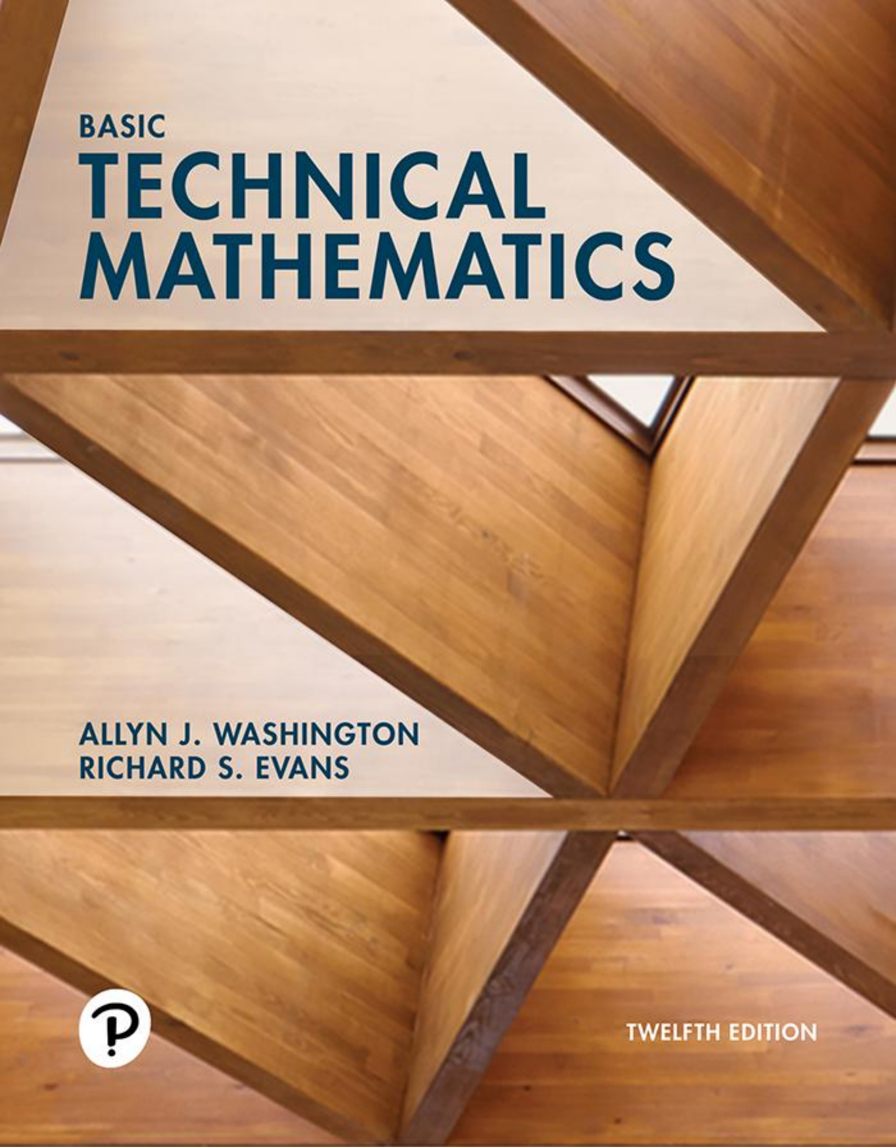

BASIC
TECHNICAL MATHEMATICS

ALLYN J. WASHINGTON
RICHARD S. EVANS

TWELFTH EDITION

TWELFTH EDITION

Basic Technical Mathematics

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In Memory of Allyn Washington
1930 – 2020

Teacher, scholar, author, friend.
Warm and generous, family man.

Committed to his work, this book.
His love for life, never shook.

Baseball, bridge, wartime tales,
Brought him joy, filled his sails.

His open heart and smiling face,
Live on inside for all my days.

