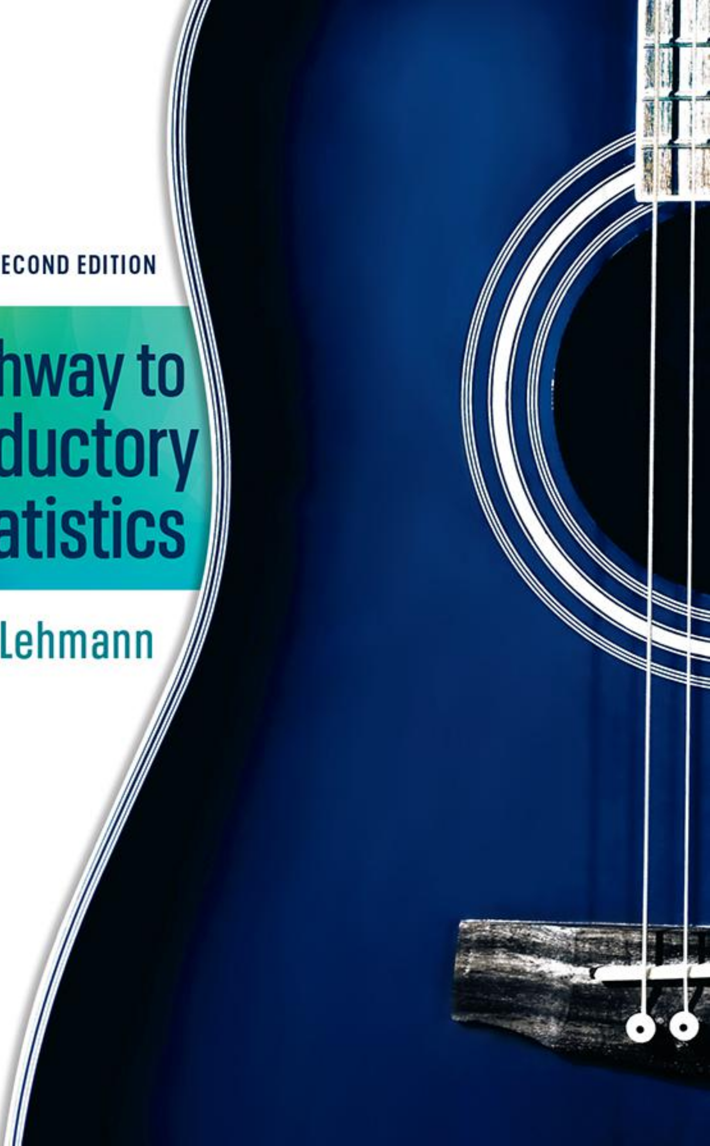
SECOND EDITION

A Pathway to Introductory Statistics

Jay Lehmann



Second Edition

A Pathway to Introductory Statistics

Jay Lehmann
College of San Mateo

Pearson

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To Keri,
For carrying my aspirations
whenever I falter.

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Preface

For a very long time, algebra has been viewed as an essential ingredient in a person's education. And for certain community college students, such as STEM majors, this is definitely true. But recently, some instructors have begun to question whether the traditional algebra sequence best serves all students. Is it the ideal preparation for a career in political science? How about psychology? Social science? Probably not.

In addition to evaluating the long-range benefits of algebra, we should also assess the short-range ones. For some non-STEM majors, the only transferable math course they need to take is statistics. But is the traditional algebra sequence the best preparation for statistics? Without question, statistics students need to have a solid understanding of certain algebra concepts. But one would be hard-pressed to argue that factoring polynomials, completing the square, and solving complicated rational equations are the most important concepts to learn before embarking on a statistics course sequence.

It is not only the *content* of the traditional algebra sequence that is misaligned with statistics. It is also the *nature* of the activities. In algebra, much attention is devoted to manipulating symbols. Statistics focuses on analyzing situations, comparing measurements, and interpreting the meaning of concepts and results. Because the nature of the activities is so different, it is not surprising that many students enter introductory statistics unprepared.

A Pathway to Introductory Statistics is meant to serve non-STEM community college students better than a traditional algebra sequence. In particular, its main goals are to

- Enhance students' ability to think statistically: analyze, compare, and interpret.
- Address descriptive statistics, including the normal distribution and regression.
- Empower students to discern good and bad practices of statistics.
- Equip students with the algebra essential for success in introductory statistics.
- Inspire students with exciting situations that are relevant to their careers.
- Foster the use of technology to enhance, rather than replace, critical thinking.
- Provide collaborative explorations in which students experience the joy of discovery.

NEW TO THE SECOND EDITION

Students will benefit from the following changes to the second edition of *A Pathway to Introductory Statistics*:

- **MyLab Math Exercises:** The number of exercises in MyLab Math has been increased. This was the number one request made by reviewers.
- **Logarithms and Systems of Linear Equations:** To support departments that want to prepare students for liberal arts mathematics courses, as well as statistics, chapters (11 and 12) on logarithms and systems of linear equations are now available online in MyLab Math or at pearsonhighered.com/mathstatsresources.
- **Workbook:** The author has written a new workbook that contains hundreds of affective domain and prestatistics activities. The workbook provides great support for collaborative learning, which research has shown is vital to students' conceptual and problem-solving development. It will be especially helpful for teaching corequisite courses.
- **Group Explorations:** Some explorations have been revised so that they are more open-ended, allowing for greater productive struggle and creative problem solving. Central themes such as center and variation now have greater emphasis.
- **Non-Time-Series Data Sets:** There is tension between providing time-series data and non-time-series data for regression. Time-series data tend to be more lively and easier to input into technology because they often consist of fewer values.

Non-time-series data sets tend to challenge students more (in a good way), provide greater complexity due to possibly having multiple data pairs sharing the same value of the explanatory variable, and better prepare students for statistics because statistics courses tend to include more non-time-series data. The percentage of non-time-series data has been increased for these latter reasons. The author has performed quite a bit of research to find interesting non-time-series data.

- **Data Sets with Multiple Columns:** To better approximate realistic data sets, the percentage of data sets with multiple columns has been increased in Chapters 6–10. This also challenges students to determine which columns they should work with.
- **Augmented Data Sets:** To make the data sets as current and relevant as possible, hundreds of data sets in examples and exercises have been augmented to include observations for recent years.
- **New Data Sets:** Hundreds of data sets in examples and exercises have been replaced with more compelling and contemporary topics such as immigration, trust in the mass media, and health care plans that cover transgender-related services.
- **Statistics versus Parameters:** Statistics students struggle to keep straight symbols and concepts for statistics and parameters. As each measure is introduced, the symbols and concepts for it are compared with previously addressed measures.
- **Statistical Emphasis:** In Chapters 1, 7, and 8, arithmetic and algebra concepts are introduced and developed with a greater statistical emphasis.
- **Complex Fractions:** Many statistics formulas involve complex fractions, so the skills of simplifying complex fractions and evaluating expressions with complex fractions have been added to Sections 1.3 and 1.7, respectively.
- **Percent Change:** Statistics requires facility in comparing values. One of those skills involves computing percent change, which has been added to Section 1.6.
- **Two Samples:** Exercises that involve two samples (or populations) have been added to Chapter 4 so that students can have more practice comparing a measure for two groups of data.
- **Empirical Rule:** To better prepare students for computing normal-curve probabilities in Section 5.5, exercises that require more intricate use of the Empirical Rule have been added to Section 5.1.
- **Discrete Random Variables:** To better prepare students for the concept binomial distribution in a statistics course, a section (5.4) on discrete random variables, including the compelling and fun concept of expected value, has been added. Of all possible concepts that could have been added, this was the top request by reviewers.
- **Residuals:** In Chapter 6 of the first edition, error was defined to be the predicted value minus the observed value, but in Chapter 9, residual was defined to be the observed value minus the predicted value. To provide consistency, error calculations have been replaced with residual calculations in Chapters 6–8. This also allows students to acclimate to residuals for several chapters before having to grapple with computing and interpreting sums of squared residuals in Chapters 9 and 10.
- **$y = a + bx$ Form:** In Chapters 7–10, the form $y = mx + b$ has been replaced with the form $y = a + bx$, which is the form typically used in statistics courses.
- **Summation Notation:** More varied forms of summation notation have been added to Section 8.4.

CONTINUED FROM THE FIRST EDITION

A Meaningful, Alternative Path This text contains the key concepts of descriptive statistics: experimental design, statistical diagrams, measures of center and spread, probability, the normal distribution, and regression. Teaching these topics along with the necessary algebra would certainly prepare students for an introductory statistics course better than the traditional algebra sequence. But to present the statistics concepts twice in the same manner—first in a *Pathway* course and again in an introductory statistics course—falls short of the sequence's highest potential. Teaching a concept from two perspectives rather than one provides students with a richer and broader learning experience.

But how can statistics be presented in a meaningful way that is different from its presentation in traditional statistics courses? There are actually many paths, but to discover the trailheads we must determine the foundational concepts with which statistics students struggle. Certainly one such concept is the normal curve, which lays the foundation for inferential statistics. How many introductory statistics students understand why the area of a region under the normal curve is equal to a probability? How many introductory statistics students understand how probability rules connect with finding such an area? And how many of them see that proportions, percentiles, and probabilities are closely related and understand why? Most instructors would agree, far too few.

What is compelling is that all three of these issues can be wonderfully addressed with one topic that is given short shrift in most, if not all, traditional statistics courses: density histograms. Because a normal curve can be viewed as a model that approximates a density histogram, students who have a firm grasp of density histograms can also gain a solid understanding of the three issues.

Many instructors' first reaction to this path is that density histograms are too difficult for students to comprehend. Actually, because density histograms are composed of rectangles, it is quite easy for students to compute areas and relate them to proportions, percentiles, and probabilities. *Pathway* takes full advantage of this by having students problem solve with rectangles in Chapter 1, construct and interpret density histograms in Chapter 3, reflect on how measures of center and spread are connected to density histograms in Chapter 4, and apply probability rules when working with density histograms in Chapter 5. After completing Chapters 1–5, the great majority of students will not only have a strong footing with the three issues mentioned earlier but also with related concepts such as probability and measures of center and spread.

Two Approaches: Acceleration versus Replacing Intermediate Algebra In terms of sequencing courses, departments will use this text in one of two ways.

Some departments plan to accelerate their non-STEM students through their math programs by replacing elementary and intermediate algebra with *Pathway*. Some of these faculty feel that the traditional algebra sequence is an unnecessary obstacle for students whose careers will not depend on a significant portion of the sequence. Others feel that presenting algebra from a statistical perspective will engage students at a higher level and be more relevant to their careers.

And then some departments plan to use *Pathway* as an alternative to intermediate algebra. This means that their non-STEM students will first take elementary algebra and then enhance their knowledge of algebra by experiencing it from a statistical perspective. This will not only broaden students' understanding of algebra but may allow some departments to put greater emphasis on statistics because their students will have seen the necessary algebra once before.

The Big Picture When the big picture is presented, students will have a map that tells them where the course's path is headed and how concepts connect. Once students have revisited many arithmetic concepts and a few simple algebra concepts in Chapter 1, they are ready for an overview of statistics in Chapter 2, which explores both good and bad experimental design. Unlike many statistics textbooks that then drop this crucial topic in subsequent chapters on descriptive statistics, *Pathway* encourages students to reflect on issues such as sampling error and sampling bias throughout the rest of the course.

At first glance, some reviewers wonder why the content of Chapter 6 was placed before the content of Chapters 7 and 8. After a closer look, they realize that Chapter 6's development of the four characteristics of an association (shape, strength, direction, and outliers) provides the big picture for the rest of this book. In fact, the four characteristics are further developed in a myriad of ways in subsequent chapters. A significant additional benefit to this organization is that Chapter 6 does not involve algebra. So, departments who want to heavily emphasize statistics can address all of Chapters 2–6 and, time permitting, pick and choose algebra topics from Chapters 7–12.

Compelling Modeling Reviewers have praised the modeling in this text because the data sets are current, authentic, and compelling. And although a homework section's modeling exercises emphasize the concepts addressed in the section, investigations prompted by the "story" of an authentic situation are also embraced. It is in part due to these excursions "off the path" that make the modeling exercises come alive.

Judiciously Selected Algebra Topics Some reviewers feel that this book contains too much algebra. Others think that the amount of algebra is just right. What is interesting is that almost all reviewers believe that for the most part, the only algebra that should be included are the concepts needed in an introductory statistics course. This suggests that instructors teach introductory statistics in different ways. For example, some instructors solve inequalities to derive confidence intervals. Others provide a more intuitive explanation. Some instructors solve equations to derive error formulas. Again, others get the idea across intuitively. And some do both.

With one possible exception, every algebra topic included in this text will be of service to *some* instructors who teach introductory statistics.

The one possible exception is the inclusion of functions. Although functions operate behind the scenes of introductory statistics courses, most textbooks do not make much, if any, use of function notation, language, and concepts. Nonetheless, keeping in mind that some departments will allow students to take *Pathway* instead of the traditional algebra sequence, functions have been included in this book in the hopes that any student who graduates from a community college will have an understanding and appreciation of a concept that is key for so much of mathematics.

Although exponential functions are definitely not included in most introductory statistics courses, they are arguably the second most important type of function (next to linear), and students can gain a significantly better understanding of linear modeling by comparing and contrasting the process with exponential modeling.

Arithmetic and Algebra Seen through a Statistics Lens To better prepare students for Chapters 2–12, some of the arithmetic and simple algebra concepts in Chapter 1 are presented with statistics in mind. For example, Section 1.3 uses fractions as a springboard for proportions and the complement rule. In Section 1.7, students will evaluate statistics expressions and work with areas of rectangles that resemble density histograms.

