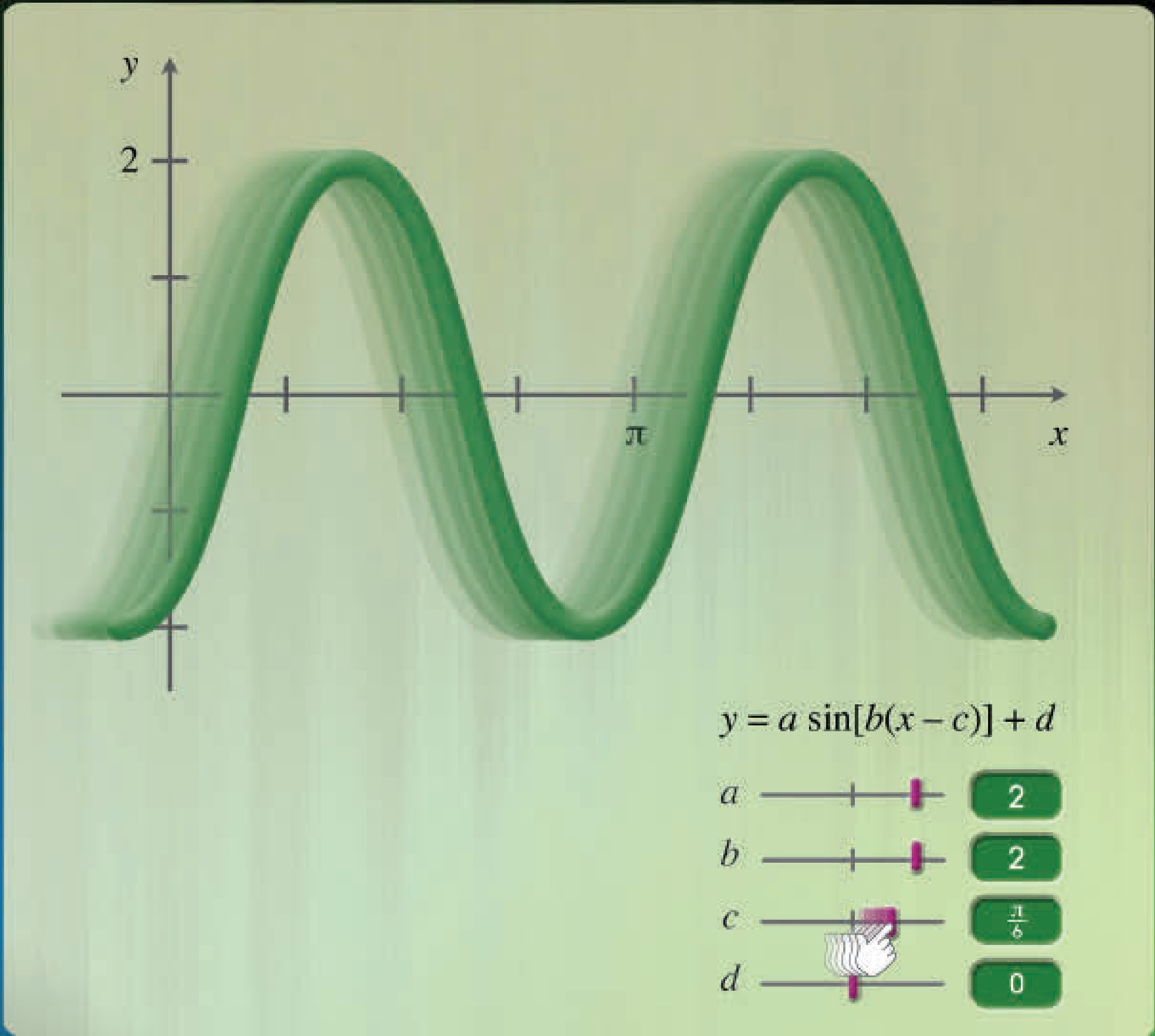
ALGEBRA and TRIGONOMETRY

Enhanced with Graphing Utilities

Eighth Edition

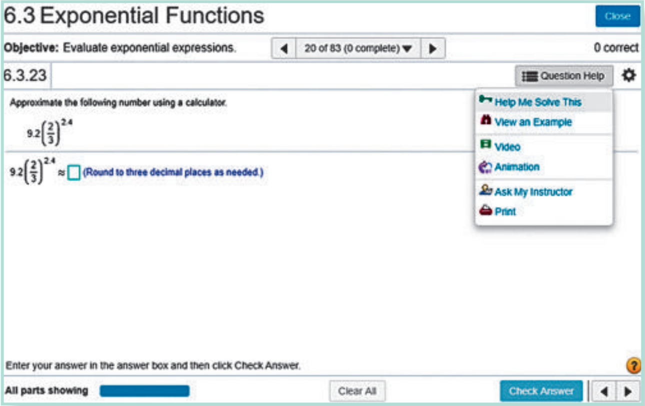


Sullivan & Sullivan

MyLab Math

*Support You **Need**, When You **Need** It*

MyLab Math has helped millions of students succeed in their math courses. Take advantage of the resources it provides.

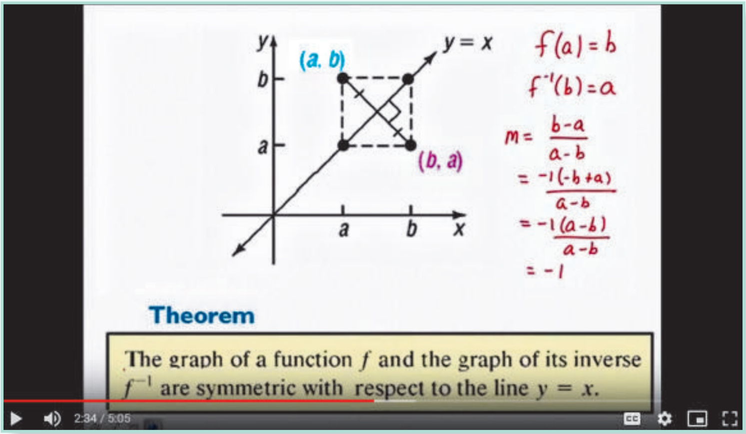


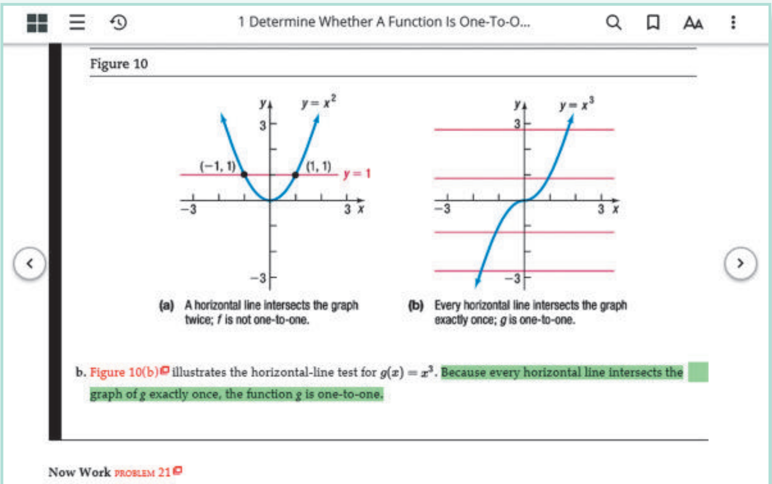
Interactive Exercises

MyLab Math's interactive exercises mirror those in the textbook but are programmed to allow you unlimited practice, leading to mastery. Most exercises include **learning aids**, such as "Help Me Solve This," "View an Example," and "Video," and they offer helpful feedback when you enter incorrect answers.

Instructional Videos

Instructional videos cover key examples from the text and can conveniently be played on any mobile device. These videos are especially helpful if you miss a class or just need further explanation.





Interactive eText

The **Pearson eText** gives you access to your textbook anytime, anywhere. In addition to note taking, highlighting, and bookmarking, the Pearson eText App offers access even when offline.

pearson.com/mylab/math

Prepare for Class: "Read the Book"

Feature	Description	Benefit	Page(s)
Every Chapter begins with . . .			
Chapter-Opening Topic & Project	Each chapter begins with a discussion of a topic of current interest and ends with a related project.	In the concluding project, you will apply what you have learned to solve a problem related to the topic.	424, 533
Internet-Based Projects	These projects allow for the integration of spreadsheet technology that you will need to be a productive member of the workforce.	The projects give you an opportunity to collaborate and use mathematics to deal with issues of current interest.	533
Every Section begins with . . .			
LEARNING OBJECTIVES 2	Each section begins with a list of objectives. Individual objectives also appear in the text where they are covered.	These objectives focus your studying by emphasizing what's most important and where to find it.	446
Sections contain . . .			
PREPARING FOR THIS SECTION	Most sections begin with a list of key concepts to review, with page numbers.	Ever forget what you've learned? This feature highlights previously learned material to be used in this section. Review it, and you'll always be prepared to move forward.	446
Now Work the 'Are You Prepared?' Problems	These problems assess whether you have the prerequisite knowledge for the upcoming section.	Work the 'Are You Prepared?' problems. If you get one wrong, you'll know exactly what you need to review and where to review it!	446, 458
Now Work PROBLEMS	These follow most examples and direct you to a related exercise.	We learn best by doing. You'll solidify your understanding of examples if you try a similar problem right away, to be sure you understand what you've just read.	454, 460
CAUTION	Words of caution are provided in the text.	These point out common mistakes and help you avoid them.	482
Explorations and Seeing the Concept	These graphing utility activities foreshadow a concept or reinforce a concept just presented.	You will obtain a deeper and more intuitive understanding of theorems and definitions.	441, 453
In Words	This feature provides alternative descriptions of select definitions and theorems.	Why didn't you say that in the first place? This feature translates math into plain English.	465
Calculus	This symbol appears next to information essential for the study of calculus.	Foreshadowing calculus now will make the material easier later.	220, 429, 454
SHOWCASE EXAMPLES	These examples provide "how to" instruction by offering a guided, step-by-step approach to solving a problem.	With each step presented on the left and the mathematics displayed on the right, you can immediately see how each step is employed.	358–359
Model It! Examples and Problems	These examples and problems require you to build a mathematical model from either a verbal description or data. The homework Model It! problems are marked by purple problem numbers.	It is rare for a problem to come in the form "*Solve the following equation*." Rather, the equation must be developed based on an explanation of the problem. These problems require you to develop models that will enable you to describe the problem mathematically and suggest a solution to the problem.	472, 504
NEW! Need to Review?	These margin notes provide a just-in-time reminder of a concept needed now, but covered in an earlier section of the book. Each note is back-referenced to the chapter, section and page where the concept was originally discussed.	Sometimes as you read, you encounter a word or concept you know you've seen before, but don't remember exactly what it means. This feature will point you to where you first learned the word or concept. A quick review now will help you see the connection to what you are learning for the first time and make remembering easier the next time.	453

Practice: "Work the Problems"

