PRECALCULUS
ESSENTIALS

BLITZER

sixth edition

A Brief Guide to Getting the Most from This Book

1 Read the Book

Feature	Description	Benefit
Section-Opening Scenarios	Every section opens with a scenario presenting a unique application of algebra or trigonometry in your life outside the classroom.	Realizing that algebra and trigonometry are everywhere will help motivate your learning. **(See page 218.)**
EXAMPLE	Examples are clearly written and provide step-by-step solutions. No steps are omitted, and each step is thoroughly explained to the right of the mathematics.	The blue annotations will help you understand the solutions by providing the reason why every algebraic or trigonometric step is true. **(See page 101.)**
Applications Using Real-World Data	Interesting applications from nearly every discipline, supported by up-to-date real-world data, are included in every section.	Ever wondered how you'll use algebra and trigonometry? This feature will show you how they can solve real problems. **(See pages 211–213.)**
GREAT QUESTION!	Answers to students' questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.	By seeing common mistakes, you'll be able to avoid them. This feature should help you not to feel anxious or threatened when asking questions in class. **(See page 326.)**
BRIEF REVIEW	Brief Reviews cover skills you already learned but may have forgotten.	Having these refresher boxes easily accessible will help ease anxiety about skills you may have forgotten. **(See page 454.)**
BLITZER BONUS	These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the algebra or trigonometry under study.	Yet even more proof that math is an interesting and dynamic discipline! **(See page 204.)**
Explanatory Voice Balloons	Voice balloons help to demystify algebra and trigonometry. They translate math into plain English, clarify problem-solving procedures, and present alternative ways of understanding.	Does math ever look foreign to you? This feature often translates math into everyday English. **(See page 440.)**
WHAT YOU'LL LEARN 1 Learning Objective	Every section begins with a list of objectives. Each objective is restated in the margin where the objective is covered.	The objectives focus your reading by emphasizing what is most important and where to find it. **(See page 618.)**
TECHNOLOGY	The screens displayed in the technology boxes show how graphing utilities verify and visualize algebraic or trigonometric results.	Even if you are not using a graphing utility in the course, this feature will help you understand different approaches to problem solving. **(See page 343.)**

2 Work the Problems

Feature	Description	Benefit
CHECK POINT	Each example is followed by a matched problem, called a Check Point, that offers you the opportunity to work a similar exercise. The answers to the Check Points are provided in the answer section.	You learn best by doing. You'll solidify your understanding of worked examples if you try a similar problem right away to be sure you understand what you've just read. **(See page 341.)**
ACHIEVING SUCCESS	Achieving Success boxes offer strategies for persistence and success in college mathematics courses.	Follow these suggestions to help achieve your full academic potential in college mathematics. **(See page 579.)**
CONCEPT AND VOCABULARY CHECK	These short-answer questions, mainly fill-in-the-blank and true/false items, assess your understanding of the definitions and concepts presented in each section.	It is difficult to learn algebra and trigonometry without knowing their special language. These exercises test your understanding of the vocabulary and concepts. **(See page 176.)**
EXERCISE SET	An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within several categories. Your instructor will usually provide guidance on which exercises to work. The exercises in the first category, Practice Exercises, follow the same order as the section's worked examples.	The parallel order of the Practice Exercises lets you refer to the worked examples and use them as models for solving these problems. **(See page 379.)**
Practice PLUS	This category of exercises contains more challenging problems that often require you to combine several skills or concepts.	It is important to dig in and develop your problem-solving skills. Practice PLUS Exercises provide you with ample opportunity to do so. **(See page 380.)**
Retaining the Concepts	Beginning with Chapter 1, each Exercise Set contains review exercises under the header "Retaining the Concepts."	These exercises improve your understanding of the topics and help maintain mastery of the material. **(See page 181.)**
Preview Exercises	Each Exercise Set concludes with three or four problems to help you prepare for the next section.	These exercises let you review previously covered material that you'll need to be successful for the forthcoming section. Some of these problems will get you thinking about concepts you'll soon encounter. **(See page 538.)**

3 Review for Quizzes and Tests

Feature	Description	Benefit
Mid-Chapter Check Point	At approximately the midway point in the chapter, an integrated set of review exercises allows you to review the skills and concepts you learned separately over several sections.	By combining exercises from the first half of the chapter, the Mid-Chapter Check Points give a comprehensive review before you move on to the material in the remainder of the chapter. **(See page 768.)**
Chapter Review Chart	Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.	Review this chart and you'll know the most important material in the chapter! **(See page 807.)**
Chapter Review Exercise Set	A comprehensive collection of review exercises for each of the chapter's sections follows the review chart.	Practice makes perfect. These exercises contain the most significant problems for each of the chapter's sections. **(See page 429.)**
Chapter Test	Each chapter contains a practice test with approximately 25 problems that cover the important concepts in the chapter. Take the practice test, check your answers, and then watch the Chapter Test Prep Videos to see worked-out solutions for any exercises you miss.	You can use the chapter test to determine whether you have mastered the material covered in the chapter. **(See page 654.)**
Chapter Test Prep Videos	These videos contain worked-out solutions to every exercise in each chapter test and can be found in MyLab Math and on YouTube at **youtube.com/user/pearsonmathstats** (playlist "Blitzer Precalculus 7e").	The videos let you review any exercises you miss on the chapter test.
Objective Videos	These fresh, interactive videos walk you through the concepts from every objective of the text.	The videos provide you with active learning at your own pace.
Cumulative Review	Beginning with Chapter 2, each chapter concludes with a comprehensive collection of mixed cumulative review exercises. These exercises combine problems from previous chapters and the present chapter, providing an ongoing cumulative review.	Ever forget what you've learned? These exercises ensure that you are not forgetting anything as you move forward. **(See page 438.)**

Precalculus Essentials

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6th EDITION

Precalculus Essentials

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About the Cover: Bob Blitzer's signature chili pepper provides a window into a vibrant nature scene rendered by Alaska-based artist Annie Brace. The cover is a metaphor for Bob's book, which provides insight into a world that is profoundly mathematical.

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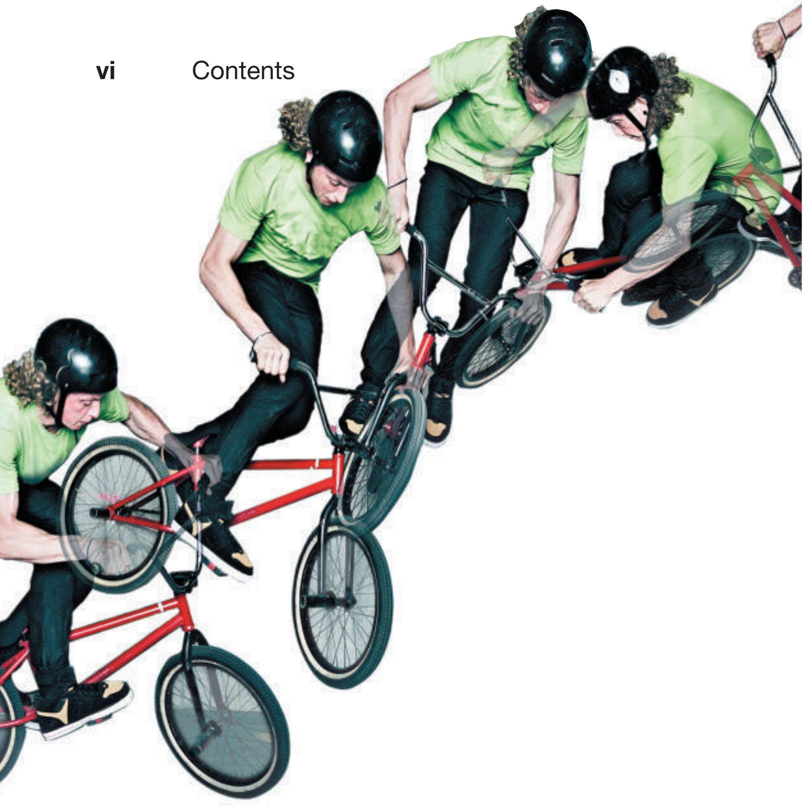
Contents



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Preface

I've written ***Precalculus Essentials*, Sixth Edition**, to help diverse groups of students, with different backgrounds and future goals, to succeed. The book has three fundamental goals:

1. To help students acquire a solid foundation in algebra and trigonometry, preparing them for other courses such as calculus, business calculus, and finite mathematics.
2. To show students how algebra and trigonometry can model and solve authentic real-world problems.
3. To enable students to develop problem-solving skills, while fostering critical thinking, within an interesting setting.

One major obstacle in the way of achieving these goals is the fact that very few students actually read their textbook. This has been a regular source of frustration for me and for my colleagues in the classroom. Anecdotal evidence gathered over years highlights two basic reasons that students do not take advantage of their textbook:

- "I'll never use this information."
- "I can't follow the explanations."

I've written every page of the Sixth Edition with the intent of eliminating these two objections. The ideas and tools I've used to do so are described for the student in "A Brief Guide to Getting the Most from This Book," which appears on the endpapers of the book.

How does *Precalculus Essentials* Differ from *Algebra and Trigonometry*?

Precalculus Essentials is not simply a condensed version of my *Algebra and Trigonometry* book. Precalculus students are different from those taking algebra and trigonometry, and this text reflects those differences. Here are a few examples:

- *Algebra and Trigonometry* devotes an entire chapter to linear equations, rational equations, quadratic equations, radical equations, linear inequalities, and developing models involving these equations and inequalities. *Precalculus Essentials* reviews these topics in three sections of the prerequisites chapter (P.7: Equations; P.8: Modeling with Equations; P.9: Linear Inequalities and Absolute Value Inequalities). Functions, the core of any precalculus course, are then introduced in Chapter 1.
- *Precalculus Essentials* contains a section on constructing functions from verbal descriptions and formulas (1.10: Modeling with Functions) that is not included in *Algebra and Trigonometry*. Modeling skills are applied to situations that students are likely to see in calculus when solving applied problems involving maximum or minimum values.
- *Precalculus Essentials* develops trigonometry from the perspective of the unit circle (4.2: Trigonometric Functions: The Unit Circle). In *Algebra and Trigonometry*, trigonometry is developed using right triangles.
- Many of the liberal arts applications in *Algebra and Trigonometry* are replaced by more scientific or higher-level applications in *Precalculus Essentials*. Some examples:
  - Black Holes in Space (Section P.2)
  - Average Velocity (Section 1.5)
  - Newton's Law of Cooling (Section 3.5)

A Note on the Essentials Version of *Precalculus*

Precalculus Essentials, Sixth Edition, is a concise version of the Seventh Edition of *Precalculus*. The essentials version differs from the Seventh Edition only in terms of length. Omitted are Chapter 7 (Systems of Equations and Inequalities), Chapter 8 (Matrices and Determinants), Chapter 9 (Conic Sections and Analytical Geometry), Chapter 10 (Sequences, Induction, and Probability), and Chapter 11 (Introduction to Calculus). The essentials version provides a more compact alternative to the Seventh Edition for instructors who do not cover the topics in Chapters 7–11.

What's New in the Sixth Edition?

The Sixth Edition contains 78 worked-out examples and exercises based on new data sets and 130 updated examples and exercises. Many of the new and updated applications involve topics relevant to college students.

New Applications

- Cost and Enrollment for Federal Social Programs (Section P.2, Exercises 115–117)
- Educational Attainment and Probability of Divorce (Section 1.1, Example 6)

- Number of Smartphone Users in the U.S. (Section 1.2, Figure 1.14)
- Spending on Pre-Primary Education and Child Care (Section 1.2, Exercises 99–100)
- Internet Plans (Section 1.3, Example 6 and Exercises 95–96)
- Trust in Government and the Media (Section 1.4, Exercises 87–88)
- Accelerating Climate Change (Blitzer Bonus in Section 1.4, p. 213)
- Living Arrangements of Young Adults (Section 1.5 opener and Example 3)
- U.S. Population Projections by Age (Section 1.7, Exercises 97–98)
- Rumbling Back: Steven Spielberg's New *West Side Story* (Blitzer Bonus in Section 1.8, p. 261)
- Addressing Leisure Time Parabolically (Blitzer Bonus in Section 2.2, p. 332)
- COVID-19 Pandemic (Section 2.3 opener; Cumulative Review for Chapters P–2, Exercise 21; Section 3.5, Example 3)
- AIDS: A Global Perspective (Blitzer Bonus in Section 2.3, p. 341)
- Area Burned by Wildfires in the U.S. (Section 2.3, Exercise 76)
- Mumps (Chapter 2 Mid-Chapter Check Point, Exercise 35)
- Costco Paid Membership (Chapter 2 Review, Exercise 82)
- Putting Off Medical Treatment Because of Expenses (Section 3.2, Exercises 115–116)
- E-commerce Sales (Chapter 3 Review, Exercise 81)
- The Electromagnetic Spectrum (Blitzer Bonus in Section 4.5, p. 600)

