fifth edition

Precalculus

A Right Triangle Approach

Ratti
McWaters
Skrzypek

with

Bernards
Fresh

This page intentionally left blank

FIFTH EDITION

Precalculus

A Right Triangle Approach

J. S. Ratti
University of South Florida

Marcus McWaters
University of South Florida

Lesław A. Skrzypek
University of South Florida

Jessica Bernards
Portland Community College

Wendy Fresh
Portland Community College

Pearson

Content Development: Eric Gregg
Content Management: Brian Fisher, Jonathan Krebs, Jeff Weidenaar
Content Production: Tamela Ambush, Nicholas Sweeny
Product Management: Jessica Darczuk
Product Marketing: Stacey Sveum, Jessica Szewczyk
Rights and Permissions: Anjali Singh

Please contact https://support.pearson.com/getsupport/s/ with any queries on this content

Cover Image: Angelo Cavalli/Stone/Getty Images; Gashgeron/Shutterstock;\KRIACHKO OLEKSII/Shutterstock; Vladimirkarp/Shutterstock

Library of Congress Cataloging-in-Publication Data

Library of Congress Control Number: 2021949815

ScoutAutomatedPrintCode

Pearson

Rental
ISBN-10: 0-13-751935-4
ISBN-13: 978-0-13-751935-4

Foreword

We're pleased to present the fifth edition of *Precalculus: A Right Triangle Approach*. Our experience in teaching this material has been exceptionally rewarding. Because students are accustomed to information being delivered by electronic media, the introduction of MyLab™ Math into our courses was, and remains, seamless. With the addition of Jessica Bernards and Wendy Fresh to the author team, the MyLab course has been given a fresh redesign that aligns with the Ratti philosophy. You will now find author created videos over every objective as well as author created assignments, quizzes, and exams. Additionally, we have included interactive figures in both the print and electronic version of the text that will allow students to get a hands on exploration of the topics.

Today's precalculus students and instructors face many challenges. Students arrive with various levels of comprehension from their previous courses. Instead of really learning the concepts presented, students often resort to memorization to pass the course. As a result, a course needs to establish a common starting point for students and engage them in becoming active learners, without sacrificing the solid mathematics essential for conceptual understanding. Instructors in this course must take on the task of providing students with an understanding of precalculus, preparing them for the next step, and ensuring that they find mathematics useful and interesting. Our efforts in this direction have been aided considerably by the many suggestions we have received from users of the previous editions of this text.

Mathematics owes it current identity to contributions from diverse cultures across the world and throughout the ages. In this text we provide references to significant improvements and achievements in mathematics and related areas from sources both ancient and modern. We place a strong emphasis on both concept development and real-life applications. Topics such as functions, graphing, the difference quotient, and limiting processes provide thorough preparation for the study of calculus and will improve students' comprehension of algebra. Just-in-time review throughout the text ensures that all students are brought to the same level before being introduced to new concepts. Numerous applications motivate students to apply the concepts and skills they learn in precalculus to other courses, including the physical and biological sciences, engineering, and economics, and to on-the-job and everyday problem solving. Students are given ample opportunities in this course to think about important mathematical ideas and to practice and apply algebraic skills.

Throughout the text, we emphasize why the material being covered is important and how it can be applied. By thoroughly developing mathematical concepts with clearly defined terminology, students see the "why" behind those concepts, paving the way for a deeper understanding, better retention, less reliance on rote memorization, and ultimately more success. The level of exposition was selected so that the material is accessible to students and provides them with an opportunity to grow.

It is our hope that once you have read through our text, you will see that we were able to fulfill the initial goals of writing for today's students and for you, the instructor.

Marcus McWaters **Lesław Skrzypek** **J. S. Ratti**

Jessica Bernards **Wendy Fresh**

Contents

Foreword iii
Preface xiii
Pearson's Commitment to Diversity, Equity, and Inclusion xx
Dedication xxi

P Basic Concepts of Algebra 1

P.1 The Real Numbers and Their Properties 2

Classifying Numbers ■ Equality of Numbers ■ Classifying Sets of Numbers ■ Rational Numbers ■ Irrational Numbers ■ Integer Exponents ■ The Real Number Line ■ Inequalities ■ Sets ■ Definition of Union and Intersection ■ Intervals ■ Absolute Value ■ Distance Between Two Points on a Real Number Line ■ Order of Operations ■ Properties of the Real Numbers ■ Subtraction and Division of Real Numbers ■ Algebraic Expressions

P.2 Integer Exponents and Scientific Notation 19

Integer Exponents ■ Rules of Exponents ■ Simplifying Exponential Expressions ■ Scientific Notation

P.3 Polynomials 29

Polynomial Vocabulary ■ Adding and Subtracting Polynomials ■ Multiplying Polynomials ■ Special Products ■ Squaring a Binomial Sum or Difference ■ The Product of the Sum and Difference of Terms

P.4 Factoring Polynomials 39

The Greatest Common Monomial Factor ■ Factoring Out a Monomial ■ Factoring Trinomials of the Form $x^2 + Bx + C$ ■ Factoring Formulas ■ Perfect-Square Trinomials ■ Difference of Squares ■ Difference and Sum of Cubes ■ Factoring by Grouping ■ Factoring Trinomials of the Form $Ax^2 + Bx + C$

P.5 Rational Expressions 49

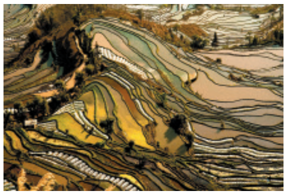
Rational Expressions ■ Lowest Terms for a Rational Expression ■ Multiplication and Division of Rational Expressions ■ Addition and Subtraction of Rational Expressions ■ Complex Fractions

P.6 Rational Exponents and Radicals 60

Square Roots ■ Simplifying Square Roots ■ Other Roots ■ Like Radicals ■ Radicals with Different Indexes ■ Rationalizing Radical Expressions ■ Conjugates ■ Rational Exponents

Chapter P Review and Tests, 73 ■ **Review Exercises,** 76
Practice Test, 78

1 Equations and Inequalities 79



1.1 Linear Equations in One Variable 80

Definitions ■ Identities, Conditional Equations, and Inconsistent Equations ■ Equivalent Equations ■ Solving Linear Equations in One Variable ■ Formulas

1.2 Applications of Linear Equations: Modeling 90

Solving Applied Problems ■ Geometry ■ Finance ■ Uniform Motion ■ Work Rate ■ Mixtures

1.3 Quadratic Equations 102

Factoring Method ■ The Square Root Method ■ Completing the Square ■ The Quadratic Formula ■ The Discriminant ■ Applications ■ Golden Rectangle

1.4 Complex Numbers: Quadratic Equations with Complex Solutions 117

Complex Numbers ■ Addition and Subtraction ■ Multiplying Complex Numbers ■ Complex Conjugates and Division ■ Quadratic Equations with Complex Solutions ■ The Discriminant

1.5 Solving Other Types of Equations 127

Solving Equations by Factoring ■ Rational Equations ■ Equations Involving Radicals ■ Equations with Rational Exponents ■ Equations That Are Quadratic in Form

1.6 Inequalities 141

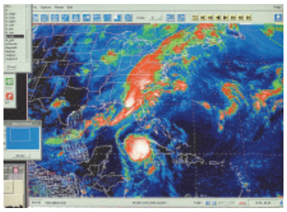
Inequalities ■ Linear Inequalities ■ Combining Two Inequalities ■ Using Test Points to Solve Inequalities

1.7 Equations and Inequalities Involving Absolute Value 154

Equations Involving Absolute Value ■ Inequalities Involving Absolute Value

Key Ideas At a Glance, 162 ■ **Chapter 1 Review and Tests,** 163 ■ **Review Exercises,** 166 ■ **Practice Test A*,** 168

2 Graphs and Functions 169



2.1 The Coordinate Plane 170

The Coordinate Plane ■ Scales on a Graphing Utility ■ Distance Formula ■ Midpoint Formula

2.2 Graphs of Equations 179

Graph of an Equation ■ Intercepts ■ Symmetry ■ Circles ■ Semicircles

2.3 Lines 192

Slope of a Line ■ Point-Slope Form ■ Slope-Intercept Form ■ Equations of Horizontal and Vertical Lines ■ General Form of the Equation of a Line ■ Parallel and Perpendicular Lines ■ Modeling Data Using Linear Regression

*Practice Test B can be found in the eText.

2.4 Functions 209

Functions ■ Function Notation ■ Representations of Functions ■ The Domain of a Function ■ The Range of a Function ■ Graphs of Functions ■ Function Information from Its Graph ■ Building Functions ■ Functions in Economics

2.5 Properties of Functions 229

Increasing and Decreasing Functions ■ Relative Maximum and Minimum Values ■ Even–Odd Functions and Symmetry ■ Average Rate of Change

2.6 A Library of Functions 243

Linear Functions ■ Square Root and Cube Root Functions ■ Piecewise Functions ■ Graphing Piecewise Functions ■ Basic Functions

2.7 Transformations of Functions 257

Transformations ■ Vertical and Horizontal Shifts ■ Reflections ■ Stretching or Compressing ■ Multiple Transformations in Sequence

2.8 Combining Functions; Composite Functions 274

Combining Functions ■ Composition of Functions ■ Domain of Composite Functions ■ Decomposition of a Function ■ Applications of Composite Functions

2.9 Inverse Functions 287

Inverses ■ Finding the Inverse Function ■ Finding the Range of a One-to-One Function ■ Applications

Key Ideas At a Glance, 300 ■ **Chapter 2 Review and Tests,** 301 ■ **Review Exercises,** 305 ■ **Practice Test A*,** 308 ■ **Cumulative Review Exercises Chapters P–2,** 309

3 Polynomial and Rational Functions 310

3.1 Quadratic Functions 311

Quadratic Functions ■ Standard Form of a Quadratic Function ■ Graphing a Quadratic Function $f(x) = ax^2 + bx + c$ ■ Applications

3.2 Polynomial Functions 325

Polynomial Functions ■ Power Functions ■ End Behavior of Polynomial Functions ■ Zeros of a Function ■ Zeros and Turning Points ■ Graphing a Polynomial Function

3.3 Dividing Polynomials 345

The Division Algorithm ■ Synthetic Division ■ The Remainder and Factor Theorems

3.4 The Real Zeros of a Polynomial Function 356

Real Zeros of a Polynomial Function ■ Rational Zeros Theorem ■ Descartes's Rule of Signs ■ Bounds on the Real Zeros ■ Find the Real Zeros of a Polynomial Function

3.5 The Complex Zeros of a Polynomial Function 367

Conjugate Pairs Theorem

3.6 Rational Functions 375

Rational Functions ■ Vertical and Horizontal Asymptotes ■ Translations of $f(x) = \frac{1}{x}$ ■ Graphing Rational Functions ■ Oblique Asymptotes ■ Graph of a Revenue Curve

3.7 Variation 393

Direct Variation ■ Inverse Variation ■ Joint and Combined Variation

Key Ideas At a Glance, 403 ■ **Chapter 3 Review and Tests,** 404 ■ **Review Exercises,** 407 ■ **Practice Test A*,** 409 ■ **Cumulative Review Exercises Chapters P–3,** 410

4 Exponential and Logarithmic Functions 411

4.1 Exponential Functions 412

Exponential Functions ■ Evaluate Exponential Functions ■ Graphing Exponential Functions ■ Transformations on Exponential Functions ■ Simple Interest ■ Compound Interest ■ Continuous Compound Interest Formula ■ The Natural Exponential Function ■ Exponential Growth and Decay

4.2 Logarithmic Functions 432

Logarithmic Functions ■ Evaluating Logarithms ■ Basic Properties of Logarithms ■ Domains of Logarithmic Functions ■ Graphs of Logarithmic Functions ■ Common Logarithm ■ Natural Logarithm ■ Investments ■ Newton's Law of Cooling

4.3 Rules of Logarithms 449

Rules of Logarithms ■ Number of Digits ■ Change of Base ■ Growth and Decay ■ Half-Life ■ Radiocarbon Dating

4.4 Exponential and Logarithmic Equations and Inequalities 462

Solving Exponential Equations ■ Applications of Exponential Equations ■ Solving Logarithmic Equations ■ Logarithmic and Exponential Inequalities

4.5 Logarithmic Scales; Modeling 475

pH Scale ■ Earthquake Intensity ■ Loudness of Sound ■ Musical Pitch ■ Star Brightness ■ Modeling

Key Ideas At a Glance, 491 ■ **Chapter 4 Review and Tests,** 492 ■ **Review Exercises,** 494 ■ **Practice Test A*,** 497 ■ **Cumulative Review Exercises Chapters P–4,** 498

5 Trigonometric Functions 499

5.1 Angles and Their Measure 500

Angles ■ Angle Measure ■ Degree Measure ■ Radian Measure ■ Relationship Between Degrees and Radians ■ Coterminal Angles ■ Complements and Supplements ■ Length of an Arc of a Circle ■ Area of a Sector ■ Linear and Angular Speed

*Practice Test B can be found in the eText.

5.2 Right-Triangle Trigonometry 514

Trigonometric Ratios and Functions ■ Relations between Trigonometric Functions ■ Function Values for Some Special Angles ■ Evaluating Trigonometric Functions Using a Calculator ■ Complements ■ Applications

5.3 Trigonometric Functions of Any Angle; The Unit Circle 528

Trigonometric Functions of Angles ■ Quadrantal Angles ■ Coterminal Angles ■ Signs of the Trigonometric Functions ■ Reference Angle ■ Using Reference Angles ■ Circular Functions

5.4 Graphs of the Sine and Cosine Functions 542

Properties of Sine and Cosine ■ Domain and Range of Sine and Cosine ■ Zeros of Sine and Cosine Functions ■ Even-Odd Properties of the Sine and Cosine Functions ■ Periodic Functions ■ Graphs of Sine and Cosine Functions ■ Graph of the Sine Function ■ Graph of the Cosine Function ■ Five Key Points ■ Symmetries of the Sine and Cosine Functions ■ Amplitude and Period ■ Phase Shift ■ Vertical Shifts ■ Modeling with Sinusoidal Curves ■ Simple Harmonic Motion

5.5 Graphs of the Other Trigonometric Functions 567

Tangent Function ■ Graph of $y = \tan x$ ■ Graphs of the Reciprocal Functions

5.6 Inverse Trigonometric Functions 577

The Inverse Sine Function ■ The Inverse Cosine Function ■ The Inverse Tangent Function ■ Other Inverse Trigonometric Functions ■ Evaluating Inverse Trigonometric Functions ■ Composition of Trigonometric and Inverse Trigonometric Functions

Key Ideas At a Glance, 593 ■ **Chapter 5 Review and Tests,** 594 ■ **Review Exercises,** 600 ■ **Practice Test A*,** 602 ■ **Cumulative Review Exercises Chapters P–5,** 602

6 Trigonometric Identities and Equations 603

6.1 Trigonometric Identities 604

Fundamental Trigonometric Identities ■ Simplifying a Trigonometric Expression ■ Trigonometric Equations and Identities ■ Process of Verifying Trigonometric Identities ■ Methods of Verifying Trigonometric Identities

6.2 Sum and Difference Formulas 616

Sum and Difference Formulas for Cosine ■ Cofunction Identities ■ Sum and Difference Formulas for Sine ■ Sum and Difference Formulas for Tangent ■ Reduction Formula

6.3 Double-Angle and Half-Angle Formulas 629

Double-Angle Formulas ■ Power-Reducing Formulas ■ Alternating Current ■ Half-Angle Formulas

6.4 Product-to-Sum and Sum-to-Product Formulas 640

Product-to-Sum Formulas ■ Sum-to-Product Formulas ■ Verify Trigonometric Identities ■ Analyzing Touch-Tone Phones

6.5 Trigonometric Equations I 648

Trigonometric Equations ■ Trigonometric Equations of the Form $a\sin(x - c) = k$, $a\cos(x - c) = k$, and $a\tan(x - c) = k$ ■ Trigonometric Equations and the Zero-Product Property ■ Equations with More Than One Trigonometric Function ■ Extraneous Solutions

6.6 Trigonometric Equations II 658

Equations Involving Multiple Angles ■ Using Sum-to-Product Identities ■ Equations Containing Inverse Functions

Key Ideas At a Glance, 668 ■ **Chapter 6 Review and Tests,** 669 ■ **Review Exercises,** 672 ■ **Practice Test A*,** 674 ■ **Cumulative Review Exercises Chapters P–6,** 675

7 Applications of Trigonometric Functions 676

7.1 The Law of Sines 677

Solving Oblique Triangles ■ The Law of Sines ■ Solving AAS and ASA Triangles ■ Solving SSA Triangles—the Ambiguous Case ■ Bearings

7.2 The Law of Cosines 693

The Law of Cosines ■ Derivation of the Law of Cosines ■ Solving SAS Triangles ■ Solving SSS Triangles

7.3 Areas of Polygons Using Trigonometry 705

Geometry Formulas ■ Area of SAS Triangles ■ Area of AAS and ASA Triangles ■ Area of SSS Triangles ■ Applications ■ Polygons

7.4 Vectors 717

Vectors ■ Geometric Vectors ■ Equivalent Vectors ■ Adding Vectors ■ Algebraic Vectors ■ Unit Vectors ■ Vectors in i, j Form ■ Vector in Terms of Magnitude and Direction ■ Applications of Vectors

7.5 The Dot Product 730

The Dot Product ■ The Angle Between Two Vectors ■ Orthogonal Vectors ■ Projection of a Vector ■ Decomposition of a Vector ■ Work

7.6 Polar Coordinates 741

Polar Coordinates ■ Multiple Representations ■ Sign of r ■ Converting Between Polar and Rectangular Forms ■ Converting Equations Between Rectangular and Polar Forms ■ The Graph of a Polar Equation ■ Limaçons

7.7 Polar Form of Complex Numbers; DeMoivre's Theorem 757

Geometric Representation of Complex Numbers ■ The Absolute Value of a Complex Number ■ Polar Form of a Complex Number ■ Product and Quotient in Polar Form ■ Powers of Complex Numbers in Polar Form ■ Roots of Complex Numbers

Key Ideas At a Glance, 767 ■ **Chapter 7 Review and Tests,** 768 ■ **Review Exercises,** 772 ■ **Practice Test A*,** 773 ■ **Cumulative Review Exercises Chapters P–7,** 774

*Practice Test B can be found in the eText.

