



PRECALCULUS

BUTLER



seventh
edition

A Brief Guide to Getting the Most from This Book

1 Read the Book

Feature	Description	Benefit
Section-Opening Scenarios	Every section opens with a scenario presenting a unique application of algebra or trigonometry in your life outside the classroom.	Realizing that algebra and trigonometry are everywhere will help motivate your learning. (See page 218.)
EXAMPLE	Examples are clearly written and provide step-by-step solutions. No steps are omitted, and each step is thoroughly explained to the right of the mathematics.	The blue annotations will help you understand the solutions by providing the reason why every algebraic or trigonometric step is true. (See page 101.)
Applications Using Real-World Data	Interesting applications from nearly every discipline, supported by up-to-date real-world data, are included in every section.	Ever wondered how you'll use algebra and trigonometry? This feature will show you how they can solve real problems. (See pages 211–213.)
> GREAT QUESTION !	Answers to students' questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.	By seeing common mistakes, you'll be able to avoid them. This feature should help you not to feel anxious or threatened when asking questions in class. (See page 326.)
BRIEF REVIEW	Brief Reviews cover skills you already learned but may have forgotten.	Having these refresher boxes easily accessible will help ease anxiety about skills you may have forgotten. (See page 454.)
BLITZER BONUS	These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the algebra or trigonometry under study.	Yet even more proof that math is an interesting and dynamic discipline! (See page 204.)
Explanatory Voice Balloons	Voice balloons help to demystify algebra and trigonometry. They translate math into plain English, clarify problem-solving procedures, and present alternative ways of understanding.	Does math ever look foreign to you? This feature often translates math into everyday English. (See page 440.)
WHAT YOU'LL LEARN	Every section begins with a list of objectives. Each objective is restated in the margin where the objective is covered.	The objectives focus your reading by emphasizing what is most important and where to find it. (See page 618.)
1 Learning Objective		
> TECHNOLOGY	The screens displayed in the technology boxes show how graphing utilities verify and visualize algebraic or trigonometric results.	Even if you are not using a graphing utility in the course, this feature will help you understand different approaches to problem solving. (See page 343.)

2 Work the Problems

Feature	Description	Benefit
 CHECK POINT	Each example is followed by a matched problem, called a Check Point, that offers you the opportunity to work a similar exercise. The answers to the Check Points are provided in the answer section.	You learn best by doing. You'll solidify your understanding of worked examples if you try a similar problem right away to be sure you understand what you've just read. (See page 341.)
 ACHIEVING SUCCESS	Achieving Success boxes offer strategies for persistence and success in college mathematics courses.	Follow these suggestions to help achieve your full academic potential in college mathematics. (See page 579.)
CONCEPT AND VOCABULARY CHECK	These short-answer questions, mainly fill-in-the-blank and true/false items, assess your understanding of the definitions and concepts presented in each section.	It is difficult to learn algebra and trigonometry without knowing their special language. These exercises test your understanding of the vocabulary and concepts. (See page 176.)
EXERCISE SET	An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within several categories. Your instructor will usually provide guidance on which exercises to work. The exercises in the first category, Practice Exercises, follow the same order as the section's worked examples.	The parallel order of the Practice Exercises lets you refer to the worked examples and use them as models for solving these problems. (See page 379.)
Practice PLUS	This category of exercises contains more challenging problems that often require you to combine several skills or concepts.	It is important to dig in and develop your problem-solving skills. Practice PLUS Exercises provide you with ample opportunity to do so. (See page 380.)
Retaining the Concepts	Beginning with Chapter 1, each Exercise Set contains review exercises under the header "Retaining the Concepts."	These exercises improve your understanding of the topics and help maintain mastery of the material. (See page 181.)
Preview Exercises	Each Exercise Set concludes with three or four problems to help you prepare for the next section.	These exercises let you review previously covered material that you'll need to be successful for the forthcoming section. Some of these problems will get you thinking about concepts you'll soon encounter. (See page 538.)

3 Review for Quizzes and Tests

Feature	Description	Benefit
Mid-Chapter Check Point	At approximately the midway point in the chapter, an integrated set of review exercises allows you to review the skills and concepts you learned separately over several sections.	By combining exercises from the first half of the chapter, the Mid-Chapter Check Points give a comprehensive review before you move on to the material in the remainder of the chapter. (See page 768.)
Chapter Review Chart	Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.	Review this chart and you'll know the most important material in the chapter! (See page 807.)
Chapter Review Exercise Set	A comprehensive collection of review exercises for each of the chapter's sections follows the review chart.	Practice makes perfect. These exercises contain the most significant problems for each of the chapter's sections. (See page 429.)
Chapter Test	Each chapter contains a practice test with approximately 25 problems that cover the important concepts in the chapter. Take the practice test, check your answers, and then watch the Chapter Test Prep Videos to see worked-out solutions for any exercises you miss.	You can use the chapter test to determine whether you have mastered the material covered in the chapter. (See page 654.)
Chapter Test Prep Videos	These videos contain worked-out solutions to every exercise in each chapter test and can be found in MyLab Math and on YouTube at youtube.com/user/pearsonmathstats (playlist "Blitzer Precalculus 7e").	The videos let you review any exercises you miss on the chapter test.
Objective Videos	These fresh, interactive videos walk you through the concepts from every objective of the text.	The videos provide you with active learning at your own pace.
Cumulative Review	Beginning with Chapter 2, each chapter concludes with a comprehensive collection of mixed cumulative review exercises. These exercises combine problems from previous chapters and the present chapter, providing an ongoing cumulative review.	Ever forget what you've learned? These exercises ensure that you are not forgetting anything as you move forward. (See page 438.)

Precalculus

This page intentionally left blank

7th EDITION

Precalculus

Robert Blitzer

Miami Dade College

Content Management: Jonathan Krebs, Jeff Weidenaar
Content Production: Tamela Ambush, Eric Gregg, Shana Siegmund
Product Management: Chelsea Kharakozova
Product Marketing: Brooke Imbornone, Stacey Sveum
Rights and Permissions: Venugopal Loganathan

Please contact <https://support.pearson.com/getsupport/s/> with any queries on this content.

Cover Image: Annie Brace; Cover Design: Jerilyn DiCarlo

About the Cover: Bob Blitzer's signature chili pepper provides a window into a vibrant nature scene rendered by Alaska-based artist Annie Brace. The cover is a metaphor for Bob's book, which provides insight into a world that is profoundly mathematical.

Copyright © 2022, 2018, 2014 by Pearson Education, Inc. or its affiliates, 221 River Street, Hoboken, NJ 07030. All Rights Reserved. Manufactured in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise. For information regarding permissions, request forms, and the appropriate contacts within the Pearson Education Global Rights and Permissions department, please visit www.pearsoned.com/permissions/.

Acknowledgments of third-party content appear on pages C1 and RC1, which constitute an extension of this copyright page.

PEARSON, ALWAYS LEARNING, and MYLAB are exclusive trademarks owned by Pearson Education, Inc. or its affiliates in the U.S. and/or other countries.

Unless otherwise indicated herein, any third-party trademarks, logos, or icons that may appear in this work are the property of their respective owners, and any references to third-party trademarks, logos, icons, or other trade dress are for demonstrative or descriptive purposes only. Such references are not intended to imply any sponsorship, endorsement, authorization, or promotion of Pearson's products by the owners of such marks, or any relationship between the owner and Pearson Education, Inc., or its affiliates, authors, licensees, or distributors.

Library of Congress Control Number: 2021931198

ScoutAutomatedPrintCode



Rental
ISBN-10: 0-13-692219-8
ISBN-13: 978-0-13-692219-3

Contents

Preface ix
To the Student xix
About the Author xx
Applications Index xxi

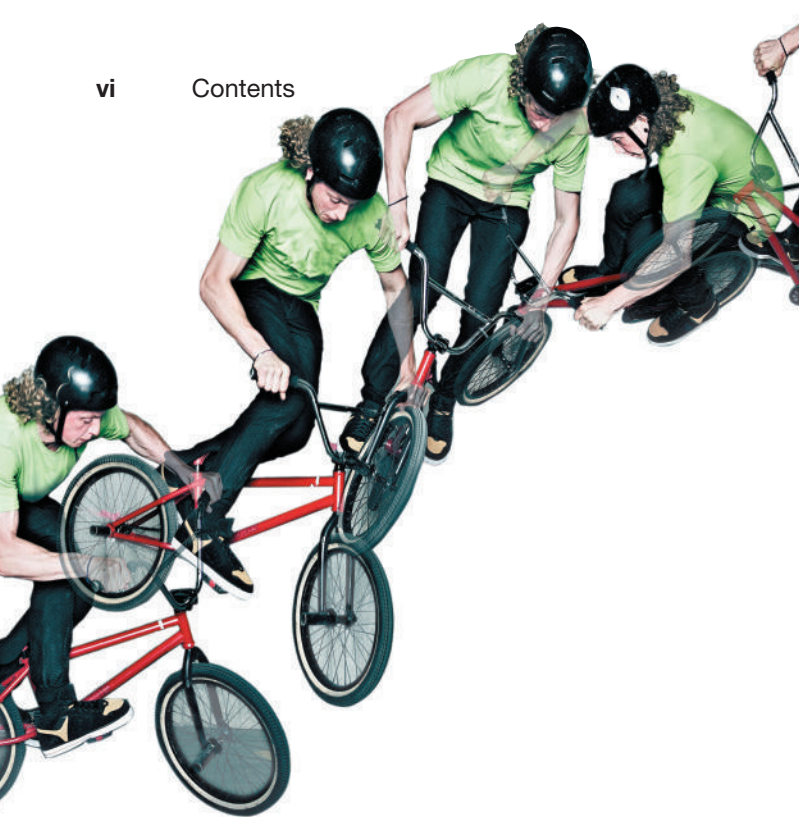
P Fundamental Concepts of Algebra 1

- P.1** Algebraic Expressions, Mathematical Models, and Real Numbers 2
- P.2** Exponents and Scientific Notation 20
- P.3** Radicals and Rational Exponents 32
- P.4** Polynomials 48
- P.5** Factoring Polynomials 57
- Mid-Chapter Check Point** 69
- P.6** Rational Expressions 70
- P.7** Equations 88
- P.8** Models and Applications 110
- P.9** Linear Inequalities and Absolute Value Inequalities 126
- Summary, Review, and Test** 142
- Review Exercises** 143
- Chapter P Test** 146

1 Functions and Graphs 149

- 1.1** Graphs and Graphing Utilities 150
- 1.2** Basics of Functions and Their Graphs 163
- 1.3** More on Functions and Their Graphs 182
- 1.4** Linear Functions and Slope 202
- 1.5** More on Slope 218
- Mid-Chapter Check Point** 229
- 1.6** Transformations of Functions 230
- 1.7** Combinations of Functions; Composite Functions 246
- 1.8** Inverse Functions 261
- 1.9** Distance and Midpoint Formulas; Circles 272
- 1.10** Modeling with Functions 282
- Summary, Review, and Test** 296
- Review Exercises** 300
- Chapter 1 Test** 305





2 Polynomial and Rational Functions 309

- 2.1 Complex Numbers 310
- 2.2 Quadratic Functions 319
- 2.3 Polynomial Functions and Their Graphs 337
- 2.4 Dividing Polynomials; Remainder and Factor Theorems 355
- 2.5 Zeros of Polynomial Functions 368

Mid-Chapter Check Point 382

- 2.6 Rational Functions and Their Graphs 384
- 2.7 Polynomial and Rational Inequalities 406
- 2.8 Modeling Using Variation 419

Summary, Review, and Test 429

Review Exercises 432

Chapter 2 Test 436

Cumulative Review Exercises (Chapters P–2) 438

3 Exponential and Logarithmic Functions 439

- 3.1 Exponential Functions 440
- 3.2 Logarithmic Functions 454
- 3.3 Properties of Logarithms 469

Mid-Chapter Check Point 479

3.4 Exponential and Logarithmic Equations 480

3.5 Exponential Growth and Decay; Modeling Data 495

Summary, Review, and Test 511

Review Exercises 513

Chapter 3 Test 517

Cumulative Review Exercises (Chapters P–3) 518

4 Trigonometric Functions 521

4.1 Angles and Radian Measure 522

4.2 Trigonometric Functions: The Unit Circle 539

4.3 Right Triangle Trigonometry 555

4.4 Trigonometric Functions of Any Angle 568

Mid-Chapter Check Point 581

4.5 Graphs of Sine and Cosine Functions 583

4.6 Graphs of Other Trigonometric Functions 604

4.7 Inverse Trigonometric Functions 618

4.8 Applications of Trigonometric Functions 636

Summary, Review, and Test 647

Review Exercises 651

Chapter 4 Test 654

Cumulative Review Exercises (Chapters P–4) 655



5 Analytic Trigonometry 657

- 5.1 Verifying Trigonometric Identities 658
- 5.2 Sum and Difference Formulas 670
- 5.3 Double-Angle, Power-Reducing, and Half-Angle Formulas 681
- Mid-Chapter Check Point** 692
- 5.4 Product-to-Sum and Sum-to-Product Formulas 693
- 5.5 Trigonometric Equations 702
- Summary, Review, and Test** 717
- Review Exercises** 718
- Chapter 5 Test** 720
- Cumulative Review Exercises (Chapters P–5)** 721

6 Additional Topics in Trigonometry 723

- 6.1 The Law of Sines 724
- 6.2 The Law of Cosines 736
- 6.3 Polar Coordinates 745
- 6.4 Graphs of Polar Equations 757
- Mid-Chapter Check Point** 768
- 6.5 Complex Numbers in Polar Form; DeMoivre's Theorem 769
- 6.6 Vectors 782
- 6.7 The Dot Product 797
- Summary, Review, and Test** 807
- Review Exercises** 810
- Chapter 6 Test** 812
- Cumulative Review Exercises (Chapters P–6)** 813

7 Systems of Equations and Inequalities 815

- 7.1 Systems of Linear Equations in Two Variables 816
- 7.2 Systems of Linear Equations in Three Variables 836
- 7.3 Partial Fractions 844
- 7.4 Systems of Nonlinear Equations in Two Variables 855
- Mid-Chapter Check Point** 865

- 7.5 Systems of Inequalities 866
- 7.6 Linear Programming 879
- Summary, Review, and Test** 886
- Review Exercises** 888
- Chapter 7 Test** 891
- Cumulative Review Exercises (Chapters P–7)** 891



8 Matrices and Determinants 893

- 8.1 Matrix Solutions to Linear Systems 894
- 8.2 Inconsistent and Dependent Systems and Their Applications 908
- 8.3 Matrix Operations and Their Applications 917
- Mid-Chapter Check Point** 932
- 8.4 Multiplicative Inverses of Matrices and Matrix Equations 933
- 8.5 Determinants and Cramer's Rule 947
- Summary, Review, and Test** 960
- Review Exercises** 962
- Chapter 8 Test** 964
- Cumulative Review Exercises (Chapters P–8)** 965

9 Conic Sections and Analytic Geometry 967

- 9.1** The Ellipse 968
- 9.2** The Hyperbola 983
- 9.3** The Parabola 998
- Mid-Chapter Check Point** 1012
- 9.4** Rotation of Axes 1014
- 9.5** Parametric Equations 1025
- 9.6** Conic Sections in Polar Coordinates 1035
- Summary, Review, and Test** 1045
- Review Exercises** 1048
- Chapter 9 Test** 1050
- Cumulative Review Exercises (Chapters P–9)** 1051

10 Sequences, Induction, and Probability 1053

- 10.1** Sequences and Summation Notation 1054
- 10.2** Arithmetic Sequences 1065
- 10.3** Geometric Sequences and Series 1076
- Mid-Chapter Check Point** 1091
- 10.4** Mathematical Induction 1092
- 10.5** The Binomial Theorem 1101
- 10.6** Counting Principles, Permutations, and Combinations 1109
- 10.7** Probability 1120
- Summary, Review, and Test** 1135
- Review Exercises** 1138
- Chapter 10 Test** 1140
- Cumulative Review Exercises (Chapters P–10)** 1141

11 Introduction to Calculus 1143

- 11.1** Finding Limits Using Tables and Graphs 1144
- 11.2** Finding Limits Using Properties of Limits 1156
- 11.3** Limits and Continuity 1170
- Mid-Chapter Check Point** 1177
- 11.4** Introduction to Derivatives 1178
- Summary, Review, and Test** 1191
- Review Exercises** 1192
- Chapter 11 Test** 1194
- Cumulative Review Exercises (Chapters P–11)** 1194

- Appendix A: Where Did That Come From? Selected Proofs 1197
- Appendix B: The Transition from Precalculus to Calculus 1203
- Answers to Selected Exercises AA1
- Subject Index I1
- Credits C1



Preface

I've written ***Precalculus, Seventh Edition***, to help diverse groups of students, with different backgrounds and future goals, to succeed. The book has three fundamental goals:

1. To help students acquire a solid foundation in algebra and trigonometry, preparing them for other courses such as calculus, business calculus, and finite mathematics.
2. To show students how algebra and trigonometry can model and solve authentic real-world problems.
3. To enable students to develop problem-solving skills, while fostering critical thinking, within an interesting setting.

One major obstacle in the way of achieving these goals is the fact that very few students actually read their textbook. This has been a regular source of frustration for me and for my colleagues in the classroom. Anecdotal evidence gathered over years highlights two basic reasons that students do not take advantage of their textbook:

- “I’ll never use this information.”
- “I can’t follow the explanations.”

I’ve written every page of the Seventh Edition with the intent of eliminating these two objections. The ideas and tools I’ve used to do so are described for the student in “A Brief Guide to Getting the Most from This Book,” which appears on the endpapers of the book.

How does *Precalculus* Differ from *Algebra and Trigonometry*?

Precalculus is not simply a condensed version of my *Algebra and Trigonometry* book. Precalculus students are different from those taking algebra and trigonometry,

and this text reflects those differences. Here are a few examples:

- *Algebra and Trigonometry* devotes an entire chapter to linear equations, rational equations, quadratic equations, radical equations, linear inequalities, and developing models involving these equations and inequalities. *Precalculus* reviews these topics in three sections of the prerequisites chapter (P.7: Equations; P.8: Modeling with Equations; P.9: Linear Inequalities and Absolute Value Inequalities). Functions, the core of any precalculus course, are then introduced in Chapter 1.
- *Precalculus* contains a section on constructing functions from verbal descriptions and formulas (1.10: Modeling with Functions) that is not included in *Algebra and Trigonometry*. Modeling skills are applied to situations that students are likely to see in calculus when solving applied problems involving maximum or minimum values.
- *Precalculus* develops trigonometry from the perspective of the unit circle (4.2: Trigonometric Functions: The Unit Circle). In *Algebra and Trigonometry*, trigonometry is developed using right triangles.
- *Precalculus* contains a chapter that takes the student into calculus with discussions of limits, continuity, and derivatives (Chapter 11: Introduction to Calculus). This chapter is not included in *Algebra and Trigonometry*.
- Many of the liberal arts applications in *Algebra and Trigonometry* are replaced by more scientific or higher-level applications in *Precalculus*. Some examples:
 - Black Holes in Space (Section P.2)
 - Average Velocity (Section 1.5)
 - Newton’s Law of Cooling (Section 3.5)

What’s New in the Seventh Edition?

The Seventh Edition contains 103 worked-out examples and exercises based on new data sets and 172 updated examples and exercises. Many of the new and updated applications involve topics relevant to college students.

