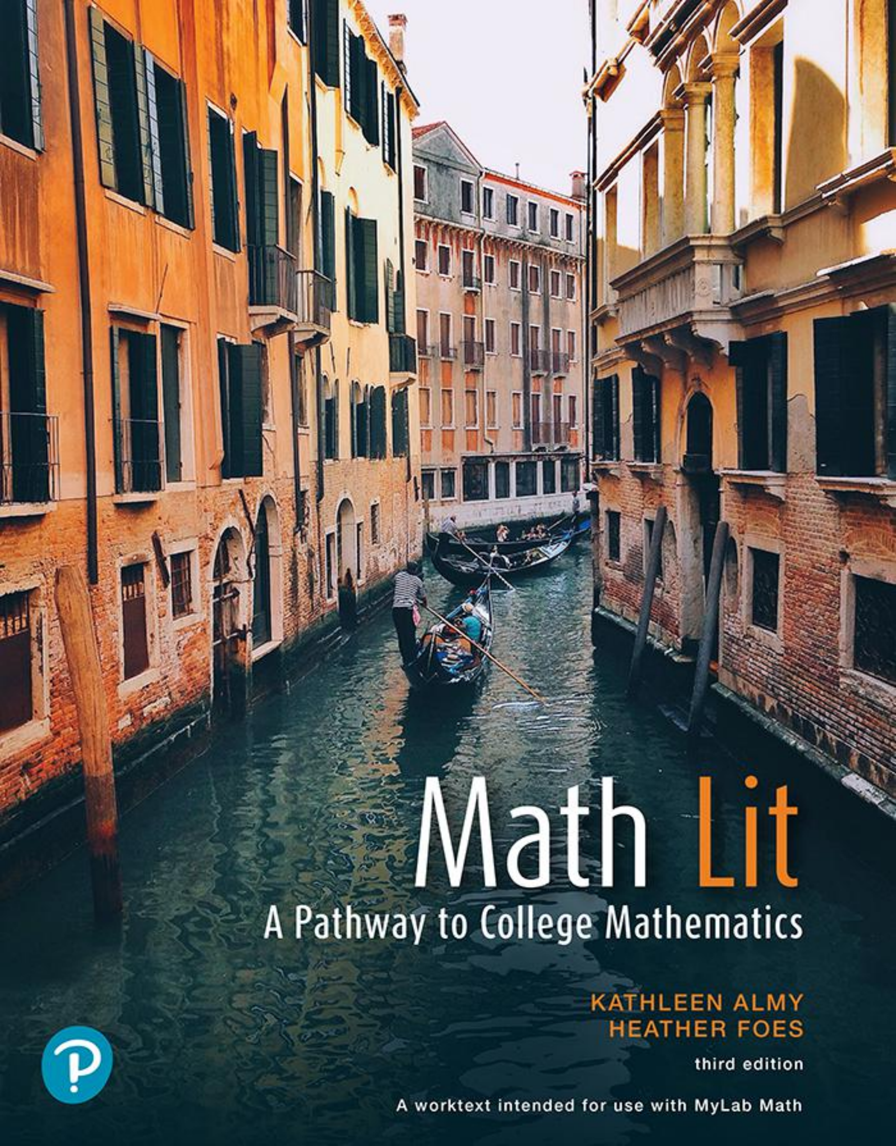

# Math Lit

## A Pathway to College Mathematics

KATHLEEN ALMY
HEATHER FOES

third edition

A worktext intended for use with MyLab Math

# Math Lit

## A Pathway to College Mathematics

third edition

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**That which we persist in doing becomes easier for us to do;
not that the nature of the thing itself is changed,
but that our power to do is increased.**

—Ralph Waldo Emerson

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# Letter to the Student

Welcome to a new pathway to college mathematics! This book is not simply a repetition of the math you had in high school. Instead, it has been written to prepare you for success in your next mathematics course, such as statistics or quantitative literacy. While *Math Lit: A Pathway to College Mathematics,* Third Edition, does include the algebra you will need to be successful in a follow-up course, it should be immediately apparent that this is not an algebra book. Instead, the focus is on solving realistic problems, gaining number sense, and building mathematical literacy. Algebra is a tool that we use when needed, but it is not the focus of the book. When we study algebraic ideas, our goal will be to understand how and when to use those ideas.

Some topics in the book might be familiar to you, but the approach to the topics and the order in which they are presented will likely not be what you have seen in the past. We have spiraled topics throughout the book, introducing a topic and then revisiting it several times, going a little more in depth each time. This approach will help you achieve a better understanding of the material, which in turn will help you to remember it longer and make use of it when needed. Most topics are explored in context before all the procedures and details are given. The goal of this book is not simply for you to gain a set of skills. We also want to work on your understanding of concepts so that they can be used to solve other problems. It's not just "Can I perform this skill?" but instead "Do I understand what I'm doing?" and "Can I use this skill?"

Our approach will require that you are an active participant in class activities and your own learning. Skill-based homework can be done in MyLab Math®, which will offer you instant feedback, multiple tries on problems, and learning aids to be used when and if you need them. Conceptual homework can be done in the worktext after you have completed the MyLab Math® problems, thus allowing you to work with the subtle details of the content and improve your understanding beyond the procedural level.

When using this book, you will likely work harder than you have before, but you may also gain more than you have in previous math classes. During your journey, you will learn more than just mathematics. You will learn how to be successful in college and beyond, and what you specifically need to do to achieve your goals.

We hope you will bring an open mind to this course and a willingness to explore how things work. If you are willing to work, your work will often be rewarded with the feeling of accomplishment that comes from truly figuring something out for yourself!

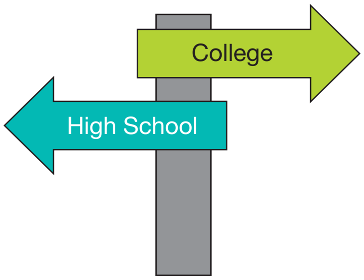
We hope you enjoy *Math Lit* and find the experience rewarding. Let's get started.

It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment.
—Carl Friedrich Gauss

Kathleen Almy

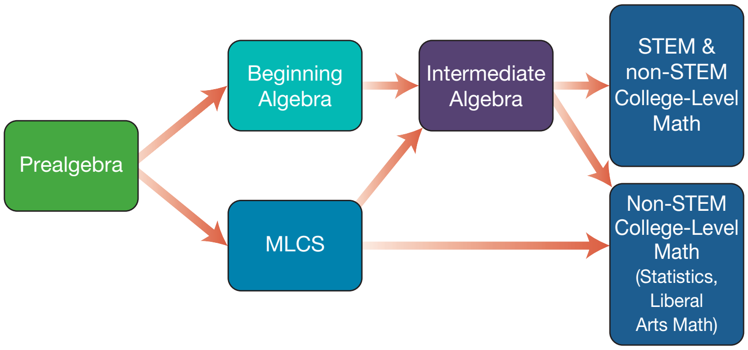
Heather Foes

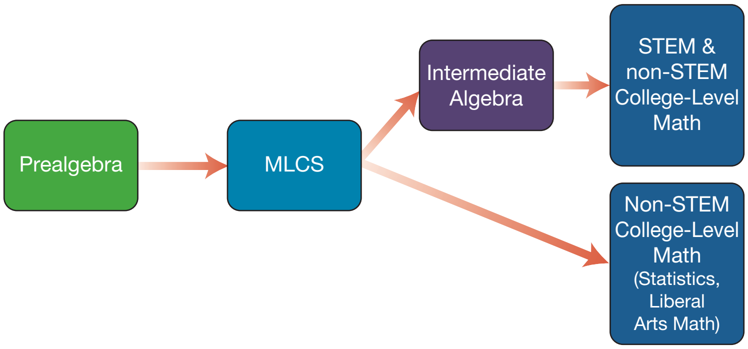
# Letter to the Instructor

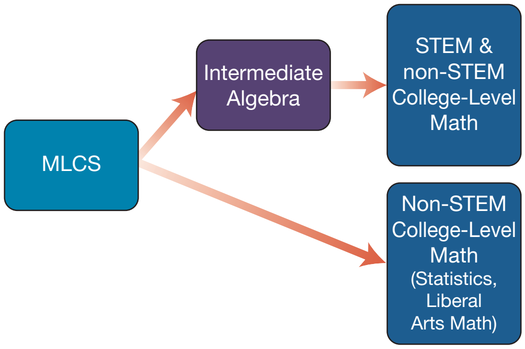



## A New Pathway That Looks Forward to College Math

This text supports pathways courses such as Mathematical Literacy for College Students (MLCS), which typically is a one-semester course for non-math and non-science majors. MLCS integrates numeracy, algebraic reasoning, proportional reasoning, and functions. The focus of this course is on developing mathematical maturity through problem solving, critical thinking, writing, and communicating mathematics. Additionally, college success components are integrated with the mathematical topics. During this course, students will develop conceptual and procedural tools that support the use of key mathematical concepts in a variety of contexts, including statistics and geometry. Upon completion of the course, students will be prepared for a statistics course, a general education mathematics course, a quantitative literacy course, or further algebra. The prerequisite to MLCS is usually basic math or prealgebra. The course can be added to the traditional developmental algebra sequence as an alternative pathway, can be used to replace beginning algebra or can be used to replace both prealgebra and beginning algebra.



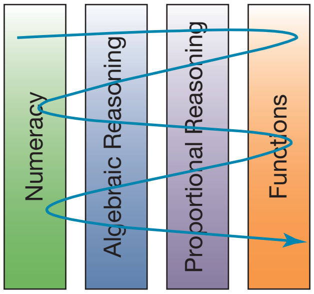



The title of this book, *Math Lit*, is an abbreviation of the course's name, "Mathematical Literacy for College Students." It is also a play on words, giving a nod to the approach taken in the text: the connection between mathematics and the written word. The majority of problems in the book are presented in a verbal format and are not overly formulated with mathematical commands like "solve" or "simplify." Additionally, each cycle contains an article to support and develop mathematical content.

# A New Paradigm

In every facet of the book, we threw out the traditional rules and wrote new ones. It's an unconventional approach but one that works. Let's explore the structure, philosophy, and features of the book.

## The Use of Cycles




Instead of linear chapters, content is developed in an integrated fashion through cycles, which increase in depth as the course progresses. Cycles are built around a central question that underlies the activities and ties the sections together into a cohesive unit. Cycles 1–4 include components from each of these four strands: numeracy, algebraic reasoning, proportional reasoning, and functions. Geometry, statistics, student success, and mathematical success are recurring themes in each cycle. A topic is usually not developed to completion the first time it is presented. Often we revisit a skill or concept but go deeper than we had previously in order to apply skills previously gained as well as to look at the topic from a fresh perspective. This layered approach helps students make connections and transfer the new knowledge into long-term memory.