Feature	Description	Benefit	Page(s)
'Are You Prepared?' Problems	These problems assess your retention of the prerequisite material. Answers are given at the end of the section exercises. This feature is related to the Preparing for This Section feature.	Do you always remember what you've learned? Working these problems is the best way to find out. If you get one wrong, you'll know exactly what you need to review and where to review it!	446, 458
Concepts and Vocabulary	These short-answer questions, mainly fill-in-the-blank, multiple-choice, and true/false items, assess your understanding of key definitions and concepts in the current section.	It is difficult to learn math without knowing the language of mathematics. These problems test your understanding of the formulas and vocabulary.	458–459
Skill Building	Correlated with section examples, these problems provide straightforward practice.	These problems give you ample opportunity to dig in and develop your skills.	459–461
Mixed Practice	These problems offer comprehensive assessment of the skills learned in the section by asking problems related to more than one concept or objective. These problems may also require you to utilize skills learned in previous sections.	Learning mathematics is a building process. Many concepts build on each other and are related. These problems help you see how mathematics builds on itself and how the concepts are linked together.	461
Applications and Extensions	These problems allow you to apply your skills to real-world problems. They also enable you to extend concepts learned in the section.	You will see that the material learned within the section has many uses in everyday life.	461–464
NEW! Challenge Problems	These problems have been added in most sections and appear at the end of the Application and Extensions exercises. They are intended to be thought-provoking, requiring some ingenuity to solve.	Challenge problems can be used for group work or to challenge your students. Solutions to Challenge Problems are in the Annotated Instructor's Edition or in the Instructor's Solution Manual (online).	464
Explaining Concepts: Discussion and Writing	"Discussion and Writing" problem numbers are colored red. They support class discussion, verbalization of mathematical ideas, and writing and research projects.	To verbalize an idea, or to describe it clearly in writing, shows real understanding. These problems nurture that understanding. Many are challenging, but you'll get out what you put in.	464
Retain Your Knowledge	These problems allow you to practice content learned earlier in the course.	Remembering how to solve all the different kinds of problems that you encounter throughout the course is difficult. This practice helps you remember previously learned skills.	464
Now Work PROBLEMS	Many examples refer you to a related homework problem. These related problems are marked by ✎ and orange problem numbers.	If you get stuck while working problems, look for the closest Now Work problem, and refer to the related example to see if it helps.	454, 456, 457
NEW! Interactive Figure Exercises	Exercises that require you manipulate an interactive figure to solve. These exercises are labeled with the icon.	These exercises help you visualize important concepts and develop a "feel" for them. The figures are housed at `bit.ly/2Mibga0` and were developed in GeoGebra by author Michael Sullivan III.	458, 459, 473, 474
Review Exercises	Every chapter concludes with a comprehensive list of exercises to practice. Use the list of objectives to determine what objective and examples correspond to each problem.	Work these problems to ensure that you understand all the skills and concepts employed in the chapter. Think of it as a comprehensive review of the chapter. All answers to Chapter Review problems appear in the back of the text.	528–531

Review: “Study for Quizzes and Tests”

Feature	Description	Benefit	Page(s)
Most Sections Contain			
Retain Your Knowledge	Keeps what you have learned at the forefront and see how topics are connected.	These problems allow content to remain fresh so you are more prepared for the final exam.	478
The Chapter Review at the end of each chapter contains . . .			
Things to Know	A detailed list of important theorems, formulas, and definitions from the chapter.	Review these and you’ll know the most important material in the chapter!	526–527
You Should Be Able to . . .	A complete list of objectives by section, examples that illustrate the objective, and practice exercises that test your understanding of the objective.	Do the recommended exercises and you’ll have mastered the key material. If you get something wrong, go back and review the example listed, and try again.	527–528
Review Exercises	These provide comprehensive review and practice of key skills, matched to the Learning Objectives for each section.	Practice makes perfect. These problems combine exercises from all sections, giving you a comprehensive review in one place.	528–531
Chapter Test	About 15–20 problems that can be taken as a Chapter Test. Be sure to take the Chapter Test under test conditions—no notes!	Be prepared. Take the sample practice test under test conditions. This will get you ready for your instructor’s test. If you get a problem wrong, you can watch the Chapter Test Prep Video.	531–532
Cumulative Review	These problem sets appear at the end of each chapter, beginning with Chapter 2. They combine problems from previous chapters, providing an ongoing cumulative review. When you use them in conjunction with the Retain Your Knowledge problems, you will be ready for the final exam.	These problem sets are really important. Completing them will ensure that you are not forgetting anything as you go. This will go a long way toward keeping you primed for the final exam.	532–533

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Algebra & Trigonometry

Enhanced with Graphing Utilities

Eighth Edition

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Pearson

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For the family

Katy (Murphy) and Pat — Shannon, Patrick, Ryan
Mike and Yola — Michael, Kevin, Marissa
Dan and Sheila — Maeve, Sean, Nolan
Colleen (O'Hara) and Bill — Kaleigh, Billy, Timmy

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Contents

Three Distinct Series to Meet Varied Instructional Needs

Students have different goals, learning styles, and levels of preparation. Instructors have different teaching philosophies, styles, and techniques. Rather than write one series to fit all, the Sullivans have written three distinct series. All share the same goal—to develop a high level of mathematical understanding and an appreciation for the way mathematics can describe the world around us. The manner of reaching that goal, however, differs from series to series.

Enhanced with Graphing Utilities Series

This series provides a thorough integration of graphing utilities into topics, allowing students to explore mathematical concepts and encounter ideas usually studied in later courses. Many examples show solutions using algebra side-by-side with graphing techniques. Using technology, the approach to solving certain problems differs from the Contemporary (Flagship) or Concepts through Functions Series, while the emphasis on understanding concepts and building strong skills is maintained. Texts in this series are *College Algebra, Algebra & Trigonometry,* and *Precalculus.*

Flagship Series

The Flagship Series is the most traditional in approach, yet modern in its treatment of precalculus mathematics. In each text, needed review material is included and is referenced when it is used. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor. Texts in this series are *College Algebra, Algebra & Trigonometry, Trigonometry: A Unit Circle Approach, and Precalculus.*

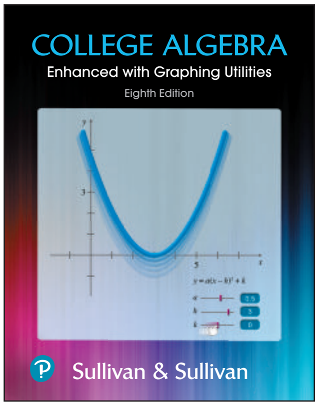
Concepts through Functions Series

This series differs from the others, utilizing a functions approach that serves as the organizing principle tying concepts together. Functions are introduced early in various formats. This approach supports the Rule of Four, which states that functions are represented symbolically, numerically, graphically, and verbally. Each chapter introduces a new type of function and then develops all concepts pertaining to that particular function. The solutions of equations and inequalities, instead of being developed as stand-alone topics, are developed in the context of the underlying functions. Graphing utility coverage is optional and can be included or excluded at the discretion of the instructor. Texts in this series are *College Algebra; Precalculus, with a Unit Circle Approach to Trigonometry; Precalculus, with a Right Triangle Approach to Trigonometry.*

The Enhanced with Graphing Utilities Series

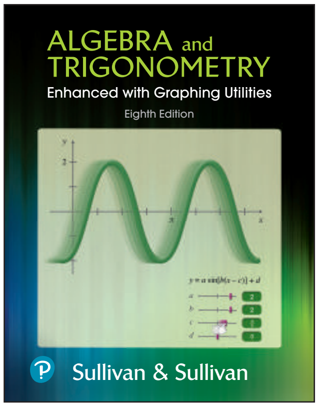
College Algebra, Eighth Edition

This text provides an approach to college algebra that completely integrates graphing technology without sacrificing mathematical analysis and conceptualization. The text has three chapters of review material preceding the chapter on functions. Graphing calculator usage is integrated throughout. After completing this text, a student will be prepared for trigonometry, finite mathematics, and business calculus.



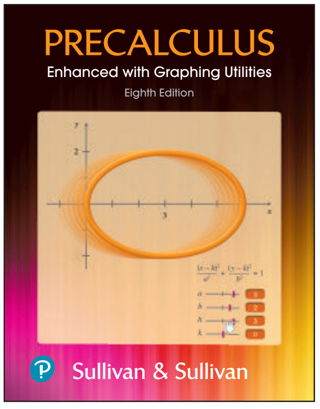
Algebra & Trigonometry, Eighth Edition

This text contains all the material in *College Algebra*, but it also develops the trigonometric functions using a right triangle approach and shows how that approach is related to the unit circle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Vectors in the plane, including the dot product, sequences, induction, and the binomial theorem are also presented. After completing this text, a student will be prepared for finite mathematics, business calculus, and engineering calculus.



Precalculus, Eighth Edition

This text contains a review chapter before covering the traditional precalculus topics of functions and their graphs, polynomial and rational functions, and exponential and logarithmic functions. The trigonometric functions are introduced using a unit circle approach and show how it is related to the right triangle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Vectors in the plane and in space, including the dot and cross products, sequences, induction, and the binomial theorem are also presented. Graphing calculator usage is integrated throughout. The final chapter provides an introduction to calculus, with a discussion of the limit, the derivative, and the integral of a function. After completing this text, a student will be prepared for finite mathematics, business calculus, and engineering calculus.



Preface to the Instructor

As professors at an urban university (Michael Sullivan) and a community college (Michael Sullivan III), we are aware of the varied needs of students in this course. Such students range from those who have little mathematical background and are fearful of mathematics courses to those with a strong mathematical education and a high level of motivation. For some of your students, this will be their last course in mathematics, whereas others will further their mathematical education. We have written this text with both groups in mind.

As a teacher, and as an author of precalculus, engineering calculus, finite mathematics, and business calculus texts, Michael Sullivan understands what students must know if they are to be focused and successful in upper-level math courses. As an instructor and an author of a developmental mathematics series, Michael's son and co-author, Michael Sullivan III, understands the trepidations and skills that students bring to the Algebra and Trigonometry course. As the father of current college students, Michael III realizes that today's college students demand a variety of media to support their education. This text addresses that demand by providing technology and video support that enhances understanding without sacrificing math skills. Together, we have taken great pains to ensure that the text offers solid, student-friendly examples and problems, as well as a clear and seamless writing style.

A tremendous benefit of authoring a successful series is the broad-based feedback we receive from teachers and students. We are sincerely grateful for their support. Virtually every change in this edition is the result of their thoughtful comments and suggestions. We are confident that, building on the success of the first seven editions and incorporating many of these suggestions, we have made *Algebra & Trigonometry Enhanced with Graphing Utilities,* 8th Edition, an even better tool for learning and teaching. We continue to encourage you to share with us your experiences teaching from this text.

Features in the Eighth Edition

A descriptive list of the many special features of *Algebra & Trigonometry* can be found in the front of this text. This list places the features in their proper context as building blocks of an overall learning system that has been carefully crafted over the years to help students get the most out of the time they put into studying. Please take the time to review this and to discuss it with your students at the beginning of your course. Our experience is that when students utilize these features, they are more successful in the course.