New Applications

- Cost and Enrollment for Federal Social Programs (Section P.2, Exercises 115–117)
- Educational Attainment and Probability of Divorce (Section 1.1, Example 6)
- Number of Smartphone Users in the U.S. (Section 1.2, Figure 1.14)
- Spending on Pre-Primary Education and Child Care (Section 1.2, Exercises 99–100)
- Internet Plans (Section 1.3, Example 6 and Exercises 95–96)

- Trust in Government and the Media (Section 1.4, Exercises 87–88)
- Accelerating Climate Change (Blitzer Bonus in Section 1.4, p. 213)
- Living Arrangements of Young Adults (Section 1.5 opener and Example 3)
- U.S. Population Projections by Age (Section 1.7, Exercises 97–98)
- Rumbling Back: Steven Spielberg’s New *West Side Story* (Blitzer Bonus in Section 1.8, p. 261)
- Addressing Leisure Time Parabolically (Blitzer Bonus in Section 2.2, p. 332)
- COVID-19 Pandemic (Section 2.3 opener; Cumulative Review for Chapters P–2, Exercise 21; Section 3.5, Example 3; Cumulative Review for Chapters P–7, Exercise 36)
- AIDS: A Global Perspective (Blitzer Bonus in Section 2.3, p. 341)
- Area Burned by Wildfires in the U.S. (Section 2.3, Exercise 76)
- Mumps (Chapter 2 Mid-Chapter Check Point, Exercise 35)
- Costco Paid Membership (Chapter 2 Review, Exercise 82)
- Putting Off Medical Treatment Because of Expenses (Section 3.2, Exercises 115–116)
- E-commerce Sales (Chapter 3 Review, Exercise 81)
- The Electromagnetic Spectrum (Blitzer Bonus in Section 4.5, p. 600)
- Modeling Body Temperature, Heart Rate, and Respiratory Rate (Chapter 7 opener; Section 7.5 opener, Example 5, and Exercises 77–82)
- Number of Men and Women in the U.S. House of Representatives (Section 7.1, Exercise 70)
- Share of U.S. Income by Top 10% and Bottom 90% of Americans (Section 7.1, Exercise 72)
- The Late Elvis Presley’s Business Machine (Section 7.2, Exercise 45)
- Use of Social Media by Age (Chapter 7 Review, Exercise 8)
- Online Classes vs. Face-to-Face Classroom Experiences (Chapter 8 opener)
- Political Party Affiliation by Generation (Section 8.1 opener)
- Interracial Married Couples in the U.S. (Cumulative Review for Chapters P–9, Exercise 21)
- Probable Majors of College Freshmen (Section 10.2, Exercises 61–62)
- Nonbinary Gender Options (Blitzer Bonus in Section 10.7, p. 1130)
- Electrical Charging Stations (Chapter 10 Review, Exercise 28)
- Regular Marijuana Use among 18- to 25-Year-Olds (Cumulative Review for Chapters P–10, Exercise 43)

Updated Applications

- Cost of Tuition and Fees at Public and Private Colleges (Section P.1, Example 2 and Exercises 131–132)
- The National Debt (Section P.2 opener, Example 6, and Exercises 118–120)
- Student Loan Debt (Chapter P Mid-Chapter Check Point, Exercise 42)
- Grade Inflation (Section P.7, Exercises 137–138)
- Toll Options (Section P.8 opener, Example 3, and Exercises 11–12)
- Median Earnings by Educational Attainment (Section P.8, Example 1)
- Attitudes of College Freshmen (Section P.8, Example 2)
- Car Prices and Age of Cars on U.S. Roads (Section P.8, Exercises 5–6)
- Different Race or Ethnicity for Two Randomly Selected Americans (Chapter P Review, Exercise 23)
- Average Price of a Movie Ticket (Chapter P Review, Exercise 144)
- Highest-Paid TV Actors and Actresses (Section 1.2, Figure 1.13)
- The Wage Gap between Men and Women (Section 1.2, Exercises 103–104)
- Interest Rates (Section 1.3, Example 5; Section 3.1, Example 7; Section 3.4, Example 10; Section 10.3, Example 7 and Exercises 79–82)
- Fuel Efficiency of New U.S. Cars (Section 1.3 opener)
- Number of Births and Deaths in the U.S. (Section 1.7 opener and Example 4)
- Political Orientation of U.S. College Freshmen (Chapter 1 Review, Exercise 67)
- One-Person Households as a Percentage of the U.S. Total (Chapter 1 Test, Exercise 28)
- AIDS Cases in the U.S. (Section 2.3, Example 3)
- World Tiger Population (Section 2.3, Exercises 73–74)
- Federal Budget Expenditures on Human Resources (Section 2.6, Exercise 107)
- Amazon Deforestation (Chapter 2 Review, Exercise 28)
- Gray Wolf Population (Section 3.1, Example 6)
- Percentage of High School Seniors Applying to More Than Three Colleges (Section 3.1, Exercises 71–72)
- Number of Pages in the Federal Tax Code (Section 3.1, Exercise 85)
- Percentage of GDP Going Toward Health Care (Section 3.4, Exercises 115–116)
- U.S. Population (Section 3.5, Example 1; Section 10.3, Example 3)
- World Population (Section 3.5, Examples 6 and 7)
- Populations of Various Countries (Section 3.5, Exercises 1–14)
- Marital Status of U.S. Adults (Section 7.1, Exercise 67; Section 10.7, Example 9 and Exercises 1–10)

- Rate of Violent Crime and Imprisonment in the U.S. (Section 7.4, Exercise 63)
- Percentage of Men and Women Completing the Transition to Adulthood (Section 8.3, Exercise 61)
- Hours per Day Spent on Digital Media (Section 10.1, Exercise 69)
- Giving Up U.S. Citizenship (Section 10.1, Exercise 70)
- Dormitory Charges (Section 10.2, Exercises 65–66)

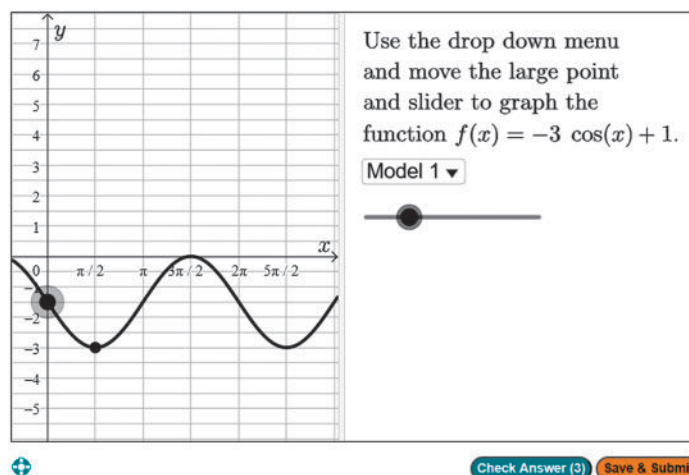
Other Textbook Changes

- Prior to the exercises in each section, the Annotated Instructor's Edition provides a list of resources available for that section in MyLab Math.
- The list of each section's objectives, previously headed "What am I supposed to learn?" (which annoyed some reviewers) has been renamed "What You'll Learn."
- Section P.6 includes new examples and exercises involving adding and subtracting rational expressions with different monomial denominators. (Section P.6, Example 8 and Exercises 43–50)
- In Chapter 2, the standard form of a quadratic function, $f(x) = a(x - h)^2 + k$, has been renamed the *vertex form*.
- Section 4.7 includes a new objective covering the definitions, properties, and graphs of the inverse cotangent, inverse cosecant, and inverse secant functions. (Section 4.7, Example 5 and Exercises 19–26, 57–62)
- Section 5.1 has a new objective on rewriting expressions that contain both trigonometric and logarithmic expressions as equivalent expressions using trigonometric identities and properties of logarithms. (Section 5.1, Example 9 and Exercises 61–66)
- Section 5.5 builds on the new material in Section 5.1 with a new objective on solving equations that contain both trigonometric and logarithmic expressions. (Section 5.5, Example 13 and Exercises 117–128)
- Section 6.5 introduces the notation $r \operatorname{cis} \theta$ as an abbreviation for $r(\cos \theta + i \sin \theta)$. (Section 6.5, Great Question! on p. 772)
- Section 6.5 provides a second explanation of finding complex roots using DeMoivre's Theorem. The new approach relies more on the relationships among the roots and less on the use of a formula. (Section 6.5, Great Question! on p. 772 and Example 9)
- Section 8.5 presents an alternative to expansion by minors for evaluating a third-order determinant. (Section 8.5, pp. 953–954)

New in MyLab Math

- **Corequisite Support Resources** provide all the content and assessment resources necessary for students and instructors. MyLab Math supports various corequisite course models, including Concurrent (aka just-in-time) and Consecutive (aka front-loaded) models. For more details, see page xiii or the Corequisite Implementation Guide at bit.ly/3il85pn.

- **Integrated Review Activities** for selected topics provide hands-on work with important prerequisites.



▲ **GeoGebra Graphing Exercises** are gradable graphing exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.

- **Setup & Solve Exercises** – We added more of these popular exercises, which require students to first describe how they will set up and approach the problem. This mirrors what students will be expected to do on a test.
- **Interactive Figures** – For this revision, we added many more interactive figures (in editable GeoGebra format) to the Video & Resource Library.
- **Enhanced Assignments** – These section-level assignments have three unique properties (and are fully editable):
 1. They help keep skills fresh with *spaced practice* of previously learned concepts.
 2. They have learning aids strategically turned off for some exercises to ensure that students understand how to work the exercises independently.
 3. They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.
- **Video Assignments** – These section-level assignments are especially helpful for online classes or “flipped” classes, where some or all learning takes place independently.
- **PowerPoint slides** are now animated. They also utilize Microsoft’s Equation Editor, making them more easily editable.
- **Personal Inventory Assessments** are a collection of online exercises designed to promote self-reflection and engagement in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.

What Familiar Features Have Been Retained in the Seventh Edition?

- **Graphing and Functions.** Graphing and functions are introduced in Chapter 1, with an integrated graphing functional approach emphasized throughout the book. Graphs and functions that model data appear in nearly every section and Exercise Set. Examples and exercises use graphs of functions to explore relationships between data and to provide ways of visualizing a problem's solution. Because functions are the core of this course, students are repeatedly shown how functions relate to equations and graphs.
- **Learning Objectives.** Learning objectives are clearly stated at the beginning of each section under the heading "What You'll Learn." These objectives help students recognize and focus on the section's most important ideas. The objectives are restated in the margin at their point of use.
- **Chapter-Opening and Section-Opening Scenarios.** Every chapter and every section open with a scenario presenting a unique application of mathematics in students' lives outside the classroom. These scenarios are revisited in the course of the chapter or section in an example, discussion, or exercise.
- **Innovative Applications.** A wide variety of interesting applications, supported by up-to-date, real-world data, are included in every section.

Explanatory Voice Balloons

- **Explanatory Voice Balloons.** Voice balloons are used in a variety of ways to demystify mathematics. They translate algebraic and trigonometric ideas into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect problem solving to concepts students have already learned.
- **Detailed Worked-Out Examples.** Each example is titled, making the purpose of the example clear. Examples are clearly written and provide students with detailed step-by-step solutions. No steps are omitted and each step is thoroughly explained to the right of the mathematics.

✓ CHECK POINT

- **Check Point Examples.** Each example is followed by a similar matched problem, called a Check Point, offering students the opportunity to test their understanding of the example by working a similar exercise. The answers to the Check Points are provided in the answer section.

- **Great Question!** This feature presents a variety of study tips in the context of students' questions. Answers to questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions. As a secondary benefit, this feature should help students not to feel anxious or threatened when asking questions in class.

BLITZER BONUS

- **Blitzer Bonuses.** These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the algebra and trigonometry under study, showing students that math is an interesting and dynamic discipline.
- **Concept and Vocabulary Checks.** This feature offers short-answer exercises, mainly fill-in-the-blank and true/false items, that assess students' understanding of the definitions and concepts presented in each section. The Concept and Vocabulary Checks precede the section Exercise Sets and have the prefix "C."
- **Extensive and Varied Exercise Sets.** An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within nine category types: Practice Exercises, Practice PLUS Exercises, Application Exercises, Explaining the Concepts, Technology Exercises, Critical Thinking Exercises, Group Exercises, Retaining the Concepts, and Preview Exercises. This format makes it easy to create well-rounded homework assignments. The order of the Practice Exercises is exactly the same as the order of the section's worked examples. This parallel order enables students to refer to the titled examples and their detailed explanations to achieve success working the Practice Exercises.
- **Practice PLUS Problems.** This category of exercises contains more challenging practice problems that often require students to combine several skills or concepts. With an average of ten Practice PLUS problems per Exercise Set, instructors are provided with the option of creating assignments that take Practice Exercises to a more challenging level.
- **Retaining the Concepts.** Beginning with Chapter 2, each Exercise Set contains three or four review exercises under the header "Retaining the Concepts." These exercises are intended for students to review previously covered objectives in order to improve their understanding of the topics and to help maintain their mastery of the material. If students are not certain how to solve a review exercise, they can turn to the section and worked example given in parentheses at the end of each exercise.

- **Mid-Chapter Check Points.** At approximately the midway point in each chapter, an integrated set of Review Exercises allows students to review and assimilate the skills and concepts they learned separately over several sections.
- **Integration of Technology Using Graphic and Numerical Approaches to Problems.** Side-by-side features in the technology boxes connect algebraic and trigonometric solutions to graphic and numerical approaches to problems. Although the use of graphing utilities is optional, students can use the explanatory voice balloons to understand different approaches to problems even if they are not using a graphing utility in the course.
- **Brief Reviews.** Beginning with Chapter 1, the Brief Review boxes that appear throughout the book summarize mathematical skills, many of which are course prerequisites that students have learned but which many students need to review. This feature appears whenever a particular skill is first needed and eliminates the need for you to reteach that skill. For more detail, students are referred to the appropriate section and objective in a previous chapter where the topic is fully developed.

> ACHIEVING SUCCESS

- **Achieving Success.** The Achieving Success boxes, appearing at the end of many sections in Chapters P through 6, offer strategies for persistence and success in college mathematics courses.
- **Discovery.** Discovery boxes, found throughout the text, encourage students to further explore algebraic and trigonometric concepts. These explorations are optional and their omission does not interfere with the continuity of the topic under consideration.
- **Chapter Summaries.** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.
- **End-of-Chapter Materials.** A comprehensive collection of Review Exercises for each of the chapter's sections follows the Summary. This is followed by a Chapter Test that enables students to test their understanding of the material covered in the chapter. Beginning with Chapter 2, each chapter concludes with a comprehensive collection of mixed Cumulative Review Exercises.

MyLab™ Math Resources for Success

MyLab Math (pearson.com/mylab/math) is available to accompany Pearson's market-leading text options, including this text (access code required). MyLab Math is the teaching and learning platform that empowers you to reach every student. It combines trusted author content—including full eText and online homework with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

NEW! Corequisite Course Support

MyLab Math supports various corequisite course models, including Concurrent (aka just-in-time) and Consecutive (aka front-loaded) models. MyLab Math for this text contains all of these learning and assessment resources to support corequisite courses:

1. **Complete Corequisite eText** built from Bob Blitzer's developmental mathematics texts so that it matches the features and pedagogy of this text.
2. **Instructional videos** for each corequisite objective.
3. **Assignable algorithmic exercises** for each corequisite objective.
4. **Worksheets** with instruction and exercises for each corequisite objective (also available in print).
5. **Activities** for selected corequisite objectives.
6. **Study Skills** support with self-help materials for time management, mindset, stress management, college transition, and more.

7. **Corequisite Implementation Guide** with specific guidelines for using the materials to teach various corequisite models. (Download at bit.ly/3il85pn.)

To help target instruction on corequisite objectives, MyLab includes these pre-made assessments:

- **Readiness Quiz 1** addresses key arithmetic topics and is designed to be administered prior to beginning College Algebra topics.
- **Readiness Quiz 2** addresses basic Introductory Algebra topics and is designed to be administered prior to beginning College Algebra topics.
- **Skills Check Quiz for Each Chapter** addresses the pre-requisite skills needed for each chapter in *Precalculus*.
- Based on the results of these quizzes, students can receive *personalized assignments* to address objectives that are not mastered. This way, students can focus on just the topics they need help with.

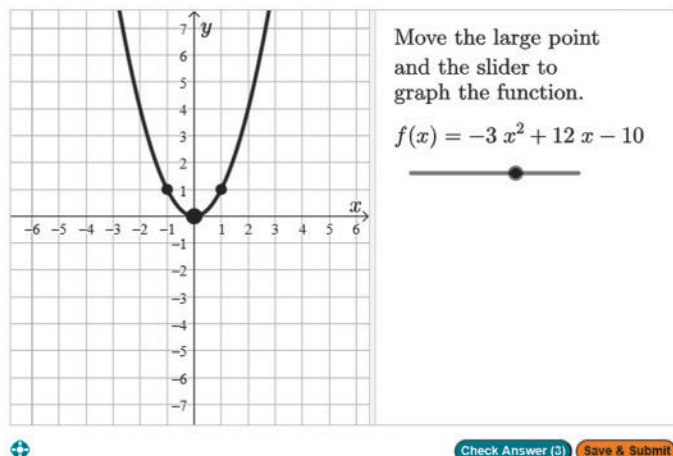
Note that the above resources are also designed to provide just-in-time help for students in your regular (non-corequisite) courses. (We understand that almost all students at some point need targeted refreshers on specific prerequisite skills.)

MyLab Math Student Resources

Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student

needs practice, giving all students the support they need—when and where they need it—to be successful.

Exercises with Immediate Feedback – The exercises in MyLab Math reflect the approach and learning style of this text and regenerate algorithmically to give students unlimited opportunity for practice and mastery. Most exercises include learning aids, such as guided solutions and sample problems, and they offer helpful feedback when students enter incorrect answers.



▲ **NEW!** **GeoGebra Exercises** are gradable graphing exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.

- **Setup & Solve** exercises require students to first describe how they will set up and approach a problem. This reinforces conceptual understanding of the process applied in approaching the problem, promotes long-term retention of the skill, and mirrors what students will be expected to do on a test.
- **Concept & Vocabulary** exercises require students to demonstrate understanding of key ideas.

Solve. Find each solution set and then use a calculator to obtain a decimal approximation to two decimal places for the solution.

a. $5^x = 134$

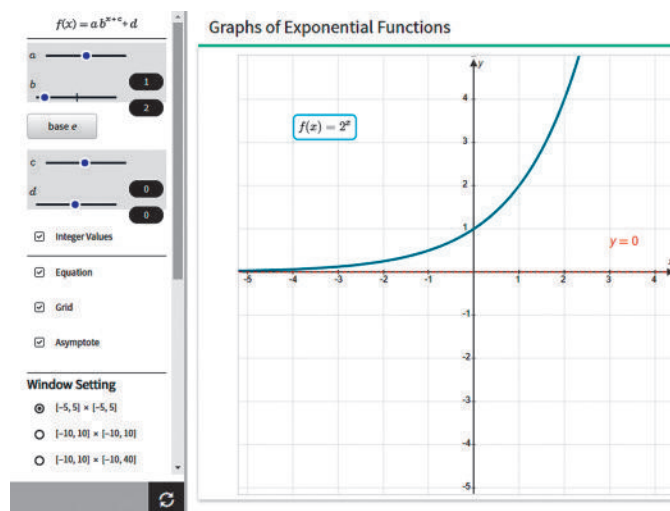


▲ **Instructional Videos** – High-quality instructional videos are included for every objective in the text. Many of these feature built-in interactive quizzes.

Chapter Test Prep Videos correspond to each exercise in the Chapter Test in the textbook, enabling students to effectively prepare for high-stakes testing. These are available in MyLab Math and www.youtube.com/user/pearsonmathstats (playlist “Blitzer Precalculus 7e”).

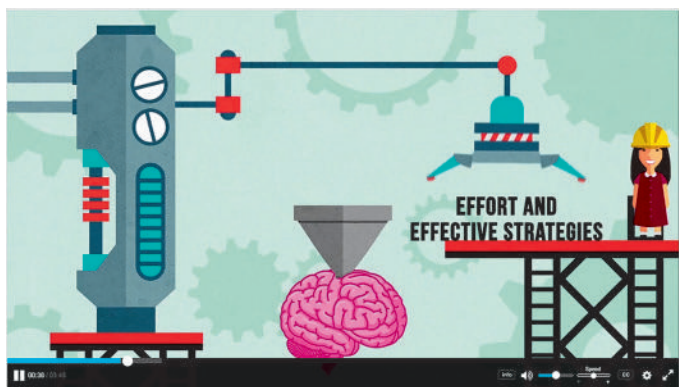
Learning Guide (also available in print format) consists of four parts:

1. **Learning Guide worksheets** for each section of the text. These worksheets start with a catchy headline and motivating real-world connection followed by numerous “Solved Problems” and accompanying “Pencil Problems.”
2. **Classroom Activities** for selected sections contain recommended group size, material needed, and time to complete.
3. **Integrated Review worksheets** for every prerequisite objective. These feature both instruction and practice.
4. **NEW!** **Integrated Review Activities** for selected topics provide hands-on work with important prerequisites.



▲ **Interactive Figures** bring mathematical concepts to life, helping students see the concepts through directed explorations and purposeful manipulation. These figures are assignable in MyLab Math and encourage active learning, critical thinking, and conceptual understanding.

NEW! For this revision, we added many more interactive figures (in editable GeoGebra format) to the Video & Resource Library.



▲ **Mindset videos** and assignable, open-ended exercises foster a growth mindset in students. This material encourages them to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as learning opportunities—so often a hurdle for math students. These videos are one of many **Study Skills and Career-Readiness Resources** that address the non-math-related issues that can affect student success.

NEW! **Personal Inventory Assessments** are a collection of online exercises designed to promote self-reflection and engagement in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.

eText – Available in two formats: one that matches the textbook page-for-page and another that is “reflowable” for use on tablets and smartphones. The latter eText is also fully accessible using screen-readers.

Student Solutions Manual – Fully worked solutions to odd-numbered exercises. Available for download from within MyLab Math.

MyLab Math Instructor Resources

Your course is unique. So whether you’d like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

Pre-Built Assignments are designed to make the homework experience as effective as possible for students. All of these assignments are *fully editable*.

- **NEW!** **Enhanced Assignments** – These section-level assignments have three unique properties:
 1. They help keep skills fresh with *spaced practice* of previously learned concepts.

2. They have learning aids strategically turned off for some exercises to ensure that students understand how to work the exercises independently.
 3. They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.
- **NEW!** **Video Assignments** – These section-level assignments are especially helpful for online classes or “flipped” classes, where some or all learning takes place independently.

Learning Catalytics – With Learning Catalytics™, you’ll hear from every student when it matters most. You pose a variety of questions in class (choosing from pre-loaded questions or questions of your own making) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops.

Accessibility – Pearson works continuously to ensure our products are as accessible as possible to all students. Currently we work toward achieving WCAG 2.0 AA for our existing products (2.1 AA for future products) and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media (<https://www.pearson.com/us/accessibility.html>).

Other instructor resources include:

- **Mini Lecture Notes** contain additional examples and helpful teaching tips for each section of the text.
- **Instructor Solution Manual** contains worked-out solutions for every exercise in the text.
- **PowerPoint Lecture Slides** are fully editable and included for each section of the text. **UPDATED!** Slides are now animated. They also utilize Microsoft’s Equation Editor, making them more easily editable.
- **TestGen®** enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson’s online catalog.
- **Test Bank** features printable PDFs containing all of the test exercises available in TestGen.

Acknowledgments

An enormous benefit of authoring a successful series is the broad-based feedback I receive from the students, dedicated users, and reviewers. Every change to this edition is the result of their thoughtful comments and suggestions. I would like to express my appreciation to all the reviewers, whose collective insights form the backbone of this revision. In particular, I would like to thank the following people for reviewing *College Algebra*, *Algebra and Trigonometry*, *Precalculus*, and *Trigonometry*. (An asterisk * indicates reviewers for the current edition.)