## A New Approach

> The difficulty lies not so much in developing new ideas as in escaping from old ones.
> —John Maynard Keynes

Most students in developmental math have had years of mathematics and, often, algebra. Our goal with this text is to approach developmental math in a way that is different from the traditional, skill-based model. To do so, we have used a different section structure, topic ordering, and content development than a traditional text uses. While this approach will be unfamiliar at first, students will quickly adjust to seeing mathematics differently, and many times they end up liking what they see.

The content, techniques, and contexts chosen are meant to be relevant for an adult learner who needs more mathematical skills before taking a college-level math class. Instead of repeating topics seen in high school in the same way and order, we approach each topic with two questions: how does it work and how can I use it?

## An Intentional Development: Less Is More

The text develops fewer topics than a traditional developmental math text, but many topics are revisited more than once. To develop number sense, we begin solving many problems with a numeric approach, working with numbers as long as it makes sense to do so. When that becomes cumbersome, we move on to other techniques such as algebraic approaches using equations or graphs. Proportional reasoning is used early and often. Regardless of the mathematical topic, we emphasize three core components: skills, concepts, and applications.

The goal of the text is not to present students with a list of skills that can be checked off during the course but instead the larger, holistic goals of mathematical maturity, problem-solving prowess, critical thinking skills, and college readiness. We want students to solve real problems and see the beauty and power of mathematics instead of seeing math as a discrete list of skills to be gained. To this end, we address the ideas of student success and mathematical success often.

These tenets are embodied throughout the text:

- Depth over breadth
- Quality over quantity
- Why before how

Having knowledge but lacking the power to express it clearly is no better than never having any ideas at all.
—Pericles

Processes are valued, but answers and methods need not be "one size fits all." This theme will appear often in sections and assessments. Metacognition is emphasized throughout the text so that students understand how they think and learn. To do this and solidify mathematical understanding, we tackle most problems from multiple perspectives. This way, students learn more about the skills and processes and also find the techniques that make the most sense to them.

## The Role of Algebra

Students need not only enough new and different concepts and/or contexts to remain engaged and interested but also some familiar concepts to ease their anxieties. With this in mind, all algebra topics are approached in novel ways. Algebra itself is not the goal but is rather another method that can be used to solve problems. This philosophy is very inviting to students. When they are not required to use algebra, students will often choose to use it because of its organizational and time-saving strengths.

If covered in its totality, the text will expose students to nearly all beginning algebra concepts. Students will also see many intermediate algebra ideas like quadratic and exponential functions, logarithmic scales, and function concepts like domain and range. With this approach, students are prepared to transition to a traditional intermediate algebra course if needed.

Factoring is included with a focus on using it for rewriting expressions with the greatest common factor or understanding the process of factoring trinomials. Additional factoring practice is available in MyLab Math to support students heading to intermediate algebra or non-STEM college-level courses that require factoring.

## The Role of Technology

The approach to technology in this text is the same approach used in the workplace: use the tool that makes the most sense for the job at hand. Sometimes a scientific or graphing calculator is needed. Other times Microsoft Excel® is used for a spreadsheet. When mental calculations are possible and faster, we encourage students to use them. And sometimes the best tool is just a pencil.

*Tech Tips* are provided throughout the text to guide students with the use of technology.

A spreadsheet icon is included next to problems that are well suited to be solved with Excel. However, the use of spreadsheets is optional and can be easily omitted.

Instructions that are provided for graphing calculators reference TI calculators. Spreadsheet instructions support Microsoft Excel®. You can adjust the instructions if you are using a different brand of calculator or software.

## Thinking Outside the Box, Writing Inside It

The text uses a conversational style that is designed to be readable and inviting. It is presented in worktext format to fulfill a primary goal of the course: student engagement. We have found through years of classroom testing that students enjoy using a worktext that seamlessly integrates theory with their notes. This format allows students to write only what is needed and thus to pay more attention to doing math than taking notes. Since key terms, examples, and procedures are called out in boxes and in a summary at the end of each cycle, students have a useful reference when studying. Instead of a book that is not opened or used, the worktext becomes a dynamic document that is added to in each class period.

## A Flexible Approach

The text supports activity-based courses as well as courses that include all direct instruction or some direct instruction with some group work. The Instructor Guide that is included with the Instructor's Edition provides more information on ways to use the text with particular formats. MyLab Math is used for homework and supports classroom activities, but it does not replace instruction. It does, however, provide the additional practice with skills that some students need but may not get in the classroom due to time constraints.

## Productive Struggle

All truths are easy to understand once they are discovered; the point is to discover them.

—Galileo Galilei

We need to allow students to think, struggle, and question. It's at those moments that they make connections and develop understanding. Countries with successful mathematics education regularly encourage this process instead of reducing each topic to a set of procedures. Allowing students to struggle in a productive way will also increase their persistence and perseverance, two components necessary for success not only in this course but also in college as a whole.

We are asking students to accept a new way of learning and a new interaction between them and the instructor. Likewise, we ask the instructor to learn a new dynamic with students, which at times is unfamiliar. Similar to what we ask of students, we ask you to accept this temporary discomfort because the rewards will be plentiful and evident in short order.

## How Will Students Transition to a Traditional Environment?

Students usually transition very well from problem-solving environments like the one encouraged by this book to more traditional classrooms. After an experience like an MLCS course, students will have strategies for facing challenges, evaluating their work, and solving nonroutine problems. A traditional environment, with a linear progression of topics, can be easier, although less engaging, than what they experience in MLCS. Further, that traditional environment is what they have likely had in most of their previous classes. Thus this transition is a return to the familiar, not the unfamiliar.

## Built by Faculty for Faculty

This textbook presents a new way of teaching and learning, so both the instructor and the student might have an initial period of adjustment. Acknowledging this, we designed the text to assist the instructor and the student with its nontraditional structure. Each section has a predictable rhythm, as does each cycle. Additionally, all sections are annotated with answers as well as notes to the instructor regarding pedagogy, common issues, and strategies to consider. We have also created an Instructor Guide, which is included in the Instructor's Edition of this text, to assist you with teaching your course.

# Organization of the Book and Sections

This book is divided into cycles instead of chapters because there is a rhythm to each cycle of introducing a focus problem, building up skills and understanding, connecting concepts, and then wrapping up with a test and a solution to the focus problem. Instead of a linear, skill-based approach, we move through a variety of problems while simultaneously completing a cycle. Composed of open-ended problems, active examples, interactive problems, and assessments, the cycles provide a rich and interesting arena for developing mathematical muscle.

Each of cycles 1–4 is divided into halves. Each half contains a set of cohesive sections to develop mathematical skills and concepts. A *Mid-Cycle Recap* is provided at the halfway point of the cycle as a way for students to check on their progress. This approach is used to provide predictability and structure. Also at the midpoint of the cycle, an *In the News* is provided to allow students the opportunity to read several articles and answer questions pertaining to the mathematics involved. At the end of the cycle, the *Cycle Wrap-Up* helps students to reflect on what they have learned to date and to practice questions that are at test-level difficulty. Cycle 0 has been included to provide review of prealgebra topics. The cycle is the length of half a cycle and includes all cycle components except for the *Mid-Cycle Recap*.

The cycles and their components help students increase their mathematical and academic success. This is accomplished by carefully designed tasks, explorations, and instruction that are assembled and paced in an effective way for the developmental learner.

## Section Format

Because the goal of the course is for students to develop mathematical maturity more than a specific skill set, the goal of the sections is the creation of an experience. Over time, content is spiraled to elicit connections and deepen understanding across contexts. By the end of the course students should feel that they have partaken in an adventure whose journey was just as valuable as the destination.

Many sections economize on precious class time to accomplish more than just mathematical goals. Often, a skill will be developed while exploring additional concepts or problem-solving strategies. Students in developmental classes usually need time to work on and make sense of ideas. Depth, not breadth, is the goal.

The sections within the cycles are divided into the following:

**Explore** An interesting problem opens the section and sets the stage for the new material.

**Discover** A new theory is presented, with examples and practice problems.

**Connect** A problem connects the content of the section to past or future sections.

**Reflect** An opportunity is given to look back at what has been learned.

## Focus Problems

The value of a problem is not so much coming up with the answer as in the ideas and attempted ideas it forces on the would-be solver.
—I. N. Herstein

For each cycle, there is a focus problem that students can solve in groups. The focus problems are current, real-world issues that are not boiled down and formulated for students. They are challenging but ultimately engaging and accessible. A section at the start of each cycle introduces students to the focus problem and how it relates to the content of the cycle. Periodic sticky notes are provided to help students make progress solving the focus problem, writing up a solution, and making sense of the solution prior to the cycle test. Focus problems allow students to apply knowledge gained during the cycle to a larger, more involved problem that does not have just one correct solution. Students learn to work with a group, understand new contexts, and write a solution in a coherent way.

Additional focus problems are available in MyLab Math. Writing templates for students and grading rubrics are included along with a detailed sample solution. More information about using the focus problems is provided in the Instructor Guide.

## Section Features

Within the sections, the following special features recur:

**HOW IT WORKS**

*How It Works* boxes summarize procedures and provide worked-out examples of them.

**To evaluate a formula:**

1. Replace the variables in the formula with the appropriate numerical values.
   a. When replacing a variable with a negative number, use parentheses around the number.
   b. State the units with the numerical values if appropriate.
2. Use the order of operations rules to simplify the expression.
3. State the result, with units if appropriate.

**EXAMPLE:** Use the formula $F = \frac{9}{5}C + 32°$ to convert 100° Celsius to Fahrenheit.

Begin by replacing $C$ with 100° and then simplify.

$$F = \frac{9}{5}(100°) + 32°$$

$$F = 180° + 32°$$

$$F = 212°$$

So 100° Celsius is the same as 212° Fahrenheit.

LOOK IT UP

*Look It Up* boxes present key terms, along with an example.

## Relative Frequency

Frequency is the number of times an event occurs. **Relative frequency** is the ratio formed when the frequency is divided by the total number.

**FOR EXAMPLE,** if 15 women in a group of 75 were diagnosed with cancer, then the relative frequency of getting cancer in that group is $\frac{15}{75} = 0.2 = 20\%$. A relative frequency can be expressed as a fraction, decimal, or percent.

