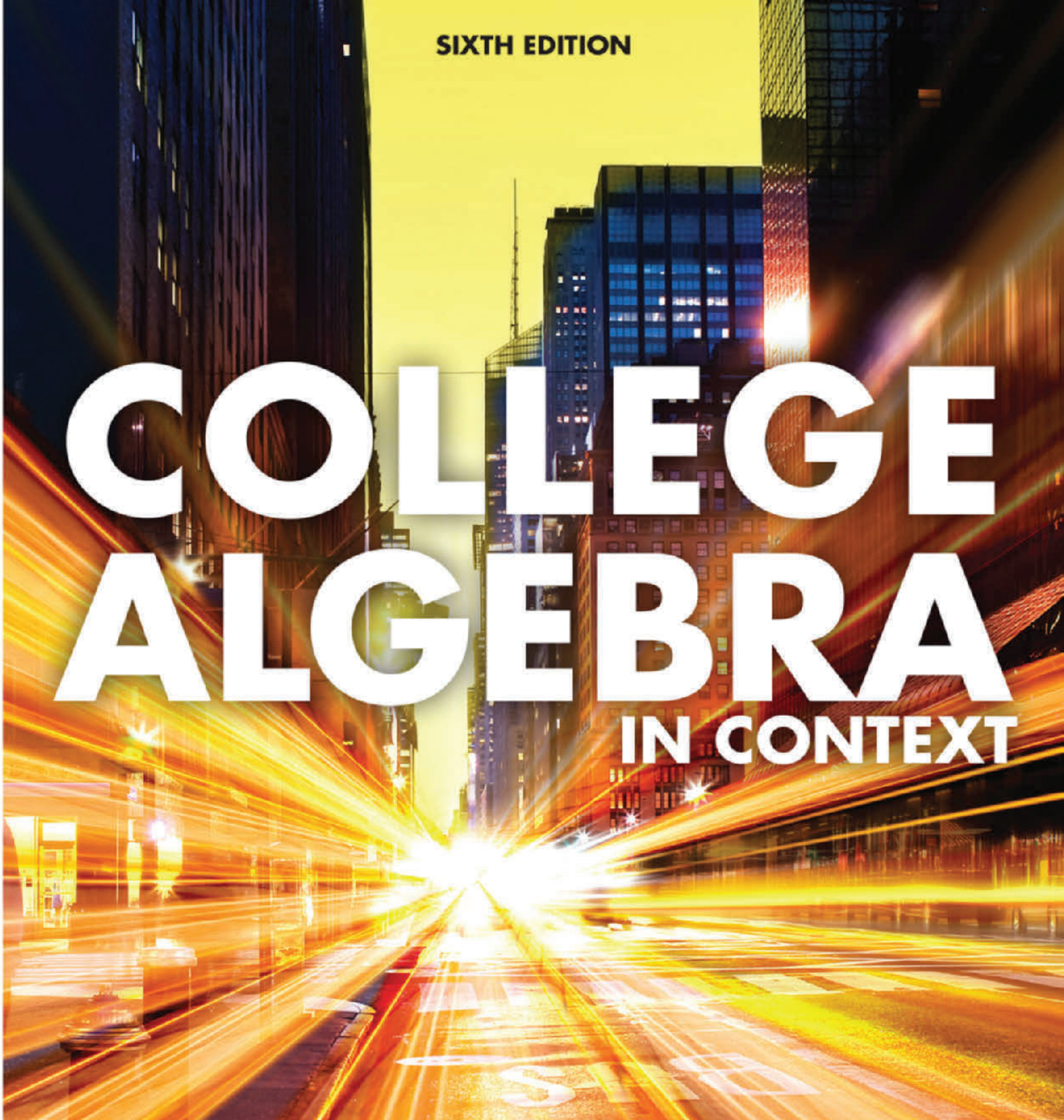
SIXTH EDITION



# COLLEGE ALGEBRA IN CONTEXT

## WITH APPLICATIONS FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES

HARSHBARGER YOCCO

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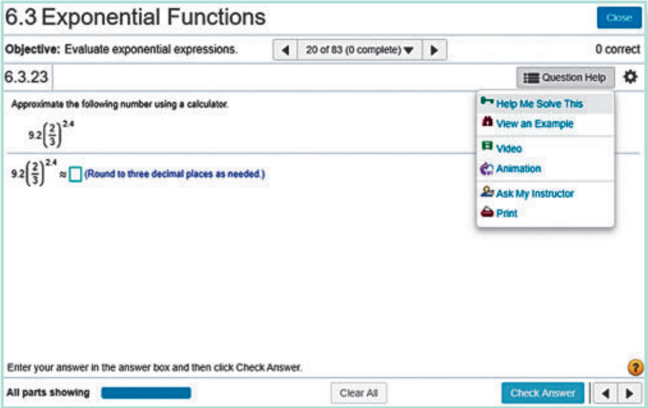


