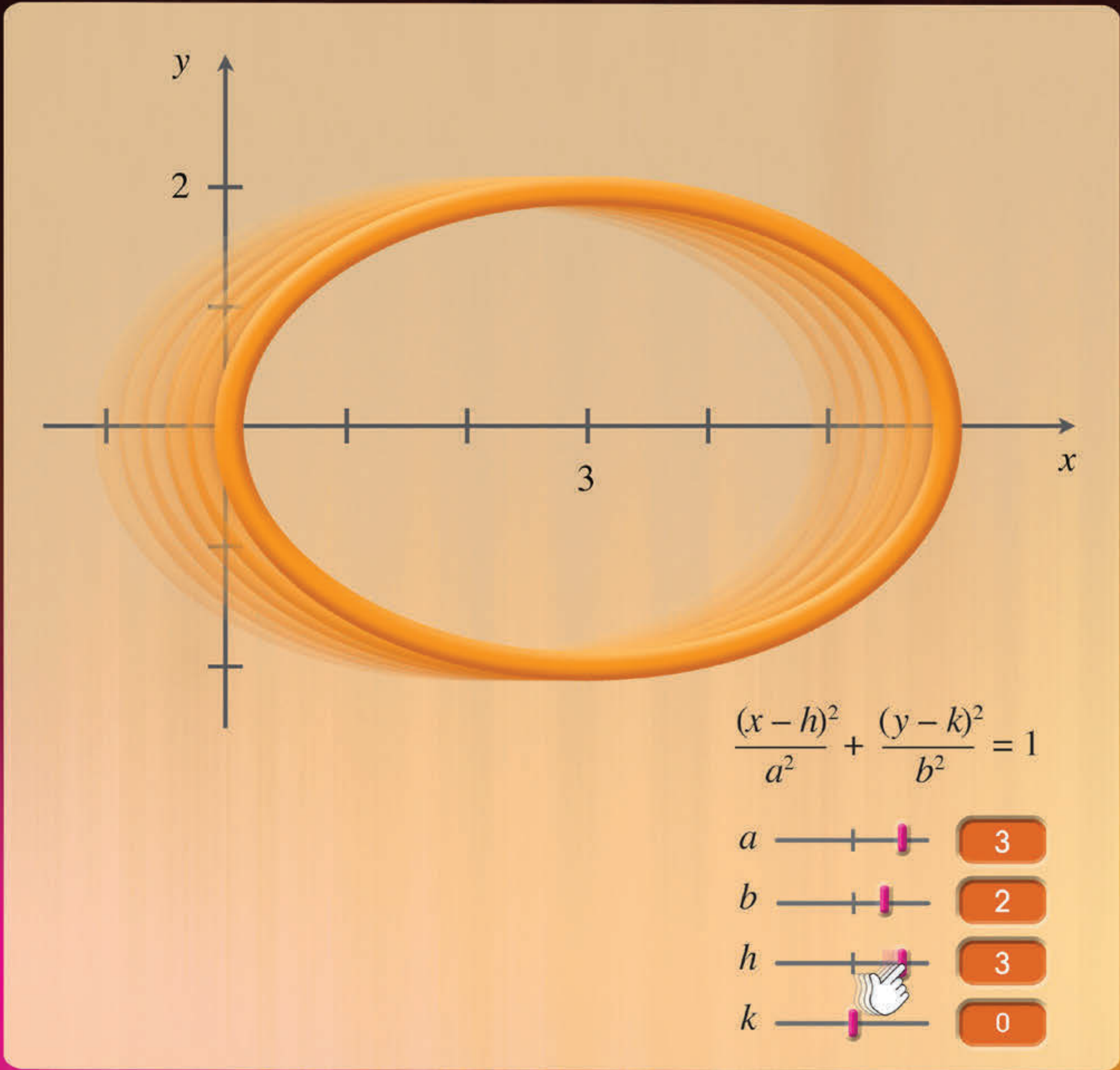
PRECALCULUS

Enhanced with Graphing Utilities

Eighth Edition



Sullivan & Sullivan

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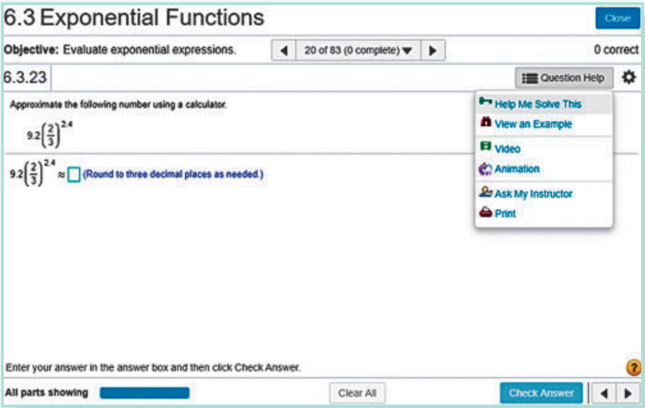
MyLab Math

*Support You **Need**, When You **Need** It*

MyLab Math has helped millions of students succeed in their math courses. Take advantage of the resources it provides.

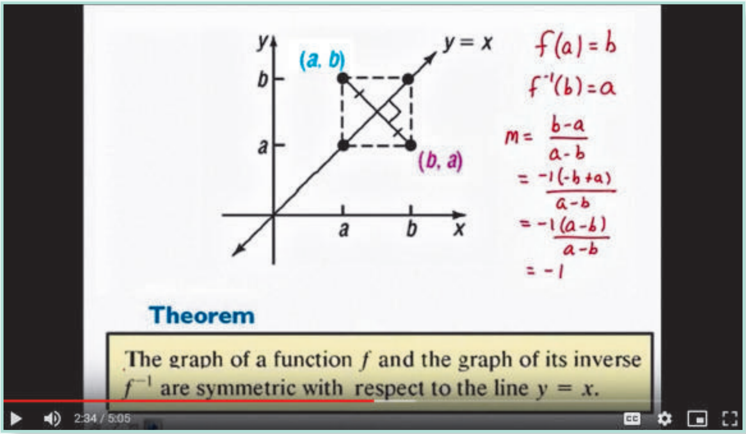


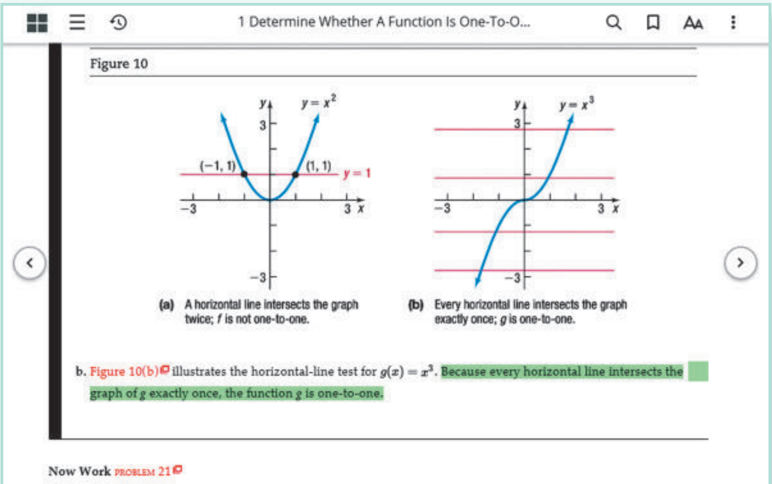
Interactive Exercises

MyLab Math's interactive exercises mirror those in the textbook but are programmed to allow you unlimited practice, leading to mastery. Most exercises include **learning aids**, such as "Help Me Solve This," "View an Example," and "Video," and they offer helpful feedback when you enter incorrect answers.

Instructional Videos

Instructional videos cover key examples from the text and can conveniently be played on any mobile device. These videos are especially helpful if you miss a class or just need further explanation.





Interactive eText

The **Pearson eText** gives you access to your textbook anytime, anywhere. In addition to note taking, highlighting, and bookmarking, the Pearson eText App offers access even when offline.

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Prepare for Class: "Read the Book"

Feature	Description	Benefit	Page(s)
Every Chapter begins with . . .			
Chapter-Opening Topic & Project	Each chapter begins with a discussion of a topic of current interest and ends with a related project.	In the concluding project, you will apply what you have learned to solve a problem related to the topic.	271, 380
Internet-Based Projects	These projects allow for the integration of spreadsheet technology that you will need to be a productive member of the workforce.	The projects give you an opportunity to collaborate and use mathematics to deal with issues of current interest.	380
Every Section begins with . . .			
LEARNING OBJECTIVES 2	Each section begins with a list of objectives. Individual objectives also appear in the text where they are covered.	These objectives focus your studying by emphasizing what's most important and where to find it.	293
Sections contain . . .			
PREPARING FOR THIS SECTION	Most sections begin with a list of key concepts to review, with page numbers.	Ever forget what you've learned? This feature highlights previously learned material to be used in this section. Review it, and you'll always be prepared to move forward.	293
Now Work the 'Are You Prepared?' Problems	These problems assess whether you have the prerequisite knowledge for the upcoming section.	Work the 'Are You Prepared?' problems. If you get one wrong, you'll know exactly what you need to review and where to review it!	293, 305
Now Work PROBLEMS	These follow most examples and direct you to a related exercise.	We learn best by doing. You'll solidify your understanding of examples if you try a similar problem right away, to be sure you understand what you've just read.	301, 307
CAUTION	Words of caution are provided in the text.	These point out common mistakes and help you avoid them.	329
Explorations and Seeing the Concept	These graphing utility activities foreshadow a concept or reinforce a concept just presented.	You will obtain a deeper and more intuitive understanding of theorems and definitions.	288, 300
In Words	This feature provides alternative descriptions of select definitions and theorems.	Why didn't you say that in the first place? This feature translates math into plain English.	312
Calculus	This symbol appears next to information essential for the study of calculus.	Foreshadowing calculus now will make the material easier later.	67, 276, 301
SHOWCASE EXAMPLES	These examples provide "how to" instruction by offering a guided, step-by-step approach to solving a problem.	With each step presented on the left and the mathematics displayed on the right, you can immediately see how each step is employed.	205–206
Model It! Examples and Problems	These examples and problems require you to build a mathematical model from either a verbal description or data. The homework Model It! problems are marked by purple problem numbers.	It is rare for a problem to come in the form "*Solve the following equation*." Rather, the equation must be developed based on an explanation of the problem. These problems require you to develop models that will enable you to describe the problem mathematically and suggest a solution to the problem.	319, 351
NEW! Need to Review?	These margin notes provide a just-in-time reminder of a concept needed now, but covered in an earlier section of the book. Each note is back-referenced to the chapter, section and page where the concept was originally discussed.	Sometimes as you read, you encounter a word or concept you know you've seen before, but don't remember exactly what it means. This feature will point you to where you first learned the word or concept. A quick review now will help you see the connection to what you are learning for the first time and make remembering easier the next time.	300

Practice: "Work the Problems"