Updated Applications

- Cost of Tuition and Fees at Public and Private Colleges (Section P.1, Example 2 and Exercises 131–132)
- The National Debt (Section P.2 opener, Example 6, and Exercises 118–120)
- Student Loan Debt (Chapter P Mid-Chapter Check Point, Exercise 42)
- Grade Inflation (Section P.7, Exercises 137–138)
- Toll Options (Section P.8 opener, Example 3, and Exercises 11–12)
- Median Earnings by Educational Attainment (Section P.8, Example 1)
- Attitudes of College Freshmen (Section P.8, Example 2)
- Car Prices and Age of Cars on U.S. Roads (Section P.8, Exercises 5–6)
- Different Race or Ethnicity for Two Randomly Selected Americans (Chapter P Review, Exercise 23)
- Average Price of a Movie Ticket (Chapter P Review, Exercise 144)
- Highest-Paid TV Actors and Actresses (Section 1.2, Figure 1.13)
- The Wage Gap between Men and Women (Section 1.2, Exercises 103–104)
- Interest Rates (Section 1.3, Example 5; Section 3.1, Example 7; Section 3.4, Example 10)
- Fuel Efficiency of New U.S. Cars (Section 1.3 opener)
- Number of Births and Deaths in the U.S. (Section 1.7 opener and Example 4)
- Political Orientation of U.S. College Freshmen (Chapter 1 Review, Exercise 67)
- One-Person Households as a Percentage of the U.S. Total (Chapter 1 Test, Exercise 28)
- AIDS Cases in the U.S. (Section 2.3, Example 3)
- World Tiger Population (Section 2.3, Exercises 73–74)
- Federal Budget Expenditures on Human Resources (Section 2.6, Exercise 107)
- Amazon Deforestation (Chapter 2 Review, Exercise 28)
- Gray Wolf Population (Section 3.1, Example 6)
- Percentage of High School Seniors Applying to More Than Three Colleges (Section 3.1, Exercises 71–72)
- Number of Pages in the Federal Tax Code (Section 3.1, Exercise 85)
- Percentage of GDP Going Toward Health Care (Section 3.4, Exercises 115–116)
- U.S. Population (Section 3.5, Example 1; Section 10.3, Example 3)
- World Population (Section 3.5, Examples 6 and 7)
- Populations of Various Countries (Section 3.5, Exercises 1–14)

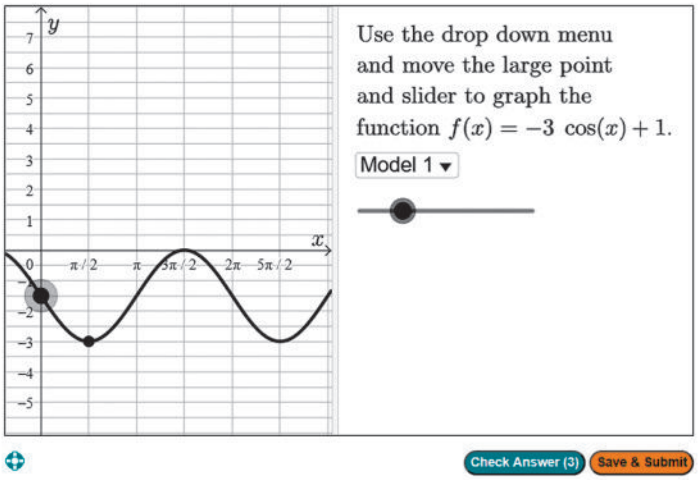
Other Textbook Changes

- Prior to the exercises in each section, the Annotated Instructor's Edition provides a list of resources available for that section in MyLab Math.
- The list of each section's objectives, previously headed "What am I supposed to learn?" (which annoyed some reviewers) has been renamed "What You'll Learn."
- Section P.6 includes new examples and exercises involving adding and subtracting rational expressions with different monomial denominators. (Section P.6, Example 8 and Exercises 43–50)
- In Chapter 2, the standard form of a quadratic function, $f(x) = a(x - h)^2 + k$, has been renamed the *vertex form*.
- Section 4.7 includes a new objective covering the definitions, properties, and graphs of the inverse cotangent, inverse cosecant, and inverse secant functions. (Section 4.7, Example 5 and Exercises 19–26, 57–62)

- Section 5.1 has a new objective on rewriting expressions that contain both trigonometric and logarithmic expressions as equivalent expressions using trigonometric identities and properties of logarithms. (Section 5.1, Example 9 and Exercises 61–66)
- Section 5.5 builds on the new material in Section 5.1 with a new objective on solving equations that contain both trigonometric and logarithmic expressions. (Section 5.5, Example 13 and Exercises 117–128)
- Section 6.5 introduces the notation $r \text{ cis } \theta$ as an abbreviation for $r(\cos\theta + i\sin\theta)$. (Section 6.5, Great Question! on p. 772)
- Section 6.5 provides a second explanation of finding complex roots using DeMoivre's Theorem. The new approach relies more on the relationships among the roots and less on the use of a formula. (Section 6.5, Great Question! on p. 772 and Example 9)

New in MyLab Math

- **Corequisite Support Resources** provide all the content and assessment resources necessary for students and instructors. MyLab Math supports various corequisite course models, including Concurrent (aka just-in-time) and Consecutive (aka front-loaded) models. For more details, see page xiii or the Corequisite Implementation Guide at **bit.ly/3il85pn**.
- **Integrated Review Activities** for selected topics provide hands-on work with important prerequisites.



GeoGebra Graphing Exercises are gradable graphing exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.

- **Setup & Solve Exercises** – We added more of these popular exercises, which require students to first describe how they will set up and approach the problem. This mirrors what students will be expected to do on a test.
- **Interactive Figures** – For this revision, we added many more interactive figures (in editable GeoGebra format) to the Video & Resource Library.
- **Enhanced Assignments** – These section-level assignments have three unique properties (and are fully editable):

  1. They help keep skills fresh with *spaced practice* of previously learned concepts.
  2. They have learning aids strategically turned off for some exercises to ensure that students understand how to work the exercises independently.
  3. They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.

- **Video Assignments** – These section-level assignments are especially helpful for online classes or "flipped" classes, where some or all learning takes place independently.
- **PowerPoint slides** are now animated. They also utilize Microsoft's Equation Editor, making them more easily editable.
- **Personal Inventory Assessments** are a collection of online exercises designed to promote self-reflection and engagement in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.

What Familiar Features Have Been Retained in the Sixth Edition?

- **Graphing and Functions.** Graphing and functions are introduced in Chapter 1, with an integrated graphing functional approach emphasized throughout the book. Graphs and functions that model data appear in nearly every section and Exercise Set. Examples and exercises use graphs of functions to explore relationships between data and to provide ways of visualizing a problem's solution. Because functions are the core of this course, students are repeatedly shown how functions relate to equations and graphs.
- **Learning Objectives.** Learning objectives are clearly stated at the beginning of each section under the heading "What You'll Learn." These objectives help students recognize and focus on the section's most important ideas. The objectives are restated in the margin at their point of use.
- **Chapter-Opening and Section-Opening Scenarios.** Every chapter and every section open with a scenario presenting a unique application of mathematics in students' lives outside the classroom. These scenarios are revisited in the course of the chapter or section in an example, discussion, or exercise.
- **Innovative Applications.** A wide variety of interesting applications, supported by up-to-date, real-world data, are included in every section.

Explanatory Voice Balloons

- **Explanatory Voice Balloons.** Voice balloons are used in a variety of ways to demystify mathematics. They translate algebraic and trigonometric ideas into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect problem solving to concepts students have already learned.
- **Detailed Worked-Out Examples.** Each example is titled, making the purpose of the example clear. Examples are clearly written and provide students with detailed step-by-step solutions. No steps are omitted and each step is thoroughly explained to the right of the mathematics.

CHECK POINT

- **Check Point Examples.** Each example is followed by a similar matched problem, called a Check Point, offering students the opportunity to test their understanding of the example by working a similar exercise. The answers to the Check Points are provided in the answer section.
- **Great Question!** This feature presents a variety of study tips in the context of students' questions. Answers to questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions. As a secondary benefit, this feature should help students not to feel anxious or threatened when asking questions in class.

BLITZER BONUS

- **Blitzer Bonuses.** These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the algebra and trigonometry under study, showing students that math is an interesting and dynamic discipline.
- **Concept and Vocabulary Checks.** This feature offers short-answer exercises, mainly fill-in-the-blank and true/false items, that assess students' understanding of the definitions and concepts presented in each section. The Concept and Vocabulary Checks precede the section Exercise Sets and have the prefix "C."
- **Extensive and Varied Exercise Sets.** An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within nine category types: Practice Exercises, Practice PLUS Exercises, Application Exercises, Explaining the Concepts, Technology Exercises, Critical Thinking Exercises, Group Exercises, Retaining the Concepts, and Preview Exercises. This format makes it easy to create well-rounded homework assignments. The order of the Practice Exercises is exactly the same as the order of the section's worked examples. This parallel order enables students to refer to the titled examples and their detailed explanations to achieve success working the Practice Exercises.
    - **Practice PLUS Problems.** This category of exercises contains more challenging practice problems that often require students to combine several skills or concepts. With an average of ten Practice PLUS problems per Exercise Set, instructors are provided with the option of creating assignments that take Practice Exercises to a more challenging level.
    - **Retaining the Concepts.** Beginning with Chapter 2, each Exercise Set contains three or four review exercises under the header "Retaining the Concepts." These exercises are intended for students to review previously covered objectives in order to improve their understanding of the topics and to help maintain their mastery of the material. If students are not certain how to solve a review exercise, they can turn to the section and worked example given in parentheses at the end of each exercise.

- **Mid-Chapter Check Points.** At approximately the midway point in each chapter, an integrated set of Review Exercises allows students to review and assimilate the skills and concepts they learned separately over several sections.
- **Integration of Technology Using Graphic and Numerical Approaches to Problems.** Side-by-side features in the technology boxes connect algebraic and trigonometric solutions to graphic and numerical approaches to problems. Although the use of graphing utilities is optional, students can use the explanatory voice balloons to understand different approaches to problems even if they are not using a graphing utility in the course.
- **Brief Reviews.** Beginning with Chapter 1, the Brief Review boxes that appear throughout the book summarize mathematical skills, many of which are course prerequisites that students have learned but which many students need to review. This feature appears whenever a particular skill is first needed and eliminates the need for you to reteach that skill. For more detail, students are referred to the appropriate section and objective in a previous chapter where the topic is fully developed.

ACHIEVING SUCCESS

- **Achieving Success.** The Achieving Success boxes, appearing at the end of many sections in Chapters P through 6, offer strategies for persistence and success in college mathematics courses.
- **Discovery.** Discovery boxes, found throughout the text, encourage students to further explore algebraic and trigonometric concepts. These explorations are optional and their omission does not interfere with the continuity of the topic under consideration.
- **Chapter Summaries.** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.
- **End-of-Chapter Materials.** A comprehensive collection of Review Exercises for each of the chapter's sections follows the Summary. This is followed by a Chapter Test that enables students to test their understanding of the material covered in the chapter. Beginning with Chapter 2, each chapter concludes with a comprehensive collection of mixed Cumulative Review Exercises.

MyLab™ Math Resources for Success

MyLab Math (**pearson.com/mylab/math**) is available to accompany Pearson's market-leading text options, including this text (access code required). MyLab Math is the teaching and learning platform that empowers you to reach every student. It combines trusted author content—including full eText and online homework with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

NEW! Corequisite Course Support

MyLab Math supports various corequisite course models, including Concurrent (aka just-in-time) and Consecutive (aka front-loaded) models. MyLab Math for this text contains all of these learning and assessment resources to support corequisite courses:

1. **Complete Corequisite eText** built from Bob Blitzer's developmental mathematics texts so that it matches the features and pedagogy of this text.
2. **Instructional videos** for each corequisite objective.
3. **Assignable algorithmic exercises** for each corequisite objective.
4. **Worksheets** with instruction and exercises for each corequisite objective (also available in print).
5. **Activities** for selected corequisite objectives.
6. **Study Skills** support with self-help materials for time management, mindset, stress management, college transition, and more.
7. **Corequisite Implementation Guide** with specific guidelines for using the materials to teach various corequisite models. (Download at **bit.ly/3il85pn**.)

To help target instruction on corequisite objectives, MyLab includes these pre-made assessments:

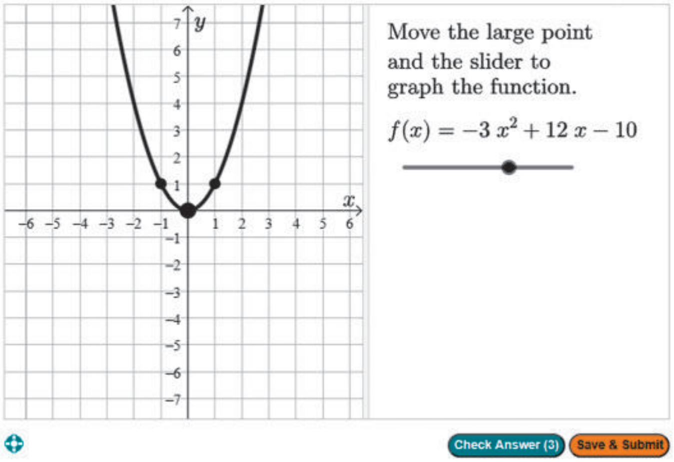
- **Readiness Quiz 1** addresses key arithmetic topics and is designed to be administered prior to beginning College Algebra topics.
- **Readiness Quiz 2** addresses basic Introductory Algebra topics and is designed to be administered prior to beginning College Algebra topics.
- **Skills Check Quiz for Each Chapter** addresses the prerequisite skills needed for each chapter in *Precalculus Essentials*.
- Based on the results of these quizzes, students can receive *personalized assignments* to address objectives that are not mastered. This way, students can focus on just the topics they need help with.

