



**LIAL HORNSBY**

**SCHNEIDER DANIELS**

# COLLEGE ALGEBRA



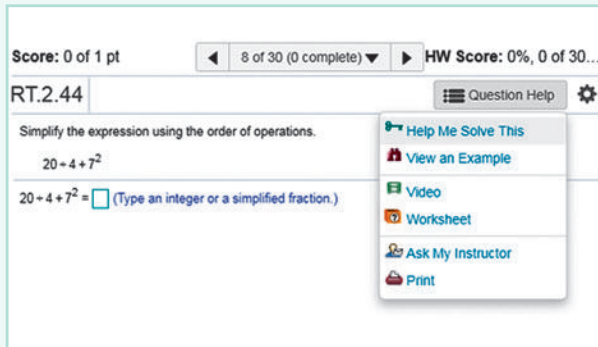
thirteenth edition

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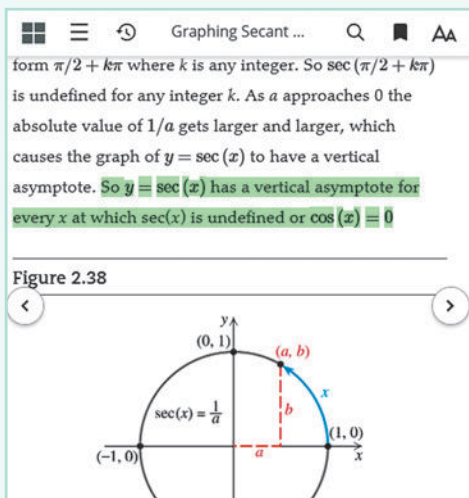
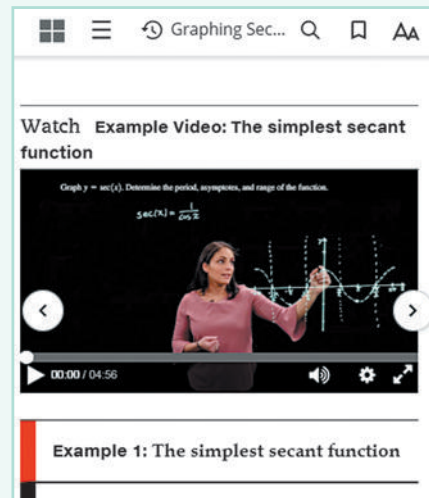


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## Interactive eText

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# College Algebra

THIRTEENTH EDITION

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# College Algebra

THIRTEENTH EDITION

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**This text is dedicated to you—the student. We hope that it helps you achieve your goals. Remember to show up, work hard, and stay positive. Everything else will take care of itself.**

**The Lial Author Team**

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## WELCOME TO THE 13TH EDITION

In the thirteenth edition of *College Algebra*, we continue our ongoing commitment to providing the best possible text to help instructors teach and students succeed. In this edition, we have remained true to the pedagogical style of the past while staying focused on the needs of today's students. Support for all classroom types (traditional, corequisite, flipped, hybrid, and online) may be found in this classic text and its supplements backed by the power of Pearson's MyLab Math.

In this edition, we have drawn on the extensive teaching experience of the Lial team, with special consideration given to reviewer suggestions. General updates include enhanced readability as we continually strive to make math understandable for students, updates to our extensive list of applications and real-world mathematics problems, use of color in displays and side comments, and coordination of exercises and their related examples.

The authors understand that teaching and learning mathematics today can be a challenging task. Some students are prepared for the challenge, while other students require more review and supplemental material. This text is written so that college algebra students with varying abilities and backgrounds will all have an opportunity for a successful learning experience.

The Lial team believes this to be our best edition of *College Algebra* yet, and we sincerely hope that you enjoy using it as much as we have enjoyed writing it. Additional texts in this series are as follows.

*Trigonometry*, Twelfth Edition  
*College Algebra and Trigonometry*, Seventh Edition  
*Precalculus*, Seventh Edition

## HIGHLIGHTS OF NEW CONTENT

- **Chapter R** has been expanded to include more of the basic concepts many college algebra students struggle with. It begins with new **Section R.1 Fractions, Decimals, and Percents**. Additional new topics have been inserted throughout the chapter, including operations with signed numbers (**Section R.3**), dividing a polynomial by a monomial (**Section R.5**), and factoring expressions with negative and rational exponents (**Section R.6**). Topics throughout the chapter have been reorganized for improved flow.

Instructors may choose to cover review topics from **Chapter R** at the beginning of a course or to insert these topics as-needed in a just-in-time fashion. Either way, students who are under-prepared for the demands of college algebra, as well as those who need a quick review, will benefit from the material contained here.

- Careful attention was given in **Chapters 1 and 2** to inserting additional exercises where needed. Specifically, **Section 2.3 Functions** has new exercises that use analytic methods to determine maximum and minimum values of a function. **Section 2.6 Graphs of Basic Functions** has new exercises and applications using the greatest integer function. **Section 2.4 Linear Functions** includes enhanced discussion of the average rate of change of a linear function. This topic is then related to the difference quotient and the average rate of change of a nonlinear function in **Section 2.8 Function Operations and Composition**.


- **Chapter 3** includes new **Section 3.6 Polynomial and Rational Inequalities**. This section features a visual approach to solving such inequalities by interpreting the graphs of related functions.
- In response to reviewer suggestions, **Section 4.3 Logarithmic Functions** has new exercises that relate exponential and logarithmic functions as inverses. **Section 5.2 Matrix Solution of Linear Systems** now includes a new example and related exercises that use Gaussian elimination to solve linear systems of equations.
- **Section 6.2 Ellipses** and **Section 6.3 Hyperbolas** each include a new example and related exercises in which completing the square is used to find the standard form of an ellipse or a hyperbola.

## FEATURES OF THIS TEXT

### SUPPORT FOR LEARNING CONCEPTS


We provide a variety of features to support students' learning of the essential topics of college algebra. Explanations that are written in understandable terms, figures and graphs that illustrate examples and concepts, graphing technology that supports and enhances algebraic manipulations, and real-life applications that enrich the topics with meaning all provide opportunities for students to deepen their understanding of mathematics. These features help students make mathematical connections and expand their own knowledge base.

- **Examples** Numbered examples that illustrate the techniques for working exercises are found in every section. We use traditional explanations, side comments, and pointers to describe the steps taken—and to warn students about common pitfalls. Some examples provide additional graphing calculator solutions, although these can be omitted if desired.
- **Now Try Exercises** Following each numbered example, the student is directed to try a corresponding odd-numbered exercise (or exercises). This feature allows for quick feedback to determine whether the student understands the principles illustrated in the example.
- **Real-Life Applications** We have included hundreds of real-life applications, many with data updated from the previous edition. They come from fields such as business, entertainment, sports, biology, astronomy, geology, and environmental studies.
- **Function Boxes** Beginning in Chapter 2, functions provide a unifying theme throughout the text. Special function boxes offer a comprehensive, visual introduction to each type of function and also serve as an excellent resource for reference and review. Each function box includes a table of values, traditional and calculator-generated graphs, the domain, the range, and other special information about the function. These boxes are assignable in MyLab Math.
- **Figures and Photos** Today's students are more visually oriented than ever before, and we have updated the figures and photos in this edition to promote visual appeal. Guided Visualizations with accompanying exercises and explorations are available and assignable in MyLab Math.
- **Cautions and Notes** Text that is marked **CAUTION** warns students of common errors, and **NOTE** comments point out explanations that should receive particular attention.

- **Looking Ahead to Calculus** These margin notes offer glimpses of how the topics currently being studied are used in calculus.
- **Use of Graphing Technology** We have integrated the use of graphing calculators where appropriate, although *this technology is completely optional and can be omitted without loss of continuity*. We continue to stress that graphing calculators support understanding but that students must first master the underlying mathematical concepts. Exercises that require the use of a graphing calculator are marked with the icon .

## SUPPORT FOR PRACTICING CONCEPTS

This text offers a wide variety of exercises to help students master college algebra. The extensive exercise sets provide ample opportunity for practice and increase in difficulty so that students at every level of understanding are challenged. The variety of exercise types promotes mastery of the concepts and reduces the need for rote memorization.

- **Concept Preview** Each exercise set begins with a group of **CONCEPT PREVIEW** exercises designed to promote understanding of vocabulary and basic concepts of each section. These new exercises are assignable in MyLab Math and provide support especially for hybrid, online, and flipped courses.
- **Exercise Sets** In addition to traditional drill exercises, this text includes writing exercises, optional graphing calculator exercises , and multiple-choice, matching, true/false, and completion exercises. Those marked *Concept Check* focus on conceptual thinking. *Connecting Graphs with Equations* exercises challenge students to write equations that correspond to given graphs. Video solutions for select problems are available in MyLab Math.
- **Relating Concepts Exercises** Appearing at the end of selected exercise sets, these groups of exercises are designed so that students who work them in numerical order will follow a line of reasoning that leads to an understanding of how various topics and concepts are related. All answers to these exercises appear in the student answer section, and these exercises are assignable in MyLab Math.

## SUPPORT FOR REVIEW AND TEST PREP

Ample opportunities for review are found both within the chapters and at the ends of chapters. Quizzes interspersed within chapters provide a quick assessment of students' understanding of the material presented up to that point in the chapter. Chapter Test Preps provide comprehensive study aids to help students prepare for tests.

- **Quizzes** Students can periodically check their progress with in-chapter quizzes that appear in all chapters, beginning with Chapter 1. All answers, with corresponding section references, appear in the student answer section. These quizzes are assignable in MyLab Math.
- **Summary Exercises** These sets of in-chapter exercises give students the all-important opportunity to work *mixed* review exercises, requiring them to synthesize concepts and select appropriate solution methods.

- **End-of-Chapter Test Prep** Following the final numbered section in each chapter, the Test Prep provides a list of **Key Terms**, a list of **New Symbols** (if applicable), and a two-column **Quick Review** that includes a section-by-section summary of concepts and examples. This feature concludes with a comprehensive set of **Review Exercises** and a **Chapter Test**. The Test Prep, Review Exercises, and Chapter Test are assignable in MyLab Math.

# Get the *most* out of MyLab Math



## MyLab Math for College Algebra 13e (access code required)

MyLab Math is tightly integrated with each author's style, offering a range of author-created resources, to give your students a consistent experience.

## Preparedness

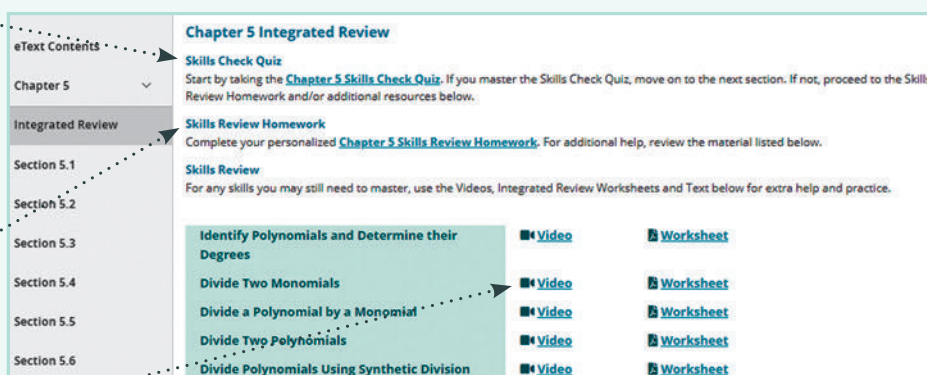
Preparedness is one of the biggest challenges in many math courses. Pearson offers a variety of content and course options to support students with just-in-time remediation and key-concept review as needed.

## Integrated Review in MyLab Math

Integrated Review can be used in corequisite courses or simply to help students who enter a course without a full understanding of prerequisite skills and concepts. Premade, editable Integrated Review assignments are available to assign in the Assignment Manager. Integrated Review landing pages (shown below) are visible by default at the start of most chapters, providing objective-level review.

- Students begin each chapter by completing a Skills Check to pinpoint which topics, if any, they need to review.

- Personalized review homework provides extra support for students who need it on just the topics they didn't master in the preceding Skills Check.