*Margaret Adams, *South Georgia South College*
 Karol Albus, *South Plains College*
 *Alvina Atkinson, *Georgia Gwinnett College*
 Kayoko Yates Barnhill, *Clark College*
 *Melissa Battis, *Greenhill School*
 Timothy Beaver, *Isothermal Community College*
 Jaromir Becan, *University of Texas, San Antonio*
 Imad Benjelloun, *Delaware Valley College*
 Lloyd Best, *Pacific Union College*
 *Joseph Bittner, *Fort Wayne North Side High School*
 *Elisa Bolotin, *Saint Stephen's Episcopal School*
 *Marie Borrazzo, *College Academy*
 David Bramlett, *Jackson State University*
 Natasha Brewley-Corbin, *Georgia Gwinnett College*
 Denise Brown, *Collin College, Spring Creek Campus*
 *Julie Brown, *University of Texas of the Permian Basin*
 David Britz, *Raritan Valley Community College*
 Bill Burgin, *Gaston College*
 Jennifer Cabaniss, *Central Texas College*
 *Ryan Caimano, *Oklahoma Bible Academy*
 *Kate Calendrillo, *Northwest Catholic High School*
 *Jaime Castrataro, *Tri-West High School*
 Jimmy Chang, *St. Petersburg College*
 Teresa Chasing Hawk, *University of South Dakota*
 *Janie Coker Pritchard, *Tyler Junior College*
 Diana Colt, *University of Minnesota, Duluth*
 Shannon Cornell, *Amarillo College*
 Wendy Davidson, *Georgia Perimeter College, Newton*
 Donna Densmore, *Bossier Parish Community College*
 *Kevin Dibert, *NSU University School*
 *Barbara Dobbs, *Academy of the Holy Cross*
 *Juan Du, *Broward College*
 *Marcial Echenique, *College of the Florida Keys*
 Disa Enegren, *Rose State College*
 Keith A. Erickson, *Georgia Gwinnett College*

*Deanna Ettore, *Washington Township High School*
 *Rafat Ewais, *Clifton High School*
 *Rebecca Faber, *Notre Dame High School*
 *Ed Fischer, *Northern Nash High School*
 Nancy Fisher, *University of Alabama*
 *Vickie Flanders, *Baton Rouge Community College*
 *Sheila Fleming, *Marble Falls High School*
 Donna Gerken, *Miami Dade College*
 *David Ghazvini, *Lee College*
 Cynthia Glickman, *Community College of Southern Nevada*
 Sudhir Kumar Goel, *Valdosta State University*
 Donald Gordon, *Manatee Community College*
 David L. Gross, *University of Connecticut*
 Jason W. Groves, *South Plains College*
 Joel K. Haack, *University of Northern Iowa*
 Jeremy Haefner, *University of Colorado*
 Joyce Hague, *University of Wisconsin at River Falls*
 *Tawfik Haj, *San Jacinto College*
 Mike Hall, *University of Mississippi*
 *Leo Hartsock, *Northland Preparatory Academy*
 Mahshid Hassani, *Hillsborough Community College*
 Tom Hayes, *Montana State University*
 Christopher N. Hay-Jahans, *University of South Dakota*
 Angela Heiden, *St. Clair Community College*
 *Baron Heinemann, *Episcopal High School*
 Celeste Hernandez, *Richland College*
 *Ann Ho, *Taipei American School*
 Alysmae Hodges, *Eastfield College*
 Amanda Hood, *Copiah-Lincoln Community College*
 Jo Beth Horney, *South Plains College*
 Heidi Howard, *Florida State College at Jacksonville, South Campus*
 *S. Larue Huckaby, *Shorter University*
 Winfield A. Ihlow, *SUNY College at Oswego*
 *Dale Johanson, *Northeast Community College*
 Nancy Raye Johnson, *Manatee Community College*
 *Kimberly Jones, *Dakota State University*
 *Kevin Jones, Jr., *McCluer High School*
 *Cheryl Kerns, *Blue Valley West High School*
 *Clayton Kitchings, *University of North Georgia*
 *Theo Koupelis, *Broward College*
 *Anthony Lamanna, *St. John's Preparatory*
 Dennine Larue, *Fairmont State University*

*Tina Lee, *Haywood Community College*
 Mary Leesburg, *Manatee Community College*
 Christine Heinecke Lehman, *Purdue University North Central*
 Alexander Levichev, *Boston University*
 *Qingxia Li, *Fisk University*
 *Ethan Lightfoot, *Gateway STEM High School*
 Zongzhu Lin, *Kansas State University*
 Benjamin Marlin, *Northwestern Oklahoma State University*
 Marilyn Massey, *Collin County Community College*
 Yvelyne McCarthy-Germaine, *University of New Orleans*
 *Kayri McCartin, *Grafton High School*
 David McMann, *Eastfield College*
 Owen Mertens, *Missouri State University, Springfield*
 James Miller, *West Virginia University*
 Martha Nega, *Georgia Perimeter College, Decatur*
 *Youssef Oumanar, *Greenhill School*
 Shahla Peterman, *University of Missouri, St. Louis*
 Debra A. Pharo, *Northwestern Michigan College*
 *Janice Phillipp, *Texas Southmost College*
 Gloria Phoenix, *North Carolina Agricultural and Technical State University*
 Katherine Pinzon, *Georgia Gwinnett College*
 David Platt, *Front Range Community College*
 Juha Pohjanpelto, *Oregon State University*
 *David Quesnell, *Immaculate High School*
 Brooke Quinlan, *Hillsborough Community College*
 *Corrie Ramage, *Kosciusko High School*
 Janice Rech, *University of Nebraska at Omaha*
 Joseph W. Rody, *Arizona State University*
 *Lee Ann Roberts, *Georgia Gwinnett College*
 Behnaz Rouhani, *Georgia Perimeter College, Dunwoody*
 Judith Salmon, *Fitchburg State University*
 *Ryan Sasaki, *Iolani School*
 Michael Schramm, *Indian River State College*
 Cynthia Schultz, *Illinois Valley Community College*
 *Brittney Seale, *Colorado Mountain College*
 *Juanita Self, *Central Texas College*
 *Patricia Senn, *Lurleen B. Wallace Community College*
 *Olimpia Simeón Monet, *Miami Dade College*
 *Muhammad Naeem Sharif, *Kuwait Technical College*
 Pat Shelton, *North Carolina Agricultural and Technical State University*
 Jed Soifer, *Atlantic Cape Community College*

*Brian Southworth, *Independence Community College*
 Caroline Spillman, *Georgia Perimeter College, Clarkston*
 *Laura Spunt, *Posnack School*
 Jonathan Stadler, *Capital University*
 Franotis R. Stallworth, *Gwinnett Technical College*
 John David Stark, *Central Alabama Community College*
 Chris Stump, *Bethel College*
 *Cynthia Lee Suplizio, *Colorado Mountain College*
 Scott Sykes, *University of West Georgia*
 Richard Townsend, *North Carolina Central University*
 Pamela Trim, *Southwest Tennessee Community College*
 *Ruth Trubnik, *Delaware Valley University*
 Chris Turner, *Arkansas State University*
 Richard E. Van Lommel, *California State University, Sacramento*
 Dan Van Peursem, *University of South Dakota*
 Philip Van Veldhuizen, *University of Nevada at Reno*
 Jeffrey Weaver, *Baton Rouge Community College*
 *Felice Weiner, *South Mountain Community College*
 *Aaron Wernet, *McLennan Community College*
 Amanda Wheeler, *Amarillo College*
 David White, *The Victoria College*
 *Molly Whittington, *Holy Cross School*
 Tracy Wienckowski, *University of Buffalo*
 *Stacy Yarnell, *Colorado Mountain College*

Additional acknowledgments are extended to:

- Brad Davis, for contributing new and updated data, providing the book's annos and answer section, and serving as (an amazing!) accuracy checker.
- Dan Miller for preparing the solution manuals and the Learning Guide.
- Brian Morris at Scientific Illustrators for superbly illustrating the book.
- Jerilyn DiCarlo and Annie Brace for their brilliant design work.
- Francesca Monaco, project manager, and Tamela Ambush, production editor, whose collective talents kept every aspect of this complex project moving through its many stages. (Special thanks to Sharon Cahill for seamlessly stepping in during Francesca's absence.)
- Dawn Murrin, Jeff Weidenaar, and Jon Krebs, my editors at Pearson, who guided and coordinated the book from manuscript through production.
- Chelsea Kharakozova, product manager, for the steady financial guidance for the project.

- Stacey Sveum and Peggy Lucas, marketing managers, for their innovative marketing efforts.
- Shana Siegmund, MyLab producer, for coordinating the many digital aspects of the project.
- Eric Gregg and Dominick Frank for splendid work on the MyLab exercises that support the texts.
- Finally, thanks to the Pearson sales force, for their confidence and enthusiasm about the book.

I hope that my passion for teaching, as well as my respect for the diverse population of students I have taught and learned from over the years, is apparent throughout this new edition. By connecting algebra and trigonometry to the whole spectrum of learning, it is my intent to show students that their world is profoundly mathematical, and indeed, π is in the sky.

Bob Blitzer

The bar graph shows some of the qualities that students say make a great teacher. It was my goal to incorporate each of these qualities throughout the pages of this book.

Explains Things Clearly

I understand that your primary purpose in reading *Precalculus* is to acquire a solid understanding of the required topics in this course. In order to achieve this goal, I've carefully explained each topic. Important definitions and procedures are set off in boxes, and worked-out examples that present solutions in a step-by-step manner appear in every section. Each example is followed by a similar matched problem, called a Check Point, for you to try so that you can actively participate in the learning process as you read the book. (Answers to all Check Points appear in the back of the book.)

Funny & Entertaining

Who says that a precalculus textbook can't be entertaining? From our unusual cover to the photos in the chapter and section openers, prepare to expect the unexpected. I hope some of the book's enrichment essays, called Blitzzer Bonuses, will put a smile on your face from time to time.

Helpful

I designed the book's features to help you acquire knowledge of algebra and trigonometry, as well as to show you how algebra and trigonometry can solve authentic problems that apply to your life. These helpful features include:

- **Explanatory Voice Balloons:** Voice balloons are used in a variety of ways to make math less intimidating. They translate algebraic and trigonometric language into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect new concepts to concepts you have already learned.
- **Great Question!:** The book's Great Question! boxes are based on questions students ask in class. The answers to these questions give suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.
- **Achieving Success:** The book's Achieving Success boxes give you helpful strategies for success in learning algebra and trigonometry, as well as suggestions that can be applied for achieving your full academic potential in future college coursework.
- **Chapter Summaries:** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples from the chapter that illustrate these key concepts are also referenced in the chart. Review these summaries and you'll know the most important material in the chapter!

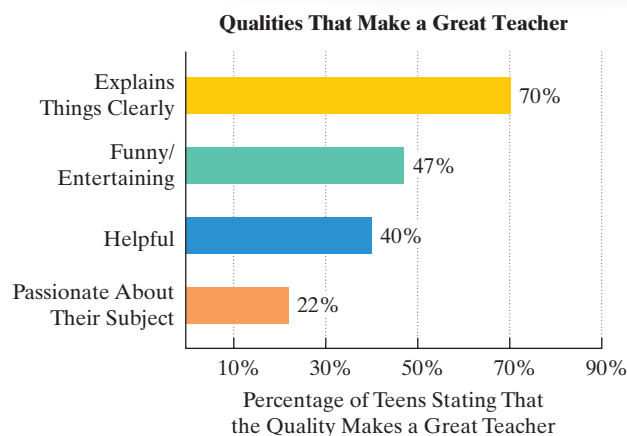
Passionate about the Subject

I passionately believe that no other discipline comes close to math in offering a more extensive set of tools for application and development of your mind. I wrote the book in Point Reyes National Seashore, 40 miles north of San Francisco. The park consists of 75,000 acres with miles of pristine surf-washed beaches, forested ridges, and bays bordered by white cliffs. It was my hope to convey the beauty and excitement of mathematics using nature's unspoiled beauty as a source of inspiration and creativity. Enjoy the pages that follow as you empower yourself with the algebra and trigonometry needed to succeed in college, your career, and your life.

Regards,

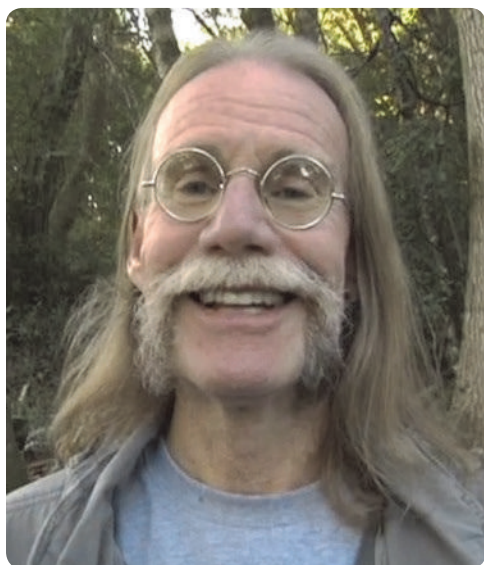
Bob Blitzzer

To the Student



Source: Avanta Learning System

ABOUT THE AUTHOR



Bob Blitzler is a native of Manhattan and received a Bachelor of Arts degree with dual majors in mathematics and psychology (minor: English literature) from the City College of New York. His unusual combination of academic interests led him toward a Master of Arts in mathematics from the University of Miami and a doctorate in behavioral sciences from Nova University. Bob's love for teaching mathematics was nourished for nearly 30 years at Miami Dade College, where he received numerous teaching awards, including Innovator of the Year from the League for Innovations in the Community College and an endowed chair based on excellence in the classroom. In addition to *Precalculus*, Bob has written textbooks covering developmental mathematics, introductory algebra, intermediate algebra, college algebra, algebra and trigonometry, trigonometry, and liberal arts mathematics, all published by Pearson. When not secluded in his Northern California writer's cabin, Bob can be found hiking the beaches and trails of Point Reyes National Seashore and tending to the chores required by his beloved entourage of horses, chickens, and irritable roosters.

APPLICATIONS INDEX

A

Accidents, automobile
 accidents per day, age of driver and, 964
 probability of accident while intoxicated, 1135
 Acid rain, 492
 Actors, selection of, 1118, 1139
 Adulthood, transition to, 931
 Adult residential community costs, 1065, 1072
 Advertising
 online spending, 1142
 sales and price and, 424–425
 African Americans, percentage with high school diploma, 516
 African life span, AIDS and, 843
 Age(s)
 body-mass index and, 877
 calories needed to maintain energy by, 86
 chances of surviving to various, 180
 of driver, accidents per day and, 964
 of driver, and rate of fatal crashes, 108
 and heart rates of children, 866, 870
 height as function of, 222, 225, 244, 1183
 and Implicit Association Test, 56
 and intellectual ability, 1154
 and lack of political affiliation, 894
 and living arrangements, 218
 marriage and, 156–158
 perceived length of time period and, 428
 and percentage of Americans receiving
 a tax refund, 46
 percentage of U.S. population never married,
 ages 25–29, 217
 percent body fat in adults by, 200
 and respiratory rate of children, 876
 and sleep, 161
 and social media use, 888
 Aging rate, space travel and, 32, 44, 47
 AIDS/HIV
 African life span and, 843
 cases diagnosed (U.S.), 341
 T cell count and, 163, 172–173
 Aircraft/airplanes
 approaching runway, vector describing, 794
 direction angle of, given speed, 796
 distance and angle of elevation of, 616
 distance flown by, 566
 ground speed of, 796
 height of, 582
 leaving airport at same time, distance between,
 735, 739–740, 810
 linear speed of propeller, 651
 Mach speed of, 691
 runway departure lineup, 1140
 true bearing of, 795–796
 vector describing flight of, 794
 velocity vector of, 792
 weight/volume constraints, 880–882
 wind direction and, 828–829, 834
 wind speed and direction angle exerted on,
 795–796
 Airports, distance between, 742
 Alcohol use
 by high school seniors, 160
 moderate wine consumption and heart
 disease, 216–217
 number of moderate users in U.S., 517
 and risk of accident, 487–488, 493
 Alligator, tail and body length of, 427
 Altitude
 atmospheric pressure and, 515

gained by hiker climbing incline, 652
 increase on inclined road of, 566
American Idol, ratings of, 335–336
 Amusia (tone deafness), sound quality and, 670, 672
 Angle(s)
 in architecture, 522
 clock hands forming, 522, 523
 of elevation, 561–564, 566–567, 582, 616, 645,
 646, 652, 655, 733–734
 Angular speed
 of audio records, 535
 of carousel, 534
 of hard drive in computer, 534
 of propeller on wind generator, 651
 Annuities
 compound interest on, 1082–1083, 1089
 value of, 1089, 1139
 Antenna, height on top of building, 653
 Apogee/perigee of satellite's orbit, 982
 Applause, decibel level of, 204
 Arch bridge, 1048
 Archer's arrow, path of, 329
 Architecture
 angles in, 522
 conic sections in, 983, 993
 Archway. *See* Semielliptical archway
 and truck clearance
 Area, of a football field, 117–118
 Area code possibilities, 1118
 Artists, in documentary, 1113–1114
 Aspirin, half-life of, 508, 997
 Asteroid detection, 855
 Atmospheric pressure and altitude, 515
 Attitudes, of college freshmen, 112–113
 Attractiveness, and success, 148
 Audio records, angular speed and linear
 speed of, 535
 Autism cases diagnosed, 1063
 Automobiles
 accidents per day, age of driver and, 964
 computing work of pushing, 803, 805
 depreciation, 180
 fatal crashes, age of driver and, 108
 leaving city at same time, distance
 between, 810
 possible race finishes, 1118
 prices of, and average age, 123–124
 probability of accident while intoxicated, 1135
 purchase options, 1118
 rentals, 126–127, 130
 required stopping distance, 406, 416–417
 stopping distances, 406, 416–417
 value over time, 1064
 Average cost function, 397–398, 402
 Average rate of change, 222–223, 244

B

Ball
 angle of elevation and throwing distance of, 720
 location of thrown, 1025
 maximum height and throwing distance of, 844
 thrown upward and outward, 333–334
 Ball (attached to spring)
 finding amplitude and period of motion of, 679
 simple harmonic motion of, 640–641, 716, 720
 Ball (height above ground), 842, 844, 896, 1142
 baseball, 519
 bounce height, 427
 football, 17, 327–328, 896
 maximum height, 813, 844

when thrown from rooftop, 416
 when thrown from top of Leaning Tower
 of Pisa, 414
 Ballots, 1118
 Banking angle and turning radius of bicycle, 427
 Baseball
 angle of elevation and throwing distance of, 716
 height of ball above ground, 519
 path of, 1025, 1033–1034
 pitcher's angle of turn to throw ball, 744
 position as function of time, 1033–1034
 Baseball contract, 143, 1053, 1088
 Baseball diamond, distance from pitcher's mound
 to bases on, 743
 Bearings, 639–640, 655
 of boat, 640, 646, 743
 distance at certain, 646, 653
 to fire from two fire stations, 731, 733, 1045
 of jet from control tower, 646
 true, of plane, 795–796
 between two cities, 653
 Beauty, and symmetry, 185
 Benefit concert lineup possibilities, 1118
 Berlin Airlift, 879, 885
 Bias, Implicit Association Test for, 48, 56
 Bicycles
 banking angle, 427
 manufacturing, 180, 402, 831, 885
 as mode of travel, 270
 Biorhythms, 521, 539, 553, 602–604
 Bird species population decline, 508
 Birth(s)
 out-of-wedlock, 291
 in U.S. from 2000 through 2009, 246, 251–252
 Birthday, probability of sharing same, 271,
 693, 1135
 Birthday cake, 47
 Black holes, 28–29
 Blood-alcohol concentration, 15–16, 19,
 487–488, 493
 Blood volume and body weight, 420–421
 Boats/ships
 bearing of, 640, 646
 changing, 743
 distance traveled at certain, 646
 to sail into harbor, 646
 direction angle of, 812
 distance from lighthouse, 654, 735
 distance from pier, 743
 ground speed, 812
 leaving harbor at same time, distance between
 after three hours, 742
 location between two radio towers, 1048
 on tilted ramp, vector components of force on,
 801, 805
 velocity of, 812
 velocity vector of, 792
 Body fat in adults by age and gender, percent, 200
 Body-mass index, 427, 877
 Body temperature, variation in, 653, 815
 Books
 arranging on shelf, 1113
 selections, 1118, 1141
 Bouquet, mixture of flowers in, 865
 Box dimensions, 366
 Brain
 growth of the human, 503
 modeling brain activity, 616
 Break-even analysis, 655, 824–825, 831, 865.
See also Cost and revenue functions/
 break-even points