Note that the above resources are also designed to provide just-in-time help for students in your regular (non-corequisite) courses. (We understand that almost all students at some point need targeted refreshers on specific prerequisite skills.)

MyLab Math Student Resources

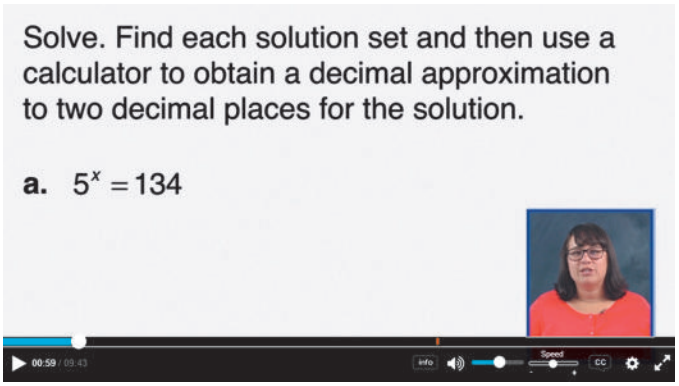
Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student needs practice, giving all students the support they need—when and where they need it—to be successful.

Exercises with Immediate Feedback – The exercises in MyLab Math reflect the approach and learning style of this text and regenerate algorithmically to give students unlimited opportunity for practice and mastery. Most exercises include learning aids, such as guided solutions and sample problems, and they offer helpful feedback when students enter incorrect answers.



▲ NEW! **GeoGebra Exercises** are gradable graphing exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.

- **Setup & Solve** exercises require students to first describe how they will set up and approach a problem. This reinforces conceptual understanding of the process applied in approaching the problem, promotes long-term retention of the skill, and mirrors what students will be expected to do on a test.
- **Concept & Vocabulary** exercises require students to demonstrate understanding of key ideas.

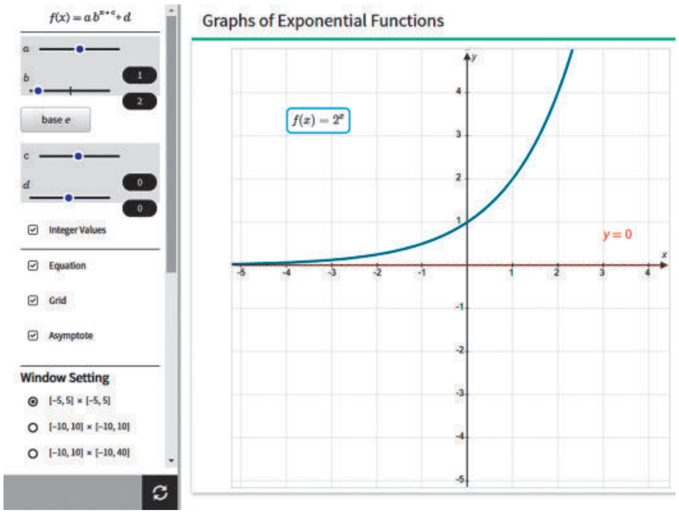


▲ **Instructional Videos** – High-quality instructional videos are included for every objective in the text. Many of these feature built-in interactive quizzes.

Chapter Test Prep Videos correspond to each exercise in the Chapter Test in the textbook, enabling students to effectively prepare for high-stakes testing. These are available in MyLab Math and **www.youtube.com/user/pearsonmathstats** (playlist "Blitzer Precalculus 7e").

Learning Guide (also available in print format) consists of four parts:

1. **Learning Guide worksheets** for each section of the text. These worksheets start with a catchy headline and motivating real-world connection followed by numerous "Solved Problems" and accompanying "Pencil Problems."
2. **Classroom Activities** for selected sections contain recommended group size, material needed, and time to complete.
3. **Integrated Review worksheets** for every prerequisite objective. These feature both instruction and practice.
4. NEW! **Integrated Review Activities** for selected topics provide hands-on work with important prerequisites.



▲ **Interactive Figures** bring mathematical concepts to life, helping students see the concepts through directed explorations and purposeful manipulation. These figures are assignable in MyLab Math and encourage active learning, critical thinking, and conceptual understanding. NEW! For this revision, we added many more interactive figures (in editable GeoGebra format) to the Video & Resource Library.

▲ **Mindset videos** and assignable, open-ended exercises foster a growth mindset in students. This material encourages them to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as learning opportunities—so often a hurdle for math students. These videos are one of many **Study Skills and Career-Readiness Resources** that address the non-math-related issues that can affect student success.

NEW! **Personal Inventory Assessments** are a collection of online exercises designed to promote self-reflection and engagement in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.

eText – Available in two formats: one that matches the textbook page-for-page and another that is "reflowable" for use on tablets and smartphones. The latter eText is also fully accessible using screen-readers.

Student Solutions Manual – Fully worked solutions to odd-numbered exercises. Available for download from within MyLab Math.

MyLab Math Instructor Resources

Your course is unique. So whether you'd like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

Pre-Built Assignments are designed to make the homework experience as effective as possible for students. All of these assignments are *fully editable*.

- NEW! **Enhanced Assignments** – These section-level assignments have three unique properties:
  1. They help keep skills fresh with *spaced practice* of previously learned concepts.
  2. They have learning aids strategically turned off for some exercises to ensure that students understand how to work the exercises independently.
  3. They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.
- NEW! **Video Assignments** – These section-level assignments are especially helpful for online classes or "flipped" classes, where some or all learning takes place independently.

Learning Catalytics – With Learning Catalytics™, you'll hear from every student when it matters most. You pose a variety of questions in class (choosing from pre-loaded questions or questions of your own making) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops.

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Other instructor resources include:

- **Mini Lecture Notes** contain additional examples and helpful teaching tips for each section of the text.
- **Instructor Solution Manual** contains worked-out solutions for every exercise in the text.
- **PowerPoint Lecture Slides** are fully editable and included for each section of the text. UPDATED! Slides are now animated. They also utilize Microsoft's Equation Editor, making them more easily editable.
- **TestGen®** enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson's online catalog.
- **Test Bank** features printable PDFs containing all of the test exercises available in TestGen.

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- Finally, thanks to the Pearson sales force, for their confidence and enthusiasm about the book.

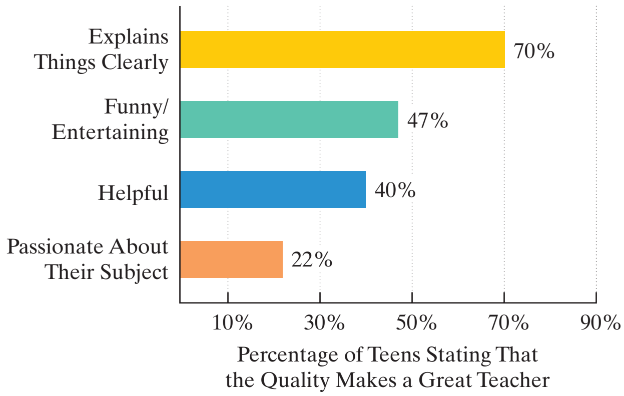
I hope that my passion for teaching, as well as my respect for the diverse population of students I have taught and learned from over the years, is apparent throughout this new edition. By connecting algebra and trigonometry to the whole spectrum of learning, it is my intent to show students that their world is profoundly mathematical, and indeed, π is in the sky.

Bob Blitzer

To the Student

The bar graph shows some of the qualities that students say make a great teacher. It was my goal to incorporate each of these qualities throughout the pages of this book.

Qualities That Make a Great Teacher



Source: Avanta Learning System

Explains Things Clearly

I understand that your primary purpose in reading *Precalculus Essentials* is to acquire a solid understanding of the required topics in this course. In order to achieve this goal, I've carefully explained each topic. Important definitions and procedures are set off in boxes, and worked-out examples that present solutions in a step-by-step manner appear in every section. Each example is followed by a similar matched problem, called a Check Point, for you to try so that you can actively participate in the learning process as you read the book. (Answers to all Check Points appear in the back of the book.)

Funny & Entertaining

Who says that a precalculus textbook can't be entertaining? From our unusual cover to the photos in the chapter and section openers, prepare to expect the unexpected. I hope some of the book's enrichment essays, called Blitzer Bonuses, will put a smile on your face from time to time.

Helpful

I designed the book's features to help you acquire knowledge of algebra and trigonometry, as well as to show you how algebra and trigonometry can solve authentic problems that apply to your life. These helpful features include:

- **Explanatory Voice Balloons:** Voice balloons are used in a variety of ways to make math less intimidating. They translate algebraic and trigonometric language into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect new concepts to concepts you have already learned.
- **Great Question!:** The book's Great Question! boxes are based on questions students ask in class. The answers to these questions give suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.
- **Achieving Success:** The book's Achieving Success boxes give you helpful strategies for success in learning algebra and trigonometry, as well as suggestions that can be applied for achieving your full academic potential in future college coursework.
- **Chapter Summaries:** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples from the chapter that illustrate these key concepts are also referenced in the chart. Review these summaries and you'll know the most important material in the chapter!

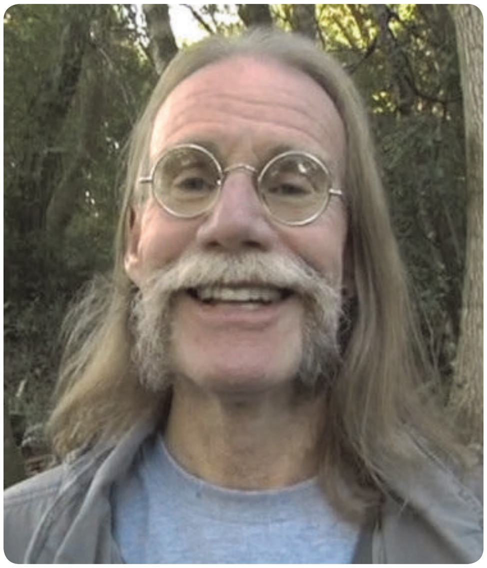
Passionate about the Subject

I passionately believe that no other discipline comes close to math in offering a more extensive set of tools for application and development of your mind. I wrote the book in Point Reyes National Seashore, 40 miles north of San Francisco. The park consists of 75,000 acres with miles of pristine surf-washed beaches, forested ridges, and bays bordered by white cliffs. It was my hope to convey the beauty and excitement of mathematics using nature's unspoiled beauty as a source of inspiration and creativity. Enjoy the pages that follow as you empower yourself with the algebra and trigonometry needed to succeed in college, your career, and your life.

Regards,

Bob Blitzer

ABOUT THE AUTHOR



Bob Blitzer is a native of Manhattan and received a Bachelor of Arts degree with dual majors in mathematics and psychology (minor: English literature) from the City College of New York. His unusual combination of academic interests led him toward a Master of Arts in mathematics from the University of Miami and a doctorate in behavioral sciences from Nova University. Bob's love for teaching mathematics was nourished for nearly 30 years at Miami Dade College, where he received numerous teaching awards, including Innovator of the Year from the League for Innovations in the Community College and an endowed chair based on excellence in the classroom. In addition to *Precalculus Essentials*, Bob has written textbooks covering developmental mathematics, introductory algebra, intermediate algebra, college algebra, algebra and trigonometry, trigonometry, and liberal arts mathematics, all published by Pearson. When not secluded in his Northern California writer's cabin, Bob can be found hiking the beaches and trails of Point Reyes National Seashore and tending to the chores required by his beloved entourage of horses, chickens, and irritable roosters.

APPLICATIONS INDEX

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Precalculus Essentials

PREREQUISITES

P Fundamental Concepts of Algebra

▶ What can algebra possibly have to tell me about

- the skyrocketing cost of a college education?
- student-loan debt?
- my workouts?
- the effects of alcohol?
- the meaning of the national debt that is more than $25 trillion?
- time dilation on a futuristic high-speed journey to a nearby star?
- racial bias?
- ethnic diversity in the United States?
- the widening imbalance between numbers of women and men on college campuses?

This chapter reviews fundamental concepts of algebra that are prerequisites for the study of college algebra. Throughout the chapter, you will see how the special language of algebra describes your world.

Here's where you'll find these applications:

- College costs: Section P.1, Example 2; Exercise Set P.1, Exercises 131–132
- Student-loan debt: Mid-Chapter Check Point, Exercise 42
- Workouts: Exercise Set P.1, Exercises 129–130
- The effects of alcohol: Blitzer Bonus beginning on page 15
- The national debt: Section P.2, Example 6
- Time dilation: Blitzer Bonus on page 44
- Racial bias: Exercise Set P.4, Exercises 91–92
- U.S. ethnic diversity: Chapter P Review, Exercise 23
- College gender imbalance: Chapter P Test, Exercise 32.

SECTION P.1 Algebraic Expressions, Mathematical Models, and Real Numbers

WHAT YOU'LL LEARN

1. Evaluate algebraic expressions.
2. Use mathematical models.
3. Find the intersection of two sets.
4. Find the union of two sets.
5. Recognize subsets of the real numbers.
6. Use inequality symbols.
7. Evaluate absolute value.
8. Use absolute value to express distance.
9. Identify properties of the real numbers.
10. Simplify algebraic expressions.

How would your lifestyle change if a gallon of gas cost \$9.15? Or if the price of a staple such as milk was \$15? That's how much those products would cost if their prices had increased at the same rate college tuition has increased since 1980. (*Source:* Center for College Affordability and Productivity) In this section, you will learn how the special language of algebra describes your world, including the skyrocketing cost of a college education.