Feature	Description	Benefit	Page(s)
'Are You Prepared?' Problems	These problems assess your retention of the prerequisite material. Answers are given at the end of the section exercises. This feature is related to the Preparing for This Section feature.	Do you always remember what you've learned? Working these problems is the best way to find out. If you get one wrong, you'll know exactly what you need to review and where to review it!	293, 305
Concepts and Vocabulary	These short-answer questions, mainly fill-in-the-blank, multiple-choice, and true/false items, assess your understanding of key definitions and concepts in the current section.	It is difficult to learn math without knowing the language of mathematics. These problems test your understanding of the formulas and vocabulary.	305–306
Skill Building	Correlated with section examples, these problems provide straightforward practice.	These problems give you ample opportunity to dig in and develop your skills.	306–308
Mixed Practice	These problems offer comprehensive assessment of the skills learned in the section by asking problems related to more than one concept or objective. These problems may also require you to utilize skills learned in previous sections.	Learning mathematics is a building process. Many concepts build on each other and are related. These problems help you see how mathematics builds on itself and how the concepts are linked together.	308
Applications and Extensions	These problems allow you to apply your skills to real-world problems. They also enable you to extend concepts learned in the section.	You will see that the material learned within the section has many uses in everyday life.	308–311
NEW! Challenge Problems	These problems have been added in most sections and appear at the end of the Application and Extensions exercises. They are intended to be thought-provoking, requiring some ingenuity to solve.	Challenge problems can be used for group work or to challenge your students. Solutions to Challenge Problems are in the Annotated Instructor's Edition or in the Instructor's Solution Manual (online).	311
Explaining Concepts: Discussion and Writing	"Discussion and Writing" problem numbers are colored red. They support class discussion, verbalization of mathematical ideas, and writing and research projects.	To verbalize an idea, or to describe it clearly in writing, shows real understanding. These problems nurture that understanding. Many are challenging, but you'll get out what you put in.	311
Retain Your Knowledge	These problems allow you to practice content learned earlier in the course.	Remembering how to solve all the different kinds of problems that you encounter throughout the course is difficult. This practice helps you remember previously learned skills.	311
Now Work PROBLEMS	Many examples refer you to a related homework problem. These related problems are marked by and orange problem numbers.	If you get stuck while working problems, look for the closest Now Work problem, and refer to the related example to see if it helps.	301, 303, 304
NEW! Interactive Figure Exercises	Exercises that require you manipulate an interactive figure to solve. These exercises are labeled with the icon .	These exercises help you visualize important concepts and develop a "feel" for them. The figures are housed at `bit.ly/2Mibga0` and were developed in GeoGebra by author Michael Sullivan III.	305, 306, 320, 321
Review Exercises	Every chapter concludes with a comprehensive list of exercises to practice. Use the list of objectives to determine what objective and examples correspond to each problem.	Work these problems to ensure that you understand all the skills and concepts employed in the chapter. Think of it as a comprehensive review of the chapter. All answers to Chapter Review problems appear in the back of the text.	375–378

Review: "Study for Quizzes and Tests"

Feature	Description	Benefit	Page(s)
Most Sections Contain			
Retain Your Knowledge	Keeps what you have learned at the forefront and see how topics are connected.	These problems allow content to remain fresh so you are more prepared for the final exam.	325
The Chapter Review at the end of each chapter contains . . .			
Things to Know	A detailed list of important theorems, formulas, and definitions from the chapter.	Review these and you'll know the most important material in the chapter!	373–374
You Should Be Able to . . .	A complete list of objectives by section, examples that illustrate the objective, and practice exercises that test your understanding of the objective.	Do the recommended exercises and you'll have mastered the key material. If you get something wrong, go back and review the example listed, and try again.	374–375
Review Exercises	These provide comprehensive review and practice of key skills, matched to the Learning Objectives for each section.	Practice makes perfect. These problems combine exercises from all sections, giving you a comprehensive review in one place.	375–378
Chapter Test	About 15–20 problems that can be taken as a Chapter Test. Be sure to take the Chapter Test under test conditions—no notes!	Be prepared. Take the sample practice test under test conditions. This will get you ready for your instructor's test. If you get a problem wrong, you can watch the Chapter Test Prep Video.	378–379
Cumulative Review	These problem sets appear at the end of each chapter, beginning with Chapter 2. They combine problems from previous chapters, providing an ongoing cumulative review. When you use them in conjunction with the Retain Your Knowledge problems, you will be ready for the final exam.	These problem sets are really important. Completing them will ensure that you are not forgetting anything as you go. This will go a long way toward keeping you primed for the final exam.	379–380

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Precalculus

Enhanced with Graphing Utilities

Eighth Edition

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Pearson

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For the Next Generation

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Ryan (Murphy)

Maeve, Sean, and
Nolan (Sullivan)

Michael S., Kevin, and
Marissa (Sullivan)

Kaleigh, Billy, and
Timmy (O'Hara)

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Contents

Three Distinct Series to Meet Varied Instructional Needs

Students have different goals, learning styles, and levels of preparation. Instructors have different teaching philosophies, styles, and techniques. Rather than write one series to fit all, the Sullivans have written three distinct series. All share the same goal—to develop a high level of mathematical understanding and an appreciation for the way mathematics can describe the world around us. The manner of reaching that goal, however, differs from series to series.

Enhanced with Graphing Utilities Series

This series provides a thorough integration of graphing utilities into topics, allowing students to explore mathematical concepts and encounter ideas usually studied in later courses. Many examples show solutions using algebra side-by-side with graphing techniques. Using technology, the approach to solving certain problems differs from the Contemporary (Flagship) or Concepts through Functions Series, while the emphasis on understanding concepts and building strong skills is maintained. Texts in this series are *College Algebra, Algebra & Trigonometry*, and *Precalculus.*

Flagship Series

The Flagship Series is the most traditional in approach, yet modern in its treatment of precalculus mathematics. In each text, needed review material is included and is referenced when it is used. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor. Texts in this series are *College Algebra, Algebra & Trigonometry, Trigonometry: A Unit Circle Approach*, and *Precalculus.*

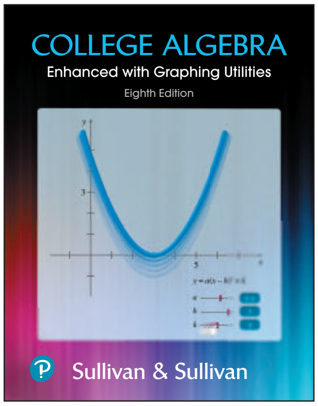
Concepts through Functions Series

This series differs from the others, utilizing a functions approach that serves as the organizing principle tying concepts together. Functions are introduced early in various formats. This approach supports the Rule of Four, which states that functions are represented symbolically, numerically, graphically, and verbally. Each chapter introduces a new type of function and then develops all concepts pertaining to that particular function. The solutions of equations and inequalities, instead of being developed as stand-alone topics, are developed in the context of the underlying functions. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor. Texts in this series are *College Algebra; Precalculus, with a Unit Circle Approach to Trigonometry; Precalculus, with a Right Triangle Approach to Trigonometry.*

The Enhanced with Graphing Utilities Series

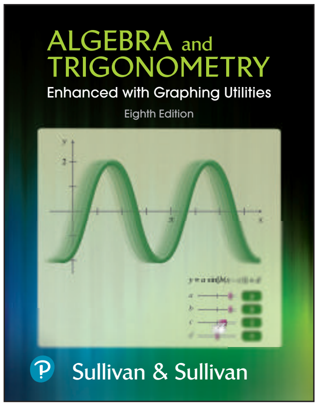
College Algebra, Eighth Edition

This text provides an approach to college algebra that completely integrates graphing technology without sacrificing mathematical analysis and conceptualization. The text has three chapters of review material preceding the chapter on functions. Graphing calculator usage is integrated throughout. After completing this text, a student will be prepared for trigonometry, finite mathematics, and business calculus.



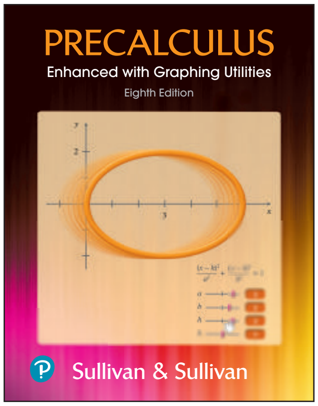
Algebra & Trigonometry, Eighth Edition

This text contains all the material in *College Algebra*, but it also develops the trigonometric functions using a right triangle approach and shows how that approach is related to the unit circle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Vectors in the plane, including the dot product, sequences, induction, and the binomial theorem are also presented. After completing this text, a student will be prepared for finite mathematics, business calculus, and engineering calculus.



Precalculus, Eighth Edition

This text contains a review chapter before covering the traditional precalculus topics of functions and their graphs, polynomial and rational functions, and exponential and logarithmic functions. The trigonometric functions are introduced using a unit circle approach and show how it is related to the right triangle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Vectors in the plane and in space, including the dot and cross products, sequences, induction, and the binomial theorem are also presented. Graphing calculator usage is integrated throughout. The final chapter provides an introduction to calculus, with a discussion of the limit, the derivative, and the integral of a function. After completing this text, a student will be prepared for finite mathematics, business calculus, and engineering calculus.



Preface to the Instructor

As professors at an urban university (Michael Sullivan) and a community college (Michael Sullivan III), we are aware of the varied needs of students in this course. Such students range from those who have little mathematical background and are fearful of mathematics courses to those with a strong mathematical education and a high level of motivation. For some of your students, this will be their last course in mathematics, whereas others will further their mathematical education. We have written this text with both groups in mind.

As a teacher, and as an author of precalculus, engineering calculus, finite mathematics, and business calculus texts, Michael Sullivan understands what students must know if they are to be focused and successful in upper-level math courses. As an instructor and an author of a developmental mathematics series, Michael's son and co-author, Michael Sullivan III, understands the trepidations and skills that students bring to the Precalculus course. As the father of current college students, Michael III realizes that today's college students demand a variety of media to support their education. This text addresses that demand by providing technology and video support that enhances understanding without sacrificing math skills. Together, we have taken great pains to ensure that the text offers solid, student-friendly examples and problems, as well as a clear and seamless writing style.

A tremendous benefit of authoring a successful series is the broad-based feedback we receive from teachers and students. We are sincerely grateful for their support. Virtually every change in this edition is the result of their thoughtful comments and suggestions. We are confident that, building on the success of the first seven editions and incorporating many of these suggestions, we have made *Precalculus Enhanced with Graphing Utilities*, 8th Edition, an even better tool for learning and teaching. We continue to encourage you to share with us your experiences teaching from this text.

Features in the Eighth Edition

A descriptive list of the many special features of *Precalculus* can be found in the front of this text. This list places the features in their proper context as building blocks of an overall learning system that has been carefully crafted over the years to help students get the most out of the time they put into studying. Please take the time to review this and to discuss it with your students at the beginning of your course. Our experience is that when students utilize these features, they are more successful in the course.

New to the Eighth Edition

New Within the Textbook

All of the exercises and examples in the text have been reviewed and analyzed, and we have incorporated feedback from users of the text. All time-sensitive problems have been updated to the most recent information available. Here are the new features of this edition:

- **Challenge Problems** – These problems appear in the Applications and Extensions part of the section exercises and are designed to challenge students. Full solutions are in the back of the Annotated Instructor's Edition and in the Instructor's Solution Manual.
- **"Need to Review?" feature** – We placed reminders in the margin for key review topics. The reminders point students to the location of the review material in the textbook.
- **Chapter Projects** – The projects have been enhanced to give students an up-to-the-minute experience. Many of these projects require the student to research information online in order to solve problems.
- **Interactive Figure Exercises** – We have added this new category of exercises that require students to manipulate an interactive figure to solve. The interactive figures may be found at `bit.ly/2Mibga0` or in the Video and Resource Library of MyLab Math, and were created by author Michael Sullivan III in GeoGebra. These exercises are labeled with the icon .
- **Expanded! Retain Your Knowledge Problems** – These problems, which were new to the previous edition, are based on learning research, including a study of precalculus students at University of Louisville entitled "Spaced retrieval practice increases college students' short- and long-term retention of mathematics knowledge" (Hopkins et al, 2016). The Retain Your Knowledge problems were so well received that we have expanded them in this edition. Moreover, while the focus remains to help students maintain their skills, in most sections, problems were chosen that preview skills required to succeed in subsequent sections or in calculus (). All answers to Retain Your Knowledge problems are given in the back of the text and these problems are available in the prebuilt assignments in the Assignment Manager in MyLab Math.
- **Key to Exercise Types** – To help you navigate the features of the exercise sets, we've included a key at the bottom of the first page of each section's exercises.

Now Work 1. Modeling 1. Writing/Discussion
Calculus Preview Interactive Figure

- **Graphing Utility Screen Captures** – In several instances we have added Desmos screen captures along with the TI-84 Plus CE screen captures. These updated screen captures provide alternative ways of visualizing concepts and making connections between equations, data, and graphs in full color.

Content Changes

Chapter 1

- Section 1.1 has been reorganized to only include an introduction to graphing and graphing utilities.
- NEW Section 1.2 The Distance and Midpoint Formulas
- NEW Section 1.3 Example 5 Testing an Equation for Symmetry

Chapter 2

- NEW Section 2.1 Objective 1 Describe a Relation
  - NEW Example 1 Describing a Relation demonstrates using the Rule of Four to express a relation numerically, as a mapping, and graphically given a verbal description.
- NEW Section 2.2 Example 4 Energy Expended

Chapter 3

- Section 3.3 now introduces the concept of concavity for a quadratic function.
- NEW Section 3.3 Example 3 Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts
- Section 3.3 Example 8 Analyzing the Motion of a Projectile (formerly in Section 3.4)
- NEW Section 3.4 Example 4 Fitting a Quadratic Function to Data

Chapter 4

- Previous Section 4.1 has been revised and split into two sections:
  - 4.1 Polynomial Functions
  - 4.2 Graphing Polynomial Functions; Models
- NEW Section 4.2 Example 2 Graphing a Polynomial Function (a 4th degree polynomial function)

Chapter 5

- NEW Section 5.2 Objective Verify a Function Defined by an Equation is an Inverse Function

Chapter 6

- NEW Section 6.1 Example 6 Field Width of a Digital Lens Reflex Camera Lens
- Sections 6.4 and 6.5 were reorganized for increased clarity. Two new objectives were added to Section 6.5.