- Breast cancer, 1121–1122
 Breathing cycle, 583
 modeling, 597–598
 velocity of air flow in, 715
 Brick-and-mortar stores, 516
 Bridge
 arch, 1048
 George Washington Bridge, 1049
 Golden Gate Bridge, 260
 suspension, parabolas formed by, 1049
 Budgeting, groceries vs. health care, 227
 Building
 height of, 562, 645, 646, 653, 655, 735
 shadow cast by, 716
 Business ventures, 831
 Butterflies, symmetry of, 745
- C**
- Cable car, distance covered by, 734–735
 Cable service, 1051
 Cable television deals, 1051
Call of Duty video game, retail sales of, 494
 Calorie-nutrient information, 889
 Calories
 needed by age groups and activity levels, 931
 needed to maintain energy balance, 86
 Camera
 price reductions, 115, 1135
 viewing angle for, 634–635
 Cancer, breast, 1121–1122
 Canoe manufacturing, 831
 Car(s). *See* Automobiles
 Carbon-14 dating, 498, 507, 508
 Carbon dioxide, atmospheric global warming and, 149, 211–213, 604, 617
 Cardboard, length/width for box, 864
 Cards. *See* Deck of 52 cards, probability and
 Carousel, linear speed and angular speed of animals on, 534, 538
 Cave paintings, prehistoric, 508
 CD selection for vacation trip, 1139
 Celebrity earnings, 163–165
 Cell phones, 164, 181, 832
 Celsius/Fahrenheit temperature interconversions, 17
 Centrifugal force, 425–426
 Chaos, patterns of, 723, 769
 Charging stations, for electric vehicles, 1138
 Chernobyl nuclear power plant accident, 451
 Chess moves, 1110
 Child care, public spending on, 179
 Child mortality, literacy and, 202, 216
 Children
 heart rate and age of, 866, 870
 modeling height of, 461, 467, 468, 489
 respiratory rate and age of, 876
 Cholesterol
 and dietary restrictions, 876
 intake, 876, 891
 Cigarette consumption. *See* Smoking
 Cigarette tax, 1063
 Circle, finding length of arc on, 653, 700
 Class structure of the United States, 962–963
 Cliff, distance of ship from base of, 645
 Climate change, 213. *See also* Global warming
 Clock(s)
 angles formed by hands of, 522, 523
 degrees moved by minute hand on, 537
 distance between tip of hour hand and ceiling, 602
 distance between tips of hands at 10:00, 744
 minute hand movement in terms of π , 537
 Club officers, choosing, 1118, 1139
 Coding, 933, 942–943, 945, 946
 Coffee, temperature of, 1156, 1166
 Coffee consumption, sleep and, 519
 Coin tosses, 1123, 1130–1131, 1133, 1134
 College(s)
 attendance, 1140
 average dormitory changes at, 1074
 percentage of U.S. high school seniors applying to more than three, 452
 College education
 cost of, 2, 4–5, 19
 and success, 122
 women vs. men, 147
 College graduates, salaries of, 300
 College students
 excuses of, for not meeting assignment deadlines, 145
 freshmen
 attitudes of, 112–113
 claiming no religious affiliation, 164–167
 political orientation, 303
 probable majors of, 1074
 grade inflation among, 108
 hours of study per week, by major, 842–843
 interactive online games played by, 834
 loan debt, 70
 majors of, 834
 procrastination and symptoms of physical illness among, 816, 833
 sleep and, 834
 study abroad destinations, 997
 women, 824
 College tuition, and student loan debt, 70
 Collinear points, 959
 Comedians, net worth of, 468
 Comedy act schedule, 1118
 Comets
 Halley's Comet, 978, 993, 1044
 intersection of planet paths and, 863, 993
 Committee formation, 1114, 1116, 1118
 Commuters, toll discount passes, 260
 Compound interest, 447–451, 453, 488, 491–493, 1064, 1089, 1139
 on annuity, 1082–1083, 1089
 continuously compounded, 479, 488, 492, 516, 518, 721, 892
 investments, 514, 647
 Computer(s)
 angular speed of hard drive in, 534
 computer-generated animation, 230
 discounts, 252–253, 260
 price before reduction, 511
 prices, 262
 ratio of students to computers in U.S. public schools, 341
 sale price, 68
 Computer graphics, 893, 917, 926, 927
 Concentration of mixture. *See* Mixture problems
 Cone volume, 426
 Conference attendees, choosing, 1116, 1118
 Constraints, 880–884, 890
 Continuously compounded interest, 479, 488, 492, 516, 518, 721, 892
 Cookies, supply and demand for, 832
 Coronary heart disease, 509
 Coronavirus, new cases of, 337–339, 438, 499–500, 892
 Corporation officers, choosing, 1113, 1118
 Cost(s). *See also* Manufacturing costs
 of college education, 2, 4–5, 19
 of medical treatment, 467
 minimizing, 885
 of raising child born in U.S., 1058–1059
 of text messaging plans, 283–285
 truck rental, 140, 1051
 of yacht, 121–122
 Cost and revenue functions/break-even points, 824–825, 831, 865, 891
 average, 397–398, 402
 bike manufacturing, 402
 break-even points, 831, 891
 customers and revenue, 285–286
 for PDA manufacturing, 865
 radio manufacturing, 259
 running shoe manufacturing, 402, 845
 virtual reality headset manufacturing, 397–398
 wheelchair manufacturing, 398
 Costco memberships, 435
 Course schedule, options in planning, 1110
 Cove, distance across, 743
 COVID-19. *See* Coronavirus
 Crane lifting boulder, computing work of, 805
 Crate, computing work of dragging, 812
 Credit cards, interest rates on, 19
 Crime, violent crime rate and imprisonment, 864
 Cryptograms, 942–943, 946. *See also* Coding
 Cups, disposable vs. reusable, 617
 Cycles, 568
 Cycloid, 1034
- D**
- Daylight, number of hours of, 539, 553, 600, 602, 616, 715
 Dead Sea Scrolls, carbon-14 dating of, 498
 Death penalty, sentences rendered by U.S. juries, 354
 Death rate, hours of sleep and, 836, 840
 Deaths
 in the 20th century, 889
 from 2000 through 2009, 246, 251–252
 by snakes, mosquitoes, and snails, 181
 Debt
 national, 20, 27–28, 31, 32
 student loan, 70
 Decay model for carbon-14, 507, 508
 Decibels. *See* Sound intensity
 Deck of 52 cards, probability and, 1124–1125, 1127–1128, 1133, 1140, 1141
 Decoding a word or message, 943, 945, 946
 Defense budget, 122
 Deforestation, Amazon, 433
 Degree-days, 1076
 Delicate Arch, angle of elevation to determine height of, 567
 Depreciation, 180
 Depression, exercise and, 230
 Desk manufacturing, 896
 Die rolling outcomes, 1123–1124, 1133, 1134, 1140
 Digital camera, price reduction for, 115, 1135
 Digital photography, 917, 926–927, 930, 931, 963
 Dinosaur bones and potassium-40 dating, 508
 Dinosaur footprints, pace angle and stride indicated by, 736, 742
 Direction, 782–783
 Disposable cups, travel mugs vs., 617
 Distance
 across cove, 743
 across lake, 562, 566, 652, 742, 743
 from base to top of Leaning Tower of Pisa, 733
 braking, 842
 between cars leaving city at same time, 810
 of forest ranger from fire, 646
 between houses at closest point, 996
 of island from coast, 645
 of marching band from person filming it, 616
 of oil platform from ends of beach, 733
 between pairs of cities, 280–281
 of rotating beam of light from point, 615, 616
 safe, expressway speed and, 143
 of ship from base of cliff, 645
 of ship from base of Statue of Liberty, 645
 of ship from lighthouse, 654
 of ship from radio towers on coast, 996
 of stolen car from point directly below helicopter, 645

that skydiver falls in given time, 1091
 traveled by plane, 566
 between two points on Earth, 537
 between two points on opposite banks
 of river, 733
 between two trains leaving station at
 same time, 768
 Distance traveled, combined walking and bus
 travel, 19
 Diver's height above water, 416
 Diversity index, 143
 Diving board motion, modeling, 616
 Divorce, marriage age and probability of,
 156–158
 DNA, structure of, 555
 Doctor, visits to, 201
 Domed ceiling, light reflectance and parabolic
 surface of, 1013
 “Don't ask, don't tell” policy, 227–228
 Drinks, order possibilities for, 1118
 Drivers, age of. *See under* Age(s)
 Driving accidents. *See* Accidents, automobile
 Driving rate and time for trip, 422
 Drug concentration, 224, 402
 Drug dosage, child vs. adult, 721
 Drug experiment volunteer selection, 1118
 Drug tests, mandatory, probability of accurate
 results, 1134
 Drug use among teenagers, 510
 Drunk driving, probability of accidents, 1135
 Dual investments, 116, 201, 892

E

Eagle, height and time in flight, 302
 Earnings. *See* Salary(-ies)
 Earth
 age of, 24
 angular velocity of point on, 538
 distance between two points on, 537
 finding radius of, 647
 motion of Moon relative to, 555
 Earthquake
 epicenter, 281
 intensity, 515
 intensity of, 454, 462
 relief from, 879–882
 simple harmonic motion from, 643
 Eating, lifetime hours spent, 123
 E-commerce, 516
 Economic impact of factory on town, 1090, 1139
 Education. *See also* College education
 level of, and earnings, 111–112, 123
 level of, and marriages, 158
 level of, U.S. population, 1133
 percentage of U.S. adults completing
 high school, 516
 public spending on pre-primary, 179
 Election ballots, 1118
 Electrical resistance, 86, 428, 1142
 Electric vehicles, 1138
 Electromagnetic spectrum, 600
 Elephant's weight, 493
 Elevation, angle of, 561–564, 566–567, 582, 616,
 645–646, 652, 655, 733–734
 Elevator capacity, 876
 Elk population, 518
 Elliptical ceiling, 981
 Elliptic pool, 981
 Encoding a message, 933, 942–943, 945, 946
 Endangered species, 508
 Environmental disasters, cleanup costs of, 70–71
 Equator, linear velocity of point on, 537
 Equilibrium, forces in, 795
 Ethnic diversity, 143
 Exam grades, 931

Exercise
 depression and, 230
 heart rate and, 3
 target heart rate ranges for, 18
 Explosions, location of
 arrival of sound, 997
 when recorded by two microphones, 993–994,
 996, 1013
 Exponential decay model, 508, 516, 518, 965
 Exponential growth, 655
 Expressway speeds and safe distances, 143
 Eye color and gender, 1141

F

Factory, economic impact on town, 1090
 Fahrenheit/Celsius temperature interconversions, 17
 Family, independent events in, 1131, 1133, 1140
 Federal budget
 deficit (*See* National debt)
 expenditures on human resources, 403
 Federal Express, aircraft purchase decisions by, 885
 Federal income tax, 181
 Federal social programs, 31
 Fencing
 for enclosure, 860–861
 maximum area inside, 331, 334
 for plot of land, 891
 Ferris wheel, 281
 height above ground when riding, 554
 linear speed of, 538
 Field, dimensions of, 890, 1142
 Films. *See* Movies
 Fire
 area burned by wildfires, 353
 distance of fire station from, 1045
 distance of forest ranger from, 646
 locating, from tower, 724, 731, 733, 768, 812, 1045
 Flagpole
 height of, finding, 721
 leaning, angle made with ground, 735
 on top of building, height of, 646
 Flood, probability of, 1140
 Floor dimensions, 863
 Flu
 inoculation costs, 86
 outbreak in town, 452
 outbreak on campus, 1090
 time-temperature scenario, 182–183
 vaccine mixture, 180, 826–828
 Focal length, 93
 Food cost per item, 227, 843
 Food stamps program, 31
 Food truck, costs of, 334–335
 Football
 area of field, 117–118
 height above ground, 17, 327–328, 896
 maximum height of, 1049
 position as function of time, 1049
 vector describing thrown, 794
 Force(s)
 on body leaning against wall, 782, 785
 in equilibrium, 795
 pulling cart up incline, 782
 resultant, 795, 811, 812
Freedom 7 spacecraft flight, 272
 Free-falling object's position, 413–414, 416
 Frequency, length of violin string and, 424
 Freshmen. *See under* College students
Friendship 7, distance from Earth's center, 1044
 Fuel efficiency, 182

G

Galaxies, elliptical, 1101
 Games, online, college students and, 834

Garbage, daily per-pound production of, 70
 Gasoline
 gallons of premium sold, 805, 831–832
 gallons of regular sold, 805
 Gas pressure
 in can, 423
 and volume, 1195
 Gay marriage, U.S. public opinion on, 509, 833
 Gay service members discharged from military,
 227–228
 Gender
 bachelor's degrees awarded and, 147
 calories needed to maintain energy by, 86
 and careers, 894
 educational level and earnings by, 111–112
 eye color and, 1141
 first-year U.S. college students claiming no
 religious affiliation by, 164–167
 jobs in U.S. labor force by, 109
 life expectancy by year of birth and, 215
 of members of House of Representatives, 833
 percentage of United States population never
 married, ages 25–29 and, 217
 percent body fat in adults by, 200
 wage gap by, 180
 George Washington Bridge, height of cable
 between towers of, 1049
 Global warming, 149, 211–213
 Golden Gate Bridge, tolls on, 260
 Golden rectangles, 46
 Government, trust in, 215
 Grade inflation, 108
 Gravitational force, 425
 Gravity model, 428
 Groceries, budgeting for, 227
 Ground speed, 796
 Groups fitting into van, 1118
 Gutter cross-sectional area, 334
 Guy wire attached to pole, angle made with
 ground and, 638

H

Half-life
 aspirin, 997
 radioactive elements, 508, 516, 965
 Xanax, 1035
 Halley's Comet, 978, 993, 1044
 Hamachiphobia, 509
 Happiness
 average level of, at different times of day, 271
 per capita income and national, 216
 Harmonic motion, simple. *See* Simple harmonic
 motion
 Headlight
 parabolic surface of, 1049, 1050
 unit design, 1049, 1050
 Headset manufacturing costs, 384, 397–398
 Health care
 budgeting for, 227
 gross domestic product (GDP) spent on, 492
 savings needed for expenses during
 retirement, 509
 Heart beats over lifetime, 32
 Heart disease
 coronary, 509
 moderate wine consumption and, 216–217
 smoking and, 404
 Heart rate
 and age of children, 866, 870
 exercise and, 3, 18
 of mammals, and life span, 436
 before and during panic attack, 353
 Heat generated by stove, 428
 Heating and cooling systems, 756
 Heat loss of a glass window, 428

Height

- of antenna on top of building, 653
 - of ball above ground (*See* Ball [height above ground])
 - of building, 562, 645, 646, 653, 655, 735
 - child's height modeled, 461, 467, 468, 489
 - diver's height above water, 416
 - of eagle, in terms of time in flight, 302
 - on Ferris wheel while riding, 554
 - of flagpole, 646, 721
 - as function of age, 222, 225, 244, 1183
 - healthy weight region for, 877
 - of leaning wall, finding, 734
 - maximum, 813, 1050, 1142
 - of Mt. Rushmore sculpture, 639
 - percentage of adult height attained by girl of given age, 467, 489
 - of plane, 582
 - of tower, finding, 637, 645, 781
 - of tree, finding, 768
 - weight and height recommendations/ calculations, 427
- Higher education costs, 1075
- High school education, percentage of U.S. adults completing, 516
- High school seniors, marijuana/alcohol use by, 160
- Hiking trails, finding bearings on, 640
- Hill, magnitude of force required to keep car from sliding down, 795
- Hispanic Americans, population growth, 516
- HIV/AIDS
- African life span and, 843
 - cases diagnosed (U.S.), 341
 - T cell count and, 163, 172–173
- Hot-air balloon, distance traveled by ascending, 638, 646, 680
- Hotel room types, 834
- House of Representatives, gender of members of, 833
- House sales prices, 181, 218, 1091
- House value, inflation rate and, 452
- Human resources, federal budget expenditures on, 403
- Hurricanes
- barometric air pressure and, 493
 - probability, 1134
- HVAC service contract, 405
- Hydrogen ion concentration, 493

I

- Ice cream flavor combinations, 1114, 1118
- Identical twins, distinguishing between, 835
- Illumination intensity, 427, 428
- Implicit Association Test, 48, 56
- Imprisonment, violent crime rate and, 864
- Income
- distribution of, among Americans, 833
 - of highest-paid TV celebrities, 163–165
 - median yearly, 28
- Income taxes, federal, 181
- Individual Retirement Account (IRA), 1082–1083, 1089, 1090, 1139
- Inflation
- grade, 108
 - rate of, 452
- Influenza. *See* Flu
- Inoculation costs for flu, 86
- Insulation, rate of heat lost through, 655
- Insurance, pet, 200
- Intellectual ability, age and, 1154
- Intelligence quotient (IQ) and mental/ chronological age, 427
- Interactive online games, college students and, 834
- Interfaith marriages, 140
- Interracial marriages, 1051–1052

Investment(s)

- accumulated value of, 448, 451, 488
 - amounts invested per rate, 843
 - choosing between, 449
 - compound interest, 447–451, 453, 479, 488, 491–493, 514, 516, 518, 647, 721, 892, 1089
 - dual, 116, 201, 892
 - in greeting cards, 831
 - and interest rates, 19
 - maximizing expected returns, 886
 - money divided between high- and low-risk, 876
 - in play, 831
 - possibility of stock changes, 1139
- IQ (intelligence quotient) and mental/ chronological age, 427
- IRA. *See* Individual Retirement Account
- Island, distance from coast of, 645

J

- Jeans, price of, 260
- Jet ski manufacturing, 891
- Job applicants, filling positions with, 1141
- Job offers, 1075, 1076, 1088
- Jobs, in U.S. labor force, by gender, 109
- Jokes about books, 1119

K

- Kidney stone disintegration, 978, 1013
- Kidney stones, lithotriper treatment of, 997
- Kinetic energy, 428
- Kite, angle made with ground of flying, 638, 1024

L

- Labrador retrievers, color of, 54
- Lake, distance across, 562, 566, 652, 742, 743
- Land
- fencing for (*See* Fencing)
 - rectangular plot, 891
 - triangular plot, 744, 810
- Landscape design, 118–119
- Leaning Tower of Pisa, distance from base to top of, 733
- Leaning wall, finding height of, 734
- Learning rate and amount learned, measuring, 813
- Learning theory project, 500
- Leisure time, and stress levels, 332
- Length of violin string and frequency, 424
- Lens, focal length of a, 93
- Letter arrangements, 1118
- License plates, 1111
- Life expectancy, 215, 436
- Light intensity of sunlight beneath ocean's surface, 491
- Light reflectance and parabolic surface, 1013, 1049, 1050
- Light waves, modeling, 653
- Linear speed, 538
- of airplane propeller, 651
 - of animals on carousel, 534, 538
 - of wind machine propeller, 535
- Line formation, 1119
- Literacy, child mortality and, 202, 216
- Lithotriper, 997
- Little League baseball team batting order, 1111–1112
- Living alone, number of Americans, 221–222, 306
- Living arrangements, and age, 218
- Lottery
- numbers selection, 1118, 1125–1126
 - probability of winning, 1109, 1125–1126, 1133, 1134, 1140, 1141

LOTTO

- numbers selection, 1118
 - probability of winning, 1134
- Loudness, 204, 428, 467, 478, 509, 518
- Love, components of, 139
- Luggage, volume of carry-on, 380–381
- Lunch menus, 885, 1118

M

- Mach speed of aircraft, 691
- Mailing costs, 200
- Mall browsing time and average amount spent, 440, 441
- Mammals, heart rate of, 436
- Mammography screening data, 1121–1122
- Mandatory drug testing, probability of accurate results, 1134
- Mandelbrot set, 769, 779, 781
- Manufacturing and testing, hours needed for, 916
- Manufacturing constraints, 880, 881, 882
- Manufacturing costs. *See also* Cost and revenue functions/break-even points
- bicycles, 180
 - PDAs, 865
 - tents, 890
 - virtual reality headsets, 384, 397–398
 - wheelchair, 398
- Maps, making, 564
- Marching band, 835
- Marijuana use, by high school seniors, 160
- Marital status
- unmarried Americans (ages 25–29), 217
 - U.S. adults (1970–2013), 832
 - U.S. population, ages 15 or older (2010), 1031, 1129–1130
- Marriage age, and probability of divorce, 156–158
- Marriage equality, U.S. public opinion on, 509, 833
- Marriages
- education and, 158
 - interfaith, 140
 - interracial, 1051–1052
- Mass attached to spring, simple harmonic motion of, 642–643
- Mathematics department personnel, random selection from, 1134
- Mathematics exam problems, 1119
- Maximum area, 331, 334, 383
- Maximum height, 813, 1050, 1142
- Maximum product, 334, 383
- Maximum profit, 383, 882, 891
- Maximum scores, 885
- Media, trust in, 215
- Median age. *See under* Age(s)
- Medicaid, 31
- Medical treatment, delaying due to cost, 467
- Medicare, 31
- Medication dosage, adult vs. child/infant, 721
- Memberships, 435, 917
- Memory retention, 467, 468, 492, 515
- Men, members of House of Representatives, 833
- Mental illness, number of U.S. adults with, 517
- Merry-go-round
- linear speed of horse on, 582
 - polar coordinates of horses on, 755
- Miles per gallon, 182
- Military, gay service members discharged from, 227–228
- Minimum product, 330
- Miscarriages, by age, 509
- Mixture problems, 180, 826, 846–848, 891, 917
- Modernistic painting consisting of geometric figures, 844
- Moiré patterns, 996
- Moon, weight of person on, 427
- Moth eggs, and abdominal width, 366

Mt. Rushmore sculpture, height of, 639
 Mountain, measuring height of, 555, 564, 734–735
 Movies
 ranking, 1118
 ticket prices, 145
 top ten Oscar-winning, 261
 Movie theater, finding best viewing angle in, 618, 634, 636

Multiple-choice test, 1110–1111, 1118, 1141
 Multiplier effect, 1086

Mumps, vaccination for, 383

Music

 amplitude and frequency of note's sine wave, 697
 amusia and, 670, 672
 modeling musical sounds, 642, 647

N

National debt, 20, 27–28, 31, 32
 National diversity index, 143
 Natural disaster relief, 885
 Nature, Fibonacci numbers found in, 1054
 Navajo sand painting, 554
 Navigation, 555. *See also* Bearings
 Neurons, human brain vs. gorilla brain, 70
 Newton's Law of Cooling, 511
 Nutritional content, 896, 916