8 Systems of Equations and Inequalities 775

8.1 Systems of Linear Equations in Two Variables 776

System of Equations ■ Graphical Method ■ Substitution Method ■ Elimination Method ■ Applications

8.2 Systems of Linear Equations in Three Variables 790

Systems of Linear Equations ■ Number of Solutions of a Linear System ■ Nonsquare Systems ■ Geometric Interpretation ■ An Application to CAT Scans

8.3 Partial-Fraction Decomposition 802

Partial Fractions ■ $Q(x)$ Has Only Distinct Linear Factors ■ $Q(x)$ Has Repeated Linear Factors ■ $Q(x)$ Has Distinct Irreducible Quadratic Factors ■ $Q(x)$ Has Repeated Irreducible Quadratic Factors

8.4 Systems of Nonlinear Equations 813

Systems of Nonlinear Equations ■ Solving Systems of Nonlinear Equations by Substitution ■ Solving Systems of Nonlinear Equations by Elimination ■ Applications

8.5 Systems of Inequalities 820

Graph of a Linear Inequality in Two Variables ■ Systems of Linear Inequalities in Two Variables ■ Nonlinear Inequality ■ Nonlinear Systems

8.6 Linear Programming 832

Linear Programming ■ Solving Linear Programming Problems ■ Applications

Key Ideas At a Glance, 841 ■ **Chapter 8 Review and Tests,** 842 ■ **Review Exercises,** 846 ■ **Practice Test A*,** 848 ■ **Cumulative Review Exercises Chapters P–8,** 849

9 Matrices and Determinants 850

9.1 Matrices and Systems of Equations 851

Definition of a Matrix ■ Using Matrices to Solve Linear Systems ■ Gaussian Elimination ■ Gauss–Jordan Elimination

9.2 Matrix Algebra 867

Equality of Matrices ■ Matrix Addition and Scalar Multiplication ■ Matrix Multiplication ■ Computer Graphics

9.3 The Matrix Inverse 882

The Multiplicative Inverse of a Matrix ■ Finding the Inverse of a Matrix ■ A Rule for Finding the Inverse of a 2×2 Matrix ■ Solving Systems of Linear Equations by Using Matrix Inverses ■ Applications of Matrix Inverses ■ Cryptography

9.4 Determinants and Cramer's Rule 896

The Determinant of a 2×2 Matrix ■ Minors and Cofactors ■ The Determinant of an $n \times n$ Matrix ■ Cramer's Rule ■ Applications

Key Ideas At a Glance, 908 ■ **Chapter 9 Review and Tests,** 909 ■ **Review Exercises,** 913 ■ **Practice Test A*,** 916 ■ **Cumulative Review Exercises Chapters P–9,** 917

10 Conic Sections 918

10.1 Conic Sections: Overview 919

10.2 The Parabola 921

Geometric Definition of a Parabola ■ Equation of a Parabola ■ Translations of Parabolas ■ Reflecting Property of Parabolas

10.3 The Ellipse 935

Definition of Ellipse ■ Equation of an Ellipse ■ Translations of Ellipses ■ Applications

10.4 The Hyperbola 948

Definition of Hyperbola ■ The Asymptotes of a Hyperbola ■ Graphing a Hyperbola with Center (0, 0) ■ Translations of Hyperbolas ■ Applications

Key Ideas At a Glance, 965 ■ **Chapter 10 Review and Tests,** 966 ■ **Review Exercises,** 969 ■ **Practice Test A*,** 970 ■ **Cumulative Review Exercises Chapters P–10,** 971

11 Further Topics in Algebra 972

11.1 Sequences and Series 973

Sequences ■ Recursive Formulas ■ Factorial Notation ■ Summation Notation ■ Series

11.2 Arithmetic Sequences; Partial Sums 985

Arithmetic Sequence ■ Sum of a Finite Arithmetic Sequence

11.3 Geometric Sequences and Series 994

Geometric Sequence ■ Finding the Sum of a Finite Geometric Sequence ■ Annuities ■ Infinite Geometric Series

11.4 Mathematical Induction 1010

Mathematical Induction ■ Determining the Statement P_{k+1} from the Statement P_k

*Practice Test B can be found in the eText.

11.5 The Binomial Theorem 1016

Binomial Expansions ■ Pascal's Triangle ■ The Binomial Theorem ■ Binomial Coefficients

11.6 Counting Principles 1024

Fundamental Counting Principle ■ Permutations ■ Combinations ■ Distinguishable Permutations ■ Deciding Whether to Use Permutations, Combinations, or the Fundamental Counting Principle

11.7 Probability 1035

The Probability of an Event ■ The Additive Rule ■ Mutually Exclusive Events ■ The Complement of an Event ■ Experimental Probabilities

Key Ideas At a Glance, 1045 ■ **Chapter 11 Review and Tests,** 1046 ■ **Review Exercises,** 1049 ■ **Practice Test A*,** 1050 ■ **Cumulative Review Exercises Chapters P–11,** 1051

Answers to Selected Exercises A-1
Credits C-1
Index I-1
Practice Test B for all Chapters (available in eText) T-1

Preface

Students begin precalculus classes with widely varying backgrounds. Some haven't taken a math course in several years and may need to spend time reviewing prerequisite topics, while others are ready to jump right into new and challenging material. In Chapter P and in some of the early sections of other chapters, we have provided review material in such a way that it can be used or omitted as appropriate for your course. In addition, students may follow several paths after completing a precalculus course. Many will continue their study of mathematics in courses such as finite mathematics, statistics, and calculus. For others, precalculus may be their last mathematics course.

Responding to the current and future needs of all these students was essential in creating this text. We introduce each exercise set with several concept and vocabulary exercises, consisting of fill-in-the-blank and true-false exercises. They are not computation-reliant, but rather test whether students have absorbed the basic concepts and vocabulary of the section. Exercises asking students to extrapolate information from a given graph now appear in much greater number and depth throughout the course. We continue to present our content in a systematic way that illustrates how to study and what to review. We believe that if students use this material well, they will succeed in this course. The changes in this edition result from the thoughtful feedback we have received from students and instructors who have used previous editions of the text. This feedback crucially enhances our own experiences, and we are extremely grateful to the many contributors whose insights are reflected in this new edition.

Key Content Changes

New! Authors. We would like to welcome two new coauthors to the author team, Jessica Bernards and Wendy Fresh! Both Jessica and Wendy are instructors at Portland Community College and have provided wonderful additions to the text and the accompanying MyLab Math course. Jessica and Wendy have had national recognition as instructors and have received several awards for excellence in teaching mathematics.

Revised! Getting Ready for the Next Section. This feature combines two features of previous editions, "Getting Ready for the Next Section" exercises and "Before Starting This Section, Review" objectives. The new structure lists Review Concepts and Review Skills for students to brush up on before beginning the next section. Both the Review Concepts and Review Skills contain section and page number references to make looking up these topics easy. The Review Concepts are meant to be broader topics that students should understand, while Review Skills give exercises from objectives that students will need in the upcoming section.

New! Key Ideas at a Glance. A new feature added to the text for this edition is a single page at the end of each chapter designed to highlight some key concepts for the chapter. In some chapters, this will serve as a comparison between two similar or parallel topics. In other chapters, this will sum up many of the ideas presented in the chapter. This page can serve as a reference to students to look back on when studying or doing exercises. There are also exercises to accompany this feature so that students may test their understanding of the ideas summarized there.

New and Revised! Exercises. We continue to improve the balance of exercises, providing a smooth transition from the less challenging to the more challenging exercises.

Overall, approximately 20% of the exercises have been updated, and more than 500 brand-new exercises have been added. These new exercises primarily consist of applications that connect with students' everyday experiences and enhance students' understanding of graphing.

Revised! Application Exercises. Every section opens with discussion of an application that relates to the topics introduced in that section. This edition continues the trend of pairing an example with this application, but we have also made an effort to include problems in the exercise sets that also tie to this application so that students have an opportunity to apply the mathematics to a real world problem. The section opening, example, and these exercises are easily identified by an accompanying icon.

New! Active Learning Exercises. In many sections throughout the text, exercise sets will end with an Active Learning exercise. This exercise is accompanied by an Interactive Figure powered by GeoGebra, which is accessed through a bit.ly link or by scanning the given QR code. Students will manipulate the figure to explore mathematics in a new way and will use the figure to answer the accompanying exercises.

New! Videos. Videos in the MyLab course have been completely re-made by the authors. Videos can be found at the section and objective level. The videos are available in the MyLab course within the Video & Resource Library, but can also be accessed directly from the text. QR codes can be found at the beginning of each section, and users can use their phone to scan the QR codes and watch the videos.

We have also created videos for select exercises in each section. The text has QR codes next to the beginning of each exercise set, and users can scan the code to find the videos for exercises in that section.

Revised! Diversity, Equity, and Inclusion. We conducted an external review of the text's content to determine how it could be improved to address issues related to diversity, equity, and inclusion. The results of that review informed the revision.

Chapter 1.

- Added a graphic to detail the process of factoring a trinomial.
- Added more Procedure by Action type examples which carefully breakdown factoring trinomials.
- Created a new section opener for section 1.5 highlighting female NASA engineers' current Mars Exploration Program

Chapter 2.

- Updated several examples and with newer application context.
- Expanded step by step transformation of functions procedure.
- Split a Procedure in Action example into two parts to give individual attention to finding a parallel line vs a perpendicular line to a given line.
- Included new Active Learning Exercises.

Chapter 3.

- Added more visual aid to the examples on graphing a polynomial, giving a more complete picture of the behavior of graphs of polynomials.
- Added more visual aid to the examples on graphing a rational function, giving a more complete picture of the behavior of graphs of rational functions.
- Included new Active Learning Exercises.

Chapter 4.

- Expanded on the basic properties of logarithms to give some derivations of these properties.
- Added color to the table of logarithmic functions to make it easier to see how changes in the function change the graph itself.
- Included new Active Learning Exercises.

Chapter 5.

- Updated graphs for sine and cosine functions to give additional detail around key points.
- Created new graphics to show the symmetries of the sine and cosine functions.
- Updated graphics showing stretch and compression of sine and cosine graphs.
- Updated graphics showing phase shifts and vertical shifts of sine and cosine graphs.
- Added additional graphics for the inverse sine, cosine, and tangent functions.
- Included new Active Learning Exercises.

Chapter 6.

- Added a graphic to demonstrate one of the Pythagorean identities.
- Added graphics to support trigonometric identities using the symmetries of the sine and cosine functions.
- Included new Active Learning Exercises.

Chapter 7

- Updated and added graphics and explanation for the ambiguous case of SSA triangles.
- Updated solution process for SSA triangles.
- Included new Active Learning Exercises.

Chapter 8.

- Wrote all new section opener discussions for sections 8.1 and 8.5.
- Added a new example in section 8.5 showing an application involving cell tower triangulation.
- Included new Active Learning Exercises.

Chapter 9.

- Added a link between the section opener discussion and an example in section 9.4.
- Included new Active Learning Exercises.

Chapter 10.

- Added many examples of conics seen and used in the real world.
- Included new Active Learning Exercises.

Chapter 11.

- Changed terms used in the sum of the first n terms of an arithmetic sequence so that the derivation of the formula is clearer.
- Added graphics to demonstrate the derivation of the sum of the first n terms of arithmetic and geometric sequences.
- Included new Active Learning Exercises.

Features

Chapter Opener. Each chapter opener includes a description of applications (one of them illustrated) relevant to the content of the chapter and the list of topics that will be covered. In one page, students see what they are going to learn and why they are learning it.

Getting Ready for the Next Section. Each section is immediately preceded by a set of concepts and skills that serve as a transition from one section to the next. These sets of problems provide a review of concepts and skills that will be used in the upcoming section.

Section Opener With Application. Each section opens with a list of clearly stated and numbered **Objectives** defined for the section. These objectives are then referenced again in the margin of the lesson at the point where the objective's topic is taught. An **Application** containing a motivating anecdote or an interesting problem then follows. An example later in the section relating to this application and identified by the same icon ( ) is then solved using the mathematics covered in the section. These applications utilize material from a variety of fields: the physical and biological sciences (including health sciences), economics, art and architecture, history, and more.

Examples and Practice Problems. **Examples** include a wide range of computational, conceptual, and modern applied problems carefully selected to build confidence, competency, and understanding. Every example has a title indicating its purpose and presents a detailed solution containing annotated steps. All examples are followed by a **Practice Problem** for students to try so that they can check their understanding of the concept covered. Answers to the Practice Problems are provided in the back of the book.

Procedure in Action Examples. These types of examples, interspersed throughout the text, present important procedures in numbered steps. Special **Procedure in Action** examples present important multistep procedures, such as the steps for doing synthetic division, in a two-column format. The steps of the procedure are given in the left column, and an example is worked, following these steps, in the right column. This approach provides students with a clear model with which they can compare when encountering difficulty in their work.

Additional Pedagogical Features

Definitions, ***Theorems***, ***Properties***, and ***Rules*** are all boxed and titled for emphasis and ease of reference.

Warnings appear as appropriate throughout the text to apprise students of common errors and pitfalls that can trip them up in their thinking or calculations.

Summary of Main Facts boxes summarize information related to equations and their graphs, such as those of the conic sections.

A Calculus Symbol appears next to information in the text that is essential for the study of calculus.

Margin Notes

Side Notes provide hints for handling newly introduced concepts.

Recall notes remind students of a key idea learned earlier in the text that will help them work through a current problem.

Technology Connections give students tips on using calculators to solve problems, check answers, and reinforce concepts. Note that the use of graphing calculators is optional in this text.

Do You Know? Features provide students with additional interesting information on topics to keep them engaged in the mathematics presented.

Exercises. The heart of any textbook is its exercises, so we have tried to ensure that the quantity, quality, and variety of exercises meet the needs of all students. Exercises are carefully graded to strengthen the skills developed in the section and are organized using the following categories.

Concepts and Vocabulary exercises begin each exercise set with problems that assess the student's grasp of the definitions and ideas introduced in that section. These true-false and fill-in-the-blank exercises help to rapidly identify gaps in comprehension of the material in that section.

Building Skills exercises develop fundamental skills—each odd-numbered exercise is closely paired with its consecutive even-numbered exercise.

Applying the Concepts exercises use the section's material to solve real-world problems—all are titled and relevant to the topics of the section.

Beyond the Basics exercises provide more challenging problems that give students an opportunity to reach beyond the material covered in the section—these are generally more theoretical in nature and are suitable for honors students, special assignments, or extra credit.

Critical Thinking/Discussion/Writing exercises, appearing as appropriate, are designed to develop students' higher-level thinking skills. Calculator problems, identified by , are included where needed.

Active Learning exercises allow students to explore mathematical concepts in new ways. Students have the chance to manipulate Interactive Figures and answer accompanying questions.

Key Ideas at a Glance. This one page feature found at the end of each chapter highlights some key concepts in each chapter. In some chapters, this will serve as a comparison between two similar or parallel topics. In other chapters, this will sum up many of the ideas presented in the chapter. This page can serve as a reference to students to look back on when studying or doing exercises. There are also exercises to accompany this feature so that students may test their understanding of the ideas summarized there.

Chapter Review and Tests. The chapter-ending material begins with an extensive **Review** featuring a two-column, section-by-section summary of the definitions, concepts, and formulas covered in that chapter, with corresponding examples. This review provides a description and examples of key topics indicating where the material occurs in the text, and encourages students to reread sections rather than memorize definitions out of context. **Review Exercises** provide students with an opportunity to practice what they have learned in the chapter. Then students are given two chapter test options. They can take **Practice Test A** in the usual open-ended format and/or **Practice Test B**, covering the same topics, in a multiple-choice format. Practice Test B has been moved online for this edition, and can be found in the eText. All tests are designed to increase student comprehension and verify that students have mastered the skills and concepts in the chapter. Mastery of these materials should indicate a true comprehension of the chapter and the likelihood of success on the associated in-class examination. **Cumulative Review Exercises** appear at the end of every chapter, starting with Chapter 2, to remind students that mathematics is not modular and that what is learned in the first part of the book will be useful in later parts of the book and on the final examination.

MyLab Math Resources for Success

MyLab Math is available to accompany Pearson's market-leading text options, including ***Precalculus: A Right Triangle Approach*, 5th Edition** (access code required).

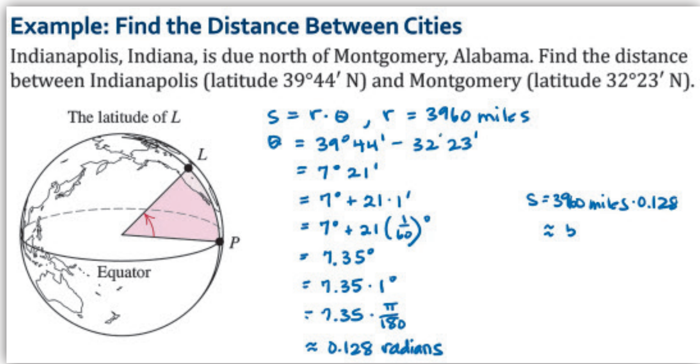
MyLab™ is the teaching and learning platform that empowers you to reach every student. MyLab Math combines trusted author content—including full eText and assessment with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

MyLab Math supports all learners, regardless of their ability and background, in order to provide an equal opportunity for success. Accessible resources support learners for a more equitable experience no matter their abilities. And options to personalize learning and address individual gaps help to provide each learner with the specific resources they need in order to achieve success.

Student Resources

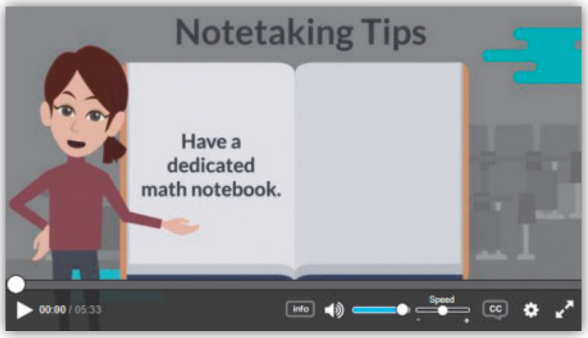
Motivate Your Students—Students are motivated to succeed when they're engaged in the learning experience and understand the relevance and power of math.

▼ **NEW!** **Section Lecture Videos**—Co-authors Jessica Bernards and Wendy Fresh (Portland Community College) have created all new Section videos, segmented and assignable by objective, using their years of teaching experience for online courses and flipped classrooms. Instructors can assign a full objective or only the segment that is needed. The videos allow students an opportunity to learn from experienced master teachers breaking down complex topics in an easy-to-understand manner.



- **NEW!** **Video Notebook** is a note-taking guide that gives students a structured place to take notes and work the example problems as they watch the videos. Definitions and important concepts are highlighted, helpful tips are pointed out along the way. Jessica Bernards and Wendy Fresh author this supplement to make sure students are actively engaged with their learning. The Video Notebook is available as PDFs and customizable Word files in MyLab Math. Instructors can also use the video notebook as a guide when creating their own lecture notes in a traditional lecture class. This way an instructor has pre-built lecture notes ready to go, or an easily adaptable set of lecture notes ready to be modified for the needs of their students.