Chapter 7

- Sections 7.1 and 7.2 were reorganized for increased clarity. Four new objectives were added to Section 7.1. The objectives in Section 7.2 were reordered.

Chapter 9

- Section 9.3 DeMoivre's Theorem was rewritten to support the exponential form of a complex number.
  - Euler's Formula is introduced to express a complex number in exponential form. The exponential form is used to compute products and quotients.
  - DeMoivre's Theorem is expressed using the exponential form of a complex number. The exponential form is used to find complex roots.

Chapter 11

- NEW Section 11.5 Example 1 Identifying Proper and Improper Rational Expressions

Chapter 12

- NEW Section 12.3 Objective 5 Solving Annuity Problems Using Formulas

Appendix A

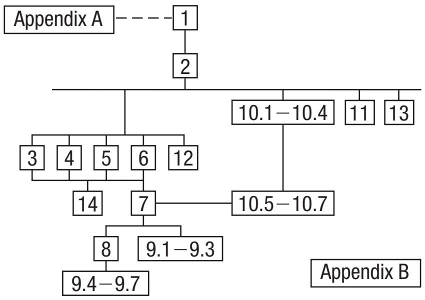
- Section A.10 Objective 3 now includes rationalizing the numerator. Problems 69–76 provide practice.

New Within MyLab Math

- **Setup & Solve Exercises** require students to show how they set up a problem as well as the solution, better mirroring what is required of them on tests. We have included both the "traditional" and Setup & Solve versions of exercise within MyLab to provide you with more options for assessing students.
- **Integrated Review** content and assessments help you provide students with the remediation they need, when they need it. Integrated Review consists of:
  - **Skills Check Quizzes** by chapter assess the prerequisite skills students need for that chapter.
  - **Skills Review Homework**, again by chapter, is personalized (based on the results of the Skills Check Quiz) to provide students with help on the prerequisite skills they are lacking. Students receive just the help they need—no more, no less.
  - **Intermediate Algebra eText, Exercises, Videos, and Worksheets**—For students who need more help (or for co-requisite courses), we've included the contents of a streamlined Intermediate Algebra course within this MyLab course. There's no need to go elsewhere for remediation.
- **Interactive Figures** (formerly titled Guided Visualizations) have been expanded to support teaching and learning. The figures (created in GeoGebra by author Michael Sullivan III) illustrate key concepts and allow manipulation. They have been designed to be used in lecture as well as by students independently.
- **Enhanced Sample Assignments** are pre-made section-level assignments that address key concepts within the section and help keep previously learned skills fresh with Retain Your Knowledge questions. They are assignable and editable.

Using the Eighth Edition Effectively with Your Syllabus

To meet the varied needs of diverse syllabi, this text contains more content than is likely to be covered in a Precalculus course. As the chart illustrates, this text has been organized with flexibility of use in mind. Within a given chapter, certain sections are optional (see the details that follow the accompanying figure) and can be omitted without loss of continuity.



Chapter 1 Graphs

A quick coverage of this chapter, which is mainly review material, will enable you to get to Chapter 2, "Functions and Their Graphs," earlier.

Chapter 2 Functions and Their Graphs

This is perhaps the most important chapter. Section 2.6 is optional.

Chapter 3 Linear and Quadratic Functions

Topic selection depends on your syllabus. Sections 3.2 and 3.4 may be omitted without loss of continuity.

Chapter 4 Polynomial and Rational Functions

Topic selection depends on your syllabus.

Chapter 5 Exponential and Logarithmic Functions

Sections 5.1–5.6 follow in sequence. Sections 5.7, 5.8, and 5.9 are optional.

Chapter 6 Trigonometric Functions

Section 6.6 may be omitted in a brief course.

Chapter 7 Analytic Trigonometry

Sections 7.2 and 7.7 may be omitted in a brief course.

Chapter 8 Applications of Trigonometric Functions

Sections 8.4 and 8.5 may be omitted in a brief course.

Chapter 9 Polar Coordinates; Vectors

Sections 9.1–9.3 and Sections 9.4–9.7 are independent and may be covered separately.

Chapter 10 Analytic Geometry

Sections 10.1–10.4 follow in sequence. Sections 10.5, 10.6, and 10.7 are independent of each other, but each requires Sections 10.1–10.4.

Chapter 11 Systems of Equations and Inequalities

Sections 11.2–11.7 may be covered in any order, but each requires Section 11.1. Section 11.8 requires Section 11.7.

Chapter 12 Sequences; Induction; The Binomial Theorem

There are three independent parts: Sections 12.1–12.3, Section 12.4, and Section 12.5.

Chapter 13 Counting and Probability

The sections follow in sequence.

Chapter 14 A Preview of Calculus: The Limit, Derivative, and Integral of a Function

If time permits, coverage of this chapter will provide your students with a beneficial head start in calculus. The sections follow in sequence.

Appendix A Review

This appendix consists of review material. It may be used as the first part of the course or later as a just-in-time review when the content is required. Specific references to this appendix occur throughout the text to assist in the review process.

Appendix B The Limit of a Sequence; Infinite Series

This section represents a more thorough treatment of sequences and series.

Acknowledgments

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- Accuracy checkers: Roger Lipsett read the entire manuscript and checked the accuracy of answers. Timothy Britt created the Solutions Manuals and accuracy-checked answers.
- Michael Sullivan III would like to thank his colleagues at Joliet Junior College for their support and feedback.

Finally, we offer our sincere thanks to the dedicated users and reviewers of our texts, whose collective insights form the backbone of each text revision.

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Paul Crittenden, University of Nebraska at Lincoln
John Davenport, East Texas State University
Faye Dang, Joliet Junior College
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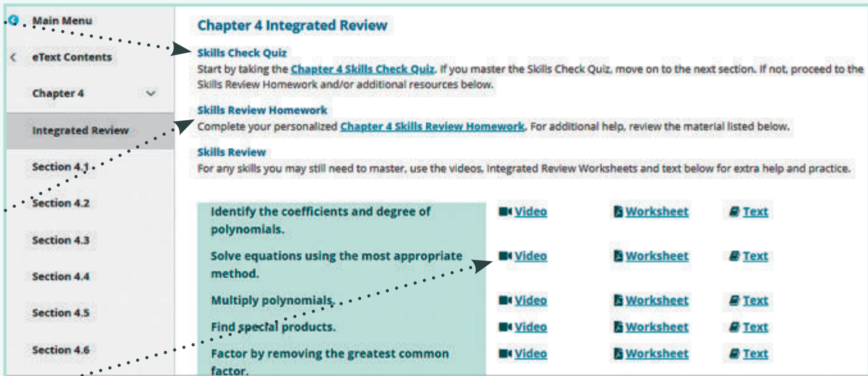

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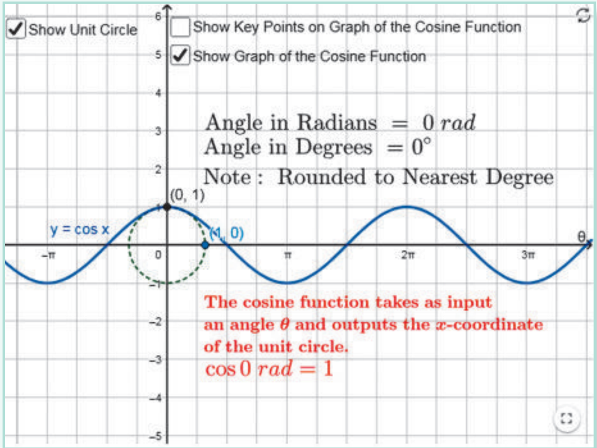

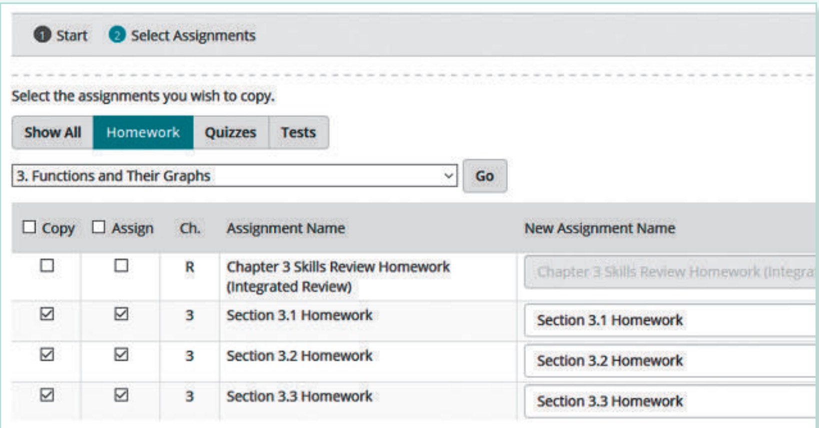

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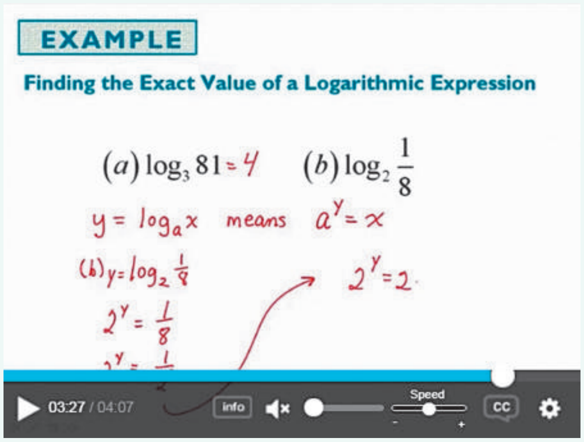

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Applications Index

To the Student

As you begin, you may feel anxious about the number of theorems, definitions, procedures, and equations you encounter. You may wonder if you can learn it all in time. Don't worry, your concerns are normal. This text was written with you in mind. If you attend class, work hard, and read and study effectively, you will build the knowledge and skills you need to be successful. Here's how you can use the text to your benefit.

Read Carefully

When you get busy, it's easy to skip reading and go right to the problems. Don't! The text provides a large number of examples and clear explanations to help you break down the mathematics into easy-to-understand steps. Reading will provide you with a clearer understanding, beyond simple memorization. Read before class (not after) so you can ask questions about anything you didn't understand. You'll be amazed at how much more you'll get out of class when you do this.

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We use many different methods in the classroom to communicate. Those methods, when incorporated into the text, are called "features." The features serve many purposes, from supplying a timely review of material you learned before (just when you need it), to providing organized review sessions to help you prepare for quizzes and tests. Take advantage of the features and you will master the material.

To make this easier, we've provided a brief guide to getting the most from this book. Refer to the "Prepare for Class," "Practice," and "Review" guidelines on the first three pages of this book. Spend fifteen minutes reviewing the guide and familiarizing yourself with the features by flipping to the page numbers provided. Then, as you read, use them. This is the best way to make the most of your text. In this edition, we've also added a handy key to the labeling of the homework exercises so that you know what the colors and icons mean:

Now Work | 1. Modeling | 1. Writing/Discussion | Calculus Preview | Interactive Figure

Please do not hesitate to contact us via Math@Pearson.com with any questions, comments, or suggestions about ways to improve this text. We look forward to hearing from you, and good luck with all of your studies.

Best Wishes!

Michael Sullivan
Michael Sullivan III

Graphs

1

How to Value a House

Two things to consider in valuing a home: (1) How does it compare to similar nearby homes that have sold recently? (2) What value do you place on the advertised features and amenities?

The Zestimate® home value is a good starting point in figuring out the value of a home. It shows you how the home compares relative to others in the area, but you then need to add in all the other qualities that only someone who has seen the house knows.