*Tech Tip* notes contain helpful hints for using calculators or other technologies.

TECH TIP

You may need two sets of parentheses to enter the formula in #5 into your calculator in one step. One set is shown in the formula, and the other set goes around the exponent if it is written as a product.

*Remember* notes are included for skills that have been presented already but might need to be reviewed.

**Remember?**

To increase a number by 20%, multiply by 1.20.

**New!**

*Active Examples* provide a framework for student solutions that can be used with teacher guidance or by students on their own.

## ACTIVE EXAMPLE

Simplify $\frac{18 - 8 \cdot 4^2}{3}$ using the order of operations.

### GUIDED SOLUTION

To begin, notice that the division bar is a grouping symbol. It groups the numerator as one expression and the denominator as another expression. Each of these expressions is simplified separately before dividing.

Apply the exponent. $\frac{18 - 8 \cdot 16}{3}$

Perform the multiplication. $\frac{18 - 128}{3}$

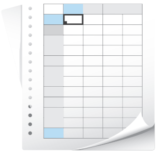
Subtract in the numerator. $-\frac{110}{3}$

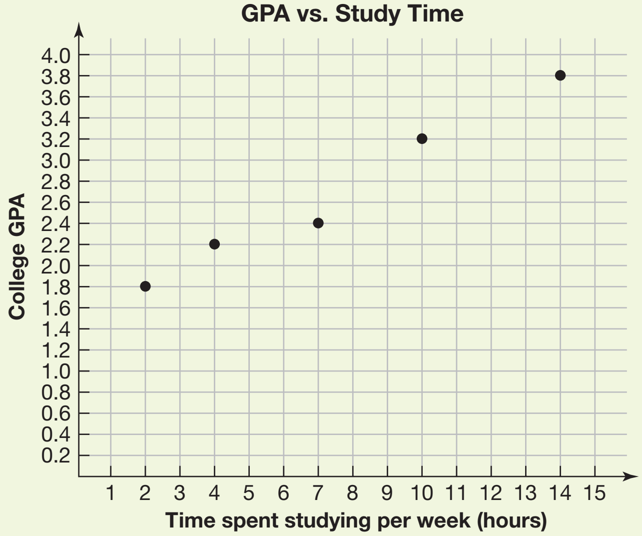
Because there are no common factors in the numerator and denominator, other than 1, the fraction is simplified.

*Sticky Notes* are written to students with study tips and other information to address both student and mathematical success.

Remember that the commutative property involves changing order, and the associative property involves regrouping.

*Spreadsheet* icons identify problems well suited for use with Microsoft Excel®.






The scatterplot shows that college students who spend more hours per week studying tend to have higher GPAs.

# Changes for the Third Edition

The overall structure of the book will be very familiar to long-term users. The four cycles and their content are largely intact, although a new Cycle 0 has been added to address the review of prealgebra content. The main changes to this edition have been made at the section level, where we have improved flow and consistency across sections as detailed below. In some sections, development of certain topics was expanded for better clarity, while development of others was condensed for improved flow. Nearly all content from the second edition is included in the third edition, but many contexts, topics, and problems have been updated to reflect current events.

## Section Changes

To improve the experience for both the student and the instructor, the following changes were made in each section.

- The *Worked-out Examples* that were added in the second edition have been converted into *Active Examples*. Full solutions were removed from the student text so that students in class can be guided by their instructors, actively working through them rather than passively reading them. Complete text and video solutions to these are available for students in the eText in MyLab Math or available at www.pearson.com/math-stats-resources.
- Icons suggesting group, whole class, or individual work along with time estimates were added to the *Explore, Discover, Connect,* and *Reflect* parts of a section, visible only to the instructor. These icons were in the first edition and some instructors expressed interest in having them again, although they are fewer in number to be less intrusive to those who do not need them.
- The amount of practice was evaluated and increased if needed by the addition of problems in the section or the addition of *Need More Practice?* margin notes.

## Cycle Changes

- The *Getting Ready* feature from the previous edition was replaced with *In the News* that appears after the *Mid-Cycle Recap*. Because the *In the News* feature is not tied to a particular section, instructors have more flexibility to use it at any point in the cycle. Instead of including a full article in the text, citations for multiple articles are provided so instructors have options. Questions are included for students to answer.
- A new focus problem has been written for each cycle, but all previous focus problems are available to instructors in MyLab Math. New focus problems include the package with rubric, writing template, and sample solution for instructors.

## Changes to Content

- A new Cycle 0 was written to cover the following prealgebra topics: types of real numbers, place value and rounding, integers, fractions, decimals, and percents. Options for using Cycle 0 are included in the Instructor Guide. Some content from Cycle 1 was moved to Cycle 0. Specifically, the following changes were made.
  - The details of fraction operations are now covered in Cycle 0. Section 1.2 focuses on applications of fractions with only a brief review of operations.
  - Two sections on integers and their operations were moved from Cycle 1 to Cycle 0.
  - The content on finding percent change was moved to Cycle 0. Application of percent change and a deeper discussion of the multiplier remain in Cycle 1 (Section 1.10).

- In Cycle 3, *More or Less: Linear Relationships* has been moved after *Get in Line: Slope-Intercept Form*.
- The three sections in Cycle 3 on quadratics have been grouped together. This change places the sections on systems of equations together at the end of the cycle. These changes make it easier to omit either topic if needed.
- With removal of the two sections on integers, Cycle 1 now contains 14 sections instead of 16. The halfway point is now after pie and bar graphs.
- Other more minor changes were made to placement and treatment of content within sections. The following changes were made:
  - Line graphs are now taught in Section 1.3 with the Cartesian Coordinate System instead of with bar and pie graphs.
  - A metric converter has been added to Section 1.9 to aid with the conversion of metric units. The section flow has been improved to more clearly convert English units, then metric units, and then between the two systems.
  - The topic of generalizing sequences in Section 1.12 has been improved by the addition of practice problems that are neither arithmetic or geometric. A clearer summary of the *n*th term formulas for arithmetic and geometric sequences and extra *Active Examples* were included to help students with this challenging content.
  - Fractional exponents have been removed from Section 2.3 and now appear in the Section 4.4 homework.
  - Rational and radical equations have been clearly defined in Section 3.9, and more practice solving them has been included.

## MyLab Math Changes

- The eText includes text and video solutions to all *Active Examples*.
- All of the conceptual homework problems were added to the MyLab homework bank.

# MyLab Math and Conceptual Homework: The Best of Both Worlds

This book and the accompanying MyLab Math course contain two different types of homework problems: exercises that address skills and problems that address concepts and applications. The two types of homework problems work together to help students develop the skills necessary to solve applied problems. A common approach to homework involves two steps:

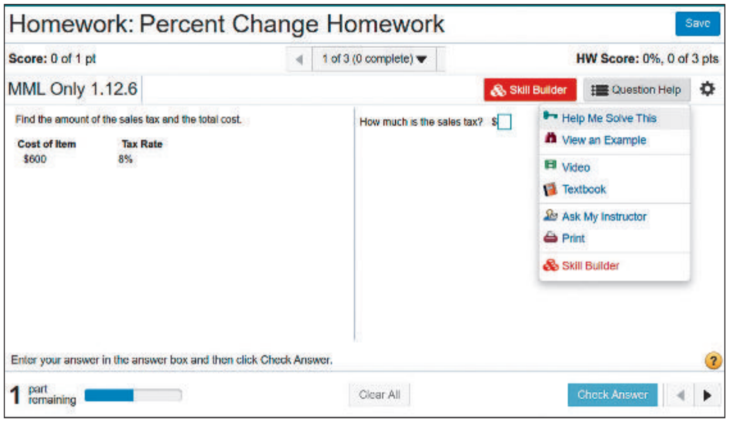
1. Complete skill exercises in MyLab Math to master skills
2. Solve *Skills* exercises and *Concepts and Applications* problems in the worktext homework

Another possible approach to homework is to simply assign *Skills* exercises and *Concepts and Applications* problems in MyLab Math and not use the textbook homework.

## Exercises in MyLab Math Only

MyLab Math provides carefully chosen exercises to allow students to practice all the skills necessary for solving applied problems. Students get immediate feedback, allowing them to master the skills of the section. We recommend that students begin each homework assignment by completing these exercises in MyLab Math.

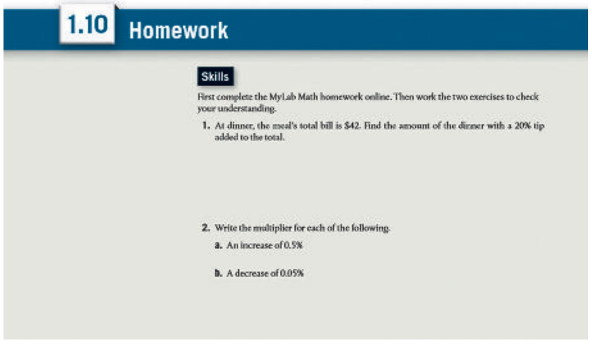
These problems are available only in MyLab Math and not the worktext. You will find these exercises in the MyLab Math assignment manager, designated with "MLM Only" (e.g., MLM Only 1.2.1). Additionally, sample homework assignments have been created for each section and are available in MyLab Math for instructor convenience.




## *Skills* Exercises in the Worktext

Two *Skills* exercises are available at the start of each homework assignment in the worktext. These exercises provide the opportunity for students to work a few exercises like those in MyLab Math without learning aids. They also allow students to have a written record of their work on paper to review for a test.

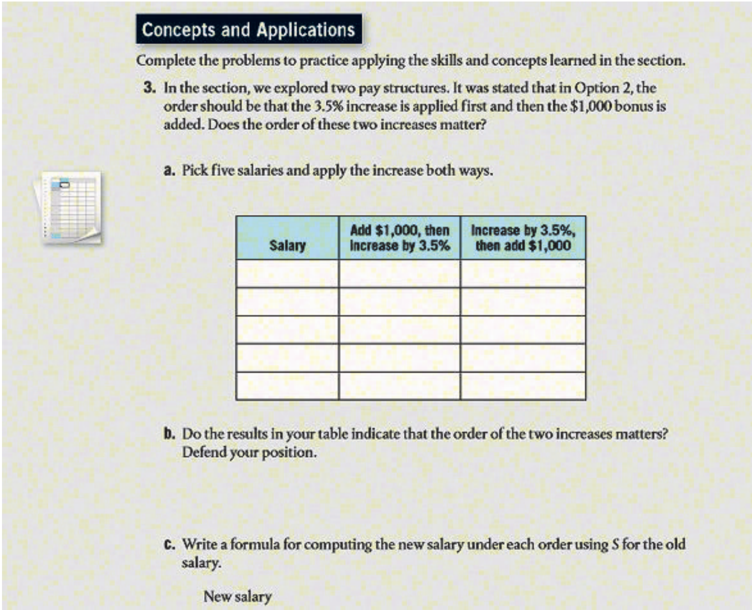
We recommend that students complete the *Skills* exercises after they complete the MyLab Math exercises. Algorithmic versions of these problems are available for instructors to assign via MyLab Math. They are designated with a "Text Skills" (e.g., Text Skills 1.2.1).



