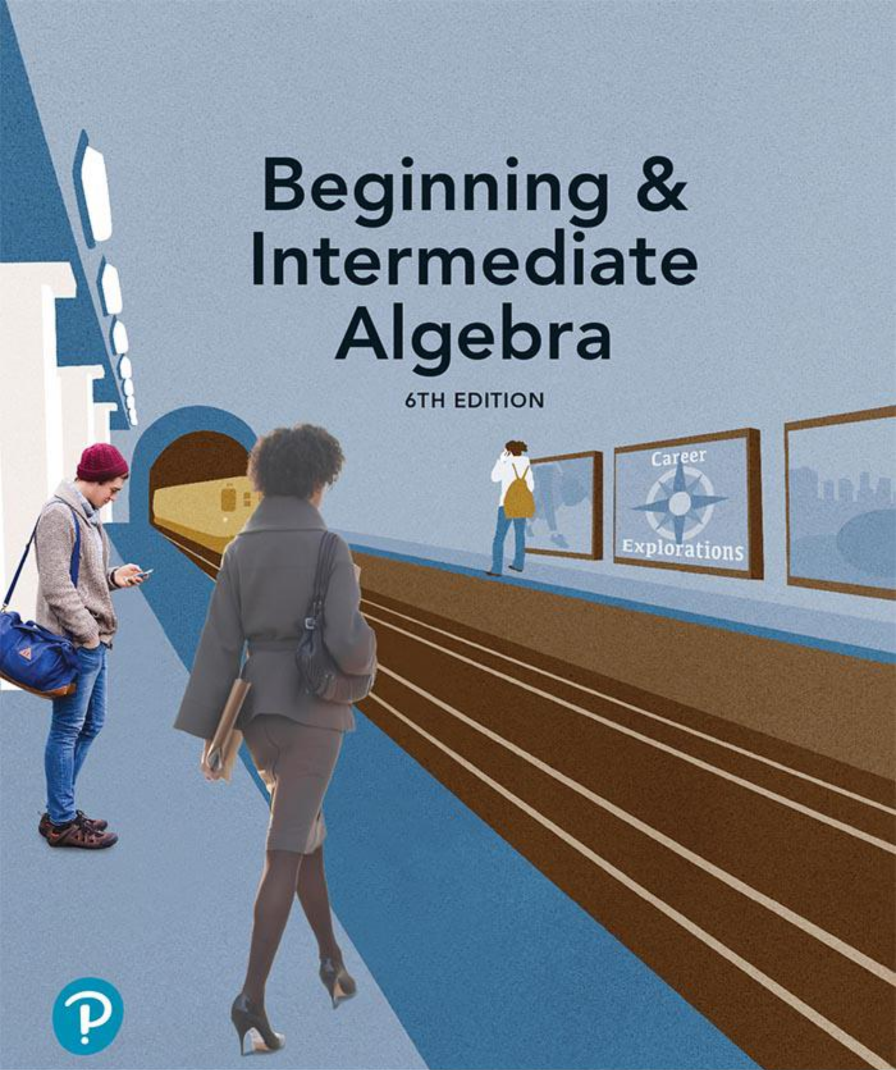
Beginning & Intermediate Algebra

6TH EDITION



TOBEY SLATER BLAIR CRAWFORD FISCHER

Beginning and Intermediate Algebra

Sixth Edition

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This book is dedicated to my husband, Nate Crawford.
Thank you for your support and patience while I worked.
You were the voice of reason countless times, exactly when I needed it.

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STEPS TO SUCCESS INDEX: Beginning & Intermediate Algebra

Preface

TO THE INSTRUCTOR

As the authors, we want to let you, the faculty member teaching this course, know that in this book you will find all the resources needed for your students to be successful. We have spent our careers in the classroom talking to students and hearing from them what they need to find success in mathematics. This revision reflects their ideas and suggestions. You can honestly tell students that this book was constructed to really help them.

Developmental mathematics course structures, trends, and dynamics continue to evolve and change, as **course redesign trends** continue to evolve and change, including the introduction of **new pathways-type courses.** Developmental mathematics instructors are increasingly challenged with helping their students **navigate career-oriented math tracks (including non-STEM and STEM pathways)** plus helping students think about **selecting a major** and **workforce readiness.** To help instructors on this front, you'll find an **emphasis on and integration of Career Explorations** throughout the text and MyLab Math course.

Additionally, the program retains its hallmark characteristics that have always made the text so easy to learn and teach from, including its building-block organization. Each section is written to stand on its own, and every homework set is completely self-testing. Exercises are paired and graded and are of varying levels and types to ensure that all skills and concepts are covered. As a result, the text offers students an effective and proven learning program suitable for a variety of course formats—including lecture-based classes; computer-lab based or hybrid classes; discussion-oriented, activity-driven classes; modular and/or self-paced programs; and distance-learning, online programs.

We have visited and listened to teachers across the country and have incorporated a number of suggestions into this edition to help you with the particular learning-delivery system at your school. The following pages describe the key changes in this sixth edition.

KEY ELEMENTS OF THE SIXTH EDITION

New Look

We have updated this title to now have a hard cover for a more durable text experience. We are also asking students to work problems on separate sheets of paper rather than in the text. We encourage the use of MyLab Math, so each student will have their own algorithmically generated version of the question to answer.

New Course Organizer

To learn more about each supporting element of this text and how to use them in all modes of learning (online, hybrid, lecture, self paced) without losing classroom time, view a video provided by the author team in the Course Organizer in MyLab Math.

Becoming a Study Skills Coach with Just 2 to 3 Minutes of Class Time

Steps to Success have been integrated throughout the text. You can display these study skills on the classroom screen as students enter and settle in class. In just a few minutes you can encourage and coach your students on the Steps to Success. If you teach online placing the Steps to Success Box at the beginning of your online material will help the student see the importance of these skills.

STEPS TO SUCCESS What Are the Absolute Essentials to Succeed in This Course?

Students who are successful in this course find there are six absolute essentials.
Here they are:

1. Attend every class session.
2. Read the textbook for every assigned section.
3. Take notes in class.
4. Do the assigned homework for every class.
5. Get help immediately when you need assistance.
6. Review what you are learning.

Making it personal: Which of the six suggestions above is the one you have the greatest trouble actually doing? Will you make a personal commitment to doing that one thing consistently for the next two weeks? You will be amazed at the results.

If you are in an online class or a nontraditional class:

Students in an online class or a self-paced class find it really helps to spread the homework and the reading over five different days each week.

Making it personal: Will you try to do your homework for five days each week over the next two weeks? You will be amazed at the results. Write out a study plan for the next two weeks.

Attend Class → Read Text → Take Notes → Do Homework → Get Help → Review

New Midterm Material to Bridge the Gap between Elementary and Intermediate Algebra

Based on faculty feedback, Appendix A: *Foundations of Intermediate Algebra: A Transition from Beginning Algebra to Intermediate Algebra* was moved to the middle of the book immediately after Chapter 7. In this transition chapter you will notice objectives that are similar to those studied in earlier chapters. Although the objective names and content may seem similar, the material in this chapter not only reviews some of the key objectives, but also raises the skill level and introduces additional objectives necessary for the upcoming chapters.

A Transition from Beginning to Intermediate Algebra

T.1 Integer Exponents, Square Roots, Order of Operations, and Scientific Notation

Graphing Organizer

Chapter 3 includes a **Graphing Organizer.** This organizer summarizes the graphing methods presented in the chapter and notes the advantages and disadvantages of each. The complete Graphing Organizer can be found on page 232.

Graphing Organizer

Method 1: Find Any Three Ordered Pairs	Method 2: Find the x- and y-Intercepts and One Additional Ordered Pair	Method 3: Write the Equation in Slope–Intercept Form, $y = mx + b$
Graph $2y - x = 2$ by plotting three points. **Choose any value for either x or y.** x | y 1 | 2 | −4 |	**Graph $2y - x = 2$. State the x- and y-intercepts.** **Choose $x = 0$ and $y = 0$, and any other value for either x or y.** x | y 0 | | 0 −1 | Notice we are still choosing 3 values, but to get the intercepts we must pick $x = 0$ and $y = 0$.	**Graph $2y - x = 2$. State m and b. Solve for y.** $2y = x + 2$ $y = \frac{1}{2}x + 1$ *Once we solve for y we can choose any method to graph.* **Option 1. Use the slope–intercept method, $y = \frac{1}{2}x + 1$.**

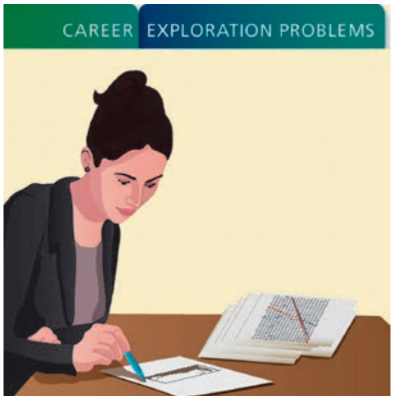
Factoring Organizer

Chapter 6 includes a **Factoring Organizer** that summarizes all of the factoring methods covered in the chapter. The complete Factoring Organizer can be found on page 406.

Factoring Organizer

Number of Terms in the Polynomial	Identifying Name and/or Formula	Example
A. Any number of terms	**Common factor** The terms have a common factor consisting of a number, a variable, or both.	$2x^2 - 16x = 2x(x - 8)$ $3x^2 + 9y - 12 = 3(x^2 + 3y - 4)$ $4x^2y + 2xy^2 - wxy + xyz = xy(4x + 2y - w + z)$
B. Two terms	**Difference of two squares** First and last terms are perfect squares. $a^2 - b^2 = (a + b)(a - b)$	$16x^2 - 1 = (4x + 1)(4x - 1)$ $25y^2 - 9x^2 = (5y + 3x)(5y - 3x)$
C. Three terms	**Perfect-square trinomial** First and last terms are perfect squares. $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$25x^2 - 10x + 1 = (5x - 1)^2$ $16x^2 + 24x + 9 = (4x + 3)^2$

Career Explorations Interactions for Students



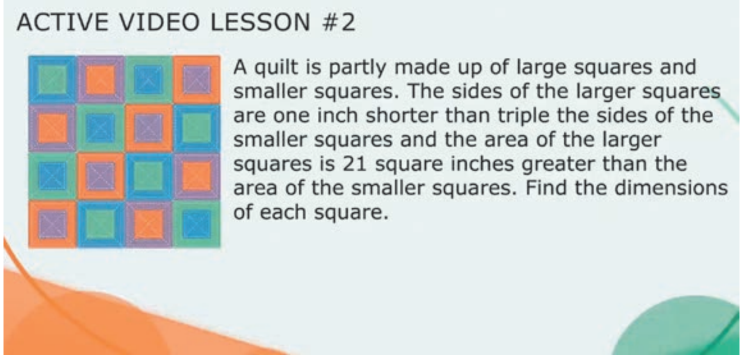
Each chapter begins with a **Career Opportunities** feature that enables students to personally investigate possible future career options while putting the math into context. Students are asked simple, interactive questions prompting them to consider employment opportunities that perhaps they had never thought possible.

Then students are directed to the corresponding **Career Exploration Problems** where they can actually solve problems that help them visualize what work would be like in that career field. This feature opens up possibilities for personal success in future employment.

The Career Exploration Problems are also assignable in MyLab Math, allowing this feature to be seamlessly integrated with the technology. The problems help to foster active learning and better understanding of the math concepts.

Guided Learning Videos

Faculty have asked for specific interactive videos that will clearly show each step of the **key concepts** of each chapter. With this revision, you'll find a new series of **Guided Learning Videos** that show in a powerful, interactive way **how to solve the most important types of problems contained in each chapter.** For student ease, icons throughout the eText indicate where the videos are available. The eText is clickable, opening the videos on the spot.



Extensive Video Program

In addition to the Guided Learning Videos with icons throughout the eText, objective-level video clips have also been added to the MyLab Math course with accompanying icons throughout the eText. These video additions expand upon an already complete video lecture series available in MyLab Math. Students and instructors will also find complete Section Lecture Videos, Math Coach Videos, and Chapter Test Prep Videos.

- **The Math Coach** is available within the MyLab Math course, with even more stepped-out, guided Math Coach problems assignable in MyLab Math. Within the text, following each Chapter Test, the **Math Coach** provides students with a personal office-hour experience by walking them through problems step-by-step and pointing out some helpful hints to keep them from making common errors on test problems. For additional help, students can also watch the authors work through these problems on the accompanying Math Coach videos in the MyLab Math course. Instructors can also assign the Math Coach problems in MyLab Math.
- Fifteen percent of the exercises throughout the text have been refreshed.
- Real-world application problems have been updated throughout the text.
- **Use Math to Save Money Animations** are available in the MyLab Math course. The animations expand upon a favorite feature from the text, allowing students to put the math they just learned into context. These newly created animations are set to music and depict real-life scenarios and real-life people using math to cut costs and spend less. To ensure that students watch and understand the animations, there are accompanying Use Math to Save Money homework assignments available in MyLab Math, which are prebuilt for instructor convenience.

  To make sure you and your students are getting the most out of the text *and* the MyLab Math course, see the following MyLab Math feature descriptions.

MyLab Math Resources for Success

Pearson MyLab

MyLab Math is available to accompany Pearson's market-leading text options, including ***Beginning and Intermediate Algebra*, 6th Edition,** 9780135839881.

MyLab™ is the teaching and learning platform that empowers you to reach every student. MyLab Math combines trusted author content—including full eText and assessment with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

Student Resources

Each student learns at a different pace. Personalized learning though MyLab Math pinpoints the precise areas where each student needs practice, giving all students the support they need—when and where they need it—to be successful.

New Steps to Success Index Found before the table of contents this index highlights study skills to help students successfully complete the course.

Real-World Application Problems and Examples have been updated throughout the text.

New Assignable MyLab Math Questions bring video content to life for easy concept mastery.

Student Success Module, an interactive module built into the MyLab navigation bar, includes videos, activities, and post-tests for Math-Read Connections, Study Skill, and College Success.

Exercises with Immediate Feedback in MyLab Math reflect the approach and learning style of this text and regenerate algorithmically to give students unlimited opportunity for practice and mastery. Most exercises include learning aids, such as guided solutions and sample problems, and they offer helpful feedback when students enter incorrect answers.

Exercises and Mixed Practice Problems have been revised in the textbook and in MyLab Math, so that students have adequate practice on all objectives. All concepts are fully represented, with every Example from the section covered by a group of exercises. The Mixed Practice problems require students to identify the type of problem and the best method they should use to solve it so students have a better understanding of the concepts in the section.

Skill Builder offers adaptive practice that is designed to increase students' ability to complete their assignments. By monitoring student performance on their homework, Skill Builder adapts to each student's needs and provides just-in-time, in-assignment practice to help them improve their proficiency of key learning objectives.

NEW Personal Inventory Assessments are a collection of online exercises designed to promote self-reflection and engagement in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.

With **Learning Catalytics™,** you'll hear from every student when it matters most. You pose a variety of questions that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops.

MyLab Math Resources for Success

Pearson MyLab

The **Student Solutions Manual** provides worked-out solutions to all odd-numbered section exercises, even and odd exercises in the Quick Review, mid-chapter reviews, chapter reviews, chapter tests, Math Coach, and cumulative reviews.

Instructor Resources

Your course is unique. So whether you'd like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

Enhanced Assignments, created at the section level, are geared to maximize students' performance with just-in-time prerequisite review. They help keep skills fresh with spaced practice of key concepts and provide opportunities to work exercises without learning aids, so students can check their understanding.