Rich Evans

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Preface

Scope of the Book

Basic Technical Mathematics, Twelfth Edition, is intended primarily for students in technical and pre-engineering technical programs or other programs for which coverage of mathematics is required. Chapters 1 through 20 provide the necessary background for further study with an integrated treatment of algebra and trigonometry. Chapter 21 covers the basic topics of analytic geometry, and Chapter 22 gives an introduction to statistics. In the examples and exercises, numerous applications from the various fields of technology are included, primarily to indicate where and how mathematical techniques are used. However, it is not necessary that the student have a specific knowledge of the technical area from which any given problem is taken. Most students using this text will have a background that includes some algebra and geometry. However, the material is presented in adequate detail for those who may need more study in these areas. The material presented here is sufficient for two to three semesters. One of the principal reasons for the arrangement of topics in this text is to present material in an order that allows a student to take courses concurrently in allied technical areas, such as physics and electricity. These allied courses normally require a student to know certain mathematics topics by certain definite times; yet the traditional order of topics in mathematics courses makes it difficult to attain this coverage without loss of continuity. However, the material in this book can be rearranged to fit any appropriate sequence of topics. The approach used in this text is not unduly rigorous mathematically, although all appropriate terms and concepts are introduced as needed and given an intuitive or algebraic foundation. The aim is to help the student develop an understanding of mathematical methods without simply providing a collection of formulas. The text material is developed recognizing that it is essential for the student to have a sound background in algebra and trigonometry in order to understand and succeed in any subsequent work in mathematics.

New to This Edition

The focus of this revision was to address feedback from our many MyLab Math users. Here are details of what has been updated in the text and online:

- Within the text, we addressed feedback received from users and reviewers. We also updated real-world data and scenarios to bring them up to date.
- We conducted an external review of the text's content to determine how it could be improved to address issues related to diversity, equity, and inclusion. The results of that review informed the revision.
- At the request of users, we added Appendix E, which covers Binary and Hexadecimal Numbers. This appendix is available online at `bit.ly/3h1t3lt`.
- The exercises in MyLab Math were improved as follows:
  - We increased the number of assignable algorithmic exercises by about 220. Of these new exercises, about 175 are based on existing textbook exercises. The remaining 45 or so are applications written especially for this revision by the author. These new problems (which do not appear in the book) are labeled EXTRA in MyLab.
  - We modified existing MyLab exercises to improve clarity. These modifications were suggested by the author.
- We greatly increased video coverage, adding 165 new example videos to bring the total to 600. Many of the new videos feature author Rich Evans. Additionally, Rich re-correlated all of the videos to the exercises in MyLab, so when a student chooses "Video" as a learning aid within a MyLab exercise, they can be confident that the video will be helpful.
  - We created a *Guide to Video-Based Assignments*, which makes it easier for you to create assignments containing video by showing which MyLab questions relate to each video.

Continuing Features

PAGE LAYOUT

Special attention has been given to the page layout. We specifically tried to avoid breaking examples or important discussions across pages. Also, all figures are shown immediately adjacent to the material in which they are discussed. Finally, whenever possible, equations or formulas needed for a particular problem are restated rather than referring to formula numbers.

CHAPTER INTRODUCTIONS

Each chapter introduction illustrates specific examples of how the development of technology has been related to the development of mathematics. In these introductions, it is shown that these past discoveries in technology led to some of the methods in mathematics, whereas in other cases mathematical topics already known were later very useful in bringing about advances in technology. Also, each chapter introduction contains a photo that refers to an example that is presented within that chapter.

WORKED-OUT EXAMPLES

EXAMPLE 3 Symbol in capital and in lowercase—forces on a beam

In the study of the forces on a certain beam, the equation $W = \frac{L(wL + 2P)}{8}$ is used. Solve for P.

$$8W = \frac{8L(wL + 2P)}{8} \quad \text{multiply both sides by 8}$$

$$8W = L(wL + 2P) \quad \text{simplify right side}$$

$$8W = wL^2 + 2LP \quad \text{remove parentheses}$$

$$8W - wL^2 = 2LP \quad \text{subtract } wL^2 \text{ from both sides}$$

$$P = \frac{8W - wL^2}{2L} \quad \text{divide both sides by } 2L \text{ and switch sides}$$

■

- **"Help Text" –** Throughout the book, special explanatory comments in blue type have been used in the examples to emphasize and clarify certain important points. Arrows are often used to indicate clearly the part of the example to which reference is made.
- **Example Descriptions –** A brief descriptive title is given for each example. This gives an easy reference for the example, particularly when reviewing the contents of the section.
- **Application Problems –** There are over 300 applied examples throughout the text that show complete solutions of application problems. Many relate to modern technology such as computer design, electronics, solar energy, lasers, fiber optics, the environment, and space technology. Other examples and exercises relate to technologies such as aeronautics, architecture, automotive, business, chemical, civil, construction, energy, environmental, fire science, machine, medical, meteorology, navigation, police, refrigeration, seismology, and wastewater. The Index of Applications at the end of the book shows the breadth of applications in the text.