New to the Eighth Edition

New Within the Textbook

All of the exercises and examples in the text have been reviewed and analyzed, and we have incorporated feedback from users of the text. All time-sensitive problems have been updated to the most recent information available. Here are the new features of this edition:

- **Challenge Problems** – These problems appear in the Applications and Extensions part of the section exercises and are designed to challenge students. Full solutions are in the back of the Annotated Instructor's Edition and in the Instructor's Solution Manual.
- **"Need to Review?" feature** – We placed reminders in the margin for key review topics. The reminders point students to the location of the review material in the textbook.
- **Chapter Projects** – The projects have been enhanced to give students an up-to-the-minute experience. Many of these projects require the student to research information online in order to solve problems.
- **Interactive Figure Exercises** – We have added this new category of exercises that require students to manipulate an interactive figure to solve. The interactive figures may be found at `bit.ly/2MibgaO` or in the Video and Resource Library of MyLab Math, and were created by author Michael Sullivan III in GeoGebra. These exercises are labeled with the icon .
- **Expanded! Retain Your Knowledge Problems** – These problems, which were new to the previous edition, are based on learning research, including a study of precalculus students at University of Louisville entitled "Spaced retrieval practice increases college students' short- and long-term retention of mathematics knowledge" (Hopkins et al, 2016). The Retain Your Knowledge problems were so well received that we have expanded them in this edition. Moreover, while the focus remains to help students maintain their skills, in most sections, problems were chosen that preview skills required to succeed in subsequent sections or in calculus ( ). All answers to Retain Your Knowledge problems are given in the back of the text and these problems are available in the prebuilt assignments in the Assignment Manager in MyLab Math.
- **Key to Exercise Types** – To help you navigate the features of the exercise sets, we've included a key at the bottom of the first page of each section's exercises.

Now Work 1. Modeling 1. Writing/Discussion
Calculus Preview Interactive Figure

- **Graphing Utility Screen Captures** – In several instances we have added Desmos screen captures along with the TI-84 Plus CE screen captures. These updated screen captures provide alternative ways of visualizing concepts and making connections between equations, data, and graphs in full color.

Content Changes

Chapter R

- Section R.8 Objective 3 now includes rationalizing the numerator. Problems 69–76 provide practice.

Chapter 1

- Section 1.1 has been reorganized to only include an introduction to graphing and graphing utilities.

Chapter 2

- NEW Section 2.1 The Distance and Midpoint Formulas
- NEW Section 2.2 Example 5 Testing an Equation for Symmetry

Chapter 3

- NEW Section 3.1 Objective 1 Describe a Relation
  - NEW Example 1 Describing a Relation demonstrates using the Rule of Four to express a relation numerically, as a mapping, and graphically given a verbal description.
- NEW Section 3.2 Example 4 Expending Energy

Chapter 4

- Section 4.3 now introduces the concept of concavity for a quadratic function.
- NEW Section 4.3 Example 3 Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts
- Section 4.3 Example 8 Analyzing the Motion of a Projectile (formerly in Section 4.4)
- NEW Section 4.4 Example 4 Fitting a Quadratic Function to Data

Chapter 5

- Previous Section 5.1 has been revised and split into two sections:
  - 5.1 Polynomial Functions
  - 5.2 Graphing Polynomial Functions; Models
- NEW Section 5.2 Example 2 Graphing a Polynomial Function (a 4th degree polynomial function)

Chapter 6

- NEW Section 6.2 Objective Verify a Function Defined by an Equation is an Inverse Function

Chapter 7

- NEW Section 7.1 Example 6 Field Width of a Digital Lens Reflex Camera Lens
- NEW Section 7.5 Example 5 Using Symmetry to Find Exact Values of Trigonometric Functions
- Sections 7.6 and 7.7 were reorganized for increased clarity. Two new objectives were added to Section 7.7.

Chapter 8

- Sections 8.1 and 8.2 were reorganized for increased clarity. Four new objectives were added to Section 8.1. The objectives in Section 8.2 were reordered.

Chapter 10

- Section 10.3 DeMoivre's Theorem was rewritten to support the exponential form of a complex number.
  - Euler's Formula is introduced to express a complex number in exponential form. The exponential form is used to compute products and quotients.
  - DeMoivre's Theorem is expressed using the exponential form of a complex number. The exponential form is used to find complex roots.

Chapter 12

- NEW Section 12.5 Example 1 Identifying Proper and Improper Rational Expressions

Chapter 13

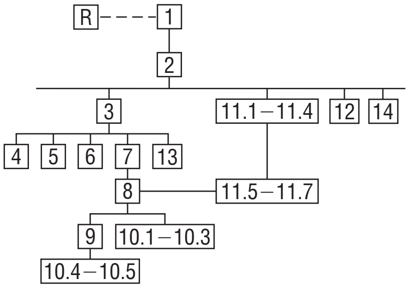
- NEW Section 13.3 Objective 5 Solving Annuity Problems Using Formulas

New Within MyLab Math

- **Setup & Solve Exercises** require students to show how they set up a problem as well as the solution, better mirroring what is required of them on tests. We have included both the "traditional" and Setup & Solve versions of exercise within MyLab to provide you with more options for assessing students.
- **Integrated Review** content and assessments help you provide students with the remediation they need, when they need it. Integrated Review consists of:
  - **Skills Check Quizzes** by chapter assess the prerequisite skills students need for that chapter.
  - **Skills Review Homework**, again by chapter, is personalized (based on the results of the Skills Check Quiz) to provide students with help on the prerequisite skills they are lacking. Students receive just the help they need—no more, no less.
  - **Intermediate Algebra eText, Exercises, Videos, and Worksheets**—For students who need more help (or for co-requisite courses), we've included the contents of a streamlined Intermediate Algebra course within this MyLab course. There's no need to go elsewhere for remediation.
- **Interactive Figures** (formerly titled Guided Visualizations) have been expanded to support teaching and learning. The figures (created in GeoGebra by author Michael Sullivan III) illustrate key concepts and allow manipulation. They have been designed to be used in lecture as well as by students independently.
- **Enhanced Sample Assignments** are pre-made section-level assignments that address key concepts within the section and help keep previously learned skills fresh with Retain Your Knowledge questions. They are assignable and editable.

Using the Eighth Edition Effectively with Your Syllabus

To meet the varied needs of diverse syllabi, this text contains more content than is likely to be covered in an *Algebra and Trigonometry* course. As the chart illustrates, this text has been organized with flexibility of use in mind. Within a given chapter, certain sections are optional (see the details that follow the accompanying figure) and can be omitted without loss of continuity.



Chapter R Review

This chapter consists of review material. It may be used as the first part of the course or later as a just-in-time review when the content is required. Specific references to this chapter occur throughout the text to assist in the review process.

Chapter 1 Equations and Inequalities

Primarily a review of intermediate algebra topics, this material is a prerequisite for later topics. The coverage of complex numbers and quadratic equations with a negative discriminant is optional and may be postponed or skipped entirely without loss of continuity.

Chapter 2 Graphs

This chapter lays the foundation for functions. Section 2.5 is optional.

Chapter 3 Functions and Their Graphs

This is perhaps the most important chapter. Section 3.6 is optional.

Chapter 4 Linear and Quadratic Functions

Topic selection depends on your syllabus. Sections 4.2 and 4.4 may be omitted without loss of continuity.

Chapter 5 Polynomial and Rational Functions

Topic selection depends on your syllabus.

Chapter 6 Exponential and Logarithmic Functions

Sections 6.1–6.6 follow in sequence. Sections 6.7, 6.8, and 6.9 are optional.

Chapter 7 Trigonometric Functions

Section 7.8 may be omitted in a brief course.

Chapter 8 Analytic Trigonometry

Sections 8.2, 8.6, and 8.7 may be omitted in a brief course.

Chapter 9 Applications of Trigonometric Functions

Sections 9.4 and 9.5 may be omitted in a brief course.

Chapter 10 Polar Coordinates; Vectors

Sections 10.1–10.3 and Sections 10.4–10.5 are independent and may be covered separately.

Chapter 11 Analytic Geometry

Sections 11.1–11.4 follow in sequence. Sections 11.5, 11.6, and 11.7 are independent of each other, but each requires Sections 11.1–11.4.

Chapter 12 Systems of Equations and Inequalities

Sections 12.2–12.7 may be covered in any order, but each requires Section 12.1. Section 12.8 requires Section 12.7.

Chapter 13 Sequences; Induction; The Binomial Theorem

There are three independent parts: Sections 13.1–13.3, Section 13.4, and Section 13.5.

Chapter 14 Counting and Probability

The sections follow in sequence.

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Thanks are due to the following people for their assistance and encouragement during the preparation of this edition:

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- Accuracy checkers: Roger Lipsett read the entire manuscript and checked the accuracy of answers. Timothy Britt created the Solutions Manuals and accuracy-checked answers.
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The list of those to whom we are indebted continues to grow. If we've forgotten anyone, please accept our apology. Thank you to all.

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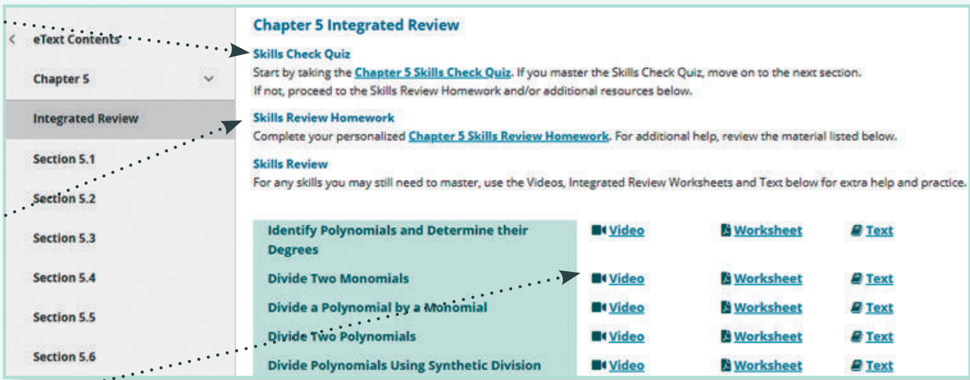

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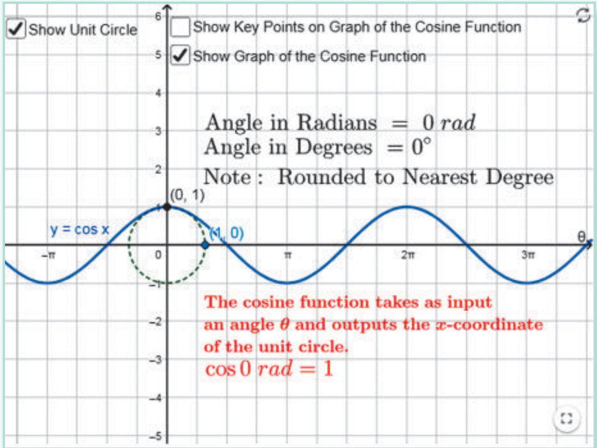