Pearson and **ProctorU** have partnered to provide customers with the first seamless, integrated proctoring service within MyLab™. This artificially intelligent proctoring service gives students the flexibility to take MyLab quizzes and tests on their own schedule and at their location of choice, while allowing the institution to maintain academic integrity. This exclusive opportunity is available to both new or existing partners of ProctorU.

Performance Analytics enable instructors to see and analyze student performance across multiple courses. Based on their current course progress, individuals' performance is identified above, at, or below expectations through a variety of graphs and visualizations. Now included with Performance Analytics, **Early Alerts** use predictive analytics to identify struggling students—even if their assignment scores are not a cause for concern. In both Performance Analytics and Early Alerts, instructors can email students individually or by group to provide feedback.

Accessibility Pearson works continuously to ensure our products are as accessible as possible to all students. Currently we work toward achieving WCAG 2.0 AA for our existing products (2.1 AA for future products) and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media.

Instructor's Solution Manual includes detailed step-by-step solutions to the even-numbered section exercises as well as solutions to every exercise in the Classroom Quiz, mid-chapter reviews, chapter reviews, chapter tests, cumulative tests, and practice final.

Instructor's Resource Manual with Tests and Mini Lectures includes a mini lecture for each text section, two short group activities per chapter, three forms of additional practice exercises, two pretests, six tests, and two final exams for every chapter, both free response and multiple choice, as well as two cumulative tests for every even-numbered chapter.

Course Organizer to provide key teaching tools and guidance to maximize the many resources.

PowerPoint Lecture Slides are fully editable and include definitions, key concepts, and examples for use in a lecture setting.

TestGen® enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions.

To learn more about each supporting element of this text, view a video provided by the author team in the the *Course Organizer* in MyLab Math.

pearson.com/mylab/math

Diagnostic Pretest:
Beginning and Intermediate Algebra

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Answer the following test questions on a separate sheet of paper. Follow the directions for each problem. Simplify each answer.

Chapter 0

1. $\frac{7}{8} + \frac{2}{3} - \frac{1}{4}$

2. $5\frac{2}{7} + 2\frac{1}{14}$

3. $\frac{15}{18} \times \frac{36}{25}$

4. $5\frac{1}{4} \div 4\frac{3}{8}$

5. $2.3 + 7.522 + 0.088$

6. 81.4×0.05

7. $0.2496 \div 0.12$

8. What is 4% of 120.8?

9. 55 is what percent of 220?

Chapter 1

10. Add. $-3 + (-4) + (12)$

11. Subtract. $-20 - (-23)$

12. Combine. $5x - 6xy - 12x - 8xy$

13. Evaluate $2x^2 - 3x - 4$ when $x = -3$.

14. Remove the grouping symbol and simplify. $2 - 3\{5 + 2[x - 4(3 - x)]\}$

15. Evaluate. $-3(2 - 6)^2 + (-12) \div (-4)$

Chapter 2

In questions 16–19, solve each equation for x.

16. $40 + 2x = 60 - 3x$

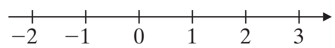
17. $7(3x - 1) = 5 + 4(x - 3)$

18. $\frac{2}{3}x - \frac{3}{4} = \frac{1}{6}x + \frac{21}{4}$

19. $\frac{4}{5}(3x + 4) = 20$

20. Solve for x. $-16x \geq 9 + 2x$

21. Solve for x and graph the result on a sheet of paper, using a number line similar to the one shown. $42 - 18x < 48x - 24$



22. The length of a rectangle is 7 meters longer than the width. The perimeter is 46 meters. Find the dimensions.

23. The drama club put on a play for Thursday, Friday, and Saturday nights. The total attendance for the three nights was 6210. Thursday night had 300 fewer people than Friday night. Saturday night had 510 more people than Friday night. How many people came each night?

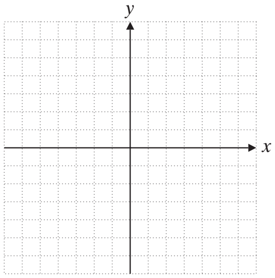
24. Two men travel in separate trucks. They each travel a distance of 225 miles on a country road. Art travels at exactly 60 mph and Lester travels at 50 mph. How much time did the trip take each man? (Use the formula distance = rate · time or $d = rt$.)

25. Each of the equal angles of an isosceles triangle is twice as large as the third angle. What is the measure of each angle?

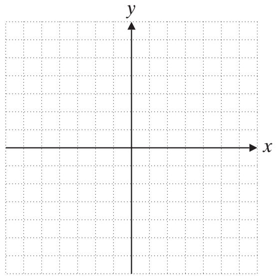
Chapter 3

Graph each of the following on a sheet of graph paper, using a coordinate system similar to the one shown.

26. $y = 2x - 4$



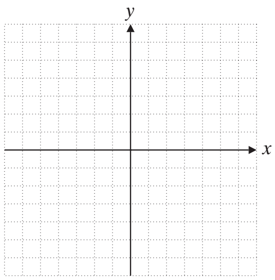
27. $3x + 4y = -12$



28. What is the slope of a line passing through $(6, -2)$ and $(-3, 4)$?

29. If $f(x) = 2x^2 - 3x + 1$, find $f(3)$.

30. Graph the region on a sheet of graph paper using a coordinate system similar to the one shown. $y \geq -\frac{1}{3}x + 2$



31. Find an equation of the line with a slope of $\frac{3}{5}$ that passes through the point $(-1, 3)$.

Chapter 4

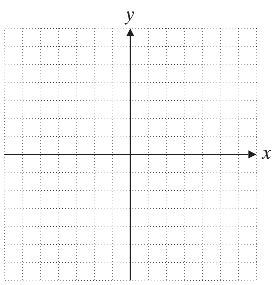
Solve each system of equations.

32. $3x + 5y = 30$
$5x + 3y = 34$

33. $2x - y + 2z = 8$
$x + y + z = 7$
$4x + y - 3z = -6$

34. A speedboat can travel 90 miles with the current in 2 hours. It can travel upstream 105 miles against the current in 3 hours. How fast is the boat in still water? How fast is the current?

35. Graph the system of inequalities. Use a sheet of graph paper and a coordinate system similar to the one shown.
$x - y \leq -42$
$2x + y \leq 0$



Chapter 5

36. Multiply. $(-2xy^2)(-4x^3y^4)$

37. Divide. $\dfrac{36x^5y^6}{-18x^3y^{10}}$

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38. Raise to the indicated power. $(-2x^3y^4)^5$

39. Evaluate. $(-3)^{-4}$

40. Multiply. $(3x^2 + 2x - 5)(4x - 1)$

41. Divide. $(x^3 + 6x^2 - x - 30) \div (x - 2)$

Chapter 6

Factor completely.

42. $5x^2 - 5$

43. $x^2 - 12x + 32$

44. $8x^2 - 2x - 3$

45. $3ax - 8b - 6a + 4bx$

Solve for x.

46. $16x^2 - 24x + 9 = 0$

47. $\dfrac{x^2 + 8x}{5} = -3$

Chapter 7

48. Simplify. $\dfrac{x^2 + 3x - 18}{2x - 6}$

49. Multiply. $\dfrac{6x^2 - 14x - 12}{6x + 4} \cdot \dfrac{x + 3}{2x^2 - 2x - 12}$

50. Divide and simplify. $\dfrac{x^2}{x^2 - 4} \div \dfrac{x^2 - 3x}{x^2 - 5x + 6}$

51. Add. $\dfrac{3}{x^2 - 7x + 12} + \dfrac{4}{x^2 - 9x + 20}$

52. Solve for x. $2 - \dfrac{5}{2x} = \dfrac{2x}{x + 1}$

53. Simplify. $\dfrac{3 + \dfrac{1}{x}}{\dfrac{9}{x} + \dfrac{3}{x^2}}$

Transition Chapter T

54. Simplify. Write your answer with positive exponents only. $\left(\dfrac{3x^{-3}y}{2xy^{-4}}\right)^{-2}$

55. Multiply. $(-2r^2 + 3)(7r - 2)$

56. For the polynomial $p(x) = -5x^3 + x^2 - 3x + 2$, evaluate $p(2)$.

Completely factor each of the following.

57. $4x^2 + 11xy + 7y^2$

58. $2x^3 - 12x^2 + 18x$

59. $125x^3 + 64$

Chapter 8

Assume that all expressions under radicals represent nonnegative numbers.

60. Multiply and simplify. $(\sqrt{3} + \sqrt{2x})(\sqrt{7} - \sqrt{2x^3})$

61. Rationalize the denominator. $\dfrac{3\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}}$

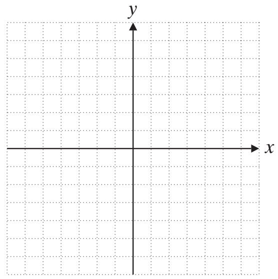
62. Solve and check your solutions. $2\sqrt{x - 1} = x - 4$

Chapter 9

63. Solve for x. $x^2 - 2x - 4 = 0$

64. Solve for x. $x^4 - 12x^2 + 20 = 0$

65. Graph $f(x) = (x - 2)^2 + 3$ on a sheet of graph paper using a coordinate system similar to the one shown. Label the vertex.



66. Solve for x. $\left|3\left(\frac{2}{3}x - 4\right)\right| \leq 12$

67. Solve for y. $|3y - 2| + 5 = 8$

Chapter 10

68. Write in standard form the equation of the circle with center at $(5, -2)$ and a radius of 6.

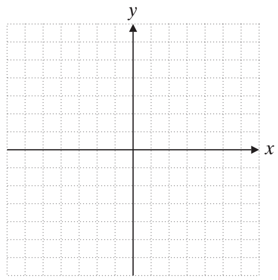
69. Write in standard form the equation of the ellipse whose center is at $(0, 0)$ and whose intercepts are at $(3, 0), (-3, 0), (0, 4)$, and $(0, -4)$.

70. Solve the following nonlinear system of equations.
$$x^2 + 4y^2 = 9$$
$$x + 2y = 3$$

Chapter 11

71. If $f(x) = 2x^2 - 3x + 4$, find $f(a + 2)$.

72. Graph on one set of axes. Use a sheet of graph paper and a coordinate system similar to the one shown. $f(x) = |x + 3|$ and $g(x) = |x + 3| - 3$



73. If $f(x) = \dfrac{3}{x + 2}$ and $g(x) = 3x^2 - 1$, find $g[f(x)]$.

74. If $f(x) = -\dfrac{1}{2}x - 5$, find $f^{-1}(x)$.

Chapter 12

75. Find y if $\log_5 125 = y$.

76. Find b if $\log_b 4 = \dfrac{2}{3}$.

77. What is $\log 10{,}000$?

78. Solve for x. $\log_6(5 + x) + \log_6 x = 2$

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CHAPTER 0

Prealgebra Review

CAREER OPPORTUNITIES

Banking and Financial Services

Careers in the banking and financial services industries offer challenging and enriching experiences. Whether it is providing information to consumers about checking account plans best suited to their needs, qualifying someone for a mortgage, or guiding a client through the intricacies of financial planning, these services require math skills of varying levels.

To investigate how the mathematics in this chapter can help with this field, see the Career Exploration Problems on page **55**.

0.1 Simplifying Fractions

Student Learning Objectives

After studying this section, you will be able to:

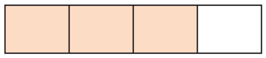
1. Understand basic mathematical definitions.
2. Simplify fractions to lowest terms using prime numbers.
3. Convert between improper fractions and mixed numbers.
4. Change a fraction to an equivalent fraction with a given denominator.

Chapter 0 is designed to give you a mental "warm-up." In this chapter you'll be able to step back a bit and tone up your math skills. This brief review of prealgebra will increase your math flexibility and give you a good running start into algebra.

1 Understanding Basic Mathematical Definitions

Whole numbers are the set of numbers 0, 1, 2, 3, 4, 5, 6, 7, ... They are used to describe whole objects, or entire quantities.

Fractions are a set of numbers that are used to describe parts of whole quantities. In the object shown in the figure there are four equal parts. The *three* of the *four* parts that are shaded are represented by the fraction $\frac{3}{4}$. In the fraction $\frac{3}{4}$ the number 3 is called the **numerator** and the number 4, the **denominator.**



$\frac{3}{4}$

3 ← *Numerator* is on the top

4 ← *Denominator* is on the bottom

The *denominator* of a fraction shows the number of equal parts in the whole and the *numerator* shows the number of these parts being talked about or being used.

Numerals are symbols we use to name numbers. There are many different numerals that can be used to describe the same number. We know that $\frac{1}{2} = \frac{2}{4}$. The fractions $\frac{1}{2}$ and $\frac{2}{4}$ both describe the same number.

Usually, we find it more useful to use fractions that are simplified. A fraction is considered to be in **simplest form** or **reduced form** when the numerator (top) and the denominator (bottom) have no common divisor other than 1, and the denominator is greater than 1.

$\frac{1}{2}$ is in simplest form.

$\frac{2}{4}$ is *not* in simplest form, since the numerator and the denominator can both be divided by 2.

If you get the answer $\frac{2}{4}$ to a problem, you should state it in simplest form, $\frac{1}{2}$. The process of changing $\frac{2}{4}$ to $\frac{1}{2}$ is called **simplifying** or **reducing** the fraction.

2 Simplifying Fractions to Lowest Terms Using Prime Numbers

Natural numbers or **counting numbers** are the set of whole numbers excluding 0. Thus the natural numbers are the numbers 1, 2, 3, 4, 5, 6, ...

When two or more numbers are multiplied, each number that is multiplied is called a **factor.** For example, when we write $3 \times 7 \times 5$, each of the numbers 3, 7, and 5 is called a factor.

Prime numbers are natural numbers greater than 1 whose only natural number factors are 1 and themselves. The number 5 is prime. The only natural number factors of 5 are 5 and 1.

$$5 = 5 \times 1$$

The number 6 is not prime. The natural number factors of 6 are 3 and 2 or 6 and 1.

$$6 = 3 \times 2 \qquad 6 = 6 \times 1$$

The first 15 prime numbers are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

Any natural number greater than 1 either is prime or can be written as the product of prime numbers. For example, we can take each of the numbers 12, 30, 14, 19, and 29 and either indicate that they are prime or, if they are not prime, write them as the product of prime numbers. We write as follows:

$12 = 2 \times 2 \times 3$ $\quad 30 = 2 \times 3 \times 5$ $\quad 14 = 2 \times 7$

19 is a prime number. 29 is a prime number.

To reduce a fraction, we use prime numbers to factor the numerator and the denominator. Write each part of the fraction (numerator and denominator) as a product of prime numbers. Note any *factors* that appear in both the *numerator* (top) and *denominator* (bottom) of the fraction. If we divide numerator and denominator by these values we will obtain an equivalent fraction in *simplest form*. When the new fraction is simplified, it is said to be in **lowest terms.** Throughout this text, to *simplify* a fraction will always mean to write the fraction in lowest terms.

Example 1 Simplify each fraction.

(a) $\frac{14}{21}$ **(b)** $\frac{15}{35}$ **(c)** $\frac{20}{70}$

Solution

(a) $\frac{14}{21} = \frac{\cancel{7} \times 2}{\cancel{7} \times 3} = \frac{2}{3}$ We factor 14 and factor 21. Then we divide numerator and denominator by 7.

(b) $\frac{15}{35} = \frac{\cancel{5} \times 3}{\cancel{5} \times 7} = \frac{3}{7}$ We factor 15 and factor 35. Then we divide numerator and denominator by 5.

(c) $\frac{20}{70} = \frac{2 \times \cancel{2} \times \cancel{5}}{7 \times \cancel{2} \times \cancel{5}} = \frac{2}{7}$ We factor 20 and factor 70. Then we divide numerator and denominator by both 2 and 5. □

Student Practice 1 Simplify each fraction.