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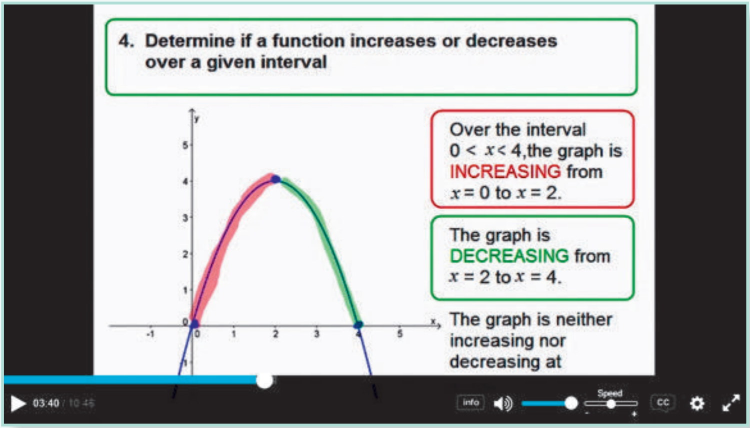


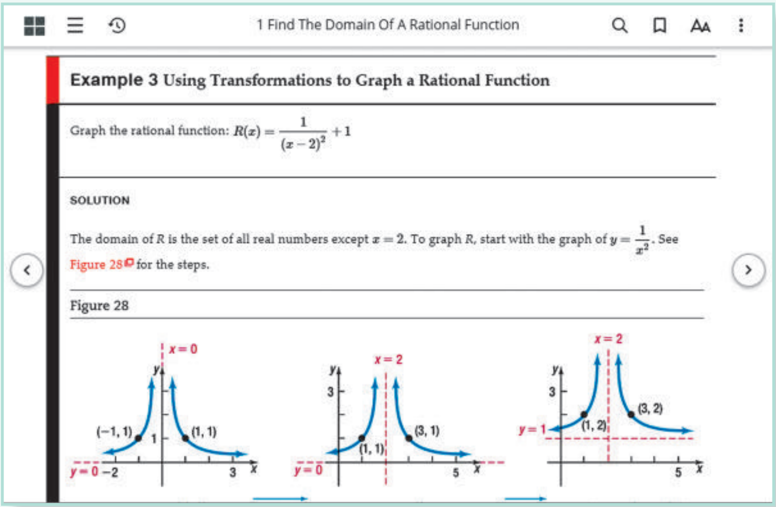


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6th Edition

# College Algebra
## IN CONTEXT

with applications for the managerial, life, and social sciences


Ronald J. Harshbarger
University of South Carolina

Lisa S. Yocco
East Georgia State College


Pearson

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# Contents

# Preface

*College Algebra in Context* is designed for a course in algebra that is based on data analysis, modeling, and real-life applications from the management, life, and social sciences. The text is intended to show students how to analyze, solve, and interpret problems in this course, in future courses, and in future careers. At the heart of this text is its emphasis on problem solving in meaningful contexts.

The text is application-driven and uses real data problems that motivate interest in the skills and concepts of algebra. Modeling is introduced early, in the discussion of linear functions and in the discussion of quadratic and power functions. Additional models are introduced when exponential, logarithmic, logistic, cubic, and quartic functions are discussed. Mathematical concepts are introduced informally with an emphasis on applications. Each chapter contains real data problems and extended application projects that can be solved by students working collaboratively.

The text features a constructive chapter-opening Algebra Toolbox, which reviews previously learned algebra concepts by presenting the prerequisite skills needed for successful completion of the chapter. In addition, Preparing for Calculus sections in each of the first six chapters provide problems that demonstrate how students can use their new knowledge in a calculus course.

## Changes to the Sixth Edition

Based on valuable suggestions from our users and reviewers of our fifth edition, as well as our own classroom experiences and student input, we have made a number of changes in the sixth edition.

- To keep the real data applications current, more than 250 problems have been replaced with new or updated problems.
- **Algebra Toolbox** content, which opens Chapters 1–7, has been expanded and strengthened to accommodate corequisite scenarios, with more than 80 new exercises. This content has been added to the Integrated Review chapter in MyLab Math. Additional topics that have been added include the following: calculations with and properties of real numbers; operations with fractions; evaluating algebraic expressions; solving basic linear equations; evaluating formulas for given values; operations with radicals; rationalizing denominators; operations with complex numbers.
- A **Corequisite Notebook**, written by Lisa Yocco, is now available as a complement to this text. It includes all **Algebra Toolbox** topics with unique exercises for practice followed by small group activities, written by Trisha Sholar, to enhance classroom time. This resource can be used for corequisite courses or simply for students who are underprepared.
- **Preparing for Calculus** sections have been added before the Chapter Summaries in each of Chapters 1 through 6. These sections present problems that use the skills of the chapters in calculus contexts to show students how the algebra topics of each chapter apply to the development and application of calculus. This is important for those who will be taking calculus. Coverage of these exercises have also been added to MyLab Math.
- A new type of exercise call Setup & Solve has been added to MyLab Math. These multi-part exercises require students to show the setup of the solution for a particular exercise as well as the solution to gauge the students' conceptual understanding of the topic. Look for them with the label "Setup & Solve."