Looking at "Comps"

Knowing whether an asking price is fair will be important when you're ready to make an offer on a house. It will be even more important when your mortgage lender hires an appraiser to determine whether the house is worth the loan you're after.

Check on Zillow to see recent sales of similar, or comparable, homes in the area. Print them out and keep these "comps." You'll be referring to them quite a bit.

Note that "recent sales" usually means within the past six months. A sales price from a year ago probably bears little or no relation to what is going on in your area right now. In fact, some lenders will not accept comps older than three months.

Market activity also determines how easy or difficult it is to find accurate comps. In a "hot" or busy market, you're likely to have lots of comps to choose from. In a less active market finding reasonable comps becomes harder. And if the home you're looking at has special design features, finding a comparable property is harder still. It's also necessary to know what's going on in a given sub-segment. Maybe large, high-end homes are selling like hotcakes, but owners of smaller houses are staying put, or vice versa.

Source: *http://luthersanchez.com/2016/03/09/how-to-value-a-house/*

—See the Internet-based Chapter Project—

A Look Back

Appendix A reviews algebra essentials, geometry essentials, and equations in one variable.

A Look Ahead

Here we connect algebra and geometry using the rectangular coordinate system. In the 1600s, algebra had developed sufficiently so that René Descartes (1596–1650) and Pierre de Fermat (1601–1665) were able to use rectangular coordinates to translate geometry problems into algebra problems, and vice versa. This enabled both geometers and algebraists to gain new insights into their subjects, which had been thought to be separate but now were seen as connected.

Outline

1.1 Graphing Utilities; Introduction to Graphing Equations

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Algebra Essentials (Section A.1, pp. A1–A10)

Now Work the **'Are You Prepared?'** problems on page 10.

OBJECTIVES
1. Graph Equations by Plotting Points (p. 4)
2. Graph Equations Using a Graphing Utility (p. 6)
3. Use a Graphing Utility to Create Tables (p. 8)
4. Find Intercepts from a Graph (p. 9)
5. Use a Graphing Utility to Approximate Intercepts (p. 9)

Rectangular Coordinates

We locate a point on the real number line by assigning it a single real number, called the *coordinate of the point*. For work in a two-dimensional plane, we locate points by using two numbers.

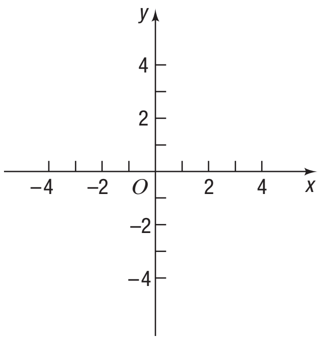


Figure 1 *xy*-Plane

Begin with two real number lines located in the same plane: one horizontal and the other vertical. The horizontal line is called the ***x*-axis**, the vertical line the ***y*-axis**, and the point of intersection the **origin *O***. See Figure 1. Assign coordinates to every point on these number lines using a convenient scale. In mathematics, we usually use the same scale on each axis, but in applications, different scales appropriate to the application may be used.

The origin O has a value of 0 on both the x-axis and the y-axis. Points on the x-axis to the right of O are associated with positive real numbers, and those to the left of O are associated with negative real numbers. Points on the y-axis above O are associated with positive real numbers, and those below O are associated with negative real numbers. In Figure 1, the x-axis and y-axis are labeled as x and y, respectively, and an arrow at the end of each axis is used to denote the positive direction.

The coordinate system described here is called a **rectangular** or **Cartesian*** **coordinate system**. The x-axis and y-axis lie in a *plane* called the ***xy*-plane**, and the x-axis and y-axis are referred to as the **coordinate axes**.

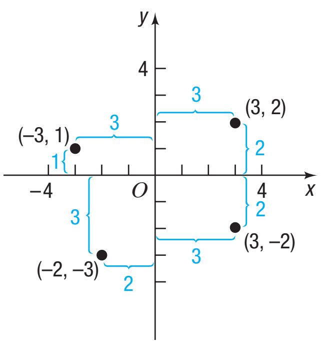


Figure 2

Any point P in the xy-plane can be located by using an **ordered pair** (x, y) of real numbers. Let x denote the signed distance of P from the y-axis (*signed* means that if P is to the right of the y-axis, then $x > 0$, and if P is to the left of the y-axis, then $x < 0$); and let y denote the signed distance of P from the x-axis. The ordered pair (x, y), also called the **coordinates** of P, gives us enough information to locate the point P in the plane.

For example, to locate the point whose coordinates are $(-3, 1)$, go 3 units along the x-axis to the left of O and then go straight up 1 unit. We **plot** this point by placing a dot at this location. See Figure 2, in which the points with coordinates $(-3, 1)$, $(-2, -3)$, $(3, -2)$, and $(3, 2)$ are plotted.

The origin has coordinates $(0, 0)$. Any point on the x-axis has coordinates of the form $(x, 0)$, and any point on the y-axis has coordinates of the form $(0, y)$.

If (x, y) are the coordinates of a point P, then x is called the ***x*-coordinate,** or **abscissa**, of P, and y is the ***y*-coordinate**, or **ordinate**, of P. We identify the point P by its coordinates (x, y) by writing $P = (x, y)$. Usually, we will simply say "the point (x, y)" rather than "the point whose coordinates are (x, y)."

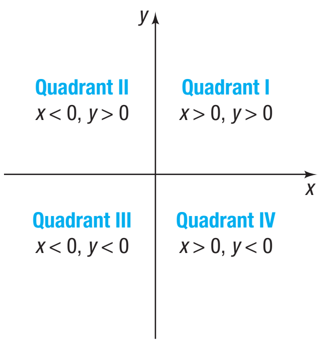


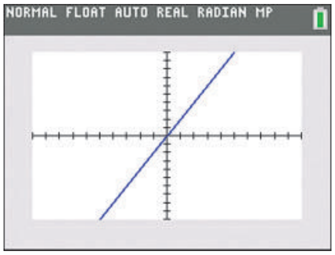
Figure 3

The coordinate axes partition the xy-plane into four sections called **quadrants**, as shown in Figure 3. In quadrant I, both the x-coordinate and the y-coordinate of all points are positive; in quadrant II, x is negative and y is positive; in quadrant III, both x and y are negative; and in quadrant IV, x is positive and y is negative. Points on the coordinate axes belong to no quadrant.

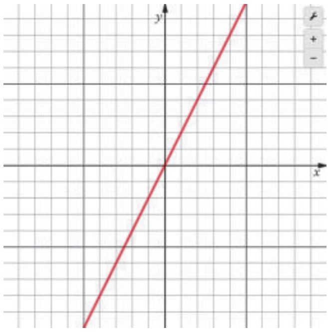
Now Work PROBLEM 7

*Named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian.

Graphing Utilities



(a) $y = 2x$ on a TI-84 Plus CE



(b) $y = 2x$ using Desmos

Figure 4

All graphing utilities (that is, all graphing calculators and all computer software graphing packages) graph equations by plotting points on a screen. The screen itself actually consists of small rectangles called **pixels**. The more pixels the screen has, the better the resolution. Most newer graphing calculators have at least 100 pixels per inch; most newer smartphones have at least 300 pixels per inch. When a point to be plotted lies inside a pixel, the pixel is turned on (lights up). The graph of an equation is a collection of pixels. Figure 4(a) shows how the graph of $y = 2x$ looks on a TI-84 Plus CE graphing calculator, and Figure 4(b) shows the same graph using Desmos.

The screen of a graphing utility displays the coordinate axes of a rectangular coordinate system. However, the scale must be set on each axis. The smallest and largest values of x and y to be included in the graph must also be set. This is called **setting the viewing rectangle** or **viewing window**. Figure 5 shows a typical viewing window on a TI-84 Plus CE.

To set the viewing window, values must be given to the following expressions:

Xmin:	the smallest value of x
Xmax:	the largest value of x
Xscl:	the number of units per tick mark on the x-axis
Ymin:	the smallest value of y
Ymax:	the largest value of y
Yscl:	the number of units per tick mark on the y-axis

Figure 6 illustrates these settings and their relation to the Cartesian coordinate system.

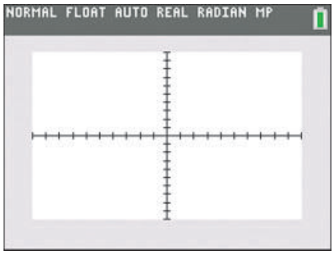


Figure 5 Viewing window on a TI-84 Plus CE

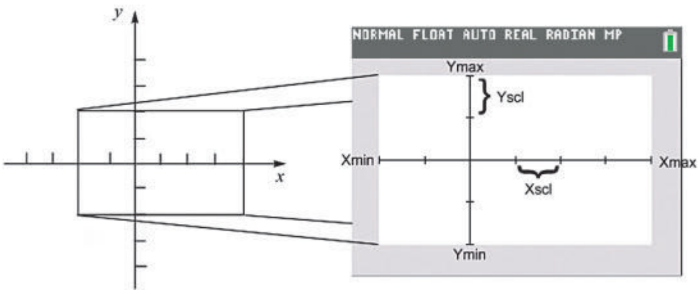


Figure 6

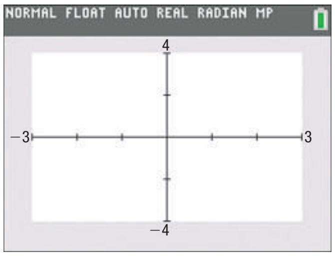


Figure 7

If the scale used on each axis is known, the minimum and maximum values of x and y shown on the screen can be determined by counting the tick marks. Look again at Figure 5. For a scale of 1 on each axis, the minimum and maximum values of x are -10 and 10, respectively; the minimum and maximum values of y are also -10 and 10. If the scale is 2 on each axis, then the minimum and maximum values of x are -20 and 20, respectively; and the minimum and maximum values of y are -20 and 20, respectively.

Conversely, if the minimum and maximum values of x and y are known, the scales can be determined by counting the tick marks displayed. This text follows the practice of showing the minimum and maximum values of x and y in illustrations so that the reader will know how the viewing window was set. See Figure 7. The numbers outside of the viewing window stand for

$$X\text{min} = -3, \quad X\text{max} = 3, \quad X\text{scl} = 1$$
$$Y\text{min} = -4, \quad Y\text{max} = 4, \quad Y\text{scl} = 2$$

Now Work PROBLEMS 11 AND 21

1 Graph Equations by Plotting Points

An **equation in two variables**, say x and y, is a statement in which two expressions involving x and y are equal. The expressions are called the **sides** of the equation. Since an equation is a statement, it may be true or false, depending on the values of the variables. Any values of x and y that result in a true statement are said to **satisfy** the equation.

For example, the following are all equations in two variables x and y:

$$x^2 + y^2 = 5 \qquad 2x - y = 6 \qquad y = 2x + 5 \qquad x^2 = y$$

The first of these, $x^2 + y^2 = 5$, is satisfied for $x = 1, y = 2$, since $1^2 + 2^2 = 5$. Other choices of x and y, such as $x = -1, y = -2$, also satisfy this equation. It is not satisfied for $x = 2$ and $y = 3$, since $2^2 + 3^2 = 4 + 9 = 13 \neq 5$.

The **graph of an equation in two variables** x and y consists of the set of points in the xy-plane whose coordinates (x, y) satisfy the equation.

Graphs play an important role in helping us to visualize the relationships that exist between two variables or quantities. Figure 8 shows the relation between the level of risk in a stock portfolio and the average annual rate of return. The graph shows that, when 30% of a portfolio of stocks is invested in foreign companies, risk is minimized.

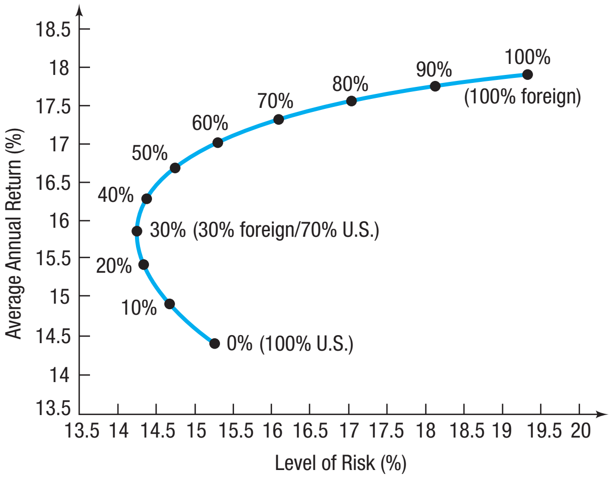


Figure 8

Source: *T. Rowe Price*

EXAMPLE 1 Determining Whether a Point Is on the Graph of an Equation

Determine whether each of the following points is on the graph of the equation $2x - y = 6$.

