Engineering Mechanics

STATICS

Fifteenth Edition

R. C. HIBBELER

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FIFTEENTH EDITION

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To the Student

With the hope that this work will stimulate
an interest in Engineering Mechanics
and provide an acceptable guide to its understanding.

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PREFACE

The main purpose of this book is to provide the student with a clear and thorough presentation of the theory and application of engineering mechanics. To achieve this objective, this work has been shaped by the comments and suggestions of hundreds of reviewers in the teaching profession, as well as many of the author's students.

New to this Edition

Expanded Answer Section. The answer section in the back of the book now includes additional information related to the solution of select Fundamental Problems in order to offer the student some guidance in solving the problems.

Re-writing of Text Material. Further clarification of some concepts has been included in this edition, and throughout the book the accuracy has been enhanced, and important definitions are now in boldface throughout the text to highlight their importance.

Additional Fundamental Problems. Some new fundamental problems have been added along with their partial solutions which are given in the back of the book.

New Photos. The relevance of knowing the subject matter is reflected by the real-world applications depicted in the over 15 new or updated photos placed throughout the book. These photos generally are used to explain how the relevant principles apply to real-world situations and how materials behave under load.

New Problems. There are approximately 30% new problems that have been added to this edition, which involve applications to many different fields of engineering.

Hallmark Features

Besides the new features mentioned, other outstanding features that define the contents of the book include the following:

Organization and Approach. Each chapter is organized into well-defined sections that contain an explanation of specific topics, illustrative example problems, and a set of homework problems. The topics within each section are placed into subgroups defined by boldface titles. The purpose of this is to present a structured method for introducing each new definition or concept and to make the book convenient for later reference and review.

Chapter Contents. Each chapter begins with an illustration demonstrating a broad-range application of the material within the chapter. A bulleted list of the chapter contents is provided to give a general overview of the material that will be covered.

Emphasis on Free-Body Diagrams. Drawing a free-body diagram is particularly important when solving problems, and for this reason this step is strongly emphasized throughout the book. In particular, special sections and examples are devoted to show how to draw free-body diagrams. Specific homework problems have also been added to develop this practice.

Procedures for Analysis. A general procedure for analyzing any mechanics problem is presented at the end of the first chapter. Then this procedure is customized to relate to specific types of problems that are covered throughout the book. This unique feature provides the student with a logical and orderly method to follow when applying the theory. The example problems are solved using this outlined method in order to clarify its numerical application. Realize, however, that once the relevant principles have been mastered and enough confidence and judgment have been obtained, the student can then develop his or her own procedures for solving problems.

Important Points. This feature provides a review or summary of the most important concepts in a section and highlights the most significant points that should be known when applying the theory to solve problems.

Fundamental Problems. These problem sets are selectively located just after most of the example problems. They provide students with simple applications of the concepts, and therefore, the chance to develop their problem-solving skills before attempting to solve any of the standard problems that follow. In addition, they can be used for preparing for exams, and they can be used at a later time when preparing for the Fundamentals in Engineering Exam. The partial solutions are given in the back of the book.

Conceptual Understanding. Through the use of photographs placed throughout the book, the theory is applied in a simplified way in order to illustrate some of its more important conceptual features and instill the physical meaning of many of the terms used in the equations.

Homework Problems. Apart from the Fundamental and Conceptual type problems mentioned previously, other types of problems contained in the book include the following:

- **Free-Body Diagram Problems.** Some sections of the book contain introductory problems that only require drawing the free-body diagram for the specific problems within a problem set. These assignments will impress upon the student the importance of mastering this skill as a requirement for a complete solution of any equilibrium problem.

- **General Analysis and Design Problems.** The majority of problems in the book depict realistic situations encountered in engineering practice. Some of these problems come from actual products used in industry. It is hoped that this realism will both stimulate the student's interest in engineering mechanics and provide a means for developing the skill to reduce any such problem from its physical description to a model or symbolic representation to which the principles of mechanics may be applied.

  Throughout the book, there is an approximate balance of problems using either SI or FPS units. Furthermore, in any set, an attempt has been made to arrange the problems in order of increasing difficulty except for the end of chapter review problems, which are presented in random order.

- **Computer Problems.** An effort has been made to include a few problems that may be solved using a numerical procedure executed on either a desktop computer or a programmable pocket calculator. The intent here is to broaden the student's capacity for using other forms of mathematical analysis without sacrificing the time needed to focus on the application of the principles of mechanics. Problems of this type, which either can or must be solved using numerical procedures, are identified by a "square" symbol (■) preceding the problem number.

The many homework problems in this edition, have been placed into two different categories. Problems that are simply indicated by a problem number have an answer and in some cases an additional numerical result given in the back of the book. An asterisk (*) before every fourth problem number indicates a problem without an answer.

Accuracy. As with the previous editions, apart from the author, the accuracy of the text and problem solutions has been thoroughly checked by Kai Beng Yap, a practicing engineer, and a team of specialists at EPAM, including Georgii Kolobov, Ekaterina Radchenko, and Artur Akberov. Thanks are also due to Keith Steuer from Snow College and Mike Freeman, Professor Emeritus at the University of Alabama.

Contents

The book is divided into 11 chapters, in which the principles are first applied to simple, then to more complicated situations. In a general sense, each principle is applied first to a particle, then a rigid body subjected to a coplanar system of forces, and finally to three-dimensional force systems acting on a rigid body.

Chapter 1 begins with an introduction to mechanics and a discussion of units. The vector properties of a concurrent force system are introduced in Chapter 2. This theory is then applied to the equilibrium of a particle in Chapter 3. Chapter 4 contains a general discussion of both concentrated and distributed force systems and the methods used to simplify them. The principles of rigid-body equilibrium are developed in Chapter 5 and then applied to specific problems involving the equilibrium of trusses, frames, and machines in Chapter 6, and to the analysis of internal forces in beams and cables in Chapter 7. Applications to problems involving frictional forces are discussed in Chapter 8, and topics related to the center of gravity and centroid are treated in Chapter 9. If time permits, sections involving more advanced topics, indicated by stars (★), may be covered. Most of these topics are included in Chapter 10 (area and mass moments of inertia) and Chapter 11 (virtual work and potential energy). Note that this material also provides a suitable reference for basic principles when it is discussed in more advanced courses. Finally, Appendix A provides a review and list of mathematical formulas needed to solve the problems in the book.

Alternative Coverage. At the discretion of the instructor, some of the material may be presented in a different sequence with no loss of continuity. For example, it is possible to introduce the concept of a force and all the necessary methods of vector analysis by first covering Chapter 2 and Section 4.2 (the cross product). Then after covering the rest of Chapter 4 (force and moment systems), the equilibrium methods of Chapters 3 and 5 can be discussed.

Acknowledgments

The author has endeavored to write this book so that it will appeal to both the student and instructor. Through the years, many people have helped in its development, and I will always be grateful for their valued suggestions and comments. Specifically, I wish to thank all the individuals who have sent comments to me. These include J. Aurand, D. Boyajian, J. Callahan, D. Dikin, I. Elishakoff, R. Hendricks, F. Herrera, J. Hilton, H. Kuhlman, K. Leipold, C. Roche, M. Rosengren, R. Scott, and J. Tashbar.

A long-time friend and associate, Kai Beng Yap, was of great help to me in preparing and checking problem solutions. During the production process I am thankful for the assistance of Rose Kernan, my production editor, and Marta

Samsel, who worked on the cover of the book. And finally, to my wife, Conny, who helped in the proofreading of the manuscript for publication.

Lastly, many thanks are extended to all my students and to members of the teaching profession who have freely taken the time to offer their suggestions and comments. Since this list is too long to mention, it is hoped that those who have given help in this manner will accept this anonymous recognition.

I would greatly appreciate hearing from you if at any time you have any comments, suggestions, or issues related to any matters regarding this edition.

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This online tutorial and assessment program allows you to integrate dynamic homework and practice problems with automated grading of exercises from the textbook. Tutorials and many end-of-section problems provide enhanced student feedback and optional hints. Mastering Engineering™ allows you to easily track the performance of your entire class on an assignment-by-assignment basis, or the detailed work of an individual student. For more information visit www.masteringengineering.com.

Resources for Instructors

Instructor's Solutions Manual This supplement provides complete solutions supported by problem statements and problem figures. The Instructor's Solutions Manual is available in PDF format on Pearson Higher Education website: www.pearson.com.

PowerPoint Slides A complete set of all the figures and tables from the textbook are available in PowerPoint format.

Resources for Students

Videos Developed by the author, three different types of videos are now available to reinforce learning the basic theory and applying the principles. The first set provides a self-test of the material related to the theory and concepts presented in the book. The second set provides a self-test of the example problems and the basic procedures used for their solution. And the third set provides an engagement for solving the Fundamental Problems throughout the book. For more information on how to access these videos visit www.pearson.com/hibbeler.

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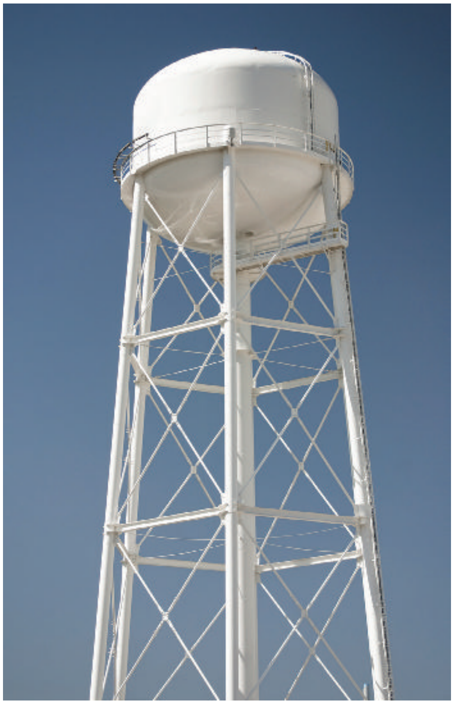

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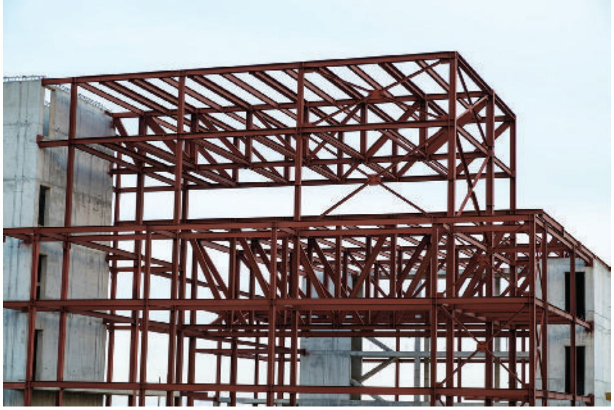

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ENGINEERING MECHANICS

STATICS

FIFTEENTH EDITION

CHAPTER 1

Cranes such as this one are required to lift extremely large loads. Their design is based on the basic principles of statics and dynamics, which form the subject matter of engineering mechanics.