O

Oculus Rift headset manufacturing costs, 384, 397–398
 Officers for Internet marketing consulting firm, choosing, 1112
 One-person households. *See* Living alone, number of Americans
 Online games, college students and, 834
 Orbit(s)
 of comets, 863, 978, 993, 997, 1044
 modeling, 1035
 perigee/apogee of satellite's orbit, 982
 of planets, 863, 977, 982
 Oscar-winning films, top ten, 261

P

Package, forces exerted on held, 791
 Pads, cost of, 1142
 Palindromic numbers, 1134
 Panic attack, heart rate before and during, 353
 Paragraph formation, 1118
 Parthenon at Athens, as golden rectangle, 46
 Passwords, 1118, 1119
 Pay. *See* Salary(-ies)
 Payroll spent in town, 1139
 PDA manufacturing costs and revenues, 865
 Pendulum swings, 1089
 Pens
 color choices, 1118
 cost of, 1142
 Per capita income and national happiness, 216
 Perceived length of time period and age, 428
 Perigee/apogee of satellite's orbit, 982
 Periodic rhythms, 702
 Pest-eradication program, 1090
 Pets
 insurance for, 200
 spending on, 1066
 pH
 of human mouth after eating sugar, 402
 pH scale, 492
 Phone calls between cities, 419, 428
 Phonograph records, angular speed and linear speed of, 535
 Photography. *See* Digital photography
 Physician visits, 201

Piano keyboard, Fibonacci numbers on, 1054
 Pier, finding length of, 734
 Planes. *See* Aircraft/airplanes
 Planets
 elliptical orbits, 977, 982
 modeling motion of, 1042, 1044
 Playground, dimensions of, 334
 Playing cards. *See* Deck of 52 cards, probability and
 Play production, break-even analysis of, 655
 Poker hands, 1116
 Pole, angle made by rope anchoring circus tent and, 655
 Political affiliation
 academic major and, 1134
 lack of, and age, 894
 Political identification, by college freshmen, 303
 Pollution
 air, 962
 removal costs, 70–71
 Pool table, elliptical, 1048
 Population
 Africa, 497
 Asia, 518
 Belgium, 124
 bird species in danger of extinction, 508
 Bulgaria, 507
 California, 491, 1088
 Canada, 511
 Colombia, 507
 elk, 518
 Europe, 865
 exponential growth modeling, 507, 508
 Florida, 878, 1139
 foreign-born (U.S.), 844
 geometric growth in, 1078
 Germany, 507, 517
 gray wolf, 446–447
 Greece, 124
 Hispanic, 516
 Hungary, 494
 India, 451, 507
 Iraq, 507
 Israel, 507
 Japan, 507
 Madagascar, 507
 Mexico, 508
 New Zealand, 508
 Nigeria, 510
 Pakistan, 507
 Palestinian, 507
 Philippines, 507
 Russia, 507
 in scientific notation, 26
 single, 221–222
 Texas, 491, 1089
 tigers, worldwide, 352
 Uganda, 511
 United States, 496–497
 age 65 and older, 510–511, 554
 by gender, 259
 percentage never married, ages 25–29, 217
 by race/ethnicity, 1069
 and walking speed, 503
 world, 147, 260, 494, 504–505, 508, 509, 517, 896–897, 1127
 Population projections, 46–47, 507
 Powerball, probability of winning, 1125–1126
 Price(s)
 advertising and, 424–425
 computer, 262, 511
 of a house, 181, 1091
 jeans, 260
 ticket, 145
 Price reductions, 115, 262, 1135
 Prisons, violent crime rate and imprisonment, 864
 Problem solving, time for, 425

Profit function, 831, 845–846, 865, 879

Profits

 department store branches, 259–260
 maximizing, 383, 885, 890, 891
 maximum daily, 882, 907
 maximum monthly, 885
 on newsprint/writing paper, 890
 total monthly, 885

Projectiles, paths of, 319, 1033–1034, 1049. *See also*
 Ball (height above ground); Free-falling
 object's position

Propeller

 of airplane, linear speed of, 651
 on wind generator, angular speed of, 651

Public transportation infrastructure, cost of maintaining, 217

R

Race, and marriage, 1051–1052
 Racial diversity, 143
 Racial prejudice, Implicit Association Test for, 48, 56
 Radiation intensity and distance of radiation machine, 427
 Radio(s), production/sales of, 831
 Radio show programming, 1118
 Radio stations
 call letters of, 1118
 locating illegal, 733
 Radio towers on coast, distance of ship from, 996
 Radio waves, simple harmonic motion of, 646
 Raffle prizes, 1118, 1119
 Railway crossing sign, length of arcs formed by cross on, 537
 Rain gutter cross-sectional area, 334
 Ramp
 computing work of pulling box along, 805
 force and weight of box being pulled along, 795
 magnitude of force required to keep object from sliding down, 795
 vector components of force on boat on tilted, 801, 805
 wheelchair, angle of elevation of, 646
 Rate of travel
 average rate and time traveled, 180
 average rate on a round-trip commute, 86
 and time for trip, 422
 walking speed and city population, 503
 Razor blades sold, 843
 Real-estate sales and prices (U.S.), 1091
 Records, angular speed and linear speed of, 535
 Rectangle
 area of, 47
 dimensions of, 245, 417, 860–861, 863, 864, 889, 891, 892, 960, 1091
 dimensions of, maximizing enclosed area, 331
 golden, 46
 perimeter of, 47, 86
 Rectangular box dimensions, 366
 Rectangular solid, volume of, 56
 Redwood trees, finding height of, 734
 Reflections, 244
 Relativity theory, space exploration and, 32, 44, 47, 1168
 Religious affiliation, first-year U.S. college students claiming no, 164–167
 Rentals
 car, 126–127, 130
 truck, 140, 1051
 Residential community costs, adult, 1065, 1072
 Resistance, electrical, 86, 428, 1142
 Respiratory rate
 and age of children, 876
 Restaurant tables and maximum occupancy, 834
 Resultant forces, 795, 796, 811, 812

Revenue functions. *See* Cost and revenue functions/break-even points

Reversibility of thought, 57

Rivers, removing pollution from, 70–71

Roller coasters, 1187

Rolling motion, 1031

Roof of A-frame cabin, finding length of, 810

Rotating beam of light, distance from point, 615, 616

Roulette wheel, independent events on, 1131

Rowing, 834

Royal flush (poker hand), probability of, 1118

Rug's length and width, 863

Runner, speed of, 291

Runner's pulse, 493

S

Sailing angle to 10-knot wind, sailing speed and, 755, 766

Salary(-ies)

anticipated earnings, 1089

of baseball players, 143

comparing, 1074–1075

earnings with overtime, 518

educational level and, 111–112, 123

first job after college, 300

increases in average, 305

lifetime computation, 1082, 1089

of nurses, 143

salesperson's earnings/commissions, 1142

in sixth year, 1139

total, 1075, 1089, 1139, 1141

total weekly earnings, 885

wage gap in, by gender, 180

for women, 894

Sale prices, 68. *See also* Price reductions

Salesperson's earnings, 1142

Sales volume/figures

brick-and-mortar stores vs. E-commerce, 516

price/advertising and, 424–425

real estate, 1091

theater ticket, 843

video games, 494

Sand painting, 554

Satellite, perigee/apogee of orbit, 982

Satellite dish, placement of receiver for, 1049

Savings

and compound interest, 491–492

geometric sequencing, 1088, 1089

needed for health-care expenses during retirement, 509

total, 1089

Scattering experiments, 996

Scheduling appearances, ways of, 1118, 1119

Semielliptical archway and truck clearance, 978, 981, 997, 1013, 1048

Shaded region areas, 56, 68

Shading process, 1090

Shadow, length of, 794

Shipping costs. *See* Mailing costs

Ships. *See* Boats/ships

Ship tracking system, 863

"Shortest time" problems, 1031

Shot put

angle and height of, 333–334

throwing distance, 691, 734

Shower, water used when taking, 421, 997

Simple harmonic motion, 813, 1142

ball attached to spring, 640–641, 716, 720

earthquake, 643

modeling, 640–643, 646, 654, 655

radio waves, 646

tuning fork, 646

Skeletons, carbon-14 dating of, 508

Skydiver's fall, 421–422, 1091

Sled, pulling

computing work of, 804

forces exerted, 794

Sleep

and age, 161

coffee consumption and, 519

college students and, 834

death rate and hours of, 836, 840

hours of, lifetime, 123

hours of, on typical night, 1120

Smoking

deaths and disease incidence ratios, 403, 1106–1107

and heart disease, 404

Snow, water supply from melting, 436

Social media use, and age, 888

Social Security, 31

Soda consumption, modeling, 282–283

Soft drink can, surface area of, 289–290

Solar energy industry, number of U.S. jobs in, 517

Sonic boom, hyperbolic shape of, 993

Sound

amplitude and frequency of, 697

locating explosion by arrival of, 997

from touching buttons on touch-tone phone, 693, 699

Sound intensity, 204, 428, 467, 478, 509, 518, 697

Sound quality, amusia and, 670, 672

Space flight/travel

aging rate and, 32, 44, 47

Freedom 7 spacecraft, 272

relativity theory and, 32, 44, 47

Spaceguard Survey, 997

Speed. *See also* Rate of travel

angular, 534, 651

linear, 538

of airplane propeller, 651

of animals on carousel, 534, 538

of wind machine propeller, 535

Mach speed of aircraft, 691

Speed skating, winning time for, 307

Spinner, probability of pointer landing in specific way, 1129, 1133, 1140, 1141

Spring(s)

force required to stretch, 427

simple harmonic motion of object attached to, 640–643

ball, 640–641, 716, 720

distance from rest position, 644, 654

frequency of, 644

maximum displacement of, 644

phase shift of motion, 644

time required for one cycle, 644

Stadium seats, 1075

Standbys for airline seats, 1118

Statue of Liberty, distance of ship from base of, 645

Stomach acid, pH of, 492

Stonehenge, raising stones of, 567

Stopping distances

for cars, 406, 416–417

for trucks, 417

Stress levels, and leisure time, 332

String length and frequency, 424

Strontium-90, 499

Student government elections, 1114

Student loan debt, 70

Students, probability of selecting specific, 1141

Study, hours per week, 842–843

Success, attractiveness and, 148

Sun, finding angle of elevation of, 563, 566–567, 583, 646, 652

Sunlight, intensity beneath ocean's surface, 491

Sunscreen, exposure time without burning and, 2

Supply and demand, 831–832

Supply-side economics, 367

Surface sunlight, intensity beneath ocean's surface, 491

Surveying

bearings in, 639–640

to find distance between two points on opposite banks of river, 733

Sushi, population who won't try, 509

Suspension bridges, parabolas formed by, 1049

Synthesizers, musical sounds modeled by, 636, 642

T

Table tennis top, dimensions of, 889

Talent contest, picking winner and runner-up in, 1119

Target, probability of hitting, 1134

Target heart rate for exercise, 18

Task mastery, 478, 515

Tax code, U.S., increase in number of pages in, 453

Taxes

age and percentage of Americans receiving a refund, 46

cigarette, 1063

federal tax rate schedule for tax owed, 200

income, 181

rebate and multiplier effect, 1086, 1090

tax rate percentage and revenue, 367

Taxi rates, 124

Teacher's aide, hourly pay for, 884

Teenage drug use, 510

Telephone(s)

calls between cities, 419, 428

land lines vs. cell phones, 832

rates for international calls, 125

sound from touching buttons on, 693, 699

Telephone numbers

seven-digit, 1141

total possible, in United States, 1111

Telephone plans

cellular, 181

per-minute costs, 192–193, 199

Telephone pole

angle between guy wire and, 566

tilted, finding length of, 734

Television

lifetime hours spent watching, 123

manufacturing profits and constraints, 884

programming of movies, 1118

sale prices, 68

screen dimensions, 121, 863

Temperature

average monthly, 602, 603

body, variation in, 653

of cooling cup of coffee, 514

degree-days, 1076

and depth of water, 427

in enclosed vehicle, increase in, 463–464

Fahrenheit-Celsius interconversions, 17

global warming, 149, 211–213

home temperature as function of time, 244–245

increase in an enclosed vehicle, 509

as magnitude, 782

Newton's Law of Cooling, 511

time-temperature flu scenario, 182–183

Text messaging plans, modeling costs of, 283–285

Theater attendance, maximizing revenue from, 885

Theater seats, 1075

Theater ticket sales, 843

Throwing distance, 681, 691

angle of elevation of, 716, 721

maximum height of thrown ball, 813
 shot put, 691, 734
 Ticket prices/sales, 145, 843
 Tides, behavior of, 539, 548, 553
 modeling cycle of, 599
 modeling water depth and, 602
 Tigers, worldwide population, 352
 Time, perceived length of, 428
 Time traveled, average rate and, 180
 Tobacco use. *See* Smoking
 Tolls, 110, 114–115, 124, 136, 137, 140, 260
 Touch-tone phone, sounds from touching buttons
 on, 693, 699
 Tower
 angle of elevation between point on ground
 and top of, 655, 781
 height of, finding, 637, 645, 646, 781
 length of two guy wires anchoring, 743
 Traffic control, 908, 912–917, 963
 Trains leaving station at same time, distance
 between, 768
 Transformations of an image, 927–928, 930, 963
 Travel, modes of, 270
 Travel cups, disposable vs., 617
 Tree(s)
 Amazon deforestation, 433
 annual yield of fruit/nut, 307, 335, 433
 finding height of, 768
 Triangle
 area of, 730, 740, 959
 dimensions of, 932, 946, 982, 1035, 1135
 isosceles, 834–835
 oblique, 730
 right, 854
 Triangular piece of land
 cost of, 744, 810
 length of sides of, 810
 Trucks
 clearance under semielliptical archway, 978,
 981, 997, 1013, 1048
 rental costs, 140, 1051
 stopping distances required for, 417
 Tugboats towing ship, resultant force of two,
 795, 796
 Tuning fork
 eardrum vibrations from, 680
 simple harmonic motion on, 646
 Tutoring, hourly pay for, 884
 TV. *See* Television

V

Vacation, yearly spending on, 31
 Vacation lodgings, 876
 Vacation plan packages, cost of, 889
 Vaccine
 mixture for flu, 180, 826–828
 for mumps, 383
 Value of an annuity, 1089, 1139
 Van, groups fitting into, 1118
 Vehicles, electric, 1138
 Velocity vector
 of boat, 792
 of plane, 792
 of wind, 791, 792, 795–796
 Video games, retail sales of, 494
 Videos rented, number of one-day and
 three-day, 805
 Violent crime, imprisonment and, 864
 Violin string, length and frequency of, 424
 Virtual reality headset manufacturing costs, 384,
 397–398
 Vitamin content, 896, 916
 Volume
 of carry-on luggage, 380–381
 of gas, 1195
 of sound (*See* Sound intensity)
 Voters, age and gender of, 931

W

Wage gap, 180
 Wages. *See* Salary(-ies)
 Wagon, computing work of pulling, 803, 805, 812
 Walking speed and city population, 503
 Wardrobe selection, 1109
 Washington Monument, angle of elevation to
 top of, 566
 Water
 pressure and depth, 419–420
 temperature and depth, 427
 used in a shower, 421, 997
 Water pipe diameter, number of houses served
 and size of, 427
 Water supply, from melting snow, 436
 Water wheel, linear speed of, 538
 Weight
 blood volume and body, 420–421
 of elephant, 493

 of great white shark, cube of its length and, 422
 healthy, for height and age, 877
 and height recommendations/calculations, 427
 of person on Moon, 427
 Weightlifting, 510, 797, 806
 Wheelchair business
 manufacturing costs, 398
 profit function for, 846
 revenue and cost functions for, 824–825
 Wheelchair ramp, angle of elevation of, 646
 Wheel rotation, centimeters moved with, 537
 Whispering gallery, 977, 982, 1050
 White House, rooms, bathrooms, fireplaces and
 elevators in, 907
 Wildfires, area burned by, 353
 Wind
 airplane speed and direction of, 828–829, 834
 velocity vector of, 791, 792, 795–796
 Wind force, 428
 Wind generator
 angular speed of propeller on, 651
 linear speed of propeller of, 535
 Wind pressure, 428
 Wine consumption, heart disease and, 216–217
 Wing span of jet fighter, finding, 735
 Witch of Agnesi, 1034
 Women. *See also* Gender
 average level of happiness at different times
 of day, 271
 college students, 824
 members of House of Representatives, 833
 in U.S. workforce, 894
 Work, 803–805
 crane lifting boulder, 805
 dragging crate, 812
 pulling box up ramp, 805
 pulling wagon, 803, 805, 812
 pushing car, 803, 805
 of weightlifter, 797, 806
 Writing pads, cost of, 1142

X

Xanax, half-life of, 508, 1035

Y

Yacht, dividing cost of a, 121–122

This page intentionally left blank

A baby wearing a black graduation cap and a white gown is the central figure. The baby is holding a one-dollar bill in their right hand and looking upwards. The background is white, and numerous one-dollar bills are floating or falling around the baby, creating a sense of abundance or financial focus.

PREREQUISITES

P

Fundamental Concepts of Algebra

- What can algebra possibly have to tell me about
- the skyrocketing cost of a college education?
 - student-loan debt?
 - my workouts?
 - the effects of alcohol?
 - the meaning of the national debt that is more than \$25 trillion?
 - time dilation on a futuristic high-speed journey to a nearby star?
 - racial bias?
 - ethnic diversity in the United States?
 - the widening imbalance between numbers of women and men on college campuses?

This chapter reviews fundamental concepts of algebra that are prerequisites for the study of college algebra. Throughout the chapter, you will see how the special language of algebra describes your world.

Here's where you'll find these applications:

- College costs: Section P.1, Example 2; Exercise Set P.1, Exercises 131–132
- Student-loan debt: Mid-Chapter Check Point, Exercise 42
- Workouts: Exercise Set P.1, Exercises 129–130
- The effects of alcohol: Blitzer Bonus beginning on page 15
- The national debt: Section P.2, Example 6
- Time dilation: Blitzer Bonus on page 44
- Racial bias: Exercise Set P.4, Exercises 91–92
- U.S. ethnic diversity: Chapter P Review, Exercise 23
- College gender imbalance: Chapter P Test, Exercise 32.

SECTION P.1

Algebraic Expressions, Mathematical Models, and Real Numbers

WHAT YOU'LL LEARN

- 1 Evaluate algebraic expressions.
- 2 Use mathematical models.
- 3 Find the intersection of two sets.
- 4 Find the union of two sets.
- 5 Recognize subsets of the real numbers.
- 6 Use inequality symbols.
- 7 Evaluate absolute value.
- 8 Use absolute value to express distance.
- 9 Identify properties of the real numbers.
- 10 Simplify algebraic expressions.

How would your lifestyle change if a gallon of gas cost \$9.15? Or if the price of a staple such as milk was \$15? That's how much those products would cost if their prices had increased at the same rate college tuition has increased since 1980. (Source: Center for College Affordability and Productivity) In this section, you will learn how the special language of algebra describes your world, including the skyrocketing cost of a college education.



Algebraic Expressions

Algebra uses letters, such as x and y , to represent numbers. If a letter is used to represent various numbers, it is called a **variable**. For example, imagine that you are basking in the sun on the beach. We can let x represent the number of minutes that you can stay in the sun without burning with no sunscreen. With a number 6 sunscreen, exposure time without burning is six times as long, or 6 times x . This can be written $6 \cdot x$, but it is usually expressed as $6x$. Placing a number and a letter next to one another indicates multiplication.

Notice that $6x$ combines the number 6 and the variable x using the operation of multiplication. A combination of variables and numbers using the operations of addition, subtraction, multiplication, or division, as well as powers or roots, is called an **algebraic expression**. Here are some examples of algebraic expressions:

$$x + 6, \quad x - 6, \quad 6x, \quad \frac{x}{6}, \quad 3x + 5, \quad x^2 - 3, \quad \sqrt{x} + 7.$$

Many algebraic expressions involve *exponents*. For example, the algebraic expression

$$-x^2 + 361x + 3193$$

approximates the average cost of tuition and fees at public U.S. colleges for the school year ending x years after 2000. The expression x^2 means $x \cdot x$ and is read “ x to the second power” or “ x squared.” The exponent, 2, indicates that the base, x , appears as a factor two times. The negative sign in front of x^2 indicates that x^2 is multiplied by -1 .

Exponential Notation

If n is a counting number (1, 2, 3, and so on),

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{b \text{ appears as a factor } n \text{ times.}}$$

Exponent or Power

Base

b^n is read “the n th power of b ” or “ b to the n th power.” Thus, the n th power of b is defined as the product of n factors of b . The expression b^n is called an **exponential expression**. Furthermore, $b^1 = b$.

For example,

$$8^2 = 8 \cdot 8 = 64, \quad 5^3 = 5 \cdot 5 \cdot 5 = 125, \quad \text{and} \quad 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16.$$

1 Evaluate algebraic expressions.

Evaluating Algebraic Expressions

Evaluating an algebraic expression means to find the value of the expression for a given value of the variable.

Many algebraic expressions involve more than one operation. Evaluating an algebraic expression without a calculator involves carefully applying the following order of operations agreement:

The Order of Operations Agreement

1. Perform operations within the innermost parentheses and work outward. If the algebraic expression involves a fraction, treat the numerator and the denominator as if they were each enclosed in parentheses.
2. Evaluate all exponential expressions.
3. Perform multiplications and divisions **as they occur**, working **from left to right**.
4. Perform additions and subtractions **as they occur**, working **from left to right**.

EXAMPLE 1 Evaluating an Algebraic Expression

Evaluate $7 + 5(x - 4)^3$ for $x = 6$.