Likewise, algebra concepts addressed in Chapters 7–12 have been developed from a statistics perspective. For example, rate of change is investigated in Section 7.2 before slope of a line is introduced. Evaluating linear functions in Section 7.4 is parlayed into using linear models to make predictions. And rather than have students work with geometry and science formulas, Section 8.4 requires students to solve probability and statistics formulas for a variable.

Group Explorations Every section of *Pathway* contains at least one exploration that supports student investigation of a concept. Instructors can use explorations as **collaborative activities** during class time or as part of homework assignments. Section Opener Explorations are directed-discovery activities that are meant to be used at the start of class. Near the end of class, teams of students can work on additional explorations meant to deepen their understanding of key concepts. Both types of explorations empower students to become active explorers of mathematics and can open the door to the wonder and beauty of the subject.

Balanced Raw Data and Visual Approach Most statistics textbooks devote an entire chapter to constructing statistical diagrams but then make little use of such diagrams in homework exercises of subsequent chapters. This is unfortunate because students learn best when new concepts are integrated with previously learned ones. For example, to gain a solid understanding of the measures of center and spread, students should analyze some exercises that supply raw data and others that supply statistical diagrams. Throughout this book, homework sections contain a good balance of both types of exercises.

Technology Back in the '80s, statistics students were expected to construct large numbers of statistical diagrams by hand and perform copious calculations with their calculators. Currently, most statistics instructors believe students should perform a limited number of such activities to get the idea and from then on use technology. The freed-up class time is devoted to enhancing students' ability to analyze authentic situations, compare measures of center and spread, and interpret concepts and results.

Pathway assumes students have access to technology. With so many packages to choose from, this text's technological support would be spread thin if it attempted to address all of them. This book focuses on the TI-84 graphing calculator and StatCrunch because the vast majority of community college instructors use one of these two technologies in their introductory statistics courses.

However, in the homework sections, the word *technology* is used rather than specifying the TI-84 or StatCrunch to accommodate classes using other technologies, unless an algebraic command specific to the TI-84 (such as *intersect*) is required.

Appendices A and B: TI-84 and StatCrunch Instructions Appendices A and B contain instructions on how to use a TI-84 and StatCrunch, respectively. A subset of either appendix can serve as a tutorial early in the course. In addition, each time this text introduces a command from either technology, students are referred to a section of the appropriate appendix.

DATA

"Data" Icon To support the appropriate use of technology, data sets in exercises and explorations that involve approximately 12 or more data values are available to download at MyLab Math and at the Pearson Downloadable Student Resources for Math and Statistics website: http://www.pearsonhighered.com/mathstatsresources. Such exercises are flagged in the text by the icon DATA.

Big Data It can make a significant, positive impression on students the first time they use technology to construct a histogram of about 100 observations when up to that point they have constructed histograms of only about 20 observations by hand. They are understandably struck by the ease, speed, and accuracy of using technology. But students can gain an even higher level of appreciation by using technology to describe a data set that consists of entries in thousands of rows and multiple columns.

Such an activity is especially relevant in today's age of big data. Although most *Pathway* students will not perform statistics in their careers, some *will* work with big data. And as part of their general education, all students should have some sense of what statisticians do.

To meet this end, exercises that involve large data sets are sprinkled throughout this text. Identified by the heading "Big Data," they are positioned at the end of homework sections. Some of these data sets contain thousands of rows and tens of columns.

Hands-On Research Even though every authentic data set in this book provides a source, some students still think that the data is fabricated. Having students find data sets themselves drives home the point that the concepts they are learning can truly be applied to real-life situations. Students begin to see that statistics can be used not only to inform but also to persuade.

To guide students in this process, this text contains exercises that direct students to analyze data found by online searches of blogs, newspapers, magazines, and scientific journals. These exercises are at the end of select homework sections, directly following the heading, "Hands-On Research."

Hands-On Projects Compelling project assignments are positioned near the end of most chapters. Some of the assignments are similar to the Hands-On Research exercises, but they are more extensive and challenging. These projects reinforce the idea that statistics is a powerful tool that can be used to analyze authentic situations. They are also an excellent opportunity for more in-depth writing assignments.

Some of the projects are about climate change and have been written at a higher reading level than the rest of this text to give students a sense of what it is like to perform research. Students will find that by carefully reading (and possibly rereading) the background information, they can comprehend the information and apply concepts they have learned in the course to make meaningful estimates about this compelling, current, and authentic situation.

Level of Difficulty As was discussed earlier, some departments plan on using *Pathway* to accelerate non-STEM students through their math program. This is a worthy goal, provided it is done well. But some instructors have collapsed the notion of acceleration with making the course easier. The line of reasoning is that if certain students would not succeed in a traditional algebra course, then those students would not succeed in an alternative course that is just as challenging. This logic does not hold up because the nature of the two courses can differ greatly. We should not rob students of the knowledge and self-esteem that result from diligent study.

Furthermore, employers in search of college graduates certainly want a college degree to mean that students have succeeded at courses that are just as demanding as those in the past.

It is for these reasons that this text has been written to challenge students as much as they are challenged in traditional algebra courses. This is primarily achieved in two ways. First, exercises and projects require the interpretation of concepts and results, which causes significant growing pains in most students. Second, many exercises contain at least one part (often out of five parts) that challenges students to apply concepts in new ways.

Warnings Throughout this text, the word WARNING in the margins flags paragraphs that describe common student misconceptions and the correct meanings or applications of concepts.

Tips for Success Many sections close with practical study tips to help students succeed in the course. A complete list of these tips is included in the Index.

GETTING IN TOUCH

I would love to hear from you and would greatly appreciate receiving your comments regarding *Pathway*. If you have any questions, please ask them, and I will respond.

Jay Lehmann
MathNerdJay@aol.com

Get the *Most* Out of MyLab Math

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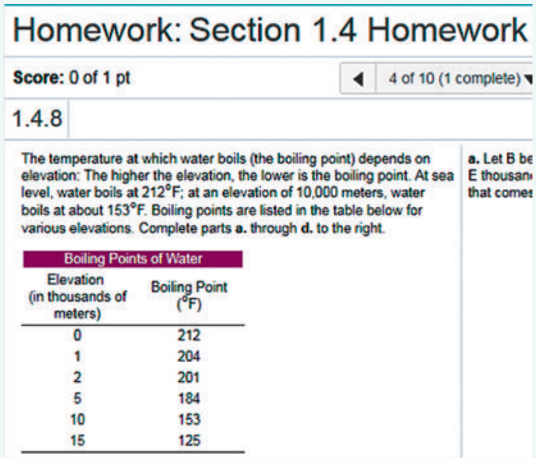
MyLab Math for A PATHWAY TO INTRODUCTORY STATISTICS, 2ND EDITION, by JAY LEHMANN

(access code required)

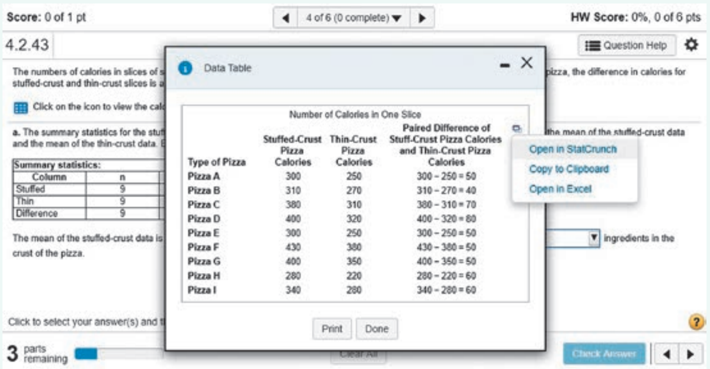
Jay Lehmann's *A Pathway to Introductory Statistics* offers market-leading content written by an author-educator, tightly integrated with the #1 choice in digital learning - MyLab Math. MyLab Math courses can be tailored to the needs of instructors and students, while weaving the author's voice and unique approach into all elements of the course.

Take advantage of the following resources to get the most out of your MyLab Math course.



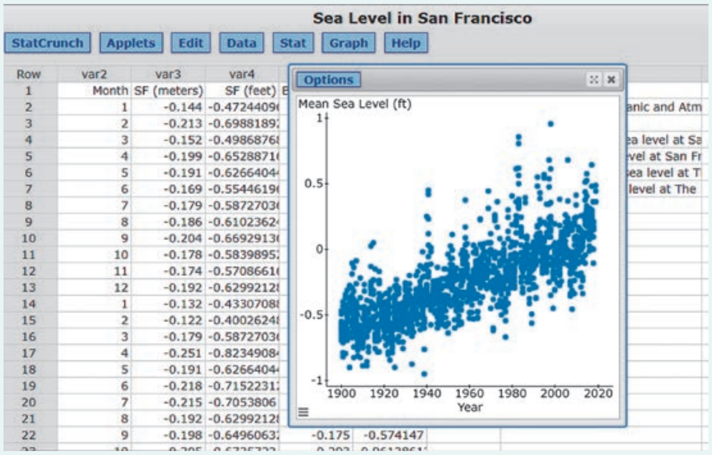
Updated! MyLab Math coverage of **exercises from the text** has been expanded in this revision, offering instructors more options when creating assignments. Many of the exercises entail data which has been augmented or updated to be as current as possible. New! Select exercises now retain their authentic data sets, even when regenerating algorithmically, so that students don't sacrifice working with real data when doing homework exercises with different values.

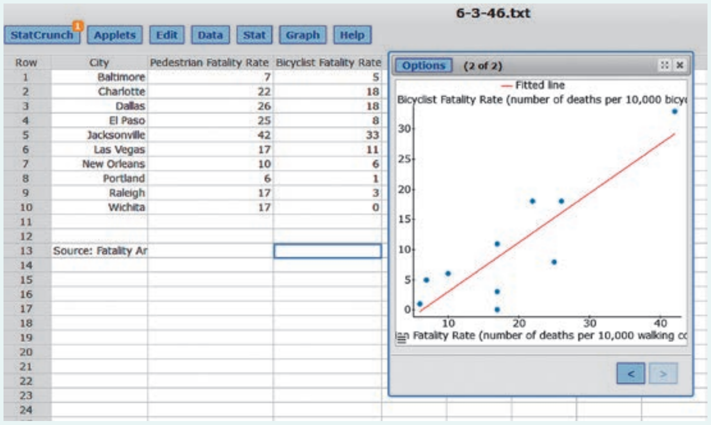
Data sets in exercises and explorations that involve approximately 12 or more data values are available to download in MyLab Math to support the appropriate use of technology. These exercises are ideal for using technology, like StatCrunch or Microsoft Excel, to analyze the data and synthesize concepts.



pearson.com/mylab/math

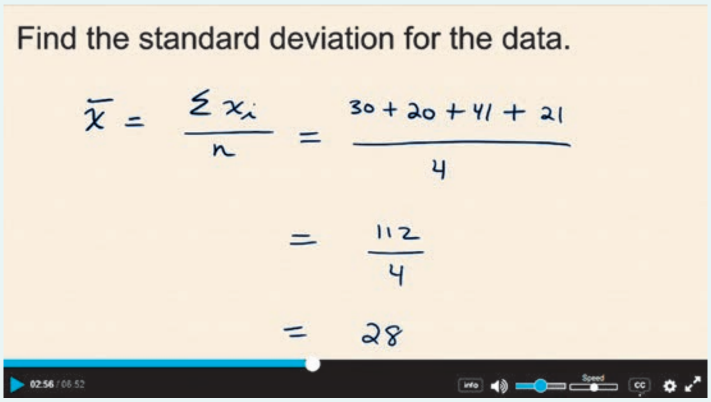
Big data sets, sprinkled throughout the text and noted with "Big Data," contain hundreds of rows of data to give students a hands-on opportunity to work with large, realistic data. In today's age of "Big Data," it can be compelling for students to see how technology can efficiently and accurately help when working with large data sets.





StatCrunch is a powerful web-based statistical software that allows users to collect, crunch, and communicate with data. Integrated into this MyLab course, StatCrunch can be used to analyze data, understand statistical concepts, and get students comfortable with statistical software early.

Instructional videos provide students with extra help for objectives from the textbook. Students can get support on topics and examples anytime, anywhere. Able to be played on any device, all videos are closed captioned.



Resources for Success

Pearson MyLab

Instructor Resources

The following instructor resources are available to download from the Instructor Resource Center at **www.pearson.com***, or in your MyLab Math course.*

Instructor's Resource Manual

This manual, written by the author, contains suggestions for pacing the course and creating homework assignments. It discusses how to incorporate technology and how to structure project assignments. The manual also contains section-by-section suggestions for presenting lectures and for undertaking the explorations in the text.

PowerPoints

These fully editable lecture slides include definitions, key concepts, and examples for use in a lecture setting. Accessible versions of these PowerPoints are also available.

Instructor's Solutions Manual

This manual includes complete solutions to the even-numbered exercises in the homework sections of the text.

TestGen

TestGen enables instructors to build, edit, print, and administer tests by using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple, but equivalent, versions of the same question or test with the click of a button. Instructors can also modify test-bank questions or add new questions. Tests can be printed or administered online. Download the software and this title's test bank from pearson.com.

Student Resources

Video Series

The video program provides students with extra help for objectives of the text. Videos highlight key examples and exercises from the text to facilitate student understanding.

NEW! Workbook

The author has written a new workbook that contains hundreds of affective domain and prestatistics activities. The workbook provides great support for collaborative learning, which research has shown is vital to students' conceptual and problem-solving development. It will be especially helpful for teaching corequisite courses.

Student's Solutions Manual

This manual contains the complete solutions to the odd-numbered exercises in the Homework sections of the text. The Student Solutions Manual (ISBN 9780136553984) is available electronically in MyLab Math.

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1 Performing Operations and Evaluating Expressions

Although the number of AIDS deaths in the United States greatly decreased from 37,787 deaths in 1996 to only 12,333 deaths in 2014, the number of AIDS deaths increased to 15,807 in 2016 (see Table 1). In Example 9 of Section 1.5, we will calculate how the number of deaths has changed in various years.