- Previous Section 3.3 has been divided into two new sections. New Section 3.3 now has expanded coverage of power functions and root functions, and new Section 3.4 now has expanded coverage of piecewise-defined functions and absolute value functions. These sections have many more examples and exercises than were in old Section 3.3.
- Additional modeling problems that involve decision making and critical thinking have been added throughout the text.
- Specific steps for additional topics have been added to Appendix A.
- More specific Excel steps have been added within the Spreadsheet Solutions in the text, with references to Appendix B for more details.
- Specific steps for additional topics have been added to Appendix B.
- Discussion of loan amortization using graphing calculators has been added to Chapter 5, and the detailed steps for solving loan amortization problems with graphing calculators has been added to Appendix A.
- Discussion of loan amortization using Excel has been added to Chapter 5, and the detailed steps for solving loan amortization problems with Excel has been added to Appendix B.

## Continued Features

Features of the text include the following:

- The development of algebra is motivated by the need to use algebra to find the solutions to **real data–based applications**.

  Real-life problems demonstrate the need for specific algebraic concepts and techniques. Each section begins with a motivational problem couched in a real-life setting. The problem is solved after the necessary skills have been presented in that section. The aim is to prepare students to solve problems of all types by first introducing them to various functions and then encouraging them to take advantage of available technology. Special business and finance models are included to demonstrate the application of functions to the business world.
- **Technology** has been integrated into the text.

  The text discusses the use of graphing calculators and Excel to solve problems, but there are no specific technology requirements. When a new calculator or spreadsheet skill becomes useful in a section, students can find the required keystrokes or commands in the text and in the *Graphing Calculator and Excel® Manual*, located within MyLab Math, as well as in Appendixes A and B discussed below. Technology is used to enhance and support learning when appropriate—not to supplant learning.
- The text contains two technology appendixes: a **Basic Graphing Calculator Guide** and a **Basic Guide to Excel**. Footnotes throughout the text refer students to these guides for a detailed exposition when a new use of technology is introduced. Additional Excel solution procedures have been added, but, as before, they can be omitted without loss of continuity in the text.
- Each of the first seven chapters begins with an **Algebra Toolbox** section that reviews the prerequisite skills needed for successful completion of the chapter.

  Topics discussed in the Toolbox are topics that are prerequisite to a college algebra course (they are often found in a Chapter R or appendix of a college algebra text). Key objectives are listed at the beginning of each Toolbox, and topics are introduced "just in time" to be used in the chapter under consideration.

- Many problems posed in the text are **multi-part and multi-level problems**.

  Many problems require thoughtful, real-world answers adapted to varying conditions, rather than numerical answers. Questions such as "When will this model no longer be valid?" "What additional limitations must be placed on your answer?" and "Interpret your answer in the context of the application" are commonplace in the text.

- Chapter objectives are listed at the beginning of each chapter, and key objectives are given at the beginning of each section.

- Each chapter has a **Chapter Summary**, a **Chapter Skills Check**, and a **Chapter Review**.

  The Chapter Summary lists the key terms and formulas discussed in the chapter, with section references. Review problems include Chapter Skills Check, which strictly reinforce methods or procedures, and Review Exercises, which are applications.

- The text encourages **collaborative learning**.

  Each chapter ends with one or more Group Activities/Extended Applications that require students to solve multi-level problems involving real data or situations, making it desirable for students to collaborate in their solutions. These activities provide opportunities for students to work together to solve real problems that involve the use of technology and that frequently require modeling.

- The text encourages students to improve **communication skills and research skills**.

  The Group Activities/Extended Applications require written reports and frequently require use of the Internet or a library. Some Extended Applications call for students to use literature or the Internet to find a graph or table of discrete data describing an issue. They are then required to make a scatter plot of the data, determine the function type that is the best fit for the data, create the model, discuss how well the model fits the data, and discuss how it can be used to analyze the issue.

- **Answers to Selected Exercises** include answers to all Chapter Skills Checks, Chapter Reviews, and odd-numbered application exercises so students have feedback regarding the exercises they work.

- **Supplements** are provided that will help students and instructors use technology to improve the learning and teaching experience. See the instructor and student resources lists.