(a) $(2, 3)$ (b) $(2, -2)$

Solution

(a) For the point $(2, 3)$, check to see whether $x = 2, y = 3$ satisfies the equation $2x - y = 6$.

$$2x - y = 2 \cdot 2 - 3 = 4 - 3 = 1 \neq 6$$

The equation is not satisfied, so the point $(2, 3)$ is not on the graph of $2x - y = 6$.

(b) For the point $(2, -2)$,

$$2x - y = 2 \cdot 2 - (-2) = 4 + 2 = 6$$

The equation is satisfied, so the point $(2, -2)$ is on the graph of $2x - y = 6$. ■

Now Work PROBLEM 27

EXAMPLE 2 How to Graph an Equation by Plotting Points

Graph the equation: $y = -2x + 3$

Step-by-Step Solution

Step 1: Find some points (x, y) that satisfy the equation. To find these points, choose values of x and use the equation to find the corresponding values for y. See Table 1.

Table 1

x	$y = -2x + 3$	(x, y)
-2	$-2\cdot(-2) + 3 = 7$	$(-2, 7)$
-1	$-2\cdot(-1) + 3 = 5$	$(-1, 5)$
0	$-2\cdot 0 + 3 = 3$	$(0, 3)$
1	$-2\cdot 1 + 3 = 1$	$(1, 1)$
2	$-2\cdot 2 + 3 = -1$	$(2, -1)$

Step 2: Plot the points listed in the table as shown in Figure 9(a). Now connect the points to obtain the graph of the equation (a line), as shown in Figure 9(b).

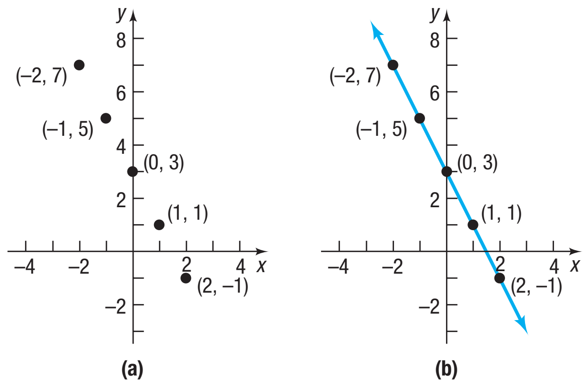


Figure 9 $y = -2x + 3$

EXAMPLE 3 Graphing an Equation by Plotting Points

Graph the equation: $y = x^2$

Solution

Table 2 provides several points on the graph. Plotting these points and connecting them with a smooth curve gives the graph (a *parabola*) shown in Figure 10.

Table 2

x	$y = x^2$	(x, y)
-4	16	$(-4, 16)$
-3	9	$(-3, 9)$
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$
3	9	$(3, 9)$
4	16	$(4, 16)$

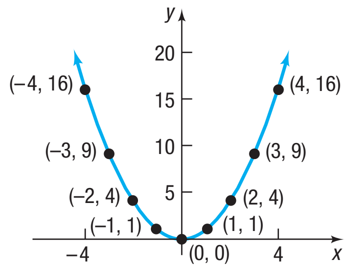


Figure 10 $y = x^2$

The graphs of the equations shown in Figures 9(b) and 10 do not show all the points that are on the graph. For example, in Figure 9(b), the point $(20, -37)$ is on the graph of $y = -2x + 3$, but it is not shown. Since the graph of $y = -2x + 3$ could be extended out as far as we please, we use arrows to indicate that the pattern shown continues. When constructing a graph, it is important to present enough of the graph so that any viewer of the illustration will "see" the rest of it as an obvious continuation of what is shown. This is referred to as a **complete graph**.

One way to obtain a complete graph of an equation is to continue plotting points on the graph until a pattern becomes evident. Then these points are connected with a smooth curve following the suggested pattern. But how many points are sufficient? Sometimes knowledge about the equation tells us. For example, we will learn in Section 1.5 that if an equation is of the form $y = mx + b$, then its graph is a line. In this case, only two points are needed to obtain the complete graph.

One purpose of this text is to investigate the properties of equations in order to decide whether a graph is complete. Sometimes we shall graph equations by plotting points on the graph until a pattern becomes evident and then connect these points with a smooth curve, following the suggested pattern. (Shortly, we shall investigate various techniques that will enable us to graph an equation without plotting so many points.) Other times we shall graph equations using a graphing utility.

2 Graph Equations Using a Graphing Utility

From Examples 2 and 3, we see that a graph can be obtained by plotting points in a rectangular coordinate system and connecting them. Graphing utilities perform these same steps when graphing an equation. For example, the TI-84 Plus CE determines 265 evenly spaced input values (starting at Xmin and ending at Xmax),* uses the equation to determine the output values, plots these points on the screen, and finally (if in the connected mode) draws a line between consecutive points.

To graph an equation in two variables x and y using a graphing utility often requires that the equation be written in the form $y = \{\text{expression in } x\}$. If the original equation is not in this form, rewrite it using equivalent equations until the form $y = \{\text{expression in } x\}$ is obtained.

Procedures That Result in Equivalent Equations

- Interchange the two sides of the equation:

$$3x + 5 = y \quad \text{is equivalent to} \quad y = 3x + 5$$

- Simplify the sides of the equation by combining like terms, eliminating parentheses, and so on:

$$2y + 2 + 6 = 2x + 5(x + 1) \quad \text{is equivalent to} \quad 2y + 8 = 7x + 5$$

- Add or subtract the same expression on both sides of the equation:

$$y + 3x - 5 = 4 \quad \text{is equivalent to} \quad y + 3x - 5 + 5 = 4 + 5$$

- Multiply or divide both sides of the equation by the same nonzero expression:

$$3y = 6 - 2x \quad \text{is equivalent to} \quad \frac{1}{3} \cdot 3y = \frac{1}{3}(6 - 2x)$$

EXAMPLE 4 Expressing an Equation in the Form $y = \{\text{expression in } x\}$

Solve for y: $2y + 3x - 5 = 4$

Solution Replace the original equation by a succession of equivalent equations.

$$2y + 3x - 5 = 4$$

$$2y + 3x - 5 + 5 = 4 + 5 \qquad \text{Add 5 to both sides.}$$

$$2y + 3x = 9 \qquad \text{Simplify.}$$

$$2y + 3x - 3x = 9 - 3x \qquad \text{Subtract } 3x \text{ from both sides.}$$

$$2y = 9 - 3x \qquad \text{Simplify.}$$

$$\frac{2y}{2} = \frac{9 - 3x}{2} \qquad \text{Divide both sides by 2.}$$

$$y = \frac{9 - 3x}{2} \qquad \text{Simplify.}$$

CAUTION Be careful when entering the expression $\frac{9 - 3x}{2}$. Use a fraction template $\frac{\square}{\square}$ or use parentheses as follows:

$(9 - 3x)/2$. ■

*These input values depend on the values of Xmin and Xmax. For example, if $X\text{min} = -10$ and $X\text{max} = 10$, then the first input value will be -10 and the next input value will be $-10 + (10 - (-10))/264 = -9.9242$, and so on.

We are now ready to graph equations using a graphing utility.

EXAMPLE 5 How to Graph an Equation Using a Graphing Utility

Use a graphing utility to graph the equation: $6x^2 + 3y = 36$

Step-by-Step Solution

Step 1: Solve the equation for y in terms of x.

$$6x^2 + 3y = 36$$

$$3y = -6x^2 + 36 \quad \text{Subtract } 6x^2 \text{ from both sides.}$$

$$y = -2x^2 + 12 \quad \text{Divide both sides by 3 and simplify.}$$

Step 2: Enter the equation to be graphed into your graphing utility. Figure 11 shows the equation to be graphed entered on a TI-84 Plus CE.

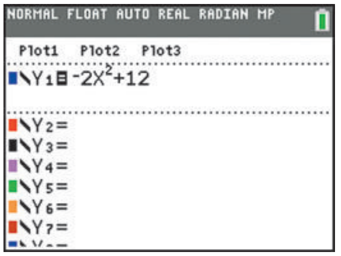


Figure 11

Step 3: Choose an initial viewing window. Without any knowledge about the behavior of the graph, it is common to choose the standard viewing window as the initial viewing window. The standard viewing window is

Xmin = −10 Ymin = −10
Xmax = 10 Ymax = 10
Xscl = 1 Yscl = 1

See Figure 12.

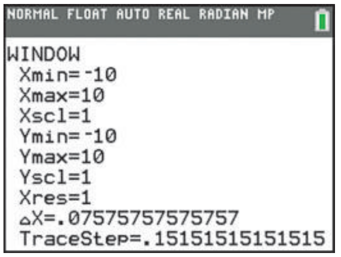


Figure 12 Standard viewing window

Step 4: Graph the equation. See Figure 13.

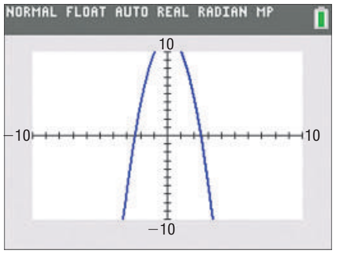


Figure 13 $Y_1 = -2x^2 + 12$

Step 5: Adjust the viewing window until a complete graph is obtained.

The graph of $y = -2x^2 + 12$ is not complete. The value of Ymax must be increased so that the top portion of the graph is visible. After increasing the value of Ymax to 12, we obtain the graph in Figure 14. The graph is now complete.

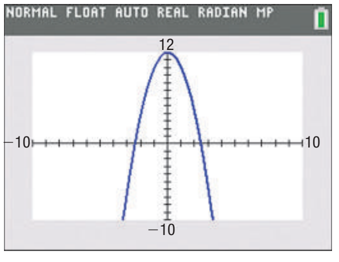


Figure 14 $Y_1 = -2x^2 + 12$

NOTE: Some graphing utilities have a ZOOM-STANDARD feature that automatically sets the viewing window to the standard viewing window. In addition, some graphing utilities have a ZOOM-FIT feature that determines the appropriate Ymin and Ymax for a given Xmin and Xmax. Consult your user's manual for the appropriate keystrokes. ■

Now Work PROBLEM 45

Look again at Figure 14 on the previous page. Although a complete graph is shown, the graph might be improved by adjusting the values of Xmin and Xmax. Figure 15 shows the graph of $y = -2x^2 + 12$ using $X\text{min} = -4$ and $X\text{max} = 4$.

Some graphing utilities do not require the equation be written in the form "$y = \{\text{expression in } x\}$." Figure 16 shows the graph of $6x^2 + 3y = 36$ using Desmos.

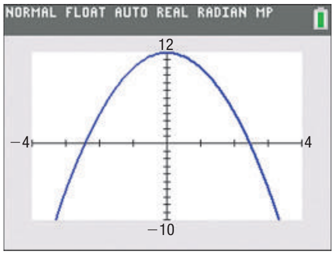


Figure 15 $Y_1 = -2x^2 + 12$

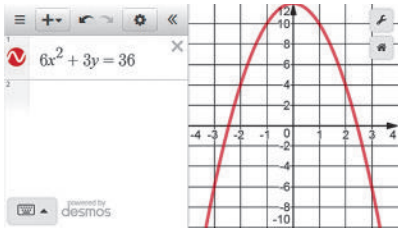


Figure 16 $6x^2 + 3y = 36$

3 Use a Graphing Utility to Create Tables

In addition to graphing equations, graphing utilities can also be used to create a table of values that satisfy the equation. This feature is especially useful in determining an appropriate viewing window when graphing an equation.

EXAMPLE 6 How to Create a Table Using a Graphing Utility

Create a table that displays the points on the graph of $6x^2 + 3y = 36$ for $x = -3, -2, -1, 0, 1, 2$, and 3.

Step-by-Step Solution

Step 1: Solve the equation for y in terms of x.

We solved the equation for y in terms of x in Example 5 and obtained $y = -2x^2 + 12$.

Step 2: Enter the expression in x following the "$Y =$" prompt of the graphing utility.

See Figure 11 on the previous page.