▼ **NEW!** **Mathematical study skills videos**, created by co-authors Jessica Bernards and Wendy Fresh, motivate students to stick with their math course and offer practical tips to succeed. The animated character, Polly Nomial, guides students through topics such as How Learning Math is Different and Having a Growth Mindset in Math that any math student could benefit from watching. These ten study skills videos have pre-built assignments that include assessment questions that test students' understanding of the content.



- **NEW!** **Personal Inventory Assessments** are a collection of online exercises designed to promote self-reflection and metacognition in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.

Address Underpreparedness—Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student needs practice, giving all students the support they need—when and where they need it—to be successful.

NEW! **Integrated Review** can be used in corequisite courses, or simply to help students who enter Precalculus without a full understanding of prerequisite skills and concepts.

- Integrated Review at the chapter level provides a Skills Check assessment to pinpoint which prerequisites topics, if any, students need to review.
- Students who require additional review proceed to a personalized homework assignment to remediate.
- Integrated Review videos and worksheets provide additional instruction.

Instructors who prefer to review at the section level can assign the Enhanced Assignments instead.

Personalized Homework—With Personalized Homework, students take a quiz or test and receive a subsequent homework assignment that is personalized based on their performance. This way, students can focus on just the topics they have not yet mastered.

Other student resources include the following:

- NEW! **Interactive Figures** bring mathematical concepts to life, helping students see the concepts through directed explorations and purposeful manipulation. For this revision, we added many more interactive figures (in editable GeoGebra format) to the Video & Resource Library. The instructional videos that accompany the text now include Interactive Figures to teach key concepts. These figures are assignable in MyLab Math and encourage active learning, critical thinking, and conceptual understanding.
- **Solution Manual**—Written by Beverly Fusfield, the Student's Solution Manual provides detailed worked-out solutions to the odd-numbered end-of-section and Chapter Review exercises as well as solutions to all the Practice Problems, Practice Tests, and Cumulative Review problems. Available in MyLab Math.

Instructor Resources

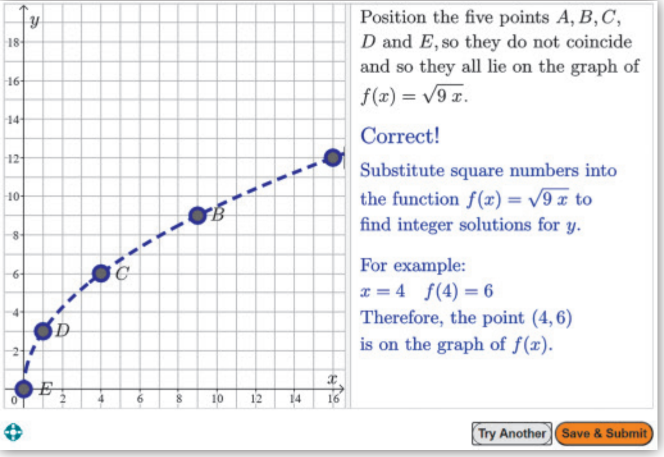
Your course is unique. So whether you'd like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

Pre-Built Assignments are designed to maximize students' performance. All assignments are *fully editable* to make your course your own.

- NEW! **Enhanced Assignments**—These section-level assignments have three unique properties:
  1. They help keep skills fresh with *spaced practice* of previously learned concepts.
  2. Learning aids are strategically turned off for some exercises to ensure students understand how to work the exercises independently.
  3. They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.
- NEW! **Learning Assignments**—Section-level assignments are especially helpful for online classes or flipped classes, where some or all learning takes place independently. These assignments include objective-level videos and interactive figures for student exploration followed by corresponding MyLab questions to ensure engagement and understanding. Instructors can assign the video notebook for students to fill out as they complete these video assignments.

MyLab Math Question Library is correlated to the exercises in the text, reflecting each author's approach and learning style. They regenerate algorithmically to give students unlimited opportunity for practice and mastery. Below are a few exercise types available to assign:

- NEW! **GeoGebra Exercises** are gradable graphing and computational exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.



- **Setup & Solve Exercises** require students to first describe how they will set up and approach the problem. This reinforces conceptual understanding of the process applied in approaching the problem, promotes long-term retention of the skill, and mirrors what students will be expected to do on a test.
- **Concept and Vocabulary**—Each exercise section begins with exercises that assess the student's grasp of the definitions and ideas introduced in that section. These true-false and fill-in-the-blank exercises help to rapidly identify gaps in comprehension and are assignable in MyLab Math and Learning Catalytics.

Learning Catalytics—With Learning Catalytics, you'll hear from every student when it matters most. You pose a variety of questions in class (choosing from pre-loaded questions or your own) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops.

Performance Analytics enable instructors to see and analyze student performance across multiple courses. Based on their current course progress, individuals' performance is identified above, at, or below expectations through a variety of graphs and visualizations.

Now included with Performance Analytics, **Early Alerts** use predictive analytics to identify struggling students—even if their assignment scores are not a cause for concern. In both Performance Analytics and Early Alerts, instructors can email students individually or by group to provide feedback.

Accessibility—Pearson works continuously to ensure our products are as accessible as possible to all students. Currently we work toward achieving WCAG 2.0 AA for our existing products (2.1 AA for future products) and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media (https://wps.pearsoned.com/accessibility/).

Other instructor resources include the following:

- **Instructor Solution Manual**—Written by Bevery Fusfield, the Instructor's Solutions Manual provides complete solutions for all end-of-section exercises, including the Critical Thinking/Discussion/Writing Projects, Practice Problems, Chapter Review exercises, Practice Tests, and Cumulative Review problems.
- **PowerPoint Lecture Slides** feature presentations written and designed specifically for this text, including figures and examples from the text. Accessible versions of the PowerPoints are also available.
- **TestGen** enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same questions or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download at pearson.com.
- **Test Bank** features a printable PDF containing all the test exercises available in TestGen. The current version contains 6 forms of tests per chapter in PDF format. Forms A-D are open-ended. Forms E and F are multiple choice.

Acknowledgments

We would like to express our gratitude to the reviewers of this fifth edition (marked with an asterisk) and previous editions, who provided such invaluable insights and comments. Their contributions helped shape the development of the text and carry out the vision stated in the Preface.

Alison Ahlgren, University of Illinois at Urbana-Champaign
Mohammad Aslam, Georgia Perimeter College
Mario Barrientos, Angelo State University
Ratan Barua, Miami-Dade College
Sam Bazzi, Henry Ford Community College
Elizabeth Bowman, The University of Alabama at Huntsville
Diane Burleson, Central Piedmont Community College
*Christina C, Northern Arizona University
Maureen T. Carroll, University of Scranton
Melissa Cass, State University of New York–New Paltz
Jason Cates, Lone Star College–Montgomery
Charles Conrad, Volunteer State Community College
Irene Coriette, Cameron University
Douglas Culler, Midlands Technical College
*Ryan DeFronzo, New Hampshire Technical Institute
Baiqiao Deng, Columbus State University
Hussain Elaloui-Talibi, Tuskegee University
Kathleen M. Fick, Methodist University–Fayetteville
Junko Forbes, El Camino College
Gene Garza, Samford University
Steven Gonzales, San Antonio College
Bobbie Jo Hill, Coastal Bend College
Fran Hopf, University of South Florida
Natalie Hutchinson, Old Dominion University
Nyeita Irish, Coastal Carolina University
Yvette Janecek, Coastal Bend College
Rose M. Jenkins, Midlands Technical College
Mohammad Kazemi, University of North Carolina at Charlotte
David Keller, Kirkwood Community College
Diana Klimek, New Jersey Institute of Technology
Mile Krajcevski, University of South Florida
*Jennifer La Fleur, Harford Community College
Lance Lana, University of Colorado–Denver
Alexander Lavrentiev, University of Wisconsin–Fox Valley
Rebecca Leefers, Michigan State University
*Weston Loucks, Northern Arizona University
Paul Morgan, College of Southern Idaho
Kathy Nickell, College of DuPage
Catherine Pellish, Front Range Community College
Betty Peterson, Mercer County Community College
Edward K. Pigg, Southern Union Community College
*Carrie Quesnell, Weber State University
Marshall Ransom, Georgia Southern University
Janice F. Rech, University of Nebraska–Omaha
Traci Reed, St. Johns River Community College
Linda Reist, Macomb Community College
Jeri Rogers, Seminole Community College–Oviedo Campus
Jason Rose, College of Southern Idaho
Mehdi Sadatmousavi, Pima Community College–West
Christy Schmidt, Northwest Vista College
Nyeita Schult, Coastal Carolina University
Delphy Shaulis, University of Colorado–Boulder
Cindy Shaw, Moraine Valley Community College
Cynthia Sikes, Georgia Southern University

*Mary Alice Smeal, University of Alabama at Huntsville
James Smith, Columbia State Community College
Linda Snellings-Neal, Wright State University
Jacqueline Stone, University of Maryland–College Park
Kay Stroope, Phillips County Community College
Robert J. Strozak, Old Dominion University
Helen Timberlake, Rochester Institute of Technology
Frances Tishkevich, Massachusetts Maritime Academy
Jo Tucker, Tarrant County College–Southeast
Violeta Vasilevska, Utah Valley University
Roger Werbylo, Pima Community College–West
Denise Williams, Coastal Carolina University
Tom Worthing, Hutchinson Community College
Vivian Zabrocki, Montana State University–Billings
Marti Zimmerman, University of Louisville

We would like to thank the following faculty members who provided input on the MyLab Math course:

Don Busenbark
Mariana Durgheu
Pamela Fails
Yvette Janecek
Minsu Kim
Paul Kimble
C.J. Kish
David Kroon
Robert Parr
Brendan Santangelo
Curtis White

Our sincerest thanks go to the legion of dedicated individuals who worked tirelessly to make this book possible. We would like to express our gratitude to our typist, Beverly DeVine-Hoffmeyer, for her amazing patience and skill. We must also thank Dr. Praveen Rohatgi, Dr. Nalini Rohatgi, and Dr. Bhupinder Bedi for the consulting they provided on all material relating to medicine. We particularly want to thank Professors Mile Krajcevski, Viktor Maymeskul, and Scott Rimbey for many helpful discussions and suggestions, particularly for improving the exercise sets. Further gratitude is due to Irena Andreevska, Gokarna Aryal, Ferene Tookos, and Christine Fitch for their assistance on the answers to the exercises in the text. In addition, we would like to thank De Cook, Doug Ewert, John Samons, and Beverly Fusfield, for their meticulous accuracy in checking the text. Thanks are due as well to Sharon Cahill and Straive for their excellent production work. Finally, our thanks are extended to the professional and remarkable staff at Pearson Education. In particular, we would like to thank Brian Fisher, Jeff Weidenaar, and Jon Krebs for Content Management; Jessica Darczuk, Product Manager; Tamela Ambush, Content Producer; Sandra Rodriguez, Managing Producer; Stacey Sveum, Product Marketing Manager; Eric Gregg, Manager of Content Development for MathXL; Nick Sweeney, Media Producer; and Joe Vetere, Senior Author Support Technology Specialist.

We invite all who use this book to send suggestions for improvements to Marcus McWaters at mmm@usf.edu.

Pearson's Commitment to Diversity, Equity, and Inclusion

Pearson is dedicated to creating bias-free content that reflects the diversity, depth, and breadth of all learners' lived experiences.

We embrace the many dimensions of diversity, including but not limited to race, ethnicity, gender, sex, sexual orientation, socioeconomic status, ability, age, and religious or political beliefs.

Education is a powerful force for equity and change in our world. It has the potential to deliver opportunities that improve lives and enable economic mobility. As we work with authors to create content for every product and service, we acknowledge our responsibility to demonstrate inclusivity and incorporate diverse scholarship so that everyone can achieve their potential through learning. As the world's leading learning company, we have a duty to help drive change and live up to our purpose to help more people create a better life for themselves and to create a better world.

Our ambition is to purposefully contribute to a world where:

- Everyone has an equitable and lifelong opportunity to succeed through learning.
- Our educational content accurately reflects the histories and lived experiences of the learners we serve.
- Our educational products and services are inclusive and represent the rich diversity of learners.
- Our educational content prompts deeper discussions with students and motivates them to expand their own learning (and worldview).

Accessibility

We are also committed to providing products that are fully accessible to all learners. As per Pearson's guidelines for accessible educational Web media, we test and retest the capabilities of our products against the highest standards for every release, following the WCAG guidelines in developing new products for copyright year 2022 and beyond.

You can learn more about Pearson's commitment to accessibility at **https://www.pearson.com/us/accessibility.html**.

Pearson

Contact Us

While we work hard to present unbiased, fully accessible content, we want to hear from you about any concerns or needs with this Pearson product so that we can investigate and address them.

Please contact us with concerns about any potential bias at **https://www.pearson.com/report-bias.html**.

For accessibility-related issues, such as using assistive technology with Pearson products, alternative text requests, or accessibility documentation, email the Pearson Disability Support team at **disability.support@pearson.com.**

DEDICATION

To Our Spouses,

Lata, Debra, Leslie, Kevin, and Jon

This page intentionally left blank

P

Basic Concepts of Algebra

TOPICS

P.1 The Real Numbers and Their Properties

P.2 Integer Exponents and Scientific Notation

P.3 Polynomials

P.4 Factoring Polynomials

P.5 Rational Expressions

P.6 Rational Exponents and Radicals

Many fascinating patterns in human and natural processes can be described in the language of algebra. We investigate events ranging from chirping crickets to the behavior of falling objects.

P.1 The Real Numbers and Their Properties

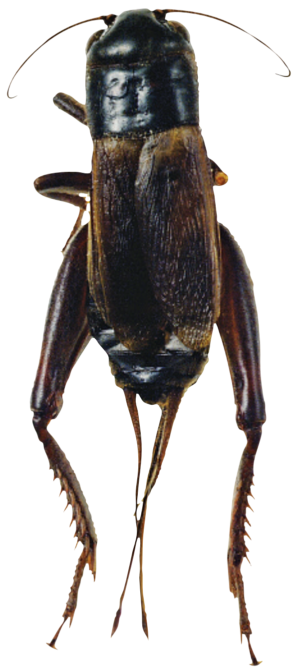
Videos for this Section

Objectives

1 ▸ Classify sets of real numbers.
2 ▸ Use exponents.
3 ▸ Use the ordering of the real numbers.
4 ▸ Specify sets of numbers in roster or set-builder notation.
5 ▸ Use interval notation.
6 ▸ Relate absolute value and distance on the real number line.
7 ▸ Use the order of operations in arithmetic expressions.
8 ▸ Identify and use properties of real numbers.
9 ▸ Evaluate algebraic expressions.

Cricket Chirps and Temperature

Crickets are sensitive to changes in air temperature; their chirps speed up as the temperature gets warmer and slow down as it gets cooler. It is possible to use the chirps of the male snowy tree cricket (*Oecanthus fultoni*), common throughout the United States, to gauge temperature. (The insect is found in every U.S. state except Hawaii, Alaska, Montana, and Florida.) By counting the chirps of this cricket, which lives in bushes a few feet from the ground, you can gauge temperature. Snowy tree crickets are more accurate than most cricket species; their chirps are slow enough to count, and they synchronize their singing. To convert cricket chirps to degrees Fahrenheit, count the number of chirps in 14 seconds and then add 40 to get the temperature. To convert cricket chirps to degrees Celsius, count the number of chirps in 25 seconds, divide by 3, and then add 4 to get the temperature. In Example 12, we evaluate algebraic expressions to learn the temperature from the number of cricket chirps.



Objective 1 ▸

Classifying Numbers

In algebra, we use letters such as a, b, x, y, and so on, to represent numbers. A letter that is used to represent one or more numbers is called a **variable**. A **constant** is a specific number such as 3 or $\frac{1}{2}$ or a letter that represents a fixed (but not necessarily specified) number. Physicists use the letter c as a constant to represent the speed of light ($c \approx 300{,}000{,}000$ meters per second).

SIDE NOTE

Here is one difficulty with attempting to divide by 0: If, for example, $\frac{5}{0} = a$, then $5 = a \cdot 0$. However, $a \cdot 0 = 0$ for all numbers a. So we would have $5 = 0$; this contradiction demonstrates that there is no appropriate choice for $\frac{5}{0}$.

We use two variables, a and b, to denote the results of the operations of addition $(a + b)$, subtraction $(a - b)$, multiplication $(a \times b$ or $a \cdot b)$, and division $\left(a \div b \text{ or } \frac{a}{b}\right)$. These operations are called **binary operations** because each is performed on two numbers.

We frequently omit the multiplication sign when writing a product involving two variables (or a constant and a variable) so that $a \cdot b$ and ab indicate the same product. Both a and b are called **factors** in the product $a \cdot b$. This is a good time to recall that we never divide by zero. For $\frac{a}{b}$ to represent a real number, b cannot be zero.

Equality of Numbers

The **equal sign**, $=$, is used much like we use the word *is* in English. The equal sign means that the number or expression on the left side is equal or equivalent to the number or expression on the right side. We write $a \neq b$ to indicate that a is not equal to b.

Classifying Sets of Numbers

The idea of a set is familiar to us. We regularly refer to "a set of baseball cards," a "set of CDs," or "a set of dishes." In mathematics, as in everyday life, a **set** is a collection of objects. The objects in the set are called the **elements**, or **members**, of the set. Capital letters are usually used to name a set. In the study of algebra, we are interested primarily in sets of numbers.

In listing the elements of a set, it is customary to enclose the listed elements in braces, { }, and separate them with commas.

We distinguish among various sets of numbers.

The numbers we use to count with constitute the set N of **natural numbers**: $\text{N} = \{1, 2, 3, 4, \ldots\}$.

The three dots . . . (called ellipsis) may be read as "and so on" and indicate that the pattern continues indefinitely.

The set W of **whole numbers** is formed by including the number 0 with the natural numbers to obtain the set: $\text{W} = \{0, 1, 2, 3, 4, \ldots\}$.

The set Z of **integers** consists of the set N of natural numbers together with their opposites and 0: $\text{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$.

Rational Numbers

The **rational numbers** consist of all numbers that *can* be expressed as the quotient or ratio, $\frac{a}{b}$, of two integers, where $b \neq 0$. The letter Q is often used to represent the set of rational numbers.

Examples of rational numbers are $\frac{1}{2}, \frac{5}{3}, \frac{-4}{17}$, and $0.07 = \frac{7}{100}$. Any integer a can be expressed as the quotient of two integers by writing $a = \frac{a}{1}$. Consequently, every integer is also a rational number. In particular, 0 is a rational number because $0 = \frac{0}{1}$.

