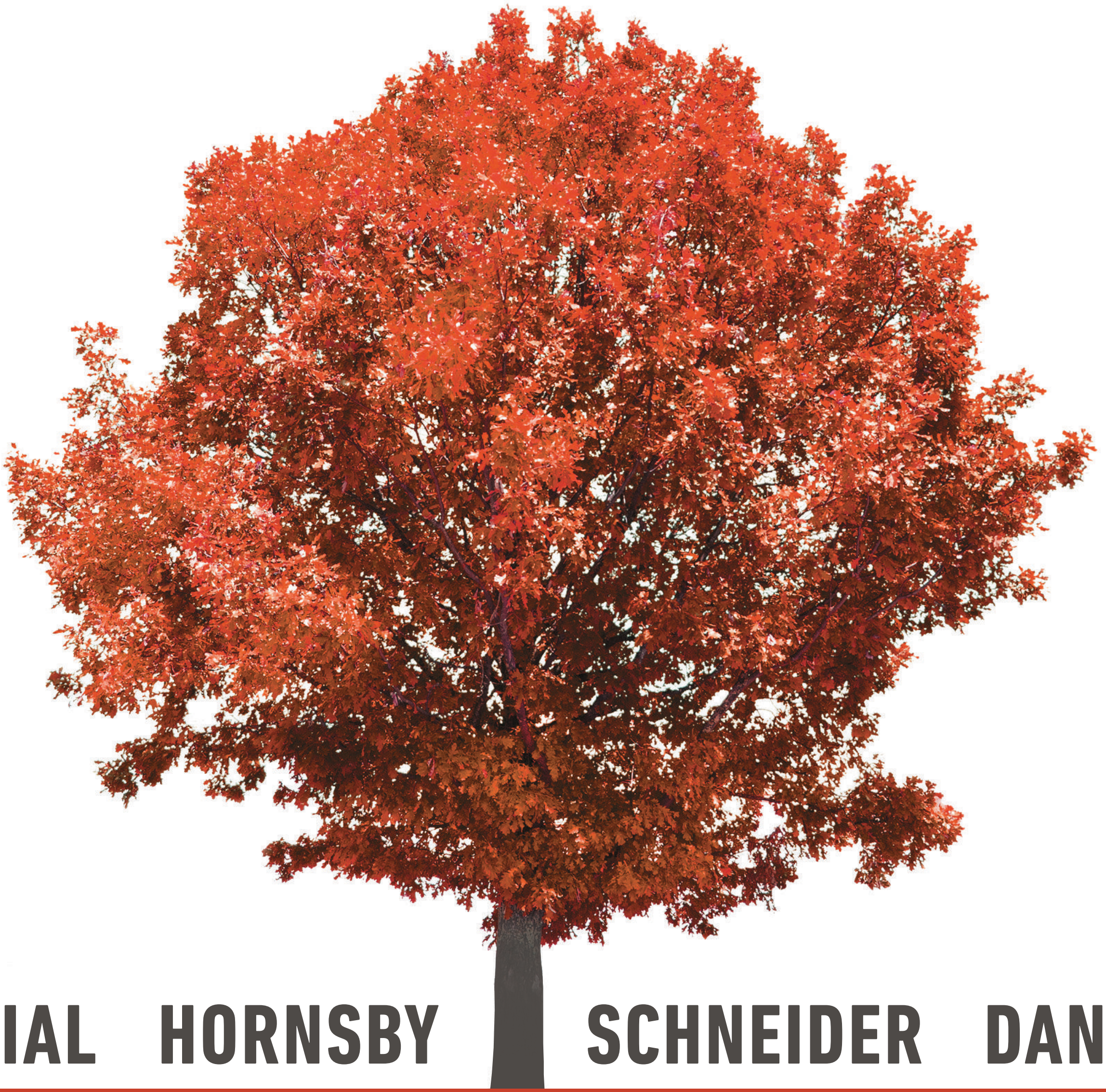

LIAL HORNSBY SCHNEIDER DANIELS

COLLEGE ALGEBRA AND TRIGONOMETRY

seventh edition

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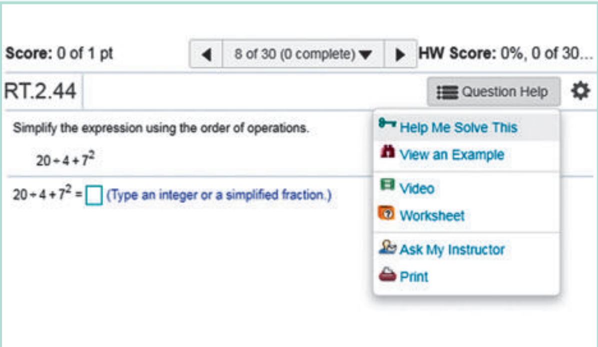

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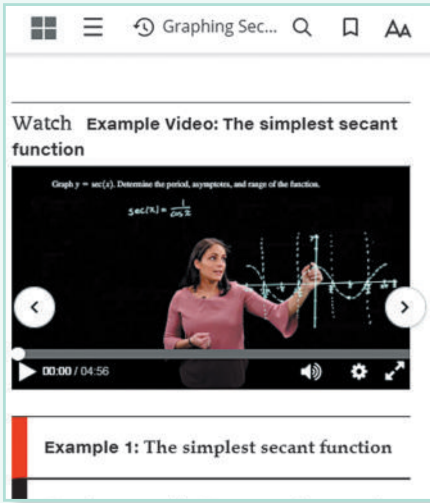
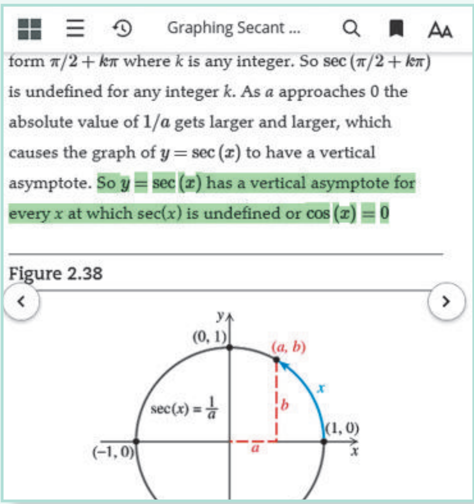
Instructional videos cover all examples from the text and can conveniently be played on any mobile device. These videos are especially helpful if you miss a class or just need further explanation.





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The **Pearson eText** gives you access to your textbook anytime, anywhere. In addition to note taking, highlighting, and bookmarking, the Pearson eText App offers access even when offline.

College Algebra and Trigonometry

SEVENTH EDITION

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College Algebra and Trigonometry

SEVENTH EDITION

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Library of Congress Cataloging-in-Publication Data

Names: Lial, Margaret L., author. | Hornsby, John, author. | Schneider, David I., author. | Daniels, Callie J., author.
Title: College algebra and trigonometry / Margaret L. Lial, American River College, John Hornsby University of New Orleans, David I. Schneider, University of Maryland, Callie J. Daniels, St. Charles Community College.
Description: Seventh edition. | Hoboken, NJ : Pearson, [2021] | Includes index.
Identifiers: LCCN 2019053777 | ISBN 9780135924549 (hardback)
Subjects: LCSH: Algebra. | Trigonometry.
Classification: LCC QA154.3 .L54 2021 | DDC 512/.13--dc23
LC record available at https://lccn.loc.gov/2019053777

ScoutAutomatedPrintCode

Pearson

ISBN 13: 978-0-13-592454-9
ISBN 10: 0-13-592454-5

This text is dedicated to you—the student. We hope that it helps you achieve your goals. Remember to show up, work hard, and stay positive. Everything else will take care of itself.

The Lial Author Team

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Contents

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Preface

WELCOME TO THE 7TH EDITION

In the seventh edition of *College Algebra and Trigonometry,* we continue our ongoing commitment to providing the best possible text to help instructors teach and students succeed. In this edition, we have remained true to the pedagogical style of the past while staying focused on the needs of today's students. Support for all classroom types (traditional, corequisite, flipped, hybrid, and online) may be found in this classic text and its supplements backed by the power of Pearson's MyLab Math.

In this edition, we have drawn on the extensive teaching experience of the Lial team, with special consideration given to reviewer suggestions. General updates include enhanced readability as we continually strive to make math understandable for students, updates to our extensive list of applications and real-world mathematics problems, use of color in displays and side comments, and coordination of exercises and their related examples.

The authors understand that teaching and learning mathematics today can be a challenging task. Some students are prepared for the challenge, while other students require more review and supplemental material. This text is written so that students with varying abilities and backgrounds will all have an opportunity for a successful learning experience.

The Lial team believes this to be our best edition of *College Algebra and Trigonometry* yet, and we sincerely hope that you enjoy using it as much as we have enjoyed writing it. Additional textbooks in this series are

College Algebra, Thirteenth Edition
Trigonometry, Twelfth Edition
Precalculus, Seventh Edition.

HIGHLIGHTS OF NEW CONTENT

- **Chapter R** has been expanded to include more of the basic concepts many students struggle with. It begins with new **Section R.1 Fractions, Decimals, and Percents.** Additional new topics have been inserted throughout the chapter, including operations with signed numbers **(Section R.3),** dividing a polynomial by a monomial **(Section R.5),** and factoring expressions with negative and rational exponents **(Section R.6).** Topics throughout the chapter have been reorganized for improved flow.

  Instructors may choose to cover review topics from **Chapter R** at the beginning of a course or to insert these topics as-needed in a just-in-time fashion. Either way, students who are under-prepared for the demands of college algebra and trigonometry, as well as those who need a quick review, will benefit from the material contained here.

- The exercise sets were a key focus of this revision, and **Chapters 1 and 2** are among the chapters that have benefitted. Specifically, **Section 1.7 Inequalities** has new exercises on solving quadratic and rational inequalities, and **Section 1.8 Absolute Value Equations and Inequalities** contains new exercises that involve the absolute value of a quadratic polynomial. **Section 2.3 Functions** has new exercises that use analytic methods to determine maximum and minimum values of a function.

Section 2.6 Graphs of Basic Functions contains new exercises and applications using the greatest integer function. **Section 2.4 Linear Functions** includes enhanced discussion of the average rate of change of a linear function. This topic is then related to the difference quotient and the average rate of change of a nonlinear function in **Section 2.8 Function Operations and Composition.**

- **Chapter 3** includes new **Section 3.6 Polynomial and Rational Inequalities.** This section features a visual approach to solving such inequalities by interpreting the graphs of related functions.
- In response to reviewer suggestions, **Section 4.3 Logarithmic Functions** has new exercises that relate exponential and logarithmic functions as inverses. **Chapter 6** includes additional exercises devoted to finding arc length and area of a sector of a circle **(Section 6.1),** as well as new applications of linear and angular speed **(Section 6.2)** and harmonic motion **(Section 6.7).**
- Proofs of identities in **Chapter 7** now feature a drop-down style for increased clarity and student understanding. Based on reviewer requests, **Section 7.7 Equations Involving Inverse Trigonometric Functions** includes new exercises in which solutions of inverse trigonometric equations are found.
- Based on reviewer feedback, **Section 8.4 Algebraically Defined Vectors and the Dot Product** has new exercises on finding the angle between two vectors, determining magnitude and direction angle for a vector, and identifying orthogonal vectors. Additionally, **Chapter 8** contains new exercises requiring students to graph polar and parametric equations **(Section 8.7)** and give parametric representations of plane curves **(Section 8.8).**
- **Section 9.2 Matrix Solution of Linear Systems** now includes a new example and related exercises that use Gaussian elimination to solve linear systems of equations. **Section 10.2 Ellipses** and **Section 10.3 Hyperbolas** include new examples and exercises in which completing the square is used to find the standard form of an ellipse or a hyperbola.

FEATURES OF THIS TEXT

SUPPORT FOR LEARNING CONCEPTS

We provide a variety of features to support students' learning of the essential topics of college algebra and trigonometry. Explanations that are written in understandable terms, figures and graphs that illustrate examples and concepts, graphing technology that supports and enhances algebraic manipulations, and real-life applications that enrich the topics with meaning all provide opportunities for students to deepen their understanding of mathematics. These features help students make mathematical connections and expand their own knowledge base.

- **Examples** Numbered examples that illustrate the techniques for working exercises are found in every section. We use traditional explanations, side comments, and pointers to describe the steps taken—and to warn students about common pitfalls. Some examples provide additional graphing calculator solutions, although these can be omitted if desired.
- **Now Try Exercises** Following each numbered example, the student is directed to try a corresponding odd-numbered exercise (or exercises). This feature allows for quick feedback to determine whether the student understands the principles illustrated in the example.