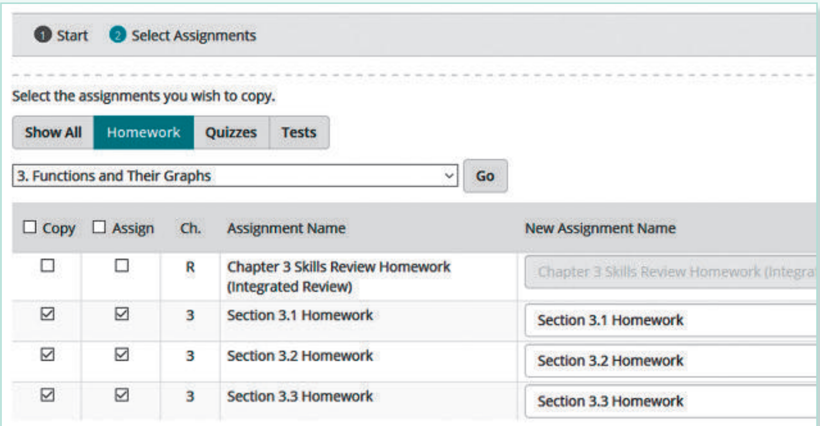

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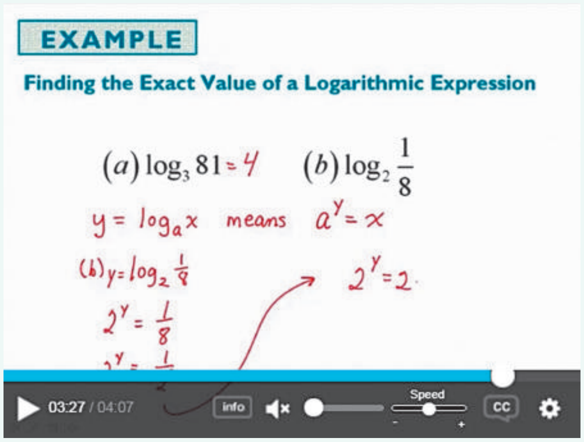

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To the Student

As you begin, you may feel anxious about the number of theorems, definitions, procedures, and equations you encounter. You may wonder if you can learn it all in time. Don't worry, your concerns are normal. This text was written with you in mind. If you attend class, work hard, and read and study effectively, you will build the knowledge and skills you need to be successful. Here's how you can use the text to your benefit.

Read Carefully

When you get busy, it's easy to skip reading and go right to the problems. Don't! The text provides a large number of examples and clear explanations to help you break down the mathematics into easy-to-understand steps. Reading will provide you with a clearer understanding, beyond simple memorization. Read before class (not after) so you can ask questions about anything you didn't understand. You'll be amazed at how much more you'll get out of class when you do this.

Use the Features

We use many different methods in the classroom to communicate. Those methods, when incorporated into the text, are called "features." The features serve many purposes, from supplying a timely review of material you learned before (just when you need it), to providing organized review sessions to help you prepare for quizzes and tests. Take advantage of the features and you will master the material.

To make this easier, we've provided a brief guide to getting the most from this book. Refer to the "Prepare for Class," "Practice," and "Review" guidelines on the first three pages of this book. Spend fifteen minutes reviewing the guide and familiarizing yourself with the features by flipping to the page numbers provided. Then, as you read, use them. This is the best way to make the most of your text. In this edition, we've also added a handy key to the labeling of the homework exercises so that you know what the colors and icons mean:

Now Work   1. Modeling   1. Writing/Discussion   Calculus Preview   Interactive Figure

Please do not hesitate to contact us via Math@Pearson.com with any questions, comments, or suggestions about ways to improve this text. We look forward to hearing from you, and good luck with all of your studies.

Best Wishes!

Michael Sullivan
Michael Sullivan III

Review

R

A Look Ahead

Chapter R, as the title states, contains review material. Your instructor may choose to cover all or part of it as a regular chapter at the beginning of your course or later as a just-in-time review when the content is required. Regardless, when information in this chapter is needed, a specific reference to this chapter will be made so you can review.

Outline

R.1 Real Numbers

PREPARING FOR THIS TEXT *Before getting started, read "To the Student" at the front of this text.*

1 Work with Sets

A **set** is a well-defined collection of distinct objects. The objects of a set are called its **elements**. By **well-defined**, we mean that there is a rule that enables us to determine whether a given object is an element of the set. If a set has no elements, it is called the **empty set**, or **null set**, and is denoted by the symbol $\varnothing$.

For example, the set of **digits** consists of the collection of numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. If we use the symbol D to denote the set of digits, then we can write

$$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

In this notation, the braces $\{\quad\}$ are used to enclose the objects, or **elements**, in the set. This method of denoting a set is called the **roster method**. A second way to denote a set is to use **set-builder notation**, where the set D of digits is written as

$$D = \{ \quad x \quad | \quad x \text{ is a digit} \}$$

Read as "D is the set of all x such that x is a digit."

EXAMPLE 1 Using Set-builder Notation and the Roster Method

(a) $E = \{x | x \text{ is an even digit}\} = \{0, 2, 4, 6, 8\}$
(b) $O = \{x | x \text{ is an odd digit}\} = \{1, 3, 5, 7, 9\}$

Because the elements of a set are distinct, we never repeat elements. For example, we would never write $\{1, 2, 3, 2\}$; the correct listing is $\{1, 2, 3\}$. Because a set is a collection, the order in which the elements are listed is immaterial. $\{1, 2, 3\}$, $\{1, 3, 2\}$, $\{2, 1, 3\}$, and so on, all represent the same set.

If every element of a set A is also an element of a set B, then A is a **subset** of B, which is denoted $A \subseteq B$. If two sets A and B have the same elements, then A **equals** B, which is denoted $A = B$.

For example, $\{1, 2, 3\} \subseteq \{1, 2, 3, 4, 5\}$ and $\{1, 2, 3\} = \{2, 3, 1\}$.

DEFINITION Intersection and Union of Two Sets

If A and B are sets, the **intersection** of A with B, denoted $A \cap B$, is the set consisting of elements that belong to both A and B. The **union** of A with B, denoted $A \cup B$, is the set consisting of elements that belong to either A or B, or both.

EXAMPLE 2 Finding the Intersection and Union of Sets

Let $A = \{1, 3, 5, 8\}$, $B = \{3, 5, 7\}$, and $C = \{2, 4, 6, 8\}$. Find:

(a) $A \cap B$ (b) $A \cup B$ (c) $B \cap (A \cup C)$

Solution

(a) $A \cap B = \{1, 3, 5, 8\} \cap \{3, 5, 7\} = \{3, 5\}$

(b) $A \cup B = \{1, 3, 5, 8\} \cup \{3, 5, 7\} = \{1, 3, 5, 7, 8\}$

(c) $B \cap (A \cup C) = \{3, 5, 7\} \cap [\{1, 3, 5, 8\} \cup \{2, 4, 6, 8\}]$
$= \{3, 5, 7\} \cap \{1, 2, 3, 4, 5, 6, 8\} = \{3, 5\}$

Now Work PROBLEM 15

Usually, in working with sets, we designate a **universal set** U, the set consisting of all the elements that we wish to consider. Once a universal set has been designated, we can consider elements of the universal set not found in a given set.

DEFINITION Complement of a Set

If A is a set, the **complement** of A, denoted $\overline{A}$, is the set consisting of all the elements in the universal set that are not in A.*

EXAMPLE 3 Finding the Complement of a Set

If the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and if $A = \{1, 3, 5, 7, 9\}$, then $\overline{A} = \{2, 4, 6, 8\}$.

It follows from the definition of complement that $A \cup \overline{A} = U$ and $A \cap \overline{A} = \varnothing$. Do you see why?

Now Work PROBLEM 19

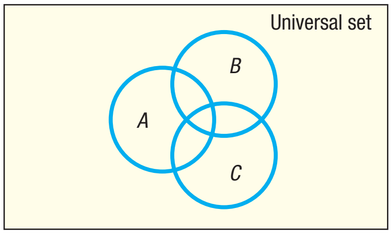
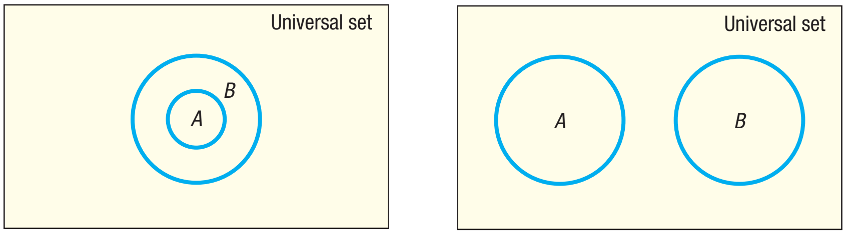


Figure 1 Venn diagram

It is often helpful to draw pictures of sets. Such pictures, called **Venn diagrams**, represent sets as circles enclosed in a rectangle, which represents the universal set. Such diagrams often help us to visualize various relationships among sets. See Figure 1.

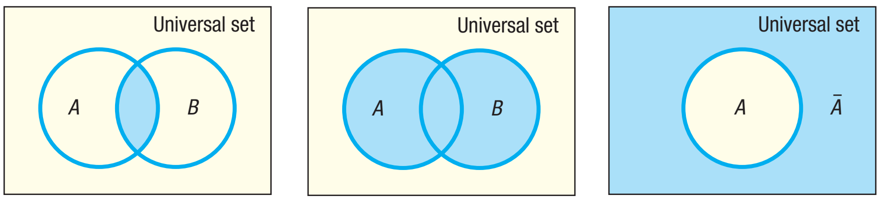
If we know that $A \subseteq B$, we might use the Venn diagram in Figure 2(a). If we know that A and B have no elements in common—that is, if $A \cap B = \varnothing$—we might use the Venn diagram in Figure 2(b). The sets A and B in Figure 2(b) are said to be **disjoint**.



(a) $A \subseteq B$ subset **(b)** $A \cap B = \varnothing$ disjoint sets

Figure 2

Figures 3(a), 3(b), and 3(c) use Venn diagrams to illustrate intersection, union, and complement, respectively.



(a) $A \cap B$ intersection **(b)** $A \cup B$ union **(c)** $\overline{A}$ complement

Figure 3

*Some texts use the notation A' or A^c for the complement of A.

2 Classify Numbers

It is helpful to classify the various kinds of numbers that we deal with as sets. The **counting numbers**, or **natural numbers**, are the numbers in the set $\{1, 2, 3, 4, \ldots\}$. (The three dots, called an **ellipsis**, indicate that the pattern continues indefinitely.) As their name implies, these numbers are often used to count things. For example, there are 26 letters in our alphabet; there are 100 cents in a dollar. The **whole numbers** are the numbers in the set $\{0, 1, 2, 3, \ldots\}$—that is, the counting numbers together with 0. The set of counting numbers is a subset of the set of whole numbers.

DEFINITION Integers

The **integers** are the set of numbers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

These numbers are useful in many situations. For example, if your checking account has \$10 in it and you write a check for \$15, you can represent the current balance as $-\$5$.

Each time we expand a number system, such as from the whole numbers to the integers, we do so in order to be able to handle new, and usually more complicated, problems. The integers enable us to solve problems requiring both positive and negative counting numbers, such as profit/loss, height above/below sea level, temperature above/below 0°F, and so on.