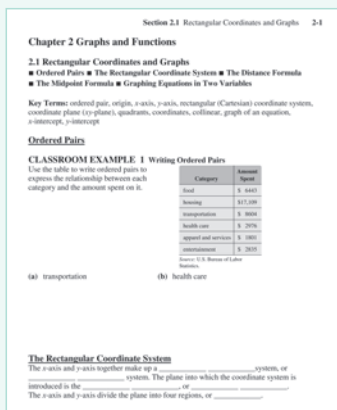
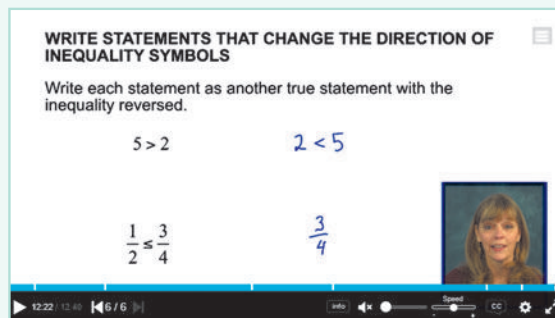
- Additional review materials, including worksheets and videos, are available.

# Get the *most* out of MyLab Math



## Updated Videos

Updated videos cover all topics in the text to support students outside of the classroom. **Quick Review** videos cover key definitions and procedures. **Example Solution** videos offer a detailed solution process for every example in the textbook.



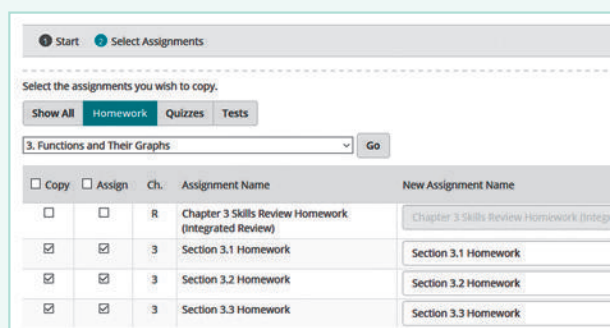
## Updated! MyNotes and MyClassroomExamples

**MyNotes** give students a note-taking structure to use while they read the text or watch the MyLab Math videos. **MyClassroomExamples** offer structure for notes taken during lecture, and are for use with the ClassroomExamples found in the Annotated Instructor Edition.

Both sets of notes are available in MyLab Math and can be customized by the instructor.

## New! Enhanced Sample Assignments

Author Callie Daniels makes course set-up easier by giving instructors a starting point for each section. Following Callie's best practices in the classroom, Enhanced Sample Assignments maximize students' performance.



- **Section Prep Assignments** include Example Videos with assessment questions. This assignment pairs with **MyNotes**. Students actively participate while taking notes from the Example Video and then work the related exercises.
- **Section Homework** includes author-selected problems and increases in difficulty.
- **Cumulative Review Homework Assignments** draw from section homework questions covered to that point in the course—helping students prepare for a final exam.

# Resources for Success



## Instructor Resources

Online resources can be downloaded at [pearson.com/mylab/math](http://pearson.com/mylab/math) or from **www.pearson.com**.

### Annotated Instructor's Edition

ISBN: 0135922003 / 9780135922002

Answers are included on the same page beside the text exercises where possible for quick reference. It includes helpful Teaching Tips and Classroom Examples.

### Online Instructor's Solution Manual

*By Beverly Fusfield*

Provides complete solutions to all text exercises

### Online Instructor's Testing Manual

Includes diagnostic pretests, grouped by section, with answers provided

### Testgen®

TestGen ([www.pearsoned.com/testgen](http://www.pearsoned.com/testgen)) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

### PowerPoint® Lecture Slides and Classroom Example PowerPoints

- The PowerPoint Lecture Slides feature presentations written and designed specifically for this text, including figures and examples from the text.
- Classroom Example PowerPoints include fully worked-out solutions to all Classroom Examples.

### Learning Catalytics™

With MyLab Math, you and your students have access to Learning Catalytics, which you can use to generate class discussion, guide your lecture, and actively engage students. Prebuilt Learning Catalytics questions have been created specifically for this text. Simply search the tag "LialCollegeAlgebra" within the Learning Catalytics Question Library.

## Student Resources

Additional resources enhance student success.

### Student's Solution Manual

*By Beverly Fusfield*

Provides detailed solutions to all odd-numbered text exercises

### Video Lectures

- Quick Reviews cover key definitions and procedures from each section.
- Example Solutions walk students through the detailed solution process for every example in the textbook.

### MyNotes with Integrated Review Worksheets

ISBN: 0135822130 / 9780135822135

**MyNotes** offer structure for students as they watch videos or read the text. These are available as a printed supplement and in MyLab Math.

- Includes textbook examples along with ample space for students to write solutions and notes
- Includes key concepts along with prompts for students to read, write, and reflect on what they have just learned
- **Customizable**—instructors can add their own examples or remove examples that are not covered in their course.

**Integrated Review Worksheets** prepare students for the College Algebra material.

- Includes key terms, guided examples with ample space for students to work, and references to extra help in MyLab Math

### MyClassroomExamples

- Available in MyLab Math and offer structure for classroom lecture
- Includes Classroom Examples along with ample space for students to write solutions and notes
- Includes key concepts along with fill-in-the-blank opportunities to keep students engaged
- **Customizable**—instructors can add their own examples or remove Classroom Examples that are not covered in their course.



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As an author team, we are committed to providing the best possible college algebra course to help instructors teach and students succeed. As we continue to work toward this goal, we welcome any comments or suggestions you might send, via e-mail, to [math@pearson.com](mailto:math@pearson.com).

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# College Algebra

THIRTEENTH EDITION

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# R

## Review of Basic Concepts

Positive and negative numbers, used to represent gains and losses on a board such as this one, are examples of *real numbers* encountered in applications of mathematics.

- R.1** Fractions, Decimals, and Percents
- R.2** Sets and Real Numbers
- R.3** Real Number Operations and Properties
- R.4** Integer and Rational Exponents
- R.5** Polynomials
- R.6** Factoring Polynomials
- R.7** Rational Expressions
- R.8** Radical Expressions

## R.1 Fractions, Decimals, and Percents

- **Lowest Terms of a Fraction**
- **Improper Fractions and Mixed Numbers**
- **Operations with Fractions**
- **Decimals as Fractions**
- **Operations with Decimals**
- **Fractions as Decimals**
- **Percents as Decimals and Decimals as Percents**
- **Percents as Fractions and Fractions as Percents**

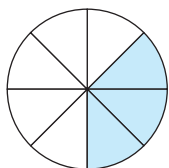
Recall that **fractions** are a way to represent parts of a whole. See **Figure 1**. In a fraction, the **numerator** gives the number of parts being represented. The **denominator** gives the total number of equal parts in the whole. The fraction bar represents division ( $\frac{a}{b} = a \div b$ ).

Fraction bar  $\rightarrow$   $\frac{3}{8}$   $\leftarrow$  Numerator  
 $\leftarrow$  Denominator

A fraction is classified as either a **proper fraction** or an **improper fraction**.

Proper fractions  $\frac{1}{5}, \frac{2}{7}, \frac{9}{10}, \frac{23}{25}$  Numerator is less than denominator.  
 Value is less than 1.

Improper fractions  $\frac{3}{2}, \frac{5}{5}, \frac{11}{7}, \frac{28}{4}$  Numerator is greater than or equal  
 to denominator. Value is greater  
 than or equal to 1.



The shaded region represents  $\frac{3}{8}$  of the circle.

Figure 1

### Lowest Terms of a Fraction

A fraction is in **lowest terms** when the numerator and denominator have no factors in common (other than 1).

### Writing a Fraction in Lowest Terms

**Step 1** Write the numerator and denominator in factored form.

**Step 2** Replace each pair of factors common to the numerator and denominator with 1.

**Step 3** Multiply the remaining factors in the numerator and in the denominator.

(This procedure is sometimes called “**simplifying the fraction**.”)

### EXAMPLE 1 Writing Fractions in Lowest Terms

Write each fraction in lowest terms.

(a)  $\frac{10}{15}$

(b)  $\frac{15}{45}$

(c)  $\frac{150}{200}$

#### SOLUTION

(a)  $\frac{10}{15} = \frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3} \cdot \frac{5}{5} = \frac{2}{3} \cdot 1 = \frac{2}{3}$  5 is the greatest common factor of 10 and 15.

(b)  $\frac{15}{45} = \frac{1 \cdot 15}{3 \cdot 15} = \frac{1}{3} \cdot 1 = \frac{1}{3}$  Remember to write 1 in the numerator.

(c)  $\frac{150}{200} = \frac{3 \cdot 50}{4 \cdot 50} = \frac{3}{4} \cdot 1 = \frac{3}{4}$  50 is the greatest common factor of 150 and 200.

Another strategy is to choose *any* common factor and work in stages.

$\frac{150}{200} = \frac{15 \cdot 10}{20 \cdot 10} = \frac{3 \cdot 5 \cdot 10}{4 \cdot 5 \cdot 10} = \frac{3}{4} \cdot 1 \cdot 1 = \frac{3}{4}$  The same answer results.

✓ Now Try Exercises 7 and 15.

**Improper Fractions and Mixed Numbers**

A **mixed number** is a single number that represents the sum of a natural (counting) number and a proper fraction.

$$\text{Mixed number} \rightarrow 2\frac{3}{4} = 2 + \frac{3}{4}$$

**EXAMPLE 2** Converting an Improper Fraction to a Mixed Number

Write  $\frac{59}{8}$  as a mixed number.

**SOLUTION** Because the fraction bar represents division ( $\frac{a}{b} = a \div b$ , or  $b \overline{)a}$ ), divide the numerator of the improper fraction by the denominator.

$$\begin{array}{r} \text{Denominator of fraction} \rightarrow 8 \overline{)59} \begin{array}{l} \text{7} \leftarrow \text{Quotient} \\ \text{56} \leftarrow \text{Remainder} \end{array} \end{array} \quad \frac{59}{8} = 7\frac{3}{8}$$

✓ Now Try Exercise 17.

**EXAMPLE 3** Converting a Mixed Number to an Improper Fraction

Write  $6\frac{4}{7}$  as an improper fraction.

**SOLUTION** Multiply the denominator of the fraction by the natural number, and then add the numerator to obtain the numerator of the improper fraction.

$$7 \cdot 6 = 42 \quad \text{and} \quad 42 + 4 = 46$$

The denominator of the improper fraction is the same as the denominator in the mixed number, which is 7 here.

$$6\frac{4}{7} = \frac{7 \cdot 6 + 4}{7} = \frac{46}{7}$$

✓ Now Try Exercise 21.

**Operations with Fractions**

Figure 2 illustrates multiplying fractions.

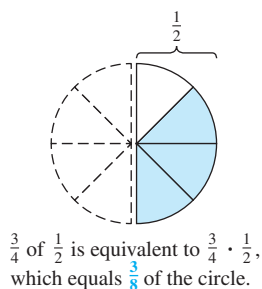


Figure 2

**Multiplying Fractions**

If  $\frac{a}{b}$  and  $\frac{c}{d}$  are fractions ( $b \neq 0, d \neq 0$ ), then  $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ .

That is, to multiply two fractions, multiply their numerators and then multiply their denominators.

**EXAMPLE 4** Multiplying Fractions

Multiply  $\frac{3}{8} \cdot \frac{4}{9}$ . Write the answer in lowest terms.

**SOLUTION**

$$\begin{aligned}\frac{3}{8} \cdot \frac{4}{9} &= \frac{3 \cdot 4}{8 \cdot 9} && \begin{array}{l} \text{Multiply numerators.} \\ \text{Multiply denominators.} \end{array} \\ &= \frac{12}{72} && \text{Multiply.} \\ &= \frac{1 \cdot 12}{6 \cdot 12} && \begin{array}{l} \text{The greatest common factor} \\ \text{of 12 and 72 is 12.} \end{array}\end{aligned}$$

Make sure the product is in lowest terms.

$$= \frac{1}{6} \quad \frac{1 \cdot 12}{6 \cdot 12} = \frac{1}{6} \cdot 1 = \frac{1}{6}$$

✓ Now Try Exercise 27.

Two numbers are **reciprocals** of each other if their product is 1. For example,

$$\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12}, \text{ or } 1.$$

Division is the inverse or opposite of multiplication, and as a result, we use reciprocals to divide fractions. **Figure 3** illustrates dividing fractions.