- **Real-Life Applications** We have included hundreds of real-life applications, many with data updated from the previous edition. They come from fields such as business, entertainment, sports, biology, astronomy, geology, and environmental studies.
- **Function Boxes** Beginning in Chapter 2, functions provide a unifying theme throughout the text. Special function boxes offer a comprehensive, visual introduction to each type of function and also serve as an excellent resource for reference and review. Each function box includes a table of values, traditional and calculator-generated graphs, the domain, the range, and other special information about the function. These boxes are assignable in MyLab Math.
- **Figures and Photos** Today's students are more visually oriented than ever before, and we have updated the figures and photos in this edition to promote visual appeal. Guided Visualizations with accompanying exercises and explorations are available and assignable in MyLab Math.
- **Cautions and Notes** Text that is marked **CAUTION** warns students of common errors, and **NOTE** comments point out explanations that should receive particular attention.
- **Looking Ahead to Calculus** These margin notes offer glimpses of how the topics currently being studied are used in calculus.
- **Use of Graphing Technology** We have integrated the use of graphing calculators where appropriate, although ***this technology is completely optional and can be omitted without loss of continuity.*** We continue to stress that graphing calculators support understanding but that students must first master the underlying mathematical concepts. Exercises that require the use of a graphing calculator are marked with the icon.

SUPPORT FOR PRACTICING CONCEPTS

This text offers a wide variety of exercises to help students master college algebra and trigonometry. The extensive exercise sets provide ample opportunity for practice and increase in difficulty so that students at every level of understanding are challenged. The variety of exercise types promotes mastery of the concepts and reduces the need for rote memorization.

- **Concept Preview** Each exercise set begins with a group of **CONCEPT PREVIEW** exercises designed to promote understanding of vocabulary and basic concepts of each section. These new exercises are assignable in MyLab Math and provide support, especially for hybrid, online, and flipped courses.
- **Exercise Sets** In addition to traditional drill exercises, this text includes writing exercises, optional graphing calculator exercises, and multiple-choice, matching, true/false, and completion exercises. Those marked ***Concept Check*** focus on conceptual thinking. ***Connecting Graphs with Equations*** exercises challenge students to write equations that correspond to given graphs. Video solutions for select problems are available in MyLab Math.
- **Relating Concepts Exercises** Appearing at the end of selected exercise sets, these groups of exercises are designed so that students who work them in numerical order will follow a line of reasoning that leads to an understanding of how various topics and concepts are related. All answers to these exercises appear in the student answer section, and these exercises are assignable in MyLab Math.

SUPPORT FOR REVIEW AND TEST PREP

Ample opportunities for review are found both within the chapters and at the ends of chapters. Quizzes and Summary Exercises, interspersed within chapters, provide a quick assessment of students' understanding of the material presented up to that point in the chapter. Chapter Test Preps provide comprehensive study aids to help students prepare for tests.

- **Quizzes** Students can periodically check their progress with in-chapter quizzes that appear in all chapters, beginning with Chapter 1. All answers, with corresponding section references, appear in the student answer section. These quizzes are assignable in MyLab Math.
- **Summary Exercises** These sets of in-chapter exercises give students the all-important opportunity to work *mixed* review exercises, requiring them to synthesize concepts and select appropriate solution methods.
- **End-of-Chapter Test Prep** Following the final numbered section in each chapter, the Test Prep provides a list of **Key Terms,** a list of **New Symbols** (if applicable), and a two-column **Quick Review** that includes a section-by-section summary of concepts with corresponding examples. This feature concludes with a comprehensive set of **Review Exercises** and a **Chapter Test.** The Test Prep, Review Exercises, and Chapter Test are assignable in MyLab Math.

*Get the **most** out of*

MyLab Math

Pearson MyLab

MyLab Math for College Algebra and Trigonometry 7e

(access code required)

MyLab Math is tightly integrated with author style, offering a range of author-created resources, to give students a consistent experience.

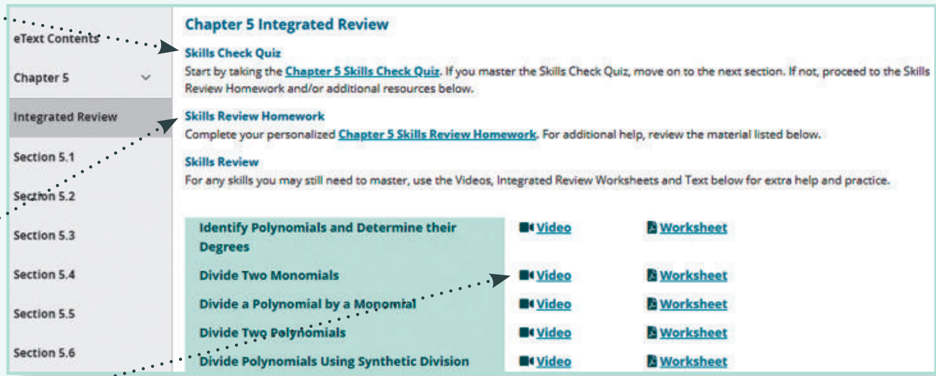
Preparedness

Preparedness is one of the biggest challenges in many math courses. Pearson offers a variety of content and course options to support students with just-in-time remediation and key-concept review as needed.

Integrated Review in MyLab Math

Integrated Review can be used in corequisite courses or simply to help students who enter a course without a full understanding of prerequisite skills and concepts. Premade, editable Integrated Review assignments are available to assign in the Assignment Manager. Integrated Review landing pages (shown below) are visible by default at the start of most chapters, providing objective-level review.

- Students begin each chapter by completing a Skills Check to pinpoint which topics, if any, they need to review.
- Personalized review homework provides extra support for students who need it on just the topics they didn't master in the preceding Skills Check.



- Additional review materials, including worksheets and videos, are available.

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*Get the **most** out of* MyLab Math

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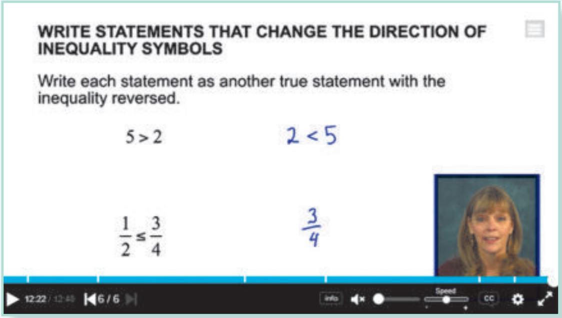
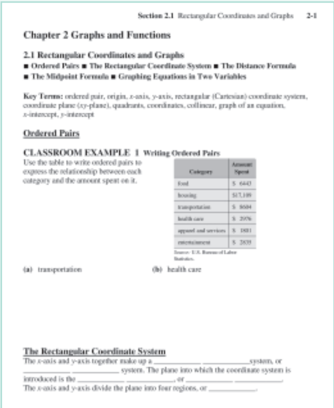
Updated! Videos

Updated videos cover all topics in the text to support students outside of the classroom. **Quick Review** videos cover key definitions and procedures. **Example Solution** videos offer a detailed solution process for every example in the textbook.





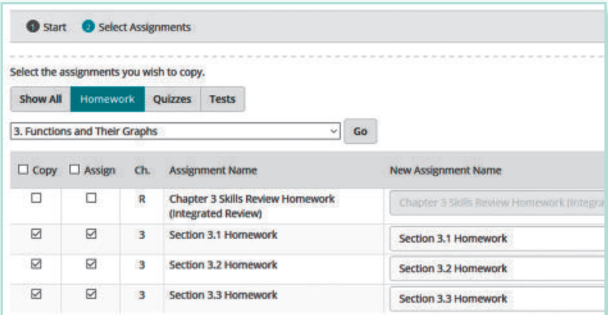
Updated! MyNotes and MyClassroomExamples

MyNotes give students a note-taking structure to use while they read the text or watch the MyLab Math videos. **MyClassroomExamples** offer structure for notes taken during lecture and are for use with the ClassroomExamples found in the Annotated Instructor Edition.

Both sets of notes are available in MyLab Math and can be customized by the instructor.

New! Enhanced Sample Assignments

Author Callie Daniels makes course set-up easier by giving instructors a starting point for each section. Following Callie's best practices in the classroom, Enhanced Sample Assignments maximize students' performance.



- **Section Prep Assignments** include Example Videos with assessment questions. This assignment pairs with **MyNotes**. Students actively participate while taking notes from the Example Video and then work the related exercises.
- **Section Homework** includes author-selected problems and increases in difficulty.
- **Cumulative Review Homework Assignments** draw from section homework questions covered to that point in the course—helping students prepare for a final exam.

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Resources for Success

Pearson MyLab

Instructor Resources

Online resources can be downloaded at pearson.com/mylab/math or from **www.pearson.com**.

Annotated Instructor's Edition

ISBN: 0135924499 / 9780135924495

Answers are included on the same page beside the text exercises where possible for quick reference. Helpful Teaching Tips and Classroom Examples are also provided.

Online Instructor's Solution Manual

By Beverly Fusfield

Provides complete solutions to all text exercises

Online Instructor's Testing Manual

Includes diagnostic pretests, grouped by section, with answers provided

Testgen®

TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides and Classroom Example PowerPoints

- The PowerPoint Lecture Slides feature presentations written and designed specifically for this text, including figures and examples from the text.
- Classroom Example PowerPoints include fully worked-out solutions to all Classroom Examples.

Learning Catalytics™

With MyLab Math, instructors and students have access to Learning Catalytics, which instructors can use to generate class discussion, guide lectures, and actively engage students. Prebuilt Learning Catalytics questions have been created specifically for this text. Simply search the tag "LialPrecalculus" within the Learning Catalytics Question Library.

Student Resources

Additional resources enhance student success.

Student's Solution Manual

By Beverly Fusfield

Provides detailed solutions to all odd-numbered text exercises

Video Lectures

- Quick Reviews cover key definitions and procedures from each section.
- Example Solutions walk students through the detailed solution process for every example in the textbook.

MyNotes with Integrated Review Worksheets

MyNotes offer structure for students as they watch videos or read the text. These are available as a printed supplement and in MyLab Math.

- Includes textbook examples along with ample space for students to write solutions and notes
- Includes key concepts along with prompts for students to read, write, and reflect on what they have just learned
- **Customizable**—instructors can add their own examples or remove examples that are not covered in their course.

Integrated Review Worksheets prepare students for the College Algebra and Trigonometry material.

- Includes key terms, guided examples with ample space for students to work, and references to extra help in MyLab Math

MyClassroomExamples

- Available in MyLab Math and offer structure for classroom lecture
- Includes Classroom Examples along with ample space for students to write solutions and notes
- Includes key concepts along with fill-in-the-blank opportunities to keep students engaged
- **Customizable**—instructors can add their own examples or remove Classroom Examples that are not covered in their course.

ACKNOWLEDGMENTS

We wish to thank the following individuals who provided valuable input into this edition of the text.

Zalmond Abbondanza – Palm Beach State College
Beyza Aslan – University of North Florida
Kathy Autrey – Northwestern State University, University of Louisiana
Shirley Brown – Weatherford College
Jolina Cadilli – Cypress College
Betty Collins – Hinds Community College
Daniela Johnson – Valencia College
Catelin Peay-Britt – Coahoma Community College
Leslie Plumlee – Western Kentucky University
Anthony Precella – Del Mar College
Luminita Razaila – University of North Florida
Jiahui Yao – Mt. San Antonio College
Paula Young – Mt. San Antonio College
Robert Young – Eastern Florida State College

Our sincere thanks to those individuals at Pearson Education who have supported us throughout this revision: Dawn Murrin, Anne Kelly, Lauren Morse, Joe Vetere, Mary Catherine Connors, and Jonathan Krebs. Terry McGinnis continues to provide behind-the-scenes guidance for both content and production. We have come to rely on her expertise during all phases of the revision process. Carol Merrigan provided excellent production work. Special thanks go out to Paul Lorczak, Hal Whipple, and Sarah Sponholz for their excellent accuracy-checking. We thank Lucie Haskins, who provided an accurate index. We appreciate the valuable suggestions for Chapter 5 from Mary Hill of *College of Dupage* and the detailed suggestions from Kyle Linden of *St. Charles Community College*.

As an author team, we are committed to providing the best possible college algebra and trigonometry course to help instructors teach and students succeed. As we continue to work toward this goal, we welcome any comments or suggestions you might send, via e-mail, to math@pearson.com.

Margaret L. Lial
John Hornsby
David I. Schneider
Callie J. Daniels

R Review of Basic Concepts

Positive and negative numbers, used to represent gains and losses on a board such as this one, are examples of *real numbers* encountered in applications of mathematics.