Solution

$$\begin{aligned}
 7 + 5(x - 4)^3 &= 7 + 5(6 - 4)^3 && \text{Replace } x \text{ with } 6. \\
 &= 7 + 5(2)^3 && \text{First work inside parentheses: } 6 - 4 = 2. \\
 &= 7 + 5(8) && \text{Evaluate the exponential expression: } 2^3 = 2 \cdot 2 \cdot 2 = 8. \\
 &= 7 + 40 && \text{Multiply: } 5(8) = 40. \\
 &= 47 && \text{Add.}
 \end{aligned}$$

✓ CHECK POINT 1 Evaluate $8 + 6(x - 3)^2$ for $x = 13$.

2 Use mathematical models.

Formulas and Mathematical Models

An **equation** is formed when an equal sign is placed between two algebraic expressions. One aim of algebra is to provide a compact, symbolic description of the world. These descriptions involve the use of *formulas*. A **formula** is an equation that uses variables to express a relationship between two or more quantities.

Here are two examples of formulas related to heart rate and exercise.



Couch-Potato Exercise

$$H = \frac{1}{5}(220 - a)$$

Heart rate, in beats per minute,

is

$\frac{1}{5}$ of

the difference between 220 and your age.



Working It

$$H = \frac{9}{10}(220 - a)$$

Heart rate, in beats per minute,

is

$\frac{9}{10}$ of

the difference between 220 and your age.

The process of finding formulas to describe real-world phenomena is called **mathematical modeling**. Such formulas, together with the meaning assigned to the variables, are called **mathematical models**. We often say that these formulas model, or describe, the relationships among the variables.

EXAMPLE 2 Modeling the Cost of Attending a Public College

The bar graph in **Figure P.1** shows the average cost of tuition and fees for public four-year colleges, adjusted for inflation. The formula

$$T = -x^2 + 361x + 3193$$

models the average cost of tuition and fees, T , for public U.S. colleges for the school year ending x years after 2000.

- Use the formula to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2020.
- By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

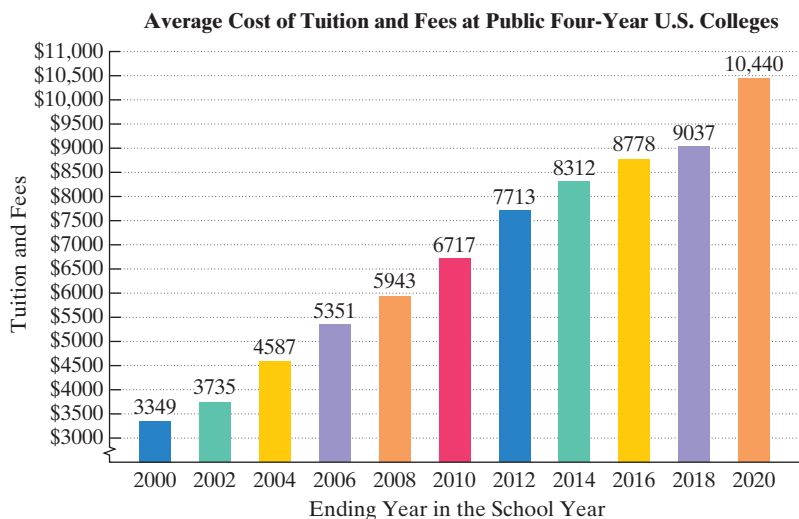


Figure P.1
Source: The College Board

Solution

- Because 2020 is 20 years after 2000, we substitute 20 for x in the given formula. Then we use the order of operations to find T , the average cost of tuition and fees for the school year ending in 2020.

$$T = -x^2 + 361x + 3193$$

This is the given mathematical model.

$$T = -(20)^2 + 361(20) + 3193$$

Replace each occurrence of x with 20.

$$T = -(400) + 361(20) + 3193$$

Evaluate the exponential expression:

$$20^2 = 20 \cdot 20 = 400.$$

$$T = -400 + 7220 + 3193$$

Multiply from left to right: $-(400) = -1(400)$
 $= -400$ and $361(20) = 7220$.

$$T = 10,013$$

Add.

The formula indicates that for the school year ending in 2020, the average cost of tuition and fees at public U.S. colleges was \$10,013.

- Figure P.1** shows that the average cost of tuition and fees for the school year ending in 2020 was \$10,440.

The cost obtained from the formula, \$10,013, underestimates the actual data value by $\$10,440 - \$10,013$, or by \$427.

Only 20 is affected by the exponent. Square 20 and copy the negative.

BLITZER BONUS**Is College Worthwhile?**

“Questions have intensified about whether going to college is worthwhile,” says *Education Pays*, released by the College Board Advocacy & Policy Center. “For the typical student, the investment pays off very well over the course of a lifetime, even considering the expense.”

Among the findings in *Education Pays*:

- Median (middlemost) full-time earnings with a bachelor’s degree in 2018 were \$65,400, which is \$24,900 more than high school graduates.
- Compared with a high school graduate, a four-year college graduate who enrolled in a public university at age 18 will break even by age 33. The college graduate will have earned enough by then to compensate for being out of the labor force for four years and for borrowing enough to pay tuition and fees, shown in **Figure P.1**.

> DISCOVERY

Using the formula from Example 2 and Check Point 2, find T for $x = 100$, $x = 200$, $x = 300$, and $x = 400$. What happens to the values of T over time? Do you see how model breakdown has occurred?

> GREAT QUESTION !

Can I use symbols other than braces when writing sets using the roster method?

No. Grouping symbols such as parentheses, $()$, and square brackets, $[]$, are not used to represent sets in the roster method. Furthermore, only commas are used to separate the elements of a set. Separators such as colons or semicolons are not used.

- 3** Find the intersection of two sets.

✓ CHECK POINT 2

- Use the formula $T = -x^2 + 361x + 3193$, described in Example 2, to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2016.
- By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

Sometimes a mathematical model gives an estimate that is not a good approximation or is extended to include values of the variable that do not make sense. In these cases, we say that **model breakdown** has occurred. For example, it is not likely that the formula in Example 2 would give a good estimate of tuition and fees in 2050 because it is too far in the future. Thus, model breakdown would occur.

Sets

Before we describe the set of real numbers, let’s be sure you are familiar with some basic ideas about sets. A **set** is a collection of objects whose contents can be clearly determined. The objects in a set are called the **elements** of the set. For example, the set of numbers used for counting can be represented by

$$\{1, 2, 3, 4, 5, \dots\}.$$

The braces, $\{ \}$, indicate that we are representing a set. This form of representation, called the **roster method**, uses commas to separate the elements of the set. The symbol consisting of three dots after the 5, called an *ellipsis*, indicates that there is no final element and that the listing goes on forever.

A set can also be written in **set-builder notation**. In this notation, the elements of the set are described but not listed. Here is an example:

$$\{x | x \text{ is a counting number less than } 6\}.$$

The set of all x

such that

x is a counting number less than 6.

The same set written using the roster method is

$$\{1, 2, 3, 4, 5\}.$$

If A and B are sets, we can form a new set consisting of all elements that are in both A and B . This set is called the *intersection* of the two sets.

Definition of the Intersection of Sets

The **intersection** of sets A and B , written $A \cap B$, is the set of elements common to both set A and set B . This definition can be expressed in set-builder notation as follows:

$$A \cap B = \{x | x \text{ is an element of } A \text{ AND } x \text{ is an element of } B\}.$$

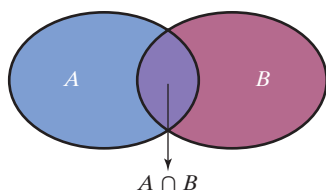


Figure P.2 Picturing the intersection of two sets

Figure P.2 shows a useful way of picturing the intersection of sets A and B . The figure indicates that $A \cap B$ contains those elements that belong to both A and B at the same time.

EXAMPLE 3 Finding the Intersection of Two Sets

Find the intersection: $\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\}$.

Solution The elements common to $\{7, 8, 9, 10, 11\}$ and $\{6, 8, 10, 12\}$ are 8 and 10. Thus,

$$\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\} = \{8, 10\}.$$

CHECK POINT 3 Find the intersection: $\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\}$.

If a set has no elements, it is called the **empty set**, or the **null set**, and is represented by the symbol \emptyset . Here is an example that shows how the empty set can result when finding the intersection of two sets:

$$\{2, 4, 6\} \cap \{3, 5, 7\} = \emptyset.$$

These sets have no common elements.

Their intersection has no elements and is the empty set.

4 Find the union of two sets.

Another set that we can form from sets A and B consists of elements that are in A or B or in both sets. This set is called the *union* of the two sets.

Definition of the Union of Sets

The **union** of sets A and B , written $A \cup B$, is the set of elements that are members of set A or of set B or of both sets. This definition can be expressed in set-builder notation as follows:

$$A \cup B = \{x | x \text{ is an element of } A \text{ OR } x \text{ is an element of } B\}.$$

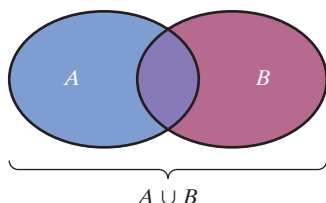


Figure P.3 Picturing the union of two sets

Figure P.3 shows a useful way of picturing the union of sets A and B . The figure indicates that $A \cup B$ is formed by joining the sets together.

We can find the union of set A and set B by listing the elements of set A . Then we include any elements of set B that have not already been listed. Enclose all elements that are listed with braces. This shows that the union of two sets is also a set.

EXAMPLE 4 Finding the Union of Two Sets

Find the union: $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$.

Solution To find $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$, start by listing all the elements from the first set, namely, 7, 8, 9, 10, and 11. Now list all the elements from the second set that are not in the first set, namely, 6 and 12. The union is the set consisting of all these elements. Thus,

$$\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\} = \{6, 7, 8, 9, 10, 11, 12\}.$$

Although 8 and 10 appear in both sets,

do not list 8 and 10 twice.

CHECK POINT 4 Find the union: $\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\}$.

> GREAT QUESTION !

How can I use the words *union* and *intersection* to help me distinguish between these two operations?

Union, as in a marriage union, suggests joining things or uniting them. Intersection, as in the intersection of two crossing streets, brings to mind the area common to both, suggesting things that overlap.

5 Recognize subsets of the real numbers.

The Set of Real Numbers

The sets that make up the real numbers are summarized in **Table P.1**. We refer to these sets as **subsets** of the real numbers, meaning that all elements in each subset are also elements in the set of real numbers.

Table P.1 Important Subsets of the Real Numbers

Name/Symbol	Description	Examples
Natural numbers \mathbb{N}	$\{1, 2, 3, 4, 5, \dots\}$ These are the numbers that we use for counting.	2, 3, 5, 17
Whole numbers \mathbb{W}	$\{0, 1, 2, 3, 4, 5, \dots\}$ The set of whole numbers includes 0 and the natural numbers.	0, 2, 3, 5, 17
Integers \mathbb{Z}	$\{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$ The set of integers includes the negatives of the natural numbers and the whole numbers.	-17, -5, -3, -2, 0, 2, 3, 5, 17
Rational numbers \mathbb{Q}	$\left\{\frac{a}{b} \mid a \text{ and } b \text{ are integers and } b \neq 0\right\}$ <div>This means that b is not equal to zero.</div> The set of rational numbers is the set of all numbers that can be expressed as a quotient of two integers, with the denominator not 0. Rational numbers can be expressed as terminating or repeating decimals.	$-17 = \frac{-17}{1}$, $-5 = \frac{-5}{1}$, $-3, -2,$ $0, 2, 3, 5, 17,$ $\frac{2}{5} = 0.4,$ $\frac{-2}{3} = -0.6666\dots = -0.\overline{6}$
Irrational numbers \mathbb{I}	The set of irrational numbers is the set of all numbers whose decimal representations are neither terminating nor repeating. Irrational numbers cannot be expressed as a quotient of integers.	$\sqrt{2} \approx 1.414214$ $-\sqrt{3} \approx -1.73205$ $\pi \approx 3.142$ $-\frac{\pi}{2} \approx -1.571$

Notice the use of the symbol \approx in the examples of irrational numbers. The symbol means “is approximately equal to.” Thus,

$$\sqrt{2} \approx 1.414214.$$

We can verify that this is only an approximation by multiplying 1.414214 by itself. The product is very close to, but not exactly, 2:

$$1.414214 \times 1.414214 = 2.000001237796.$$

Not all square roots are irrational. For example, $\sqrt{25} = 5$ because $5^2 = 5 \cdot 5 = 25$. Thus, $\sqrt{25}$ is a natural number, a whole number, an integer, and a rational number ($\sqrt{25} = \frac{5}{1}$).

The set of *real numbers* is formed by taking the union of the sets of rational numbers and irrational numbers. Thus, every real number is either rational or irrational, as shown in **Figure P.4**.

TECHNOLOGY

A calculator with a square root key gives a decimal approximation for $\sqrt{2}$, not the exact value.

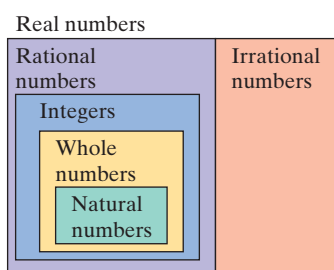


Figure P.4 Every real number is either rational or irrational.

Real Numbers

The set of **real numbers** is the set of numbers that are either rational or irrational:

$$\{x \mid x \text{ is rational or } x \text{ is irrational}\}.$$

The symbol \mathbb{R} is used to represent the set of real numbers. Thus,

$$\mathbb{R} = \{x \mid x \text{ is rational}\} \cup \{x \mid x \text{ is irrational}\}.$$

EXAMPLE 5 Recognizing Subsets of the Real Numbers

Consider the following set of numbers:

$$\left\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\right\}.$$

List the numbers in the set that are

- | | | |
|----------------------|------------------------|------------------|
| a. natural numbers. | b. whole numbers. | c. integers. |
| d. rational numbers. | e. irrational numbers. | f. real numbers. |

Solution

- a. Natural numbers: The natural numbers are the numbers used for counting. The only natural number in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ is $\sqrt{81}$ because $\sqrt{81} = 9$. (9 multiplied by itself, or 9^2 , is 81.)
- b. Whole numbers: The whole numbers consist of the natural numbers and 0. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are whole numbers are 0 and $\sqrt{81}$.
- c. Integers: The integers consist of the natural numbers, 0, and the negatives of the natural numbers. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are integers are $\sqrt{81}$, 0, and -7 .
- d. Rational numbers: All numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that can be expressed as the quotient of integers are rational numbers. These include -7 ($-7 = \frac{-7}{1}$), $-\frac{3}{4}$, 0 ($0 = \frac{0}{1}$), and $\sqrt{81}$ ($\sqrt{81} = \frac{9}{1}$). Furthermore, all numbers in the set that are terminating or repeating decimals are also rational numbers. These include $0.\overline{6}$ and 7.3 .
- e. Irrational numbers: The irrational numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are $\sqrt{5}$ ($\sqrt{5} \approx 2.236$) and π ($\pi \approx 3.14$). Both $\sqrt{5}$ and π are only approximately equal to 2.236 and 3.14, respectively. In decimal form, $\sqrt{5}$ and π neither terminate nor have blocks of repeating digits.
- f. Real numbers: All the numbers in the given set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are real numbers.

**✓ CHECK POINT 5** Consider the following set of numbers:

$$\left\{-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}.$$

List the numbers in the set that are

- | | | |
|----------------------|------------------------|------------------|
| a. natural numbers. | b. whole numbers. | c. integers. |
| d. rational numbers. | e. irrational numbers. | f. real numbers. |

The Real Number Line

The **real number line** is a graph used to represent the set of real numbers. An arbitrary point, called the **origin**, is labeled 0. Select a point to the right of 0 and label it 1. The distance from 0 to 1 is called the **unit distance**. Numbers to the right of the origin are **positive** and numbers to the left of the origin are **negative**. The real number line is shown in **Figure P.5**.

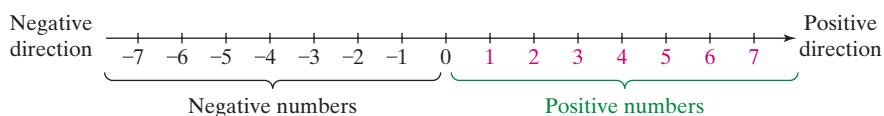
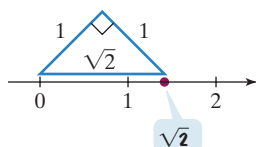


Figure P.5 The real number line

> GREAT QUESTION !

How did you locate $\sqrt{2}$ as a precise point on the number line in Figure P.6?

We used a right triangle with two legs of length 1. The remaining side has a length measuring $\sqrt{2}$.



We'll have lots more to say about right triangles later in the book.

Real numbers are **graphed** on a number line by placing a dot at the correct location for each number. The integers are easiest to locate. In **Figure P.6**, we've graphed six rational numbers and three irrational numbers on a real number line.

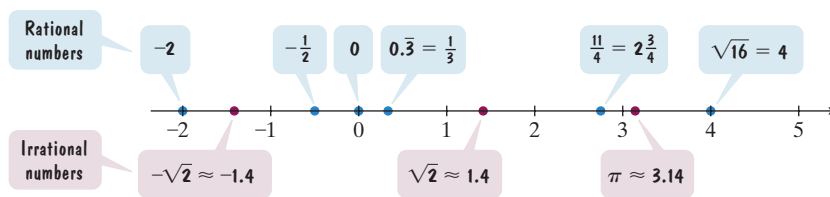


Figure P.6 Graphing numbers on a real number line

Every real number corresponds to a point on the number line and every point on the number line corresponds to a real number. We say that there is a **one-to-one correspondence** between all the real numbers and all points on a real number line.

6 Use inequality symbols.

Ordering the Real Numbers

On the real number line, the real numbers increase from left to right. The lesser of two real numbers is the one farther to the left on a number line. The greater of two real numbers is the one farther to the right on a number line.

Look at the number line in **Figure P.7**. The integers -4 and -1 are graphed.

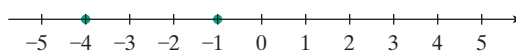


Figure P.7

Observe that -4 is to the left of -1 on the number line. This means that -4 is less than -1 .

$$-4 < -1$$

-4 is less than -1 because -4 is to the left of -1 on the number line.

In **Figure P.7**, we can also observe that -1 is to the right of -4 on the number line. This means that -1 is greater than -4 .

$$-1 > -4$$

-1 is greater than -4 because -1 is to the right of -4 on the number line.

The symbols $<$ and $>$ are called **inequality symbols**. These symbols always point to the lesser of the two real numbers when the inequality statement is true.

-4 is less than -1 .

$$-4 < -1$$

The symbol points to -4 , the lesser number.

-1 is greater than -4 .

$$-1 > -4$$

The symbol still points to -4 , the lesser number.

The symbols $<$ and $>$ may be combined with an equal sign, as shown in the following table:

	Symbols	Meaning	Examples	Explanation
This inequality is true if either the $<$ part or the $=$ part is true.	$a \leq b$	a is less than or equal to b .	$2 \leq 9$ $9 \leq 9$	Because $2 < 9$ Because $9 = 9$
	$b \geq a$	b is greater than or equal to a .	$9 \geq 2$ $2 \geq 2$	Because $9 > 2$ Because $2 = 2$

7 Evaluate absolute value.

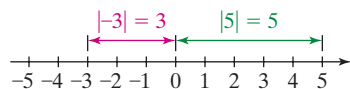


Figure P.8 Absolute value as the distance from 0

Absolute Value

The **absolute value** of a real number a , denoted by $|a|$, is the distance from 0 to a on the number line. **This distance is always taken to be nonnegative.** For example, the real number line in **Figure P.8** shows that

$$|-3| = 3 \quad \text{and} \quad |5| = 5.$$

The absolute value of -3 is 3 because -3 is 3 units from 0 on the number line. The absolute value of 5 is 5 because 5 is 5 units from 0 on the number line. The absolute value of a positive real number or 0 is the number itself. The absolute value of a negative real number, such as -3 , is the number without the negative sign.

We can define the absolute value of the real number x without referring to a number line. The algebraic definition of the absolute value of x is given as follows:

Definition of Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

If x is nonnegative (that is, $x \geq 0$), the absolute value of x is the number itself. For example,

$$|5| = 5 \quad |\pi| = \pi \quad \left|\frac{1}{3}\right| = \frac{1}{3} \quad |0| = 0.$$

Zero is the only number whose absolute value is 0.

If x is a negative number (that is, $x < 0$), the absolute value of x is the opposite of x . This makes the absolute value positive. For example,

$$|-3| = -(-3) = 3 \quad |-\pi| = -(-\pi) = \pi \quad \left|-\frac{1}{3}\right| = -\left(-\frac{1}{3}\right) = \frac{1}{3}.$$

This middle step is usually omitted.

Observe that **the absolute value of any nonzero number is always positive.**

EXAMPLE 6 Evaluating Absolute Value

Rewrite each expression without absolute value bars:

a. $|\sqrt{3} - 1|$ b. $|2 - \pi|$ c. $\frac{|x|}{x}$ if $x < 0$.

Solution

- a. Because $\sqrt{3} \approx 1.7$, the number inside the absolute value bars, $\sqrt{3} - 1$, is positive. The absolute value of a positive number is the number itself. Thus,

$$|\sqrt{3} - 1| = \sqrt{3} - 1.$$

- b. Because $\pi \approx 3.14$, the number inside the absolute value bars, $2 - \pi$, is negative. The absolute value of x when $x < 0$ is $-x$. Thus,

$$|2 - \pi| = -(2 - \pi) = \pi - 2.$$

- c. If $x < 0$, then $|x| = -x$. Thus,

$$\frac{|x|}{x} = \frac{-x}{x} = -1.$$

✓ CHECK POINT 6 Rewrite each expression without absolute value bars:

a. $|1 - \sqrt{2}|$ b. $|\pi - 3|$ c. $\frac{|x|}{x}$ if $x > 0$.