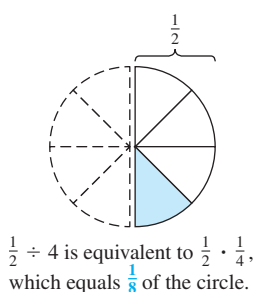


Figure 3

**Dividing Fractions**

If  $\frac{a}{b}$  and  $\frac{c}{d}$  are fractions ( $b \neq 0, d \neq 0, c \neq 0$ ), then  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ .

That is, to divide by a fraction, multiply by its reciprocal.

**EXAMPLE 5** Dividing Fractions

Divide. Write answers in lowest terms as needed.

(a)  $\frac{3}{4} \div \frac{8}{5}$

(b)  $\frac{5}{8} \div 10$

(c)  $1\frac{2}{3} \div 4\frac{1}{2}$

**SOLUTION**

(a)  $\frac{3}{4} \div \frac{8}{5}$

$$= \frac{3}{4} \cdot \frac{5}{8} \quad \begin{array}{l} \text{Multiply by } \frac{5}{8}, \text{ the} \\ \text{reciprocal of } \frac{8}{5}. \end{array}$$

$$= \frac{3 \cdot 5}{4 \cdot 8} \quad \begin{array}{l} \text{Multiply numerators.} \\ \text{Multiply denominators.} \end{array}$$

$$= \frac{15}{32} \quad \begin{array}{l} \text{Make sure the answer} \\ \text{is in lowest terms.} \end{array}$$

(b)  $\frac{5}{8} \div 10$

Think of 10 as  $\frac{10}{1}$  here.

$$= \frac{5}{8} \cdot \frac{1}{10} \quad \begin{array}{l} \text{Multiply by } \frac{1}{10}, \\ \text{the reciprocal of 10.} \end{array}$$

$$= \frac{5 \cdot 1}{8 \cdot 2 \cdot 5} \quad \text{Multiply and factor.}$$

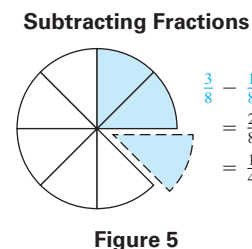
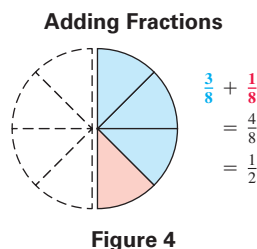
$$= \frac{1}{16} \quad \begin{array}{l} \text{Remember to write 1} \\ \text{in the numerator.} \end{array}$$



$$\begin{aligned}
 \text{(c)} \quad & 1\frac{2}{3} \div 4\frac{1}{2} \\
 &= \frac{5}{3} \div \frac{9}{2} \quad \text{Write each mixed number as an improper fraction.} \\
 &= \frac{5}{3} \cdot \frac{2}{9} \quad \text{Multiply by } \frac{2}{9}, \text{ the reciprocal of } \frac{9}{2}. \\
 &= \frac{10}{27} \quad \text{Multiply. The quotient is in lowest terms.}
 \end{aligned}$$

✓ Now Try Exercises 37, 43, and 49.

Figures 4 and 5 illustrate adding and subtracting fractions.



### Adding and Subtracting Fractions

If  $\frac{a}{b}$  and  $\frac{c}{b}$  are fractions ( $b \neq 0$ ), then add or subtract as follows.

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

That is, to add or subtract two fractions having the same denominator, add or subtract the numerators and keep the same denominator.

**If the denominators are different, first find the least common denominator (LCD).** Write each fraction as an equivalent fraction with this denominator. Then add or subtract as above.

### EXAMPLE 6 Adding and Subtracting Fractions

Add or subtract as indicated. Write answers in lowest terms as needed.

$$\text{(a)} \quad \frac{2}{10} + \frac{3}{10} \qquad \text{(b)} \quad \frac{4}{15} + \frac{5}{9} \qquad \text{(c)} \quad \frac{15}{6} - \frac{4}{9} \qquad \text{(d)} \quad 4\frac{1}{2} - 1\frac{3}{4}$$

#### SOLUTION

$$\begin{aligned}
 \text{(a)} \quad & \frac{2}{10} + \frac{3}{10} \\
 &= \frac{2+3}{10} \quad \text{Add numerators. Keep the same denominator.} \\
 &= \frac{5}{10} \\
 &= \frac{1}{2} \quad \text{Write in lowest terms.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{4}{15} + \frac{5}{9} \\
 &= \frac{4}{15} \cdot \frac{3}{3} + \frac{5}{9} \cdot \frac{5}{5} \quad \begin{array}{l} 15 = 3 \cdot 5 \text{ and } \\ 9 = 3 \cdot 3, \text{ so the } \\ \text{LCD is } 3 \cdot 3 \cdot 5 = 45. \end{array} \\
 &= \frac{12}{45} + \frac{25}{45} \quad \text{Write equivalent fractions with the common denominator.} \\
 &= \frac{37}{45} \quad \text{Add numerators. Keep the same denominator.}
 \end{aligned}$$

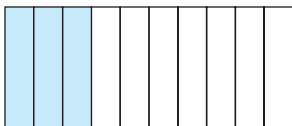
$$\begin{aligned}
 \text{(c)} \quad & \frac{15}{16} - \frac{4}{9} \\
 &= \frac{15}{16} \cdot \frac{9}{9} - \frac{4}{9} \cdot \frac{16}{16} \quad \text{Because 16 and 9 have no common factors except 1, the LCD is } 16 \cdot 9 = 144. \\
 &= \frac{135}{144} - \frac{64}{144} \quad \text{Write equivalent fractions with the common denominator.} \\
 &= \frac{71}{144} \quad \text{Subtract numerators.} \\
 & \quad \text{Keep the common denominator.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d) Method 1} \quad & 4\frac{1}{2} - 1\frac{3}{4} \\
 &= \frac{9}{2} - \frac{7}{4} \quad \text{Write each mixed number as an improper fraction.} \\
 &= \frac{18}{4} - \frac{7}{4} \quad \text{Find a common denominator. The LCD is 4.} \\
 &= \frac{11}{4}, \text{ or } 2\frac{3}{4} \quad \text{Subtract. Write as a mixed number.}
 \end{aligned}$$

Think:  $\frac{9}{2} \cdot \frac{2}{2} = \frac{18}{4}$

$$\begin{aligned}
 \text{Method 2} \quad & 4\frac{1}{2} = 4\frac{2}{4} = 3\frac{6}{4} \quad \text{The LCD is 4.} \\
 & 4\frac{2}{4} = 3 + 1 + \frac{2}{4} = 3 + \frac{4}{4} + \frac{2}{4} = 3\frac{6}{4} \\
 & -1\frac{3}{4} = -1\frac{3}{4} = -1\frac{3}{4} \\
 & \hline
 & 2\frac{3}{4}, \text{ or } \frac{11}{4} \quad \text{The same answer results.}
 \end{aligned}$$

✓ Now Try Exercises 55, 57, 71, and 75.



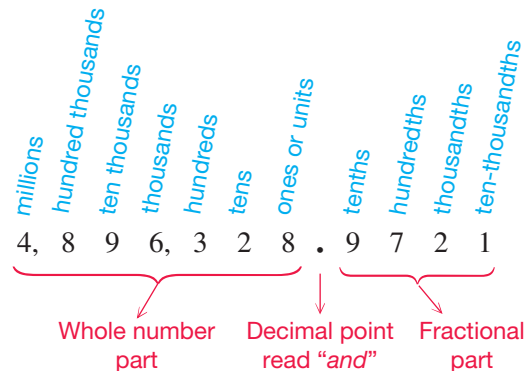
3 parts of the whole 10 are shaded. As a fraction,  $\frac{3}{10}$  of the figure is shaded. As a decimal, 0.3 is shaded. Both of these numbers are read “three-tenths.”

FIGURE 6

Fractions are one way to represent parts of a whole. Another way is with a decimal fraction or **decimal**, a number written with a decimal point.

9.25, 14.001, 0.3 Decimal numbers

See Figure 6. Each digit in a decimal number has a place value, as shown below.



*Each successive place value is ten times greater than the place value to its right and one-tenth as great as the place value to its left.*



## Decimals as Fractions

## Converting a Decimal to a Fraction

Read the decimal using the correct place value. Write it in fractional form just as it is read.

- The numerator will be the digits to the right of the decimal point.
- The denominator will be a power of 10—that is, 10 for tenths, 100 for hundredths, and so on.

## EXAMPLE 7 Writing Decimals as Fractions

Write each decimal as a fraction. (Do not write in lowest terms.)

- (a) 0.95                      (b) 0.056                      (c) 4.2095

## SOLUTION

- (a) We read 0.95 as “*ninety-five hundredths*.”

$$0.95 = \frac{95}{100} \leftarrow \text{For hundredths}$$

- (b) We read 0.056 as “*fifty-six thousandths*.”

Do not confuse 0.056 with 0.56,  
read “fifty-six hundredths,”  
which is the fraction  $\frac{56}{100}$ .

$$0.056 = \frac{56}{1000} \leftarrow \text{For thousandths}$$

- (c) We read 4.2095, which is greater than 1, as “*Four and two thousand ninety-five ten-thousandths*.”

$$4.2095 = 4 \frac{2095}{10,000} \quad \text{Write the decimal number as a mixed number.}$$

Think:  $10,000 \cdot 4 + 2095$

$$= \frac{42,095}{10,000} \quad \text{Write the mixed number as an improper fraction.}$$

✓ Now Try Exercises 85, 89, and 91.

## Operations with Decimals

## EXAMPLE 8 Adding and Subtracting Decimals

Add or subtract as indicated.

- (a)  $6.92 + 14.8 + 3.217$                       (b)  $47.6 - 32.509$

## SOLUTION

- (a) Place the digits of the decimal numbers in columns by place value. Attach zeros as placeholders.

$$\begin{array}{r} 6.92 \\ 14.8 \\ + 3.217 \\ \hline \end{array} \quad \text{Be sure to line up decimal points.} \quad \text{becomes} \quad \begin{array}{r} 6.920 \\ 14.800 \\ + 3.217 \\ \hline 24.937 \end{array} \quad \begin{array}{l} \text{Attach 0s.} \\ 6.92 \text{ is equivalent to } 6.920. \\ 14.8 \text{ is equivalent to } 14.800. \end{array}$$

- (b)  $\begin{array}{r} 47.6 \\ - 32.509 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 47.600 \\ - 32.509 \\ \hline 15.091 \end{array} \quad \text{Write the decimal numbers in columns, attaching 0s to } 47.6.$

✓ Now Try Exercises 93 and 97.

**EXAMPLE 9** Multiplying and Dividing Decimals

Multiply or divide as indicated. In part (c), round the answer to two decimal places.

(a)  $29.3 \times 4.52$

(b)  $0.05 \times 0.3$

(c)  $8.949 \div 1.25$

**SOLUTION**

- (a) Multiply normally. Place the decimal point in the answer as shown.

$$\begin{array}{r} 29.3 \\ \times 4.52 \\ \hline 586 \\ 1465 \\ 1172 \\ \hline 132.436 \end{array}$$

1 decimal place  
2 decimal places  
 $1 + 2 = 3$   
3 decimal places

- (b) Here  $5 \times 3 = 15$ . Be careful placing the decimal point.

$$\begin{array}{r} 2 \text{ decimal places} \quad 1 \text{ decimal place} \\ 0.05 \quad \times \quad 0.3 \end{array}$$

Do not write 0.150.  $= 0.015$

$2 + 1 = 3$  decimal places  
Attach 0 as a placeholder in the tenths place.

- (c)  $1.25 \overline{)8.949}$  Move each decimal point two places to the right.

$$\begin{array}{r} 7.159 \\ 125 \overline{)894.900} \\ \underline{875} \phantom{00} \\ 199 \phantom{00} \\ \underline{125} \phantom{00} \\ 740 \phantom{00} \\ \underline{625} \phantom{00} \\ 1150 \phantom{00} \\ \underline{1125} \phantom{00} \\ 25 \phantom{00} \end{array}$$

Move the decimal point straight up, and divide as with whole numbers. Attach 0s as placeholders.

We carried out the division to three decimal places so that we could round the answer to two decimal places, obtaining 7.16.

✓ Now Try Exercises 105 and 113.

**NOTE** To round 7.159 in **Example 9(c)** to two decimal places (that is, to the nearest hundredth), we look at the digit to the *right* of the hundredths place.

- If this digit is 5 or greater, we round up.
- If this digit is less than 5, we drop the digit(s) beyond the desired place.