KEY FORMULAS AND PROCEDURES

Throughout the book, important formulas are set off and displayed so that they can be easily referenced for use. Similarly, summaries of techniques and procedures consistently appear in color-shaded boxes. The back endpapers of the text provide a handy reference of key formulas and facts.

"CAUTION" AND "NOTE" INDICATORS

CAUTION This heading is used to identify errors students commonly make or places where they frequently have difficulty. ■

NOTE ▸ The NOTE label in the side margin, along with [accompanying blue brackets in the main body of the text,] points out material that is of particular importance in developing or understanding the topic under discussion.

GRAPHING CALCULATOR HANDBOOK

The margins of the text contain short URLs that take students directly to the relevant content in the Graphing Calculator Handbook (which was written by Benjamin Rushing of Northwestern State University). If you'd like to see a complete listing of entries for the online graphing calculator handbook, go to bit.ly/2NFYzDK.

CHAPTER AND SECTION CONTENTS

A listing of learning outcomes for each chapter is given on the introductory page of the chapter. Also, a listing of the key topics of each section is given below the section number and title on the first page of the section. This gives the student and instructor a quick preview of the chapter and section contents.

PRACTICE EXERCISES

Most sections include some practice exercises in the margin. They are included so that a student is more actively involved in the learning process and can check his or her understanding of the material. They can also be used for classroom exercises. The answers to these exercises are given at the end of the exercises set for the section. There are over 450 of these exercises.

FEATURES OF EXERCISES

- **Exercises Directly Referenced to Text Examples –** The first few exercises in most of the text sections are referenced directly to a specific example of the section. These exercises are worded so that it is necessary for the student to refer to the example in order to complete the required solution. In this way, the student should be able to better review and understand the text material before attempting to solve the exercises that follow.
- **Writing Exercises –** There are over 270 writing exercises through the book (at least eight in each chapter) that require at least a sentence or two of explanation as part of the answer. These are noted by a pencil icon next to the exercise number.
- **Application Problems –** There are about 2000 application exercises in the text that represent the breadth of applications that students will encounter in their chosen professions. The Index of Applications at the end of the book shows the breadth of applications in the text.

CHAPTER ENDMATTER

- **Key Formulas & Equations –** Here, all important formulas and equations are listed together with their corresponding equation numbers for easy reference.
- **Chapter Review Exercises –** These exercises consist of (a) Concept Check Exercises (a set of true/false exercises) and (b) Practice and Applications.
- **Chapter Test –** These are designed to mirror what students might see on the actual chapter test. Complete step-by-step solutions to all practice test problems are given in the back of the book.

MARGIN NOTES

Throughout the text, some margin notes point out relevant historical events in mathematics and technology. Other margin notes are used to make specific comments related to the text material. Also, where appropriate, equations from earlier material are shown for reference in the margin.

ANSWERS TO EXERCISES

The answers to odd-numbered exercises are given near the end of the book. The Student's Solution Manual contains solutions to every other odd-numbered exercise and the Instructor's Solution Manual contains solutions to all section exercises.

FLEXIBILITY OF COVERAGE

The order of coverage can be changed in many places and certain sections may be omitted without loss of continuity of coverage. Users of earlier editions have indicated successful use of numerous variations in coverage. Any changes will depend on the type of course and completeness required.

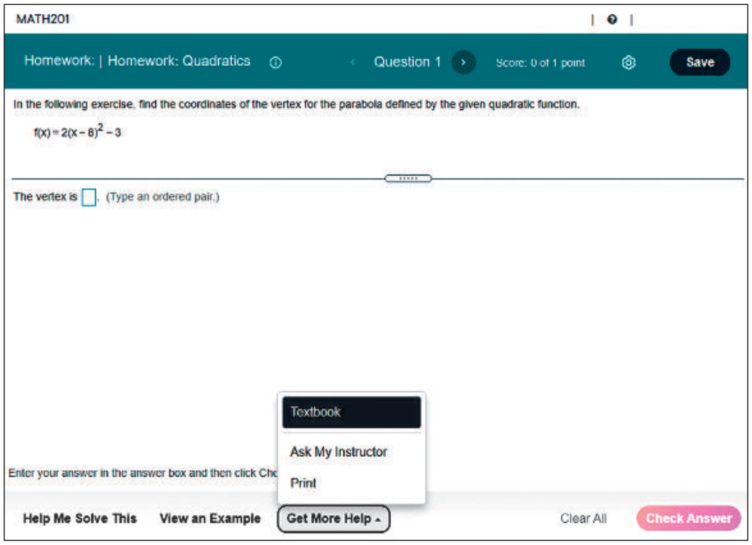
MyLab™ Math Resources for Success

MyLab Math (pearson.com/mylab/math) is available to accompany Pearson's market-leading text options, including this text (access code required). MyLab Math is the teaching and learning platform that empowers you to reach every student. It combines trusted author content—including full eText and online homework with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

