

ELEVENTH EDITION

OPTIONS, FUTURES, AND OTHER DERIVATIVES



John Hull



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John C. Hull

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Library of Congress Cataloging-in-Publication Data

Names: Hull, John, author.

Title: Options, futures, and other derivatives / John C. Hull, Maple

Financial Group, Professor of Derivatives and Risk Management, Joseph L.

Rotman School of Management, University of Toronto.

Description: Eleventh edition. | New York, NY: Pearson, [2022] | Revised

edition of the author's Options, futures, and other derivatives, [2018]

Identifiers: LCCN 2021002151 | ISBN 9780136939979 (hardcover)

Subjects: LCSH: Futures. | Stock options. | Derivative securities.

Classification: LCC HG6024.A3 H85 2022 | DDC 332.64/5-dc23

LC record available at https://lccn.loc.gov/2021002151

ScoutAutomatedPrintCode



ISBN 10: 0-13-693997-X ISBN 13: 978-0-13-693997-9



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- 2. Properties of the Lognormal Distribution
- 3. Warrant Valuation When Value of Equity plus Warrants Is Lognormal
- 4. Exact Procedure for Valuing American Calls on Stocks Paying a Single Dividend
- 5. Calculation of the Cumulative Probability in a Bivariate Normal Distribution
- 6. Differential Equation for Price of a Derivative on a Stock Paying a Known Dividend Yield
- 7. Differential Equation for Price of a Derivative on a Futures Price
- 8. Analytic Approximation for Valuing American Options
- 9. Generalized Tree-Building Procedure
- 10. The Cornish–Fisher Expansion to Estimate VaR
- 11. Manipulation of Credit Transition Matrices
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- 14. The Hull-White Two-Factor Model
- 15. Valuing Options on Coupon-Bearing Bonds in a One-Factor Interest Rate Model
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- 30. The Return of a Security Dependent on Multiple Sources of Uncertainty
- 31. Properties of Ho-Lee and Hull-White Interest Rate Models

PREFACE

Derivatives markets have seen many changes over the last 30 years. Successive editions of *Options, Futures, and Other Derivatives* have managed to keep up to date. The book has an applied approach. It is a very popular college text, but it can also be found on trading-room desks throughout the world. (Indeed, I receive emails from derivatives practitioners about the book almost every day.) The blending of material useful for practitioners with material appropriate for university courses is what makes the book unique.

NEW TO THIS EDITION

- A major change in financial markets will be the phase-out of LIBOR. This has led to important changes throughout the 11th edition. The overnight reference rates that will replace LIBOR, and the way they are used to determine zero curves, are discussed carefully.
- Within-chapter examples and end-of-chapter problems that were previously based on LIBOR have been largely replaced by examples based on the new reference rates or by generic examples.
- The likely impact of the new reference rates on valuation models is discussed.
- The new reference rates are considered to be risk-free whereas LIBOR incorporates
 a time-varying credit spread. The book discusses the desire on the part of banks to
 augment the new reference rates with a measure of the level of credit spreads in the
 market.
- The chapter on Wiener processes now covers fractional Brownian motion. This is becoming increasingly used in modeling volatility.
- Rough volatility models which have in the last few years been found to fit volatility surfaces well are added to the models considered in Chapter 27.
- Machine learning is becoming increasingly used in pricing and hedging derivatives. The reader is introduced to these applications at various points in the book.
- Changes in the regulatory environment, including Basel IV, are covered.
- To help students determine whether key ideas have been understood, short concept questions are included at the ends of the first 20 chapters.
- The end-of-chapter problems have been updated. To make the book as easy to use as possible, solutions to all end-of-chapter problems are now on www.pearson.com and www-2.rotman.utoronto.ca/~hull.

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 Instructor support material has been revised. In particular, there are now many more suggestions on assignment questions that can be used in conjunction with chapters.

- The DerivaGem software is less LIBOR-focused and is available for download from www-2.rotman.utoronto.ca/~hull/software.
- Tables, charts, market data, and examples have been updated throughout the book.

SOLVING TEACHING AND LEARNING CHALLENGES

Most instructors find that courses in derivatives are fun to teach. There is not a big gap between theory and practice. Most students know a little about the subject and are motivated to learn more. Usually there is some current news that can be discussed in class, e.g., the level of the VIX index or events that affect particular option prices.

Math Knowledge

Math is the key challenge for many students taking a course in derivatives. I have kept this in mind in the way material is presented throughout the book. Instructors are often faced with a trade-off between mathematical rigor and the simplicity with which an idea is explained. My preference is always to look for the simplest way of explaining an idea in the first instance. Sometimes using words rather than equations is effective. I avoid using notation that has lots of subscripts, superscripts, and function arguments as far as possible because this can be off-putting to a reader who is new to the material. Nonessential mathematical material has been either eliminated or included in technical notes on my website.

The reality is that many students only understand an equation when they have seen numbers substituted into it. For that reason, many numerical examples have been included in the text. The software DerivaGem (discussed below) allows students to get a feel for equations by trying different inputs.

I am often asked about the math prerequisites for *Options, Futures and Other Derivatives*. Students will be able to cope with a course based on this book if they are comfortable with algebra and understand probabilities and probability distributions. A knowledge of calculus concepts is useful for parts of the book. But no knowledge of stochastic calculus is assumed. The basic knowledge of stochastic processes that is needed for a more advanced understanding of derivatives is explained carefully in Chapter 14.

End of Chapter Problems

After reading a chapter, students find it useful to have a quick check of whether they have understood the ideas that have been presented. With that in mind, an innovation in the 11th edition is the inclusion of "Short Concept Questions" at the end of each of the first 20 chapters.

As in earlier editions, there are many other end-of-chapter problems to help students apply the ideas presented in the chapters. These have been updated. The distinction between "practice questions" and "further questions" has been eliminated. Answers to all end-of-chapter problems and short concept questions are on my website and available through www.pearson.com.

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Designing a Course

There are many ways in which the material in the book can be used. Instructors teaching an introductory course in derivatives tend to spend most time on the first 20 chapters, and often choose to omit Chapter 14 and Section 15.6. Instructors teaching a more advanced course find that many different combinations of chapters in the second half of the book can be used. I find that Chapter 37 is a fun chapter that works well at the end of either an introductory or an advanced course.

Software

The DerivaGem software is an important part of the book. Students get comfortable with the models presented in the book when they use DerivaGem to value transactions under different assumptions. The use of the software is explained at the end of the book.

I recommend giving students assignments that involve using the basic DG400a.xls software. There are many types of assignments that can be developed. For example, students can be asked to compare American or European option prices given by a binomial model with those from the Black–Scholes–Merton model. They can be asked to report what happens as the number of time steps is increased in a binomial model and can use the software to display trees. (DerivaGem can display trees with up to 10 time steps and can calculate prices and Greek letters using up to 500 time steps.)

Many charts can be produced using the software and students can include those charts in reports produced for the instructor. The calculation of zero curves and swap valuation is made easy with DerivaGem. I like to use DerivaGem in class when I illustrate some key concepts.

Students taking a more advanced course in derivatives can be asked to compare prices given by different models using the *Alternative Models* worksheet in DG400a.xls. Alternatives to Black–Scholes that are covered include CEV, Merton mixed jump–diffusion, variance gamma, Heston, and SABR. Students can also be asked to carry out assignments concerned with the use of different models for pricing bond options. The CDS and CDO worksheets can be used in conjunction with each other for an assignment if CDOs are covered.

DerivaGem can be used in conjunction with current market data that can be downloaded from Yahoo Finance or other providers. For example, students can be asked to compare implied volatilities for options on different stocks that have been in the news. They can also be asked to calculate volatility term structures and volatility smiles for stock indices. Assignments such as these can be important because they make the underlying concepts more "real" and lead to interesting classroom discussions.

The *DG400 Applications* software enables students to carry out assignments where they are asked investigate issues such as how the performance of delta hedging is improved as the interval between rebalancing is decreased or how managing gamma can improve the performance of delta hedging. Assuming students have a basic knowledge of Excel, they should have no difficulty using this software and changing instructions as necessary.

The *DG400 Functions* software is a little more challenging. It contains the functions used by DG400a.xls. Students can use these functions to develop their own Excel worksheets in order to investigate particular issues and answer assignment questions.

Many instructors find DerivaGem to be a really useful resource. DerivaGem can be downloaded from www-2.rotman.utoronto.ca/~hull/software.

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Slides

Several hundred PowerPoint slides accompany this book. They can be a useful starting point for instructors. Those who adopt the text are welcome to adapt the slides to meet their needs. These slides are available on www.pearson.com and www-2.rotman.utoronto.ca/~hull.

Technical Notes

There are over 30 technical notes available. They are referred to in the text and can be downloaded from www-2.rotman.utoronto.ca/~hull/TechnicalNotes.

By not including the Technical Notes in the book, I am able to streamline the presentation of material so that it is more reader-friendly.

EMPLOYABILITY

A natural question for students is: "Will a course in derivatives improve my chances of a getting a job in finance?" The answer is an overwhelming yes. Probably the first thing many students think about when considering options or other derivatives is an exchange such as the CBOE. In fact, as Chapter 1 makes clear, the over-the-counter (OTC) market is much larger than the exchange-traded market and likely to be much more important to students in their first job (or subsequent jobs). *Options, Futures, and Other Derivatives* has a much bigger focus on the OTC market than most other derivatives texts.

Derivatives have steadily increased in importance. Potential employers can be classified as "buy side" and "sell side". The buy side includes nonfinancial corporations, insurance companies, fund managers, and some other financial institutions. The sell side consists of large financial institutions who act as market makers. Many students who take courses in derivatives may not become derivatives traders or derivatives analysts. However, derivatives now permeate all aspects of finance. If you work in investment banking, there is likely to be a derivatives component to some of the deals you are involved in; if you work in fund management, you will probably find derivatives to be convenient tools for some purposes; if you work for a nonfinancial corporation, you may be involved in using derivative contracts for hedging and negotiating with a sell-side institution; and so on. Whatever your role in finance, it is important that you be able to talk about derivatives knowledgeably, use the right words, and understand the motivations of a counterparty to a transaction. A course based on *Options, Futures, and Other Derivatives* will help you do this.

What about those of you who want to specialize in derivatives? I have literally lost count of the many successful derivative executives who have told me "Thank you for your book. I read it before the interview, and it got me my first job in derivatives." (My joking response has typically been: "Great, but you realize that means you owe me 20% of your first year's salary.") The people I am talking about typically had engineering, physics, or other quantitative backgrounds at the time of the interview but had never taken a course in finance! So, while the book is important for those planning a career in finance, it is absolutely essential reading for all those aspiring to a career in derivatives. As mentioned earlier, it is found on trading-room desks throughout the world.