GENERAL PRINCIPLES

CHAPTER OBJECTIVES

- To provide an introduction to the basic quantities and idealizations of mechanics.
- To state Newton's Laws of Motion and Gravitation.
- To review the principles for applying the SI system of units.
- To examine the standard procedures for performing numerical calculations.
- To present a general guide for solving problems.

1.1 MECHANICS

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces. In general, this subject can be subdivided into three branches: *rigid-body mechanics, deformable-body mechanics*, and *fluid mechanics*. In this book we will study rigid-body mechanics since it is a basic requirement for the study of the mechanics of deformable bodies and the mechanics of fluids. Furthermore, rigid-body mechanics is essential for the design and analysis of many types of structural members, mechanical components, or electrical devices encountered in engineering.

Rigid-body mechanics is divided into two areas: statics and dynamics. ***Statics*** deals with the equilibrium of bodies, that is, those that are either at rest or move with a constant velocity; whereas ***dynamics*** is concerned with the accelerated motion of bodies. We can consider statics as a special case of dynamics, in which the acceleration is zero; however, statics deserves separate treatment in engineering education since many objects are designed with the intention that they remain in equilibrium.

Historical Development. The subject of statics developed at a very early time because its principles can be formulated simply from measurements of geometry and force. For example, the writings of Archimedes (287–212 B.C.) deal with the principle of the lever. Studies of the pulley, inclined plane, and wrench are also recorded in ancient writings—at times when the requirements for engineering were limited primarily to building construction.

Since the principles of dynamics depend on an accurate measurement of time, this subject developed much later. Galileo Galilei (1564–1642) was one of the first major contributors to this field. His work consisted of experiments using pendulums and falling bodies. The most significant contributions in dynamics, however, were made by Isaac Newton (1642–1727), who is noted for his formulation of the three fundamental laws of motion and the law of universal gravitational attraction. Shortly after these laws were postulated, important techniques for their application were developed by other scientists and engineers, some of whom will be mentioned throughout the book.

1.2 FUNDAMENTAL CONCEPTS

Before we begin our study of engineering mechanics, it is important to understand the meaning of certain fundamental concepts and principles.

Basic Quantities. The following four quantities are used throughout mechanics.

Length. ***Length*** is used to locate the position of a point in space and thereby describe the size of a physical system. Once a standard unit of length is defined, one can then use it to define distances and geometric properties of a body as multiples of this unit.

Time. ***Time*** is conceived as a succession of events. Although the principles of statics are time independent, this quantity plays an important role in the study of dynamics.

Mass. ***Mass*** is a measure of a quantity of matter that is used to compare the action of one body with that of another. This property manifests itself as a gravitational attraction between two bodies and provides a measure of the resistance of matter to a change in velocity.

Force. In general, ***force*** is considered as a "push" or "pull" exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall, or it can occur through a distance when the bodies are physically separated. Examples of the latter type include gravitational, electrical, and magnetic forces. In any case, a force is completely characterized by its magnitude, direction, and point of application.

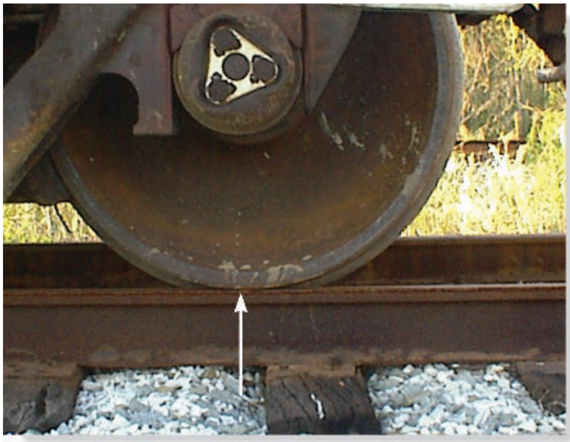
Idealizations. Models or idealizations are used in mechanics in order to simplify application of the theory. Here we will consider three important idealizations.

Particle. A ***particle*** has a mass, but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit, and therefore the earth can be modeled as a particle when studying its orbital motion. When a body is idealized as a particle, the principles of mechanics reduce to a rather simplified form since the geometry of the body *will not be involved* in the analysis of the problem.

Three forces act on the ring. Since these forces all meet at a point, then for any force analysis, we can assume the ring to be represented as a particle.

Rigid Body. A ***rigid body*** can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying a load. This model is important because the body's shape does not change when a load is applied, and so we do not have to consider the type of material from which the body is made. In most cases the actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and the rigid-body assumption is suitable for analysis.

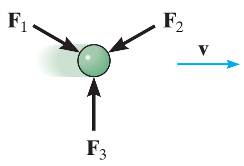
Concentrated Force. A ***concentrated force*** represents the effect of a loading which is assumed to act at a point on a body. We can represent a load by a concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body. An example would be the contact force between a wheel and the ground.



Steel is a common engineering material that does not deform very much under load. Therefore, we can consider this railroad wheel to be a rigid body acted upon by the concentrated force of the rail.

Newton's Three Laws of Motion. Engineering mechanics is formulated on the basis of Newton's three laws of motion, the validity of which is based on experimental observation. These laws apply to the motion of a particle as measured from a *nonaccelerating* reference frame. They may be briefly stated as follows.

First Law. A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this equilibrium state provided the particle is *not* subjected to an unbalanced force, Fig. 1–1*a*.

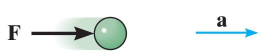


Equilibrium

(a)

Second Law. A particle acted upon by an *unbalanced force* **F** experiences an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force, Fig. 1–1*b*.* If the particle has a mass m, this law may be expressed mathematically as

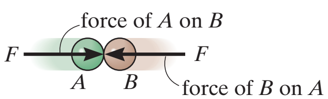
$$\mathbf{F} = m\mathbf{a} \qquad (1\text{–}1)$$



Accelerated motion

(b)

Third Law. The mutual forces of action and reaction between two particles are equal, opposite, and collinear, Fig. 1–1*c*.



Action–reaction

(c)

Fig. 1–1

*Stated another way, the unbalanced force acting on the particle is proportional to the time rate of change of the particle's linear momentum.

Newton's Law of Gravitational Attraction. Shortly after formulating his three laws of motion, Newton postulated a law governing the gravitational attraction between any two particles. Stated mathematically,

$$F = G\frac{m_1 m_2}{r^2} \tag{1–2}$$

where

F = force of gravitation between the two particles

G = universal constant of gravitation; according to experimental evidence, $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$

m_1, m_2 = mass of each of the two particles

r = distance between the two particles

Weight. According to Eq. 1–2, any two particles or bodies have a mutual attractive (gravitational) force acting between them. In the case of a particle located at or near the surface of the earth, however, the only gravitational force having any sizable magnitude is that between the earth, because of its very large mass, and the particle. Consequently, this force, called the ***weight***, will be the only gravitational force we will consider.

From Eq. 1–2, if the particle has a mass $m_1 = m$, and we assume the earth is a nonrotating sphere of constant density and having a mass $m_2 = M_e$, then if r is the distance between the earth's center and the particle, the weight W of the particle becomes

$$W = G\frac{mM_e}{r^2}$$

If we let $g = GM_e/r^2$, we have

$$\boxed{W = mg} \tag{1–3}$$

If we allow the particle to fall downward, then neglecting air resistance, the only force acting on the particle is its weight, and so Eq. 1–1 becomes $W = ma$. Comparing this result with Eq. 1–3, we see that $a = g$. In other words, g is the acceleration due to gravity. Since it depends on r, then the weight of the particle or body is *not* an absolute quantity. Instead, its magnitude depends upon the elevation where the measurement was made. For most engineering calculations, however, g is determined at sea level and at a latitude of 45°, which is considered the "standard location."

The astronaut's weight is diminished since she is far removed from the gravitational field of the earth.

1.3 UNITS OF MEASUREMENT

The four basic quantities—length, time, mass, and force—are not all independent from one another; in fact, they are *related* by Newton's second law of motion, $\mathbf{F} = m\mathbf{a}$. Because of this, the *units* used to measure these quantities cannot *all* be selected arbitrarily. The equality $\mathbf{F} = m\mathbf{a}$ is maintained only if three of the four units, called ***base units***, are *defined* and the fourth unit is then *derived* from the equation.

Fig. 1-2

SI Units. The International System of units, abbreviated SI after the French "Système International d'Unités," is a modern version of the metric system which has received worldwide recognition. As shown in Table 1–1, the SI system defines length in meters (m), time in seconds (s), and mass in kilograms (kg).† The unit of force, called a ***newton*** (N), is *derived* from $\mathbf{F} = m\mathbf{a}$. Thus, 1 newton is equal to a force required to give 1 kilogram of mass an acceleration of $1\ \text{m/s}^2$ $(\text{N} = \text{kg} \cdot \text{m/s}^2)$. Think of this force as the weight of a small apple.

If the weight of a body located at the "standard location" is to be determined in newtons, then Eq. 1–3 must be applied. Here measurements give $g = 9.806\,65\ \text{m/s}^2$; however, for calculations, the value $g = 9.81\ \text{m/s}^2$ will be used. Thus,

$$W = mg \qquad (g = 9.81\ \text{m/s}^2) \tag{1–4}$$

Therefore, a body of mass 1 kg has a weight of 9.81 N, a 2-kg body weighs 19.62 N, and so on, Fig. 1–2*a*.

U.S. Customary. In the U.S. Customary system of units (FPS) length is measured in feet (ft), time in seconds (s), and force in pounds (lb), Table 1–1. The unit of mass, called a ***slug***, is *derived* from $\mathbf{F} = m\mathbf{a}$. Hence, 1 slug is equal to the amount of matter accelerated at $1\ \text{ft/s}^2$ when acted upon by a force of 1 lb $(\text{slug} = \text{lb} \cdot \text{s}^2/\text{ft})$.

Therefore, if the measurements are made at the "standard location," where $g = 32.2\ \text{ft/s}^2$, then from Eq. 1–3,

$$m = \frac{W}{g} \qquad (g = 32.2\ \text{ft/s}^2) \tag{1–5}$$

And so a body weighing 32.2 lb has a mass of 1 slug, a 64.4-lb body has a mass of 2 slugs, and so on, Fig. 1–2*b*.

TABLE 1–1 Systems of Units

Name	Length	Time	Mass	Force
International System of Units	meter	second	kilogram	newton*
SI	m	s	kg	N $\left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)$
U.S. Customary FPS	foot	second	slug*	pound
	ft	s	$\left(\frac{\text{lb} \cdot \text{s}^2}{\text{ft}}\right)$	lb

*Derived unit.

†Historically, the meter was defined as 1/10,000,000 the distance from the Equator to the North Pole, and the kilogram is 1/1000 of a cubic meter of water.