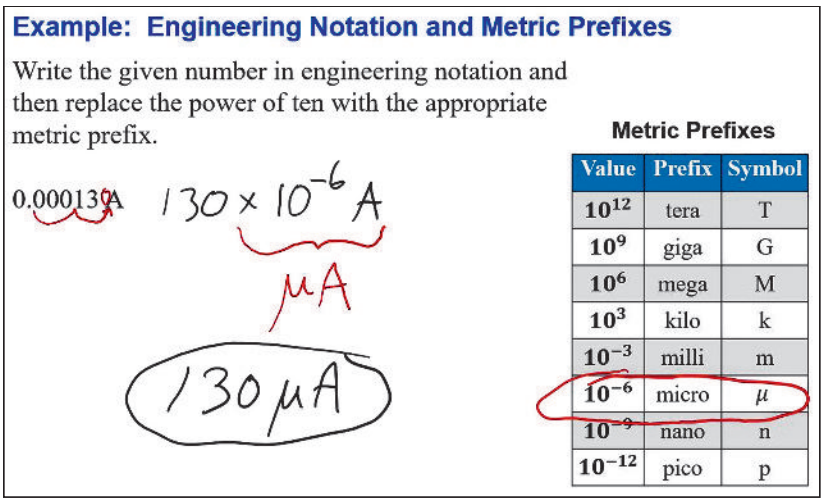
MYLAB MATH STUDENT RESOURCES

Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student needs practice, giving all students the support they need — when and where they need it — to be successful.

- **Exercises with Immediate Feedback –** The 2570 exercises in MyLab Math (220 of them new for this edition) reflect the approach and learning style of this text, and regenerate algorithmically to give students unlimited opportunity for practice and mastery. Most exercises include learning aids, such as guided solutions, videos, and sample problems, and they offer helpful feedback when students enter incorrect answers.



- **Instructional videos** – The 600 videos (165 of them new for this edition) in the 12th edition MyLab Math course provide help for students outside of the classroom. These videos are also available as learning aids within the homework exercises, for students to refer to at point-of-use.



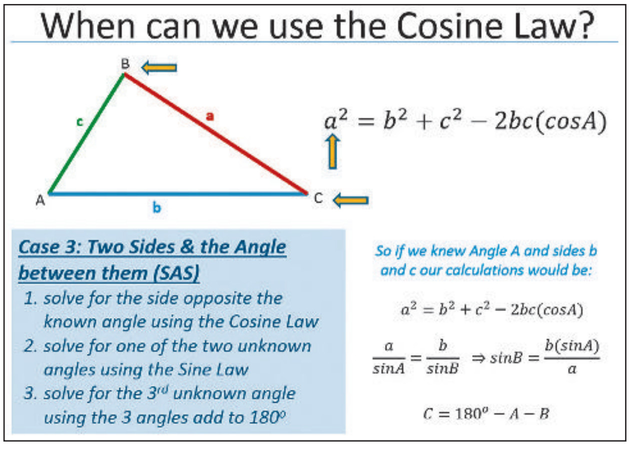
- **The complete eText** is available to students through their MyLab Math course. The eText includes links to videos. The eText is also available for stand-alone purchase.
- **Online Graphing Calculator Handbook**, created specifically for this text by Benjamin Rushing (Northwestern State University), features instructions for the TI-84 and TI-89 family of calculators. Links to specific parts of the handbook are included as short URLs that appear throughout the textbook. If you'd like to see a complete listing of entries for the online graphing calculator handbook, go to bit.ly/2NFYzDK.
- **Mindset videos** and corresponding assignable, open-ended exercises foster a growth mindset in students. This material encourages them to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as learning opportunities — so often a hurdle for math students.

- **Personal Inventory Assessments** are a collection of online exercises designed to promote self-reflection and engagement in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.
- **Student Solutions Manual** – Contains fully worked solutions to odd-numbered exercises. Available for download from within MyLab Math.