Hundredths place



7.159    9, the digit to the right of the hundredths place, is 5 or greater.  
 $\approx 7.16$     Round 5 up to 6.  $\approx$  means *is approximately equal to*.

**Multiplying and Dividing by Powers of 10 (Shortcuts)**

- To *multiply* by a power of 10, *move the decimal point to the right* as many places as the number of zeros.
- To *divide* by a power of 10, *move the decimal point to the left* as many places as the number of zeros.

*In both cases, insert 0s as placeholders if necessary.*

**EXAMPLE 10** Multiplying and Dividing by Powers of 10

Multiply or divide as indicated.

(a)  $48.731 \times 10$

(b)  $48.731 \times 1000$

(c)  $48.731 \div 10$

(d)  $48.731 \div 1000$

**SOLUTION**

(a)  $48.731 \times 10$

$$= 48.731$$

Move the decimal point one place to the right.

$$= 487.31$$

(b)  $48.731 \times 1000$

$$= 48.731$$

Move the decimal point three places to the right.

$$= 48,731$$

(c)  $48.731 \div 10$

$$= 48.731$$

Move the decimal point one place to the left.

$$= 4.8731$$

(d)  $48.731 \div 1000$

$$= 048.731$$

Move the decimal point three places to the left.

$$= 0.048731$$

✓ **Now Try Exercises 117 and 125.****Fractions as Decimals****Converting a Fraction to a Decimal**

Because a fraction bar indicates division, write a fraction as a decimal by dividing the numerator by the denominator.

**EXAMPLE 11** Writing Fractions as Decimals

Write each fraction as a decimal.

(a)  $\frac{19}{8}$

(b)  $\frac{2}{3}$

**SOLUTION**

(a)  $\frac{19}{8}$

$$\begin{array}{r} 2.375 \\ 8 \overline{)19.000} \\ \underline{16} \phantom{000} \\ 30 \phantom{00} \\ \underline{24} \phantom{00} \\ 60 \phantom{00} \\ \underline{56} \phantom{00} \\ 40 \phantom{00} \\ \underline{40} \phantom{00} \\ 0 \end{array}$$

Divide 19 by 8. Add a decimal point and as many 0s as necessary to 19.

(b)  $\frac{2}{3}$

$$\begin{array}{r} 0.6666 \dots \\ 3 \overline{)2.0000 \dots} \\ \underline{18} \phantom{0000} \\ 20 \phantom{000} \\ \underline{18} \phantom{000} \\ 20 \phantom{000} \\ \underline{18} \phantom{000} \\ 20 \phantom{000} \\ \underline{18} \phantom{000} \\ 2 \end{array}$$

$$\frac{19}{8} = 2.375 \leftarrow \text{Terminating decimal}$$

$$\frac{2}{3} = 0.6666 \dots \leftarrow \text{Repeating decimal}$$

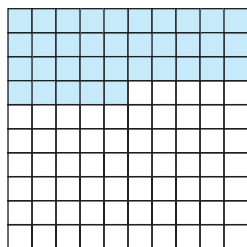
$$= 0.\overline{6}$$

A bar is written over the repeating digit(s).

$$\approx 0.667$$

Nearest thousandth

✓ **Now Try Exercises 141 and 147.**



35 of the 100 squares are shaded. That is,  $\frac{35}{100}$ , or **35%**, of the figure is shaded.

Figure 7

### Percents as Decimals and Decimals as Percents

The word **percent** means “*per 100*.” Percent is written with the symbol **%**. “*One percent*” means “*one per one hundred*,” or “*one one-hundredth*.” See Figure 7.

### Percent, Fraction, and Decimal Equivalents

$$1\% = \frac{1}{100} = 0.01, \quad 10\% = \frac{10}{100} = 0.10, \quad 100\% = \frac{100}{100} = 1$$

For example, 73% means “73 *per one hundred*.”

$$73\% = \frac{73}{100} = 0.73$$

Essentially, we are dropping the % symbol from 73% and dividing 73 by 100. Doing this moves the decimal point, which is understood to be after the 3, two places to the *left*.

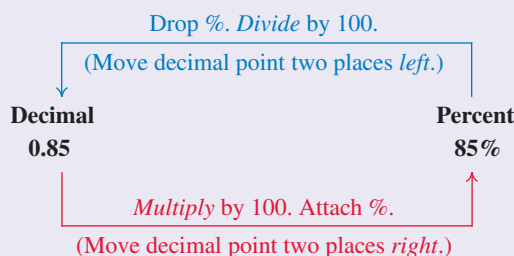
Writing 0.73 as a percent is the opposite process.

$$0.73 = 0.73 \cdot 100\% = 73\% \quad 100\% = 1$$

Moving the decimal point two places to the *right* and attaching a % symbol gives the same result.

### Converting Percents and Decimals (Shortcuts)

- To convert a percent to a decimal, move the decimal point two places to the *left* and drop the % symbol.
- To convert a decimal to a percent, move the decimal point two places to the *right* and attach a % symbol.



### EXAMPLE 12 Converting Percents and Decimals by Moving the Decimal Point

Convert each percent to a decimal and each decimal to a percent.

- (a) 45%    (b) 250%    (c) 9%    (d) 0.57    (e) 1.5    (f) 0.007

#### SOLUTION

- (a)  $45\% = 0.45$     (b)  $250\% = 2.50$ , or 2.5    (c)  $9\% = 0.09$   
 (d)  $0.57 = 57\%$     (e)  $1.5 = 1.50 = 150\%$     (f)  $0.007 = 0.7\%$

✓ Now Try Exercises 151 and 161.

## Percents as Fractions and Fractions as Percents

**EXAMPLE 13** Writing Percents as Fractions

Write each percent as a fraction. Give answers in lowest terms.

(a) 8%

(b) 175%

(c) 13.5%

**SOLUTION**(a) We use the fact that *percent* means “per one hundred,” and convert as follows.

$$8\% = \frac{8}{100} = \frac{2 \cdot 4}{25 \cdot 4} = \frac{2}{25}$$

As with converting percents to decimals, drop the % symbol and divide by 100.

Write in lowest terms.

$$(b) 175\% = \frac{175}{100} = \frac{7 \cdot 25}{4 \cdot 25} = \frac{7}{4}, \text{ or } 1\frac{3}{4}$$

A number greater than 1 is more than 100%.

Write in lowest terms.

(c) 13.5%

$$= \frac{13.5}{100}$$

Drop the % symbol. Divide by 100.

$$= \frac{13.5}{100} \cdot \frac{10}{10}$$

Multiply by 1 in the form  $\frac{10}{10}$  to eliminate the decimal in the numerator.

$$= \frac{135}{1000}$$

Multiply.

$$= \frac{27}{200}$$

Write in lowest terms.  
 $\frac{135}{1000} = \frac{27 \cdot 5}{200 \cdot 5} = \frac{27}{200}$

✓ Now Try Exercises 171, 177, and 179.

**EXAMPLE 14** Writing Fractions as Percents

Write each fraction as a percent.

(a)  $\frac{2}{5}$

(b)  $\frac{1}{6}$

**SOLUTION**

(a)  $\frac{2}{5}$

$$= \frac{2}{5} \cdot 100\%$$

Multiply by 1 in the form 100%.

$$= \frac{2}{5} \cdot \frac{100}{1} \%$$

$$= \frac{2 \cdot 5 \cdot 20}{5 \cdot 1} \%$$

Multiply and factor.

$$= \frac{2 \cdot 20}{1} \%$$

Divide out the common factor.

$$= 40\%$$

Simplify.

(b)  $\frac{1}{6}$

$$= \frac{1}{6} \cdot 100\%$$

$$= \frac{1}{6} \cdot \frac{100}{1} \%$$

$$= \frac{1 \cdot 2 \cdot 50}{2 \cdot 3 \cdot 1} \%$$

$$= \frac{50}{3} \%$$

$$= 16\frac{2}{3} \%, \text{ or } 16.\overline{6}\%$$

✓ Now Try Exercises 181 and 185.

# R.1 Exercises

**CONCEPT PREVIEW** Determine whether each statement is true or false. If it is false, explain why.

1. In the fraction  $\frac{5}{8}$ , 5 is the numerator and 8 is the denominator.
2. The mixed number equivalent of the improper fraction  $\frac{31}{5}$  is  $6\frac{1}{5}$ .
3. The fraction  $\frac{7}{7}$  is proper.
4. The reciprocal of  $\frac{6}{2}$  is  $\frac{3}{1}$ .

**CONCEPT PREVIEW** Choose the letter of the correct response.

5. Which choice shows the correct way to write  $\frac{16}{24}$  in lowest terms?  
 A.  $\frac{16}{24} = \frac{8+8}{8+16} = \frac{8}{16} = \frac{1}{2}$       B.  $\frac{16}{24} = \frac{4 \cdot 4}{4 \cdot 6} = \frac{4}{6}$       C.  $\frac{16}{24} = \frac{8 \cdot 2}{8 \cdot 3} = \frac{2}{3}$
6. Which fraction is *not* equal to  $\frac{5}{9}$ ?  
 A.  $\frac{15}{27}$       B.  $\frac{30}{54}$       C.  $\frac{40}{74}$       D.  $\frac{55}{99}$

Write each fraction in lowest terms. See Example 1.

7.  $\frac{8}{16}$
8.  $\frac{4}{12}$
9.  $\frac{15}{18}$
10.  $\frac{16}{20}$
11.  $\frac{90}{150}$
12.  $\frac{100}{140}$
13.  $\frac{18}{90}$
14.  $\frac{16}{64}$
15.  $\frac{120}{144}$
16.  $\frac{77}{132}$

Write each improper fraction as a mixed number. See Example 2.

17.  $\frac{12}{7}$
18.  $\frac{16}{9}$
19.  $\frac{77}{12}$
20.  $\frac{67}{13}$

Write each mixed number as an improper fraction. See Example 3.

21.  $2\frac{3}{5}$
22.  $5\frac{6}{7}$
23.  $12\frac{2}{3}$
24.  $10\frac{1}{5}$

Multiply or divide as indicated. Write answers in lowest terms as needed. See Examples 4 and 5.

25.  $\frac{4}{5} \cdot \frac{6}{7}$
26.  $\frac{5}{9} \cdot \frac{2}{7}$
27.  $\frac{2}{15} \cdot \frac{3}{8}$
28.  $\frac{3}{20} \cdot \frac{5}{21}$
29.  $\frac{1}{10} \cdot \frac{12}{5}$
30.  $\frac{1}{8} \cdot \frac{10}{7}$
31.  $\frac{15}{4} \cdot \frac{8}{25}$
32.  $\frac{21}{8} \cdot \frac{4}{7}$
33.  $21 \cdot \frac{3}{7}$
34.  $36 \cdot \frac{4}{9}$
35.  $3\frac{1}{4} \cdot 1\frac{2}{3}$
36.  $2\frac{2}{3} \cdot 1\frac{3}{5}$
37.  $\frac{7}{9} \div \frac{3}{2}$
38.  $\frac{6}{11} \div \frac{5}{4}$
39.  $\frac{5}{4} \div \frac{3}{8}$
40.  $\frac{7}{5} \div \frac{3}{10}$
41.  $\frac{32}{5} \div \frac{8}{15}$
42.  $\frac{24}{7} \div \frac{6}{21}$
43.  $\frac{3}{4} \div 12$
44.  $\frac{2}{5} \div 30$
45.  $6 \div \frac{3}{5}$
46.  $8 \div \frac{4}{9}$
47.  $6\frac{3}{4} \div \frac{3}{8}$
48.  $5\frac{3}{5} \div \frac{7}{10}$
49.  $2\frac{1}{2} \div 1\frac{5}{7}$
50.  $2\frac{2}{9} \div 1\frac{2}{5}$
51.  $2\frac{5}{8} \div 1\frac{15}{32}$
52.  $2\frac{3}{10} \div 1\frac{4}{5}$

Add or subtract as indicated. Write answers in lowest terms as needed. See Example 6.