## *Concepts and Applications* Problems in the Worktext

For each section in the worktext, the *Skills* problems are followed by *Concepts and Applications* problems that focus more on applications and help students connect and apply ideas. Most conceptual homework assignments include only 3–10 problems, which increases the importance of each and every problem in the assignment.

All of the *Concepts and Applications* problems are now available for instructors to assign via MyLab Math. Some of these problems have been formatted to allow for automatic grading while others have been written in an essay format which will require the instructor to grade them. You will find these problems in the MyLab Math assignment manager, designated with a "Text C&Apps" (e.g., Text C&Apps 1.2.3).



**Concepts and Applications**

Complete the problems to practice applying the skills and concepts learned in the section.

**3.** In the section, we explored two pay structures. It was stated that in Option 2, the order should be that the 3.5% increase is applied first and then the $1,000 bonus is added. Does the order of these two increases matter?

**a.** Pick five salaries and apply the increase both ways.

| Salary | Add $1,000, then Increase by 3.5% | Increase by 3.5%, then add $1,000 |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

**b.** Do the results in your table indicate that the order of the two increases matters? Defend your position.

**c.** Write a formula for computing the new salary under each order using *S* for the old salary.

New salary

## Sample Assignments

There is a sample homework assignment for each section. These sample assignments initially contain the MyLab Math Only exercises. If you would like to have students complete their homework assignment entirely in MyLab Math, these sample homework assignments can easily be modified by adding any of the following:

- Additional MyLab Math Only exercises ("MLM Only")
- *Skills* exercises from the worktext ("Text Skills")
- *Concepts and Applications* problems from the worktext ("Text C&Apps")

Creating an assignment in this manner emulates the approach used when combining online and text homework, but it has the added convenience of all grading being completed in MyLab Math.

With this new edition, the MyLab Math course now contains a larger pool of exercises, which supports a variety of course formats.

## Sample Quizzes and Tests

In MyLab Math, there is a sample quiz to accompany each *Mid-Cycle Recap*. There is also a sample test for each cycle to support instructors who want to give tests online.

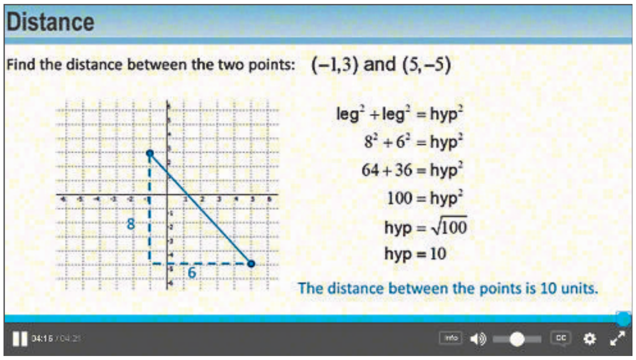
# Resources for Success

MyLab Math is available to accompany Pearson's market-leading text offerings. This text's flavor and approach are tightly integrated throughout the accompanying MyLab Math course, giving students a consistent tone, voice, and teaching method that make learning the material as seamless as possible.

## Videos for Students

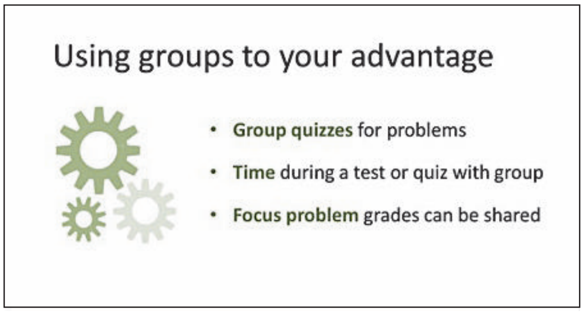
Videos in MyLab Math give students even more resources for outside the classroom and give instructors added flexibility and resources for various class formats. These videos can be used to supplement learning as students practice, or they can be assigned to prepare students for classroom activities.

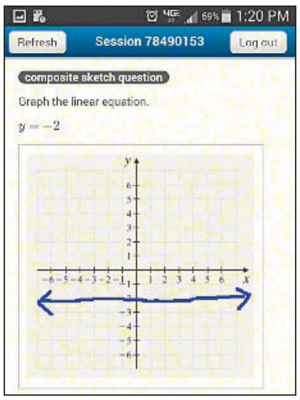
- Videos walk students through *Active Examples*, leading them to a complete solution as an instructor would in class.
- Videos cover some *Look It Up* and *How It Works* features, allowing students to follow along with the content from the text.




## Instructor Resource Videos

These videos, produced by the authors, present helpful suggestions based on their own experiences teaching Math Literacy for College Students. Including topics such as a typical class, focus problems, and group work, these videos provide the guidance an instructor might find useful in teaching this course and/or teaching with this text for the first time.




## Learning Catalytics

Integrated into MyLab Math, Learning Catalytics uses students' devices, such as tablets, smart phones, or computers, for an engagement, assessment, and classroom response system. In Learning Catalytics, instructors can do the following.

- Pose a variety of open-ended questions to help develop critical thinking skills.
- Use real-time data to adjust instructional strategy and improve engagement.
- Manage student interactions by automatically grouping students for discussion, teamwork, and peer-to-peer learning.

Pearson-created questions for developmental math topics are available to allow you to take advantage of this exciting technology immediately. “Explore” questions related to the worktext are also available to give instructors a starting point for using this feature with Learning Catalytics. Search the question library for MLAF and the section number, for example MLAF312 for Section 3.12.

# Instructor Resources

## Annotated Instructor’s Edition

This version of the text includes answers to all exercises presented in the book, as well as helpful teaching tips and the Instructor Guide.

## Instructor’s Solutions Manual

This online manual contains fully worked-out solutions to all text homework exercises. It is available in MyLab Math and to download from www.pearsonhighered.com.

## Instructor’s Resource Manual

To provide additional support for instructors, this manual includes the following resources:

- Two quizzes for each half cycle (Cycles 1-4)
- One test for each half cycle (Cycles 1-4)
- Two tests for each whole cycle
- Two final exams

Available in MyLab Math and to download from www.pearsonhighered.com.

## PowerPoint Lecture Slides

Available in MyLab Math and to download from www.pearsonhighered.com, these fully editable lecture slides include key concepts and examples for use in a lecture setting. Accessible versions are now also available.

## TestGen®

TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from www.pearsonhighered.com.

# Student Resources

In addition to the Learning Catalytics questions, instructional videos, and Active Examples noted above, the following resources are available in MyLab Math.

## Mindset Module

This module has growth mindset-focused videos and exercises that encourage students to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as a learning opportunity.

## Personal Inventory Assessments

This collection of online activities is designed to promote self-reflection in students. The 33 assessments include topics such as a Stress Management Assessment and Time Management Assessment.

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CYCLE **ZERO**

# WHICH TYPE OF NUMBER?

fractions
negative
integers
signed numbers
positive
real numbers
decimals
whole
percent
rational
percent change
imaginary
irrational
COMPLEX

# SELF-ASSESSMENT: **PREVIEW**

Below is a list of objectives for this cycle. For each objective, use the boxes provided to indicate your current level of expertise.

| SKILL or CONCEPT | LOW 1 | | | | HIGH 5 |
|---|---|---|---|---|---|
| **1.** Identify types of real numbers. | | | | | |
| **2.** Round whole numbers. | | | | | |
| **3.** Add and subtract integers. | | | | | |
| **4.** Multiply and divide integers. | | | | | |
| **5.** Solve application problems involving integers. | | | | | |
| **6.** Interpret fractions. | | | | | |
| **7.** Add and subtract fractions. | | | | | |
| **8.** Multiply and divide fractions. | | | | | |
| **9.** Interpret decimals. | | | | | |
| **10.** Round decimals. | | | | | |
| **11.** Add and subtract decimals. | | | | | |
| **12.** Multiply and divide decimals. | | | | | |
| **13.** Convert between fractions, decimal values, and percents. | | | | | |
| **14.** Calculate the base, amount, or percent in a percent problem. | | | | | |
| **15.** Calculate percent change. | | | | | |

I have a vision of the world as a global village, a world without boundaries.
—Christa McAuliffe

CYCLE ZERO

# 0.1 If the World Were a Village

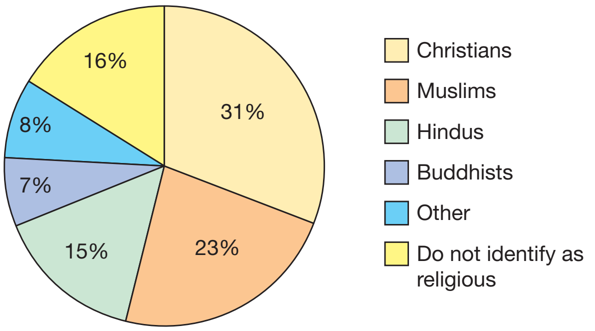
## Focus Problem

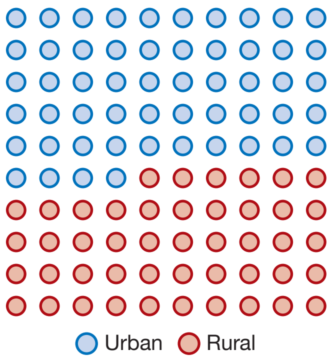
At nearly 8 billion people, the world's population is challenging to visualize and comprehend. We are inundated with survey results, polling data, and demographic trends, but we must be able to interpret them. Thinking of the population of the world as a village of 100 people allows us to interpret percentages in a more personal, concrete way.