But integers alone are not sufficient for *all* problems. For example, they do not answer the question "What part of a dollar is 38 cents?" To answer such a question, we enlarge our number system to include *rational numbers*. For example, $\frac{38}{100}$ answers the question "What part of a dollar is 38 cents?"

DEFINITION Rational Number

A **rational number** is a number that can be expressed as a quotient $\frac{a}{b}$ of two integers. The integer a is called the **numerator**, and the integer b, which cannot be 0, is called the **denominator**. The rational numbers are the numbers in the set $\left\{x \middle| x = \frac{a}{b}, \text{ where } a, b \text{ are integers and } b \neq 0\right\}$.

Examples of rational numbers are $\frac{3}{4}, \frac{5}{2}, \frac{0}{4}, -\frac{2}{3}$, and $\frac{100}{3}$. Since $\frac{a}{1} = a$ for any integer a, it follows that the set of integers is a subset of the set of rational numbers.

Rational numbers may be represented as **decimals**. For example, the rational numbers $\frac{3}{4}, \frac{5}{2}, -\frac{2}{3}$, and $\frac{7}{66}$ may be represented as decimals by merely carrying out the indicated division:

$$\frac{3}{4} = 0.75 \qquad \frac{5}{2} = 2.5 \qquad -\frac{2}{3} = -0.666\ldots = -0.\overline{6} \qquad \frac{7}{66} = 0.1060606\ldots = 0.1\overline{06}$$

Notice that the decimal representations of $\frac{3}{4}$ and $\frac{5}{2}$ terminate, or end. The decimal representations of $-\frac{2}{3}$ and $\frac{7}{66}$ do not terminate, but they do exhibit a pattern of repetition. For $-\frac{2}{3}$, the 6 repeats indefinitely, as indicated by the bar over the 6; for $\frac{7}{66}$, the block 06 repeats indefinitely, as indicated by the bar over the 06. It can be shown that every rational number may be represented by a decimal that either terminates or is nonterminating with a repeating block of digits, and vice versa.

On the other hand, some decimals do not fit into either of these categories. Such decimals represent **irrational numbers**. Every irrational number may be represented by a decimal that neither repeats nor terminates. In other words, irrational numbers cannot be written in the form $\frac{a}{b}$, where a, b are integers and $b \neq 0$.

Irrational numbers occur naturally. For example, consider the isosceles right triangle whose legs are each of length 1. See Figure 4. The length of the hypotenuse is $\sqrt{2}$, an irrational number.

Also, the number that equals the ratio of the circumference C to the diameter d of any circle, denoted by the symbol π (the Greek letter pi), is an irrational number. See Figure 5.

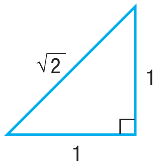


Figure 4

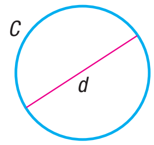


Figure 5 $\pi = \frac{C}{d}$

DEFINITION Real Numbers

The set of **real numbers** is the union of the set of rational numbers with the set of irrational numbers.

Figure 6 shows the relationship of various types of numbers.*

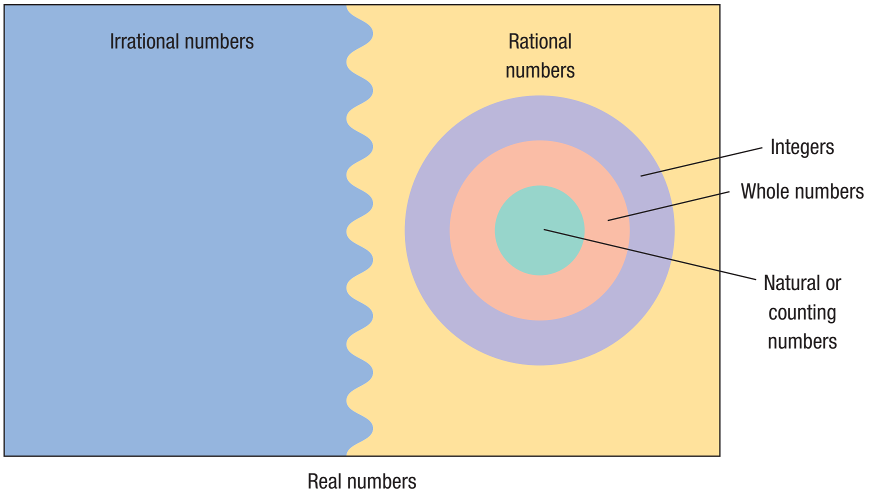


Figure 6

EXAMPLE 4 Classifying the Numbers in a Set

List the numbers in the set

$$\left\{-3, \frac{4}{3}, 0.12, \sqrt{2}, \pi, 10, 2.151515\ldots \text{ (where the block 15 repeats)}\right\}$$

that are

(a) Natural numbers (b) Integers (c) Rational numbers

(d) Irrational numbers (e) Real numbers

Solution

(a) 10 is the only natural number.

(b) -3 and 10 are integers.

(c) $-3, 10, \frac{4}{3}, 0.12$, and $2.151515\ldots$ are rational numbers.

(d) $\sqrt{2}$ and π are irrational numbers.

(e) All the numbers listed are real numbers.

Now Work PROBLEM 25

*The set of real numbers is a subset of the set of complex numbers. We discuss complex numbers in Chapter 1, Section 1.4.

Approximations

Every decimal may be represented by a real number (either rational or irrational), and every real number may be represented by a decimal.

In practice, the decimal representation of an irrational number is given as an approximation. For example, using the symbol $\approx$ (read as "approximately equal to"), we can write

$$\sqrt{2} \approx 1.4142 \qquad \pi \approx 3.1416$$

In approximating decimals, we either *round* or *truncate* to a given number of decimal places.* The number of places establishes the location of the *final digit* in the decimal approximation.

Truncation: Drop all of the digits that follow the specified final digit in the decimal.

Rounding: Identify the specified final digit in the decimal. If the next digit is 5 or more, add 1 to the final digit; if the next digit is 4 or less, leave the final digit as it is. Then truncate following the final digit.

EXAMPLE 5 Approximating a Decimal to Two Places

Approximate 20.98752 to two decimal places by

(a) Truncating
(b) Rounding

Solution For 20.98752, the final digit is 8, since it is two decimal places from the decimal point.

(a) To truncate, we remove all digits following the final digit 8. The truncation of 20.98752 to two decimal places is 20.98.

(b) The digit following the final digit 8 is the digit 7. Since 7 is 5 or more, we add 1 to the final digit 8 and truncate. The rounded form of 20.98752 to two decimal places is 20.99.

EXAMPLE 6 Approximating a Decimal to Two and Four Places

	Number	Rounded to Two Decimal Places	Rounded to Four Decimal Places	Truncated to Two Decimal Places	Truncated to Four Decimal Places
(a)	3.14159	3.14	3.1416	3.14	3.1415
(b)	0.056128	0.06	0.0561	0.05	0.0561
(c)	893.46125	893.46	893.4613	893.46	893.4612

Now Work PROBLEM 29

Significant Digits

There are two types of numbers—*exact* and *approximate*. **Exact numbers** are numbers whose value is known with 100% certainty and accuracy. For example, there are 12 donuts in a dozen donuts, or there are 50 states in the United States.

*Sometimes we say "correct to a given number of decimal places" instead of "truncate."

Approximate numbers are numbers whose value is not known with 100% certainty or whose measurement is inexact. When values are determined from measurements they are typically approximate numbers because the exact measurement is limited by the accuracy of the measuring device and the skill of the individual obtaining the measurement. The **number of significant digits** in a number represents the level of accuracy of the measurement.

The following rules are used to determine the number of significant digits in approximate numbers.

The Number of Significant Digits

- Leading zeros are not significant. For example, 0.0034 has two significant digits.
- Embedded zeros are significant. For example, 208 has three significant digits.
- Trailing zeros are significant only if the decimal point is specified. For example, 2800 has two significant digits. However, if we specify the measurement is accurate to the ones digit, then 2800 has four significant digits.

When performing computations with approximate numbers, it is important not to report the result with more accuracy than the measurements used in the computation.

When performing computations using significant digits, proceed with the computation as you normally would, then round the final answer to the number of significant digits as the least accurately known number. For example, suppose we want to find the area of a rectangle whose width is 1.94 inches (three significant digits) and whose length is 2.7 inches (two significant digits). Because the length has two significant digits, we report the area to two significant digits. The area, $(1.94 \text{ inches})(2.7 \text{ inches}) = 5.238 \text{ square inches}$, can only be written to two significant digits and is reported as 5.2 square inches.

Calculators and Graphing Utilities

Calculators are incapable of displaying decimals that contain a large number of digits. For example, some calculators are capable of displaying only eight digits. When a number requires more than eight digits, the calculator either truncates or rounds. To see how your calculator handles decimals, divide 2 by 3. How many digits do you see? Is the last digit a 6 or a 7? If it is a 6, your calculator truncates; if it is a 7, your calculator rounds.

There are different kinds of calculators. An **arithmetic** calculator can only add, subtract, multiply, and divide numbers; therefore, this type is not adequate for this course. **Scientific** calculators have all the capabilities of arithmetic calculators and contain **function keys** labeled ln, log, sin, cos, tan, x^y, inv, and so on. **Graphing** calculators have all the capabilities of scientific calculators and contain a screen on which graphs can be displayed. As you proceed through this text, you will discover how to use many of the function keys. We use the term **graphing utilities** to refer to all graphing calculators and computer software graphing packages.

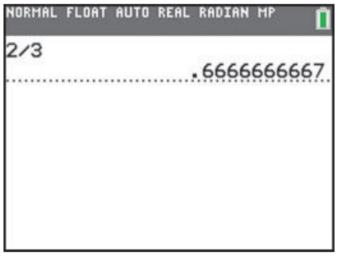


Figure 7

Figure 7 shows $\dfrac{2}{3}$ on a TI-84 Plus CE graphing calculator. How many digits are displayed? Does a TI-84 Plus CE round or truncate? What does your calculator do?

Operations

In algebra, we use letters such as x, y, a, b, and c to represent numbers. The symbols used in algebra for the operations of addition, subtraction, multiplication, and division are $+$, $-$, $\cdot$, and /. The words used to describe the results of these operations are **sum**, **difference**, **product**, and **quotient**. Table 1 on the next page summarizes these ideas.

Table 1

Operation	Symbol	Words
Addition	$a + b$	Sum: a plus b
Subtraction	$a - b$	Difference: a minus b
Multiplication	$a \cdot b$, $(a) \cdot b$, $a \cdot (b)$, $(a) \cdot (b)$, ab, $(a)b$, $a(b)$, $(a)(b)$	Product: a times b
Division	a/b or $\frac{a}{b}$	Quotient: a divided by b

In algebra, we generally avoid using the multiplication sign $\times$ and the division sign $\div$ so familiar in arithmetic. Notice also that when two expressions are placed next to each other without an operation symbol, as in ab, or in parentheses, as in $(a)(b)$, it is understood that the expressions, called **factors**, are to be multiplied.

We also prefer not to use mixed numbers in algebra. When mixed numbers are used, addition is understood; for example, $2\frac{3}{4}$ means $2 + \frac{3}{4}$. In algebra, use of a mixed number may be confusing because the absence of an operation symbol between two terms is generally taken to mean multiplication. The expression $2\frac{3}{4}$ is therefore written instead as 2.75 or as $\frac{11}{4}$.

The symbol $=$, called an **equal sign** and read as "equals" or "is," is used to express the idea that the number or expression on the left of the equal sign is equivalent to the number or expression on the right.

EXAMPLE 7 Writing Statements Using Symbols

(a) The sum of 2 and 7 equals 9. In symbols, this statement is written as $2 + 7 = 9$.

(b) The product of 3 and 5 is 15. In symbols, this statement is written as $3 \cdot 5 = 15$.

Now Work PROBLEM 41

3 Evaluate Numerical Expressions

Consider the expression $2 + 3 \cdot 6$. It is not clear whether we should add 2 and 3 to get 5, and then multiply by 6 to get 30; or first multiply 3 and 6 to get 18, and then add 2 to get 20. To avoid this ambiguity, we have the following agreement.

In Words
Multiply first, then add.

We agree that whenever the two operations of addition and multiplication separate three numbers, the multiplication operation is always performed first, followed by the addition operation.

For $2 + 3 \cdot 6$, then, we have

$$2 + 3 \cdot 6 = 2 + 18 = 20$$

EXAMPLE 8 Finding the Value of an Expression

Evaluate each expression.

(a) $3 + 4 \cdot 5$ (b) $8 \cdot 2 + 1$ (c) $2 + 2 \cdot 2$

Solution

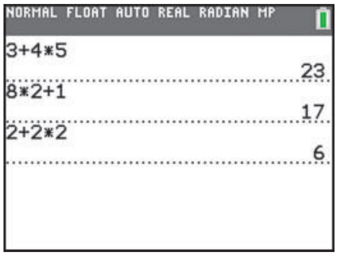


Figure 8

(a) $3 + 4 \cdot 5 = 3 + 20 = 23$ ↑ Multiply first.

(b) $8 \cdot 2 + 1 = 16 + 1 = 17$ ↑ Multiply first.

(c) $2 + 2 \cdot 2 = 2 + 4 = 6$

Figure 8 shows the solution to Example 8 using a TI-84 Plus CE graphing calculator. Notice that the calculator follows the agreed order of operations.

Now Work PROBLEM 53

When we want to indicate adding 3 and 4 and then multiplying the result by 5, we use parentheses and write $(3 + 4) \cdot 5$. Whenever parentheses appear in an expression, it means "perform the operations within the parentheses first!"

EXAMPLE 9 Finding the Value of an Expression

(a) $(5 + 3) \cdot 4 = 8 \cdot 4 = 32$

(b) $(4 + 5) \cdot (8 - 2) = 9 \cdot 6 = 54$

When we divide two expressions, as in

$$\frac{2 + 3}{4 + 8}$$

it is understood that the division bar acts like parentheses; that is,

$$\frac{2 + 3}{4 + 8} = \frac{(2 + 3)}{(4 + 8)}$$

Rules for the Order of Operations

1. Begin with the innermost parentheses and work outward. Remember that in dividing two expressions, we treat the numerator and denominator as if they were enclosed in parentheses.
2. Perform multiplications and divisions, working from left to right.
3. Perform additions and subtractions, working from left to right.

EXAMPLE 10 Finding the Value of an Expression

Evaluate each expression.

(a) $8 \cdot 2 + 3$

(b) $5 \cdot (3 + 4) + 2$

(c) $\dfrac{2 + 5}{2 + 4 \cdot 7}$

(d) $2 + [4 + 2 \cdot (10 + 6)]$

Solution

(a) $8 \cdot 2 + 3 = 16 + 3 = 19$ ↑ Multiply first.

(b) $5 \cdot (3 + 4) + 2 = 5 \cdot 7 + 2 = 35 + 2 = 37$ ↑ Parentheses first ↑ Multiply before adding.

(c) $\dfrac{2 + 5}{2 + 4 \cdot 7} = \dfrac{2 + 5}{2 + 28} = \dfrac{7}{30}$

(d) $2 + [4 + 2 \cdot (10 + 6)] = 2 + [4 + 2 \cdot (16)]$
$= 2 + [4 + 32] = 2 + [36] = 38$

Be careful if you use a calculator. For Example 10(c), you need to use parentheses. See Figure 9(a).* If you don't, the calculator will compute the expression

$$2 + \frac{5}{2} + 4 \cdot 7 = 2 + 2.5 + 28 = 32.5$$

giving a wrong answer.

Another option, when using a TI-84 Plus CE graphing calculator, is to use the fraction template. See Figure 9(b).

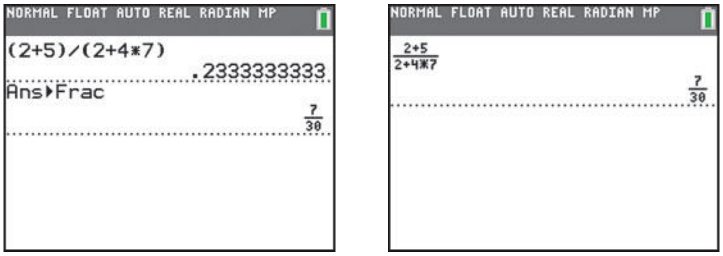


Figure 9 (a) (b)

Now Work PROBLEMS 59 AND 67

4 Work with Properties of Real Numbers

An equal sign is used to mean that one expression is equivalent to another. Four important properties of equality are listed next. In this list, a, b, and c represent real numbers.

- The **reflexive property** states that a number equals itself; that is, $a = a$.
- The **symmetric property** states that if $a = b$, then $b = a$.
- The **transitive property** states that if $a = b$ and $b = c$, then $a = c$.
- The **principle of substitution** states that if $a = b$, then we may substitute b for a in any expression containing a.

Now, let's consider some other properties of real numbers.

EXAMPLE 11 Commutative Properties

(a) $3 + 5 = 8$
$5 + 3 = 8$
$3 + 5 = 5 + 3$

(b) $2 \cdot 3 = 6$
$3 \cdot 2 = 6$
$2 \cdot 3 = 3 \cdot 2$

This example illustrates the *commutative property* of real numbers, which states that the order in which addition or multiplication takes place does not affect the final result.

Commutative Properties

$$a + b = b + a \qquad \textbf{(1a)}$$

$$a \cdot b = b \cdot a \qquad \textbf{(1b)}$$

Here, and in the properties listed on pages 11–14, a, b, and c represent real numbers.

*Notice that we converted the decimal to its fraction form in Figure 9(a). Consult your manual to see how to enter such expressions on your calculator.

EXAMPLE 12 Associative Properties

(a) $2 + (3 + 4) = 2 + 7 = 9$
$(2 + 3) + 4 = 5 + 4 = 9$
$2 + (3 + 4) = (2 + 3) + 4$

(b) $2 \cdot (3 \cdot 4) = 2 \cdot 12 = 24$
$(2 \cdot 3) \cdot 4 = 6 \cdot 4 = 24$
$2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$

The way we add or multiply three real numbers does not affect the final result. Expressions such as $2 + 3 + 4$ and $3 \cdot 4 \cdot 5$ present no ambiguity, even though addition and multiplication are performed on one pair of numbers at a time. This property is called the *associative property*.

Associative Properties

$$a + (b + c) = (a + b) + c = a + b + c \quad \textbf{(2a)}$$
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c = a \cdot b \cdot c \quad \textbf{(2b)}$$

Distributive Property

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad \textbf{(3a)}$$
$$(a + b) \cdot c = a \cdot c + b \cdot c \quad \textbf{(3b)}$$

The distributive property may be used in two different ways.

EXAMPLE 13 Distributive Property

(a) $2 \cdot (x + 3) = 2 \cdot x + 2 \cdot 3 = 2x + 6$ **Use to remove parentheses.**
(b) $3x + 5x = (3 + 5)x = 8x$ **Use to combine two expressions.**
(c) $(x + 2)(x + 3) = x(x + 3) + 2(x + 3) = (x^2 + 3x) + (2x + 6)$
$= x^2 + (3x + 2x) + 6 = x^2 + 5x + 6$

Now Work PROBLEM 89

The real numbers 0 and 1 have unique properties called the *identity properties*.

EXAMPLE 14 Identity Properties

(a) $4 + 0 = 0 + 4 = 4$ (b) $3 \cdot 1 = 1 \cdot 3 = 3$

Identity Properties

$$0 + a = a + 0 = a \quad \textbf{(4a)}$$
$$a \cdot 1 = 1 \cdot a = a \quad \textbf{(4b)}$$

We call 0 the **additive identity** and 1 the **multiplicative identity**.

For each real number a, there is a real number $-a$, called the **additive inverse** of a, having the following property:

Additive Inverse Property

$$a + (-a) = -a + a = 0 \quad \textbf{(5a)}$$

EXAMPLE 15

Finding an Additive Inverse

(a) The additive inverse of 6 is -6, because $6 + (-6) = 0$.

(b) The additive inverse of -8 is 8, because $-8 + 8 = 0$.

The additive inverse of a, that is, $-a$, is often called the *negative* of a. The use of the term 'negative' can be dangerous, because it suggests that the additive inverse is a negative number, which may not be the case. For example, the additive inverse of -3 equals 3, a positive number.

For each *nonzero* real number a, there is a real number $\frac{1}{a}$, called the **multiplicative inverse** of a, having the following property:

Multiplicative Inverse Property

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \qquad \text{if } a \neq 0 \quad \textbf{(5b)}$$

The multiplicative inverse $\frac{1}{a}$ of a nonzero real number a is also referred to as the **reciprocal** of a. The number 0 has no reciprocal.

EXAMPLE 16

Finding a Reciprocal

(a) The reciprocal of 6 is $\frac{1}{6}$, because $6 \cdot \frac{1}{6} = 1$.

(b) The reciprocal of -3 is $\frac{1}{-3}$, because $-3 \cdot \frac{1}{-3} = 1$.

(c) The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, because $\frac{2}{3} \cdot \frac{3}{2} = 1$.

With these properties for adding and multiplying real numbers, we can define the operations of subtraction and division as follows:

DEFINITION Difference

The **difference** $a - b$, also read "a less b" or "a minus b," is defined as

$$a - b = a + (-b) \quad \textbf{(6)}$$

In Words

To subtract b from a, add the additive inverse of b to a.

DEFINITION Quotient

If b is a nonzero real number, the **quotient** $\frac{a}{b}$, also read as "a divided by b" or "the ratio of a to b," is defined as

$$\frac{a}{b} = a \cdot \frac{1}{b} \qquad \text{if } b \neq 0 \tag{7}$$

EXAMPLE 17 Working with Differences and Quotients

(a) $8 - 5 = 8 + (-5) = 3$

(b) $4 - 9 = 4 + (-9) = -5$

(c) $\frac{5}{8} = 5 \cdot \frac{1}{8}$

For any number a, the product of a times 0 is always 0; that is,

In Words
The result of multiplying by zero is zero.