53.  $\frac{7}{15} + \frac{4}{15}$

54.  $\frac{2}{9} + \frac{5}{9}$

55.  $\frac{7}{12} + \frac{1}{12}$

56.  $\frac{3}{16} + \frac{5}{16}$

57.  $\frac{5}{9} + \frac{1}{3}$

58.  $\frac{4}{15} + \frac{1}{5}$

59.  $\frac{3}{8} + \frac{5}{6}$

60.  $\frac{5}{6} + \frac{2}{9}$

61.  $\frac{5}{9} + \frac{3}{16}$

62.  $\frac{3}{4} + \frac{6}{25}$

63.  $3\frac{1}{8} + 2\frac{1}{4}$

64.  $4\frac{2}{3} + 2\frac{1}{6}$

65.  $\frac{7}{9} - \frac{2}{9}$

66.  $\frac{8}{11} - \frac{3}{11}$

67.  $\frac{13}{15} - \frac{3}{15}$

68.  $\frac{11}{12} - \frac{3}{12}$

69.  $\frac{7}{12} - \frac{1}{3}$

70.  $\frac{5}{6} - \frac{1}{2}$

71.  $\frac{7}{12} - \frac{1}{9}$

72.  $\frac{11}{16} - \frac{1}{12}$

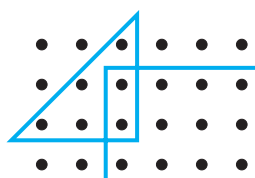
73.  $4\frac{3}{4} - 1\frac{2}{5}$

74.  $3\frac{4}{5} - 1\frac{4}{9}$

75.  $8\frac{2}{9} - 4\frac{2}{3}$

76.  $7\frac{5}{12} - 4\frac{5}{6}$

Work each problem involving fractions.



77. Refer to the figure in the margin. For each description, write a fraction in lowest terms that represents the region described.

- (a) The dots in the large rectangle as a part of the dots in the entire figure
- (b) The dots in the triangle as a part of the dots in the entire figure
- (c) The dots in the overlapping region of the triangle and the rectangle as a part of the dots in the triangle alone; As a part of the rectangle alone

78. At the conclusion of the Pearson softball league season, batting statistics for five players were as shown in the table.

Player	At-Bats	Hits	Home Runs
Benita	36	12	3
Christine	40	9	2
Chase	11	5	1
Chin	16	8	0
Greg	20	10	2

Use this information to answer each question. Estimate as necessary.

- (a) Which player got a hit in exactly  $\frac{1}{3}$  of their at-bats?
- (b) Which player got a home run in just less than  $\frac{1}{10}$  of their at-bats?
- (c) Which player got a hit in just less than  $\frac{1}{4}$  of their at-bats?
- (d) Which two players got hits in exactly the same fractional part of their at-bats? What was that fractional part, expressed in lowest terms?

**Concept Check** Provide the correct response.

79. In the decimal number 367.9412, name the digit that has each place value.

- (a) tens    (b) tenths    (c) thousandths    (d) ones or units    (e) hundredths

80. Write a decimal number that has 5 in the thousands place, 0 in the tenths place, and 4 in the ten-thousandths place.

81. For the decimal number 46.249, round to the place value indicated.

- (a) hundredths    (b) tenths    (c) ones or units    (d) tens

82. Round each decimal to the nearest thousandth.

- (a)  $0.\bar{8}$     (b)  $0.\bar{4}$     (c) 0.9762    (d) 0.8645

Write each decimal as a fraction. (Do not write in lowest terms.) See Example 7.

83. 0.4      84. 0.6      85. 0.64      86. 0.82      87. 0.138  
 88. 0.104      89. 0.043      90. 0.087      91. 3.805      92. 5.166

Add or subtract as indicated. See Example 8.

93.  $25.32 + 109.2 + 8.574$       94.  $90.527 + 32.43 + 589.8$       95.  $28.73 - 3.12$   
 96.  $46.88 - 13.45$       97.  $43.5 - 28.17$       98.  $345.1 - 56.31$   
 99.  $3.87 + 15 + 2.9$       100.  $8.2 + 1.09 + 12$       101.  $32.56 + 47.356 + 1.8$   
 102.  $75.2 + 123.96 + 3.897$       103.  $18 - 2.789$       104.  $29 - 8.582$

Multiply or divide as indicated. See Example 9.

105.  $12.8 \times 9.1$       106.  $34.04 \times 0.56$       107.  $22.41 \times 33$       108.  $55.76 \times 72$   
 109.  $0.2 \times 0.03$       110.  $0.07 \times 0.004$       111.  $78.65 \div 11$       112.  $73.36 \div 14$   
 113.  $32.48 \div 11.6$       114.  $85.26 \div 17.4$       115.  $19.967 \div 9.74$       116.  $44.4788 \div 5.27$

Multiply or divide as indicated. See Example 10.

117.  $123.26 \times 10$       118.  $785.91 \times 10$       119.  $57.116 \times 100$       120.  $82.053 \times 100$   
 121.  $0.094 \times 1000$       122.  $0.025 \times 1000$       123.  $1.62 \div 10$       124.  $8.04 \div 10$   
 125.  $124.03 \div 100$       126.  $490.35 \div 100$       127.  $23.29 \div 1000$       128.  $59.8 \div 1000$

**Concept Check** Complete the table of fraction, decimal, and percent equivalents.

	Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
129.	$\frac{1}{100}$	0.01	
130.	$\frac{1}{50}$		2%
131.		0.05	5%
132.	$\frac{1}{10}$		
133.	$\frac{1}{8}$	0.125	
134.			20%
135.	$\frac{1}{4}$		
136.	$\frac{1}{3}$		
137.			50%
138.	$\frac{2}{3}$		$66\frac{2}{3}\%$ , or $66.\overline{6}\%$
139.		0.75	
140.	1	1.0	

Write each fraction as a decimal. For repeating decimals, write the answer by first using bar notation and then rounding to the nearest thousandth. See Example 11.

141.  $\frac{21}{5}$       142.  $\frac{9}{5}$       143.  $\frac{9}{4}$       144.  $\frac{15}{4}$       145.  $\frac{3}{8}$   
 146.  $\frac{7}{8}$       147.  $\frac{5}{9}$       148.  $\frac{8}{9}$       149.  $\frac{1}{6}$       150.  $\frac{5}{6}$



Write each percent as a decimal. See Examples 12(a)–12(c).

151. 54%      152. 39%      153. 7%      154. 4%      155. 117%  
 156. 189%      157. 2.4%      158. 3.1%      159.  $6\frac{1}{4}\%$       160.  $5\frac{1}{2}\%$

Write each decimal as a percent. See Examples 12(d)–12(f).

161. 0.79      162. 0.83      163. 0.02      164. 0.08      165. 0.004  
 166. 0.005      167. 1.28      168. 2.35      169. 0.4      170. 0.6

Write each percent as a fraction. Give answers in lowest terms. See Example 13.

171. 51%      172. 47%      173. 15%      174. 35%      175. 2%  
 176. 8%      177. 140%      178. 180%      179. 7.5%      180. 2.5%

Write each fraction as a percent. See Example 14.

181.  $\frac{4}{5}$       182.  $\frac{3}{25}$       183.  $\frac{7}{50}$       184.  $\frac{9}{20}$       185.  $\frac{2}{11}$   
 186.  $\frac{4}{9}$       187.  $\frac{9}{4}$       188.  $\frac{8}{5}$       189.  $\frac{13}{6}$       190.  $\frac{31}{9}$

## R.2 Sets and Real Numbers

- Basic Definitions
- Operations on Sets
- Sets of Numbers and the Number Line

### Basic Definitions

A **set** is a collection of objects. The objects that belong to a set are its **elements**, or **members**. In algebra, the elements of a set are usually numbers. Sets are commonly written using **set braces**,  $\{ \}$ .

$\{1, 2, 3, 4\}$     The set containing the elements 1, 2, 3, and 4

The order in which the elements are listed is not important. This set can also be written  $\{4, 3, 2, 1\}$  or with any other arrangement of the four numbers.

To show that 4 is an element of the set  $\{1, 2, 3, 4\}$ , we use the symbol  $\in$ .

$$4 \in \{1, 2, 3, 4\}$$

Since 5 is *not* an element of this set, we place a slash through the symbol  $\in$ .

$$5 \notin \{1, 2, 3, 4\}$$

It is customary to name sets with capital letters.

$$S = \{1, 2, 3, 4\} \quad S \text{ is used to name the set.}$$

Set  $S$  was written above by listing its elements. Set  $S$  might also be described as “the set containing the first four counting numbers.”

The set  $F$ , consisting of all fractions between 0 and 1, is an example of an **infinite set**—one that has an unending list of distinct elements. A **finite set** is one that has a limited number of elements. The process of counting its elements comes to an end. Some infinite sets can be described by listing. For example, the set of numbers  $N$  used for counting, which are the **natural numbers** or the **counting numbers**, can be written as follows.

$$N = \{1, 2, 3, 4, \dots\} \quad \text{Natural (counting) numbers}$$

The three dots (*ellipsis points*) show that the list of elements of the set continues according to the established pattern.



Every element of the set  $S = \{1, 2, 3, 4\}$  is a natural number.  $S$  is an example of a *subset* of the set  $N$  of natural numbers. This relationship is written using the symbol  $\subseteq$ .

$$S \subseteq N$$

By definition, set  $A$  is a **subset** of set  $B$  if every element of set  $A$  is also an element of set  $B$ . For example, if  $A = \{2, 5, 9\}$  and  $B = \{2, 3, 5, 6, 9, 10\}$ , then  $A \subseteq B$ . However, there are some elements of  $B$  that are not in  $A$ , so  $B$  is not a subset of  $A$ . This relationship is written using the symbol  $\not\subseteq$ .

$$B \not\subseteq A$$

Every set is a subset of itself. Also,  $\emptyset$  is a subset of every set.

**If  $A$  is any set, then  $A \subseteq A$  and  $\emptyset \subseteq A$ .**

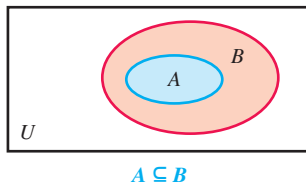


Figure 8

**Figure 8** shows a set  $A$  that is a subset of set  $B$ . The rectangle in the drawing represents the universal set  $U$ . Such a diagram is a **Venn diagram**.

Two sets  $A$  and  $B$  are equal whenever  $A \subseteq B$  and  $B \subseteq A$ . Equivalently,  $A = B$  if the two sets contain exactly the same elements. For example,

$$\{1, 2, 3\} = \{3, 1, 2\}$$

is true because both sets contain exactly the same elements. However,

$$\{1, 2, 3\} \neq \{0, 1, 2, 3\}$$

because the set  $\{0, 1, 2, 3\}$  contains the element 0, which is not an element of  $\{1, 2, 3\}$ .

### EXAMPLE 3 Examining Subset Relationships

Let  $U = \{1, 3, 5, 7, 9, 11, 13\}$ ,  $A = \{1, 3, 5, 7, 9, 11\}$ ,  $B = \{1, 3, 7, 9\}$ ,  $C = \{3, 9, 11\}$ , and  $D = \{1, 9\}$ . Determine whether each statement is *true* or *false*.

- (a)  $D \subseteq B$       (b)  $B \subseteq D$       (c)  $C \not\subseteq A$       (d)  $U = A$

### SOLUTION

- (a) All elements of  $D$ , namely 1 and 9, are also elements of  $B$ , so  $D$  is a subset of  $B$ , and  $D \subseteq B$  is true.
- (b) There is at least one element of  $B$  (for example, 3) that is not an element of  $D$ , so  $B$  is *not* a subset of  $D$ . Thus,  $B \subseteq D$  is false.
- (c)  $C$  is a subset of  $A$ , because every element of  $C$  is also an element of  $A$ . Thus,  $C \subseteq A$  is true, and as a result,  $C \not\subseteq A$  is false.
- (d)  $U$  contains the element 13, but  $A$  does not. Therefore,  $U = A$  is false.

✓ **Now Try Exercises 53, 55, 63, and 65.**

### Operations on Sets

Given a set  $A$  and a universal set  $U$ , the set of all elements of  $U$  that do *not* belong to set  $A$  is the **complement** of set  $A$ . For example, if set  $A$  is the set of all students in a class 30 years old or older, and set  $U$  is the set of all students in the class, then the complement of  $A$  would be the set of all students in the class younger than age 30.

The complement of set  $A$  is written  $A'$  (read “*A-prime*”). The Venn diagram in **Figure 9** shows a set  $A$ . Its complement,  $A'$ , is in color. Using set-builder notation, the complement of set  $A$  is described as follows.

$$A' = \{x | x \in U, x \notin A\}$$

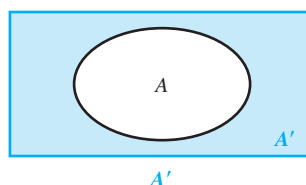


Figure 9

**EXAMPLE 4** Finding Complements of Sets

Let  $U = \{1, 2, 3, 4, 5, 6, 7\}$ ,  $A = \{1, 3, 5, 7\}$ , and  $B = \{3, 4, 6\}$ . Find each set.