(a) $\frac{10}{16}$ **(b)** $\frac{24}{36}$ **(c)** $\frac{36}{42}$

Sometimes when we simplify a fraction, all the prime factors in the top (numerator) are divided out. When this happens, we must remember that a 1 is left in the numerator.

Example 2 Simplify each fraction.

(a) $\frac{7}{21}$ **(b)** $\frac{15}{105}$

Solution

(a) $\frac{7}{21} = \frac{\cancel{7} \times 1}{\cancel{7} \times 3} = \frac{1}{3}$ **(b)** $\frac{15}{105} = \frac{\cancel{5} \times \cancel{3} \times 1}{7 \times \cancel{5} \times \cancel{3}} = \frac{1}{7}$ □

Student Practice 2 Simplify each fraction.

(a) $\frac{4}{12}$ **(b)** $\frac{25}{125}$ **(c)** $\frac{73}{146}$

If all the prime numbers in the bottom (denominator) are divided out, we do not need to leave a 1 in the denominator, since we do not need to express the answer as a fraction. The answer is then a whole number and is not usually expressed as a fraction.

Example 3 Simplify each fraction.

(a) $\dfrac{35}{7}$ **(b)** $\dfrac{70}{10}$

Solution

(a) $\dfrac{35}{7} = \dfrac{5 \times \cancel{7}}{\cancel{7} \times 1} = 5$ **(b)** $\dfrac{70}{10} = \dfrac{7 \times \cancel{5} \times \cancel{2}}{\cancel{5} \times \cancel{2} \times 1} = 7$

Student Practice 3 Simplify each fraction.

(a) $\dfrac{18}{6}$ **(b)** $\dfrac{146}{73}$ **(c)** $\dfrac{28}{7}$

Sometimes the fraction we use represents how many of a certain thing are successful. For example, if a baseball player was at bat 30 times and achieved 12 hits, we could say that he had a hit $\frac{12}{30}$ of the time. If we reduce the fraction, we could say he had a hit $\frac{2}{5}$ of the time.

Example 4 Cindy got 48 out of 56 questions correct on a test. Write this as a fraction in simplest form.

Solution Express as a fraction in simplest form the number of correct responses out of the total number of questions on the test.

$$48 \text{ out of } 56 \longrightarrow \frac{48}{56} = \frac{2 \times 3 \times \cancel{2} \times \cancel{2} \times \cancel{2}}{7 \times \cancel{2} \times \cancel{2} \times \cancel{2}} = \frac{6}{7}$$

Cindy answered the questions correctly $\frac{6}{7}$ of the time.

Student Practice 4 The major league pennant winner in 1917 won 56 games out of 154 games played. Express as a fraction in simplest form the number of games won in relation to the number of games played.

The number *one* can be expressed as $1, \frac{1}{1}, \frac{2}{2}, \frac{6}{6}, \frac{8}{8}$, and so on, since

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{6}{6} = \frac{8}{8}.$$

We say that these numerals are *equivalent ways* of writing the number *one* because they all express the same quantity even though they appear to be different.

Sidelight: The Multiplicative Identity

When we simplify fractions, we are actually using the fact that we can multiply any number by 1 without changing the value of that number. (Mathematicians call the number 1 the **multiplicative identity** because it leaves any number it multiplies with the same identical value as before.)

Let's look again at one of the previous examples.

$$\frac{14}{21} = \frac{7 \times 2}{7 \times 3} = \frac{7}{7} \times \frac{2}{3} = 1 \times \frac{2}{3} = \frac{2}{3}$$

So we see that

$$\frac{14}{21} = \frac{2}{3}$$

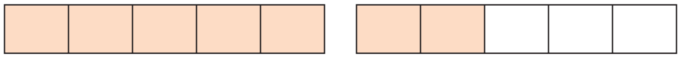
When we simplify fractions, we are using this property of multiplying by 1.

3 Converting Between Improper Fractions and Mixed Numbers

If the numerator is less than the denominator, the fraction is a **proper fraction.** A proper fraction is used to describe a quantity smaller than a whole.

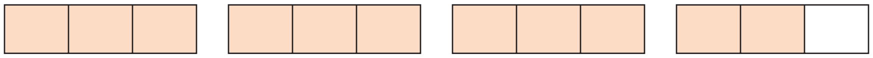
Fractions can also be used to describe quantities larger than a whole. The following figure shows two bars that are equal in size. Each bar is divided into 5 equal pieces. The first bar is shaded completely. The second bar has 2 of the 5 pieces shaded.

The shaded-in region can be represented by $\frac{7}{5}$ since 7 of the pieces (each of which is $\frac{1}{5}$ of a whole box) are shaded. The fraction $\frac{7}{5}$ is called an improper fraction. An **improper fraction** is one in which the numerator is larger than or equal to the denominator.



The shaded-in region can also be represented by 1 whole added to $\frac{2}{5}$ of a whole, or $1 + \frac{2}{5}$. This is written as $1\frac{2}{5}$. The fraction $1\frac{2}{5}$ is called a mixed number. A **mixed number** consists of a whole number added to a proper fraction (the numerator is smaller than the denominator). The addition is understood but not written. When we write $1\frac{2}{5}$, it represents $1 + \frac{2}{5}$. The numbers $1\frac{7}{8}$, $2\frac{3}{4}$, $8\frac{1}{3}$, and $126\frac{1}{10}$ are all mixed numbers. From the preceding figure it seems clear that $\frac{7}{5} = 1\frac{2}{5}$. This suggests that we can change from one form to the other without changing the value of the fraction.

From a picture it is easy to see how to *change improper fractions to mixed numbers*. For example, suppose we start with the fraction $\frac{11}{3}$ and represent it by the following figure (where 11 of the pieces, each of which is $\frac{1}{3}$ of a box, are shaded). We see that $\frac{11}{3} = 3\frac{2}{3}$, since 3 whole boxes and $\frac{2}{3}$ of a box are shaded.



Changing Improper Fractions to Mixed Numbers You can follow the same procedure without a picture. For example, to change $\frac{11}{3}$ to a mixed number, we can do the following:

$$\frac{11}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3}$$ Use the rule for adding fractions (which is discussed in detail in Section 0.2).

$$= 1 + 1 + 1 + \frac{2}{3}$$ Write 1 in place of $\frac{3}{3}$, since $\frac{3}{3} = 1$.

$$= 3 + \frac{2}{3}$$ Write 3 in place of $1 + 1 + 1$.

$$= 3\frac{2}{3}$$ Use the notation for mixed numbers.

Now that you know how to change improper fractions to mixed numbers and why the procedure works, here is a shorter method.

TO CHANGE AN IMPROPER FRACTION TO A MIXED NUMBER

1. Divide the denominator into the numerator.
2. The quotient is the whole-number part of the mixed number.
3. The remainder from the division will be the numerator of the fraction. The denominator of the fraction remains unchanged.

We can write the fraction as a division statement and divide. The arrows show how to write the mixed number.

$$\frac{7}{5} \qquad \begin{array}{r} 1 \\ 5\overline{)7} \\ \underline{5} \\ 2 \end{array}$$

Whole-number part

$1\frac{2}{5}$ Numerator of fraction

Remainder

Thus, $\frac{7}{5} = 1\frac{2}{5}$.

$\dfrac{11}{3}$ $\qquad 3\overline{)11}$, 9, 2

Whole-number part → $3\dfrac{2}{3}$ ← Numerator of fraction

Remainder

Thus, $\dfrac{11}{3} = 3\dfrac{2}{3}$.

Sometimes the remainder is 0. In this case, the improper fraction changes to a whole number.

Example 5 Change to a mixed number or to a whole number.

(a) $\dfrac{7}{4}$ **(b)** $\dfrac{15}{3}$

Solution

(a) $\dfrac{7}{4} = 7 \div 4$ $\qquad \begin{array}{r} 1 \\ 4\overline{)7} \\ 4 \\ \hline 3 \end{array}$ Remainder

Thus $\dfrac{7}{4} = 1\dfrac{3}{4}$.

(b) $\dfrac{15}{3} = 15 \div 3$ $\qquad \begin{array}{r} 5 \\ 3\overline{)15} \\ 15 \\ \hline 0 \end{array}$ Remainder

Thus $\dfrac{15}{3} = 5$. ◻

Student Practice 5 Change to a mixed number or to a whole number.

(a) $\dfrac{12}{7}$ **(b)** $\dfrac{20}{5}$

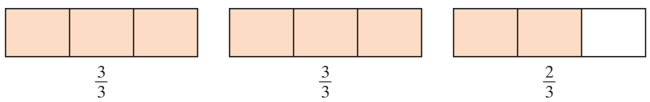
Changing Mixed Numbers to Improper Fractions It is not difficult to see how to change mixed numbers to improper fractions. Suppose that you wanted to write $2\frac{2}{3}$ as an improper fraction.

$$2\frac{2}{3} = 2 + \frac{2}{3} \qquad \text{The meaning of mixed number notation}$$

$$= 1 + 1 + \frac{2}{3} \qquad \text{Since } 1 + 1 = 2$$

$$= \frac{3}{3} + \frac{3}{3} + \frac{2}{3} \qquad \text{Since } 1 = \frac{3}{3}$$

When we draw a picture of $\frac{3}{3} + \frac{3}{3} + \frac{2}{3}$, we have this figure:



If we count the shaded parts, we see that

$$\frac{3}{3} + \frac{3}{3} + \frac{2}{3} = \frac{8}{3}. \qquad \text{Thus} \quad 2\frac{2}{3} = \frac{8}{3}.$$

Now that you have seen how this change can be done, here is a shorter method.

TO CHANGE A MIXED NUMBER TO AN IMPROPER FRACTION

1. Multiply the whole number by the denominator.
2. Add this to the numerator. The result is the new numerator. The denominator does not change.

Example 6 Change to an improper fraction.

(a) $3\frac{1}{7}$ **(b)** $5\frac{4}{5}$

Solution

(a) $3\frac{1}{7} = \frac{(3 \times 7) + 1}{7} = \frac{21 + 1}{7} = \frac{22}{7}$

(b) $5\frac{4}{5} = \frac{(5 \times 5) + 4}{5} = \frac{25 + 4}{5} = \frac{29}{5}$

Student Practice 6 Change to an improper fraction.

(a) $3\frac{2}{5}$ **(b)** $1\frac{3}{7}$ **(c)** $2\frac{6}{11}$ **(d)** $4\frac{2}{3}$

4 Changing a Fraction to an Equivalent Fraction with a Given Denominator

A fraction can be changed to an equivalent fraction with a different denominator by multiplying both numerator and denominator by the same number.

$$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12} \quad \text{and} \quad \frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21} \text{ so}$$

$$\frac{5}{6} \text{ is equivalent to } \frac{10}{12} \quad \text{and} \quad \frac{3}{7} \text{ is equivalent to } \frac{9}{21}.$$

We often multiply in this way to obtain an equivalent fraction with a *particular denominator.*

Example 7 Find the missing numerator.

(a) $\frac{3}{5} = \frac{?}{25}$ **(b)** $\frac{4}{7} = \frac{?}{14}$ **(c)** $\frac{2}{9} = \frac{?}{36}$

Solution

(a) $\frac{3}{5} = \frac{?}{25}$ Observe that we need to multiply the denominator by 5 to obtain 25. So we multiply the numerator 3 by 5 also.

$\frac{3 \times 5}{5 \times 5} = \frac{15}{25}$ The desired numerator is 15.

(b) $\frac{4}{7} = \frac{?}{14}$ Observe that $7 \times 2 = 14$. We need to multiply the numerator by 2 to get the new numerator.

$\frac{4 \times 2}{7 \times 2} = \frac{8}{14}$ The desired numerator is 8.

(c) $\frac{2}{9} = \frac{?}{36}$ Observe that $9 \times 4 = 36$. We need to multiply the numerator by 4 to get the new numerator.

$\frac{2 \times 4}{9 \times 4} = \frac{8}{36}$ The desired numerator is 8.

Student Practice 7 Find the missing numerator.

(a) $\frac{3}{8} = \frac{?}{24}$ **(b)** $\frac{5}{6} = \frac{?}{30}$ **(c)** $\frac{2}{7} = \frac{?}{56}$

0.1 Exercises MyLab™ Math

Verbal and Writing Skills, Exercises 1–4

1. In the fraction $\frac{12}{13}$, what number is the numerator?

2. In the fraction $\frac{13}{17}$, what number is the denominator?

3. What is a factor? Give an example.

4. Give some examples of the number 1 written as a fraction.

5. Draw a diagram to illustrate $2\frac{2}{3}$.

6. Draw a diagram to illustrate $3\frac{3}{4}$.

Simplify each fraction.

7. $\frac{9}{15}$ **8.** $\frac{20}{24}$ **9.** $\frac{12}{36}$ **10.** $\frac{8}{48}$ **11.** $\frac{60}{12}$ **12.** $\frac{72}{18}$

13. $\frac{24}{36}$ **14.** $\frac{30}{85}$ **15.** $\frac{32}{64}$ **16.** $\frac{33}{55}$ **17.** $\frac{42}{7}$ **18.** $\frac{63}{9}$

Change to a mixed number.

19. $\frac{17}{6}$ **20.** $\frac{19}{5}$ **21.** $\frac{47}{5}$ **22.** $\frac{54}{7}$ **23.** $\frac{38}{7}$ **24.** $\frac{41}{6}$

25. $\frac{41}{2}$ **26.** $\frac{25}{3}$ **27.** $\frac{32}{5}$ **28.** $\frac{79}{7}$ **29.** $\frac{111}{9}$ **30.** $\frac{124}{8}$

Change to an improper fraction or whole number.

31. $3\frac{1}{5}$ **32.** $4\frac{2}{5}$ **33.** $6\frac{3}{5}$ **34.** $5\frac{1}{12}$ **35.** $1\frac{2}{9}$ **36.** $1\frac{5}{6}$

37. $8\frac{3}{7}$ **38.** $6\frac{2}{3}$ **39.** $24\frac{1}{4}$ **40.** $10\frac{1}{9}$ **41.** $\frac{72}{9}$ **42.** $\frac{78}{6}$

Find the missing numerator.

43. $\frac{3}{8} = \frac{?}{64}$ **44.** $\frac{5}{9} = \frac{?}{54}$ **45.** $\frac{3}{5} = \frac{?}{35}$

46. $\frac{5}{9} = \frac{?}{45}$ **47.** $\frac{4}{13} = \frac{?}{39}$ **48.** $\frac{13}{17} = \frac{?}{51}$

49. $\frac{3}{7} = \frac{?}{49}$ **50.** $\frac{10}{15} = \frac{?}{60}$ **51.** $\frac{3}{4} = \frac{?}{20}$

52. $\frac{7}{8} = \frac{?}{40}$ **53.** $\frac{35}{40} = \frac{?}{80}$ **54.** $\frac{45}{50} = \frac{?}{100}$

Applications

Solve.

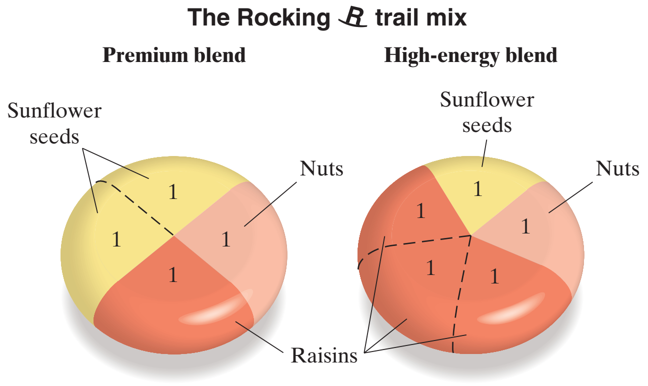
55. ***Women's Professional Basketball*** During a WNBA basketball season, Maya Moore of the Minnesota Lynx scored 799 points in 34 games. Express as a mixed number in simplified form how many points she averaged per game.