DISCOVERY

Verify the triangle inequality if $a = 4$ and $b = 5$. Verify the triangle inequality if $a = 4$ and $b = -5$.

When does equality occur in the triangle inequality and when does inequality occur? Verify your observation with additional number pairs.

8 Use absolute value to express distance.

Listed below are several basic properties of absolute value. Each of these properties can be derived from the definition of absolute value.

Properties of Absolute Value

For all real numbers a and b ,

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $a \leq |a|$
4. $|ab| = |a||b|$
5. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \quad b \neq 0$
6. $|a + b| \leq |a| + |b|$ (called the triangle inequality).

Distance Between Points on a Real Number Line

Absolute value is used to find the distance between two points on a real number line. If a and b are any real numbers, the **distance between a and b** is the absolute value of their difference. For example, the distance between 4 and 10 is 6. Using absolute value, we find this distance in one of two ways:

$$|10 - 4| = |6| = 6 \quad \text{or} \quad |4 - 10| = |-6| = 6.$$

The distance between 4 and 10 on the real number line is 6.

Notice that we obtain the same distance regardless of the order in which we subtract.

Distance Between Two Points on the Real Number Line

If a and b are any two points on a real number line, then the distance between a and b is given by

$$|a - b| \quad \text{or} \quad |b - a|,$$

where $|a - b| = |b - a|$.

EXAMPLE 7 Distance Between Two Points on a Number Line

Find the distance between -5 and 3 on the real number line.

Solution Because the distance between a and b is given by $|a - b|$, the distance between -5 and 3 is

$$|-5 - 3| = |-8| = 8.$$

$$a = -5 \quad b = 3$$

Figure P.9 verifies that there are 8 units between -5 and 3 on the real number line. We obtain the same distance if we reverse the order of the subtraction:

$$|3 - (-5)| = |8| = 8.$$

CHECK POINT 7 Find the distance between -4 and 5 on the real number line.

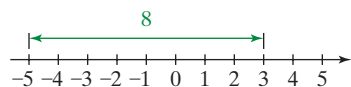


Figure P.9 The distance between -5 and 3 is 8.

9 Identify properties of the real numbers.

Properties of Real Numbers and Algebraic Expressions

When you use your calculator to add two real numbers, you can enter them in any order. The fact that two real numbers can be added in any order is called the **commutative property of addition**. You probably use this property, as well as other

properties of real numbers listed in **Table P.2**, without giving it much thought. The properties of the real numbers are especially useful when working with algebraic expressions. For each property listed in **Table P.2**, a , b , and c represent real numbers, variables, or algebraic expressions.

Table P.2 Properties of the Real Numbers

Name	Meaning	Examples
Commutative Property of Addition	Changing order when adding does not affect the sum. $a + b = b + a$	<ul style="list-style-type: none"> $13 + 7 = 7 + 13$ $13x + 7 = 7 + 13x$
Commutative Property of Multiplication	Changing order when multiplying does not affect the product. $ab = ba$	<ul style="list-style-type: none"> $\sqrt{2} \cdot \sqrt{5} = \sqrt{5} \cdot \sqrt{2}$ $x \cdot 6 = 6x$
Associative Property of Addition	Changing grouping when adding does not affect the sum. $(a + b) + c = a + (b + c)$	<ul style="list-style-type: none"> $3 + (8 + x) = (3 + 8) + x = 11 + x$
Associative Property of Multiplication	Changing grouping when multiplying does not affect the product. $(ab)c = a(bc)$	<ul style="list-style-type: none"> $-2(3x) = (-2 \cdot 3)x = -6x$
Distributive Property of Multiplication over Addition	Multiplication distributes over addition. $a \cdot (b + c) = a \cdot b + a \cdot c$	<ul style="list-style-type: none"> $7(4 + \sqrt{3}) = 7 \cdot 4 + 7 \cdot \sqrt{3} = 28 + 7\sqrt{3}$ $5(3x + 7) = 5 \cdot 3x + 5 \cdot 7 = 15x + 35$
Identity Property of Addition	Zero can be deleted from a sum. $a + 0 = a$ $0 + a = a$	<ul style="list-style-type: none"> $\sqrt{3} + 0 = \sqrt{3}$ $0 + 6x = 6x$
Identity Property of Multiplication	One can be deleted from a product. $a \cdot 1 = a$ $1 \cdot a = a$	<ul style="list-style-type: none"> $1 \cdot \pi = \pi$ $13x \cdot 1 = 13x$
Inverse Property of Addition	The sum of a real number and its additive inverse gives 0, the additive identity. $a + (-a) = 0$ $(-a) + a = 0$	<ul style="list-style-type: none"> $\sqrt{5} + (-\sqrt{5}) = 0$ $-\pi + \pi = 0$ $6x + (-6x) = 0$ $(-4y) + 4y = 0$
Inverse Property of Multiplication	The product of a nonzero real number and its multiplicative inverse gives 1, the multiplicative identity. $a \cdot \frac{1}{a} = 1, a \neq 0$ $\frac{1}{a} \cdot a = 1, a \neq 0$	<ul style="list-style-type: none"> $7 \cdot \frac{1}{7} = 1$ $\left(\frac{1}{x-3}\right)(x-3) = 1, x \neq 3$

The properties of the real numbers in **Table P.2** apply to the operations of addition and multiplication. Subtraction and division are defined in terms of addition and multiplication.

> GREAT QUESTION !

Do the commutative and associative properties work for subtraction and division?

No. Subtraction and division are not commutative operations.

$$a - b \neq b - a \quad \frac{a}{b} \neq \frac{b}{a}$$

Furthermore, subtraction and division are not associative operations.

$$(a - b) - c \neq a - (b - c)$$

$$(a \div b) \div c \neq a \div (b \div c)$$

Verify each of these four statements using $a = 10$, $b = 5$, and $c = 2$.

Definitions of Subtraction and Division

Let a and b represent real numbers.

Subtraction: $a - b = a + (-b)$

We call $-b$ the **additive inverse** or **opposite** of b .

Division: $a \div b = a \cdot \frac{1}{b}$, where $b \neq 0$

We call $\frac{1}{b}$ the **multiplicative inverse** or **reciprocal** of b . The quotient of a and b , $a \div b$, can be written in the form $\frac{a}{b}$, where a is the **numerator** and b is the **denominator** of the fraction.

Because subtraction is defined in terms of adding an inverse, the distributive property can be applied to subtraction:

$$a(b - c) = ab - ac$$

$$(b - c)a = ba - ca$$

For example,

$$4(2x - 5) = 4 \cdot 2x - 4 \cdot 5 = 8x - 20.$$

10 Simplify algebraic expressions.

Simplifying Algebraic Expressions

The **terms** of an algebraic expression are those parts that are separated by addition. For example, consider the algebraic expression

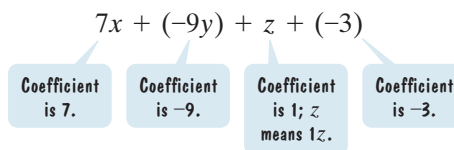
$$7x - 9y + z - 3,$$

which can be expressed as

$$7x + (-9y) + z + (-3).$$

This expression contains four terms, namely, $7x$, $-9y$, z , and -3 .

The numerical part of a term is called its **coefficient**. In the term $7x$, the 7 is the coefficient. If a term containing one or more variables is written without a coefficient, the coefficient is understood to be 1. Thus, z means $1z$. If a term is a **constant**, its coefficient is that constant. Thus, the coefficient of the constant term -3 is -3 .



The parts of each term that are multiplied are called the **factors** of the term. The **factors** of the term $7x$ are 7 and x .

Like terms are terms that have exactly the same variable factors. For example, $3x$ and $7x$ are like terms. The distributive property in the form

$$ba + ca = (b + c)a$$

enables us to add or subtract like terms. For example,

$$3x + 7x = (3 + 7)x = 10x$$

$$7y^2 - y^2 = 7y^2 - 1y^2 = (7 - 1)y^2 = 6y^2.$$

This process is called **combining like terms**.

> GREAT QUESTION !

What is the bottom line for combining like terms?

To combine like terms mentally, add or subtract the coefficients of the terms. Use this result as the coefficient of the terms' variable factor(s).

An algebraic expression is **simplified** when parentheses have been removed and like terms have been combined.

EXAMPLE 8 Simplifying an Algebraic Expression

Simplify: $6(2x^2 + 4x) + 10(4x^2 + 3x)$.

Solution

$$\begin{aligned}
 & 6(2x^2 + 4x) + 10(4x^2 + 3x) \\
 &= 6 \cdot 2x^2 + 6 \cdot 4x + 10 \cdot 4x^2 + 10 \cdot 3x && \text{Use the distributive property to remove the parentheses.} \\
 &= 12x^2 + 24x + 40x^2 + 30x && \text{Multiply.} \\
 &= (12x^2 + 40x^2) + (24x + 30x) && \text{Group like terms.} \\
 &= 52x^2 + 54x && \text{Combine like terms.}
 \end{aligned}$$

$52x^2$ and $54x$ are not like terms. They contain different variable factors, x^2 and x , and cannot be combined.

✓ **CHECK POINT 8** Simplify: $7(4x^2 + 3x) + 2(5x^2 + x)$.

Properties of Negatives

The distributive property can be extended to cover more than two terms within parentheses. For example,

$$\begin{aligned}
 -3(4x - 2y + 6) &= -3 \cdot 4x - (-3) \cdot 2y - 3 \cdot 6 \\
 &= -12x - (-6y) - 18 \\
 &= -12x + 6y - 18.
 \end{aligned}$$

This sign represents subtraction.

This sign tells us that the number is negative.

The voice balloons illustrate that negative signs can appear side by side. They can represent the operation of subtraction or the fact that a real number is negative. Here is a list of properties of negatives and how they are applied to algebraic expressions:

Properties of Negatives

Let a and b represent real numbers, variables, or algebraic expressions.

Property

1. $(-1)a = -a$
2. $-(-a) = a$
3. $(-a)b = -ab$
4. $a(-b) = -ab$
5. $-(a + b) = -a - b$
6. $-(a - b) = -a + b$
 $= b - a$

Examples

$$\begin{aligned}
 (-1)4xy &= -4xy \\
 -(-6y) &= 6y \\
 (-7)4xy &= -7 \cdot 4xy = -28xy \\
 5x(-3y) &= -5x \cdot 3y = -15xy \\
 -(7x + 6y) &= -7x - 6y \\
 -(3x - 7y) &= -3x + 7y \\
 &= 7y - 3x
 \end{aligned}$$

It is not uncommon to see algebraic expressions with parentheses preceded by a negative sign or subtraction. Properties 5 and 6 in the box, $-(a + b) = -a - b$ and $-(a - b) = -a + b$, are related to this situation. An expression of the form $-(a + b)$ can be simplified as follows:

$$-(a + b) = -1(a + b) = (-1)a + (-1)b = -a + (-b) = -a - b.$$

Do you see a fast way to obtain the simplified expression on the right in the preceding equation? **If a negative sign or a subtraction symbol appears outside parentheses, drop the parentheses and change the sign of every term within the parentheses.** For example,

$$-(3x^2 - 7x - 4) = -3x^2 + 7x + 4.$$

EXAMPLE 9 Simplifying an Algebraic Expression

Simplify: $8x + 2[5 - (x - 3)]$.

Solution

$$8x + 2[5 - (x - 3)]$$

$$= 8x + 2[5 - x + 3]$$

Drop parentheses and change the sign of each term in parentheses: $-(x - 3) = -x + 3$.

$$= 8x + 2[8 - x]$$

Simplify inside brackets: $5 + 3 = 8$.

$$= 8x + 16 - 2x$$

Apply the distributive property:

$$2[8 - x] = 2 \cdot 8 - 2x = 16 - 2x.$$

$$= (8x - 2x) + 16$$

Group like terms.

$$= (8 - 2)x + 16$$

Apply the distributive property.

$$= 6x + 16$$

Simplify.

CHECK POINT 9 Simplify: $6 + 4[7 - (x - 2)]$.

BLITZER BONUS

Using Algebra to Measure Blood-Alcohol Concentration

The amount of alcohol in a person's blood is known as blood-alcohol concentration (BAC), measured in grams of alcohol per deciliter of blood. A BAC of 0.08, meaning 0.08%, indicates that a person has 8 parts alcohol per 10,000 parts blood. In every state in the United States, it is illegal to drive with a BAC of 0.08 or higher.

How Do I Measure My Blood-Alcohol Concentration?

Here's a formula that models BAC for a person who weighs w pounds and who has n drinks* per hour.

$$\text{BAC} = \frac{600n}{w(0.6n + 169)}$$

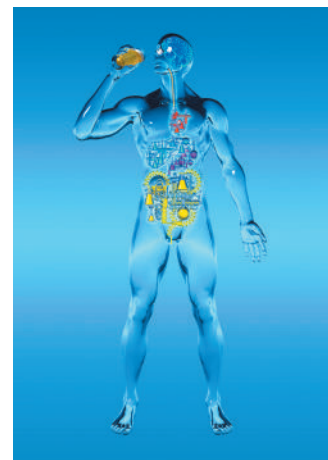
Blood-alcohol concentration
Body weight, in pounds
Number of drinks consumed in an hour

*A drink can be a 12-ounce can of beer, a 5-ounce glass of wine, or a 1.5-ounce shot of liquor. Each contains approximately 14 grams, or $\frac{1}{2}$ ounce, of alcohol.

Blood-alcohol concentration can be used to quantify the meaning of “tipsy.”

BAC	Effects on Behavior
0.05	Feeling of well-being; mild release of inhibitions; absence of observable effects
0.08	Feeling of relaxation; mild sedation; exaggeration of emotions and behavior; slight impairment of motor skills; increase in reaction time
0.12	Muscle control and speech impaired; difficulty performing motor skills; uncoordinated behavior
0.15	Euphoria; major impairment of physical and mental functions; irresponsible behavior; some difficulty standing, walking, and talking
0.35	Surgical anesthesia; lethal dosage for a small percentage of people
0.40	Lethal dosage for 50% of people; severe circulatory and respiratory depression; alcohol poisoning/overdose

Source: National Clearinghouse for Alcohol and Drug Information



(continues on next page)

Keeping in mind the meaning of “tipsy,” we can use our model to compare blood-alcohol concentrations of a 120-pound person and a 200-pound person for various numbers of drinks.

We determined each BAC using a calculator, rounding to three decimal places.

Blood-Alcohol Concentrations of a 120-Pound Person

$$\text{BAC} = \frac{600n}{120(0.6n + 169)}$$

n (number of drinks per hour)	1	2	3	4	5	6	7	8	9	10
BAC (blood-alcohol concentration)	0.029	0.059	0.088	0.117	0.145	0.174	0.202	0.230	0.258	0.286

Illegal to drive

Blood-Alcohol Concentrations of a 200-Pound Person

$$\text{BAC} = \frac{600n}{200(0.6n + 169)}$$

n (number of drinks per hour)	1	2	3	4	5	6	7	8	9	10
BAC (blood-alcohol concentration)	0.018	0.035	0.053	0.070	0.087	0.104	0.121	0.138	0.155	0.171

Illegal to drive

Like all mathematical models, the formula for BAC gives approximate rather than exact values. There are other variables that influence blood-alcohol concentration that are not contained

in the model. These include the rate at which an individual’s body processes alcohol, how quickly one drinks, sex, age, physical condition, and the amount of food eaten prior to drinking.

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

- C1. A combination of numbers, variables, and operation symbols is called an algebraic _____.

C2. If n is a counting number, b^n , read _____, indicates that there are n factors of b . The number b is called the _____ and the number n is called the _____.

C3. An equation that expresses a relationship between two or more variables, such as $H = \frac{9}{10}(220 - a)$, is called a/an _____. The process of finding such equations to describe real-world phenomena is called mathematical _____. Such equations, together with the meaning assigned to the variables, are called mathematical _____.

C4. The set of elements common to both set A and set B is called the _____ of sets A and B and is symbolized by _____.

C5. The set of elements that are members of set A or set B or of both sets is called the _____ of sets A and B and is symbolized by _____.

C6. The set $\{1, 2, 3, 4, 5, \dots\}$ is called the set of _____ numbers.

C7. The set $\{0, 1, 2, 3, 4, 5, \dots\}$ is called the set of _____ numbers.

C8. The set $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ is called the set of _____.
- C9. The set of numbers in the form $\frac{a}{b}$, where a and b belong to the set in Exercise C8 and $b \neq 0$, is called the set of _____ numbers.

C10. The set of numbers whose decimal representations are neither terminating nor repeating is called the set of _____ numbers.

C11. Every real number is either a/an _____ number or a/an _____ number.

C12. The notation $|x|$ is read the _____ of x . If $x \geq 0$, then $|x| = \rule{1cm}{0.4pt}$. If $x < 0$, then $|x| = \rule{1cm}{0.4pt}$.

C13. The commutative properties state that $a + b = \rule{1cm}{0.4pt}$ and $ab = \rule{1cm}{0.4pt}$.

C14. The associative properties state that $(a + b) + c = \rule{1cm}{0.4pt}$ and $\rule{1cm}{0.4pt} = a(bc)$.

C15. The distributive property states that $a(b + c) = \rule{1cm}{0.4pt}$.

C16. $a + (-a) = \rule{1cm}{0.4pt}$: The sum of a real number and its additive _____ is _____, the additive _____.

C17. $a \cdot \frac{1}{a} = 1, a \neq 0$: The product of a nonzero real number and its multiplicative _____ is _____, the multiplicative _____.

C18. An algebraic expression is _____ when parentheses have been removed and like terms have been combined.

C19. $-(-a) = \rule{1cm}{0.4pt}$.

P.1 EXERCISE SET

Practice Exercises

In Exercises 1–16, evaluate each algebraic expression for the given value or values of the variable(s).

- $7 + 5x$, for $x = 10$
- $8 + 6x$, for $x = 5$
- $6x - y$, for $x = 3$ and $y = 8$
- $8x - y$, for $x = 3$ and $y = 4$
- $x^2 + 3x$, for $x = 8$
- $x^2 + 5x$, for $x = 6$
- $x^2 - 6x + 3$, for $x = 7$
- $x^2 - 7x + 4$, for $x = 8$
- $4 + 5(x - 7)^3$, for $x = 9$
- $6 + 5(x - 6)^3$, for $x = 8$
- $x^2 - 3(x - y)$, for $x = 8$ and $y = 2$
- $x^2 - 4(x - y)$, for $x = 8$ and $y = 3$
- $\frac{5(x + 2)}{2x - 14}$, for $x = 10$
- $\frac{7(x - 3)}{2x - 16}$, for $x = 9$
- $\frac{2x + 3y}{x + 1}$, for $x = -2$ and $y = 4$
- $\frac{2x + y}{xy - 2x}$, for $x = -2$ and $y = 4$

The formula

$$C = \frac{5}{9}(F - 32)$$

expresses the relationship between Fahrenheit temperature, F , and Celsius temperature, C . In Exercises 17–18, use the formula to convert the given Fahrenheit temperature to its equivalent temperature on the Celsius scale.

- 50°F
- 86°F

A football was kicked vertically upward from a height of 4 feet with an initial speed of 60 feet per second. The formula

$$h = 4 + 60t - 16t^2$$

describes the ball's height above the ground, h , in feet, t seconds after it was kicked. Use this formula to solve Exercises 19–20.

- What was the ball's height 2 seconds after it was kicked?
- What was the ball's height 3 seconds after it was kicked?

In Exercises 21–28, find the intersection of the sets.

- $\{1, 2, 3, 4\} \cap \{2, 4, 5\}$
- $\{1, 3, 7\} \cap \{2, 3, 8\}$
- $\{s, e, t\} \cap \{t, e, s\}$
- $\{r, e, a, l\} \cap \{l, e, a, r\}$
- $\{1, 3, 5, 7\} \cap \{2, 4, 6, 8, 10\}$
- $\{0, 1, 3, 5\} \cap \{-5, -3, -1\}$
- $\{a, b, c, d\} \cap \emptyset$
- $\{w, y, z\} \cap \emptyset$

In Exercises 29–34, find the union of the sets.

- $\{1, 2, 3, 4\} \cup \{2, 4, 5\}$
- $\{1, 3, 7, 8\} \cup \{2, 3, 8\}$
- $\{1, 3, 5, 7\} \cup \{2, 4, 6, 8, 10\}$
- $\{0, 1, 3, 5\} \cup \{2, 4, 6\}$
- $\{a, e, i, o, u\} \cup \emptyset$
- $\{e, m, p, t, y\} \cup \emptyset$

In Exercises 35–38, list all numbers from the given set that are **a. natural numbers**, **b. whole numbers**, **c. integers**, **d. rational numbers**, **e. irrational numbers**, **f. real numbers**.

- $\{-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}\}$
- $\{-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}\}$
- $\{-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}\}$
- $\{-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}\}$

- Give an example of a whole number that is not a natural number.
- Give an example of a rational number that is not an integer.
- Give an example of a number that is an integer, a whole number, and a natural number.
- Give an example of a number that is a rational number, an integer, and a real number.

Determine whether each statement in Exercises 43–50 is true or false.

- $-13 \leq -2$
- $-6 > 2$
- $4 \geq -7$
- $-13 < -5$
- $-\pi \geq -\pi$
- $-3 > -13$
- $0 \geq -6$
- $0 \geq -13$

In Exercises 51–60, rewrite each expression without absolute value bars.

- $|300|$
- $|-203|$
- $|12 - \pi|$
- $|7 - \pi|$
- $|\sqrt{2} - 5|$
- $|\sqrt{5} - 13|$
- $\frac{-3}{|-3|}$
- $\frac{-7}{|-7|}$
- $||-3| - |-7||$
- $||-5| - |-13||$

In Exercises 61–66, evaluate each algebraic expression for $x = 2$ and $y = -5$.