Algebraic Expressions

Algebra uses letters, such as x and y, to represent numbers. If a letter is used to represent various numbers, it is called a **variable**. For example, imagine that you are basking in the sun on the beach. We can let x represent the number of minutes that you can stay in the sun without burning with no sunscreen. With a number 6 sunscreen, exposure time without burning is six times as long, or 6 times x. This can be written $6 \cdot x$, but it is usually expressed as $6x$. Placing a number and a letter next to one another indicates multiplication.

Notice that $6x$ combines the number 6 and the variable x using the operation of multiplication. A combination of variables and numbers using the operations of addition, subtraction, multiplication, or division, as well as powers or roots, is called an **algebraic expression**. Here are some examples of algebraic expressions:

$$x + 6, \quad x - 6, \quad 6x, \quad \frac{x}{6}, \quad 3x + 5, \quad x^2 - 3, \quad \sqrt{x} + 7.$$

Many algebraic expressions involve *exponents*. For example, the algebraic expression

$$-x^2 + 361x + 3193$$

approximates the average cost of tuition and fees at public U.S. colleges for the school year ending x years after 2000. The expression x^2 means $x \cdot x$ and is read "x to the second power" or "x squared." The exponent, 2, indicates that the base, x, appears as a factor two times. The negative sign in front of x^2 indicates that x^2 is multiplied by -1.

Exponential Notation

If n is a counting number (1, 2, 3, and so on),

Exponent or Power

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{b \text{ appears as a factor } n \text{ times.}}.$$

Base

b^n is read "the nth power of b" or "b to the nth power." Thus, the nth power of b is defined as the product of n factors of b. The expression b^n is called an **exponential expression**. Furthermore, $b^1 = b$.

For example,

$$8^2 = 8 \cdot 8 = 64, \quad 5^3 = 5 \cdot 5 \cdot 5 = 125, \quad \text{and} \quad 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16.$$

1 Evaluate algebraic expressions.

Evaluating Algebraic Expressions

Evaluating an algebraic expression means to find the value of the expression for a given value of the variable.

Many algebraic expressions involve more than one operation. Evaluating an algebraic expression without a calculator involves carefully applying the following order of operations agreement:

The Order of Operations Agreement

1. Perform operations within the innermost parentheses and work outward. If the algebraic expression involves a fraction, treat the numerator and the denominator as if they were each enclosed in parentheses.
2. Evaluate all exponential expressions.
3. Perform multiplications and divisions **as they occur**, working **from left to right**.
4. Perform additions and subtractions **as they occur**, working **from left to right**.

EXAMPLE 1 Evaluating an Algebraic Expression

Evaluate $7 + 5(x - 4)^3$ for $x = 6$.

Solution

$$\begin{aligned} 7 + 5(x - 4)^3 &= 7 + 5(6 - 4)^3 && \text{Replace } x \text{ with 6.} \\ &= 7 + 5(2)^3 && \text{First work inside parentheses: } 6 - 4 = 2. \\ &= 7 + 5(8) && \text{Evaluate the exponential expression: } 2^3 = 2\cdot2\cdot2 = 8. \\ &= 7 + 40 && \text{Multiply: } 5(8) = 40. \\ &= 47 && \text{Add.} \end{aligned}$$

CHECK POINT 1 Evaluate $8 + 6(x - 3)^2$ for $x = 13$.

2 Use mathematical models.

Formulas and Mathematical Models

An **equation** is formed when an equal sign is placed between two algebraic expressions. One aim of algebra is to provide a compact, symbolic description of the world. These descriptions involve the use of *formulas*. A **formula** is an equation that uses variables to express a relationship between two or more quantities.

Here are two examples of formulas related to heart rate and exercise.

Couch-Potato Exercise

$$H = \frac{1}{5}(220 - a)$$

Heart rate, in beats per minute, | is | $\frac{1}{5}$ of | the difference between 220 and your age.

Working It

$$H = \frac{9}{10}(220 - a)$$

Heart rate, in beats per minute, | is | $\frac{9}{10}$ of | the difference between 220 and your age.

The process of finding formulas to describe real-world phenomena is called **mathematical modeling**. Such formulas, together with the meaning assigned to the variables, are called **mathematical models**. We often say that these formulas model, or describe, the relationships among the variables.

EXAMPLE 2 Modeling the Cost of Attending a Public College

The bar graph in **Figure P.1** shows the average cost of tuition and fees for public four-year colleges, adjusted for inflation. The formula

$$T = -x^2 + 361x + 3193$$

models the average cost of tuition and fees, T, for public U.S. colleges for the school year ending x years after 2000.

a. Use the formula to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2020.

b. By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

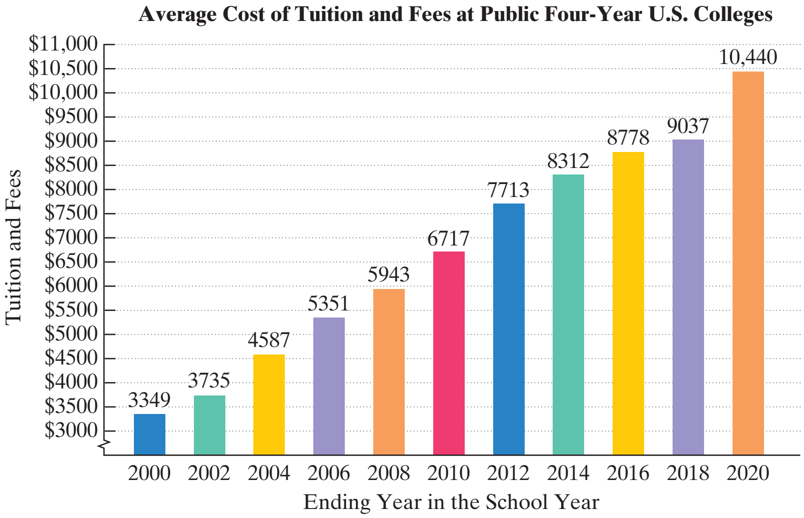


Figure P.1
Source: The College Board

Solution

a. Because 2020 is 20 years after 2000, we substitute 20 for x in the given formula. Then we use the order of operations to find T, the average cost of tuition and fees for the school year ending in 2020.

$T = -x^2 + 361x + 3193$ — This is the given mathematical model.

$T = -(20)^2 + 361(20) + 3193$ — Replace each occurrence of x with 20.

Only 20 is affected by the exponent. Square 20 and copy the negative.

$T = -(400) + 361(20) + 3193$ — Evaluate the exponential expression: $20^2 = 20 \cdot 20 = 400$.

$T = -400 + 7220 + 3193$ — Multiply from left to right: $-(400) = -1(400) = -400$ and $361(20) = 7220$.

$T = 10{,}013$ — Add.

The formula indicates that for the school year ending in 2020, the average cost of tuition and fees at public U.S. colleges was $10,013.

b. **Figure P.1** shows that the average cost of tuition and fees for the school year ending in 2020 was $10,440.

The cost obtained from the formula, $10,013, underestimates the actual data value by $10,440 − $10,013, or by $427.

BLITZER BONUS Is College Worthwhile?

"Questions have intensified about whether going to college is worthwhile," says *Education Pays*, released by the College Board Advocacy & Policy Center. "For the typical student, the investment pays off very well over the course of a lifetime, even considering the expense."

Among the findings in *Education Pays*:

- Median (middlemost) full-time earnings with a bachelor's degree in 2018 were $65,400, which is $24,900 more than high school graduates.
- Compared with a high school graduate, a four-year college graduate who enrolled in a public university at age 18 will break even by age 33. The college graduate will have earned enough by then to compensate for being out of the labor force for four years and for borrowing enough to pay tuition and fees, shown in **Figure P.1**.

DISCOVERY

Using the formula from Example 2 and Check Point 2, find T for $x = 100$, $x = 200$, $x = 300$, and $x = 400$. What happens to the values of T over time? Do you see how model breakdown has occurred?

CHECK POINT 2

a. Use the formula $T = -x^2 + 361x + 3193$, described in Example 2, to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2016.

b. By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

Sometimes a mathematical model gives an estimate that is not a good approximation or is extended to include values of the variable that do not make sense. In these cases, we say that **model breakdown** has occurred. For example, it is not likely that the formula in Example 2 would give a good estimate of tuition and fees in 2050 because it is too far in the future. Thus, model breakdown would occur.

Sets

Before we describe the set of real numbers, let's be sure you are familiar with some basic ideas about sets. A **set** is a collection of objects whose contents can be clearly determined. The objects in a set are called the **elements** of the set. For example, the set of numbers used for counting can be represented by

$$\{1, 2, 3, 4, 5, \ldots\}.$$

The braces, { }, indicate that we are representing a set. This form of representation, called the **roster method**, uses commas to separate the elements of the set. The symbol consisting of three dots after the 5, called an *ellipsis*, indicates that there is no final element and that the listing goes on forever.

A set can also be written in **set-builder notation**. In this notation, the elements of the set are described but not listed. Here is an example:

$$\{x \mid x \text{ is a counting number less than } 6\}.$$

The set of all x | such that | x is a counting number less than 6.

GREAT QUESTION!

Can I use symbols other than braces when writing sets using the roster method?

No. Grouping symbols such as parentheses, ( ), and square brackets, [ ], are not used to represent sets in the roster method. Furthermore, only commas are used to separate the elements of a set. Separators such as colons or semicolons are not used.

The same set written using the roster method is

$$\{1, 2, 3, 4, 5\}.$$

3 Find the intersection of two sets.

If A and B are sets, we can form a new set consisting of all elements that are in both A and B. This set is called the *intersection* of the two sets.

Definition of the Intersection of Sets

The **intersection** of sets A and B, written $A \cap B$, is the set of elements common to both set A **and** set B. This definition can be expressed in set-builder notation as follows:

$$A \cap B = \{x \mid x \text{ is an element of } A \text{ AND } x \text{ is an element of } B\}.$$

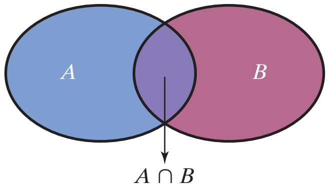


Figure P.2 Picturing the intersection of two sets

Figure P.2 shows a useful way of picturing the intersection of sets A and B. The figure indicates that $A \cap B$ contains those elements that belong to both A and B at the same time.

EXAMPLE 3 Finding the Intersection of Two Sets

Find the intersection: $\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\}$.

Solution The elements common to $\{7, 8, 9, 10, 11\}$ and $\{6, 8, 10, 12\}$ are 8 and 10. Thus,

$$\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\} = \{8, 10\}.$$

CHECK POINT 3 Find the intersection: $\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\}$.

If a set has no elements, it is called the **empty set**, or the **null set**, and is represented by the symbol $\varnothing$. Here is an example that shows how the empty set can result when finding the intersection of two sets:

$$\{2, 4, 6\} \cap \{3, 5, 7\} = \varnothing.$$

These sets have no common elements.

Their intersection has no elements and is the empty set.

4 Find the union of two sets.

Another set that we can form from sets A and B consists of elements that are in A or B or in both sets. This set is called the *union* of the two sets.

Definition of the Union of Sets

The **union** of sets A and B, written $A \cup B$, is the set of elements that are members of set A **or** of set B or of both sets. This definition can be expressed in set-builder notation as follows:

$$A \cup B = \{x \mid x \text{ is an element of } A \text{ OR } x \text{ is an element of } B\}.$$

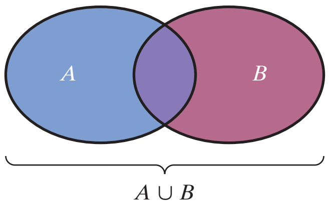


Figure P.3 Picturing the union of two sets

Figure P.3 shows a useful way of picturing the union of sets A and B. The figure indicates that $A \cup B$ is formed by joining the sets together.

We can find the union of set A and set B by listing the elements of set A. Then we include any elements of set B that have not already been listed. Enclose all elements that are listed with braces. This shows that the union of two sets is also a set.

GREAT QUESTION!

How can I use the words *union* and *intersection* to help me distinguish between these two operations?

Union, as in a marriage union, suggests joining things or uniting them. Intersection, as in the intersection of two crossing streets, brings to mind the area common to both, suggesting things that overlap.

EXAMPLE 4 Finding the Union of Two Sets

Find the union: $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$.

Solution To find $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$, start by listing all the elements from the first set, namely, 7, 8, 9, 10, and 11. Now list all the elements from the second set that are not in the first set, namely, 6 and 12. The union is the set consisting of all these elements. Thus,

$$\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\} = \{6, 7, 8, 9, 10, 11, 12\}.$$

Although 8 and 10 appear in both sets, do not list 8 and 10 twice.

CHECK POINT 4 Find the union: $\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\}$.

5 Recognize subsets of the real numbers.

The Set of Real Numbers

The sets that make up the real numbers are summarized in **Table P.1**. We refer to these sets as **subsets** of the real numbers, meaning that all elements in each subset are also elements in the set of real numbers.