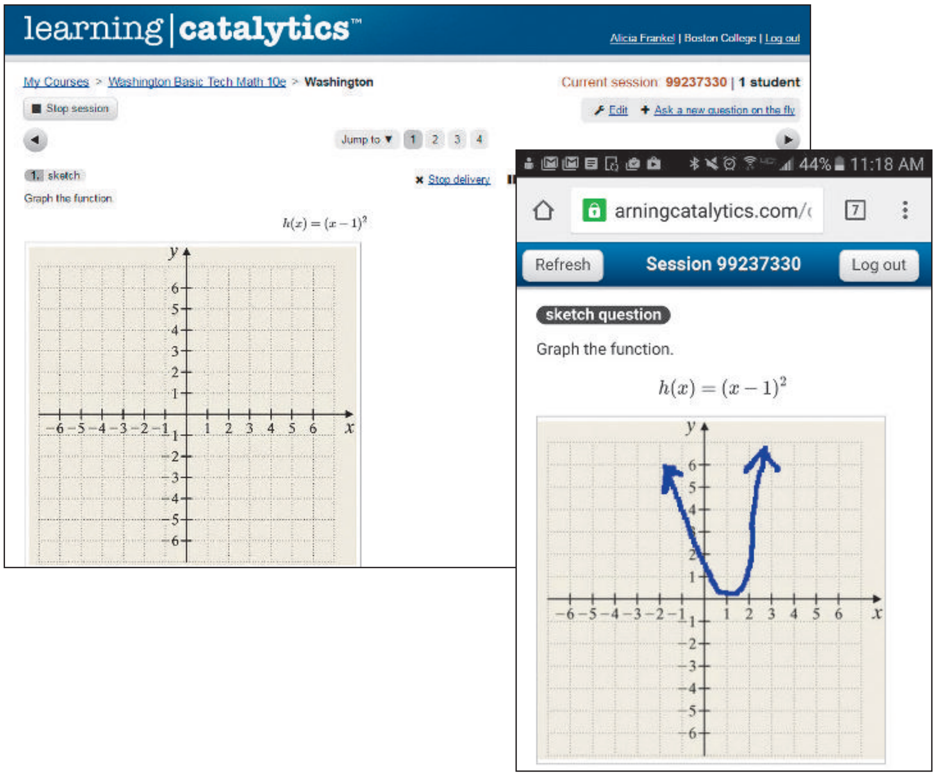
MYLAB MATH INSTRUCTOR RESOURCES

Your course is unique. Whether you'd like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

- **PowerPoint®** files feature animations that are designed to help you better teach key concepts.



- **Learning Catalytics** – With Learning Catalytics™, you'll hear from every student when it matters most. You pose a variety of questions in class (choosing from pre-loaded questions or questions of your own making) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops.



- **Guide to Video-Based Assignments** – Makes it easier for you to create assignments containing video by showing which MyLab questions relate to each video.
- **Instructor's Solution Manual** by Matthew Hudelson (Washington State University) contains detailed solutions to every section exercise, including review exercises.
- **Accessibility** – Throughout our development process for every release, we test and retest the capabilities of our products against the highest standards. Many existing products meet the Web Content Accessibility Guidelines (WCAG) 2.0 guidelines. As of Summer 2020, new product development has worked to meet the WCAG 2.1 AA guidelines. More information can be found at pearson.com/us/accessibility.
- **TestGen** enables instructors to build, edit, print, and administer tests using a bank of questions developed to cover all objectives in the text. TestGen is algorithmically based, allowing you to create multiple but equivalent versions of the same question or test. Instructors can also modify test bank questions or add new questions. The TestGen software and accompanying test bank are available to qualified instructors for download from Pearson's online catalog.

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Finally, I wish to sincerely thank again each of the over 410 reviewers of the twelve editions of this text. Their comments have helped further the education of more than two million students since this text was first published in 1964.

Rich Evans

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Basic Algebraic Operations

1

Interest in things such as the land on which they lived, the structures they built, and the motion of the planets led people in early civilizations to keep records and to create methods of counting and measuring.

In turn, some of the early ideas of arithmetic, geometry, and trigonometry were developed. From such beginnings, mathematics has played a key role in the great advances in science and technology.

Often, mathematical methods were developed from scientific studies made in particular areas, such as astronomy and physics. Many people were interested in the math itself and added to what was then known. Although this additional mathematical knowledge may not have been related to applications at the time it was developed, it often later became useful in applied areas.

In the chapter introductions that follow, examples of the interaction of technology and mathematics are given. From these examples and the text material, it is hoped you will better understand the important role that math has had and still has in technology. In this text, there are applications from technologies including (but not limited to) aeronautical, business, communications, electricity, electronics, engineering, environmental, heat and air conditioning, mechanical, medical, meteorology, petroleum, product design, solar, and space.

