Introduction to Engineering Analysis

Kirk D. Hagen

Fifth Edition

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Pearson

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About This Book

Introduction to Engineering Analysis is designed to teach first-year engineering students how to perform engineering analyses using a systematic problem-solving method. Written for students embarking on any engineering major, the book introduces the fundamental principles of a variety of engineering subjects and then applies the problem-solving method to those subjects. Following introductory chapters on analysis, design, and dimensions and units, the book outlines and illustrates the problem-solving method in detail. The problem-solving method is then used throughout the rest of the book. Chapters include topics traditionally introduced in the first or second year of an engineering curriculum: engineering mechanics, electrical circuits, thermodynamics, and fluid mechanics. The last three chapters cover fundamental principles of renewable energy followed by chapters on graphing and statistics. Approximately 40 percent of the end-of-chapter problems in the fifth edition are revised or new. Studied conscientiously, this book will help students get a good start in their engineering coursework.

CHAPTER

1

The Role of Analysis in Engineering

Objectives

After reading this chapter, you will have learned

- What engineering analysis is
- That analysis is a major component of the engineering curriculum
- How analysis is used in engineering design
- How analysis helps engineers prevent and diagnose failures

1.1 INTRODUCTION

What is ***analysis***? A dictionary definition of analysis might read something like this:

> the separation of a whole into its component parts, or an examination of a complex system, its elements, and their relationships.

Based on this general definition, analysis may refer to everything from the study of a person's mental state (psychoanalysis) to the determination of the amount of certain elements in an unknown metal alloy (elemental analysis). ***Engineering analysis***, however, has a specific meaning. A concise working definition is:

> analytical solution of an engineering problem, using mathematics and principles of science.

Engineering analysis relies heavily on ***basic mathematics*** such as algebra, geometry, trigonometry, calculus, and statistics. ***Higher level mathematics*** such as linear algebra, differential equations, and complex variables may also be used. Principles and laws from the ***physical sciences***, particularly physics and chemistry, are key ingredients of engineering analysis.

Engineering analysis involves more than searching for an equation that fits a problem, plugging numbers into the equation, and "turning the crank" to generate an answer. It is not a simple "plug and chug" procedure. Engineering analysis requires logical and systematic thinking about the engineering problem. The engineer must first be able to state the problem clearly, logically, and concisely. The engineer must understand the physical behavior of the system being analyzed and know which scientific principles to apply. He or she must recognize which mathematical tools to use and how to implement them by hand or on a computer. The engineer must be able to generate a solution that is consistent with the stated problem and any simplifying assumptions. The engineer must then ascertain that the solution is reasonable and contains no errors.

Engineering analysis may be regarded as a type of ***modeling*** or *simulation*. For example, suppose that a civil engineer wants to know the tensile stress in a cable of a suspension bridge that is being designed. The bridge exists only on paper, so a direct stress measurement cannot be made. A scale model of the bridge could be constructed, and a stress measurement taken on the model, but models are expensive and very time-consuming to develop. A better approach is to create an analytical model of the bridge or a portion of the bridge containing the cable. From this model, the tensile stress can be calculated.

Engineering courses that focus on analysis, such as statics, dynamics, mechanics of materials, thermodynamics, and electrical circuits, are considered *core* courses in the engineering curriculum. Because you will be taking many of these courses, it is vital that you gain a fundamental understanding of what analysis is and, more importantly, how to do analysis properly. As the bridge example illustrates, analysis is an integral part of engineering design. Analysis is also a key part of the study of engineering failures.

Engineers who perform engineering analyses on a regular basis are referred to as *engineering analysts* or *analytic engineers*. These functional titles are used to differentiate analysis from the other engineering functions such as research and development (R&D), design, testing, production, sales, and marketing. In some engineering companies, clear distinctions are made between the various engineering functions and the people who work in them. Depending on the organizational structure and the type of products involved, large companies may dedicate a separate department or group of engineers to be analysts. Engineers whose work is dedicated to analysis are considered specialists. In this capacity, the engineering analyst usually works in a support role for design engineering. It is not uncommon, however, for design and analysis functions to be combined in a single department because design and analysis are so closely related. In small firms that employ only a few engineers, the engineers often bear the responsibility of many technical functions, including analysis.

PROFESSIONAL SUCCESS—CHOOSING AN ENGINEERING MAJOR

Perhaps the biggest question facing the new engineering student (besides "How much money will I make after I graduate?") is "In which field of engineering should I major?" Engineering is a broad area, so the beginning student has numerous options. The new engineering student should be aware of a few facts. First, all engineering majors have the potential for preparing the student for a satisfying and rewarding engineering career. As a profession, engineering has historically enjoyed a fairly stable and well-paid market.

There have been fluctuations in the engineering market in recent decades, but the demand for engineers in all the major disciplines is high, and the future looks bright for engineers. Second, all engineering majors are academically challenging, but some engineering majors may be more challenging than others. Study the differences between the various engineering programs. Compare the course requirements of each program by examining the course listings in your college or university catalog. Ask department chairs or advisors to discuss the similarities and differences between their engineering programs and the programs in other departments. (Just keep in mind that professors may be eager to tell you that *their* engineering discipline is the best.) Talk with people who are practicing engineers in the various disciplines and ask them about their educational experiences. Learn all you can from as many sources as you can about the various engineering disciplines. Third, and this is the most important point, try to answer the following question: "What kind of engineering will be the most gratifying for me?" It makes little sense to devote four or more years of intense study of X engineering just because it happens to be the highest paid discipline, because your uncle Vinny is an X engineer, because X engineering is the easiest program at your school, or because someone tells you that they are an X engineer, so you should be one too.

Engineering disciplines may be broadly categorized as either mainstream or narrowly focused. Mainstream disciplines are the broad-based, traditional disciplines that have been in existence for decades (or even centuries) and in which degrees are offered by most of the larger colleges and universities. Many colleges and universities do not offer engineering degrees in some of the narrowly focused disciplines. Chemical, civil, computer, electrical, and mechanical engineering are considered the core mainstream disciplines. These mainstream disciplines are broad in subject content and represent the majority of practicing engineers. Narrowly focused disciplines concentrate on a particular engineering subject by combining specific components from the mainstream disciplines. For example, biomedical engineering may combine portions of electrical and mechanical engineering plus components from biology. Construction engineering may combine elements from civil engineering and business or construction trades. Other narrowly focused disciplines include materials, aeronautical and aerospace, environmental, nuclear, ceramic, geological, manufacturing, automotive, metallurgical, corrosion, ocean, and cost and safety engineering.

Should you major in a mainstream area or a narrowly focused area? The safest thing to do, especially if you are uncertain about which discipline to study, is to major in one of the mainstream disciplines. By majoring in a mainstream area, you will graduate with a general engineering education that will make you marketable in a broad engineering industry. On the other hand, majoring in a narrowly focused discipline may lead you into an extremely satisfying career, particularly if your area of expertise, narrow as it may be, is in high demand. Perhaps your decision will be largely governed by geographical issues. The narrowly focused majors may not be offered at the school you wish to attend. These are important issues to consider when selecting an engineering major.

1.2 ANALYSIS AND ENGINEERING DESIGN

Design is the heart of engineering. In ancient times, people recognized a need for protection against the natural elements, for collecting and utilizing water, for finding and growing food, for transportation, and for defending themselves against other people with unfriendly intentions. Today, even though our world is much more advanced and complex than that of our ancestors, our basic needs are essentially the same. Throughout history, engineers have designed various devices and systems that met the changing needs of society. The following is a concise definition of ***engineering design***:

> a process of devising a component, system, or operation that meets a specific need.

The key word in this definition is *process*. The design process is like a road map that guides the designer from need recognition to problem solution. Design engineers make decisions based on a thorough understanding of engineering fundamentals, design constraints, cost, reliability, manufacturability, and human factors. A knowledge of design *principles* can be learned in school from professors and books, but in order to become a good design engineer, you must *practice* design. Design engineers are like artists and architects who harness their creative powers and skills to produce sculptures and buildings. The end products made by design engineers may be more functional than artistic, but their creation still requires knowledge, imagination, and creativity.

Engineering design is a process by which engineers meet the needs of society. This process may be described in a variety of ways, but it typically consists of the systematic sequence of steps shown in Figure 1.1.

Design has always been a key element of engineering programs in colleges and universities. Traditionally, engineering students take a “senior design” or a “capstone design project” course in their senior year. Recognizing that design is indeed the heart of engineering and that students need an earlier introduction to the subject, many schools integrate design experiences earlier in the curriculum, perhaps as early as the introductory course. By introducing design at the level that introductory mathematics and science courses are taught, engineering programs provide students a meaningful context within which mathematics and science are applied.

What is the relationship between engineering analysis and engineering design? As we defined it earlier, engineering analysis is the *analytical solution of an engineering problem, using mathematics and principles of science*. The false notion that engineering is merely mathematics and applied science is widely held by many beginning engineering students. This may lead a student to believe that engineering design is the equivalent of a “story problem” found in high school algebra books. However, unlike math problems, design problems are “open ended.” This means, among other things, that such problems do not have a single “correct” solution. Design problems have many possible solutions, depending on the *decisions* made by the design engineer. The main goal of engineering design is to obtain the *best* or *optimum* solution within the specifications and constraints of the problem.

So, how does analysis fit in? One of the steps in the design process is to obtain a preliminary concept of the *design*. (Note that the word *design* here refers to the actual component, system, or operation that is being created.) At this point, the engineer begins to investigate design alternatives. Alternatives are different approaches, or options,

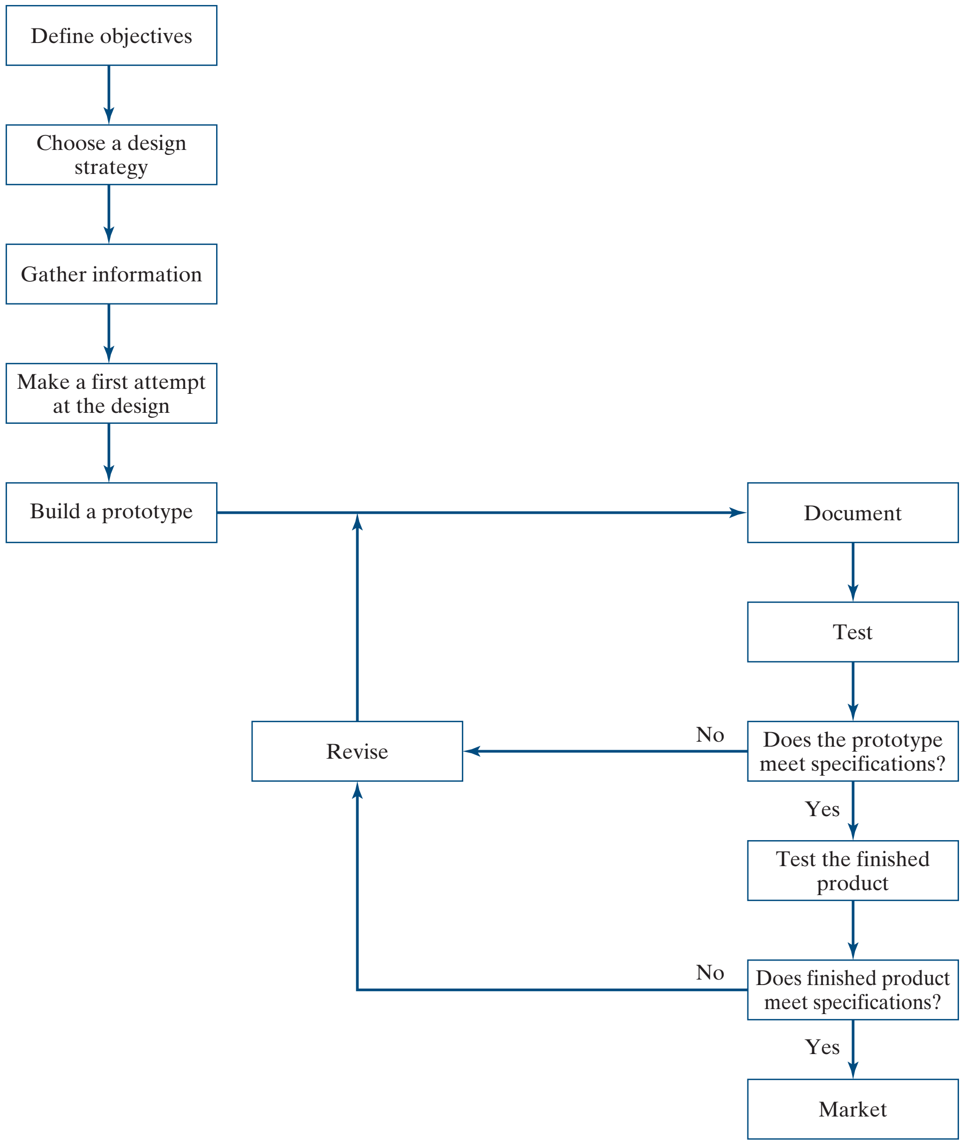

Figure 1.1
The engineering design process.

that the design engineer considers to be viable at the conceptual stage of the design. For example, some of these concepts may be used to design a better mousetrap:

- use a mechanical or an electronic sensor;
- insert cheese or peanut butter as bait;
- construct a wood, plastic, or metal cage;
- install an audible or a visible alarm;
- kill or catch and release the mouse.