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*Ronald J. Harshbarger*
*Lisa S. Yocco*

* Denotes reviewers of the sixth edition.

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# MyLab Math

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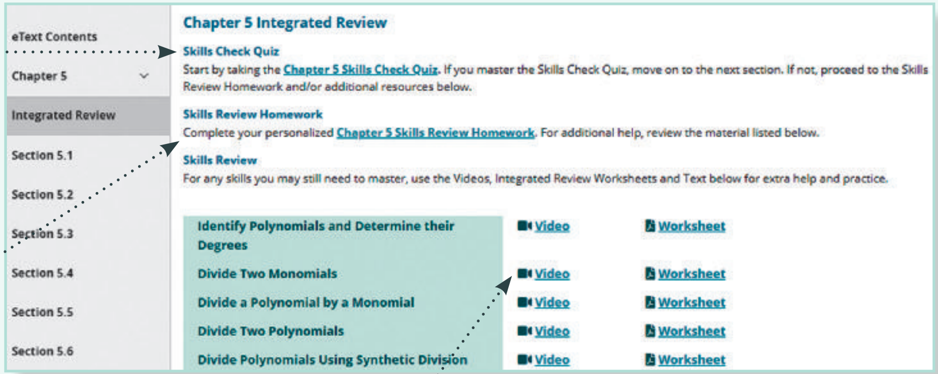
## Preparedness

Preparedness is one of the biggest challenges in many math courses. Pearson offers a variety of content and course options to support students with just-in-time remediation and key-concept review as needed.

## Integrated Review in MyLab Math

Integrated Review can be used in corequisite courses or simply to help students who enter a course without a full understanding of prerequisite skills and concepts.




- Students begin each chapter by completing a Skills Check to pinpoint which topics, if any, they need to review.
- Personalized review homework provides extra support for students who need it on just the topics they didn't master in the preceding Skills Check.
- Additional review materials, including **Updated Worksheets** and **New Videos**, are available.

## Corequisite Notebook

**New! The Corequisite Notebook,** created by author Lisa Yocco, expands on the Algebra Toolbox topics within the text, which align to the Integrated Review videos/objectives. Students actively participate in learning the *how* and *why* of fundamental topics to ensure they are prepared for their college-level work.

- **Learning Tips** begin each chapter to set students in a learning mindset.
- **Worksheets** cover all Algebra Toolbox prerequisite topics and include unique examples and exercises for practice.
- **Small Group Activities** round out the chapter to be completed in class.

# *Get the **most** out of*
# MyLab Math

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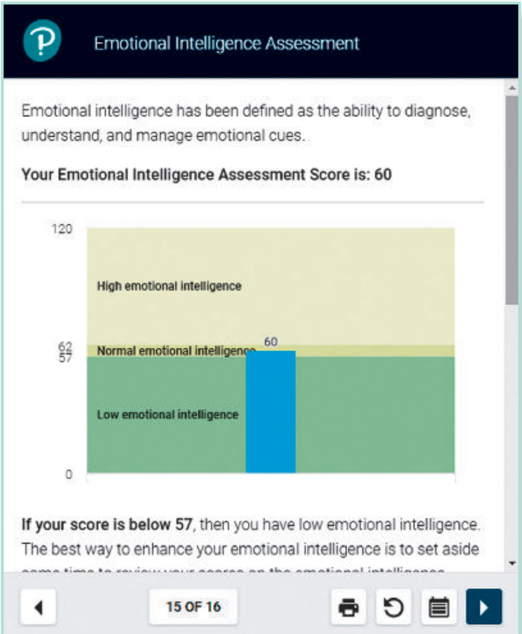
**MyLab Math for COLLEGE ALGEBRA IN CONTEXT 6e**
**by Ronald J. Harshbarger and Lisa S. Yocco**
*(access code required)*

## Motivation Resources

Motivate and inspire your students to succeed by encouraging a growth mindset and opportunities for self-reflection.




**New! Personal Assessment Inventories** is a collection of online activities designed to promote self-reflection and engagement in students.

**New! Mindset videos** encourage students to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as learning opportunities—so often a hurdle for math students.


## Preparing for Calculus Exercises

Mastering prerequisite content requires both a positive growth mindset *and* an understanding of why algebra is needed.

**New! Preparing for Calculus Exercises** place skills from each chapter in calculus contexts to demonstrate the importance of algebra for future calculus courses. Exercises are assignable in MyLab Math.

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*Resources for*

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## Instructor Resources

Online resources can be downloaded at pearson.com/mylab/math MyLab Math or from www.pearson.com.

### Instructor's Edition

ISBN - 0135757770 / 9780135757772

The instructor's edition includes all answers to the exercise sets. Shorter answers are presented on the same page as the exercise; longer answers are in the back of the text.