The rational number $\frac{a}{b}$ can be written as a decimal by using long division. When any integer a is divided by an integer b, $b \neq 0$, the result is always a **terminating decimal** such as $\frac{1}{2} = 0.5$ or a **nonterminating repeating decimal** such as $\frac{2}{3} = 0.666\ldots$.

We sometimes place a bar over the repeating digits in a nonterminating repeating decimal. Thus, $\frac{2}{3} = 0.666\ldots = 0.\overline{6}$ and $\frac{141}{110} = 1.2818181\ \ldots = 1.2\overline{81}$.

EXAMPLE 1 Converting Decimal Rationals to a Quotient

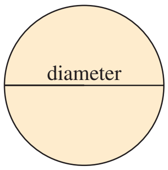
Write the rational number $7.\overline{45}$ as the ratio of two integers in lowest terms.

Solution

Let $x = 7.454545 \ldots$. Then

$$
\begin{array}{rrcll}
 & 100x & = & 745.4545\ldots & \text{Multiply both sides by 100.} \\
\text{subtract} & x & = & 7.4545\ldots & \\ \hline
 & 99x & = & 738 & 100x - x = 99x \\
 & x & = & \dfrac{738}{99} & \text{Divide both sides by 99.} \\
 & & = & \dfrac{82 \times \cancel{9}}{11 \times \cancel{9}} & \text{Common factor} \\
 & x & = & \dfrac{82}{11}. & \text{Reduce to lowest terms.}
\end{array}
$$

Practice Problem 1. Repeat Example 1 for $2.132132132 \ldots$. ■



$\pi = \dfrac{\text{circumference}}{\text{diameter}}$

Figure P.1 ▸ Definition of π

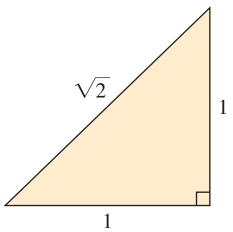


Figure P.2

RECALL

An integer is a *perfect square* if it is a product $a \cdot a$, where a is an integer. For example, $9 = 3 \cdot 3$ is a perfect square.

Irrational Numbers

An **irrational number** is a real number that cannot be written as a ratio of two integers. This means that its decimal representation must be nonrepeating and nonterminating. We can construct such a decimal using only the digits 0 and 1, such as $0.01001000100001\ldots$. Because each group of zeros contains one more zero than the previous group, no group of digits repeats. Other numbers such as π ("pi"; see Figure P.1) and $\sqrt{2}$ (the square root of 2; see Figure P.2) can also be expressed as decimals that neither terminate nor repeat; so they are irrational numbers as well. We can obtain an approximation of an irrational number by using an initial portion of its decimal representation. For example, we can write $\pi \approx 3.14159$ or $\sqrt{2} \approx 1.41421$, where the symbol $\approx$ is read "is approximately equal to."

No familiar process, such as long division, is available for obtaining the decimal representation of an irrational number. However, your calculator can provide a useful approximation for irrational numbers such as $\sqrt{2}$. (Try it!) Because a calculator displays a fixed number of decimal places, it gives a **rational approximation** of an irrational number.

It is usually not easy to determine whether a specific number is irrational. One helpful fact in this regard is that *the square root of any natural number that is not a perfect square is irrational.* So $\sqrt{6}$ is irrational but $\sqrt{16} = \sqrt{4^2} = 4$ is rational.

Because rational numbers have decimal representations that either terminate or repeat, whereas irrational numbers do not have such representations, *no number is both rational and irrational.*

The rational numbers together with the irrational numbers form the set R of **real numbers**.

The diagram in Figure P.3 shows how various sets of numbers are related. For example, every natural number is also a whole number, an integer, a rational number, and a real number.

Real Numbers

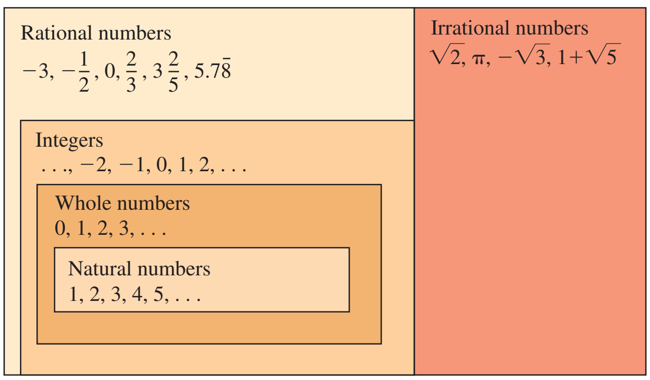


Figure P.3 ▸ Relationships among sets of real numbers

EXAMPLE 2 Identifying Sets of Numbers

Let $A = \left\{-17, -5, -\frac{6}{3}, -\frac{2}{3}, 0, \frac{5}{12}, \frac{1}{2}, \sqrt{2}, \pi, \sqrt{35}, 7, 18\right\}$.

Identify all the elements of the set A that are

a. Natural numbers **b.** Whole numbers **c.** Integers
d. Rational numbers **e.** Irrational numbers **f.** Real numbers

Solution

a. Natural numbers: 7 and 18

b. Whole numbers: 0, 7, and 18

c. Integers: $-17, -5, -\frac{6}{3}$ (or -2), 0, 7, and 18

d. Rational numbers: $-17, -5, -\frac{6}{3}, -\frac{2}{3}, 0, \frac{5}{12}, \frac{1}{2}$, 7, and 18

e. Irrational numbers: $\sqrt{2}$, π, and $\sqrt{35}$

f. Real numbers: All numbers in the set A are real numbers.

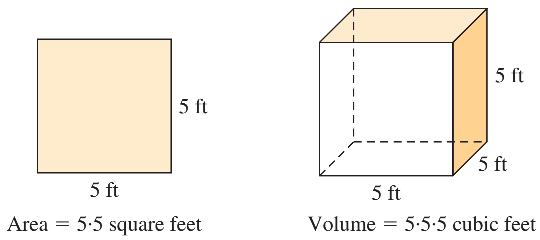
Practice Problem 2. Repeat Example 2 for the following set:

$B = \left\{-6, -\frac{21}{7}, -\frac{1}{2}, 0, \frac{4}{3}, \sqrt{3}, 2, \sqrt{17}, 7\right\}$. ■

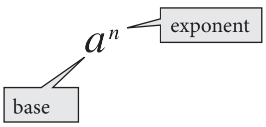
Objective 2 ▸

Integer Exponents

The area of a square with sides of five feet each is $5 \cdot 5 = 25$ square feet.
The volume of a cube that has sides of 5 feet each is $5 \cdot 5 \cdot 5 = 125$ cubic feet.



A shorter notation for $5 \cdot 5$ is 5^2 and for $5 \cdot 5 \cdot 5$ is 5^3. The number 5 is called the *base* for both 5^2 and 5^3. The number 2 is called the *exponent* in the expression 5^2 and indicates that the base 5 appears as a factor twice.



Positive Integer Exponent

If a is a real number and n is a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}.$$

The number a^n is called the nth power of a and is read "a to the nth power," or "a to the n." The number a is called the **base**; n, the **exponent**. We adopt the convention that $a^1 = a$.

EXAMPLE 3 Evaluating Expressions That Use Exponents

Evaluate each expression.

a. 5^3 **b.** $(-3)^2$ **c.** -3^2 **d.** $(-2)^3$

Solution

a. $5^3 = 5 \cdot 5 \cdot 5 = 125$ 5 is the base, and 3 is the exponent.

b. $(-3)^2 = (-3)(-3) = 9$ $(-3)^2$ is the opposite of 3, squared.

c. $-3^2 = -3 \cdot 3 = -9$ Multiplication of $3 \cdot 3$ occurs first. (-3^2 is the opposite of 3^2.)

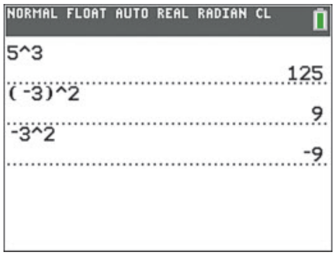
d. $(-2)^3 = (-2)(-2)(-2) = -8$

Practice Problem 3. Evaluate the following.

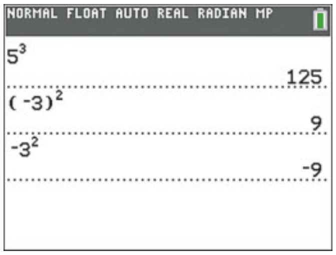
a. 2^3 **b.** $(3a)^2$ **c.** $\left(\frac{1}{2}\right)^4$ ■

TECHNOLOGY CONNECTION

Type 5^3 for 5^3 on a graphing calculator. Any expression on a calculator enclosed in parentheses and followed by ^n is raised to the nth power. A common error is to forget parentheses when computing an expression such as $(-3)^2$, with the result being -3^2. Newer graphing calculators, such as the TI-84 series, provide display options called CLASSIC and MATHPRINT. The MATHPRINT option displays 5^3 as 5^3 but still requires you to type 5, then ^, then 3.



CLASSIC Mode



MATHPRINT Mode

In this text we will display screens in MATHPRINT mode.

In Example 3, pay careful attention to the fact (from parts **b** and **c**) that $(-3)^2 \neq -3^2$. In $(-3)^2$, the parentheses indicate that the exponent 2 applies to the base -3, whereas in -3^2, the absence of parentheses indicates that the exponent applies only to the base 3. When n is even, $(-a)^n \neq -a^n$ for $a \neq 0$.

The Real Number Line

We associate the real numbers with points on a geometric line (imagined to be extended indefinitely in both directions) in such a way that each real number corresponds to exactly one point and each point corresponds to exactly one real number. The point is called the **graph** of the corresponding real number, and the real number is called the **coordinate** of the point. By agreement, ***positive numbers*** lie to the right of the point corresponding to 0 and ***negative numbers*** lie to the left of 0. See Figure P.4.

Notice that $\frac{1}{2}$ and $-\frac{1}{2}$, 2 and -2, and π and $-\pi$ correspond to pairs of points exactly the same distance from 0 but on opposite sides of 0.

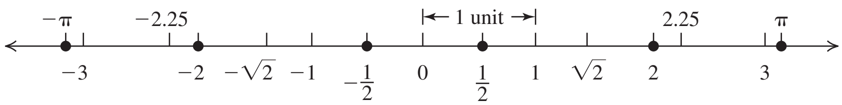


Figure P.4 ▸ The real number line

When coordinates have been assigned to points on a line in the manner just described, the line is called a **real number line**, a **coordinate line**, a **real line**, or simply a **number line**. The point corresponding to 0 is called the **origin**.

Objective 3 ▸

Inequalities

The real numbers are **ordered** by their size. We say that ***a* is less than *b*** and write $a < b$, provided that $b = a + c$ for some ***positive*** number c. We also write $b > a$, meaning the same thing as $a < b$, and say that ***b* is greater than *a***. On the real line, the numbers increase from left to right. Consequently, ***a is to the left of b on the number line when a < b***. Similarly, a is to the right of b on the number line when $a > b$. We sometimes want to indicate that at least one of two conditions is correct: Either $a < b$ or $a = b$. In this case, we write $a \leq b$ or $b \geq a$. The four symbols $<, >, \leq,$ and $\geq$ are called **inequality symbols**.

EXAMPLE 4 Identifying Inequalities

Decide whether each of the following is true or false from their position on a number line.

a. $5 > 0$ **b.** $-2 < -3$ **c.** $2 \leq 3$ **d.** $4 \leq 4$

SIDE NOTE

Notice that the inequality sign always points to the smaller number.

$2 < 7$, 2 is smaller.

$5 > 1$, 1 is smaller.

Solution

a. $5 > 0$ is true because 5 is to the right of 0 on the number line. See Figure P.5.

b. $-2 < -3$ is false because -2 is to the right of -3 on the number line.

c. $2 \leq 3$ is true because 2 is to the left of 3 on the number line. (Recall that $2 \leq 3$ is true if either $2 < 3$ or $2 = 3$.)

d. $4 \leq 4$ is true because $4 \leq 4$ is true if either $4 < 4$ or $4 = 4$.

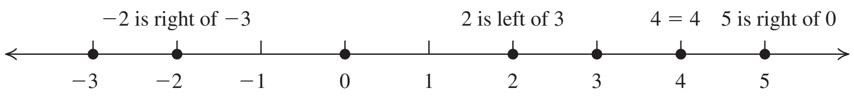


Figure P.5

Practice Problem 4. Decide whether each of the following is true or false.

a. $-2 < 0$ **b.** $5 \leq 7$ **c.** $-4 > -1$ ■

The following properties of inequalities for real numbers are used throughout this text. Here a, b, and c represent real numbers.

Trichotomy Property: Exactly one of the following is true:

$$a < b,\ a = b, \text{ or } a > b.$$

Transitive Property: If $a < b$ and $b < c$, then $a < c$.

The trichotomy property says that if two real numbers are not equal, then one is larger than the other. The transitive property says that "less than" works like "smaller than" or "lighter than." Frequently, we read $a > 0$ as "***a* is positive**" instead of "a is greater than 0." We can also read $a < 0$ as "***a* is negative**." If $a \geq 0$, then either $a > 0$ or $a = 0$, and we may say that "***a* is nonnegative**."

Objective 4 ▸

Sets

To specify a set, we do one of the following:

1. List the elements of the set (**roster method**).
2. Describe the elements of the set (often using **set-builder notation**).

Variables are helpful in describing sets when we use set-builder notation. The notation $\{x|x$ is a natural number less than $6\}$ is in set-builder notation and describes the set $\{1, 2, 3, 4, 5\}$ using the roster method.

We read $\{x|x$ is a natural number less than $6\}$ as "the set of all x such that x is a natural number less than six." Generally, $\{x|x$ has property $P\}$ designates the set of all x such that (the vertical bar is read "such that") x has property P.

It may happen that a description fails to describe any number. For example, consider $\{x|x < 2$ and $x > 7\}$. Of course, no number can be simultaneously less than 2 and greater than 7, so this set has no members. We refer to a set with no elements as the **empty set**, or **null set**, and use the special symbol $\varnothing$, or sometimes $\{\ \}$, to denote it.

Definition of Union and Intersection

The **union** of two sets A and B, denoted $A \cup B$, is the set consisting of all elements that are in A *or* B (or both). See Figure P.6a. The **intersection** of A and B, denoted $A \cap B$, is the set consisting of all elements that are in both A *and* B. In other words, $A \cap B$ consists of the elements common to A and B. See Figure P.6b.

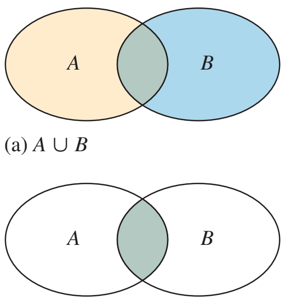


Figure P.6 ▸ Picturing union and intersection

EXAMPLE 5 Forming Set Unions and Intersections

Find $A \cap B$, $A \cup B$, and $A \cap C$, if $A = \{-2, -1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2, 4\}$, and $C = \{-3, 3\}$.

Solution

$A \cap B = \{-2, 0, 2\}$, the set of elements common to both A and B.

$A \cup B = \{-4, -2, -1, 0, 1, 2, 4\}$, the set of elements that are in A or B (or both).

$A \cap C = \varnothing$

Practice Problem 5. Find $A \cap B$ and $A \cup B$, if $A = \{-3, -1, 0, 1, 3\}$ and $B = \{-4, -2, 0, 2, 4\}$. ■

Objective 5 ▸

Intervals

We now turn our attention to graphing certain sets of numbers. That is, we graph each number in a given set. We are particularly interested in sets of real numbers, called **intervals**, whose graphs correspond to special sections of the number line.

If $a < b$, then the set of real numbers between a and b, but not including either a or b, is called the **open interval** from a to b and is denoted by (a, b). See Figure P.7. Using set-builder notation, we can write

$(a, b) = \{x \mid a < x < b\}$

Figure P.7 ▸ An open interval

$$(a, b) = \{x | a < x < b\}.$$

We indicate graphically that the endpoints a and b are excluded from the open interval by drawing a left parenthesis at a and a right parenthesis at b. These parentheses enclose the numbers between a and b.

The **closed interval** from a to b is the set

$[a, b] = \{x \mid a \le x \le b\}$

Figure P.8 ▸ A closed interval

$$[a, b] = \{x | a \le x \le b\}.$$

The closed interval includes both endpoints a and b. We replace the parentheses with square brackets in the interval notation and on the graph. See Figure P.8. Sometimes we want to include only one endpoint of an interval and exclude the other. Table P.1 below shows how this is done.

An alternative notation for indicating whether endpoints are included uses open circles to show exclusion and closed circles to show inclusion. See Figure P.9.

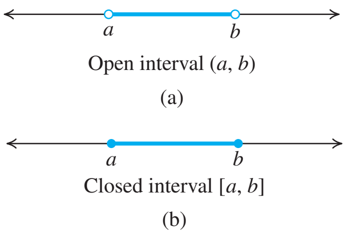


Figure P.9 ▸ Endpoint inclusion and exclusion

If an interval extends indefinitely in one or both directions, it is called an **unbounded interval**. For example, the set of all numbers to the right of 2,

$$\{x | x > 2\},$$

is an unbounded interval denoted by $(2, \infty)$. See Figure P.10.

Interval $(2, \infty)$

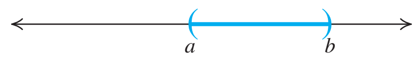
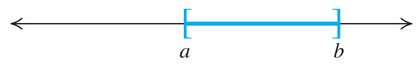
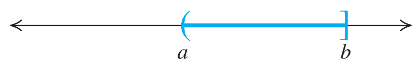
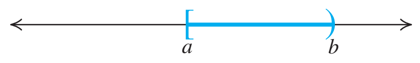
Figure P.10 ▸ An unbounded interval

The symbol ∞ ("infinity") is not a number, but is used to indicate all numbers to the right of 2.

The symbol $-\infty$ is another symbol that does not represent a number. The notation $(-\infty, a)$ is used to indicate the set of all real numbers that are less than a. The notation $(-\infty, \infty)$ represents the set of all real numbers.

Table P.1 lists various types of intervals that we use in this text. In the table, when two points a and b are given, we assume that $a < b$. This is because if $a > b$, then (a, b) is the empty set.

TABLE P.1

Interval Notation	Set Notation	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \ge a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\{x \mid x \text{ is a real number}\}$	

SIDE NOTE

The symbols ∞ and $-\infty$ are always used with parentheses, not square brackets. Also note that $<$ and $>$ are used with parentheses and that $\le$ and $\ge$ are used with square brackets.