Step 3: Set up the table. Graphing utilities typically have two modes for creating tables. In the AUTO mode, the user determines a starting point for the table (TblStart) and ΔTbl (pronounced "delta table"). The ΔTbl feature determines the increment for x in the table. The ASK mode requires the user to enter values of x, and then the utility determines the corresponding value of y.

Create a table using AUTO mode. The table we wish to create starts at -3, so TblStart $= -3$. The increment for x is 1, so ΔTbl $= 1$. See Figure 17.

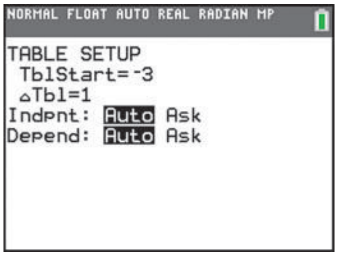
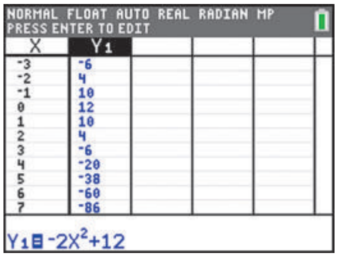


Figure 17 Table setup

Step 4: Create the table. See Table 3.

Table 3



X	Y1
-3	-6
-2	4
-1	10
0	12
1	10
2	4
3	-6
4	-20
5	-38
6	-60
7	-86

In AUTO mode, the user can scroll forward or backward within the table to find additional values.

In looking at Table 3 on the previous page, notice that $y = 12$ when $x = 0$. This information could have been used to help create the initial viewing window by letting us know that Ymax needs to be at least 12 in order to get a complete graph.

4 Find Intercepts from a Graph

The points, if any, at which a graph crosses or touches the coordinate axes are called the **intercepts**. See Figure 18. The x-coordinate of a point at which the graph crosses or touches the x-axis is an ***x*-intercept**, and the y-coordinate of a point at which the graph crosses or touches the y-axis is a ***y*-intercept**. For a graph to be complete, all its intercepts must be displayed.

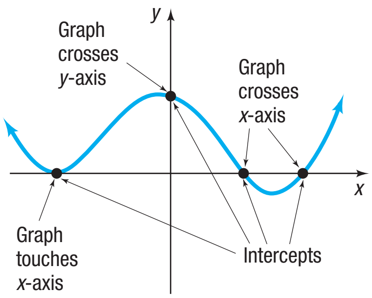


Figure 18 Intercepts

EXAMPLE 7 Finding Intercepts from a Graph

Find the intercepts of the graph in Figure 19. What are its x-intercepts? What are its y-intercepts?

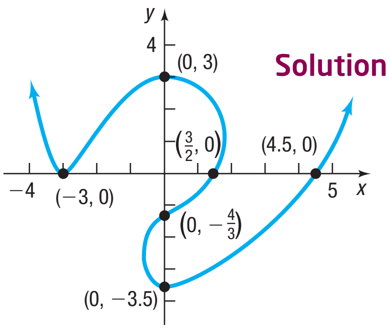


Figure 19

Solution The intercepts of the graph are the points

$$(-3, 0), \quad (0, 3), \quad \left(\frac{3}{2}, 0\right), \quad \left(0, -\frac{4}{3}\right), \quad (0, -3.5), \quad (4.5, 0)$$

The x-intercepts are $-3, \frac{3}{2}$, and 4.5; the y-intercepts are $-3.5, -\frac{4}{3}$, and 3.

In Example 7 notice the following usage: If the type of intercept is not specified (x- versus y-), then report the intercept as an ordered pair. However, if the type of intercept is specified, then report the coordinate of the specified intercept. For x-intercepts, report the x-coordinate of the intercept; for y-intercepts, report the y-coordinate of the intercept.

In Words
Intercepts are points (ordered pairs). An x-intercept or a y-intercept is a number. For example, the point $(3, 0)$ is an intercept; the number 3 is an x-intercept.

Now Work PROBLEM 37

5 Use a Graphing Utility to Approximate Intercepts

We can use a graphing utility to approximate the intercepts of the graph of an equation.

EXAMPLE 8 Approximating Intercepts Using a Graphing Utility

Use a graphing utility to approximate the intercepts of the equation $y = x^3 - 16$.

Solution On the next page, Figure 20(a) shows the graph of $y = x^3 - 16$.

The eVALUEate feature of a TI-84 Plus CE graphing calculator accepts as input a value for x and determines the value of y. If we let $x = 0$, the y-intercept is found to be -16. See Figure 20(b) on the next page. *(continued)*

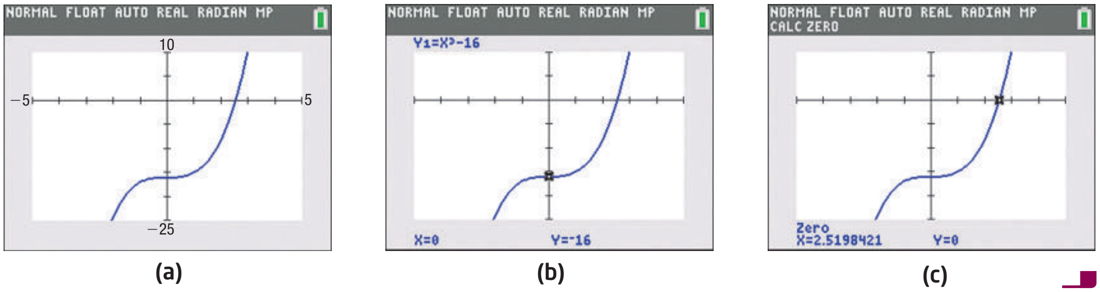


Figure 20

The ZERO feature of a TI-84 Plus CE is used to find the x-intercept(s). See Figure 20(c). Rounded to two decimal places, the x-intercept is 2.52.

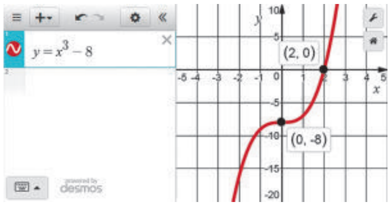


Figure 21 $y = x^3 - 8$

Now Work PROBLEM 55

Some graphing utilities automatically identify key points such as intercepts and intersection points. For example, Figure 21 shows the graph of $y = x^3 - 8$ using Desmos where the intercepts are already identified.

To find the intercepts algebraically requires the ability to solve equations, the subject of the following four sections.

1.1 Assess Your Understanding

'Are You Prepared?'

Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

1. On a real number line the origin is assigned the number ______. (p. A4)

2. On a real number line, graph all numbers x for which $x < 5$. (pp. A4–A5)

Concepts and Vocabulary

3. If (x, y) are the coordinates of a point P in the xy-plane, then x is called the ____________ of P and y is the ____________ of P. The coordinate axes divide the xy-plane into four sections called ____________.

4. ***True or False*** The point $(-1, 4)$ lies in quadrant IV of the Cartesian plane.

5. ***Multiple Choice*** Given that the intercepts of a graph are $(-4, 0)$ and $(0, 5)$, choose the statement that is true.

(a) The y-intercept is -4 and the x-intercept is 5.
(b) The y-intercepts are -4 and 5.
(c) The x-intercepts are -4 and 5.
(d) The x-intercept is -4 and the y-intercept is 5.

6. ***Multiple Choice*** Which of the following points does not satisfy the equation $2x^2 - 5y = 20$?

(a) $(0, -4)$
(b) $(5, 6)$
(c) $(-5, -14)$
(d) $(10, 36)$

Skill Building

In Problems 7 and 8, plot each point in the xy-plane. Tell in which quadrant or on what coordinate axis each point lies.

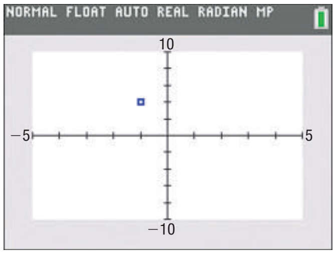
7. **(a)** $A = (-3, 2)$ **(b)** $B = (6, 0)$ **(c)** $C = (-2, -2)$ **(d)** $D = (6, 5)$ **(e)** $E = (0, -3)$ **(f)** $F = (6, -3)$

8. **(a)** $A = (1, 4)$ **(b)** $B = (-3, -4)$ **(c)** $C = (-3, 4)$ **(d)** $D = (4, 1)$ **(e)** $E = (0, 1)$ **(f)** $F = (-3, 0)$

9. Plot the points $(2, 0)$, $(2, -3)$, $(2, 4)$, $(2, 1)$, and $(2, -1)$. Describe the set of all points of the form $(2, y)$, where y is a real number.

10. Plot the points $(0, 3)$, $(1, 3)$, $(-2, 3)$, $(5, 3)$, and $(-4, 3)$. Describe the set of all points of the form $(x, 3)$, where x is a real number.

In Problems 11–14, determine the coordinates of the points shown. Tell in which quadrant each point lies. Assume the coordinates are integers.

11.


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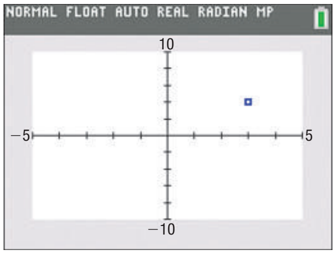

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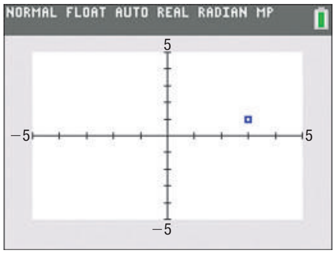

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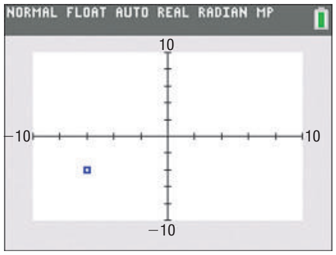


In Problems 15–20, select a setting so that each given point will lie within the viewing window.

15. $(-10, 5), (3, -2), (4, -1)$

16. $(5, 0), (6, 8), (-2, -3)$

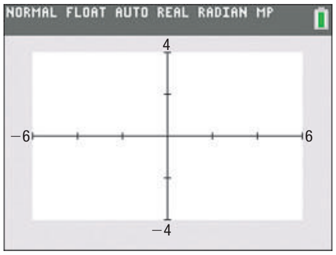
17. $(40, 20), (-20, -80), (10, 40)$

18. $(-80, 60), (20, -30), (-20, -40)$

19. $(0, 0), (100, 5), (5, 150)$

20. $(0, -1), (100, 50), (-10, 30)$

In Problems 21–26, determine the viewing window used.

21.


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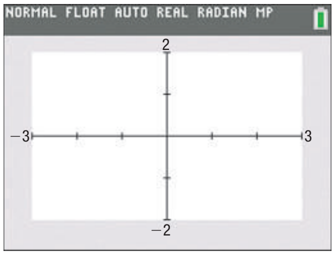

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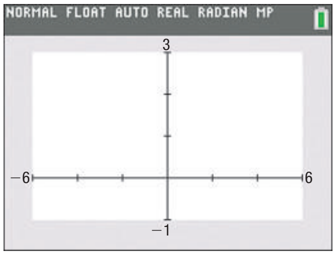

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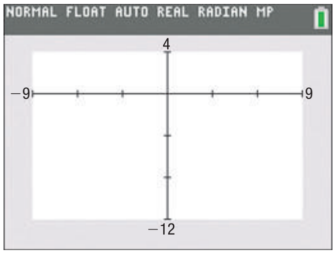
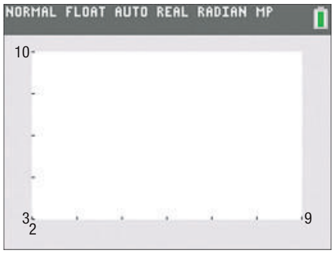


25.


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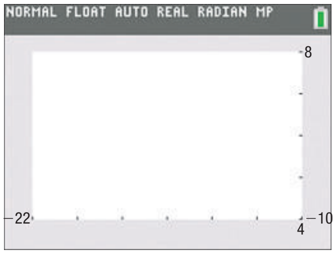


In Problems 27–32, tell whether the given points are on the graph of the equation.