This book will help you develop your quant skills so that you become more marketable in finance. But other skills are of course important. Good communication skills are

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necessary. Many instructors ask students to present the results of projects in class. Students should take full advantage of these opportunities to practice and improve. If presentations are recorded, they should review the recording carefully.

At my business school, we used to run optional mock interviews and other self-development activities for students. Interestingly, the students that took advantage of them tended to be the ones that already had fairly good skills. The students that really needed help did not participate. (We have since made the activities mandatory.) I would urge all students to take advantage of all opportunities to improve their soft skills. Do not dismiss them as unimportant.

What are other important skills? The book discusses the regulatory environment for derivatives which changed a lot following the 2008 financial crisis. Make sure you understand the issues and are familiar with the latest developments. You should also use a derivatives course to help develop your critical thinking skills. Ask questions in class and do not be afraid to express an opinion about an issue.

A potential employer will want to be convinced that you can work well with others. While at university you will be involved in many group projects and should take this opportunity to develop good collaboration skills. You may find some members of your group difficult to work with, but this is also likely to be true in your first full-time job. Go to an interview prepared to talk about your experiences working with other students.

In addition to quant skills and knowledge of derivatives, I have mentioned that communication skills, the ability to work collaboratively, and critical thinking are soft skills that you should try and develop to make sure you get that first job. Another I might add is social responsibility. It is not an accident that most successful corporate executives are actively involved in community activities. Be prepared to talk about sustainable finance, which is an aspect of social responsibility and becoming an increasingly important area within finance.

ACKNOWLEDGMENTS

Many people have played a part in the development of successive editions of this book. Indeed, the list of people who have provided me with feedback on the book is now so long that it is not possible to mention everyone. I have benefited from the advice of many academics who have taught from the book and from the comments of many derivatives practitioners. I would like to thank the students in my courses at the University of Toronto who have made many suggestions on how the material can be improved. Eddie Mizzi from The Geometric Press did an excellent job editing the final manuscript and handling page composition. Emilio Barone from Luiss Guido Carli University in Rome provided many detailed comments. Andrés Olivé provided valuable research assistance.

Alan White, a colleague at the University of Toronto, deserves a special acknowledgment. Alan and I have been carrying out joint research and consulting in the areas of derivatives and risk management for over 30 years. During that time, we have spent many hours discussing key issues. Many of the new ideas in this book, and many of the new ways used to explain old ideas, are as much Alan's as mine. Alan has done most of the development work on the DerivaGem software.

Special thanks are due to the many people at Pearson I have worked with for over 30 years. Those who have worked with me on the 11th edition include Neeraj Bhalla,

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Sugandh Juneja, and Emily Biberger. I would like to thank them for their enthusiasm, advice, and encouragement.

I welcome comments on the book from readers. My e-mail address is:

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John Hull

About the Author

John Hull is the Maple Financial Professor of Derivatives and Risk Management at the Joseph L. Rotman School of Management, University of Toronto. He was in 2016 awarded the title of University Professor (an honor granted to only 2% of faculty at the University of Toronto). He is an internationally recognized authority on derivatives and risk management and has many publications in this area. His work has an applied focus. He has acted as consultant to many financial institutions throughout the world and has won many teaching awards, including the University of Toronto's prestigious Northrop Frye award. His research and teaching activities include risk management, regulation, and machine learning, as well as derivatives. He is co-director of Rotman's Master of Finance and Master of Financial Risk Management programs.





In the last 40 years, derivatives have become increasingly important in finance. Futures and options are actively traded on many exchanges throughout the world. Many different types of forward contracts, swaps, options, and other derivatives are entered into by financial institutions, fund managers, and corporate treasurers in the over-the-counter market. Derivatives are added to bond issues, used in executive compensation plans, embedded in capital investment opportunities, used to transfer risks in mortgages from the original lenders to investors, and so on. We have now reached the stage where those who work in finance, and many who work outside finance, need to understand how derivatives work, how they are used, and how they are priced.

Whether you love derivatives or hate them, you cannot ignore them! The derivatives market is huge—much bigger than the stock market when measured in terms of underlying assets. The value of the assets underlying outstanding derivatives transactions is several times the world gross domestic product. As we shall see in this chapter, derivatives can be used for hedging or speculation or arbitrage. They can transfer a wide range of risks in the economy from one entity to another.

A *derivative* involves two parties agreeing to a future transaction. Its value depends on (or derives from) the values of other underlying variables. Very often the variables underlying derivatives are the prices of traded assets. A stock option, for example, is a derivative whose value is dependent on the price of a stock. However, derivatives can be dependent on almost any variable, from the price of hogs to the amount of snow falling at a certain ski resort.

Since the first edition of this book was published in 1988 there have been many developments in derivatives markets. For example:

- Many new instruments such as credit derivatives, electricity derivatives, weather derivatives, and insurance derivatives have been developed.
- Many new types of interest rate, foreign exchange, and equity derivatives now trade.
- There have been many new ideas in risk management and risk measurement.
- Real option methods for capital investment appraisal have been developed.
- The financial crisis of 2008 occurred, with derivatives (perhaps unfairly) getting much of the blame.

 Many regulations affecting the over-the-counter derivatives market have been introduced.

- The "risk-free" discount rate used to value derivatives has changed and the decision has been taken to phase out LIBOR.
- Derivatives dealers now adjust the way they price derivatives to allow for credit risks, funding costs, and capital requirements.
- Collateral and credit issues are now given much more attention and have led to changes in the way derivatives are traded.
- Machine learning is now becoming widely used for managing derivatives portfolios.

The book has evolved to keep up to date with these developments. For example: the 2008 financial crisis is discussed in Chapter 8; changes in the interest rates used for derivatives pricing are discussed in Chapter 4; valuation adjustments are covered in Chapter 9; real options are explained in Chapter 36; credit derivatives are covered in Chapter 25; energy, weather, and insurance derivatives are covered in Chapter 35. Machine learning applications are discussed at various points in the book.

In this opening chapter, we take a first look at derivatives markets and how they are changing. We contrast exchange-traded and over-the-counter derivatives markets and review recent regulatory changes affecting the markets. We describe forward, futures, and options markets and provide examples of how they are used by hedgers, speculators, and arbitrageurs. Later in the book we will elaborate on many of the points made in this chapter.

1.1 EXCHANGE-TRADED MARKETS

A derivatives exchange is a market where individuals and companies trade standardized contracts that have been defined by the exchange. Derivatives exchanges have existed for a long time. The Chicago Board of Trade (CBOT) was established in 1848 to bring farmers and merchants together. Initially its main task was to standardize the quantities and qualities of the grains that were traded. Within a few years, the first futures-type contract was developed. It was known as a *to-arrive contract*. Speculators soon became interested in the contract and found trading the contract to be an attractive alternative to trading the grain itself. A rival futures exchange, the Chicago Mercantile Exchange (CME), was established in 1919. Now futures exchanges exist all over the world. (See table at the end of the book.) The CME and CBOT have merged to form the CME Group (www.cmegroup.com), which also includes the New York Mercantile Exchange (NYMEX), and the Kansas City Board of Trade (KCBT).

The Chicago Board Options Exchange (CBOE, www.cboe.com) started trading call option contracts on 16 stocks in 1973. Options had traded prior to 1973, but the CBOE succeeded in creating an orderly market with well-defined contracts. Put option contracts started trading on the exchange in 1977. The CBOE now trades options on thousands of stocks and many different stock indices. Like futures, options have proved to be very popular contracts. Many other exchanges throughout the world now trade

options. (See table at the end of the book.) The underlying assets include foreign currencies and futures contracts as well as stocks and stock indices.

Once two traders have agreed to trade a product offered by an exchange, it is handled by the exchange clearing house. This stands between the two traders and manages the risks. Suppose, for example, that trader A enters into a futures contract to buy 100 ounces of gold from trader B in six months for \$1,750 per ounce. The result of this trade will be that A has a contract to buy 100 ounces of gold from the clearing house at \$1,750 per ounce in six months and B has a contract to sell 100 ounces of gold to the clearing house for \$1,750 per ounce in six months. The advantage of this arrangement is that traders do not have to worry about the creditworthiness of the people they are trading with. The clearing house takes care of credit risk by requiring each of the two traders to deposit funds (known as margin) with the clearing house to ensure that they will live up to their obligations. Margin requirements and the operation of clearing houses are discussed in more detail in Chapter 2.

Electronic Markets

Traditionally derivatives exchanges have used what is known as the *open outcry system*. This involves traders physically meeting on the floor of the exchange, shouting, and using a complicated set of hand signals to indicate the trades they would like to carry out. Exchanges have largely replaced the open outcry system by *electronic trading*. This involves traders entering their desired trades at a keyboard and a computer being used to match buyers and sellers. The open outcry system has its advocates, but, as time passes, it is becoming less and less used.

Electronic trading has led to a growth in high-frequency trading. This involves the use of algorithms to initiate trades, often without human intervention, and has become an important feature of derivatives markets.

1.2 OVER-THE-COUNTER MARKETS

Not all derivatives trading is on exchanges. Many trades take place in the *over-the-counter* (OTC) market. Banks, other large financial institutions, fund managers, and corporations are the main participants in OTC derivatives markets. Once an OTC trade has been agreed, the two parties can either present it to a central counterparty (CCP) or clear the trade bilaterally. A CCP is like an exchange clearing house. It stands between the two parties to the derivatives transaction so that one party does not have to bear the risk that the other party will default. When trades are cleared bilaterally, the two parties have usually signed an agreement covering all their transactions with each other. The issues covered in the agreement include the circumstances under which outstanding transactions can be terminated, how settlement amounts are calculated in the event of a termination, and how the collateral (if any) that must be posted by each side is calculated. CCPs and bilateral clearing are discussed in more detail in Chapter 2.

Large banks often act as market makers for the more commonly traded instruments. This means that they are always prepared to quote a bid price (at which they are prepared to take one side of a derivatives transaction) and an ask price (at which they are prepared to take the other side).

Business Snapshot 1.1 The Lehman Bankruptcy

On September 15, 2008, Lehman Brothers filed for bankruptcy. This was the largest bankruptcy in U.S. history and its ramifications were felt throughout derivatives markets. Almost until the end, it seemed as though there was a good chance that Lehman would survive. A number of companies (e.g., the Korean Development Bank, Barclays Bank in the United Kingdom, and Bank of America) expressed interest in buying it, but none of these was able to close a deal. Many people thought that Lehman was "too big to fail" and that the U.S. government would have to bail it out if no purchaser could be found. This proved not to be the case.