Multiplication by Zero

$$a \cdot 0 = 0 \tag{8}$$

For a nonzero number a,

Division Properties

$$\frac{0}{a} = 0 \qquad \frac{a}{a} = 1 \quad \text{if } a \neq 0 \tag{9}$$

NOTE Division by 0 is not defined. One reason is to avoid the following difficulty: $\frac{2}{0} = x$ means to find x such that $0 \cdot x = 2$. But $0 \cdot x$ equals 0 for all x, so there is no number x for which $\frac{2}{0} = x$.

Rules of Signs

$$a(-b) = -(ab) \qquad (-a)b = -(ab) \qquad (-a)(-b) = ab$$
$$-(-a) = a \qquad \frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b} \text{ if } b \neq 0 \qquad \frac{-a}{-b} = \frac{a}{b} \text{ if } b \neq 0 \tag{10}$$

EXAMPLE 18 Applying the Rules of Signs

(a) $2(-3) = -(2 \cdot 3) = -6$

(b) $(-3)(-5) = 3 \cdot 5 = 15$

(c) $\frac{3}{-2} = \frac{-3}{2} = -\frac{3}{2}$

(d) $\frac{-4}{-9} = \frac{4}{9}$

(e) $\frac{x}{-2} = \frac{1}{-2} \cdot x = -\frac{1}{2}x$

Reduction Properties

$$ac = bc \quad \text{implies} \quad a = b \qquad \text{if } c \neq 0$$
$$\frac{ac}{bc} = \frac{a}{b} \qquad \text{if } b \neq 0,\ c \neq 0 \qquad \textbf{(11)}$$

EXAMPLE 19 Using the Reduction Properties

(a) If $2x = 6$, then

$$2x = 6$$
$$2x = 2 \cdot 3 \qquad \text{Factor 6.}$$
$$x = 3 \qquad \text{Divide out the 2's.}$$

NOTE We follow the common practice of using slash marks to indicate factors dividing out. ■

(b) $\dfrac{18}{12} = \dfrac{3 \cdot \cancel{6}}{2 \cdot \cancel{6}} = \dfrac{3}{2}$

↑ Divide out the 6's. ■

Zero-Product Property

In Words
If a product equals 0, then one or both of the factors is 0.

If $ab = 0$, then $a = 0$, or $b = 0$, or both. **(12)**

EXAMPLE 20 Using the Zero-Product Property

If $2x = 0$, then either $2 = 0$ or $x = 0$. Since $2 \neq 0$, it follows that $x = 0$. ■

Arithmetic of Quotients

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd} \qquad \text{if } b \neq 0,\ d \neq 0 \qquad \textbf{(13)}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \text{if } b \neq 0,\ d \neq 0 \qquad \textbf{(14)}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \qquad \text{if } b \neq 0,\ c \neq 0,\ d \neq 0 \qquad \textbf{(15)}$$

EXAMPLE 21 Adding, Subtracting, Multiplying, and Dividing Quotients

(a) $\dfrac{2}{3} + \dfrac{5}{2} = \dfrac{2 \cdot 2}{3 \cdot 2} + \dfrac{3 \cdot 5}{3 \cdot 2} = \dfrac{2 \cdot 2 + 3 \cdot 5}{3 \cdot 2} = \dfrac{4 + 15}{6} = \dfrac{19}{6}$

↑ By equation (13)

(b) $\dfrac{3}{5} - \dfrac{2}{3} = \dfrac{3}{5} + \left(-\dfrac{2}{3}\right) = \dfrac{3}{5} + \dfrac{-2}{3}$

↑ By equation (6) ↑ By equation (10)

$= \dfrac{3 \cdot 3 + 5 \cdot (-2)}{5 \cdot 3} = \dfrac{9 + (-10)}{15} = \dfrac{-1}{15} = -\dfrac{1}{15}$

↑ By equation (13)

(continued)

NOTE Slanting the slash marks in different directions for different factors, as shown here, is a good practice to follow, since it will help in checking for errors.

(c) $\frac{8}{3}\cdot\frac{15}{4} = \frac{8\cdot 15}{3\cdot 4} = \frac{2\cdot \cancel{4}\cdot \cancel{3}\cdot 5}{\cancel{3}\cdot \cancel{4}\cdot 1} = \frac{2\cdot 5}{1} = 10$

By equation (14) — By equation (11)

(d) $\dfrac{\frac{3}{5}}{\frac{7}{9}} = \frac{3}{5}\cdot\frac{9}{7} = \frac{3\cdot 9}{5\cdot 7} = \frac{27}{35}$

By equation (15) — By equation (14)

NOTE In writing quotients, we follow the usual convention and write the quotient in lowest terms. That is, we write it so that any common factors of the numerator and the denominator have been removed using the Reduction Properties, equation (11). As examples,

$$\frac{90}{24} = \frac{15\cdot \cancel{6}}{4\cdot \cancel{6}} = \frac{15}{4}$$

$$\frac{24x^2}{18x} = \frac{4\cdot \cancel{6}\cdot x\cdot \cancel{x}}{3\cdot \cancel{6}\cdot \cancel{x}} = \frac{4x}{3} \qquad x \neq 0$$

Now Work PROBLEMS 69, 73, AND 83

Sometimes it is easier to add two fractions using the *least common multiple* (LCM). The LCM of two numbers is the smallest number that each has as a common multiple.

EXAMPLE 22 Finding the Least Common Multiple of Two Numbers

Find the least common multiple of 15 and 12.

Solution To find the LCM of 15 and 12, we look at multiples of 15 and 12.

15, 30, 45, **60**, 75, 90, 105, **120**, . . .

12, 24, 36, 48, **60**, 72, 84, 96, 108, **120**, . . .

The *common* multiples are in blue. The *least* common multiple is 60.

EXAMPLE 23 Using the Least Common Multiple to Add Two Fractions

Find: $\frac{8}{15} + \frac{5}{12}$

Solution We use the LCM of the denominators of the fractions and rewrite each fraction using the LCM as a common denominator. The LCM of the denominators (12 and 15) is 60. Rewrite each fraction using 60 as the denominator.

$$\begin{aligned}\frac{8}{15} + \frac{5}{12} &= \frac{8}{15}\cdot\frac{4}{4} + \frac{5}{12}\cdot\frac{5}{5}\\ &= \frac{32}{60} + \frac{25}{60}\\ &= \frac{32 + 25}{60}\\ &= \frac{57}{60}\\ &= \frac{19}{20} \qquad \text{Reduce the fraction.}\end{aligned}$$

Now Work PROBLEM 77

Historical Feature

The real number system has a history that stretches back at least to the ancient Babylonians (1800 BC). It is remarkable how much the ancient Babylonian approach resembles our own. As we stated in the text, the fundamental difficulty with irrational numbers is that they cannot be written as quotients of integers or, equivalently, as repeating or terminating decimals. The Babylonians wrote their numbers in a system based on 60 in the same way that we write ours based on 10. They would carry as many places for π as the accuracy of the problem demanded, just as we now use

$$\pi \approx 3\frac{1}{7} \quad \text{or} \quad \pi \approx 3.1416 \quad \text{or} \quad \pi \approx 3.14159$$

$$\text{or} \quad \pi \approx 3.14159265358979$$

depending on how accurate we need to be.

Things were very different for the Greeks, whose number system allowed only rational numbers. When it was discovered that $\sqrt{2}$ was not a rational number, this was regarded as a fundamental flaw in the number concept. So serious was the matter that the Pythagorean Brotherhood (an early mathematical society) is said to have drowned one of its members for revealing this terrible secret. Greek mathematicians then turned away from the number concept, expressing facts about whole numbers in terms of line segments.

In astronomy, however, Babylonian methods, including the Babylonian number system, continued to be used. Simon Stevin (1548–1620), probably using the Babylonian system as a model, invented the decimal system, complete with rules of calculation, in 1585. [Others, for example, al-Kashi of Samarkand (d. 1429), had made some progress in the same direction.] The decimal system so effectively conceals the difficulties that the need for more logical precision began to be felt only in the early 1800s. Around 1880, Georg Cantor (1845–1918) and Richard Dedekind (1831–1916) gave precise definitions of real numbers. Cantor's definition, although more abstract and precise, has its roots in the decimal (and hence Babylonian) numerical system.

Sets and set theory were a spin-off of the research that went into clarifying the foundations of the real number system. Set theory has developed into a large discipline of its own, and many mathematicians regard it as the foundation upon which modern mathematics is built. Cantor's discoveries that infinite sets can also be counted and that there are different sizes of infinite sets are among the most astounding results of modern mathematics.

R.1 Assess Your Understanding

Concepts and Vocabulary

1. The numbers in the set $\left\{x \middle| x = \frac{a}{b}, \text{ where } a, b \text{ are integers and } b \neq 0\right\}$ are called ________ numbers.
2. The value of the expression $4 + 5 \cdot 6 - 3$ is ____.
3. The fact that $2x + 3x = (2 + 3)x$ is a consequence of the ____________ Property.
4. ***Multiple Choice*** Which of the following represents "the product of 5 and $x + 3$ equals 6"?
   (a) $5 + (x + 3)$ **(b)** $5 \cdot x + 3 = 6$
   (c) $5(x + 3) = 6$ **(d)** None of these
5. ***Multiple Choice*** The intersection of sets A and B is denoted by which of the following?
   (a) $A \cap B$ **(b)** $A \cup B$ **(c)** $A \subseteq B$ **(d)** $A \varnothing B$
6. ***Multiple Choice*** Choose the correct name for the set of numbers $\{0, 1, 2, 3, \ldots\}$.
   (a) Counting numbers **(b)** Whole numbers
   (c) Integers **(d)** Irrational numbers
7. ***True or False*** Rational numbers have decimals that either terminate or are nonterminating with a repeating block of digits.
8. ***True or False*** The Zero-Product Property states that the product of any number and zero equals zero.
9. ***True or False*** The least common multiple of 12 and 18 is 6.
10. ***True or False*** No real number is both rational and irrational.

Skill Building

In Problems 11–22, use $U = \text{universal set} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 4, 5, 9\}$, $B = \{2, 4, 6, 7, 8\}$, *and* $C = \{1, 3, 4, 6\}$ *to find each set.*

11. $A \cup B$ **12.** $A \cup C$ **13.** $A \cap B$ **14.** $A \cap C$

15. $(A \cup B) \cap C$ **16.** $(A \cap B) \cup C$ **17.** $\overline{A}$ **18.** $\overline{C}$

19. $\overline{A \cap B}$ **20.** $\overline{B \cup C}$ **21.** $\overline{A} \cup \overline{B}$ **22.** $\overline{B} \cap \overline{C}$

In Problems 23–28, list the numbers in each set that are (a) Natural numbers, (b) Integers, (c) Rational numbers, (d) Irrational numbers, (e) Real numbers.

23. $A = \left\{-6, \frac{1}{2}, -1.333\ldots \text{ (the 3's repeat)}, \pi, 2, 5\right\}$

24. $B = \left\{-\frac{5}{3}, 2.060606\ldots \text{ (the block 06 repeats)}, 1.25, 0, 1, \sqrt{5}\right\}$

25. $C = \left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

26. $D = \{-1, -1.1, -1.2, -1.3\}$

27. $E = \left\{\sqrt{2}, \pi, \sqrt{2} + 1, \pi + \frac{1}{2}\right\}$

28. $F = \left\{-\sqrt{2}, \pi + \sqrt{2}, \frac{1}{2} + 10.3\right\}$

In Problems 29–40, approximate each number (a) rounded and (b) truncated to three decimal places.