27. Equation: $y = x^4 - \sqrt{x}$
Points: $(0, 0); (1, 1); (-1, 0)$

28. Equation: $y = x^3 - 2\sqrt{x}$
Points: $(0, 0); (1, 1); (1, -1)$

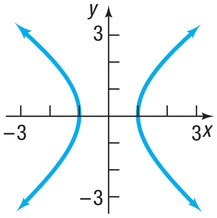
29. Equation: $y^2 = x^2 + 9$
Points: $(0, 3); (3, 0); (-3, 0)$

30. Equation: $y^3 = x + 1$
Points: $(1, 2); (0, 1); (-1, 0)$

31. Equation: $x^2 + y^2 = 4$
Points: $(0, 2); (-2, 2); (\sqrt{2}, \sqrt{2})$

32. Equation: $x^2 + 4y^2 = 4$
Points: $(0, 1); (2, 0); \left(2, \frac{1}{2}\right)$

In Problems 33–40, the graph of an equation is given. List the intercepts of the graph.

33.


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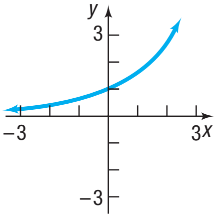

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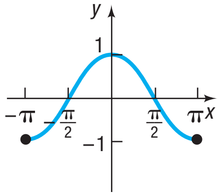
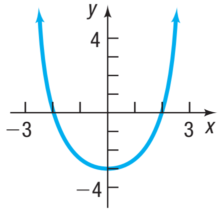


36.


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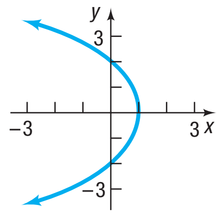

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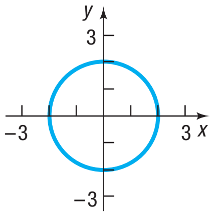

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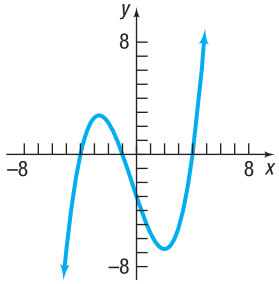

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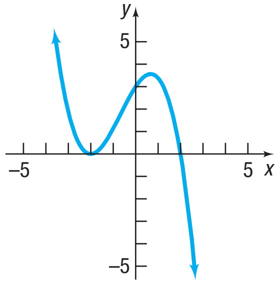


In Problems 41–52, graph each equation by plotting points. Verify your results using a graphing utility.

41. $y = x + 2$ **42.** $y = x - 6$ **43.** $y = 2x + 8$ **44.** $y = 3x - 9$ **45.** $y = x^2 - 1$ **46.** $y = x^2 - 9$

47. $y = -x^2 + 4$ **48.** $y = -x^2 + 1$ **49.** $2x + 3y = 6$ **50.** $5x + 2y = 10$ **51.** $9x^2 + 4y = 36$ **52.** $4x^2 + y = 4$

In Problems 53–60, graph each equation using a graphing utility. Use a graphing utility to approximate the intercepts rounded to two decimal places. Use the TABLE feature to help to establish the viewing window.

53. $y = 2x - 13$ **54.** $y = -3x + 14$ **55.** $y = 2x^2 - 15$ **56.** $y = -3x^2 + 19$

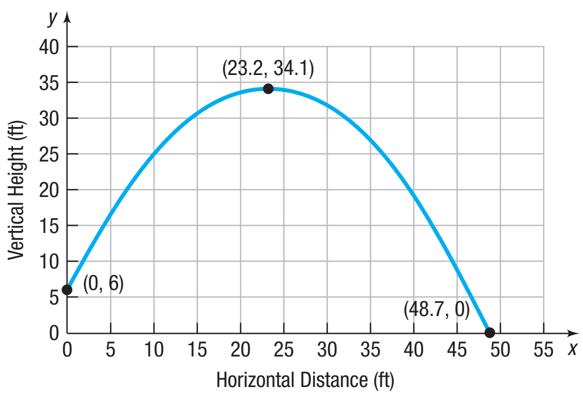
57. $3x - 2y = 43$ **58.** $4x + 5y = 82$ **59.** $5x^2 + 3y = 37$ **60.** $2x^2 - 3y = 35$

61. If the point $(2, 5)$ is shifted 3 units right and 2 units down, what are its new coordinates?

62. If the point $(-1, 6)$ is shifted 2 units left and 4 units up, what are its new coordinates?

Applications and Extensions

63. Shot-put Throw The graph below shows the height y, in feet, of a shot (metal ball) thrown by a shot-putter after it has traveled x feet horizontally.

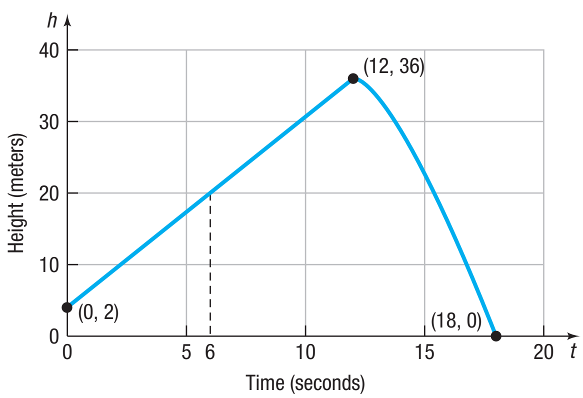


(a) What is the height of the shot after it has traveled 10 feet horizontally?

(b) How far has the shot traveled when its height is at a maximum? What is the maximum height?

(c) Identify and interpret the intercepts.

64. Discus Throw The graph below shows the height h, in meters, of a discus t seconds after it is thrown.

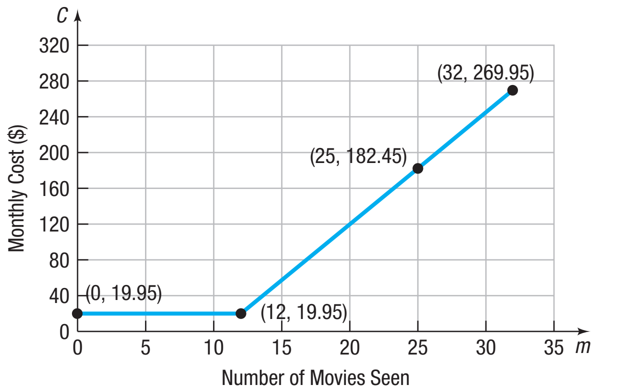


(a) What is the height of the discus after 6 seconds?

(b) When does the discus reach its maximum height? What is the maximum height?

(c) Identify and interpret the intercepts.

65. Movie Membership A movie theater offers a monthly membership for avid movie lovers. The graph below shows the relation between the monthly cost C and the number of movies seen m.

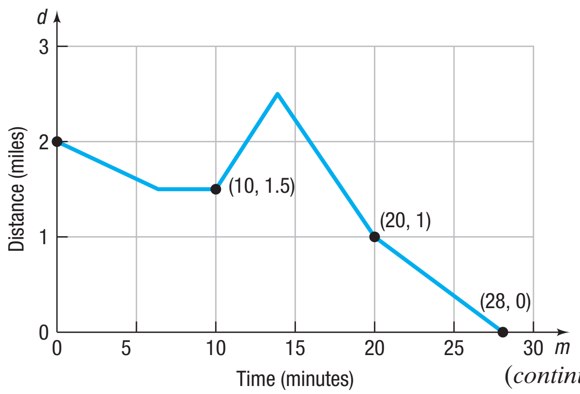


(a) What is the cost of watching 5 movies in a month? 10 movies?

(b) What is the cost of watching 25 movies in a month?

(c) Identify and interpret the intercept.

66. Bicycle Motion Caleb rides home from his friend's house on his bicycle. The graph below shows his distance d from his house after m minutes.



(continued)

(a) How far was Caleb from home after 10 minutes?
(b) How far was Caleb from home after 20 minutes?
(c) Identify and interpret the intercepts.

67. ***Challenge Problem*** Use a graphing utility that does not require the equation to be written in the form $y = \{\text{expression in } x\}$ to find the intercepts of the graph of

$$(x^2 + y^2 - 1)^3 - x^2y^3 = 0$$

68. ***Challenge Problem*** Use a graphing utility that does not require the equation to be written in the form $y = \{\text{expression in } x\}$ to find the intercepts of the graph of

$$y^2(y^2 - 4) = x^2(x^2 - x - 6)$$

Explaining Concepts: Discussion and Writing

69. Make up an equation satisfied by the ordered pairs $(2, 0)$, $(4, 0)$, and $(0, 1)$. Compare your equation with a friend's equation. Comment on any similarities.

In Problem 70, you may use a graphing utility, but it is not required.

70. **(a)** Graph $y = \sqrt{x^2}$, $y = x$, $y = |x|$, and $y = (\sqrt{x})^2$, noting which graphs are the same.
(b) Explain why the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.
(c) Explain why the graphs of $y = x$ and $y = (\sqrt{x})^2$ are not the same.
(d) Explain why the graphs of $y = \sqrt{x^2}$ and $y = x$ are not the same.

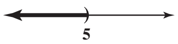
71. Draw a graph that contains the points $(-2, -1)$, $(0, 1)$, $(1, 3)$, and $(3, 5)$. Compare your graph with those of other students. Are most of the graphs almost straight lines? How many are "curved"? Discuss the various ways that these points might be connected.

72. Explain what is meant by a complete graph.

73. Write a paragraph that describes a Cartesian plane. Then write a second paragraph that describes how to plot points in the Cartesian plane. Your paragraphs should include the terms "coordinate axes," "ordered pair," "coordinates," "plot," "x-coordinate," and "y-coordinate."

'Are You Prepared?' Answers

1. 0

2. 

1.2 The Distance and Midpoint Formulas

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Algebra Essentials (Section A.1, pp. A1–A10)
- Rectangular Coordinates (Section 1.1, p. 2)
- Geometry Essentials (Section A.2, pp. A14–A19)

Now Work the **'Are You Prepared?'** problems on page 17.

OBJECTIVES
1 Use the Distance Formula (p. 13)
2 Use the Midpoint Formula (p. 16)

1 Use the Distance Formula

If the same units of measurement (such as inches, centimeters, and so on) are used for both the x-axis and y-axis, then all distances in the xy-plane can be measured using this unit of measurement.

EXAMPLE 1 **Finding the Distance between Two Points**

Find the distance d between the points $(1, 3)$ and $(5, 6)$.

Solution First plot the points $(1, 3)$ and $(5, 6)$ and connect them with a line segment. See Figure 22(a) on the next page. To find the length d, begin by drawing a horizontal line segment from $(1, 3)$ to $(5, 3)$ and a vertical line segment from $(5, 3)$ to $(5, 6)$, forming a right triangle, as shown in Figure 22(b) on the next page. One leg of the

(continued)

Need to Review?
The Pythagorean Theorem and its converse are discussed in Section A.2, pp. A14–A15.

triangle is of length 4 (since $|5 - 1| = 4$), and the other is of length 3 (since $|6 - 3| = 3$). By the Pythagorean Theorem, the square of the distance d that we seek is

$$d^2 = 4^2 + 3^2 = 16 + 9 = 25$$
$$d = \sqrt{25} = 5$$

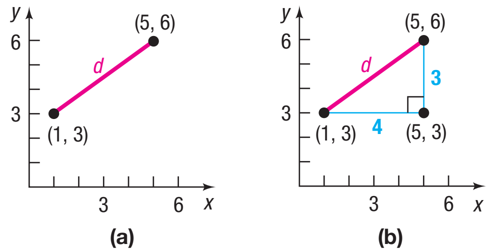


Figure 22

The **distance formula** provides a straightforward method for computing the distance between two points.

THEOREM Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \textbf{(1)}$$

In Words
To compute the distance between two points, find the difference of the x-coordinates, square it, and add this to the square of the difference of the y-coordinates. The square root of this sum is the distance.

Proof of the Distance Formula Let (x_1, y_1) denote the coordinates of point P_1 and let (x_2, y_2) denote the coordinates of point P_2.