Conversion of Units. Table 1–2 provides a set of direct conversion factors between FPS and SI units for the basic quantities. Also, in the FPS system, recall that 1 ft = 12 in. (inches), 5280 ft = 1 mi (mile), 1000 lb = 1 kip (kilo-pound), and 2000 lb = 1 ton.

TABLE 1–2 Conversion Factors

Quantity	Unit of Measurement (FPS)	Equals	Unit of Measurement (SI)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

1.4 THE INTERNATIONAL SYSTEM OF UNITS

The SI system of units is used extensively in this book since it is intended to become the worldwide standard for measurement. Therefore, we will now present some of the rules for its use and some of its terminology relevant to engineering mechanics.

Prefixes. When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix. Some of the prefixes used in the SI system are shown in Table 1–3. Each represents a multiple or submultiple of a unit which, if applied successively, moves the decimal point of a numerical quantity to every third place.* For example, 4 000 000 N = 4000 kN (kilo-newton) = 4 MN (mega-newton), or 0.005 m = 5 mm (milli-meter). Notice that the SI system does not include the multiple deca (10) or the submultiple centi (0.01), which form part of the metric system. Except for some volume and area measurements, the use of these prefixes is generally avoided in science and engineering.

TABLE 1–3 Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

*The kilogram is the only base unit that is defined with a prefix.

1

Rules for Use. Here are a few of the important rules that describe the proper use of the various SI symbols:

- Quantities defined by several units which are multiples of one another are separated by a *dot* to avoid confusion with prefix notation, as indicated by $\text{N} = \text{kg}\cdot\text{m/s}^2 = \text{kg}\cdot\text{m}\cdot\text{s}^{-2}$. Also, $\text{m}\cdot\text{s}$ (meter-second), whereas ms (milli-second).
- The exponential power on a unit having a prefix refers to both the unit *and* its prefix. For example, $\mu\text{N}^2 = (\mu\text{N})^2 = \mu\text{N}\cdot\mu\text{N}$. Likewise, mm^2 represents $(\text{mm})^2 = \text{mm}\cdot\text{mm}$.
- With the exception of the base unit the kilogram, in general avoid the use of a prefix in the denominator of composite units. For example, do not write N/mm, but rather kN/m; also, m/mg should be written as Mm/kg.
- When performing calculations, represent the numbers in terms of their *base or derived units* by converting all prefixes to powers of 10. The final result should then be expressed using a *single prefix*. Also, after calculation, it is best to keep numerical values between 0.1 and 1000; otherwise, a suitable prefix should be chosen. For example,

$$\begin{aligned}(50\text{ kN})(60\text{ nm}) &= \left[50(10^3)\text{ N}\right]\left[60(10^{-9})\text{ m}\right]\\ &= 3000(10^{-6})\text{ N}\cdot\text{m} = 3(10^{-3})\text{ N}\cdot\text{m} = 3\text{ mN}\cdot\text{m}\end{aligned}$$

1.5 NUMERICAL CALCULATIONS

Numerical work in engineering practice is most often performed by using handheld calculators and computers. It is important, however, that the answers to any problem be reported with justifiable accuracy using appropriate significant figures. In this section we will discuss these topics together with some other important aspects involved in all engineering calculations.

Dimensional Homogeneity. The terms of any equation used to describe a physical process must be ***dimensionally homogeneous***; that is, each term must be expressed in the same units. Provided this is the case, all the terms of an equation can then be combined if numerical values are substituted for the variables. Consider, for example, the equation $s = vt + \frac{1}{2}at^2$, where, in SI units, s is the position in meters, m, t is time in seconds, s, v is velocity in m/s and a is acceleration in m/s^2. Regardless of how this equation is evaluated, it maintains its dimensional homogeneity. In the form stated, each of the three terms is expressed in meters $\left[\text{m}, (\text{m}/\cancel{\text{s}})\cancel{\text{s}}, (\text{m}/\cancel{\text{s}^2})\cancel{\text{s}^2}\right]$ or solving for a, $a = 2s/t^2 - 2v/t$, the terms are each expressed in units of m/s^2 $\left[\text{m/s}^2, \text{m/s}^2, (\text{m/s})/\text{s}\right]$.

Keep in mind that problems in mechanics always involve the solution of dimensionally homogeneous equations, and so this fact can then be used as a partial check for algebraic manipulations of an equation.

Significant Figures. The number of significant figures contained in any number determines the accuracy of the number. For instance, the number 4981 contains four significant figures. However, if zeros occur at the end of a whole number, it may be unclear as to how many significant figures the number represents. For example, 23 400 might have three (234), four (2340), or five (23 400) significant figures. To avoid these ambiguities, we will use ***engineering notation*** to report a result. This requires that numbers be rounded off to the appropriate number of significant digits and then expressed in multiples of (10^3), such as (10^3), (10^6), or (10^{-9}). For instance, if 23 400 has five significant figures, it is written as $23.400(10^3)$, but if it has only three significant figures, it is written as $23.4(10^3)$.

If zeros occur at the beginning of a number that is less than one, then the zeros are not significant. For example, 0.008 21 has three significant figures. Using engineering notation, this number is expressed as $8.21\left(10^{-3}\right)$. Likewise, 0.000 582 can be expressed as $0.582\left(10^{-3}\right)$ or $582\left(10^{-6}\right)$.

Rounding Off Numbers. Rounding off a number is necessary so that the accuracy of the result will be the same as that of the problem data. As a general rule, any numerical figure ending in a number greater than five is rounded up and a number less than five is not rounded up. The rules for rounding off numbers are best illustrated by examples. Suppose the number 3.5587 is to be rounded off to *three* significant figures. Because the fourth digit (8) is *greater than* 5, the third number is rounded up to 3.56. Likewise 0.5896 becomes 0.590 and 9.3866 becomes 9.39. If we round off 1.341 to three significant figures, because the fourth digit (1) is *less than* 5, then we get 1.34. Likewise 0.3762 becomes 0.376 and 9.871 becomes 9.87. There is a special case for any number that ends in a 5. As a general rule, if the digit preceding the 5 is an *even number*, then this digit is *not* rounded up. If the digit preceding the 5 is an *odd number*, then it is rounded up. For example, 75.25 rounded off to three significant digits becomes 75.2, 0.1275 becomes 0.128, and 0.2555 becomes 0.256.

Calculations. When a sequence of calculations is performed, it is best to store the intermediate results in the calculator. In other words, do not round off calculations until expressing the final result. This procedure maintains precision throughout the series of steps to the final solution. In this book we will generally round off the answers to *three significant figures* since most of the data in engineering mechanics, such as geometry and loads, may be reliably measured to this accuracy.

1

When solving problems, do the work as neatly as possible. Being neat will stimulate clear and orderly thinking, and vice versa.

1.6 GENERAL PROCEDURE FOR ANALYSIS

Attending a lecture, reading this book, and studying the example problems helps, but **the most effective way of learning the principles of engineering mechanics is to *solve problems***. To be successful at this, it is important to always present the work in a *logical* and *orderly manner*, as suggested by the following sequence of steps:

- Read the problem carefully and try to correlate the actual physical situation with the theory studied.
- Tabulate the problem data and *draw to a large scale* any necessary diagrams.
- Apply the relevant principles, generally in mathematical form. When writing any equations, be sure they are dimensionally homogeneous.
- Solve the necessary equations, and report the answer with no more than three significant figures.
- Study the answer with technical judgment and common sense to determine whether or not it seems reasonable.

IMPORTANT POINTS

- Statics is the study of bodies that are at rest or move with constant velocity.
- A particle has a mass but a size that can be neglected, and a rigid body does not deform under load.
- A force is considered as a "push" or "pull" of one body on another.
- Concentrated forces are assumed to act at a point on a body.
- Newton's three laws of motion should be memorized.
- Mass is measure of a quantity of matter that does not change from one location to another. Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located.
- In the SI system the unit of force, the newton, is a derived unit. The meter, second, and kilogram are base units.
- Prefixes G, M, k, m, μ, and n are used to represent large and small numerical quantities. Their exponential size should be known, along with the rules for using the SI units.
- Perform numerical calculations with several significant figures, and then report the final answer to three significant figures.
- Algebraic manipulations of an equation can be checked in part by verifying that the equation remains dimensionally homogeneous.
- Know the rules for rounding off numbers.

1

EXAMPLE 1.1

Convert 2 km/h to m/s. How many ft/s is this?

SOLUTION

Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of the units can be applied:

$$2\text{ km/h} = \frac{2\cancel{\text{km}}}{\cancel{\text{h}}}\left(\frac{1000\text{ m}}{\cancel{\text{km}}}\right)\left(\frac{1\cancel{\text{h}}}{3600\text{ s}}\right)$$

$$= \frac{2000\text{ m}}{3600\text{ s}} = 0.556\text{ m/s} \qquad \textit{Ans.}$$

From Table 1–2, 1 ft = 0.3048 m. Thus,

$$0.556\text{ m/s} = \left(\frac{0.556\cancel{\text{m}}}{\text{s}}\right)\left(\frac{1\text{ ft}}{0.3048\cancel{\text{m}}}\right)$$

$$= 1.82\text{ ft/s} \qquad \textit{Ans.}$$

Note: Remember to round off the final answer to three significant figures.

EXAMPLE 1.2

Convert the quantities 300 lb · s and 52 slug/ft^3 to appropriate SI units.

SOLUTION

Using Table 1–2, 1 lb = 4.448 N.

$$300\text{ lb}\cdot\text{s} = 300\cancel{\text{lb}}\cdot\text{s}\left(\frac{4.448\text{ N}}{1\cancel{\text{lb}}}\right)$$

$$= 1334.4\text{ N}\cdot\text{s} = 1.33\text{ kN}\cdot\text{s} \qquad \textit{Ans.}$$

Since 1 slug = 14.59 kg and 1 ft = 0.3048 m, then

$$52\text{ slug/ft}^3 = \frac{52\cancel{\text{slug}}}{\cancel{\text{ft}^3}}\left(\frac{14.59\text{ kg}}{1\cancel{\text{slug}}}\right)\left(\frac{1\cancel{\text{ft}}}{0.3048\text{ m}}\right)^3$$

$$= 26.8(10^3)\text{ kg/m}^3$$

$$= 26.8\text{ Mg/m}^3 \qquad \textit{Ans.}$$

1

EXAMPLE 1.3

Evaluate each of the following and express with SI units having an appropriate prefix: (a) (50 mN)(6 GN), (b) (400 mm)(0.6 MN)^2, (c) 45 MN^3/900 Gg.

SOLUTION

First convert each number to base units, perform the indicated operations, then choose an appropriate prefix.