### Instructor's Solution Manual

Includes fully worked solutions to all exercises in the text (download only).

### Learning Catalytics Question Library

Questions written for this text are available to deliver through Learning Catalytics to engage students in your course. Search "Harshbarger" within Learning Catalytics to view questions.

### PowerPoint® Lecture Slides

Written specifically for this text, these fully editable lecture slides provide an outline for presenting definitions, figures, and key examples. Accessible PowerPoints, which are designed to be used with screen readers, are also available.

### Instructor's Testing Manual

This manual contains three alternative forms of tests per chapter with answer keys that contain more applications (download only).

### TestGen®

TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

## Student Resources

Additional resources to enhance student success.

### Corequisite Notebook:

**New! The Corequisite Notebook,** created by author Lisa Yocco, expands on the Algebra Toolbox topics within the text, which align to the Integrated Review videos/objectives found in MyLab Math. Students actively participate in learning the *how* and *why* of fundamental topics to ensure they are prepared for their college-level work.

### Additional Skill and Exercise Manual

This manual provides additional practice and test preparation for students (download only).

### Student's Solutions Manual

ISBN - 0135757665 / 9780135757666

Provides detailed worked-out solutions to odd-numbered exercises.


# 1

# Functions, Graphs, and Models; Linear Functions

With digital TV becoming more affordable by the day, the demand for high-definition home entertainment is growing rapidly, with 180 million Americans having digital TV in 2019. The worldwide total was 553 million in 2018. Cell phone use is also on the rise, not just in the United States but throughout the world. By the middle of 2014, the number of subscribers to cell phone carriers had dramatically increased, and the number of total users had reached 4.7 billion. If the numbers continue to increase at a steady rate, the number of subscribers is expected to reach into the tens of billions over the next few years. These projections and others are made by collecting real-world data and creating mathematical models. The goal of this chapter, and future chapters, is to use real data and mathematical models to make predictions and solve meaningful problems.

| sections | objectives | applications |
|---|---|---|
| **1.1** Functions and Models | Determine graphs, tables, and equations that represent functions; find domains and ranges; evaluate functions and mathematical models; align data | Body temperature, female physicians, stock market, U.S. industrial shipments, older men in the workforce, medical 3-D printing |
| **1.2** Graphs of Functions | Graph and evaluate functions with technology; graph mathematical models; graph data points; scale data | Global EV/PHEV sales, aging workers, cost-benefit, U.S. diabetes |
| **1.3** Linear Functions | Identify and graph linear functions; find and interpret intercepts and slopes; find constant rates of change; model revenue, cost, and profit; find marginal revenue, marginal cost, and marginal profit; identify special linear functions | Hispanics in the United States, loan balances, revenue, cost, profit, marginal cost, marginal revenue, marginal profit |
| **1.4** Equations of Lines | Write equations of lines; identify parallel and perpendicular lines; find average rates of change; compute the difference quotient; model approximately linear data | Service call charges, depreciation, prison population, telecom artificial intelligence, public school enrollment, average velocity |

# Algebra TOOLBOX

**KEY OBJECTIVES**

- Write sets of numbers using description or elements
- Find intersection and union of sets
- Identify sets of real numbers as being integers, rational numbers, and/or irrational numbers
- Calculate with real numbers
- Use properties of real numbers
- Express inequalities as intervals and graph inequalities
- Identify the coefficients of terms and constraints in algebraic expressions
- Evaluate algebraic expressions
- Combine like terms
- Remove parentheses and simplify expressions
- Solve basic linear equations
- Plot points on a coordinate system
- Use subscripts to represent fixed points

The Algebra Toolbox is designed to review prerequisite skills needed for success in each chapter. In this Toolbox, we discuss sets, the real numbers, the coordinate system, algebraic expressions, equations, inequalities, absolute values, and subscripts.

## Sets

In this chapter we will use sets to write domains and ranges of functions, and in future chapters we will find solution sets to equations and inequalities. A **set** is a well-defined collection of objects including, but not limited to, numbers. In this section, we will discuss sets of real numbers, including natural numbers, integers, and rational numbers, and later in the text we will discuss the set of complex numbers. There are two ways to define a set. One way is by listing the **elements** (or **members**) of the set (usually between braces). For example, we may say that a set $A$ contains 2, 3, 5, and 7 by writing $A = \{2, 3, 5, 7\}$. To say that 5 is an element of the set $A$, we write $5 \in A$. To indicate that 6 is not an element of the set, we write $6 \notin A$. Domains of functions and solutions to equations are sometimes given in sets with the elements listed.