Table P.1 Important Subsets of the Real Numbers

Name/Symbol	Description	Examples
Natural numbers $\mathbb{N}$	$\{1, 2, 3, 4, 5, \ldots\}$ These are the numbers that we use for counting.	$2, 3, 5, 17$
Whole numbers $\mathbb{W}$	$\{0, 1, 2, 3, 4, 5, \ldots\}$ The set of whole numbers includes 0 and the natural numbers.	$0, 2, 3, 5, 17$
Integers $\mathbb{Z}$	$\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$ The set of integers includes the negatives of the natural numbers and the whole numbers.	$-17, -5, -3, -2, 0, 2, 3, 5, 17$
Rational numbers $\mathbb{Q}$	$\left\{\frac{a}{b} \,\middle\vert\, a \text{ and } b \text{ are integers and } b \neq 0\right\}$ This means that b is not equal to zero. The set of rational numbers is the set of all numbers that can be expressed as a quotient of two integers, with the denominator not 0. Rational numbers can be expressed as terminating or repeating decimals.	$-17 = \frac{-17}{1}, -5 = \frac{-5}{1}, -3, -2,$ $0, 2, 3, 5, 17,$ $\frac{2}{5} = 0.4,$ $\frac{-2}{3} = -0.6666\ldots = -0.\overline{6}$
Irrational numbers $\mathbb{I}$	The set of irrational numbers is the set of all numbers whose decimal representations are neither terminating nor repeating. Irrational numbers cannot be expressed as a quotient of integers.	$\sqrt{2} \approx 1.414214$ $-\sqrt{3} \approx -1.73205$ $\pi \approx 3.142$ $-\frac{\pi}{2} \approx -1.571$

Notice the use of the symbol $\approx$ in the examples of irrational numbers. The symbol means "is approximately equal to." Thus,

$$\sqrt{2} \approx 1.414214.$$

We can verify that this is only an approximation by multiplying 1.414214 by itself. The product is very close to, but not exactly, 2:

$$1.414214 \times 1.414214 = 2.000001237796.$$

TECHNOLOGY

A calculator with a square root key gives a decimal approximation for $\sqrt{2}$, not the exact value.

Not all square roots are irrational. For example, $\sqrt{25} = 5$ because $5^2 = 5 \cdot 5 = 25$. Thus, $\sqrt{25}$ is a natural number, a whole number, an integer, and a rational number $\left(\sqrt{25} = \frac{5}{1}\right)$.

The set of *real numbers* is formed by taking the union of the sets of rational numbers and irrational numbers. Thus, every real number is either rational or irrational, as shown in **Figure P.4**.

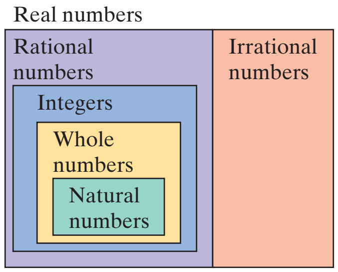


Figure P.4 Every real number is either rational or irrational.

Real Numbers

The set of **real numbers** is the set of numbers that are either rational or irrational:

$$\{x \mid x \text{ is rational or } x \text{ is irrational}\}.$$

The symbol $\mathbb{R}$ is used to represent the set of real numbers. Thus,

$$\mathbb{R} = \{x \mid x \text{ is rational}\} \cup \{x \mid x \text{ is irrational}\}.$$

EXAMPLE 5 Recognizing Subsets of the Real Numbers

Consider the following set of numbers:

$$\left\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\right\}.$$

List the numbers in the set that are

a. natural numbers. **b.** whole numbers. **c.** integers.
d. rational numbers. **e.** irrational numbers. **f.** real numbers.

Solution

a. Natural numbers: The natural numbers are the numbers used for counting. The only natural number in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ is $\sqrt{81}$ because $\sqrt{81} = 9$. (9 multiplied by itself, or 9^2, is 81.)

b. Whole numbers: The whole numbers consist of the natural numbers and 0. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are whole numbers are 0 and $\sqrt{81}$.

c. Integers: The integers consist of the natural numbers, 0, and the negatives of the natural numbers. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are integers are $\sqrt{81}$, 0, and -7.

d. Rational numbers: All numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that can be expressed as the quotient of integers are rational numbers. These include $-7\left(-7 = \frac{-7}{1}\right)$, $-\frac{3}{4}$, $0\left(0 = \frac{0}{1}\right)$, and $\sqrt{81}\left(\sqrt{81} = \frac{9}{1}\right)$. Furthermore, all numbers in the set that are terminating or repeating decimals are also rational numbers. These include $0.\overline{6}$ and 7.3.

e. Irrational numbers: The irrational numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are $\sqrt{5}\left(\sqrt{5} \approx 2.236\right)$ and $\pi(\pi \approx 3.14)$. Both $\sqrt{5}$ and π are only approximately equal to 2.236 and 3.14, respectively. In decimal form, $\sqrt{5}$ and π neither terminate nor have blocks of repeating digits.

f. Real numbers: All the numbers in the given set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are real numbers.

CHECK POINT 5 Consider the following set of numbers:

$$\left\{-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}.$$

List the numbers in the set that are

a. natural numbers. **b.** whole numbers. **c.** integers.
d. rational numbers. **e.** irrational numbers. **f.** real numbers.

The Real Number Line

The **real number line** is a graph used to represent the set of real numbers. An arbitrary point, called the **origin**, is labeled 0. Select a point to the right of 0 and label it 1. The distance from 0 to 1 is called the **unit distance**. Numbers to the right of the origin are **positive** and numbers to the left of the origin are **negative**. The real number line is shown in **Figure P.5**.

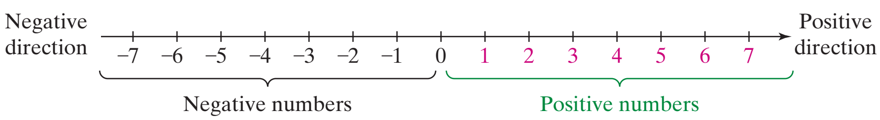
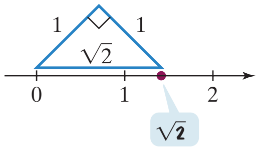


Figure P.5 The real number line

> **GREAT QUESTION!**
>
> **How did you locate $\sqrt{2}$ as a precise point on the number line in Figure P.6?**
>
> We used a right triangle with two legs of length 1. The remaining side has a length measuring $\sqrt{2}$.
>
> 
>
>
> We'll have lots more to say about right triangles later in the book.

Real numbers are **graphed** on a number line by placing a dot at the correct location for each number. The integers are easiest to locate. In **Figure P.6**, we've graphed six rational numbers and three irrational numbers on a real number line.

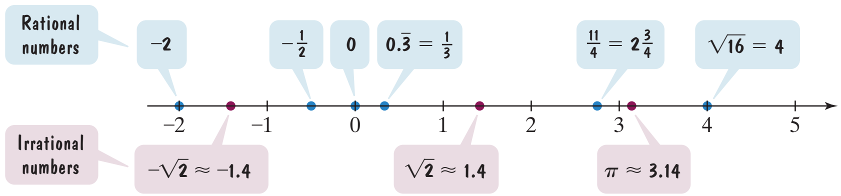


Figure P.6 Graphing numbers on a real number line

Every real number corresponds to a point on the number line and every point on the number line corresponds to a real number. We say that there is a **one-to-one correspondence** between all the real numbers and all points on a real number line.

6 Use inequality symbols.

Ordering the Real Numbers

On the real number line, the real numbers increase from left to right. The lesser of two real numbers is the one farther to the left on a number line. The greater of two real numbers is the one farther to the right on a number line.

Look at the number line in **Figure P.7**. The integers -4 and -1 are graphed.

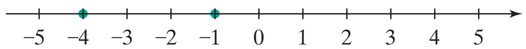


Figure P.7

Observe that -4 is to the left of -1 on the number line. This means that -4 is less than -1.

$$-4 < -1$$

−4 is less than −1 because −4 is to the left of −1 on the number line.

In **Figure P.7**, we can also observe that -1 is to the right of -4 on the number line. This means that -1 is greater than -4.

$$-1 > -4$$

−1 is greater than −4 because −1 is to the right of −4 on the number line.

The symbols $<$ and $>$ are called **inequality symbols**. These symbols always point to the lesser of the two real numbers when the inequality statement is true.

−4 is less than −1. $\quad -4 < -1 \quad$ The symbol points to -4, the lesser number.

−1 is greater than −4. $\quad -1 > -4 \quad$ The symbol still points to -4, the lesser number.

The symbols $<$ and $>$ may be combined with an equal sign, as shown in the following table:

Symbols	Meaning	Examples	Explanation
$a \le b$	a is less than or equal to b.	$2 \le 9$ $9 \le 9$	Because $2 < 9$ Because $9 = 9$
$b \ge a$	b is greater than or equal to a.	$9 \ge 2$ $2 \ge 2$	Because $9 > 2$ Because $2 = 2$

This inequality is true if either the $<$ part or the $=$ part is true.

This inequality is true if either the $>$ part or the $=$ part is true.

7 Evaluate absolute value.

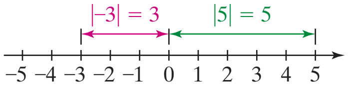


Figure P.8 Absolute value as the distance from 0

Absolute Value

The **absolute value** of a real number a, denoted by $|a|$, is the distance from 0 to a on the number line. **This distance is always taken to be nonnegative.** For example, the real number line in **Figure P.8** shows that

$$|-3| = 3 \quad \text{and} \quad |5| = 5.$$

The absolute value of -3 is 3 because -3 is 3 units from 0 on the number line. The absolute value of 5 is 5 because 5 is 5 units from 0 on the number line. The absolute value of a positive real number or 0 is the number itself. The absolute value of a negative real number, such as -3, is the number without the negative sign.

We can define the absolute value of the real number x without referring to a number line. The algebraic definition of the absolute value of x is given as follows:

Definition of Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

If x is nonnegative (that is, $x \geq 0$), the absolute value of x is the number itself. For example,

$$|5| = 5 \qquad |\pi| = \pi \qquad \left|\frac{1}{3}\right| = \frac{1}{3} \qquad |0| = 0.$$

Zero is the only number whose absolute value is 0.

If x is a negative number (that is, $x < 0$), the absolute value of x is the opposite of x. This makes the absolute value positive. For example,

$$|-3| = -(-3) = 3 \qquad |-\pi| = -(-\pi) = \pi \qquad \left|-\frac{1}{3}\right| = -\left(-\frac{1}{3}\right) = \frac{1}{3}.$$

This middle step is usually omitted.

Observe that **the absolute value of any nonzero number is always positive**.

EXAMPLE 6 Evaluating Absolute Value

Rewrite each expression without absolute value bars:

a. $|\sqrt{3} - 1|$ **b.** $|2 - \pi|$ **c.** $\dfrac{|x|}{x}$ if $x < 0$.

Solution

a. Because $\sqrt{3} \approx 1.7$, the number inside the absolute value bars, $\sqrt{3} - 1$, is positive. The absolute value of a positive number is the number itself. Thus,

$$|\sqrt{3} - 1| = \sqrt{3} - 1.$$

b. Because $\pi \approx 3.14$, the number inside the absolute value bars, $2 - \pi$, is negative. The absolute value of x when $x < 0$ is $-x$. Thus,

$$|2 - \pi| = -(2 - \pi) = \pi - 2.$$

c. If $x < 0$, then $|x| = -x$. Thus,

$$\frac{|x|}{x} = \frac{-x}{x} = -1.$$

CHECK POINT 6 Rewrite each expression without absolute value bars:

a. $|1 - \sqrt{2}|$ **b.** $|\pi - 3|$ **c.** $\dfrac{|x|}{x}$ if $x > 0$.

DISCOVERY

Verify the triangle inequality if $a = 4$ and $b = 5$. Verify the triangle inequality if $a = 4$ and $b = -5$.

When does equality occur in the triangle inequality and when does inequality occur? Verify your observation with additional number pairs.

Listed below are several basic properties of absolute value. Each of these properties can be derived from the definition of absolute value.

Properties of Absolute Value

For all real numbers a and b,

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $a \leq |a|$
4. $|ab| = |a||b|$
5. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \quad b \neq 0$
6. $|a + b| \leq |a| + |b|$ (called the triangle inequality).

8 Use absolute value to express distance.

Distance Between Points on a Real Number Line

Absolute value is used to find the distance between two points on a real number line. If a and b are any real numbers, the **distance between *a* and *b*** is the absolute value of their difference. For example, the distance between 4 and 10 is 6. Using absolute value, we find this distance in one of two ways:

$$|10 - 4| = |6| = 6 \qquad \text{or} \qquad |4 - 10| = |-6| = 6.$$

The distance between 4 and 10 on the real number line is 6.

Notice that we obtain the same distance regardless of the order in which we subtract.

Distance Between Two Points on the Real Number Line

If a and b are any two points on a real number line, then the distance between a and b is given by

$$|a - b| \quad \text{or} \quad |b - a|,$$

where $|a - b| = |b - a|$.

EXAMPLE 7 **Distance Between Two Points on a Number Line**

Find the distance between -5 and 3 on the real number line.

Solution Because the distance between a and b is given by $|a - b|$, the distance between -5 and 3 is

$$|-5 - 3| = |-8| = 8.$$

$a = -5$ $b = 3$

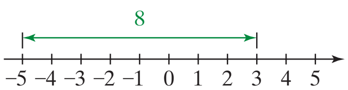


Figure P.9 The distance between -5 and 3 is 8.

Figure P.9 verifies that there are 8 units between -5 and 3 on the real number line. We obtain the same distance if we reverse the order of the subtraction:

$$|3 - (-5)| = |8| = 8.$$

CHECK POINT 7 Find the distance between -4 and 5 on the real number line.