HIV infection can lead to contracting AIDS. In 2010, President Obama released the National HIV/AIDS Strategy to lower the number of new HIV infections. But how can we determine what will make a difference?

Table 1 Numbers of AIDS Deaths in the United States

Year	Number of AIDS Deaths
2010	14,399
2011	14,122
2012	13,984
2013	12,963
2014	12,333
2015	12,497
2016	15,807

Source: *Centers for Disease Control and Prevention*

In this course, we will address such questions by practicing statistics. We will learn to form *precise* questions such as "Does raising public awareness about the ways HIV can be contracted decrease the HIV infection rate?" Next, we will discuss how to develop a careful plan to answer a precise question, which will include taking a close look at how to collect the relevant information. Then we will construct tables and diagrams and perform calculations to analyze the information. Throughout much of the course, we will determine what types of conclusions we can draw about questions raised.

In this chapter, we will discuss the arithmetic and algebra that form the foundation of statistics.

1.1 Variables, Constants, Plotting Points, and Inequalities

Objectives

- » Describe the meaning of *variable* and *constant*.
- » Identify and graph types of numbers.
- » Graph data on a number line.
- » Plot points on a coordinate system.
- » Graph an inequality on a number line.

In this section, we will work with *variables* and *constants*, two extremely important building blocks of algebra and statistics. We will also discuss various types of numbers and how to describe numbers and pairs of numbers visually. Finally, we will compare the sizes of quantities.

Variables and Constants

In arithmetic, we work with numbers. In algebra and statistics, we work with *variables* as well as numbers.

Definition Variable

A **variable** is a symbol that represents a quantity that can vary.

» Use inequality notation, interval notation, and graphs to describe possible values of a variable for an authentic situation.

» Describe a concept or procedure.

For example, we can define h to be the height (in feet) of a specific child. Height is a quantity that varies: As time passes, the child's height will increase. So, h is a variable. When we say $h = 4$, we mean the child's height is 4 feet.

This definition of variable is typically used in algebra and will be extremely useful in this chapter. In Section 2.1, we will discuss a definition that is typically used in statistics. At the heart of both definitions is the fact that a variable describes something that can vary.

Example 1 Using a Variable to Represent a Quantity

1. Let s be a car's speed (in miles per hour). What is the meaning of $s = 60$?
2. Let n be the number of employees (in millions) who work from home at least half the time. For the year 2017, $n = 3.7$ (Source: *Fundera*). What does that mean in this situation?
3. Let t be the number of years since 2015. What is the meaning of $t = 5$?

Solution

1. The speed of the car is 60 miles per hour.
2. In 2017, 3.7 million people worked from home at least half the time.
3. $2015 + 5 = 2020$; so, $t = 5$ represents the year 2020.

There are many benefits to using variables. For example, in Problem 2 of Example 1, we found that the simple equation "$n = 3.7$" means the same thing as the wordy sentence "3.7 million people worked from home at least half the time." Variables can help us describe some situations with a small amount of writing.

In Problem 3 of Example 1, we described the year 2020 by using $t = 5$. So, our definition of t allows us to use smaller numbers to describe various years—an approach that will be especially helpful in Chapters 6–10.

We will see other benefits of variables as we proceed through the course.

Example 2 Using a Variable to Represent a Quantity

Choose a symbol to represent the given quantity. Explain why the symbol is a variable. Give two numbers that the variable can represent and two numbers that it cannot represent.

1. The weight (in pounds) of a baby at birth
2. The number of people who live in a two-bedroom house

Solution

1. Let be the weight (in pounds) of a baby at birth. The weight of a baby at birth can vary, so is a variable. For example, can represent the numbers 6 and 8 because babies can weigh 6 or 8 pounds at birth. The variable does not represent 0 or 300 because babies cannot weigh 0 or 300 pounds at birth!
2. Let n be the number of people who live in a two-bedroom house. The number of people who live in a two-bedroom house can vary, so n is a variable. For example, n can represent the numbers 2 and 3 because 2 or 3 people can live in a two-bedroom house. The variable n cannot represent the numbers 5000 or $\frac{1}{2}$ because 5000 people cannot live in a two-bedroom house and half of a person doesn't make sense.

In Problem 1 of Example 2, we stated that the units of are pounds. Without stating the units of , "$w = 10$" could mean the baby's weight was 10 ounces, 10 pounds, or 10 tons! In defining a variable, it is important to describe the variable's units.

A variable is a symbol—typically a letter—that represents a quantity that can vary. When we use a symbol to represent a quantity that does *not* vary, we call that symbol a *constant*. So, 2, 0, 4.8, and π are constants. The constant π is approximately equal to 3.14.

> **Definition** Constant
>
> A **constant** is a symbol that represents a specific number (a quantity that does *not* vary).

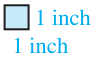


Figure 1 One square inch

In the next example, we will compare the meanings of a variable and a constant while we consider the widths, lengths, and areas of some rectangles. The **area** (in square inches) of a flat surface is the number of square inches that it takes to cover the surface (see Fig. 1). **The area of a rectangle is equal to the rectangle's length times its width.**

Example 3 Comparing Constants and Variables

A rectangle has an area of 12 square inches. Let W be the width (in inches), L be the length (in inches), and A be the area (in square inches).

1. Sketch three possible rectangles of area 12 square inches.
2. Which of the symbols W, L, and A are variables? Explain.
3. Which of the symbols W, L, and A are constants? Explain.

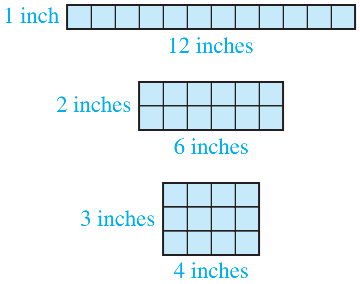


Figure 2 Three possible rectangles of area 12 square inches

Solution

1. We sketch three rectangles for which the width times the length is equal to 12 square inches (see Fig. 2).
2. The symbols W and L are variables because they represent quantities that vary.
3. The symbol A is a constant because in this problem the area does not vary—the area is always 12 square inches.

Identify and Graph Types of Numbers

When we describe people, it often helps to describe them in terms of certain categories, such as gender, ethnicity, and employment. In mathematics, it helps to describe numbers in terms of categories, too. We begin by describing the *counting numbers*, which are the numbers 1, 2, 3, 4, 5, and so on.

> **Definition** Counting numbers (natural numbers)
>
> The **counting numbers**, or **natural numbers**, are the numbers
>
> $$1, 2, 3, 4, 5, \ldots$$

The three dots mean that the pattern of the numbers shown continues without ending. In this case, the pattern continues with 6, 7, 8, and so on. When a list of numbers goes on forever, we say that there are an *infinite* number of numbers.

Next, we describe the *integers*, which include the counting numbers and other numbers.

> **Definition** Integers
>
> The **integers** are the numbers
>
> $$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

The three dots on both sides mean that the pattern of the numbers shown continues without ending in both directions. In this case, the pattern continues with $-4, -5, -6$, and so on, and with 4, 5, 6, and so on.

If you write a check for more money than is in your checking account, you will have a negative balance. A balance (in dollars) of -60 is an integer.

The **positive integers** are the numbers $1, 2, 3, \ldots$. The **negative integers** are the numbers $-1, -2, -3, \ldots$. The integer 0 is neither positive nor negative. So, the integers consist of the counting numbers (which are positive integers), the negative integers, and 0.

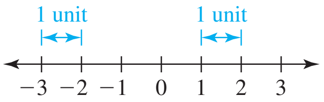


Figure 3 The number line

We can visualize numbers on a *number line* (see Fig. 3). Each point (location) on the number line represents a number. The numbers increase from left to right. We refer to the distance between two consecutive integers on the number line as 1 *unit* (see Fig. 3).

Example 4 Graphing Integers on a Number Line

Draw dots on a number line to represent the integers between −2 and 3, inclusive.

Solution

The integers between −2 and 3, inclusive, are −2, −1, 0, 1, 2, and 3. "Inclusive" means to include the first and last numbers, which in this case are −2 and 3. We sketch a number line and draw dots at the appropriate locations for the numbers −2, −1, 0, 1, 2, and 3 (see Fig. 4).

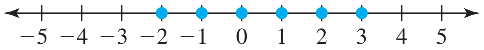


Figure 4 Graphing the numbers −2, −1, 0, 1, 2, and 3

When we draw dots on a number line, we say that we are "plotting points" or "graphing numbers."

WARNING

In Example 4, we worked with the integers between −2 and 3, inclusive: −2, −1, 0, 1, 2, and 3. Here are the integers between −2 and 3: −1, 0, 1, and 2. We did not include −2 or 3 because the word "inclusive" was not used. When working with such problems, it is important to check whether the word "inclusive" is used.

For a fraction $\frac{n}{d}$, we call n the **numerator** and d the **denominator**. The dash between the numerator and the denominator is the **fraction bar**:

Numerator ⟶ n
Denominator ⟶ d ⟵ Fraction bar

A fraction can be used to describe a part of a whole. For example, consider the meaning of $\frac{5}{8}$ of a pizza. If we divide the pizza into 8 slices of equal area, 5 of the slices make up $\frac{5}{8}$ of the pizza (see Fig. 5).

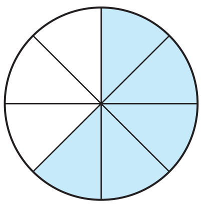


Figure 5 $\frac{5}{8}$ of a pizza

The number $\frac{5}{8}$ is called a *rational number*.

Definition Rational numbers

The **rational numbers** are the numbers that can be written in the form $\frac{n}{d}$, where n and d are integers and d is nonzero.

We specify that d is nonzero because, as we shall see later, division by zero does not make sense.

Here are some examples of rational numbers:

$$\frac{3}{7} \qquad \frac{-2}{5} \qquad 4 = \frac{4}{1}$$

Rational numbers include all the integers because any integer n can be written as $\frac{n}{1}$.

There are numbers represented on the number line that are *not* rational. These numbers are called **irrational numbers**. An irrational number *cannot* be written in the form $\frac{n}{d}$, where n and d are integers and d is nonzero. The number $\sqrt{2}$ is the number greater than zero that we multiply by itself to get 2. The number $\sqrt{2}$ is an irrational number. Here are some more examples of irrational numbers:

$$\pi \qquad \sqrt{3} \qquad \sqrt{5}$$

We know that $\sqrt{9} = 3$ because $3 \times 3 = 9$. So $\sqrt{9} = 3 = \frac{3}{1}$. Therefore, $\sqrt{9}$ is rational (not irrational).

The list price of an Xbox One X console is \$399.99, which is a decimal number. Any rational number or irrational number can be written as a decimal number.

A rational number can be written as a decimal number that either terminates or repeats:

$$\frac{3}{4} = \underbrace{0.75}_{\text{terminates}} \qquad \frac{3}{11} = 0.\underbrace{27272727\ldots}_{\text{repeats}}$$

We can use an overbar to write the repeating decimal $0.272727\ldots = 0.\overline{27}$.

An irrational number can be written as a decimal number that neither terminates nor repeats. It is impossible to write all the digits of an irrational number, but we can approximate the number by rounding. For example, earlier we *approximated* π by rounding to the second decimal place: $\pi \approx 3.14$.

Recall that each point on the number line represents a number. We call all the numbers represented by all the points on the number line the *real numbers*.

Definition Real numbers

The **real numbers** are all the numbers represented on the number line.

The real numbers are made up of the rational numbers and the irrational numbers. Here are some real numbers:

$$-1.8 \qquad -1 \qquad -\frac{7}{10} \qquad 0 \qquad 0.4 \qquad \frac{6}{5} \qquad \pi$$

We graph these real numbers in Fig. 6.

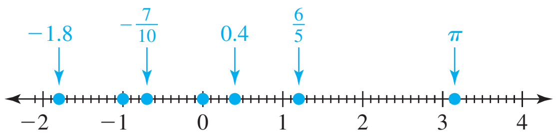


Figure 6 Graphing the real numbers -1.8, -1, $-\frac{7}{10}$, 0, 0.4, $\frac{6}{5}$, and π

We use an arrow to label points that do not fall on a labeled tick mark.

Example 5 Graphing Real Numbers on a Number Line

Graph the number on a number line.

1. $-\frac{7}{4}$

2. 2.3

Solution

1. We draw a number line so that the distance between tick marks is $\frac{1}{4}$ unit (see Fig. 7). To graph $-\frac{7}{4}$, we draw a dot at the seventh tick mark to the left of 0.

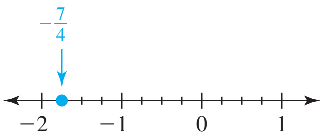


Figure 7 Graphing the number $-\frac{7}{4}$

2. We draw a number line so that the distance between tick marks is $0.1 = \frac{1}{10}$ unit (see Fig. 8). To graph 2.3, we draw a dot at the third tick mark to the right of 2.

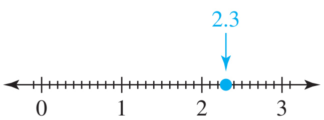
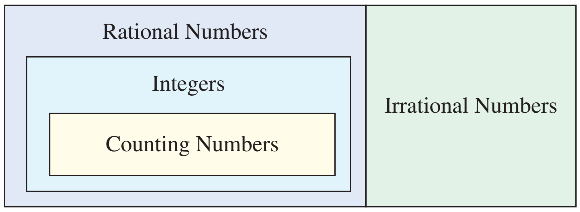


Figure 8 Graphing the number 2.3

Figure 9 illustrates how the various types of numbers we have discussed so far are related. In particular, it shows that every counting number is an integer, every integer is a rational number, and every rational number is a real number. It also shows that **irrational numbers are the real numbers that are not rational.**



All numbers shown here are real numbers.