We begin by reviewing the concepts that deal with numbers and symbols. This will enable us to develop topics in algebra, an understanding of which is essential for progress in other areas such as geometry, trigonometry, and calculus.

LEARNING OUTCOMES

After completion of this chapter, the student should be able to:

- Identify real, imaginary, rational, and irrational numbers
- Perform mathematical operations on integers, decimals, fractions, and radicals
- Use the fundamental laws of algebra in numeric and algebraic expressions
- Employ mathematical order of operations
- Understand technical measurement, approximation, the use of significant digits, and rounding
- Use scientific and engineering notations
- Convert units of measurement
- Rearrange and solve basic algebraic equations
- Interpret word problems using algebraic symbols

◀ **From the Great Pyramid of Giza, built in Egypt 4500 years ago, to the modern technology of today, mathematics has played a key role in the advancement of civilization. Along the way, important discoveries have been made in areas such as architecture, navigation, transportation, electronics, communication, and astronomy. Mathematics will continue to pave the way for new discoveries.**

1.1 Numbers

Real Number System • Number Line • Absolute Value • Signs of Inequality • Reciprocal • Denominate Numbers • Literal Numbers

In technology and science, as well as in everyday life, we use the very familiar **counting numbers,** or **natural numbers** 1, 2, 3, and so on. The **whole numbers** include 0 as well as all the natural numbers. Because it is necessary and useful to use negative numbers as well as positive numbers in mathematics and its applications, the natural numbers are called the **positive integers**, and the numbers -1, -2, -3, and so on are the **negative integers.**

Therefore, *the* **integers** *include the positive integers, the negative integers*, and **zero,** *which is neither positive nor negative*. This means that the integers are the numbers ..., -3, -2, -1, 0, 1, 2, 3, ... and so on.

A **rational number** *is a number that can be expressed as the division of one integer* ***a*** *by another nonzero integer b, and can be represented by the fraction* ***a*/*b*.** Here ***a*** *is the* **numerator** *and* ***b*** *is the* **denominator.** Here we have used algebra by letting letters represent numbers.

■ Irrational numbers were discussed by the Greek mathematician Pythagoras in about 540 B.C.E.

Another type of number, an **irrational number,** *cannot be written in the form of a fraction that is the division of one integer by another integer*. The following example illustrates integers, rational numbers, and irrational numbers.

EXAMPLE 1 Identifying rational numbers and irrational numbers

The numbers 5 and -19 are integers. They are also rational numbers because they can be written as $\frac{5}{1}$ and $\frac{-19}{1}$, respectively. Normally, we do not write the 1's in the denominators.

■ For reference, $\pi = 3.14159265\ldots$

The numbers $\frac{5}{8}$ and $\frac{-11}{3}$ are rational numbers because the numerator and the denominator of each are integers.

The numbers $\sqrt{2}$ and π are irrational numbers. It is not possible to find two integers, one divided by the other, to represent either of these numbers. In decimal form, irrational numbers are nonterminating, nonrepeating decimals. It can be shown that square roots (and other roots) that cannot be expressed exactly in decimal form are irrational. Also, $\frac{22}{7}$ is sometimes used as an *approximation* for π, but it is not equal *exactly* to π. We must remember that $\frac{22}{7}$ is rational and π is irrational.

■ A notation that is often used for repeating decimals is to place a bar over the digits that repeat. Using this notation we can write $\frac{1121}{1665} = 0.6\overline{732}$ and $\frac{2}{3} = 0.\overline{6}$.

The decimal number 1.5 is rational since it can be written as $\frac{3}{2}$. Any such *terminating decimal* is rational. The number 0.6666..., where the 6's continue on indefinitely, is rational because we may write it as $\frac{2}{3}$. In fact, any *repeating decimal* (in decimal form, a specific sequence of digits is repeated indefinitely) is rational. The decimal number 0.6732732732... is a repeating decimal where the sequence of digits 732 is repeated indefinitely $\left(0.6732732732\ldots = \frac{1121}{1665}\right)$. ■

The rational numbers together with the irrational numbers, including all such numbers that are positive, negative, or zero, make up the **real number system** (see Fig. 1.1). There are times we will encounter an **imaginary number,** *the name given to the square root of a*