56. ***Kentucky Derby Nominations*** In a recent year, 424 horses were nominated to compete in the Kentucky Derby. Only 20 horses were actually chosen to compete in the Derby. What simplified fraction shows what portions of the nominated horses actually competed?

57. ***Income Tax*** Last year, my parents had a combined income of $64,000. They paid $13,200 in federal income taxes. What simplified fraction shows how much my parents spent on their federal taxes?

58. ***Employment*** A large employment agency was able to find jobs within 6 months for 1400 people out of 2420 applicants who applied at one of its branches. What simplified fraction shows what portion of applicants gained employment?

Trail Mix *The following chart gives recipes for two trail mix blends.*



59. What fractional part of the premium blend is nuts?

60. What fractional part of the high-energy blend is raisins?

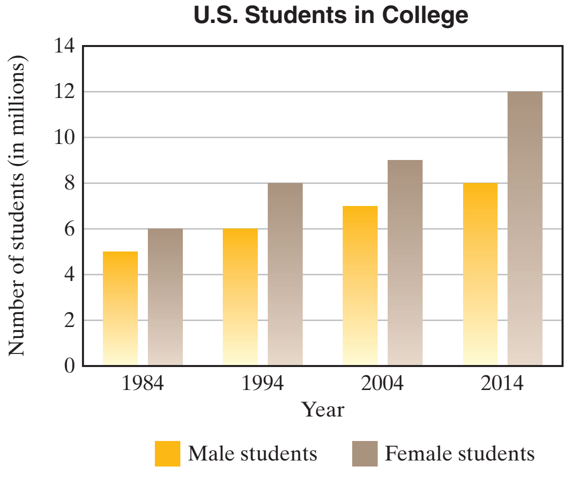
61. What fractional part of the premium blend is not sunflower seeds?

62. What fractional part of the high-energy blend does not contain nuts?

College Enrollment *The following chart provides statistics about the total enrollment of male and female students in U.S. colleges for specific years during the period from 1984 to 2014.*

63. What fractional part of the number of students enrolled in 2014 are female?

64. What fractional part of the number of students enrolled in 1994 are male?



Source: Digest of Education statistics, 2014

Quick Quiz 0.1

1. Simplify. $\dfrac{84}{92}$

2. Write as an improper fraction. $6\dfrac{9}{11}$

3. Write as a mixed number. $\dfrac{103}{21}$

4. **Concept Check** Explain in your own words how to change a mixed number to an improper fraction.

0.2 Adding and Subtracting Fractions

Student Learning Objectives

After studying this section, you will be able to:

1. Add or subtract fractions with a common denominator.
2. Use prime factors to find the least common denominator of two or more fractions.
3. Add or subtract fractions with different denominators.
4. Add or subtract mixed numbers.

1 Adding or Subtracting Fractions with a Common Denominator

If fractions have the same denominator, the numerators may be added or subtracted. The denominator remains the same.

TO ADD OR SUBTRACT TWO FRACTIONS WITH A COMMON DENOMINATOR

1. Add or subtract the numerators.
2. Keep the same (common) denominator.
3. Simplify the answer whenever possible.

Example 1 Add the fractions. Simplify your answer whenever possible.

(a) $\frac{5}{7}+\frac{1}{7}$ **(b)** $\frac{2}{3}+\frac{1}{3}$ **(c)** $\frac{1}{8}+\frac{3}{8}+\frac{2}{8}$ **(d)** $\frac{3}{5}+\frac{4}{5}$

Solution

(a) $\frac{5}{7}+\frac{1}{7}=\frac{5+1}{7}=\frac{6}{7}$

(b) $\frac{2}{3}+\frac{1}{3}=\frac{2+1}{3}=\frac{3}{3}=1$

(c) $\frac{1}{8}+\frac{3}{8}+\frac{2}{8}=\frac{1+3+2}{8}=\frac{6}{8}=\frac{3}{4}$

(d) $\frac{3}{5}+\frac{4}{5}=\frac{3+4}{5}=\frac{7}{5}$ or $1\frac{2}{5}$ □

Student Practice 1 Add the fractions. Simplify your answer whenever possible.

(a) $\frac{3}{6}+\frac{2}{6}$ **(b)** $\frac{3}{11}+\frac{8}{11}$ **(c)** $\frac{1}{8}+\frac{2}{8}+\frac{1}{8}$ **(d)** $\frac{5}{9}+\frac{8}{9}$

Example 2 Subtract the fractions. Simplify your answer whenever possible.

(a) $\frac{9}{11}-\frac{2}{11}$ **(b)** $\frac{5}{6}-\frac{1}{6}$

Solution

(a) $\frac{9}{11}-\frac{2}{11}=\frac{9-2}{11}=\frac{7}{11}$

(b) $\frac{5}{6}-\frac{1}{6}=\frac{5-1}{6}=\frac{4}{6}=\frac{2}{3}$ □

Student Practice 2 Subtract the fractions. Simplify your answer whenever possible.

(a) $\frac{11}{13}-\frac{6}{13}$ **(b)** $\frac{8}{9}-\frac{2}{9}$

Although adding and subtracting fractions with the same denominator is fairly simple, most problems involve fractions that do not have a common denominator. Fractions and mixed numbers such as halves, fourths, and eighths are often used. To add or subtract such fractions, we begin by finding a common denominator.

2 Using Prime Factors to Find the Least Common Denominator of Two or More Fractions

Before you can add or subtract fractions, they must have the same denominator. To save work, we select the smallest possible common denominator. This is called the **least common denominator** or LCD (also known as the *lowest common denominator*).

The LCD of two or more fractions is the smallest whole number that is exactly divisible by each denominator of the fractions.

Example 3 Find the LCD. $\frac{2}{3}$ and $\frac{1}{4}$

Solution The numbers are small enough to find the LCD by inspection. The LCD is 12, since 12 is exactly divisible by 4 and by 3. There is no smaller number that is exactly divisible by 4 and 3.

Student Practice 3 Find the LCD. $\frac{1}{8}$ and $\frac{5}{7}$

In some cases, the LCD cannot easily be determined by inspection. If we write each denominator as the product of prime factors, we will be able to find the LCD. We will use $(\cdot)$ to indicate multiplication. For example, $30 = 2 \cdot 3 \cdot 5$. This means $30 = 2 \times 3 \times 5$.

PROCEDURE TO FIND THE LCD USING PRIME FACTORS

1. Write each denominator as the product of prime factors.
2. The LCD is a product containing each different factor.
3. If a factor occurs more than once in any one denominator, the LCD will contain that factor repeated the greatest number of times that it occurs in any one denominator.

Example 4 Find the LCD of $\frac{5}{6}$ and $\frac{1}{15}$ using the prime factor method.

Solution

$6 = 2 \cdot 3$ Write each denominator as the product of prime factors.

$15 = \phantom{2 \cdot{}} 3 \cdot 5$

$\text{LCD} = 2 \cdot 3 \cdot 5$ The LCD is a product containing each different prime factor.

$\text{LCD} = 2 \cdot 3 \cdot 5 = 30$ The different factors are 2, 3, and 5, and each factor appears at most once in any one denominator.

Student Practice 4 Find the LCD of $\frac{8}{35}$ and $\frac{6}{15}$ using the prime factor method.

Great care should be used to determine the LCD in the case of repeated factors.

Example 5 Find the LCD of $\frac{4}{27}$ and $\frac{5}{18}$.

Solution

$27 = 3 \cdot 3 \cdot 3$ Write each denominator as the product of prime factors. We observe that the factor 3 occurs three times in the factorization of 27.

$18 = \phantom{3 \cdot{}} 3 \cdot 3 \cdot 2$

$\text{LCD} = 3 \cdot 3 \cdot 3 \cdot 2$

$\text{LCD} = 3 \cdot 3 \cdot 3 \cdot 2 = 54$

The LCD is a product containing each different factor. The factor 3 *occurred most* in the factorization of 27, where it occurred *three* times. Thus the LCD will be the product of *three* 3s and *one* 2.

Student Practice 5 Find the LCD of $\frac{5}{12}$ and $\frac{7}{30}$.

Example 6 Find the LCD of $\frac{5}{12}, \frac{1}{15}$, and $\frac{7}{30}$.

Solution

$$\begin{aligned} 12 &= 2 \cdot 2 \cdot 3 \\ 15 &= 3 \cdot 5 \\ 30 &= 2 \cdot 3 \cdot 5 \\ \text{LCD} &= 2 \cdot 2 \cdot 3 \cdot 5 \end{aligned}$$

Write each denominator as the product of prime factors. Notice that the only repeated factor is 2, which occurs twice in the factorization of 12.

$$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot 5 = 60$$

The LCD is the product of each different factor, with the factor 2 appearing twice since it occurred twice in one denominator.

Student Practice 6 Find the LCD of $\frac{2}{27}, \frac{1}{18}$, and $\frac{5}{12}$.

3 Adding or Subtracting Fractions with Different Denominators

Before you can add or subtract them, fractions must have the same denominator. Using the LCD will make your work easier. First you must find the LCD. Then change each fraction to a fraction that has the LCD as the denominator. Sometimes one of the fractions will already have the LCD as the denominator. Once all the fractions have the same denominator, you can add or subtract. Be sure to simplify the fraction in your answer if this is possible.

TO ADD OR SUBTRACT FRACTIONS THAT DO NOT HAVE A COMMON DENOMINATOR

1. Find the LCD of the fractions.
2. Change each fraction to an equivalent fraction with the LCD for a denominator.
3. Add or subtract the fractions.
4. Simplify the answer whenever possible.

Let us return to the two fractions of Example 3. We have previously found that the LCD is 12.

Example 7 Bob picked $\frac{2}{3}$ of a bushel of apples on Monday and $\frac{1}{4}$ of a bushel of apples on Tuesday. How much did he pick in total?

Solution To solve this problem we need to add $\frac{2}{3}$ and $\frac{1}{4}$, but before we can do so, we must change $\frac{2}{3}$ and $\frac{1}{4}$ to fractions with the same denominator. We change each fraction to an equivalent fraction with a common denominator of 12, the LCD.

$$\frac{2}{3} = \frac{?}{12} \qquad \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \quad \text{so} \quad \frac{2}{3} = \frac{8}{12}$$

$$\frac{1}{4} = \frac{?}{12} \qquad \frac{1 \times 3}{4 \times 3} = \frac{3}{12} \quad \text{so} \quad \frac{1}{4} = \frac{3}{12}$$

Then we rewrite the problem with common denominators and add.

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{8+3}{12} = \frac{11}{12}$$

In total Bob picked $\frac{11}{12}$ of a bushel of apples.

Student Practice 7 Carol planted corn in $\frac{5}{7}$ of the farm fields at the Old Robinson Farm. Connie planted soybeans in $\frac{1}{8}$ of the farm fields. What fractional part of the farm fields of the Old Robinson Farm was planted in corn or soybeans?

Sometimes one of the denominators is the LCD. In such cases the fraction that has the LCD for the denominator will not need to be changed. If every other denominator divides into the largest denominator, the largest denominator is the LCD.

Example 8 Find the LCD and then add. $\frac{3}{5} + \frac{7}{20} + \frac{1}{2}$

Solution We can see by inspection that both 5 and 2 divide exactly into 20. Thus 20 is the LCD. Now add.

$$\frac{3}{5} + \frac{7}{20} + \frac{1}{2}$$

We change $\frac{3}{5}$ and $\frac{1}{2}$ to equivalent fractions with a common denominator of 20, the LCD.

$$\frac{3}{5} = \frac{?}{20} \qquad \frac{3 \times 4}{5 \times 4} = \frac{12}{20} \quad \text{so} \quad \frac{3}{5} = \frac{12}{20}$$

$$\frac{1}{2} = \frac{?}{20} \qquad \frac{1 \times 10}{2 \times 10} = \frac{10}{20} \quad \text{so} \quad \frac{1}{2} = \frac{10}{20}$$

Then we rewrite the problem with common denominators and add.

$$\frac{3}{5} + \frac{7}{20} + \frac{1}{2} = \frac{12}{20} + \frac{7}{20} + \frac{10}{20} = \frac{12 + 7 + 10}{20} = \frac{29}{20} \quad \text{or} \quad 1\frac{9}{20}$$

Student Practice 8 Find the LCD and then add.

$$\frac{4}{5} + \frac{6}{25} + \frac{1}{50}$$

Now we turn to examples where the selection of the LCD is not so obvious. In Examples 9 through 11 we will use the prime factorization method to find the LCD.

Example 9 Add. $\frac{7}{18} + \frac{5}{12}$

Solution First we find the LCD.

$$18 = 3 \cdot 3 \cdot 2$$
$$12 = \quad\ \ 3 \cdot 2 \cdot 2$$
$$\downarrow \ \ \downarrow \ \ \downarrow \ \ \downarrow$$
$$\text{LCD} = 3 \cdot 3 \cdot 2 \cdot 2 = 36$$

Now we change $\frac{7}{18}$ and $\frac{5}{12}$ to equivalent fractions that have the LCD.

$$\frac{7}{18} = \frac{?}{36} \qquad \frac{7 \times 2}{18 \times 2} = \frac{14}{36}$$

$$\frac{5}{12} = \frac{?}{36} \qquad \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$$

Now we add the fractions.

$$\frac{7}{18} + \frac{5}{12} = \frac{14}{36} + \frac{15}{36} = \frac{29}{36}$$ This fraction cannot be simplified.

Student Practice 9 Add.

$$\frac{1}{49} + \frac{3}{14}$$

Example 10 Subtract. $\frac{25}{48} - \frac{5}{36}$

Solution First we find the LCD.

$$\begin{aligned} 48 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \\ 36 &= \downarrow \quad \downarrow \quad 2 \cdot 2 \cdot 3 \cdot 3 \\ \text{LCD} &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 144 \end{aligned}$$

Now we change $\frac{25}{48}$ and $\frac{5}{36}$ to equivalent fractions that have the LCD.

$$\frac{25}{48} = \frac{?}{144} \qquad \frac{25 \times 3}{48 \times 3} = \frac{75}{144}$$

$$\frac{5}{36} = \frac{?}{144} \qquad \frac{5 \times 4}{36 \times 4} = \frac{20}{144}$$

Now we subtract the fractions.

$$\frac{25}{48} - \frac{5}{36} = \frac{75}{144} - \frac{20}{144} = \frac{55}{144}$$ This fraction cannot be simplified.

Student Practice 10 Subtract.

$$\frac{1}{12} - \frac{1}{30}$$

Example 11 Combine. $\frac{1}{5} + \frac{1}{6} - \frac{3}{10}$

Solution First we find the LCD.

$$\begin{aligned} 5 &= 5 \\ 6 &= \quad\; 2 \cdot 3 \\ 10 &= 5 \cdot 2 \quad \downarrow \\ \text{LCD} &= 5 \cdot 2 \cdot 3 = 30 \end{aligned}$$

Now we change $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{3}{10}$ to equivalent fractions that have the LCD for a denominator.

$$\frac{1}{5} = \frac{?}{30} \qquad \frac{1 \times 6}{5 \times 6} = \frac{6}{30}$$

$$\frac{1}{6} = \frac{?}{30} \qquad \frac{1 \times 5}{6 \times 5} = \frac{5}{30}$$

$$\frac{3}{10} = \frac{?}{30} \qquad \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$$

Now we combine the three fractions.

$$\frac{1}{5} + \frac{1}{6} - \frac{3}{10} = \frac{6}{30} + \frac{5}{30} - \frac{9}{30} = \frac{2}{30} = \frac{1}{15}$$

Note the important step of simplifying the fraction to obtain the final answer.