How did this happen? It was a combination of high leverage, risky investments, and liquidity problems. Commercial banks that take deposits are subject to regulations on the amount of capital they must keep. Lehman was an investment bank and not subject to these regulations. By 2007, its leverage ratio had increased to 31:1, which means that a 3–4% decline in the value of its assets would wipe out its capital. Dick Fuld, Lehman's Chairman and Chief Executive Officer, encouraged an aggressive deal-making, risk-taking culture. He is reported to have told his executives: "Every day is a battle. You have to kill the enemy." The Chief Risk Officer at Lehman was competent, but did not have much influence and was even removed from the executive committee in 2007. The risks taken by Lehman included large positions in the instruments created from subprime mortgages, which will be described in Chapter 8. Lehman funded much of its operations with short-term debt. When there was a loss of confidence in the company, lenders refused to renew this funding, forcing it into bankruptcy.

Lehman was very active in the over-the-counter derivatives markets. It had over a million transactions outstanding with about 8,000 different counterparties. Lehman's counterparties were often required to post collateral and this collateral had in many cases been used by Lehman for various purposes. Litigation aimed at determining who owes what to whom continued for many years after the bankruptcy filing.

Prior to the financial crisis, which started in 2007 and is discussed in some detail in Chapter 8, OTC derivatives markets were largely unregulated. Following the financial crisis and the failure of Lehman Brothers (see Business Snapshot 1.1), we have seen the development of many new regulations affecting the operation of OTC markets. The main objectives of the regulations are to improve the transparency of OTC markets and reduce systemic risk (see Business Snapshot 1.2). The over-the-counter market in some respects is being forced to become more like the exchange-traded market. Three important changes are:

- 1. Standardized OTC derivatives between two financial institutions in the United States must, whenever possible, be traded on what are referred to a *swap execution facilities* (SEFs). These are platforms similar to exchanges where market participants can post bid and ask quotes and where market participants can trade by accepting the quotes of other market participants.
- **2.** There is a requirement in most parts of the world that a CCP be used for most standardized derivatives transactions between financial institutions.
- **3.** All trades must be reported to a central repository.

Business Snapshot 1.2 Systemic Risk

Systemic risk is the risk that a default by one financial institution will create a "ripple effect" that leads to defaults by other financial institutions and threatens the stability of the financial system. There are huge numbers of over-the-counter transactions between banks. If Bank A fails, Bank B may take a huge loss on the transactions it has with Bank A. This in turn could lead to Bank B failing. Bank C that has many outstanding transactions with both Bank A and Bank B might then take a large loss and experience severe financial difficulties; and so on.

The financial system has survived defaults such as Drexel in 1990 and Lehman Brothers in 2008, but regulators continue to be concerned. During the market turmoil of 2007 and 2008, many large financial institutions were bailed out, rather than being allowed to fail, because governments were concerned about systemic risk.

Market Size

Both the over-the-counter and the exchange-traded market for derivatives are huge. The number of derivatives transactions per year in OTC markets is smaller than in exchange-traded markets, but the average size of the transactions is much greater. Although the statistics that are collected for the two markets are not exactly comparable, it is clear that the volume of business in the over-the-counter market is much larger than in the exchange-traded market. The Bank for International Settlements (www.bis.org) started collecting statistics on the markets in 1998. Figure 1.1 compares (a) the estimated total principal amounts underlying transactions that were outstanding in the over-the-counter markets between June 1998 and December 2019 and (b) the estimated total value of the assets underlying exchange-traded contracts during the same period. Using these measures, the size of the over-the-counter market in December 2019 was \$558.5 trillion

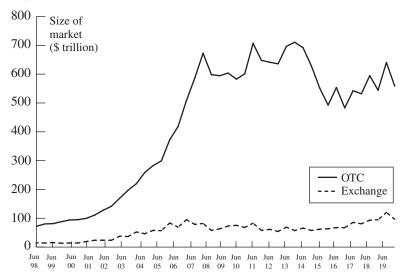


Figure 1.1 Size of over-the-counter and exchange-traded derivatives markets.

and the size of the exchange-traded market was \$96.5 trillion. Figure 1.1 shows that the OTC market grew rapidly up to 2007, but has seen very little net growth since then. One reason for the lack of growth is the popularity of *compression*. This is a procedure where two or more counterparties restructure transactions with each other with the result that the underlying principal is reduced.

In interpreting Figure 1.1, we should bear in mind that the principal underlying an over-the-counter transaction is not the same as its value. An example of an over-the-counter transaction is an agreement to buy 100 million U.S. dollars with British pounds at a predetermined exchange rate in 1 year. The total principal amount underlying this transaction is \$100 million. However, the value of the transaction might be only \$1 million. The Bank for International Settlements estimates the gross market value of all over-the-counter transactions outstanding in December 2019 to be about \$11.6 trillion.²

1.3 FORWARD CONTRACTS

A relatively simple derivative is a *forward contract*. It is an agreement to buy or sell an asset at a certain future time for a certain price. It can be contrasted with a *spot contract*, which is an agreement to buy or sell an asset almost immediately. A forward contract is traded in the over-the-counter market—usually between two financial institutions or between a financial institution and one of its clients.

One of the parties to a forward contract assumes a *long position* and agrees to buy the underlying asset on a certain specified future date for a certain specified price. The other party assumes a *short position* and agrees to sell the asset on the same date for the same price.

Forward contracts on foreign exchange are very popular. Most large banks employ both spot and forward foreign-exchange traders. As we shall see in Chapter 5, there is a relationship between forward prices, spot prices, and interest rates in the two currencies. Table 1.1 provides quotes for the exchange rate between the British pound (GBP) and the U.S. dollar (USD) that might be made by a large international bank on May 21, 2020. The quote is for the number of USD per GBP. The first row indicates that the

Table 1.1 Spot and forward quotes for the exchange rate between USD and GBP on May 21, 2020 (GBP = British pound; USD = U.S. dollar; quote is number of USD per GBP).

	Bid	Ask
Spot	1.2217	1.2220
1-month forward	1.2218	1.2222
3-month forward	1.2220	1.2225
6-month forward	1.2224	1.2230

¹ When a CCP stands between two sides in an OTC transaction, two transactions are considered to have been created for the purposes of the BIS statistics.

² A contract that is worth \$1 million to one side and -\$1 million to the other side would be counted as having a gross market value of \$1 million.

bank is prepared to buy GBP (also known as sterling) in the spot market (i.e., for virtually immediate delivery) at the rate of \$1.2217 per GBP and sell sterling in the spot market at \$1.2220 per GBP. The second, third, and fourth rows indicate that the bank is prepared to buy sterling in 1, 3, and 6 months at \$1.2218, \$1.2220, and \$1.2224 per GBP, respectively, and to sell sterling in 1, 3, and 6 months at \$1.2222, \$1.2225, and \$1.2230 per GBP, respectively.

Forward contracts can be used to hedge foreign currency risk. Suppose that, on May 21, 2020, the treasurer of a U.S. corporation knows that the corporation will pay £1 million in 6 months (i.e., on November 21, 2020) and wants to hedge against exchange rate moves. Using the quotes in Table 1.1, the treasurer can agree to buy £1 million 6 months forward at an exchange rate of 1.2230. The corporation then has a long forward contract on GBP. It has agreed that on November 21, 2020, it will buy £1 million from the bank for \$1.2230 million. The bank has a short forward contract on GBP. It has agreed that on November 21, 2020, it will sell £1 million for \$1.2230 million. Both sides have made a binding commitment.

Payoffs from Forward Contracts

Consider the position of the corporation in the trade we have just described. What are the possible outcomes? The forward contract obligates the corporation to buy £1 million for \$1,223,000. If the spot exchange rate rose to, say, 1.3000, at the end of the 6 months, the forward contract would be worth \$77,000 (= \$1,300,000 - \$1,223,000) to the corporation. It would enable £1 million to be purchased at an exchange rate of 1.2230 rather than 1.3000. Similarly, if the spot exchange rate fell to 1.2000 at the end of the 6 months, the forward contract would have a negative value to the corporation of \$23,000 because it would lead to the corporation paying \$23,000 more than the market price for the sterling.

In general, the payoff from a long position in a forward contract on one unit of an asset is

$$S_T - K$$

where K is the delivery price and S_T is the spot price of the asset at maturity of the contract. This is because the holder of the contract is obligated to buy an asset worth S_T for K. Similarly, the payoff from a short position in a forward contract on one unit of an asset is

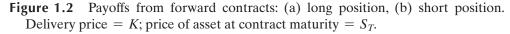
$$K - S_T$$

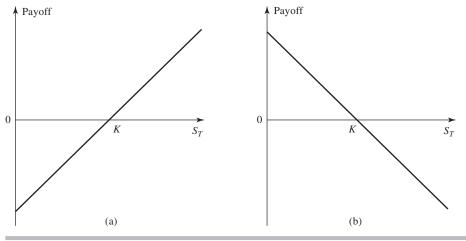
These payoffs can be positive or negative. They are illustrated in Figure 1.2. Because it costs nothing to enter into a forward contract, the payoff from the contract is also the trader's total gain or loss from the contract.

In the example just considered, K=1.2230 and the corporation has a long contract. When $S_T=1.3000$, the payoff is \$0.077 per £1; when $S_T=1.2000$, it is -\$0.023 per £1.

Forward Prices and Spot Prices

We shall be discussing in some detail the relationship between spot and forward prices in Chapter 5. For a quick preview of why the two are related, consider a stock that pays no dividend and is worth \$60. You can borrow or lend money for 1 year at 5%. What should the 1-year forward price of the stock be?





The answer is \$60 grossed up at 5% for 1 year, or \$63. If the forward price is more than this, say \$67, you could borrow \$60, buy one share of the stock, and sell it forward for \$67. After paying off the loan, you would net a profit of \$4 in 1 year. If the forward price is less than \$63, say \$58, an investor owning the stock as part of a portfolio would sell the stock for \$60 and enter into a forward contract to buy it back for \$58 in 1 year. The proceeds of investment would be invested at 5% to earn \$3. The investor would end up \$5 better off than if the stock were kept in the portfolio for the year.

1.4 FUTURES CONTRACTS

Like a forward contract, a futures contract is an agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. Unlike forward contracts, futures contracts are normally traded on an exchange. To make trading possible, the exchange specifies certain standardized features of the contract. As the two parties to the contract do not necessarily know each other, the exchange clearing house stands between them as mentioned earlier.