EXAMPLE 6 Union and Intersection of Intervals

Consider the two intervals $I_1 = (-3, 4)$ and $I_2 = [2, 6]$.

Find: **a.** $I_1 \cup I_2$. **b.** $I_1 \cap I_2$.

Solution

Use the to (exclude −3.

Use the] to include 6.

a. From Figure P.11, we see that $I_1 \cup I_2 = (-3, 6]$. We note that every number in the interval $(-3, 6]$ is in either I_1 or I_2 or in both I_1 and I_2.

b. We see in Figure P.11 that $I_1 \cap I_2 = [2, 4)$. Every number in the interval $[2, 4)$ is in both I_1 and I_2. Notice that while 4 is in I_2, 4 is not in I_1.

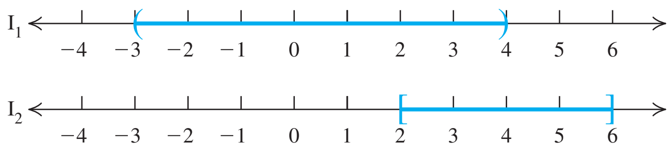


Figure P.11

Practice Problem 6. Let $I_1 = (-\infty, 5)$ and $I_2 = [-2, \infty)$. Find the following.

a. $I_1 \cup I_2$ **b.** $I_1 \cap I_2$ ■

Objective 6 ▸

Absolute Value

The ***absolute value*** of a number a, denoted by $|a|$, is the distance between the origin and the point on the number line with coordinate a. The point with coordinate -3 is 3 units from the origin, so we write $|-3| = 3$ and say that the absolute value of -3 is 3. See Figure P.12.

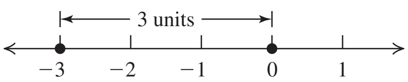


Figure P.12 ▸ Absolute value

Absolute Value

For any real number a, the **absolute value** of a, denoted $|a|$, is defined by

$$|a| = a \quad \text{if } a \geq 0 \qquad \text{and} \qquad |a| = -a \quad \text{if } a < 0.$$

SIDE NOTE

Finding the absolute value requires knowing whether the number or expression inside the absolute value bars is positive, zero, or negative. If it is positive or zero, you can simply remove the absolute value bars. If it is negative, you remove the bars and change the sign of the number or expression inside the absolute value bars.

EXAMPLE 7 Determining Absolute Value

Find the value of each of the following expressions.

a. $|4|$ **b.** $|-4|$ **c.** $|0|$ **d.** $|(-3) + 1|$

Solution

a. $|4| = 4$ Because the number inside the absolute value bars is 4 and $4 \geq 0$, just remove the absolute value bars.

b. $|-4| = -(-4) = 4$ Because the number inside the absolute value bars is -4 and $-4 < 0$, remove the bars and change the sign.

c. $|0| = 0$ Because the number inside the absolute value bars is 0 and $0 \geq 0$, just remove the absolute value bars.

d. $|(-3) + 1| = |-2|$
$= -(-2) = 2$ Because the expression inside the absolute value bars is $(-3) + 1 = -2$ and $-2 < 0$, remove the bars and change the sign.

Practice Problem 7. Find the value of each of the following.

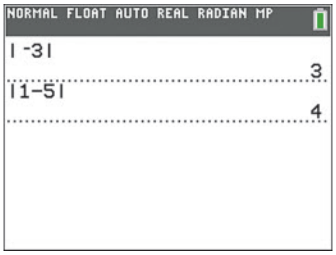
a. $|-10|$ **b.** $|3 - 4|$ **c.** $|2(-3) + 7|$ ■

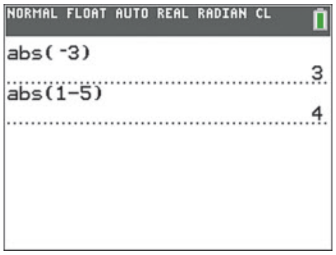
WARNING

The absolute value of a number represents a distance. Because distance can never be negative, the absolute value of a number a is never negative. So, $|a| \geq 0$. However, if a is not 0, $-|a|$ is always negative. Thus, $-|5.3| = -5.3$, $-|-4| = -4$, and $-|1.\overline{18}| = -1.\overline{18}$.

TECHNOLOGY CONNECTION

The absolute value function on your graphing calculator will find the value of the expression entered and then compute its absolute value. The first screen is in CLASSIC mode.





Distance Between Two Points on a Real Number Line

The absolute value is used to define the distance between two points on a number line.

Distance Formula on a Number Line

If a and b are the coordinates of two points on a number line, then the distance between a and b, denoted by $d(a, b)$, is $|a - b|$. In symbols, $d(a, b) = |a - b|$.

EXAMPLE 8 Finding the Distance Between Two Points

Find the distance between -3 and 4 on the number line.

Solution

Figure P.13 shows that the distance between -3 and 4 is 7 units. The distance formula gives the same answer.

$$d(-3, 4) = |-3 - 4| = |-7| = -(-7) = 7.$$

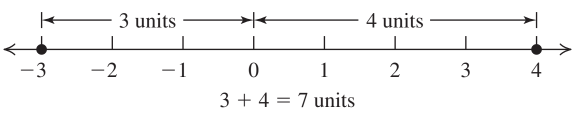


Figure P.13 ▸ Distance on the number line

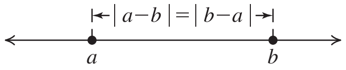


Figure P.14

Notice that reversing the order of -3 and 4 in this computation gives the same answer. That is, the distance between 4 and -3 is $|4-(-3)| = |4+3| = |7| = 7$. It is always true that $|a-b| = |b-a|$. See Figure P.14.

Practice Problem 8. Find the distance between -7 and 2 on the number line. ■

We summarize the properties of absolute value next.

Properties of Absolute Value

If a and b are any real numbers, the following properties apply.

Property	Example
1. $\lvert a \rvert \geq 0$	$\lvert -5 \rvert = 5$ and $5 \geq 0$
2. $\lvert a \rvert = \lvert -a \rvert$	$\lvert 3 \rvert = \lvert -3 \rvert$
3. $\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$	$\lvert 3(-5) \rvert = \lvert 3 \rvert \lvert -5 \rvert$
4. $\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a \rvert}{\lvert b \rvert}, b \neq 0$	$\left\lvert \frac{-7}{3} \right\rvert = \frac{\lvert -7 \rvert}{\lvert 3 \rvert}$
5. $\lvert a-b \rvert = \lvert b-a \rvert$	$\lvert 2-7 \rvert = \lvert 7-2 \rvert$
6. $a \leq \lvert a \rvert$	$-2 \leq \lvert -2 \rvert, 3 \leq \lvert 3 \rvert$
7. $\lvert a+b \rvert \leq \lvert a \rvert + \lvert b \rvert$ (the triangle inequality)	$\lvert -2+5 \rvert \leq \lvert -2 \rvert + \lvert 5 \rvert$

Objective 7 ▸

Order of Operations

When we write numbers in a meaningful combination of the basic operations of arithmetic, the result is called an **arithmetic expression**. The real number that results from performing all operations in the expression is called the **value** of the expression. In arithmetic and algebra, parentheses ( ) are **grouping symbols** used to indicate which operations are to be performed first. Other common grouping symbols are square brackets [ ], braces { }, fraction bars — or /, and absolute value bars | |.

Evaluating (finding the value of) arithmetic expressions requires carefully applying the following conventions for the order in which the operations are performed.

SIDE NOTE

You can remember the order of operations by the acronym **PEMDAS** (Please Excuse My Dear Aunt Sally):

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction.

The Order of Operations

Step 1 ▸ Whenever a **fraction bar** is encountered, work separately above and below it.

Step 2 ▸ When **parentheses or other grouping symbols** are present, start inside the innermost pair of grouping symbols and work outward.

Step 3 ▸ Do operations involving **exponents**.

Step 4 ▸ Do **multiplications** and **divisions** in the order in which they occur, **working from left to right**.

Step 5 ▸ Do **additions** and **subtractions** in the order in which they occur, **working from left to right**.

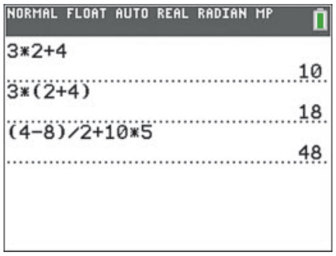
EXAMPLE 9 Evaluating Arithmetic Expressions

Use order of operations to evaluate each of the following.

a. $3 \cdot 2^5 + 4$ **b.** $5 - (3-1)^2$

TECHNOLOGY CONNECTION

The order of operations is built into your graphing calculator.



Solution

a. $3 \cdot 2^5 + 4 = 3 \cdot 32 + 4$ Do operations involving exponents first, $2^5 = 32$.

$= 96 + 4$ Multiplication is done next, $3 \cdot 32 = 96$.

$= 100$ Addition is done next, $96 + 4 = 100$.

b. $5 - (3 - 1)^2 = 5 - (2)^2$ Work inside the parentheses first, $3 - 1 = 2$.

$= 5 - 4$ Next, do operations involving exponents, $2^2 = 4$.

$= 1$ Subtraction is done next, $5 - 4 = 1$.

Practice Problem 9. Evaluate.

a. $(-3) \cdot 5 + 20$ **b.** $5 - 12 \div 6 \cdot 2$ **c.** $\frac{9-1}{4} - 5 \cdot 7$

d. $-3 + (x - 4)^2$ for $x = 6$ ■

Objective 8 ▸

Properties of the Real Numbers

When doing arithmetic, we intuitively use important properties of the real numbers. We know, for example, that if we add or multiply two real numbers, the result is a real number. This fact is known as the **closure** property of real numbers. For each property listed below, *a, b, and c represent real numbers.*

Properties of the Real Numbers

Name	Addition (A)	Multiplication (M)	Examples
Closure	$a + b$ is a real number	$a \cdot b$ is a real number	A: $1 + \sqrt{2}$ is a real number. M: $2 \cdot \pi$ is a real number.
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$	A: $4 + 7 = 7 + 4$ M: $3 \cdot 8 = 8 \cdot 3$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	A: $(2 + 1) + 7 = 2 + (1 + 7)$ M: $(5 \cdot 9) \cdot 13 = 5 \cdot (9 \cdot 13)$
Identity	There is a unique real number 0, called the **additive identity**, such that $a + 0 = a$ and $0 + a = a$.	There is a unique real number 1, called the **multiplicative identity**, such that $a \cdot 1 = a$ and $1 \cdot a = a$.	A: $3 + 0 = 3$ and $0 + 3 = 3$ M: $7 \cdot 1 = 7$ and $1 \cdot 7 = 7$
Inverse	There is a unique real number $-a$, called the **opposite of *a***, such that $a + (-a) = 0$ and $(-a) + a = 0$.	If $a \neq 0$, there is a unique real number $\frac{1}{a}$, called the **reciprocal of *a***, such that $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$.	A: $5 + (-5) = 0$ and $(-5) + 5 = 0$ M: $4 \cdot \frac{1}{4} = 1$ and $\frac{1}{4} \cdot 4 = 1$
Distributive	Multiplication distributes over addition.	$a \cdot (b + c) = a \cdot b + a \cdot c$ $(a + b) \cdot c = a \cdot c + b \cdot c$	$3 \cdot (2 + 5) = 3 \cdot 2 + 3 \cdot 5$ $(2 + 5) \cdot 3 = 2 \cdot 3 + 5 \cdot 3$

Two other useful properties of 0 involve multiplication rather than addition.

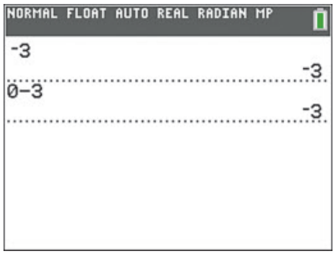
Zero-Product Properties

$$0 \cdot a = 0 \text{ and } a \cdot 0 = 0$$

$$\text{If } a \cdot b = 0, \text{ then } a = 0 \quad \text{or} \quad b = 0.$$

TECHNOLOGY CONNECTION

On your graphing calculator, the ***opposite*** of a is denoted with a shorter horizontal hyphen than that used to denote subtraction. It is also placed somewhat higher than the subtraction symbol.



Subtraction and Division of Real Numbers

You may have noticed that all of the properties discussed to this point apply to the operations of addition and multiplication. What about subtraction and division? We see next that subtraction and division can be defined in terms of addition and multiplication.

Subtraction of the number b from the number a is defined with the use of a and $-b$; **to subtract b from a, add the opposite of b to a.**

Subtraction

$$a - b = a + (-b)$$

If a and b are real numbers and $b \neq 0$, **division** of a by b is defined with the use of a and $\frac{1}{b}$; **to divide a by b, multiply a by the reciprocal of b.**

Division

$$\text{If } b \neq 0, \qquad a \div b = \frac{a}{b} = a \cdot \frac{1}{b}$$

$$\frac{a}{0} \text{ is undefined.}$$

For $b \neq 0$, $\frac{a}{b}$ is called the **quotient**, "the **ratio** of a to b," or the **fraction** with **numerator** a and **denominator** b. Here are some useful properties involving opposites, subtraction, and division.

Throughout, we assume that the denominator of each fraction is nonzero.

Properties of Opposites

$(-1)a = -a$	$-(-a) = a$	$(-a)b = a(-b) = -(ab)$
$(-a)(-b) = ab$	$-(a + b) = -a - b$	$-(a - b) = b - a$
$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$	$\frac{-a}{-b} = \frac{a}{b}$	$a(b - c) = ab - ac$

To combine real numbers by means of the division operation, we use the following properties. We write "$x \pm y$" as shorthand for "$x + y$ or $x - y$."

Properties of Fractions

All denominators are assumed to be nonzero.

$\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$	$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
$\frac{ac}{bc} = \frac{a}{b}$	$\frac{a}{b} = \frac{c}{d}$ means $a \cdot d = b \cdot c$

The property $\frac{ac}{bc} = \frac{a}{b}$, equivalently $\frac{a}{b} = \frac{ac}{bc}$, can be used to produce a common denominator when adding or subtracting fractions.

EXAMPLE 10 **Adding, Subtracting, Multiplying, and Dividing Fractions**

Perform the indicated operations.

a. $\frac{5}{3} + \frac{3}{2}$ **b.** $\frac{5}{6} - \frac{2}{3}$ **c.** $\frac{4}{3} \cdot \frac{15}{8}$ **d.** $\frac{\frac{2}{5}}{\frac{9}{25}}$

SIDE NOTE

Part **a** of Example 10 can also be done as follows:

$$\frac{5}{3} + \frac{3}{2} = \frac{5 \cdot 2 + 3 \cdot 3}{3 \cdot 2}$$

using $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$ directly. Part **b** can be done similarly.

Solution

a. $\frac{5}{3} + \frac{3}{2} = \frac{5 \cdot 2}{3 \cdot 2} + \frac{3 \cdot 3}{2 \cdot 3}$ Using $\frac{a}{b} = \frac{ac}{bc}$

$= \frac{5 \cdot 2 + 3 \cdot 3}{3 \cdot 2} = \frac{19}{6}$ Using $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

b. $\frac{5}{6} - \frac{2}{3} = \frac{5}{6} - \frac{2 \cdot 2}{3 \cdot 2}$ Using $\frac{a}{b} = \frac{ac}{bc}$

$= \frac{5 - 2 \cdot 2}{6} = \frac{1}{6}$ Using $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

c. $\frac{4}{3} \cdot \frac{15}{8} = \frac{4 \cdot 15}{3 \cdot 8}$ Using $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

$= \frac{\cancel{4} \cdot \cancel{3} \cdot 5}{\cancel{3} \cdot \cancel{4} \cdot 2} = \frac{5}{2}$ Using $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$

d. $\frac{\frac{2}{5}}{\frac{9}{25}} = \frac{2}{5} \cdot \frac{25}{9}$ Using $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

$= \frac{2 \cdot 25}{5 \cdot 9}$ Using $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

$= \frac{2 \cdot \cancel{5} \cdot 5}{\cancel{5} \cdot 9} = \frac{10}{9}$ Using $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$

Practice Problem 10. Perform the indicated operations.

a. $\frac{7}{4} + \frac{3}{8}$ **b.** $\frac{8}{3} - \frac{2}{5}$ **c.** $\frac{9}{14} \cdot \frac{7}{3}$ **d.** $\frac{\frac{5}{8}}{\frac{15}{16}}$ ■

Objective 9 ▸

Algebraic Expressions

In algebra, any number, constant, variable, or parenthetical group (or any product of them) is called a **term**. A combination of terms using the ordinary operations of addition, subtraction, multiplication, and division (as well as exponentiation and roots, which are discussed later in this chapter) is called an **algebraic expression**, or simply an **expression**. If we replace each variable in an expression with a specific number and get a real number, the resulting number is called the **value of the algebraic expression**. Of course, this value depends on the numbers we use to replace the variables in the expression. Here are some examples of algebraic expressions:

$$x - 2 \qquad \frac{1}{x} + \sqrt{7} \qquad \frac{10}{y+3} \qquad \sqrt{x} + |y| \div 5$$

WARNING

Incorrect	Correct
~~$5(x+1) = 5x + 1$~~	$5(x+1) = 5x + 5$
~~$\left(\frac{1}{3}x\right)\left(\frac{1}{3}y\right) = \frac{1}{3}xy$~~	$\left(\frac{1}{3}x\right)\left(\frac{1}{3}y\right) = \frac{1}{9}xy$
~~$x - (4y + 3) = x - 4y + 3$~~	$x - (4y + 3) = x - 4y - 3$

EXAMPLE 11 Evaluating an Algebraic Expression

Evaluate the following expressions for the given values of the variables.

a. $[(9 + x) \div 7] \cdot 3 - x$ for $x = 5$

b. $|x| - \dfrac{2}{y}$ for $x = 2$ and $y = -2$

Solution

a $[(9 + x) \div 7] \cdot 3 - x = [(9 + 5) \div 7] \cdot 3 - 5$ Replace x with 5.

$= [14 \div 7] \cdot 3 - 5$ Work inside the parentheses.

$= 2 \cdot 3 - 5$ Work inside the brackets.

$= 6 - 5$ Multiply.

$= 1$ The value of the expression

b. $|x| - \dfrac{2}{y} = |2| - \dfrac{2}{-2}$ Replace x with 2 and y with -2.

$= 2 - \dfrac{2}{-2}$ Eliminate the absolute value, $|2| = 2$.

$= 2 - (-1)$ $\dfrac{2}{-2} = -1$; division occurs before subtraction.