Student Practice 11 Combine.

$$\frac{2}{3} + \frac{3}{4} - \frac{3}{8}$$

4 Adding or Subtracting Mixed Numbers

If your addition or subtraction problem has mixed numbers, change them to improper fractions first and then combine (add or subtract). As a convention in this book, if the original problem contains mixed numbers, express the result as a mixed number rather than as an improper fraction.

Example 12 Combine. Simplify your answer whenever possible.

(a) $5\frac{1}{2} + 2\frac{1}{3}$ **(b)** $2\frac{1}{5} - 1\frac{3}{4}$ **(c)** $1\frac{5}{12} + \frac{7}{30}$

Solution

(a) First we change the mixed numbers to improper fractions.

$$5\frac{1}{2} = \frac{5 \times 2 + 1}{2} = \frac{11}{2} \qquad 2\frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3}$$

Next we change each fraction to an equivalent form with the common denominator of 6.

$$\frac{11}{2} = \frac{?}{6} \qquad \frac{11 \times 3}{2 \times 3} = \frac{33}{6}$$

$$\frac{7}{3} = \frac{?}{6} \qquad \frac{7 \times 2}{3 \times 2} = \frac{14}{6}$$

Finally, we add the two fractions and change our answer to a mixed number.

$$\frac{33}{6} + \frac{14}{6} = \frac{47}{6} = 7\frac{5}{6}$$

Thus $5\frac{1}{2} + 2\frac{1}{3} = 7\frac{5}{6}$.

(b) First we change the mixed numbers to improper fractions.

$$2\frac{1}{5} = \frac{2 \times 5 + 1}{5} = \frac{11}{5} \qquad 1\frac{3}{4} = \frac{1 \times 4 + 3}{4} = \frac{7}{4}$$

Next we change each fraction to an equivalent form with the common denominator of 20.

$$\frac{11}{5} = \frac{?}{20} \qquad \frac{11 \times 4}{5 \times 4} = \frac{44}{20}$$

$$\frac{7}{4} = \frac{?}{20} \qquad \frac{7 \times 5}{4 \times 5} = \frac{35}{20}$$

Now we subtract the two fractions.

$$\frac{44}{20} - \frac{35}{20} = \frac{9}{20}$$

Thus $2\frac{1}{5} - 1\frac{3}{4} = \frac{9}{20}$.

Note: It is not necessary to use these exact steps to add and subtract mixed numbers. If you know another method and can use it to obtain the correct answers, it is all right to continue to use that method throughout this chapter.

(c) Now we add $1\frac{5}{12} + \frac{7}{30}$.

The LCD of 12 and 30 is 60. Why? Change the mixed number to an improper fraction. Then change each fraction to an equivalent form with a common denominator.

$$1\frac{5}{12} = \frac{17 \times 5}{12 \times 5} = \frac{85}{60} \qquad \frac{7 \times 2}{30 \times 2} = \frac{14}{60}$$

Continued on next page

Then add the fractions, simplify, and write the answer as a mixed number.

$$\frac{85}{60} + \frac{14}{60} = \frac{99}{60} = \frac{33}{20} = 1\frac{13}{20}$$

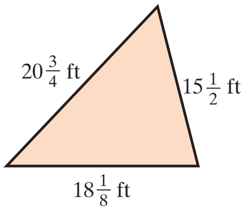
Thus $1\frac{5}{12} + \frac{7}{30} = 1\frac{13}{20}$.

Student Practice 12 Combine. Simplify your answer whenever possible.

(a) $1\frac{2}{3} + 2\frac{4}{5}$ **(b)** $5\frac{1}{4} - 2\frac{2}{3}$

▲ Example 13 Manuel is enclosing a triangle-shaped exercise yard for his new dog. He wants to determine how many feet of fencing he will need. The sides of the yard measure $20\frac{3}{4}$ feet, $15\frac{1}{2}$ feet, and $18\frac{1}{8}$ feet. What is the perimeter of (total distance around) the triangle?

Solution ***Understand the problem.*** Begin by drawing a picture.



We want to add up the lengths of all three sides of the triangle. This distance around the triangle is called the **perimeter.**

$$20\frac{3}{4} + 15\frac{1}{2} + 18\frac{1}{8} = \frac{83}{4} + \frac{31}{2} + \frac{145}{8}$$

$$= \frac{166}{8} + \frac{124}{8} + \frac{145}{8} = \frac{435}{8} = 54\frac{3}{8}$$

He will need $54\frac{3}{8}$ feet of fencing.

▲ Student Practice 13 Find the perimeter of a rectangle with sides of $4\frac{1}{5}$ cm and $6\frac{1}{2}$ cm. Begin by drawing a picture. Label the picture by including the measure of *each* side.

STEPS TO SUCCESS Faithful Class Attendance Is Well Worth It.

If you attend a traditional mathematics class that meets one or more times each week:

Get started in the right direction. Make a personal commitment to attend class every day, beginning with the first day of class. Teachers and students all over the country have discovered that faithful class attendance and good grades go together.

The vital content of class. What goes on in class is designed to help you learn more quickly. Each day significant information is given that will truly help you to understand concepts. There is no substitute for this firsthand learning experience.

Meet a friend. You will soon discover that other students are also coming to class every single class period. It is easy to strike up a friendship with students like you who have this common commitment. They will usually be available to answer a question after class and give you an additional source of help when you encounter difficulty.

Making it personal: Write down what you think is the most compelling reason to come to every class meeting. Make that commitment and see how much it helps you.

0.2 Exercises MyLab™ Math

Verbal and Writing Skills, Exercises 1 and 2

1. Explain why the denominator 8 is the least common denominator of $\frac{3}{4}$ and $\frac{5}{8}$.

2. What must you do before you add or subtract fractions that do not have a common denominator?

Find the LCD (least common denominator) of each set of fractions. Do not combine the fractions; only find the LCD.

3. $\frac{4}{9}$ and $\frac{5}{12}$

4. $\frac{21}{30}$ and $\frac{17}{20}$

5. $\frac{7}{10}$ and $\frac{1}{4}$

6. $\frac{5}{18}$ and $\frac{1}{24}$

7. $\frac{5}{18}$ and $\frac{7}{54}$

8. $\frac{5}{16}$ and $\frac{7}{48}$

9. $\frac{1}{15}$ and $\frac{4}{21}$

10. $\frac{11}{12}$ and $\frac{7}{20}$

11. $\frac{17}{40}$ and $\frac{13}{60}$

12. $\frac{7}{30}$ and $\frac{8}{45}$

13. $\frac{2}{5}, \frac{3}{8}$, and $\frac{5}{12}$

14. $\frac{1}{7}, \frac{3}{14}$, and $\frac{9}{35}$

15. $\frac{5}{6}, \frac{9}{14}$, and $\frac{17}{26}$

16. $\frac{3}{8}, \frac{5}{12}$, and $\frac{11}{42}$

17. $\frac{1}{2}, \frac{1}{18}$, and $\frac{13}{30}$

18. $\frac{5}{8}, \frac{3}{14}$, and $\frac{11}{16}$

Combine. Be sure to simplify your answer whenever possible.

19. $\frac{3}{8} + \frac{2}{8}$

20. $\frac{3}{11} + \frac{5}{11}$

21. $\frac{5}{14} - \frac{1}{14}$

22. $\frac{11}{15} - \frac{2}{15}$

23. $\frac{5}{12} + \frac{5}{8}$

24. $\frac{3}{20} + \frac{13}{15}$

25. $\frac{5}{7} - \frac{2}{9}$

26. $\frac{4}{5} - \frac{3}{7}$

27. $\frac{1}{3} + \frac{2}{5}$

28. $\frac{3}{8} + \frac{1}{3}$

29. $\frac{5}{9} + \frac{5}{12}$

30. $\frac{2}{15} + \frac{7}{10}$

31. $\frac{11}{15} - \frac{31}{45}$

32. $\frac{21}{12} - \frac{23}{24}$

33. $\frac{16}{24} - \frac{1}{6}$

34. $\frac{13}{15} - \frac{1}{5}$

35. $\frac{3}{8} + \frac{4}{7}$

36. $\frac{7}{4} + \frac{5}{9}$

37. $\frac{2}{3} + \frac{7}{12} + \frac{1}{4}$

38. $\frac{4}{7} + \frac{7}{9} + \frac{1}{3}$

39. $\frac{5}{30} + \frac{3}{40} + \frac{1}{8}$

40. $\frac{1}{12} + \frac{3}{14} + \frac{4}{21}$

41. $\frac{1}{3} + \frac{1}{12} - \frac{1}{6}$

42. $\frac{1}{5} + \frac{2}{3} - \frac{11}{15}$

43. $\frac{5}{36} + \frac{7}{9} - \frac{5}{12}$

44. $\frac{5}{24} + \frac{3}{8} - \frac{1}{3}$

45. $4\frac{1}{3} + 3\frac{2}{5}$

46. $3\frac{1}{8} + 2\frac{1}{6}$

47. $1\frac{5}{24} + \frac{5}{18}$

48. $6\frac{2}{3} + \frac{3}{4}$

49. $7\frac{1}{6} - 2\frac{1}{4}$

50. $7\frac{2}{5} - 3\frac{3}{4}$

51. $8\frac{5}{7} - 2\frac{1}{4}$

52. $7\frac{8}{15} - 2\frac{3}{5}$

53. $2\frac{1}{8} + 3\frac{2}{3}$

54. $3\frac{1}{7} + 4\frac{1}{3}$

55. $11\frac{1}{7} - 6\frac{5}{7}$

56. $12\frac{1}{3} - 5\frac{2}{3}$

57. $3\frac{5}{12} + 5\frac{7}{12}$

58. $9\frac{12}{13} + 9\frac{1}{13}$

Mixed Practice

59. $\frac{7}{8} + \frac{1}{12}$

60. $\frac{19}{30} + \frac{3}{10}$

61. $3\frac{3}{16} + 4\frac{3}{8}$

62. $5\frac{2}{3} + 7\frac{2}{5}$

63. $\frac{16}{21} - \frac{2}{7}$

64. $\frac{15}{24} - \frac{3}{8}$

65. $5\frac{1}{5} - 2\frac{1}{2}$

66. $6\frac{1}{3} - 4\frac{1}{4}$

67. $\frac{5}{7} - \frac{1}{7}$

68. $\frac{3}{8} - \frac{1}{8}$

69. $1\frac{1}{6} + \frac{3}{8}$

70. $1\frac{2}{3} + \frac{5}{18}$

71. $\frac{1}{6} + \frac{5}{6}$

72. $\frac{3}{15} + \frac{2}{15}$

73. $36 - 2\frac{4}{7}$

74. $28 - 3\frac{5}{8}$

Applications

75. ***Inline Skating*** Nancy and Sarah meet three mornings a week to skate. They skated $8\frac{1}{4}$ miles on Monday, $10\frac{2}{3}$ miles on Wednesday, and $5\frac{3}{4}$ miles on Friday. What was their total distance for those three days?

76. ***Marathon Training*** Paco and Eskinder are training for the Boston Marathon. Their coach gave them the following schedule: a medium run of $10\frac{1}{2}$ miles on Thursday, a short run of $5\frac{1}{4}$ miles on Friday, a rest day on Saturday, and a long run of $18\frac{2}{3}$ miles on Sunday. How many miles did they run over these four days?

77. ***Restaurant Management*** The manager of a Boston restaurant must have his staff replace unsafe and rusted knives and replace tables and chairs in the dining area on Monday when the restaurant is closed. He has scheduled the staff for $15\frac{1}{2}$ hours of work. He estimates it will take $3\frac{2}{3}$ hours to replace the unsafe and rusted knives. He estimates it will take $9\frac{1}{4}$ hours to replace the tables and chairs in the dining area. In the time remaining, he wants them to wash the front windows. How much time will be available for washing the front windows?

78. ***Aquariums*** Carl bought a 20-gallon aquarium. He put $17\frac{3}{4}$ gallons of water into the aquarium, but it looked too low, so he added $1\frac{1}{4}$ more gallons of water. He then put in the artificial plants and the gravel but now the water was too high, so he siphoned off $2\frac{2}{3}$ gallons of water. How many gallons of water are now in the aquarium?

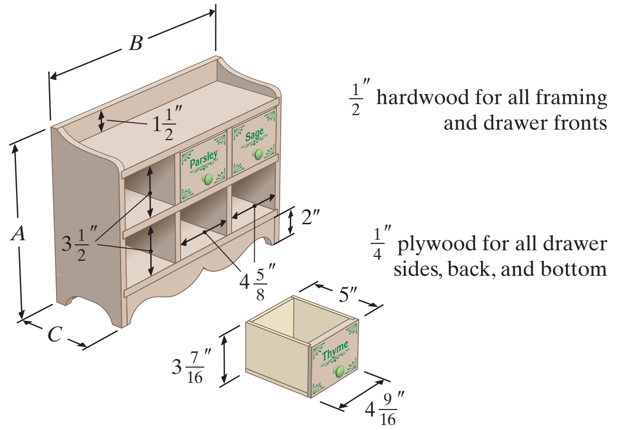
To Think About

Carpentry *Carpenters use fractions in their work. The picture below is a diagram of a spice cabinet. The symbol ″ means inches. Use the picture to answer exercises 79 and 80.*

79. Before you can determine where the cabinet will fit, you need to calculate the height, A, and the width, B. Don't forget to include the $\frac{1}{2}$-inch thickness of the wood where needed.

80. Look at the close-up of the drawer. The width is $4\frac{9}{16}''$. In the diagram, the width of the opening for the drawer is $4\frac{5}{8}''$. What is the difference?

Why do you think the drawer is smaller than the opening?



81. ***Facilities Management*** The Falmouth Country Club maintains the putting greens with a grass height of $\frac{7}{8}$ inch. The grass on the fairways is maintained at a height of $2\frac{1}{2}$ inches. How much must the mower blade be lowered by a person mowing the fairways if that person will be using the same mowing machine on the putting greens?

82. ***Facilities Management*** The director of facilities maintenance at the club in Exercise 81 discovered that due to slippage in the adjustment lever, the lawn mower actually cuts the grass $\frac{1}{16}$ of an inch too long or too short on some days. What is the maximum height that the fairway grass could be after being mowed with this machine? What is the minimum height that the putting greens could be after being mowed with this machine?

Cumulative Review

83. **[0.1.2]** Simplify. $\dfrac{36}{44}$

84. **[0.1.3]** Change to an improper fraction. $26\dfrac{3}{5}$

Quick Quiz 0.2 *Perform the operations indicated. Simplify your answers whenever possible.*

1. $\dfrac{3}{4} + \dfrac{1}{2} + \dfrac{5}{12}$

2. $2\dfrac{3}{5} + 4\dfrac{14}{15}$

3. $6\dfrac{1}{9} - 3\dfrac{5}{6}$

4. **Concept Check** Explain how you would find the LCD of the fractions $\frac{4}{21}$ and $\frac{5}{18}$.

0.3 Multiplying and Dividing Fractions

Student Learning Objectives

After studying this section, you will be able to:

1. Multiply fractions, whole numbers, and mixed numbers.
2. Divide fractions, whole numbers, and mixed numbers.

1 Multiplying Fractions, Whole Numbers, and Mixed Numbers

Multiplying Fractions During a recent snowstorm, the runway at Beverly Airport was plowed. However, the plow cleared only $\frac{3}{5}$ of the width and $\frac{2}{7}$ of the length. What fraction of the total runway area was cleared? To answer this question, we need to multiply $\frac{3}{5} \times \frac{2}{7}$.

The answer is that $\frac{6}{35}$ of the total runway area was cleared.

The multiplication rule for fractions states that to multiply two fractions, we multiply the two numerators and multiply the two denominators.

TO MULTIPLY ANY TWO FRACTIONS

1. Multiply the numerators.
2. Multiply the denominators.

Example 1 Multiply.

(a) $\frac{3}{5} \times \frac{2}{7}$ **(b)** $\frac{1}{3} \times \frac{5}{4}$ **(c)** $\frac{7}{3} \times \frac{1}{5}$ **(d)** $\frac{6}{5} \times \frac{2}{3}$

Solution

(a) $\frac{3}{5} \times \frac{2}{7} = \frac{3 \cdot 2}{5 \cdot 7} = \frac{6}{35}$

(b) $\frac{1}{3} \times \frac{5}{4} = \frac{1 \cdot 5}{3 \cdot 4} = \frac{5}{12}$

(c) $\frac{7}{3} \times \frac{1}{5} = \frac{7 \cdot 1}{3 \cdot 5} = \frac{7}{15}$

(d) $\frac{6}{5} \times \frac{2}{3} = \frac{6 \cdot 2}{5 \cdot 3} = \frac{12}{15} = \frac{4}{5}$

Note that we must simplify this fraction. □

Student Practice 1 Multiply.

(a) $\frac{2}{7} \times \frac{5}{11}$ **(b)** $\frac{1}{5} \times \frac{7}{10}$ **(c)** $\frac{9}{5} \times \frac{1}{4}$ **(d)** $\frac{8}{9} \times \frac{3}{10}$

It is possible to avoid having to simplify a fraction as the last step. In many cases we can divide by a value that appears as a factor in both a numerator and a denominator. Often it is helpful to write the numbers as products of prime factors in order to do this.

Example 2 Multiply.

(a) $\frac{3}{5} \times \frac{5}{7}$ **(b)** $\frac{4}{11} \times \frac{5}{2}$ **(c)** $\frac{15}{8} \times \frac{10}{27}$

Solution

(a) $\frac{3}{5} \times \frac{5}{7} = \frac{3 \cdot 5}{5 \cdot 7} = \frac{3 \cdot \overset{1}{\cancel{5}}}{7 \cdot \underset{1}{\cancel{5}}} = \frac{3}{7}$ Note that here we divided numerator and denominator by 5.

If we factor each number, we can see the common factors.

(b) $\frac{4}{11} \times \frac{5}{2} = \frac{2 \cdot \overset{1}{\cancel{2}}}{11} \times \frac{5}{\underset{1}{\cancel{2}}} = \frac{10}{11}$ **(c)** $\frac{15}{8} \times \frac{10}{27} = \frac{\overset{1}{\cancel{3}} \cdot 5}{2 \cdot 2 \cdot \underset{1}{\cancel{2}}} \times \frac{5 \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}} \cdot 3 \cdot 3} = \frac{25}{36}$

After dividing out common factors, the resulting multiplication problem involves smaller numbers and the answers are in simplified form. □

Student Practice 2 Multiply.

(a) $\frac{3}{5} \times \frac{4}{3}$ **(b)** $\frac{9}{10} \times \frac{5}{12}$

Sidelight: Dividing Out Common Factors

Why does this method of dividing out a value that appears as a factor in both numerator and denominator work? Let's reexamine one of the examples we solved previously.

$$\frac{3}{5} \times \frac{5}{7} = \frac{3 \cdot 5}{5 \cdot 7} = \frac{3 \cdot \cancelto{1}{5}}{7 \cdot \cancelto{1}{5}} = \frac{3}{7}$$

Consider the following steps and reasons.

$$\frac{3}{5} \times \frac{5}{7} = \frac{3 \cdot 5}{5 \cdot 7}$$ Definition of multiplication of fractions.

$$= \frac{3 \cdot 5}{7 \cdot 5}$$ Change the order of the factors in the denominator, since $5 \cdot 7 = 7 \cdot 5$. This is called the commutative property of multiplication.

$$= \frac{3}{7} \cdot \frac{5}{5}$$ Definition of multiplication of fractions.

$$= \frac{3}{7} \cdot 1$$ Write 1 in place of $\frac{5}{5}$, since 1 is another name for $\frac{5}{5}$.

$$= \frac{3}{7}$$ $\frac{3}{7} \cdot 1 = \frac{3}{7}$, since any number can be multiplied by 1 without changing the value of the number.

Think about this concept. It is an important one that we will use again when we discuss rational expressions.

Multiplying a Fraction by a Whole Number Whole numbers can be named using fractional notation. 3, $\frac{9}{3}$, $\frac{6}{2}$, and $\frac{3}{1}$ are ways of expressing the number *three*. Therefore,

$$3 = \frac{9}{3} = \frac{6}{2} = \frac{3}{1}.$$

When we multiply a fraction by a whole number, we merely express the whole number as a fraction whose denominator is 1 and follow the multiplication rule for fractions.

Example 3 Multiply.

(a) $7 \times \frac{3}{5}$ **(b)** $\frac{3}{16} \times 4$

Solution

(a) $7 \times \frac{3}{5} = \frac{7}{1} \times \frac{3}{5} = \frac{21}{5}$ or $4\frac{1}{5}$

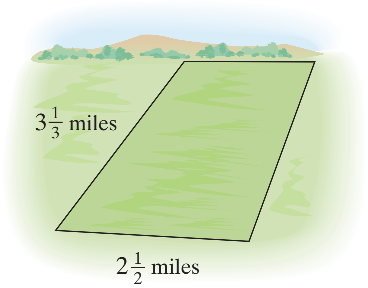
(b) $\frac{3}{16} \times 4 = \frac{3}{16} \times \frac{4}{1} = \frac{3}{4 \cdot \cancel{4}} \times \frac{\cancel{4}}{1} = \frac{3}{4}$

Notice that in **(b)** we did not use *prime* factors to factor 16. We recognized that $16 = 4 \cdot 4$. This is a more convenient factorization of 16 for this problem. Choose the factorization that works best for each problem. If you cannot decide what is best, factor into primes. □

Student Practice 3 Multiply.

(a) $4 \times \frac{2}{7}$ **(b)** $12 \times \frac{3}{4}$

Multiplying Mixed Numbers When multiplying mixed numbers, we first change them to improper fractions and then follow the multiplication rule for fractions.



▲ **Example 4** How do we find the area of a rectangular field $3\frac{1}{3}$ miles long and $2\frac{1}{2}$ miles wide?

Solution To find the area, we multiply length times width.

$$3\frac{1}{3} \times 2\frac{1}{2} = \frac{10}{3} \times \frac{5}{2} = \frac{\cancel{2} \cdot 5}{3} \times \frac{5}{\cancel{2}} = \frac{25}{3} = 8\frac{1}{3}$$

The area is $8\frac{1}{3}$ square miles. □

▲ **Student Practice 4** Delbert Robinson has a farm with a rectangular field that measures $5\frac{3}{5}$ miles long and $3\frac{3}{4}$ miles wide. What is the area of that field?

Example 5 Multiply. $2\frac{2}{3} \times \frac{1}{4} \times 6$

Solution

$$2\frac{2}{3} \times \frac{1}{4} \times 6 = \frac{8}{3} \times \frac{1}{4} \times \frac{6}{1} = \frac{\cancel{4} \cdot 2}{\cancel{3}} \times \frac{1}{\cancel{4}} \times \frac{2 \cdot \cancel{3}}{1} = \frac{4}{1} = 4$$ □

Student Practice 5 Multiply.

$$3\frac{1}{2} \times \frac{1}{14} \times 4$$

2 Dividing Fractions, Whole Numbers, and Mixed Numbers

Dividing Fractions To divide two fractions, we invert the second fraction (that is, the divisor) and then multiply the two fractions.

> **TO DIVIDE TWO FRACTIONS**
>
> 1. Invert the second fraction (that is, the divisor).
> 2. Now multiply the two fractions.

Example 6 Divide.

(a) $\frac{1}{3} \div \frac{1}{2}$ **(b)** $\frac{2}{5} \div \frac{3}{10}$ **(c)** $\frac{2}{3} \div \frac{7}{5}$

Solution

(a) $\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$ Note that we always invert the *second* fraction.

(b) $\frac{2}{5} \div \frac{3}{10} = \frac{2}{5} \times \frac{10}{3} = \frac{2}{\cancel{5}} \times \frac{\cancel{5} \cdot 2}{3} = \frac{4}{3}$ or $1\frac{1}{3}$ **(c)** $\frac{2}{3} \div \frac{7}{5} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$ □

Student Practice 6 Divide.

(a) $\frac{2}{5} \div \frac{1}{3}$ **(b)** $\frac{12}{13} \div \frac{4}{3}$

Dividing a Fraction and a Whole Number The process of inverting the second fraction and then multiplying the two fractions should be done very carefully when one of the original values is a whole number. Remember, a whole number such as 2 is equivalent to $\frac{2}{1}$.

Example 7 Divide.

(a) $\frac{1}{3} \div 2$ **(b)** $5 \div \frac{1}{3}$

Solution

(a) $\frac{1}{3} \div 2 = \frac{1}{3} \div \frac{2}{1} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

(b) $5 \div \frac{1}{3} = \frac{5}{1} \div \frac{1}{3} = \frac{5}{1} \times \frac{3}{1} = \frac{15}{1} = 15$

Student Practice 7 Divide.

(a) $\frac{3}{7} \div 6$ **(b)** $8 \div \frac{2}{3}$

Sidelight: Number Sense

Look at the answers to the problems in Example 7. In part (a), you will notice that $\frac{1}{6}$ is less than the original number $\frac{1}{3}$. Does this seem reasonable? Let's see. If $\frac{1}{3}$ is divided by 2, it means that $\frac{1}{3}$ will be divided into two equal parts. We would expect that each part would be less than $\frac{1}{3}$. $\frac{1}{6}$ is a reasonable answer to this division problem.

In part **(b)**, 15 is greater than the original number 5. Does this seem reasonable? Think of what $5 \div \frac{1}{3}$ means. It means that 5 will be divided into thirds. Let's think of an easier problem. What happens when we divide 1 into thirds? We get *three* thirds. We would expect, therefore, that when we divide 5 into thirds, we would get 5×3 or 15 thirds. 15 is a reasonable answer to this division problem.

Complex Fractions Sometimes division is written in the form of a **complex fraction** with one fraction in the numerator and one fraction in the denominator. It is best to write this in standard division notation first; then complete the problem using the rule for division.

Example 8 Divide.

(a) $\dfrac{\frac{3}{7}}{\frac{3}{5}}$ **(b)** $\dfrac{\frac{2}{9}}{\frac{5}{7}}$

Solution

(a) $\dfrac{\frac{3}{7}}{\frac{3}{5}} = \frac{3}{7} \div \frac{3}{5} = \frac{\cancel{3}}{7} \times \frac{5}{\cancel{3}} = \frac{5}{7}$ **(b)** $\dfrac{\frac{2}{9}}{\frac{5}{7}} = \frac{2}{9} \div \frac{5}{7} = \frac{2}{9} \times \frac{7}{5} = \frac{14}{45}$

Student Practice 8 Divide.

(a) $\dfrac{\frac{3}{11}}{\frac{5}{7}}$ **(b)** $\dfrac{\frac{12}{5}}{\frac{8}{15}}$

Sidelight: Invert and Multiply

Why does the method of "invert and multiply" work? The division rule really depends on the property that any number can be multiplied by 1 without changing the value of the number. Let's look carefully at an example of division of fractions:

$$\frac{2}{5} \div \frac{3}{7} = \frac{\frac{2}{5}}{\frac{3}{7}}$$ We can write the problem using a complex fraction.

$$= \frac{\frac{2}{5}}{\frac{3}{7}} \times 1$$ We can multiply by 1, since any number can be multiplied by 1 without changing the value of the number.

$$= \frac{\frac{2}{5}}{\frac{3}{7}} \times \frac{\frac{7}{3}}{\frac{7}{3}}$$ We write 1 in the form $\frac{\frac{7}{3}}{\frac{7}{3}}$, since any nonzero number divided by itself equals 1. We choose this value as a multiplier because it will help simplify the denominator.

$$= \frac{\frac{2}{5} \times \frac{7}{3}}{\frac{3}{7} \times \frac{7}{3}}$$ Definition of multiplication of fractions.

$$= \frac{\frac{2}{5} \times \frac{7}{3}}{1} = \frac{2}{5} \times \frac{7}{3}$$ The product in the denominator equals 1.

Thus we have shown that $\frac{2}{5} \div \frac{3}{7}$ is equivalent to $\frac{2}{5} \times \frac{7}{3}$ and have shown justification for the "invert and multiply rule."

Dividing Mixed Numbers This method for division of fractions can be used with mixed numbers. However, we first must change the mixed numbers to improper fractions and then use the rule for dividing fractions.

Example 9 Divide.

(a) $2\frac{1}{3} \div 3\frac{2}{3}$ **(b)** $\dfrac{2}{3\frac{1}{2}}$

Solution

(a) $2\frac{1}{3} \div 3\frac{2}{3} = \frac{7}{3} \div \frac{11}{3} = \frac{7}{\cancel{3}} \times \frac{\cancel{3}}{11} = \frac{7}{11}$

(b) $\dfrac{2}{3\frac{1}{2}} = 2 \div 3\frac{1}{2} = \frac{2}{1} \div \frac{7}{2} = \frac{2}{1} \times \frac{2}{7} = \frac{4}{7}$ □

Student Practice 9 Divide.

(a) $1\frac{2}{5} \div 2\frac{1}{3}$ **(b)** $4\frac{2}{3} \div 7$

(c) $\dfrac{1\frac{1}{5}}{1\frac{2}{7}}$

For all Student Practice problems, write out the steps on a separate sheet of paper. Check the step-by-step solutions in the back of the textbook.

Example 10

A chemist has 96 fluid ounces of a solution. She pours the solution into test tubes. Each test tube holds $\frac{3}{4}$ fluid ounce. How many test tubes can she fill?

Solution We need to divide the total number of ounces, 96, by the number of ounces in each test tube, $\frac{3}{4}$.

$$96 \div \frac{3}{4} = \frac{96}{1} \div \frac{3}{4} = \frac{96}{1} \times \frac{4}{3} = \frac{\cancel{3} \cdot 32}{1} \times \frac{4}{\cancel{3}} = \frac{128}{1} = 128$$

She will be able to fill 128 test tubes.

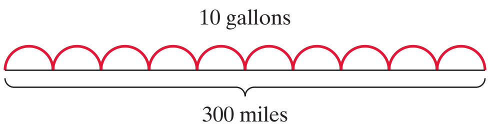
Check: Pause for a moment to think about the answer. Does 128 test tubes filled with solution seem like a reasonable answer? Did you perform the correct operation? □

Student Practice 10 A chemist has 64 fluid ounces of a solution. He wishes to fill several jars, each holding $5\frac{1}{3}$ fluid ounces. How many jars can he fill?

Sometimes when solving word problems involving fractions or mixed numbers, it is helpful to solve the problem using simpler numbers first. Once you understand what operation is involved, you can go back and solve using the original numbers in the word problem.

Example 11 A car traveled 301 miles on $10\frac{3}{4}$ gallons of gas. How many miles per gallon did it get?

Solution Use simpler numbers: 300 miles on 10 gallons of gas. We want to find out how many miles the car traveled on 1 gallon of gas. You may want to draw a picture.



Divide. $300 \div 10 = 30$.

Now use the original numbers given in the problem.

$$301 \div 10\frac{3}{4} = \frac{301}{1} \div \frac{43}{4} = \frac{301}{1} \times \frac{4}{43} = \frac{1204}{43} = 28$$

The car got 28 miles per gallon. □

Student Practice 11 A car traveled 126 miles on $5\frac{1}{4}$ gallons of gas. How many miles per gallon did it get?

STEPS TO SUCCESS Doing Homework for Each Class Is Critical.

Many students in the class ask the question, "Is homework really that important? Do I actually have to do it?"

You learn by doing. It really makes a difference. Mathematics involves mastering a set of skills that you learn by practicing, not by watching someone else do it. Your instructor may make solving a mathematics problem look very easy, but for you to learn the necessary skills, you must practice them over and over.

The key to success is practice. Learning mathematics is like learning to play a musical instrument, to type, or to play a sport. No matter how much you watch someone else do mathematical calculations, no matter how many books you read on "how to" do it, and no matter how easy it appears to be, the key to success in mathematics is practice on each homework set.

Do each kind of problem. Some exercises in a homework set are more difficult than others. Some stress different concepts. Usually you need to work at least all the odd-numbered problems in the exercise set. This allows you to cover the full range of skills in the problem set. Remember, the more exercises you do, the better you will become in your mathematical skills.

Making it personal: Write down your personal reason for why you think doing the homework in each section is very important for success. Which of the three points given do you find is the most convincing?

0.3 Exercises

MyLab™ Math

Verbal and Writing Skills, Exercises 1 and 2

1. Explain in your own words how to multiply two mixed numbers.

2. Explain in your own words how to divide two proper fractions.

Multiply. Simplify your answer whenever possible.

3. $\frac{28}{5} \times \frac{6}{35}$

4. $\frac{5}{7} \times \frac{28}{15}$

5. $\frac{17}{18} \times \frac{3}{5}$

6. $\frac{17}{26} \times \frac{13}{34}$

7. $\frac{4}{5} \times \frac{3}{10}$

8. $\frac{3}{11} \times \frac{5}{7}$

9. $\frac{24}{25} \times \frac{5}{2}$

10. $\frac{15}{24} \times \frac{8}{9}$

11. $\frac{7}{12} \times \frac{8}{28}$

12. $\frac{6}{21} \times \frac{9}{18}$

13. $\frac{6}{35} \times 5$

14. $\frac{2}{21} \times 15$

15. $9 \times \frac{2}{5}$

16. $\frac{8}{11} \times 3$

Divide. Simplify your answer whenever possible.

17. $\frac{8}{5} \div \frac{8}{3}$

18. $\frac{13}{9} \div \frac{13}{7}$

19. $\frac{3}{7} \div 3$

20. $\frac{7}{8} \div 4$

21. $10 \div \frac{5}{7}$

22. $18 \div \frac{2}{9}$

23. $\frac{6}{14} \div \frac{3}{8}$

24. $\frac{8}{12} \div \frac{5}{6}$

25. $\frac{7}{24} \div \frac{9}{8}$

26. $\frac{9}{28} \div \frac{4}{7}$

27. $\dfrac{\frac{7}{8}}{\frac{3}{4}}$

28. $\dfrac{\frac{5}{6}}{\frac{10}{13}}$

29. $\dfrac{\frac{5}{6}}{\frac{7}{9}}$

30. $\dfrac{\frac{3}{4}}{\frac{11}{12}}$

31. $1\frac{3}{7} \div 6\frac{1}{4}$

32. $4\frac{1}{2} \div 3\frac{3}{8}$

33. $3\frac{1}{3} \div 2\frac{1}{2}$

34. $5\frac{1}{2} \div 3\frac{3}{4}$

35. $6\frac{1}{2} \div \frac{3}{4}$

36. $\frac{1}{4} \div 1\frac{7}{8}$

37. $\dfrac{15}{2\frac{2}{5}}$

38. $\dfrac{18}{4\frac{1}{2}}$

39. $\dfrac{\frac{2}{3}}{1\frac{1}{4}}$

40. $\dfrac{\frac{5}{6}}{2\frac{1}{2}}$

Mixed Practice *Perform the proper calculations. Simplify your answer whenever possible.*

41. $\frac{4}{7} \times \frac{21}{2}$

42. $\frac{12}{18} \times \frac{9}{2}$

43. $\frac{5}{14} \div \frac{2}{7}$

44. $\frac{5}{6} \div \frac{11}{18}$

45. $10\frac{3}{7} \times 5\frac{1}{4}$

46. $10\frac{2}{9} \div 2\frac{1}{3}$

47. $25 \div \frac{5}{8}$

48. $15 \div 1\frac{2}{3}$

49. $6 \times 4\frac{2}{3}$

50. $6\frac{1}{2} \times 12$

51. $2\frac{1}{2} \times \frac{1}{10} \times \frac{3}{4}$

52. $2\frac{1}{3} \times \frac{2}{3} \times \frac{3}{5}$

53. **(a)** $\frac{1}{15} \times \frac{25}{21}$
(b) $\frac{1}{15} \div \frac{25}{21}$

54. **(a)** $\frac{1}{6} \times \frac{24}{15}$
(b) $\frac{1}{6} \div \frac{24}{15}$

55. **(a)** $\frac{2}{3} \div \frac{12}{21}$
(b) $\frac{2}{3} \times \frac{12}{21}$

56. **(a)** $\frac{3}{7} \div \frac{21}{25}$
(b) $\frac{3}{7} \times \frac{21}{25}$

Applications

57. ***Shirt Manufacturing*** A denim shirt at the Gap requires $2\frac{3}{4}$ yards of material. How many shirts can be made from $71\frac{1}{2}$ yards of material?

58. ***Pullover Manufacturing*** A fleece pullover requires $1\frac{5}{8}$ yards of material. How many fleece pullovers can be made from $29\frac{1}{4}$ yards of material?

▲ **59.** ***Window Construction*** Jesse needs to find the area of a large window he has been hired to build so that he can order enough glass. The window measures $11\frac{1}{3}$ feet long and 12 feet wide. What is the area of this window?

▲ **60.** ***Gardening*** Sara must find the area of her flower garden so that she can determine how much fertilizer to purchase. What is the area of her rectangular garden, which measures 15 feet long and $10\frac{1}{5}$ feet wide?

Cumulative Review

In exercises 61 and 62, find the missing numerator.

61. **[0.1.4]** $\frac{11}{15} = \frac{?}{75}$

62. **[0.1.4]** $\frac{7}{9} = \frac{?}{63}$

Quick Quiz 0.3

Perform the operations indicated. Simplify answers whenever possible.

1. $\frac{7}{15} \times \frac{25}{14}$

2. $3\frac{1}{4} \times 4\frac{1}{2}$

3. $3\frac{3}{10} \div 2\frac{1}{2}$

4. **Concept Check** Explain the steps you would take to perform the calculation $3\frac{1}{4} \div 2\frac{1}{2}$.

0.4 Using Decimals

Student Learning Objectives

After studying this section, you will be able to:

1. Understand the meaning of decimals.
2. Change a fraction to a decimal.
3. Change a decimal to a fraction.
4. Add and subtract decimals.
5. Multiply decimals.
6. Divide decimals.
7. Multiply and divide a decimal by a multiple of 10.

1 Understanding the Meaning of Decimals

We can express a part of a whole as a fraction or as a decimal. A **decimal** is another way of writing a fraction whose denominator is 10, 100, 1000, and so on.

$$\frac{3}{10} = 0.3 \qquad \frac{5}{100} = 0.05 \qquad \frac{172}{1000} = 0.172 \qquad \frac{58}{10{,}000} = 0.0058$$

The period in decimal notation is known as the **decimal point.** The number of digits in a number to the right of the decimal point is known as the number of **decimal places** of the number. The place value of decimals is shown in the following chart.

Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones	← Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths
100,000	10,000	1000	100	10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10{,}000}$	$\frac{1}{100{,}000}$

Example 1 Write each of the following decimals as a fraction or mixed number. State the number of decimal places. Write out in words the way the number would be spoken.

(a) 0.6 **(b)** 0.29 **(c)** 0.527 **(d)** 1.38 **(e)** 0.00007

Solution

Decimal Form	*Fraction Form*	*Number of Decimal Places*	*The Words Used to Describe the Number*
(a) 0.6	$\frac{6}{10}$	one	six tenths
(b) 0.29	$\frac{29}{100}$	two	twenty-nine hundredths
(c) 0.527	$\frac{527}{1000}$	three	five hundred twenty-seven thousandths
(d) 1.38	$1\frac{38}{100}$	two	one and thirty-eight hundredths
(e) 0.00007	$\frac{7}{100{,}000}$	five	seven hundred-thousandths

□

Student Practice 1 State the number of decimal places. Write each decimal as a fraction or mixed number and in words.

(a) 0.9 **(b)** 0.09 **(c)** 0.731 **(d)** 1.371 **(e)** 0.0005

You have seen that a given fraction can be written in several different but equivalent ways. There are also several different equivalent ways of writing the decimal form of a fraction. The decimal 0.18 can be written in the following equivalent ways:

$$\text{Fractional form:}\quad \frac{18}{100} = \frac{180}{1000} = \frac{1800}{10{,}000} = \frac{18{,}000}{100{,}000}$$

$$\text{Decimal form:}\quad 0.18 = 0.180 = 0.1800 = 0.18000$$

Thus we see that *any number of terminal zeros may be added to the right-hand side of a decimal* without changing its value.

$$0.13 = 0.1300 \qquad 0.162 = 0.162000$$

Similarly, *any number of terminal zeros may be removed from the right-hand side of a decimal* without changing its value.

2 Changing a Fraction to a Decimal

A fraction can be changed to a decimal by dividing the denominator into the numerator.

Example 2 Write each of the following fractions as a decimal.

(a) $\frac{3}{4}$ **(b)** $\frac{21}{20}$ **(c)** $\frac{1}{8}$ **(d)** $\frac{3}{200}$

Solution

(a) $\frac{3}{4} = 0.75$ since
$$\begin{array}{r} 0.75 \\ 4\overline{)\,3.00} \\ \underline{2\;8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

(b) $\frac{21}{20} = 1.05$ since
$$\begin{array}{r} 1.05 \\ 20\overline{)\,21.00} \\ \underline{20} \\ 1\;00 \\ \underline{1\;00} \\ 0 \end{array}$$

(c) $\frac{1}{8} = 0.125$ since
$$\begin{array}{r} 0.125 \\ 8\overline{)\,1.000} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

(d) $\frac{3}{200} = 0.015$ since
$$\begin{array}{r} 0.015 \\ 200\overline{)\,3.000} \\ \underline{2\;00} \\ 1\;000 \\ \underline{1\;000} \\ 0 \end{array}$$

Student Practice 2 Write each of the following fractions as a decimal.

(a) $\frac{3}{8}$ **(b)** $\frac{7}{200}$ **(c)** $\frac{33}{20}$

Sometimes division yields an infinite repeating decimal. We use three dots to indicate that the pattern continues forever. For example,

$$\frac{1}{3} = 0.3333\ldots \qquad \begin{array}{r} 0.333 \\ 3\,\overline{)\,1.000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

An alternative notation is to place a bar over the repeating digit(s):

$$0.3333\ldots = 0.\overline{3} \qquad 0.575757\ldots = 0.\overline{57}$$

Example 3 Write each fraction as a decimal.

(a) $\dfrac{2}{11}$ **(b)** $\dfrac{5}{6}$

Solution

(a) $\dfrac{2}{11} = 0.181818\ldots$ or $0.\overline{18}$

$$\begin{array}{r} 0.1818 \\ 11\,\overline{)\,2.0000} \\ \underline{1\;1} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 2 \end{array}$$

(b) $\dfrac{5}{6} = 0.8333\ldots$ or $0.8\overline{3}$

$$\begin{array}{r} 0.8333 \\ 6\,\overline{)\,5.0000} \\ \underline{4\,8} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

Note that the 8 does not repeat. Only the digit 3 is repeating.

Student Practice 3 Write each fraction as a decimal.

(a) $\dfrac{1}{6}$ **(b)** $\dfrac{5}{11}$

Calculator

Fraction to Decimal

You can use a calculator to change $\frac{3}{5}$ to a decimal.
Enter:

3 $\boxed{\div}$ 5 $\boxed{=}$

The display should read

$\boxed{0.6}$

Try the following.

(a) $\dfrac{17}{25}$ **(b)** $\dfrac{2}{9}$

(c) $\dfrac{13}{10}$ **(d)** $\dfrac{15}{19}$

Sometimes division must be carried out to many places in order to observe the repeating pattern. This is true in the following example:

$$\frac{2}{7} = 0.285714285714285714\ldots \qquad \text{This can also be written as } \frac{2}{7} = 0.\overline{285714}.$$

It can be shown that the denominator determines the maximum number of decimal places that might repeat. So $\frac{2}{7}$ must repeat in the seventh decimal place or sooner.

3 Changing a Decimal to a Fraction

To convert from a decimal to a fraction, merely write the decimal as a fraction with a denominator of 10, 100, 1000, 10,000, and so on, and simplify the result when possible.

Example 4 Write each decimal as a fraction and simplify whenever possible.

(a) 0.2 **(b)** 0.35 **(c)** 0.516 **(d)** 0.74 **(e)** 0.138 **(f)** 0.008

Solution

(a) $0.2 = \frac{2}{10} = \frac{1}{5}$ **(b)** $0.35 = \frac{35}{100} = \frac{7}{20}$

(c) $0.516 = \frac{516}{1000} = \frac{129}{250}$ **(d)** $0.74 = \frac{74}{100} = \frac{37}{50}$

(e) $0.138 = \frac{138}{1000} = \frac{69}{500}$ **(f)** $0.008 = \frac{8}{1000} = \frac{1}{125}$ □

Student Practice 4 Write each decimal as a fraction and simplify whenever possible.

(a) 0.8 **(b)** 0.88 **(c)** 0.45 **(d)** 0.148 **(e)** 0.612 **(f)** 0.016

All repeating decimals can also be converted to fractional form. In practice, however, repeating decimals are usually rounded to a few places. It will not be necessary, therefore, to learn how to convert $0.\overline{033}$ to $\frac{11}{333}$ for this course.

4 Adding and Subtracting Decimals

Last week Bob spent $19.83 on lunches purchased at the cafeteria at work. During this same period, Sally spent $24.76 on lunches. How much did the two of them spend on lunches last week?

Adding and subtracting decimals is similar to adding and subtracting whole numbers, except that it is necessary to line up decimal points. To perform the operation 19.83 + 24.76, we line up the numbers in column form and add the digits:

$$\begin{array}{r} 19.83 \\ +\ 24.76 \\ \hline 44.59 \end{array}$$

Thus Bob and Sally spent $44.59 on lunches last week.

ADDITION AND SUBTRACTION OF DECIMALS

1. Write in column form and line up the decimal points.
2. Add or subtract the digits.

Example 5 Add or subtract.

(a) 3.6 + 2.3 **(b)** 127.32 − 38.48 **(c)** 3.1 + 42.36 + 9.034 **(d)** 5.0006 − 3.1248

Solution

(a) $\begin{array}{r} 3.6 \\ +\ 2.3 \\ \hline 5.9 \end{array}$ **(b)** $\begin{array}{r} 127.32 \\ -\ 38.48 \\ \hline 88.84 \end{array}$ **(c)** $\begin{array}{r} 3.1 \\ 42.36 \\ +\ 9.034 \\ \hline 54.494 \end{array}$ **(d)** $\begin{array}{r} 5.0006 \\ -\ 3.1248 \\ \hline 1.8758 \end{array}$ □

Student Practice 5 Add or subtract.

(a) 3.12 + 5.08 **(b)** 152.003 − 136.118

(c) 1.1 + 3.16 + 5.123 **(d)** 1.0052 − 0.1234

Sidelight: Adding Zeros to the Right-Hand Side of the Decimal

When we added fractions, we had to have common denominators. Since decimals are really fractions, why can we add them without having common denominators? Actually, we have to have common denominators to add any fractions, whether they are in decimal form or fraction form. However, sometimes the notation does not show this. Let's examine Example 5**(c).**

Original Problem

$$\begin{array}{r} 3.1 \\ 42.36 \\ +\ 9.034 \\ \hline 54.494 \end{array}$$

We are adding the three numbers:

$$3\tfrac{1}{10} + 42\tfrac{36}{100} + 9\tfrac{34}{1000}$$

$$3\tfrac{100}{1000} + 42\tfrac{360}{1000} + 9\tfrac{34}{1000}$$

$$3.100 + 42.360 + 9.034$$ This is the new problem.

Original Problem	***New Problem***
$\begin{array}{r} 3.1 \\ 42.36 \\ +\ 9.034 \\ \hline 54.494 \end{array}$	$\begin{array}{r} 3.100 \\ 42.360 \\ +\ 9.034 \\ \hline 54.494 \end{array}$

We notice that the results are the same. The only difference is the notation. We are using the property that any number of zeros may be added to the right-hand side of a decimal without changing its value.

This shows the convenience of adding and subtracting fractions in decimal form. Little work is needed to change the decimals so that they have a common denominator. All that is required is to add zeros to the right-hand side of the decimal (and we usually do not even write out that step except when subtracting).

As long as we line up the decimal points, we can add or subtract any decimal fractions.

In the following example we will find it useful to add zeros to the right-hand side of the decimal.

Example 6 Perform the following operations.

(a) $1.0003 + 0.02 + 3.4$ **(b)** $12 - 0.057$

Solution We will add zeros so that each number shows the same number of decimal places.

(a) $\begin{array}{r} 1.0003 \\ 0.0200 \\ +\ 3.4000 \\ \hline 4.4203 \end{array}$ **(b)** $\begin{array}{r} 12.000 \\ -\ 0.057 \\ \hline 11.943 \end{array}$

Student Practice 6 Perform the following operations.

(a) $0.061 + 5.0008 + 1.3$ **(b)** $18 - 0.126$

5 Multiplying Decimals

MULTIPLICATION OF DECIMALS

To multiply decimals, you first multiply as with whole numbers. To determine the position of the decimal point, you count the total number of decimal places in the two numbers being multiplied. This will determine the number of decimal places that should appear in the answer.

Example 7 Multiply. 0.8×0.4

Solution

$$\begin{array}{rl} 0.8 & (\text{one decimal place}) \\ \times\, 0.4 & (\text{one decimal place}) \\ \hline 0.32 & (\text{two decimal places}) \end{array}$$

Student Practice 7 Multiply. 0.5×0.3

Note that you will often have to add zeros to the left of the digits obtained in the product so that you obtain the necessary number of decimal places.

Example 8 Multiply. 0.123×0.5

Solution

$$\begin{array}{rl} 0.123 & (\text{three decimal places}) \\ \times\quad 0.5 & (\text{one decimal place}) \\ \hline 0.0615 & (\text{four decimal places}) \end{array}$$

Student Practice 8 Multiply. 0.12×0.4

Here are some examples that involve more decimal places.

Example 9 Multiply.

(a) 2.56×0.003 **(b)** 0.0036×0.008

Solution

(a)
$$\begin{array}{rl} 2.56 & (\text{two decimal places}) \\ \times\, 0.003 & (\text{three decimal places}) \\ \hline 0.00768 & (\text{five decimal places}) \end{array}$$

(b)
$$\begin{array}{rl} 0.0036 & (\text{four decimal places}) \\ \times\quad 0.008 & (\text{three decimal places}) \\ \hline 0.0000288 & (\text{seven decimal places}) \end{array}$$

Student Practice 9 Multiply.

(a) 1.23×0.005 **(b)** 0.003×0.00002

Sidelight: Counting the Number of Decimal Places

Why do we count the number of decimal places? The rule really comes from the properties of fractions. If we write the problem in Example 8 in fraction form, we have

$$0.123 \times 0.5 = \frac{123}{1000} \times \frac{5}{10} = \frac{615}{10{,}000} = 0.0615.$$

6 Dividing Decimals

When discussing division of decimals, we frequently refer to the three primary parts of a division problem. Be sure you know the meaning of each term.

The **divisor** is the number you divide into another.
The **dividend** is the number to be divided.
The **quotient** is the result of dividing one number by another.

In the problem $6 \div 2 = 3$ we represent each of these terms as follows:

$$\text{divisor} \longrightarrow 2\overline{)6} \qquad \begin{array}{l} 3 \leftarrow \text{quotient} \\ 6 \leftarrow \text{dividend} \end{array}$$

$$\text{divisor}\overline{)\text{dividend}}^{\;\text{quotient}}$$

> When dividing two decimals, count *the number of decimal places* in the divisor. Then *move the decimal point to the right* that *same number of places* in both *the divisor* and *the dividend.* Mark that position with a caret ($_\wedge$). Finally, perform the division. Be sure to line up the decimal point in the quotient with the position indicated by the caret.

Example 10 Four friends went out for lunch. The total bill, including tax, was \$32.68. How much did each person pay if they shared the cost equally?

Solution To answer this question, we must calculate $32.68 \div 4$.

$$\begin{array}{r} 8.17 \\ 4\overline{)32.68} \\ \underline{32} \\ 06 \\ \underline{4} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

Since there are no decimal places in the divisor, we do not need to move the decimal point. We must be careful, however, to place the decimal point in the quotient directly above the decimal point in the dividend.

Thus $32.68 \div 4 = 8.17$, and each friend paid \$8.17. □

Student Practice 10 Sally Keyser purchased 6 boxes of paper for a laser printer. The cost was \$31.56. There was no tax since she purchased the paper for a charitable organization. How much did she pay for each box of paper?

Note that sometimes we will need to place extra zeros in the dividend in order to move the decimal point the required number of places.

Example 11 Divide. $16.2 \div 0.027$

Solution

$$0.027_\wedge\overline{)16.200_\wedge}$$

There are **three** decimal places in the divisor, so we move the decimal point **three places** to **the right** in the **divisor** and **dividend** and mark the new position by a caret. Note that we must add two zeros to 16.2 in order to do this.

three decimal places

$$\begin{array}{r} 600. \\ 0.027_\wedge\overline{)16.200_\wedge} \\ \underline{16\ 2} \\ 000 \end{array}$$

Now perform the division as with whole numbers.
The decimal point in the answer is directly above the caret.

Thus $16.2 \div 0.027 = 600.$ □

Student Practice 11 Divide. $1800. \div 0.06$

Special care must be taken to line up the digits in the quotient. Note that sometimes we will need to place zeros in the quotient after the decimal point.

Example 12 Divide. $0.04288 \div 3.2$

Solution

$3.2_{\wedge}\overline{)\,0.0_{\wedge}4288}$ There is **one** decimal place in the divisor, so we move the decimal point **one place** to **the right** in the **divisor** and **dividend** and mark the new position by a caret.

one decimal place

$$
\begin{array}{r}
0.0134 \\
3.2_{\wedge}\overline{)\,0.0_{\wedge}4288} \\
\underline{32} \\
108 \\
\underline{96} \\
128 \\
\underline{128} \\
0
\end{array}
$$

Now perform the division as for whole numbers. The decimal point in the answer is directly above the caret. Note the need for the initial zero after the decimal point in the answer.

Thus $0.04288 \div 3.2 = 0.0134$. □

Student Practice 12 Divide. $0.01764 \div 4.9$

Sidelight: Dividing Decimals by Another Method

Why does this method of dividing decimals work? Essentially, we are using the steps we used in Section 0.1 to change a fraction to an equivalent fraction by multiplying both the numerator and denominator by the same number. Let's reexamine Example 12.

$0.04288 \div 3.2 = \dfrac{0.04288}{3.2}$ Write the original problem using fraction notation.

$= \dfrac{0.04288 \times 10}{3.2 \times 10}$ Multiply the numerator and denominator by 10. Since this is the same as multiplying by 1, we are not changing the fraction.

$= \dfrac{0.4288}{32}$ Write the result of multiplication by 10.

$= 0.4288 \div 32$ Rewrite the fraction as an equivalent problem with division notation.

Notice that we have obtained a new problem that is the same as the problem in Example 12 when we moved the decimal one place to the right in the divisor and dividend. We see that the reason we can move the decimal point as many places as necessary to the right in the divisor and dividend is that this is the same as multiplying the numerator and denominator of a fraction by a power of 10 to obtain an equivalent fraction.

7 Multiplying and Dividing a Decimal by a Multiple of 10

When multiplying by 10, 100, 1000, and so on, a simple rule may be used to obtain the answer. For every zero in the multiplier, move the decimal point one place to the right.

Example 13 Multiply.

(a) 3.24×10 **(b)** 15.6×100 **(c)** 0.0026×1000

Solution

(a) $3.24 \times 10 = 32.4$ One zero—move decimal point one place to the right.

(b) $15.6 \times 100 = 1560$ Two zeros—move decimal point two places to the right.

(c) $0.0026 \times 1000 = 2.6$ Three zeros—move decimal point three places to the right.

Student Practice 13 Multiply.

(a) 0.0016×100 **(b)** 2.34×1000 **(c)** $56.75 \times 10{,}000$

The reverse rule is true for division. When dividing by 10, 100, 1000, 10,000, and so on, move the decimal point one place to the left for every zero in the divisor.

Example 14 Divide.

(a) $52.6 \div 10$ **(b)** $0.0038 \div 100$ **(c)** $5936.2 \div 1000$

Solution

(a) $\dfrac{52.6}{10} = 5.26$ Move decimal point one place to the left.

(b) $\dfrac{0.0038}{100} = 0.000038$ Move decimal point two places to the left.

(c) $\dfrac{5936.2}{1000} = 5.9362$ Move decimal point three places to the left.

Student Practice 14 Divide.

(a) $\dfrac{5.82}{10}$ **(b)** $123.4 \div 1000$ **(c)** $\dfrac{0.00614}{10{,}000}$

STEPS TO SUCCESS Do You Realize How Valuable Friendship Is?

In a math class a friend is a person of fantastic value. Robert Louis Stevenson once wrote "A friend is a gift you give yourself." This is especially true when you take a mathematics class and make a friend in the class. You will find that you enjoy sitting together and drawing support and encouragement from each other. You may want to exchange phone numbers or e-mail addresses. You may want to study together or review together before a test.

How do you get started? Try talking to the students seated around you. Ask someone for help about something you did not understand in class. Take the time to listen to them and their interests and concerns. You may discover you have a lot in common. If the first few people you talk to seem uninterested, try sitting in a different part of the room and talk to those students who are seated around you in the new location. Don't force a friendship on anyone but just look for chances to open up a good channel of communication.

Making it personal: What do you think is the best way to make a friend of someone in your class? Which of the given suggestions do you find the most helpful? Will you take the time to reach out to someone in your class this week and try to begin a new friendship?

0.4 Exercises MyLab™ Math

Verbal and Writing Skills, Exercises 1–4

1. A decimal is another way of writing a fraction whose denominator is ______________________.

2. We write 0.42 in words as ______________________.

3. When dividing 7432.9 by 1000 we move the decimal point _______ places to the _________.

4. When dividing 96.3 by 10,000 we move the decimal point _______ places to the _________.

Write each fraction as a decimal.

5. $\frac{7}{8}$ **6.** $\frac{18}{25}$ **7.** $\frac{3}{15}$ **8.** $\frac{9}{15}$ **9.** $\frac{7}{11}$ **10.** $\frac{1}{6}$

Write each decimal as a fraction in simplified form.

11. 0.8 **12.** 0.5 **13.** 0.25 **14.** 0.35 **15.** 0.625 **16.** 0.775

17. 0.06 **18.** 0.08 **19.** 3.4 **20.** 4.8 **21.** 5.5 **22.** 6.25

Add or subtract.

23. $1.71 + 0.38$ **24.** $4.64 + 0.23$ **25.** $2.5 + 3.42 + 4.9$ **26.** $6.31 + 4.2 + 8.5$

27. $46.03 + 215.1 + 0.078$ **28.** $33.01 + 0.38 + 175.401$ **29.** $147.18 - 15.39$ **30.** $121.52 - 79.85$

31. $6.0054 - 2.0257$ **32.** $5.0032 - 3.0036$ **33.** $125.43 - 2.8$ **34.** $212.54 - 3.6$

Multiply or divide.

35. 7.21×4.2 **36.** 6.12×3.4 **37.** 0.04×0.08 **38.** 6.32×1.31

39. 4.23×0.025 **40.** 3.84×0.0017 **41.** $58{,}200 \times 0.0015$ **42.** $23{,}000 \times 0.0042$

43. $3.616 \div 64$ **44.** $12.6672 \div 39$ **45.** $7.9728 \div 3.02$ **46.** $6.519 \div 2.05$

47. $0.5230 \div 0.002$ **48.** $0.031 \div 0.005$ **49.** $0.03048 \div 0.06$ **50.** $0.00855 \div 0.09$

Multiply or divide by moving the decimal point.

51. 3.45×1000 **52.** 1.36×1000 **53.** $0.76 \div 100$ **54.** $175{,}318 \div 1000$

55. $7.36 \times 10{,}000$ **56.** $0.00243 \times 100{,}000$ **57.** $73{,}892 \div 100{,}000$ **58.** $3.52 \div 1000$

59. 0.1498×100 **60.** $85.54 \times 10{,}000$ **61.** $1.931 \div 100$ **62.** $96.12 \div 10{,}000$

Mixed Practice *Perform the calculations indicated.*

63. 54.8×0.15

64. 8.252×0.005

65. $13.75 + 2.55 + 0.078$

66. $1.109 + 0.088 + 16.4$

67. $0.05724 \div 0.027$

68. $77.136 \div 0.003$

69. 0.7683×1000

70. $25.62 \times 10{,}000$

71. $56.37 - 4.29$

72. $14.3 - 0.68$

73. $153.7 \div 100$

74. $0.58 \div 1000$

Applications

75. ***Measurement*** While mixing solutions in her chemistry lab, Mia needed to change the measured data from pints to liters. There is 0.4732 liter in one pint and the original measurement was 5.5 pints. What is the measured data in liters?

76. ***Mileage of Hybrid Cars*** In order to minimize fuel costs, Chris Smith purchased a used Honda Civic Hybrid that averages 44 miles per gallon in the city. The gas tank holds 13.2 gallons of gas. How many miles can Chris drive the car in the city on a full tank of gas?

77. ***Wages*** Harry has a part-time job at Stop and Shop. He earns \$9 an hour. He requested enough hours of work each week so that he could earn at least \$185 a week. How many hours will he have to work to achieve his goal? By how much will he exceed his earning goal of \$185 per week?

78. ***Drinking Water*** The EPA standard for safe drinking water is a maximum of 1.3 milligrams of copper per liter of water. A water testing firm found 6.8 milligrams of copper in a 5-liter sample drawn from Jim and Sharon LeBlanc's house. Is the water safe or not? By how much does the amount of copper exceed or fall short of the maximum allowed?

Cumulative Review *Perform each operation. Simplify all answers.*

79. **[0.3.2]** $3\frac{1}{2} \div 5\frac{1}{4}$

80. **[0.3.1]** $\frac{3}{8} \cdot \frac{12}{27}$

81. **[0.2.3]** $\frac{12}{25} + \frac{9}{20}$

82. **[0.2.4]** $1\frac{3}{5} - \frac{1}{2}$

Quick Quiz 0.4 *Perform the calculations indicated.*

1. $8.0567 - 2.3489$

2. 58.7×0.06

3. $4.608 \div 0.16$

4. **Concept Check** Explain how you would place the decimal points when performing the calculation $0.252 \div 0.0035$.