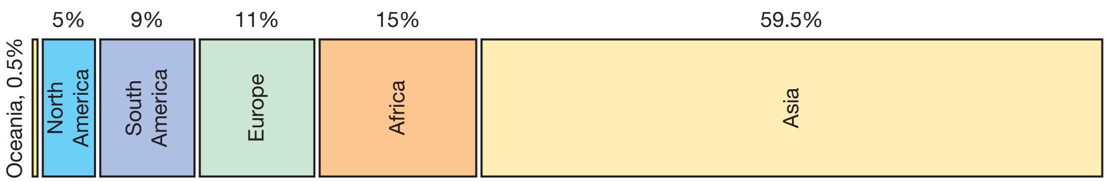
The following graphs illustrate the approximate percentage of the world population in 2020 that lives in urban or rural environments, belongs to various religions, and lives on the six populated continents. Determine how many people are represented by each category if the world population was approximately 7.8 billion in 2020.

At first glance, the urban/rural divide might not seem huge. How many more people actually live in urban environments than rural environments? If two categories differ by only one percentage point, how many people are represented by that 1%?

Besides geography and religion, there are many additional ways to describe the world's population. Research the percentages of the population in various age ranges and the percentages of the population who speak various languages. Create a table that shows the percentages as well as the number of people in each category for ages and languages. Approximately what fraction of the popula-



tion speaks the most common language? Approximately what fraction of people are in the most common age bracket? Comment on at least one detail of the data that surprised you.

Do an analysis to illustrate your city or state as a village of 100 people. Choose at least five variables that can be used to categorize the population. You can use religion, geography, and age again or research other variables of interest to you. Use your data to create a one-page infographic that illustrates the information in a visually interesting way. Include a brief summary on your infographic that shares the name of the city or state, the total population, and the year the data was collected.

As we tried to describe the global population, we had to consider . . .

# WHICH TYPE OF NUMBER?

This cycle is intended not only as a review of different types of numbers and how you might use them but also *when* you might prefer one over another. Certain demographic information is best illustrated with percents, while other data is illuminated by actual numbers of people. This will be the case in other contexts within this cycle and this book. You will need to move fluidly between whole numbers, fractions, decimals, and percents. This opening cycle is provided to give you the foundation needed to do this, and the focus problem is included to give you a context to apply what you learn.

# 0.2 Keeping It Real: Types of Numbers and Rounding

## Explore

1. Calculating pay based on an hourly rate often involves rounding the time worked to an amount more easily used in calculations. Rounding might be necessary because of a policy chosen or due to the software used. How a time is rounded can have an impact, negative or positive, for the employee and the employer.

   a. Round each amount of time shown in the table according to the rounding rules provided. Calculate the total number of hours for each rule.

| Time Clocked by an Employee | Rounded to the Nearest Hour | Rounded up to the Nearest Hour | Rounded to the Nearest Quarter Hour | Rounded to the Nearest Half Hour |
|---|---|---|---|---|
| 5 hours, 20 minutes | | | | |
| 5 hours, 33 minutes | | | | |
| 5 hours, 48 minutes | | | | |
| 6 hours, 3 minutes | | | | |
| **Total time** | | | | |

   b. Which rounding rule is best for the employees? Which rounding rule is best for the employer in this case? Why did that rounding rule favor the employer for these particular times?

## Discover

In the *Explore*, the problem involved fractions, decimals, and whole numbers that occurred naturally in the context. It is worth knowing particular types of numbers, what they are called, and how they relate to form a number system. In this section, we will review the real number system and the sets of numbers within it. We will also consider some attributes of the real number system, such as place value. Throughout this book, you will need to round answers to problems as requested, so rounding rules will also be reviewed in this section.

As you work through this section, think about these questions:

1. Can you list different types of numbers and when they might be used?
2. Do you know how to round a number in different ways?

Let's begin with the review of real numbers.

LOOK IT UP

## Real Numbers

The set of **real numbers** includes all the numbers that can be represented on a number line. For example, 3, 0, 1/2, −5.7, and $\pi$ are real numbers. The set of real numbers contains each of the following sets of numbers:

The set of **counting or natural numbers**, {1, 2, 3, ...}, is used to count objects.

The set of **whole numbers**, {0, 1, 2, 3, ...}, contains the counting numbers and zero.

The set of **integers**, {..., −3, −2, −1, 0, 1, 2, 3, ...}, contains the whole numbers and their opposites.

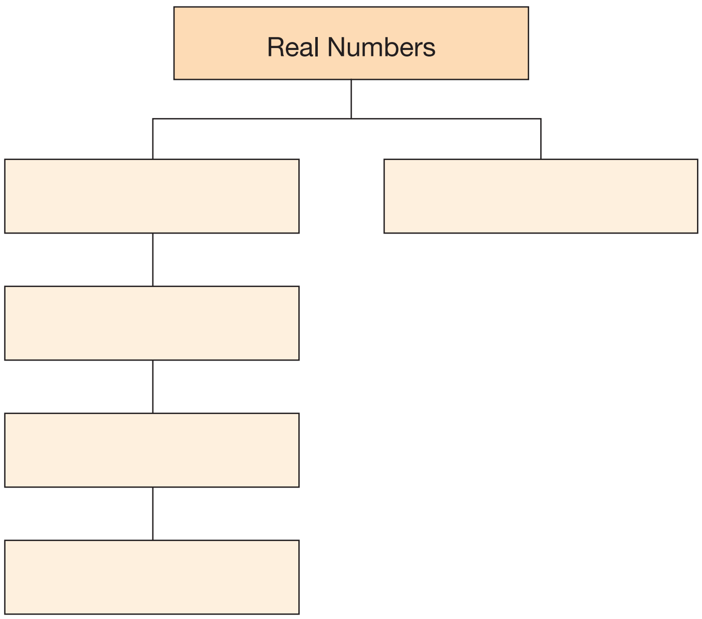
The set of **rational numbers** contains numbers that can be written as a quotient of two integers. Decimals that terminate or repeat are rational numbers.

Examples include: $\frac{2}{3}$, −3, 1.2, $4\frac{1}{2}$, 0, and 0.33333333...

The set of **irrational numbers** contains numbers that cannot be written as a quotient of two integers. Decimals that neither terminate nor repeat are irrational numbers. Examples include: $\sqrt{5}$, $\pi$, and 0.123123412345...

To make sense of the types of real numbers, we can organize them into a flowchart to see the relationships between them. It is also helpful to think of examples in each set and examples of numbers that are in one set but not another.

**2.** Fill in the boxes in the following flowchart with the types of real numbers from the *Look It Up* so that each set is included in the sets above it.




**3.** **a.** State an integer that is not a whole number.

**b.** State a rational number that is not an integer.

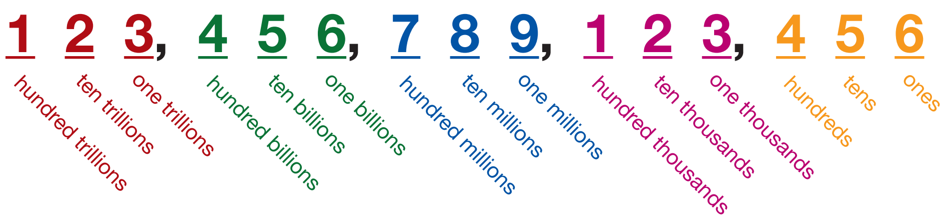
**c.** List all the integers between −10 and −5, inclusive.

**d.** List all the integers between −10 and −5, exclusive.

**e.** State an integer that is also a counting number.

**f.** Write a decimal that is irrational other than the examples provided.

The real number system is positional, and digits have different values depending on where they are placed in the numeral. Place values differ by powers of ten, with each place value getting ten times larger as we move to the left in the numeral and ten times smaller as we move to the right. Place values are named starting on the left, in groups of three separated by commas, as shown below.



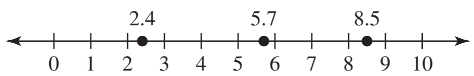

**Place Value**

Sometimes large numbers can be interpreted in more meaningful ways using place value to read them differently. For example, one million can be thought of as each of the following.

| | |
|---|---|
| 1,000,000 | 1 million |
| 1,000,000 | 10 hundred thousands |
| 1,000,000 | 100 ten thousands |
| 1,000,000 | 1,000 thousands |

One million can be interpreted as ten hundred thousands by noticing there is a 10 in the hundred thousands position.

Another way to make a number easier to report and understand is to round it. When we round a number, we report it as approximately equal to a number that it is near. If we want to round 2.4 to the nearest whole number, we would round it down to 2 because 2 is the closest whole number on a number line. If we want to round 5.7 to the nearest whole number, we would round it up to 6 because 6 is the closest integer on a number line. For numbers that are exactly halfway between two whole numbers, we can arbitrarily decide to round them up to the nearest whole number. So, 8.5 would be rounded up to 9 because it is halfway between 8 and 9. Numbers can be rounded to any specified place value, as you will learn in the *Active Example*.



**Need the complete solution? Check out the etext.**

## ACTIVE EXAMPLE

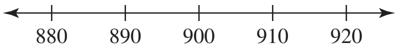
Round each of the numbers as requested.

**a.** Round 906 to the nearest ten.
**b.** Round 1,999 to the nearest hundred.
**c.** Round 2,876,500 to the nearest ten thousand.

### GUIDED SOLUTION

**a.** Since 906 is to be rounded to the nearest ten, it must be reported as a multiple of ten.

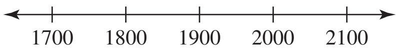
Graph 906 on the following number line, which includes only nearby multiples of ten.




Which multiple of ten is closest to 906?

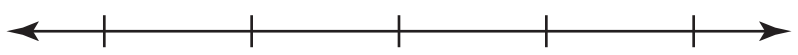
**b.** Since 1,999 is to be rounded to the nearest hundred, it must be reported as a multiple of one hundred.

Graph 1,999 on the following number line, which includes only nearby multiples of one hundred.




Which multiple of one hundred is closest to 1,999?

**c.** Label the number line below to assist in the rounding of 2,876,500 to the nearest ten thousand.



Which multiple of ten thousand is closest to 2,876,500?

The rules that you will see in the following *How It Works* are based on the visual idea developed here. We "round" numbers by reporting them to the closest appropriate multiple. If a number is exactly halfway between two such multiples, we usually and arbitrarily decide to round it to the higher one.

## HOW IT WORKS

**To round whole numbers:**

1. Identify the target digit in the requested place-value position.
2. Look at the digit one position to the right. If this digit is 0, 1, 2, 3, or 4, the target digit will stay the same. If this digit is 5, 6, 7, 8, or 9, the target digit will increase by 1.
3. Replace all digits to the right of the target digit with zeros.

**EXAMPLE:** 15,489 rounded to the nearest ten is 15,490.

15,489 rounded to the nearest hundred is 15,500.