$= 2 + 1$ $-(-1) = 1$

$= 3$ The value of the expression

Practice Problem 11. Evaluate.

a. $(x - 2) \div 3 + x$ for $x = 3$ **b.** $7 - \dfrac{x}{|y|}$ for $x = -1, y = 3$ ■

EXAMPLE 12 Finding the Temperature from Cricket Chirps

Write an algebraic expression for converting cricket chirps to degrees Celsius. Assuming that you count 48 chirps in 25 seconds, what is the Celsius temperature?

Solution

Recall from the introduction to this section that to convert cricket chirps to degrees Celsius, you count the chirps in 25 seconds, divide by 3, and then add 4 to get the temperature.

$$\boxed{\text{Temperature in degrees Celsius}} = \left(\frac{\text{Number of chirps in 25 seconds}}{3}\right) + 4$$

If we let C = temperature in degrees Celsius and N = number of chirps in 25 seconds, we get the expression

$$C = \frac{N}{3} + 4.$$

Suppose we count 48 chirps in 25 seconds. The Celsius temperature is

$$C = \frac{48}{3} + 4$$
$$= 16 + 4 = 20 \text{ degrees.}$$

Practice Problem 12. Use the temperature expression in Example 12 to find the Celsius temperature, assuming that 39 chirps are counted in 25 seconds. ■

P.1 Exercises

Concepts and Vocabulary

1. Whole numbers are formed by adding the number ____________ to the set of natural numbers.

2. The number -3 is an integer, but it is also a(n) ____________ and a(n) ____________.

3. If $a < b$, then a is to the ____________ of b on the number line.

4. If a real number is not a rational number, it is a(n) ____________ number.

5. True or False. If $-x$ is positive, then $-x > 0$.

6. True or False. $\frac{-5}{2} < -2\frac{1}{2}$

7. True or False. The sum of two rational numbers is a rational number.

8. True or False. The sum of two irrational numbers is always an irrational number.

Building Skills

In Exercises 9–16, write each of the following rational numbers as a decimal and state whether the decimal is repeating or terminating.

9. $\frac{1}{3}$

10. $\frac{2}{3}$

11. $-\frac{4}{5}$

12. $-\frac{3}{12}$

13. $\frac{3}{11}$

14. $\frac{11}{33}$

15. $\frac{95}{30}$

16. $\frac{41}{15}$

In Exercises 17–24, convert each decimal to a quotient of two integers in lowest terms.

17. 3.75

18. -2.35

19. $-5.\overline{3}$

20. $9.\overline{6}$

21. $2.\overline{13}$

22. $3.\overline{23}$

23. $4.5\overline{23}$

24. $1.4\overline{235}$

In Exercises 25–32, classify each of the following numbers as rational or irrational.

25. -207

26. -114

27. $\sqrt{81}$

28. $-\sqrt{25}$

29. $\frac{7}{2}$

30. $-\frac{15}{12}$

31. $\sqrt{12}$

32. $\sqrt{3}$

In Exercises 33–38, list all the elements of the set

$$A = \left\{-19, -\frac{12}{3}, 0, \sqrt{3}, 2, \sqrt{10}, \frac{17}{4}, 11\right\}$$

that are

33. natural numbers

34. whole numbers

35. integers

36. rational numbers

37. irrational numbers

38. real numbers

In Exercises 39–50, each expression contains a base with an exponent greater than 1. Name the exponent and the base and evaluate the expression.

39. 10^3

40. 5^4

41. $\left(\frac{2}{3}\right)^3$

42. $\left(\frac{5}{2}\right)^4$

43. $(-2)^3$

44. $\left(-\frac{1}{2}\right)^4$

45. $2 \cdot 3^4$

46. $5 \cdot \left(\frac{1}{3}\right)^3$

47. $-2 \cdot 3^4$

48. $-3 \cdot (-2)^4$

49. $-3 \cdot (-2)^5$

50. $-5 \cdot (-3)^2$

In Exercises 51–60, use inequality symbols to write the given statements symbolically.

51. 3 is greater than -2.

52. -3 is less than -2.

53. $\frac{1}{2}$ is greater than or equal to $\frac{1}{2}$.

54. x is less than $x + 1$.

55. 5 is less than or equal to $2x$.

56. $x - 1$ is greater than 2.

57. $-x$ is positive.

58. x is negative.

59. $2x + 7$ is less than or equal to 14.

60. $2x + 3$ is not greater than 5.

In Exercises 61–64, fill in the blank with one of the symbols $=$, $<$, or $>$ to produce a true statement.

61. 4 _______ $\frac{24}{6}$

62. -5 _______ -2

63. -4 _______ 0

64. $-\frac{9}{2}$ _______ $-4\frac{1}{2}$

In Exercises 65–72, find each set if $A = \{-4, -2, 0, 2, 4\}$, $B = \{-3, 0, 1, 2, 3, 4\}$, and $C = \{-4, -3, -2, -1, 0, 2\}$.

65. $A \cup B$

66. $A \cap B$

67. $A \cap C$

68. $B \cup C$

69. $(B \cap C) \cup A$

70. $(A \cup C) \cap B$

71. $(A \cup B) \cap C$

72. $(A \cup B) \cup C$

In Exercises 73–80, find the union and the intersection of the given pairs of intervals.

73. $I_1 = (-2, 3]$; $I_2 = [1, 5)$

74. $I_1 = [1, 7]$; $I_2 = (3, 5)$

75. $I_1 = (-6, 2)$; $I_2 = [2, 10)$

76. $I_1 = (-\infty, -3]$; $I_2 = (-3, \infty)$

77. $I_1 = (-\infty, 5)$; $I_2 = [2, \infty)$

78. $I_1 = (-2, \infty)$; $I_2 = (0, \infty)$

79. $I_1 = (-\infty, 3) \cup [5, \infty)$; $I_2 = [-1, 7]$

80. $I_1 = (-\infty, 2) \cup (6, \infty)$; $I_2 = [-3, 0]$

In Exercises 81–92, rewrite each expression without absolute value bars.

81. $|-4|$ **82.** $-|-17|$

83. $\left|\frac{5}{-7}\right|$ **84.** $\left|\frac{-3}{5}\right|$

85. $|5 - \sqrt{2}|$ **86.** $|\sqrt{2} - 5|$

87. $|\sqrt{3} - 2|$ **88.** $|3 - \pi|$

89. $\frac{8}{|-8|}$ **90.** $\frac{-8}{|8|}$

91. $|5 - |-7||$ **92.** $||4| - |7||$

In Exercises 93–100, use the absolute value to express the distance between the points with coordinates *a* and *b* on the number line. Then determine this distance by evaluating the absolute value expression.

93. $a = 3$ and $b = 8$ **94.** $a = 2$ and $b = 14$

95. $a = -6$ and $b = 9$ **96.** $a = -12$ and $b = 3$

97. $a = -20$ and $b = -6$ **98.** $a = -14$ and $b = -1$

99. $a = \frac{22}{7}$ and $b = -\frac{4}{7}$ **100.** $a = \frac{16}{5}$ and $b = -\frac{3}{5}$

In Exercises 101–108, graph each of the given intervals on a number line and write the inequality notation for each.

101. $(-3, 1]$ **102.** $[-6, -2)$

103. $[-3, \infty)$ **104.** $[0, \infty)$

105. $(-\infty, 5]$ **106.** $(-\infty, -1]$

107. $\left(-\frac{3}{4}, \frac{9}{4}\right)$ **108.** $\left(-3, -\frac{1}{2}\right)$

In Exercises 109–112, use the distributive property to write each expression without parentheses.

109. $4(x + 1)$ **110.** $(-3)(2 - x)$

111. $5(x - y + 1)$ **112.** $2(3x + 5 - y)$

In Exercises 113–116, find the additive inverse and reciprocal of each number.

Number	Additive Inverse	Reciprocal
113. 5		
114. $-\frac{2}{3}$		
115. 0		
116. 1.7		

In Exercises 117–128, name the property of real numbers that justifies the given equality. All variables represent real numbers.

117. $(-7) + 7 = 0$ **118.** $5 + (-5) = 0$

119. $(x + 2) = 1 \cdot (x + 2)$ **120.** $3a = 1 \cdot 3a$

121. $7(xy) = (7x)y$ **122.** $3 \cdot (6x) = (3 \cdot 6)x$

123. $\frac{3}{2}\left(\frac{2}{3}\right) = 1$ **124.** $2\left(\frac{1}{2}\right) = 1$

125. $(3 + x) + 0 = 3 + x$

126. $x(2 + y) + 0 = x(2 + y)$

127. $(x + 5) + 2y = x + (5 + 2y)$

128. $(3 + x) + 5 = x + (3 + 5)$

In Exercises 129–150, perform the indicated operations.

129. $\frac{3}{5} + \frac{4}{3}$ **130.** $\frac{7}{10} + \frac{3}{4}$

131. $\frac{6}{5} + \frac{5}{7}$ **132.** $\frac{9}{2} + \frac{5}{12}$

133. $\frac{5}{6} + \frac{3}{10}$ **134.** $\frac{8}{15} + \frac{2}{9}$

135. $\frac{5}{8} - \frac{9}{10}$ **136.** $\frac{7}{8} - \frac{1}{5}$

137. $\frac{5}{9} - \frac{7}{11}$ **138.** $\frac{5}{8} - \frac{7}{12}$

139. $\frac{2}{5} - \frac{1}{2}$ **140.** $\frac{1}{4} - \frac{1}{6}$

141. $\frac{3}{4} \cdot \frac{8}{27}$ **142.** $\frac{9}{7} \cdot \frac{14}{27}$

143. $\dfrac{\frac{8}{5}}{\frac{16}{15}}$ **144.** $\dfrac{\frac{5}{6}}{\frac{15}{6}}$

145. $\dfrac{\frac{7}{8}}{\frac{21}{16}}$ **146.** $\dfrac{\frac{3}{10}}{\frac{7}{15}}$

147. $5 \cdot \frac{3}{10} - \frac{1}{2}$ **148.** $2 \cdot \frac{7}{2} - \frac{3}{2}$

149. $3 \cdot \frac{2}{15} - \frac{1}{3}$ **150.** $2 \cdot \frac{5}{3} - \frac{3}{2}$

In Exercises 151–160, evaluate each expression for $x = 3$ and $y = -5$.

151. $2(x + y) - 3y$ **152.** $-2(x + y) + 5y$

153. $3|x| - 2|y|$ **154.** $7|x - y|$

155. $\frac{x - 3y}{2} + xy$ **156.** $\frac{y + 3}{x} - xy$

157. $\frac{2(1 - 2x)}{y} - (-x)y$ **158.** $\frac{3(2 - x)}{y} - (1 - xy)$

159. $\dfrac{\frac{14}{x} + \frac{1}{2}}{\frac{-y}{4}}$ **160.** $\dfrac{\frac{4}{-y} + \frac{8}{x}}{\frac{y}{2}}$

In Exercises 161–170, correct the error in each formula.

161. $\frac{x}{y} + \frac{x}{3} = \frac{x}{(y + 3)}$

162. $(x + 2)(x + 3) = x + 2x + 3$

163. $5(x + 3) = 5x + 3$

164. $(25x)(4x) = 100x$

165. $x - (3y + 2) = x - 3y + 2$

166. $2x - (4y - 5) = 2x - 4y - 5$

167. $\frac{x + y}{x} = 1 + y$

168. $\frac{x + y}{x + z} = 1 + \frac{y}{z}$

169. $(x + 1)(y + 1) = xy + 1$

170. $\dfrac{\frac{1}{x}}{\frac{1}{y}} = \dfrac{1}{xy}$

171. Let P and Q be two points on a number line with coordinates a and b, respectively. Show that the point M on the number line with coordinate $\dfrac{a + b}{2}$ is the midpoint of the line segment PQ. [*Hint:* Show that $d(P, M) = d(Q, M)$.]

172. Find 100 rational numbers between $-\dfrac{4}{13}$ and $\dfrac{7}{13}$.
(*Hint:* $-\dfrac{4}{13} = -\dfrac{40}{130}$ and $\dfrac{7}{13} = \dfrac{70}{130}$;
$-40 < -31 < -30 < \ldots < 0 < 1 < \ldots < 69$.)

Applying the Concepts

173. Media players. Let A = the set of people who own MP3 players and B = the set of people who own DVD players.
a. Describe the set $A \cup B$.
b. Describe the set $A \cap B$.

174. Optional car features. The table below indicates whether certain features are offered for each of three types of 2021 cars.

	Android Auto	Manual Transmission	Apple CarPlay
Ford Bronco	yes	yes	yes
Honda Accord	yes	no	yes
Audi A4	no	no	yes

Source: http://motortrend.com

Use the roster method to describe each of the following sets.
a. A = cars which offer Android Auto
b. B = cars which offer manual transmission
c. C = cars which offer Apple CarPlay
d. $A \cap B$
e. $A \cap C$
f. $A \cup B$
g. $A \cup C$

175. Blood pressure. A group of college students had systolic blood pressure readings that ranged from a low value of 119.5 to a high value of 134.5 inclusive. Let x represent the value of the systolic-blood pressure readings. Use inequalities to describe this range of values and graph the corresponding interval on a number line.

176. Population projections. Population projections suggest that by 2060, the number of people 65 years and older in the United States will be about 95 million. In 2020, the number of people 65 years and older in the United States was 49 million. Let x represent the number (in millions) of people in the United States who are 65 years and older. Use inequalities to describe this population range from 2020 to 2060 and graph the corresponding interval on a number line. (*Source:* U.S. Census Bureau)

177. Heart rate. For exercise to be most beneficial, the optimum heart rate for a 20-year-old person is 120 beats per minute. Use absolute value notation to write an expression that describes the difference between the heart rate achieved by each of the following 20-year-old people and the ideal exercise heart rate. Then evaluate that expression.
a. Latasha: 124 beats per minute
b. Frances: 137 beats per minute
c. Ignacio: 114 beats per minute

178. Streaming music and video. Normal-quality music streaming uses 72MB per hour on average, and standard definition video uses about 700MB per hour on average. Use absolute value notation to write an expression that describes the difference between this average time and the actual time Dewayne used to stream some music and video. Then evaluate that expression. (*Source:* https://www.androidcentral.com)
a. The music used 56MB per hour.
b. The video used 380MB per hour.

179. Use the formula from Example 12 for finding the temperature in degrees Celsius from the number of cricket chirps counted in 25 seconds to find the Celsius temperature if you count 42 chirps in 25 seconds.

180. Use the formula from Example 12 for finding the temperature in degrees Celsius from the number of cricket chirps counted in 25 seconds to find number of chirps that would be counted if the Celsius temperature is 22 degrees.

Negative calories. In Exercises 181 and 182, use the fact that eating 100 grams of broccoli (a negative calorie food) actually results in a net *loss* of 55 calories.

181. If a cheeseburger has 522.5 calories, how many grams of broccoli would a person have to consume to have a net gain of zero calories?

182. Maria ate 600 grams of broccoli and now wants to eat just enough French fries to get a net calorie intake of zero. If a small order of fries has 165 calories, how many orders are required?

Beyond the Basics

In Exercises 183–190, state whether each statement is true or false. Justify your assertion.

183. The opposite of an irrational number is also an irrational number.

184. The sum of a rational number and an irrational number is an irrational number.

185. a. The product of a rational number and an irrational number is an irrational number.
b. If your answer to part(a) is "false," modify the statement to make it a true statement.

186. The product of two irrational numbers is an irrational number.

187. The difference of two irrational numbers is an irrational number.

188. The quotient of two irrational numbers is an irrational number.

189. The product of two rational numbers is a rational number.

190. The quotient of two rational numbers is a rational number.

In Exercises 191 and 192, use the following definition: An integer p is *even* if $p = 2n$ for some integer n; an integer q is *odd* if $q = 2k + 1$, for some integer k.

191. a. Show that if an integer a is odd, then a^2 is also an odd integer.
b. Show that if b^2 is an even integer, then b is also an even integer.

192. Show that if $p^2 = 2q^2$ for two integers p and q, then q is an even integer.

GETTING READY ▸ for the Next Section

REVIEW CONCEPTS

Variables **(Section P.1, page 2)**

Properties of opposites **(Section P.1, page 13)**

REVIEW SKILLS

Integer exponents **(Section P.1, page 5)**

In Exercises GR1–GR4, use the rules of exponents to simplify each expression. Here *a*, *b*, *m*, and *n* are natural numbers.

GR1. **a.** $a^2 \cdot a^3$ **b.** $a^4 \cdot a^7$ **c.** $a^m \cdot a^n$

GR2. **a.** $\frac{b^3}{b}$ **b.** $\frac{b^7}{b^3}$ **c.** $\frac{b^m}{b^n}$

GR3. **a.** $(a^2)^3$ **b.** $(a^4)^2$ **c.** $(a^m)^n$

GR4. **a.** $(ab)^2$ **b.** $(ab)^4$ **c.** $(ab)^n$

Arithmetic of fractions **(Section P.1, pages 13 and 14)**

In Exercises GR5–GR8, perform the indicated operation.

GR5. $\frac{7}{3} + \frac{3}{2}$ **GR6.** $\frac{3}{6} - \frac{2}{3}$

GR7. $\frac{3}{2} \cdot \frac{12}{7}$ **GR8.** $\frac{3}{5} \div \frac{12}{15}$

P.2 Integer Exponents and Scientific Notation

Videos for this Section

Objectives

1 ▸ Use integer exponents.

2 ▸ Use the rules of exponents.

3 ▸ Simplify exponential expressions.

4 ▸ Use scientific notation.

Coffee and Candy Consumption in America

In 2020, Americans drank about 146 billion cups of coffee and spent more than $21 billion on candy and snacks. The American population at that time was about 328 million. Because our day-to-day activities bring us into contact with more manageable quantities, most people find large numbers like those just cited a bit difficult to understand. Using exponents, the method of scientific notation allows us to write large and small quantities in a manner that makes comparing such quantities fairly easy. In turn, these comparisons help us get a better perspective on these quantities. In Example 9 of this section, without relying on a calculator, we see that if we distribute the cost of the candy and snacks used in 2020 evenly among all individuals in the United States, each person will spend over $64. Distributing the coffee used in 2020 evenly among all individuals in the United States gives each person about 445 cups of coffee.

Objective 1 ▸

Integer Exponents

In Section P.1, we introduced the following notation:

If a is a real number and n is a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}.$$

We now define a^n when the exponent is 0 or a negative integer.