- $|x + y|$
- $|x - y|$
- $|x| + |y|$
- $|x| - |y|$
- $\frac{y}{|y|}$
- $\frac{|x|}{x} + \frac{|y|}{y}$

In Exercises 67–74, express the distance between the given numbers using absolute value. Then find the distance by evaluating the absolute value expression.

- 2 and 17
- 4 and 15
- 2 and 5
- 6 and 8
- 19 and -4
- 26 and -3
- 3.6 and -1.4
- 5.4 and -1.2

In Exercises 75–84, state the name of the property illustrated.

- $6 + (-4) = (-4) + 6$
- $11 \cdot (7 + 4) = 11 \cdot 7 + 11 \cdot 4$
- $6 + (2 + 7) = (6 + 2) + 7$
- $6 \cdot (2 \cdot 3) = 6 \cdot (3 \cdot 2)$
- $(2 + 3) + (4 + 5) = (4 + 5) + (2 + 3)$
- $7 \cdot (11 \cdot 8) = (11 \cdot 8) \cdot 7$

81. $2(-8 + 6) = -16 + 12$
 82. $-8(3 + 11) = -24 + (-88)$
 83. $\frac{1}{(x + 3)}(x + 3) = 1, x \neq -3$
 84. $(x + 4) + [-(x + 4)] = 0$

In Exercises 85–96, simplify each algebraic expression.

85. $5(3x + 4) - 4$ 86. $2(5x + 4) - 3$
 87. $5(3x - 2) + 12x$ 88. $2(5x - 1) + 14x$
 89. $7(3y - 5) + 2(4y + 3)$
 90. $4(2y - 6) + 3(5y + 10)$
 91. $5(3y - 2) - (7y + 2)$
 92. $4(5y - 3) - (6y + 3)$
 93. $7 - 4[3 - (4y - 5)]$
 94. $6 - 5[8 - (2y - 4)]$
 95. $18x^2 + 4 - [6(x^2 - 2) + 5]$
 96. $14x^2 + 5 - [7(x^2 - 2) + 4]$

In Exercises 97–102, write each algebraic expression without parentheses.

97. $-(-14x)$ 98. $-(-17y)$
 99. $-(2x - 3y - 6)$ 100. $-(5x - 13y - 1)$
 101. $\frac{1}{3}(3x) + [(4y) + (-4y)]$ 102. $\frac{1}{2}(2y) + [(-7x) + 7x]$

Practice PLUS

In Exercises 103–110, insert either $<$, $>$, or $=$ in the shaded area to make a true statement.

103. $|-6|$ $|-3|$ 104. $|-20|$ $|-50|$
 105. $\left|\frac{3}{5}\right|$ $|-0.6|$ 106. $\left|\frac{5}{2}\right|$ $|-2.5|$
 107. $\frac{30}{40} - \frac{3}{4}$ $\frac{14}{15} \cdot \frac{15}{14}$ 108. $\frac{17}{18} \cdot \frac{18}{17}$ $\frac{50}{60} - \frac{5}{6}$
 109. $\frac{8}{13} \div \frac{8}{13}$ $|-1|$ 110. $|-2|$ $\frac{4}{17} \div \frac{4}{17}$

In Exercises 111–120, use the order of operations to simplify each expression.

111. $8^2 - 16 \div 2^2 \cdot 4 - 3$ 112. $10^2 - 100 \div 5^2 \cdot 2 - 3$
 113. $\frac{5 \cdot 2 - 3^2}{[3^2 - (-2)]^2}$ 114. $\frac{10 \div 2 + 3 \cdot 4}{(12 - 3 \cdot 2)^2}$
 115. $8 - 3[-2(2 - 5) - 4(8 - 6)]$
 116. $8 - 3[-2(5 - 7) - 5(4 - 2)]$
 117. $\frac{2(-2) - 4(-3)}{5 - 8}$ 118. $\frac{6(-4) - 5(-3)}{9 - 10}$
 119. $\frac{(5 - 6)^2 - 2|3 - 7|}{89 - 3 \cdot 5^2}$ 120. $\frac{12 \div 3 \cdot 5|2^2 + 3^2|}{7 + 3 - 6^2}$

In Exercises 121–128, write each English phrase as an algebraic expression. Then simplify the expression. Let x represent the number.

121. A number decreased by the sum of the number and four
 122. A number decreased by the difference between eight and the number
 123. Six times the product of negative five and a number
 124. Ten times the product of negative four and a number
 125. The difference between the product of five and a number and twice the number

126. The difference between the product of six and a number and negative two times the number
 127. The difference between eight times a number and six more than three times the number
 128. Eight decreased by three times the sum of a number and six

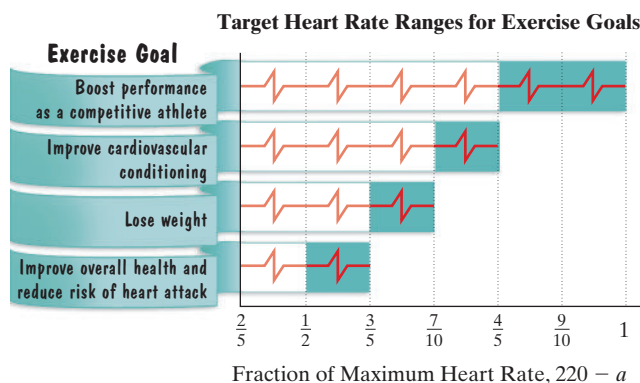
Application Exercises

The maximum heart rate, in beats per minute, that you should achieve during exercise is 220 minus your age:

$$220 - a.$$

This algebraic expression gives maximum heart rate in terms of age, a .

The following graph shows the target heart rate ranges for four types of exercise goals. The lower and upper limits of these ranges are fractions of the maximum heart rate, $220 - a$. Exercises 129–130 are based on the information in the graph.



129. If your exercise goal is to improve cardiovascular conditioning, the graph shows the following range for target heart rate, H , in beats per minute:

Lower limit of range $H = \frac{7}{10}(220 - a)$

Upper limit of range $H = \frac{4}{5}(220 - a).$

- a. What is the lower limit of the heart rate range, in beats per minute, for a 20-year-old with this exercise goal?
 b. What is the upper limit of the heart rate range, in beats per minute, for a 20-year-old with this exercise goal?

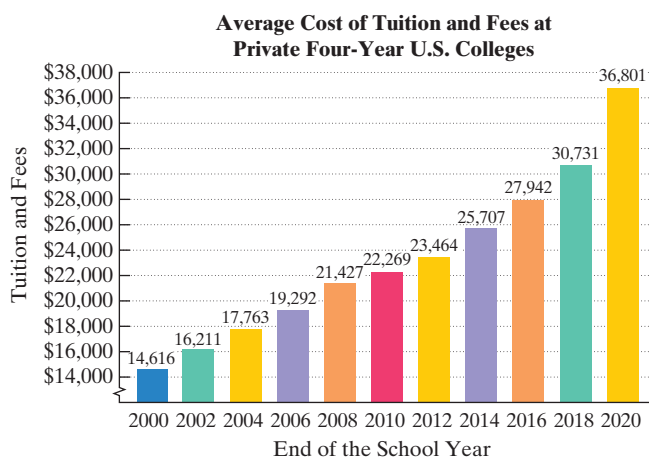
130. If your exercise goal is to improve overall health, the graph shows the following range for target heart rate, H , in beats per minute:

Lower limit of range $H = \frac{1}{2}(220 - a)$

Upper limit of range $H = \frac{3}{5}(220 - a).$

- a. What is the lower limit of the heart rate range, in beats per minute, for a 30-year-old with this exercise goal?
 b. What is the upper limit of the heart rate range, in beats per minute, for a 30-year-old with this exercise goal?

The bar graph shows the average cost of tuition and fees at private four-year colleges in the United States.



Source: The College Board

Here are two formulas that model the data shown in the graph. In each formula, T represents the average cost of tuition and fees at private U.S. colleges for the school year ending x years after 2000.

Model 1 $T = 975x + 13,547$

Model 2 $T = 32x^2 + 331x + 15,479$

Use this information to solve Exercises 131–132.

- 131. a.** Use each formula to find the average cost of tuition and fees at private U.S. colleges for the school year ending in 2018. By how much does each model underestimate or overestimate the actual cost shown for the school year ending in 2018?
- b.** Use model 2 to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2030.
- 132. a.** Use each formula to find the average cost of tuition and fees at private U.S. colleges for the school year ending in 2010. By how much does each underestimate or overestimate the actual cost shown for the school year ending in 2010?
- b.** Use model 2 to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2025.
- 133.** This month you have a total of \$6000 in interest-bearing credit card debt, split between a card charging 18% annual interest and a card charging 21% annual interest. If the interest-bearing balance on the card charging 18% is x dollars, then the total interest for the month is given by the algebraic expression

$$0.015x + 0.0175(6000 - x).$$

- a.** Simplify the algebraic expression.
- b.** Use each form of the algebraic expression to determine the total interest for the month if the balance on the card charging 18% is \$4400.
- c.** Use the simplified form of the algebraic expression to determine the total interest for the month if the \$6000 debt is split evenly between the two cards.

- 134.** It takes you 50 minutes to get to campus. You spend t minutes walking to the bus stop and the rest of the time riding the bus. Your walking rate is 0.06 mile per minute and the bus travels at a rate of 0.5 mile per minute. The total distance walking and traveling by bus is given by the algebraic expression

$$0.06t + 0.5(50 - t).$$

- a.** Simplify the algebraic expression.
- b.** Use each form of the algebraic expression to determine the total distance that you travel if you spend 20 minutes walking to the bus stop.
- c.** Use the simplified form of the algebraic expression to determine the total distance you travel if the 50 minutes is split evenly between walking and riding the bus.
- 135.** Read the Blitzer Bonus beginning on page 15. Use the formula

$$\text{BAC} = \frac{600n}{w(0.6n + 169)}$$

and replace w with your body weight. Using this formula and a calculator, compute your BAC for integers from $n = 1$ to $n = 10$. Round to three decimal places. According to this model, how many drinks can you consume in an hour without exceeding the legal measure of drunk driving?

Explaining the Concepts

> ACHIEVING SUCCESS

An effective way to understand something is to explain it to someone else. You can do this by using the Explaining the Concepts exercises that ask you to respond with verbal or written explanations. Speaking or writing about a new concept uses a different part of your brain than thinking about the concept. Explaining new ideas verbally will quickly reveal any gaps in your understanding. It will also help you to remember new concepts for longer periods of time.

- 136.** What is an algebraic expression? Give an example with your explanation.
- 137.** If n is a natural number, what does b^n mean? Give an example with your explanation.
- 138.** What does it mean when we say that a formula models real-world phenomena?
- 139.** What is the intersection of sets A and B ?
- 140.** What is the union of sets A and B ?
- 141.** How do the whole numbers differ from the natural numbers?
- 142.** Can a real number be both rational and irrational? Explain your answer.
- 143.** If you are given two real numbers, explain how to determine which is the lesser.

Critical Thinking Exercises

Make Sense? In Exercises 144–147, determine whether each statement makes sense or does not make sense, and explain your reasoning.

- 144.** My mathematical model describes the data for tuition and fees at public four-year colleges for the past 20 years extremely well, so it will serve as an accurate prediction for the cost of public colleges in 2050.

145. A model that describes the average cost of tuition and fees at private U.S. colleges for the school year ending x years after 2000 cannot be used to estimate the cost of private education for the school year ending in 2000.
146. Regardless of what real numbers I substitute for x and y , I will always obtain zero when evaluating $2x^2y - 2yx^2$.
147. Just as the commutative properties change groupings, the associative properties change order.

In Exercises 148–155, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

148. Every rational number is an integer.
149. Some whole numbers are not integers.
150. Some rational numbers are not positive.
151. Irrational numbers cannot be negative.
152. The term x has no coefficient.
153. $5 + 3(x - 4) = 8(x - 4) = 8x - 32$
154. $-x - x = -x + (-x) = 0$
155. $x - 0.02(x + 200) = 0.98x - 4$

In Exercises 156–158, insert either $<$ or $>$ in the shaded area between the numbers to make the statement true.

156. $\sqrt{2}$ 1.5
157. $-\pi$ -3.5
158. $-\frac{3.14}{2}$ $-\frac{\pi}{2}$

Preview Exercises

Exercises 159–161 will help you prepare for the material covered in the next section.

159. In parts (a) and (b), complete each statement.
- a. $b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b) = b^?$
- b. $b^5 \cdot b^5 = (b \cdot b \cdot b \cdot b \cdot b)(b \cdot b \cdot b \cdot b \cdot b) = b^?$
- c. Generalizing from parts (a) and (b), what should be done with the exponents when multiplying exponential expressions with the same base?
160. In parts (a) and (b), complete each statement.
- a. $\frac{b^7}{b^3} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b} = b^?$
- b. $\frac{b^8}{b^2} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b^?$
- c. Generalizing from parts (a) and (b), what should be done with the exponents when dividing exponential expressions with the same base?
161. If 6.2 is multiplied by 10^3 , what does this multiplication do to the decimal point in 6.2?

SECTION P.2

Exponents and Scientific Notation

WHAT YOU'LL LEARN

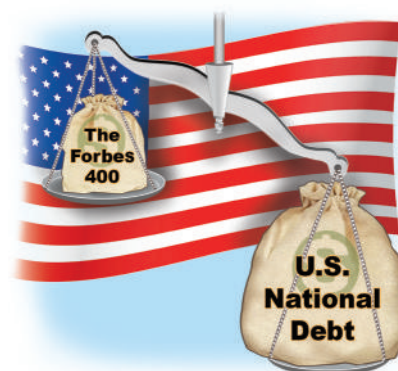
- 1 Use properties of exponents.
- 2 Simplify exponential expressions.
- 3 Use scientific notation.

Bigger than the biggest thing ever and then some. Much bigger than that in fact, really amazingly immense, a totally stunning size, real 'wow, that's big', time. . . . Gigantic multiplied by colossal multiplied by staggeringly huge is the sort of concept we're trying to get across here.

Douglas Adams, *The Restaurant at the End of the Universe*

In October 2019, *Forbes* published a list of the 400 wealthiest Americans, who had a total net worth of \$2.96 trillion. Amazon founder and CEO Jeff Bezos topped the list with a net worth of \$114 billion. That sounds like a lot of money, but consider this: At the end of 2019, the national debt was approximately \$22.7 trillion, rising to over \$25.5 trillion by midyear 2020. This \$2.8 trillion increase in the national debt over just a few months is nearly the total net worth of the 400 wealthiest Americans.

One of the best ways to put “staggeringly huge” numbers into perspective is by making comparisons. In this section, we will compare large and small numbers using exponents and scientific notation.



1 Use properties of exponents.

Properties of Exponents

The major properties of exponents are summarized in the box that follows.

> GREAT QUESTION !

Cut to the chase. What do I do with negative exponents?

When a negative integer appears as an exponent, switch the position of the base (from numerator to denominator or from denominator to numerator) and make the exponent positive.

> GREAT QUESTION !

What's the difference between $\frac{4^3}{4^5}$ and $\frac{4^5}{4^3}$?

These quotients represent different numbers:

$$\frac{4^3}{4^5} = 4^{3-5} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$\frac{4^5}{4^3} = 4^{5-3} = 4^2 = 16.$$

Properties of Exponents

Property

The Negative-Exponent Rule

If b is any real number other than 0 and n is a natural number, then

$$b^{-n} = \frac{1}{b^n}.$$

The Zero-Exponent Rule

If b is any real number other than 0,

$$b^0 = 1.$$

Examples

- $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$
- $\frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 4^2 = 16$

- $7^0 = 1$
- $(-5)^0 = 1$
- $-5^0 = -1$

Only 5 is raised to the zero power.

The Product Rule

If b is a real number or algebraic expression, and m and n are integers,

$$b^m \cdot b^n = b^{m+n}.$$

- $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$
- $x^{-3} \cdot x^7 = x^{-3+7} = x^4$

When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.

The Power Rule

If b is a real number or algebraic expression, and m and n are integers,

$$(b^m)^n = b^{mn}.$$

- $(2^2)^3 = 2^{2 \cdot 3} = 2^6 = 64$
- $(x^{-3})^4 = x^{-3 \cdot 4} = x^{-12} = \frac{1}{x^{12}}$

When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.

The Quotient Rule

If b is a nonzero real number or algebraic expression, and m and n are integers,

$$\frac{b^m}{b^n} = b^{m-n}.$$

- $\frac{2^8}{2^4} = 2^{8-4} = 2^4 = 16$
- $\frac{x^3}{x^7} = x^{3-7} = x^{-4} = \frac{1}{x^4}$

When dividing exponential expressions with the same nonzero base, subtract the exponent in the denominator from the exponent in the numerator. Use this difference as the exponent of the common base.

Products Raised to Powers

If a and b are real numbers or algebraic expressions, and n is an integer,

$$(ab)^n = a^n b^n.$$

- $(-2y)^4 = (-2)^4 y^4 = 16y^4$
- $(-2xy)^3 = (-2)^3 x^3 y^3 = -8x^3 y^3$

When a product is raised to a power, raise each factor to that power.

(continued)

Property

Quotients Raised to Powers


If a and b are real numbers, $b \neq 0$, or algebraic expressions, and n is an integer,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

Examples

- $\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{16}{625}$
- $\left(-\frac{3}{x}\right)^3 = \frac{(-3)^3}{x^3} = -\frac{27}{x^3}$

When a quotient is raised to a power, raise the numerator to that power and divide by the denominator to that power.

 Simplify exponential expressions.

Simplifying Exponential Expressions

Properties of exponents are used to simplify exponential expressions. An exponential expression is **simplified** when

- No parentheses appear.
- No powers are raised to powers.
- Each base occurs only once.
- No negative or zero exponents appear.

Simplifying Exponential Expressions

	Example
<p>1. If necessary, remove parentheses by using</p> $(ab)^n = a^n b^n \quad \text{or} \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$	$(xy)^3 = x^3 y^3$
<p>2. If necessary, simplify powers to powers by using</p> $(b^m)^n = b^{mn}.$	$(x^4)^3 = x^{4 \cdot 3} = x^{12}$
<p>3. If necessary, be sure that each base appears only once by using</p> $b^m \cdot b^n = b^{m+n} \quad \text{or} \quad \frac{b^m}{b^n} = b^{m-n}.$	$x^4 \cdot x^3 = x^{4+3} = x^7$
<p>4. If necessary, rewrite exponential expressions with zero powers as 1 ($b^0 = 1$). Furthermore, write the answer with positive exponents by using</p> $b^{-n} = \frac{1}{b^n} \quad \text{or} \quad \frac{1}{b^{-n}} = b^n.$	$\frac{x^5}{x^8} = x^{5-8} = x^{-3} = \frac{1}{x^3}$

The following example shows how to simplify exponential expressions. Throughout the example, assume that no variable in a denominator is equal to zero.

EXAMPLE 1 Simplifying Exponential Expressions

Simplify:

a. $(-3x^4y^5)^3$ b. $(-7xy^4)(-2x^5y^6)$ c. $\frac{-35x^2y^4}{5x^6y^{-8}}$ d. $\left(\frac{4x^2}{y}\right)^{-3}$.

Solution

a. $(-3x^4y^5)^3 = (-3)^3(x^4)^3(y^5)^3$ **Raise each factor inside the parentheses to the third power.**

$$= (-3)^3 x^{4 \cdot 3} y^{5 \cdot 3}$$

Multiply the exponents when raising powers to powers.

$$= -27x^{12}y^{15} \quad (-3)^3 = (-3)(-3)(-3) = -27$$

b. $(-7xy^4)(-2x^5y^6) = (-7)(-2)xx^5y^4y^6$ **Group factors with the same base.**

$$= 14x^{1+5}y^{4+6}$$

When multiplying expressions with the same base, add the exponents.

$$= 14x^6y^{10}$$

Simplify.

c. $\frac{-35x^2y^4}{5x^6y^{-8}} = \left(\frac{-35}{5}\right)\left(\frac{x^2}{x^6}\right)\left(\frac{y^4}{y^{-8}}\right)$ **Group factors with the same base.**

$$= -7x^{2-6}y^{4-(-8)}$$

When dividing expressions with the same base, subtract the exponents.

$$= -7x^{-4}y^{12}$$

Simplify. Notice that $4 - (-8) = 4 + 8 = 12$.

$$= \frac{-7y^{12}}{x^4}$$

Write as a fraction and move the base with the negative exponent, x^{-4} , to the other side of the fraction bar and make the negative exponent positive.

d. $\left(\frac{4x^2}{y}\right)^{-3} = \frac{(4x^2)^{-3}}{y^{-3}}$ **Raise the numerator and the denominator to the -3 power.**

$$= \frac{4^{-3}(x^2)^{-3}}{y^{-3}}$$

Raise each factor inside the parentheses to the -3 power.

$$= \frac{4^{-3}x^{-6}}{y^{-3}}$$

Multiply the exponents when raising a power to a power:
 $(x^2)^{-3} = x^{2(-3)} = x^{-6}$.

$$= \frac{y^3}{4^3x^6}$$

Move each base with a negative exponent to the other side of the fraction bar and make each negative exponent positive.

$$= \frac{y^3}{64x^6} \quad 4^3 = 4 \cdot 4 \cdot 4 = 64$$

✓ CHECK POINT 1 Simplify:

a. $(2x^3y^6)^4$ b. $(-6x^2y^5)(3xy^3)$

c. $\frac{100x^{12}y^2}{20x^{16}y^{-4}}$ d. $\left(\frac{5x}{y^4}\right)^{-2}$