15,489 rounded to the nearest thousand is 15,000.

15,489 rounded to the nearest ten thousand is 20,000.

**4.** Round the number to the place value indicated. 67,259

**a.** Ten thousands

**b.** Thousands

**c.** Hundreds

**d.** Tens

In some applications, it makes sense to round your answer always up or round it always down and not necessarily round to the *nearest* whole number. When instructed to round down, you must keep the target digit the same or lower it by one. When instructed to round up, you must keep the target digit the same or increase it by one.

**5.** Round each of the numbers as requested.

**a.** Round 349 *down* to the nearest ten.

**b.** Round 805 *up* to the nearest hundred.

**c.** Round 3,485,901 *up* to the nearest million.

## Connect

Not all numbers are included in the real number system discussed in this section. Square roots of negative numbers are not real numbers but are instead a set of numbers called **imaginary numbers**. Imaginary numbers are expressed using $i$ for the square root of $-1$. Imaginary numbers are part of the **complex numbers**, which have the form $a + bi$, where $a$ and $b$ are real numbers.

This book focuses primarily on real numbers, but you might encounter imaginary numbers later in a college algebra course.

**TECH TIP**

Check to see if your calculator has different modes for real numbers and complex numbers. The real number mode will be sufficient for work in this book.

**6.** Use your calculator to find the values of the following square roots. State the result shown on your calculator. Classify your answer as rational, irrational, or imaginary.

**a.** $\sqrt{9}$

**b.** $\sqrt{10}$

**c.** $\sqrt{\frac{1}{9}}$

**d.** $\sqrt{-9}$

## Reflect

## WRAP-UP

### What's the point?

Understanding the names of different types of numbers as well as some important qualities of them will be useful throughout the course. Rounding is a skill that will be used regularly, particularly in MyLab Math homework.

### What did you learn?

How to identify types of real numbers
How to round whole numbers

# 0.2 Homework

## Skills

**1.** Identify the indicated types of numbers in the set.

$$\left\{-3, 0, \frac{1}{2}, 1, \sqrt{2}, \frac{5}{3}, 7.8, \sqrt{-3}\right\}$$

**a.** Real numbers

**b.** Counting numbers

**c.** Rational numbers

**d.** Irrational numbers

**e.** Integers

**f.** Whole numbers

**g.** Imaginary numbers

**2.** Round the number to the place value indicated. 989,400,902

**a.** Millions

**b.** Hundred thousands

**c.** Ten thousands

**d.** Thousands

**e.** Hundreds

**f.** Tens

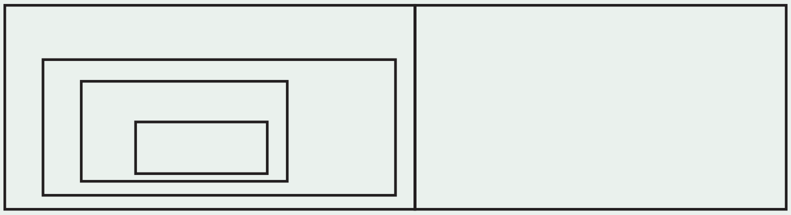
## Concepts and Applications

**3.** **a.** Name a rational number that is not a whole number.

**b.** Name a real number that is not a rational number.

**c.** Name an integer that is not a natural number.

**4.** The following graphic is a Venn diagram that can be used to illustrate relationships between sets. Each rectangle represents a set of real numbers. If a rectangle is completely inside another rectangle, then it illustrates that the inner set is a subset of the outer set. Label each rectangle with one of the types of real numbers to illustrate their relationships.



**5.** Write the square root of each natural number from 1 to 20. Identify which are rational and which are irrational numbers.

**6.** When a whole number ends in 5, it is exactly halfway between two tens. If we want to round it to the nearest ten, the normal rounding rules tell us to round it up to the next ten. So 15 would be rounded to 20. By arbitrarily deciding to round up in this case, a bias is created. Another method of rounding addresses this bias by rounding to the even tens place if the ones digit is a 5. So 35 would be rounded to 40, but 45 would also be rounded to 40.

Round the following numbers to the tens place, using the method of "round half to the even" as needed.

**a.** 293

**b.** 247

**c.** 285

**d.** 235

**7. a.** Find the following sum by first rounding each number to the nearest hundred and then adding.

$$32{,}549 + 63{,}649$$

**b.** Find the same sum by first adding and then rounding the sum to the nearest hundred.

**c.** Compare the answers to #7a and #7b. Does it matter when you round?

**8.** Write 7,000,000,000 in each of the following ways.

**a.** ________billions

**b.** ________________millions

**c.** ______________thousands

**d.** _______________hundreds

**9.** Write the following numbers in words and then add. Make exchanges as necessary.

62,745 = _____ten thousands_____thousands_____hundreds_____tens_____ones

+8,622 = _____ten thousands_____thousands_____hundreds_____tens_____ones

_____ten thousands______thousands______hundreds_____tens_____ones

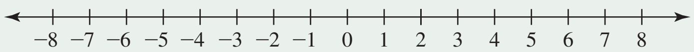
_____ten thousands______thousands_____hundreds_____tens_____ones

_____ten thousands_____thousands_____hundreds_____tens_____ones

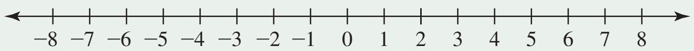
=

**10. Looking Forward, Looking Back**

**a.** Use the following number line to illustrate the subtraction and its result. 8 – 5




**b.** Use the following number line to illustrate the subtraction and its result. 5 – 8



# 0.3 It's All Relative: Integers and Their Operations

## Explore

Atoms contain protons, neutrons, and electrons. Each proton has a positive one charge, each electron has a negative one charge, and neutrons have no charge. An atom, by definition, is neutral and has the same number of protons and electrons. An atom can gain or lose electrons and become a charged particle called an ion. A positive ion has more protons than electrons, while a negative ion has more electrons than protons.

The abbreviation for an ion includes either a positive or a negative exponent that represents the net charge. If the net charge of an ion is +1 or −1, only a positive or negative sign is shown because the 1 is implied. Because the net charge of an atom is 0, no charge is indicated in its abbreviation.

**1.** Complete the last column in the following table by determining the net charge for each atom or ion.

| | Symbol | Number of Protons | Number of Electrons | Net Charge |
|---|---|---|---|---|
| Chlorine atom | Cl | 17 | 17 | |
| Chlorine ion | $Cl^{-}$ | 17 | 18 | |
| Aluminum atom | Al | 13 | 13 | |
| Aluminum ion | $Al^{3+}$ | 13 | 10 | |

**2.** An atom can become a charged ion only by gaining or losing electrons, not protons. An atom and an ion of the same element always have the same number of protons. Use this information to complete the following chart.

| | Symbol | Number of Protons | Number of Electrons | Net Charge |
|---|---|---|---|---|
| Hydrogen atom | H | 1 | | 0 |
| Hydrogen ion | $H^{+}$ | | 0 | +1 |
| Oxygen atom | O | | 8 | |
| Oxygen ion | $O^{2-}$ | | | −2 |
| Calcium atom | Ca | 20 | | |
| Calcium ion | $Ca^{2+}$ | | | |

## Discover

Negative numbers may seem abstract because they don't represent physical counts. However, there are naturally occurring situations in which we have less than zero, such as debt, and negative numbers allow us to describe such situations. In addition to describing situations with negative numbers, we also need to be able to perform calculations involving them and accurately determine the sign of the answer. This section will discuss both the interpretation of negative numbers and the rules for computing with them.

As you work through this section, think about these questions:

1. Do you know the difference between the sign and size of a number?
2. Do you know when two negatives make a positive?
3. Do you know why the result of a calculation is positive or negative?

When a situation requires a negative number, there are many ways to express it. On a budget sheet, you might see negative quantities written in red or parentheses, and positive numbers written in black. When describing altitudes, below sea level might be written as a negative number. In other cases, the negative may be implied but not written.

**3.** The following situations contain negative numbers used in real-life situations. Interpret what each one means.

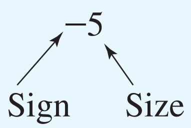
**a.** −30°F

**b.** A poll reports that 47% ±3% of Americans would vote for the incumbent for president.

LOOK IT UP

## Absolute Value

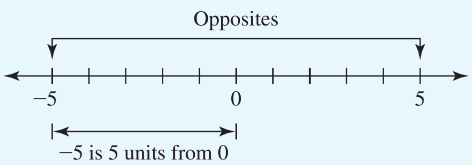
Every real number other than zero has a sign. Think of each real number as consisting of two parts: its sign and its size.




The "size of a number" is the informal way of saying how far the number is from zero on a number line. In mathematics, we refer to the size of a number as its **absolute value**. To indicate absolute value, we draw vertical bars before and after the number.

Two numbers that have the same size but opposite signs are called **opposites** because they are on opposite sides of zero on a number line.

**FOR EXAMPLE,** the absolute value of −5 is 5 because −5 is five units from zero. The absolute value of −5 is written as |−5|. The numbers 5 and −5 are also opposites.




**4. a.** What is the opposite of −8?

**b.** Find |−11|.

**c.** What is the sign of 24? The size?

d. Read and simplify: $-(-6)$

e. Read and simplify: $-|-6|$

Being able to visualize negative numbers on a number line is helpful not only for interpreting them but also for using them in calculations. In the *Active Example*, number lines will be used to help you make sense of the rules that are developed for signed-number operations.

**Need the complete solution? Check out the etext.**

## ACTIVE EXAMPLE

Solve each of the following problems by drawing a number line picture and writing a calculation.

a. It is 34° below zero, but the temperature will increase 12° throughout the day. What will the temperature be at the end of the day?

b. If a civilization began in 100 BC and lasted 450 years, when did it end?

c. The stock market has already lost 12 points for the day but, in the next hour, it drops 43 points more. What is the total loss so far?

### GUIDED SOLUTION

a. Label −34° on the number line and draw an arrow to indicate a 12° increase. What's the final temperature?

Write and perform a calculation to get the same result.

b. Represent 100 BC on the number line and draw an arrow to indicate the 450-year period. What year did the civilization end?

Write and perform a calculation to get the same result.

c. Represent the stock market's current 12-point loss as a point on the number line and draw an arrow to indicate the further decline. What is the total loss?

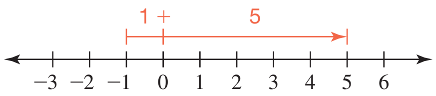
Write and perform a calculation to get the same result.