- (a)  $A'$       (b)  $B'$       (c)  $\emptyset'$       (d)  $U'$

**SOLUTION**

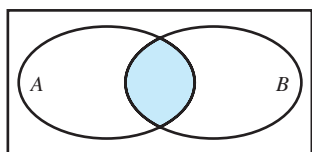
(a) Set  $A'$  contains the elements of  $U$  that are not in  $A$ . Thus,  $A' = \{2, 4, 6\}$ .

- (b)  $B' = \{1, 2, 5, 7\}$       (c)  $\emptyset' = U$       (d)  $U' = \emptyset$

✓ Now Try Exercise 89.

Given two sets  $A$  and  $B$ , the set of all elements belonging to both set  $A$  and set  $B$  is the **intersection** of the two sets, written  $A \cap B$ . For example, if  $A = \{1, 2, 4, 5, 7\}$  and  $B = \{2, 4, 5, 7, 9, 11\}$ , then we have the following.

$$A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 4, 5, 7, 9, 11\} = \{2, 4, 5, 7\}$$



$A \cap B$

Figure 10

The Venn diagram in **Figure 10** shows two sets  $A$  and  $B$ . Their intersection,  $A \cap B$ , is in color. Using set-builder notation, the intersection of sets  $A$  and  $B$  is described as follows.

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

Two sets that have no elements in common are **disjoint sets**. If  $A$  and  $B$  are any two disjoint sets, then  $A \cap B = \emptyset$ . For example, there are no elements common to both  $\{50, 51, 54\}$  and  $\{52, 53, 55, 56\}$ , so these two sets are disjoint.

$$\{50, 51, 54\} \cap \{52, 53, 55, 56\} = \emptyset$$

**EXAMPLE 5** Finding Intersections of Two Sets

Find each of the following. Identify any disjoint sets.

- (a)  $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\}$   
 (b)  $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\}$       (c)  $\{1, 3, 5\} \cap \{2, 4, 6\}$

**SOLUTION**

- (a)  $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\} = \{15, 25\}$

The elements 15 and 25 are the only ones belonging to both sets.

- (b)  $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\} = \{2, 3, 4\}$

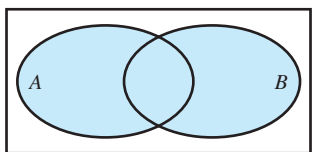
- (c)  $\{1, 3, 5\} \cap \{2, 4, 6\} = \emptyset$  Disjoint sets

✓ Now Try Exercises 69, 75, and 85.

The set of all elements belonging to set  $A$  or to set  $B$  (or to both) is the **union** of the two sets, written  $A \cup B$ . For example, if  $A = \{1, 3, 5\}$  and  $B = \{3, 5, 7, 9\}$ , then we have the following.

$$A \cup B = \{1, 3, 5\} \cup \{3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}$$

The Venn diagram in **Figure 11** shows two sets  $A$  and  $B$ . Their union,  $A \cup B$ , is in color. Using set-builder notation, the union of sets  $A$  and  $B$  is described as follows.



$A \cup B$

Figure 11

$$A \cup B = \{x | x \in A \text{ or } x \in B \text{ or } x \in \text{both } A \text{ and } B\}$$

**EXAMPLE 6** Finding Unions of Two Sets

Find each of the following.

(a)  $\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\}$       (b)  $\{1, 3, 5, 7\} \cup \{2, 4, 6\}$

(c)  $\{1, 3, 5, 7, \dots\} \cup \{2, 4, 6, \dots\}$

**SOLUTION**

(a) Begin by listing the elements of the first set,  $\{1, 2, 5, 9, 14\}$ . Then include any elements from the second set that are not already listed.

$$\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\} = \{1, 2, 3, 4, 5, 8, 9, 14\}$$

(b)  $\{1, 3, 5, 7\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 5, 6, 7\}$

(c)  $\{1, 3, 5, 7, \dots\} \cup \{2, 4, 6, \dots\} = N$  Natural numbers

✓ Now Try Exercises 71 and 83.

The **set operations** are summarized below.

**Set Operations**

Let  $A$  and  $B$  define sets, with universal set  $U$ .

The **complement** of set  $A$  is the set  $A'$  of all elements in the universal set that do *not* belong to set  $A$ .

$$A' = \{x | x \in U, x \notin A\}$$

The **intersection** of sets  $A$  and  $B$ , written  $A \cap B$ , is made up of all the elements belonging to both set  $A$  and set  $B$ .

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

The **union** of sets  $A$  and  $B$ , written  $A \cup B$ , is made up of all the elements belonging to set  $A$  or to set  $B$  (or to both).

$$A \cup B = \{x | x \in A \text{ or } x \in B \text{ or } x \in \text{both } A \text{ and } B\}$$

**Sets of Numbers and the Number Line**

As mentioned previously, the set of **natural numbers** is written in set notation as follows.

$$\{1, 2, 3, 4, \dots\} \quad \text{Natural numbers}$$

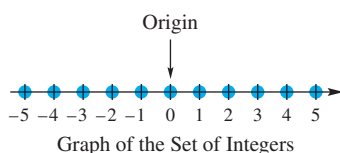
Including 0 with the set of natural numbers gives the set of **whole numbers**.

$$\{0, 1, 2, 3, 4, \dots\} \quad \text{Whole numbers}$$

Including the negatives of the natural numbers with the set of whole numbers gives the set of **integers**.

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\} \quad \text{Integers}$$

Integers can be graphed on a **number line**. See **Figure 12**. Every number corresponds to one and only one point on the number line, and each point corresponds to one and only one number. The number associated with a given point is the **coordinate** of the point. This correspondence forms a **coordinate system**.



**Figure 12**

The result of dividing two integers (with a nonzero divisor) is a *rational number*, or *fraction*. A **rational number** is an element of the set defined as follows.

$$\left\{ \frac{p}{q} \mid p \text{ and } q \text{ are integers and } q \neq 0 \right\} \quad \text{Rational numbers}$$

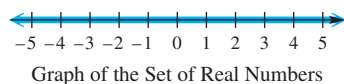
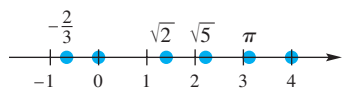


Figure 13



Graph of the Set

$$\left\{ -\frac{2}{3}, 0, \sqrt{2}, \sqrt{5}, \pi, 4 \right\}$$

$\sqrt{2}$ ,  $\sqrt{5}$ , and  $\pi$  are irrational. Because  $\sqrt{2}$  is approximately equal to 1.41, it is located between 1 and 2, slightly closer to 1.

Figure 14

The set of rational numbers includes the natural numbers, the whole numbers, and the integers. For example, the integer  $-3$  is a rational number because it can be written as  $\frac{-3}{1}$ . Numbers that can be written as repeating or terminating decimals are also rational numbers. For example,  $0.\overline{6} = 0.66666 \dots$  represents a rational number that can be expressed as the fraction  $\frac{2}{3}$ .

The set of all numbers that correspond to points on a number line is the **real numbers**, shown in **Figure 13**. Real numbers can be represented by decimals. Because every fraction has a decimal form—for example,  $\frac{1}{4} = 0.25$ —real numbers include rational numbers.

Some real numbers cannot be represented by quotients of integers. These numbers are **irrational numbers**. The set of irrational numbers includes  $\sqrt{2}$  and  $\sqrt{5}$ . Another irrational number is  $\pi$ , which is *approximately* equal to 3.14159. Some rational and irrational numbers are graphed in **Figure 14**.

The sets of numbers discussed so far are summarized as follows.

### Sets of Numbers

Set	Description
<b>Natural numbers</b>	$\{1, 2, 3, 4, \dots\}$
<b>Whole numbers</b>	$\{0, 1, 2, 3, 4, \dots\}$
<b>Integers</b>	$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
<b>Rational numbers</b>	$\left\{ \frac{p}{q} \mid p \text{ and } q \text{ are integers and } q \neq 0 \right\}$
<b>Irrational numbers</b>	$\{x \mid x \text{ is real but not rational}\}$
<b>Real numbers</b>	$\{x \mid x \text{ corresponds to a point on a number line}\}$

### EXAMPLE 7 Identifying Sets of Numbers

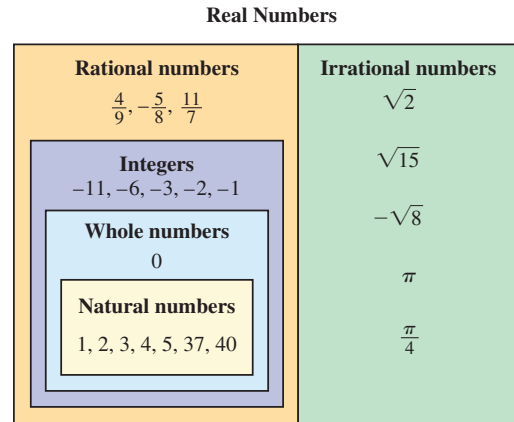
Let  $A = \left\{ -8, -6, -\frac{12}{4}, -\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}, 1, \sqrt{2}, \sqrt{5}, 6 \right\}$ . List all the elements of  $A$  that belong to each set.

- (a) Natural numbers      (b) Whole numbers      (c) Integers  
 (d) Rational numbers      (e) Irrational numbers      (f) Real numbers

### SOLUTION

- (a) Natural numbers: 1 and 6      (b) Whole numbers: 0, 1, and 6  
 (c) Integers:  $-8, -6, -\frac{12}{4}$  (or  $-3$ ), 0, 1, and 6  
 (d) Rational numbers:  $-8, -6, -\frac{12}{4}$  (or  $-3$ ),  $-\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}, 1$ , and 6  
 (e) Irrational numbers:  $\sqrt{2}$  and  $\sqrt{5}$   
 (f) All elements of  $A$  are real numbers.

**Figure 15** shows the relationships among the subsets of the real numbers. As shown, the natural numbers are a subset of the whole numbers, which are a subset of the integers, which are a subset of the rational numbers. The union of the rational numbers and irrational numbers is the set of real numbers.



**Figure 15**

## R.2 Exercises

**CONCEPT PREVIEW** Fill in the blank to correctly complete each sentence.

1. Set  $A$  is a(n) \_\_\_\_\_ of set  $B$  if every element of set  $A$  is also an element of set  $B$ .
2. The set of all elements of the universal set  $U$  that do not belong to set  $A$  is the \_\_\_\_\_ of set  $A$ .
3. The \_\_\_\_\_ of sets  $A$  and  $B$  is made up of all the elements belonging to both set  $A$  and set  $B$ .
4. The \_\_\_\_\_ of sets  $A$  and  $B$  is made up of all the elements belonging to set  $A$  or to set  $B$  (or to both).

**CONCEPT PREVIEW** Work each problem.

5. Identify the set  $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots\right\}$  as *finite* or *infinite*.
6. Use set notation and write the elements belonging to the set  $\{x \mid x \text{ is a natural number less than } 6\}$ .
7. Let  $U = \{1, 2, 3, 4, 5\}$  and  $A = \{1, 2, 3\}$ . Find  $A'$ .
8. Find  $\{16, 18, 21, 50\} \cap \{15, 16, 17, 18\}$ .
9. Find  $\{16, 18, 21, 50\} \cup \{15, 16, 17, 18\}$ .
10. **CONCEPT PREVIEW** Match each number from Column I with the letter or letters of the sets of numbers from Column II to which the number belongs. There may be more than one choice, so give all choices.

I		II	
(a) 0	(b) 34	A. Natural numbers	B. Whole numbers
(c) $-\frac{9}{4}$	(d) $\sqrt{36}$	C. Integers	D. Rational numbers
(e) $\sqrt{13}$	(f) 2.16	E. Irrational numbers	F. Real numbers

Identify each set as finite or infinite. Then determine whether 10 is an element of the set. See Example 1.