Figure 9 The real numbers

The **negative real numbers** are the real numbers less than 0, and the **positive real numbers** are the real numbers greater than 0 (see Fig. 10).

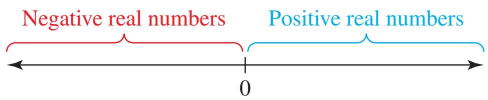


Figure 10 The location of the negative real numbers and the positive real numbers on the number line

Some examples of negative real numbers are -13, -5.2, $-\frac{3}{4}$, and $-\sqrt{2}$. Some examples of positive real numbers are 13, 5.2, $\frac{3}{4}$, and π. As we discussed earlier, the number 0 is neither positive nor negative.

We say that the *sign* of a negative real number is negative and that the *sign* of a positive real number is positive. To include zero, we define the **nonnegative real numbers** as the positive real numbers together with 0. Likewise, we define the **nonpositive real numbers** as the negative real numbers together with 0.

Data are quantities or categories that describe people, animals, or things. For example, the following heights of six people, all in inches, are data: 64, 71, 75, 68, 71, and 69. The following genres of music of the top 5 grossing singles this week are data: pop, rock, pop, hip-hop, and country.

Example 6 Identifying Types of Data

Among the following groups of numbers, determine the smallest group that contains any possible data for the given situation: counting numbers, integers, nonnegative real numbers, and real numbers. Explain.

1. The number of students enrolled in Harvard in some year between 1900 and 2019, inclusive
2. The elevation (in feet) at some location
3. The volume (in gallons) of gasoline in a car's gas tank

Solution

1. Students have been enrolled in Harvard every year since 1990, and there cannot be just a portion of a student enrolled. So, among the given choices, the counting numbers is the smallest group of numbers that contains possible data.
2. A location's elevation can be a fraction of a foot. It can also be negative. For example, the elevation at Death Valley is -282 feet. So, among the given choices, the real numbers is the smallest group of numbers that contains possible data.
3. There can be a fraction of a gallon of gasoline in the tank, but the volume of gasoline cannot be negative. So, among the given choices, the nonnegative real numbers is the smallest group of numbers that contains possible data.

Graphing Data on a Number Line

We often can get a better sense of data that are quantities by graphing them on a number line.

Example 7 Graphing Data

The total amounts (in billions of dollars) of Goldman Sachs loans for the years 2012, 2013, 2014, 2015, and 2016 are 65, 80, 87, 91, and 94, respectively (Source: *Goldman Sachs*). Let L be the total amount (in billions of dollars) of Goldman Sachs loans in a given year.

1. Graph the data.
2. Did the total amount of the loans increase, decrease, stay approximately constant, or none of these from 2012 to 2016, inclusive? Explain.
3. Did the *increases* in the total amounts of the loans increase, decrease, stay approximately constant, or none of these from 2012 to 2016, inclusive? Explain.

Solution

1. We sketch a number line and write "L" to the right of the number line and the units "Billions of dollars" underneath the number line (see Fig. 11). Because the data values are between 65 and 94, inclusive, we write the numbers 60, 65, 70, 75, 80, 85, 90, 95 equally spaced on the number line. Then we graph the numbers 65, 80, 87, 91, and 94.

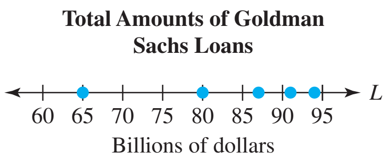


Figure 11 Graphing the data

2. From the opening paragraph, we know that the total amount of the loans is increasing. (From the graph alone, we cannot tell this because the years are not included.)
3. As we look from left to right at the points plotted on the graph, we see that the distance between adjacent points decreases. This means that the increases in the total amounts of the loans decreased. That is, the jump from 65 to 80 is greater than the jump from 80 to 87, and so on.

WARNING In Fig. 11, we wrote the numbers 60, 65, 70, 75, 80, 85, 90, and 95 on the number line. **When we write numbers on a number line, they should increase by a fixed amount and be equally spaced.**

Example 8 Graphing a Negative Quantity

A person bounces several checks and, as a result, is charged service fees. If b is the balance (in dollars) of the checking account, what value of b means the person owes \$50? Graph the number on a number line.

Solution

Because the person *owes* money, the value of b is negative: $b = -50$. We graph -50 on a number line in Fig. 12.

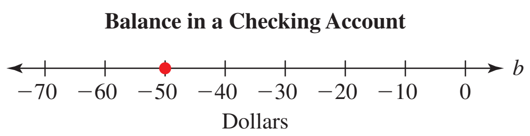


Figure 12 Graphing the number $b = -50$

So far, we have discussed how to describe the values of a *single* variable. Now we will discuss how to describe pairs of values of *two* variables.

Plotting Points on a Coordinate System

The quiz scores of a student in one of the author's prestatistics classes are shown in Table 2. We define n to be the quiz number and s to be the quiz score (in points). From Table 2, we see the student's score on Quiz 1 was 7 points. So, when $n = 1$, $s = 7$. If we agree to write the quiz number first and the quiz score second, we can use the **ordered pair** $(1, 7)$ to mean that when $n = 1$, $s = 7$. We call each of the numbers in an ordered pair a **coordinate**. For $(1, 7)$ in this situation, we call 1 the *n-coordinate* and 7 the *s-coordinate*.

Table 2 A Student's Quiz Scores

Quiz Number n	Quiz Score (points) s
1	7
2	6
3	9
4	8
5	9

Source: *J. Lehmann*

The ordered pair $(2, 6)$ means that when $n = 2$, $s = 6$. This indicates that the student's score on Quiz 2 was 6 points, which agrees with the second row of Table 2.

We graph the ordered pairs by using *two* number lines, which are called **axes** (singular: **axis**). To start, we draw a horizontal number line called the n-axis and a vertical number line called the s-axis (see Fig. 13). We refer to such a pair of axes as a **coordinate system**. The **origin** is the intersection point of the axes. The axes divide the coordinate system into four regions called **quadrants**, which we call Quadrants I, II, III, and IV. The quadrants do not include the axes.

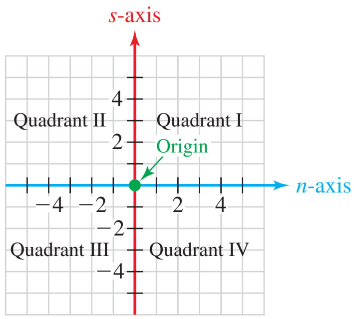


Figure 13 Coordinate system

Next, we plot the ordered pair (3, 9) shown in the third row of Table 2. To do so, we start at the origin, look 3 units to the right and 9 units up, and then draw a dot (see Fig. 14). In Fig. 15, we plot all the ordered pairs listed in Table 2.

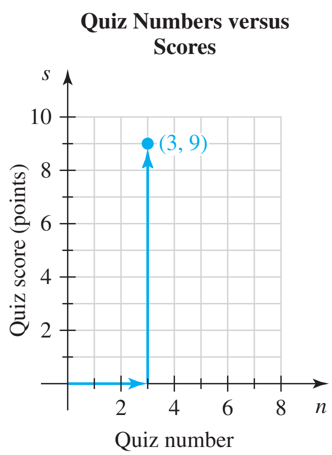


Figure 14 Plot (3, 9)

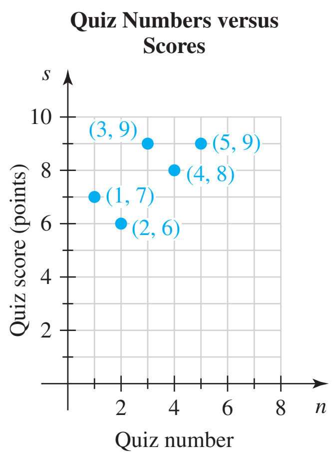


Figure 15 Plot the ordered pairs from Table 2

As we look at the plotted points in Fig. 15 from left to right, the points, in general, go upward. This means the quiz scores, in general, are increasing.

When we plot points that are not being used to describe authentic situations, we call the horizontal axis the *x-axis* and the vertical axis the *y-axis*. The ordered pair $(6, 3)$ means $x = 6$ and $y = 3$. So, the x-coordinate is 6 and the y-coordinate is 3.

Example 9 Plotting Points

Plot the points $(3, 4)$, $(-5, -3)$, $(-4, 2)$, and $(5, -4)$ on a coordinate system.

Solution

We plot the ordered pairs $(3, 4)$ and $(-5, -3)$ in Fig. 16, and we plot the ordered pairs $(-4, 2)$ and $(5, -4)$ in Fig. 17.

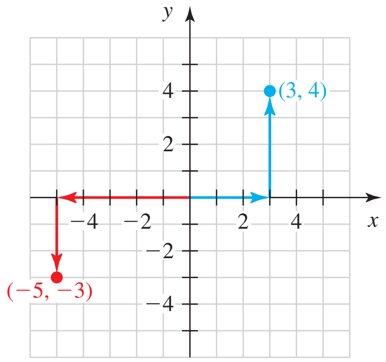


Figure 16 Plotting the ordered pairs $(3, 4)$ and $(-5, -3)$

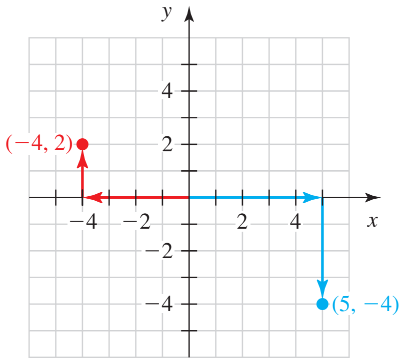


Figure 17 Plotting the ordered pairs $(-4, 2)$ and $(5, -4)$

Graphing an Inequality on a Number Line

In statistics, we often compare the sizes of two quantities. We can do this using the **inequality symbols** $<$, $\le$, $>$, and $\ge$. Here are the meanings of these symbols and some examples of *inequalities*:

Symbol	Meaning	Examples of Inequalities
$<$	Is less than	$2 < 5, 0 < 5, -6 < -1$
$\le$	Is less than or equal to	$4 \le 7, 2 \le 2, -3 \le 0$
$>$	Is greater than	$9 > 2, -4 > -6, 2 > 0$
$\ge$	Is greater than or equal to	$8 \ge 3, 5 \ge 5, -2 \ge -8$

An **inequality** contains one of the symbols $<$, $\le$, $>$, and $\ge$ with a constant or variable on one side and a constant or variable on the other side. Here are some more examples of inequalities:

$$x < -3 \qquad -4 \le 5 \qquad 7 > 2 \qquad x \ge 6$$

Example 10 Inequalities

Decide whether the inequality statement is true or false.

1. $3 \le 6$ **2.** $-5 > -2$ **3.** $8 \ge 8$ **4.** $9 < 9$

Solution

1. Because 3 is less than 6, the statement $3 \le 6$ is true.
2. Because -5 lies to the left of -2 on the number line, -5 is less than -2. So, -5 is *not* greater than -2, and the statement $-5 > -2$ is false.
3. Because 8 is equal to itself, the statement $8 \ge 8$ is true.
4. Because 9 is not less than itself, the statement $9 < 9$ is false.

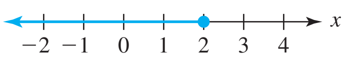


Figure 18 Graph of $x \le 2$

Consider the inequality $x \le 2$. This inequality says the values of x are less than or equal to 2. We can represent these values graphically on a number line by shading the part of the number line that lies to the left of 2 (see Fig. 18). We draw a *filled-in* circle at 2 to indicate that 2 is a value of x, too.

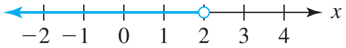


Figure 19 Graph of $x < 2$

To graph the inequality $x < 2$, we shade the part of the number line that lies to the left of 2 but draw an *open* circle at 2 to indicate that 2 is *not* a value of x (see Fig. 19).

We use **interval notation** to describe a set of numbers. For example, we describe the numbers greater than 3 by $(3, \infty)$. We describe the numbers greater than or equal to 3 by $[3, \infty)$. We describe the set of real numbers by $(-\infty, \infty)$. More examples of inequalities and interval notation are shown in Fig. 20.

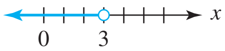
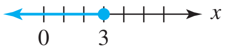
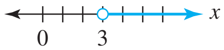
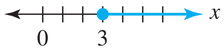
In Words	Inequality	Graph	Interval Notation
Numbers less than 3	$x < 3$		$(-\infty, 3)$
Numbers less than or equal to 3	$x \leq 3$		$(-\infty, 3]$
Numbers greater than 3	$x > 3$		$(3, \infty)$
Numbers greater than or equal to 3	$x \geq 3$		$[3, \infty)$

Figure 20 Words, inequalities, graphs, and interval notation

Example 11 Graphing an Inequality

Write the inequality $x > -2$ in interval notation, and graph the values of x.

Solution

The inequality $x > -2$ means that the values of x are greater than -2. We describe these numbers in interval notation by $(-2, \infty)$. To graph the values of x, we shade the part of the number line that lies to the right of -2 and draw an open circle at -2 (see Fig. 21).

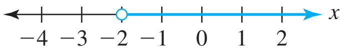


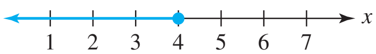
Figure 21 Graph of $x > -2$

The phrase "less than or equal to 5" means the same thing as "at most 5" and "no more than 5." Similarly, the phrase "greater than or equal to 5" means the same thing as "at least 5" and "no less than 5."

More examples of such phrases with matching inequalities and graphs are shown in Fig. 22.