29. 18.9526 **30.** 25.86134 **31.** 28.65319 **32.** 99.05249 **33.** 0.06291 **34.** 0.05388

35. 9.9985 **36.** 1.0006 **37.** $\frac{3}{7}$ **38.** $\frac{5}{9}$ **39.** $\frac{521}{15}$ **40.** $\frac{81}{5}$

In Problems 41–50, write each statement using symbols.

41. The sum of 3 and 2 equals 5

42. The product of 5 and 2 equals 10.

43. The sum of x and 2 is the product of 3 and 4.

44. The sum of 3 and y is the sum of 2 and 2.

45. The product of 3 and y is the sum of 1 and 2.

46. The product of 2 and x is the product of 4 and 6.

47. The difference x less 2 equals 6.

48. The difference 2 less y equals 6.

49. The quotient x divided by 2 is 6.

50. The quotient 2 divided by x is 6.

In Problems 51–88, evaluate each expression.

51. $9 - 4 + 2$ **52.** $6 - 4 + 3$ **53.** $-6 + 4 \cdot 3$ **54.** $8 - 4 \cdot 2$

55. $18 - 5 \cdot 2$ **56.** $100 - 10 \cdot 2$ **57.** $4 + \frac{1}{3}$ **58.** $2 - \frac{1}{2}$

59. $6 - [3 \cdot 5 + 2 \cdot (3 - 2)]$ **60.** $2 \cdot [8 - 3(4 + 2)] - 3$ **61.** $4 \cdot (9 + 5) - 6 \cdot 7 + 3$ **62.** $1 - (4 \cdot 3 - 2 + 2)$

63. $10 - [6 - 2 \cdot 2 + (8 - 3)] \cdot 2$ **64.** $2 - 5 \cdot 4 - [6 \cdot (3 - 4)]$

65. $(5 - 3)\frac{1}{2}$ **66.** $(5 + 4)\frac{1}{3}$ **67.** $\frac{4 + 8}{5 - 3}$ **68.** $\frac{2 - 4}{5 - 3}$

69. $\frac{3}{5} \cdot \frac{10}{21}$ **70.** $\frac{5}{9} \cdot \frac{3}{10}$ **71.** $\frac{6}{25} \cdot \frac{10}{27}$ **72.** $\frac{21}{25} \cdot \frac{100}{3}$

73. $\frac{3}{4} + \frac{2}{5}$ **74.** $\frac{4}{3} + \frac{1}{2}$ **75.** $\frac{7}{8} + \frac{4}{7}$ **76.** $\frac{8}{9} + \frac{15}{2}$

77. $\frac{5}{18} + \frac{1}{12}$ **78.** $\frac{2}{15} + \frac{8}{9}$ **79.** $\frac{5}{24} - \frac{8}{15}$ **80.** $\frac{3}{14} - \frac{2}{21}$

81. $\frac{3}{20} - \frac{2}{15}$ **82.** $\frac{6}{35} - \frac{3}{14}$ **83.** $\dfrac{\frac{5}{18}}{\frac{11}{27}}$ **84.** $\dfrac{\frac{5}{21}}{\frac{2}{35}}$

85. $\frac{1}{3} \cdot \frac{4}{7} + \frac{17}{21}$ **86.** $\frac{2}{3} + \frac{4}{5} \cdot \frac{1}{6}$ **87.** $2 \cdot \frac{3}{4} + \frac{3}{8}$ **88.** $3 \cdot \frac{5}{6} - \frac{1}{2}$

In Problems 89–100, use the Distributive Property to remove the parentheses.

89. $6(x + 4)$ **90.** $4(2x - 1)$ **91.** $x(x - 4)$ **92.** $4x(x + 3)$

93. $2\left(\frac{3}{4}x - \frac{1}{2}\right)$ **94.** $3\left(\frac{2}{3}x + \frac{1}{6}\right)$ **95.** $(x + 2)(x + 4)$ **96.** $(x + 5)(x + 1)$

97. $(x + 9)(2x - 7)$ **98.** $(3x - 1)(x + 5)$ **99.** $(x - 8)(x - 2)$ **100.** $(x - 4)(x - 2)$

101. ***Challenge Problem*** Find k if $3x(x - 5k) = 3x^2 - 60x$.

102. ***Challenge Problem*** Find k if $(x - k)(x + 3k) = x^2 + 4x - 12$.

Explaining Concepts: Discussion and Writing

103. Explain to a friend how the Distributive Property is used to justify the fact that $2x + 3x = 5x$.

104. Explain to a friend why $2 + 3 \cdot 4 = 14$, whereas $(2 + 3) \cdot 4 = 20$.

105. Explain why $2(3 \cdot 4)$ is not equal to $(2 \cdot 3) \cdot (2 \cdot 4)$.

106. Explain why $\frac{4 + 3}{2 + 5}$ is not equal to $\frac{4}{2} + \frac{3}{5}$.

107. Is subtraction commutative? Support your conclusion with an example.

108. Is subtraction associative? Support your conclusion with an example.

109. Is division commutative? Support your conclusion with an example.

110. Is division associative? Support your conclusion with an example.

111. If $2 = x$, why does $x = 2$?

112. If $x = 5$, why does $x^2 + x = 30$?

113. Are there any real numbers that are both rational and irrational? Are there any real numbers that are neither? Explain your reasoning.

114. Explain why the sum of a rational number and an irrational number must be irrational.

115. A rational number is defined as the quotient of two integers. When written as a decimal, the decimal will either repeat or terminate. By looking at the denominator of the rational number, there is a way to tell in advance whether its decimal representation will repeat or terminate. Make a list of rational numbers and their decimals. See if you can discover the pattern. Confirm your conclusion by consulting books on number theory at the library. Write a brief essay on your findings.

116. The current time is 12 noon CST. What time (CST) will it be 12,997 hours from now?

117. Both $\frac{a}{0}\ (a \neq 0)$ and $\frac{0}{0}$ are undefined, but for different reasons. Write a paragraph or two explaining the different reasons.

R.2 Algebra Essentials

OBJECTIVES
1. Graph Inequalities (p. 19)
2. Find Distance on the Real Number Line (p. 20)
3. Evaluate Algebraic Expressions (p. 21)
4. Determine the Domain of a Variable (p. 22)
5. Use the Laws of Exponents (p. 23)
6. Evaluate Square Roots (p. 24)
7. Use a Calculator to Evaluate Exponents (p. 25)
8. Use Scientific Notation (p. 25)

The Real Number Line

Real numbers can be represented by points on a line called the **real number line**. There is a one-to-one correspondence between real numbers and points on a line. That is, every real number corresponds to a point on the line, and each point on the line has a unique real number associated with it.

Pick a point on a line somewhere in the center, and label it O. This point, called the **origin**, corresponds to the real number 0. See Figure 10. The point 1 unit to the right of O corresponds to the number 1. The distance between 0 and 1 determines the **scale** of the number line. For example, the point associated with the number 2 is twice as far from O as 1. Notice that an arrowhead on the right end of the line indicates the direction in which the numbers increase. Points to the left of the origin correspond to the real numbers -1, -2, and so on. Figure 10 also shows the points associated with the rational numbers $-\frac{1}{2}$ and $\frac{1}{2}$ and with the irrational numbers $\sqrt{2}$ and π.

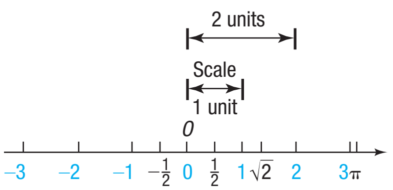


Figure 10 Real number line

> **DEFINITION Coordinate; Real Number Line**
>
> The real number associated with a point P is called the **coordinate** of P, and the line whose points have been assigned coordinates is called the **real number line**.

Now Work PROBLEM 13

The real number line consists of three classes of real numbers, as shown in Figure 11.

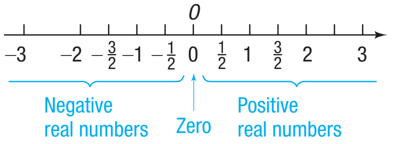


Figure 11

- The **negative real numbers** are the coordinates of points to the left of the origin O.
- The real number **zero** is the coordinate of the origin O.
- The **positive real numbers** are the coordinates of points to the right of the origin O.

Multiplication Properties of Positive and Negative Numbers

- The product of two positive numbers is a positive number.
- The product of two negative numbers is a positive number.
- The product of a positive number and a negative number is a negative number.

1 Graph Inequalities

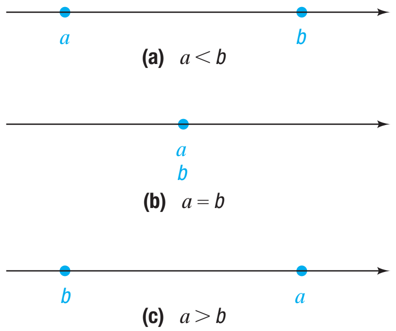


Figure 12

An important property of the real number line follows from the fact that, given two numbers a and b, either a is to the left of b, or a is at the same location as b, or a is to the right of b. See Figure 12.

If a is to the left of b, then "a is less than b," which is written $a < b$. If a is to the right of b, then "a is greater than b," which is written $a > b$. If a is at the same location as b, then $a = b$. If a is either less than or equal to b, then $a \leq b$. Similarly, $a \geq b$ means that a is either greater than or equal to b. Collectively, the symbols $<$, $>$, $\leq$, and $\geq$ are called **inequality symbols**.

Note that $a < b$ and $b > a$ mean the same thing. It does not matter whether we write $2 < 3$ or $3 > 2$.

Furthermore, if $a < b$ or if $b > a$, then the difference $b - a$ is positive. Do you see why?

EXAMPLE 1 **Using Inequality Symbols**

(a) $3 < 7$ (b) $-8 > -16$ (c) $-6 < 0$

(d) $-8 < -4$ (e) $4 > -1$ (f) $8 > 0$

In Example 1(a), we conclude that $3 < 7$ either because 3 is to the left of 7 on the real number line or because the difference, $7 - 3 = 4$, is a positive real number.

Similarly, we conclude in Example 1(b) that $-8 > -16$ either because -8 lies to the right of -16 on the real number line or because the difference, $-8 - (-16) = -8 + 16 = 8$, is a positive real number.

Look again at Example 1. Note that the inequality symbol always points in the direction of the smaller number.

An **inequality** is a statement in which two expressions are related by an inequality symbol. The expressions are referred to as the **sides** of the inequality. Inequalities of the form $a < b$ or $b > a$ are called **strict inequalities**, whereas inequalities of the form $a \leq b$ or $b \geq a$ are called **nonstrict inequalities**.

Based on the discussion so far, we conclude that

$a > 0$ is equivalent to a is positive

$a < 0$ is equivalent to a is negative

We sometimes read $a > 0$ by saying that "a is positive." If $a \geq 0$, then either $a > 0$ or $a = 0$, and we may read this as "a is nonnegative."

Now Work PROBLEMS 17 AND 27