Two large exchanges on which futures contracts are traded are the Chicago Board of Trade (CBOT) and the Chicago Mercantile Exchange (CME), which have now merged to form the CME Group. On these and other exchanges throughout the world, a very wide range of commodities and financial assets form the underlying assets in the various contracts. The commodities include pork bellies, live cattle, sugar, wool, lumber, copper, aluminum, gold, and tin. The financial assets include stock indices, currencies, and Treasury bonds. Futures prices are regularly reported in the financial press. Suppose that, on September 1, the December futures price of gold is quoted as \$1,750. This is the price, exclusive of commissions, at which traders can agree to buy or sell gold for December delivery. It is determined in the same way as other prices (i.e., by the laws of supply and demand). If more traders want to go long than to go short, the price goes up; if the reverse is true, then the price goes down.

Further details on issues such as margin requirements, daily settlement procedures, delivery procedures, bid-ask spreads, and the role of the exchange clearing house are given in Chapter 2.

1.5 OPTIONS

Options are traded both on exchanges and in the over-the-counter market. There are two types of option. A *call option* gives the holder the right to buy the underlying asset by a certain date for a certain price. A *put option* gives the holder the right to sell the underlying asset by a certain date for a certain price. The price in the contract is known as the *exercise price* or *strike price*; the date in the contract is known as the *expiration date* or *maturity*. *American options* can be exercised at any time up to the expiration date. *European options* can be exercised only on the expiration date itself.³ Most of the options that are traded on exchanges are American. In the exchange-traded equity option market, one contract is usually an agreement to buy or sell 100 shares. European options are generally easier to analyze than American options, and some of the properties of an American option are frequently deduced from those of its European counterpart.

It should be emphasized that an option gives the holder the right to do something. The holder does not have to exercise this right. This is what distinguishes options from forwards and futures, where the holder is obligated to buy or sell the underlying asset. Whereas it costs nothing to enter into a forward or futures contract, except for margin requirements which will be discussed in Chapter 2, there is a cost to acquiring an option.

The largest exchange in the world for trading stock options is the Chicago Board Options Exchange (CBOE; www.cboe.com). Table 1.2 gives the bid and ask quotes for some of the call options trading on Apple (ticker symbol: AAPL), on May 21, 2020. Table 1.3 does the same for put options trading on Apple on that date. The quotes are taken from the CBOE website. The Apple stock price at the time of the quotes was bid 316.23, ask 316.50. The bid–ask spread for an option (as a percent of the price) is usually

Table 1.2	Prices of call options on Apple, May 21, 2020; stock price: bid \$316.23, ask
\$316.50 (Source: CBOE).

Strike price	June 2020		September 2020		December 2020	
(\$)	Bid	Ask	Bid	Ask	Bid	Ask
290	29.80	30.85	39.35	40.40	46.20	47.60
300	21.55	22.40	32.50	33.90	40.00	41.15
310	14.35	15.30	26.35	27.25	34.25	35.65
320	8.65	9.00	20.45	21.70	28.65	29.75
330	4.20	5.00	15.85	16.25	23.90	24.75
340	1.90	2.12	11.35	12.00	19.50	20.30

³ Note that the terms *American* and *European* do not refer to the location of the option or the exchange. Some options trading on North American exchanges are European.

	Strike price	June 2020		September 2020		December 2020	
	(\$)	Bid	Ask	Bid	Ask	Bid	Ask
Ī	290	3.00	3.30	12.70	13.65	20.05	21.30
	300	4.80	5.20	15.85	16.85	23.60	24.90
	310	7.15	7.85	19.75	20.50	28.00	28.95
	320	11.25	12.05	24.05	24.80	32.45	33.35
	330	17.10	17.85	28.75	29.85	37.45	38.40
	340	24.40	25.45	34.45	35.65	42.95	44.05

Table 1.3 Prices of put options on Apple, May 21, 2020; stock price: bid \$316.23, ask \$316.50 (*Source*: CBOE).

much greater than that for the underlying stock and depends on the volume of trading. The option strike prices in Tables 1.2 and 1.3 are \$290, \$300, \$310, \$320, \$330, and \$340. The maturities are June 2020, September 2020, and December 2020. The precise expiration day is the third Friday of the expiration month. The June options expire on June 19, 2020, the September options on September 18, 2020, and the December options on December 18, 2020.

The tables illustrate a number of properties of options. The price of a call option decreases as the strike price increases, while the price of a put option increases as the strike price increases. Both types of option tend to become more valuable as their time to maturity increases. These properties of options will be discussed further in Chapter 11.

Suppose a trader instructs a broker to buy one December call option contract on Apple with a strike price of \$340. The broker will relay these instructions to a trader at the CBOE and the deal will be done. The (ask) price indicated in Table 1.2 is \$20.30. This is the price for an option to buy one share. In the United States, an option contract is a contract to buy or sell 100 shares. Therefore, the trader must arrange for \$2,030 to be remitted to the exchange through the broker. The exchange will then arrange for this amount to be passed on to the party on the other side of the transaction.

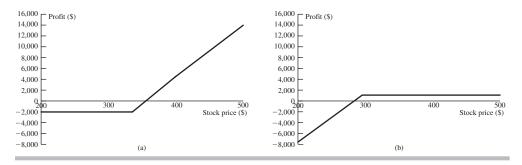
In our example, the trader has obtained at a cost of \$2,030 the right to buy 100 Apple shares for \$340 each. If the price of Apple does not rise above \$340 by December 18, 2020, the option is not exercised and the trader loses \$2,030.⁴ But if Apple does well and the option is exercised when the bid price for the stock is \$400, the trader is able to buy 100 shares at \$340 and immediately sell them for \$400 for a profit of \$6,000, or \$3,970 when the initial cost of the option contract is taken into account.⁵

An alternative trade would be to sell one September put option contract with a strike price of \$290 at the bid price of \$12.70. The trader receives $100 \times 12.70 = \$1,270$. If the Apple stock price stays above \$290, the option is not exercised and the trader makes a \$1,270 profit. However, if stock price falls and the option is exercised when the stock price is \$250, there is a loss. The trader must buy 100 shares at \$290 when they are worth only \$250. This leads to a loss of \$4,000, or \$2,730 when the initial amount received for the option contract is taken into account.

⁴ The calculations here ignore any commissions paid by the trader.

⁵ The calculations here ignore the effect of discounting. The \$6,000 should be discounted from the time of exercise to the purchase date when calculating the profit.

Figure 1.3 Net profit from (a) purchasing a contract consisting of 100 Apple December call options with a strike price of \$340 and (b) selling a contract consisting of 100 Apple September put options with a strike price of \$290.



The stock options trading on the CBOE are American. If we assume for simplicity that they are European, so that they can be exercised only at maturity, the trader's profit as a function of the final stock price for the two trades we have considered is shown in Figure 1.3.

Further details about the operation of options markets and how prices such as those in Tables 1.2 and 1.3 are determined by traders are given in later chapters. At this stage we note that there are four types of participants in options markets:

- 1. Buyers of calls
- 2. Sellers of calls
- 3. Buyers of puts
- 4. Sellers of puts.

Buyers are referred to as having *long positions*; sellers are referred to as having *short positions*. Selling an option is also known as *writing the option*.

1.6 TYPES OF TRADERS

Derivatives markets have been outstandingly successful. The main reason is that they have attracted many different types of traders and have a great deal of liquidity. When a trader wants to take one side of a contract, there is usually no problem in finding someone who is prepared to take the other side.

Three broad categories of traders can be identified: hedgers, speculators, and arbitrageurs. Hedgers use derivatives to reduce the risk that they face from potential future movements in a market variable. Speculators use them to bet on the future direction of a market variable. Arbitrageurs take offsetting positions in two or more instruments to lock in a profit. As described in Business Snapshot 1.3, hedge funds have become big users of derivatives for all three purposes.

In the next few sections, we will consider the activities of each type of trader in more detail.

Business Snapshot 1.3 Hedge Funds

Hedge funds have become major users of derivatives for hedging, speculation, and arbitrage. They are similar to mutual funds in that they invest funds on behalf of clients. However, they accept funds only from professional fund managers or financially sophisticated individuals and do not publicly offer their securities. Mutual funds are subject to regulations requiring that the shares be redeemable at any time, that investment policies be disclosed, that the use of leverage be limited, and so on. Hedge funds are relatively free of these regulations. This gives them a great deal of freedom to develop sophisticated, unconventional, and proprietary investment strategies. The fees charged by hedge fund managers are dependent on the fund's performance and are relatively high—typically 1 to 2% of the amount invested plus 20% of the profits. Hedge funds have grown in popularity, with about \$2 trillion being invested in them throughout the world. "Funds of funds" have been set up to invest in a portfolio of hedge funds.

The investment strategy followed by a hedge fund manager often involves using derivatives to set up a speculative or arbitrage position. Once the strategy has been defined, the hedge fund manager must:

- 1. Evaluate the risks to which the fund is exposed
- 2. Decide which risks are acceptable and which will be hedged
- **3.** Devise strategies (usually involving derivatives) to hedge the unacceptable risks.

Here are some examples of the labels used for hedge funds together with the trading strategies followed:

Long/Short Equities: Purchase securities considered to be undervalued and short those considered to be overvalued in such a way that the exposure to the overall direction of the market is small.

Convertible Arbitrage: Take a long position in a thought-to-be-undervalued convertible bond combined with an actively managed short position in the underlying equity.

Distressed Securities: Buy securities issued by companies in, or close to, bankruptcy.

Emerging Markets: Invest in debt and equity of companies in developing or emerging countries and in the debt of the countries themselves.

Global Macro: Carry out trades that reflect anticipated global macroeconomic trends.

Merger Arbitrage: Trade after a possible merger or acquisition is announced so that a profit is made if the announced deal takes place.

1.7 HEDGERS

In this section we illustrate how hedgers can reduce their risks with forward contracts and options.

Hedging Using Forward Contracts

Suppose that it is May 21, 2020, and ImportCo, a company based in the United States, knows that it will have to pay £10 million on August 21, 2020, for goods it has

purchased from a British supplier. The GBP/USD exchange rate quotes made by a financial institution are shown in Table 1.1. ImportCo could hedge its foreign exchange risk by buying pounds (GBP) from the financial institution in the 3-month forward market at 1.2225. This would have the effect of fixing the price to be paid to the British exporter at \$12,225,000.

Consider next another U.S. company, which we will refer to as ExportCo, that is exporting goods to the United Kingdom and, on May 21, 2020, knows that it will receive £30 million 3 months later. ExportCo can hedge its foreign exchange risk by selling £30 million in the 3-month forward market at an exchange rate of 1.2220. This would have the effect of locking in the U.S. dollars to be realized for the sterling at \$36,660,000.