If all the elements of the set can be listed, the set is said to be a **finite set**. If all elements of a set cannot be listed, the set is called an **infinite set**. To indicate that a set continues with the established pattern, we use three dots. For example, $B = \{1, 2, 3, 4, 5, \ldots, 100\}$ describes the finite set of whole numbers from 1 through 100, and $N = \{1, 2, 3, 4, 5, \ldots\}$ describes the infinite set of all whole numbers beginning with 1. This set is called the **natural numbers**.

Another way to define a set is to give its description. For example, we may write $\{x | x \text{ is a math book}\}$ to define the set of math books. This is read as "the set of all $x$ such that $x$ is a math book." $N = \{x | x \text{ is a natural number}\}$ defines the set of natural numbers, which was also defined by $N = \{1, 2, 3, 4, 5, \ldots\}$ above.

The set that contains no elements is called the **empty set** and is denoted by $\varnothing$.

### EXAMPLE 1 ▶ Sets

Write the following sets in two ways.

**a.** The set $A$ containing the natural numbers less than 7.

**b.** The set $B$ of natural numbers that are at least 7.

**SOLUTION**

**a.** $A = \{1, 2, 3, 4, 5, 6\}, A = \{x | x \in N, x < 7\}$

**b.** $B = \{7, 8, 9, 10, \ldots\}, B = \{x | x \in N, x \geq 7\}$

---

The relations that can exist between two sets follow.

**Relations Between Sets**

1. Sets $X$ and $Y$ are **equal** if they contain exactly the same elements.
2. Set $A$ is called a **subset** of set $B$ if each element of $A$ is an element of $B$. This is denoted $A \subseteq B$.

3. If sets $C$ and $D$ have no elements in common, they are called **disjoint**.
4. The set containing the elements that are common to two sets is said to be the **intersection** of the two sets. The intersection of $A$ and $B$ is written $A \cap B$.
5. The **union** of two sets is the set that contains all the elements of both sets. The union of A and B is written $A \cup B$.

## EXAMPLE 2 ▶ Relations Between Sets

For the sets $A = \{x | x \leq 9, x \text{ is a natural number}\}, B = \{2, 4, 6\}, C = \{3, 5, 8, 10\}$:

**a.** Which of the sets $A$, $B$, and $C$ are subsets of $A$?

**b.** Which pairs of sets are disjoint?

**c.** Are any of these three sets equal?

### SOLUTION

**a.** Every element of $B$ is contained in $A$. Thus, set $B$ is a subset of $A$. Because every element of $A$ is contained in $A$, $A$ is a subset of $A$.

**b.** Sets $B$ and $C$ have no elements in common, so they are disjoint.

**c.** None of these sets have exactly the same elements, so none are equal.

## EXAMPLE 3 ▶ Intersection and Union of Sets

Given sets $A = \{5, 6, 7, 8, 9, 10, 11\}$ and $B = \{6, 8, 10, 12, 14, 16\}$, find

**a.** the intersection of $A$ and $B$

**b.** the union of $A$ and $B$

### SOLUTION

**a.** The intersection of $A$ and $B$, $A \cap B$, contains all elements that are common to both sets:

$$A \cap B = \{6, 8, 10\}$$

**b.** The union of $A$ and $B$, $A \cup B$, contains all elements that are in both sets:

$$A \cup B = \{5, 6, 7, 8, 9, 10, 11, 12, 14, 16\}$$

## The Real Numbers

Because most of the mathematical applications you will encounter in an applied nontechnical setting use real numbers, the emphasis in this text is the **real number system**.* Real numbers can be rational or irrational. **Rational numbers** include integers, fractions containing only integers (with no 0 in a denominator), and decimals that either terminate or repeat. Some examples of rational numbers are

$$-9, \quad \frac{1}{2}, \quad 0, \quad 12, \quad -\frac{4}{7}, \quad 6.58, \quad -7.\overline{3}$$

*The complex number system will be discussed in the Chapter 3 Toolbox.

**Irrational numbers** are real numbers that are not rational. Some examples of irrational numbers are $\pi$ (a number familiar to us from the study of circles), $\sqrt{2}$, $\sqrt[3]{5}$, and $\sqrt[3]{-10}$.

The types of real numbers are described in Table 1.1.

**Table 1.1**

| Types of Real Numbers | Descriptions |
|---|---|
| Natural numbers | 1, 2, 3, 4, . . . |
| Integers | Natural numbers, zero, and the negatives of the natural numbers: . . . , −3, −2, −1, 0, 1, 2, 3, . . . |
| Rational numbers | All numbers that can be written in the form $\frac{p}{q}$, where $p$ and $q$ are both integers with $q \neq 0$. Rational numbers can be written as terminating or repeating decimals. |
| Irrational numbers | All real numbers that are not rational numbers. Irrational numbers cannot be written as terminating or repeating decimals. |

We can represent real numbers on a **real number line**. Exactly one real number is associated with each point on the line, and we say there is a one-to-one correspondence between the real numbers and the points on the line. That is, the real number line is a graph of the real numbers (see Figure 1.1).