Part (a)

$$(50\text{ mN})(6\text{ GN}) = \left[50(10^{-3})\text{ N}\right]\left[6(10^{9})\text{ N}\right]$$
$$= 300(10^{6})\text{ N}^2$$
$$= 300(10^{6})\text{ N}^2\left(\frac{1\text{ kN}}{10^3\text{ N}}\right)\left(\frac{1\text{ kN}}{10^3\text{ N}}\right)$$
$$= 300\text{ kN}^2 \qquad \textit{Ans.}$$

Note: Keep in mind the convention $\text{kN}^2 = (\text{kN})^2 = 10^6\text{ N}^2$.

Part (b)

$$(400\text{ mm})(0.6\text{ MN})^2 = \left[400(10^{-3})\text{ m}\right]\left[0.6(10^{6})\text{ N}\right]^2$$
$$= \left[400(10^{-3})\text{ m}\right]\left[0.36(10^{12})\text{ N}^2\right]$$
$$= 144(10^{9})\text{ m}\cdot\text{N}^2$$
$$= 144\text{ Gm}\cdot\text{N}^2 \qquad \textit{Ans.}$$

We can also write

$$144(10^{9})\text{ m}\cdot\text{N}^2 = 144(10^{9})\text{ m}\cdot\text{N}^2\left(\frac{1\text{ MN}}{10^6\text{ N}}\right)\left(\frac{1\text{ MN}}{10^6\text{ N}}\right)$$
$$= 0.144\text{ m}\cdot\text{MN}^2 \qquad \textit{Ans.}$$

Part (c)

$$\frac{45\text{ MN}^3}{900\text{ Gg}} = \frac{45(10^6\text{ N})^3}{900(10^6)\text{ kg}}$$
$$= 50(10^{9})\text{ N}^3/\text{kg}$$
$$= 50(10^{9})\text{ N}^3\left(\frac{1\text{ kN}}{10^3\text{ N}}\right)^3\frac{1}{\text{kg}}$$
$$= 50\text{ kN}^3/\text{kg} \qquad \textit{Ans.}$$

PROBLEMS

The answers to all but every fourth problem (asterisk) are given in the back of the book.

1–1. Round off the following numbers to three significant figures: (a) 3.455 55 m, (b) 45.556 s, (c) 5555 N, (d) 4525 kg.

1–2. Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) $kN/\mu s$, (b) Mg/mN, (c) $MN/(kg \cdot ms)$.

1–3. Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) Mg/mm, (b) $mN/\mu s$, (c) $\mu m \cdot Mg$.

***1–4.** What is the weight in newtons of an object that has a mass of (a) 8 kg, (b) 0.04 mg, (c) 760 Mg?

1–5. Represent each of the following as a number between 0.1 and 1000 using an appropriate prefix: (a) 45 320 kN, (b) $568(10^5)$ mm, (c) 0.005 63 mg.

1–6. Round off the following numbers to three significant figures: (a) 58 342 m, (b) 68.534 s, (c) 2553 N, (d) 7555 kg.

1–7. Represent each of the following quantities in the correct SI form using an appropriate prefix: (a) 0.000 431 kg, (b) $35.3(10^3)$ N, (c) 0.005 32 km.

***1–8.** Represent each of the following combinations of units in the correct SI form: (a) Mg/ms, (b) N/mm, (c) $mN/(kg \cdot \mu s)$.

1–9. Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) m/ms, (b) μkm, (c) ks/mg, (d) $km \cdot \mu N$.

1–10. Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) $GN \cdot \mu m$, (b) $kg/\mu m$, (c) N/ks^2, (d) $kN/\mu s$.

1–11. Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) μMN, (b) $N/\mu m$, (c) MN/ks^2, (d) kN/ms.

***1–12.** Water has a density of 1.94 $slug/ft^3$. What is the density expressed in SI units? Express the answer to three significant figures.

1–13. The density (mass/volume) of aluminum is 5.26 $slug/ft^3$. Determine its density in SI units. Use an appropriate prefix.

1–14. Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) $(212\ mN)^2$, (b) $(52\ 800\ ms)^2$, (c) $[548(10^6)]^{1/2}$ ms.

1–15. Using the SI system of units, show that Eq. 1–2 is a dimensionally homogeneous equation which gives F in newtons. Determine to three significant figures the gravitational force acting between two spheres that are touching each other. The mass of each sphere is 200 kg and the radius is 300 mm.

***1–16.** Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) $(200\ kN)^2$, (b) $(0.005\ mm)^2$, (c) $(400\ m)^3$.

1–17. If a car is traveling at 55 mi/h, determine its speed in kilometers per hour and meters per second.

1–18. Evaluate (204 mm)(0.004 57 kg)/(34.6 N) to three significant figures and express the answer in SI units using an appropriate prefix.

1–19. The specific weight (wt./vol.) of brass is 520 lb/ft^3. Determine its density (mass/vol.) in SI units. Use an appropriate prefix.

***1–20.** If a man weighs 155 lb on earth, specify (a) his mass in slugs, (b) his mass in kilograms, and (c) his weight in newtons. If the man is on the moon, where the acceleration due to gravity is $g_m = 5.30\ ft/s^2$, determine (d) his weight in pounds, and (e) his mass in kilograms.

1–21. Two particles have a mass of 8 kg and 12 kg, respectively. If they are 800 mm apart, determine the force of gravity acting between them. Compare this result with the weight of each particle.

CHAPTER 2

This electric transmission tower is stabilized by cables that exert forces on the tower at their points of connection. In this chapter we will show how to express these forces as Cartesian vectors, and then determine their resultant.

FORCE VECTORS

CHAPTER OBJECTIVES

- To show how to add forces and resolve them into components using the parallelogram law.
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction.
- To introduce the dot product in order to use it to find the angle between two vectors or the projection of one vector onto another.

2.1 SCALARS AND VECTORS

Many physical quantities in engineering mechanics are measured using either scalars or vectors.

Scalar. A ***scalar*** is any positive or negative physical quantity that can be completely specified by its *magnitude*. Examples of scalar quantities include length, mass, and time.

Vector. A ***vector*** is any physical quantity that requires both a *magnitude* and a *direction* for its complete description. Examples of vectors encountered in statics are force, position, and moment. A vector is shown graphically by an arrow, Fig. 2–1. The length of the arrow represents the *magnitude* of the vector, and the angle θ between the vector and a fixed axis defines the *direction of its line of action*. The head or tip of the arrow indicates the *sense of direction* of the vector.

In print, vector quantities are represented by boldface letters such as **A**, and the magnitude of a vector is italicized, *A*. For handwritten work, it is often convenient to denote a vector quantity by simply drawing an arrow above it, $\vec{A}$.

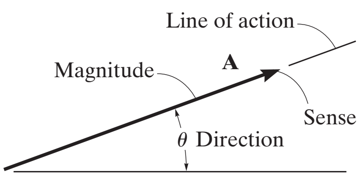
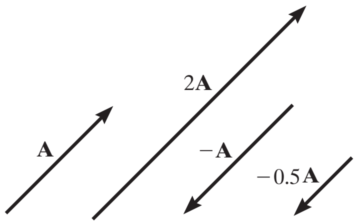


Fig. 2–1

2.2 VECTOR OPERATIONS

Multiplication and Division of a Vector by a Scalar. If a vector is multiplied or divided by a positive scalar, its magnitude is changed by that amount. Multiplying or dividing by a negative scalar will also change the directional sense of the vector. Graphic examples of these operations are shown in Fig. 2–2.

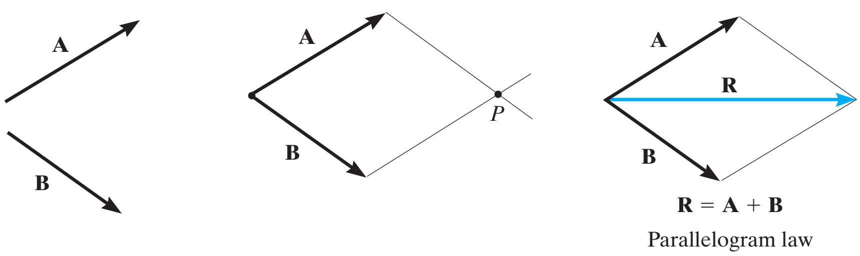


Scalar multiplication and division

Fig. 2–2

Vector Addition. When adding two vectors together it is important to account for both their magnitudes and their directions. To do this we must use the ***parallelogram law of addition.*** To illustrate, the two ***component vectors*** **A** and **B** in Fig. 2–3*a* are added to form a ***resultant vector*** $\mathbf{R} = \mathbf{A} + \mathbf{B}$ using the following procedure:

- First join the tails of the components at a point to make them concurrent, Fig. 2–3*b*.
- From the head of **B**, draw a line parallel to **A**. Draw another line from the head of **A** that is parallel to **B**. These two lines intersect at point *P* to form the adjacent sides of a parallelogram.
- The diagonal of this parallelogram that extends to *P* forms **R**, which then represents the resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$, Fig. 2–3*c*.



(a) (b) (c)

Fig. 2–3

We can also add **B** to **A**, Fig. 2–4*a*, using the ***triangle rule***, which is a special case of the parallelogram law, whereby vector **B** is added to vector **A** in a "head-to-tail" fashion, i.e., by connecting the tail of **B** to the head of **A**, Fig. 2–4*b*. The resultant **R** extends from the tail of **A** to the head of **B**. In a similar manner, **R** can also be obtained by adding **A** to **B**, Fig. 2–4*c*. By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either order, i.e., $\mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.

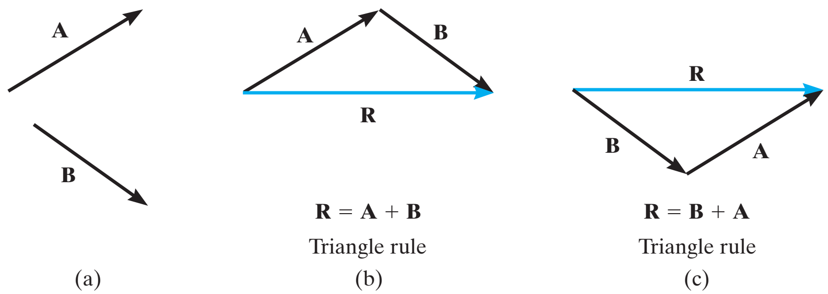
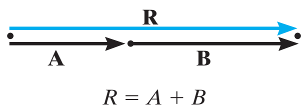


Fig. 2–4

As a special case, if the two vectors **A** and **B** are ***collinear***, i.e., both have the same line of action, the parallelogram law reduces to an *algebraic* or *scalar addition* $R = A + B$, as shown in Fig. 2–5.