11.  $\{4, 5, 6, \dots, 15\}$
12.  $\{1, 2, 3, 4, 5, \dots, 75\}$
13.  $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots\right\}$
14.  $\{4, 5, 6, \dots\}$
15.  $\{x \mid x \text{ is a natural number greater than } 11\}$
16.  $\{x \mid x \text{ is a natural number greater than or equal to } 10\}$
17.  $\{x \mid x \text{ is a fraction between } 8 \text{ and } 9\}$
18.  $\{x \mid x \text{ is an even natural number}\}$

Use set notation, and list all the elements of each set. See Example 2.

19.  $\{12, 13, 14, \dots, 20\}$
20.  $\{8, 9, 10, \dots, 17\}$
21.  $\left\{1, \frac{1}{2}, \frac{1}{4}, \dots, \frac{1}{32}\right\}$
22.  $\{3, 9, 27, \dots, 729\}$
23.  $\{17, 22, 27, \dots, 47\}$
24.  $\{74, 68, 62, \dots, 38\}$
25.  $\{x \mid x \text{ is a natural number greater than } 8 \text{ and less than } 15\}$
26.  $\{x \mid x \text{ is a natural number not greater than } 4\}$

Insert  $\in$  or  $\notin$  in each blank to make the resulting statement true. See Examples 1 and 2.

27.  $6 \text{ } \underline{\hspace{1cm}} \{3, 4, 5, 6\}$
28.  $9 \text{ } \underline{\hspace{1cm}} \{2, 3, 5, 9, 8\}$
29.  $5 \text{ } \underline{\hspace{1cm}} \{4, 6, 8, 10\}$
30.  $13 \text{ } \underline{\hspace{1cm}} \{3, 5, 12, 14\}$
31.  $0 \text{ } \underline{\hspace{1cm}} \{0, 2, 3, 4\}$
32.  $0 \text{ } \underline{\hspace{1cm}} \{0, 5, 6, 7, 8, 10\}$
33.  $\{3\} \text{ } \underline{\hspace{1cm}} \{2, 3, 4, 5\}$
34.  $\{5\} \text{ } \underline{\hspace{1cm}} \{3, 4, 5, 6, 7\}$
35.  $\{0\} \text{ } \underline{\hspace{1cm}} \{0, 1, 2, 5\}$
36.  $\{2\} \text{ } \underline{\hspace{1cm}} \{2, 4, 6, 8\}$
37.  $0 \text{ } \underline{\hspace{1cm}} \emptyset$
38.  $\emptyset \text{ } \underline{\hspace{1cm}} \emptyset$

Determine whether each statement is true or false. See Examples 1–3.

39.  $3 \in \{2, 5, 6, 8\}$
40.  $6 \in \{2, 5, 8, 9\}$
41.  $1 \in \{11, 5, 4, 3, 1\}$
42.  $12 \in \{18, 17, 15, 13, 12\}$
43.  $9 \notin \{8, 5, 2, 1\}$
44.  $3 \notin \{7, 6, 5, 4\}$
45.  $\{2, 5, 8, 9\} = \{2, 5, 9, 8\}$
46.  $\{3, 0, 9, 6, 2\} = \{2, 9, 0, 3, 6\}$
47.  $\{5, 8, 9\} = \{5, 8, 9, 0\}$
48.  $\{3, 7, 12, 14\} = \{3, 7, 12, 14, 0\}$
49.  $\{x \mid x \text{ is a natural number less than } 3\} = \{1, 2\}$
50.  $\{x \mid x \text{ is a natural number greater than } 10\} = \{11, 12, 13, \dots\}$

Let  $A = \{2, 4, 6, 8, 10, 12\}$ ,  $B = \{2, 4, 8, 10\}$ ,  $C = \{4, 10, 12\}$ ,  $D = \{2, 10\}$ , and  $U = \{2, 4, 6, 8, 10, 12, 14\}$ .

Determine whether each statement is true or false. See Example 3.

51.  $A \subseteq U$
52.  $C \subseteq U$
53.  $D \subseteq B$
54.  $D \subseteq A$
55.  $A \subseteq B$
56.  $B \subseteq C$
57.  $\emptyset \subseteq A$
58.  $\emptyset \subseteq \emptyset$
59.  $\{4, 8, 10\} \subseteq B$
60.  $\{0, 2\} \subseteq D$
61.  $B \subseteq D$
62.  $A \subseteq C$



Insert  $\subseteq$  or  $\not\subseteq$  in each blank to make the resulting statement true. See Example 3.

63.  $\{2, 4, 6\}$  \_\_\_\_\_  $\{2, 3, 4, 5, 6\}$       64.  $\{1, 5\}$  \_\_\_\_\_  $\{0, 1, 2, 3, 5\}$   
 65.  $\{0, 1, 2\}$  \_\_\_\_\_  $\{1, 2, 3, 4, 5\}$       66.  $\{5, 6, 7, 8\}$  \_\_\_\_\_  $\{1, 2, 3, 4, 5, 6, 7\}$   
 67.  $\emptyset$  \_\_\_\_\_  $\{1, 4, 6, 8\}$       68.  $\emptyset$  \_\_\_\_\_  $\emptyset$

Determine whether each statement is true or false. See Examples 4–6.

69.  $\{5, 7, 9, 19\} \cap \{7, 9, 11, 15\} = \{7, 9\}$   
 70.  $\{8, 11, 15\} \cap \{8, 11, 19, 20\} = \{8, 11\}$   
 71.  $\{1, 2, 7\} \cup \{1, 5, 9\} = \{1\}$   
 72.  $\{6, 12, 14, 16\} \cup \{6, 14, 19\} = \{6, 14\}$   
 73.  $\{2, 3, 5, 9\} \cap \{2, 7, 8, 10\} = \{2\}$   
 74.  $\{6, 8, 9\} \cup \{9, 8, 6\} = \{8, 9\}$   
 75.  $\{3, 5, 9, 10\} \cap \emptyset = \{3, 5, 9, 10\}$   
 76.  $\{3, 5, 9, 10\} \cup \emptyset = \{3, 5, 9, 10\}$   
 77.  $\{1, 2, 4\} \cup \{1, 2, 4\} = \{1, 2, 4\}$   
 78.  $\{1, 2, 4\} \cap \{1, 2, 4\} = \emptyset$   
 79.  $\emptyset \cup \emptyset = \emptyset$       80.  $\emptyset \cap \emptyset = \emptyset$

Let  $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ ,  $M = \{0, 2, 4, 6, 8\}$ ,  
 $N = \{1, 3, 5, 7, 9, 11, 13\}$ ,  $Q = \{0, 2, 4, 6, 8, 10, 12\}$ , and  $R = \{0, 1, 2, 3, 4\}$ .

Use these sets to find each of the following. Identify any disjoint sets. See Examples 4–6.

- |                                             |                                             |                         |
|---------------------------------------------|---------------------------------------------|-------------------------|
| 81. $M \cap R$                              | 82. $M \cap U$                              | 83. $M \cup N$          |
| 84. $M \cup R$                              | 85. $M \cap N$                              | 86. $U \cap N$          |
| 87. $N \cup R$                              | 88. $M \cup Q$                              | 89. $N'$                |
| 90. $Q'$                                    | 91. $M' \cap Q$                             | 92. $Q \cap R'$         |
| 93. $\emptyset \cap R$                      | 94. $\emptyset \cap Q$                      | 95. $N \cup \emptyset$  |
| 96. $R \cup \emptyset$                      | 97. $(M \cap N) \cup R$                     | 98. $(N \cup R) \cap M$ |
| 99. $(Q \cap M) \cup R$                     | 100. $(R \cup N) \cap M'$                   |                         |
| 101. $(M' \cup Q) \cap R$                   | 102. $Q \cap (M \cup N)$                    |                         |
| 103. $Q' \cap (N' \cap U)$                  | 104. $(U \cap \emptyset') \cup R$           |                         |
| 105. $\{x   x \in U, x \notin M\}$          | 106. $\{x   x \in U, x \notin R\}$          |                         |
| 107. $\{x   x \in M \text{ and } x \in Q\}$ | 108. $\{x   x \in Q \text{ and } x \in R\}$ |                         |
| 109. $\{x   x \in M \text{ or } x \in Q\}$  | 110. $\{x   x \in Q \text{ or } x \in R\}$  |                         |

Let  $A = \left\{-6, -\frac{12}{4}, -\frac{5}{8}, -\sqrt{3}, 0, \frac{1}{4}, 1, 2\pi, 3, \sqrt{12}\right\}$ . List all the elements of  $A$  that belong to each set. See Example 7.

- |                       |                         |                   |
|-----------------------|-------------------------|-------------------|
| 111. Natural numbers  | 112. Whole numbers      | 113. Integers     |
| 114. Rational numbers | 115. Irrational numbers | 116. Real numbers |

## R.3 Real Number Operations and Properties

- Order on the Number Line
- Absolute Value
- Operations on Real Numbers
- Exponents
- Order of Operations
- Properties of Real Numbers

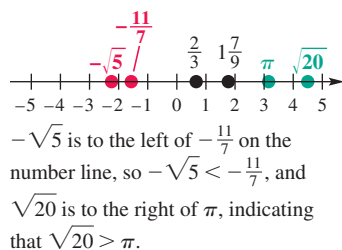


Figure 16

**Order on the Number Line** If the real number  $a$  is to the left of the real number  $b$  on a number line, then

$a$  is less than  $b$ , written  $a < b$ .

If  $a$  is to the right of  $b$ , then

$a$  is greater than  $b$ , written  $a > b$ .

The inequality symbol must point toward the lesser number.

See Figure 16. Statements involving these symbols, as well as the symbols less than or equal to,  $\leq$ , and greater than or equal to,  $\geq$ , are **inequalities**. The inequality  $a < b < c$  says that  $b$  is **between**  $a$  and  $c$  because  $a < b$  and  $b < c$ .

### Inequality Symbols

Symbol	Meaning	Example
$\neq$	is not equal to	$3 \neq 7$
$<$	is less than	$-4 < -1$
$>$	is greater than	$3 > -2$
$\leq$	is less than or equal to	$6 \leq 6$
$\geq$	is greater than or equal to	$-8 \geq -10$

**Absolute Value** The undirected distance on a number line from a number to 0 is the **absolute value** of that number. The absolute value of the number  $a$  is written  $|a|$ . For example, the distance on a number line from 5 to 0 is 5, as is the distance from  $-5$  to 0. See Figure 17. Therefore,  $|5| = 5$  and  $|-5| = 5$ .

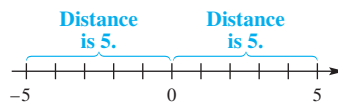


Figure 17

**NOTE** Because distance cannot be negative, the absolute value of a number is always positive or 0.

The algebraic definition of absolute value follows.

### Absolute Value

Let  $a$  represent a real number.

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

That is, the absolute value of a positive number or 0 equals that number, while the absolute value of a negative number equals its negative (or opposite).

**EXAMPLE 1** Evaluating Absolute Values

Evaluate each expression.

$$(a) \left| -\frac{5}{8} \right| \quad (b) -|8| \quad (c) -|-8| \quad (d) |2x|, \text{ for } x = \pi$$

**SOLUTION**

$$(a) \left| -\frac{5}{8} \right| = \frac{5}{8} \quad (b) -|8| = -(8) = -8$$

$$(c) -|-8| = -(8) = -8 \quad (d) |2\pi| = 2\pi$$

✓ Now Try Exercises 11 and 15.

Consider the following properties of absolute value.

**Properties of Absolute Value**Let  $a$  and  $b$  represent real numbers.**Property****Description**

1.  $|a| \geq 0$

The absolute value of a real number is positive or 0.

2.  $|-a| = |a|$

The absolute values of a real number and its opposite are equal.

3.  $|a| \cdot |b| = |ab|$

The product of the absolute values of two real numbers equals the absolute value of their product.

4.  $\frac{|a|}{|b|} = \left| \frac{a}{b} \right| \quad (b \neq 0)$

The quotient of the absolute values of two real numbers equals the absolute value of their quotient.

5.  $|a + b| \leq |a| + |b|$   
(triangle inequality)

The absolute value of the sum of two real numbers is less than or equal to the sum of their absolute values.

**LOOKING AHEAD TO CALCULUS**

One of the most important definitions in calculus, that of the **limit**, uses absolute value. The symbols  $\epsilon$  (epsilon) and  $\delta$  (delta) are often used to represent small quantities in mathematics.