Note that a company might do better if it chooses not to hedge than if it chooses to hedge. Alternatively, it might do worse. Consider ImportCo. If the exchange rate is 1.2000 on August 21 and the company has not hedged, the £10 million that it has to pay will cost \$12,000,000, which is less than \$12,225,000. On the other hand, if the exchange rate is 1.3000, the £10 million will cost \$13,000,000—and the company will wish that it had hedged! The position of ExportCo if it does not hedge is the reverse. If the exchange rate in August proves to be less than 1.2220, the company will wish that it had hedged; if the rate is greater than 1.2220, it will be pleased that it has not done so.

This example illustrates a key aspect of hedging. The purpose of hedging is to reduce risk. There is no guarantee that the outcome with hedging will be better than the outcome without hedging.

Hedging Using Options

Options can also be used for hedging. Consider an investor who in May of a particular year owns 1,000 shares of a particular company. The share price is \$28 per share. The investor is concerned about a possible share price decline in the next 2 months and wants protection. The investor could buy ten July put option contracts on the

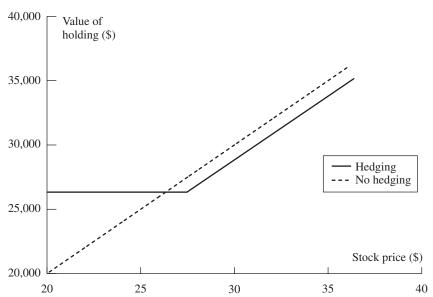


Figure 1.4 Value of the stock holding in 2 months with and without hedging.

company's stock with a strike price of \$27.50. Each contract is on 100 shares, so this would give the investor the right to sell a total of 1,000 shares for a price of \$27.50. If the quoted option price is \$1, then each option contract would cost $100 \times 100 \times 100$ and the total cost of the hedging strategy would be $10 \times 100 = 1,000$.

The strategy costs \$1,000 but guarantees that the shares can be sold for at least \$27.50 per share during the life of the option. If the market price of the stock falls below \$27.50, the options will be exercised, so that \$27,500 is realized for the entire holding. When the cost of the options is taken into account, the amount realized is \$26,500. If the market price stays above \$27.50, the options are not exercised and expire worthless. However, in this case the value of the holding is always above \$27,500 (or above \$26,500 when the cost of the options is taken into account). Figure 1.4 shows the net value of the portfolio (after taking the cost of the options into account) as a function of the stock price in 2 months. The dotted line shows the value of the portfolio assuming no hedging.

A Comparison

There is a fundamental difference between the use of forward contracts and options for hedging. Forward contracts are designed to neutralize risk by fixing the price that the hedger will pay or receive for the underlying asset. Option contracts, by contrast, provide insurance. They offer a way for investors to protect themselves against adverse price movements in the future while still allowing them to benefit from favorable price movements. Unlike forwards, options involve the payment of an up-front fee.

1.8 SPECULATORS

We now move on to consider how futures and options markets can be used by speculators. Whereas hedgers want to avoid exposure to adverse movements in the price of an asset, speculators wish to take a position in the market. Either they are betting that the price of the asset will go up or they are betting that it will go down.

Speculation Using Futures

Consider a U.S. speculator who in May thinks that the British pound will strengthen relative to the U.S. dollar over the next 2 months and is prepared to back that hunch to the tune of £250,000. One thing the speculator can do is purchase £250,000 in the spot market in the hope that the sterling can be sold later at a higher price. (The sterling once purchased would be kept in an interest-bearing account.) Another possibility is to take a long position in four CME July futures contracts on sterling. (Each futures contract is for the purchase of £62,500 in July.) Table 1.4 summarizes the two alternatives on the assumption that the current exchange rate is 1.2220 dollars per pound and the July futures price is 1.2223 dollars per pound. If the exchange rate turns out to be 1.3000 dollars per pound in July, the futures contract alternative enables the speculator to realize a profit of $(1.3000 - 1.2223) \times 250,000 = \$19,425$. The spot market alternative leads to 250,000 units of an asset being purchased for \$1.2220 in May and sold for \$1.3000 in July, so that a profit of $(1.3000 - 1.2220) \times 250,000 = \$19,500$ is made. If the exchange rate falls to 1.2000 dollars per pound, the futures contract gives rise to a $(1.2223 - 1.2000) \times 250,000 = \$5,575$ loss, whereas the spot market alternative gives

Table 1.4 Speculation using spot and futures contracts. One futures contract is on £62,500. Initial margin on four futures contracts = \$20,000.

	Possible trades			
	Buy £250,000 Buy 4 futures co			
	$Spot\ price = 1.2220$	$Futures\ price=1.2223$		
Investment	\$305,500	\$20,000		
Profit if July spot $= 1.3000$	\$19,500	\$19,425		
Profit if July spot $= 1.2000$	-\$5,500	-\$5,575		

rise to a loss of $(1.2220 - 1.2000) \times 250,000 = \$5,500$. The futures market alternative appears to give rise to slightly worse outcomes for both scenarios. But this is because the calculations do not reflect the interest that is earned or paid.

What then is the difference between the two alternatives? The first alternative of buying sterling requires an up-front investment of $250,000 \times 1.2220 = \$305,500$. In contrast, the second alternative requires only a small amount of cash to be deposited by the speculator in what is termed a "margin account". (The operation of margin accounts is explained in Chapter 2.) In Table 1.4, the initial margin requirement is assumed to be \$5,000 per contract, or \$20,000 in total. The futures market allows the speculator to obtain leverage. With a relatively small initial outlay, a large speculative position can be taken.

Speculation Using Options

Options can also be used for speculation. Suppose that it is October and a speculator considers that a stock is likely to increase in value over the next 2 months. The stock price is currently \$20, and a 2-month call option with a \$22.50 strike price is currently selling for \$1. Table 1.5 illustrates two possible alternatives, assuming that the speculator is willing to invest \$2,000. One alternative is to purchase 100 shares; the other involves the purchase of 2,000 call options (i.e., 20 call option contracts). Suppose that the speculator's hunch is correct and the price of the stock rises to \$27 by December. The first alternative of buying the stock yields a profit of

$$100 \times (\$27 - \$20) = \$700$$

However, the second alternative is far more profitable. A call option on the stock with a strike price of \$22.50 gives a payoff of \$4.50, because it enables something worth \$27 to

Table 1.5 Comparison of profits from two alternative strategies for using \$2,000 to speculate on a stock worth \$20 in October.

	December stock price		
Speculator's strategy	\$15	\$27	
Buy 100 shares	-\$500	\$700	
Buy 2,000 call options	-\$2,000	\$7,000	

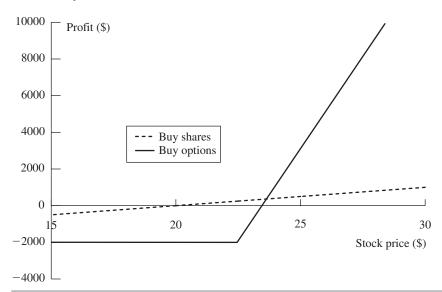


Figure 1.5 Profit or loss from two alternative strategies for speculating on a stock currently worth \$20.

be bought for \$22.50. The total payoff from the 2,000 options that are purchased under the second alternative is

$$2,000 \times \$4.50 = \$9,000$$

Subtracting the original cost of the options yields a net profit of

$$$9,000 - $2,000 = $7,000$$

The options strategy is, therefore, 10 times more profitable than directly buying the stock. Options also give rise to a greater potential loss. Suppose the stock price falls to \$15 by December. The first alternative of buying stock yields a loss of

$$100 \times (\$20 - \$15) = \$500$$

Because the call options expire without being exercised, the options strategy would lead to a loss of \$2,000—the original amount paid for the options. Figure 1.5 shows the profit or loss from the two strategies as a function of the stock price in 2 months.

Options like futures provide a form of leverage. For a given investment, the use of options magnifies the financial consequences. Good outcomes become very good, while bad outcomes result in the whole initial investment being lost.

A Comparison

Futures and options are similar instruments for speculators in that they both provide a way in which a type of leverage can be obtained. However, there is an important difference between the two. When a speculator uses futures, the potential loss as well as the potential gain is very large. When options are purchased, no matter how bad things get, the speculator's loss is limited to the amount paid for the options.

1.9 ARBITRAGEURS

Arbitrageurs are a third important group of participants in futures, forward, and options markets. Arbitrage involves locking in a riskless profit by simultaneously entering into transactions in two or more markets. In later chapters we will see how arbitrage is sometimes possible when the futures price of an asset gets out of line with its spot price. We will also examine how arbitrage can be used in options markets. This section illustrates the concept of arbitrage with a very simple example.

Let us consider a stock that is traded on both the New York Stock Exchange (www.nyse.com) and the London Stock Exchange (www.londonstockexchange.com). Suppose that the stock price is \$120 in New York and £100 in London at a time when the exchange rate is \$1.2300 per pound. An arbitrageur could simultaneously buy 100 shares of the stock in New York and sell them in London to obtain a risk-free profit of

$$100 \times [(\$1.23 \times 100) - \$120]$$

or \$300 in the absence of transactions costs. Transactions costs would probably eliminate the profit for a small trader. However, a large investment bank faces very low transactions costs in both the stock market and the foreign exchange market. It would find the arbitrage opportunity very attractive and would try to take as much advantage of it as possible.

Arbitrage opportunities such as the one just described cannot last for long. As arbitrageurs buy the stock in New York, the forces of supply and demand will cause the dollar price to rise. Similarly, as they sell the stock in London, the sterling price will be driven down. Very quickly the two prices will become equivalent at the current exchange rate. Indeed, the existence of profit-hungry arbitrageurs makes it unlikely that a major disparity between the sterling price and the dollar price could ever exist in the first place. Generalizing from this example, we can say that the very existence of arbitrageurs means that in practice only very small arbitrage opportunities are observed in the prices that are quoted in most financial markets. In this book most of the arguments concerning futures prices, forward prices, and the values of option contracts will be based on the assumption that no arbitrage opportunities exist.

1.10 DANGERS

Derivatives are very versatile instruments. As we have seen, they can be used for hedging, for speculation, and for arbitrage. It is this very versatility that can cause problems. Sometimes traders who have a mandate to hedge risks or follow an arbitrage strategy become (consciously or unconsciously) speculators. The results can be disastrous. One example of this is provided by the activities of Jérôme Kerviel at Société Général (see Business Snapshot 1.4).