$x \leq 4$

Numbers less than or equal to 4
Numbers that are at most 4
Numbers that are no more than 4



$x \geq 4$

Numbers greater than or equal to 4
Numbers that are at least 4
Numbers that are no less than 4

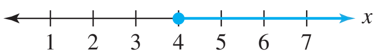


Figure 22 Inequalities, words, and graphs

Example 12 Describe Values of a Variable and Graph an Inequality

1. The values of x are at most -4. Describe the values of x as an inequality, in interval notation, and as a graph.
2. The values of x are no less than 1. Describe the values as an inequality, in interval notation, and as a graph.

Solution

1. The phrase "at most -4" means less than or equal to -4. So, we can describe the values of x in inequality notation as $x \leq -4$ and in interval notation as $(-\infty, -4]$. To graph the values of x, we shade the part of the number line that lies to the left of -4 and draw a filled-in circle at -4 (see Fig. 23).

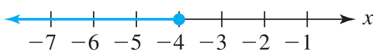


Figure 23 Graph of $x \leq -4$

2. The phrase "no less than 1" means greater than or equal to 1. So, we can describe the values of x in inequality notation as $x \geq 1$ and in interval notation as $[1, \infty)$. To graph the values of x, we shade the part of the number line that lies to the right of 1 and draw a filled-in circle at 1 (see Fig. 24).

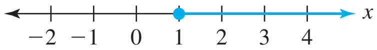


Figure 24 Graph of $x \geq 1$

Figure 25 Graph of $3 \leq x \leq 7$

Now we will work with *compound inequalities in one variable,* such as $3 \leq x \leq 7$, which means the values of x are *both* greater than or equal to 3 *and* less than or equal to 7. In other words, all values of x are between 3 and 7, inclusive. To graph the solutions, we shade the part of the number line that lies between 3 and 7 (see Fig. 25). We draw filled-in circles at 3 and 7 to indicate that 3 and 7 are solutions, too.

We describe the numbers between 3 and 7, inclusive, in interval notation by [3, 7]. More examples of compound inequalities, with matching graphs and interval notation, are shown in Fig. 26.

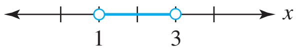
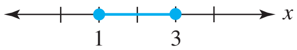
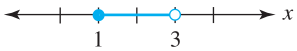
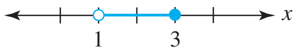
In Words	Inequality	Graph	Interval Notation
Numbers between 1 and 3	$1 < x < 3$		$(1, 3)$
Numbers between 1 and 3, inclusive	$1 \leq x \leq 3$		$[1, 3]$
Numbers between 1 and 3, as well as 1	$1 \leq x < 3$		$[1, 3)$
Numbers between 1 and 3, as well as 3	$1 < x \leq 3$		$(1, 3]$

Figure 26 Words, inequalities, graphs, and interval notations

WARNING

We use notation such as $(3, 7)$ in two ways: When we work with one variable, the *interval* $(3, 7)$ is the set of numbers between 3 and 7; when we work with two variables, such as x and y, the *ordered pair* $(3, 7)$ means $x = 3$ and $y = 7$.

Describing Values of Variables for an Authentic Situation

When analyzing authentic situations, we will often use inequality notation, interval notation, and graphs to describe possible values of variables.

Example 13 Describe Values of Variables and Graph a Compound Inequality

1. In a Category 5 hurricane, the highest-category hurricane, winds are no less than 157 miles per hour. Let s be the wind speed (in miles per hour) of a Category 5 hurricane. Describe the hurricane's wind speed using inequality notation, interval notation, and a graph.
2. For a United Airlines economy ticket, a checked bag that is not overweight weighs at most 50 pounds (Source: *United Airlines*). Let be the weight (in pounds) of a checked bag that is not overweight for a United Airlines economy ticket. Describe the bag's weight using inequality notation, interval notation, and a graph.
3. Let be the weight (in pounds) of a man. Interpret and graph the inequality $173 < w < 177$.

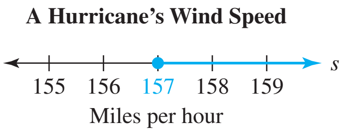


Figure 27 Graph of $s \geq 157$

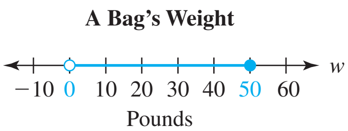


Figure 28 Graph of $0 < w \leq 50$

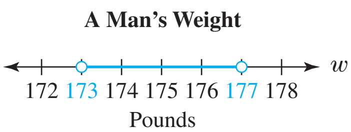


Figure 29 Graph of $173 < w < 177$

Solution

1. The phrase "no less than 157" means greater than or equal to 157. So, we can describe the hurricane's wind speed in inequality notation as $s \geq 157$ and in interval notation as $[157, \infty)$. To graph the values of s, we shade the part of the number line that lies to the right of 157 and draw in a filled-in circle at 157 (see Fig. 27).
2. The phrase "at most 50" means less than or equal to 50. So, the weight of the bag is less than or equal to 50 pounds. Because the bag has some weight, the weight is also greater than 0 pounds. So, we can describe the weight in inequality notation as $0 < w \leq 50$ and in interval notation as $(0, 50]$. To graph the values of , we shade the part of the number line that lies between 0 and 50 (see Fig. 28). We draw an open circle at 0 and a filled-in circle at 50.
3. The inequality $173 < w < 177$ means the values of are between 173 and 177. So, the man's weight is between 173 and 177 pounds. To graph the values of , we shade the part of the number line that lies between 173 and 177 (see Fig. 29). We draw open circles at 173 and 177.

Describing a Concept or Procedure

In some homework exercises, you will be asked to describe, in general, a concept or procedure.

Guidelines on Writing a Good Response

- Create an example that illustrates the concept or outlines the procedure. Looking at examples or exercises may jump-start you into creating your own example.
- Using complete sentences and correct terminology, describe the key ideas or steps of your example. You can review the text for ideas, but write your description in your own words.
- Describe also the concept or the procedure in general without referring to your example. It may help to reflect on several examples and what they all have in common.
- In some cases, it will be helpful to point out the similarities and the differences between the concept or the procedure and other concepts or procedures.
- Describe the benefits of knowing the concept or the procedure.
- If you have described the steps in a procedure, explain why it is permissible to follow these steps.
- Clarify any common misunderstandings about the concept, or discuss how to avoid making common mistakes when following the procedure.

Example 14 Responding to a General Question about a Concept

Describe the meaning of *variable.*

Solution

Let t be the number of hours that a person works at Starbucks in a week. The symbol t is an example of a variable because the value of t can vary. In general, a variable is a symbol that stands for an amount that can vary. A symbol that stands for an amount that does *not* vary is called a constant.

There are many benefits to using variables. We can use a variable to concisely describe a quantity; using the earlier definition of t, we see that the equation $t = 32$ means a person works at Starbucks for 32 hours in a week. By using a variable, we can also use smaller numbers to describe various years.

In defining a variable, it is important to describe its units.

GROUP EXPLORATION
Reasonable values of a variable

1. Let u be the number of units (credits or hours) a student is currently taking at your college.
   a. Which of the following values of u are reasonable in this situation? Explain.
      i. $u = 15$
      ii. $u = -5$
      iii. $u = 200$
      iv. $u = 15.5$
      v. $u = 15.1$
      vi. $u = 0$
   b. Describe all the real numbers that are reasonable values of u. Use a number line, a list of numbers, words, or some other way to describe these numbers.
2. A few months ago, a person bought a Porsche 911 Carrera Turbo for $160,700. It has a 17.7-gallon fuel tank. Let g be the amount of gasoline (in gallons) that is in the tank.
   a. Which of the following values of g are reasonable in this situation? Explain.
      i. $g = 7$
      ii. $g = 19$
      iii. $g = -4$
      iv. $g = 17.7$
      v. $g = 0$
      vi. $g = 10.392$
   b. Describe all the real numbers that are reasonable values of g.
3. The legal capacity of a club is 180 people. Let n be the number of people who are at the club. You may assume that the number of people in the club never exceeds the legal limit. Describe all the reasonable values of n.
4. Define a variable for an authentic situation in which all the reasonable values of the variable are counting numbers.
5. Define a variable for an authentic situation in which all the reasonable values of the variable are nonnegative real numbers and some of the values are not counting numbers.
6. Define a variable for an authentic situation in which all the reasonable values of the variable are real numbers and some of the values are not integers.

Tips for Success Take Notes

It is always a good idea to take notes during classroom activities. Not only will you have something to refer to later when doing the homework, but also you will have something to help you prepare for tests. In addition, taking notes makes you become even more involved with the material, which will likely increase your understanding and retention of it.

HOMEWORK 1.1

For extra help ▶ MyLab Math Watch the videos in MyLab Math

1. A(n) ________ is a symbol that represents a quantity that can vary.
2. A(n) ________ is a symbol that represents a specific number.
3. The ________ numbers are all the numbers represented on the number line.
4. ________ are quantities or categories that describe people, animals, or things.

Respond to the questions in Exercises 5–10 by using complete sentences.

5. Let s be the annual iPad® sales (in millions). The value of s is 44 for 2018 (Source: *Apple*). What does $s = 44$ mean in this situation?
6. Let p be the percentage of children ages 6–12 who participate in a team sport (organized or unorganized) on a regular basis. The value of p is about 37 for 2017 (Source: *National Physical Activity Plan Alliance*). What does $p = 37$ mean in this situation?
7. Let p be Uber's second-quarter profit (in millions of dollars). For 2018, the value of p is -891 (Source: *Bloomberg*). What does $p = -891$ mean in this situation?
8. Let T be the temperature (in degrees Fahrenheit). What does $T = -10$ mean in this situation?
9. Let t be the number of years since 2010. What does $t = 9$ mean in this situation?
10. Let t be the number of years since 2015. What does $t = -3$ mean in this situation?

For Exercises 11–16, choose a variable name for the given quantity. Give two numbers that the variable can represent and two numbers that it cannot represent.

11. The height (in inches) of a person

12. The annual salary (in thousands of dollars) of a person

13. The price (in dollars) of a video game

14. The number of students enrolled in a prestatistics class

15. The total time (in hours) a person exercises in a week

16. The temperature (in degrees Fahrenheit) in an oven

17. A rectangle has an area of 24 square inches. Let W be the width (in inches), L be the length (in inches), and A be the area (in square inches).

a. Sketch three possible rectangles of area 24 square inches.
b. Which of the symbols W, L, and A are variables? Explain.
c. Which of the symbols W, L, and A are constants? Explain.

18. A rectangle has an area of 36 square feet. Let W be the width (in feet), L be the length (in feet), and A be the area (in square feet).

a. Sketch three possible rectangles of area 36 square feet.
b. Which of the symbols W, L, and A are variables? Explain.
c. Which of the symbols W, L, and A are constants? Explain.

19. The length of a rectangle is 3 inches more than the width. Let W be the width (in inches), L be the length (in inches), and A be the area (in square inches).

a. Sketch three possible rectangles in which the length is 3 inches more than the width.
b. Which of the symbols W, L, and A are variables? Explain.
c. Which of the symbols W, L, and A are constants? Explain.

20. The length of a rectangle is twice the width. Let W be the width (in inches), L be the length (in inches), and A be the area (in square inches). [**Hint:** *Twice* means to multiply by 2.]

a. Sketch three possible rectangles in which the length is twice the width.
b. Which of the symbols W, L, and A are variables? Explain.
c. Which of the symbols W, L, and A are constants? Explain.

Graph all the given numbers on one number line.

21. $5, -2, 0, -3, 4, -1$

22. $-4, 1, -6, 2, 7, -3$

23. $-\frac{2}{3}, -1, \frac{7}{3}, 1, -\frac{5}{3}, 2$

24. $\frac{1}{4}, 0, -2, -\frac{5}{4}, \frac{9}{4}, 1$

25. $-2, 3.1, 1.2, -1.8, 0.5, 1$

26. $1, 0.2, -2.4, -0.7, 1.9, -1$

Graph the numbers on a number line.

27. Counting numbers between 3 and 8

28. Counting numbers between 1 and 5

29. Integers between -2 and 2, inclusive

30. Integers between -6 and 3, inclusive

31. Counting numbers that are at most 4

32. Negative integers that are at least -3

33. Negative integers between -4 and 4

34. Positive integers between -4 and 4

Give three examples of the following types of numbers.

35. Negative integers less than -7

36. Integers that are not counting numbers

37. Rational numbers that are not integers

38. Real numbers between -3 and -2

Among the following groups of numbers, which is the smallest group that contains any possible data for the given situation: nonnegative real numbers, integers, real numbers, and counting numbers? Explain.

39. The volume (in gallons) of water in a lake

40. The temperature (in degrees Fahrenheit) at the top of a skyscraper

41. The number of people in a household

42. The commute time (in minutes) of an employee

43. The annual profit (in dollars) of a company

44. The total number of hamburgers sold at McDonald's in some year between 2000 and 2019

For Exercises 45–50, use points on a number line to describe the given values of a variable.

45. A student goes to a college for six semesters. Here are the numbers of units (credits or hours) taken per semester: 10, 12, 6, 9, 15, 14. Let u be the number of units taken in one semester.

46. The percentages of airline flights that are on time for various years are 79%, 82%, 80%, 77%, and 76%. Let p be the percentage of flights in a year that are on time.

47. The number (in thousands) of firearm suicides for various years is 19.4, 21.3, 20.7, 21.2, and 20.0. Let n be the number (in thousands) of firearm suicides in a year.

48. The U.S. average annual per person consumption of sports drinks (in gallons) for various years is 1.9, 2.5, 2.1, 2.3, and 2.2. Let c be the U.S. average per person consumption (in gallons per year) of sports drinks in a year.

49. The low temperatures (in degrees Fahrenheit) for three days in December in Chicago are 5°F above zero, 4°F below zero, and 6°F below zero. Let F be the low temperature (in degrees Fahrenheit) for one day.

50. Here are a company's annual profits and losses for various years: loss of \$5 million, profit of \$3 million, and loss of \$8 million. Let p be the company's annual profit (in millions of dollars).