9 Identify properties of the real numbers.

Properties of Real Numbers and Algebraic Expressions

When you use your calculator to add two real numbers, you can enter them in any order. The fact that two real numbers can be added in any order is called the **commutative property of addition**. You probably use this property, as well as other properties of real numbers listed in **Table P.2**, without giving it much thought. The properties of the real numbers are especially useful when working with algebraic expressions. For each property listed in **Table P.2**, a, b, and c represent real numbers, variables, or algebraic expressions.

Table P.2 Properties of the Real Numbers

Name	Meaning	Examples
Commutative Property of Addition	Changing order when adding does not affect the sum. $a + b = b + a$	• $13 + 7 = 7 + 13$ • $13x + 7 = 7 + 13x$
Commutative Property of Multiplication	Changing order when multiplying does not affect the product. $ab = ba$	• $\sqrt{2} \cdot \sqrt{5} = \sqrt{5} \cdot \sqrt{2}$ • $x \cdot 6 = 6x$
Associative Property of Addition	Changing grouping when adding does not affect the sum. $(a + b) + c = a + (b + c)$	• $3 + (8 + x) = (3 + 8) + x$ $= 11 + x$
Associative Property of Multiplication	Changing grouping when multiplying does not affect the product. $(ab)c = a(bc)$	• $-2(3x) = (-2 \cdot 3)x = -6x$
Distributive Property of Multiplication over Addition	Multiplication distributes over addition. $a \cdot (b + c) = a \cdot b + a \cdot c$	• $7(4 + \sqrt{3}) = 7 \cdot 4 + 7 \cdot \sqrt{3}$ $= 28 + 7\sqrt{3}$ • $5(3x + 7) = 5 \cdot 3x + 5 \cdot 7$ $= 15x + 35$
Identity Property of Addition	Zero can be deleted from a sum. $a + 0 = a$ $0 + a = a$	• $\sqrt{3} + 0 = \sqrt{3}$ • $0 + 6x = 6x$
Identity Property of Multiplication	One can be deleted from a product. $a \cdot 1 = a$ $1 \cdot a = a$	• $1 \cdot \pi = \pi$ • $13x \cdot 1 = 13x$
Inverse Property of Addition	The sum of a real number and its additive inverse gives 0, the additive identity. $a + (-a) = 0$ $(-a) + a = 0$	• $\sqrt{5} + (-\sqrt{5}) = 0$ • $-\pi + \pi = 0$ • $6x + (-6x) = 0$ • $(-4y) + 4y = 0$
Inverse Property of Multiplication	The product of a nonzero real number and its multiplicative inverse gives 1, the multiplicative identity. $a \cdot \frac{1}{a} = 1, \quad a \neq 0$ $\frac{1}{a} \cdot a = 1, \quad a \neq 0$	• $7 \cdot \frac{1}{7} = 1$ • $\left(\frac{1}{x - 3}\right)(x - 3) = 1, \quad x \neq 3$

The properties of the real numbers in **Table P.2** apply to the operations of addition and multiplication. Subtraction and division are defined in terms of addition and multiplication.

GREAT QUESTION!

Do the commutative and associative properties work for subtraction and division?

No. Subtraction and division are not commutative operations.

$$a - b \neq b - a \qquad \frac{a}{b} \neq \frac{b}{a}$$

Furthermore, subtraction and division are not associative operations.

$$(a - b) - c \neq a - (b - c)$$
$$(a \div b) \div c \neq a \div (b \div c)$$

Verify each of these four statements using $a = 10$, $b = 5$, and $c = 2$.

Definitions of Subtraction and Division

Let a and b represent real numbers.

Subtraction: $a - b = a + (-b)$

We call $-b$ the **additive inverse** or **opposite** of b.

Division: $a \div b = a \cdot \frac{1}{b}$, where $b \neq 0$

We call $\frac{1}{b}$ the **multiplicative inverse** or **reciprocal** of b. The quotient of a and b, $a \div b$, can be written in the form $\frac{a}{b}$, where a is the **numerator** and b is the **denominator** of the fraction.

Because subtraction is defined in terms of adding an inverse, the distributive property can be applied to subtraction:

$$a(b - c) = ab - ac$$
$$(b - c)a = ba - ca.$$

For example,

$$4(2x - 5) = 4 \cdot 2x - 4 \cdot 5 = 8x - 20.$$

10 Simplify algebraic expressions.

Simplifying Algebraic Expressions

The **terms** of an algebraic expression are those parts that are separated by addition. For example, consider the algebraic expression

$$7x - 9y + z - 3,$$

which can be expressed as

$$7x + (-9y) + z + (-3).$$

This expression contains four terms, namely, $7x$, $-9y$, z, and -3.

The numerical part of a term is called its **coefficient**. In the term $7x$, the 7 is the coefficient. If a term containing one or more variables is written without a coefficient, the coefficient is understood to be 1. Thus, z means $1z$. If a term is a **constant**, its coefficient is that constant. Thus, the coefficient of the constant term -3 is -3.

$$7x + (-9y) + z + (-3)$$

Coefficient is 7. | Coefficient is −9. | Coefficient is 1; z means $1z$. | Coefficient is −3.

The parts of each term that are multiplied are called the **factors** of the term. The **factors** of the term $7x$ are 7 and x.

Like terms are terms that have exactly the same variable factors. For example, $3x$ and $7x$ are like terms. The distributive property in the form

$$ba + ca = (b + c)a$$

enables us to add or subtract like terms. For example,

$$3x + 7x = (3 + 7)x = 10x$$
$$7y^2 - y^2 = 7y^2 - 1y^2 = (7 - 1)y^2 = 6y^2.$$

This process is called **combining like terms**.

GREAT QUESTION!

What is the bottom line for combining like terms?

To combine like terms mentally, add or subtract the coefficients of the terms. Use this result as the coefficient of the terms' variable factor(s).

An algebraic expression is **simplified** when parentheses have been removed and like terms have been combined.

EXAMPLE 8 **Simplifying an Algebraic Expression**

Simplify: $6(2x^2 + 4x) + 10(4x^2 + 3x)$.

Solution

$$6(2x^2 + 4x) + 10(4x^2 + 3x)$$

$= 6 \cdot 2x^2 + 6 \cdot 4x + 10 \cdot 4x^2 + 10 \cdot 3x$ — Use the distributive property to remove the parentheses.

$= 12x^2 + 24x + 40x^2 + 30x$ — Multiply.

$= (12x^2 + 40x^2) + (24x + 30x)$ — Group like terms.

$= 52x^2 + 54x$ — Combine like terms.

$52x^2$ and $54x$ are not like terms. They contain different variable factors, x^2 and x, and cannot be combined.

CHECK POINT 8 Simplify: $7(4x^2 + 3x) + 2(5x^2 + x)$.

Properties of Negatives

The distributive property can be extended to cover more than two terms within parentheses. For example,

This sign represents subtraction. This sign tells us that the number is negative.

$$-3(4x - 2y + 6) = -3 \cdot 4x - (-3) \cdot 2y - 3 \cdot 6$$
$$= -12x - (-6y) - 18$$
$$= -12x + 6y - 18.$$

The voice balloons illustrate that negative signs can appear side by side. They can represent the operation of subtraction or the fact that a real number is negative. Here is a list of properties of negatives and how they are applied to algebraic expressions:

Properties of Negatives

Let a and b represent real numbers, variables, or algebraic expressions.

Property	Examples
1. $(-1)a = -a$	$(-1)4xy = -4xy$
2. $-(-a) = a$	$-(-6y) = 6y$
3. $(-a)b = -ab$	$(-7)4xy = -7 \cdot 4xy = -28xy$
4. $a(-b) = -ab$	$5x(-3y) = -5x \cdot 3y = -15xy$
5. $-(a + b) = -a - b$	$-(7x + 6y) = -7x - 6y$
6. $-(a - b) = -a + b = b - a$	$-(3x - 7y) = -3x + 7y = 7y - 3x$

It is not uncommon to see algebraic expressions with parentheses preceded by a negative sign or subtraction. Properties 5 and 6 in the box, $-(a + b) = -a - b$ and $-(a - b) = -a + b$, are related to this situation. An expression of the form $-(a + b)$ can be simplified as follows:

$$-(a + b) = -1(a + b) = (-1)a + (-1)b = -a + (-b) = -a - b.$$

Do you see a fast way to obtain the simplified expression on the right in the preceding equation? **If a negative sign or a subtraction symbol appears outside parentheses, drop the parentheses and change the sign of every term within the parentheses.** For example,

$$-(3x^2 - 7x - 4) = -3x^2 + 7x + 4.$$

EXAMPLE 9 **Simplifying an Algebraic Expression**

Simplify: $8x + 2[5 - (x - 3)]$.

Solution

$8x + 2[5 - (x - 3)]$	
$= 8x + 2[5 - x + 3]$	Drop parentheses and change the sign of each term in parentheses: $-(x - 3) = -x + 3$.
$= 8x + 2[8 - x]$	Simplify inside brackets: $5 + 3 = 8$.
$= 8x + 16 - 2x$	Apply the distributive property: $2[8 - x] = 2 \cdot 8 - 2x = 16 - 2x$.
$= (8x - 2x) + 16$	Group like terms.
$= (8 - 2)x + 16$	Apply the distributive property.
$= 6x + 16$	Simplify.

CHECK POINT 9 Simplify: $6 + 4[7 - (x - 2)]$.

BLITZER BONUS **Using Algebra to Measure Blood-Alcohol Concentration**

The amount of alcohol in a person's blood is known as blood-alcohol concentration (BAC), measured in grams of alcohol per deciliter of blood. A BAC of 0.08, meaning 0.08%, indicates that a person has 8 parts alcohol per 10,000 parts blood. In every state in the United States, it is illegal to drive with a BAC of 0.08 or higher.

How Do I Measure My Blood-Alcohol Concentration?

Here's a formula that models BAC for a person who weighs w pounds and who has n drinks* per hour.

$$\text{BAC} = \frac{600n}{w(0.6n + 169)}$$

Number of drinks consumed in an hour

Blood-alcohol concentration

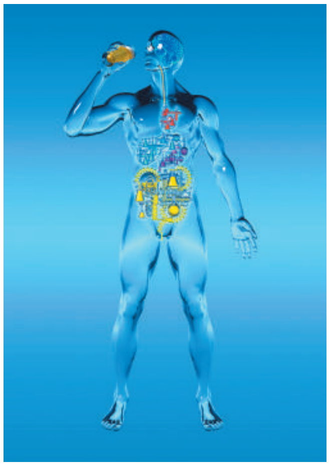
Body weight, in pounds

*A drink can be a 12-ounce can of beer, a 5-ounce glass of wine, or a 1.5-ounce shot of liquor. Each contains approximately 14 grams, or $\frac{1}{2}$ ounce, of alcohol.

Blood-alcohol concentration can be used to quantify the meaning of "tipsy."

BAC	Effects on Behavior
0.05	Feeling of well-being; mild release of inhibitions; absence of observable effects
0.08	Feeling of relaxation; mild sedation; exaggeration of emotions and behavior; slight impairment of motor skills; increase in reaction time
0.12	Muscle control and speech impaired; difficulty performing motor skills; uncoordinated behavior
0.15	Euphoria; major impairment of physical and mental functions; irresponsible behavior; some difficulty standing, walking, and talking
0.35	Surgical anesthesia; lethal dosage for a small percentage of people
0.40	Lethal dosage for 50% of people; severe circulatory and respiratory depression; alcohol poisoning/overdose

Source: National Clearinghouse for Alcohol and Drug Information



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Keeping in mind the meaning of "tipsy," we can use our model to compare blood-alcohol concentrations of a 120-pound person and a 200-pound person for various numbers of drinks. We determined each BAC using a calculator, rounding to three decimal places.

Blood-Alcohol Concentrations of a 120-Pound Person

$$\text{BAC} = \frac{600n}{120(0.6n + 169)}$$

***n* (number of drinks per hour)**	1	2	3	4	5	6	7	8	9	10
BAC (blood-alcohol concentration)	0.029	0.059	0.088	0.117	0.145	0.174	0.202	0.230	0.258	0.286

Illegal to drive (n = 3 to 10)

Blood-Alcohol Concentrations of a 200-Pound Person

$$\text{BAC} = \frac{600n}{200(0.6n + 169)}$$

***n* (number of drinks per hour)**	1	2	3	4	5	6	7	8	9	10
BAC (blood-alcohol concentration)	0.018	0.035	0.053	0.070	0.087	0.104	0.121	0.138	0.155	0.171

Illegal to drive (n = 5 to 10)

Like all mathematical models, the formula for BAC gives approximate rather than exact values. There are other variables that influence blood-alcohol concentration that are not contained in the model. These include the rate at which an individual's body processes alcohol, how quickly one drinks, sex, age, physical condition, and the amount of food eaten prior to drinking.

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

C1. A combination of numbers, variables, and operation symbols is called an algebraic ________.

C2. If n is a counting number, b^n, read ________, indicates that there are n factors of b. The number b is called the ________ and the number n is called the ________.

C3. An equation that expresses a relationship between two or more variables, such as $H = \frac{9}{10}(220 - a)$, is called a/an ________. The process of finding such equations to describe real-world phenomena is called mathematical ________. Such equations, together with the meaning assigned to the variables, are called mathematical ________.

C4. The set of elements common to both set A and set B is called the ________ of sets A and B and is symbolized by ________.

C5. The set of elements that are members of set A or set B or of both sets is called the ________ of sets A and B and is symbolized by ________.