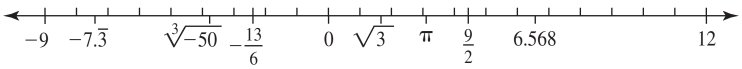



**Figure 1.1**

Notice the number $\pi$ on the real number line in Figure 1.1. This special number, which can be approximated by 3.14, results when the circumference of (distance around) any circle is divided by the diameter of the circle. Another special real number is $e$; it is denoted by

$$e \approx 2.71828$$

We will discuss this number, which is important in financial and biological applications, later in the text.

## Calculating with Real Numbers

We use a standard order of operations to calculate expressions when grouping symbols are not clear:

1. Perform all operations inside parentheses or other grouping symbols before removing them.
2. Raise numbers to indicated powers and take indicated roots.
3. Do all multiplications and divisions, in the order they occur from left to right.
4. Do all additions and subtractions, in the order they occur from left to right.

To *add* two signed numbers with the *same sign*, add their absolute values (the numerical values, disregarding the sign), and keep their common sign. For example, $(-4) + (-5) = -9$.

To *add* two signed numbers with *unlike signs*, subtract the smaller absolute value from the larger absolute value, and keep the sign of the number with the larger absolute value. For example, $(-7) + (3) = -(7 - 3) = -4$.

To *subtract* two signed numbers, change the sign of the number being subtracted and proceed as in addition. For example, $(-9) - (-3) = (-9) + 3 = -6$.

To *add three or more signed numbers*, add them two at a time. If no grouping symbols are present, add from left to right. If grouping symbols are present, add inside the grouping symbols first. For example, $-8 + (-6) - 4 - 3 = (-8 + (-6)) - 4 - 3 = -14 - 4 - 3 = -18 - 3 = -21$.

When *multiplying or dividing* two numbers with the *same* sign, the product or quotient is positive. For example, $(-2)(-3) = 6$; $12 \div 3 = 4$

When *multiplying or dividing* two numbers with *unlike* signs, the product or quotient is negative. For example, $(-5)(3) = -15$; $\frac{16}{-2} = -8$.

When *multiplying or dividing more than two signed numbers*, the product will be positive if there is an even number of negative signs. The product will be negative if there is an odd number of negative signs. A number **multiplied by 0** is equal to 0. **Division by 0** is undefined. Zero divided by a nonzero number is defined.

## Properties of Real Numbers

1. The *Commutative Property of Addition* states that the sum of two real numbers is the same even if the order of the numbers is changed. That is, addition of real numbers is commutative: $a + b = b + a$ for all real numbers.

2. The *Commutative Property of Multiplication* states that the product of two real numbers is the same even if the order of the numbers is changed. That is, multiplication of real numbers is commutative: $a \cdot b = b \cdot a$ for all real numbers.

3. The *Associative Property of Addition* states that the sum of three numbers is the same if the first pair or the last pair is added first. That is, $a + (b + c) = (a + b) + c$.

4. The *Associative Property of Multiplication* states that the product of three numbers is the same if the first pair or the last pair is multiplied first. That is, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.

5. The *Distributive Property of Multiplication over Addition* states that multiplying a sum of two numbers by a third number gives the same result as multiplying the third number by each of the two numbers in the sum and adding the two products. That is, $a \cdot (b + c) = a \cdot b + a \cdot c$.

6. 0 is the *additive identity*. For any real number $a$, $a + 0 = a$.

7. 1 is the *multiplicative identity*. For any real number $a$, $a \cdot 1 = a$.

8. For every real number $a$, there exists a number $b$ such that $a + b = 0$. The *additive inverse* of $a$ is $-a$.

9. For every real number $a \neq 0$, there exists a number $b$ such that $a \cdot b = 1$. The *multiplicative inverse* of $a$ is also called the *reciprocal* of $a$ and denoted as $\frac{1}{a}$.

## Inequalities and Intervals on the Number Line

In this chapter, we will sometimes use inequalities and interval notation to describe domains and ranges of functions. An **inequality** is a statement that one quantity is greater (or less) than another quantity. We say that $a$ is less than $b$ (written $a < b$) if the point representing $a$ is to the left of the point representing $b$ on the real number line. We may indicate that the number $a$ is greater than or equal to $b$ by writing $a \geq b$. The subset of real numbers $x$ that lie between $a$ and $b$ (excluding $a$ and $b$) can be denoted by the **double inequality** $a < x < b$ or by the **open interval** $(a, b)$. This is called an open interval because neither of the endpoints is included in the interval. The **closed interval** $[a, b]$ represents the set of all real numbers satisfying $a \le x \le b$. Intervals containing one endpoint, such as $[a, b)$ or $(a, b]$, are called **half-open intervals**. We can represent the inequality $x \ge a$ by the interval $[a, \infty)$, and we can represent the inequality $x < a$ by the interval $(-\infty, a)$. Note that $\infty$ and $-\infty$ are not numbers, but $\infty$ is used in $[a, \infty)$ to represent the fact that $x$ increases without bound and $-\infty$ is used in $(-\infty, a)$ to indicate that $x$ decreases without bound. Table 1.2 shows the graphs of different types of intervals.