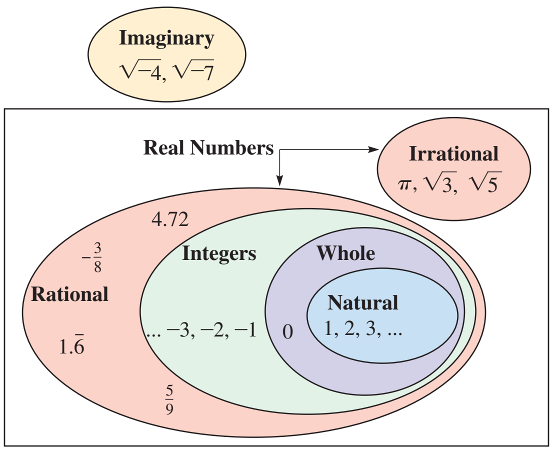


Fig. 1.1

negative number. Imaginary numbers are not real numbers and will be discussed in Chapter 12. However, unless specifically noted, we will use real numbers. Until Chapter 12, it will be necessary to only *recognize* imaginary numbers when they occur.

Also in Chapter 12, we will consider **complex numbers,** which include both the real numbers and imaginary numbers. See Exercise 39 of this section.

■ Real numbers and imaginary numbers are both included in the *complex number system*. See Exercise 39.

EXAMPLE 2 Identifying real numbers and imaginary numbers

(a) The number 7 is an integer. It is also rational because $7 = \frac{7}{1}$, and it is a real number since the real numbers include all the rational numbers.

(b) The number 3π is irrational, and it is real because the real numbers include all the irrational numbers.

(c) The numbers $\sqrt{-10}$ and $-\sqrt{-7}$ are imaginary numbers.

(d) The number $\frac{-3}{7}$ is rational and real. The number $-\sqrt{7}$ is irrational and real.

(e) The number $\frac{\pi}{6}$ is irrational and real. The number $\frac{\sqrt{-3}}{2}$ is imaginary. ■

■ Fractions were used by early Egyptians and Babylonians. They were used for calculations that involved parts of measurements, property, and possessions.

A **fraction** *may contain any number or symbol representing a number in its numerator or in its denominator*. The fraction indicates the division of the numerator by the denominator, as we previously indicated in writing rational numbers. Therefore, a fraction may be a number that is rational, irrational, or imaginary.

EXAMPLE 3 Fractions

(a) The numbers $\frac{2}{7}$ and $\frac{-3}{2}$ are fractions, and they are rational.

(b) The numbers $\frac{\sqrt{2}}{9}$ and $\frac{6}{\pi}$ are fractions, but they are not rational numbers. It is not possible to express either as one integer divided by another integer.

(c) The number $\frac{\sqrt{-5}}{6}$ is a fraction, and it is an imaginary number. ■

THE NUMBER LINE

Real numbers may be represented by points on a line. We draw a horizontal line and designate some point on it by O, which we call the **origin** (see Fig. 1.2). The integer *zero* is located at this point. Equal intervals are marked to the right of the origin, and the positive integers are placed at these positions. The other positive rational numbers are located between the integers. The points that cannot be defined as rational numbers represent irrational numbers. We cannot tell whether a given point represents a rational number or an irrational number unless it is specifically marked to indicate its value.

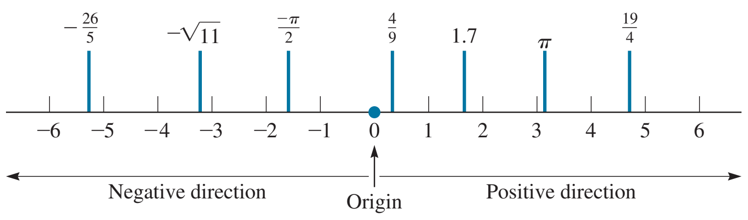


Fig. 1.2

The negative numbers are located on the number line by starting at the origin and marking off equal intervals *to the left, which is the* **negative direction.** As shown in Fig. 1.2, *the positive numbers are to the right of the origin and the negative numbers are to the left of the origin*. Representing numbers in this way is especially useful for graphical methods.

We next define another important concept of a number. *The* **absolute value** *of a positive number is the number itself, and the absolute value of a negative number is the corresponding positive number*. On the number line, we may interpret the absolute value of a number as the distance (which is always positive) between the origin and the number. Absolute value is denoted by writing the number between vertical lines, as shown in the following example.

EXAMPLE 4 Absolute value

The absolute value of 6 is 6, and the absolute value of -7 is 7. We write these as $|6| = 6$ and $|-7| = 7$. See Fig. 1.3.

