y = 1$
(g) $x^2 + y^2 + 9x + 10 = 0$
(h) $x^2 + y^2 - 9x - 10 = 0$



137. **(a)** Graph $y = \sqrt{x^2}$, $y = x$, $y = |x|$, and $y = (\sqrt{x})^2$, noting which graphs are the same.

(b) Explain why the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.

(c) Explain why the graphs of $y = x$ and $y = (\sqrt{x})^2$ are not the same.

(d) Explain why the graphs of $y = \sqrt{x^2}$ and $y = x$ are not the same.

138. Draw a graph of an equation that contains two x-intercepts; at one the graph crosses the x-axis, and at the other the graph touches the x-axis.

139. Make up an equation with the intercepts $(2, 0)$, $(4, 0)$, and $(0, 1)$. Compare your equation with a friend's equation. Comment on any similarities.

140. Draw a graph that contains the points $(-2, -1)$, $(0, 1)$, $(1, 3)$, and $(3, 5)$. Compare your graph with those of other students. Are most of the graphs almost straight lines? How many are "curved"? Discuss the various ways in which these points might be connected.

141. An equation is being tested for symmetry with respect to the x-axis, the y-axis, and the origin. Explain why, if two of these symmetries are present, the remaining one must also be present.

142. Draw a graph that contains the points $(-2, 5)$, $(-1, 3)$, and $(0, 2)$ and is symmetric with respect to the y-axis. Compare your graph with those of other students; comment on any similarities. Can a graph contain these points and be symmetric with respect to the x-axis? the origin? Why or why not?

143. Explain how the center and radius of a circle can be used to graph the circle.

144. **What Went Wrong?** A student stated that the center and radius of the graph whose equation is $(x + 3)^2 + (y - 2)^2 = 16$ are $(3, -2)$ and 4, respectively. Why is this incorrect?

'Are You Prepared?' Answers

1. add; 25

2. $\{-1, 5\}$

1.3 Functions and Their Graphs

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Interval Notation (Section A.6, pp. A45–A47)
- Solving Inequalities (Section A.6, pp. A49–A51)
- Evaluating Algebraic Expressions, Domain of a Variable (Section A.1, pp. A6–A7)
- Rationalizing Denominators and Numerators (Section A.7, pp. A58–A59)

Now Work the **'Are You Prepared?'** problems on page 37.

OBJECTIVES
1 Describe a Relation (p. 24)
2 Determine Whether a Relation Represents a Function (p. 26)
3 Use Function Notation; Find the Value of a Function (p. 29)
4 Find the Difference Quotient of a Function (p. 31)
5 Find the Domain of a Function Defined by an Equation (p. 32)
6 Identify the Graph of a Function (p. 34)
7 Obtain Information from or about the Graph of a Function (p. 35)

1 Describe a Relation

Often there are situations where one variable is somehow linked to another variable. For example, the price of a gallon of gas is linked to the price of a barrel of oil. A person can be associated to her telephone number(s). The volume V of a sphere depends on its radius R. The force F exerted by an object corresponds to its acceleration a. These are examples of a *relation*, a correspondence between two sets called the *domain* and the *range*.

DEFINITION **Relation**

A **relation** is a correspondence between two sets: a set X, called the **domain**, and a set Y, called the **range**. In a relation, each element from the domain corresponds to at least one element from the range.

If x is an element of the domain and y is an element of the range, and if a relation exists from x to y, then we say that y **corresponds** to x or that y **depends on** x, and we write $x \rightarrow y$. It is often helpful to think of x as the **input** and y as the **output** of the relation. See Figure 28.

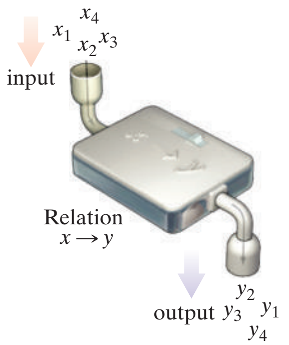


Figure 28

Suppose an astronaut standing on the Moon throws a rock 20 meters up and starts a stopwatch as the rock begins to fall back down. The astronaut measures the height of the rock at 1, 2, 2.5, 3, 4, and 5 seconds and obtains heights of 19.2, 16.8, 15, 12.8, 7.2, and 0 meters, respectively. This is an example of a relation expressed **verbally**. The domain of the relation is the set $\{0, 1, 2, 2.5, 3, 4, 5\}$ and the range of the relation is the set $\{20, 19.2, 16.8, 15, 12.8, 7.2, 0\}$.

The astronaut could also express this relation *numerically*, *graphically*, or *algebraically*.

The relation can be expressed **numerically** using a table of numbers, as in Table 5, or by using a **set of ordered pairs**. Using ordered pairs, the relation is

$$\{(0, 20), (1, 19.2), (2, 16.8), (2.5, 15), (3, 12.8), (4, 7.2), (5, 0)\}$$

where the first element of each pair denotes the time and the second element denotes the height.

Table 5

Time (in seconds)	Height (in meters)
0	20
1	19.2
2	16.8
2.5	15
3	12.8
4	7.2
5	0

Suppose x represents the number of seconds on the stopwatch and y represents the height of the rock in meters. Then the relation can be expressed **graphically** by plotting the points (x, y). See Figure 29.

The relation can be represented as a **mapping** by drawing an arrow from an element in the domain to the corresponding element in the range. See Figure 30.