51. The revenue (in billions of dollars) of coffee in the years 2013, 2014, 2015, 2016, and 2017 is 67, 70, 75, 79, and 83, respectively (Source: *Euromonitor International*). Let r be the annual revenue (in billions of dollars) of coffee.

a. Use points on a number line to describe the given values of r.
b. Did the annual revenue increase, decrease, stay approximately constant, or none of these between 2013 and 2017, inclusive?
c. Did the annual *increases* in the annual revenue increase, decrease, stay approximately constant, or none of these between 2013 and 2017, inclusive? Explain.

52. The number of hours of video uploaded to YouTube per minute in the years 2009, 2010, 2011, 2012, 2013, and 2014 is 14, 25, 48, 73, 100, and 300, respectively (Source: *YouTube*). Let t be the number of hours of video uploaded to YouTube per minute.

a. Use points on a number line to describe the given values of t.
b. Did the number of hours of video uploaded to YouTube per minute increase, decrease, stay approximately constant, or none of these from 2009 to 2014?

c. Did the *increases* in the number of hours of video uploaded to YouTube per minute increase, decrease, stay approximately constant, or none of these from 2009 to 2014? Explain.

53. The U.S. government's share (in percent) of outstanding student debt in the years 2011, 2012, 2013, 2014, 2015, and 2016 is 90.1, 91.0, 91.6, 92.1, 92.4, and 92.5, respectively (Source: *Measure One; Education Department*). Let p be the government's share (in percent) of outstanding student debt.

a. Use points on a number line to describe the given values of p.
b. Did the U.S. government's share of outstanding student debt increase, decrease, stay approximately constant, of none of these from 2011 to 2016, inclusive?
c. Did the *increases* in the U.S. government's share of outstanding student debt increase, decrease, stay approximately constant, of none of these from 2011 to 2016, inclusive? Explain.

54. The number (in thousands) of microbreweries in the years 2013, 2014, 2015, 2016, and 2017 is 1.5, 2.1, 2.6, 3.2, and 3.8, respectively (Source: *Brewers Association*). Let n be the number (in thousands) of microbreweries.

a. Use points on a number line to describe the given values of n.
b. Did the number of microbreweries increase, decrease, stay approximately constant, or none of these from 2013 to 2017?
c. Did the *increases* in the number of microbreweries increase, decrease, stay approximately constant, or none of these from 2013 to 2017? Explain.

For Exercises 55–68, plot the given points in a coordinate system.

55. $(5, 1)$ **56.** $(2, 3)$
57. $(4, -2)$ **58.** $(3, -4)$
59. $(-5, 4)$ **60.** $(-1, 3)$
61. $(-3, -6)$ **62.** $(-5, -2)$
63. $(0, 2)$ **64.** $(0, -4)$
65. $(-3, 0)$ **66.** $(1, 0)$
67. $(2.5, -4.5)$ **68.** $(-3.5, 1.5)$

69. What is the x-coordinate of the ordered pair $(2, -4)$?

70. What is the y-coordinate of the ordered pair $(2, -4)$?

71. Find the coordinates of points A, B, C, D, E, and F shown in Fig. 30.

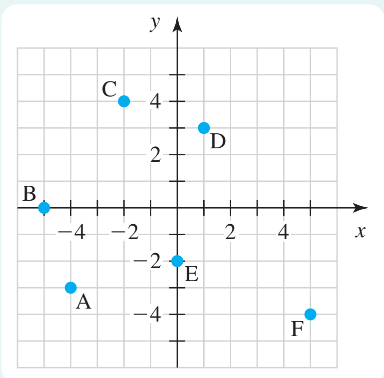


Figure 30 Exercise 71

72. Find the coordinates of points A, B, C, D, E, and F shown in Fig. 31.

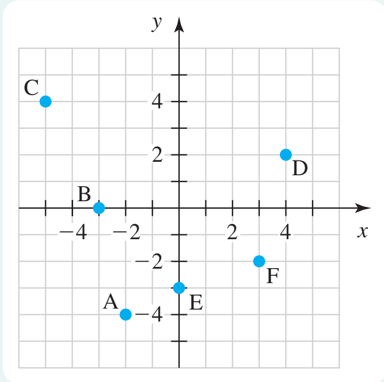


Figure 31 Exercise 72

Decide whether the given inequality is true or false.

73. $-3 > -5$ **74.** $-6 \leq -2$
75. $4 \geq 4$ **76.** $-5 < -5$

Sketch the graph of the given inequality.

77. $x < 4$ **78.** $x < -5$
79. $x \geq -1$ **80.** $x \geq 1$
81. $x \leq -2$ **82.** $x \leq 4$
83. $x > 6$ **84.** $x > -3$

For Exercises 85–90, describe the given values of x as an inequality, in interval notation, and as a graph.

85. The values of x are less than 2.
86. The values of x are greater than -5.
87. The values of x are at least -4.
88. The values of x are at most 3.
89. The values of x are no more than 6.
90. The values of x are no less than -1.
91. Use words, inequalities, graphs, and interval notation to complete Fig. 32.

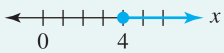
In Words	Inequality	Graph	Interval Notation
			
Numbers less than or equal to -2			
			$(-\infty, 1)$
	$x > -5$		

Figure 32 Exercise 91

92. Use words, inequalities, graphs, and interval notation to complete Fig. 33.

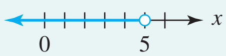
In Words	Inequality	Graph	Interval Notation
	$x \leq -6$		
Numbers greater than 1			
			$[-4, \infty)$
			

Figure 33 Exercise 92

For Exercises 93–98, sketch the graph of the given inequality.

93. $2 \le x \le 4$ **94.** $1 \le x \le 6$

95. $-2 < x < 3$ **96.** $-5 < x < 1$

97. $-6 \le x < -3$ **98.** $-5 < x \le -2$

99. Use words, inequalities, graphs, and interval notation to complete Fig. 34.

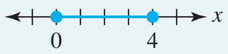
In Words	Inequality	Graph	Interval Notation
Numbers between 1 and 5			
			$[-5, 2)$
	$-2 < x \le 4$		
			

Figure 34 Exercise 99

100. Use words, inequalities, graphs, and interval notation to complete Fig. 35.

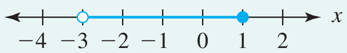
In Words	Inequality	Graph	Interval Notation
			$(-3, 0)$
	$1 \le x < 4$		
			
Numbers between -4 and -1, inclusive			

Figure 35 Exercise 100

101. Let be the average daily coffee consumption (in ounces) of a person. Graph and interpret the inequality $w > 8$.

102. Let t be the time (in minutes) it takes for a student to complete a homework assignment. Graph and interpret the inequality $t \ge 30$.

103. A person's height must be at least 54 inches to ride Six Flags roller coaster Scream (Source: *Six Flags*). Let h be the height (in inches) of a person who meets the height requirement to ride Scream. Describe the person's height using inequality notation, interval notation, and a graph.

104. A person's height must be at least 44 inches to ride Walt Disney World's roller coaster Space Mountain (Source: *Disney*). Let h be the height (in inches) of a person who meets the height requirement to ride Space Mountain. Describe the person's height using inequality notation, interval notation, and a graph.

105. Musical instruments that weigh at most 165 pounds are accepted on United Airlines flights (Source: *United Airlines*). Let be the weight (in pounds) of a musical instrument that is accepted on a United Airlines flight. Describe the instrument's weight using inequality notation, interval notation, and a graph.

106. According to the U.S. Occupational Safety and Health Administration standard, a person can listen to a sound level of 100 decibels for at most 2 hours without experiencing hearing loss. If a person listens to music at a sound level of 100 decibels without experiencing hearing loss, let T be the amount of time (in hours) the person listens to the music. Describe the amount of time the person listens to music using inequality notation, interval notation, and a graph.

107. For California roads, a vehicle must weigh no more than 80 thousand pounds (Source: *State of California*). Let be a vehicle's weight (in thousands of pounds) that meets California's weight regulation. Describe the vehicle's weight using inequality notation, interval notation, and a graph.

108. On April 12, 2018, no less than 4.2 thousand gallons of oil fuel spilled into the Mississippi River due to a vessel hitting a pier (Source: WDSU News, *4/12/18*). Let V be the volume (in thousands of gallons) of the oil spill. Describe the oil spill's volume using inequality notation, interval notation, and a graph.

109. Let be the weight of a hamburger (in ounces) served at a fast-food restaurant. Graph and interpret the inequality $1 \le w \le 3$.

110. Let M be the average gas mileage (in miles per gallon) of a car on highways. Graph and interpret the inequality $35 < M < 40$.

111. The percentage of teenagers who will have more than one episode of depression within two years is between 20% and 40%, inclusive (Source: *TeenHelp*). Let p be the percentage of teenagers who will have more than one episode of depression within two years. Describe the values of p using inequality notation, interval notation, and a graph.

112. In November 2018, the daily high temperature in Chicago was between 41 and 56 degrees Fahrenheit, inclusive (Source: *AccuWeather*). Let T be the high temperature (in degrees Fahrenheit) of a day in November 2018 in Chicago. Describe the high temperature using inequality notation, interval notation, and a graph.

113. Let d be the distance (in miles) of a person's work commute that is between 15 and 20 miles. Describe the work commute distance using inequality notation, interval notation, and a graph.

114. Let be the weight (in pounds) of a woman who weighs between 140 and 145 pounds. Describe the woman's weight using inequality notation, interval notation, and a graph.

Concepts

115. Let T be the temperature in degrees Fahrenheit.

a. What value of T represents the temperature that is 5°F below zero?

b. A student says that T represents only positive numbers and zero because there is no negative sign. Is the student correct? Explain.

116. A student says the integers between 2 and 5 are the numbers 2, 3, 4, and 5. Is the student correct? Explain.

117. How is a variable different from a constant?

118. List five ordered pairs whose y-coordinate is 2. Then plot the ordered pairs in a coordinate system. What do you notice about the arrangement of the points? Explain why this makes sense.

119. How many numbers does the inequality $2 < x < 4$ describe? List three of those numbers.

120. A student says the inequality $4 \le 4$ is false because 4 is not less than 4. What would you tell the student?

121. A student says the sentence "x is at least 5" means $x < 5$. What would you tell the student?

122. List the various types of numbers discussed in this section and describe the meanings of each type. (See page 12 for guidelines on writing a good response.)

123. Describe how to graph a negative quantity. (See page 12 for guidelines on writing a good response.)

1.2 Expressions

Objectives

- » Describe the meaning of *expression* and *evaluate an expression*.
- » Use expressions to describe authentic quantities.
- » Evaluate expressions.
- » Translate English phrases to and from mathematical expressions.
- » Evaluate expressions with more than one variable.

In this section, we will work with expressions—a very important concept in algebra and statistics.

Expressions

Addition, subtraction, multiplication, and division are examples of *operations*. In arithmetic, we perform operations with numbers. Because variables represent numbers, we can perform operations with variables, too.

Example 1 Using Operations with Variables and Numbers

Each employee at a small company receives a \$500 bonus at the end of the year. For each employee's annual salary shown, find the employee's annual salary plus bonus.

1. \$28,000 **2.** \$32,000 **3.** s dollars

Solution

1. The employee's annual salary plus bonus is $28{,}000 + 500 = 28{,}500$ dollars.
2. The employee's annual salary plus bonus is $32{,}000 + 500 = 32{,}500$ dollars.
3. In Problems 1 and 2, we added the annual salary and \$500, the bonus, to find the results. So, the employee's annual salary plus bonus (in dollars) is $s + 500$.

In Example 1, we took s to be an employee's annual salary and $s + 500$ to be the employee's annual salary plus bonus. We call s and $s + 500$ *expressions*.

Definition Expression

An **expression** is a constant, a variable, or a combination of constants, variables, operation symbols, and grouping symbols, such as parentheses.

Here are some more examples of expressions:

$$t + 6 \qquad \pi \qquad L + W - 9 \qquad y \qquad 4 \qquad 5 \div (x + 2)$$

In Example 1, we used a variable to represent a quantity from an authentic situation. Sometimes we use variables to represent numbers in a math problem that is not being used to describe an authentic situation. In this case, we often use x for the variable. For example, we could let x represent a number. In this case, x could be *any* number.

To avoid confusing the multiplication symbol $\times$ and the variable name x, we use $\cdot$ or no operation symbol to indicate multiplication. For example, each of the following expressions describes multiplying 2 by 3:

$$2 \cdot 3 \qquad 2(3) \qquad (2)3 \qquad (2)(3)$$

And each of the following expressions describes multiplying 2 by k:

$$2 \cdot k \qquad 2k \qquad 2(k) \qquad (2)k \qquad (2)(k)$$

Using Expressions to Describe Authentic Quantities

We can use expressions to describe authentic quantities. In Example 2, we will find such an expression by noticing a pattern as we calculate values of a quantity.

Example 2 Describing a Quantity

A hot-dog stand sells hot dogs for \$3 apiece. Find the total cost of buying the given number of hot dogs.

1. 2 hot dogs **2.** 5 hot dogs **3.** 8 hot dogs **4.** n hot dogs

Solution

1. Two hot dogs cost $2(3) = 6$ dollars.
2. Five hot dogs cost $5(3) = 15$ dollars.
3. Eight hot dogs cost $8(3) = 24$ dollars.
4. In Problems 1–3, we found the total cost by multiplying the number of hot dogs by 3, the cost (in dollars) per hot dog. So, if there are n hot dogs, the total cost (in dollars) is $n(3)$. We can also write the expression as $3(n)$ or $3n$.

In Example 3, we will use a table to help us find an expression that describes an authentic quantity.