C6. The set $\{1, 2, 3, 4, 5, \ldots\}$ is called the set of ________ numbers.

C7. The set $\{0, 1, 2, 3, 4, 5, \ldots\}$ is called the set of ________ numbers

C8. The set $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ is called the set of ________.

C9. The set of numbers in the form $\frac{a}{b}$, where a and b belong to the set in Exercise C8 and $b \neq 0$, is called the set of ________ numbers.

C10. The set of numbers whose decimal representations are neither terminating nor repeating is called the set of ________ numbers.

C11. Every real number is either a/an ________ number or a/an ________ number.

C12. The notation $|x|$ is read the ________ of x. If $x \geq 0$, then $|x| =$ ________. If $x < 0$, then $|x| =$ ________.

C13. The commutative properties state that $a + b =$ ________ and $ab =$ ________.

C14. The associative properties state that $(a + b) + c =$ ________ and ________ $= a(bc)$.

C15. The distributive property states that $a(b + c) =$ ________.

C16. $a + (-a) =$ ________: The sum of a real number and its additive ________ is ________, the additive ________.

C17. $a \cdot \frac{1}{a} = 1, a \neq 0$: The product of a nonzero real number and its multiplicative ________ is ________, the multiplicative ________.

C18. An algebraic expression is ________ when parentheses have been removed and like terms have been combined.

C19. $-(-a) =$ ________.

P.1 EXERCISE SET

Practice Exercises

In Exercises 1–16, evaluate each algebraic expression for the given value or values of the variable(s).

1. $7 + 5x$, for $x = 10$
2. $8 + 6x$, for $x = 5$
3. $6x - y$, for $x = 3$ and $y = 8$
4. $8x - y$, for $x = 3$ and $y = 4$
5. $x^2 + 3x$, for $x = 8$
6. $x^2 + 5x$, for $x = 6$
7. $x^2 - 6x + 3$, for $x = 7$
8. $x^2 - 7x + 4$, for $x = 8$
9. $4 + 5(x - 7)^3$, for $x = 9$
10. $6 + 5(x - 6)^3$, for $x = 8$
11. $x^2 - 3(x - y)$, for $x = 8$ and $y = 2$
12. $x^2 - 4(x - y)$, for $x = 8$ and $y = 3$
13. $\frac{5(x + 2)}{2x - 14}$, for $x = 10$
14. $\frac{7(x - 3)}{2x - 16}$, for $x = 9$
15. $\frac{2x + 3y}{x + 1}$, for $x = -2$ and $y = 4$
16. $\frac{2x + y}{xy - 2x}$, for $x = -2$ and $y = 4$

The formula

$$C = \frac{5}{9}(F - 32)$$

expresses the relationship between Fahrenheit temperature, F, and Celsius temperature, C. In Exercises 17–18, use the formula to convert the given Fahrenheit temperature to its equivalent temperature on the Celsius scale.

17. 50°F
18. 86°F

A football was kicked vertically upward from a height of 4 feet with an initial speed of 60 feet per second. The formula

$$h = 4 + 60t - 16t^2$$

describes the ball's height above the ground, h, in feet, t seconds after it was kicked. Use this formula to solve Exercises 19–20.

19. What was the ball's height 2 seconds after it was kicked?
20. What was the ball's height 3 seconds after it was kicked?

In Exercises 21–28, find the intersection of the sets.

21. $\{1, 2, 3, 4\} \cap \{2, 4, 5\}$
22. $\{1, 3, 7\} \cap \{2, 3, 8\}$
23. $\{s, e, t\} \cap \{t, e, s\}$
24. $\{r, e, a, l\} \cap \{l, e, a, r\}$
25. $\{1, 3, 5, 7\} \cap \{2, 4, 6, 8, 10\}$
26. $\{0, 1, 3, 5\} \cap \{-5, -3, -1\}$
27. $\{a, b, c, d\} \cap \varnothing$
28. $\{w, y, z\} \cap \varnothing$

In Exercises 29–34, find the union of the sets.

29. $\{1, 2, 3, 4\} \cup \{2, 4, 5\}$
30. $\{1, 3, 7, 8\} \cup \{2, 3, 8\}$
31. $\{1, 3, 5, 7\} \cup \{2, 4, 6, 8, 10\}$
32. $\{0, 1, 3, 5\} \cup \{2, 4, 6\}$
33. $\{a, e, i, o, u\} \cup \varnothing$
34. $\{e, m, p, t, y\} \cup \varnothing$

In Exercises 35–38, list all numbers from the given set that are **a.** *natural numbers,* **b.** *whole numbers,* **c.** *integers,* **d.** *rational numbers,* **e.** *irrational numbers,* **f.** *real numbers.*

35. $\{-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}\}$
36. $\{-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}\}$
37. $\{-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}\}$
38. $\{-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}\}$
39. Give an example of a whole number that is not a natural number.
40. Give an example of a rational number that is not an integer.
41. Give an example of a number that is an integer, a whole number, and a natural number.
42. Give an example of a number that is a rational number, an integer, and a real number.

Determine whether each statement in Exercises 43–50 is true or false.

43. $-13 \leq -2$
44. $-6 > 2$
45. $4 \geq -7$
46. $-13 < -5$
47. $-\pi \geq -\pi$
48. $-3 > -13$
49. $0 \geq -6$
50. $0 \geq -13$

In Exercises 51–60, rewrite each expression without absolute value bars.

51. $|300|$
52. $|-203|$
53. $|12 - \pi|$
54. $|7 - \pi|$
55. $|\sqrt{2} - 5|$
56. $|\sqrt{5} - 13|$
57. $\frac{-3}{|-3|}$
58. $\frac{-7}{|-7|}$
59. $||-3| - |-7||$
60. $||-5| - |-13||$

In Exercises 61–66, evaluate each algebraic expression for $x = 2$ and $y = -5$.

61. $|x + y|$
62. $|x - y|$
63. $|x| + |y|$
64. $|x| - |y|$
65. $\frac{y}{|y|}$
66. $\frac{|x|}{x} + \frac{|y|}{y}$

In Exercises 67–74, express the distance between the given numbers using absolute value. Then find the distance by evaluating the absolute value expression.

67. 2 and 17
68. 4 and 15
69. −2 and 5
70. −6 and 8
71. −19 and −4
72. −26 and −3
73. −3.6 and −1.4
74. −5.4 and −1.2

In Exercises 75–84, state the name of the property illustrated.

75. $6 + (-4) = (-4) + 6$
76. $11 \cdot (7 + 4) = 11 \cdot 7 + 11 \cdot 4$
77. $6 + (2 + 7) = (6 + 2) + 7$
78. $6 \cdot (2 \cdot 3) = 6 \cdot (3 \cdot 2)$
79. $(2 + 3) + (4 + 5) = (4 + 5) + (2 + 3)$
80. $7 \cdot (11 \cdot 8) = (11 \cdot 8) \cdot 7$

81. $2(-8 + 6) = -16 + 12$

82. $-8(3 + 11) = -24 + (-88)$

83. $\frac{1}{(x + 3)}(x + 3) = 1, x \neq -3$

84. $(x + 4) + [-(x + 4)] = 0$

In Exercises 85–96, simplify each algebraic expression.

85. $5(3x + 4) - 4$ **86.** $2(5x + 4) - 3$

87. $5(3x - 2) + 12x$ **88.** $2(5x - 1) + 14x$

89. $7(3y - 5) + 2(4y + 3)$

90. $4(2y - 6) + 3(5y + 10)$

91. $5(3y - 2) - (7y + 2)$

92. $4(5y - 3) - (6y + 3)$

93. $7 - 4[3 - (4y - 5)]$

94. $6 - 5[8 - (2y - 4)]$

95. $18x^2 + 4 - [6(x^2 - 2) + 5]$

96. $14x^2 + 5 - [7(x^2 - 2) + 4]$

In Exercises 97–102, write each algebraic expression without parentheses.

97. $-(-14x)$ **98.** $-(-17y)$

99. $-(2x - 3y - 6)$ **100.** $-(5x - 13y - 1)$

101. $\frac{1}{3}(3x) + [(4y) + (-4y)]$ **102.** $\frac{1}{2}(2y) + [(-7x) + 7x]$

Practice PLUS

In Exercises 103–110, insert either <, >, or = in the shaded area to make a true statement.

103. $|-6| \;\square\; |-3|$ **104.** $|-20| \;\square\; |-50|$

105. $\left|\frac{3}{5}\right| \;\square\; |-0.6|$ **106.** $\left|\frac{5}{2}\right| \;\square\; |-2.5|$

107. $\frac{30}{40} - \frac{3}{4} \;\square\; \frac{14}{15} \cdot \frac{15}{14}$ **108.** $\frac{17}{18} \cdot \frac{18}{17} \;\square\; \frac{50}{60} - \frac{5}{6}$

109. $\frac{8}{13} \div \frac{8}{13} \;\square\; |-1|$ **110.** $|-2| \;\square\; \frac{4}{17} \div \frac{4}{17}$

In Exercises 111–120, use the order of operations to simplify each expression.

111. $8^2 - 16 \div 2^2 \cdot 4 - 3$ **112.** $10^2 - 100 \div 5^2 \cdot 2 - 3$

113. $\frac{5 \cdot 2 - 3^2}{[3^2 - (-2)]^2}$ **114.** $\frac{10 \div 2 + 3 \cdot 4}{(12 - 3 \cdot 2)^2}$

115. $8 - 3[-2(2 - 5) - 4(8 - 6)]$

116. $8 - 3[-2(5 - 7) - 5(4 - 2)]$

117. $\frac{2(-2) - 4(-3)}{5 - 8}$ **118.** $\frac{6(-4) - 5(-3)}{9 - 10}$

119. $\frac{(5 - 6)^2 - 2|3 - 7|}{89 - 3 \cdot 5^2}$ **120.** $\frac{12 \div 3 \cdot 5|2^2 + 3^2|}{7 + 3 - 6^2}$

In Exercises 121–128, write each English phrase as an algebraic expression. Then simplify the expression. Let x represent the number.

121. A number decreased by the sum of the number and four

122. A number decreased by the difference between eight and the number

123. Six times the product of negative five and a number

124. Ten times the product of negative four and a number

125. The difference between the product of five and a number and twice the number

126. The difference between the product of six and a number and negative two times the number

127. The difference between eight times a number and six more than three times the number

128. Eight decreased by three times the sum of a number and six

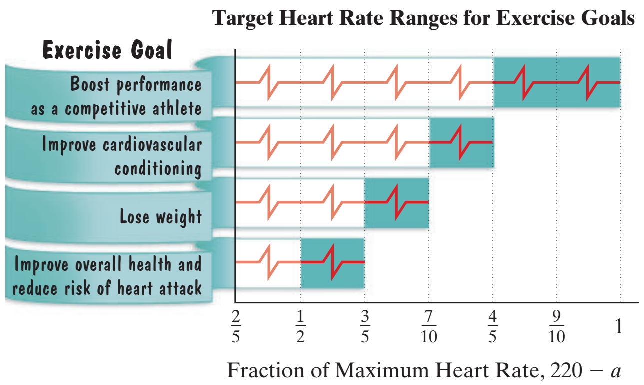
Application Exercises

The maximum heart rate, in beats per minute, that you should achieve during exercise is 220 minus your age:

$$220 - a.$$

This algebraic expression gives maximum heart rate in terms of age, a.

The following graph shows the target heart rate ranges for four types of exercise goals. The lower and upper limits of these ranges are fractions of the maximum heart rate, 220 − a. Exercises 129–130 are based on the information in the graph.



129. If your exercise goal is to improve cardiovascular conditioning, the graph shows the following range for target heart rate, H, in beats per minute:

Lower limit of range $H = \frac{7}{10}(220 - a)$

Upper limit of range $H = \frac{4}{5}(220 - a).$

a. What is the lower limit of the heart rate range, in beats per minute, for a 20-year-old with this exercise goal?

b. What is the upper limit of the heart rate range, in beats per minute, for a 20-year-old with this exercise goal?

130. If your exercise goal is to improve overall health, the graph shows the following range for target heart rate, H, in beats per minute:

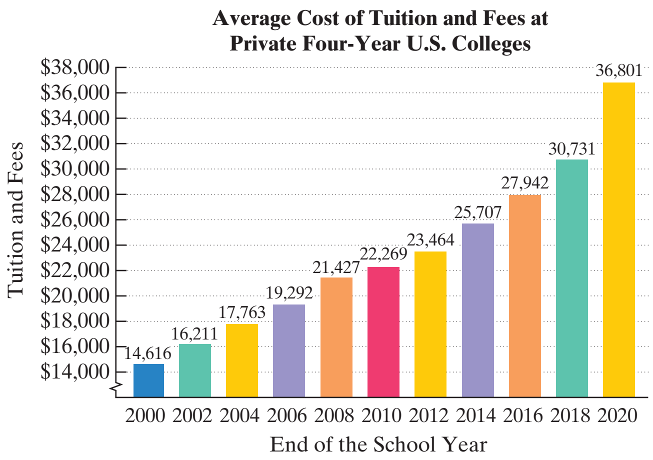
Lower limit of range $H = \frac{1}{2}(220 - a)$

Upper limit of range $H = \frac{3}{5}(220 - a).$

a. What is the lower limit of the heart rate range, in beats per minute, for a 30-year-old with this exercise goal?

b. What is the upper limit of the heart rate range, in beats per minute, for a 30-year-old with this exercise goal?