SIDE NOTE

The laws of exponents suggest why, for $a \neq 0$, $a^0 = 1$.

$$2^0 = 2^{1-1} = 2 \cdot 2^{-1} = 2 \cdot \frac{1}{2} = \frac{2}{2} = 1$$

Zero and Negative Integer Exponents

For any nonzero number a and any positive integer n,

$$a^0 = 1 \text{ and } a^{-n} = \frac{1}{a^n}.$$

WARNING

Negative exponents indicate the ***reciprocal*** of a number. Zero cannot be used as a base with a negative exponent because zero does not have a reciprocal. Furthermore, $\mathbf{0^0}$ **is not defined**. We assume throughout this text that the base is not equal to zero if any of the exponents are negative or zero.

EXAMPLE 1 Evaluating Expressions That Use Zero or Negative Exponents

Evaluate.

a. $(-5)^{-2}$ **b.** -5^{-2} **c.** 8^0 **d.** $\left(\frac{2}{3}\right)^{-3}$

Solution

a. $(-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$ The exponent -2 applies to the base -5.

b. $-5^{-2} = -\frac{1}{5^2} = -\frac{1}{25}$ The exponent -2 applies to the base 5.

c. $8^0 = 1$ By definition

d. $\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\frac{2}{3}\right)^3} = \frac{1}{\frac{8}{27}} = \frac{27}{8}$ $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$

Practice Problem 1. Evaluate.

a. 2^{-1} **b.** $\left(\frac{4}{5}\right)^0$ **c.** $\left(\frac{3}{2}\right)^{-2}$ ■

In Example 1, we see that parts **a** and **b** give different results. In part **a**, the base is -5 and the exponent is -2, whereas in part **b**, we evaluate the ***opposite*** of the expression with base 5 and exponent -2.

Objective 2 ▸

Rules of Exponents

We now review the rules of exponents.

Let's see what happens when we multiply a^5 by a^3. We have

$$a^5 \cdot a^3 = \underbrace{(a \cdot a \cdot a \cdot a \cdot a)}_{5 \text{ factors}} \cdot \underbrace{(a \cdot a \cdot a)}_{3 \text{ factors}} = \underbrace{a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a}_{5+3=8 \text{ factors}} = a^8.$$

Similarly, if you multiply m factors of a by n factors of a, you get $m + n$ factors of a.

RECALL

When using the product rule, remember that a must be nonzero if the exponent is zero or negative.

Product Rule of Exponents

If a is a real number and m and n are integers, then

$$a^m \cdot a^n = a^{m+n}.$$

To multiply exponential expressions with the same base, *keep the base and add exponents*.

EXAMPLE 2 Using the Product Rule of Exponents

Simplify. Use the product rule and (if necessary) the definition of a negative exponent or reciprocal to write each answer without negative exponents.

a. $2x^3 \cdot x^5$ **b.** $x^7 \cdot x^{-7}$ **c.** $(-4y^2)(3y^7)$

Solution

a. $2x^3 \cdot x^5 = 2x^{3+5} = 2x^8$ Add exponents: $3 + 5 = 8$.

b. $x^7 \cdot x^{-7} = x^{7+(-7)} = x^0 = 1$ Add exponents: $7 + (-7) = 0$; simplify.

c. $(-4y^2)(3y^7) = (-4)3y^2y^7$ Group the factors with variable bases.

$= -12y^{2+7}$ Add exponents.

$= -12y^9$ $2 + 7 = 9$

Practice Problem 2. Simplify. Write each answer without using negative exponents.

a. $x^2 \cdot 3x^7$ **b.** $(2^2x^3)(4x^{-3})$ ■

Look what happens when we divide a^5 by a^3. We have

$$\frac{a^5}{a^3} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a} = \frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}} \cdot \frac{a \cdot a}{1} = a \cdot a = a^2.$$

So $\frac{a^5}{a^3} = a^5 \cdot \frac{1}{a^3} = a^5 \cdot a^{-3} = a^{5-3} = a^2$. You subtract exponents because the three factors in the denominator eliminate three of the factors in the numerator.

Quotient Rule for Exponents

If a is a nonzero real number and m and n are integers, then

$$\frac{a^m}{a^n} = a^{m-n}.$$

To divide two exponential expressions with the same base, *keep the base and subtract exponents*.

SIDE NOTE

Remember that if $a \neq 0$, then $a^0 = 1$; so it is okay for a denominator to contain a nonzero base with a zero exponent. For example, $\frac{3}{5^0} = \frac{3}{1} = 3$.

EXAMPLE 3 Using the Quotient Rule of Exponents

Simplify. Use the quotient rule to write each answer without negative exponents.

a. $\frac{5^{10}}{5^{10}}$ **b.** $\frac{2^{-1}}{2^3}$ **c.** $\frac{x^{-3}}{x^5}$

Solution

a. $\dfrac{5^{10}}{5^{10}} = 5^{10-10} = 5^0 = 1$

b. $\dfrac{2^{-1}}{2^3} = 2^{-1-3} = 2^{-4} = \dfrac{1}{2^4} = \dfrac{1}{16}$

c. $\dfrac{x^{-3}}{x^5} = x^{-3-5} = x^{-8} = \dfrac{1}{x^8}$

Practice Problem 3. Simplify. Write each answer without negative exponents.

a. $\dfrac{3^4}{3^0}$ **b.** $\dfrac{5}{5^{-2}}$ **c.** $\dfrac{2x^3}{3x^{-4}}$ ■

To introduce the next rule of exponents, we consider $(2^3)^4$.

$$\begin{aligned}(2^3)^4 &= 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 && a^4 = a \cdot a \cdot a \cdot a;\text{ here } a = 2^3.\\ &= (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2) && \text{12 factors of 2}\\ &= 2^{12}\end{aligned}$$

So $(2^3)^4 = 2^{3 \cdot 4} = 2^{12}$.

This suggests the following rule.

Power-of-a-Power Rule for Exponents

If a is a real number and m and n are integers, then

$$(a^m)^n = a^{mn}.$$

To find the power of a power, *keep the base and multiply exponents.*

EXAMPLE 4 Using the Power-of-a-Power Rule of Exponents

Simplify. Write each answer without negative exponents.

a. $(5^2)^0$ **b.** $[(-3)^2]^3$ **c.** $(x^3)^{-1}$ **d.** $(x^{-2})^{-3}$

Solution

a. $(5^2)^0 = 5^{2 \cdot 0} = 5^0 = 1$

b. $[(-3)^2]^3 = (-3)^{2 \cdot 3} = (-3)^6 = 729$ For $(-3)^6$, the base is -3.

c. $(x^3)^{-1} = x^{3(-1)} = x^{-3} = \dfrac{1}{x^3}$

d. $(x^{-2})^{-3} = x^{(-2)(-3)} = x^6$

Practice Problem 4. Simplify. Write each answer without negative exponents.

a. $(7^{-5})^0$ **b.** $(7^0)^{-5}$ **c.** $(x^{-1})^8$ **d.** $(x^{-2})^{-5}$ ■

We now consider the power of a product.

$$\begin{aligned}(2 \cdot 3)^5 &= (2 \cdot 3)(2 \cdot 3)(2 \cdot 3)(2 \cdot 3)(2 \cdot 3) && a^5 = a \cdot a \cdot a \cdot a \cdot a;\text{ here } a = 2 \cdot 3.\\ &= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) && \text{Use associative and commutative properties.}\\ &= 2^5 \cdot 3^5\end{aligned}$$

So $(2 \cdot 3)^5 = 2^5 \cdot 3^5$.

This suggests the following rule.

Power-of-a-Product Rule

If a and b are real numbers and n is an integer, then

$$(a \cdot b)^n = a^n \cdot b^n.$$

EXAMPLE 5 Using the Power-of-a-Product Rule

Simplify. Use the power-of-a-product rule to write each expression without negative exponents.

a. $(3x)^2$ **b.** $(-3x)^{-2}$ **c.** $(-3^2)^3$ **d.** $(xy)^{-4}$ **e.** $(x^2y)^3$

Solution

a. $(3x)^2 = 3^2x^2 = 9x^2$ Note that $(3x)^2 \neq 3x^2$

b. $(-3x)^{-2} = \dfrac{1}{(-3x)^2} = \dfrac{1}{(-3)^2x^2} = \dfrac{1}{9x^2}$ Negative exponents denote reciprocals.

c. $(-3^2)^3 = (-1 \cdot 3^2)^3 = (-1)^3(3^2)^3 = (-1)(3^6) = -729$

d. $(xy)^{-4} = \dfrac{1}{(xy)^4} = \dfrac{1}{x^4y^4}$ Negative exponents denote reciprocals.

e. $(x^2y)^3 = (x^2)^3y^3 = x^{2\cdot 3}y^3 = x^6y^3$ Recall that $(x^2)^3 = x^{2\cdot 3}$.

Practice Problem 5. Simplify. Write each answer without negative exponents.

a. $\left(\dfrac{1}{2}x\right)^{-1}$ **b.** $(5x^{-1})^2$ **c.** $(xy^2)^3$ **d.** $(x^{-2}y)^{-3}$ ■

To see the last rule, we consider $\left(\dfrac{3}{2}\right)^5$. We have

$$\left(\frac{3}{2}\right)^5 = \frac{3}{2}\cdot\frac{3}{2}\cdot\frac{3}{2}\cdot\frac{3}{2}\cdot\frac{3}{2}$$
$$= \frac{3\cdot 3\cdot 3\cdot 3\cdot 3}{2\cdot 2\cdot 2\cdot 2\cdot 2}$$
$$= \frac{3^5}{2^5}.$$

More generally, we have the following rules.

Power-of-Quotient Rules

If a and b are nonzero real numbers and n is an integer, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}.$$

EXAMPLE 6 Using the Power-of-Quotient Rules

Simplify. Use the power-of-quotient rules to write each answer without negative exponents.

a. $\left(\dfrac{3}{5}\right)^3$ **b.** $\left(\dfrac{2}{3}\right)^{-2}$

Solution

a. $\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{27}{125}$ Equation (2)

b. $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$ Equation (3) followed by equation (2)

Practice Problem 6. Simplify. Write each answer without negative exponents.

a. $\left(\frac{1}{3}\right)^2$ **b.** $\left(\frac{10}{7}\right)^{-2}$

WARNING

Incorrect	Correct
~~$(3x)^2 = 3x^2$~~	$(3x)^2 = 3^2x^2 = 9x^2$
~~$(x^2)^5 = x^7$~~	$(x^2)^5 = x^{2\cdot5} = x^{10}$
~~$x^3x^5 = x^{15}$~~	$x^3x^5 = x^{3+5} = x^8$

Objective 3 ▸

Simplifying Exponential Expressions

Rules for Simplifying Exponential Expressions

An exponential expression is considered **simplified** when

(i) each base appears only once,
(ii) each exponent is a positive number, and
(iii) no power is raised to a power.

EXAMPLE 7 Simplifying Exponential Expressions

Simplify the following.

a. $(-4x^2y^3)(7x^3y)$ **b.** $\left(\frac{x^5}{2y^{-3}}\right)^{-3}$

Solution

a. $(-4x^2y^3)(7x^3y) = (-4)(7)x^2x^3y^3y$ Group factors with the same base.

$= -28x^{2+3}y^{3+1}$ Apply the product rule to add exponents; remember that $y = y^1$.

$= -28x^5y^4$

b. $\left(\frac{x^5}{2y^{-3}}\right)^{-3} = \frac{(x^5)^{-3}}{(2y^{-3})^{-3}}$ The exponent -3 is applied to both the numerator and the denominator.

$= \frac{x^{5(-3)}}{2^{-3}(y^{-3})^{-3}}$ Multiply exponents in the numerator; apply the exponent -3 to each factor in the denominator.

$= \frac{x^{-15}}{2^{-3}y^{(-3)(-3)}}$ Power rule for exponents; $5(-3) = -15$.

$= \frac{x^{-15}}{2^{-3}y^9}$ $(-3)(-3) = 9$

$= \frac{x^{-15}x^{15}2^3}{x^{15}2^3 2^{-3}y^9}$ Multiply numerator and denominator by $x^{15}2^3$.

$= \frac{2^3}{x^{15}y^9}$ $x^{-15}x^{15} = x^0 = 1$; $2^32^{-3} = 2^0 = 1$

$= \frac{8}{x^{15}y^9}$ $2^3 = 8$

SIDE NOTE

There are many correct ways to simplify exponential expressions. The order in which you apply the rules for exponents is a matter of personal preference.

Practice Problem 7. Simplify each expression.

a. $(2x^4)^{-2}$ **b.** $\dfrac{x^2(-y)^3}{(xy^2)^3}$ ■

Objective 4 ▸

Scientific Notation

Scientific measurements and calculations often involve very large or very small positive numbers. For example, 1 gram of oxygen contains approximately

$$37{,}600{,}000{,}000{,}000{,}000{,}000{,}000 \text{ atoms}$$

and the mass of one oxygen atom is approximately

$$0.0000000000000000000000266 \text{ gram.}$$

Such numbers contain so many zeros that they are awkward to work with in calculations. Fortunately, scientific notation provides a better way to write and work with such large and small numbers.

Scientific notation consists of the product of a number less than 10, and greater than or equal to 1, and an integer power of 10. That is, scientific notation of a number has the form

$$c \times 10^n,$$

where c is a real number in decimal notation with $1 \le c < 10$ and n is an integer.

Converting a Decimal Number to Scientific Notation

1. Count the number, n, of places the decimal point in the given number must be moved to obtain a number c with $1 \le c < 10$.
2. If the decimal point is moved n places to the left, the scientific notation is $c \times 10^n$. If the decimal point is moved n places to the right, the scientific notation is $c \times 10^{-n}$.
3. If the decimal point does not need to be moved, the scientific notation is $c \times 10^0$.

EXAMPLE 8 Converting a Decimal Number to Scientific Notation

Write each decimal number in scientific notation.

a. 421,000 **b.** 10 **c.** 3.621 **d.** 0.000561

Solution

a. Because $421{,}000 = 421000.0$, count five spaces to move the decimal point between the 4 and the 2 and produce the number 4.21.

$$421{,}000 = 4\,2\,1\,0\,0\,0.$$

5 places

Because the decimal point is moved five places to the *left*, the exponent is positive and we write

$$421{,}000 = 4.21 \times 10^5.$$

b. The decimal point for 10 is to the right of the units digit. Count one place to move the decimal point between the 1 and the 0 and produce the number 1.0.

$$10 = 1\,0\,.\,0$$

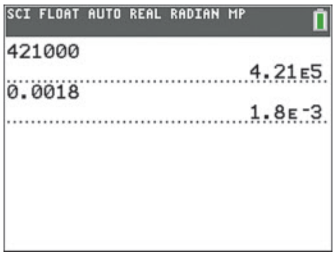
1 places

Because the decimal point is moved one place to the *left*, the exponent is positive and we write

$$10 = 1.0 \times 10^1.$$

TECHNOLOGY CONNECTION

To write numbers in scientific notation on a graphing calculator, you first must change the mode to *scientific*. Then on most graphing calculators, you will see the number displayed as a decimal number, followed by the letter *E*, followed by the exponent for 10. So 421,000 is shown as 4.21E5 and 0.0018 is shown as 1.8E-3. Some calculators omit the *E*.



c. The number 3.621 is already between 1 and 10, so the decimal does not need to be moved. We write

$$3.621 = 3.621 \times 10^0.$$

d. The decimal point in 0.000561 must be moved between the 5 and the 6 to produce the number 5.61. We count four places as follows:

$$0.0005\,61. \quad \text{(4 places)}$$

Because the decimal point is moved four places to the *right*, the exponent is negative and we write

$$0.000561 = 5.61 \times 10^{-4}.$$

SIDE NOTE

Note that in scientific notation, "small" numbers (positive numbers less than 1) have negative exponents and "large" numbers (numbers greater than 10) have positive exponents.

Practice Problem 8. Write 732,000 in scientific notation. ■

EXAMPLE 9 Distributing Coffee and Candy in America

At the beginning of this section, we mentioned that in 2020, Americans drank about 146 billion cups of coffee and spent more than $21 billion on candy and snacks. To see how these products would be evenly distributed among the population, we first convert those numbers to scientific notation.

$$146 \text{ billion is } 146{,}000{,}000{,}000 = 1.46 \times 10^{11}.$$
$$21 \text{ billion is } 21{,}000{,}000{,}000 = 2.1 \times 10^{10}.$$

The U.S. population in 2020 was about 328 million, and 328 million is $328{,}000{,}000 = 3.28 \times 10^8$.

To distribute the coffee evenly among the population, we divide:

$$\frac{1.46 \times 10^{11}}{3.28 \times 10^8} = \frac{1.46}{3.28} \times \frac{10^{11}}{10^8} = \frac{146}{328} \times 10^{11-8} \approx 0.445 \times 10^3 = 445 \text{ cups per person.}$$

To distribute the cost of the candy evenly among the population, we divide:

$$\frac{2.1 \times 10^{10}}{3.28 \times 10^8} = \frac{2.10}{3.28} \times \frac{10^{10}}{10^8} = \frac{210}{328} \times 10^{10-8} \approx 0.64 \times 10^2 = 64,$$

or about $64 per person.

Practice Problem 9. If the amount spent on candy and snacks consumption remains unchanged when the U.S. population reaches 350 million, what is the cost per person when cost is distributed evenly throughout the population? ■

Main Facts about Exponents

$$a^n = \underbrace{a \cdot a \ldots a}_{n \text{ factors}} \text{ for } a \neq 0, \quad a^0 = 1, \quad \text{and} \quad a^{-n} = \frac{1}{a^n}$$

Product Rule: $a^m \cdot a^n = a^{m+n}$ **Quotient Rule:** $\frac{a^m}{a^n} = a^{m-n}$

Power-of-a-Power Rule: $(a^m)^n = a^{mn}$ **Power-of-a-Product Rule:** $(ab)^m = a^m b^m$

Power-of-Quotient Rules: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$; $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$, $b \neq 0$

P.2 Exercises

Concepts and Vocabulary

1. In the expression 7^{-2}, the number -2 is called the ________.
2. In the expression -3^7, the base is ________.
3. The number $\frac{1}{4^{-2}}$ simplifies to the positive integer ________.
4. The power-of-a-product rule allows us to rewrite $(5a)^3$ as ________.
5. True or False. $(-11)^{10} = -11^{10}$.
6. True or False. The exponential expression $(x^2)^3$ is considered simplified.
7. True or False. $(ab)^n = a^n b^n$
8. True or False. $(a + b)^n = a^n + b^n$

Building Skills

In Exercises 9–46, evaluate each expression.

9. 3^{-2}
10. 2^{-3}
11. $\left(\frac{1}{2}\right)^{-4}$
12. $\left(-\frac{1}{2}\right)^{-2}$
13. 7^0
14. $(-8)^0$
15. $-(-7)^0$
16. $(\sqrt{2})^0$
17. $(2^3)^2$
18. $(3^2)^3$
19. $(3^2)^{-2}$
20. $(7^2)^{-1}$
21. $(5^{-2})^3$
22. $(5^{-1})^3$
23. $(4^{-3}) \cdot (4^5)$
24. $(7^{-2}) \cdot (7^3)$
25. $(\sqrt{3})^0 + 10^0$
26. $(\sqrt{5})^0 - 9^0$
27. $3^{-2} + \left(\frac{1}{3}\right)^2$
28. $5^{-2} + \left(\frac{1}{5}\right)^2$
29. -2^{-3}
30. -3^{-2}
31. $(-3)^{-2}$
32. $(-2)^{-3}$
33. $\frac{2^{11}}{2^{10}}$
34. $\frac{3^6}{3^8}$
35. $\frac{(5^3)^4}{5^{12}}$
36. $\frac{(9^5)^2}{9^8}$
37. $\frac{2^5 \cdot 3^{-2}}{2^4 \cdot 3^{-3}}$
38. $\frac{4^{-2} \cdot 5^3}{4^{-3} \cdot 5}$
39. $\frac{-5^{-2}}{2^{-1}}$
40. $\frac{-7^{-2}}{3^{-1}}$
41. $\left(\frac{2}{3}\right)^{-1}$
42. $\left(\frac{1}{5}\right)^{-1}$
43. $\left(\frac{2}{3}\right)^{-2}$
44. $\left(\frac{3}{2}\right)^{-2}$
45. $\left(\frac{11}{7}\right)^{-2}$
46. $\left(\frac{13}{5}\right)^{-2}$

In Exercises 47–86, simplify each expression. Write your answers without negative exponents. Whenever an exponent is negative or zero, assume that the base is not zero.

47. $x^4 y^0$
48. $x^{-1} y^0$
49. $x^{-1} y$
50. $x^2 y^{-2}$
51. $x^{-1} y^{-2}$
52. $x^{-3} y^{-2}$
53. $(x^{-3})^4$
54. $(x^{-5})^2$
55. $(x^{-11})^{-3}$
56. $(x^{-4})^{-12}$
57. $-3(xy)^5$
58. $-8(xy)^6$
59. $4(xy^{-1})^2$
60. $6(x^{-1}y)^3$
61. $3(x^{-1}y)^{-5}$
62. $-5(xy^{-1})^{-6}$
63. $\frac{(x^3)^2}{(x^2)^5}$
64. $\frac{x^2}{(x^3)^4}$
65. $\left(\frac{2xy}{x^2}\right)^3$
66. $\left(\frac{5xy}{x^3}\right)^4$
67. $\left(\frac{-3x^2y}{x}\right)^5$
68. $\left(\frac{-2xy^2}{y}\right)^3$
69. $\left(\frac{-3x}{5}\right)^{-2}$
70. $\left(\frac{-5y}{3}\right)^{-4}$
71. $\left(\frac{4x^{-2}}{xy^5}\right)^3$
72. $\left(\frac{3x^2y}{y^3}\right)^5$
73. $\frac{x^3y^{-3}}{x^{-2}y}$
74. $\frac{x^2y^{-2}}{x^{-1}y^2}$
75. $\frac{27x^{-3}y^5}{9x^{-4}y^7}$
76. $\frac{15x^5y^{-2}}{3x^7y^{-3}}$
77. $\frac{1}{x^3}(x^2)^3 x^{-4}$
78. $(8a^3b)^{-4}\left(\frac{2a}{b}\right)^{12}$
79. $(-xy^2)^3(-2x^2y^2)^{-4}$
80. $\left[\frac{(-x^2y)^3 y^{-4}}{(xy)^5}\right]^{-2}$
81. $(4x^3y^2z)^2(x^3y^2z)^{-7}$
82. $\frac{(2xyz)^2}{(x^3y)^2(xz)^{-1}}$
83. $\frac{5a^{-2}bc^2}{a^4b^{-3}c^2}$
84. $\frac{(-3)^2a^5(bc)^2}{(-2)^3a^2b^3c^4}$
85. $\left(\frac{xy^{-3}z^{-2}}{x^2y^{-4}z^3}\right)^{-3}$
86. $\left(\frac{xy^{-2}z^{-1}}{x^{-5}yz^{-8}}\right)^{-1}$

In Exercises 87–94, write each number in scientific notation.

87. 125
88. 247
89. 850,000
90. 205,000
91. 0.007
92. 0.0019
93. 0.00000275
94. 0.0000038

Applying the Concepts

Uniform Scaling. In Exercises 95 and 96, use the fact that when you uniformly stretch or shrink a three-dimensional object in every direction by a factor of a, the volume of the resulting figure is scaled by a factor of a^3. For example, when you uniformly scale (stretch or shrink) a three-dimensional object by a factor of 2, the volume of the resulting figure is scaled by a factor of 2^3, or 8.

95. A display in the shape of a baseball bat has a volume of 135 cubic feet. Find the volume of the display that results by uniformly scaling the figure by a factor of 2.

96. A display in the shape of a football has a volume of 675 cubic inches. Find the volume of the display that results by uniformly scaling the figure by a factor of $\frac{1}{3}$.

97. The area A of a square with side of length x is given by $A = x^2$. Use this relationship to

a. verify that doubling the length of the side of a square floor increases the area of the floor by a factor of 2^2.

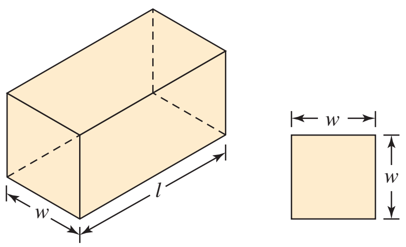
b. verify that tripling the length of the side of a square floor increases the area of the floor by a factor of 3^2.

98. The area A of a circle with diameter d is given by $A = \pi\left(\frac{d}{2}\right)^2$. Use this relationship to

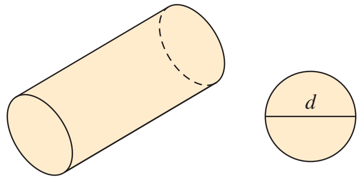
a. verify that doubling the length of the diameter of a circular skating rink increases the area of the rink by a factor of 2^2.

b. verify that tripling the length of the diameter of a circular skating rink increases the area of the rink by a factor of 3^2.

99. Weight-bearing supports. The cross section of one type of weight-bearing rod (the surface you would get if you sliced the rod perpendicular to its axis) used in the Olympics is a square, as shown in the accompanying figure. The rod can handle a stress of 25,000 pounds per square inch (psi). The relationship between the stress S the rod can handle, the load F the rod can carry, and the width w of the cross section is given by the equation $Sw^2 = F$. Assuming that $w = 0.25$ inch, find the load the rod can support.



100. Weight-bearing supports. The cross section of one type of weight-bearing rod (the surface you would get if you sliced the rod perpendicular to its axis) used in the Olympics is a circle, as shown in the figure. The rod can handle a stress of 10,000 pounds per square inch (psi). The relationship between the stress S the rod can handle, the load F the rod can carry, and the diameter d of the cross section is given by the equation $S\pi(d/2)^2 = F$. Assuming that $d = 1.5$ inches, find the load the rod can support. Use $\pi \approx 3.14$ in your calculation.



101. Pet expenditures. Assume that pet industry expenditures in the United States is approximately 99 billion dollars and the U.S. population is 330 million. If this expense is distributed evenly across the population, what is the cost per person?

102. Sporting event costs. Assume that Americans spend \$56 billion dollars on attending sporting events and the U.S. population is 330 million. If this expense is distributed evenly across the population, what is the cost per person?

103. Unit conversion. A year has 365.25 days. Write the number of seconds in one year in scientific notation.

104. Unit conversion. Repeat Exercise 103 for a leap year (366 days).

105. Celestial bodies. Complete the following table.

Celestial Body	Equatorial Diameter (km)	Scientific Notation
Earth	12,700	
Moon		3.48×10^3 (km)
Sun	1,390,000	
Jupiter		1.34×10^5 (km)
Mercury	4800	

Scientific notation. In Exercises 106–112, express the number in each statement in scientific notation.

106. One gram of oxygen contains about

37,600,000,000,000,000,000,000 atoms.

107. One gram of hydrogen contains about

602,000,000,000,000,000,000 atoms.

108. One oxygen atom weighs about

0.0000000000000000000000266 kilogram.

109. One hydrogen atom weighs about

0.00000000000000000000000167 kilogram.

110. The distance from Earth to the moon is about

380,000,000 meters.

111. The mass of Earth is about

5,980,000,000,000,000,000,000,000 kilograms.

112. The mass of the sun is about

2,000,000,000,000,000,000,000,000,000,000 kilograms.

Beyond the Basics

113. If $2^x = 32$, find
a. 2^{x+2}.
b. 2^{x-1}.

114. If $3^x = 81$, find
a. 3^{x+1}.
b. 3^{x-2}.

115. If $5^x = 11$, find
a. 5^{x+1}. **b.** 5^{x-2}.

116. If $a^x = b$, find
a. a^{x+2}. **b.** a^{x-1}.

117. Simplify: $\dfrac{3^{2-n} \cdot 9^{2n-2}}{3^{3n}}$.

118. Simplify: $\dfrac{2^{m+1} \cdot 3^{2m-n} \cdot 5^{m+n+2} \cdot 6^{n}}{6^{m} \cdot 10^{n+1} \cdot 15^{m}}$.

[*Hint:* $6 = 2 \cdot 3, 10 = 2 \cdot 5, 15 = 3 \cdot 5$.]

119. Simplify: $\dfrac{2^{x(y-z)}}{2^{y(x-z)}} \div \left(\dfrac{2^y}{2^x}\right)^z$.

120. Simplify: $\left(\dfrac{a^x}{a^y}\right)^{\frac{1}{xy}} \cdot \left(\dfrac{a^y}{a^z}\right)^{\frac{1}{yz}} \cdot \left(\dfrac{a^z}{a^x}\right)^{\frac{1}{xz}}$.

GETTING READY ▸ for the Next Section

REVIEW CONCEPTS

Definition of subtraction (Section P.1, page 13)

Properties of opposites (Section P.1, page 13)

Rules for exponents (Section P.2, page 26)

REVIEW SKILLS

Integer exponents (Section P.1, page 5)

GR1. Simplify:

a. $x^2 \cdot x^5$

b. $(2x)(-5x^2)$

c. $(2y^2)(3y^3)(4y^5)$

GR2. Simplify:

a. $2x^2 + 5x^2$. **b.** $3x^2 - 4x^2$.

c. $3x^3 - 5x^3 + 11x^3$.

Associative, commutative, and distributive properties (Section P.1, page 12)

GR3. $x(2x + 5)$ **GR4.** $2x(7 - 3x)$

GR5. $-5x(x^2 + 1)$

GR6. True or False. $(6x + 4)(x + 9) = (x + 9)(6x + 4)$

GR7. True or False. $5x^2 + 3x^3 = 8x^5$

GR8. Use the distributive property to simplify: $2x^2(5x^3 - 3x + 4)$.

P.3 Polynomials

Objectives

1 ▸ Learn polynomial vocabulary.

2 ▸ Add and subtract polynomials.

3 ▸ Multiply polynomials.

4 ▸ Use special-product formulas.

Videos for this Section

Galileo and Free-Falling Objects

At one time, it was widely believed that heavy objects should fall faster than lighter objects, or, more exactly, that the speed of falling objects ought to be proportional to their weights. So if one object is four times as heavy as another, the heavier object should fall four times as fast as the lighter one. Legend has it that Galileo Galilei (1564–1642) disproved this theory by dropping two balls of equal size, one made of lead and the other of balsa wood, from the top of Italy's Leaning Tower of Pisa. Both balls are said to have reached the ground at the same instant after they were released simultaneously from the top of the tower.

Leaning Tower of Pisa

In fact, such experiments "work" properly only in a vacuum, where objects of different mass do fall at the same rate. In the absence of a vacuum, air resistance may greatly affect the rate at which different objects fall. Whether Galileo's experiment was actually performed is unknown, but astronaut David R. Scott successfully performed a version of Galileo's experiment (using a feather and a hammer) on the surface of the moon on August 2, 1971. It turns out that on Earth (ignoring air resistance), regardless of the weight of the object we hold at rest and then drop, the expression $16t^2$ gives the distance in feet the object falls in t seconds. If we throw the object down with an initial velocity of v_0 feet per second, the distance the object travels in t seconds is given by the expression

$$16t^2 + v_0 t.$$

In Example 1, we evaluate such expressions.

Objective 1 ▸

Polynomial Vocabulary

The height (in feet) of a golf ball above a driving range t seconds after being driven from the tee (at 70 feet per second) is given by the polynomial $70t - t^2$. After two seconds $(t = 2)$, the ball is $70(2) - (2)^2 = 136$ feet above the ground. *Polynomials* such as $70t - t^2$ appear frequently in applications.

We begin by reviewing the basic vocabulary of polynomials. A **monomial** is the simplest polynomial; it contains one term. In the variable x it has the form ax^k, where a is a constant and k is either a positive integer or zero. The constant a is called the **coefficient** of the monomial. For $a \neq 0$, the integer k is called the **degree** of the monomial. If $a = 0$, the monomial is 0 and has **no degree**. Any variable may be used in place of x to form a monomial in that variable.

The following are examples of monomials.

$2x^5$	The coefficient is 2, and the degree is 5.
$-3x^2$	The coefficient is -3, and the degree is 2.
-7	$-7 = -7(1) = -7x^0$. The coefficient is -7, and the degree is 0.
$8x$	The coefficient is 8, and the degree is $1(x = x^1)$.
x^{12}	The coefficient is 1 $(x^{12} = 1 \cdot x^{12})$, and the degree is 12.
$-x^4$	The coefficient is $-1(-x^4 = (-1) \cdot x^4)$, and the degree is 4.

The expression $5x^{-3}$ is not a monomial because the exponent on x is negative.

Two monomials in the same variable with the same degree can be combined into one monomial using the distributive property. For example, $-3x^5$ and $9x^5$ can be added: $-3x^5 + 9x^5 = (-3 + 9)x^5 = 6x^5$. Or $-3x^5$ and $9x^5$ can be subtracted: $-3x^5 - 9x^5 = (-3 - 9)x^5 = -12x^5$. Monomials in the same variable with the same degree are called **like terms**.

Polynomials in One Variable

A **polynomial** in x is any sum of monomials in x. By combining like terms, we can write any polynomial in the form

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0,$$

where n is either a positive integer or zero and $a_n, a_{n-1}, \ldots a_1, a_0$ are constants, called the **coefficients** of the polynomial. If $a_n \neq 0$, then n, the largest exponent on x, is called the **degree** and a_n is called the **leading coefficient** of the polynomial. The monomials $a_nx^n, a_{n-1}x^{n-1}, \ldots, a_2x^2, a_1x$, and a_0 are the **terms** of the polynomial. The monomial a_nx^n is the **leading term** of the polynomial, and a_0 is the **constant term**.

The symbols $a_0, a_1, a_2, \ldots a_n$ in the general notation for a polynomial

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

are just constants; the numbers to the lower right of a are called subscripts. The notation a_2 is read "a sub 2", a_1 is read "a sub 1", and a_0 is read "a sub 0". This type of notation is used when a large or indefinite number of constants are required. The terms of the polynomial $5x^2 + 3x + 1$ are $5x^2$, $3x$, and 1; the coefficients are 5, 3, and 1; and its degree is 2.

Note that the terms in the general form for a polynomial are connected by the addition symbol $+$. How do we handle $5x^2 - 3x + 1$? Because subtraction is defined in terms of addition, we rewrite

$$5x^2 - 3x + 1 = 5x^2 + (-3)x + 1$$

and see that the terms of $5x^2 - 3x + 1$ are $5x^2$, $-3x$, and 1 and that the coefficients are 5, -3, and 1. (The degree is 2.) Once we recognize this fact, we no longer rewrite the polynomial with the $+$ sign separating terms; we just remember that the "sign" is part of both the term and the coefficient. Polynomials and their properties will be discussed in more detail in Chapter 3.

Polynomials can be classified according to the number of terms they have. Polynomials with one term are called **monomials**, polynomials with two *unlike* terms are called **binomials**, and polynomials with three *unlike* terms are called **trinomials**. Polynomials with more than three unlike terms do not have special names.

By agreement, the only polynomial that has *no* degree is the **zero polynomial**, which results when all of the coefficients are 0. It is easiest to find the degree of a polynomial when it is written in **descending order**—that is, when the exponents decrease from left to right. In this case, the degree is the exponent of the leftmost term. A polynomial written in descending order is said to be in **standard form**.

EXAMPLE 1 Examining Free-Falling Objects

In our introductory discussion about Galileo and free-falling objects, we mentioned two well-known polynomials.

1. $16t^2$ gives the distance in feet a free-falling object falls in t seconds.
2. With $v_0 = 10$, the expression $16t^2 + 10t$ gives the distance an object falls in t seconds when it is thrown down with an initial velocity of 10 feet per second. Use these polynomials to
   - **a.** Find how far a wallet dropped from the 86th-floor observatory of the Empire State Building will fall in 5 seconds.
   - **b.** Find how far a quarter thrown down with an initial velocity of 10 feet per second from a hot air balloon will travel after 5 seconds.

Solution

a. The value of $16t^2$ for $t = 5$ is $16(5)^2 = 400$. The wallet falls 400 feet.

b. The value of $16t^2 + 10t$ for $t = 5$ is $16(5)^2 + 10(5) = 450$. The quarter travels 450 feet on its downward path after five seconds.

Practice Problem 1. If $16t^2 + 15t$ gives the distance an object falls in t seconds when it is thrown down at an initial velocity of 15 feet per second, find how far a quarter thrown down with an initial velocity of 15 feet per second from a hot air balloon has traveled after 7 seconds. ■

Objective 2 ▸

Adding and Subtracting Polynomials

Like monomials, polynomials are added and subtracted by combining like terms. By convention, we write polynomials in standard form.

EXAMPLE 2 Adding Polynomials

Find the sum of the polynomials

$$-4x^3 + 5x^2 + 7x - 2 \quad \text{and} \quad 6x^3 - 2x^2 - 8x - 5.$$

Horizontal Method: Group like terms and then combine them.

$$(-4x^3 + 5x^2 + 7x - 2) + (6x^3 - 2x^2 - 8x - 5)$$
$$= (-4x^3 + 6x^3) + (5x^2 - 2x^2) + (7x - 8x) + (-2 - 5) \quad \text{Group like terms.}$$
$$= 2x^3 + 3x^2 - x - 7 \quad \text{Combine like terms.}$$