To avoid the sort of problems Société Général encountered, it is very important for both financial and nonfinancial corporations to set up controls to ensure that derivatives are being used for their intended purpose. Risk limits should be set and the activities of traders should be monitored daily to ensure that these risk limits are adhered to.

Unfortunately, even when traders follow the risk limits that have been specified, big mistakes can happen. Some of the activities of traders in the derivatives market during

Business Snapshot 1.4 SocGen's Big Loss in 2008

Derivatives are very versatile instruments. They can be used for hedging, speculation, and arbitrage. One of the risks faced by a company that trades derivatives is that an employee who has a mandate to hedge or to look for arbitrage opportunities may become a speculator.

Jérôme Kerviel joined Société Général (SocGen) in 2000 to work in the compliance area. In 2005, he was promoted and became a junior trader in the bank's Delta One products team. He traded equity indices such as the German DAX index, the French CAC 40, and the Euro Stoxx 50. His job was to look for arbitrage opportunities. These might arise if a futures contract on an equity index was trading for a different price on two different exchanges. They might also arise if equity index futures prices were not consistent with the prices of the shares constituting the index. (This type of arbitrage is discussed in Chapter 5.)

Kerviel used his knowledge of the bank's procedures to speculate while giving the appearance of arbitraging. He took big positions in equity indices and created fictitious trades to make it appear that he was hedged. In reality, he had large bets on the direction in which the indices would move. The size of his unhedged position grew over time to tens of billions of euros.

In January 2008, his unauthorized trading was uncovered by SocGen. Over a three-day period, the bank unwound his position for a loss of 4.9 billion euros. This was at the time the biggest loss created by fraudulent activity in the history of finance. (Later in the year, a much bigger loss from Bernard Madoff's Ponzi scheme came to light.)

Rogue trader losses were not unknown at banks prior to 2008. For example, in the 1990s, Nick Leeson, who worked at Barings Bank, had a mandate similar to that of Jérôme Kerviel. His job was to arbitrage between Nikkei 225 futures quotes in Singapore and Osaka. Instead he found a way to make big bets on the direction of the Nikkei 225 using futures and options, losing \$1 billion and destroying the 200-year old bank in the process. In 2002, it was found that John Rusnak at Allied Irish Bank had lost \$700 million from unauthorized foreign exchange trading. The lessons from these losses are that it is important to define unambiguous risk limits for traders and then to monitor what they do very carefully to make sure that the limits are adhered to.

the period leading up to the start of the financial crisis in July 2007 proved to be much riskier than they were thought to be by the financial institutions they worked for. As will be discussed in Chapter 8, house prices in the United States had been rising fast. Most people thought that the increases would continue—or, at worst, that house prices would simply level off. Very few were prepared for the steep decline that actually happened. Furthermore, very few were prepared for the high correlation between mortgage default rates in different parts of the country. Some risk managers did express reservations about the exposures of the companies for which they worked to the U.S. real estate market. But, when times are good (or appear to be good), there is an unfortunate tendency to ignore risk managers and this is what happened at many financial institutions during the 2006–2007 period. The key lesson from the financial crisis is that financial institutions should always be dispassionately asking "What can go wrong?", and they should follow that up with the question "If it does go wrong, how much will we lose?"

SUMMARY

One of the exciting developments in finance over the last 40 years has been the growth of derivatives markets. In many situations, both hedgers and speculators find it more attractive to trade a derivative on an asset than to trade the asset itself. Some derivatives are traded on exchanges; others are traded by financial institutions, fund managers, and corporations in the over-the-counter market, or added to new issues of debt and equity securities. Much of this book is concerned with the valuation of derivatives. The aim is to present a unifying framework within which all derivatives—not just options or futures—can be valued.

In this chapter we have taken a first look at forward, futures, and option contracts. A forward or futures contract involves an obligation to buy or sell an asset at a certain time in the future for a certain price. There are two types of options: calls and puts. A call option gives the holder the right to buy an asset by a certain date for a certain price. A put option gives the holder the right to sell an asset by a certain date for a certain price. Forwards, futures, and options trade on a wide range of different underlying assets.

The success of derivatives can be attributed to their versatility. They can be used by: hedgers, speculators, and arbitrageurs. Hedgers are in the position where they face risk associated with the price of an asset. They use derivatives to reduce or eliminate this risk. Speculators wish to bet on future movements in the price of an asset. They use derivatives to get extra leverage. Arbitrageurs are in business to take advantage of a discrepancy between prices in two different markets. If, for example, they see the futures price of an asset getting out of line with the cash price, they will take offsetting positions in the two markets to lock in a profit.

FURTHER READING

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- Zingales, L., "Causes and Effects of the Lehman Bankruptcy," Testimony before Committee on Oversight and Government Reform, United States House of Representatives, October 6, 2008.

Short Concept Questions

- 1.1. What is a derivative?
- 1.2. Explain the two main ways that derivatives trade?
- 1.3. What are the changes that have taken place in the regulation of the OTC market since the 2008 financial crisis?
- 1.4. Which is bigger, the exchange-traded market or the OTC market?

1.5. What is the difference between a long forward position and a short forward position?

- 1.6. How do (a) forward and (b) futures contracts trade?
- 1.7. What is the difference between a forward contract to buy an asset at \$30 and a call option to buy the same asset for \$30?
- 1.8. Explain carefully the difference between hedging, speculation, and arbitrage.
- 1.9. Explain the meaning of bid and ask quotes.
- 1.10. What gave rise to Jerome Kerviel's big loss?

Practice Questions

- 1.11. Explain carefully the difference between selling a call option and buying a put option.
- 1.12. An investor enters into a short forward contract to sell 100,000 British pounds for U.S. dollars at an exchange rate of 1.3000 USD per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is (a) 1.2900 and (b) 1.3200?
- 1.13. A trader enters into a short cotton futures contract when the futures price is 50 cents per pound. The contract is for the delivery of 50,000 pounds. How much does the trader gain or lose if the cotton price at the end of the contract is (a) 48.20 cents per pound and (b) 51.30 cents per pound?
- 1.14. Suppose that you write a put contract with a strike price of \$40 and an expiration date in 3 months. The current stock price is \$41 and the contract is on 100 shares. What have you committed yourself to? How much could you gain or lose?
- 1.15. You would like to speculate on a rise in the price of a certain stock. The current stock price is \$29 and a 3-month call with a strike price of \$30 costs \$2.90. You have \$5,800 to invest. Identify two alternative investment strategies, one in the stock and the other in an option on the stock. What are the potential gains and losses from each?
- 1.16. Suppose that you own 5,000 shares worth \$25 each. How can put options be used to provide you with insurance against a decline in the value of your holding over the next 4 months?
- 1.17. When first issued, a stock provides funds for a company. Is the same true of a stock option? Discuss.
- 1.18. Explain why a futures contract can be used for either speculation or hedging.
- 1.19. Suppose that a March call option to buy a share for \$50 costs \$2.50 and is held until March. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.
- 1.20. Suppose that a June put option to sell a share for \$60 costs \$4 and is held until June. Under what circumstances will the seller of the option (i.e., the party with the short position) make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.
- 1.21. It is May and a trader writes a September call option with a strike price of \$20. The stock price is \$18 and the option price is \$2. Describe the trader's cash flows if the option is held until September and the stock price is \$25 at that time.

1.22. A trader writes a December put option with a strike price of \$30. The price of the option is \$4. Under what circumstances does the trader make a gain?

- 1.23. A company knows that it is due to receive a certain amount of a foreign currency in 4 months. What type of option contract is appropriate for hedging?
- 1.24. A U.S. company expects to have to pay 1 million Canadian dollars in 6 months. Explain how the exchange rate risk can be hedged using (a) a forward contract and (b) an option.
- 1.25. A trader enters into a short forward contract on 100 million yen. The forward exchange rate is \$0.0090 per yen. How much does the trader gain or lose if the exchange rate at the end of the contract is (a) \$0.0084 per yen and (b) \$0.0101 per yen?
- 1.26. The CME Group offers a futures contract on long-term Treasury bonds. Characterize the traders likely to use this contract.
- 1.27. "Options and futures are zero-sum games." What do you think is meant by this?
- 1.28. Describe the profit from the following portfolio: a long forward contract on an asset and a long European put option on the asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up.
- 1.29. In the 1980s, Bankers Trust developed *index currency option notes* (ICONs). These were bonds in which the amount received by the holder at maturity varied with a foreign exchange rate. One example was its trade with the Long Term Credit Bank of Japan. The ICON specified that if the yen/USD exchange rate, S_T , is greater than 169 yen per dollar at maturity (in 1995), the holder of the bond receives \$1,000. If it is less than 169 yen per dollar, the amount received by the holder of the bond is

$$1,000 - \max \left[0, 1,000 \left(\frac{169}{S_T} - 1 \right) \right]$$

When the exchange rate is below 84.5, nothing is received by the holder at maturity. Show that this ICON is a combination of a regular bond and two options.

- 1.30. On July 1, 2021, a company enters into a forward contract to buy 10 million Japanese yen on January 1, 2022. On September 1, 2021, it enters into a forward contract to sell 10 million Japanese yen on January 1, 2022. Describe the payoff from this strategy.
- 1.31. Suppose that USD/sterling spot and forward exchange rates are as follows:

Spot	1.2580
90-day forward	1.2556
180-day forward	1.2518

What opportunities are open to an arbitrageur in the following situations?

- (a) A 180-day European call option to buy £1 for \$1.22 costs 2 cents.
- (b) A 90-day European put option to sell £1 for \$1.29 costs 2 cents.
- 1.32. A trader buys a call option with a strike price of \$30 for \$3. Does the trader ever exercise the option and lose money on the trade? Explain your answer.
- 1.33. A trader sells a put option with a strike price of \$40 for \$5. What is the trader's maximum gain and maximum loss? How does your answer change if it is a call option?
- 1.34. "Buying a put option on a stock when the stock is owned is a form of insurance." Explain this statement.

1.35. On May 21, 2020, as indicated in Table 1.2, the spot ask price of Apple stock is \$316.50 and the ask price of a call option with a strike price of \$320 and a maturity date of September is \$21.70. A trader is considering two alternatives: buy 100 shares of the stock and buy 100 September call options. For each alternative, what is (a) the upfront cost, (b) the total gain if the stock price in September is \$400, and (c) the total loss if the stock price in September is \$300. Assume that the option is not exercised before September and positions are unwound at option maturity.