The bar graph shows the average cost of tuition and fees at private four-year colleges in the United States.



Source: The College Board

Here are two formulas that model the data shown in the graph. In each formula, T represents the average cost of tuition and fees at private U.S. colleges for the school year ending x years after 2000.

Model 1 $T = 975x + 13{,}547$

Model 2 $T = 32x^2 + 331x + 15{,}479$

Use this information to solve Exercises 131–132.

131. a. Use each formula to find the average cost of tuition and fees at private U.S. colleges for the school year ending in 2018. By how much does each model underestimate or overestimate the actual cost shown for the school year ending in 2018?

b. Use model 2 to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2030.

132. a. Use each formula to find the average cost of tuition and fees at private U.S. colleges for the school year ending in 2010. By how much does each underestimate or overestimate the actual cost shown for the school year ending in 2010?

b. Use model 2 to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2025.

133. This month you have a total of $6000 in interest-bearing credit card debt, split between a card charging 18% annual interest and a card charging 21% annual interest. If the interest-bearing balance on the card charging 18% is x dollars, then the total interest for the month is given by the algebraic expression

$$0.015x + 0.0175(6000 - x).$$

a. Simplify the algebraic expression.

b. Use each form of the algebraic expression to determine the total interest for the month if the balance on the card charging 18% is $4400.

c. Use the simplified form of the algebraic expression to determine the total interest for the month if the $6000 debt is split evenly between the two cards.

134. It takes you 50 minutes to get to campus. You spend t minutes walking to the bus stop and the rest of the time riding the bus. Your walking rate is 0.06 mile per minute and the bus travels at a rate of 0.5 mile per minute. The total distance walking and traveling by bus is given by the algebraic expression

$$0.06t + 0.5(50 - t).$$

a. Simplify the algebraic expression.

b. Use each form of the algebraic expression to determine the total distance that you travel if you spend 20 minutes walking to the bus stop.

c. Use the simplified form of the algebraic expression to determine the total distance you travel if the 50 minutes is split evenly between walking and riding the bus.

135. Read the Blitzer Bonus beginning on page 15. Use the formula

$$\text{BAC} = \frac{600n}{w(0.6n + 169)}$$

and replace w with your body weight. Using this formula and a calculator, compute your BAC for integers from $n = 1$ to $n = 10$. Round to three decimal places. According to this model, how many drinks can you consume in an hour without exceeding the legal measure of drunk driving?

Explaining the Concepts

ACHIEVING SUCCESS

An effective way to understand something is to explain it to someone else. You can do this by using the Explaining the Concepts exercises that ask you to respond with verbal or written explanations. Speaking or writing about a new concept uses a different part of your brain than thinking about the concept. Explaining new ideas verbally will quickly reveal any gaps in your understanding. It will also help you to remember new concepts for longer periods of time.

136. What is an algebraic expression? Give an example with your explanation.

137. If n is a natural number, what does b^n mean? Give an example with your explanation.

138. What does it mean when we say that a formula models real-world phenomena?

139. What is the intersection of sets A and B?

140. What is the union of sets A and B?

141. How do the whole numbers differ from the natural numbers?

142. Can a real number be both rational and irrational? Explain your answer.

143. If you are given two real numbers, explain how to determine which is the lesser.

Critical Thinking Exercises

Make Sense? *In Exercises 144–147, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

144. My mathematical model describes the data for tuition and fees at public four-year colleges for the past 20 years extremely well, so it will serve as an accurate prediction for the cost of public colleges in 2050.

145. A model that describes the average cost of tuition and fees at private U.S. colleges for the school year ending x years after 2000 cannot be used to estimate the cost of private education for the school year ending in 2000.

146. Regardless of what real numbers I substitute for x and y, I will always obtain zero when evaluating $2x^2y - 2yx^2$.

147. Just as the commutative properties change groupings, the associative properties change order.

In Exercises 148–155, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

148. Every rational number is an integer.

149. Some whole numbers are not integers.

150. Some rational numbers are not positive.

151. Irrational numbers cannot be negative.

152. The term x has no coefficient.

153. $5 + 3(x - 4) = 8(x - 4) = 8x - 32$

154. $-x - x = -x + (-x) = 0$

155. $x - 0.02(x + 200) = 0.98x - 4$

In Exercises 156–158, insert either $<$ or $>$ in the shaded area between the numbers to make the statement true.

156. $\sqrt{2}\ \square\ 1.5$

157. $-\pi\ \square\ -3.5$

158. $-\frac{3.14}{2}\ \square\ -\frac{\pi}{2}$

Preview Exercises

Exercises 159–161 will help you prepare for the material covered in the next section.

159. In parts (a) and (b), complete each statement.

a. $b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b) = b^?$

b. $b^5 \cdot b^5 = (b \cdot b \cdot b \cdot b \cdot b)(b \cdot b \cdot b \cdot b \cdot b) = b^?$

c. Generalizing from parts (a) and (b), what should be done with the exponents when multiplying exponential expressions with the same base?

160. In parts (a) and (b), complete each statement.

a. $\frac{b^7}{b^3} = \frac{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot b \cdot b \cdot b \cdot b}{\cancel{b} \cdot \cancel{b} \cdot \cancel{b}} = b^?$

b. $\frac{b^8}{b^2} = \frac{\cancel{b} \cdot \cancel{b} \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{\cancel{b} \cdot \cancel{b}} = b^?$

c. Generalizing from parts (a) and (b), what should be done with the exponents when dividing exponential expressions with the same base?

161. If 6.2 is multiplied by 10^3, what does this multiplication do to the decimal point in 6.2?

SECTION P.2 Exponents and Scientific Notation

WHAT YOU'LL LEARN

1. Use properties of exponents.
2. Simplify exponential expressions.
3. Use scientific notation.

Bigger than the biggest thing ever and then some. Much bigger than that in fact, really amazingly immense, a totally stunning size, real 'wow, that's big', time. . . . Gigantic multiplied by colossal multiplied by staggeringly huge is the sort of concept we're trying to get across here.

Douglas Adams, *The Restaurant at the End of the Universe*

In October 2019, *Forbes* published a list of the 400 wealthiest Americans, who had a total net worth of \$2.96 trillion. Amazon founder and CEO Jeff Bezos topped the list with a net worth of \$114 billion. That sounds like a lot of money, but consider this: At the end of 2019, the national debt was approximately \$22.7 trillion, rising to over \$25.5 trillion by midyear 2020. This \$2.8 trillion increase in the national debt over just a few months is nearly the total net worth of the 400 wealthiest Americans.

One of the best ways to put "staggeringly huge" numbers into perspective is by making comparisons. In this section, we will compare large and small numbers using exponents and scientific notation.

1 Use properties of exponents.

Properties of Exponents

The major properties of exponents are summarized in the box that follows.

GREAT QUESTION!

Cut to the chase. What do I do with negative exponents?

When a negative integer appears as an exponent, switch the position of the base (from numerator to denominator or from denominator to numerator) and make the exponent positive.

Properties of Exponents

Property	Examples
The Negative-Exponent Rule If b is any real number other than 0 and n is a natural number, then $$b^{-n} = \frac{1}{b^n}.$$	• $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ • $\frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 4^2 = 16$
The Zero-Exponent Rule If b is any real number other than 0, $$b^0 = 1.$$	• $7^0 = 1$ • $(-5)^0 = 1$ • $-5^0 = -1$ Only 5 is raised to the zero power.
The Product Rule If b is a real number or algebraic expression, and m and n are integers, $$b^m \cdot b^n = b^{m+n}.$$	• $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$ • $x^{-3} \cdot x^7 = x^{-3+7} = x^4$

When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.

The Power Rule If b is a real number or algebraic expression, and m and n are integers, $$(b^m)^n = b^{mn}.$$	• $(2^2)^3 = 2^{2\cdot 3} = 2^6 = 64$ • $(x^{-3})^4 = x^{-3\cdot 4} = x^{-12} = \frac{1}{x^{12}}$

When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.

The Quotient Rule If b is a nonzero real number or algebraic expression, and m and n are integers, $$\frac{b^m}{b^n} = b^{m-n}.$$	• $\frac{2^8}{2^4} = 2^{8-4} = 2^4 = 16$ • $\frac{x^3}{x^7} = x^{3-7} = x^{-4} = \frac{1}{x^4}$

When dividing exponential expressions with the same nonzero base, subtract the exponent in the denominator from the exponent in the numerator. Use this difference as the exponent of the common base.

Products Raised to Powers If a and b are real numbers or algebraic expressions, and n is an integer, $$(ab)^n = a^n b^n.$$	• $(-2y)^4 = (-2)^4 y^4 = 16y^4$ • $(-2xy)^3 = (-2)^3 x^3 y^3 = -8x^3y^3$

When a product is raised to a power, raise each factor to that power.

GREAT QUESTION!

What's the difference between $\frac{4^3}{4^5}$ and $\frac{4^5}{4^3}$?

These quotients represent different numbers:

$$\frac{4^3}{4^5} = 4^{3-5} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$\frac{4^5}{4^3} = 4^{5-3} = 4^2 = 16.$$

(continued)

Property	**Examples**
Quotients Raised to Powers If a and b are real numbers, $b \neq 0$, or algebraic expressions, and n is an integer, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.	• $\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{16}{625}$ • $\left(-\frac{3}{x}\right)^3 = \frac{(-3)^3}{x^3} = -\frac{27}{x^3}$

When a quotient is raised to a power, raise the numerator to that power and divide by the denominator to that power.

Simplifying Exponential Expressions

2 Simplify exponential expressions.

Properties of exponents are used to simplify exponential expressions. An exponential expression is **simplified** when

- No parentheses appear.
- No powers are raised to powers.
- Each base occurs only once.
- No negative or zero exponents appear.

Simplifying Exponential Expressions

	Example
1. If necessary, remove parentheses by using $(ab)^n = a^n b^n$ or $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.	$(xy)^3 = x^3y^3$
2. If necessary, simplify powers to powers by using $(b^m)^n = b^{mn}$.	$(x^4)^3 = x^{4\cdot 3} = x^{12}$
3. If necessary, be sure that each base appears only once by using $b^m \cdot b^n = b^{m+n}$ or $\frac{b^m}{b^n} = b^{m-n}$.	$x^4 \cdot x^3 = x^{4+3} = x^7$
4. If necessary, rewrite exponential expressions with zero powers as 1 ($b^0 = 1$). Furthermore, write the answer with positive exponents by using $b^{-n} = \frac{1}{b^n}$ or $\frac{1}{b^{-n}} = b^n$.	$\frac{x^5}{x^8} = x^{5-8} = x^{-3} = \frac{1}{x^3}$

The following example shows how to simplify exponential expressions. Throughout the example, assume that no variable in a denominator is equal to zero.

EXAMPLE 1 **Simplifying Exponential Expressions**

Simplify:

a. $(-3x^4y^5)^3$ **b.** $(-7xy^4)(-2x^5y^6)$ **c.** $\frac{-35x^2y^4}{5x^6y^{-8}}$ **d.** $\left(\frac{4x^2}{y}\right)^{-3}$.

Solution

a. $(-3x^4y^5)^3 = (-3)^3(x^4)^3(y^5)^3$ — Raise each factor inside the parentheses to the third power.

$= (-3)^3x^{4\cdot3}y^{5\cdot3}$ — Multiply the exponents when raising powers to powers.

$= -27x^{12}y^{15}$ — $(-3)^3 = (-3)(-3)(-3) = -27$

b. $(-7xy^4)(-2x^5y^6) = (-7)(-2)xx^5y^4y^6$ — Group factors with the same base.

$= 14x^{1+5}y^{4+6}$ — When multiplying expressions with the same base, add the exponents.

$= 14x^6y^{10}$ — Simplify.

c. $\dfrac{-35x^2y^4}{5x^6y^{-8}} = \left(\dfrac{-35}{5}\right)\left(\dfrac{x^2}{x^6}\right)\left(\dfrac{y^4}{y^{-8}}\right)$ — Group factors with the same base.

$= -7x^{2-6}y^{4-(-8)}$ — When dividing expressions with the same base, subtract the exponents.

$= -7x^{-4}y^{12}$ — Simplify. Notice that $4 - (-8) = 4 + 8 = 12$.

$= \dfrac{-7y^{12}}{x^4}$ — Write as a fraction and move the base with the negative exponent, x^{-4}, to the other side of the fraction bar and make the negative exponent positive.

d. $\left(\dfrac{4x^2}{y}\right)^{-3} = \dfrac{(4x^2)^{-3}}{y^{-3}}$ — Raise the numerator and the denominator to the -3 power.

$= \dfrac{4^{-3}(x^2)^{-3}}{y^{-3}}$ — Raise each factor inside the parentheses to the -3 power.

$= \dfrac{4^{-3}x^{-6}}{y^{-3}}$ — Multiply the exponents when raising a power to a power: $(x^2)^{-3} = x^{2(-3)} = x^{-6}$.

$= \dfrac{y^3}{4^3x^6}$ — Move each base with a negative exponent to the other side of the fraction bar and make each negative exponent positive.

$= \dfrac{y^3}{64x^6}$ — $4^3 = 4\cdot4\cdot4 = 64$

CHECK POINT 1 Simplify:

a. $(2x^3y^6)^4$

b. $(-6x^2y^5)(3xy^3)$

c. $\dfrac{100x^{12}y^2}{20x^{16}y^{-4}}$

d. $\left(\dfrac{5x}{y^4}\right)^{-2}$.