Addition of collinear vectors

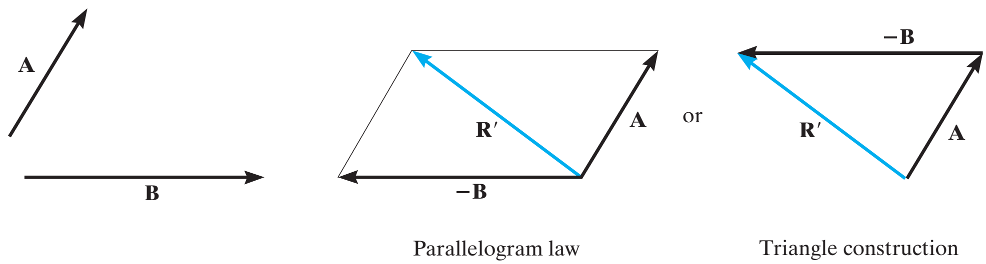
Fig. 2–5

Vector Subtraction.

The resultant of the *difference* between two vectors **A** and **B** of the same type may be expressed as

$$\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

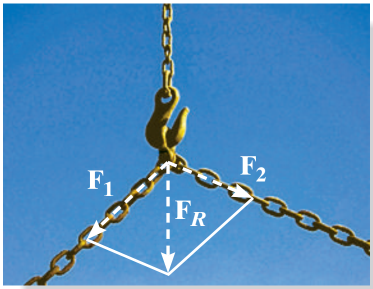
This vector sum is shown graphically in Fig. 2–6. Subtraction is therefore defined as a special case of addition, so the rules of vector addition also apply to vector subtraction.



Vector subtraction

Fig. 2–6

2.3 VECTOR ADDITION OF FORCES



The parallelogram law must be used to determine the resultant of the two forces acting on the hook.

Experimental evidence has shown that a force is a vector quantity since it has a specified magnitude, direction, and sense and it adds according to the parallelogram law. Two common problems in statics involve either finding the resultant force, knowing its components, or resolving a known force into two components. We will now describe how each of these problems is solved using the parallelogram law.

Finding a Resultant Force. The two component forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the pin in Fig. 2–7*a* are added together to form the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$, using the parallelogram law as shown in Fig. 2–7*b*. From this construction, or using the triangle rule, Fig. 2–7*c*, we can apply the law of cosines or the law of sines to the triangle in order to obtain the magnitude of the resultant force and its direction.

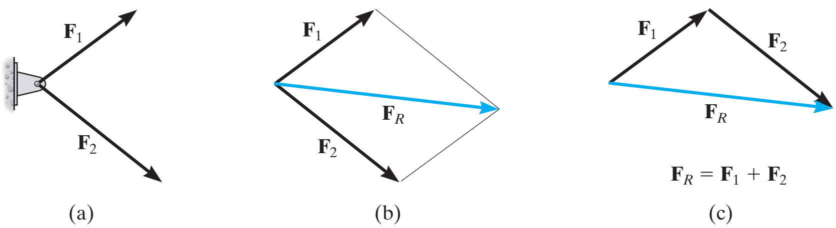


Fig. 2–7

Using the parallelogram law the supporting force **F** can be resolved into components acting along the u and axes.

Finding the Components of a Force. Sometimes it is necessary to resolve a force into two *components* in order to study its pulling or pushing effect in two specific directions. For example, in Fig. 2–8*a*, **F** is to be resolved into two components along the two members, defined by the u and axes. In order to determine the magnitude of each component, a parallelogram is constructed first, by drawing lines starting from the tip of **F**, one line parallel to u, and the other line parallel to . These lines intersect the and u axes, forming a parallelogram. The force components $\mathbf{F}_u$ and **F** are established by simply joining them to the tail of **F**, to the intersection points on the u and axes, Fig. 2–8*b*. This parallelogram can be reduced to a triangle, which represents the triangle rule, Fig. 2–8*c*. From this, the law of sines can be applied to determine the unknown magnitudes of the components.

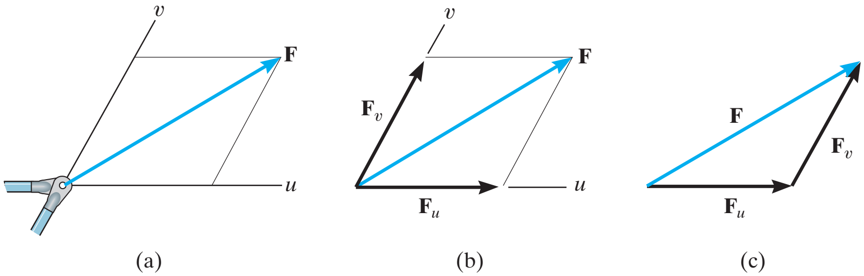


Fig. 2–8

Addition of Several Forces. If more than two forces are to be added, successive applications of the parallelogram law can be carried out in order to obtain the resultant force. For example, if three forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ act at a point O, Fig. 2–9, the resultant of any two of the forces is found, say, $\mathbf{F}_1 + \mathbf{F}_2$, and then this resultant is added to the third force, yielding the resultant of all three forces; i.e., $\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3$. Using the parallelogram law to add more than two forces, as shown here, generally requires extensive geometric and trigonometric calculation to determine the magnitude and direction of the resultant. Instead, problems of this type are easily solved by using the "rectangular-component method," which is explained in the next section.

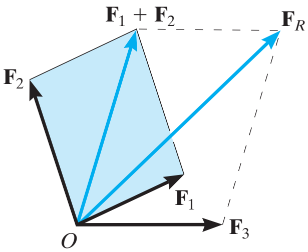
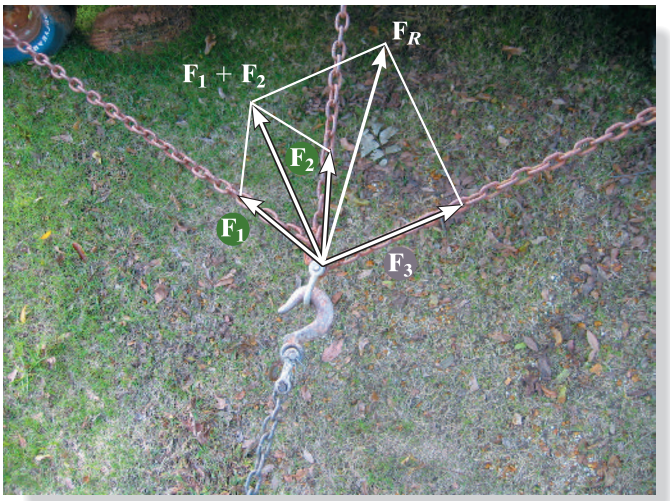


Fig. 2–9



The resultant force $\mathbf{F}_R$ on the hook requires the addition of $\mathbf{F}_1 + \mathbf{F}_2$, then this resultant is added to $\mathbf{F}_3$.

IMPORTANT POINTS

- A scalar is a positive or negative number.
- A vector is a quantity that has a magnitude, direction, and sense.
- Multiplication or division of a vector by a scalar will change the magnitude of the vector. The sense of the vector will change if the scalar is negative.
- Vectors are added or subtracted using the parallelogram law or the triangle rule.
- As a special case, if the vectors are collinear, the resultant is formed by an algebraic or scalar addition.

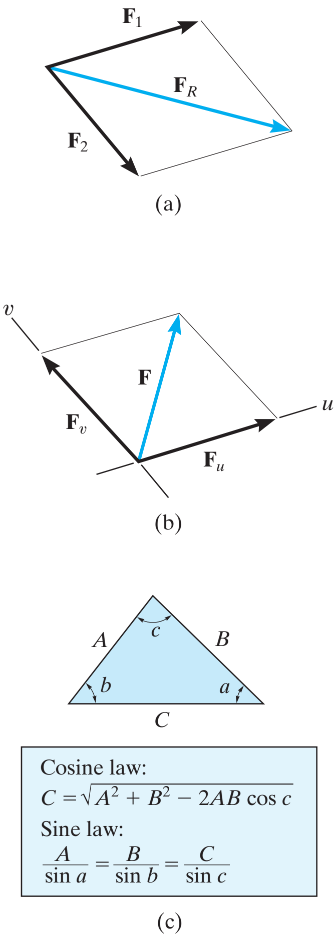


Fig. 2–10

PROCEDURE FOR ANALYSIS

Problems that involve the addition of two forces can be solved as follows:

Parallelogram Law.

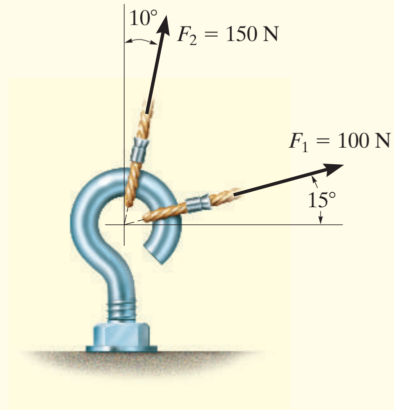
- Sketch the two "component" forces $\mathbf{F}_1$ and $\mathbf{F}_2$ added together according to the parallelogram law, yielding the *resultant* force $\mathbf{F}_R$ that forms the diagonal of the parallelogram, Fig. 2–10*a*.
- If a force **F** is to be resolved into *components* along two axes u and v, then start at the head of force **F** and construct lines parallel to the axes, thereby forming the parallelogram, Fig. 2–10*b*. The sides of the parallelogram represent the components, $\mathbf{F}_u$ and **F** .
- Label all the known and unknown force magnitudes and the angles on the sketch and identify the two unknowns as the magnitude and direction of $\mathbf{F}_R$, or the magnitudes of its components.

Trigonometry.

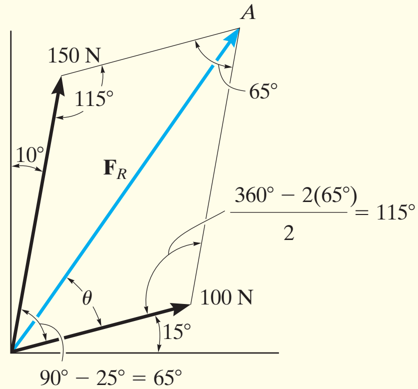
- Redraw a half portion of the parallelogram to illustrate the triangular head-to-tail addition of the components.
- From this triangle, the magnitude of the resultant force can be determined using the law of cosines, and its direction is determined from the law of sines. The magnitudes of two force components are determined from the law of sines. The formulas are given in Fig. 2–10*c*.

EXAMPLE 2.1

The screw eye in Fig. 2–11*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



(a)



(b)

SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of $\mathbf{F}_1$ that is parallel to $\mathbf{F}_2$, and another line from the head of $\mathbf{F}_2$ that is parallel to $\mathbf{F}_1$. The resultant force $\mathbf{F}_R$ extends to where these lines intersect at point A, Fig. 2–11*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2–11*c*. Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115°}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$

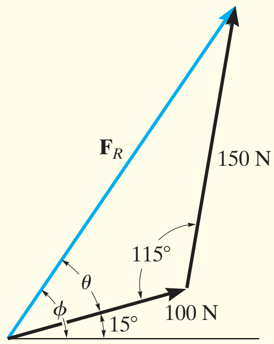
Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115°} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115°)$$
$$\theta = 39.8°$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8° + 15.0° = 54.8° \qquad \textit{Ans.}$$

NOTE: The results seem reasonable, since Fig. 2–11*b* shows $\mathbf{F}_R$ to have a magnitude larger than its components and a direction that is between them.