Analysis is a *decision-making tool* for evaluating a set of design alternatives. By performing analysis, the design engineer zeroes in on the alternatives that yield the optimum solution, while eliminating alternatives that either violate design constraints or yield inferior solutions. In the mousetrap design, a dynamics analysis may show that a mechanical sensor is too slow, resulting in delaying the closing of a trap door and therefore freeing the mouse. Thus, an electronic sensor is chosen because it yields a superior solution.

The application that follows illustrates how analysis is used to design a machine component.

APPLICATION

DESIGNING A MACHINE COMPONENT

One of the major roles for mechanical engineers is the design of machines. Machines can be very complex systems consisting of numerous moving components. In order for a machine to work properly, each component must be designed so that it performs a specific function in unison with the other components. The components must be designed to withstand specified forces, vibrations, temperatures, corrosion, and other mechanical and environmental factors. An important aspect of machine design is determining the *dimensions* of the mechanical components.

Consider a machine component consisting of a 20-cm-long circular rod, as shown in Figure 1.2. As the machine operates, the rod is subjected to a 100-kN tensile force. (The unit "kN" stands for "kilo newton," which denotes 1000 newton. A newton is a unit of force). One of the design constraints is that the axial deformation (change in length) of the rod cannot exceed 0.5 mm if the rod is to interface properly with a mating component. Taking the rod length and the applied tensile force as given, what is the minimum diameter required for the rod?

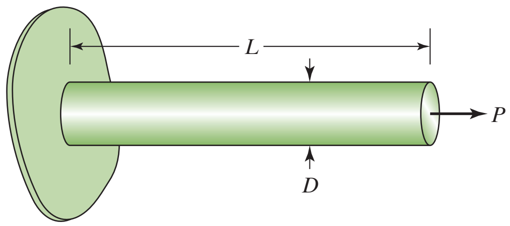

Figure 1.2
A machine component.

To solve this problem, we use an equation from mechanics of materials,

$$\delta = \frac{PL}{AE}$$

where

δ = axial deformation (m)
P = axial tensile force (N)
L = original length of rod (m)
$A = \pi D^2/4$ = cross-sectional area of rod (m^2)
E = modulus of elasticity (N/m^2).

The use of this equation assumes that the material behaves elastically (i.e., it does not undergo permanent deformation when subjected to a force). Upon substituting the formula for the rod's cross-sectional area into the equation and solving for the rod diameter D, we obtain

$$D = \sqrt{\frac{4PL}{\pi\delta E}}.$$

We know the tensile force P, the original rod length L, and the maximum axial deformation δ. But to find the diameter D, we must also know the modulus of elasticity E. The modulus of elasticity is a material property, a constant defined by the ratio of stress to strain. Suppose we choose 7075-T6 aluminum for the rod. This material has a modulus of elasticity of $E = 72$ GPa. (Note: a unit of stress, which is force divided by area, is the pascal (Pa). $1\ \text{Pa} = 1\ \text{N/m}^2$ and $1\ \text{GPa} = 10^9$ Pa.) Substituting values into the equation gives the following diameter:

$$D = \sqrt{\frac{4(100 \times 10^3\ \text{N})(0.20\ \text{m})}{\pi(0.0005\ \text{m})(72 \times 10^9\ \text{N/m}^2)}}$$
$$= 0.0266\ \text{m} = 26.6\ \text{mm}.$$

As part of the design process, we wish to consider other materials for the rod. Let's find the diameter for a rod made of structural steel ($E = 200$ GPa). For structural steel, the rod diameter is

$$D = \sqrt{\frac{4(100 \times 10^3\ \text{N})(0.20\ \text{m})}{\pi(0.0005\ \text{m})(200 \times 10^9\ \text{N/m}^2)}}$$
$$= 0.0160\ \text{m} = 16.0\ \text{mm}.$$

Our analysis shows that the minimum diameter for the rod depends on the material we choose. Either 7075-T6 aluminum or structural steel will work as far as the axial deformation is concerned, but other design issues such as weight, strength, wear, corrosion, and cost should be considered. The important point to be learned here is that analysis is a fundamental step in machine design.

As the application example illustrates, analysis is used to ascertain what design features are required to make the component or system *functional*. Analysis is used to size the cable of a suspension bridge, to select a cooling fan for a computer, to size the heating elements for curing a plastic part in a manufacturing plant, and to design the solar panels that convert solar energy to electrical energy for a spacecraft. Analysis is a crucial part of virtually every design task because it guides the design engineer through a sequence of decisions that ultimately lead to the optimum design. It is important to point out that in design work, it is not enough to produce a *drawing* or CAD (computer-aided design) model of the component or system. A drawing by itself, while revealing the visual and dimensional characteristics of the design, may say little, or nothing, about the functionality of the design. Analysis must be included in the design process if the engineer is to know whether the design will actually work when it is placed into service. Also, once a working prototype of the design is constructed, testing is performed to validate analysis and to aid in the refinement of the design.

1.3 ANALYSIS AND ENGINEERING FAILURE

With the possible exception of farmers, engineers are probably the most taken-for-granted people in the world. Virtually all the man-made products and devices that people use in their personal and professional lives were designed by engineers. Think for a moment. What is the first thing you did when you arose from bed this morning? Did you hit the snooze button on your alarm clock? Your alarm clock was designed by engineers. What did you do next, go into the bathroom, perhaps? The bathroom fixtures — the sink, bathtub, shower, and toilet — were designed by engineers. Did you use an electrical appliance to fix breakfast? Your toaster, waffle maker, microwave oven, refrigerator, and other kitchen appliances were designed by engineers. Even if you ate cold cereal for breakfast, you still took advantage of engineering because engineers designed the processes by which the cereal and milk were produced, and they even designed the machinery for making the cereal box and milk container! What did you do after breakfast? If you brushed your teeth, you can thank engineers for designing the toothpaste tube and toothbrush and even formulating the toothpaste. Before leaving for school, you got dressed; engineers designed the machines that manufactured your clothes. Did you drive a car to school or ride a bicycle? In either case, engineers designed both transportation devices. What did you do when you arrived at school? You sat down in your favorite chair in a classroom, removed a pen or pencil and a note pad from your backpack, and began another day of learning. The chair you sat in, the writing instrument you used to take notes, the notepad you wrote on, and the bulging backpack you use to carry books, binders, paper, pens, and pencils, plus numerous other devices were designed by engineers.

We take engineers for granted, but we expect a lot from them. We expect everything they design, including alarm clocks, plumbing, toasters, automobiles, chairs, and pencils, to work and to work all the time. Unfortunately, they don't. We experience a relatively minor inconvenience when the heating coil in our toaster burns out, but when a bridge collapses, a commercial airliner crashes, or a space shuttle explodes, and people are injured or die, the story makes headline news, and engineers are suddenly thrust into the spotlight of public scrutiny. Are engineers to blame for every failure that occurs? Some failures occur because people misuse the products. For example, if you persist in using a screwdriver to pry lids off cans, to dig weeds from the garden, and to chisel masonry, it may soon stop functioning as a screwdriver. Although engineers try to design products that are "people proof," the types of failures that engineers take primary responsibility for are those caused by various types of errors during the design phase. After all, engineering is a human enterprise, and humans make mistakes.

Whether we like it or not, ***failure*** is part of engineering. It is part of the design process. When engineers design a new product, it seldom works exactly as expected the first time. Mechanical components may not fit properly, electrical components may be connected incorrectly, software glitches may occur, or materials may be incompatible. The list of potential causes of failure is long, and the cause of a specific failure in a design is probably unexpected because otherwise the design engineer would have accounted for it. Failure will always be part of engineering, because engineers cannot anticipate every mechanism by which failures *can* occur. Engineers should make a concerted effort to design systems that do not fail. If failures do arise, ideally they are revealed during the design phase and can be corrected before the product goes into service. One of the hallmarks of a good design engineer is one who turns failure into success.

The role of analysis in engineering failure is twofold. First, as discussed earlier, analysis is a crucial part of engineering design. It is one of the main decision-making tools the design engineer uses to explore alternatives. Analysis helps establish the functionality of the design. Analysis may therefore be regarded as a *failure prevention* tool. People expect kitchen appliances, automobiles, airplanes, televisions, and other systems to work as they are supposed to work, so engineers make every reasonable attempt to design products that are reliable. As part of the design phase, engineers use analysis to ascertain what the physical characteristics of the system must be in order to prevent system failure within a specified period of time. Do engineers ever design products to fail on purpose? Surprisingly, the answer is yes. Some devices rely on failure for their proper operation. For example, a fuse "fails" when the electrical current flowing through it exceeds a specified amperage. When this amperage is exceeded, a metallic element in the fuse melts, breaking the circuit, thereby protecting personnel or a piece of electrical equipment. Shear pins in transmission systems protect shafts, gears, and other components when the shear force exceeds a certain value. Some utility poles and highway signs are designed to safely break away when struck by an automobile.

The second role of failure analysis in engineering pertains to situations where design flaws escaped detection during the design phase, only to reveal themselves after the product was placed into service. In this role, analysis is utilized to address the questions "Why did the failure occur?" and "How can it be avoided in the future?" This type of detective work in engineering is sometimes referred to as *forensic engineering*. In failure investigations, analysis is used as a diagnostic tool of reevaluation and reconstruction. Following the explosion of the Space Shuttle *Challenger* in 1986, engineers at Thiokol used analysis (and testing) to reevaluate the joint design of the solid rocket boosters. Their analyses and tests showed that, under the unusually cold conditions on the day of launch, the rubber O-rings responsible for maintaining a seal between the segments of one of the solid rocket boosters lost resiliency and therefore the ability to contain the high-pressure gases inside the booster. Hot gases leaking past the O-rings developed into an impinging jet directed against the external (liquid hydrogen) tank and a lower strut attaching the booster to the external tank. Within seconds, the entire aft dome of the tank fell away, releasing massive amounts of liquid hydrogen. *Challenger* was immediately enveloped in the explosive burn, destroying the vehicle and killing all seven astronauts. In the aftermath of the *Challenger* disaster, engineers used analysis extensively to redesign the solid rocket booster joint.

APPLICATION

FAILURE OF THE TACOMA NARROWS BRIDGE

The collapse of the Tacoma Narrows Bridge was one of the most sensational failures in the history of engineering. This suspension bridge was the first of its kind spanning the Puget Sound, connecting Washington State with the Olympic Peninsula. Compared with existing suspension bridges, the Tacoma Narrows Bridge had an unconventional design. It had a narrow two-lane deck, and the stiffened-girder road structure was not very deep. This unusual design gave the bridge a slender, graceful appearance. Although the bridge was visually appealing, it had a problem: it oscillated in the wind. During the four months following its opening to traffic on July 1, 1940, the bridge earned the nickname "Galloping Gertie" from motorists who felt as though they were riding a giant roller coaster as they crossed the 2800-ft center span. (See Figure 1.3.) The design engineers failed to recognize that their bridge might behave more like the wing of an airplane subjected to severe turbulence than an earth-bound structure subjected to a steady load. The engineers' failure to consider the aerodynamic aspects of the design led to the destruction of the bridge on November 7, 1940, during a 42-mile-per-hour wind storm. (See Figure 1.4.) Fortunately, no people were injured or killed. A newspaper editor, who lost control of his car between the towers due to the violent undulations, managed to stumble and crawl his way to safety, only to look back to see the road rip away from the suspension cables and plunge, along with his car and presumably his dog, which he could not save, into the Narrows below.

Even as the bridge was being torn apart by the windstorm, engineers were testing a scale model of the bridge at the University of Washington in an attempt to understand the problem. Within a few days following the bridge's demise, Theodore von Karman, a world renowned fluid dynamicist, who worked at the California Institute of

Figure 1.3
The Tacoma Narrows Bridge twisting in the wind. (AP Images)

Figure 1.4
The center span of the Tacoma Narrows Bridge plunges into Puget Sound. (AP Images)

Technology, submitted a letter to *Engineering News-Record* outlining an aerodynamic analysis of the bridge. In the analysis, he used a differential equation for an idealized bridge deck twisting like an airplane wing as the lift forces of the wind tend to twist the deck one way, while the steel in the bridge tends to twist it in another way. His analysis showed that the Tacoma Narrows Bridge should indeed have exhibited an aerodynamic instability more pronounced than any existing suspension bridge. Remarkably, von Karman's "back of the envelope" calculations predicted dangerous levels of vibration for a wind speed less than 10 miles per hour over the wind speed measured on the morning of November 7, 1940. The dramatic failure of Galloping Gertie forever established the importance of aerodynamic analysis in the design of suspension bridges.

The bridge was eventually redesigned with a deeper and stiffer open-truss structure that allowed the wind to pass through. The new and safer Tacoma Narrows Bridge was opened on October 14, 1950.

PROFESSIONAL SUCCESS—LEARN FROM FAILURE

The Tacoma Narrows Bridge and countless other engineering failures teach engineers a valuable lesson:

Learn from your own failures and the failures of other engineers.

Unfortunately, the designers of the Tacoma Narrows Bridge did not learn from the failures of others. Had they studied the history of suspension bridges dating back to the early nineteenth century, they would have discovered that 10 suspension bridges suffered severe damage or destruction by winds.

NASA and Thiokol learned that the pressure-seal design in the solid rocket-booster joint of the Space Shuttle *Challenger* was overly sensitive to a variety of factors such as temperature, physical dimensions, reusability, and joint loading. Not only did they learn some hard-core technical lessons, they also learned some lessons in engineering judgment. They learned that the decision-making process culminating in the launch of *Challenger* was flawed. To correct both types of errors, during the two-year period following the *Challenger* catastrophe, the joint was redesigned, additional safety-related measures were implemented, and the decision-making process leading to shuttle launches was improved.

In another catastrophic failure, NASA determined that fragments of insulation that broke away from the external fuel tank during the launch of the Space Shuttle *Columbia* impacted the left wing of the vehicle, severely damaging the wing's leading edge. The damage caused a breach in the wing's surface which, upon reentry of *Columbia*, precipitated a gradual burn-through of the wing, resulting in a loss of vehicle control. *Columbia* broke apart over the southwestern part of the United States, killing all seven astronauts aboard.

If we are to learn from engineering failures, the *history* of engineering becomes as relevant to our education as design, analysis, science, mathematics, and the liberal arts. Lessons learned not only from our own experiences, but also from those who have gone before us, contribute enormously to the improvement of our technology and the advancement of engineering as a profession. Errors in judgment made by Roman and Egyptian engineers are still relevent in modern times, notwithstanding a greatly improved chest of scientific and mathematical tools. Engineers have and will continue to make mistakes. We should learn from these mistakes.

KEY TERMS

analysis
basic mathematics
engineering analysis
engineering design
failure
higher level mathematics
modeling
physical sciences

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PROBLEMS

Analysis and engineering design

1.1 The following basic devices are commonly found in a typical home or office. Discuss how analysis might be used to design these items.

a. tape dispenser
b. scissors
c. fork
d. mechanical pencil
e. door hinge
f. refrigerator
g. toilet
h. incandescent light bulb
i. microwave oven
j. waste basket
k. three-ring binder
l. light switch
m. doorknob
n. stapler
o. can opener
p. flashlight
q. kitchen sink
r. electrical outlet
s. soft drink can
t. toaster
u. screwdriver
v. chair
w. table
x. mailbox
y. drawer slide
z. padlock

1.2 A 1-m-long cantilevered beam of rectangular cross section carries a uniform load of $w = 15$ kN/m. The design specification calls for a 5-mm maximum deflection of the end of the beam. The beam is to be constructed of fir ($E = 13$ GPa). By analysis, determine at least five combinations of beam height h and beam width b that meet the specification. Use the equation

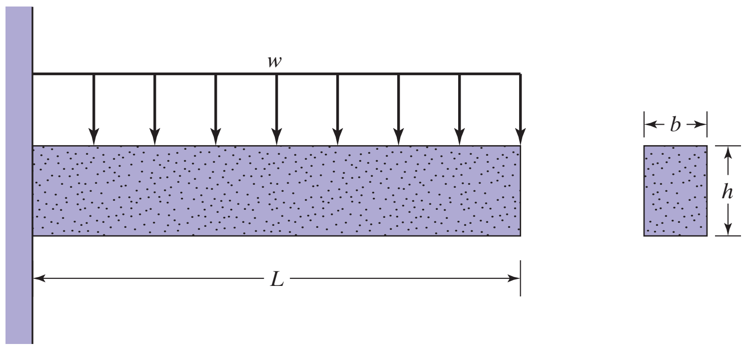
$$y_{\max} = \frac{wL^4}{8EI}$$

where

$y_{\max}$ = deflection of end of beam (m)
w = uniform loading (N/m)
L = beam length (m)
E = modulus of elasticity of beam (Pa)
$I = bh^3/12$ = moment of inertia of beam cross section (m^4).

Note: $1 \text{ Pa} = 1 \text{ N/m}^2$, $1 \text{ kN} = 10^3$ N, and $1 \text{ GPa} = 10^9$ Pa.
What design conclusions can you draw about the influence of beam height and width on the maximum deflection? Is the deflection more sensitive to h or b? If the beam were constructed of a different material, how would the deflection change? See Figure P1.2 for an illustration of the beam.

Figure P1.2



Analysis and engineering failure

1.3 Identify a device from your own experience that has failed. Discuss how it failed and how analysis might be used to redesign it.

1.4 Research the following notable engineering failures. Discuss how analysis was used or could have been used to investigate the failure.

a. Dee bridge, England, 1847
b. Boiler explosions, North America, 1870–1910
c. *Titanic*, North Atlantic, 1912
d. *Hindenburg* airship, New Jersey, 1937
e. *Apollo I* capsule fire, Cape Canaveral, Florida, 1967
f. *Apollo 13*, 1970
g. Ford Pinto gas tanks, 1970s
h. Teton dam, Idaho, 1976

i. Hartford Civic Center, Connecticut, 1978
j. Skylab, 1979
k. Three Mile Island nuclear power plant, Pennsylvania, 1979
l. American Airlines DC-10, Chicago, 1979
m. Hyatt Hotel, Kansas City, 1981
n. Union Carbide plant, India, 1984
o. Space shuttle *Challenger*, 1986
p. Chernobyl nuclear power plant, Soviet Union, 1986
q. Highway I-880, Loma Prieta, California earthquake, 1989
r. Green Bank radio telescope, West Virginia, 1989
s. Hubble space telescope, 1990
t. ValuJet Airlines DC-9, Miami, 1996
u. Mars Climate Orbiter, 1999
v. Space shuttle *Columbia*, 2003
w. Levees, New Orleands, Louisiana, 2005
x. BP oil spill, Gulf of Mexico, 2010
y. Nuclear power plant, Okuma, Fukushima, Japan, 2011
z. Florida International University pedestrian bridge collapse, Sweetwater, Florida, 2018

CHAPTER

2 Dimensions and Units

Objectives

After reading this chapter, you will have learned

- How to check equations for dimensional consistency
- The physical standards on which units are based
- Rules for proper usage of SI units
- Rules for proper usage of English units
- The difference between mass and weight
- How to do unit conversions between the SI and English unit systems

2.1 INTRODUCTION

Suppose for a moment that someone asks you to hurry to the grocery store to buy a few items for tonight's dinner. You get in your car, turn the ignition on, and drive down the road. Immediately you notice something strange. There are no numbers or divisions on your speedometer! As you accelerate and decelerate, the speedometer indicator changes position, but you do not know your speed because there are no markings to read. Bewildered, you notice that the speed limit and other road signs between your house and the store also lack numerical information. Realizing that you were instructed to arrive home with the groceries by 6 pm, you glance at your digital watch only to discover that the display is blank. Upon arriving at the store, you check your list: 1 pound of lean ground beef, 4 ounces of fresh mushrooms, and a 12-ounce can of tomato paste. You go to the meat counter first but the label on each package does not indicate the weight of the product. You grab what appears to be a 1-pound package and proceed to the produce section. Scooping up a bunch of mushrooms, you place them on the scale to weigh them, but the scale looks like your speedometer—it has no markings either! Once again, you estimate. One item is left: the tomato paste. The canned goods aisle contains many cans, but the labels on the cans have no numerical information—no weight, no volume, nothing to let you know the amount of tomato paste in the can. You make your purchase, drive home, and deliver the items mystified and shaken by the whole experience.

The preceding *Twilight Zone*-like story is, of course, fictitious, but it dramatically illustrates how strange our world would be without measures of physical quantities. Speed is a physical quantity that is measured by the speedometers in our automobiles and the radar gun of a traffic officer. Time is a physical quantity that is measured by the watch on our wrist and the clock on the wall. Weight is a physical quantity that is measured by the scale in the grocery store or at the health spa. The need for measurement was recognized by the ancients, who based standards of length on the breadth of the hand or palm, the length of the foot, or the distance from the elbow to the tip of the middle finger (referred to as a cubit). Such measurement standards were both changeable and perishable because they were based on human dimensions. In modern times, definite and unchanging standards of measurement have been adopted to help us quantify the physical world. These measurement standards are used by engineers and scientists to analyze physical phenomena by applying the laws of nature such as conservation of energy, the laws of thermodynamics, and the law of universal gravitation. As engineers design new products and processes by utilizing these laws, they use dimensions and units to describe the physical quantities involved. For instance, the design of a bridge primarily involves the dimensions of length and force. The units used to express the magnitudes of these quantities are usually either the meter and newton or the foot and pound. The thermal design of a boiler primarily involves the dimensions of pressure, temperature, and heat transfer, which are expressed in units of pascal, degrees Celsius, and watt, respectively. Dimensions and units are as important to engineers as the physical laws they describe. It is vitally important that engineering students learn how to work with dimensions and units. Without dimensions and units, analyses of engineering systems have little meaning.

2.2 DIMENSIONS

To most people, the term *dimension* denotes a measurement of length. Certainly, length is one type of *dimension*, but the term dimension has a broader meaning. A ***dimension*** is a *physical variable that is used to describe or specify the nature of a measurable quantity*. For example, the mass of a gear in a machine is a dimension of the gear. Obviously, the diameter is also a dimension of the gear. The compressive force in a concrete column holding up a bridge is a structural dimension of the column. The pressure and temperature of a liquid in a hydraulic cylinder are thermodynamic dimensions of the liquid. The velocity of a space probe orbiting a distant planet is also a dimension. Many other examples could be given. Any variable that engineers use to specify a physical quantity is, in the general sense, a dimension of the physical quantity. Hence, there are as many dimensions as there are physical quantities. Engineers always use dimensions in their analytical and experimental work. In order to specify a dimension fully, two characteristics must be given. First, the *numerical value* of the dimension is required. Second, the appropriate *unit* must be assigned. A dimension missing either of these two elements is incomplete and therefore cannot be fully used by the engineer. If the diameter of a gear is given as 3.85, we would ask the question, "3.85 what? Inches? Meters?" Similarly, if the compressive force in a concrete column is given as 150,000, we would ask, "150,000 what? Newtons? Pounds?"

Dimensions are categorized as either *base* or *derived*. A ***base dimension***, sometimes referred to as a *fundamental* dimension, is a dimension that has been internationally accepted as the most basic dimension of a physical quantity. There are seven base dimensions that have been formally defined for use in science and engineering:

1. length L
2. mass M
3. time t
4. temperature T
5. electric current I
6. amount of substance n
7. luminous intensity i.

A ***derived dimension*** is obtained by any combination of the base dimensions. For example, volume is length cubed, density is mass divided by length cubed, and velocity is length divided by time. Obviously, there are numerous derived dimensions. Table 2.1 lists some of the most commonly used derived dimensions in engineering, expressed in terms of base dimensions.

The single letters in Table 2.1 are symbols that designate each base dimension. These symbols are useful for checking the dimensional consistency of equations. Every mathematical relation used in science and engineering must be ***dimensionally consistent***, or *dimensionally homogeneous*. This means that the dimension on the left side of the equal sign must be the same as the dimension on the right side of the

Table 2.1 Derived Dimensions Expressed in Terms of Base Dimensions

Quantity	Variable Name	Base Dimensions
Area	A	L^2
Volume	V	L^3
Velocity	v	Lt^{-1}
Acceleration	a	Lt^{-2}
Density	ρ	ML^{-3}
Force	F	MLt^{-2}
Pressure	P	$ML^{-1}t^{-2}$
Stress	σ	$ML^{-1}t^{-2}$
Energy	E	ML^2t^{-2}
Work	W	ML^2t^{-2}
Power	P	ML^2t^{-3}
Mass flow rate	$\dot{m}$	Mt^{-1}
Specific heat	c	$L^2t^{-2}T^{-1}$
Dynamic viscosity	μ	$ML^{-1}t^{-1}$
Molar mass	M	Mn^{-1}
Voltage	V	$ML^2t^{-3}T^{-1}$
Resistance	R	$ML^2t^{-3}T^{-2}$

equal sign. The equality in any equation denotes not only a numerical equivalency but also a dimensional equivalency. To use a simple analogy, you cannot say that five apples equals four apples, nor can you say that five apples equals five oranges. You can only say that five apples equals five apples.

The following examples illustrate the concept of dimensional consistency.

EXAMPLE 2.1

Dynamics is a branch of engineering mechanics that deals with the motion of particles and rigid bodies. The straight-line motion of a particle, under the influence of gravity, may be analyzed by using the equation

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

where

y = height of particle at time t
y_0 = initial height of particle (at $t = 0$)
v_0 = initial velocity of particle (at $t = 0$)
t = time
g = gravitational acceleration.

Verify that this equation is dimensionally consistent.

Solution

We check the dimensional consistency of the equation by determining the dimensions on both sides of the equal sign. The heights, y_0 and y, are one-dimensional coordinates of the particle, so these quantities have a dimension of length L. The initial velocity v_0 is a derived dimension consisting of a length L divided by a time t. Gravitational acceleration g is also a derived dimension consisting of a length L, divided by time squared t^2. Of course, time t is a base dimension. Writing the equation in its dimensional form, we have

$$\mathrm{L} = \mathrm{L} + \mathrm{L}\mathrm{t}^{-1}\mathrm{t} - \mathrm{L}\mathrm{t}^{-2}\mathrm{t}^2.$$

Note that the factor, $\frac{1}{2}$, in front of the gt^2 term is a pure number, and therefore has no dimension. In the second term on the right side of the equal sign, the dimension t cancels, leaving length L. Similarly, in the third term on the right side of the equal sign, the dimension t^2 cancels, leaving length L. This equation is dimensionally consistent because all terms have the dimension of length L.

EXAMPLE 2.2

Aerodynamics is the study of forces acting on bodies moving through air. An aerodynamics analysis could be used to determine the lift force on an airplane wing or the drag force on an automobile. A commonly used equation in aerodynamics relates the total drag force acting on a body to the velocity of the air approaching it. This equation is

$$F_D = \frac{1}{2} C_D A \rho U^2$$

where

$$
\begin{aligned}
F_D &= \text{drag force} \\
C_D &= \text{drag coefficient} \\
A &= \text{frontal area of body} \\
\rho &= \text{air density} \\
U &= \text{upstream air velocity.}
\end{aligned}
$$

Determine the dimensions of the drag coefficient, C_D.

Solution

The dimension of the drag coefficient C_D may be found by writing the equation in dimensional form and simplifying the equation by combining like dimensions. Using the information in Table 2.1, we write the dimensional equation as

$$
\begin{aligned}
\mathrm{MLt^{-2}} &= C_D \mathrm{L^2 ML^{-3} L^2 t^{-2}} \\
&= C_D \ \mathrm{MLt^{-2}}.
\end{aligned}
$$

Compare the combination of base dimensions on the left and right sides of the equal sign. They are identical. This can only mean that the drag coefficient C_D has no dimension. If it did, the equation would not be dimensionally consistent. Thus, we say that C_D is *dimensionless*. In other words, the drag coefficient C_D has a numerical value, but no dimensional value. This is not as strange as it may sound. In engineering, there are many instances, particularly, in the disciplines of fluid mechanics and heat transfer, where a physical quantity is dimensionless. Dimensionless quantities enable engineers to form special ratios that reveal certain physical insights into properties and processes. In this instance, the drag coefficient is physically interpreted as a "shear stress" at the surface of the body, which means that there is an aerodynamic force acting on the body parallel to its surface that tends to retard the body's motion through the air. If you take a course in fluid mechanics, you will learn more about this important concept.

EXAMPLE 2.3

For the following dimensional equation, find the dimensions of the quantity k:

$$\mathrm{MLt^{-2}} = k \ \mathrm{Lt}.$$

Solution

To find the dimensions of k, we multiply both sides of the equation by $\mathrm{L^{-1}t^{-1}}$ to eliminate the dimensions on the right side of the equation, leaving k by itself. Thus, we obtain

$$\mathrm{MLt^{-2}L^{-1}t^{-1}} = k$$

which, after applying a law of exponents, reduces to

$$\mathrm{Mt^{-3}} = k.$$

A closer examination of the given dimensional equation reveals that it is Newton's second law of motion:

$$F = ma.$$

Here F is force, m is mass, and a is acceleration. Referring to Table 2.1, force has dimensions of $\mathrm{MLt^{-2}}$, which is a mass M multiplied by acceleration $\mathrm{Lt^{-2}}$.

PRACTICE!

1. For the following dimensional equation, find the base dimensions of the parameter k:
$$\mathrm{ML^2} = k\,\mathrm{LtM^2}.$$
Answer: $\mathrm{LM^{-1}t^{-1}}$.

2. For the following dimensional equation, find the base dimensions of the parameter g:
$$\mathrm{T^{-1}tL} = g\,\mathrm{L^{-2}}.$$
Answer: $\mathrm{L^3tT^{-1}}$.

3. For the following dimensional equation, find the base dimensions of the parameter h:
$$\mathrm{It^{-1}}h = \mathrm{N}.$$
Answer: $\mathrm{NI^{-1}t}$.

4. For the following dimensional equation, find the base dimensions of the parameter f:
$$\mathrm{MM^{-3}} = a\cos(f\,\mathrm{L}).$$
Answer: $\mathrm{L^{-1}}$.

5. For the following dimensional equation, find the base dimensions of the parameter p:
$$\mathrm{T} = \mathrm{T}\log(\mathrm{T^{-2}t}\,p).$$
Answer: $\mathrm{T^2t^{-1}}$.

2.3 UNITS

A ***unit*** is *a standard measure of the magnitude of a dimension.* For example, the dimension length L may be expressed in units of meter (m), feet (ft), mile (mi), millimeter (mm), and many others. The dimension temperature T is expressed in units of degrees Celsius (°C), degrees Fahrenheit (°F), degrees Rankine (°R), or kelvin (K). (By convention, the degree symbol (°) is not used for the Kelvin temperature scale.) In the United States, there are two unit systems commonly in use. The first unit system, and the one that is internationally accepted as the standard, is the ***SI*** (System International d'Unites) ***unit system***, commonly referred to as the *metric* system. The second unit system is the ***English*** (or **British)** ***unit system***, sometimes referred to as the *United States Customary System (USCS).* With the exception of the United States, most of the industrialized nations of the world use the SI system exclusively. The SI system is preferred over the English system, because it is an internationally accepted standard and is based on simple powers of 10. To a limited extent, a transition to the SI system has been federally mandated in the United States. Unfortunately, this transition to total SI usage has been a slow one, but many American companies are using the SI system to remain internationally competitive. Until the United States makes a complete adaptation to the SI system, U.S. engineering students need to be conversant in both unit systems and know how to make unit conversions.

The seven base dimensions are expressed in terms of SI units that are based on ***physical standards***. These standards are defined such that, the corresponding SI units, except the mass unit, can be reproduced in a laboratory anywhere in the world. The reproducibility of these standards is important, because everyone with a suitably equipped laboratory has access to the same standards. Hence, all physical quantities, regardless of where in the world they are measured, are based on identical standards. This universality of physical standards eliminates the ancient problem of basing dimensions on the changing physical attributes of kings, rulers, and magistrates who reigned for a finite time. Modern standards are based on constants of nature and physical attributes of matter and energy.

The seven base dimensions and their associated SI units are summarized in Table 2.2. Note the symbol for each unit. These symbols are the accepted conventions for science and engineering. The discussion that follows outlines the physical standards by which the base units are defined.

Length

The unit of length in the SI system is the *meter* (m). As illustrated in Figure 2.1, the meter is defined as the distance traveled by light in a vacuum, during a time interval of 1/299,792,458 s. The definition is based on a physical standard, the speed of light in a vacuum. The speed of light in a vacuum is 299,792,458 m/s. Thus, light travels one meter during a time interval of the reciprocal of this number. Of course, the unit of time, the *second* (s), is itself a base unit.

Table 2.2 Base Dimensions and Their SI Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A
Amount of substance	mole	mol
Luminous intensity	candela	cd

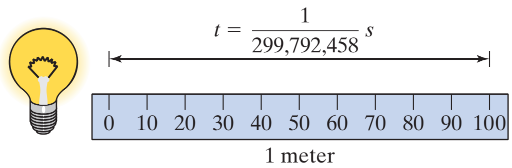


Figure 2.1
The physical standard for the meter is based on the speed of light in a vacuum.

Mass

Prior to 2019, mass was the only base dimension that was defined by an artifact, a cylinder of platinum-iridium alloy maintained by the International Bureau of Weights and Measures in Paris, France. Because an artifact is not as easily reproduced as the other laboratory-based standards, the *kilogram* (kg) has been redefined in terms of the Planck constant, one of the fundamental constants in quantum physics. The Planck constant, denoted by the symbol h, has been set by the international scientific community as exactly $6.62607015 \times 10^{-34}\ J \cdot s$. The Planck constant can be used to define mass because the unit joule (J), when broken into its base units, contains the mass unit kilogram.

Time

The unit of time in the SI system is the *second* (s). The second is defined as the duration of 9,192,631,770 cycles of radiation of the cesium atom. An atomic clock incorporating this standard is maintained by NIST. (See Figure 2.2.)

Temperature

The unit of temperature in the SI system is the *kelvin* (K). The kelvin is defined as the fraction 1/273.16 of the temperature of the triple point of water. The triple point of water is the combination of pressure and temperature at which water exists as a solid, liquid, and gas at the same time. (See Figure 2.3.) This temperature is 273.16 K, 0.01°C, or 32.002°F. Absolute zero is the temperature at which all molecular activity ceases and has a value of 0 K.

Electric Current

The unit of electric current in the SI system is the *ampere* (A). As shown in Figure 2.4, the ampere is defined as the steady current, which, if maintained in

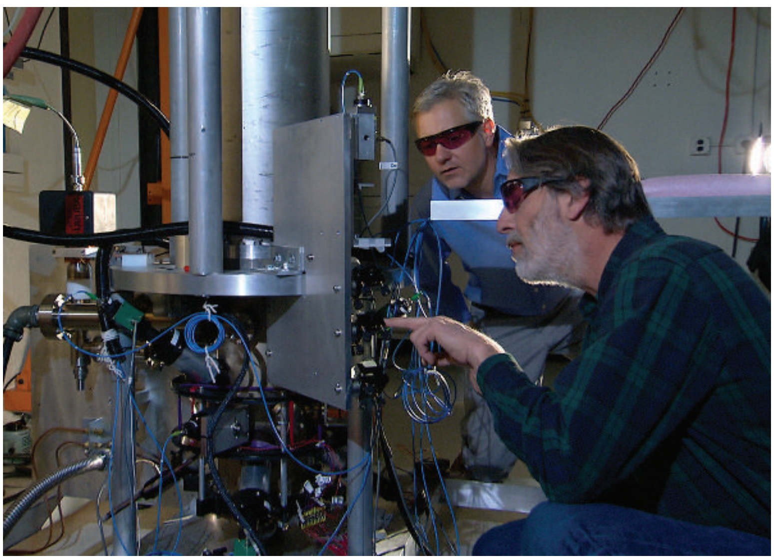


Figure 2.2
The NIST-F2 cesium fountain atomic clock keeps time to an accuracy of one second in 300 million years. (Source: National Institute of Standards and Technology, Boulder, CO.)

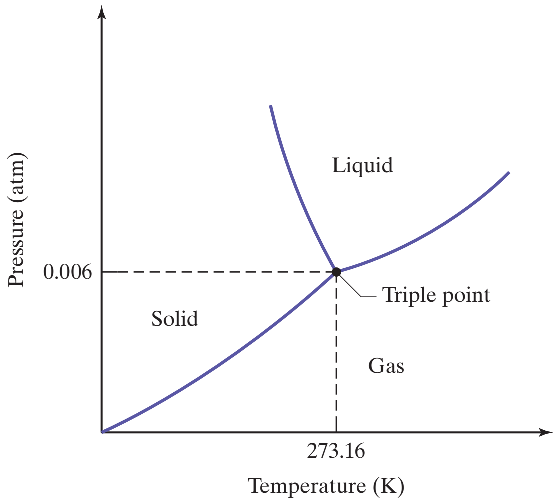


Figure 2.3
A phase diagram for water shows the triple point on which the kelvin temperature standard is based.

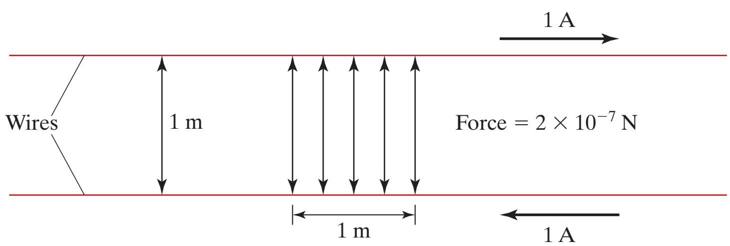


Figure 2.4
The standard for the ampere is based on the electrical force produced between two parallel wires, each carrying 1 A, located 1 m apart.

two straight parallel wires of infinite length and negligible circular cross section and placed one meter apart in a vacuum, produces a force of 2×10^{-7} newton per meter of wire length. Using Ohm's law, $I = V/R$, one ampere may also be denoted as the current that flows when one volt is applied across a 1-ohm resistor.

Amount of Substance

The unit used to denote the amount of substance is the *mole* (mol). One mole contains the same number of elements as there are atoms in 0.012 kg of carbon-12. This number is called Avogadro's number and has a value of approximately 6.022×10^{23}. (See Figure 2.5.)

Luminous Intensity

The unit for luminous intensity is the *candela* (cd). As illustrated in Figure 2.6, one candela is the luminous intensity of a source emitting light radiation at a frequency

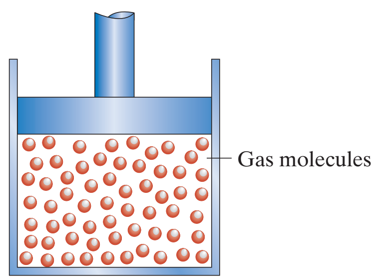


Figure 2.5
A mole of gas molecules in a piston-cylinder device contains 6.022×10^{23} molecules.

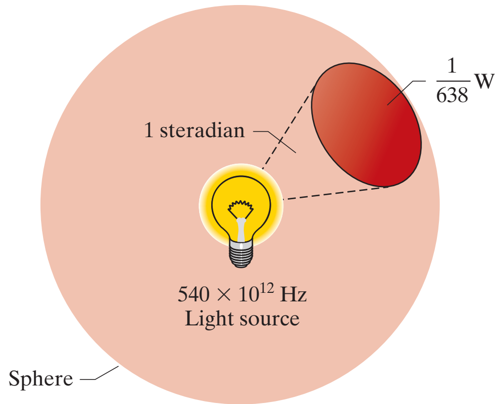


Figure 2.6
The candela standard for luminous intensity.

Table 2.3 Supplementary Dimensions

Quantity	Unit	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

of 540×10^{12} Hz, that provides a power of $1/683$ watt (W) per steradian. A steradian is a solid angle, which, having its vertex in the center of a sphere, subtends (cuts off) an area of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

The unit for luminous intensity, the candela, utilizes the steradian, a dimension that may be unfamiliar to most students. The *radian* and *steradian* are called *supplementary* dimensions. These quantities, summarized in Table 2.3, refer to plane and solid angles, respectively. The radian is frequently used in engineering, and it is defined as the plane angle between two radii of a circle that subtends on the circumference of an arc equal in length to the radius. From trigonometry, you may recall that there are 2π radians in a circle (i.e., 2π radians equals $360°$). Thus, one radian equals approximately $57.3°$. The steradian, defined earlier, is used primarily for expressing radiation quantities such as light intensity and other electromagnetic parameters. These units appear dimensionless in measurements.

2.4 SI UNITS

Throughout the civilized world there are thousands of engineering companies that design and manufacture products for the benefit of society. The international buying and selling of these products is an integral part of a global network of industrialized countries, and the economic health of these countries, including the United States, depends to a large extent on international trade. Industries such as the automotive and electronics industries are heavily involved in international trade, so these industries have readily embraced the SI unit system in order to be economically competitive. The general adoption of the SI unit system by U.S. companies has been slow, but global economic imperatives are driving them to fall into step with the other industrialized nations of the world. SI units are now commonplace on food and beverage containers, gasoline pumps, and automobile speedometers.

The SI unit system is the internationally accepted standard. In the United States, however, the English unit system is still widely used. Perhaps it is only a matter of time before all U.S. companies use SI units exclusively. Until that time, the burden is upon you, the engineering student, to learn both unit systems. You will gladly discover, however, that most engineering textbooks emphasize SI units, but provide a list of unit conversions between the SI and English systems.

Table 2.2 summarizes the seven base dimensions and their SI units, and Table 2.3 summarizes the supplementary dimensions and their units. Derived dimensions consist of a combination of base and supplementary dimensions. Sometimes, the units of a derived dimension are given a specific name. For example, the derived dimension *force* consists of the SI base units $kg \cdot m \cdot s^{-2}$. This combination of SI base units is called a *newton* and is abbreviated N. Note that the unit name, in honor of Isaac Newton, is not capitalized when spelled out as a unit name. The same rule applies to other units named after people such as hertz (Hz), kelvin (K), and pascal (Pa). Another example is the *joule*, the SI unit for energy, work, and heat. The joule unit is abbreviated J and consists of the SI base units $kg \cdot m^2 \cdot s^{-2}$. A summary of the most commonly used SI derived dimensions and the corresponding SI unit names is given in Table 2.4.

Most derived dimensions do not have specific SI unit names, but their units may contain specific SI unit names. For example, the dimension *mass flow rate* is the mass of a fluid that flows past a point in a given time. The SI units for mass flow rate are $kg \cdot s^{-1}$, which we state as "kilograms per second." Note that units that are located in the denominator, that is, those that have a negative sign on their exponent, may also be written using a divisor line. Thus, the units for mass flow rate may be written as kg/s. Caution must be exercised, however, when utilizing this type of notation for some units. For example, the SI units for thermal conductivity, a quantity used in heat transfer, are $W \cdot m^{-1} \cdot K^{-1}$. How do we write these

Table 2.4 Derived Dimensions and SI Units with Specific Names

Quantity	SI Unit	Unit Name	Base Units
Frequency	Hz	hertz	s^{-1}
Force	N	newton	$kg \cdot m \cdot s^{-2}$
Pressure	Pa	pascal	$kg \cdot m^{-1} \cdot s^{-2}$
Stress	Pa	pascal	$kg \cdot m^{-1} \cdot s^{-2}$
Energy	J	joule	$kg \cdot m^2 \cdot s^{-2}$
Work	J	joule	$kg \cdot m^2 \cdot s^{-2}$
Heat	J	joule	$kg \cdot m^2 \cdot s^{-2}$
Power	W	watt	$kg \cdot m^2 \cdot s^{-3}$
Electric charge	C	coulomb	$A \cdot s$
Electric potential (voltage)	V	volt	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$
Electric resistance	Ω	ohm	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$
Magnetic flux	Wb	weber	$kg^{-1} \cdot m \cdot s^{-2} \cdot A^{-1}$
Luminous flux	lm	lumen	$cd \cdot sr$

units with a divisor line? Do we write these units as W/m/K? How about $W/m \cdot K$? Either choice can cause some confusion. Does a "watt per meter per kelvin" mean that the kelvin unit is inverted twice and therefore goes above the divisor line? One glance at the units written as $W \cdot m^{-1} \cdot K^{-1}$ tells us that the temperature unit belongs "downstairs" because K has a negative exponent. If the kelvin unit were placed above the divisor line, and the thermal conductivity were used in an equation, a dimensional inconsistency would result. The second choice requires agreeing that multiplication takes precedence over division. Because the meter and kelvin units are located to the right of the divisor line and they are separated by a dot, both units are interpreted as being in the denominator. But to avoid all ambiguity, parentheses are used to group units above or below the divisor line. Units for thermal conductivity would then be written as $W/(m \cdot K)$. In any case, a dot or a dash should always be placed between adjacent units to separate them regardless of whether the units are above or below the divisor line. Some derived dimensions and their SI units are given in Table 2.5.

When a physical quantity has a numerical value that is very large or very small, it is cumbersome to write the number in standard decimal form. The general practice in engineering is to express numerical values between 0.1 and 1000 in standard decimal form. If a value cannot be expressed within this range, a *prefix* should be used. Because the SI unit system is based on powers of 10, it is more convenient to express such numbers by using prefixes. A prefix is a letter in front of a number that denotes multiples of powers of 10. For example, if the internal force in an I-beam is three million seven hundred and fifty thousand newtons, it would be awkward to write this number as 3,750,000 N. It is preferred to write the force as 3.75MN, which is stated as "3.75 mega newtons." The prefix "M" denotes a multiple of a million. Hence, 3.75 MN equals 3.75×10^6 N. Electrical current is a good example of a quantity represented by a small number. Suppose the current flowing in a wire is 0.0082 A. This quantity would be expressed as 8.2 mA, which is stated as "8.2 milliamperes." The prefix "m" denotes a multiple of one-thousandth, or 1×10^{-3}.

A term we often hear in connection with computers is the storage capacity of hard disks. When personal computers first appeared in the early 1980s, most hard disks could hold around 10 or 20 MB (megabytes) of information. Nowadays, the typical storage capacity of a personal computer's hard disk is on the order of TB (terabytes). The standard prefixes for SI units are given in Table 2.6.

As indicated in Table 2.6, the most widely used SI prefixes for science and engineering quantities come in multiples of one thousand. For example, stress and pressure, which are generally large quantities for most structures and pressure vessels, are normally expressed in units of kPa, MPa, or GPa. Frequencies of electromagnetic waves such as radio, television, and telecommunications are also large numbers. Hence, they are generally expressed in units of kHz, MHz, or GHz. Electrical currents, on the other hand, are often small quantities, so they are usually expressed in units of μA or mA. Because frequencies of most electromagnetic waves are large quantities, the wavelengths of these waves are small. For example, the wavelength range of the visible light region of the electromagnetic spectrum is approximately 0.4 μm to 0.75 μm. It should be noted that the SI mass unit kilogram (kg) is the only base unit that has a prefix.

Here are some rules on how to use SI units properly that every beginning engineering student should know:

1. A unit symbol is never written as a plural with an "s." If a unit is pluralized, the "s" may be confused with the unit second (s).

Table 2.5 Derived Dimensions and SI Units

Quantity	SI Units
Acceleration	$m \cdot s^{-2}$
Angular acceleration	$rad \cdot s^{-2}$
Angular velocity	$rad \cdot s^{-1}$
Area	m^2
Concentration	$mol \cdot m^{-3}$
Density	$kg \cdot m^{-3}$
Electric field strength	$V \cdot m^{-1}$
Energy	$N \cdot m$
Entropy	$J \cdot k^{-1}$
Heat	J
Heat transfer	W
Magnetic field strength	$A \cdot m^{-1}$
Mass flow rate	$kg \cdot s^{-1}$
Moment of force	$N \cdot m$
Radiant intensity	$W \cdot sr^{-1}$
Specific energy	$J \cdot kg^{-1}$
Surface tension	$N \cdot m^{-1}$
Thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
Velocity	$m \cdot s^{-1}$
Viscosity, dynamic	$Pa \cdot s$
Viscosity, kinematic	$m^2 \cdot s^{-1}$
Volume	m^3
Volume flow rate	$m^3 \cdot s^{-1}$
Wavelength	m
Weight	N

2. A period is never used after a unit symbol, unless the symbol is at the end of a sentence.

3. Do not use invented unit symbols. For example, the unit symbol for "second" is (s), not (sec), and the unit symbol for "ampere" is (A), not (amp).

4. A unit symbol is always written by using lowercase letters, with two exceptions. The first exception applies to units named after people, such as the newton (N), joule (J), and watt (W). The second exception applies to units with the prefixes M, G, and T. (See Table 2.6.)

5. A quantity consisting of several units must be separated by dots or dashes to avoid confusion with prefixes. For example, if a dot is not used to express the units of "meter-second" (m · s), the units could be interpreted as "millisecond" (ms).

6. An exponential power for a unit with a prefix refers to both the prefix and the unit; for example, $ms^2 = (ms)^2 = ms \cdot ms$.
7. Do not use compound prefixes. For example, a "kilo MegaPascal" (kMPa) should be written as GPa, because the product of "kilo" (10^3) and "mega" (10^6) equals "giga" (10^9).
8. Put a space between the numerical value and the unit symbol.
9. Do not put a space between a prefix and a unit symbol.
10. Do not use prefixes in the denominator of composite units. For example, the units N/mm should be written as kN/m.

Table 2.7 provides some additional examples of these rules.

Table 2.6 Standard Prefixes for SI Units

Multiple	Exponential Form	Prefix	Prefix Symbol
1,000,000,000,000,000	10^{15}	peta	P
1,000,000,000,000	10^{12}	tera	T
1,000,000,000	10^{9}	giga	G
1,000,000	10^{6}	mega	M
1000	10^{3}	kilo	k
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
0.000 000 000 000 001	10^{-15}	femto	f

Table 2.7 Correct and Incorrect Ways of Using SI Units

Correct	Incorrect	Rules
47.7 kg	47.7 kgs	1
1056 J	1056 Js	1
140 kPa	140 kPa.	2
1.25 A	1.25 Amps	1, 3
3.2 s	3.2 sec	3
60.0 kg	60.0 Kg	4
75 W	75 w.	2, 4
8.25 kg/m·s	8.25 kg/ms	5
550 GN	550 MkN	7
8 ms	8 kμs	7
430 Pa·s	430Pa·s	8
1.5 MΩ	1.5 M Ω	9
9 MN/m	9 N/μm	10

APPLICATION

DERIVING FORMULAS FROM UNIT CONSIDERATIONS

To the beginning engineering student, it can seem as if there is an infinite number of formulas to learn. Formulas contain physical quantities that have numerical values plus units. Because formulas are written as equalities, formulas must be numerically and dimensionally equivalent across the equal sign. Can this feature be used to help us derive formulas that we do not know or have forgotten? Suppose that we want to know the mass of gasoline in an automobile's gas tank. The tank has a volume of 70 L, and a handbook of fluid properties states that the density of gasoline is 736 kg/m^3. (Note: 1 L $= 10^{-3}$ m^3). Thus, we write

$$\rho = 736 \text{ kg/m}^3, \quad V = 70 \text{ L} = 0.070 \text{ m}^3.$$

If the tank is completely filled with gasoline, what is the mass of the gasoline? Suppose that we have forgotten that density is defined as mass per volume, $\rho = m/V$. Because our answer will be a mass, the unit of our answer must be kilogram (kg). Looking at the units of the input quantities, we see that if we multiply density ρ by volume V, the volume unit (m^3) divides out, leaving mass (kg). Hence, the formula for mass in terms of ρ and V is

$$m = \rho V$$

so the mass of gasoline is

$$m = (736 \text{ kg/m}^3)(0.070 \text{ m}^3) = 51.5 \text{ kg}.$$

PROFESSIONAL SUCCESS–USING SI UNITS IN EVERYDAY LIFE

The SI unit system is used commercially to a limited extent in the United States, so the average person does not know the highway speed limit in kilometers per hour, his or her weight in newtons, atmospheric pressure in kilopascals, or the outdoor air temperature in kelvin or degrees Celsius. It is ironic that the leading industrialized nation on earth has yet to embrace this international standard. Admittedly, American beverage containers routinely show the volume of the liquid product in liters (L) or milliliters (mL), gasoline pumps often show liters of gasoline delivered, speedometers may indicate speed in kilometers per hour (km/h), and automobile tires indicate the proper inflation pressure in kilopascals (kPa) on the sidewall. On each of these products, and many others like them, a corresponding English unit is written along side the SI unit. The beverage container shows pints or quarts, the gasoline pump shows gallons, speedometers show miles per hour, and tires show pounds per square inch. Dual labeling of SI and English units on U.S. products are supposed to help people learn the SI system, "weaning" them from the antiquated English system in anticipation of the time when a full conversion to SI units occurs. This transition is analogous to the process of incrementally quitting smoking. Rather than quitting "cold turkey," we employ nicotine patches, gums, and other substitutes until our habit is broken. So, you may ask, "Why don't we make the total conversion now? Is it as painful as quitting smoking suddenly?" It probably is. As you might guess, the problem is largely an economic one. A complete conversion to SI units may not occur until we are willing to pay the price in actual dollars. People could learn the SI unit system fairly quickly if the conversion were done suddenly, but an enormous financial commitment would have to be made.

As long as dual product labeling of units is employed in the United States, most people will tend to ignore the SI unit and look only at the English unit, the unit with which they are most familiar. In U.S. engineering schools, SI units are emphasized. Therefore, the engineering student is not the average person on the street who does not know, or know how to calculate, his or her weight in newtons. So, what can engineering students in the United States do to accelerate the conversion process? A good place to start is with yourself. Start using SI units in your everyday life. When you make a purchase at the grocery store, look only at the SI unit on the label. Learn by inspection how many milliliters of liquid product are packaged in your favorite sized container. Abandon the use of inches, feet, yards, and miles as much as possible. How many kilometers lie between your home and school? What is 65 miles per hour in kilometers per hour? What is the mass of your automobile in kilograms? Determine your height in meters, your mass in kilograms, and your weight in newtons. How long is your arm in centimeters? What is your waist size in centimeters? What is the current outdoor air temperature in degrees Celsius? Most fast-food restaurants offer a "quarter pounder" on their menu. It turns out that 1 N = 0.2248 lb, almost a quarter pound. On the next visit to your favorite fast-food place, order a "newton burger" and fries. (See Figure 2.7.)

Figure 2.7
An engineering student orders lunch (art by Kathryn Colton).

PRACTICE!

1. A structural engineer states that an I-beam in a truss has a design stress of "five million, six hundred thousand pascals." Write this stress, using the appropriate SI unit prefix.
   Answer: 5.6 MPa.
2. The power cord on an electric string trimmer carries a current of 5.2 A. How many milliamperes is this? How many microamperes?
   Answer: 5.2×10^3 mA, 5.2×10^6 μA.
3. Write the pressure 7.2 GPa in scientific notation.
   Answer: 7.2×10^9 Pa.
4. Write the voltage 0.000875 V, using the appropriate SI unit prefix.
   Answer: 0.875 mV or 875 μV.
5. In the following list, various quantities are written using SI units incorrectly. Write the quantities, using the correct form of SI units.
   a. 4.5 mw
   b. 8.75 M pa
   c. 200 Joules/sec
   d. 20 W/m^2 K
   e. 3 Amps.

   Answer:
   a. 4.5 mW
   b. 8.75 MPa
   c. 200 J/s
   d. 20 W/m$^2 \cdot$ K
   e. 3 A.

2.5 ENGLISH UNITS

The English unit system is known by various names. Sometimes it is referred to as the United States Customary System (USCS), the British System or the Foot-Pound-Second (FPS) system. The English unit system is still used extensively in the United States even though the rest of the industrialized world, including Great Britain, has adopted the SI unit system. English units have a long and colorful history. In ancient times, measures of length were based on human dimensions. The foot started out as the actual length of a man's foot. Because not all men were the same size, the foot varied in length by as much as three or four inches. Once the ancients started using feet and arms for measuring distance, it was only a matter of time before they began using hands and fingers. The unit of length that we refer to today as the inch was originally the width of a man's thumb. The inch was also once defined as the distance between the tip to the first joint of the forefinger. Twelve times that distance made one foot. Three times the length of a foot was the distance from the tip of a man's nose to the end of his outstretched arm. This distance closely approximates what we refer to today as the yard. Two yards equaled a fathom, which was defined as the distance across a man's outstretched arms. Half a yard was the 18-inch cubit, which was called a span. Half a span was referred to as a hand.

The pound, which uses the symbol lb, is named after the ancient Roman unit of weight called the libra. The British Empire retained this symbol into modern times. Today, there are actually two kinds of pound units, one for mass and one for weight and force. The first unit is called pound-mass (lb_m), and the second is called pound-force (lb_f). Because mass and weight are not the same quantity, the units lb_f and lb_m are different.

As discussed previously, the seven base dimensions are length, mass, time, temperature, electric current, amount of substance, and luminous intensity. These base dimensions, along with their corresponding English units, are given in Table 2.8. As with SI units, English units are not capitalized. The slug, which has no abbreviated symbol, is the mass unit in the English system, but the pound-mass (lb_m) is frequently used. Electric current is based on SI units of meter and newton, and luminous intensity is based on SI units of watt. Hence, these two base dimensions do not have English units per se, and these quantities are rarely used in combination with other English units.

Recall that derived dimensions consist of a combination of base and supplementary dimensions. Table 2.9 summarizes some common derived dimensions expressed in English units. Note that Table 2.9 is the English counterpart of the SI version given by Table 2.5. The most notable English unit with a special name is the British thermal unit (Btu), a unit of energy. One Btu is defined as the energy required to change the temperature of 1 lb_m of water at a temperature of 68°F by 1°F. One Btu is approximately the energy released by the complete burning of a single kitchen match. The magnitudes of the kilojoule and Btu are almost equal (1 Btu = 1.055 KJ). Unlike the kelvin (K), the temperature unit in the SI system, the rankine (°R) employs a degree symbol as do the Celsius (°C) and Fahrenheit (°F) units. The same rules for writing SI units apply for English units with one major exception: *prefixes are generally not used with English units*. Thus, units such as kft (kilo foot), Mslug (megaslug), and GBtu (gigaBtu) should not be used. Prefixes are reserved for SI units. Two exceptions are the units ksi, which refers to a stress of 1000 psi (pounds per square inch), and kip, which is a special name for a force of 1000 lb_f (pound-force).

Table 2.8 Base Dimensions and Their English Units

Quantity	Unit	Symbol
Length	foot	ft
Mass	slug(1)	slug
Time	second	s
Temperature	rankine	°R
Electric current	ampere(2)	A
Amount of substance	mole	mol
Luminous intensity	candela(2)	cd

(1) The unit poind-mass (lb_m) is also used. 1 slug = 32.174 lb_m.
(2) There are no English units for electrical current and luminous intensity. The SI units are given here for completeness only.

Table 2.9 Derived Dimensions and English Units

Quantity	English Units
Acceleration	$ft \cdot s^{-2}$
Angular acceleration	$rad \cdot s^{-2}$
Angular velocity	$rad \cdot s^{-1}$
Area	ft^2
Concentration	$mol \cdot ft^{-3}$
Density	$slug \cdot ft^{-3}$
Electric field strength	$V \cdot ft^{-1}$
Energy	Btu
Entropy	$Btu \cdot slug^{-1} \cdot {}^{\circ}R^{-1}$
Force	lb_f
Heat	Btu
Heat transfer	$Btu \cdot s^{-1}$
Magnetic field strength	$A \cdot ft^{-1}$
Mass flow rate	$slug \cdot s^{-1}$
Moment of force	$lb_f \cdot ft$
Radiant intensity	$Btu \cdot s^{-1} \cdot sr^{-1}$
Specific energy	$Btu \cdot slug^{-1}$
Surface tension	$lb_f \cdot ft^{-1}$
Thermal conductivity	$Btu \cdot s^{-1} \cdot ft^{-1} \cdot {}^{\circ}R$
Velocity	$ft \cdot s^{-1}$
Viscosity, dynamic	$slug \cdot ft^{-1} \cdot s^{-1}$
Viscosity, kinematic	$ft^2 \cdot s^{-1}$
Volume	ft^3
Volume flow rate	$ft^3 \cdot s^{-1}$
Wavelength	ft

There are some non-SI units that are routinely used in the United States and elsewhere. Table 2.10 summarizes some of these units and provides an equivalent value in the SI system. The inch is a common length unit, being found on virtually every student's ruler and carpenter's tape measure in the United States. There are exactly 2.54 centimeters per inch. Inches are still used as the primary length unit in many engineering companies. The yard is commonly used for measuring cloth, carpets, and loads of concrete (cubic yards), as well as ball advancement on the American football field. The ton is used in numerous industries, including shipping, construction, and transportation. Time subdivisions on clocks are measured

Table 2.10 Non-SI Units Commonly Used in the United States

Quantity	Unit Name	Symbol	SI Equivalent
Length	inch	in	0.0254 m[1]
	yard	yd	0.9144 m (36 in)
Mass	metric ton	t	1000 kg
	short ton	t	907.18 kg $(2000\ \text{lb}_m)$
Time	minute	min	60 s
	hour	h	3600 s
	day	d	86,400 s
Plane angle	degree	°	$\pi/180$ rad
	minute	′	$\pi/10{,}800$ rad
	second	″	$\pi/648{,}000$ rad
Volume	liter	L	$10^{-3}\ \text{m}^3$
Land area	hectare	ha	$10^4\ \text{m}^2$
Energy	electron-volt	eV	1.602177×10^{-19} J

(1) Exact conversion.

in hours, minutes, and seconds. Radians and degrees are the most commonly used units for plane angles, whereas minutes and seconds are primarily used in navigational applications when referring to latitude and longitude on the earth's surface. The liter has made a lot of headway into the American culture, being found on beverage and food containers and many gasoline pumps. Virtually every American has seen the liter unit on a product, and many know that there are about four liters in a gallon (actually, 1 gal = 3.7854 L), but fewer people know that 1000 L = 1 m^3.

2.6 MASS AND WEIGHT

The concepts of *mass* and *weight* are fundamental to the proper use of dimensions and units in engineering analysis. Mass is one of the seven base dimensions used in science and engineering. Mass is a base dimension because it cannot be broken down into more fundamental dimensions. ***Mass*** is defined as a *quantity of matter.* This simple definition of mass may be expanded by exploring its basic properties. All matter possesses mass. The magnitude of a given mass is a measure of its resistance to a change in velocity. This property of matter is called *inertia.* A large mass offers more resistance to a change in velocity than a small mass, so a large mass has a greater inertia than a small mass. Mass may be considered in another way. Because all matter has mass, all matter exerts a gravitational attraction on other matter. Shortly after formulating his three laws of motion, Sir Isaac Newton postulated a law governing the gravitational attraction between two masses. Newton's law of universal gravitation is stated mathematically as

$$F = G\frac{m_1 m_2}{r^2} \tag{2.1}$$

where

F = gravitational force between masses (N)
G = universal gravitational constant $= 6.673 \times 10^{-11} \mathrm{m^3/kg \cdot s^2}$
m_1 = mass of body 1 (kg)
m_2 = mass of body 2 (kg)
r = distance between the centers of the two masses (m).

According to Equation (2.1), between any two masses there exists an attractive gravitational force whose magnitude varies inversely as the square of the distance between the masses. Because Newton's law of universal gravitation applies to *any* two masses, let's apply Equation (2.1) to a body resting on the surface of the earth. Accordingly, we let $m_1 = m_e$, the mass of the earth, and $m_2 = m$, the mass of the body. The distance, r, between the body and the earth may be taken as the mean radius of the earth, r_e. The quantities m_e and r_e have the approximate values

$$m_e = 5.979 \times 10^{24} \text{ kg} \qquad r_e = 6.378 \times 10^6 \text{ m}.$$

Thus, we have

$$\begin{aligned} F &= G\,\frac{m_e m}{r_e^2} \\ &= \frac{(6.673 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2)(5.979 \times 10^{24} \text{ kg})}{(6.378 \times 10^6 \text{ m})^2}\, m \\ &= (9.808 \text{ m/s}^2)\, m. \end{aligned}$$

We can see that upon substituting values, the term Gm_e/r_e^2 yields approximately 9.81 m/s^2, the standard acceleration of gravity on the earth's surface. Redefining this term as g, and letting $F = W$, we express the law of universal gravitation in a special form as

$$W = mg \tag{2.2}$$

where

W = weight of body (N)
m = mass of body (kg)
g = standard gravitational acceleration $= 9.81 \text{ m/s}^2$.

This derivation clearly shows the difference between mass and weight. We may therefore state the definition of ***weight*** as *a gravitational force exerted on a body by the earth.* Because mass is defined as a quantity of matter, the mass of a body is independent of its location in the universe. A body has the same mass whether it is located on the earth, the moon, Mars, or in outer space. The weight of the body, however, depends on its location. The mass of an 80 kg astronaut is the same whether or not he is on earth or in orbit above the earth. The astronaut weighs approximately 785 N on the earth, but while in orbit he is "weightless." His weight is zero while he orbits the earth, because he is continually "falling" toward earth. A similar weightless or "zero-g" condition is experienced by a skydiver as he begins falling.

The greatest source of confusion about mass and weight to the beginning engineering student is not the physical concept, but the units used to express each quantity. To see how units of mass and weight relate to each other, we employ a well-known scientific principle, ***Newton's second law*** of motion. Newton's second law of motion states that *a body of mass, m, acted upon by an unbalanced force, F, experiences an acceleration, a, that has the same direction of the force and a magnitude that is directly proportional to the force.* Stated mathematically, this law is

$$F = ma \tag{2.3}$$

where

F = force (N)

m = mass (Kg)

a = acceleration $(\mathrm{m/s^2})$.

Note that this relation resembles Equation (2.2). Weight is a particular type of force, and acceleration due to gravity is a particular type of acceleration, so Equation (2.2) is a special case of Newton's second law, given by Equation (2.3). In the SI unit system, the newton (N) is *defined* as the force that will accelerate a 1-kg mass at a rate of 1 $\mathrm{m/s^2}$. Hence, we may write Newton's second law dimensionally as

$$1\ \mathrm{N} = 1\ \mathrm{Kg \cdot m/s^2}.$$

In the English unit system, the pound-force ($\mathrm{lb_f}$) is *defined* as the force that will accelerate a 1-slug mass at a rate of 1 $\mathrm{ft/s^2}$. Hence, we may write Newton's second law dimensionally as

$$1\ \mathrm{lb_f} = 1\ \mathrm{slug \cdot ft/s^2}.$$

See Figure 2.8 for an illustration of Newton's second law. Confusion arises from the careless interchange of the English mass unit, pound-mass ($\mathrm{lb_m}$), with the English force unit, pound-force ($\mathrm{lb_f}$). These units are not the same thing! In accordance with our definitions of mass and weight, pound-mass refers to a quantity of matter, whereas pound-force refers to a force or weight. In order to write Newton's second law in terms of pound-mass instead of slug, we rewrite Equation (2.3) as

$$F = \frac{ma}{g_c} \tag{2.4}$$

where g_c is a constant that is required to make Newton's second law dimensionally consistent when mass, m, is expressed in $\mathrm{lb_m}$, rather than slug. As stated previously,

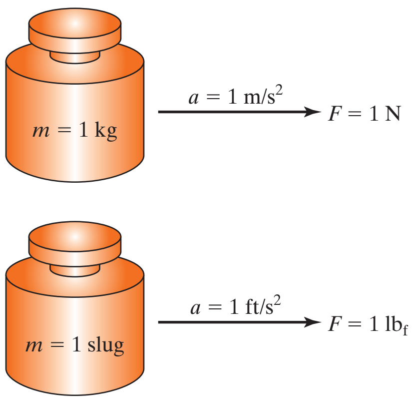


Figure 2.8
Definitions of the force units newton (N) and pound-force ($\mathrm{lb_f}$).

the English unit for force is lb_f, the English unit for acceleration is ft/s^2, and, as indicated in Table 2.8, 1 slug $= 32.174\ lb_m$. Thus, the constant g_c is

$$\begin{aligned} g_c &= \frac{ma}{F} \\ &= \frac{(32.174\ \mathrm{lb_m})(\mathrm{ft/s^2})}{\mathrm{lb_f}} \\ &= 32.174 \frac{\mathrm{lb_m \cdot ft}}{\mathrm{lb_f \cdot s^2}}. \end{aligned}$$

This value is usually rounded to

$$g_c = 32.2 \frac{\mathrm{lb_m \cdot ft}}{\mathrm{lb_f \cdot s^2}}.$$

Note that g_c has the same numerical value as g, the standard acceleration of gravity on the earth's surface. Newton's second law as expressed by Equation (2.4) is dimensionally consistent when the English unit of mass, lb_m, is used.

To verify that Equation (2.4) works, we recall that the pound-force is defined as the force that will accelerate a 1-slug mass at a rate of 1 ft/s^2. Recognizing that 1 slug $= 32.2\ lb_m$, we have

$$\begin{aligned} f &= \frac{ma}{g_c} \\ &= \frac{(32.2\ \mathrm{lb_m})(1\ \mathrm{ft/s^2})}{32.2 \dfrac{\mathrm{lb_m \cdot ft}}{\mathrm{lb_f \cdot s^2}}} = 1\ \mathrm{lb_f}. \end{aligned}$$

Note that in this expression, all the units, except lb_f, cancel. Hence, the pound-force (lb_f) is *defined* as the force that will accelerate a 32.2-lb_m mass at a rate of 1 ft/s^2. Therefore, we may write Newton's second law dimensionally as

$$1\ \mathrm{lb_f} = 32.2\ \mathrm{lb_m \cdot ft/s^2}.$$

To have dimensional consistency when English units are involved, Equation (2.4) *must* be used when mass, m, is expressed in lb_m. When mass is expressed in slug, however, the use of g_c in Newton's second law is not required for dimensional consistency because 1 lb_f is already defined as the force that will accelerate a 1-slug mass at a rate of 1 ft/s^2. Furthermore, because 1 N is already defined as the force that will accelerate a 1-kg mass at a rate of 1 m/s^2, the use of g_c is not required for dimensional consistency in the SI unit system. *Thus, Equation (2.3) suffices for all calculations, except for those in which mass is expressed in lb_m; in that case, Equation (2.4) must be used.* However, Equation (2.4) may be universally used when recognizing that the numerical value and units for g_c can be defined such that any consistent unit system will work. For example, substituting $F = 1$ N, $m = 1$ kg, and $a = 1\ m/s^2$ into Equation (2.4) and solving for g_c, we obtain

$$g_c = \frac{1\ \mathrm{kg \cdot m}}{\mathrm{N \cdot s^2}}.$$

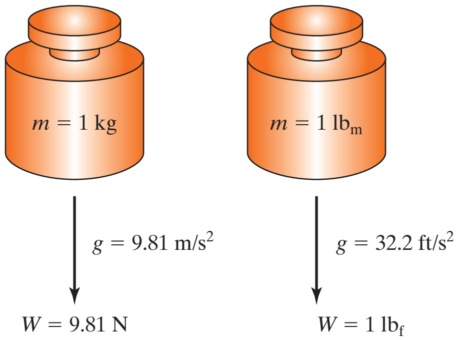


Figure 2.9
Definitions of weight for the standard value of gravitational acceleration.

Since the numerical value of g_c is 1, we can successfully use Equation (2.3) as long as we recognize that 1 N is the force that will accelerate a 1-kg mass at a rate of 1 m/s^2.

Sometimes, the units pound-mass (lb_m) and pound-force (lb_f) are casually interchanged because a body with a mass of 1 lb_m has a weight of 1 lb_f (i.e., the mass and weight are *numerically equivalent*). Let's see how this works: By definition, a body with a mass of 32.2 lb_m (1 slug) when accelerated at a rate of 1 ft/s^2 has a weight of 1 lb_f. Therefore, using Newton's second law in the form, $W = mg$, we can also state that a body with a mass of 1 lb_m, when accelerated at a rate of 32.2 ft/s^2 (the standard value of g), has a weight of 1 lb_f. Our rationale for making such a statement is that we maintained the same numerical value on the right side of Newton's second law by assigning the mass, m, a value of 1 lb_m and the gravitational acceleration, g, the standard value of 32.2 ft/s^2. The numerical values of the mass and weight are equal even though a pound-mass and a pound-force are conceptually different quantities. It must be emphasized, however, that mass in pound-mass and weight in pound-force are numerically equivalent only when the standard value, $g = 32.2$ ft/s^2, is used. See Figure 2.9 for an illustration. The next example illustrates the use of g_c.

EXAMPLE 2.4

Find the weight of some objects with the following masses:

a. 50 slug
b. 50 lb_m
c. 75 kg.

Solution

To find weight, we use Newton's second law, where the acceleration a is the standard acceleration of gravity, $g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2$.

a. The mass unit slug is the standard unit for mass in the English unit system. The weight is

$$\begin{aligned} W &= mg \\ &= (50 \text{ slug})(32.2 \text{ ft/s}^2) = 1{,}610 \text{ lb}_f. \end{aligned}$$

b. When mass is expressed in terms of lb_m, we must use Equation (2.4):

$$W = \frac{mg}{g_c} = \frac{(50\ \text{lb}_\text{m})(32.2\ \text{ft}\ /\ \text{s}^2))}{32.2\dfrac{\text{lb}_\text{m}\cdot\text{ft}}{\text{lb}_\text{f}\cdot\text{s}^2}} = 50\ \text{lb}_\text{f}.$$

Note that the mass and weight are numerically equivalent. This is true only in cases where the standard value of g is used, which means that an object with a mass of x lb_m will always have a weight of x lb_f on the earth's surface.

c. The mass unit kg is the standard unit for mass in the SI unit system. The weight is

$$\begin{aligned} W &= mg \\ &= (75\ \text{kg})(9.81\ \text{m/s}^2) = 736\ \text{N}. \end{aligned}$$

Alternatively, we can find weight by using Equation (2.4):

$$W = \frac{mg}{g_c} = \frac{(75\ \text{kg})(9.81\ \text{m/s}^2)}{1\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}} = 736\ \text{N}.$$

Now that we understand the difference between mass and weight and know how to use mass and weight units in the SI and English systems, let's revisit the astronaut we discussed earlier. (See Figure 2.10.) The mass of the astronaut is 80 kg, which equals about 5.48 slug. His mass does not change, regardless of where he ventures. Prior to departing on a trip to the moon, he weighs in at 785 N (176 lb_f). What is the mass of the astronaut in pound-mass? Three days later, his vehicle lands on the moon, and he begins constructing a permanent base for future planetary missions. The value of the gravitational acceleration on the moon is only 1.62 m/s^2(5.31 ft/s^2). The astronaut's mass is still 80 kg, but his weight is only 130 N (29.1 lb_f) due to the smaller value of g. Is the mass and weight of the astronaut in pound-mass and pound-force numerically equivalent? No, because the standard value of g is not used.

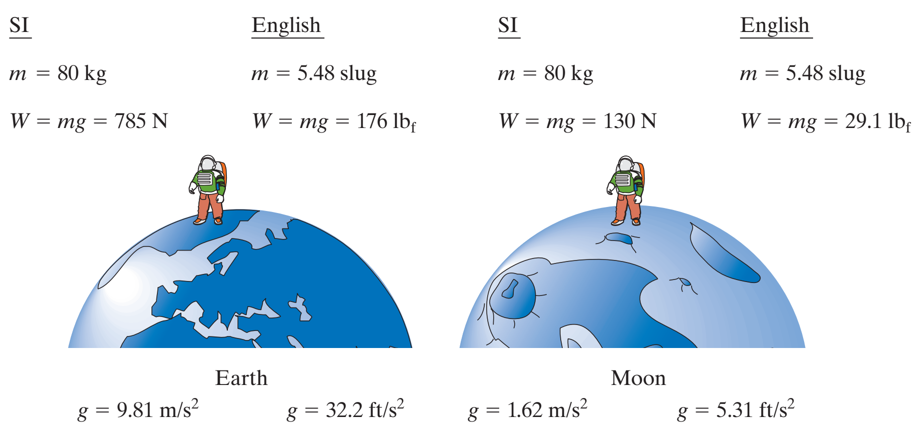


Figure 2.10
An astronaut's mass and weight on the earth and moon.

EXAMPLE 2.5

Special hoists are used in automotive repair shops to lift engines. As illustrated in Figure 2.11, a 200 kg engine is suspended in a fixed position by a chain attached to the cross member of an engine hoist. Neglecting the weight of the chain itself, what is the tension in portion AD of the chain?

Solution

This example is a simple problem in engineering statics. Statics is the branch of engineering mechanics that deals with forces acting on bodies at rest. The engine is held by the chain in a fixed position, so clearly the engine is at rest; that is, it is not in motion. This problem can be solved by recognizing that the entire weight of the engine is supported by portion AD of the chain. (The tension in portions AB and AC could also be calculated, but a thorough equilibrium analysis would be required.) Hence, the tension, which is a force that tends to elongate the chain, is equivalent to the weight of the engine. Using Equation (2.2), we have

$$\begin{aligned} F &= mg \\ &= (200 \text{ kg})(9.81 \text{ m/s}^2) = 1962 \text{ N}. \end{aligned}$$

Therefore, the tension in portion AD of the chain is 1962 N, the weight of the engine.

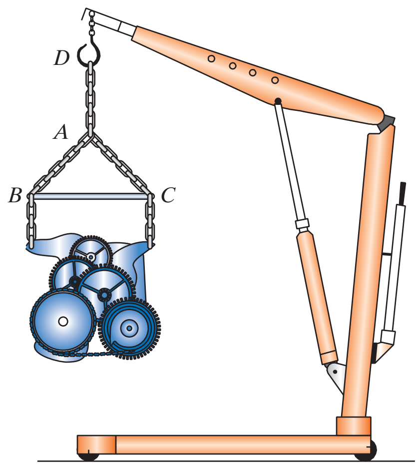


Figure 2.11 Engine hoist for Example 2.5.

PRACTICE!

1. It has been said that you do not fully understand a basic technical concept, unless you can explain it in terms simple enough that a second grader can understand it. Write an explanation of the difference between mass and weight for a second grader.
2. Which is larger, a slug or a pound-mass?
   Answer: slug.

3. Consider a professional linebacker who weighs 310 lb_f. What is his mass in slugs?
Answer: 9.63 slug.
4. A rock ($\rho = 2300\ kg/m^3$) is suspended by a single rope. Assuming the rock to be spherical, with a radius of 20 cm, what is the tension in the rope?
Answer: 756 N.

2.7 UNIT CONVERSIONS

Although the SI unit system is the international standard, English units are in widespread use in the United States. Americans as a whole are much more familiar with English units than SI units. Students of science and engineering in U.S. schools primarily use SI units in their course work because most textbooks and the professors who teach out of them stress SI units. Unfortunately, when students of these disciplines go about their day-to-day activities outside of the academic environment, they tend to slip back into the English unit mode along with everyone else. It seems as if students have a "unit switch" in their brains. When they are in the classroom or laboratory, the switch is turned to the "SI position." When they are at home, in the grocery store, or driving their car, the switch is turned to the "English position." Ideally, there should be no unit switch at all, but as long as science and engineering programs at colleges and universities stress SI units and American culture stresses English units, our cerebral unit switch toggles. In this section, a systematic method for converting units between the SI and English systems is given.

A ***unit conversion*** enables us to convert from one unit system to the other by using ***conversion factors***. A conversion factor is an equivalency ratio that has a *unit value* of 1. Stated another way, a conversion factor simply relates the same physical quantity in two different unit systems. For example, 0.0254 m and 1 in are equivalent length quantities because 0.0254 m = 1 in. The ratio of these two quantities has a unit value of 1 because they are physically the same quantity. Obviously, the numerical value of the ratio is not 1, but depends on the numerical value of each individual quantity. Thus, when we multiply a given quantity by one or more conversion factors, we alter only the numerical value of the result and not its dimension. Table 2.11 summarizes some common conversion factors used in engineering analysis. A more extensive listing of unit conversions is given in Appendix B.

A systematic procedure for converting a quantity from one unit system to the other is as follows:

2.7.1 Unit Conversion Procedure

1. Write the given quantity in terms of its numerical value and units. Use a horizontal line to divide units in the numerator (upstairs) from those in the denominator (downstairs).
2. Determine the units *to* which you want to make the conversion.
3. Multiply the given quantity by one or more conversion factors that, upon cancellation of units, leads to the desired units. Use a horizontal line to divide the units in the numerator and denominator of each conversion factor.
4. Draw a line through all canceled units.
5. Perform the numerical computations on a calculator, retaining maximum decimal place accuracy until the end of the computations.

Table 2.11 Some Common SI-to-English Unit Conversions

Quantity	Unit Conversion
Acceleration	$1\ m/s^2 = 3.2808\ ft/s^2$
Area	$1 m^2 = 10.7636\ ft^2 = 1550\ in^2$
Density	$1 kg/m^3 = 0.06243\ lb_m/ft^3$
Energy, work, heat	$1055.06\ J = 1\ Btu = 252\ cal$
Force	$1 N = 0.22481\ lb_f$
Length	$1 m = 3.2808\ ft = 39.370\ in$
	$0.0254 m = 1\ in^{(1)}$
Mass	$1 kg = 2.20462\ lb_m = 0.06852\ slug$
Power	$1 W = 3.4121\ Btu/h$
	$745.7\ W = 1\ hp$
Pressure	$1 kPa = 20.8855\ lb_f/ft^2 = 0.14504\ lb_f/in^2$
Specific heat	$1 kj/kg \cdot °C = 0.2388\ Btu/lb_m \cdot °F$
Temperature	$T(K) = T(°C) + 273.16 = T(°R)/1.8 = [T(°F) + 459.67]/1.8$
Velocity	$1 m/s = 2.2369\ mi/h$

(1) Exact conversion.

6. Write the numerical value of the converted quantity by using the desired number of significant figures (three significant figures is standard practice for engineering) with the desired units.

Examples 2.6, 2.7, and 2.8 illustrate the unit conversion procedure.

EXAMPLE 2.6

An engineering student is late for an early morning class, so she runs across campus at a speed of 9 mi/h. Determine her speed in units of m/s.

Solution

The given quantity, expressed in English units, is 9 mi/h, but we want our answer to be in SI units of m/s. Thus, we need a conversion factor between mi and m and a conversion factor between h and s. To better illustrate the unit conversion procedure, we will use two length conversion factors rather than one. Following the procedure outlined, we have

$$\underbrace{9\frac{\cancel{mi}}{\cancel{h}}}_{\uparrow \text{ given quantity}} \times \underbrace{\frac{5280\ \cancel{ft}}{1\ \cancel{mi}} \times \frac{1\ m}{3.2808\ \cancel{ft}} \times \frac{1\ \cancel{h}}{3600\ s}}_{\uparrow \text{ conversion factors}} = \underbrace{4.02\frac{m}{s}}_{\uparrow \text{ answer}}.$$

The key aspect of the unit conversion process is that the conversion factors must be written such that the appropriate units in the conversion factors cancel those in the given quantity. If we had inverted the conversion factor between ft and mi, writing it instead as 1 mi/5280 ft, the mi unit would not cancel and our unit conversion exercise would not work, because we would end up with units of mi^2 in the numerator. Similarly, the conversion factor between m and ft was written such that the ft unit canceled the ft unit in the first conversion factor. Also, the conversion factor between h and s was written such that the h unit canceled with the h unit in the given quantity. Writing conversion factors with the units in the proper locations, "upstairs" or "downstairs," requires some practice, but after doing several conversion problems, the correct placement of units will become second nature to you. Note that our answer is expressed in three significant figures.

EXAMPLE 2.7

Lead has one of the highest densities of all the pure metals. The density of lead is 11,340 kg/m^3. What is the density of lead in units of lb_m/in^3?

Solution

A direct conversion factor from kg/m^3 to lb_m/in^3 may be available, but to illustrate an important aspect of converting units with exponents, we will use a series of conversion factors for each length and mass unit. Thus, we write our unit conversion as

$$11{,}340\,\frac{\cancel{\text{kg}}}{\cancel{\text{m}^3}} \times \left(\frac{1\ \cancel{\text{m}}}{3.2808\ \cancel{\text{ft}}}\right)^3 \times \left(\frac{1\ \cancel{\text{ft}}}{12\ \text{in}}\right)^3 \times \frac{2.20462\ \text{lb}_\text{m}}{1\ \cancel{\text{kg}}} = 0.410\ \text{lb}_\text{m}/\text{in}^3.$$

We used two length conversion factors, one factor between m and ft and the other between ft and in. But the given quantity is a density that has a volume unit. When performing unit conversions involving exponents, *both* the numerical value and the unit must be raised to the exponent. A common error that students make is to raise the unit to the exponent, which properly cancels units, but to forget to raise the numerical value also. Failure to raise the numerical value to the exponent will lead to the wrong numerical answer even though the units in the answer will be correct. Using the direct conversion factor obtained from Appendix B, we obtain the same result:

$$11{,}340\ \cancel{\text{kg}}/\cancel{\text{m}}^3 \times \frac{3.6127\times10^{-5}\ \text{lb}_\text{m}/\text{in}^3}{1\ \cancel{\text{kg}}/\cancel{\text{m}}^3} = 0.410\ \text{lb}_\text{m}/\text{in}^3.$$

EXAMPLE 2.8

Specific heat is defined as the energy required to raise the temperature of a unit mass of a substance by one degree. Pure aluminum has a specific heat of approximately $900\ \text{J/Kg}\cdot{}^\circ\text{C}$. Convert this value to units of $\text{Btu/lb}_\text{m}\cdot{}^\circ\text{F}$.

Solution

By following the unit conversion procedure, we write the given quantity and then multiply it by the appropriate conversion factors, which can be found in Appendix B:

$$\frac{900\ \cancel{\text{J}}}{\cancel{\text{kg}}\cdot{}^\circ\cancel{\text{C}}}\times\frac{1\ \text{Btu}}{1055.06\ \cancel{\text{J}}}\times\frac{1\ \cancel{\text{kg}}}{2.20462\ \text{lb}_\text{m}}\times\frac{1^\circ\cancel{\text{C}}}{1.8^\circ\text{F}}=0.215\ \text{Btu/lb}_\text{m}\cdot{}^\circ\text{F}.$$

The temperature unit °C in the original quantity has a unique interpretation. Because specific heat is the energy required to raise a unit mass of a substance by one degree, the temperature unit in this quantity denotes a temperature *change*, not an absolute temperature value. A temperature change of 1°C is equivalent to a temperature change of 1.8°F. Other thermal properties, such as thermal conductivity, involve the same temperature change interpretation.

This example can also be done by applying a single conversion factor $1\ \text{kJ/Kg}\cdot{}^\circ\text{C} = 0.2388\ \text{Btu/lb}_\text{m}\cdot{}^\circ\text{F}$, which yields the same result.

PROFESSIONAL SUCCESS—UNIT CONVERSIONS AND CALCULATORS

Scientific pocket calculators have evolved from simple electronic versions of adding machines to complex portable computers. Today's high-end scientific calculators have numerous capabilities, including programming, graphing, numerical methods, and symbolic mathematics. Most scientific calculators also have an extensive compilation of conversion factors. Why, then, should students learn to do unit conversions by hand when calculators will do the work? This question lies at the root of a more fundamental question: why should students learn to do *any* computational task by hand when calculators or computers will do the work? Is it because "in the old days" engineers did not have the luxury of highly sophisticated computational tools, so professors, who perhaps lived in the "old days," forced their students to do things the old fashioned way? Not really.

Students will always need to learn engineering by *thinking* and *reasoning* their way through a problem, regardless of whether that problem is a unit conversion or a stress calculation in a machine component. Computers, and the software that runs on them, do not replace the thinking process. The calculator, like the computer, should never become a "black box" to the student. A black box is a mysterious device whose inner workings are largely unknown, but that, nonetheless, provides output for every input supplied. By the time you graduate with an engineering degree, or certainly by the time you have a few years of professional engineering practice, you will come to realize that a calculator program or computer software package exists for solving many types of engineering problems. This does not mean that you need to learn every one of these programs and software packages. It means that you should become proficient in the use of those computational tools that pertain to your particular engineering field *after* learning the underlying basis for each. By all means, use a calculator or a unit conversion application to perform unit conversions, but *first* know how to do them by hand, so you gain confidence in your own computational skills and have a way to verify the results of computer-based tools.

PRACTICE!

1. A microswitch is an electrical switch that requires only a small force to operate it. If a microswitch is activated by a 0.25-oz force, what is the force in units of N that will activate it?
   Answer: 0.0695 N.
2. At room temperature, water has a density of about $62.4\ \mathrm{lb_m/ft^3}$. Convert this value to units of $\mathrm{slug/in^3}$ and $\mathrm{kg/m^3}$.
   Answer: $1.12 \times 10^{-3}\ \mathrm{slug/in^3}$, $999.5\ \mathrm{kg/m^3}$.
3. At launch, the Saturn V rocket that carried astronauts to the moon developed five million pounds of thrust. What is the thrust in units of MN?
   Answer: 22.2 MN.
4. Standard incandescent light bulbs produce more heat than light. Assuming that a typical house has twenty 60-W bulbs that are continuously on, how much heat in units of Btu/h is supplied to the house from light bulbs if 90 percent of the energy produced by the bulbs is in the form of heat?
   Answer: 3685 Btu/h.
5. Certain properties of animal (including human) tissue can be approximated by using those of water. Using the density of water at room temperature, $\rho = 62.4\ \mathrm{lb_m/ft^3}$, calculate the weight of a human male by approximating him as a cylinder with a length and diameter of 6 ft and 10 in, respectively.
   Answer: $204\ \mathrm{lb_f}$.
6. The standard frequency for electrical power in the United States is 60 Hz. For an electrical device that operates on this power, how many times does the current alternate during a year?
   Answer: 1.89×10^9.

KEY TERMS

base dimension	English unit system	unit
conversion factors	mass	unit conversion
derived dimension	Newton's second law	weight
dimension	physical standards	
dimensionally consistent	SI unit system	

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PROBLEMS

Dimensions

2.1 For the following dimensional equations, find the base dimensions of the parameter k:

a. $\mathrm{MLt} = k\,\mathrm{ML^{-1}t^{-2}}$
b. $\mathrm{MLt^{-2}L^{-1}} = k\ \mathrm{Lt^{-3}}$
c. $\mathrm{L^3t^{-2}} = k\,\mathrm{M^3T}$
d. $\mathrm{ML^2t^{-3}} = k\ \mathrm{LT}$
e. $\mathrm{nLL^3}k = \mathrm{T^2M^{-2}L}$
f. $\mathrm{MI^2}k = \mathrm{nTM^{-3}L^{-1}}$
g. $\mathrm{IL^{-2}t} = k^2\,\mathrm{M^4t^2}$
h. $k^3\mathrm{T^6M^3L^{-5}} = \mathrm{T^{-3}t^{-6}L}$
i. $\mathrm{T^{-1/2}L^{-1}I^2} = k^{-1/2}\mathrm{t^4T^{-5/2}L^{-3}}$
j. $\mathrm{MLt^{-2}} = \mathrm{MLt^{-2}}\ \sin(k\mathrm{L^{-2}M^{-1}})$
k. $\mathrm{T^2n} = T^2\mathrm{n}\ \ln(k\mathrm{nT^{-1}})$

2.2 Is the following dimensional equation dimensionally consistent? Explain.

$$\mathrm{ML} = \mathrm{ML}\cos(\mathrm{Lt}).$$

2.3 Is the following dimensional equation dimensionally consistent? Explain.

$$\mathrm{t^2LT} = \mathrm{tLT}\log\left(\mathrm{tt^{-1}}\right).$$

2.4 Is the following dimensional equation dimensionally consistent? Explain.

$$\mathrm{TnT} = \mathrm{TnT}\exp(\mathrm{MM^{-1}}).$$

Units

2.5 In the following list, various quantities are written using SI units incorrectly: Write the quantities, using the correct form of SI units.

a. 10.6 secs
b. 4.75 amps
c. 120 M hz
d. 2.5 Kw
e. 0.00846 kg/μs
f. 90 W/m^2 K