When you solve problems involving signed numbers, it is important to interpret your answer carefully. A negative result might be stated as a negative number or as a decrease, but you'll want to be careful not to be redundant by stating it as a negative decrease.

Notice in the *Active Example* that we are adding signed numbers in each scenario. When we add two negative numbers, the result is even more negative. The size of the result is the sum of the sizes of the numbers in the calculation. When we add numbers with different signs, the sign of the result is determined by the sign of the number in the calculation that has the largest size. To get the size of the result, we subtract the sizes.

In general, subtracting a number is the same as adding its opposite. That is, $a - b = a + (-b)$. Consider the context of money to make sense of this. When you gain debt (add a negative amount), you lose money.

We can use this idea to rewrite $5 - (-1)$ as $5 + (+1) = 6$. A number line can also be used to obtain this result by noting that the directed distance from −1 to 5 is 6. To find the total distance, we can find the distance from −1 to 0 and from 0 to 5 and add them.




$5 - (-1) = 5 + (+1) = 6$

Generalizing these ideas, we have the following rules.

## HOW IT WORKS

**To add two numbers with the same sign:**

Add the sizes; keep the sign.

**EXAMPLE:** Add −15 and −40.

To find the sum, add the sizes (15 + 40) and keep the sign (negative). The result is −55.

$$-15 + (-40) = -(15 + 40)$$
$$= -55$$

**To add two numbers with different signs:**

Subtract the sizes (larger − smaller); keep the sign of the number that has the larger size.

**EXAMPLE:** Add −22 and 30.

To find the sum, subtract the sizes (30 − 22) and keep the sign of the number whose size is larger. Since 30 is larger than 22, keep its sign, which is positive. The final result is 8.

$$-22 + 30 = +(30 - 22)$$
$$= +8$$

**To change subtraction into addition:**

Instead of subtracting, add the opposite. That is, rewrite $a - b$ as $a + (-b)$, or rewrite $a - (-b)$ as $a + b$. Then add.

**EXAMPLES:** $-5 - 10$ can be written as $-5 + (-10) = -15$

$5 - (-10)$ can be written as $5 + (+10) = 15$

Remember that the negative sign can be thought of interchangeably as negative or opposite or subtraction.

When adding signed numbers, remember:

1. Different signs, think difference
2. Same signs, think sum

**5.** For each problem, compute the result without a calculator. If you are unsure how to proceed, put the problem in a familiar context, such as temperatures, or use a number line. Also, convert subtraction problems to addition when helpful.

**a.** $-58 + (-18)$

**b.** $58 + 18$

**c.** $-58 + 18$

**d.** $58 + (-18)$

**e.** $120 - 7$

**f.** $120 - (-7)$

**g.** $-72 - 12$

**h.** $-72 - (-12)$

The following are correct ways to indicate the product of 5 and –6.

$5 \times -6$
$5 \cdot -6$
$5(-6)$
$(5)(-6)$

Now that we understand how to add and subtract signed numbers, we will look at the operations of multiplication and division. We can begin by looking at a negative number multiplied by a positive number. Because multiplication is repeated addition, the calculation 3(–4) can be changed to an addition problem and then simplified.

$$3(-4) = (-4) + (-4) + (-4) = -12$$

Generalizing, we can multiply a positive number and a negative number by multiplying the sizes and making the product negative.

The first four multiples of –7 can be found by using the rule above. Then the pattern can be continued by simply adding seven to the previous multiple.

$$3(-7) = -21$$
$$2(-7) = -14$$
$$1(-7) = -7$$
$$0(-7) = 0$$
$$(-1)(-7) = 7$$
$$(-2)(-7) = 14$$
$$(-3)(-7) = 21$$

The last three calculations indicate that a negative number multiplied by a negative number results in a positive number.

The sign rules for multiplying numbers can be used to divide them as well. Every division problem can be rewritten as a multiplication problem. For example, to calculate $-35 \div -7$, we can determine what number multiplied by –7 will produce –35. The answer must be a positive number, specifically 5, based on the sign rules for multiplication. Let's look at some examples.

$$-42 \div -6 = ? \longrightarrow -6 \times ? = -42 \longrightarrow 7$$
$$-42 \div 6 = ? \longrightarrow 6 \times ? = -42 \longrightarrow -7$$

Notice that, to get the result, we could simply divide the sizes of the numbers and determine the sign using the same sign rules as we did for multiplication. Simply put, if two numbers with the same sign are multiplied or divided, then the result is positive. If two numbers with different signs are multiplied or divided, then the result is negative.

## HOW IT WORKS

**To multiply or divide two numbers with the same sign:**
Multiply or divide the sizes. The result is positive.

**EXAMPLES:** $(3)(6) = 18$ $\quad$ $35 \div 7 = 5$

$(-3)(-6) = 18$ $\quad$ $-35 \div (-7) = 5$

**To multiply or divide two numbers with different signs:**
Multiply or divide the sizes. The result is negative.

**EXAMPLES:** $(3)(-6) = -18$ $\quad$ $-35 \div 7 = -5$

$(-3)(6) = -18$ $\quad$ $35 \div (-7) = -5$

Notice that $-1(-7) = 7$, which is the opposite of $-7$. Multiplying any number by $-1$ produces its opposite.

**6.** Compute each result without a calculator.

**a.** $(-12)(-5)$

**b.** $(12)(-5)$

**c.** $(-13)(4)$

**d.** $(13)(4)$

**e.** $-64 \div (-16)$

**f.** $-64 \div (16)$

**g.** $81 \div (3)$

**h.** $81 \div (-3)$

## Connect

The rules for signed-number operations allow you sometimes to predict the sign on the answer to a calculation. Sometimes the sign on the answer depends on the relative sizes of the numbers involved and can't be easily predicted from the rules.

**7.** For each type of calculation, state whether the answer is positive, negative, or it depends. If you state "it depends," give two examples to support your answer.

**a.** Negative + negative

**b.** Positive + negative

**c.** Positive − negative

**d.** Negative − negative

**e.** Positive × negative

**f.** Negative − positive

**g.** Negative ÷ negative

**Reflect**

## WRAP-UP

### What's the point?

Signed numbers are commonly used in daily life. It is important to be able to interpret them in context as well as to calculate with them. When performing a calculation involving signed numbers, we can use familiar contexts and pictures to make sense of the problem. To perform the computation, we have to find both the size of the result and its sign.

- When adding two numbers with the same sign, add the sizes and keep the same sign.
- When adding two numbers with different signs, subtract the sizes and keep the sign of the number with the larger size.
- When multiplying or dividing two numbers with the same sign, the result is positive.
- When multiplying or dividing two numbers with different signs, the result is negative.

### What did you learn?

How to interpret integers
How to add, subtract, multiply, and divide integers
How to solve application problems involving integers

# 0.3 Homework

## Skills

**1.** The following situations contain negative numbers used in real-life situations. Interpret what each one means.

**a.** −20 rushing yards in a football game

**b.** Stock market performance for the day: −120.62 points

**2.** Calculate each of the following.

**a.** $-45 + 9$

**b.** $-45 - 9$

**c.** $-45(9)$

**d.** $\frac{-45}{9}$

## Concepts and Applications

**3. a.** What is the opposite of 7?

**b.** What is the opposite of your result from part a?

**c.** So the opposite of the opposite of 7 is_____. That is, $-(-7) =$_____.

**d.** Generalize this by completing the statement: For any number $a$, $-(-a) =$_____.

**4.** For each situation, write a computation and find the result. Interpret your answer.

**a.** Beth already owes Joe \$10 but asks to borrow \$25 more. How much will she owe him in total?

**b.** Lora has \$11,500 in debt and receives \$20,000 through an inheritance. How much will she have after the debt is paid?

**5.** A football team gained 6 yards on the first down, lost 15 yards on the second down, and gained 12 yards on the third down. How many yards does it need to gain on the fourth down to have a 10-yard gain (known as getting a first down) from its starting position?

**6. a.** If it is 22° Fahrenheit in the morning but rises to 47° by noon, what is the change in temperature? Write a calculation, find the result, and interpret it. Your result should reflect the numerical change and direction—in this case, rising.

**b.** Generalize how you wrote the calculation. That is, in what order was the subtraction done? You will need this idea to complete the next two problems.

**c.** If it is 5° Fahrenheit below zero when you wake up in the morning, but the temperature is 26° above zero by the time you leave work, what is the change in temperature? Write a calculation, find the result, and interpret it.

**d.** If it is 26° Fahrenheit now but the temperature will drop to 5° below zero overnight, what is the change in temperature? Write a calculation, find the result, and interpret it.

**7. a.** Give an example in which a mathematical operation of two negative numbers results in a positive number, and then give an example in which a mathematical operation of two negative numbers results in a negative number.

**b.** Can we always conclude that performing a mathematical operation on two negative numbers results in a positive number?

**8.** The role of parentheses is important in mathematics. Sometimes they make a big difference, and sometimes they can be removed without changing the problem. Compute each of the following. Then rewrite each calculation without parentheses, compute, and determine whether the parentheses made a difference.

| Calculation with parentheses | Answer | Calculation without parentheses | New result | Do parentheses matter? |
|---|---|---|---|---|
| $2(-4)$ | $-8$ | $2 - 4$ | $-2$ | Yes |
| $(-2) + (-4)$ | | | | |
| $2 - (4)$ | | | | |
| $(-2)(-4)$ | | | | |

**9.** When trying to find the result of the division $6 \div 3$, we can think "$3 \times ? = 6$." Let's apply this idea to the following divisions. Rewrite each division as a multiplication and then find the result. Verify your result by using your calculator. To answer the division that results in an error on the calculator, write "undefined."

**a.** $0 \div 6$

**b.** $6 \div 0$

**c.** Try a few more similar problems in which 0 is divided by a number or a number is divided by zero. What conclusions can be drawn?

**10.** **Looking Forward, Looking Back** Suppose your friend reads the following instructions to you from a book of math tricks:

Pick a number.
Subtract 1.
Multiply by 3.
Add 6.
Divide by 3.
Subtract the original number.

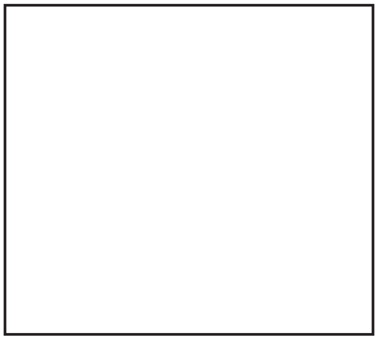
She claims that she can predict the result without knowing your initial number. She plays the trick on you twice, correctly guessing the final result each time. Explain how she is doing this and why it works.