(c)

Fig. 2–11

2

2

EXAMPLE 2.2

Resolve the horizontal 600-lb force in Fig. 2–12*a* into components acting along the *u* and *v* axes and determine the magnitudes of these components.

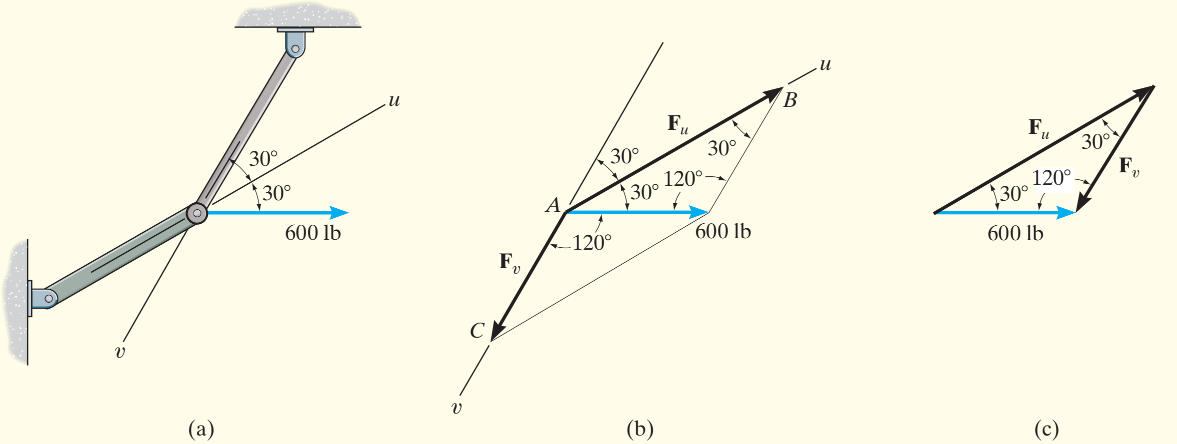


Fig. 2–12

SOLUTION

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the *v* axis until it intersects the *u* axis at point *B*, Fig. 2–12*b*. The arrow from *A* to *B* represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the *u* axis intersects the *v* axis at point *C*, which gives $\mathbf{F}_v$.

The vector addition using the triangle rule is shown in Fig. 2–12*c*. The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,

$$\frac{F_u}{\sin 120^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039 \text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_v = 600 \text{ lb} \qquad \textit{Ans.}$$

NOTE: The result for F_u shows that sometimes a component can have a greater magnitude than the resultant.

EXAMPLE 2.3

Determine the magnitude of the component force **F** in Fig. 2–13*a* and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive *y* axis.

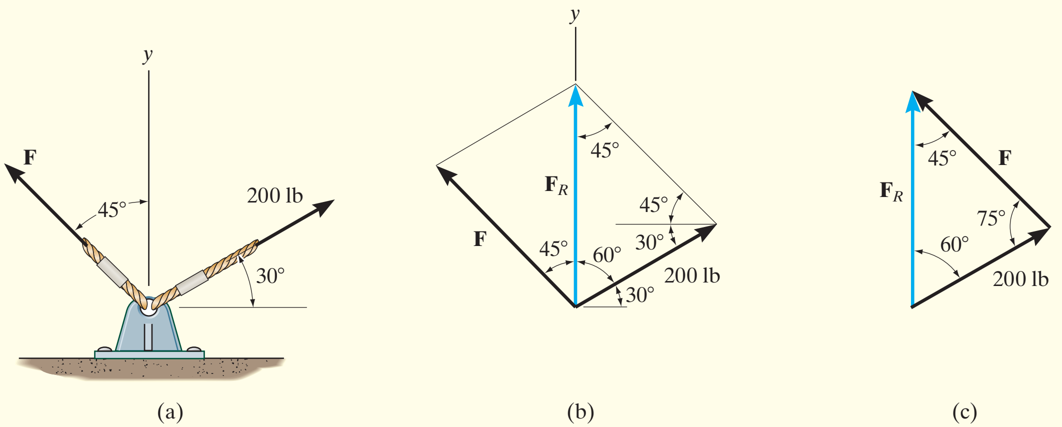


Fig. 2–13

SOLUTION

The parallelogram law of addition is shown in Fig. 2–13*b*, and the triangle rule is shown in Fig. 2–13*c*. The magnitudes of $\mathbf{F}_R$ and **F** are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F = 245 \text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F_R = 273 \text{ lb} \qquad \textit{Ans.}$$

2

EXAMPLE 2.4

It is required that the resultant force acting on the eyebolt in Fig. 2–14*a* be directed along the positive x axis and that $\mathbf{F}_2$ have a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.

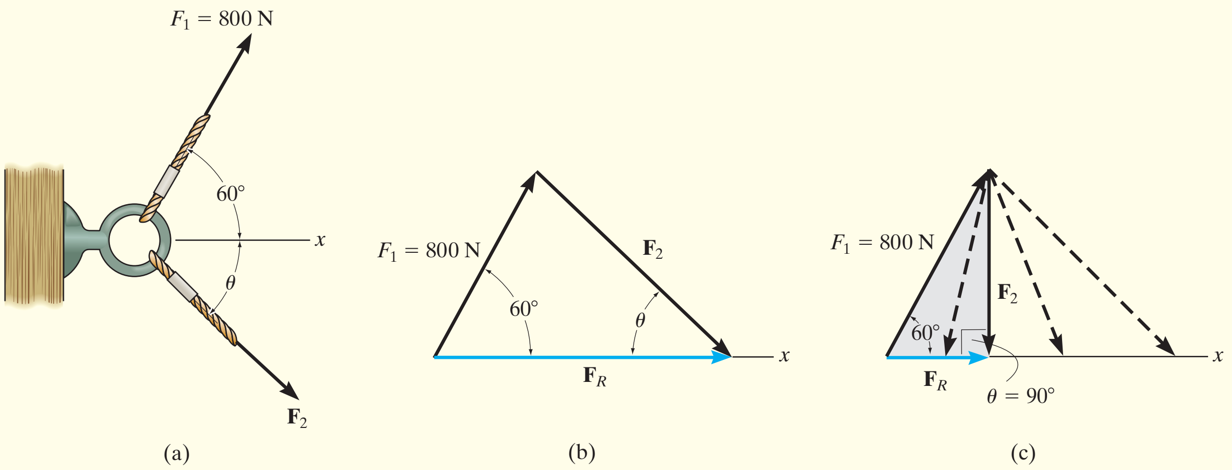


Fig. 2–14

SOLUTION

The triangle rule for $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ is shown in Fig. 2–14*b*. Since the magnitudes (lengths) of $\mathbf{F}_R$ and $\mathbf{F}_2$ are not specified, then $\mathbf{F}_2$ can actually be any vector that has its head touching the line of action of $\mathbf{F}_R$, Fig. 2–14*c*. However, as shown, the magnitude of $\mathbf{F}_2$ is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of $\mathbf{F}_R$, that is, when

$$\theta = 90^\circ \qquad \textit{Ans.}$$

Since the vector addition now forms the shaded right triangle, the two unknown magnitudes can be obtained by trigonometry.

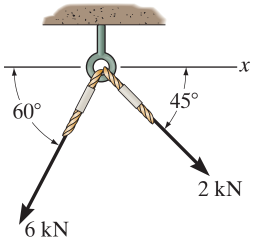
$$F_R = (800\text{ N})\cos 60^\circ = 400\text{ N} \qquad \textit{Ans.}$$

$$F_2 = (800\text{ N})\sin 60^\circ = 693\text{ N} \qquad \textit{Ans.}$$

FUNDAMENTAL PROBLEMS

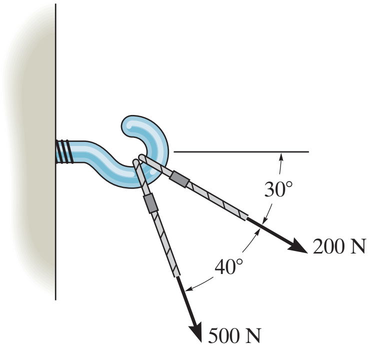
Partial solutions and answers to all Fundamental Problems are given in the back of the book.

F2–1. Determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.



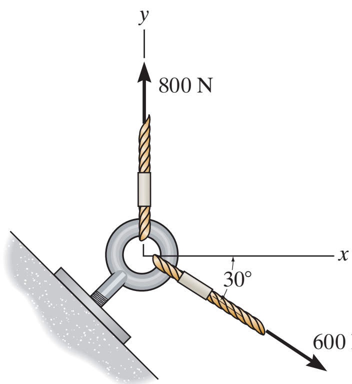
Prob. F2–1

F2–2. Two forces act on the hook. Determine the magnitude of the resultant force.



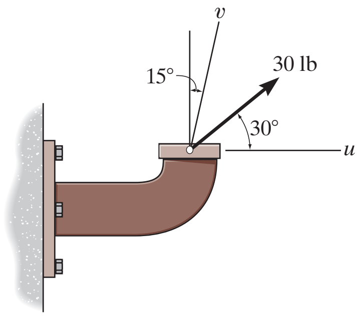
Prob. F2–2

F2–3. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



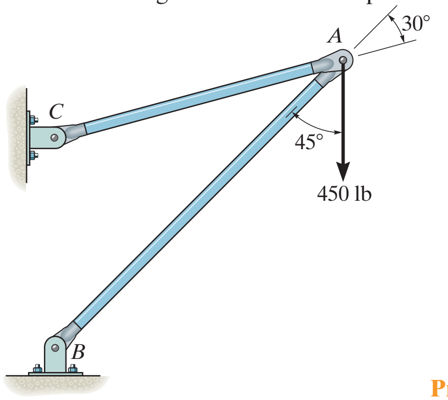
Prob. F2–3

F2–4. Resolve the 30-lb force into components along the u and v axes, and determine the magnitude of each of these components.



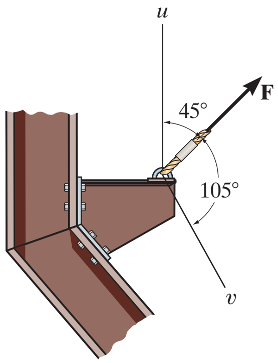
Prob. F2–4

F2–5. The force $F = 450$ lb acts on the frame. Resolve this force into components acting along members AB and AC, and determine the magnitude of each component.



Prob. F2–5

F2–6. If force **F** is to have a component along the u axis of $F_u = 6$ kN, determine the magnitude of **F** and the magnitude of its component F_v along the v axis.



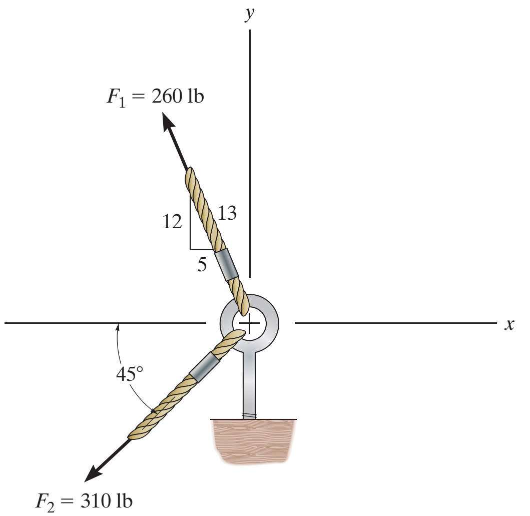
Prob. F2–6

2

PROBLEMS

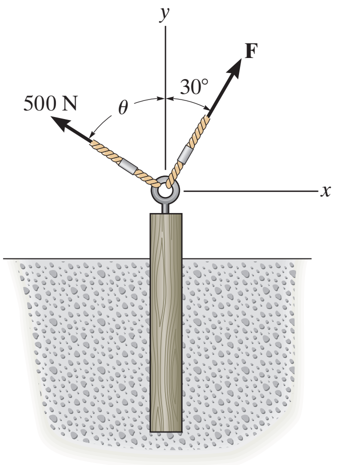
2–1. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its orientation θ, measured counterclockwise from the positive x axis.

2–2. Determine the magnitude of the resultant force $\mathbf{F}'_R = \mathbf{F}_1 - \mathbf{F}_2$ and its orientation θ, measured counterclockwise from the positive x axis.



Probs. 2–1/2

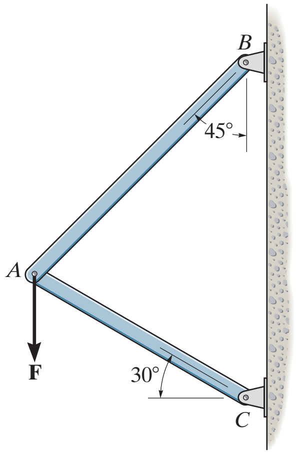
2–3. Two forces are applied at the end of a screw eye in order to remove the post. Determine the angle $\theta (0° \leq \theta \leq 90°)$ and the magnitude of force **F** so that the resultant force acting on the post is directed vertically upward and has a magnitude of 750 N.



Prob. 2–3

***2–4.** Determine the magnitudes of the two components of **F** along members AB and AC. Set $F = 500$ N.

2–5. Solve Prob. 2–4 with $F = 350$ lb.

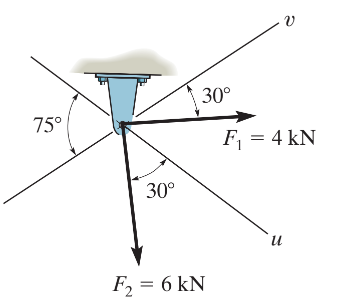


Probs. 2–4/5

2–6. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.

2–7. Resolve the force $\mathbf{F}_1$ into components along the u and v axes and determine the magnitudes of the components.

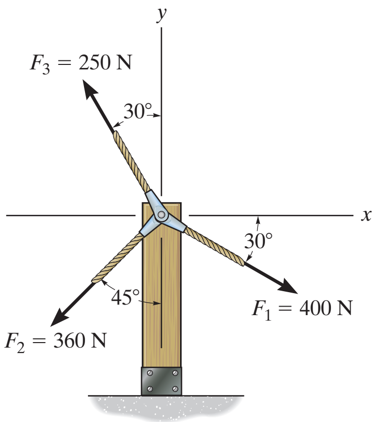
***2–8.** Resolve the force $\mathbf{F}_2$ into components along the u and v axes and determine the magnitudes of the components.



Probs. 2–6/7/8

2–9. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its orientation θ, measured clockwise from the positive x axis.

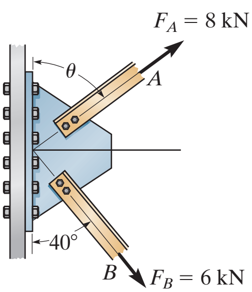
2–10. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_3$ and its orientation θ, measured counterclockwise from the positive x axis.



Probs. 2–9/10

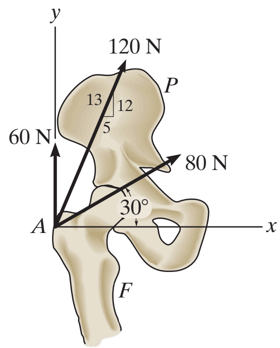
2–11. If $\theta = 60°$, determine the magnitude of the resultant force and its direction measured clockwise from the horizontal.

***2–12.** Determine the angle θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



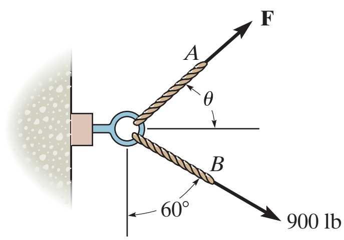
Probs. 2–11/12

2–13. The pelvis P is connected to the femur F at A using three different muscles, which exert the forces shown on the femur. Determine the resultant force and specify its orientation θ, measured counterclockwise from the positive x axis.



Prob. 2–13

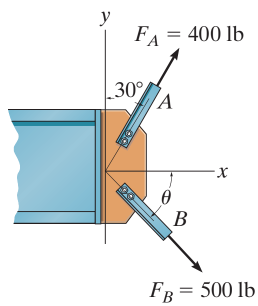
2–14. If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force $\mathbf{F}$ in rope A and the corresponding angle θ.



Prob. 2–14

2–15. The plate is subjected to the forces acting on members A and B. If $\theta = 60°$, determine the magnitude of the resultant of these forces and its direction measured clockwise from the positive x axis.

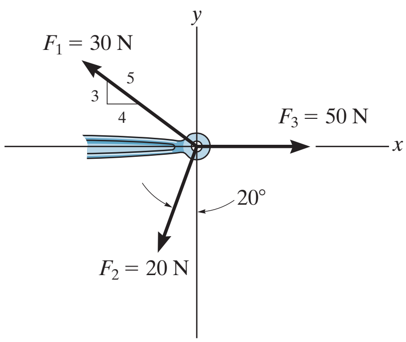
***2–16.** Determine the angle θ for connecting member B to the plate so that the resultant of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed along the positive x axis. What is the magnitude of the resultant force?



Probs. 2–15/16

2–17. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then finding $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.

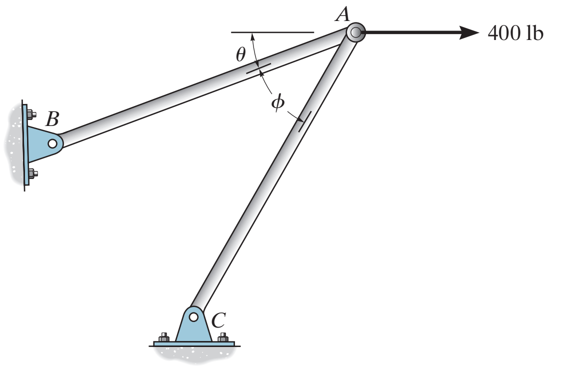
2–18. Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then finding $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



Probs. 2–17/18

2–21. Determine the design angle θ $(0° \leq \theta \leq 90°)$ for member AB so that the 400-lb horizontal force has a component of 500 lb directed from A toward C. What is the component of force acting along member AB? Take $\phi = 40°$.

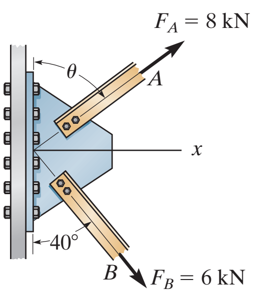
2–22. Determine the design angle ϕ $(0° \leq \phi \leq 90°)$ between members AB and AC so that the 400-lb horizontal force has a component of 600 lb which acts up to the right, in the direction from B toward A. Also calculate the magnitude of the force component along AC. Take $\theta = 30°$.



Probs. 2–21/22

2–19. The plate is subjected to the two forces at A and B. If $\theta = 60°$, determine the magnitude of the resultant force and its direction measured from the positive x axis.

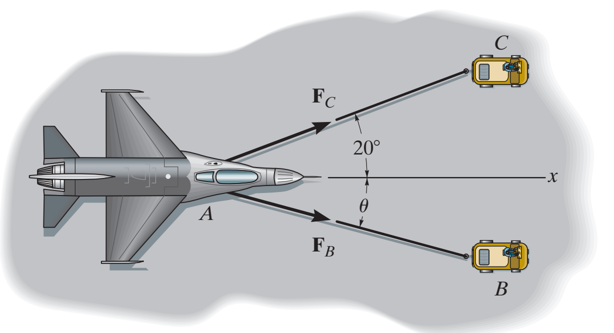
***2–20.** Determine the angle θ for connecting member A to the plate so that the resultant of F_A and F_B is directed along the positive x axis. Also, what is the magnitude of the resultant force?



Probs. 2–19/20

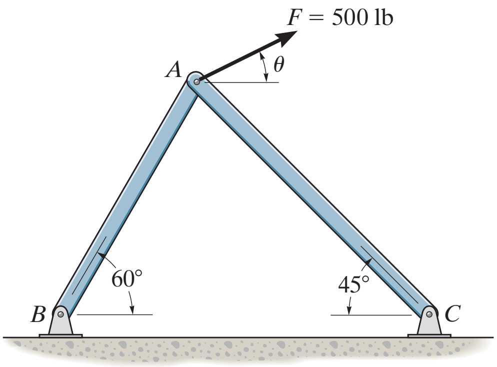
2–23. Determine the magnitude of the two towing forces $\mathbf{F}_B$ and $\mathbf{F}_C$ if the resultant force has a magnitude $F_R = 10$ kN and is directed along the positive x axis. Set $\theta = 15°$.

***2–24.** If the resultant $\mathbf{F}_R$ of the two forces acting on the jet aircraft is to be directed along the positive x axis and have a magnitude of 10 kN, determine the angle θ of the cable attached to the truck at B so that F_B is a minimum. What is the magnitude of force in each cable when this occurs?



Probs. 2–23/24

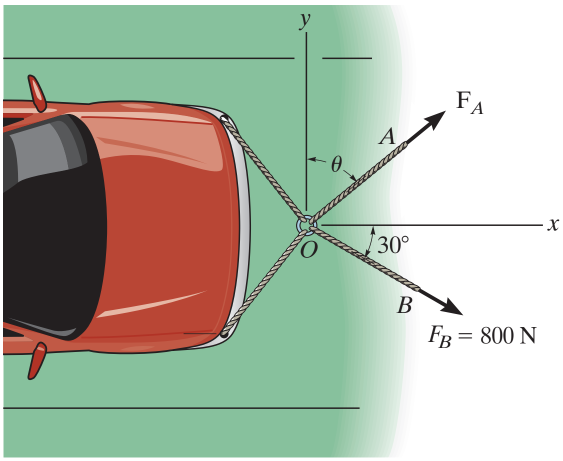
2–25. The 500-lb force is to be resolved into two components acting along the axis of the struts AB and AC. If the component of force along AC is required to be 300 lb, directed from A to C, determine the magnitude of the force acting along AB and the angle θ of the 500-lb force.



Prob. 2–25

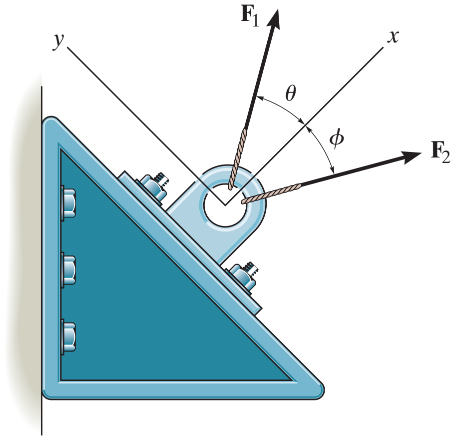
2–26. Determine the magnitude and direction θ of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.

2–27. Determine the magnitude of the resultant force acting on the ring at O, if $F_A = 750$ N and $\theta = 45°$. What is its direction, measured counterclockwise from the positive x axis?



Probs. 2–26/27

***2–28.** If $F_1 = F_2 = 30$ lb, determine the angles θ and ϕ so that the resultant force is directed along the positive x axis and has a magnitude of $F_R = 20$ lb.

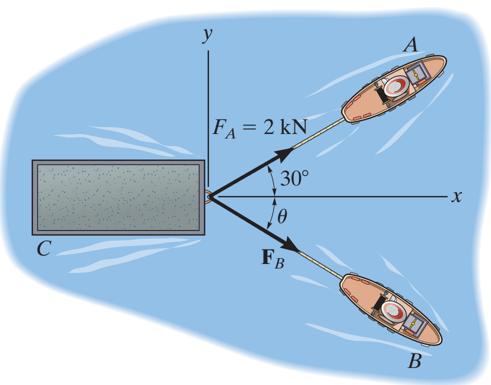


Prob. 2–28

2–29. If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction θ.

2–30. If $F_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.

2–31. If the resultant force of the two tugboats is required to be directed toward the positive x axis, and F_B is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle θ.



Probs. 2–29/30/31

2

2.4 ADDITION OF A SYSTEM OF COPLANAR FORCES

When a force is resolved into two components along the x and y axes, the components are then called ***rectangular components***. For analytical work we can represent these components in one of two ways, using either scalar or Cartesian vector notation.

Scalar Notation. The rectangular components of force $\mathbf{F}$ shown in Fig. 2–15*a* are found using the parallelogram law, so that $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$. Because these components form a right triangle, they can be determined from

$$F_x = F\cos\theta \qquad \text{and} \qquad F_y = F\sin\theta$$

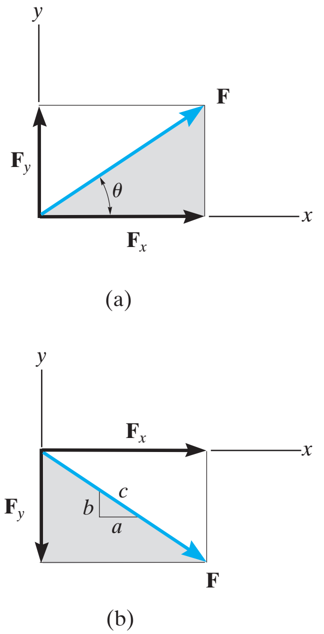


Fig. 2–15

Instead of using the angle θ, however, the direction of $\mathbf{F}$ can also be defined using a small "slope" triangle, as in the example shown in Fig. 2–15*b*. Since this triangle and the larger shaded triangle are similar, the proportional length of the sides gives

$$\frac{F_x}{F} = \frac{a}{c}$$

or

$$F_x = F\left(\frac{a}{c}\right)$$

and

$$\frac{F_y}{F} = \frac{b}{c}$$

or

$$F_y = -F\left(\frac{b}{c}\right)$$

Here the y component is a *negative scalar* since $\mathbf{F}_y$ is directed along the negative y axis.

It is important to keep in mind that this positive and negative scalar notation is to be used only for calculations, not for graphical representations in figures. Throughout the text, the *head of a vector arrow* in *any figure* indicates the sense of the vector *graphically*; algebraic signs are not used for this purpose. Thus, the vectors in Figs. 2–15*a* and 2–15*b* are designated by using boldface (vector) notation.* Whenever italic symbols are written near vector arrows in figures, they indicate the *magnitude* of the vector, which is *always* a *positive* quantity.

*Negative signs are used only in figures with boldface notation when showing equal but opposite pairs of vectors, as in Fig. 2–2.

Cartesian Vector Notation. It is also possible to represent the x and y components of a force in terms of Cartesian unit vectors $\mathbf{i}$ and $\mathbf{j}$. They are called unit vectors because they have a dimensionless magnitude of 1, and so they can be used to designate the *directions* of the x and y axes, respectively, Fig. 2–16.*

Since the *magnitude* of each component of $\mathbf{F}$ is *always a positive quantity*, which is represented by the (positive) scalars F_x and F_y, then we can express $\mathbf{F}$ as a ***Cartesian vector***,

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

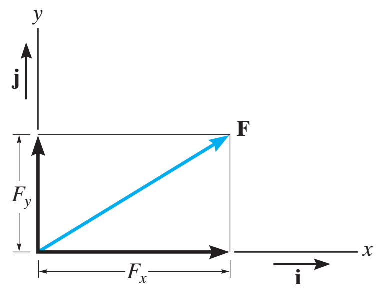


Fig. 2–16

Coplanar Force Resultants. We can use either of the two methods just described to determine the resultant of several ***coplanar forces***. To do this, each force is first resolved into its x and y components, and then the respective components are added using *scalar algebra* since they are collinear. The resultant force is then formed by adding the resultant components using the parallelogram law. For example, consider the three concurrent forces in Fig. 2–17*a*, which have x and y components shown in Fig. 2–17*b*. Using Cartesian vector notation, each force is first represented as a Cartesian vector, i.e.,

$$\mathbf{F}_1 = F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}$$

The vector resultant, Fig. 2–17*c*, is therefore

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\\ &= F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j} - F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j} + F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}\\ &= (F_{1x} - F_{2x} + F_{3x})\,\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\,\mathbf{j}\\ &= (F_R)_x\,\mathbf{i} + (F_R)_y\,\mathbf{j}\end{aligned}$$

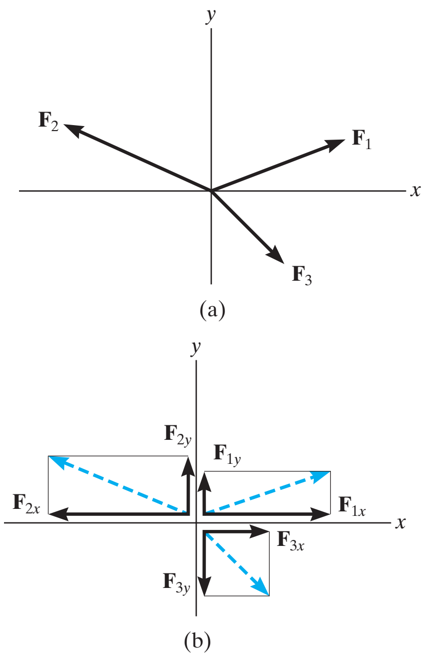
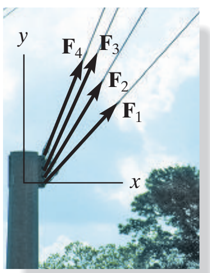


Fig. 2–17

If *scalar notation* is used, then indicating the positive directions of components along the x and y axes with symbolic arrows, we have

$$\overset{+}{\rightarrow} \qquad (F_R)_x = F_{1x} - F_{2x} + F_{3x}$$
$$+\uparrow \qquad (F_R)_y = F_{1y} + F_{2y} - F_{3y}$$

Notice that these are the *same* results as the $\mathbf{i}$ and $\mathbf{j}$ components of $\mathbf{F}_R$ determined above.



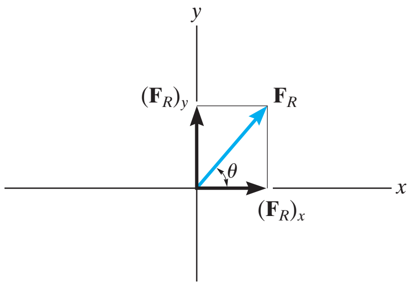
The resultant force of the four cable forces acting on the post can be determined by adding algebraically the separate x and y components of each cable force. This resultant $\mathbf{F}_R$ produces the *same pulling effect* on the post as all four cables.

*For handwritten work, unit vectors are usually indicated using a circumflex, e.g., $\hat{\imath}$ and $\hat{\jmath}$. Also, realize that F_x and F_y in Fig. 2–16 represent the *magnitudes* of the components, which are *always positive scalars*. The directions are defined by $\mathbf{i}$ and $\mathbf{j}$. If instead we used scalar notation, then F_x and F_y could be positive or negative scalars, since they would account for *both* the magnitude and direction of the components.

In general then, the components of the resultant force of any number of coplanar forces can be represented by the algebraic sum of the x and y components of all the forces, i.e.,

$$(F_R)_x = \Sigma F_x \qquad (2\text{–}1)$$
$$(F_R)_y = \Sigma F_y$$

Once these components are determined, they may be sketched along the x and y axes with their proper sense of direction, and the resultant force can be determined from vector addition, Fig. 2–17c. From this sketch, the magnitude of $\mathbf{F}_R$ is then found from the Pythagorean theorem; that is,



(c)

Fig. 2–17 (cont.)

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

Also, the angle θ, which specifies the direction of the resultant force, is determined from trigonometry:

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

The above concepts are illustrated numerically in the examples which follow.

IMPORTANT POINTS

- The resultant of several coplanar forces can easily be determined if an x, y coordinate system is established and the forces are resolved into components along the axes.
- The direction of each force is specified by the angle its line of action makes with one of the axes, or by a slope triangle.
- The orientation of the x and y axes is arbitrary, and their positive direction can be specified by the Cartesian unit vectors **i** and **j**.
- The x and y components of the *resultant force* are simply the algebraic addition of the components of all the coplanar forces.
- The magnitude of the resultant force is determined from the Pythagorean theorem, and when the resultant components are sketched on the x and y axes, Fig. 2–17c, the direction θ of the resultant can be determined from trigonometry.