Example 3 Using a Table to Find an Expression

An instructor adds 5 points to each student's test score. Find the new scores if the original scores are 60 points, 70 points, 80 points, and 90 points. Show the arithmetic to help you see the pattern. Organize the calculations in a table, and include an expression that stands for the new score if the original score is s points.

Solution

First, we construct Table 3. From the last row of the table, we see that the expression $s + 5$ represents the new score (in points) for a test with original score s points.

Table 3 Original and New Test Scores

Original Score (points)	New Score (points)
60	$60 + 5$
70	$70 + 5$
80	$80 + 5$
90	$90 + 5$
s	$s + 5$

Example 4 Using a Table to Find an Expression

A person drives at a constant speed of 75 miles per hour. Find the distance traveled in 1, 2, 3, and 4 hours of driving at that speed. Show the arithmetic to help you see the pattern. Organize the calculations in a table, and include an expression that stands for the distance traveled in t hours.

Solution

First, we construct Table 4. From the last row of the table, we see that the expression $75t$ represents the distance traveled (in miles) in t hours.

Table 4 Driving Times and Distances

Driving Time (hours)	Distance (miles)
1	$75 \cdot 1$
2	$75 \cdot 2$
3	$75 \cdot 3$
4	$75 \cdot 4$
t	$75 \cdot t$

Evaluating Expressions

In Example 4, we used $75t$ to describe the distance traveled (in miles) in t hours. This means if the driving time is 5 hours, the distance traveled is $75(5) = 375$ miles. To find the distance, we substituted 5 for t. We say we have *evaluated* the expression $75t$ for $t = 5$.

Definition Evaluate an expression

We **evaluate an expression** by substituting a number for each variable in the expression and then calculating the result. If a variable appears more than once in the expression, the same number is substituted for that variable each time.

When we evaluate an expression, it is good practice to use parentheses each time a number is substituted for a variable. For example, here we evaluate $5x$ for $x = 3$:

$$5(3) = 15$$

This strategy will be especially helpful when we evaluate an expression for a negative number, which we will begin to do in Section 1.4.

Example 5 Evaluating Expressions

1. In Example 1, we used s to represent an employee's annual salary (in dollars) and $s + 500$ to represent the employee's annual salary plus bonus (in dollars). Evaluate $s + 500$ for $s = 40{,}000$, and describe the meaning of the result.
2. In Example 2, we used n to represent the number of hot dogs bought and $n(3)$ to represent the total cost (in dollars) of n hot dogs. Evaluate $n(3)$ for $n = 4$, and describe the meaning of the result.

Solution

1. We substitute 40,000 for s in $s + 500$:

$$(40{,}000) + 500 = 40{,}500$$

So, the annual salary plus bonus is $40,500.

2. We substitute 4 for n in $n(3)$:

$$(4)(3) = 12$$

So, the total cost of 4 hot dogs is $12.

Translating English Phrases to and from Expressions

In order to use mathematics to find results for authentic situations, we must translate from English to mathematics and vice versa. To do this, the following definitions are helpful:

Definition Product, factor, and quotient

Let a and b be numbers. Then

- The **product** of a and b is ab. We call a and b **factors** of ab.
- The **quotient** of a and b is $a \div b$, where b is not zero.

For example, because $6 \cdot 3 = 18$, the number 18 is the product of 6 and 3 and the numbers 6 and 3 are factors of 18. The quotient of 6 and 3 is $6 \div 3 = 2$.

Here are some examples of English phrases or sentences and mathematical expressions that have the same meaning:

Operation	English Phrase or Sentence	Mathematical Expression
Addition	A number plus 3	$x + 3$
	The sum of a number and 3	$x + 3$
	The total of a number and 3	$x + 3$
	Add a number and 3.	$x + 3$
	3 more than a number	$x + 3$
	A number increased by 3	$x + 3$
Subtraction	A number minus 3	$x - 3$
	The difference of a number and 3	$x - 3$
	Subtract 3 from a number.	$x - 3$
	3 less than a number	$x - 3$
	A number decreased by 3	$x - 3$
Multiplication	Multiply 3 by a number.	$3x$
	3 times a number	$3x$
	The product of 3 and a number	$3x$
	Twice a number	$2x$
	One-third of a number	$\frac{1}{3}x$
Division	Divide a number by 3.	$x \div 3$
	The quotient of a number and 3	$x \div 3$
	The ratio of a number to 3	$x \div 3$

WARNING To subtract 2 from 5, we write $5 - 2$, not $2 - 5$. Suppose you have \$5 and you take \$2 from the \$5. Then you have $5 - 2 = 3$ dollars left. So, subtracting 2 from 5 is $5 - 2$.

Example 6 Translating from English to Mathematics

Let x be a number.

1. Translate the English phrase "The product of 2 and the number" into an expression.
2. Evaluate your result in Problem 1 for $x = 3$.
3. Evaluate your result in Problem 1 for $x = 7$.

Solution

1. The expression is $2x$.
2. $2(3) = 6$
3. $2(7) = 14$

Example 7 Translating from English to Mathematics

Let x be a number. Translate the English phrase or sentence into an expression. Then evaluate the expression for $x = 6$.

1. The quotient of the number and 3
2. Subtract the number from 8.

Solution

1. The expression is $x \div 3$. Next, we evaluate $x \div 3$ for $x = 6$:

$$(6) \div 3 = 2$$

2. The expression is $8 - x$. Next, we evaluate $8 - x$ for $x = 6$:

$$8 - (6) = 2$$

Example 8 Translating from Mathematics to English

Let x be a number. Translate the expression into an English phrase.

1. $6 - x$
2. $8x$

Solution

1. The difference of 6 and the number
2. The product of 8 and the number

Expressions with More Than One Variable

Some expressions used in statistics contain more than one variable. For example, in Section 4.1, we will evaluate the following key expression:

$$\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

Example 9 Translating from English to Mathematics

Write the phrase as a mathematical expression, and then evaluate the result for $x = 8$ and $y = 4$.

1. The sum of x and y
2. The quotient of x and y

Solution

1. The expression is $x + y$. Next, we evaluate $x + y$ for $x = 8$ and $y = 4$:

$$(8) + (4) = 12$$

2. The expression is $x \div y$. Next, we evaluate $x \div y$ for $x = 8$ and $y = 4$:

$$(8) \div (4) = 2$$

Example 10 Evaluating Expressions in Two Variables

1. Let and a be the numbers (in millions) of women and all people, respectively, who live alone. For 2015, the values of and a are 19.4 and 34.9, respectively (Source: *U.S. Census Bureau*). Evaluate $a -$ for $w = 19.4$ and $a = 34.9$. What does the result mean in this situation?
2. Let r be the annual revenue (in millions of dollars) of coffee and tea shops and n be the number (in millions) of customer visits, both between 2 p.m. and 4 p.m. For 2017, the values of r and n are 4259 and 876 (Source: *NPD Group*). Evaluate $r \div n$ for $r = 4259$ and $n = 876$. What does the result mean in this situation?

Solution

1. We substitute 19.4 for and 34.9 for a in the expression $a -$:

$$(34.9) - (19.4) = 15.5$$

Subtracting the number of women who live alone from the number of all people who live alone gives the number of men who live alone. So, 15.5 million men lived alone in 2015.

2. We substitute 4259 for r and 876 for n in the expression $r \div n$:

$$(4259) \div (876) \approx 4.86$$

By dividing the revenue \$4259 million into 876 million equal parts, we have found that the typical (average) customer purchase at coffee and tea shops between 2 p.m. and 4 p.m. is \$4.86.

GROUP EXPLORATION
Expressions used to describe a quantity

Consider the expression $x + 2$. Suppose that a child has grown 2 inches within the last year. We could define x to be the child's height (in inches) last year, and then $x + 2$ would be the child's current height (in inches).

Describe a situation in which x represents a meaningful quantity and the expression given describes another meaningful quantity.

1. $x + 3$
2. $x - 4$
3. $3x$
4. $x \div 2$

For each of the four expressions, evaluate it for a reasonable value of x and describe the meaning of the result.

Tips for Success Study Time

For each hour of class time, study for at least two hours outside class. If your math background is weak, you may need to spend more time studying.

One way to study is to do what you are doing now: Read the text. Class time is a great opportunity to be introduced to new concepts and to see how they fit together with previously learned ones. However, there is usually not enough time to address details as well as a textbook can. In this way, a textbook can serve as a supplement to what you learn in class.

HOMEWORK 1.2

For extra help ▶ MyLab Math Watch the videos in MyLab Math

1. A(n) ________ is a constant, a variable, or a combination of constants, variables, operation symbols, and grouping symbols, such as parentheses.
2. We ________ an expression by substituting a number for each variable in the expression and then calculating the result.
3. The product of a and b is ________.
4. The quotient of a and b is ________, where b is not zero.

For Exercises 5–16, evaluate the expression for $x = 6$.

5. $x + 2$
6. $5 + x$
7. $9 - x$
8. $x - 4$
9. $7x$
10. $x(9)$
11. $x \div 3$
12. $30 \div x$
13. $x + x$
14. $x - x$
15. $x \cdot x$
16. $x \div x$
17. For the period 2010–2016, if F is the number (in thousands) of Ford employees, then $F + 17.3$ is approximately the number (in thousands) of General Motors employees (Source: *Ford; General Motors*). There were about 60.9 thousand Ford employees in 2016. Evaluate $F + 17.3$ for $F = 60.9$. What does your result mean in this situation?
18. For the period 2007–2017, if r is the percentage of Republicans who favor gays to marry legally, then $r + 29$ is approximately the percentage of Democrats who favor gays to marry legally (Source: *Pew Research Center*). In 2017, 47% of Republicans favored gays to marry legally. Evaluate $r + 29$ for $r = 47$. What does your result mean in this situation?
19. If p percent of ballots are for a presidential candidate, then $100 - p$ percent of the ballots are for other candidates. In the 2016 presidential election, 48.2% of the ballots were for President Trump (Source: *Dave Leip's Atlas of U.S. Presidential Elections*). Evaluate $100 - p$ for $p = 48.2$. What does your result mean in this situation?
20. For the period 2012–2016, if U is the average daily shipping volume (in millions of packages) of UPS, then $U - 6$ is approximately the average daily shipping volume (in millions of packages) of FedEx (Source: *UPS; FedEx*). In 2016, UPS's average daily shipping volume was 13.5 million packages. Evaluate $U - 6$ for $U = 13.5$. What does your result mean in this situation?
21. If n million albums *Tha Carter III* are sold for \$9.99, the total revenue is $9.99n$ million dollars. In the first week, 1.05 million albums were sold (Source: *Zimbio*). Evaluate $9.99n$ for $n = 1.05$. Round your result to the second decimal place. What does your result mean in this situation?
22. If n thousand Fender Standard Jazz Electric Bass Guitars with maple fingerboards are sold for \$599.99, the total revenue is $599.99n$ thousand dollars. Evaluate $599.99n$ for $n = 17$. Round your result to the ones place. What does your result mean in this situation?
23. A researcher wants to survey an equal number of people from five age groups. If the researcher plans to survey a total of n people, then there will be $n \div 6$ people in each age group. Evaluate $n \div 6$ for $n = 300$. What does your result mean in this situation?
24. If a student earns a total of T points on four tests, then $T \div 4$ is the student's average test score (in points). Evaluate $T \div 4$ for $T = 328$. What does your result mean in this situation?
25. Each student at a community college pays a student services fee of \$20.
    a. Complete Table 5 to help find an expression that describes the total cost (in dollars) of tuition plus the services fee if a student pays t dollars for tuition. Show the arithmetic to help you see a pattern.

Table 5 Tuitions and Total Costs

Tuition (dollars)	Total Cost (dollars)
400	
401	
402	
403	
t	

    b. Evaluate the expression you found in part (a) for $t = 417$. What does your result mean in this situation?
26. A person is driving 5 miles per hour over the speed limit.
    a. Complete Table 6 to help find an expression that describes the driving speed (in miles per hour) if the speed limit is s miles per hour. Show the arithmetic to help you see a pattern.

Table 6 Speed Limits and Driving Speeds

Speed Limit (miles per hour)	Driving Speed (miles per hour)
35	
40	
45	
50	
s	

    b. Evaluate the expression you found in part (a) for $s = 65$. What does your result mean in this situation?
27. For the spring semester 2019, district residents at St. Louis Community College paid an enrollment fee of \$110.50 per credit hour (unit) (Source: *St. Louis Community College*).
    a. Complete Table 7 to help find an expression that describes the total cost (in dollars) of enrolling in n credit hours of classes. Show the arithmetic to help you see a pattern.

Table 7 Credit Hours and Total Costs

Number of Credit Hours of Courses	Total Cost (dollars)
1	
2	
3	
4	
n	

    b. Evaluate the expression you found in part (a) for $n = 15$. What does your result mean in this situation?

28. Each share of Nike Inc. stock was worth \$74.74 on January 16, 2018 (Source: *Google Finance*).

a. Complete Table 8 to help find an expression that describes the total value (in dollars) of n shares of the stock. Show the arithmetic to help you see a pattern.

Table 8 Numbers of Shares and Total Values

Number of Shares	Total Value (dollars)
1	
2	
3	
4	
n	

b. Evaluate the expression you found in part (a) for $n = 7$. What does your result mean in this situation?

29. A student has scheduled a total of 30 hours to study for her final exams and will spend an equal amount of time preparing for each one.

a. Complete Table 9 to help you find an expression that describes how much time (in hours) the student will spend studying for each final if she has n finals. Show the arithmetic to help you see a pattern.

Table 9 Numbers of Finals and Study Times

Number of Finals	Study Time (hours)
2	
3	
4	
5	
n	

b. Evaluate the expression you found in part (a) for $n = 6$. What does your result mean in this situation?

30. Some siblings contribute equal amounts of money to pay for their parents to go on a trip to Hawaii, which costs \$3000.

a. Complete Table 10 to help you find an expression that describes each sibling's share (in dollars) of the cost if there are n siblings. Show the arithmetic to help you see a pattern.