**Table 1.2**

| Interval Notation | Inequality Notation | Verbal Description | Number Line Graph |
|---|---|---|---|
| $(a, \infty)$ | $x > a$ | $x$ is greater than $a$ |   |
| $[a, \infty)$ | $x \ge a$ | $x$ is greater than or equal to $a$ |   |
| $(-\infty, b)$ | $x < b$ | $x$ is less than $b$ |   |
| $(-\infty, b]$ | $x \le b$ | $x$ is less than or equal to $b$ |   |
| $(a, b)$ | $a < x < b$ | $x$ is between $a$ and $b$, not including either $a$ or $b$ |   |
| $[a, b)$ | $a \le x < b$ | $x$ is between $a$ and $b$, including $a$ but not including $b$ |   |
| $(a, b]$ | $a < x \le b$ | $x$ is between $a$ and $b$, not including $a$ but including $b$ |   |
| $[a, b]$ | $a \le x \le b$ | $x$ is between $a$ and $b$, including both $a$ and $b$ |   |

Note that open circles may be used instead of parentheses, and solid circles may be used instead of brackets, in the number line graphs.

## EXAMPLE 4 ▶ Intervals

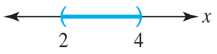
Write the interval corresponding to each of the inequalities in parts (a)–(e), and then graph the inequality.

**a.** $-1 \le x \le 2$ **b.** $2 < x < 4$ **c.** $-2 < x \le 3$ **d.** $x \ge 3$ **e.** $x < 5$

### SOLUTION

**a.** $[-1, 2]$


**b.** $(2, 4)$




**c.** $(-2, 3]$


**d.** $[3, \infty)$


**e.** $(-\infty, 5)$

## Algebraic Expressions

In algebra we deal with a combination of real numbers and letters. Generally, the letters are symbols used to represent unknown quantities or fixed but unspecified constants. Letters representing unknown quantities are usually called **variables**, and letters representing fixed but unspecified numbers are called **literal constants**. An expression created by performing additions, subtractions, or other arithmetic operations with one or more real numbers and variables is called an **algebraic expression**. Unless otherwise specified, the variables represent real numbers for which the algebraic expression is a real number. Examples of algebraic expressions include

$$5x - 2y, \quad \frac{3x - 5}{12 + 5y}, \quad \text{and} \quad 7z + 2$$

A **term** of an algebraic expression is the product of one or more variables and a real number; the real number is called a **numerical coefficient** or simply a **coefficient**. A constant is also considered a term of an algebraic expression and is called a **constant term**. For instance, the term $5yz$ is the product of the factors 5, $y$, and $z$; this term has coefficient 5.

## Evaluating Algebraic Expressions

If we give a specific value to a variable, we can **evaluate an algebraic expression**. To evaluate an algebraic expression means to find its numerical value after we know the values of the variables. Algebraic expressions are often found in real-life situations. For example, $2l + 2w$ represents the perimeter of a rectangle where $l$ is the length and $w$ is the width.

### EXAMPLE 5 ▶ Algebraic Expressions

**a.** State the terms of the algebraic expression $3x - 7yz + 4$.

**b.** State the factors of the second term in part (a).

**c.** Evaluate the expression in part (a) if $x = -2$, $y = 4$, and $z = \frac{1}{2}$.

**d.** If the length of a rectangle is 17 feet and its width is 9 feet, find the perimeter of the rectangle.

**SOLUTION**

**a.** The terms of the algebraic expression are $3x$, $-7yz$, and 4.

**b.** The factors of the term $-7yz$ are $-7$, $y$, and $z$.

**c.** Substituting $-2$ for $x$, 4 for $y$, and $\frac{1}{2}$ for $z$ gives

$$3(-2) - 7(4)\left(\frac{1}{2}\right) + 4 = -6 - 14 + 4 = -16$$

**d.** Substituting 17 for $l$ and 9 for $w$ in the algebraic expression $2l + 2w$ gives

$$2(17) + 2(9) = 52$$

so the perimeter is 52 feet.

## Combining Like Terms

Terms that contain exactly the same variables with exactly the same exponents are called **like terms**. For example, $3x^2y$ and $7x^2y$ are like terms, but $3x^2y$ and $3xy$ are not. We add or subtract (**combine**) algebraic expressions by combining the like terms.