- Assume that the line joining P_1 and P_2 is neither horizontal nor vertical. Refer to Figure 23(a). The coordinates of P_3 are (x_2, y_1). The horizontal distance from P_1 to P_3 equals the absolute value of the difference of the x-coordinates, $|x_2 - x_1|$. The vertical distance from P_3 to P_2 equals the absolute value of the difference of the y-coordinates, $|y_2 - y_1|$. See Figure 23(b). The distance $d(P_1, P_2)$ is the length of the hypotenuse of the right triangle, so, by the Pythagorean Theorem, it follows that

$$[d(P_1, P_2)]^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

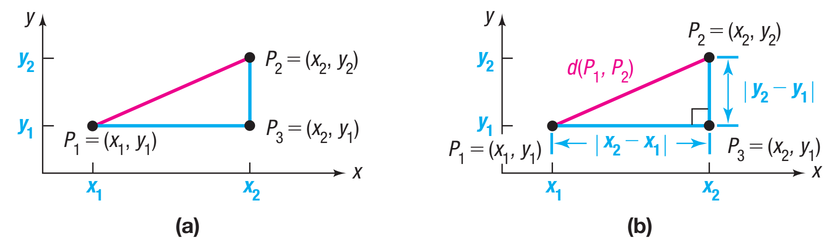


Figure 23

- If the line joining P_1 and P_2 is horizontal, then the y-coordinate of P_1 equals the y-coordinate of P_2; that is, $y_1 = y_2$. Refer to Figure 24(a) on the next page. In this case, the distance formula (1) still works, because for $y_1 = y_2$, it reduces to

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + 0^2} = \sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$$

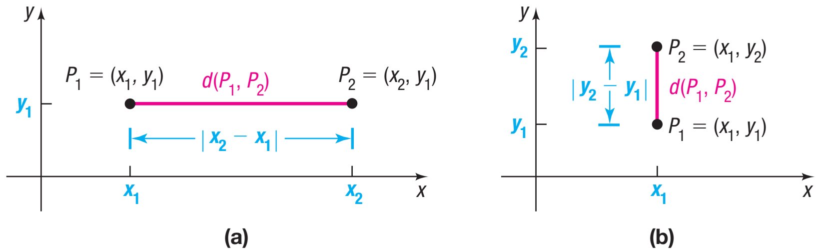


Figure 24

- A similar argument holds if the line joining P_1 and P_2 is vertical. See Figure 24(b). ■

EXAMPLE 2 Finding the Length of a Line Segment

Find the length of the line segment shown in Figure 25.

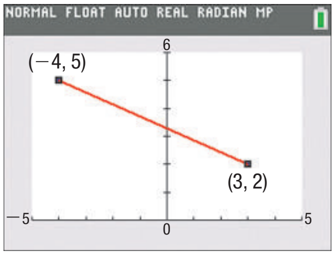


Figure 25

Solution The length of the line segment is the distance between the points $P_1 = (x_1, y_1) = (-4, 5)$ and $P_2 = (x_2, y_2) = (3, 2)$. Using the distance formula (1) with $x_1 = -4, y_1 = 5, x_2 = 3$, and $y_2 = 2$, the length d is

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[3 - (-4)]^2 + (2 - 5)^2} \\ &= \sqrt{7^2 + (-3)^2} = \sqrt{49 + 9} = \sqrt{58} \quad \text{Exact} \\ &\approx 7.62 \quad \text{Approximate} \end{aligned}$$

Now Work PROBLEMS 11 AND 15

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is never a negative number. Also, the distance between two points is 0 only when the points are identical—that is, when $x_1 = x_2$ and $y_1 = y_2$. And, because $(x_2 - x_1)^2 = (x_1 - x_2)^2$ and $(y_2 - y_1)^2 = (y_1 - y_2)^2$, it makes no difference whether the distance is computed from P_1 to P_2 or from P_2 to P_1; that is, $d(P_1, P_2) = d(P_2, P_1)$.

Rectangular coordinates enable us to translate geometry problems into algebra problems, and vice versa. The next example shows how algebra (the distance formula) can be used to solve geometry problems.

EXAMPLE 3 Using Algebra to Solve a Geometry Problem

Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

(a) Plot each point and form the triangle ABC.

(b) Find the length of each side of the triangle.

(c) Show that the triangle is a right triangle.

(d) Find the area of the triangle.

Solution

(a) Figure 26 shows the points A, B, C and the triangle ABC.

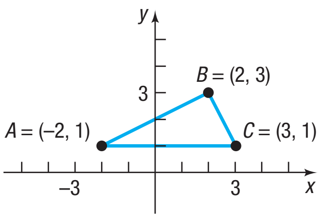


Figure 26

(b) To find the length of each side of the triangle, use the distance formula, equation (1).

$$\begin{aligned} d(A, B) &= \sqrt{[2 - (-2)]^2 + (3 - 1)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \\ d(B, C) &= \sqrt{(3 - 2)^2 + (1 - 3)^2} = \sqrt{1 + 4} = \sqrt{5} \\ d(A, C) &= \sqrt{[3 - (-2)]^2 + (1 - 1)^2} = \sqrt{25 + 0} = 5 \end{aligned}$$

(c) If the sum of the squares of the lengths of two of the sides equals the square of the length of the third side, then the triangle is a right triangle. Looking at Figure 26, it seems reasonable to conjecture that the angle at vertex B might be a right angle. We shall check to see whether

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

(continued)

Using the results in part (b) yields

$$[d(A,B)]^2 + [d(B,C)]^2 = \left(2\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2$$
$$= 20 + 5 = 25 = [d(A,C)]^2$$

It follows from the converse of the Pythagorean Theorem that triangle ABC is a right triangle.

(d) Because the right angle is at vertex B, the sides AB and BC form the base and height of the triangle. Its area is

$$\text{Area} = \frac{1}{2}\cdot\text{Base}\cdot\text{Height} = \frac{1}{2}\cdot 2\sqrt{5}\cdot\sqrt{5} = 5 \text{ square units}$$

Now Work PROBLEM 25

2 Use the Midpoint Formula

We now derive a formula for the coordinates of the **midpoint of a line segment**. Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be the endpoints of a line segment, and let $M = (x, y)$ be the point on the line segment that is the same distance from P_1 as it is from P_2. See Figure 27. The triangles P_1AM and MBP_2 are congruent. [Do you see why? $d(P_1, M) = d(M, P_2)$ is given; also, $\angle AP_1M = \angle BMP_2$* and $\angle P_1MA = \angle MP_2B$. So, we have angle–side–angle.] Because triangles P_1AM and MBP_2 are congruent, corresponding sides are equal in length. That is,

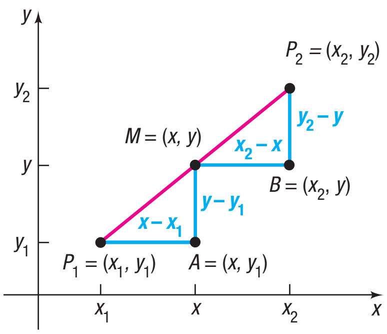


Figure 27

$$x - x_1 = x_2 - x \quad\text{and}\quad y - y_1 = y_2 - y$$
$$2x = x_1 + x_2 \qquad\qquad 2y = y_1 + y_2$$
$$x = \frac{x_1 + x_2}{2} \qquad\qquad y = \frac{y_1 + y_2}{2}$$

In Words
To find the midpoint of a line segment, average the x-coordinates of the endpoints, and average the y-coordinates of the endpoints.

THEOREM Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \textbf{(2)}$$

EXAMPLE 4 Finding the Midpoint of a Line Segment

Find the midpoint of the line segment from $P_1 = (-5, 5)$ to $P_2 = (3, 1)$. Plot the points P_1 and P_2 and their midpoint.

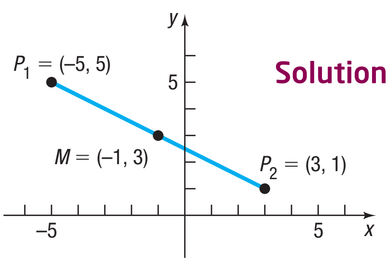


Figure 28

Solution

Use the midpoint formula (2) with $x_1 = -5$, $y_1 = 5$, $x_2 = 3$, and $y_2 = 1$. The coordinates (x, y) of the midpoint M are

$$x = \frac{x_1 + x_2}{2} = \frac{-5 + 3}{2} = -1 \quad\text{and}\quad y = \frac{y_1 + y_2}{2} = \frac{5 + 1}{2} = 3$$

That is, $M = (-1, 3)$. See Figure 28.

Now Work PROBLEM 31

*A postulate from geometry states that the transversal $\overline{P_1P_2}$ forms congruent corresponding angles with the parallel line segments $\overline{P_1A}$ and $\overline{MB}$.

1.2 Assess Your Understanding

'Are You Prepared?'

Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

1. If −3 and 5 are the coordinates of two points on the real number line, the distance between these points is _____. (pp. A5–A6)

2. If 3 and 4 are the legs of a right triangle, the hypotenuse is _____. (p. A14)

3. Use the converse of the Pythagorean Theorem to show that a triangle whose sides are of lengths 11, 60, and 61 is a right triangle. (pp. A14–A15)

4. In the rectangular coordinate system, the origin has coordinates _________. (p. 2)

5. The area A of a triangle whose base is b and whose altitude is h is $A =$ ________. (p. A15)

6. ***True or False*** Two triangles are congruent if two angles and the included side of one equals two angles and the included side of the other. (pp. A16–A17)

Concepts and Vocabulary

7. If three distinct points P, Q, and R all lie on a line, and if $d(P, Q) = d(Q, R)$, then Q is called the _________ of the line segment from P to R.

8. ***True or False*** The distance between two points is sometimes a negative number.

9. ***True or False*** The midpoint of a line segment is found by averaging the x-coordinates and averaging the y-coordinates of the endpoints.

10. ***Multiple Choice*** Choose the expression that equals the distance between two points (x_1, y_1) and (x_2, y_2).

(a) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(b) $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$

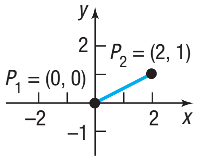
(c) $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

(d) $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$

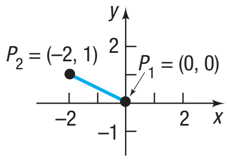
Skill Building

In Problems 11–24, find the distance d between the points P_1 and P_2.

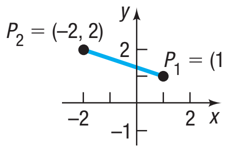
11.



12.



13.



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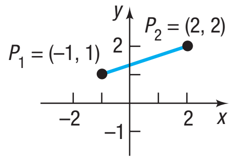

15. $P_1 = (3, -4);\quad P_2 = (5, 4)$

16. $P_1 = (-1, 0);\quad P_2 = (2, 4)$

17. $P_1 = (-7, 3);\quad P_2 = (4, 0)$

18. $P_1 = (2, -3);\quad P_2 = (4, 2)$

19. $P_1 = (5, -2);\quad P_2 = (6, 1)$

20. $P_1 = (-4, -3);\quad P_2 = (6, 2)$

21. $P_1 = (-0.2, 0.3);\quad P_2 = (2.3, 1.1)$

22. $P_1 = (1.2, 2.3);\quad P_2 = (-0.3, 1.1)$

23. $P_1 = (a, b);\quad P_2 = (0, 0)$

24. $P_1 = (a, a);\quad P_2 = (0, 0)$

In Problems 25–30, plot each point and form the triangle ABC. Show that the triangle is a right triangle. Find its area.

25. $A = (-2, 5);\quad B = (1, 3);\quad C = (-1, 0)$

26. $A = (-2, 5);\quad B = (12, 3);\quad C = (10, -11)$

27. $A = (-5, 3);\quad B = (6, 0);\quad C = (5, 5)$

28. $A = (-6, 3);\quad B = (3, -5);\quad C = (-1, 5)$

29. $A = (4, -3);\quad B = (0, -3);\quad C = (4, 2)$

30. $A = (4, -3);\quad B = (4, 1);\quad C = (2, 1)$

In Problems 31–38, find the midpoint of the line segment joining the points P_1 and P_2.

31. $P_1 = (3, -4);\quad P_2 = (5, 4)$

32. $P_1 = (-2, 0);\quad P_2 = (2, 4)$

33. $P_1 = (-1, 4);\quad P_2 = (8, 0)$

34. $P_1 = (2, -3);\quad P_2 = (4, 2)$

35. $P_1 = (7, -5);\quad P_2 = (9, 1)$

36. $P_1 = (-4, -3);\quad P_2 = (2, 2)$

37. $P_1 = (a, b);\quad P_2 = (0, 0)$

38. $P_1 = (a, a);\quad P_2 = (0, 0)$