Table 10 Numbers of Siblings and Shares of Cost

Number of Siblings	Share of Cost (dollars)
2	
3	
4	
5	
n	

b. Evaluate the expression you found in part (a) for $n = 6$. What does your result mean in this situation?

31. A person pays an \$8 cover charge to hear the band Little Muddy.

a. Find an expression for the total cost (in dollars) of the cover charge and d dollars spent on drinks. [**Hint:** If you have trouble finding the expression, construct a table of values for d and the total cost.]

b. Evaluate the expression you found in part (a) for $d = 14$. What does your result mean in this situation?

32. A person pays \$10 for parking at an arts-and-crafts fair.

a. Find an expression for the total cost (in dollars) of parking and v dollars spent on a vase. [**Hint:** If you have trouble finding the expression, construct a table of values for v and the total cost.]

b. Evaluate the expression you found in part (a) for $v = 25$. What does your result mean in this situation?

33. To make fudgelike brownies, a person bakes a brownie mix for 5 minutes less than the baking time suggested on the box.

a. Find an expression for the actual baking time (in minutes) if the suggested baking time is s minutes. [**Hint:** If you have trouble finding the expression, construct a table of values for s and the actual baking time.]

b. Evaluate the expression you found in part (a) for $s = 23$. What does your result mean in this situation?

34. A company offers a \$2 mail-in rebate on a shaver. The *retail price* of a shaver is the price paid at the store (not including the \$2 rebate). The *net price* is the price of the shaver, taking into account the \$2 rebate.

a. Find an expression for the net price (in dollars) of a shaver whose retail price is r dollars. [**Hint:** If you have trouble finding the expression, construct a table of values for r and the net price.]

b. Evaluate the expression you found in part (a) for $r = 6$. What does your result mean in this situation?

35. A Punisher 9001 Cherry Blossom Skateboard costs \$49.99.

a. Find an expression that describes the total revenue (in dollars) from selling n skateboards.

b. Evaluate the expression you found in part (a) for $n = 259$. Round your result to the ones place. What does your result mean in this situation?

36. For Oregon students, Chemeketa Community College charged \$105 per credit (unit or hour) for tuition in spring term 2019.

a. Find an expression that describes the total cost (in dollars) of tuition for enrolling in c credits of classes.

b. Evaluate the expression you found in part (a) for $c = 15$. What does your result mean in this situation?

37. A total of n friends contribute equal amounts of money to pay for a party, which costs \$400.

a. Find an expression that describes each friend's contribution (in dollars).

b. Evaluate the expression you found in part (a) for $n = 8$. What does your result mean in this situation?

38. A total of n siblings each receive an equal share of a \$420 thousand inheritance.

a. Find an expression that describes each sibling's share (in thousands of dollars) of the inheritance.

b. Evaluate the expression you found in part (a) for $n = 3$. What does your result mean in this situation?

For Exercises 39–48, let x be a number. Translate the English phrase or sentence into a mathematical expression. Then evaluate the expression for $x = 8$.

39. The number plus 4

40. 8 minus the number

41. The quotient of the number and 2

42. Add 6 and the number.

43. Subtract 5 from the number.

44. 15 more than the number

45. The product of 7 and the number

46. The difference of the number and 7

47. 16 divided by the number

48. Multiply the number by 5.

Let x be a number. Translate the expression into an English phrase.

49. $x \div 2$ **50.** $6 \div x$ **51.** $7 - x$ **52.** $x - 2$

53. $x + 5$ **54.** $4 + x$ **55.** $9x$ **56.** $x(5)$

57. $x - 7$ **58.** $x + 3$ **59.** $x(2)$ **60.** $x \div 5$

Evaluate the expression for $x = 6$ and $y = 3$.

61. $x + y$ **62.** $y + x$ **63.** $x - y$

64. xy **65.** yx **66.** $x \div y$

For Exercises 67–70, translate the phrase into a mathematical expression. Then evaluate the expression for $x = 9$ and $y = 3$.

67. The product of x and y

68. The sum of x and y

69. The difference of x and y

70. The quotient of x and y

71. Let U and I be the numbers of Dunkin' Donuts stores in the United States and outside the United States, respectively. For 2017, the values of U and I are 9141 and 3397, respectively (Source: *Dunkin' Brands*). Evaluate $U + I$ for $U = 9141$ and $I = 3397$. What does your result mean in this situation?

72. Let c and r be the average annual per-person consumptions (in pounds per person) of chicken and red meat, respectively. For 2015, the values of c and r are 90.0 and 104.8, respectively (Source: *U.S. Department of Agriculture*). Evaluate $c + r$ for $c = 90.0$ and $r = 104.8$. What does your result mean in this situation?

73. Let d and a be the annual revenues (in billions of dollars) from digital music and all music, respectively. For 2015, the values of d and a are 6.7 and 12.5, respectively (Source: *IFPI*). Evaluate $a - d$ for $d = 6.7$ and $a = 12.5$. What does your result mean in this situation?

74. Let and a be the college enrollments (in millions of students) of women and all students, respectively. For 2015, the values of and a are 11.26 and 19.98 (Source: *National Center for Education Statistics*). Evaluate $a -$ for $w = 11.26$ and $a = 19.98$. What does your result mean in this situation?

75. Let b be the average monthly cell phone bill (in dollars per month) and n be the number (in millions) of cell phone subscribers. For 2015, the values of b and n are 44.65 and 377.9, respectively (Source: *Cellular Telecommunications & Internet Association*). Evaluate bn for $b = 44.65$ and $n = 377.9$. Round your result to the ones place. What does your result mean in this situation?

76. Let N be the number (in millions) of students who take AP exams and A be the average number of AP exams taken per student (Source: *The College Board*). For 2016, the values of N and A are 2.5 and 1.8, respectively. Evaluate NA for $N = 2.5$ and $A = 1.8$. What does your result mean in this situation?

77. Let d be the total household debt (in millions of dollars) in the United States and n be the number (in millions) of households in the United States. For 2017, the values of d and n are 12,730,000 and 126.22 (Source: *New York Fed Consumer Credit; U.S. Census Bureau*). Evaluate $d \div n$ for $d = 12{,}730{,}000$ and $n = 126.22$. Round your result to the ones place. What does your result mean in this situation?

78. Let s be the total money (in millions of dollars) earned by teachers and n be the number (in millions) of teachers. For 2014, the values of s and n are 205,200 and 3.6, respectively (Source: *National Center for Education Statistics; National Education Association*). Evaluate $s \div n$ for $s = 205{,}200$ and $n = 3.6$. What does your result mean in this situation?

Concepts

79. **a.** Evaluate $6 + x$ for $x = 1, x = 2,$ and $x = 3$.
b. Evaluate $6x$ for $x = 1, x = 2,$ and $x = 3$.
c. A student says that the expressions $6 + x$ and $6x$ are the same thing. What would you tell the student?

80. **a.** Evaluate $x + 2$ for $x = 4, x = 5,$ and $x = 6$.
b. Evaluate $2x$ for $x = 4, x = 5,$ and $x = 6$.
c. A student says that the expressions $x + 2$ and $2x$ are the same thing. What would you tell the student?

81. A person gets paid $15t$ dollars for t hours of work.
a. Evaluate $15t$ for $t = 1, t = 2, t = 3,$ and $t = 4$. Describe the meaning of your results.
b. Refer to the results you found in part (a) to determine how much the person gets paid per hour. Explain.
c. Compare the result you found in part (b) with the expression $15t$. What do you notice?

82. The total price of n loaves of bread is $3n$ dollars.
a. Evaluate $3n$ for $n = 1, n = 2, n = 3,$ and $n = 4$. Describe the meaning of your results.
b. Refer to the results you found in part (a) to determine the cost per loaf of bread. Explain.
c. Compare the result you found in part (b) with the expression $3n$. What do you notice?

83. A person drives $50t$ miles in t hours.
a. Evaluate $50t$ for $t = 1, t = 2, t = 3,$ and $t = 4$. Describe the meaning of your results.
b. Refer to the results you found in part (a) to determine at what speed the person is traveling. Explain.
c. Compare the result you found in part (b) with the expression $50t$. What do you notice?

84. An elevator rises $2t$ yards in t seconds.
a. Evaluate $2t$ for $t = 1, t = 2, t = 3,$ and $t = 4$. Describe the meaning of your results.
b. Refer to the results you found in part (a) to determine at what speed the elevator is rising. Explain.
c. Compare the result you found in part (b) with the expression $2t$. What do you notice?

85. Compare the meaning of *variable* with the meaning of *expression*. (See page 12 for guidelines on writing a good response.)

86. Give an example of an expression containing a variable, and then evaluate it three times to get three different results.

87. Describe an authentic situation for the expression $8x$. Include a definition for the variable x in your description.

88. Describe an authentic situation for the expression $200 \div x$. Include a definition for the variable x in your description.

1.3 Operations with Fractions and Proportions; Converting Units

Objectives

- Describe the meaning of a fraction.
- Describe the rules for $a \cdot 1$, $\frac{a}{1}$, and $\frac{a}{a}$.
- Multiply fractions.
- Simplify fractions.
- Divide fractions.
- Add fractions.
- Subtract fractions.
- Simplify complex fractions.
- Find proportions.
- Convert units of quantities.

In this section, we will perform operations with fractions, which are used in numerous fields, including music, social science, business, engineering, and statistics.

Meaning of a Fraction

A fraction can be used to describe a part of a whole. For example, consider the meaning of $\frac{3}{4}$ of a pizza. If we divide the pizza into 4 slices of *equal* area, 3 of the slices make up $\frac{3}{4}$ of the pizza (see Fig. 36).

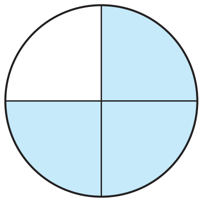


Figure 36 $\frac{3}{4}$ of a pizza

WARNING

Even though the orange region in Fig. 37 is 1 of 3 parts, it is *not* $\frac{1}{3}$ of the pizza because the 3 parts do not have equal area. The orange region *is* equal to $\frac{1}{2}$ of the pizza because it is 1 of 2 parts of equal area that make up the pizza (see Fig. 38).

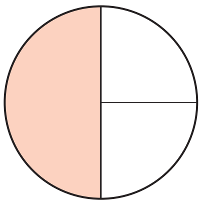


Figure 37 The 3 parts do not have equal area

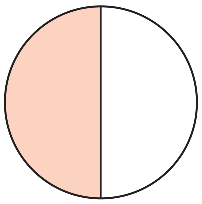


Figure 38 The 2 parts have equal area, so the orange part is $\frac{1}{2}$ of the pizza

The fraction $\frac{a}{b}$ means $a \div b$. For example, $\frac{8}{4} = 8 \div 4 = 2$. So, 8 quarters of pizza make 2 pizzas with 4 slices each (see Fig. 39).

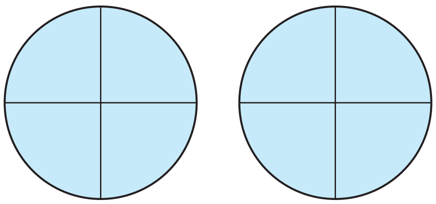


Figure 39 The 8 quarters of pizza make 2 pizzas

We can think of division in terms of repeated subtraction. For example, $17 \div 5$ is equal to 3 with a remainder of 2 (try it). This means if we subtract 5 from 17 three times, the result is 2 (the remainder):

$$17 - 5 = 12, \qquad 12 - 5 = 7, \qquad 7 - 5 = 2$$

Note that the remainder, 2, is less than the divisor, 5.

As a matter of fact, the remainder must always be less than the divisor. This rule will help us see that division by 0 is undefined. For example, consider $8 \div 0$. No matter how many times we subtract 0 from 8, the result is always 8:

$$8 - 0 = 8, \qquad 8 - 0 = 8, \qquad 8 - 0 = 8, \text{ and so on}$$

If $8 \div 0$ is defined, the remainder would have to be the repeated result 8. Because the remainder must be less than the divisor, it is implied that 8 is less than 0, which is false. So, $8 \div 0$ is undefined. In fact, any number divided by 0 is undefined.

Division by Zero

The fraction $\frac{a}{b}$ is undefined if $b = 0$. Division by 0 is undefined.

For example, $\frac{6}{0}$ is undefined. If you use a calculator to divide by 0, the screen will likely display "Error," "ERR:," "E," or "ERR: Divide by 0" to indicate that division by 0 is undefined.

WARNING

However, the fraction $\frac{0}{6}$ *is* defined. In fact, $\frac{0}{6} = 0$. For example, if a person eats zero sixths of a pizza, this means that the person didn't eat any pizza.

Rules for $a \cdot 1$, $\frac{a}{1}$, and $\frac{a}{a}$

The products $4 \cdot 1 = 4$, $5 \cdot 1 = 5$, and $8 \cdot 1 = 8$ suggest the following property:

Multiplying a Number by 1

$$a \cdot 1 = a$$

In words: A number multiplied by 1 is that same number.

When we write statements such as $a \cdot 1 = a$, we mean if we evaluate $a \cdot 1$ and a for *any* value of a in both expressions, the results will be equal. We say that the expressions $a \cdot 1$ and a are **equivalent expressions**.

The quotients $\frac{4}{1} = 4 \div 1 = 4$, $\frac{5}{1} = 5 \div 1 = 5$, and $\frac{8}{1} = 8 \div 1 = 8$ suggest the following property:

Dividing a Number by 1

$$\frac{a}{1} = a$$

In words: A number divided by 1 is that same number.

Finally, the quotients $\frac{4}{4} = 4 \div 4 = 1$, $\frac{5}{5} = 5 \div 5 = 1$, and $\frac{8}{8} = 8 \div 8 = 1$ suggest the following property: