



COLLEGE
ALGEBRA
ESSENTIALS

BUTLER

sixth
edition

A Brief Guide to Getting the Most from This Book

1 Read the Book

Feature	Description	Benefit
Section-Opening Scenarios	Every section opens with a scenario presenting a unique application of algebra in your life outside the classroom.	Realizing that algebra is everywhere will help motivate your learning. (See page 108.)
EXAMPLE	Examples are clearly written and provide step-by-step solutions. No steps are omitted, and each step is thoroughly explained to the right of the mathematics.	The blue annotations will help you understand the solutions by providing the reason why every algebraic step is true. (See page 110.)
Applications Using Real-World Data	Interesting applications from nearly every discipline, supported by up-to-date real-world data, are included in every section.	Ever wondered how you'll use algebra? This feature will show you how algebra can solve real problems. (See pages 266–268.)
> GREAT QUESTION !	Answers to students' questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.	By seeing common mistakes, you'll be able to avoid them. This feature should help you not to feel anxious or threatened when asking questions in class. (See page 111.)
BRIEF REVIEW	Brief Reviews cover skills you already learned but may have forgotten.	Having these refresher boxes easily accessible will help ease anxiety about skills you may have forgotten. (See page 480.)
BLITZER BONUS	These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the algebra under study.	Yet even more proof that math is an interesting and dynamic discipline! (See page 167.)
Explanatory Voice Balloons	Voice balloons help to demystify algebra. They translate math into plain English, clarify problem-solving procedures, and present alternative ways of understanding.	Does math ever look foreign to you? This feature often translates math into everyday English. (See page 165.)
WHAT YOU'LL LEARN	Every section begins with a list of objectives. Each objective is restated in the margin where the objective is covered.	The objectives focus your reading by emphasizing what is most important and where to find it. (See page 150.)
1 Learning Objective		
> TECHNOLOGY	The screens displayed in the technology boxes show how graphing utilities verify and visualize algebraic results.	Even if you are not using a graphing utility in the course, this feature will help you understand different approaches to problem solving. (See page 112.)

2 Work the Problems

Feature	Description	Benefit
CHECK POINT	Each example is followed by a matched problem, called a Check Point, that offers you the opportunity to work a similar exercise. The answers to the Check Points are provided in the answer section.	You learn best by doing. You'll solidify your understanding of worked examples if you try a similar problem right away to be sure you understand what you've just read. (See page 110.)
ACHIEVING SUCCESS	Achieving Success boxes offer strategies for persistence and success in college mathematics courses.	Follow these suggestions to help achieve your full academic potential in college mathematics. (See page 121.)
CONCEPT AND VOCABULARY CHECK	These short-answer questions, mainly fill-in-the-blank and true/false items, assess your understanding of the definitions and concepts presented in each section.	It is difficult to learn algebra without knowing its special language. These exercises test your understanding of the vocabulary and concepts. (See page 121.)
EXERCISE SET	An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within several categories. Your instructor will usually provide guidance on which exercises to work. The exercises in the first category, Practice Exercises, follow the same order as the section's worked examples.	The parallel order of the Practice Exercises lets you refer to the worked examples and use them as models for solving these problems. (See page 122.)
Practice PLUS	This category of exercises contains more challenging problems that often require you to combine several skills or concepts.	It is important to dig in and develop your problem-solving skills. Practice PLUS Exercises provide you with ample opportunity to do so. (See page 409.)
Retaining the Concepts	Beginning with Chapter 2, each Exercise Set contains review exercises under the header "Retaining the Concepts."	These exercises improve your understanding of the topics and help maintain mastery of the material. (See page 256.)
Preview Exercises	Each Exercise Set concludes with three problems to help you prepare for the next section.	These exercises let you review previously covered material that you'll need to be successful for the forthcoming section. Some of these problems will get you thinking about concepts you'll soon encounter. (See page 125.)

3 Review for Quizzes and Tests

Feature	Description	Benefit
Mid-Chapter Check Point	At approximately the midway point in the chapter, an integrated set of review exercises allows you to review the skills and concepts you learned separately over several sections.	By combining exercises from the first half of the chapter, the Mid-Chapter Check Points give a comprehensive review before you move on to the material in the remainder of the chapter. (See page 283.)
Chapter Review Chart	Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.	Review this chart and you'll know the most important material in the chapter! (See page 208.)
Chapter Review Exercise Set	A comprehensive collection of review exercises for each of the chapter's sections follows the review chart.	Practice makes perfect. These exercises contain the most significant problems for each of the chapter's sections. (See page 211.)
Chapter Test	Each chapter contains a practice test with approximately 25 problems that cover the important concepts in the chapter. Take the practice test, check your answers, and then watch the Chapter Test Prep Videos to see worked-out solutions for any exercises you miss.	You can use the chapter test to determine whether you have mastered the material covered in the chapter. (See page 215.)
Chapter Test Prep Videos	These videos contain worked-out solutions to every exercise in each chapter test and can be found in MyLab Math and on YouTube at youtube.com/user/pearsonmathstats (playlist "Blitzer College Algebra 8e").	The videos let you review any exercises you miss on the chapter test.
Objective Videos	These fresh, interactive videos walk you through the concepts from every objective of the text.	The videos provide you with active learning at your own pace.
Cumulative Review	Beginning with Chapter 2, each chapter concludes with a comprehensive collection of mixed cumulative review exercises. These exercises combine problems from previous chapters and the present chapter, providing an ongoing cumulative review.	Ever forget what you've learned? These exercises ensure that you are not forgetting anything as you move forward. (See page 345.)

College Algebra Essentials

This page intentionally left blank

6th EDITION

College Algebra Essentials

Robert Blitzer

Miami Dade College

Content Management: Jonathan Krebs, Jeff Weidenaar
Content Production: Tamela Ambush, Eric Gregg, Shana Siegmund
Product Management: Chelsea Kharakozova
Product Marketing: Brooke Imbornone, Stacey Sveum
Rights and Permissions: Venugopal Loganathan

Please contact <https://support.pearson.com/getsupport/s/> with any queries on this content.

Cover Image: Annie Brace; Cover Design: Jerilyn DiCarlo

About the Cover: Bob Blitzer's signature chili pepper provides a window into a vibrant nature scene rendered by Alaska-based artist Annie Brace. The cover is a metaphor for Bob's book, which provides insight into a world that is profoundly mathematical.

Copyright © 2022, 2018, 2014 by Pearson Education, Inc. or its affiliates, 221 River Street, Hoboken, NJ 07030. All Rights Reserved. Manufactured in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise. For information regarding permissions, request forms, and the appropriate contacts within the Pearson Education Global Rights and Permissions department, please visit www.pearsoned.com/permissions/.

Acknowledgments of third-party content appear on pages C1 and RC1, which constitute an extension of this copyright page.

PEARSON, ALWAYS LEARNING, and MYLAB are exclusive trademarks owned by Pearson Education, Inc. or its affiliates in the U.S. and/or other countries.

Unless otherwise indicated herein, any third-party trademarks, logos, or icons that may appear in this work are the property of their respective owners, and any references to third-party trademarks, logos, icons, or other trade dress are for demonstrative or descriptive purposes only. Such references are not intended to imply any sponsorship, endorsement, authorization, or promotion of Pearson's products by the owners of such marks, or any relationship between the owner and Pearson Education, Inc., or its affiliates, authors, licensees, or distributors.

Library of Congress Cataloging-in-Publication Data

Library of Congress Control Number: 2020947930

ScoutAutomatedPrintCode



Rental

ISBN-10: 0-13-730828-0

ISBN-13: 978-0-13-730828-6

Annotated Instructor's Edition

ISBN-10: 0-13-680461-6

ISBN-13: 978-0-13-680461-1

Contents

Preface ix
To the Student xix
About the Author xx
Applications Index xxi

P Fundamental Concepts of Algebra 1

- P.1** Algebraic Expressions, Mathematical Models, and Real Numbers 2
- P.2** Exponents and Scientific Notation 20
- P.3** Radicals and Rational Exponents 35
- P.4** Polynomials 51
- Mid-Chapter Check Point** 62
- P.5** Factoring Polynomials 63
- P.6** Rational Expressions 75
- Summary, Review, and Test** 90
- Review Exercises** 91
- Chapter P Test** 93

1 Equations and Inequalities 95

- 1.1** Graphs and Graphing Utilities 96
- 1.2** Linear Equations and Rational Equations 108
- 1.3** Models and Applications 126
- 1.4** Complex Numbers 140
- 1.5** Quadratic Equations 150
- Mid-Chapter Check Point** 173
- 1.6** Other Types of Equations 175
- 1.7** Linear Inequalities and Absolute Value Inequalities 191
- Summary, Review, and Test** 208
- Review Exercises** 211
- Chapter 1 Test** 215





2 Functions and Graphs 217

- 2.1** Basics of Functions and Their Graphs 218
- 2.2** More on Functions and Their Graphs 237
- 2.3** Linear Functions and Slope 257
- 2.4** More on Slope 273
- Mid-Chapter Check Point** 283
- 2.5** Transformations of Functions 284
- 2.6** Combinations of Functions; Composite Functions 300
- 2.7** Inverse Functions 315
- 2.8** Distance and Midpoint Formulas; Circles 326
- Summary, Review, and Test** 336
- Review Exercises** 339
- Chapter 2 Test** 343
- Cumulative Review Exercises (Chapters 1–2)** 345

3 Polynomial and Rational Functions 347

- 3.1** Quadratic Functions 348
- 3.2** Polynomial Functions and Their Graphs 366
- 3.3** Dividing Polynomials; Remainder and Factor Theorems 384
- 3.4** Zeros of Polynomial Functions 397
- Mid-Chapter Check Point** 412
- 3.5** Rational Functions and Their Graphs 413
- 3.6** Polynomial and Rational Inequalities 433
- 3.7** Modeling Using Variation 446
- Summary, Review, and Test** 456
- Review Exercises** 459
- Chapter 3 Test** 462
- Cumulative Review Exercises (Chapters 1–3)** 464





5 Systems of Equations and Inequalities 543

5.1 Systems of Linear Equations in Two Variables 544

5.2 Systems of Linear Equations in Three Variables 563

5.3 Partial Fractions 571

5.4 Systems of Nonlinear Equations in Two Variables 582

Mid-Chapter Check Point 592

5.5 Systems of Inequalities 593

5.6 Linear Programming 606

Summary, Review, and Test 613

Review Exercises 615

Chapter 5 Test 618

Cumulative Review Exercises (Chapters 1–5) 618

Appendix: Where Did That Come From? Selected Proofs 621

Answers to Selected Exercises AA1

Subject Index I1

Credits C1

4 Exponential and Logarithmic Functions 465

4.1 Exponential Functions 466

4.2 Logarithmic Functions 480

4.3 Properties of Logarithms 495

Mid-Chapter Check Point 505

4.4 Exponential and Logarithmic Equations 506

4.5 Exponential Growth and Decay; Modeling Data 521

Summary, Review, and Test 535

Review Exercises 537

Chapter 4 Test 541

Cumulative Review Exercises (Chapters 1–4) 542

This page intentionally left blank

Preface

I've written *College Algebra Essentials, Sixth Edition*, to help diverse students, with different backgrounds and future goals, to succeed. The book has three fundamental goals:

1. To help students acquire a solid foundation in algebra, preparing them for other courses such as calculus, business calculus, and finite mathematics.
2. To show students how algebra can model and solve authentic real-world problems.
3. To enable students to develop problem-solving skills, while fostering critical thinking, within an interesting setting.

One major obstacle in the way of achieving these goals is the fact that very few students actually read their textbook. This has been a regular source of frustration for me and for my colleagues in the classroom. Anecdotal

evidence gathered over years highlights two basic reasons that students do not take advantage of their textbook:

- “I’ll never use this information.”
- “I can’t follow the explanations.”

I’ve written every page of the Sixth Edition with the intent of eliminating these two objections. The ideas and tools I’ve used to do so are described for the student in “A Brief Guide to Getting the Most from This Book,” which appears on the endpapers of the book.

College Algebra Essentials, Sixth Edition, is a concise version of the **Eighth Edition** of *College Algebra*. The essentials version differs from the Eighth Edition only in terms of length: Chapter 6 (Matrices and Determinants), Chapter 7 (Conic Sections), and Chapter 8 (Sequences, Induction, and Probability) have been eliminated.

What’s New in the Sixth Edition?

The Sixth Edition contains 62 worked-out examples and exercises based on new data sets and 112 updated examples and exercises. Many of the new and updated applications involve topics relevant to college students.

New Applications

- Cost and Enrollment for Federal Social Programs (Section P.2, Exercises 115–117)
- Educational Attainment and Probability of Divorce (Section 1.1, Example 6)
- Number of Smartphone Users in the U.S. (Section 2.1, Figure 2.2)
- Spending on Pre-Primary Education and Child Care (Section 2.1, Exercises 99–100)
- Internet Plans (Section 2.2, Example 6 and Exercises 95–96)
- Trust in Government and the Media (Section 2.3, Exercises 87–88)
- Accelerating Climate Change (Blitzer Bonus in Section 2.3, p. 268)
- Living Arrangements of Young Adults (Section 2.4 opener and Example 3)
- U.S. Population Projections by Age (Section 2.6, Exercises 97–98)
- Time Spent Online (Cumulative Review for Chapters 1–2, Exercise 21)
- Rumbling Back: Steven Spielberg’s New *West Side Story* (Blitzer Bonus in Section 2.7, p. 315)
- Addressing Leisure Time Parabolically (Blitzer Bonus in Section 3.1, p. 361)
- COVID-19 Pandemic (Section 3.2 opener; Cumulative Review for Chapters 1–3, Exercise 21; Section 4.5, Example 3; Cumulative Review for Chapters 1–5, Exercise 36)
- AIDS: A Global Perspective (Blitzer Bonus in Section 3.2, p. 370)
- Area Burned by Wildfires in the U.S. (Section 3.2, Exercise 76)
- Costco Paid Membership (Chapter 3 Review, Exercise 68)
- Mumps (Chapter 3 Mid-Chapter Check Point, Exercise 28)
- Putting Off Medical Treatment Because of Expenses (Section 4.2, Exercises 115–116)
- E-commerce Sales (Cumulative Review for Chapters 1–4, Exercise 81)
- Modeling Body Temperature, Heart Rate, and Respiratory Rate (Chapter 5 opener; Section 5.5 opener, Example 5, and Exercises 72–82)
- Number of Men and Women in the U.S. House of Representatives (Section 5.1, Exercise 70)
- Share of U.S. Income by Top 10% and Bottom 90% of Americans (Section 5.1, Exercise 72)

- The Late Elvis Presley’s Business Machine (Section 5.2, Exercise 45)
- Use of Social Media by Age (Chapter 5 Review, Exercise 8)

Updated Applications

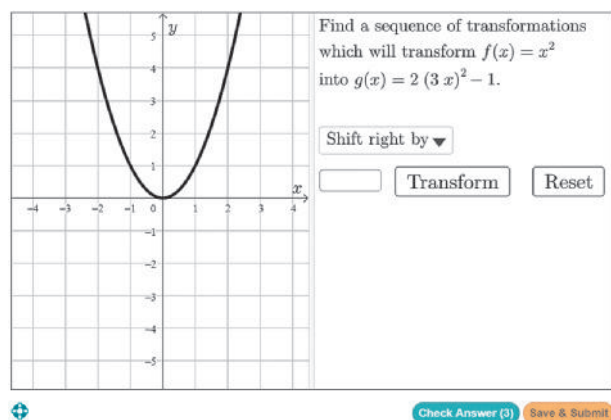
- Cost of Tuition and Fees at Public and Private Colleges (Section P.1, Example 2 and Exercises 131–132)
- The National Debt (Section P.2 opener, Example 12, and Exercises 118–120)
- Student Loan Debt (Chapter P Mid-Chapter Check Point, Exercise 31)
- Different Race or Ethnicity for Two Randomly Selected Americans (Chapter P Review, Exercise 23)
- Alcohol and Marijuana Use by High School Seniors (Section 1.1, Exercises 55–56)
- Inflation (Section 1.2, Exercises 109–112)
- Median Earnings by Educational Attainment (Section 1.3, Example 1)
- Attitudes of College Freshmen (Section 1.3, Example 2)
- Interest Rates (Section 1.3, Example 5; Section 4.1, Example 7; Section 4.4, Example 10; Section 8.3, Example 7 and Exercises 79–82)
- Car Prices and Age of Cars on U.S. Roads (Section 1.3, Exercises 5–6)
- Average Price of a Movie Ticket (Chapter 1 Review, Exercise 37)
- Toll Options (Section 1.3 opener, Example 3, and Exercises 11–12)
- Highest-Paid TV Actors and Actresses (Section 2.1, Figure 2.1)
- The Wage Gap between Men and Women (Section 2.1, Exercises 103–104)
- Fuel Efficiency of New U.S. Cars (Section 2.2 opener)
- Number of Births and Deaths in the U.S. (Section 2.6 opener and Example 4)
- Political Orientation of U.S. College Freshmen (Chapter 2 Review, Exercise 53)
- One-Person Households as a Percentage of the U.S. Total (Chapter 2 Test, Exercise 28)
- AIDS Cases in the U.S. (Section 3.2, Example 3)
- World Tiger Population (Section 3.2, Exercises 73–74)
- Federal Budget Expenditures on Human Resources (Section 3.5, Exercise 107)
- Amazon Deforestation (Chapter 3 Review, Exercise 14)
- Gray Wolf Population (Section 4.1, Example 6)
- Percentage of High School Seniors Applying to More Than Three Colleges (Section 4.1, Exercises 71–72)
- Number of Pages in the Federal Tax Code (Section 4.1, Exercise 85)
- Percentage of GDP Going Toward Health Care (Section 4.4, Exercises 115–116)
- U.S. Population (Section 4.5, Example 1; Section 8.3, Example 3)
- World Population (Section 4.5, Examples 5 and 6)
- Populations of Various Countries (Section 4.5, Exercises 1–14)
- Marital Status of U.S. Adults (Section 5.1, Exercise 67; Section 8.7, Example 9 and Exercises 1–10)
- Rate of Violent Crime and Imprisonment in the U.S. (Section 5.4, Exercise 63)

Other Textbook Changes

- Prior to the exercises in each section, the Annotated Instructor’s Edition provides a list of resources available for that section in MyLab Math.
- The list of each section’s objectives, previously headed “What am I supposed to learn?” (which annoyed some reviewers) has been renamed “What You’ll Learn.”
- Section P.6 includes new examples and exercises involving adding and subtracting rational expressions with different monomial denominators. (Section P.6, Example 8 and Exercises 43–50)
- Section 1.4 contains a new objective involving simplifying powers of i . (Section 1.4, Example 7 and Exercises 55–60)
- In Chapter 3, the standard form of a quadratic function, $f(x) = a(x - h)^2 + k$, has been renamed the *vertex form*.

New in MyLab Math

- **Corequisite Support Resources** provide all the content and assessment resources necessary for students and instructors. MyLab Math supports various corequisite course models, including Concurrent (aka just-in-time) and Consecutive (aka front-loaded) models. For more details, see page xiii or the Corequisite Implementation Guide at bit.ly/2ZXGs52.
- **Integrated Review Activities** for selected topics provide hands-on work with important prerequisites.
- **Interactive Figures** – For this revision, we added many more interactive figures (in editable GeoGebra format) to the Video & Resource Library.
- **PowerPoint slides** are now animated. They also utilize Microsoft’s Equation Editor, making them more easily editable.
- **Personal Inventory Assessments** are a collection of online exercises designed to promote self-reflection and engagement in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.



▲ **GeoGebra Graphing Exercises** are gradable graphing exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.

- **Setup & Solve Exercises** – We added more of these popular exercises, which require students to first describe how they will set up and approach the problem. This mirrors what students will be expected to do on a test.
- **Enhanced Assignments** – These section-level assignments have three unique properties (and are fully editable):
 1. They help keep skills fresh with *spaced practice* of previously learned concepts.
 2. They have learning aids strategically turned off for some exercises to ensure that students understand how to work the exercises independently.
 3. They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.
- **Video Assignments** – These section-level assignments are especially helpful for online classes or “flipped” classes, where some or all learning takes place independently.

What Familiar Features Have Been Retained in the Sixth Edition?

- **Graphing and Functions.** Graphing is introduced in Chapter 1 and functions are introduced in Chapter 2, with an integrated graphing functional approach emphasized throughout the book. Graphs and functions that model data appear in nearly every section and Exercise Set. Examples and exercises use graphs of functions to explore relationships between data and to provide ways of visualizing a problem’s solution. Because functions are the core of this course, students are repeatedly shown how functions relate to equations and graphs.
- **Learning Objectives.** Learning objectives are clearly stated at the beginning of each section under the heading “What You’ll Learn.” These objectives help students recognize and focus on the section’s most important ideas. The objectives are restated in the margin at their point of use.
- **Chapter-Opening and Section-Opening Scenarios.** Every chapter and every section open with a scenario presenting a unique application of mathematics in students’ lives outside the classroom. These scenarios are revisited in the course of the chapter or section in an example, discussion, or exercise.
- **Innovative Applications.** A wide variety of interesting applications, supported by up-to-date, real-world data, are included in every section.

Explanatory
Voice Balloons

- **Explanatory Voice Balloons.** Voice balloons are used in a variety of ways to demystify mathematics.

They translate algebraic ideas into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect problem solving to concepts students have already learned.

- **Brief Reviews.** Beginning with Chapter 1, the Brief Review boxes that appear throughout the book summarize mathematical skills, many of which are course prerequisites that students have learned but which many students need to review. This feature appears whenever a particular skill is first needed and eliminates the need for you to reteach that skill. For more detail, students are referred to the appropriate section and objective in a previous chapter where the topic is fully developed.
- **Detailed Worked-Out Examples.** Each example is titled, making the purpose of the example clear. Examples are clearly written and provide students with detailed step-by-step solutions. No steps are omitted and each step is thoroughly explained to the right of the mathematics.

✓ CHECK POINT

- **Check Point Examples.** Each example is followed by a similar matched problem, called a Check Point, offering students the opportunity to test their understanding of the example by working a similar exercise. The answers to the Check Points are provided in the answer section.

- **Great Question!** This feature presents a variety of study tips in the context of students' questions. Answers to questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions. As a secondary benefit, this feature should help students not to feel anxious or threatened when asking questions in class.

BLITZER BONUS

- **Blitzer Bonuses.** These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the algebra under study, showing students that math is an interesting and dynamic discipline.
- **Concept and Vocabulary Checks.** This feature offers short-answer exercises, mainly fill-in-the-blank and true/false items, that assess students' understanding of the definitions and concepts presented in each section. The Concept and Vocabulary Checks precede the section Exercise Sets and have the prefix "C."
- **Extensive and Varied Exercise Sets.** An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within nine category types to make it easy to create well-rounded homework assignments:
 - Practice Exercises (see details below)
 - Practice PLUS Exercises (see details below)
 - Application Exercises
 - Explaining the Concepts
 - Technology Exercises
 - Critical Thinking Exercises
 - Group Exercises
 - Retaining the Concepts (see details below)
 - Preview Exercises
- **Practice Exercises.** The order of the Practice Exercises is exactly the same as the order of the section's worked examples. This parallel order enables students to refer to the titled examples and their detailed explanations to achieve success working the Practice Exercises.
- **Practice PLUS Problems.** This category of exercises contains more challenging practice problems that often require students to combine several skills or concepts. With an average of ten Practice PLUS problems per Exercise Set, instructors are provided with the option of creating assignments that take Practice Exercises to a more challenging level.
- **Retaining the Concepts.** Beginning with Chapter 2, Section 2.1, each Exercise Set contains three review exercises under the header "Retaining the Concepts." These exercises are intended for students to review previously covered objectives in order to improve their understanding of the topics and to help maintain their mastery of the material. If students are not certain how to solve a review exercise, they can turn to the section and worked example given in parentheses at the end of each exercise.
- **Mid-Chapter Check Points.** At approximately the midway point in each chapter, an integrated set of Review Exercises allows students to review and assimilate the skills and concepts they learned separately over several sections.
- **Integration of Technology Using Graphic and Numerical Approaches to Problems.** Side-by-side features in the technology boxes connect algebraic solutions to graphic and numerical approaches to problems. Although the use of graphing utilities is optional, students can use the explanatory voice balloons to understand different approaches to problems even if they are not using a graphing utility in the course.

> ACHIEVING SUCCESS

- **Achieving Success.** The Achieving Success boxes, appearing at the end of many sections in Chapters 1 through 5, offer strategies for persistence and success in college mathematics courses.
- **Discovery.** Discovery boxes, found throughout the text, encourage students to further explore algebraic concepts. These explorations are optional and their omission does not interfere with the continuity of the topic under consideration.
- **Chapter Summaries.** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.
- **End-of-Chapter Materials.** A comprehensive collection of Review Exercises for each of the chapter's sections follows the Summary. This is followed by a Chapter Test that enables students to test their understanding of the material covered in the chapter. Beginning with Chapter 2, each chapter concludes with a comprehensive collection of mixed Cumulative Review Exercises.

MyLab™ Math Resources for Success

MyLab Math (pearson.com/mylab/math) is available to accompany Pearson's market-leading text options, including this text (access code required). MyLab Math is the teaching and learning platform that empowers you to reach every student. It combines trusted author content—including full eText and online homework with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

NEW! Corequisite Course Support

MyLab Math supports various corequisite course models, including Concurrent (aka just-in-time) and Consecutive (aka front-loaded) models. MyLab Math for this text contains all of these learning and assessment resources to support corequisite courses:

1. **Complete Corequisite eText** built from Bob Blitzer's developmental mathematics texts so that it matches the features and pedagogy of this text.
2. **Instructional videos** for each corequisite objective.
3. **Assignable algorithmic exercises** for each corequisite objective.
4. **Worksheets** with instruction and exercises for each corequisite objective (also available in print).
5. **Activities** for selected corequisite objectives.
6. **Study Skills** support with self-help materials for time management, mindset, stress management, college transition, and more.
7. **Corequisite Implementation Guide** with specific guidelines for using the materials to teach various corequisite models. (Download at bit.ly/2ZXGs52.)

To help target instruction on corequisite objectives, MyLab includes these pre-made assessments:

- **Readiness Quiz 1** addresses key arithmetic topics and is designed to be administered prior to beginning College Algebra topics.
- **Readiness Quiz 2** addresses basic Introductory Algebra topics and is designed to be administered prior to beginning College Algebra topics.
- **Skills Check Quiz for Each Chapter** addresses the pre-requisite skills needed for each chapter in College Algebra.
- Based on the results of these quizzes, students can receive *personalized assignments* to address objectives that are not mastered. This way, students can focus on just the topics they need help with.

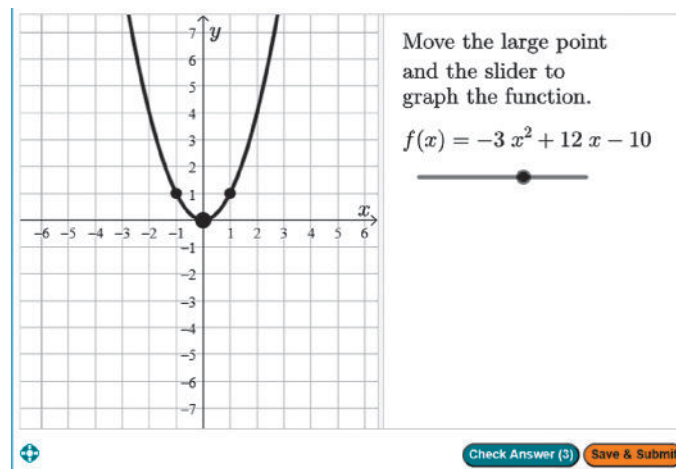
Note that the above resources are also designed to provide just-in-time help for students in your regular (non-corequisite) courses. (We understand that almost all students at some point need targeted refreshers on specific prerequisite skills.)

MyLab Math Student Resources

Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student

needs practice, giving all students the support they need—when and where they need it—to be successful.

Exercises with Immediate Feedback – The exercises in MyLab Math reflect the approach and learning style of this text and regenerate algorithmically to give students unlimited opportunity for practice and mastery. Most exercises include learning aids, such as guided solutions and sample problems, and they offer helpful feedback when students enter incorrect answers.

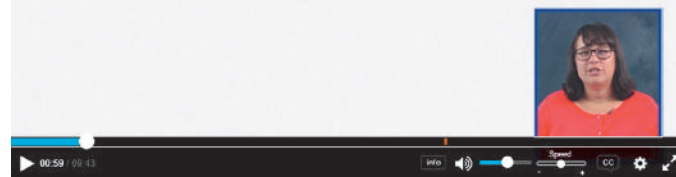


NEW! GeoGebra Exercises are gradable graphing exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.

- **Setup & Solve** exercises require students to first describe how they will set up and approach a problem. This reinforces conceptual understanding of the process applied in approaching the problem, promotes long-term retention of the skill, and mirrors what students will be expected to do on a test.
- **Concept & Vocabulary** exercises require students to demonstrate understanding of key ideas.

Solve. Find each solution set and then use a calculator to obtain a decimal approximation to two decimal places for the solution.

a. $5^x = 134$

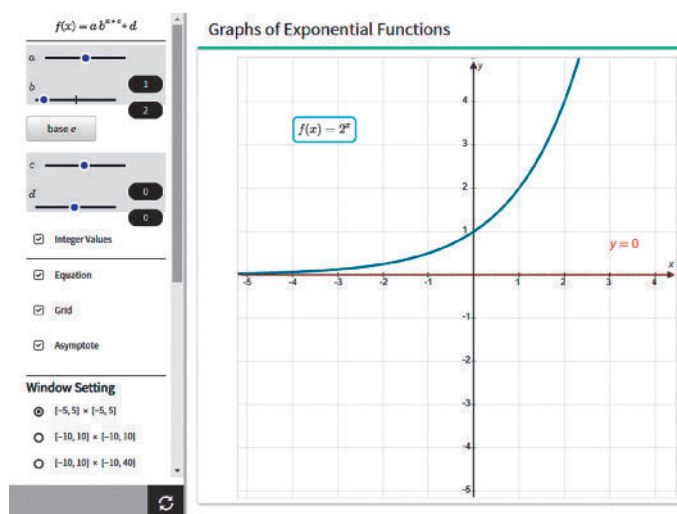


Instructional Videos – High-quality instructional videos are included for every objective in the text. Many of these feature built-in interactive quizzes.

Chapter Test Prep Videos correspond to each exercise in the Chapter Test in the textbook, enabling students to effectively prepare for high-stakes testing. These are available in MyLab Math and www.youtube.com/user/pearsonmathstats (playlist “Blitzer College Algebra 8e”).

Learning Guide (also available in print format) consists of four parts:

1. **Learning Guide worksheets** for each section of the text. These worksheets start with a catchy headline and motivating real-world connection followed by numerous “Solved Problems” and accompanying “Pencil Problems.”
2. **Classroom Activities** for selected sections contain recommended group size, material needed, and time to complete.
3. **Integrated Review worksheets** for every prerequisite objective. These feature both instruction and practice.
4. **NEW! Integrated Review Activities** for selected topics provide hands-on work with important prerequisites.



▲ **Interactive Figures** bring mathematical concepts to life, helping students see the concepts through directed explorations and purposeful manipulation. These figures are assignable in MyLab Math and encourage active learning, critical thinking, and conceptual understanding.

NEW! For this revision, we added many more interactive figures (in editable GeoGebra format) to the Video & Resource Library.



▲ **Mindset videos** and assignable, open-ended exercises foster a growth mindset in students. This material encourages them to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as learning opportunities—so often a hurdle for math students. These videos are one of many **Study Skills and Career-Readiness Resources** that address the non-math-related issues that can affect student success.

NEW! Personal Inventory Assessments are a collection of online exercises designed to promote self-reflection and engagement in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.

eText – Available in two formats: one that matches the textbook page-for-page and another that is “reflowable” for use on tablets and smartphones. The latter eText is also fully accessible using screen-readers.

Student Solutions Manual – Fully worked solutions to odd-numbered exercises. Available for download from within MyLab Math.

MyLab Math Instructor Resources

Your course is unique. So whether you’d like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

Pre-Built Assignments are designed to make the homework experience as effective as possible for students. All of these assignments are *fully editable*.

- **NEW! Enhanced Assignments** – These section-level assignments have three unique properties:
 1. They help keep skills fresh with *spaced practice* of previously learned concepts.

2. They have learning aids strategically turned off for some exercises to ensure that students understand how to work the exercises independently.
 3. They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.
- **NEW!** **Video Assignments** – These section-level assignments are especially helpful for online classes or “flipped” classes, where some or all learning takes place independently.

Learning Catalytics – With Learning Catalytics™, you’ll hear from every student when it matters most. You pose a variety of questions in class (choosing from pre-loaded questions or questions of your own making) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops.

Accessibility – Pearson works continuously to ensure our products are as accessible as possible to all students. Currently we work toward achieving WCAG 2.0 AA for our existing products (2.1 AA for future products) and Section 508 standards, as expressed in the Pearson

Guidelines for Accessible Educational Web Media (<https://wps.pearsoned.com/accessibility/>).

Other instructor resources include:

- **Mini Lecture Notes** contain additional examples and helpful teaching tips for each section of the text.
- **Instructor Solution Manual** contains worked-out solutions for every exercise in the text.
- **PowerPoint Lecture Slides** are fully editable and included for each section of the text. **UPDATED!** Slides are now animated. They also utilize Microsoft’s Equation Editor, making them more easily editable.
- **TestGen®** enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson’s online catalog.
- **Test Bank** features printable PDFs containing all of the test exercises available in TestGen.

Acknowledgments

An enormous benefit of authoring a successful series is the broad-based feedback I receive from the students, dedicated users, and reviewers. Every change to this edition is the result of their thoughtful comments and suggestions. I would like to express my appreciation to all the reviewers, whose collective insights form the backbone of this revision. In particular, I would like to thank the following people for reviewing *College Algebra*, *Algebra and Trigonometry*, *Precalculus*, and *Trigonometry*. (An asterisk * indicates reviewers for the current edition.)

*Margaret Adams, *South Georgia South College*

Karol Albus, *South Plains College*

*Alvina Atkinson, *Georgia Gwinnett College*

Kayoko Yates Barnhill, *Clark College*

*Melissa Battis, *Greenhill School*

Timothy Beaver, *Isothermal Community College*

Jaromir Becan, *University of Texas, San Antonio*

Imad Benjelloun, *Delaware Valley College*

Lloyd Best, *Pacific Union College*

*Joseph Bittner, *Fort Wayne North Side High School*

*Elisa Bolotin, *Saint Stephen’s Episcopal School*

*Marie Borrazzo, *College Academy*

David Bramlett, *Jackson State University*

Natasha Brewley-Corbin, *Georgia Gwinnett College*

Denise Brown, *Collin College, Spring Creek Campus*

*Julie Brown, *University of Texas of the Permian Basin*

David Britz, *Raritan Valley Community College*

Bill Burgin, *Gaston College*

Jennifer Cabaniss, *Central Texas College*

*Ryan Caimano, *Oklahoma Bible Academy*

*Kate Calendrillo, *Northwest Catholic High School*

*Jaime Castrataro, *Tri-West High School*

Jimmy Chang, *St. Petersburg College*

Teresa Chasing Hawk, *University of South Dakota*

*Janie Coker Pritchard, *Tyler Junior College*

Diana Colt, *University of Minnesota, Duluth*

Shannon Cornell, *Amarillo College*

Wendy Davidson, *Georgia Perimeter College, Newton*

Donna Densmore, *Bossier Parish Community College*

*Kevin Dibert, *NSU University School*

*Barbara Dobbs, *Academy of the Holy Cross*

*Juan Du, *Broward College*

*Marcial Echenique, *College of the Florida Keys*

Disa Enegren, *Rose State College*

Keith A. Erickson, *Georgia Gwinnett College*

*Deanna Ettore, *Washington Township High School*

- *Rafat Ewais, *Clifton High School*
- *Rebecca Faber, *Notre Dame High School*
- *Ed Fischer, *Northern Nash High School*
- Nancy Fisher, *University of Alabama*
- *Vickie Flanders, *Baton Rouge Community College*
- *Sheila Fleming, *Marble Falls High School*
- Donna Gerken, *Miami Dade College*
- *David Ghazvini, *Lee College*
- Cynthia Glickman, *Community College of Southern Nevada*
- Sudhir Kumar Goel, *Valdosta State University*
- Donald Gordon, *Manatee Community College*
- David L. Gross, *University of Connecticut*
- Jason W. Groves, *South Plains College*
- Joel K. Haack, *University of Northern Iowa*
- Jeremy Haefner, *University of Colorado*
- Joyce Hague, *University of Wisconsin at River Falls*
- *Tawfik Haj, *San Jacinto College*
- Mike Hall, *University of Mississippi*
- *Leo Hartsock, *Northland Preparatory Academy*
- Mahshid Hassani, *Hillsborough Community College*
- Tom Hayes, *Montana State University*
- Christopher N. Hay-Jahans, *University of South Dakota*
- Angela Heiden, *St. Clair Community College*
- *Baron Heinemann, *Episcopal High School*
- Celeste Hernandez, *Richland College*
- *Ann Ho, *Taipei American School*
- Alysmarie Hodges, *Eastfield College*
- Amanda Hood, *Copiah-Lincoln Community College*
- Jo Beth Horney, *South Plains College*
- Heidi Howard, *Florida State College at Jacksonville, South Campus*
- *S. Larue Huckaby, *Shorter University*
- Winfield A. Ihlow, *SUNY College at Oswego*
- *Dale Johanson, *Northeast Community College*
- Nancy Raye Johnson, *Manatee Community College*
- *Kimberly Jones, *Dakota State University*
- *Kevin Jones, Jr., *McCluer High School*
- *Cheryl Kerns, *Blue Valley West High School*
- *Clayton Kitchings, *University of North Georgia*
- *Theo Koupelis, *Broward College*
- *Anthony Lamanna, *St. John's Preparatory*
- Dennine Larue, *Fairmont State University*
- *Tina Lee, *Haywood Community College*
- Mary Leesburg, *Manatee Community College*
- Christine Heinecke Lehman, *Purdue University North Central*
- Alexander Levichev, *Boston University*
- *Qingxia Li, *Fisk University*
- *Ethan Lightfoot, *Gateway STEM High School*
- Zongzhu Lin, *Kansas State University*
- Benjamin Marlin, *Northwestern Oklahoma State University*
- Marilyn Massey, *Collin County Community College*
- Yvelyne McCarthy-Germaine, *University of New Orleans*
- *Kayri McCartin, *Grafton High School*
- David McMann, *Eastfield College*
- Owen Mertens, *Missouri State University, Springfield*
- James Miller, *West Virginia University*
- Martha Nega, *Georgia Perimeter College, Decatur*
- *Youssef Oumanar, *Greenhill School*
- Shahla Peterman, *University of Missouri, St. Louis*
- Debra A. Pharo, *Northwestern Michigan College*
- *Janice Phillipp, *Texas Southmost College*
- Gloria Phoenix, *North Carolina Agricultural and Technical State University*
- Katherine Pinzon, *Georgia Gwinnett College*
- David Platt, *Front Range Community College*
- Juha Pohjanpelto, *Oregon State University*
- *David Quesnell, *Immaculate High School*
- Brooke Quinlan, *Hillsborough Community College*
- *Corrie Ramage, *Kosciusko High School*
- Janice Rech, *University of Nebraska at Omaha*
- Joseph W. Rody, *Arizona State University*
- *Lee Ann Roberts, *Georgia Gwinnett College*
- Behnaz Rouhani, *Georgia Perimeter College, Dunwoody*
- Judith Salmon, *Fitchburg State University*
- *Ryan Sasaki, *Iolani School*
- Michael Schramm, *Indian River State College*
- Cynthia Schultz, *Illinois Valley Community College*
- *Brittney Seale, *Colorado Mountain College*
- *Juanita Self, *Central Texas College*
- *Patricia Senn, *Lurleen B. Wallace Community College*
- *Olimpia Simeón Monet, *Miami Dade College*
- *Muhammad Naeem Sharif, *Kuwait Technical College*
- Pat Shelton, *North Carolina Agricultural and Technical State University*
- Jed Soifer, *Atlantic Cape Community College*
- *Brian Southworth, *Independence Community College*
- Caroline Spillman, *Georgia Perimeter College, Clarkston*

*Laura Spunt, *Posnack School*

Jonathan Stadler, *Capital University*

Franotis R. Stallworth, *Gwinnett Technical College*

John David Stark, *Central Alabama Community College*

Chris Stump, *Bethel College*

*Cynthia Lee Suplizio, *Colorado Mountain College*

Scott Sykes, *University of West Georgia*

Richard Townsend, *North Carolina Central University*

Pamela Trim, *Southwest Tennessee Community College*

*Ruth Trubnik, *Delaware Valley University*

Chris Turner, *Arkansas State University*

Richard E. Van Lommel, *California State University,
Sacramento*

Dan Van Peurse, *University of South Dakota*

Philip Van Veldhuizen, *University of Nevada at Reno*

Jeffrey Weaver, *Baton Rouge Community College*

*Felice Weiner, *South Mountain Community College*

*Aaron Wernet, *McLennan Community College*

Amanda Wheeler, *Amarillo College*

David White, *The Victoria College*

*Molly Whittington, *Holy Cross School*

Tracy Wienckowski, *University of Buffalo*

*Stacy Yarnell, *Colorado Mountain College*

Additional acknowledgments are extended to:

- Brad Davis, for contributing new and updated data, providing the book's annos and answer section, and serving as (an amazing!) answer checker.
- Dan Miller for preparing the solution manuals and the Learning Guide.

- Brian Morris at Scientific Illustrators for superbly illustrating the book.
- Jerilyn DiCarlo and Annie Brace for their brilliant design work.
- Francesca Monaco, project manager, and Tamela Ambush, production editor, whose collective talents kept every aspect of this complex project moving through its many stages. (Special thanks to Sharon Cahill for seamlessly stepping in during Francesca's absence.)
- Dawn Murrin, Jeff Weidenaar, and Jon Krebs, my editors at Pearson, who guided and coordinated the book from manuscript through production.
- Chelsea Kharakozova, product manager, for the steady financial guidance for the project.
- Stacey Sveum and Peggy Lucas, marketing managers, for their innovative marketing efforts.
- Shana Siegmund, MyLab producer, for coordinating the many digital aspects of the project.
- Eric Gregg and Dominick Frank for splendid work on the MyLab exercises that support the texts.
- Finally, thanks to the Pearson sales force, for their confidence and enthusiasm about the book.

I hope that my passion for teaching, as well as my respect for the diversity of students I have taught and learned from over the years, is apparent throughout this new edition. By connecting algebra to the whole spectrum of learning, it is my intent to show students that their world is profoundly mathematical, and indeed, π is in the sky.

Bob Blitzer

This page intentionally left blank

The bar graph shows some of the qualities that students say make a great teacher. It was my goal to incorporate each of these qualities throughout the pages of this book.

Explains Things Clearly

I understand that your primary purpose in reading *College Algebra Essentials* is to acquire a solid understanding of the required topics in your algebra course. In order to achieve this goal, I've carefully explained each topic. Important definitions and procedures are set off in boxes, and worked-out examples that present solutions in a step-by-step manner appear in every section. Each example is followed by a similar matched problem, called a Check Point, for you to try so that you can actively participate in the learning process as you read the book. (Answers to all Check Points appear in the back of the book.)

Funny & Entertaining

Who says that an algebra textbook can't be entertaining? From our unusual cover to the photos in the chapter and section openers, prepare to expect the unexpected. I hope some of the book's enrichment essays, called Blitzer Bonuses, will put a smile on your face from time to time.

Helpful

I designed the book's features to help you acquire knowledge of college algebra, as well as to show you how algebra can solve authentic problems that apply to your life. These helpful features include:

- **Explanatory Voice Balloons:** Voice balloons are used in a variety of ways to make math less intimidating. They translate algebraic language into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect new concepts to concepts you have already learned.
- **Great Question!:** The book's Great Question! boxes are based on questions students ask in class. The answers to these questions give suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.
- **Achieving Success:** The book's Achieving Success boxes give you helpful strategies for success in learning algebra, as well as suggestions that can be applied for achieving your full academic potential in future college coursework.
- **Chapter Summaries:** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples from the chapter that illustrate these key concepts are also referenced in the chart. Review these summaries and you'll know the most important material in the chapter!

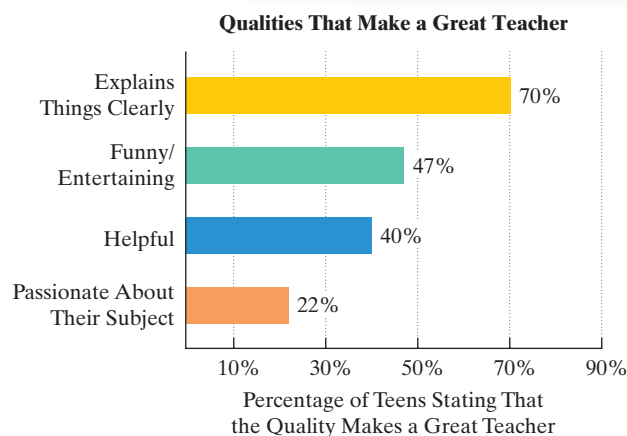
Passionate about the Subject

I passionately believe that no other discipline comes close to math in offering a more extensive set of tools for application and development of your mind. I wrote the book in Point Reyes National Seashore, 40 miles north of San Francisco. The park consists of 75,000 acres with miles of pristine surf-washed beaches, forested ridges, and bays bordered by white cliffs. It was my hope to convey the beauty and excitement of mathematics using nature's unspoiled beauty as a source of inspiration and creativity. Enjoy the pages that follow as you empower yourself with the algebra needed to succeed in college, your career, and your life.

Regards,

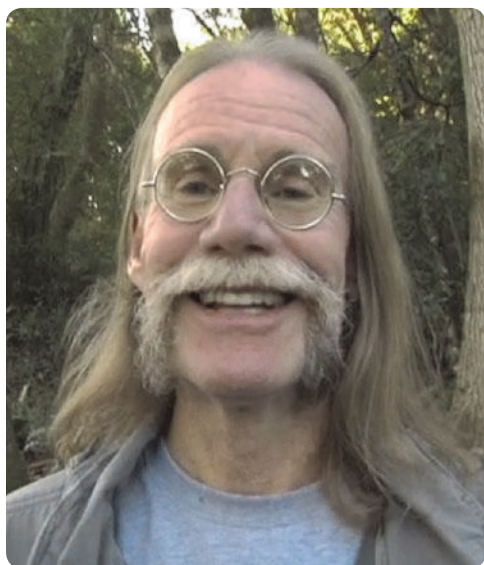
Bob Blitzer

To the Student



Source: Avanta Learning System

ABOUT THE AUTHOR



Bob Blitzer is a native of Manhattan and received a Bachelor of Arts degree with dual majors in mathematics and psychology (minor: English literature) from the City College of New York. His unusual combination of academic interests led him toward a Master of Arts in mathematics from the University of Miami and a doctorate in behavioral sciences from Nova University. Bob's love for teaching mathematics was nourished for nearly 30 years at Miami Dade College, where he received numerous teaching awards, including Innovator of the Year from the League for Innovations in the Community College and an endowed chair based on excellence in the classroom. In addition to *College Algebra Essentials*, Bob has written textbooks covering developmental mathematics, introductory algebra, intermediate algebra, trigonometry, algebra and trigonometry, precalculus, and liberal arts mathematics, all published by Pearson. When not secluded in his Northern California writer's cabin, Bob can be found hiking the beaches and trails of Point Reyes National Seashore and tending to the chores required by his beloved entourage of horses, chickens, and irritable roosters.

APPLICATIONS INDEX

A

Accidents, automobile
 age of driver and, 171
 alcohol use and, 513–514, 519
 Acid rain, 519
 Actors, highest-paid, on television, 218–221
 Advertising, 451–452, 456
 African Americans, with high school diploma, 540
 Age(s)
 arrests and drunk driving as function of, 432
 average number of awakenings during night by, 106
 body-mass index and, 604
 calories needed to maintain energy by, 89
 chances of surviving to various, 235
 fatal crashes and, 171
 and heart rate for children, 593, 597–598
 height as function of, 277, 280, 298
 perceived length of time period and, 455
 percent body fat in adults by, 255
 racial prejudice and, 61
 and respiratory rate for children, 603
 and social media use, 615
 systolic blood pressure and, 165–166
 tax refund likelihood and, 49
 U.S. population projection by, 313
 weight of human fetus and, 214
 Aging rate, space travel and, 35, 47, 50
 AIDS. *See also* HIV infection
 cases diagnosed (U.S.), 370
 global perspective on, 370
 Airplanes, weight/volume constraints, 605
 Alcohol use
 and accident risk, 513–514, 519
 drunk driving arrests, 432
 moderate wine consumption and heart disease, 271–272
 number of moderate users in U.S., 540
 by U.S. high school seniors, 106
 Alligator(s)
 population of, 174
 tail length given body length, 454
 Altitude and atmospheric pressure, 539
 Amazon deforestation, 459–460
American Idol, ratings of, 364–365
 Apartment, rent, 213
 Applause, decibel level of, 259
 Archer's arrow, path of, 358
 Area
 maximum, 360, 363, 412, 459
 of plane figure, 62
 of shaded region, 61, 74
 Arrests, drunk driving, 432
 Aspirin, half-life of, 532
 Asteroid detection, 582
 Atmospheric pressure and altitude, 539
 Automobiles
 alcohol use and accident risk, 513–514, 519
 annual price increases of, 137
 average age, on U.S. roads, 137
 depreciation, 137, 235
 drunk driving arrests as function of age, 432
 fatal accidents and driver's age, 171

rentals, 191–192, 215
 repair estimates for, 207
 required stopping distance, 443–444
 travel by bicycle vs., 324–325
 Average cost function, 426–427, 430, 461, 463
 Average rate of change, 277–278, 298

B

Backpack, price before reduction, 535
 Ball, thrown upward and outward, 363
 Ball's height above ground
 baseball, 542
 bounce height, 454
 football, 17, 356–358, 459
 maximum height, 459
 when thrown across field, 571
 when thrown from rooftop, 443, 569
 when thrown from top of Leaning Tower of Pisa, 441
 Banking angle and turning radius of bicycle, 454
 Baseball
 diamond diagonal length, 171
 height of ball above ground, 542
 Basketball, hang time in air when shooting, 189
 Basketball court, dimensions of, 134
 Bass in lake over time, 461
 Beauty, symmetry and, 240
 Berlin Airlift, 606, 612
 Bias, Implicit Association Test for, 51, 61
 Bicycle
 banking angle, 454
 manufacturing, 235, 430, 558, 612
 travel by car vs., 324–325
 Bird species population decline, 532
 Birth(s), in U.S. from 2000 through 2009, 300, 305–306
 Birthday, probability of sharing same, 325
 Birthday cake, 50
 Blood-alcohol concentration, 15–16, 19, 513–514, 519
 Blood pressure, systolic, age and, 165–166
 Blood volume and body weight, 93, 447–448
 Body fat in adults by age and gender, percent, 255
 Body-mass index, 454, 604
 Box dimensions, 395
 Brain, growth of the human, 527
 Break-even analysis, 552–553, 558, 592
 Bridge coupon book/toll passes, 126, 129–131, 137, 174, 202–203, 207, 314
 Budgeting, groceries vs. health care, 281
 Building's shadow, 214
 Business ventures, 558

C

Cable lengths between vertical poles, 190
 Calculator manufacturing costs, 461
Call of Duty video game, retail sales of, 520
 Calorie-nutrient information, 616
 Calories, needed to maintain energy balance, 89
 Camera, price before reduction, 131
 Canoe manufacturing, 558
 Car(s). *See* Automobiles

Carbon-14 dating, 524, 531–532
 Carbon dioxide, atmospheric global warming and, 217, 266–268
 Carbon dioxide, atmospheric levels of, 268
 Cardboard length/width for box, 591
 Cave paintings, prehistoric, 532
 Celebrity earnings, 218–221
 Cellular phones
 pricing of plans, 236
 replacement of land lines with, 559
 Celsius/Fahrenheit temperature interconversions, 17, 206, 341
 Centrifugal force, 452–453
 Checking accounts, 207
 Chernobyl nuclear power plant accident, 477
 Chess tournament, round-robin, 170
 Child care, spending on, 234
 Children
 heart rate and age, 593, 597–598
 respiratory rate and age, 603
 Children's height modeled, 487, 494, 515
 Cholesterol
 and dietary restrictions, 603
 intake, 603
 Cigarette consumption. *See* Smoking
 Coffee consumption, sleep and, 542
 Coin tosses, 206
 College(s)
 percentage of U.S. high school seniors applying to more than three, 478
 projected enrollment, 137, 213, 214
 salary after, 212
 College education
 availability of, to qualified students, 137
 average yearly earnings and, 136–137
 cost of, 2, 4–5, 19–20, 365
 excuses for not meeting assignment deadlines, 212
 government aid decreases, 214
 women vs. men, 93
 College graduates, median starting salaries for, 127–128
 College majors, campus mergers and, 139
 College students
 excuses for not meeting assignment deadlines, 212
 freshmen
 attitudes about life goals, 128–129
 with A average in high school, 125
 claiming no religious affiliation, 221–222
 grade inflation, 124
 political orientation, 341
 hours per week spent studying, by major, 569–570
 loan debt, 63
 music majors, 561
 percentage students playing online games, 561
 procrastination and symptoms of physical illness among, 544, 560
 sleep hours of, 561
 women as percentage of, 561
 College tuition
 government aid decreases, 214
 student loan debt, 63

Comedians, net worth of, 494
 Comets, intersection of planet paths and, 590
 Commuters, toll discount passes, 129–131, 137, 207
 Compound interest
 choosing between investments, 475
 compounding periods, 479
 continuously compounded, 514, 518, 540, 541, 619
 formula for, 514
 investments, 538
 savings accounts, 517–518
 value of Manhattan Island and, 478
 Computer(s)
 assembly, time required for, 462
 computer-generated animation, 284
 discounts, 306–307, 314
 prices, 316, 618
 ratio of students to computers in
 U.S. public schools, 370
 sale price, 74
 Concentration of mixture, 125
 Cone volume, 453
 Continuously compounded interest, 514, 518, 540, 541, 619
 Cookies, supply and demand for, 559
 Coronary heart disease, 533
 Coronavirus cases
 modeling, in U.S., 525–526
 in New York City region, 619
 in U.S., 366–368
 worldwide, 464
 Corporate income tax, 174
 Cost(s). *See also* Manufacturing costs
 of college education, 2, 4–5, 19–20, 365
 of federal social programs, 34
 of industrial clean up, 75–76
 of medical treatment, 493
 minimizing, 612
 of public transportation infrastructure
 maintenance, 272
 Cost and revenue functions/break-even points, 558, 618
 average, 426–427, 430, 461, 463
 bike manufacturing, 430
 computer desk manufacturing, 615
 graphing calculator manufacturing, 461
 roast beef sandwiches, 364
 running shoe manufacturing, 430, 553
 tablet case manufacturing, 313
 toner cartridges, 592
 virtual reality headset manufacturing, 426–427
 wheelchair manufacturing, 427, 552–553
 Costco memberships, 461
 Credit card balances, 213
 Crime, prison population and, 591

D

Deadlines, excuses for not meeting, 212
 Dead Sea Scrolls, carbon-14 dating of, 524
 Death penalty, sentences rendered by
 U.S. juries, 383
 Death rate, hours of sleep and, 563, 567
 Deaths
 in the 20th century, 616
 from 2000 through 2009, 300, 305–306
 by snakes, mosquitoes, and snails, 236

Debt
 national, 20, 31–32, 34, 35, 92
 student loan, 63
 Decay model for carbon-14, 531–532
 Decibels. *See* Sound intensity
 Deforestation, Amazon, 459–460
 Depreciation, 137, 235
 Depression
 exercise and, 284
 sense of humor and, 108–109, 120–121
 Dinosaur bones, potassium-40 dating of, 532
 Distance
 between pairs of cities, 334–335
 safe, expressway speed and, 91
 Distance traveled
 by car after brakes applied, 569
 combined walking and bus travel, 19
 Diver's height above water, 443
 Diversity index, 91
 Divorce, educational attainment and probability
 of, 102–104
 “Don’t ask, don’t tell” policy, 281–282
 Drivers, age of. *See under* Age(s)
 Driving rate and time for trip, 449
 Drug concentration, 279, 430
 Drug use among teenagers, 533
 Drunk driving arrests, age as function of, 432
 Dual investments, 19, 132, 138, 174, 256, 345, 456, 603, 619

E

Eagle, height and time in flight, 341
 Earnings. *See* Salary(-ies)
 Earth, age of, 28
 Earthquake
 epicenter, 334–335
 intensity, 480, 488, 538
 Earthquake relief, 605–610
 E-commerce sales, 539
 Education. *See also* College education
 percentage of U.S. adults completing high
 school, 540
 spending on pre-primary, 234
 unemployment and years of, 461
 Educational attainment, divorce and, 102–104
 Electrical resistance, 149, 455
 Elephant's weight, 519
 Elevator capacity, 207, 603
 Elk population, 541–542
 Endangered species, 532
 Ethnic diversity, 91
 Exam grades, 207, 215
 Excuses, for not meeting college assignment
 deadlines, 212
 Exercise
 depression and, 284
 heart rate and, 3
 target heart rate ranges for, 18
 Exponential decay model, 532, 540, 541
 Expressway speeds and safe distances, 91

F

Fahrenheit/Celsius temperature
 interconversions, 17, 206, 341
 Federal budget
 deficit, 92 (*See also* National debt)
 expenditures on human resources, 431

Federal Express aircraft purchase
 decisions, 612
 Federal income tax, 236
 Federal social programs, cost and
 enrollment, 34
 Fencing
 for enclosure, 587–588
 maximum area inside, 360, 363, 365
 Ferris wheel, 335
 Fetal weight, age and, 214
 Field's dimensions, 213, 617
 Films, Oscar-winning, 315
 Financial aid, college student, 214
 Floor dimensions, and area for pool and
 fountain, 590
 Floor space, length and width of, 214
 Flu
 epidemic, 478
 inoculation costs, 89
 time-temperature scenario, 237–238
 vaccine mixture, 235, 554–556
 Food
 cost per item, 281, 570
 lunch menus, 612
 Food stamps program, 34
 Food truck vendor, profit and cost for, 364
 Football field dimensions, 133–134
 Football's height above ground, 17, 356–358
 Foreign-born population in U.S., 174
 Frame dimensions, 138
 Freedom 7 spacecraft flight, 326
 Free-falling object's position, 440–441, 443, 462
 Frequency, length of violin string and, 451
 Freshmen. *See under* College students
 Fuel efficiency, 237

G

Garbage, daily per-pound production of, 63
 Garden, width of path around, 172
 Gasoline price, supply/demand and, 558–559
 Gas pressure in can, 450
 Gay marriage, U.S. public opinion on, 533, 560
 Gay service members discharged from military, 281–282
 Gender
 average number of awakenings during
 night by, 106
 bachelor's degrees awarded and, 93
 calories needed to maintain energy by, 89
 first-year U.S. college students claiming no
 religious affiliation by, 221–222
 of House of Representatives members', 560
 labor force participation by, 189
 life expectancy by year of birth and, 270
 percent body fat in adults by, 255
 wage gap by, 235
 Global warming, 266–268
 Golden rectangles, 49
 Government, trust in, 270
 Government financial aid, college tuition, 214
 Grade inflation, 124
 Gravitational force, 452
 Gravity model, 455
 Groceries, budgeting for, 281
 Gutter cross-sectional area, 172, 363
 Gym membership fees, 216

H

Half-life
 aspirin, 532
 radioactive elements, 532, 540
 Xanax, 532
 Hamachiphobia, 533
 Happiness
 average level of, at different times of day, 325
 per capita income and national, 271
 HDTV screen dimensions, 166–167, 445
 Health care
 budgeting for, 281
 gross domestic product (GDP) spent on, 518
 savings needed for expenses during retirement, 533
 Health club membership fees, 137
 Heart beats over lifetime, 35
 Heart disease
 coronary, 533
 moderate wine consumption and, 271–272
 smoking and, 432
 Heart rate
 and age for children, 593, 597–598
 exercise and, 3, 18
 life span and, 462
 modeling, with inequalities, 597–598
 before and during panic attack, 382
 Heat generated by stove, 455
 Heat loss of a glass window, 455
 Height. *See also* Ball's height
 above ground of building, shadow cast and, 214
 child's height modeled, 487, 494, 515
 diver's height above water, 443
 of eagle, in terms of time in flight, 341
 as function of age, 277, 280, 298
 healthy weight region for, 604
 percentage of adult height attained by girl of given age, 493, 515
 weight and height recommendations/calculations, 138, 454
 High school education, percentage of U.S. adults completing, 540
 Hispanic Americans, population growth, 540
 HIV infection
 number of Americans living with, 370
 T cell count and, 218
 HIV infection, T cell count and, 227–228
 Home ownership, 273, 276–277
 Hotel room types, 561
 Households, mixed religious beliefs in, 206
 House of Representatives, men vs. women in, 560
 House sales prices, 236
 House value, inflation rate and, 478
 Hubble Space Telescope, 456
 Human resources, federal budget expenditures on, 431
 Humor, sense of, depression and, 108–109, 120–121
 Hurricanes, barometric air pressure and, 519
 Hydrogen ion concentration, 518–519

I

Identical twins, distinguishing between, 562
 Illumination intensity, 454, 455
 Imaginary number joke, 149

Implicit Association Test, 51, 61
 Income
 highest-paid TV celebrities, 218–221
 median yearly income of full-time workers, 32
 for top 10% vs. bottom 90% of Americans, 560
 Income tax, federal, 236
 Industrial accidents, cost of cleaning up, 75–76
 Inflation, cost of, 124
 Inflation rate, 478
 Influenza. *See* Flu
 Inn charges, before tax, 138
 Inoculation costs for flu, 89
 Insurance, pet, 255
 Intelligence quotient (IQ) and mental/chronological age, 454
 Internet service providers, 213
 Internet streaming services, 175
 Interracial marriage, laws prohibiting, 214–215
 Investment(s)
 accumulated value of, 474–475, 478, 514
 amounts invested per rate, 570
 choosing between, 475
 compound interest, 474–475, 478, 479, 514, 518, 519, 538, 540, 541, 619
 for desired return, 215
 dual, 19, 132, 138, 174, 256, 345, 456, 603, 619
 in greeting cards, 558
 and interest rates, 19, 213
 maximizing expected returns, 613
 money divided between high- and low-risk, 603
 in play, 558
 IQ (intelligence quotient) and mental/chronological age, 454
 IRA. *See* Individual Retirement Account

J

Jeans, price of, 314
 Jet skis, 618

K

Kinetic energy, 455

L

Labor force, participation by gender, 189
 Labrador retrievers, color of, 59
 Ladder's reach, 171
 Land line telephones, replacement with cell phones, 559
 Lead, half-life of, 532
 Learning curve, 124
 Learning theory project, 526
 Lemon tree, maximum yield, 365
 Length of violin string and frequency, 451
 LGBTQ representation, in books, 540
 Life, most time-consuming activities during, 136
 Life events, sense of humor and response to, 108–109, 120–121
 Life expectancy, 136, 270
 Life span, heart rate and, 462
 Light intensity, 463, 517
 Literacy and child mortality, 257, 271
 Living alone, number of Americans, 344
 Living arrangements, of young adults, 273, 276–277

Long-distance telephone charges, 138
 Loudness, 259, 455, 462, 493, 504, 533, 541
 Love, course of over time, 206
 Luggage, volume of carry-on, 409–410
 Lunch menus, 612

M

Mailing costs, 255
 Mall browsing time and average amount spent, 466, 467
 Manufacturing constraints, 607, 608, 609, 611, 617
 Manufacturing costs. *See also* Cost and revenue functions/break-even points
 bicycles, 235
 calculator, 461
 portable satellite radio players, 463
 tents, 617
 toner cartridges, 592
 virtual reality headsets, 413, 426–427
 wheelchair, 427
 Marching band, 562
 Marijuana use by U.S. high school seniors, 106
 Marital status, of U.S. population, 559
 Markup, 138
 Marriage, interracial, 214–215
 Marriage equality, U.S. public opinion on, 533, 560
 Mauna Loa Observatory, 268
 Maximum area, 360, 363, 412, 459
 Maximum product, 363, 412, 462–463
 Maximum profit, 412, 462–463, 609, 618
 Maximum scores, 612
 Maximum yield, 365
 Media, trust in, 270
 Median age. *See under* Age(s)
 Median yearly income, 32
 Medicaid, 34
 Medical treatment, delaying, 493
 Medicare, 34
 Memory retention, 478, 494, 495, 518, 539
 Miles per gallon, 237
 Military, gay service members discharged from, 281–282
 Minimum product, 359, 459
 Miscarriages, by age, 533
 Mixture problems, 125, 235, 554–556, 561, 592, 615, 618
 Modernistic painting consisting of geometric figures, 571
 Moon weight of person given Earth weight, 454
 Moth eggs and abdominal width, 395
 Movies
 ticket price of, 213
 top ten Oscar-winning, 315
 Mumps, U.S. cases of, 412

N

National debt, 20, 31–32, 34, 35, 92
 National diversity index, 91
 Natural disaster relief, 612
 Negative life events, sense of humor and response to, 108–109, 120–121
 Negative numbers, square roots of, 140
 Negative square roots, 149
 Net worth, of wealthiest Americans, 20
 Neurons in human vs. gorilla brain, 63
 Newton's Law of Cooling, 535

O

Oculus Rift headset manufacturing costs, 413, 426–427
 Ohm's law, 149
 One-person households. *See* Living alone, number of Americans
 Online, time spent, 345
 Online games, percentage of U.S. college students playing, 561
 Open box lengths and widths, 172
 Orbits
 of comets, 590
 of planets, 590
 Oscar-winning films, top ten, 315

P

Panic attack, heart rate before and during, 382
 Park, pedestrian route around, 171
 Parking lot, dimensions of, 171
 Parthenon at Athens, as golden rectangle, 49
 Path around swimming pool, dimensions of, 138
 Paycheck size, 138
 Pedestrian route around park, 171
 Per capita income and national happiness, 271
 Perceived length of time period and age, 455
 Pets, insurance for, 255
 pH
 of human mouth after eating sugar, 430
 scale, 518–519
 Phone calls between cities, 446, 455
 Physician visits, 256
 Pitch of a musical tone, 462
 Planets, length of years on, 189
 Playground, dimensions of, 363
 Pliocene Epoch, 268
 Political identification, college freshmen and, 341
 Pollution removal costs, 76
 Pool dimensions, 138, 171
 Population
 Africa, 523
 alligator, 174
 Asia, 541
 bird species in danger of extinction, 532
 Bulgaria, 531
 California, 517
 Canada, 535
 Colombia, 531
 elk, 541–542
 Europe, 592
 exponential growth modeling, 531, 532
 Florida, 605
 foreign-born (U.S.), 174, 571
 Germany, 531, 541
 gray wolf, 472–473
 Hispanic, 540
 Hungary, 520
 India, 477, 531
 Iraq, 531
 Israel, 531
 Japan, 531
 Madagascar, 531
 Mexico, 532
 New Zealand, 532
 Nigeria, 534
 over age 65 (U.S.), 534
 Pakistan, 531
 Palestinian, 531
 Philippines, 531
 Russia, 531
 in scientific notation, 30
 single, 276–277
 Texas, 517
 tigers, worldwide, 381
 Uganda, 535
 United States
 by age, 313
 age 65 and older, 534
 modeling growth of, 523–524
 and national debt, 32
 and walking speed, 527
 world, 314, 521, 528–530, 533, 541
 Population projections, 49, 137, 531
 Potassium-40, 532
 Pre-primary education spending, 234
 Price(s)
 advertising and, 451–452, 456
 backpack, 535
 computer, 316
 gasoline, 558–559
 of a house, 236
 jeans, 314
 of movie ticket, 213
 supply/demand and, 558–559
 Price reductions, 131, 138, 139, 174, 216, 316
 Pricing options, 208
 Prison population, violent crime and, 591
 Problem solving, payments for, 139
 Problem-solving time, 452
 Profit function, 364, 554, 558, 592
 Profits
 department store branches, 313
 maximizing, 364, 412, 462–463, 611, 612, 617, 618
 maximum, 462–463
 maximum daily, 609
 maximum monthly, 611
 on newsprint/writing paper, 617
 production and sales for gains in, 207
 total monthly, 611
 Projectiles, paths of, 348, 462. *See also* Ball's height above ground; Free-falling object's position
 Public transportation infrastructure, cost of maintaining, 272
 Pyramid volume, 462

R

Racial diversity, 91
 Racial prejudice, Implicit Association Test for, 51, 61
 Radiation intensity and distance of radiation machine, 454
 Rain gutter cross-sectional area, 172, 363
 Rate of travel
 airplane rate, 561
 average rate and time traveled, 235
 average rate on a round-trip commute, 89
 rowing rate, 561
 and time for trip, 449
 Razor blades sold, 570
 Real estate agents, commission for, 213
 Rectangle
 area of, 49
 dimensions of, 171, 174, 214, 299, 444, 562, 587–588, 590, 616, 618, 619
 dimensions of, maximizing enclosed area, 360
 golden, 49
 perimeter of, 49, 89, 125
 Rectangular box dimensions, 395
 Rectangular carpet dimensions, 216
 Rectangular garden
 dimensions of, 345
 width of path around, 172
 Rectangular sign dimensions, 172
 Rectangular solid, volume of, 61
 Reflections, 289
 Refund, likelihood of tax, 49
 Relativity theory, space exploration and, 35, 47, 50
 Religious affiliation
 first-year U.S. college students claiming no, 221–222
 spouses with different, 206
 Rental
 car, 191–192, 215
 truck, 207
 Repair bill
 cost of parts and labor on, 138
 estimate, 207
 Resistance, electrical, 149, 455
 Respiratory rate, age for children and, 603
 Restaurant tables and maximum occupancy, 561
 Revenue functions. *See* Cost and revenue functions/break-even points
 Reversibility of thought, 63
 Right triangle, isosceles, 172
 Roads to expressway, length of, 190
 Rug's length and width, 590
 Runner's pulse, 519

S

Salary(-ies)
 after college, 212
 choosing between pay arrangements, 345
 college education and, 136–137
 college graduates with undergraduate degrees, 127–128
 earnings of Elvis Presley's estate, 570
 gross amount per paycheck, 138
 and paycheck size, 138
 summer sales job, 345
 wage gap in, by gender, 235
 weekly, 125, 611
 Sale prices, 74. *See also* Price reductions
 Sales figures
 price/advertising and, 451–452, 456
 theater ticket, 570
 Satellite radio players, manufacturing costs of, 463
 Savings
 and compound interest, 517–518
 needed for health-care expenses during retirement, 533
 Sense of humor, depression and, 108–109
 Service contract, HVAC company, 432
 Shaded region areas, 61, 74
 Shipping cost, 255
 Ship tracking system, 590

Shot put
 angle and height of, 362–363
 path of, given angle, 171
 Skeletons, carbon-14 dating of, 532
 Skydiver's fall, 448–449, 462
 Sleep
 average number of awakenings during
 night, by age and gender, 106
 coffee consumption and, 542
 college students' nightly hours of, 561
 death rate and hours of, 563, 567
 Smoking
 deaths and disease incidence ratios, 431
 and heart disease, 432
 Soccer field dimension, 138
 Social media use, by age, 615
 Social Security benefits/costs, 34, 216
 Sound intensity, 259, 455, 462, 493, 504,
 533, 541
 Space exploration and relativity theory, 35,
 47, 50
 Space flight/travel
 aging rate and, 35, 47, 50
 Freedom 7 spacecraft, 326
 Hubble Space Telescope, 456
 relativity theory and, 35, 47, 50
 Speaker loudness, 462
 Speed. *See* Rate of travel
 Spouses with different faiths, 206
 Spring, force required to stretch, 454
 Square, length of side of, 172
 Stereo speaker loudness, 462
 Stolen plants, 139
 Stomach acid, pH of, 519
 Stopping distances
 for car, 443–444
 for motorcycles at selected speeds, 461–462
 for trucks, 444
 Stories, matching graphs with, 107
 Stress levels, 361
 String length and frequency, 451
 Strontium-90, 524
 Student loan debt, 63
 Studying, hours per week by college students,
 569–570
 Sunscreen, exposure time without burning and, 2
 Supply and demand, 558–559
 Supply-side economics, 396
 Surface sunlight, intensity beneath ocean's
 surface, 517
 Sushi, population who won't try, 533
 Swimming pool
 path around, 138, 172
 tile border, 173
 Systolic blood pressure, age and, 165–166

T

Tablet case manufacturing, 313
 Target heart rate for exercise, 18
 Task mastery, 504, 539
 Taxes
 bills, 207
 federal tax rate schedule for tax owed, 255

income
 corporate, 174
 federal, 236
 inn charges before, 138
 refund likelihood and age, 49
 tax rate percentage and revenue, 396
 Taxi rates, 137
 Teenage drug use, 533
 Telephone, land lines and cell phones, 559
 Telephone plans
 cellular plans, 236
 texting plans, 136
 Television
 highest-paid actors and actresses, 218–221
 internet streaming services, 175
 manufacturing profits and constraints, 611
 sale prices, 74
 screen area, 167
 screen dimensions, 166–167, 445, 590
 viewing, by annual income, 186
 Temperature
 of cooling cup of coffee, 538
 and depth of water, 454
 in enclosed vehicle, increase in, 489–490
 Fahrenheit-Celsius interconversions, 17, 206, 341
 global warming, 217, 266–268
 home temperature as function of time, 298–299
 increase in an enclosed vehicle, 533
 Newton's Law of Cooling, 535
 normal body temperature ranges, 543, 605
 time-temperature flu scenario, 237–238
 Tennis club payment options, 139
 Tennis court dimensions, 138
 Test scores, maximum, 612
 Texting plans, 136
 Theater attendance, maximizing revenue
 from, 612
 Theater ticket sales, 570
 Thefts in U.S., 460
 Thorium-229, 532
 Ticket prices/sales
 movie ticket prices, 213
 theater ticket sales, 570
 Tigers, worldwide population, 381
 Time, perceived length of, 455
 Time spent online, in U.S., 345
 Time traveled, average rate and, 235
 Tolls, 126, 129–131, 137, 174, 202–203, 207, 314
 Toner cartridge manufacturing costs and
 revenues, 592
 Triangle, isosceles, 172, 561
 Trucks, stopping distances required for, 444
 Trust, in government and media, 270
 Tuition, government aid for, 214
 TV. *See* Television

U

Unemployment and years of education, 461

V

Vacation lodgings, 603
 Vacation plan packages, cost of, 616

Vaccine, mixture for flu, 235, 554–556
 Value
 of house, inflation rate and, 478
 of investments, 474–475, 478, 514
 Vehicle fatalities, driver's age and, 171
 Vertical pole supported by wire, 174, 216
 Video games, retail sales of, 520
 Violent crime, prison population and, 591
 Violin string length and frequency, 451
 Virtual reality headset manufacturing
 costs, 413, 426–427
 Volume (sound). *See* Sound intensity
 Volume (space)
 of carry-on luggage, 409–410
 of cone, 453
 for given regions, 74
 of open box, 61
 of solid, 411

W

Wage gap, 235, 365
 Wages. *See* Salary(-ies)
 Walking speed and city population, 527
 Water
 pressure and depth, 446–447
 temperature and depth, 454
 used in a shower, 448
 Water pipe diameter, number of houses
 served and size of, 454
 Water supply produced by snowpack, 462
 Wealthiest Americans, net worth of, 20
 Weight
 blood volume and body, 93, 447–448
 elephant's, age and, 519
 of great white shark, cube of its length
 and, 449
 healthy, for height and age, 604
 and height recommendations/calculations, 454
 of human fetus, age and, 214
 moon weight of person given Earth weight, 454
 Weightlifting, 534
West Side Story (movie), 315
 Wheelchair business
 manufacturing costs, 427
 profit function for, 554
 revenue and cost functions for, 552–553
 Wheelchair ramp, vertical distance of, 171
 White-collar jobs, in U.S., 540
 Wildfires, area burned by, 382
 Will distribution, 139
 Wind force, 455
 Wind pressure, 455
 Wine consumption, heart disease and,
 271–272
 Wire length, 172
 Women. *See also* Gender
 average level of happiness at different times
 of day, 325
 in the labor force, 189

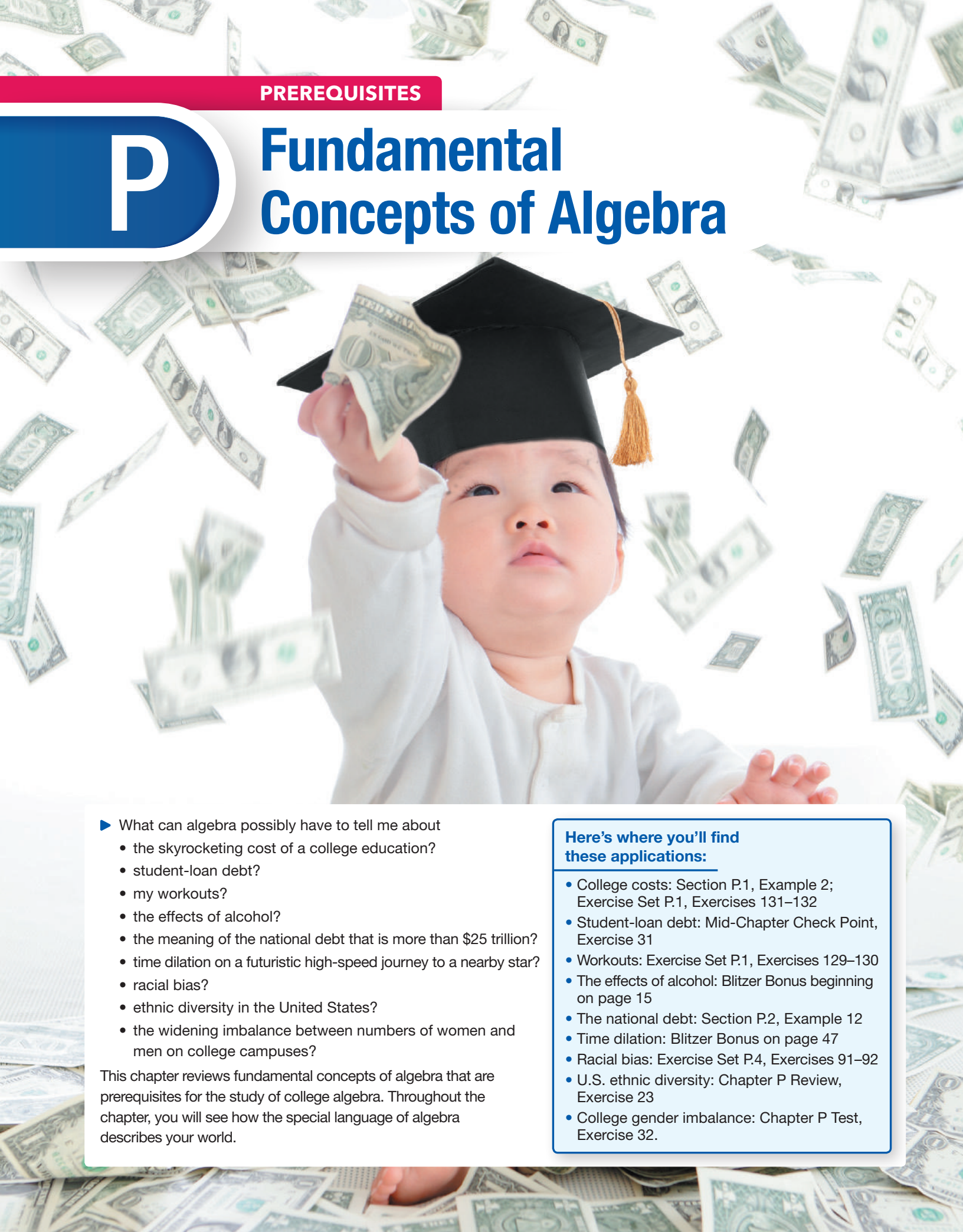
X

Xanax, half-life of, 532

This page intentionally left blank

College Algebra Essentials

This page intentionally left blank

A baby wearing a black graduation cap and a white gown is the central figure. The baby is holding a one-dollar bill in their right hand and looking upwards. The background is white, and numerous one-dollar bills are floating or falling around the baby, creating a sense of abundance or financial focus.

PREREQUISITES

P

Fundamental Concepts of Algebra

- What can algebra possibly have to tell me about
- the skyrocketing cost of a college education?
 - student-loan debt?
 - my workouts?
 - the effects of alcohol?
 - the meaning of the national debt that is more than \$25 trillion?
 - time dilation on a futuristic high-speed journey to a nearby star?
 - racial bias?
 - ethnic diversity in the United States?
 - the widening imbalance between numbers of women and men on college campuses?

This chapter reviews fundamental concepts of algebra that are prerequisites for the study of college algebra. Throughout the chapter, you will see how the special language of algebra describes your world.

Here's where you'll find these applications:

- College costs: Section P.1, Example 2; Exercise Set P.1, Exercises 131–132
- Student-loan debt: Mid-Chapter Check Point, Exercise 31
- Workouts: Exercise Set P.1, Exercises 129–130
- The effects of alcohol: Blitzer Bonus beginning on page 15
- The national debt: Section P.2, Example 12
- Time dilation: Blitzer Bonus on page 47
- Racial bias: Exercise Set P.4, Exercises 91–92
- U.S. ethnic diversity: Chapter P Review, Exercise 23
- College gender imbalance: Chapter P Test, Exercise 32.

SECTION P.1

Algebraic Expressions, Mathematical Models, and Real Numbers

WHAT YOU'LL LEARN

- 1 Evaluate algebraic expressions.
- 2 Use mathematical models.
- 3 Find the intersection of two sets.
- 4 Find the union of two sets.
- 5 Recognize subsets of the real numbers.
- 6 Use inequality symbols.
- 7 Evaluate absolute value.
- 8 Use absolute value to express distance.
- 9 Identify properties of the real numbers.
- 10 Simplify algebraic expressions.

How would your lifestyle change if a gallon of gas cost \$9.15? Or if the price of a staple such as milk was \$15? That's how much those products would cost if their prices had increased at the same rate college tuition has increased since 1980. (Source: Center for College Affordability and Productivity) In this section, you will learn how the special language of algebra describes your world, including the skyrocketing cost of a college education.



Algebraic Expressions

Algebra uses letters, such as x and y , to represent numbers. If a letter is used to represent various numbers, it is called a **variable**. For example, imagine that you are basking in the sun on the beach. We can let x represent the number of minutes that you can stay in the sun without burning with no sunscreen. With a number 6 sunscreen, exposure time without burning is six times as long, or 6 times x . This can be written $6 \cdot x$, but it is usually expressed as $6x$. Placing a number and a letter next to one another indicates multiplication.

Notice that $6x$ combines the number 6 and the variable x using the operation of multiplication. A combination of variables and numbers using the operations of addition, subtraction, multiplication, or division, as well as powers or roots, is called an **algebraic expression**. Here are some examples of algebraic expressions:

$$x + 6, \quad x - 6, \quad 6x, \quad \frac{x}{6}, \quad 3x + 5, \quad x^2 - 3, \quad \sqrt{x} + 7.$$

Many algebraic expressions involve *exponents*. For example, the algebraic expression

$$-x^2 + 361x + 3193$$

approximates the average cost of tuition and fees at public U.S. colleges for the school year ending x years after 2000. The expression x^2 means $x \cdot x$ and is read “ x to the second power” or “ x squared.” The exponent, 2, indicates that the base, x , appears as a factor two times. The negative sign in front of x^2 indicates that x^2 is multiplied by -1 .

Exponential Notation

If n is a counting number (1, 2, 3, and so on),

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{b \text{ appears as a factor } n \text{ times.}}$$

Exponent or Power

Base

b^n is read “the n th power of b ” or “ b to the n th power.” Thus, the n th power of b is defined as the product of n factors of b . The expression b^n is called an **exponential expression**. Furthermore, $b^1 = b$.

For example,

$$8^2 = 8 \cdot 8 = 64, \quad 5^3 = 5 \cdot 5 \cdot 5 = 125, \quad \text{and} \quad 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16.$$

1 Evaluate algebraic expressions.

Evaluating Algebraic Expressions

Evaluating an algebraic expression means to find the value of the expression for a given value of the variable.

Many algebraic expressions involve more than one operation. Evaluating an algebraic expression without a calculator involves carefully applying the following order of operations agreement:

The Order of Operations Agreement

1. Perform operations within the innermost parentheses and work outward. If the algebraic expression involves a fraction, treat the numerator and the denominator as if they were each enclosed in parentheses.
2. Evaluate all exponential expressions.
3. Perform multiplications and divisions **as they occur**, working **from left to right**.
4. Perform additions and subtractions **as they occur**, working **from left to right**.

EXAMPLE 1 Evaluating an Algebraic Expression

Evaluate $7 + 5(x - 4)^3$ for $x = 6$.

Solution

$$\begin{aligned}
 7 + 5(x - 4)^3 &= 7 + 5(6 - 4)^3 && \text{Replace } x \text{ with } 6. \\
 &= 7 + 5(2)^3 && \text{First work inside parentheses: } 6 - 4 = 2. \\
 &= 7 + 5(8) && \text{Evaluate the exponential expression: } 2^3 = 2 \cdot 2 \cdot 2 = 8. \\
 &= 7 + 40 && \text{Multiply: } 5(8) = 40. \\
 &= 47 && \text{Add.}
 \end{aligned}$$

✓ CHECK POINT 1 Evaluate $8 + 6(x - 3)^2$ for $x = 13$.

2 Use mathematical models.

Formulas and Mathematical Models

An **equation** is formed when an equal sign is placed between two algebraic expressions. One aim of algebra is to provide a compact, symbolic description of the world. These descriptions involve the use of *formulas*. A **formula** is an equation that uses variables to express a relationship between two or more quantities.

Here are two examples of formulas related to heart rate and exercise.



Couch-Potato Exercise

$$H = \frac{1}{5}(220 - a)$$

Heart rate, in beats per minute,

is

$\frac{1}{5}$ of

the difference between 220 and your age.



Working It

$$H = \frac{9}{10}(220 - a)$$

Heart rate, in beats per minute,

is

$\frac{9}{10}$ of

the difference between 220 and your age.

The process of finding formulas to describe real-world phenomena is called **mathematical modeling**. Such formulas, together with the meaning assigned to the variables, are called **mathematical models**. We often say that these formulas model, or describe, the relationships among the variables.

EXAMPLE 2 Modeling the Cost of Attending a Public College

The bar graph in **Figure P.1** shows the average cost of tuition and fees for public four-year colleges, adjusted for inflation. The formula

$$T = -x^2 + 361x + 3193$$

models the average cost of tuition and fees, T , for public U.S. colleges for the school year ending x years after 2000.

- Use the formula to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2020.
- By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

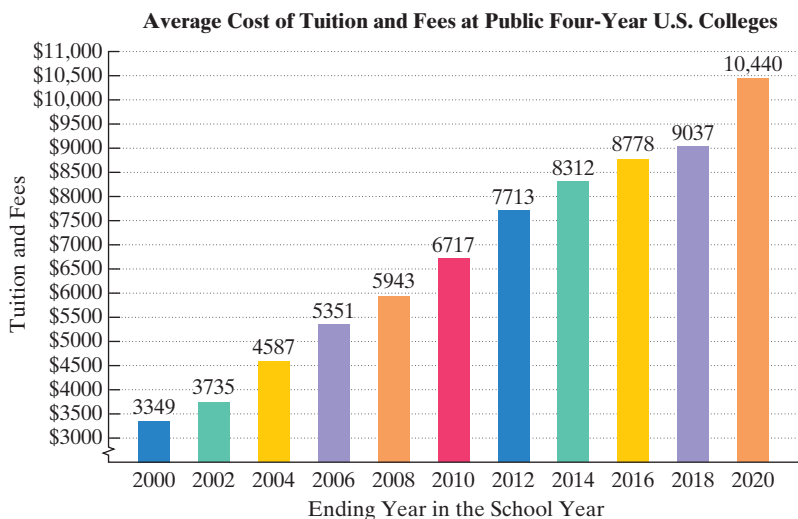


Figure P.1
Source: The College Board

Solution

- Because 2020 is 20 years after 2000, we substitute 20 for x in the given formula. Then we use the order of operations to find T , the average cost of tuition and fees for the school year ending in 2020.

$$T = -x^2 + 361x + 3193$$

This is the given mathematical model.

$$T = -(20)^2 + 361(20) + 3193$$

Replace each occurrence of x with 20.

$$T = -(400) + 361(20) + 3193$$

Evaluate the exponential expression:

$$20^2 = 20 \cdot 20 = 400.$$

$$T = -400 + 7220 + 3193$$

Multiply from left to right: $-(400) = -1(400)$
 $= -400$ and $361(20) = 7220$.

$$T = 10,013$$

Add.

The formula indicates that for the school year ending in 2020, the average cost of tuition and fees at public U.S. colleges was \$10,013.

- Figure P.1** shows that the average cost of tuition and fees for the school year ending in 2020 was \$10,440.

The cost obtained from the formula, \$10,013, underestimates the actual data value by $\$10,440 - \$10,013$, or by \$427.

Only 20 is affected by the exponent. Square 20 and copy the negative.

BLITZER BONUS**Is College Worthwhile?**

“Questions have intensified about whether going to college is worthwhile,” says *Education Pays*, released by the College Board Advocacy & Policy Center. “For the typical student, the investment pays off very well over the course of a lifetime, even considering the expense.”

Among the findings in *Education Pays*:

- Median (middlemost) full-time earnings with a bachelor’s degree in 2018 were \$65,400, which is \$24,900 more than high school graduates.
- Compared with a high school graduate, a four-year college graduate who enrolled in a public university at age 18 will break even by age 33. The college graduate will have earned enough by then to compensate for being out of the labor force for four years and for borrowing enough to pay tuition and fees, shown in **Figure P.1**.

> DISCOVERY

Using the formula from Example 2 and Check Point 2, find T for $x = 100$, $x = 200$, $x = 300$, and $x = 400$. What happens to the values of T over time? Do you see how model breakdown has occurred?

> GREAT QUESTION!

Can I use symbols other than braces when writing sets using the roster method?

No. Grouping symbols such as parentheses, $()$, and square brackets, $[]$, are not used to represent sets in the roster method. Furthermore, only commas are used to separate the elements of a set. Separators such as colons or semicolons are not used.

- 3** Find the intersection of two sets.

✓ CHECK POINT 2

- Use the formula $T = -x^2 + 361x + 3193$, described in Example 2, to find the average cost of tuition and fees at public U.S. colleges for the school year ending in 2016.
- By how much does the formula underestimate or overestimate the actual cost shown in **Figure P.1**?

Sometimes a mathematical model gives an estimate that is not a good approximation or is extended to include values of the variable that do not make sense. In these cases, we say that **model breakdown** has occurred. For example, it is not likely that the formula in Example 2 would give a good estimate of tuition and fees in 2050 because it is too far in the future. Thus, model breakdown would occur.

Sets

Before we describe the set of real numbers, let’s be sure you are familiar with some basic ideas about sets. A **set** is a collection of objects whose contents can be clearly determined. The objects in a set are called the **elements** of the set. For example, the set of numbers used for counting can be represented by

$$\{1, 2, 3, 4, 5, \dots\}.$$

The braces, $\{ \}$, indicate that we are representing a set. This form of representation, called the **roster method**, uses commas to separate the elements of the set. The symbol consisting of three dots after the 5, called an *ellipsis*, indicates that there is no final element and that the listing goes on forever.

A set can also be written in **set-builder notation**. In this notation, the elements of the set are described but not listed. Here is an example:

$$\{x | x \text{ is a counting number less than } 6\}.$$

The set of all x

such that

x is a counting number less than 6.

The same set written using the roster method is

$$\{1, 2, 3, 4, 5\}.$$

If A and B are sets, we can form a new set consisting of all elements that are in both A and B . This set is called the *intersection* of the two sets.

Definition of the Intersection of Sets

The **intersection** of sets A and B , written $A \cap B$, is the set of elements common to both set A and set B . This definition can be expressed in set-builder notation as follows:

$$A \cap B = \{x | x \text{ is an element of } A \text{ AND } x \text{ is an element of } B\}.$$

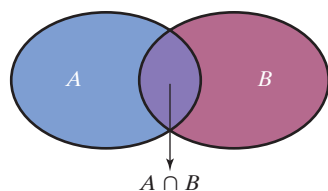


Figure P.2 Picturing the intersection of two sets

Figure P.2 shows a useful way of picturing the intersection of sets A and B . The figure indicates that $A \cap B$ contains those elements that belong to both A and B at the same time.

EXAMPLE 3 Finding the Intersection of Two Sets

Find the intersection: $\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\}$.

Solution The elements common to $\{7, 8, 9, 10, 11\}$ and $\{6, 8, 10, 12\}$ are **8** and **10**. Thus,

$$\{7, 8, 9, 10, 11\} \cap \{6, 8, 10, 12\} = \{8, 10\}.$$

✓ **CHECK POINT 3** Find the intersection: $\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\}$.

If a set has no elements, it is called the **empty set**, or the **null set**, and is represented by the symbol $\emptyset$. Here is an example that shows how the empty set can result when finding the intersection of two sets:

$$\{2, 4, 6\} \cap \{3, 5, 7\} = \emptyset.$$

These sets have no common elements.

Their intersection has no elements and is the empty set.

4 Find the union of two sets.

Another set that we can form from sets A and B consists of elements that are in A or B or in both sets. This set is called the *union* of the two sets.

Definition of the Union of Sets

The **union** of sets A and B , written $A \cup B$, is the set of elements that are members of set A **or** of set B or of both sets. This definition can be expressed in set-builder notation as follows:

$$A \cup B = \{x | x \text{ is an element of } A \text{ OR } x \text{ is an element of } B\}.$$

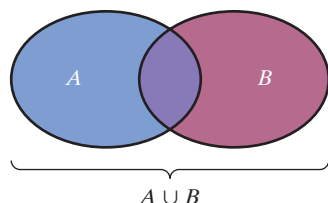


Figure P.3 Picturing the union of two sets

Figure P.3 shows a useful way of picturing the union of sets A and B . The figure indicates that $A \cup B$ is formed by joining the sets together.

We can find the union of set A and set B by listing the elements of set A . Then we include any elements of set B that have not already been listed. Enclose all elements that are listed with braces. This shows that the union of two sets is also a set.

EXAMPLE 4 Finding the Union of Two Sets

Find the union: $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$.

Solution To find $\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\}$, start by listing all the elements from the first set, namely, 7, 8, 9, 10, and 11. Now list all the elements from the second set that are not in the first set, namely, 6 and 12. The union is the set consisting of all these elements. Thus,

$$\{7, 8, 9, 10, 11\} \cup \{6, 8, 10, 12\} = \{6, 7, 8, 9, 10, 11, 12\}.$$

Although 8 and 10 appear in both sets,

do not list 8 and 10 twice.

✓ **CHECK POINT 4** Find the union: $\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\}$.

> GREAT QUESTION !

How can I use the words *union* and *intersection* to help me distinguish between these two operations?

Union, as in a marriage union, suggests joining things or uniting them. Intersection, as in the intersection of two crossing streets, brings to mind the area common to both, suggesting things that overlap.

5 Recognize subsets of the real numbers.

The Set of Real Numbers

The sets that make up the real numbers are summarized in **Table P.1**. We refer to these sets as **subsets** of the real numbers, meaning that all elements in each subset are also elements in the set of real numbers.

Table P.1 Important Subsets of the Real Numbers

Name/Symbol	Description	Examples
Natural numbers $\mathbb{N}$	$\{1, 2, 3, 4, 5, \dots\}$ These are the numbers that we use for counting.	2, 3, 5, 17
Whole numbers $\mathbb{W}$	$\{0, 1, 2, 3, 4, 5, \dots\}$ The set of whole numbers includes 0 and the natural numbers.	0, 2, 3, 5, 17
Integers $\mathbb{Z}$	$\{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$ The set of integers includes the negatives of the natural numbers and the whole numbers.	-17, -5, -3, -2, 0, 2, 3, 5, 17
Rational numbers $\mathbb{Q}$	$\left\{\frac{a}{b} \mid a \text{ and } b \text{ are integers and } b \neq 0\right\}$ This means that b is not equal to zero. The set of rational numbers is the set of all numbers that can be expressed as a quotient of two integers, with the denominator not 0. Rational numbers can be expressed as terminating or repeating decimals.	$-17 = \frac{-17}{1}$, $-5 = \frac{-5}{1}$, -3, -2, 0, 2, 3, 5, 17, $\frac{2}{5} = 0.4$, $\frac{-2}{3} = -0.6666\dots = -0.\overline{6}$
Irrational numbers $\mathbb{I}$	The set of irrational numbers is the set of all numbers whose decimal representations are neither terminating nor repeating. Irrational numbers cannot be expressed as a quotient of integers.	$\sqrt{2} \approx 1.414214$ $-\sqrt{3} \approx -1.73205$ $\pi \approx 3.142$ $-\frac{\pi}{2} \approx -1.571$

Notice the use of the symbol $\approx$ in the examples of irrational numbers. The symbol means “is approximately equal to.” Thus,

$$\sqrt{2} \approx 1.414214.$$

We can verify that this is only an approximation by multiplying 1.414214 by itself. The product is very close to, but not exactly, 2:

$$1.414214 \times 1.414214 = 2.000001237796.$$

Not all square roots are irrational. For example, $\sqrt{25} = 5$ because $5^2 = 5 \cdot 5 = 25$. Thus, $\sqrt{25}$ is a natural number, a whole number, an integer, and a rational number ($\sqrt{25} = \frac{5}{1}$).

The set of *real numbers* is formed by taking the union of the sets of rational numbers and irrational numbers. Thus, every real number is either rational or irrational, as shown in **Figure P.4**.

TECHNOLOGY

A calculator with a square root key gives a decimal approximation for $\sqrt{2}$, not the exact value.

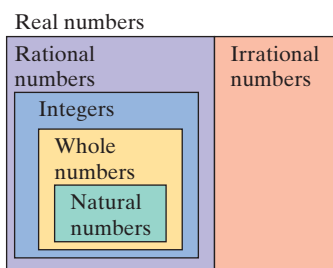


Figure P.4 Every real number is either rational or irrational.

Real Numbers

The set of **real numbers** is the set of numbers that are either rational or irrational:

$$\{x \mid x \text{ is rational or } x \text{ is irrational}\}.$$

The symbol $\mathbb{R}$ is used to represent the set of real numbers. Thus,

$$\mathbb{R} = \{x \mid x \text{ is rational}\} \cup \{x \mid x \text{ is irrational}\}.$$

EXAMPLE 5 Recognizing Subsets of the Real Numbers

Consider the following set of numbers:

$$\left\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\right\}.$$

List the numbers in the set that are

- | | | |
|----------------------|------------------------|------------------|
| a. natural numbers. | b. whole numbers. | c. integers. |
| d. rational numbers. | e. irrational numbers. | f. real numbers. |

Solution

- a. Natural numbers: The natural numbers are the numbers used for counting. The only natural number in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ is $\sqrt{81}$ because $\sqrt{81} = 9$. (9 multiplied by itself, or 9^2 , is 81.)
- b. Whole numbers: The whole numbers consist of the natural numbers and 0. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are whole numbers are 0 and $\sqrt{81}$.
- c. Integers: The integers consist of the natural numbers, 0, and the negatives of the natural numbers. The elements of the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that are integers are $\sqrt{81}$, 0, and -7 .
- d. Rational numbers: All numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ that can be expressed as the quotient of integers are rational numbers. These include -7 ($-7 = \frac{-7}{1}$), $-\frac{3}{4}$ ($-\frac{3}{4} = \frac{-3}{4}$), 0 ($0 = \frac{0}{1}$), and $\sqrt{81}$ ($\sqrt{81} = \frac{9}{1}$). Furthermore, all numbers in the set that are terminating or repeating decimals are also rational numbers. These include $0.\overline{6}$ and 7.3 .
- e. Irrational numbers: The irrational numbers in the set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are $\sqrt{5}$ ($\sqrt{5} \approx 2.236$) and π ($\pi \approx 3.14$). Both $\sqrt{5}$ and π are only approximately equal to 2.236 and 3.14, respectively. In decimal form, $\sqrt{5}$ and π neither terminate nor have blocks of repeating digits.
- f. Real numbers: All the numbers in the given set $\{-7, -\frac{3}{4}, 0, 0.\overline{6}, \sqrt{5}, \pi, 7.3, \sqrt{81}\}$ are real numbers.

**✓ CHECK POINT 5** Consider the following set of numbers:

$$\left\{-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}.$$

List the numbers in the set that are

- | | | |
|----------------------|------------------------|------------------|
| a. natural numbers. | b. whole numbers. | c. integers. |
| d. rational numbers. | e. irrational numbers. | f. real numbers. |

The Real Number Line

The **real number line** is a graph used to represent the set of real numbers. An arbitrary point, called the **origin**, is labeled 0. Select a point to the right of 0 and label it 1. The distance from 0 to 1 is called the **unit distance**. Numbers to the right of the origin are **positive** and numbers to the left of the origin are **negative**. The real number line is shown in **Figure P.5**.

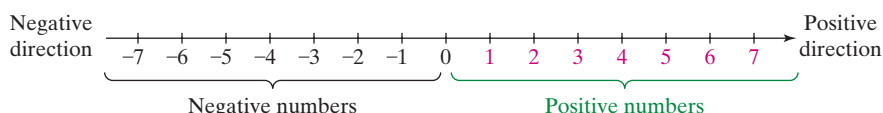
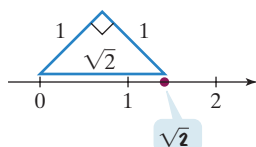


Figure P.5 The real number line

> GREAT QUESTION !

How did you locate $\sqrt{2}$ as a precise point on the number line in Figure P.6?

We used a right triangle with two legs of length 1. The remaining side has a length measuring $\sqrt{2}$.



We'll have lots more to say about right triangles later in the book.

Real numbers are **graphed** on a number line by placing a dot at the correct location for each number. The integers are easiest to locate. In **Figure P.6**, we've graphed six rational numbers and three irrational numbers on a real number line.

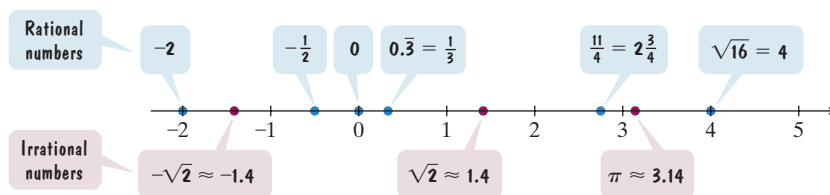


Figure P.6 Graphing numbers on a real number line

Every real number corresponds to a point on the number line and every point on the number line corresponds to a real number. We say that there is a **one-to-one correspondence** between all the real numbers and all points on a real number line.

6 Use inequality symbols.

Ordering the Real Numbers

On the real number line, the real numbers increase from left to right. The lesser of two real numbers is the one farther to the left on a number line. The greater of two real numbers is the one farther to the right on a number line.

Look at the number line in **Figure P.7**. The integers -4 and -1 are graphed.

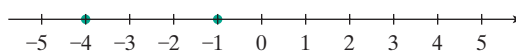


Figure P.7

Observe that -4 is to the left of -1 on the number line. This means that -4 is less than -1 .

$$-4 < -1$$

-4 is less than -1 because -4 is to the left of -1 on the number line.

In **Figure P.7**, we can also observe that -1 is to the right of -4 on the number line. This means that -1 is greater than -4 .

$$-1 > -4$$

-1 is greater than -4 because -1 is to the right of -4 on the number line.

The symbols $<$ and $>$ are called **inequality symbols**. These symbols always point to the lesser of the two real numbers when the inequality statement is true.

-4 is less than -1 .

$$-4 < -1$$

The symbol points to -4 , the lesser number.

-1 is greater than -4 .

$$-1 > -4$$

The symbol still points to -4 , the lesser number.

The symbols $<$ and $>$ may be combined with an equal sign, as shown in the following table:

Symbols	Meaning	Examples	Explanation
$a \leq b$	a is less than or equal to b .	$2 \leq 9$ $9 \leq 9$	Because $2 < 9$ Because $9 = 9$
$b \geq a$	b is greater than or equal to a .	$9 \geq 2$ $2 \geq 2$	Because $9 > 2$ Because $2 = 2$

This inequality is true if either the $<$ part or the $=$ part is true.

This inequality is true if either the $>$ part or the $=$ part is true.

7 Evaluate absolute value.

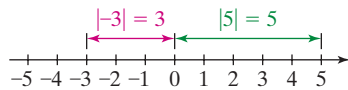


Figure P.8 Absolute value as the distance from 0

Absolute Value

The **absolute value** of a real number a , denoted by $|a|$, is the distance from 0 to a on the number line. **This distance is always taken to be nonnegative.** For example, the real number line in **Figure P.8** shows that

$$|-3| = 3 \quad \text{and} \quad |5| = 5.$$

The absolute value of -3 is 3 because -3 is 3 units from 0 on the number line. The absolute value of 5 is 5 because 5 is 5 units from 0 on the number line. The absolute value of a positive real number or 0 is the number itself. The absolute value of a negative real number, such as -3 , is the number without the negative sign.

We can define the absolute value of the real number x without referring to a number line. The algebraic definition of the absolute value of x is given as follows:

Definition of Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

If x is nonnegative (that is, $x \geq 0$), the absolute value of x is the number itself. For example,

$$|5| = 5 \quad |\pi| = \pi \quad \left|\frac{1}{3}\right| = \frac{1}{3} \quad |0| = 0.$$

Zero is the only number whose absolute value is 0.

If x is a negative number (that is, $x < 0$), the absolute value of x is the opposite of x . This makes the absolute value positive. For example,

$$|-3| = -(-3) = 3 \quad |-\pi| = -(-\pi) = \pi \quad \left|-\frac{1}{3}\right| = -\left(-\frac{1}{3}\right) = \frac{1}{3}.$$

This middle step is usually omitted.

Observe that **the absolute value of any nonzero number is always positive.**

EXAMPLE 6 Evaluating Absolute Value

Rewrite each expression without absolute value bars:

a. $|\sqrt{3} - 1|$ b. $|2 - \pi|$ c. $\frac{|x|}{x}$ if $x < 0$.

Solution

- a. Because $\sqrt{3} \approx 1.7$, the number inside the absolute value bars, $\sqrt{3} - 1$, is positive. The absolute value of a positive number is the number itself. Thus,

$$|\sqrt{3} - 1| = \sqrt{3} - 1.$$

- b. Because $\pi \approx 3.14$, the number inside the absolute value bars, $2 - \pi$, is negative. The absolute value of x when $x < 0$ is $-x$. Thus,

$$|2 - \pi| = -(2 - \pi) = \pi - 2.$$

- c. If $x < 0$, then $|x| = -x$. Thus,

$$\frac{|x|}{x} = \frac{-x}{x} = -1.$$

✓ CHECK POINT 6 Rewrite each expression without absolute value bars:

a. $|1 - \sqrt{2}|$ b. $|\pi - 3|$ c. $\frac{|x|}{x}$ if $x > 0$.

DISCOVERY

Verify the triangle inequality if $a = 4$ and $b = 5$. Verify the triangle inequality if $a = 4$ and $b = -5$.

When does equality occur in the triangle inequality and when does inequality occur? Verify your observation with additional number pairs.

8 Use absolute value to express distance.

Listed below are several basic properties of absolute value. Each of these properties can be derived from the definition of absolute value.

Properties of Absolute Value

For all real numbers a and b ,

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $a \leq |a|$
4. $|ab| = |a||b|$
5. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \quad b \neq 0$
6. $|a + b| \leq |a| + |b|$ (called the triangle inequality).

Distance Between Points on a Real Number Line

Absolute value is used to find the distance between two points on a real number line. If a and b are any real numbers, the **distance between a and b** is the absolute value of their difference. For example, the distance between 4 and 10 is 6. Using absolute value, we find this distance in one of two ways:

$$|10 - 4| = |6| = 6 \quad \text{or} \quad |4 - 10| = |-6| = 6.$$

The distance between 4 and 10 on the real number line is 6.

Notice that we obtain the same distance regardless of the order in which we subtract.

Distance Between Two Points on the Real Number Line

If a and b are any two points on a real number line, then the distance between a and b is given by

$$|a - b| \quad \text{or} \quad |b - a|,$$

where $|a - b| = |b - a|$.

EXAMPLE 7 Distance Between Two Points on a Number Line

Find the distance between -5 and 3 on the real number line.

Solution Because the distance between a and b is given by $|a - b|$, the distance between -5 and 3 is

$$|-5 - 3| = |-8| = 8.$$

$$a = -5 \quad b = 3$$

Figure P.9 verifies that there are 8 units between -5 and 3 on the real number line. We obtain the same distance if we reverse the order of the subtraction:

$$|3 - (-5)| = |8| = 8.$$

CHECK POINT 7 Find the distance between -4 and 5 on the real number line.

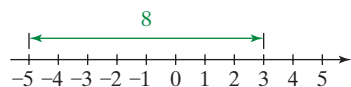


Figure P.9 The distance between -5 and 3 is 8.

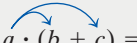
9 Identify properties of the real numbers.

Properties of Real Numbers and Algebraic Expressions

When you use your calculator to add two real numbers, you can enter them in any order. The fact that two real numbers can be added in any order is called the **commutative property of addition**. You probably use this property, as well as other

properties of real numbers listed in **Table P.2**, without giving it much thought. The properties of the real numbers are especially useful when working with algebraic expressions. For each property listed in **Table P.2**, a , b , and c represent real numbers, variables, or algebraic expressions.

Table P.2 Properties of the Real Numbers

Name	Meaning	Examples
Commutative Property of Addition	Changing order when adding does not affect the sum. $a + b = b + a$	$13 + 7 = 7 + 13$ $13x + 7 = 7 + 13x$
Commutative Property of Multiplication	Changing order when multiplying does not affect the product. $ab = ba$	$\sqrt{2} \cdot \sqrt{5} = \sqrt{5} \cdot \sqrt{2}$ $x \cdot 6 = 6x$
Associative Property of Addition	Changing grouping when adding does not affect the sum. $(a + b) + c = a + (b + c)$	$3 + (8 + x) = (3 + 8) + x = 11 + x$
Associative Property of Multiplication	Changing grouping when multiplying does not affect the product. $(ab)c = a(bc)$	$-2(3x) = (-2 \cdot 3)x = -6x$
Distributive Property of Multiplication over Addition	Multiplication distributes over addition.  $a \cdot (b + c) = a \cdot b + a \cdot c$	 $7(4 + \sqrt{3}) = 7 \cdot 4 + 7 \cdot \sqrt{3} = 28 + 7\sqrt{3}$  $5(3x + 7) = 5 \cdot 3x + 5 \cdot 7 = 15x + 35$
Identity Property of Addition	Zero can be deleted from a sum. $a + 0 = a$ $0 + a = a$	$\sqrt{3} + 0 = \sqrt{3}$ $0 + 6x = 6x$
Identity Property of Multiplication	One can be deleted from a product. $a \cdot 1 = a$ $1 \cdot a = a$	$1 \cdot \pi = \pi$ $13x \cdot 1 = 13x$
Inverse Property of Addition	The sum of a real number and its additive inverse gives 0, the additive identity. $a + (-a) = 0$ $(-a) + a = 0$	$\sqrt{5} + (-\sqrt{5}) = 0$ $-\pi + \pi = 0$ $6x + (-6x) = 0$ $(-4y) + 4y = 0$
Inverse Property of Multiplication	The product of a nonzero real number and its multiplicative inverse gives 1, the multiplicative identity. $a \cdot \frac{1}{a} = 1, a \neq 0$ $\frac{1}{a} \cdot a = 1, a \neq 0$	$7 \cdot \frac{1}{7} = 1$ $\left(\frac{1}{x-3}\right)(x-3) = 1, x \neq 3$

The properties of the real numbers in **Table P.2** apply to the operations of addition and multiplication. Subtraction and division are defined in terms of addition and multiplication.

> GREAT QUESTION !

Do the commutative and associative properties work for subtraction and division?

No. Subtraction and division are not commutative operations.

$$a - b \neq b - a \quad \frac{a}{b} \neq \frac{b}{a}$$

Furthermore, subtraction and division are not associative operations.

$$(a - b) - c \neq a - (b - c)$$

$$(a \div b) \div c \neq a \div (b \div c)$$

Verify each of these four statements using $a = 10$, $b = 5$, and $c = 2$.

Definitions of Subtraction and Division

Let a and b represent real numbers.

Subtraction: $a - b = a + (-b)$

We call $-b$ the **additive inverse** or **opposite** of b .

Division: $a \div b = a \cdot \frac{1}{b}$, where $b \neq 0$

We call $\frac{1}{b}$ the **multiplicative inverse** or **reciprocal** of b . The quotient of a and b , $a \div b$, can be written in the form $\frac{a}{b}$, where a is the **numerator** and b is the **denominator** of the fraction.

Because subtraction is defined in terms of adding an inverse, the distributive property can be applied to subtraction:

$$a(b - c) = ab - ac$$

$$(b - c)a = ba - ca$$

For example,

$$4(2x - 5) = 4 \cdot 2x - 4 \cdot 5 = 8x - 20.$$

10 Simplify algebraic expressions.

Simplifying Algebraic Expressions

The **terms** of an algebraic expression are those parts that are separated by addition. For example, consider the algebraic expression

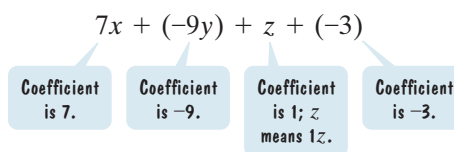
$$7x - 9y + z - 3,$$

which can be expressed as

$$7x + (-9y) + z + (-3).$$

This expression contains four terms, namely, $7x$, $-9y$, z , and -3 .

The numerical part of a term is called its **coefficient**. In the term $7x$, the 7 is the coefficient. If a term containing one or more variables is written without a coefficient, the coefficient is understood to be 1. Thus, z means $1z$. If a term does not contain a variable, it is a **constant** and its coefficient is that constant. Thus, the coefficient of the constant term -3 is -3 .



The parts of each term that are multiplied are called the **factors** of the term. The **factors** of the term $7x$ are 7 and x .

Like terms are terms that have exactly the same variable factors. For example, $3x$ and $7x$ are like terms. The distributive property in the form

$$ba + ca = (b + c)a$$

enables us to add or subtract like terms. For example,

$$3x + 7x = (3 + 7)x = 10x$$

$$7y^2 - y^2 = 7y^2 - 1y^2 = (7 - 1)y^2 = 6y^2.$$

This process is called **combining like terms**.

> GREAT QUESTION !

What is the bottom line for combining like terms?

To combine like terms mentally, add or subtract the coefficients of the terms. Use this result as the coefficient of the terms' variable factor(s).

An algebraic expression is **simplified** when parentheses have been removed and like terms have been combined.

EXAMPLE 8 Simplifying an Algebraic Expression

Simplify: $6(2x^2 + 4x) + 10(4x^2 + 3x)$.

Solution

$52x^2$ and $54x$ are not like terms. They contain different variable factors, x^2 and x , and cannot be combined.

$$\begin{aligned} & 6(2x^2 + 4x) + 10(4x^2 + 3x) \\ &= 6 \cdot 2x^2 + 6 \cdot 4x + 10 \cdot 4x^2 + 10 \cdot 3x \\ &= 12x^2 + 24x + 40x^2 + 30x \\ &= (12x^2 + 40x^2) + (24x + 30x) \\ &= 52x^2 + 54x \end{aligned}$$

Use the distributive property to remove the parentheses.

Multiply.

Group like terms.

Combine like terms.

✓ **CHECK POINT 8** Simplify: $7(4x^2 + 3x) + 2(5x^2 + x)$.

Properties of Negatives

The distributive property can be extended to cover more than two terms within parentheses. For example,

$$\begin{aligned} -3(4x - 2y + 6) &= -3 \cdot 4x - (-3) \cdot 2y - 3 \cdot 6 \\ &= -12x - (-6y) - 18 \\ &= -12x + 6y - 18. \end{aligned}$$

This sign represents subtraction.

This sign tells us that the number is negative.

The voice balloons illustrate that negative signs can appear side by side. They can represent the operation of subtraction or the fact that a real number is negative. Here is a list of properties of negatives and how they are applied to algebraic expressions:

Properties of Negatives

Let a and b represent real numbers, variables, or algebraic expressions.

Property

1. $(-1)a = -a$
2. $-(-a) = a$
3. $(-a)b = -ab$
4. $a(-b) = -ab$
5. $-(a + b) = -a - b$
6. $-(a - b) = -a + b$
 $= b - a$

Examples

$$\begin{aligned} (-1)4xy &= -4xy \\ -(-6y) &= 6y \\ (-7)4xy &= -7 \cdot 4xy = -28xy \\ 5x(-3y) &= -5x \cdot 3y = -15xy \\ -(7x + 6y) &= -7x - 6y \\ -(3x - 7y) &= -3x + 7y \\ &= 7y - 3x \end{aligned}$$

It is not uncommon to see algebraic expressions with parentheses preceded by a negative sign or subtraction. Properties 5 and 6 in the box, $-(a + b) = -a - b$ and $-(a - b) = -a + b$, are related to this situation. An expression of the form $-(a + b)$ can be simplified as follows:

$$-(a + b) = -1(a + b) = (-1)a + (-1)b = -a + (-b) = -a - b.$$

Do you see a fast way to obtain the simplified expression on the right in the preceding equation? **If a negative sign or a subtraction symbol appears outside parentheses, drop the parentheses and change the sign of every term within the parentheses.** For example,

$$-(3x^2 - 7x - 4) = -3x^2 + 7x + 4.$$

EXAMPLE 9 Simplifying an Algebraic Expression

Simplify: $8x + 2[5 - (x - 3)]$.

Solution

$$8x + 2[5 - (x - 3)]$$

$$= 8x + 2[5 - x + 3]$$

Drop parentheses and change the sign of each term in parentheses: $-(x - 3) = -x + 3$.

$$= 8x + 2[8 - x]$$

Simplify inside brackets: $5 + 3 = 8$.

$$= 8x + 16 - 2x$$

Apply the distributive property:

$$= (8x - 2x) + 16$$

$$2[8 - x] = 2 \cdot 8 - 2x = 16 - 2x.$$

Group like terms.

$$= (8 - 2)x + 16$$

Apply the distributive property.

$$= 6x + 16$$

Simplify.

CHECK POINT 9 Simplify: $6 + 4[7 - (x - 2)]$.

BLITZER BONUS

Using Algebra to Measure Blood-Alcohol Concentration

The amount of alcohol in a person's blood is known as blood-alcohol concentration (BAC), measured in grams of alcohol per deciliter of blood. A BAC of 0.08, meaning 0.08%, indicates that a person has 8 parts alcohol per 10,000 parts blood. In every state in the United States, it is illegal to drive with a BAC of 0.08 or higher.

How Do I Measure My Blood-Alcohol Concentration?

Here's a formula that models BAC for a person who weighs w pounds and who has n drinks* per hour.

$$\text{BAC} = \frac{600n}{w(0.6n + 169)}$$

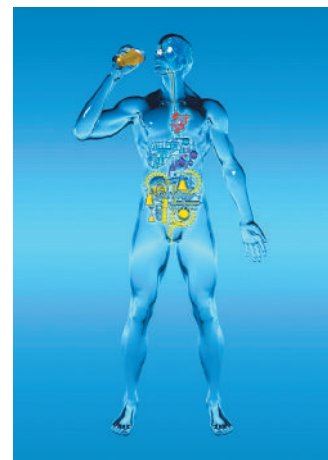
Blood-alcohol concentration
Body weight, in pounds
Number of drinks consumed in an hour

*A drink can be a 12-ounce can of beer, a 5-ounce glass of wine, or a 1.5-ounce shot of liquor. Each contains approximately 14 grams, or $\frac{1}{2}$ ounce, of alcohol.

Blood-alcohol concentration can be used to quantify the meaning of “tipsy.”

BAC	Effects on Behavior
0.05	Feeling of well-being; mild release of inhibitions; absence of observable effects
0.08	Feeling of relaxation; mild sedation; exaggeration of emotions and behavior; slight impairment of motor skills; increase in reaction time
0.12	Muscle control and speech impaired; difficulty performing motor skills; uncoordinated behavior
0.15	Euphoria; major impairment of physical and mental functions; irresponsible behavior; some difficulty standing, walking, and talking
0.35	Surgical anesthesia; lethal dosage for a small percentage of people
0.40	Lethal dosage for 50% of people; severe circulatory and respiratory depression; alcohol poisoning/overdose

Source: National Clearinghouse for Alcohol and Drug Information



(continues on next page)

Keeping in mind the meaning of “tipsy,” we can use our model to compare blood-alcohol concentrations of a 120-pound person and a 200-pound person for various numbers of drinks.

We determined each BAC using a calculator, rounding to three decimal places.

Blood-Alcohol Concentrations of a 120-Pound Person

$$\text{BAC} = \frac{600n}{120(0.6n + 169)}$$

<i>n</i> (number of drinks per hour)	1	2	3	4	5	6	7	8	9	10
BAC (blood-alcohol concentration)	0.029	0.059	0.088	0.117	0.145	0.174	0.202	0.230	0.258	0.286

Illegal to drive

Blood-Alcohol Concentrations of a 200-Pound Person

$$\text{BAC} = \frac{600n}{200(0.6n + 169)}$$

<i>n</i> (number of drinks per hour)	1	2	3	4	5	6	7	8	9	10
BAC (blood-alcohol concentration)	0.018	0.035	0.053	0.070	0.087	0.104	0.121	0.138	0.155	0.171

Illegal to drive

Like all mathematical models, the formula for BAC gives approximate rather than exact values. There are other variables that influence blood-alcohol concentration that are not contained

in the model. These include the rate at which an individual’s body processes alcohol, how quickly one drinks, sex, age, physical condition, and the amount of food eaten prior to drinking.

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

- C1. A combination of numbers, variables, and operation symbols is called an algebraic _____.

C2. If n is a counting number, b^n , read _____, indicates that there are n factors of b . The number b is called the _____ and the number n is called the _____.

C3. An equation that expresses a relationship between two or more variables, such as $H = \frac{9}{10}(220 - a)$, is called a/an _____. The process of finding such equations to describe real-world phenomena is called mathematical _____. Such equations, together with the meaning assigned to the variables, are called mathematical _____.

C4. The set of elements common to both set A and set B is called the _____ of sets A and B and is symbolized by _____.

C5. The set of elements that are members of set A or set B or of both sets is called the _____ of sets A and B and is symbolized by _____.

C6. The set $\{1, 2, 3, 4, 5, \dots\}$ is called the set of _____ numbers.

C7. The set $\{0, 1, 2, 3, 4, 5, \dots\}$ is called the set of _____ numbers.

C8. The set $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ is called the set of _____.
- C9. The set of numbers in the form $\frac{a}{b}$, where a and b belong to the set in Exercise 8 and $b \neq 0$, is called the set of _____ numbers.

C10. The set of numbers whose decimal representations are neither terminating nor repeating is called the set of _____ numbers.

C11. Every real number is either a/an _____ number or a/an _____ number.

C12. The notation $|x|$ is read the _____ of x . If $x \geq 0$, then $|x| = \rule{1cm}{0.4pt}$. If $x < 0$, then $|x| = \rule{1cm}{0.4pt}$.

C13. The commutative properties state that $a + b = \rule{1cm}{0.4pt}$ and $ab = \rule{1cm}{0.4pt}$.

C14. The associative properties state that $(a + b) + c = \rule{1cm}{0.4pt}$ and $\rule{1cm}{0.4pt} = a(bc)$.

C15. The distributive property states that $a(b + c) = \rule{1cm}{0.4pt}$.

C16. $a + (-a) = \rule{1cm}{0.4pt}$: The sum of a real number and its additive _____ is _____, the additive _____.

C17. $a \cdot \frac{1}{a} = 1, a \neq 0$: The product of a nonzero real number and its multiplicative _____ is _____, the multiplicative _____.

C18. An algebraic expression is _____ when parentheses have been removed and like terms have been combined.

C19. $-(-a) = \rule{1cm}{0.4pt}$.

P.1 EXERCISE SET

Practice Exercises

In Exercises 1–16, evaluate each algebraic expression for the given value or values of the variable(s).

- $7 + 5x$, for $x = 10$
- $8 + 6x$, for $x = 5$
- $6x - y$, for $x = 3$ and $y = 8$
- $8x - y$, for $x = 3$ and $y = 4$
- $x^2 + 3x$, for $x = 8$
- $x^2 + 5x$, for $x = 6$
- $x^2 - 6x + 3$, for $x = 7$
- $x^2 - 7x + 4$, for $x = 8$
- $4 + 5(x - 7)^3$, for $x = 9$
- $6 + 5(x - 6)^3$, for $x = 8$
- $x^2 - 3(x - y)$, for $x = 8$ and $y = 2$
- $x^2 - 4(x - y)$, for $x = 8$ and $y = 3$
- $\frac{5(x + 2)}{2x - 14}$, for $x = 10$
- $\frac{7(x - 3)}{2x - 16}$, for $x = 9$
- $\frac{2x + 3y}{x + 1}$, for $x = -2$ and $y = 4$
- $\frac{2x + y}{xy - 2x}$, for $x = -2$ and $y = 4$

The formula

$$C = \frac{5}{9}(F - 32)$$

expresses the relationship between Fahrenheit temperature, F , and Celsius temperature, C . In Exercises 17–18, use the formula to convert the given Fahrenheit temperature to its equivalent temperature on the Celsius scale.

- 50°F
- 86°F

A football was kicked vertically upward from a height of 4 feet with an initial speed of 60 feet per second. The formula

$$h = 4 + 60t - 16t^2$$

describes the ball's height above the ground, h , in feet, t seconds after it was kicked. Use this formula to solve Exercises 19–20.

- What was the ball's height 2 seconds after it was kicked?
- What was the ball's height 3 seconds after it was kicked?

In Exercises 21–28, find the intersection of the sets.

- $\{1, 2, 3, 4\} \cap \{2, 4, 5\}$
- $\{1, 3, 7\} \cap \{2, 3, 8\}$
- $\{s, e, t\} \cap \{t, e, s\}$
- $\{r, e, a, l\} \cap \{l, e, a, r\}$
- $\{1, 3, 5, 7\} \cap \{2, 4, 6, 8, 10\}$
- $\{0, 1, 3, 5\} \cap \{-5, -3, -1\}$
- $\{a, b, c, d\} \cap \emptyset$
- $\{w, y, z\} \cap \emptyset$

In Exercises 29–34, find the union of the sets.

- $\{1, 2, 3, 4\} \cup \{2, 4, 5\}$
- $\{1, 3, 7, 8\} \cup \{2, 3, 8\}$
- $\{1, 3, 5, 7\} \cup \{2, 4, 6, 8, 10\}$
- $\{0, 1, 3, 5\} \cup \{2, 4, 6\}$
- $\{a, e, i, o, u\} \cup \emptyset$
- $\{e, m, p, t, y\} \cup \emptyset$

In Exercises 35–38, list all numbers from the given set that are **a. natural numbers**, **b. whole numbers**, **c. integers**, **d. rational numbers**, **e. irrational numbers**, **f. real numbers**.

- $\{-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}\}$
- $\{-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}\}$
- $\{-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}\}$
- $\{-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}\}$

- Give an example of a whole number that is not a natural number.
- Give an example of a rational number that is not an integer.
- Give an example of a number that is an integer, a whole number, and a natural number.
- Give an example of a number that is a rational number, an integer, and a real number.

Determine whether each statement in Exercises 43–50 is true or false.

- $-13 \leq -2$
- $-6 > 2$
- $4 \geq -7$
- $-13 < -5$
- $-\pi \geq -\pi$
- $-3 > -13$
- $0 \geq -6$
- $0 \geq -13$

In Exercises 51–60, rewrite each expression without absolute value bars.

- $|300|$
- $|-203|$
- $|12 - \pi|$
- $|7 - \pi|$
- $|\sqrt{2} - 5|$
- $|\sqrt{5} - 13|$
- $\frac{-3}{|-3|}$
- $\frac{-7}{|-7|}$
- $||-3| - |-7||$
- $||-5| - |-13||$

In Exercises 61–66, evaluate each algebraic expression for $x = 2$ and $y = -5$.

- $|x + y|$
- $|x - y|$
- $|x| + |y|$
- $|x| - |y|$
- $\frac{y}{|y|}$
- $\frac{|x|}{x} + \frac{|y|}{y}$

In Exercises 67–74, express the distance between the given numbers using absolute value. Then find the distance by evaluating the absolute value expression.

- 2 and 17
- 4 and 15
- 2 and 5
- 6 and 8
- 19 and -4
- 26 and -3
- 3.6 and -1.4
- 5.4 and -1.2

In Exercises 75–84, state the name of the property illustrated.

- $6 + (-4) = (-4) + 6$
- $11 \cdot (7 + 4) = 11 \cdot 7 + 11 \cdot 4$
- $6 + (2 + 7) = (6 + 2) + 7$
- $6 \cdot (2 \cdot 3) = 6 \cdot (3 \cdot 2)$
- $(2 + 3) + (4 + 5) = (4 + 5) + (2 + 3)$
- $7 \cdot (11 \cdot 8) = (11 \cdot 8) \cdot 7$

81. $2(-8 + 6) = -16 + 12$
 82. $-8(3 + 11) = -24 + (-88)$
 83. $\frac{1}{(x + 3)}(x + 3) = 1, x \neq -3$
 84. $(x + 4) + [-(x + 4)] = 0$

In Exercises 85–96, simplify each algebraic expression.

85. $5(3x + 4) - 4$ 86. $2(5x + 4) - 3$
 87. $5(3x - 2) + 12x$ 88. $2(5x - 1) + 14x$
 89. $7(3y - 5) + 2(4y + 3)$
 90. $4(2y - 6) + 3(5y + 10)$
 91. $5(3y - 2) - (7y + 2)$
 92. $4(5y - 3) - (6y + 3)$
 93. $7 - 4[3 - (4y - 5)]$
 94. $6 - 5[8 - (2y - 4)]$
 95. $18x^2 + 4 - [6(x^2 - 2) + 5]$
 96. $14x^2 + 5 - [7(x^2 - 2) + 4]$

In Exercises 97–102, write each algebraic expression without parentheses.

97. $-(-14x)$ 98. $-(-17y)$
 99. $-(2x - 3y - 6)$ 100. $-(5x - 13y - 1)$
 101. $\frac{1}{3}(3x) + [(4y) + (-4y)]$ 102. $\frac{1}{2}(2y) + [(-7x) + 7x]$

Practice PLUS

In Exercises 103–110, insert either $<$, $>$, or $=$ in the shaded area to make a true statement.

103. $|-6|$ $|-3|$ 104. $|-20|$ $|-50|$
 105. $\left|\frac{3}{5}\right|$ $|-0.6|$ 106. $\left|\frac{5}{2}\right|$ $|-2.5|$
 107. $\frac{30}{40} - \frac{3}{4}$ $\frac{14}{15} \cdot \frac{15}{14}$ 108. $\frac{17}{18} \cdot \frac{18}{17}$ $\frac{50}{60} - \frac{5}{6}$
 109. $\frac{8}{13} \div \frac{8}{13}$ $|-1|$ 110. $|-2|$ $\frac{4}{17} \div \frac{4}{17}$

In Exercises 111–120, use the order of operations to simplify each expression.

111. $8^2 - 16 \div 2^2 \cdot 4 - 3$ 112. $10^2 - 100 \div 5^2 \cdot 2 - 3$
 113. $\frac{5 \cdot 2 - 3^2}{[3^2 - (-2)]^2}$ 114. $\frac{10 \div 2 + 3 \cdot 4}{(12 - 3 \cdot 2)^2}$
 115. $8 - 3[-2(2 - 5) - 4(8 - 6)]$
 116. $8 - 3[-2(5 - 7) - 5(4 - 2)]$
 117. $\frac{2(-2) - 4(-3)}{5 - 8}$ 118. $\frac{6(-4) - 5(-3)}{9 - 10}$
 119. $\frac{(5 - 6)^2 - 2|3 - 7|}{89 - 3 \cdot 5^2}$ 120. $\frac{12 \div 3 \cdot 5|2^2 + 3^2|}{7 + 3 - 6^2}$

In Exercises 121–128, write each English phrase as an algebraic expression. Then simplify the expression. Let x represent the number.

121. A number decreased by the sum of the number and four
 122. A number decreased by the difference between eight and the number
 123. Six times the product of negative five and a number
 124. Ten times the product of negative four and a number
 125. The difference between the product of five and a number and twice the number

126. The difference between the product of six and a number and negative two times the number
 127. The difference between eight times a number and six more than three times the number
 128. Eight decreased by three times the sum of a number and six

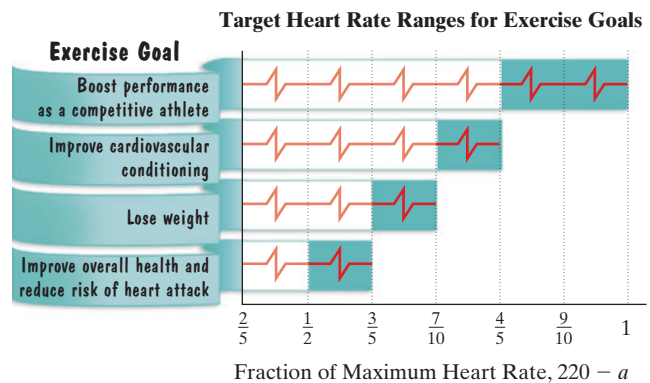
Application Exercises

The maximum heart rate, in beats per minute, that you should achieve during exercise is 220 minus your age:

$$220 - a.$$

This algebraic expression gives maximum heart rate in terms of age, a .

The following graph shows the target heart rate ranges for four types of exercise goals. The lower and upper limits of these ranges are fractions of the maximum heart rate, $220 - a$. Exercises 129–130 are based on the information in the graph.



129. If your exercise goal is to improve cardiovascular conditioning, the graph shows the following range for target heart rate, H , in beats per minute:

Lower limit of range $H = \frac{7}{10}(220 - a)$

Upper limit of range $H = \frac{4}{5}(220 - a).$

- a. What is the lower limit of the heart rate range, in beats per minute, for a 20-year-old with this exercise goal?
 b. What is the upper limit of the heart rate range, in beats per minute, for a 20-year-old with this exercise goal?

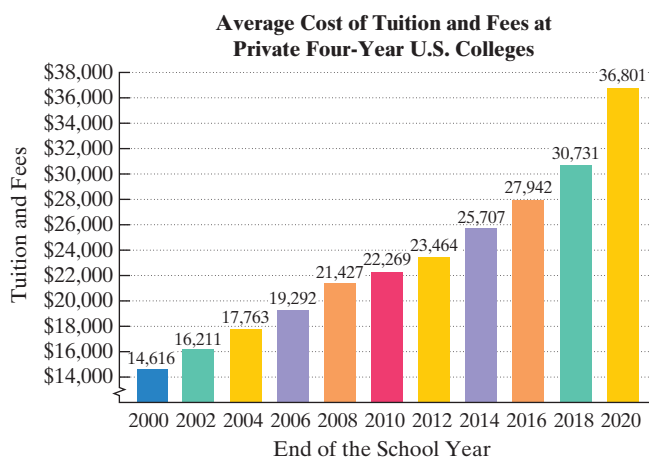
130. If your exercise goal is to improve overall health, the graph shows the following range for target heart rate, H , in beats per minute:

Lower limit of range $H = \frac{1}{2}(220 - a)$

Upper limit of range $H = \frac{3}{5}(220 - a).$

- a. What is the lower limit of the heart rate range, in beats per minute, for a 30-year-old with this exercise goal?
 b. What is the upper limit of the heart rate range, in beats per minute, for a 30-year-old with this exercise goal?

The bar graph shows the average cost of tuition and fees at private four-year colleges in the United States.



Source: The College Board

Here are two formulas that model the data shown in the graph. In each formula, T represents the average cost of tuition and fees at private U.S. colleges for the school year ending x years after 2000.

Model 1 $T = 975x + 13,547$

Model 2 $T = 32x^2 + 331x + 15,479$

Use this information to solve Exercises 131–132.

- 131. a.** Use each formula to find the average cost of tuition and fees at private U.S. colleges for the school year ending in 2018. By how much does each model underestimate or overestimate the actual cost shown for the school year ending in 2018?
- b.** Use model 2 to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2030.
- 132. a.** Use each formula to find the average cost of tuition and fees at private U.S. colleges for the school year ending in 2010. By how much does each underestimate or overestimate the actual cost shown for the school year ending in 2010?
- b.** Use model 2 to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2025.
- 133.** This month you have a total of \$6000 in interest-bearing credit card debt, split between a card charging 18% annual interest and a card charging 21% annual interest. If the interest-bearing balance on the card charging 18% is x dollars, then the total interest for the month is given by the algebraic expression

$$0.015x + 0.0175(6000 - x).$$

- a.** Simplify the algebraic expression.
- b.** Use each form of the algebraic expression to determine the total interest for the month if the balance on the card charging 18% is \$4400.
- c.** Use the simplified form of the algebraic expression to determine the total interest for the month if the \$6000 debt is split evenly between the two cards.

- 134.** It takes you 50 minutes to get to campus. You spend t minutes walking to the bus stop and the rest of the time riding the bus. Your walking rate is 0.06 mile per minute and the bus travels at a rate of 0.5 mile per minute. The total distance walking and traveling by bus is given by the algebraic expression

$$0.06t + 0.5(50 - t).$$

- a.** Simplify the algebraic expression.
- b.** Use each form of the algebraic expression to determine the total distance that you travel if you spend 20 minutes walking to the bus stop.
- c.** Use the simplified form of the algebraic expression to determine the total distance you travel if the 50 minutes is split evenly between walking and riding the bus.
- 135.** Read the Blitzer Bonus beginning on page 15. Use the formula

$$\text{BAC} = \frac{600n}{w(0.6n + 169)}$$

and replace w with your body weight. Using this formula and a calculator, compute your BAC for integers from $n = 1$ to $n = 10$. Round to three decimal places. According to this model, how many drinks can you consume in an hour without exceeding the legal measure of drunk driving?

Explaining the Concepts

> ACHIEVING SUCCESS

An effective way to understand something is to explain it to someone else. You can do this by using the Explaining the Concepts exercises that ask you to respond with verbal or written explanations. Speaking or writing about a new concept uses a different part of your brain than thinking about the concept. Explaining new ideas verbally will quickly reveal any gaps in your understanding. It will also help you to remember new concepts for longer periods of time.

- 136.** What is an algebraic expression? Give an example with your explanation.
- 137.** If n is a natural number, what does b^n mean? Give an example with your explanation.
- 138.** What does it mean when we say that a formula models real-world phenomena?
- 139.** What is the intersection of sets A and B ?
- 140.** What is the union of sets A and B ?
- 141.** How do the whole numbers differ from the natural numbers?
- 142.** Can a real number be both rational and irrational? Explain your answer.
- 143.** If you are given two real numbers, explain how to determine which is the lesser.

Critical Thinking Exercises

Make Sense? In Exercises 144–147, determine whether each statement makes sense or does not make sense, and explain your reasoning.

- 144.** My mathematical model describes the data for tuition and fees at public four-year colleges for the past 20 years extremely well, so it will serve as an accurate prediction for the cost of public colleges in 2050.

145. A model that describes the average cost of tuition and fees at private U.S. colleges for the school year ending x years after 2000 cannot be used to estimate the cost of private education for the school year ending in 2000.
146. Regardless of what real numbers I substitute for x and y , I will always obtain zero when evaluating $2x^2y - 2yx^2$.
147. Just as the commutative properties change groupings, the associative properties change order.

In Exercises 148–155, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

148. Every rational number is an integer.
149. Some whole numbers are not integers.
150. Some rational numbers are not positive.
151. Irrational numbers cannot be negative.
152. The term x has no coefficient.
153. $5 + 3(x - 4) = 8(x - 4) = 8x - 32$
154. $-x - x = -x + (-x) = 0$
155. $x - 0.02(x + 200) = 0.98x - 4$

In Exercises 156–158, insert either $<$ or $>$ in the shaded area between the numbers to make the statement true.

156. $\sqrt{2}$ 1.5
157. $-\pi$ -3.5
158. $-\frac{3.14}{2}$ $-\frac{\pi}{2}$

Preview Exercises

Exercises 159–161 will help you prepare for the material covered in the next section.

159. In parts (a) and (b), complete each statement.
- a. $b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b) = b^?$
- b. $b^5 \cdot b^5 = (b \cdot b \cdot b \cdot b \cdot b)(b \cdot b \cdot b \cdot b \cdot b) = b^?$
- c. Generalizing from parts (a) and (b), what should be done with the exponents when multiplying exponential expressions with the same base?
160. In parts (a) and (b), complete each statement.
- a. $\frac{b^7}{b^3} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b} = b^?$
- b. $\frac{b^8}{b^2} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b^?$
- c. Generalizing from parts (a) and (b), what should be done with the exponents when dividing exponential expressions with the same base?
161. If 6.2 is multiplied by 10^3 , what does this multiplication do to the decimal point in 6.2?

SECTION P.2

Exponents and Scientific Notation

WHAT YOU'LL LEARN

- 1 Use the product rule.
- 2 Use the quotient rule.
- 3 Use the zero-exponent rule.
- 4 Use the negative-exponent rule.
- 5 Use the power rule.
- 6 Find the power of a product.
- 7 Find the power of a quotient.
- 8 Simplify exponential expressions.
- 9 Use scientific notation.

Bigger than the biggest thing ever and then some. Much bigger than that in fact, really amazingly immense, a totally stunning size, real 'wow, that's big', time. . . . Gigantic multiplied by colossal multiplied by staggeringly huge is the sort of concept we're trying to get across here.

Douglas Adams, *The Restaurant at the End of the Universe*

In October 2019, *Forbes* published a list of the 400 wealthiest Americans, who had a total net worth of \$2.96 trillion. Amazon founder and CEO Jeff Bezos topped the list with a net worth of \$114 billion. That sounds like a lot of money, but consider this: At the end of 2019, the national debt was approximately \$22.7 trillion, rising to over \$25.5 trillion by midyear 2020. This \$2.8 trillion increase in the national debt over just a few months is nearly the total net worth of the 400 wealthiest Americans.

One of the best ways to put “staggeringly huge” numbers into perspective is by making comparisons. In this section, we will compare large and small numbers using exponents and scientific notation.



The Product and Quotient Rules

We have seen that exponents are used to indicate repeated multiplication. Now consider the multiplication of two exponential expressions, such as $b^4 \cdot b^3$. We are multiplying four factors of b and three factors of b . We have a total of seven factors of b :

$$b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b) = b^7.$$

4 factors of b 3 factors of b

Total: 7 factors of b

The product is exactly the same if we add the exponents:

$$b^4 \cdot b^3 = b^{4+3} = b^7.$$

The fact that $b^4 \cdot b^3 = b^7$ suggests the following rule:

1 Use the product rule.

The Product Rule

$$b^m \cdot b^n = b^{m+n}$$

When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.

EXAMPLE 1 Using the Product Rule

Multiply each expression using the product rule:

a. $2^2 \cdot 2^3$ b. $(6x^4y^3)(5x^2y^7)$.

Solution

a. $2^2 \cdot 2^3 = 2^{2+3} = 2^5$ or 32 $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

b. $(6x^4y^3)(5x^2y^7)$

$$= 6 \cdot 5 \cdot x^4 \cdot x^2 \cdot y^3 \cdot y^7$$

Use the associative and commutative properties. This step can be done mentally.

$$= 30x^{4+2}y^{3+7}$$

$$= 30x^6y^{10}$$

✓ CHECK POINT 1 Multiply each expression using the product rule:

a. $3^3 \cdot 3^2$ b. $(4x^3y^4)(10x^2y^6)$.

2 Use the quotient rule.

Now, consider the division of two exponential expressions, such as the quotient of b^7 and b^3 . We are dividing seven factors of b by three factors of b .

$$\frac{b^7}{b^3} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b} = \frac{b \cdot b \cdot b}{b \cdot b \cdot b} \cdot b \cdot b \cdot b \cdot b = 1 \cdot b \cdot b \cdot b \cdot b = b^4$$

This factor is equal to 1.

The quotient is exactly the same if we subtract the exponents:

$$\frac{b^7}{b^3} = b^{7-3} = b^4.$$

This suggests the following rule:

The Quotient Rule

$$\frac{b^m}{b^n} = b^{m-n}, \quad b \neq 0$$

When dividing exponential expressions with the same nonzero base, subtract the exponent in the denominator from the exponent in the numerator. Use this difference as the exponent of the common base.

EXAMPLE 2 Using the Quotient Rule

Divide each expression using the quotient rule:

a. $\frac{(-2)^7}{(-2)^4}$ b. $\frac{30x^{12}y^9}{5x^3y^7}$

Solution

a. $\frac{(-2)^7}{(-2)^4} = (-2)^{7-4} = (-2)^3$ or -8 $(-2)^3 = (-2)(-2)(-2) = -8$

b. $\frac{30x^{12}y^9}{5x^3y^7} = \frac{30}{5} \cdot \frac{x^{12}}{x^3} \cdot \frac{y^9}{y^7} = 6x^{12-3}y^{9-7} = 6x^9y^2$

✓ CHECK POINT 2 Divide each expression using the quotient rule:

a. $\frac{(-3)^6}{(-3)^3}$ b. $\frac{27x^{14}y^8}{3x^3y^5}$

3 Use the zero-exponent rule.

Zero as an Exponent

A nonzero base can be raised to the 0 power. The quotient rule can be used to help determine what zero as an exponent should mean. Consider the quotient of b^4 and b^4 , where b is not zero. We can determine this quotient in two ways.

$$\frac{b^4}{b^4} = 1$$

Any nonzero expression divided by itself is 1.

$$\frac{b^4}{b^4} = b^{4-4} = b^0$$

Use the quotient rule and subtract exponents.

This means that b^0 must equal 1.

The Zero-Exponent Rule

If b is any real number other than 0,

$$b^0 = 1.$$

Here are examples involving simplification using the zero-exponent rule:

$$8^0 = 1, \quad (-6)^0 = 1, \quad -6^0 = -1, \quad (5x)^0 = 1, \quad 5x^0 = 5.$$

Because there are no parentheses, only 6 is raised to the 0 power: $-6^0 = -(6^0) = -1$.

Because there are no parentheses, only x is raised to the 0 power: $5x^0 = 5 \cdot 1 = 5$.

4 Use the negative-exponent rule.

Negative Integers as Exponents

A nonzero base can be raised to a negative power. The quotient rule can be used to help determine what a negative integer as an exponent should mean. Consider the quotient of b^3 and b^5 , where b is not zero. We can determine this quotient in two ways.

$$\frac{b^3}{b^5} = \frac{\cancel{b} \cdot \cancel{b} \cdot \cancel{b}}{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot b \cdot b} = \frac{1}{b^2}$$

After dividing common factors, we have two factors of b in the denominator.

$$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$$

Use the quotient rule and subtract exponents.

Notice that $\frac{b^3}{b^5}$ equals both b^{-2} and $\frac{1}{b^2}$. This means that b^{-2} must equal $\frac{1}{b^2}$. This example is a special case of the **negative-exponent rule**.

The Negative-Exponent Rule

If b is any real number other than 0 and n is a natural number, then

$$b^{-n} = \frac{1}{b^n}.$$

EXAMPLE 3 Using the Negative-Exponent Rule

Use the negative-exponent rule to write each expression with a positive exponent. Simplify, if possible:

- a. 9^{-2} b. $(-2)^{-5}$ c. $\frac{1}{6^{-2}}$ d. $7x^{-5}y^2$.

Solution

a. $9^{-2} = \frac{1}{9^2} = \frac{1}{81}$

b. $(-2)^{-5} = \frac{1}{(-2)^5} = \frac{1}{(-2)(-2)(-2)(-2)(-2)} = \frac{1}{-32} = -\frac{1}{32}$

Only the sign of the exponent, -5 , changes.
The base, -2 , does not change sign.

c. $\frac{1}{6^{-2}} = \frac{1}{\frac{1}{6^2}} = 1 \cdot \frac{6^2}{1} = 6^2 = 36$

d. $7x^{-5}y^2 = 7 \cdot \frac{1}{x^5} \cdot y^2 = \frac{7y^2}{x^5}$

✓ CHECK POINT 3 Use the negative-exponent rule to write each expression with a positive exponent. Simplify, if possible:

- a. 5^{-2} b. $(-3)^{-3}$ c. $\frac{1}{4^{-2}}$ d. $3x^{-6}y^4$.

In Example 3 and Check Point 3, did you notice that

$$\frac{1}{6^{-2}} = 6^2 \quad \text{and} \quad \frac{1}{4^{-2}} = 4^2?$$