R.1 Fractions, Decimals, and Percents

- Lowest Terms of a Fraction
- Improper Fractions and Mixed Numbers
- Operations with Fractions
- Decimals as Fractions
- Operations with Decimals
- Fractions as Decimals
- Percents as Decimals and Decimals as Percents
- Percents as Fractions and Fractions as Percents

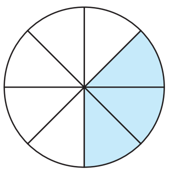
Recall that **fractions** are a way to represent parts of a whole. See **Figure 1.** In a fraction, the **numerator** gives the number of parts being represented. The **denominator** gives the total number of equal parts in the whole. The fraction bar represents division $\left(\frac{a}{b} = a \div b\right)$.

$$\text{Fraction bar} \longrightarrow \frac{3}{8} \begin{array}{l} \longleftarrow \text{Numerator} \\ \longleftarrow \text{Denominator} \end{array}$$

A fraction is classified as either a **proper fraction** or an **improper fraction.**

Proper fractions $\frac{1}{5}, \frac{2}{7}, \frac{9}{10}, \frac{23}{25}$ Numerator is less than denominator. Value is less than 1.

Improper fractions $\frac{3}{2}, \frac{5}{5}, \frac{11}{7}, \frac{28}{4}$ Numerator is greater than or equal to denominator. Value is greater than or equal to 1.



The shaded region represents $\frac{3}{8}$ of the circle.

Figure 1

Lowest Terms of a Fraction

A fraction is in **lowest terms** when the numerator and denominator have no factors in common (other than 1).

Writing a Fraction in Lowest Terms

Step 1 Write the numerator and denominator in factored form.

Step 2 Replace each pair of factors common to the numerator and denominator with 1.

Step 3 Multiply the remaining factors in the numerator and in the denominator.

(This procedure is sometimes called **"simplifying the fraction."**)

EXAMPLE 1 Writing Fractions in Lowest Terms

Write each fraction in lowest terms.

(a) $\frac{10}{15}$ **(b)** $\frac{15}{45}$ **(c)** $\frac{150}{200}$

SOLUTION

(a) $\frac{10}{15} = \frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3} \cdot \frac{5}{5} = \frac{2}{3} \cdot 1 = \frac{2}{3}$ 5 is the greatest common factor of 10 and 15.

(b) $\frac{15}{45} = \frac{1 \cdot 15}{3 \cdot 15} = \frac{1}{3} \cdot 1 = \frac{1}{3}$ Remember to write 1 in the numerator.

(c) $\frac{150}{200} = \frac{3 \cdot 50}{4 \cdot 50} = \frac{3}{4} \cdot 1 = \frac{3}{4}$ 50 is the greatest common factor of 150 and 200.

Another strategy is to choose *any* common factor and work in stages.

$$\frac{150}{200} = \frac{15 \cdot 10}{20 \cdot 10} = \frac{3 \cdot 5 \cdot 10}{4 \cdot 5 \cdot 10} = \frac{3}{4} \cdot 1 \cdot 1 = \frac{3}{4}$$ The same answer results.

✔ **Now Try Exercises 7 and 15.**

Improper Fractions and Mixed Numbers A **mixed number** is a single number that represents the sum of a natural (counting) number and a proper fraction.

$$\text{Mixed number} \longrightarrow 2\frac{3}{4} = 2 + \frac{3}{4}$$

EXAMPLE 2 Converting an Improper Fraction to a Mixed Number

Write $\frac{59}{8}$ as a mixed number.

SOLUTION Because the fraction bar represents division $\left(\frac{a}{b} = a \div b, \text{ or } b\overline{)a}\right)$, divide the numerator of the improper fraction by the denominator.

$$\begin{array}{rl} & 7 \longleftarrow \text{Quotient} \\ \text{Denominator of fraction} \longrightarrow 8 & \overline{)59} \longleftarrow \text{Numerator of fraction} \\ & \underline{56} \\ & 3 \longleftarrow \text{Remainder} \end{array} \qquad \frac{59}{8} = 7\frac{3}{8}$$

✔ **Now Try Exercise 17.**

EXAMPLE 3 Converting a Mixed Number to an Improper Fraction

Write $6\frac{4}{7}$ as an improper fraction.

SOLUTION Multiply the denominator of the fraction by the natural number, and then add the numerator to obtain the numerator of the improper fraction.

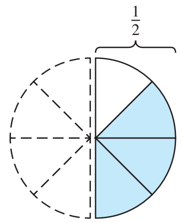
$$7 \cdot 6 = 42 \quad \text{and} \quad 42 + 4 = 46$$

The denominator of the improper fraction is the same as the denominator in the mixed number, which is 7 here.

$$6\frac{4}{7} = \frac{7 \cdot 6 + 4}{7} = \frac{46}{7}$$

✔ **Now Try Exercise 21.**

Operations with Fractions **Figure 2** illustrates multiplying fractions.



$\frac{3}{4}$ of $\frac{1}{2}$ is equivalent to $\frac{3}{4} \cdot \frac{1}{2}$, which equals $\frac{3}{8}$ of the circle.

Figure 2

Multiplying Fractions

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions $(b \neq 0, d \neq 0)$, then $$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

That is, to multiply two fractions, multiply their numerators and then multiply their denominators.

EXAMPLE 4 Multiplying Fractions

Multiply $\frac{3}{8} \cdot \frac{4}{9}$. Write the answer in lowest terms.

SOLUTION

$$\frac{3}{8} \cdot \frac{4}{9}$$

$$= \frac{3 \cdot 4}{8 \cdot 9} \quad \text{Multiply numerators. Multiply denominators.}$$

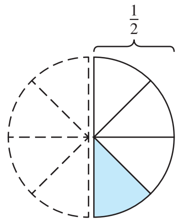
$$= \frac{12}{72} \quad \text{Multiply.}$$

$$= \frac{1 \cdot 12}{6 \cdot 12} \quad \text{The greatest common factor of 12 and 72 is 12.}$$

$$= \frac{1}{6} \quad \frac{1 \cdot 12}{6 \cdot 12} = \frac{1}{6} \cdot 1 = \frac{1}{6}$$

Make sure the product is in lowest terms.

✔ **Now Try Exercise 27.**



$\frac{1}{2} \div 4$ is equivalent to $\frac{1}{2} \cdot \frac{1}{4}$, which equals $\frac{1}{8}$ of the circle.

Figure 3

Two numbers are **reciprocals** of each other if their product is 1. For example,

$$\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12}, \quad \text{or} \quad 1.$$

Division is the inverse or opposite of multiplication, and as a result, we use reciprocals to divide fractions. **Figure 3** illustrates dividing fractions.

Dividing Fractions

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions $(b \neq 0, d \neq 0, c \neq 0)$, then $\boldsymbol{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}}$.

That is, to divide by a fraction, multiply by its reciprocal.

EXAMPLE 5 Dividing Fractions

Divide. Write answers in lowest terms as needed.

(a) $\frac{3}{4} \div \frac{8}{5}$ **(b)** $\frac{5}{8} \div 10$ **(c)** $1\frac{2}{3} \div 4\frac{1}{2}$

SOLUTION

(a) $\frac{3}{4} \div \frac{8}{5}$

$$= \frac{3}{4} \cdot \frac{5}{8} \quad \text{Multiply by } \tfrac{5}{8}\text{, the reciprocal of } \tfrac{8}{5}.$$

$$= \frac{3 \cdot 5}{4 \cdot 8} \quad \text{Multiply numerators. Multiply denominators.}$$

$$= \frac{15}{32}$$

Make sure the answer is in lowest terms.

(b) $\frac{5}{8} \div 10$

Think of 10 as $\frac{10}{1}$ here.

$$= \frac{5}{8} \cdot \frac{1}{10} \quad \text{Multiply by } \tfrac{1}{10}\text{, the reciprocal of 10.}$$

$$= \frac{5 \cdot 1}{8 \cdot 2 \cdot 5} \quad \text{Multiply and factor.}$$

$$= \frac{1}{16}$$

Remember to write 1 in the numerator.

(c) $1\frac{2}{3} \div 4\frac{1}{2}$

$= \frac{5}{3} \div \frac{9}{2}$ — Write each mixed number as an improper fraction.

$= \frac{5}{3} \cdot \frac{2}{9}$ — Multiply by $\frac{2}{9}$, the reciprocal of $\frac{9}{2}$.

$= \frac{10}{27}$ — Multiply. The quotient is in lowest terms.

✔ **Now Try Exercises 37, 43, and 49.**

Figures 4 and **5** illustrate adding and subtracting fractions.

Adding Fractions

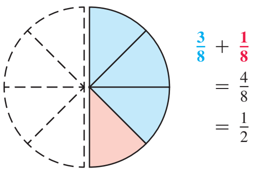


Figure 4

Subtracting Fractions

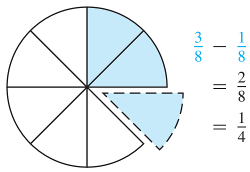


Figure 5

Adding and Subtracting Fractions

If $\frac{a}{b}$ and $\frac{c}{b}$ are fractions $(b \neq 0)$, then add or subtract as follows.

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

That is, to add or subtract two fractions having the same denominator, add or subtract the numerators and keep the same denominator.

If the denominators are different, first find the least common denominator (LCD). Write each fraction as an equivalent fraction with this denominator. Then add or subtract as above.

EXAMPLE 6 Adding and Subtracting Fractions

Add or subtract as indicated. Write answers in lowest terms as needed.

(a) $\frac{2}{10} + \frac{3}{10}$ **(b)** $\frac{4}{15} + \frac{5}{9}$ **(c)** $\frac{15}{6} - \frac{4}{9}$ **(d)** $4\frac{1}{2} - 1\frac{3}{4}$

SOLUTION

(a) $\frac{2}{10} + \frac{3}{10}$

$= \frac{2+3}{10}$ — Add numerators. Keep the same denominator.

$= \frac{5}{10}$

$= \frac{1}{2}$ — Write in lowest terms.

(b) $\frac{4}{15} + \frac{5}{9}$

$= \frac{4}{15} \cdot \frac{3}{3} + \frac{5}{9} \cdot \frac{5}{5}$ — $15 = 3 \cdot 5$ and $9 = 3 \cdot 3$, so the LCD is $3 \cdot 3 \cdot 5 = 45$.

$= \frac{12}{45} + \frac{25}{45}$ — Write equivalent fractions with the common denominator.

$= \frac{37}{45}$ — Add numerators. Keep the same denominator.

(c) $\dfrac{15}{16} - \dfrac{4}{9}$

$= \dfrac{15}{16} \cdot \dfrac{9}{9} - \dfrac{4}{9} \cdot \dfrac{16}{16}$ Because 16 and 9 have no common factors except 1, the LCD is $16 \cdot 9 = 144$.

$= \dfrac{135}{144} - \dfrac{64}{144}$ Write equivalent fractions with the common denominator.

$= \dfrac{71}{144}$ Subtract numerators. Keep the common denominator.

(d) ***Method 1*** $4\dfrac{1}{2} - 1\dfrac{3}{4}$

$= \dfrac{9}{2} - \dfrac{7}{4}$ Write each mixed number as an improper fraction.

$= \dfrac{18}{4} - \dfrac{7}{4}$ Find a common denominator. The LCD is 4.

Think: $\frac{9}{2} \cdot \frac{2}{2} = \frac{18}{4}$

$= \dfrac{11}{4}$, or $2\dfrac{3}{4}$ Subtract. Write as a mixed number.

Method 2

$$4\frac{1}{2} = 4\frac{2}{4} = 3\frac{6}{4}$$

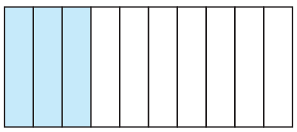
The LCD is 4. $4\frac{2}{4} = 3 + 1 + \frac{2}{4} = 3 + \frac{4}{4} + \frac{2}{4} = 3\frac{6}{4}$

$$-1\frac{3}{4} = -1\frac{3}{4} = -1\frac{3}{4}$$

$$2\frac{3}{4}, \text{ or } \frac{11}{4}$$

The same answer results.

✓ **Now Try Exercises 55, 57, 71, and 75.**



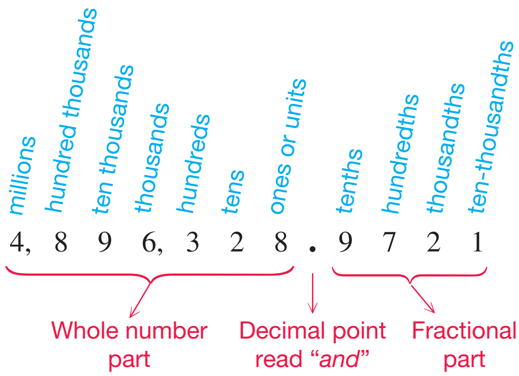
3 parts of the whole 10 are shaded. As a fraction, $\frac{3}{10}$ of the figure is shaded. As a decimal, 0.3 is shaded. Both of these numbers are read "*three-tenths*."