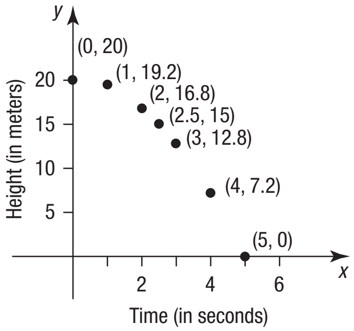


Figure 29

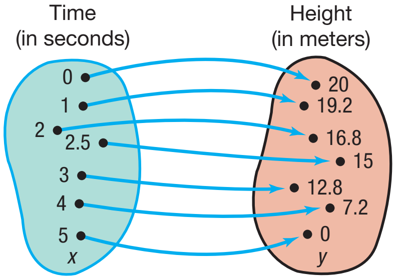


Figure 30

Finally, from physics, the relation can be expressed **algebraically** using the equation

$$y = 20 - 0.8x^2$$

EXAMPLE 1 Describing a Relation

A verbal description of a relation is given below.

The price of First Class U.S. postage stamps has changed over the years. To mail a letter in 2015 cost \$0.49. In 2016 it cost \$0.49 for part of the year and \$0.47 for the rest of the year. In 2017 it cost \$0.47 for part of the year and \$0.49 for the rest of the year. In 2018 it cost \$0.49 for part of the year and \$0.50 for the rest of the year.

Using year as input and price as output,

(a) What is the domain and the range of the relation?

(b) Express the relation as a set of ordered pairs.

(c) Express the relation as a mapping.

(d) Express the relation as a graph.

Solution The relation establishes a correspondence between the input, year, and the output, price of a First Class U.S. postage stamp.

(a) The domain of the relation is $\{2015, 2016, 2017, 2018\}$. The range of the relation is $\{\$0.47, \$0.49, \$0.50\}$.

(b) The relation expressed as a set of ordered pairs is $\{(2015, \$0.49), (2016, \$0.47), (2016, \$0.49), (2017, \$0.47), (2017, \$0.49), (2018, \$0.49), (2018, \$0.50)\}$

(c) See Figure 31 for the relation expressed as a mapping, using t for year and p for price.

(d) Figure 32 shows a graph of the relation.

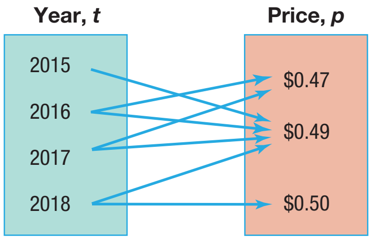


Figure 31 First Class stamp price

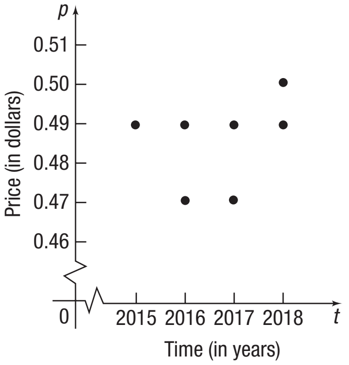


Figure 32 First Class stamp price (2015–2018)

Now Work PROBLEM 21

2 Determine Whether a Relation Represents a Function

Look back at the relation involving the height of a rock on the Moon described at the beginning of the section. Notice that each input, time, corresponds to exactly one output, height. Given a time, you could tell the exact height of the rock. But that is not the case with the price of stamps. Given the year 2018, you cannot determine the price of a stamp with certainty. It could be \$0.49, or it could be \$0.50.

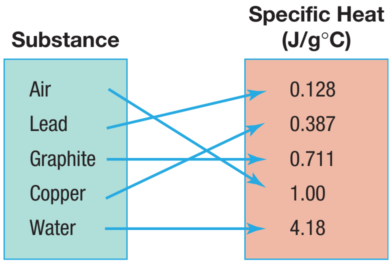


Figure 33 Specific heat of some common substances

Consider the mapping of the relation in Figure 33. It shows a correspondence between a substance and its specific heat. Notice that for each substance you can tell its specific heat with certainty.

The relation associating the time to the height of the rock is a *function*, and the relation associating a given substance to its specific heat is a function. But the relation associating the year to the price of a First Class postage stamp is *not* a function. To be a function, each input must correspond to *exactly one* output.

DEFINITION Function

Let X and Y be two nonempty sets.* A **function** from X into Y is a relation that associates with each element of X exactly one element of Y.

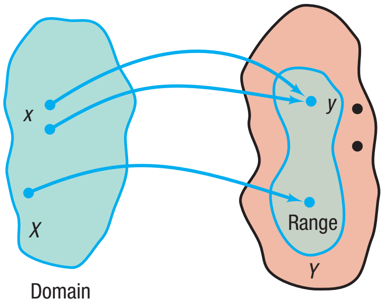


Figure 34

The set X is called the **domain** of the function. For each element x in X, the corresponding element y in Y is called the **value** of the function at x, or the **image** of x. The set of all images of the elements in the domain is called the **range** of the function. See Figure 34.

Since there may be some elements in Y that are not the image of some x in X, it follows that the range of a function may be a proper subset of Y, as shown in Figure 34.

The idea behind a function is its certainty. If an input is given, we can use the function to determine the output. This is not possible if a relation is not a function. The requirement of "one output" provides a predictable behavior that is important when using mathematics to model or analyze the real world. It allows doctors to know

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*The sets X and Y will usually be sets of real numbers, in which case a (real) function results. The two sets can also be sets of complex numbers, and then we have defined a complex function. In the broad definition (proposed by Lejeune Dirichlet), X and Y can be any two sets.