- 1.36. What is arbitrage? Explain the arbitrage opportunity when the price of a dually listed mining company stock is \$50 (USD) on the New York Stock Exchange and \$60 (CAD) on the Toronto Stock Exchange. Assume that the exchange rate is such that 1 U.S. dollar equals 1.21 Canadian dollars. Explain what is likely to happen to prices as traders take advantage of this opportunity.
- 1.37. Trader A enters into a forward contract to buy an asset for \$1,000 in one year. Trader B buys a call option to buy the asset for \$1,000 in one year. The cost of the option is \$100. What is the difference between the positions of the traders? Show the profit as a function of the price of the asset in one year for the two traders.
- 1.38. In March, a U.S. investor instructs a broker to sell one July put option contract on a stock. The stock price is \$42 and the strike price is \$40. The option price is \$3. Explain what the investor has agreed to. Under what circumstances will the trade prove to be profitable? What are the risks?
- 1.39. A U.S. company knows it will have to pay 3 million euros in three months. The current exchange rate is 1.1500 dollars per euro. Discuss how forward and options contracts can be used by the company to hedge its exposure.
- 1.40. A stock price is \$29. A trader buys one call option contract on the stock with a strike price of \$30 and sells a call option contract on the stock with a strike price of \$32.50. The market prices of the options are \$2.75 and \$1.50, respectively. The options have the same maturity date. Describe the trader's position.
- 1.41. The price of gold is currently \$1,200 per ounce. The forward price for delivery in 1 year is \$1,300 per ounce. An arbitrageur can borrow money at 3% per annum. What should the arbitrageur do? Assume that the cost of storing gold is zero and that gold provides no income.
- 1.42. On May 21, 2020, an investor owns 100 Apple shares. As indicated in Table 1.3, the share price is about \$316 and a December put option with a strike price of \$290 costs \$21.30. The investor is comparing two alternatives to limit downside risk. The first involves buying one December put option contract with a strike price of \$290. The second involves instructing a broker to sell the 100 shares as soon as Apple's price reaches \$290. Discuss the advantages and disadvantages of the two strategies.
- 1.43. A bond issued by Standard Oil some time ago worked as follows. The holder received no interest. At the bond's maturity the company promised to pay \$1,000 plus an additional amount based on the price of oil at that time. The additional amount was equal to the product of 170 and the excess (if any) of the price of a barrel of oil at maturity over \$25. The maximum additional amount paid was \$2,550 (which corresponds to a price of \$40 per barrel). Show that the bond is a combination of a regular bond, a long position in call options on oil with a strike price of \$25, and a short position in call options on oil with a strike price of \$40.

1.44. Suppose that in the situation of Table 1.1 a corporate treasurer said: "I will have £1 million to sell in 6 months. If the exchange rate is less than 1.19, I want you to give me 1.19. If it is greater than 1.25, I will accept 1.25. If the exchange rate is between 1.19 and 1.25, I will sell the sterling for the exchange rate." How could you use options to satisfy the treasurer?





Futures Markets and Central Counterparties

In Chapter 1 we explained that both futures and forward contracts are agreements to buy or sell an asset at a future time for a certain price. A futures contract is traded on an exchange, and the contract terms are standardized by that exchange. A forward contract is traded in the over-the-counter market and can be customized to meet the needs of users.

This chapter covers the details of how futures markets work. We examine issues such as the specification of contracts, the operation of margin accounts, the organization of exchanges, the regulation of markets, the way in which quotes are made, and the treatment of futures transactions for accounting and tax purposes. We explain how some of the ideas pioneered by futures exchanges have been adopted by over-the-counter markets.

2.1 BACKGROUND

Examples of large futures exchanges are the CME Group (www.cmegroup.com), the Intercontinental Exchange (www.theice.com), Eurex (www.eurexchange.com), B3, Brazil (www.b3.com.br), the National Stock Exchange of India (www.nse-india .com), the China Financial Futures Exchange (www.cffex.com.cn), and the Tokyo Financial Exchange (www.tfx.co.jp). (See the table at the end of this book for a more complete list.)

We examine how a futures contract comes into existence by considering the corn futures contract traded by the CME Group. On June 5, a trader in New York might call a broker with instructions to buy 5,000 bushels of corn for delivery in September of the same year. The broker would immediately issue instructions to a trader to buy (i.e., take a long position in) one September corn contract. (Each corn contract is for the delivery of exactly 5,000 bushels.) At about the same time, another trader in Kansas might instruct a broker to sell 5,000 bushels of corn for September delivery. This broker would then issue instructions to sell (i.e., take a short position in) one corn contract. A price would be determined and the deal would be done. Under the traditional open outcry system, floor traders representing each party would physically meet to determine the price. With electronic trading, a computer matches the traders.

Business Snapshot 2.1 The Unanticipated Delivery of a Futures Contract

This story (which may well be apocryphal) was told to the author of this book a long time ago by a senior executive of a financial institution. It concerns a new employee of the financial institution who had not previously worked in the financial sector. One of the clients of the financial institution regularly entered into a long futures contract on live cattle for hedging purposes and issued instructions to close out the position on the last day of trading. (Live cattle futures contracts are traded by the CME Group and each contract is on 40,000 pounds of cattle.) The new employee was given responsibility for handling the account.

When the time came to close out a contract the employee noted that the client was long one contract and instructed a trader at the exchange to buy (not sell) one contract. The result of this mistake was that the financial institution ended up with a long position in two live cattle futures contracts. By the time the mistake was spotted trading in the contract had ceased.

The financial institution (not the client) was responsible for the mistake. As a result, it started to look into the details of the delivery arrangements for live cattle futures contracts—something it had never done before. Under the terms of the contract, cattle could be delivered by the party with the short position to a number of different locations in the United States during the delivery month. Because it was long, the financial institution could do nothing but wait for a party with a short position to issue a *notice of intention to deliver* to the exchange and for the exchange to assign that notice to the financial institution.

It eventually received a notice from the exchange and found that it would receive live cattle at a location 2,000 miles away the following Tuesday. The new employee was sent to the location to handle things. It turned out that the location had a cattle auction every Tuesday. The party with the short position that was making delivery bought cattle at the auction and then immediately delivered them. Unfortunately the cattle could not be resold until the next cattle auction the following Tuesday. The employee was therefore faced with the problem of making arrangements for the cattle to be housed and fed for a week. This was a great start to a first job in the financial sector!

The trader in New York who agreed to buy has a *long futures position* in one contract; the trader in Kansas who agreed to sell has a *short futures position* in one contract. The price agreed to is the current *futures price* for September corn, say 600 cents per bushel. This price, like any other price, is determined by the laws of supply and demand. If, at a particular time, more traders wish to sell rather than buy September corn, the price will go down. New buyers then enter the market so that a balance between buyers and sellers is maintained. If more traders wish to buy rather than sell September corn, the price goes up. New sellers then enter the market and a balance between buyers and sellers is maintained.

Closing Out Positions

The vast majority of futures contracts do not lead to delivery. The reason is that most traders choose to close out their positions prior to the delivery period specified in the

contract. Closing out a position means entering into the opposite trade to the original one. For example, the New York trader who bought a September corn futures contract on June 5 can close out the position by selling (i.e., shorting) one September corn futures contract on, say, July 20. The Kansas trader who sold (i.e., shorted) a September contract on June 5 can close out the position by buying one September contract on, say, August 25. In each case, the trader's total gain or loss is determined by the change in the futures price between June 5 and the day when the contract is closed out.

Delivery is so unusual that traders sometimes forget how the delivery process works (see Business Snapshot 2.1). Nevertheless, we will review delivery procedures later in this chapter. This is because it is the possibility of final delivery that ties the futures price to the spot price.¹

2.2 SPECIFICATION OF A FUTURES CONTRACT

When developing a new contract, the exchange must specify in some detail the exact nature of the agreement between the two parties. In particular, it must specify the asset, the contract size (exactly how much of the asset will be delivered under one contract), where delivery can be made, and when delivery can be made.

Sometimes alternatives are specified for the grade of the asset that will be delivered or for the delivery locations. As a general rule, it is the party with the short position (the party that has agreed to sell the asset) that chooses what will happen when alternatives are specified by the exchange. When the party with the short position is ready to deliver, it files a *notice of intention to deliver* with the exchange. This notice indicates any selections it has made with respect to the grade of asset that will be delivered and the delivery location.

The Asset

When the asset is a commodity, there may be quite a variation in the quality of what is available in the marketplace. When the asset is specified, it is therefore important that the exchange stipulate the grade or grades of the commodity that are acceptable. The Intercontinental Exchange (ICE) has specified the asset in its orange juice futures contract as frozen concentrates that are U.S. Grade A with Brix value of not less than 62.5 degrees.

For some commodities a range of grades can be delivered, but the price received depends on the grade chosen. For example, in the CME Group's corn futures contract, the standard grade is "No. 2 Yellow," but substitutions are allowed with the price being adjusted in a way established by the exchange. No. 1 Yellow is deliverable for 1.5 cents per bushel more than No. 2 Yellow. No. 3 Yellow is deliverable for 2 to 4 cents per bushel less than No. 2 Yellow depending on indicators of quality.

The financial assets in futures contracts are generally well defined and unambiguous. For example, there is no need to specify the grade of a Japanese yen. However, there are

¹ As mentioned in Chapter 1, the spot price is the price for almost immediate delivery.

² There are rare exceptions. As pointed out by J. E. Newsome, G. H. F. Wang, M. E. Boyd, and M. J. Fuller in "Contract Modifications and the Basic Behavior of Live Cattle Futures," *Journal of Futures Markets*, 24, 6 (2004), 557–90, the CME gave the buyer some delivery options in live cattle futures starting in 1995.

some interesting features of the Treasury bond and Treasury note futures contracts traded on the Chicago Board of Trade. For example, the underlying asset in the Treasury bond contract is any U.S. Treasury bond that has a maturity between 15 and 25 years; in the 10-year Treasury note futures contract, the underlying asset is any Treasury note with a maturity of between 6.5 and 10 years. The exchange has a formula for adjusting the price received according to the coupon and maturity date of the bond delivered. This is discussed in Chapter 6.

The Contract Size

The contract size specifies the amount of the asset that has to be delivered under one contract. This is an important decision for the exchange. If the contract size is too large, many traders who wish to hedge relatively small exposures or who wish to take relatively small speculative positions will be unable to use the exchange. On the other hand, if the contract size is too small, trading may be expensive as there is a cost associated with each contract traded.

The correct size for a contract clearly depends on the likely user. Whereas the value of what is delivered under a futures contract on an agricultural product might be \$10,000 to \$20,000, it is much higher for some financial futures. For example, under the Treasury bond futures contract traded by the CME Group, instruments with a face value of \$100,000 are delivered.

In some cases exchanges have introduced "mini" contracts to attract smaller traders. For example, the CME Group's Mini Nasdaq 100 contract is on 20 times the Nasdaq 100 index, whereas the regular contract is on 100 times the index. (We will cover futures on indices more fully in Chapter 3.)

Delivery Arrangements

The place where delivery will be made must be specified by the exchange. This is particularly important for commodities that involve significant transportation costs. In the case of the ICE frozen concentrate orange juice contract, delivery is to exchange-licensed warehouses in Florida, New Jersey, or Delaware.

When alternative delivery locations are specified, the price received by the party with the short position is sometimes adjusted according to the location chosen by that party. The price tends to be higher for delivery locations that are relatively far from the main sources of the commodity.

Delivery Months

A futures contract is referred to by its delivery month. The exchange must specify the precise period during the month when delivery can be made. For many futures contracts, the delivery period is the whole month.

The delivery months vary from contract to contract and are chosen by the exchange to meet the needs of market participants. For example, corn futures traded by the CME Group have delivery months of March, May, July, September, and December. At any given time, contracts trade for the closest delivery month and a number of subsequent delivery months. The exchange specifies when trading in a particular month's contract will begin. The exchange also specifies the last day on which trading can take place for a

given contract. Trading generally ceases a few days before the last day on which delivery can be made.

Price Quotes

The exchange defines how prices will be quoted. For example, crude oil futures prices are quoted in dollars and cents. Treasury bond and Treasury note futures prices are quoted in dollars and thirty-seconds of a dollar.

Price Limits and Position Limits

For most contracts, daily price movement limits are specified by the exchange. If in a day the price moves down from the previous day's close by an amount equal to the daily price limit, the contract is said to be *limit down*. If it moves up by the limit, it is said to be *limit up*. A *limit move* is a move in either direction equal to the daily price limit. Normally, trading ceases for the day once the contract is limit up or limit down. However, in some instances the exchange has the authority to step in and change the limits.

The purpose of daily price limits is to prevent large price movements from occurring because of speculative excesses. However, limits can become an artificial barrier to trading when the price of the underlying commodity is advancing or declining rapidly. Whether price limits are, on balance, good for futures markets is controversial.

Position limits are the maximum number of contracts that a speculator may hold. The purpose of these limits is to prevent speculators from exercising undue influence on the market.

2.3 CONVERGENCE OF FUTURES PRICE TO SPOT PRICE

As the delivery period for a futures contract is approached, the futures price converges to the spot price of the underlying asset. When the delivery period is reached, the futures price equals—or is very close to—the spot price.

To see why this is so, we first suppose that the futures price is above the spot price during the delivery period. Traders then have a clear arbitrage opportunity:

- 1. Sell (i.e., short) a futures contract
- **2.** Buy the asset
- **3.** Make delivery.

These steps are certain to lead to a profit equal to the amount by which the futures price exceeds the spot price. As traders exploit this arbitrage opportunity, the futures price will fall. Suppose next that the futures price is below the spot price during the delivery period. Companies interested in acquiring the asset will find it attractive to enter into a long futures contract and then wait for delivery to be made. As they do so, the futures price will tend to rise.

The result is that the futures price is very close to the spot price during the delivery period. Figure 2.1 illustrates the convergence of the futures price to the spot price. In Figure 2.1a the futures price is above the spot price prior to the delivery period. In

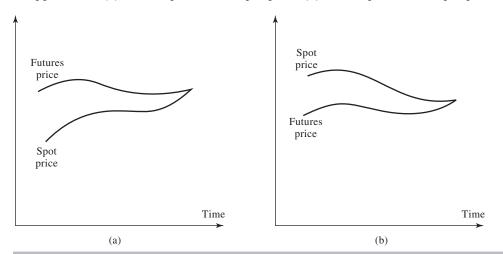


Figure 2.1 Relationship between futures price and spot price as the delivery period is approached: (a) Futures price above spot price; (b) futures price below spot price.

Figure 2.1b the futures price is below the spot price prior to the delivery period. The circumstances under which these two patterns are observed are discussed in Chapter 5.

2.4 THE OPERATION OF MARGIN ACCOUNTS

If two traders get in touch with each other directly and agree to trade an asset in the future for a certain price, there are obvious risks. One of the traders may regret the deal and try to back out. Alternatively, the trader simply may not have the financial resources to honor the agreement. One of the key roles of the exchange is to organize trading so that contract defaults are avoided. This is where margin accounts come in.

Daily Settlement

To illustrate how margin accounts work, we consider a trader who buys (i.e., takes a long position in) two December gold futures contracts. We suppose that the current futures price is \$1,750 per ounce. Because the contract size is 100 ounces, the trader has contracted to buy a total of 200 ounces at this price. The trader has to keep funds in what is known as a margin account. The amount that must be deposited at the time the contract is entered into is known as the *initial margin*. We suppose this is \$6,000 per contract, or \$12,000 in total. At the end of each trading day, the margin account is adjusted to reflect the trader's gain or loss. This practice is referred to as *daily settlement* or *marking to market*.

Suppose, for example, that by the end of the first day the futures price has dropped by \$9 from \$1,750 to \$1,741. The trader has a loss of \$1,800 (= $200 \times 9), because the 200 ounces of December gold, which the trader contracted to buy at \$1,750, can now be sold for only \$1,741. The balance in the margin account would therefore be reduced by \$1,800 to \$10,200. Similarly, if the price of December gold rose to \$1,759 by the end of

the first day, the balance in the margin account would be increased by \$1,800 to \$13,800. A trade is first settled at the close of the day on which it takes place. It is then settled at the close of trading on each subsequent day.

Daily settlement leads to funds flowing each day between traders with long positions and traders with short positions. If the futures price increases from one day to the next, funds flow from traders with short positions to traders with long positions. If the futures price decreases from one day to the next, funds flow in the opposite direction, from traders with short positions to traders with long positions. This daily flow of funds between traders to reflect gains and losses is known as *variation margin*.

Most individuals have to contact their brokers to trade. They are subject to what is termed a *maintenance margin*. This is somewhat lower than the initial margin. If the balance in the margin account falls below the maintenance margin, the trader receives a margin call and is expected to top up the margin account to the initial margin level within a short period of time. If the trader does not provide this variation margin, the broker closes out the position. In the case of the trader considered earlier, closing out the position would involve neutralizing the existing contract by selling 200 ounces of gold for delivery in December.

If the trader's contract increases in value, the balance in the margin account increases. The trader is entitled to withdraw any balance in the margin account that is in excess of the initial margin.

Table 2.1 Operation of margin account for a long position in two gold futures contracts. The initial margin is \$6,000 per contract, or \$12,000 in total; the maintenance margin is \$4,500 per contract, or \$9,000 in total. The contract is entered into on Day 1 at \$1,750 and closed out on Day 16 at \$1,726.90.

Day	Trade price (\$)	Settlement price (\$)	Daily gain (\$)	Cumulative gain (\$)	Margin account balance (\$)	Margin call (\$)
1	1,750.00				12,000	
1		1,741.00	-1,800	-1,800	10,200	
2		1,738.30	-540	-2,340	9,660	
3		1,744.60	1,260	-1,080	10,920	
4		1,741.30	-660	-1,740	10,260	
5		1,740.10	-240	-1,980	10,020	
6		1,736.20	-780	-2,760	9,240	
7		1,729.90	-1,260	-4,020	7,980	4,020
8		1,730.80	180	-3,840	12,180	
9		1,725.40	-1,080	-4,920	11,100	
10		1,728.10	540	-4,380	11,640	
11		1,711.00	-3,420	-7,800	8,220	3,780
12		1,711.00	0	-7,800	12,000	
13		1,714.30	660	-7,140	12,660	
14		1,716.10	360	-6,780	13,020	
15		1,723.00	1,380	-5,400	14,400	
16	1,726.90		780	-4,620	15,180	

Table 2.1 illustrates the operation of the margin account for one possible sequence of futures prices in the case of the trader considered earlier who buys two gold futures contracts. The maintenance margin is assumed to be \$4,500 per contract, or \$9,000 in total. On Day 7, the balance in the margin account falls \$1,020 below the maintenance margin level. This drop triggers a margin call from the broker for an additional \$4,020 to bring the account balance up to the initial margin level of \$12,000. It is assumed that the trader provides this margin by the close of trading on Day 8. On Day 11, the balance in the margin account again falls below the maintenance margin level, and a margin call for \$3,780 is sent out. The trader provides this margin by the close of trading on Day 12. On Day 16, the trader decides to close out the position by selling two contracts. The futures price on that day is \$1,726.90, and the trader has a cumulative loss of \$4,620. Note that the trader has excess margin on Days 8, 13, 14, and 15. It is assumed that the excess is not withdrawn.

Most brokers pay traders interest on the balance in a margin account. The balance in the account does not, therefore, represent a true cost, provided that the interest rate is competitive with what could be earned elsewhere. To satisfy the initial margin requirements, but not subsequent margin calls, a trader can usually deposit securities with the broker. Treasury bills are usually accepted in lieu of cash at about 90% of their market value. Shares are also sometimes accepted in lieu of cash, but at about 50% of their market value.

Whereas a forward contract is settled at the end of its life, a futures contract is, as we have seen, settled daily. At the end of each day, the trader's gain (loss) is added to (subtracted from) the margin account, bringing the value of the contract back to zero. A futures contract is in effect closed out and rewritten at a new price each day.

Minimum levels for the initial and maintenance margin are set by the exchange clearing house. Individual brokers may require greater margins from their clients than the minimum levels specified by the exchange clearing house. Minimum margin levels are determined by the variability of the price of the underlying asset and are revised when necessary. The higher the variability, the higher the margin levels. The maintenance margin is usually about 75% of the initial margin.

Note that margin requirements are the same on short futures positions as they are on long futures positions. It is just as easy to take a short futures position as it is to take a long one. The spot market does not have this symmetry. Taking a long position in the spot market involves buying the asset for immediate delivery and presents no problems. Taking a short position involves selling an asset that you do not own. This is a more complex transaction that may or may not be possible in a particular market. It is discussed further in Chapter 5.

The Clearing House and Its Members

A *clearing house* acts as an intermediary in futures transactions. It guarantees the performance of the parties to each transaction. The clearing house has a number of members. Brokers who are not members themselves must channel their business through a member and post margin with the member. The main task of the clearing house is to keep track of all the transactions that take place during a day, so that it can calculate the net position of each of its members.

The clearing house member is required to provide to the clearing house initial margin (sometimes referred to as clearing margin) reflecting the total number of contracts that