Suppose that a function  $f$  is defined at every number in an open interval  $I$  containing  $a$ , except perhaps at  $a$  itself. Then the limit of  $f(x)$  as  $x$  approaches  $a$  is  $L$ , written

$$\lim_{x \rightarrow a} f(x) = L,$$

if for every  $\epsilon > 0$  there exists a  $\delta > 0$  such that  $|f(x) - L| < \epsilon$  whenever  $0 < |x - a| < \delta$ .

*Examples of Properties 1–4:*

$$|-15| = 15 \text{ and } 15 \geq 0. \quad \text{Property 1}$$

$$|-10| = 10 \text{ and } |10| = 10, \text{ so } |-10| = |10|. \quad \text{Property 2}$$

$$|5| \cdot |-4| = 5 \cdot 4 = 20 \text{ and } |5(-4)| = |-20| = 20, \\ \text{so } |5| \cdot |-4| = |5(-4)|. \quad \text{Property 3}$$

$$\frac{|2|}{|3|} = \frac{2}{3} \text{ and } \left| \frac{2}{3} \right| = \frac{2}{3}, \text{ so } \frac{|2|}{|3|} = \left| \frac{2}{3} \right|. \quad \text{Property 4}$$

*Example of the triangle inequality:*

$$|a + b| = |3 + (-7)| = |-4| = 4$$

$$|a| + |b| = |3| + |-7| = 3 + 7 = 10 \quad \text{Let } a = 3 \text{ and } b = -7.$$

$$\text{Thus, } |a + b| \leq |a| + |b|. \quad \text{Property 5}$$

**Operations on Real Numbers**

Recall that the answer to an addition problem is a **sum**. The answer to a subtraction problem is a **difference**. The answer to a multiplication problem is a **product**, and the answer to a division problem is a **quotient**. Other phrases that indicate these operations are given in the table.

**Words or Phrases That Indicate Operations on Real Numbers**

Operation	Word or Phrase
Addition	Sum, added to, more than, increased by, plus
Subtraction	Difference of, subtracted from, less than, decreased by, minus
Multiplication	Product of, times, twice, triple, of, percent of
Division	Quotient of, divided by, ratio of

To perform operations on real numbers, we use the following sign rules.

**Sign Rules for Operations on Real Numbers**

Operation	Examples
To <b>add</b> two numbers with the <i>same</i> sign, add their absolute values. The sum has the same sign as the given numbers.	$2 + 7 = 9$ $-2 + (-7) = -9$
To <b>add</b> two numbers with <i>different</i> signs, find the absolute values of the numbers, and subtract the lesser absolute value from the greater. The sum has the same sign as the number with the greater absolute value.	$-12 + 5 = -7$ $4 + (-10) = -6$ $-3 + 8 = 5$
To <b>subtract</b> a number $b$ from another number $a$ , change to addition and replace $b$ by its <b>additive inverse (opposite)</b> , $-b$ . $a - b = a + (-b)$ Then use the sign rules for addition.	$5 - 7 = 5 + (-7) = -2$ $-8 - 4 = -8 + (-4) = -12$ $-17 - (-4) = -17 + 4 = -13$
To <b>multiply</b> or <b>divide</b> two numbers with the <i>same</i> sign, multiply or divide their absolute values. The answer will be positive.	$5(7) = 35$ , $-5(-7) = 35$ $\frac{35}{5} = 7$ , $\frac{-35}{-5} = 7$
To <b>multiply</b> or <b>divide</b> two numbers with <i>different</i> signs, multiply or divide their absolute values. The answer will be negative.	$-8(9) = -72$ , $6(-7) = -42$ $\frac{-45}{9} = -5$ , $\frac{36}{-9} = -4$

Consider the quotient  $15 \div 3$  and the product  $15 \cdot \frac{1}{3}$ .

$$15 \div 3 = \frac{15}{3} = 5 \quad \text{and} \quad 15 \cdot \frac{1}{3} = \frac{15}{3} = 5$$

Thus division can be written in terms of multiplication using the **multiplicative inverse (reciprocal)**. For all real numbers  $a$  and  $b$  ( $b \neq 0$ ),

$$a \div b = \frac{a}{b} = a \cdot \frac{1}{b}.$$

To divide  $a$  by  $b$ , multiply  $a$  (the **dividend**)  
by  $\frac{1}{b}$ , the reciprocal of  $b$  (the **divisor**).

For this reason, the sign rules for division are the same as those for multiplication.

Consider the quotient  $\frac{a}{b}$ . To find  $\frac{a}{b}$ , we look for a number  $n$  that when multiplied by  $b$  will give  $a$ . For example,  $\frac{10}{5} = 2$  because  $2 \cdot 5 = 10$ .

What happens if we try to divide a nonzero number by 0? For example, to find  $\frac{4}{0}$ , we look for some number  $n$  that when multiplied by 0 will give 4. But  $n \cdot 0$  will always give a product of 0, never 4. Therefore, division by 0 is *undefined*. On the other hand,  $\frac{0}{4} = 0$  because  $0 \cdot 4 = 0$ . Thus, dividing 0 by any nonzero number will always yield 0.

Trying to divide 0 by 0 is problematic for another reason. If  $\frac{0}{0} = n$ , then  $n \cdot 0 = 0$ . This statement is true for *all* numbers  $n$ , which means that  $\frac{0}{0}$  is not unique. We say that  $\frac{0}{0}$  is *indeterminate*—that is, there is no unique answer.

This discussion can be summarized as follows.

### Division Involving 0

For any nonzero real number  $a$ ,

$$\frac{a}{0} \text{ is undefined, } \frac{0}{a} = 0, \text{ and } \frac{0}{0} \text{ is indeterminate.}$$

### EXAMPLE 2 Adding and Subtracting Real Numbers

Find each sum or difference.

(a)  $-6 + (-3)$     (b)  $-2.3 + 5.6$     (c)  $-12 - 4$     (d)  $\frac{5}{6} - \left(-\frac{3}{8}\right)$

#### SOLUTION

(a)  $-6 + (-3)$

$$\begin{aligned} &= -(|-6| + |-3|) \quad \text{Add the absolute values. The numbers} \\ &= -(6 + 3) \quad \text{have the same sign, both negative, so the} \\ &= -9 \quad \text{sum will be negative.} \end{aligned}$$

(b)  $-2.3 + 5.6$

$$\begin{aligned} &= |5.6| - |2.3| \quad \text{The numbers have different signs. Subtract} \\ &= 5.6 - 2.3 \quad \text{the lesser absolute value from the greater.} \\ &= 3.3 \end{aligned}$$

(c)  $-12 - 4 = -12 + (-4) = -16$

Change to addition.  
The opposite of 4 is  $-4$ .

(d)  $\frac{5}{6} - \left(-\frac{3}{8}\right)$

$$= \frac{5}{6} + \frac{3}{8} \quad \text{To subtract } a - b, \text{ add the opposite of } b \text{ to } a.$$

$$= \frac{20}{24} + \frac{9}{24} \quad \text{The least common denominator is 24.}$$

$$\frac{5}{6} \cdot \frac{4}{4} = \frac{20}{24}; \quad \frac{3}{8} \cdot \frac{3}{3} = \frac{9}{24}$$

$$= \frac{29}{24} \quad \begin{array}{l} \text{Add numerators.} \\ \text{Keep the same denominator.} \end{array}$$

**EXAMPLE 3** Multiplying and Dividing Real Numbers

Find each product or quotient where possible.

(a)  $-3(-9)$       (b)  $6(-9)$       (c)  $-0.05(0.3)$       (d)  $-\frac{3}{4}\left(\frac{2}{9}\right)$

(e)  $\frac{-9}{0}$       (f)  $\frac{0}{-12}$       (g)  $\frac{-12}{4}$       (h)  $\frac{-\frac{2}{3}}{-\frac{5}{9}}$

**SOLUTION**

(a)  $-3(-9) = 27$       (b)  $6(-9) = -54$       (c)  $-0.05(0.3) = -0.015$

The numbers have the *same* sign, so the product is positive.

The numbers have *different* signs, so the product is negative.

(d)  $-\frac{3}{4}\left(\frac{2}{9}\right)$

$$= -\frac{6}{36} \quad \begin{array}{l} \text{Multiply numerators.} \\ \text{Multiply denominators.} \end{array}$$

$$= -\frac{1}{6} \quad \text{Write in lowest terms; } \frac{6}{36} = \frac{1 \cdot 6}{6 \cdot 6} = \frac{1}{6}$$

(e)  $\frac{-9}{0}$  is **undefined**.      (f)  $\frac{0}{-12} = 0$       (g)  $\frac{-12}{4} = -3$

This is true because  $0(-12) = 0$ .      The numbers have *different* signs, so the quotient is negative.

(h)  $\frac{-\frac{2}{3}}{-\frac{5}{9}}$       This is a *complex fraction*. A complex fraction has a fraction in the numerator, the denominator, or both.

$$= -\frac{2}{3}\left(-\frac{9}{5}\right) \quad \frac{a}{b} = a \cdot \frac{1}{b}; \text{ Multiply by } -\frac{9}{5}, \text{ the reciprocal of the divisor } -\frac{5}{9}.$$

$$= \frac{18}{15} \quad \begin{array}{l} \text{Multiply numerators.} \\ \text{Multiply denominators.} \end{array}$$

$$= \frac{6}{5} \quad \text{Write in lowest terms; } \frac{18}{15} = \frac{6 \cdot 3}{5 \cdot 3} = \frac{6}{5}$$

✓ Now Try Exercises 51, 57, 73, and 77.

**Exponents**

Any collection of numbers or variables joined by the basic operations of addition, subtraction, multiplication, or division (except by 0), or the operations of raising to powers or taking roots, formed according to the rules of algebra, is an **algebraic expression**.

$$-2x^2 + 3x, \quad \frac{15y}{2y-3}, \quad \sqrt{m^3-64}, \quad (3a+b)^4 \quad \text{Algebraic expressions}$$

Exponent: 3

$$2^3 = \underbrace{2 \cdot 2 \cdot 2}_{\text{Three factors of 2}} = 8$$

Base: 2

The expression  $2^3$  is an **exponential expression**, or **exponential**, where the 3 indicates that three factors of 2 appear in the corresponding product. The number 2 is the **base**, and the number 3 is the **exponent**.

### Exponential Notation

If  $n$  is any positive integer and  $a$  is any real number, then the  $n$ th power of  $a$  is written using exponential notation as follows.

$$a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{n \text{ factors of } a}$$

Read  $a^n$  as “ $a$  to the  $n$ th power” or simply “ $a$  to the  $n$ th.”

#### EXAMPLE 4 Evaluating Exponential Expressions

Evaluate each exponential expression, and identify the base and the exponent.

- (a)  $4^3$       (b)  $(-6)^2$       (c)  $-6^2$       (d)  $4 \cdot 3^2$       (e)  $(4 \cdot 3)^2$

#### SOLUTION

(a)  $4^3 = \underbrace{4 \cdot 4 \cdot 4}_{3 \text{ factors of } 4} = 64$       The base is 4 and the exponent is 3.

(b)  $(-6)^2 = (-6)(-6) = 36$       The base is  $-6$  and the exponent is 2.

(c)  $-6^2 = -(6 \cdot 6) = -36$       Notice that parts (b) and (c) are different.  
The base is 6 and the exponent is 2.

(d)  $4 \cdot 3^2 = 4 \cdot 3 \cdot 3 = 36$       The base is 3 and the exponent is 2.  
 $3^2 = 3 \cdot 3$ , NOT  $3 \cdot 2$

(e)  $(4 \cdot 3)^2 = 12^2 = 144$        $(4 \cdot 3)^2 \neq 4 \cdot 3^2$   
The base is  $4 \cdot 3$ , or 12, and the exponent is 2.

✓ Now Try Exercises 81, 83, and 87.

### Order of Operations

When an expression involves more than one operation symbol, such as  $5 \cdot 2 + 3$ , we use the following order of operations.

#### Order of Operations

If grouping symbols such as parentheses, square brackets, absolute value bars, or fraction bars are present, begin as follows.

**Step 1** Work separately above and below each **fraction bar**.

**Step 2** Use the rules below within each set of **parentheses** or **square brackets**. Start with the innermost set and work outward.

If no grouping symbols are present, follow these steps.

**Step 1** Simplify all **powers** and **roots**. *Work from left to right.*

**Step 2** Do any **multiplications** or **divisions** in order. *Work from left to right.*

**Step 3** Do any **negations**, **additions**, or **subtractions** in order. *Work from left to right.*