First, try the steps with a negative number and with a fraction.

# 0.4 Part and Whole: Rational Numbers and Their Operations

## Explore

**1.** Use the square below to find a quarter of a quarter of a half.



## Discover

Dividing a whole in a particular way comes up in a variety of situations, from cutting a cake to distributing an estate, and the resulting shares can be described with fractions. What begins simply as a quarter or a half can quickly become more complex as we work with the subtleties of fractions. This section will explore the notation used in fractions and its multiple meanings. We will then use that notation to develop the procedures, which you have likely encountered before. One goal for the section is to understand why the fraction rules work as they do; another is to know how to use them in a variety of ways, including with signed numbers and mixed numbers.

As you work through this section, think about these questions:

1. Do you know how to read a fraction in a meaningful way?
2. Do you remember the rules for adding and multiplying fractions?
3. How do the rules for integer computations from the previous section apply to rational numbers?

We know from Section 0.2 that numbers that can be written as a quotient of integers are called rational numbers. The term *fraction* applies to those rational numbers that are positive, but it is also frequently used as a term for all rational numbers. Let's begin by considering some important interpretations of fractions.

LOOK IT UP

## Interpreting Rational Numbers

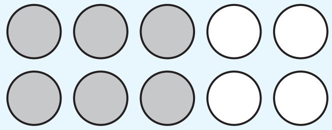
A **rational number** is any real number that can be written as a quotient of two integers. While rational numbers can be written in either fraction or decimal notation, there are three common ways they are interpreted.

- A number of equal parts of a whole
- The ratio of two integers
- A real number on a number line

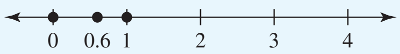
**FOR EXAMPLE,** $\frac{3}{5}$ can be interpreted in the following ways:

3 out of 5 equal parts of a whole or 3 fifths

3 out of every 5



3 divided by 5, or 0.6




TECH TIP

When entering a fraction on your calculator, you can use either the fraction mode or simply the division key.

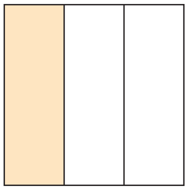
**2.** Draw two different illustrations of the meaning of the fraction $\frac{7}{4}$.

**Remember?**

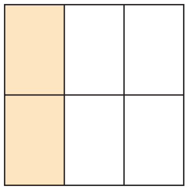
The top number in the fraction is called the numerator, and the bottom number is called the denominator.

Some fraction operations require that we can write an equivalent fraction. The part-of-a-whole interpretation is useful for understanding this process.

The fraction $\frac{1}{3}$ can be viewed as one of three equally sized pieces. If we multiply the numerator and denominator of the fraction by the same nonzero number, we create an equivalent fraction.

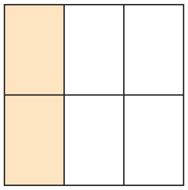


$$\frac{1}{3} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$$

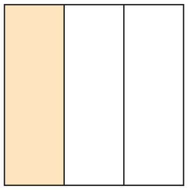


1 of 3 is equivalent to 2 of 6

Likewise, if we divide the numerator and denominator of the fraction by the same nonzero number, we create a simpler equivalent fraction. This process is known as simplifying a fraction because the numerator and denominator each become smaller, but the amount represented by the fraction remains the same.



$$\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$$



2 of 6 is equivalent to 1 of 3

## Remember?

A fraction is in simplest form when the numerator and denominator have no common factors other than 1.

**3.** Rewrite the fraction $\frac{22}{30}$ in each of the following ways.

**a.** With a denominator of 60

**b.** With a denominator of 90

**c.** In simplest form

Interpreting fractions correctly and reading them in a meaningful way can help you understand and remember the rules for fraction operations. We'll start by reading fractions in a meaningful way to remember why we add fractions with like denominators the way that we do.

**4.** Add or subtract and then write the answer as a fraction in simplest form.

**a.** $\frac{5}{9} + \frac{2}{9} =$ 5 ninths
+ 2 ninths

**b.** $\frac{8}{11} - \frac{2}{11} =$ 8 elevenths
−2 elevenths

**c.** $\frac{5}{8} + \frac{7}{8} =$ 5 eighths
+7 eighths

The last problem illustrated the rule for adding fractions with like denominators. We simply add the numerators and keep the same denominator. Next, let's consider what to do when the denominators are different.

**Need the complete solution? Check out the etext.**

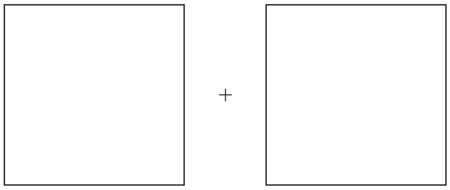
## ACTIVE EXAMPLE 1

**a.** Calculate the sum $\frac{1}{3} + \frac{2}{5}$ by drawing a model.

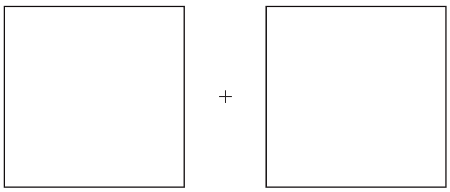
**b.** Calculate the sum $\frac{1}{3} + \frac{2}{5}$ using a common denominator.

### GUIDED SOLUTION

**a.** Use vertical lines and shading to illustrate $\frac{1}{3}$ on the first square. Use horizontal lines and shading to illustrate $\frac{2}{5}$ on the second square.




To add the shaded parts, they must be the same size. Re-create your models of $\frac{1}{3}$ and $\frac{2}{5}$ below. Then divide each image by drawing either horizontal or vertical lines to make all the shaded parts the same size.




Add the number of shaded parts in the first square to the number of shaded parts in the second square.

Count the number of parts created in each square. What is the size of each part?

Write the sum as a fraction. $\frac{1}{3} + \frac{2}{5} =$

**b.** To find the sum without pictures, start by identifying the smallest number that is a multiple of both denominators, 3 and 5.

Rewrite each fraction in the sum with this common multiple as a denominator.

$$\frac{1}{3} + \frac{2}{5}$$

Add the fractions, keeping the same common denominator.

The fractions added in this example did not initially have the same denominator and did not represent parts of the same size. To resolve this issue, the fractions must be renamed, which can be accomplished by either redrawing the fraction models or writing the fractions with a common denominator.

---

The fraction models are helpful for motivating the procedure we will use to add fractions, but this is not an efficient method simply to find the answer. The standard procedure for adding fractions is summarized in the *How It Works.*

**HOW IT WORKS**

**To add or subtract fractions:**

1. Rewrite the fractions with a common denominator if necessary.
2. Add or subtract numerators.
3. Keep the same denominator.
4. Simplify if needed.

**EXAMPLE:** $\frac{7}{8} - \frac{1}{5} = \frac{7 \cdot 5}{8 \cdot 5} - \frac{1 \cdot 8}{5 \cdot 8} = \frac{35}{40} - \frac{8}{40} = \frac{27}{40}$

**5.** Add or subtract and then write the answer as a fraction in simplest form.

**a.** $\frac{3}{10} + \frac{7}{20}$

**b.** $\frac{5}{12} + \frac{2}{15}$

**c.** $\frac{5}{6} - \frac{1}{8}$

**d.** $\frac{11}{18} - \frac{1}{27}$

**Need the complete solution? Check out the etext.**

## ACTIVE EXAMPLE 2

**a.** Calculate the product $\frac{2}{3} \cdot \frac{4}{5}$ by drawing a model.

**b.** Calculate the product $\frac{2}{3} \cdot \frac{4}{5}$ using the traditional approach.

### GUIDED SOLUTION

**a.** Interpret $\frac{2}{3} \cdot \frac{4}{5}$ as two thirds of $\frac{4}{5}$. Begin by using horizontal lines and shading to illustrate $\frac{4}{5}$ on the first square.

Re-create the model of $\frac{4}{5}$ below. Then take two-thirds of the shaded area by using vertical lines to divide it into thirds and shading two of the thirds.

Count the number of parts that were shaded twice.

Count the total number of parts created in the square.

Write the product as a fraction. $\frac{2}{3} \cdot \frac{4}{5} =$

**b.** Multiply the fractions in the traditional way by simply multiplying the numerators and multiplying the denominators.

$$\frac{2}{3} \cdot \frac{4}{5} =$$

It is not efficient to multiply fractions using models, but the process can help you make sense of the procedure normally used for this operation. Notice that the product involved fifteenths because the model was divided into thirds in one direction and fifths in the other, creating fifteenths.

While *Active Example 2* helps motivate the rule for multiplying fractions, we also need to review how to divide fractions. Consider the fact that dividing by two can be seen as multiplying by one-half (the **reciprocal** of 2).

$$10 \div 2 = 10 \times \frac{1}{2}$$

This technique can be used to rewrite any division calculation as a multiplication by the reciprocal. The procedures for multiplying and dividing fractions are summarized in the *How It Works*.

## HOW IT WORKS

**To multiply or divide fractions:**

1. If dividing, take the reciprocal of the second fraction and make the operation multiplication.
2. Multiply the numerators and multiply the denominators.
3. Simplify if possible.

**EXAMPLE:** $\frac{12}{35} \div \frac{4}{15} = \frac{12}{35} \cdot \frac{15}{4} = \frac{180}{140} = \frac{9}{7}$

Alternatively, the simplifying can be done before multiplying the numerators and multiplying the denominators.

$$\frac{12}{35} \div \frac{4}{15} = \frac{12}{35} \cdot \frac{15}{4} = \frac{\overset{3}{\cancel{12}}}{\underset{7}{\cancel{35}}} \cdot \frac{\overset{3}{\cancel{15}}}{\underset{1}{\cancel{4}}} = \frac{9}{7}$$

### Remember?

If the numerator of a fraction is greater than or equal to the denominator, the fraction is called improper. Improper fractions *can* be written as mixed numbers, but they can also be left in fraction form as long as they are simplified.

**6.** Multiply or divide and then write the answer as a fraction in simplest form.

**a.** $\frac{6}{17} \cdot \frac{3}{8}$

**b.** $\frac{2}{9} \cdot \frac{33}{5}$

**c.** $\frac{7}{36} \div \frac{2}{5}$

**d.** $\frac{18}{13} \div 4$

So far in this section, we have worked with only positive rational numbers. We also need to perform calculations with signed fractions. To do so, we use the same sign rules that we learned in Section 0.3.