FIGURE 6

Fractions are one way to represent parts of a whole. Another way is with a decimal fraction or **decimal,** a number written with a decimal point.

9.25, 14.001, 0.3 Decimal numbers

See **Figure 6.** Each digit in a decimal number has a place value, as shown below.



Each successive place value is ten times greater than the place value to its right and one-tenth as great as the place value to its left.

Decimals as Fractions

Converting a Decimal to a Fraction

Read the decimal using the correct place value. Write it in fractional form just as it is read.

- The numerator will be the digits to the right of the decimal point.
- The denominator will be a power of 10—that is, 10 for tenths, 100 for hundreds, and so on.

EXAMPLE 7 Writing Decimals as Fractions

Write each decimal as a fraction. (Do not write in lowest terms.)

(a) 0.95 **(b)** 0.056 **(c)** 4.2095

SOLUTION

(a) We read 0.95 as "*ninety-five hundredths*."

$$0.95 = \frac{95}{100} \leftarrow \text{For hundredths}$$

(b) We read 0.056 as "*fifty-six thousandths*."

Do not confuse 0.056 with 0.56, read "fifty-six *hundredths*," which is the fraction $\frac{56}{100}$.

$$0.056 = \frac{56}{1000} \leftarrow \text{For thousandths}$$

(c) We read 4.2095, which is greater than 1, as "*Four **and** two thousand ninety-five ten-thousandths*."

$$4.2095 = 4\frac{2095}{10,000}$$ Write the decimal number as a mixed number.

Think: $10,000 \cdot 4 + 2095$

$$= \frac{42,095}{10,000}$$ Write the mixed number as an improper fraction.

✔ **Now Try Exercises 85, 89, and 91.**

Operations with Decimals

EXAMPLE 8 Adding and Subtracting Decimals

Add or subtract as indicated.

(a) $6.92 + 14.8 + 3.217$ **(b)** $47.6 - 32.509$

SOLUTION

(a) Place the digits of the decimal numbers in columns by place value. Attach zeros as placeholders.

Be sure to line up decimal points.

$$\begin{array}{r} 6.92 \\ 14.8 \\ +\ 3.217 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 6.920 \\ 14.800 \\ +\ 3.217 \\ \hline 24.937 \end{array}$$

Attach 0s.

6.92 is equivalent to 6.920. 14.8 is equivalent to 14.800.

(b)

$$\begin{array}{r} 47.6 \\ -\ 32.509 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 47.600 \\ -\ 32.509 \\ \hline 15.091 \end{array}$$

Write the decimal numbers in columns, attaching 0s to 47.6.

✔ **Now Try Exercises 93 and 97.**

EXAMPLE 9 Multiplying and Dividing Decimals

Multiply or divide as indicated. In part (c), round the answer to two decimal places.

(a) 29.3×4.52 **(b)** 0.05×0.3 **(c)** $8.949 \div 1.25$

SOLUTION

(a) Multiply normally. Place the decimal point in the answer as shown.

$$\begin{array}{r} 29.3 \\ \times \quad 4.52 \\ \hline 586 \\ 1465 \\ 1172 \\ \hline 132.436 \end{array}$$

29.3 — 1 decimal place
4.52 — 2 decimal places
1 + 2 = 3
132.436 — 3 decimal places

(b) Here $5 \times 3 = 15$. Be careful placing the decimal point.

$$0.05 \times 0.3 = 0.015$$

0.05 — 2 decimal places; 0.3 — 1 decimal place

Do *not* write 0.150.

2 + 1 = 3 decimal places
Attach 0 as a placeholder in the tenths place.

(c) $1.25\overline{)8.949}$ Move each decimal point two places to the right.

$$\begin{array}{r} 7.159 \\ 125\overline{)894.900} \\ 875 \\ \hline 199 \\ 125 \\ \hline 740 \\ 625 \\ \hline 1150 \\ 1125 \\ \hline 25 \end{array}$$

Move the decimal point straight up, and divide as with whole numbers. Attach 0s as placeholders.

We carried out the division to three decimal places so that we could round the answer to two decimal places, obtaining 7.16.

✓ **Now Try Exercises 105 and 113.**

NOTE To round 7.159 in **Example 9(c)** to two decimal places (that is, to the nearest hundredth), we look at the digit to the *right* of the hundredths place.

- If this digit is 5 or greater, we round up.
- If this digit is less than 5, we drop the digit(s) beyond the desired place.

Hundredths place ↓

7.159 — 9, the digit to the right of the hundredths place, is 5 or greater.

≈ 7.16 — Round 5 up to 6. ≈ means *is approximately equal to*.

Multiplying and Dividing by Powers of 10 (Shortcuts)

- To ***multiply*** by a power of 10, ***move the decimal point to the right*** as many places as the number of zeros.
- To ***divide*** by a power of 10, ***move the decimal point to the left*** as many places as the number of zeros.

In both cases, insert 0s as placeholders if necessary.

EXAMPLE 10 Multiplying and Dividing by Powers of 10

Multiply or divide as indicated.

(a) 48.731×10

(b) 48.731×1000

(c) $48.731 \div 10$

(d) $48.731 \div 1000$

SOLUTION

(a) 48.731×10

$= 48.731$

$= 487.31$

Move the decimal point one place to the *right*.

(b) 48.731×1000

$= 48.731$

$= 48{,}731$

Move the decimal point three places to the *right*.

(c) $48.731 \div 10$

$= 48.731$

$= 4.8731$

Move the decimal point one place to the *left*.

(d) $48.731 \div 1000$

$= 048.731$

$= 0.048731$

Move the decimal point three places to the *left*.

✓ **Now Try Exercises 117 and 125.**

Fractions as Decimals

Converting a Fraction to a Decimal

Because a fraction bar indicates division, write a fraction as a decimal by dividing the numerator by the denominator.

EXAMPLE 11 Writing Fractions as Decimals

Write each fraction as a decimal.

(a) $\frac{19}{8}$

(b) $\frac{2}{3}$

SOLUTION

(a) $\frac{19}{8}$

$$
\begin{array}{r}
2.375 \\
8\overline{)19.000} \\
\underline{16} \\
30 \\
\underline{24} \\
60 \\
\underline{56} \\
40 \\
\underline{40} \\
0
\end{array}
$$

Divide 19 by 8. Add a decimal point and as many 0s as necessary to 19.

$\frac{19}{8} = 2.375 \leftarrow$ **Terminating decimal**

(b) $\frac{2}{3}$

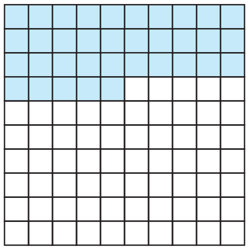
$$
\begin{array}{r}
0.6666\ldots \\
3\overline{)2.0000\ldots} \\
\underline{18} \\
20 \\
\underline{18} \\
20 \\
\underline{18} \\
20 \\
\underline{18} \\
2
\end{array}
$$

$\frac{2}{3} = 0.6666\ldots \leftarrow$ **Repeating decimal**

$= 0.\overline{6}$ A bar is written over the repeating digit(s).

≈ 0.667 Nearest thousandth

✓ **Now Try Exercises 141 and 147.**



35 of the 100 squares are shaded. That is, $\frac{35}{100}$, or 35%, of the figure is shaded.

Figure 7

Percents as Decimals and Decimals as Percents The word **percent** means ***"per 100."*** Percent is written with the symbol **%**. ***"One percent" means "one per one hundred," or "one one-hundredth."*** See **Figure 7.**

Percent, Fraction, and Decimal Equivalents

$$1\% = \frac{1}{100} = 0.01, \quad 10\% = \frac{10}{100} = 0.10, \quad 100\% = \frac{100}{100} = 1$$

For example, 73% means "73 *per one hundred*."

$$73\% = \frac{73}{100} = 0.73$$

Essentially, we are dropping the % symbol from 73% and dividing 73 by 100. Doing this moves the decimal point, which is understood to be after the 3, two places to the *left*.

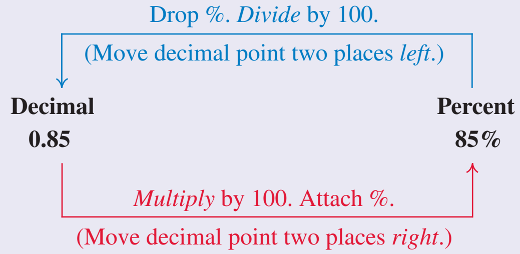
Writing 0.73 as a percent is the opposite process.

$$0.73 = 0.73 \cdot 100\% = 73\% \qquad 100\% = 1$$

Moving the decimal point two places to the *right* and attaching a % symbol gives the same result.

Converting Percents and Decimals (Shortcuts)

- To convert a percent to a decimal, move the decimal point two places to the *left* and drop the % symbol.
- To convert a decimal to a percent, move the decimal point two places to the *right* and attach a % symbol.



EXAMPLE 12 **Converting Percents and Decimals by Moving the Decimal Point**

Convert each percent to a decimal and each decimal to a percent.

(a) 45% **(b)** 250% **(c)** 9% **(d)** 0.57 **(e)** 1.5 **(f)** 0.007

SOLUTION

(a) $45\% = 0.45$ **(b)** $250\% = 2.50$, or 2.5 **(c)** $9\% = 09\% = 0.09$

(d) $0.57 = 57\%$ **(e)** $1.5 = 1.50 = 150\%$ **(f)** $0.007 = 0.7\%$

✔ **Now Try Exercises 151 and 161.**

Percents as Fractions and Fractions as Percents

EXAMPLE 13 Writing Percents as Fractions

Write each percent as a fraction. Give answers in lowest terms.

(a) 8% **(b)** 175% **(c)** 13.5%

SOLUTION

(a) We use the fact that *percent* means "*per one hundred*," and convert as follows.

$$8\% = \frac{8}{100} = \frac{2 \cdot 4}{25 \cdot 4} = \frac{2}{25}$$

As with converting percents to decimals, drop the % symbol and divide by 100.

Write in lowest terms.

(b) $175\% = \frac{175}{100} = \frac{7 \cdot 25}{4 \cdot 25} = \frac{7}{4}, \quad \text{or} \quad 1\frac{3}{4}$

A number greater than 1 is more than 100%.

Write in lowest terms.

(c) 13.5%

$= \frac{13.5}{100}$ Drop the % symbol. Divide by 100.

$= \frac{13.5}{100} \cdot \frac{10}{10}$ Multiply by 1 in the form $\frac{10}{10}$ to eliminate the decimal in the numerator.

$= \frac{135}{1000}$ Multiply.

$= \frac{27}{200}$ Write in lowest terms. $\frac{135}{1000} = \frac{27 \cdot 5}{200 \cdot 5} = \frac{27}{200}$

✓ **Now Try Exercises 171, 177, and 179.**

EXAMPLE 14 Writing Fractions as Percents

Write each fraction as a percent.

(a) $\frac{2}{5}$ **(b)** $\frac{1}{6}$

SOLUTION

(a) $\frac{2}{5}$

$= \frac{2}{5} \cdot 100\%$ Multiply by 1 in the form 100%.

$= \frac{2}{5} \cdot \frac{100}{1}\%$

$= \frac{2 \cdot 5 \cdot 20}{5 \cdot 1}\%$ Multiply and factor.

$= \frac{2 \cdot 20}{1}\%$ Divide out the common factor.

$= 40\%$ Simplify.

(b) $\frac{1}{6}$

$= \frac{1}{6} \cdot 100\%$

$= \frac{1}{6} \cdot \frac{100}{1}\%$

$= \frac{1 \cdot 2 \cdot 50}{2 \cdot 3 \cdot 1}\%$

$= \frac{50}{3}\%$

$= 16\frac{2}{3}\%, \quad \text{or} \quad 16.\overline{6}\%$

✓ **Now Try Exercises 181 and 185.**

R.1 Exercises

CONCEPT PREVIEW *Determine whether each statement is* true *or* false. *If it is false, explain why.*

1. In the fraction $\frac{5}{8}$, 5 is the numerator and 8 is the denominator.
2. The mixed number equivalent of the improper fraction $\frac{31}{5}$ is $6\frac{1}{5}$.
3. The fraction $\frac{7}{7}$ is proper.
4. The reciprocal of $\frac{6}{2}$ is $\frac{3}{1}$.

CONCEPT PREVIEW *Choose the letter of the correct response.*

5. Which choice shows the correct way to write $\frac{16}{24}$ in lowest terms?

A. $\frac{16}{24} = \frac{8+8}{8+16} = \frac{8}{16} = \frac{1}{2}$ **B.** $\frac{16}{24} = \frac{4 \cdot 4}{4 \cdot 6} = \frac{4}{6}$ **C.** $\frac{16}{24} = \frac{8 \cdot 2}{8 \cdot 3} = \frac{2}{3}$

6. Which fraction is *not* equal to $\frac{5}{9}$?

A. $\frac{15}{27}$ **B.** $\frac{30}{54}$ **C.** $\frac{40}{74}$ **D.** $\frac{55}{99}$

Write each fraction in lowest terms. ***See Example 1.***

7. $\frac{8}{16}$ 8. $\frac{4}{12}$ 9. $\frac{15}{18}$ 10. $\frac{16}{20}$ 11. $\frac{90}{150}$

12. $\frac{100}{140}$ 13. $\frac{18}{90}$ 14. $\frac{16}{64}$ 15. $\frac{120}{144}$ 16. $\frac{77}{132}$

Write each improper fraction as a mixed number. ***See Example 2.***

17. $\frac{12}{7}$ 18. $\frac{16}{9}$ 19. $\frac{77}{12}$ 20. $\frac{67}{13}$

Write each mixed number as an improper fraction. ***See Example 3.***

21. $2\frac{3}{5}$ 22. $5\frac{6}{7}$ 23. $12\frac{2}{3}$ 24. $10\frac{1}{5}$

Multiply or divide as indicated. Write answers in lowest terms as needed. ***See Examples 4 and 5.***

25. $\frac{4}{5} \cdot \frac{6}{7}$ 26. $\frac{5}{9} \cdot \frac{2}{7}$ 27. $\frac{2}{15} \cdot \frac{3}{8}$ 28. $\frac{3}{20} \cdot \frac{5}{21}$

29. $\frac{1}{10} \cdot \frac{12}{5}$ 30. $\frac{1}{8} \cdot \frac{10}{7}$ 31. $\frac{15}{4} \cdot \frac{8}{25}$ 32. $\frac{21}{8} \cdot \frac{4}{7}$

33. $21 \cdot \frac{3}{7}$ 34. $36 \cdot \frac{4}{9}$ 35. $3\frac{1}{4} \cdot 1\frac{2}{3}$ 36. $2\frac{2}{3} \cdot 1\frac{3}{5}$

37. $\frac{7}{9} \div \frac{3}{2}$ 38. $\frac{6}{11} \div \frac{5}{4}$ 39. $\frac{5}{4} \div \frac{3}{8}$ 40. $\frac{7}{5} \div \frac{3}{10}$

41. $\frac{32}{5} \div \frac{8}{15}$ 42. $\frac{24}{7} \div \frac{6}{21}$ 43. $\frac{3}{4} \div 12$ 44. $\frac{2}{5} \div 30$

45. $6 \div \frac{3}{5}$ 46. $8 \div \frac{4}{9}$ 47. $6\frac{3}{4} \div \frac{3}{8}$ 48. $5\frac{3}{5} \div \frac{7}{10}$

49. $2\frac{1}{2} \div 1\frac{5}{7}$ 50. $2\frac{2}{9} \div 1\frac{2}{5}$ 51. $2\frac{5}{8} \div 1\frac{15}{32}$ 52. $2\frac{3}{10} \div 1\frac{4}{5}$

Add or subtract as indicated. Write answers in lowest terms as needed. ***See Example 6.***

53. $\frac{7}{15}+\frac{4}{15}$ **54.** $\frac{2}{9}+\frac{5}{9}$ **55.** $\frac{7}{12}+\frac{1}{12}$ **56.** $\frac{3}{16}+\frac{5}{16}$

57. $\frac{5}{9}+\frac{1}{3}$ **58.** $\frac{4}{15}+\frac{1}{5}$ **59.** $\frac{3}{8}+\frac{5}{6}$ **60.** $\frac{5}{6}+\frac{2}{9}$

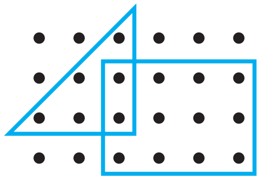
61. $\frac{5}{9}+\frac{3}{16}$ **62.** $\frac{3}{4}+\frac{6}{25}$ **63.** $3\frac{1}{8}+2\frac{1}{4}$ **64.** $4\frac{2}{3}+2\frac{1}{6}$

65. $\frac{7}{9}-\frac{2}{9}$ **66.** $\frac{8}{11}-\frac{3}{11}$ **67.** $\frac{13}{15}-\frac{3}{15}$ **68.** $\frac{11}{12}-\frac{3}{12}$

69. $\frac{7}{12}-\frac{1}{3}$ **70.** $\frac{5}{6}-\frac{1}{2}$ **71.** $\frac{7}{12}-\frac{1}{9}$ **72.** $\frac{11}{16}-\frac{1}{12}$

73. $4\frac{3}{4}-1\frac{2}{5}$ **74.** $3\frac{4}{5}-1\frac{4}{9}$ **75.** $8\frac{2}{9}-4\frac{2}{3}$ **76.** $7\frac{5}{12}-4\frac{5}{6}$

Work each problem involving fractions.



77. Refer to the figure in the margin. For each description, write a fraction in lowest terms that represents the region described.

(a) The dots in the large rectangle as a part of the dots in the entire figure

(b) The dots in the triangle as a part of the dots in the entire figure

(c) The dots in the overlapping region of the triangle and the rectangle as a part of the dots in the triangle alone; As a part of the rectangle alone

78. At the conclusion of the Pearson softball league season, batting statistics for five players were as shown in the table.

Player	At-Bats	Hits	Home Runs
Benita	36	12	3
Christine	40	9	2
Chase	11	5	1
Chin	16	8	0
Greg	20	10	2

Use this information to answer each question. Estimate as necessary.

(a) Which player got a hit in exactly $\frac{1}{3}$ of their at-bats?

(b) Which player got a home run in just less than $\frac{1}{10}$ of their at-bats?

(c) Which player got a hit in just less than $\frac{1}{4}$ of their at-bats?

(d) Which two players got hits in exactly the same fractional part of their at-bats? What was that fractional part, expressed in lowest terms?

Concept Check *Provide the correct response.*

79. In the decimal number 367.9412, name the digit that has each place value.

(a) tens **(b)** tenths **(c)** thousandths **(d)** ones or units **(e)** hundredths

80. Write a decimal number that has 5 in the thousands place, 0 in the tenths place, and 4 in the ten-thousandths place.

81. For the decimal number 46.249, round to the place value indicated.

(a) hundredths **(b)** tenths **(c)** ones or units **(d)** tens

82. Round each decimal to the nearest thousandth.

(a) $0.\overline{8}$ **(b)** $0.\overline{4}$ **(c)** 0.9762 **(d)** 0.8645

Write each decimal as a fraction. (Do not write in lowest terms.) ***See Example 7.***

83. 0.4 **84.** 0.6 **85.** 0.64 **86.** 0.82 **87.** 0.138

88. 0.104 **89.** 0.043 **90.** 0.087 **91.** 3.805 **92.** 5.166

Add or subtract as indicated. ***See Example 8.***

93. 25.32 + 109.2 + 8.574 **94.** 90.527 + 32.43 + 589.8 **95.** 28.73 − 3.12

96. 46.88 − 13.45 **97.** 43.5 − 28.17 **98.** 345.1 − 56.31

99. 3.87 + 15 + 2.9 **100.** 8.2 + 1.09 + 12 **101.** 32.56 + 47.356 + 1.8

102. 75.2 + 123.96 + 3.897 **103.** 18 − 2.789 **104.** 29 − 8.582

Multiply or divide as indicated. ***See Example 9.***

105. 12.8 × 9.1 **106.** 34.04 × 0.56 **107.** 22.41 × 33 **108.** 55.76 × 72

109. 0.2 × 0.03 **110.** 0.07 × 0.004 **111.** 78.65 ÷ 11 **112.** 73.36 ÷ 14

113. 32.48 ÷ 11.6 **114.** 85.26 ÷ 17.4 **115.** 19.967 ÷ 9.74 **116.** 44.4788 ÷ 5.27

Multiply or divide as indicated. ***See Example 10.***

117. 123.26 × 10 **118.** 785.91 × 10 **119.** 57.116 × 100 **120.** 82.053 × 100

121. 0.094 × 1000 **122.** 0.025 × 1000 **123.** 1.62 ÷ 10 **124.** 8.04 ÷ 10

125. 124.03 ÷ 100 **126.** 490.35 ÷ 100 **127.** 23.29 ÷ 1000 **128.** 59.8 ÷ 1000

Concept Check *Complete the table of fraction, decimal, and percent equivalents.*

	Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
129.	$\frac{1}{100}$	0.01	
130.	$\frac{1}{50}$		2%
131.		0.05	5%
132.	$\frac{1}{10}$		
133.	$\frac{1}{8}$	0.125	
134.			20%
135.	$\frac{1}{4}$		
136.	$\frac{1}{3}$		
137.			50%
138.	$\frac{2}{3}$		$66\frac{2}{3}\%$, or $66.\overline{6}\%$
139.		0.75	
140.	1	1.0	

Write each fraction as a decimal. For repeating decimals, write the answer by first using bar notation and then rounding to the nearest thousandth. ***See Example 11.***

141. $\frac{21}{5}$ **142.** $\frac{9}{5}$ **143.** $\frac{9}{4}$ **144.** $\frac{15}{4}$ **145.** $\frac{3}{8}$

146. $\frac{7}{8}$ **147.** $\frac{5}{9}$ **148.** $\frac{8}{9}$ **149.** $\frac{1}{6}$ **150.** $\frac{5}{6}$

Write each percent as a decimal. **See Examples 12(a)–12(c).**

151. 54% **152.** 39% **153.** 7% **154.** 4% **155.** 117%

156. 189% **157.** 2.4% **158.** 3.1% **159.** $6\frac{1}{4}\%$ **160.** $5\frac{1}{2}\%$

Write each decimal as a percent. **See Examples 12(d)–12(f).**

161. 0.79 **162.** 0.83 **163.** 0.02 **164.** 0.08 **165.** 0.004

166. 0.005 **167.** 1.28 **168.** 2.35 **169.** 0.4 **170.** 0.6

Write each percent as a fraction. Give answers in lowest terms. **See Example 13.**

171. 51% **172.** 47% **173.** 15% **174.** 35% **175.** 2%

176. 8% **177.** 140% **178.** 180% **179.** 7.5% **180.** 2.5%

Write each fraction as a percent. **See Example 14.**

181. $\frac{4}{5}$ **182.** $\frac{3}{25}$ **183.** $\frac{7}{50}$ **184.** $\frac{9}{20}$ **185.** $\frac{2}{11}$

186. $\frac{4}{9}$ **187.** $\frac{9}{4}$ **188.** $\frac{8}{5}$ **189.** $\frac{13}{6}$ **190.** $\frac{31}{9}$

R.2 Sets and Real Numbers

- Basic Definitions
- Operations on Sets
- Sets of Numbers and the Number Line

Basic Definitions

A **set** is a collection of objects. The objects that belong to a set are its **elements,** or **members.** In algebra, the elements of a set are usually numbers. Sets are commonly written using **set braces,** $\{ \}$.

$\{1, 2, 3, 4\}$ The set containing the elements 1, 2, 3, and 4

The order in which the elements are listed is not important. This set can also be written $\{4, 3, 2, 1\}$ or with any other arrangement of the four numbers.

To show that 4 is an element of the set $\{1, 2, 3, 4\}$, we use the symbol $\in$.

$$4 \in \{1, 2, 3, 4\}$$

Since 5 is *not* an element of this set, we place a slash through the symbol $\in$.

$$5 \notin \{1, 2, 3, 4\}$$

It is customary to name sets with capital letters.

$S = \{1, 2, 3, 4\}$ *S* is used to name the set.

Set *S* was written above by listing its elements. Set *S* might also be described as "the set containing the first four counting numbers."

The set *F*, consisting of all fractions between 0 and 1, is an example of an **infinite set**—one that has an unending list of distinct elements. A **finite set** is one that has a limited number of elements. The process of counting its elements comes to an end. Some infinite sets can be described by listing. For example, the set of numbers *N* used for counting, which are the **natural numbers** or the **counting numbers,** can be written as follows.

$\mathbf{N = \{1, 2, 3, 4, \ldots\}}$ Natural (counting) numbers

The three dots (*ellipsis points*) show that the list of elements of the set continues according to the established pattern.

Sets are often written in **set-builder notation,** which uses a variable, such as x, to describe the elements of the set. The following set-builder notation represents the set $\{3, 4, 5, 6\}$ and is read "*the set of all elements x such that x is a natural number between* 2 *and* 7." The numbers 2 and 7 are *not* between 2 and 7.

$$\{x \mid x \text{ is a natural number between 2 and 7}\} = \{3, 4, 5, 6\}$$ Set-builder notation

The set of all elements x — such that — x is a natural number between 2 and 7

EXAMPLE 1 Using Set Notation and Terminology

Identify each set as *finite* or *infinite*. Then determine whether 10 is an element of the set.

(a) $\{7, 8, 9, \ldots, 14\}$

(b) $\left\{1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots\right\}$

(c) $\{x \mid x \text{ is a fraction between 1 and 2}\}$

(d) $\{x \mid x \text{ is a natural number between 9 and 11}\}$

SOLUTION

(a) The set is finite, because the process of counting its elements 7, 8, 9, 10, 11, 12, 13, and 14 comes to an end. The number 10 belongs to the set.

$$10 \in \{7, 8, 9, \ldots, 14\}$$

(b) The set is infinite, because the ellipsis points indicate that the pattern continues indefinitely. In this case,

$$10 \notin \left\{1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots\right\}.$$

(c) Between any two distinct natural numbers there are infinitely many fractions, so this set is infinite. The number 10 is not an element.

(d) There is only one natural number between 9 and 11, namely 10. So the set is finite, and 10 is an element.

✔ **Now Try Exercises 11, 13, 15, and 17.**

EXAMPLE 2 Listing the Elements of a Set

Use set notation, and list all the elements of each set.

(a) $\{x \mid x \text{ is a natural number less than 5}\}$

(b) $\{x \mid x \text{ is a natural number greater than 7 and less than 14}\}$

SOLUTION

(a) The natural numbers less than 5 form the set $\{1, 2, 3, 4\}$.

(b) This is the set $\{8, 9, 10, 11, 12, 13\}$.

✔ **Now Try Exercise 25.**

When we are discussing a particular situation or problem, the **universal set** (whether expressed or implied) contains all the elements included in the discussion. The letter U is used to represent the universal set. The **empty set,** or **null set,** is the set containing no elements. We write the empty set using the special symbol $\emptyset$.

CAUTION $\{\emptyset\}$ ***is not the empty set.*** It is the set containing the symbol $\emptyset$.

Every element of the set $S = \{1, 2, 3, 4\}$ is a natural number. S is an example of a *subset* of the set N of natural numbers. This relationship is written using the symbol $\subseteq$.

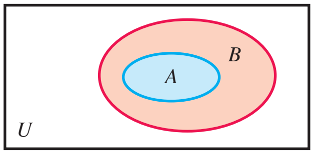
$$S \subseteq N$$

By definition, set A is a **subset** of set B if every element of set A is also an element of set B. For example, if $A = \{2, 5, 9\}$ and $B = \{2, 3, 5, 6, 9, 10\}$, then $A \subseteq B$. However, there are some elements of B that are not in A, so B is not a subset of A. This relationship is written using the symbol $\nsubseteq$.

$$B \nsubseteq A$$

Every set is a subset of itself. Also, $\emptyset$ is a subset of every set.

If A is any set, then $A \subseteq A$ and $\emptyset \subseteq A$.



$A \subseteq B$

Figure 8

Figure 8 shows a set A that is a subset of set B. The rectangle in the drawing represents the universal set U. Such a diagram is a **Venn diagram.**

Two sets A and B are equal whenever $A \subseteq B$ and $B \subseteq A$. Equivalently, $A = B$ if the two sets contain exactly the same elements. For example,

$$\{1, 2, 3\} = \{3, 1, 2\}$$

is true because both sets contain exactly the same elements. However,

$$\{1, 2, 3\} \neq \{0, 1, 2, 3\}$$

because the set $\{0, 1, 2, 3\}$ contains the element 0, which is not an element of $\{1, 2, 3\}$.

EXAMPLE 3 Examining Subset Relationships

Let $U = \{1, 3, 5, 7, 9, 11, 13\}$, $A = \{1, 3, 5, 7, 9, 11\}$, $B = \{1, 3, 7, 9\}$, $C = \{3, 9, 11\}$, and $D = \{1, 9\}$. Determine whether each statement is *true* or *false*.

(a) $D \subseteq B$ **(b)** $B \subseteq D$ **(c)** $C \nsubseteq A$ **(d)** $U = A$

SOLUTION

(a) All elements of D, namely 1 and 9, are also elements of B, so D is a subset of B, and $D \subseteq B$ is true.

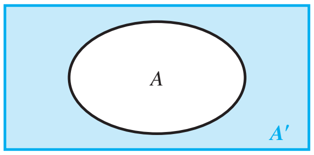
(b) There is at least one element of B (for example, 3) that is not an element of D, so B is *not* a subset of D. Thus, $B \subseteq D$ is false.

(c) C is a subset of A, because every element of C is also an element of A. Thus, $C \subseteq A$ is true, and as a result, $C \nsubseteq A$ is false.

(d) U contains the element 13, but A does not. Therefore, $U = A$ is false.

✔ **Now Try Exercises 53, 55, 63, and 65.**

Operations on Sets Given a set A and a universal set U, the set of all elements of U that do ***not*** belong to set A is the **complement** of set A. For example, if set A is the set of all students in a class 30 years old or older, and set U is the set of all students in the class, then the complement of A would be the set of all students in the class younger than age 30.



A'

Figure 9

The complement of set A is written A' (read **"*A*-*prime*"**). The Venn diagram in **Figure 9** shows a set A. Its complement, A', is in color. Using set-builder notation, the complement of set A is described as follows.

$$A' = \{x \mid x \in U,\quad x \notin A\}$$

EXAMPLE 4 Finding Complements of Sets

Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5, 7\}$, and $B = \{3, 4, 6\}$. Find each set.

(a) A' **(b)** B' **(c)** $\varnothing'$ **(d)** U'

SOLUTION

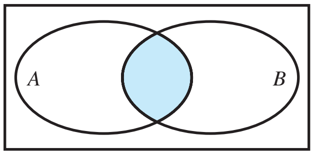
(a) Set A' contains the elements of U that are not in A. Thus, $A' = \{2, 4, 6\}$.

(b) $B' = \{1, 2, 5, 7\}$ **(c)** $\varnothing' = U$ **(d)** $U' = \varnothing$

✔ **Now Try Exercise 89.**

Given two sets A and B, the set of all elements belonging to both set A ***and*** set B is the **intersection** of the two sets, written $A \cap B$. For example, if $A = \{1, 2, 4, 5, 7\}$ and $B = \{2, 4, 5, 7, 9, 11\}$, then we have the following.

$$A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 4, 5, 7, 9, 11\} = \{2, 4, 5, 7\}$$



$A \cap B$

Figure 10

The Venn diagram in **Figure 10** shows two sets A and B. Their intersection, $A \cap B$, is in color. Using set-builder notation, the intersection of sets A and B is described as follows.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Two sets that have no elements in common are **disjoint sets.** If A and B are any two disjoint sets, then $A \cap B = \varnothing$. For example, there are no elements common to both $\{50, 51, 54\}$ and $\{52, 53, 55, 56\}$, so these two sets are disjoint.

$$\{50, 51, 54\} \cap \{52, 53, 55, 56\} = \varnothing$$

EXAMPLE 5 Finding Intersections of Two Sets

Find each of the following. Identify any disjoint sets.

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\}$

(b) $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\}$ **(c)** $\{1, 3, 5\} \cap \{2, 4, 6\}$

SOLUTION

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\} = \{15, 25\}$

The elements 15 and 25 are the only ones belonging to both sets.

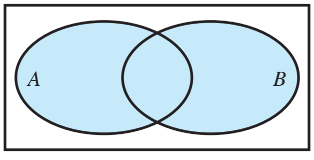
(b) $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\} = \{2, 3, 4\}$

(c) $\{1, 3, 5\} \cap \{2, 4, 6\} = \varnothing$ Disjoint sets

✔ **Now Try Exercises 69, 75, and 85.**

The set of all elements belonging to set A ***or*** to set B (or to both) is the **union** of the two sets, written $A \cup B$. For example, if $A = \{1, 3, 5\}$ and $B = \{3, 5, 7, 9\}$, then we have the following.

$$A \cup B = \{1, 3, 5\} \cup \{3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}$$



$A \cup B$

Figure 11

The Venn diagram in **Figure 11** shows two sets A and B. Their union, $A \cup B$, is in color. Using set-builder notation, the union of sets A and B is described as follows.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in \text{both } A \text{ and } B\}$$

EXAMPLE 6 **Finding Unions of Two Sets**

Find each of the following.

(a) $\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\}$ **(b)** $\{1, 3, 5, 7\} \cup \{2, 4, 6\}$

(c) $\{1, 3, 5, 7, \ldots\} \cup \{2, 4, 6, \ldots\}$

SOLUTION

(a) Begin by listing the elements of the first set, $\{1, 2, 5, 9, 14\}$. Then include any elements from the second set that are not already listed.

$$\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\} = \{1, 2, 3, 4, 5, 8, 9, 14\}$$

(b) $\{1, 3, 5, 7\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 5, 6, 7\}$

(c) $\{1, 3, 5, 7, \ldots\} \cup \{2, 4, 6, \ldots\} = N$ Natural numbers

✔ **Now Try Exercises 71 and 83.**

The **set operations** are summarized below.

Set Operations

Let A and B define sets, with universal set U.

The **complement** of set A is the set A' of all elements in the universal set that do *not* belong to set A.

$$A' = \{x \mid x \in U, \quad x \notin A\}$$

The **intersection** of sets A and B, written $A \cap B$, is made up of all the elements belonging to both set A *and* set B.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

The **union** of sets A and B, written $A \cup B$, is made up of all the elements belonging to set A *or* to set B (or to both).

$$A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in \text{both } A \text{ and } B\}$$

Sets of Numbers and the Number Line As mentioned previously, the set of **natural numbers** is written in set notation as follows.

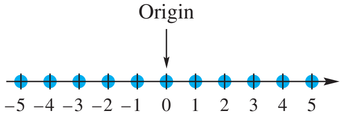
$$\{1, 2, 3, 4, \ldots\} \quad \text{Natural numbers}$$

Including 0 with the set of natural numbers gives the set of **whole numbers.**

$$\{0, 1, 2, 3, 4, \ldots\} \quad \text{Whole numbers}$$

Including the negatives of the natural numbers with the set of whole numbers gives the set of **integers.**

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \quad \text{Integers}$$



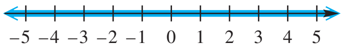
Graph of the Set of Integers

Figure 12

Integers can be graphed on a **number line.** See **Figure 12.** Every number corresponds to one and only one point on the number line, and each point corresponds to one and only one number. The number associated with a given point is the **coordinate** of the point. This correspondence forms a **coordinate system.**

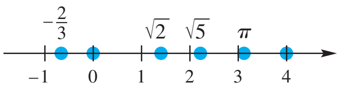
The result of dividing two integers (with a nonzero divisor) is a *rational number*, or *fraction.* A **rational number** is an element of the set defined as follows.

$$\left\{\frac{p}{q} \,\middle|\, p \text{ and } q \text{ are integers and } q \neq 0\right\} \quad \text{Rational numbers}$$



Graph of the Set of Real Numbers

Figure 13



Graph of the Set

$\left\{-\frac{2}{3}, 0, \sqrt{2}, \sqrt{5}, \pi, 4\right\}$

$\sqrt{2}$, $\sqrt{5}$, and π are irrational. Because $\sqrt{2}$ is approximately equal to 1.41, it is located between 1 and 2, slightly closer to 1.

Figure 14

The set of rational numbers includes the natural numbers, the whole numbers, and the integers. For example, the integer -3 is a rational number because it can be written as $\frac{-3}{1}$. Numbers that can be written as repeating or terminating decimals are also rational numbers. For example, $0.\overline{6} = 0.66666\ldots$ represents a rational number that can be expressed as the fraction $\frac{2}{3}$.

The set of all numbers that correspond to points on a number line is the **real numbers,** shown in **Figure 13.** Real numbers can be represented by decimals. Because every fraction has a decimal form—for example, $\frac{1}{4} = 0.25$—real numbers include rational numbers.

Some real numbers cannot be represented by quotients of integers. These numbers are **irrational numbers.** The set of irrational numbers includes $\sqrt{2}$ and $\sqrt{5}$. Another irrational number is π, which is *approximately* equal to 3.14159. Some rational and irrational numbers are graphed in **Figure 14.**

The sets of numbers discussed so far are summarized as follows.

Sets of Numbers

Set	Description
Natural numbers	$\{1, 2, 3, 4, \ldots\}$
Whole numbers	$\{0, 1, 2, 3, 4, \ldots\}$
Integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational numbers	$\left\{\frac{p}{q} \,\middle|\, p \text{ and } q \text{ are integers and } q \neq 0\right\}$
Irrational numbers	$\{x \mid x \text{ is real but not rational}\}$
Real numbers	$\{x \mid x \text{ corresponds to a point on a number line}\}$

EXAMPLE 7 Identifying Sets of Numbers

Let $A = \left\{-8, -6, -\frac{12}{4}, -\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}, 1, \sqrt{2}, \sqrt{5}, 6\right\}$. List all the elements of A that belong to each set.

(a) Natural numbers **(b)** Whole numbers **(c)** Integers

(d) Rational numbers **(e)** Irrational numbers **(f)** Real numbers

SOLUTION

(a) Natural numbers: 1 and 6

(b) Whole numbers: 0, 1, and 6

(c) Integers: $-8, -6, -\frac{12}{4}$ (or -3), 0, 1, and 6

(d) Rational numbers: $-8, -6, -\frac{12}{4}$ (or -3), $-\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}$, 1, and 6

(e) Irrational numbers: $\sqrt{2}$ and $\sqrt{5}$

(f) All elements of A are real numbers.

✔ **Now Try Exercises 111, 113, and 115.**

Figure 15 shows the relationships among the subsets of the real numbers. As shown, the natural numbers are a subset of the whole numbers, which are a subset of the integers, which are a subset of the rational numbers. The union of the rational numbers and irrational numbers is the set of real numbers.

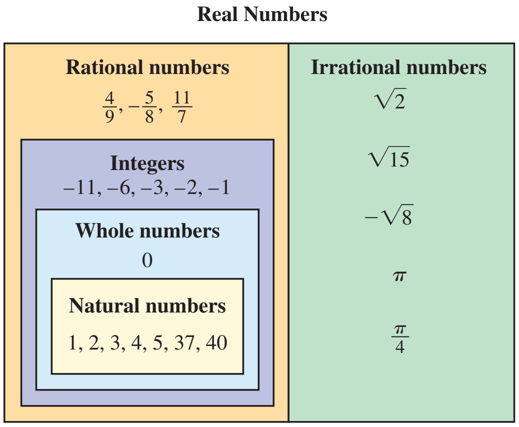


Figure 15

R.2 Exercises

CONCEPT PREVIEW *Fill in the blank to correctly complete each sentence.*

1. Set A is a(n) ________ of set B if every element of set A is also an element of set B.
2. The set of all elements of the universal set U that do not belong to set A is the ________ of set A.
3. The ________ of sets A and B is made up of all the elements belonging to both set A *and* set B.
4. The ________ of sets A and B is made up of all the elements belonging to set A *or* to set B (or to both).

CONCEPT PREVIEW *Work each problem.*

5. Identify the set $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\right\}$ as *finite* or *infinite*.
6. Use set notation and write the elements belonging to the set

$$\{x \mid x \text{ is a natural number less than } 6\}.$$

7. Let $U = \{1, 2, 3, 4, 5\}$ and $A = \{1, 2, 3\}$. Find A'.
8. Find $\{16, 18, 21, 50\} \cap \{15, 16, 17, 18\}$.
9. Find $\{16, 18, 21, 50\} \cup \{15, 16, 17, 18\}$.
10. **CONCEPT PREVIEW** Match each number from Column I with the letter or letters of the sets of numbers from Column II to which the number belongs. There may be more than one choice, so give all choices.

I		II	
(a) 0	**(b)** 34	**A.** Natural numbers	**B.** Whole numbers
(c) $-\frac{9}{4}$	**(d)** $\sqrt{36}$	**C.** Integers	**D.** Rational numbers
(e) $\sqrt{13}$	**(f)** 2.16	**E.** Irrational numbers	**F.** Real numbers

Identify each set as finite *or* infinite. *Then determine whether* 10 *is an element of the set.* ***See Example 1.***

11. $\{4, 5, 6, \ldots, 15\}$

12. $\{1, 2, 3, 4, 5, \ldots, 75\}$

13. $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\right\}$

14. $\{4, 5, 6, \ldots\}$

15. $\{x \mid x$ is a natural number greater than $11\}$

16. $\{x \mid x$ is a natural number greater than or equal to $10\}$

17. $\{x \mid x$ is a fraction between 8 and $9\}$

18. $\{x \mid x$ is an even natural number$\}$

Use set notation, and list all the elements of each set. ***See Example 2.***

19. $\{12, 13, 14, \ldots, 20\}$

20. $\{8, 9, 10, \ldots, 17\}$

21. $\left\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{32}\right\}$

22. $\{3, 9, 27, \ldots, 729\}$

23. $\{17, 22, 27, \ldots, 47\}$

24. $\{74, 68, 62, \ldots, 38\}$

25. $\{x \mid x$ is a natural number greater than 8 and less than $15\}$

26. $\{x \mid x$ is a natural number not greater than $4\}$

Insert $\in$ *or* $\notin$ *in each blank to make the resulting statement true.* ***See Examples 1 and 2.***

27. 6 _____ $\{3, 4, 5, 6\}$

28. 9 _____ $\{2, 3, 5, 9, 8\}$

29. 5 _____ $\{4, 6, 8, 10\}$

30. 13 _____ $\{3, 5, 12, 14\}$

31. 0 _____ $\{0, 2, 3, 4\}$

32. 0 _____ $\{0, 5, 6, 7, 8, 10\}$

33. $\{3\}$ _____ $\{2, 3, 4, 5\}$

34. $\{5\}$ _____ $\{3, 4, 5, 6, 7\}$

35. $\{0\}$ _____ $\{0, 1, 2, 5\}$

36. $\{2\}$ _____ $\{2, 4, 6, 8\}$

37. 0 _____ $\varnothing$

38. $\varnothing$ _____ $\varnothing$

Determine whether each statement is true *or* false. ***See Examples 1–3.***

39. $3 \in \{2, 5, 6, 8\}$

40. $6 \in \{2, 5, 8, 9\}$

41. $1 \in \{11, 5, 4, 3, 1\}$

42. $12 \in \{18, 17, 15, 13, 12\}$

43. $9 \notin \{8, 5, 2, 1\}$

44. $3 \notin \{7, 6, 5, 4\}$

45. $\{2, 5, 8, 9\} = \{2, 5, 9, 8\}$

46. $\{3, 0, 9, 6, 2\} = \{2, 9, 0, 3, 6\}$

47. $\{5, 8, 9\} = \{5, 8, 9, 0\}$

48. $\{3, 7, 12, 14\} = \{3, 7, 12, 14, 0\}$

49. $\{x \mid x$ is a natural number less than $3\} = \{1, 2\}$

50. $\{x \mid x$ is a natural number greater than $10\} = \{11, 12, 13, \ldots\}$

Let $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{2, 4, 8, 10\}$, $C = \{4, 10, 12\}$, $D = \{2, 10\}$, *and* $U = \{2, 4, 6, 8, 10, 12, 14\}$.

Determine whether each statement is true *or* false. ***See Example 3.***

51. $A \subseteq U$

52. $C \subseteq U$

53. $D \subseteq B$

54. $D \subseteq A$

55. $A \subseteq B$

56. $B \subseteq C$

57. $\varnothing \subseteq A$

58. $\varnothing \subseteq \varnothing$

59. $\{4, 8, 10\} \subseteq B$

60. $\{0, 2\} \subseteq D$

61. $B \subseteq D$

62. $A \subseteq C$

Insert $\subseteq$ *or* $\not\subseteq$ *in each blank to make the resulting statement true.* ***See Example 3.***

63. $\{2, 4, 6\}$ ______ $\{2, 3, 4, 5, 6\}$

64. $\{1, 5\}$ ______ $\{0, 1, 2, 3, 5\}$

65. $\{0, 1, 2\}$ ______ $\{1, 2, 3, 4, 5\}$

66. $\{5, 6, 7, 8\}$ ______ $\{1, 2, 3, 4, 5, 6, 7\}$

67. $\varnothing$ ______ $\{1, 4, 6, 8\}$

68. $\varnothing$ ______ $\varnothing$

Determine whether each statement is true *or* false. ***See Examples 4–6.***

69. $\{5, 7, 9, 19\} \cap \{7, 9, 11, 15\} = \{7, 9\}$

70. $\{8, 11, 15\} \cap \{8, 11, 19, 20\} = \{8, 11\}$

71. $\{1, 2, 7\} \cup \{1, 5, 9\} = \{1\}$

72. $\{6, 12, 14, 16\} \cup \{6, 14, 19\} = \{6, 14\}$

73. $\{2, 3, 5, 9\} \cap \{2, 7, 8, 10\} = \{2\}$

74. $\{6, 8, 9\} \cup \{9, 8, 6\} = \{8, 9\}$

75. $\{3, 5, 9, 10\} \cap \varnothing = \{3, 5, 9, 10\}$

76. $\{3, 5, 9, 10\} \cup \varnothing = \{3, 5, 9, 10\}$

77. $\{1, 2, 4\} \cup \{1, 2, 4\} = \{1, 2, 4\}$

78. $\{1, 2, 4\} \cap \{1, 2, 4\} = \varnothing$

79. $\varnothing \cup \varnothing = \varnothing$

80. $\varnothing \cap \varnothing = \varnothing$

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$, $M = \{0, 2, 4, 6, 8\}$, $N = \{1, 3, 5, 7, 9, 11, 13\}$, $Q = \{0, 2, 4, 6, 8, 10, 12\}$, *and* $R = \{0, 1, 2, 3, 4\}$.

Use these sets to find each of the following. Identify any disjoint sets. ***See Examples 4–6.***

81. $M \cap R$

82. $M \cap U$

83. $M \cup N$

84. $M \cup R$

85. $M \cap N$

86. $U \cap N$

87. $N \cup R$

88. $M \cup Q$

89. N'

90. Q'

91. $M' \cap Q$

92. $Q \cap R'$

93. $\varnothing \cap R$

94. $\varnothing \cap Q$

95. $N \cup \varnothing$

96. $R \cup \varnothing$

97. $(M \cap N) \cup R$

98. $(N \cup R) \cap M$

99. $(Q \cap M) \cup R$

100. $(R \cup N) \cap M'$

101. $(M' \cup Q) \cap R$

102. $Q \cap (M \cup N)$

103. $Q' \cap (N' \cap U)$

104. $(U \cap \varnothing') \cup R$

105. $\{x \mid x \in U, \quad x \notin M\}$

106. $\{x \mid x \in U, \quad x \notin R\}$

107. $\{x \mid x \in M \text{ and } x \in Q\}$

108. $\{x \mid x \in Q \text{ and } x \in R\}$

109. $\{x \mid x \in M \text{ or } x \in Q\}$

110. $\{x \mid x \in Q \text{ or } x \in R\}$

Let $A = \left\{-6, -\frac{12}{4}, -\frac{5}{8}, -\sqrt{3}, 0, \frac{1}{4}, 1, 2\pi, 3, \sqrt{12}\right\}$. *List all the elements of A that belong to each set.* ***See Example 7.***

111. Natural numbers

112. Whole numbers

113. Integers

114. Rational numbers

115. Irrational numbers

116. Real numbers

R.3 Real Number Operations and Properties

- Order on the Number Line
- Absolute Value
- Operations on Real Numbers
- Exponents
- Order of Operations
- Properties of Real Numbers

Order on the Number Line

If the real number a is to the left of the real number b on a number line, then

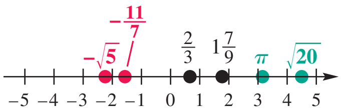
$$\textbf{\textit{a is less than b,}} \quad \text{written} \quad a < b.$$

If a is to the right of b, then

$$\textbf{\textit{a is greater than b,}} \quad \text{written} \quad a > b.$$

The inequality symbol must point toward the lesser number.

See **Figure 16.** Statements involving these symbols, as well as the symbols less than or equal to, $\leq$, and greater than or equal to, $\geq$, are **inequalities.** The inequality $a < b < c$ says that b is ***between*** a and c because $a < b$ and $b < c$.



$-\sqrt{5}$ is to the left of $-\frac{11}{7}$ on the number line, so $-\sqrt{5} < -\frac{11}{7}$, and $\sqrt{20}$ is to the right of π, indicating that $\sqrt{20} > \pi$.

Figure 16

Inequality Symbols

Symbol	Meaning	Example
$\neq$	is not equal to	$3 \neq 7$
$<$	is less than	$-4 < -1$
$>$	is greater than	$3 > -2$
$\leq$	is less than or equal to	$6 \leq 6$
$\geq$	is greater than or equal to	$-8 \geq -10$

Absolute Value

The undirected distance on a number line from a number to 0 is the **absolute value** of that number. The absolute value of the number a is written $|a|$. For example, the distance on a number line from 5 to 0 is 5, as is the distance from -5 to 0. See **Figure 17.** Therefore, $|5| = 5$ and $|-5| = 5$.

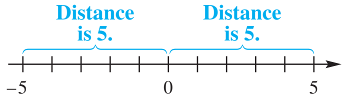


Figure 17

NOTE ***Because distance cannot be negative, the absolute value of a number is always positive or* 0.**

The algebraic definition of absolute value follows.

Absolute Value

Let a represent a real number.

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

That is, the absolute value of a positive number or 0 equals that number, while the absolute value of a negative number equals its negative (or opposite).

EXAMPLE 1 Evaluating Absolute Values

Evaluate each expression.

(a) $\left|-\frac{5}{8}\right|$ **(b)** $-|8|$ **(c)** $-|-8|$ **(d)** $|2x|$, for $x = \pi$

SOLUTION

(a) $\left|-\frac{5}{8}\right| = \frac{5}{8}$ **(b)** $-|8| = -(8) = -8$

(c) $-|-8| = -(8) = -8$ **(d)** $|2\pi| = 2\pi$

✔ **Now Try Exercises 11 and 15.**

Consider the following properties of absolute value.

Properties of Absolute Value

Let a and b represent real numbers.

Property	Description
1. $\lvert a\rvert \geq 0$	The absolute value of a real number is positive or 0.
2. $\lvert -a\rvert = \lvert a\rvert$	The absolute values of a real number and its opposite are equal.
3. $\lvert a\rvert \cdot \lvert b\rvert = \lvert ab\rvert$	The product of the absolute values of two real numbers equals the absolute value of their product.
4. $\frac{\lvert a\rvert}{\lvert b\rvert} = \left\lvert \frac{a}{b}\right\rvert \; (b \neq 0)$	The quotient of the absolute values of two real numbers equals the absolute value of their quotient.
5. $\lvert a + b\rvert \leq \lvert a\rvert + \lvert b\rvert$ (triangle inequality)	The absolute value of the sum of two real numbers is less than or equal to the sum of their absolute values.

LOOKING AHEAD TO CALCULUS

One of the most important definitions in calculus, that of the **limit,** uses absolute value. The symbols ϵ (epsilon) and δ (delta) are often used to represent small quantities in mathematics.

Suppose that a function f is defined at every number in an open interval I containing a, except perhaps at a itself. Then the limit of $f(x)$ as x approaches a is L, written

$$\lim_{x \to a} f(x) = L,$$

if for every $\epsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$.

Examples of Properties 1–4:

$|-15| = 15$ and $15 \geq 0$. Property 1

$|-10| = 10$ and $|10| = 10$, so $|-10| = |10|$. Property 2

$|5| \cdot |-4| = 5 \cdot 4 = 20$ and $|5(-4)| = |-20| = 20$,

so $|5| \cdot |-4| = |5(-4)|$. Property 3

$\frac{|2|}{|3|} = \frac{2}{3}$ and $\left|\frac{2}{3}\right| = \frac{2}{3}$, so $\frac{|2|}{|3|} = \left|\frac{2}{3}\right|$. Property 4

Example of the triangle inequality:

$|a + b| = |3 + (-7)| = |-4| = 4$
Let $a = 3$ and $b = -7$.
$|a| + |b| = |3| + |-7| = 3 + 7 = 10$

Thus, $|a + b| \leq |a| + |b|$. Property 5

Operations on Real Numbers Recall that the answer to an addition problem is a **sum.** The answer to a subtraction problem is a **difference.** The answer to a multiplication problem is a **product,** and the answer to a division problem is a **quotient.** Other phrases that indicate these operations are given in the table.

Words or Phrases That Indicate Operations on Real Numbers

Operation	Word or Phrase
Addition	Sum, added to, more than, increased by, plus
Subtraction	Difference of, subtracted from, less than, decreased by, minus
Multiplication	Product of, times, twice, triple, of, percent of
Division	Quotient of, divided by, ratio of

To perform operations on real numbers, we use the following sign rules.

Sign Rules for Operations on Real Numbers

Operation	Examples
To **add** two numbers with the *same* sign, add their absolute values. The sum has the same sign as the given numbers.	$2 + 7 = 9$ $-2 + (-7) = -9$
To **add** two numbers with *different* signs, find the absolute values of the numbers, and subtract the lesser absolute value from the greater. The sum has the same sign as the number with the greater absolute value.	$-12 + 5 = -7$ $4 + (-10) = -6$ $-3 + 8 = 5$
To **subtract** a number b from another number a, change to addition and replace b by its **additive inverse (opposite),** $-b$. $\boldsymbol{a - b = a + (-b)}$ Then use the sign rules for addition.	$5 - 7 = 5 + (-7) = -2$ $-8 - 4 = -8 + (-4) = -12$ $-17 - (-4) = -17 + 4 = -13$
To **multiply** or **divide** two numbers with the *same* sign, multiply or divide their absolute values. The answer will be positive.	$5(7) = 35, \quad -5(-7) = 35$ $\frac{35}{5} = 7, \quad \frac{-35}{-5} = 7$
To **multiply** or **divide** two numbers with *different* signs, multiply or divide their absolute values. The answer will be negative.	$-8(9) = -72, \quad 6(-7) = -42$ $\frac{-45}{9} = -5, \quad \frac{36}{-9} = -4$

Consider the quotient $15 \div 3$ and the product $15 \cdot \frac{1}{3}$.

$$15 \div 3 = \frac{15}{3} = 5 \quad \text{and} \quad 15 \cdot \frac{1}{3} = \frac{15}{3} = 5$$

Thus division can be written in terms of multiplication using the **multiplicative inverse (reciprocal).** For all real numbers a and b $(b \neq 0)$,

$$\boldsymbol{a \div b = \frac{a}{b} = a \cdot \frac{1}{b}}.$$

To divide a by b, multiply a (the **dividend**) by $\frac{1}{b}$, the reciprocal of b (the **divisor**).

For this reason, the sign rules for division are the same as those for multiplication.

Consider the quotient $\frac{a}{b}$. To find $\frac{a}{b}$, we look for a number n that when multiplied by b will give a. For example, $\frac{10}{5} = 2$ because $2 \cdot 5 = 10$.

What happens if we try to divide a nonzero number by 0? For example, to find $\frac{4}{0}$, we look for some number n that when multiplied by 0 will give 4. But $n \cdot 0$ will always give a product of 0, never 4. Therefore, division by 0 is *undefined.* On the other hand, $\frac{0}{4} = 0$ because $0 \cdot 4 = 0$. Thus, dividing 0 by any nonzero number will always yield 0.

Trying to divide 0 by 0 is problematic for another reason. If $\frac{0}{0} = n$, then $n \cdot 0 = 0$. This statement is true for *all* numbers n, which means that $\frac{0}{0}$ is not unique. We say that $\frac{0}{0}$ is *indeterminate*—that is, there is no unique answer.

This discussion can be summarized as follows.

Division Involving 0

For any nonzero real number a,

$$\frac{a}{0} \textbf{ is undefined,} \quad \frac{0}{a} = 0, \quad \text{and} \quad \frac{0}{0} \textbf{ is indeterminate.}$$

EXAMPLE 2 Adding and Subtracting Real Numbers

Find each sum or difference.

(a) $-6 + (-3)$ **(b)** $-2.3 + 5.6$ **(c)** $-12 - 4$ **(d)** $\frac{5}{6} - \left(-\frac{3}{8}\right)$

SOLUTION

(a) $-6 + (-3)$

$= -(|-6| + |-3|)$ Add the absolute values. The numbers have the *same* sign, both negative, so the sum will be negative.

$= -(6 + 3)$

$= -9$

(b) $-2.3 + 5.6$

$= |5.6| - |2.3|$ The numbers have *different* signs. Subtract the lesser absolute value from the greater.

$= 5.6 - 2.3$

$= 3.3$

Change to addition.

The opposite of 4 is -4.

(c) $-12 - 4 = -12 + (-4) = -16$

(d) $\frac{5}{6} - \left(-\frac{3}{8}\right)$

$= \frac{5}{6} + \frac{3}{8}$ To subtract $a - b$, add the opposite of b to a.

$= \frac{20}{24} + \frac{9}{24}$ The least common denominator is 24. $\frac{5}{6} \cdot \frac{4}{4} = \frac{20}{24}$; $\frac{3}{8} \cdot \frac{3}{3} = \frac{9}{24}$

$= \frac{29}{24}$ Add numerators. Keep the same denominator.

✔ **Now Try Exercises 27, 39, and 45.**

EXAMPLE 3 Multiplying and Dividing Real Numbers

Find each product or quotient where possible.

(a) $-3(-9)$ **(b)** $6(-9)$ **(c)** $-0.05(0.3)$ **(d)** $-\frac{3}{4}\left(\frac{2}{9}\right)$

(e) $\frac{-9}{0}$ **(f)** $\frac{0}{-12}$ **(g)** $\frac{-12}{4}$ **(h)** $\frac{-\frac{2}{3}}{-\frac{5}{9}}$

SOLUTION

(a) $-3(-9) = 27$ **(b)** $6(-9) = -54$ **(c)** $-0.05(0.3) = -0.015$

The numbers have the *same* sign, so the product is positive.

The numbers have *different* signs, so the product is negative.

(d) $-\frac{3}{4}\left(\frac{2}{9}\right)$

$= -\frac{6}{36}$ Multiply numerators. Multiply denominators.

$= -\frac{1}{6}$ Write in lowest terms; $\frac{6}{36} = \frac{1 \cdot 6}{6 \cdot 6} = \frac{1}{6}$

(e) $\frac{-9}{0}$ is undefined. **(f)** $\frac{0}{-12} = 0$ **(g)** $\frac{-12}{4} = -3$

This is true because $0(-12) = 0$.

The numbers have *different* signs, so the quotient is negative.

(h) $\frac{-\frac{2}{3}}{-\frac{5}{9}}$ This is a *complex fraction*. A complex fraction has a fraction in the numerator, the denominator, or both.

$= -\frac{2}{3}\left(-\frac{9}{5}\right)$ $\frac{a}{b} = a \cdot \frac{1}{b}$; Multiply by $-\frac{9}{5}$, the reciprocal of the divisor $-\frac{5}{9}$.

$= \frac{18}{15}$ Multiply numerators. Multiply denominators.

$= \frac{6}{5}$ Write in lowest terms; $\frac{18}{15} = \frac{6 \cdot 3}{5 \cdot 3} = \frac{6}{5}$

✔ **Now Try Exercises 51, 57, 73, and 77.**

Exponents Any collection of numbers or variables joined by the basic operations of addition, subtraction, multiplication, or division (except by 0), or the operations of raising to powers or taking roots, formed according to the rules of algebra, is an **algebraic expression.**

$$-2x^2 + 3x, \quad \frac{15y}{2y - 3}, \quad \sqrt{m^3 - 64}, \quad (3a + b)^4 \quad \text{Algebraic expressions}$$

Exponent: 3

$2^3 = \underbrace{2 \cdot 2 \cdot 2}_{} = 8$

Base: 2 Three factors of 2

The expression 2^3 is an **exponential expression,** or **exponential,** where the 3 indicates that three factors of 2 appear in the corresponding product. The number 2 is the **base,** and the number 3 is the **exponent.**

Exponential Notation

If n is any positive integer and a is any real number, then the nth power of a is written using exponential notation as follows.

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a}$$

Read a^n as **"*a to the* n*th power*"** or simply **"*a to the* n*th.*"**

EXAMPLE 4 Evaluating Exponential Expressions

Evaluate each exponential expression, and identify the base and the exponent.

(a) 4^3 **(b)** $(-6)^2$ **(c)** -6^2 **(d)** $4 \cdot 3^2$ **(e)** $(4 \cdot 3)^2$

SOLUTION

(a) $4^3 = \underbrace{4 \cdot 4 \cdot 4}_{3 \text{ factors of } 4} = 64$ The base is 4 and the exponent is 3.

(b) $(-6)^2 = (-6)(-6) = 36$ The base is -6 and the exponent is 2.

(c) $-6^2 = -(6 \cdot 6) = -36$ — Notice that parts (b) and (c) are different.

The base is 6 and the exponent is 2.

(d) $4 \cdot 3^2 = 4 \cdot 3 \cdot 3 = 36$ The base is 3 and the exponent is 2.

$3^2 = 3 \cdot 3$, NOT $3 \cdot 2$

(e) $(4 \cdot 3)^2 = 12^2 = 144$ — $(4 \cdot 3)^2 \neq 4 \cdot 3^2$

The base is $4 \cdot 3$, or 12, and the exponent is 2.

✓ **Now Try Exercises 81, 83, and 87.**

Order of Operations When an expression involves more than one operation symbol, such as $5 \cdot 2 + 3$, we use the following order of operations.

Order of Operations

If grouping symbols such as parentheses, square brackets, absolute value bars, or fraction bars are present, begin as follows.

Step 1 Work separately above and below each **fraction bar.**

Step 2 Use the rules below within each set of **parentheses** or **square brackets.** Start with the innermost set and work outward.

If no grouping symbols are present, follow these steps.

Step 1 Simplify all **powers** and **roots.** ***Work from left to right.***

Step 2 Do any **multiplications** or **divisions** in order. ***Work from left to right.***

Step 3 Do any **negations, additions,** or **subtractions** in order. ***Work from left to right.***