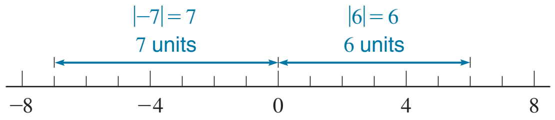


Fig. 1.3

Other examples are $\left|\frac{7}{5}\right| = \frac{7}{5}, \left|-\sqrt{2}\right| = \sqrt{2}$, $|0| = 0, -|\pi| = -\pi$, $|-5.29| = 5.29$, and $-|-9| = -9$ since $|-9| = 9$. ■

Practice Exercises

1. $|-4.2| = ?$ **2.** $-\left|-\frac{3}{4}\right| = ?$

■ The symbols $=$, $<$, and $>$ were introduced by English mathematicians in the late 1500s.

On the number line, *if a first number is to the right of a second number, then the first number is said to be* **greater than** *the second. If the first number is to the left of the second, it is* **less than** *the second number*. The symbol $>$ designates "is greater than," and the symbol $<$ designates "is less than." These are called **signs of inequality.** See Fig. 1.4.

EXAMPLE 5 Signs of inequality

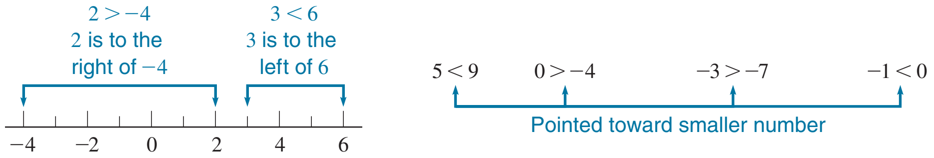


Fig. 1.4 ■

Practice Exercises

Place the correct sign of inequality ($<$ or $>$) between the given numbers.

3. $-5 \quad 4$ **4.** $0 \quad -3$

Every number, except zero, has a **reciprocal.** *The reciprocal of a number is 1 divided by the number*.

EXAMPLE 6 Reciprocal

The reciprocal of 7 is $\frac{1}{7}$. The reciprocal of $\frac{2}{3}$ is

$$\frac{1}{\frac{2}{3}} = 1 \times \frac{3}{2} = \frac{3}{2}$$ invert denominator and multiply (from arithmetic)

The reciprocal of 0.5 is $\frac{1}{0.5} = 2$. The reciprocal of $-\pi$ is $-\frac{1}{\pi}$. Note that the negative sign is retained in the reciprocal of a negative number.

We showed the multiplication of 1 and $\frac{3}{2}$ as $1 \times \frac{3}{2}$. We could also show it as $1 \cdot \frac{3}{2}$ or $1\left(\frac{3}{2}\right)$. We will often find the form with parentheses is preferable. ■

Practice Exercise

5. Find the reciprocals of

(a) -4 **(b)** $\frac{3}{8}$

In applications, *numbers that represent a measurement and are written with units of measurement are called* **denominate numbers.** The next example illustrates the use of units and the symbols that represent them.

EXAMPLE 7 Denominate numbers

■ For reference, see Appendix B for units of measurement and the symbols used for them.

(a) To show that a certain TV weighs 62 pounds, we write the weight as 62 lb.

(b) To show that a giant redwood tree is 330 feet high, we write the height as 330 ft.

(c) To show that the speed of a rocket is 1500 meters per second, we write the speed as 1500 m/s. (Note the use of s for second. We use s rather than sec.)

(d) To show that the area of a computer chip is 0.75 square inch, we write the area as 0.75 in.^2. (We will not use sq in.)

(e) To show that the volume of water in a glass tube is 25 cubic centimeters, we write the volume as 25 cm^3. (We will not use cu cm nor cc.) ■

It is usually more convenient to state definitions and operations on numbers in a general form. *To do this, we represent the numbers by letters, called* **literal numbers.** For example, if we want to say "If a first number is to the right of a second number on the number line, then the first number is greater than the second number," we can write "If a is to the right of b on the number line, then $a > b$." Another example of using a literal number is "The reciprocal of n is $1/n$."

Certain literal numbers may take on any allowable value, whereas other literal numbers represent the same value throughout the discussion. *Those literal numbers that may vary in a given problem are called* **variables,** and *those literal numbers that are held fixed are called* **constants.**

EXAMPLE 8 Variables and constants

(a) The resistance of an electric resistor is R. The current I in the resistor equals the voltage V divided by R, written as $I = V/R$. For this resistor, I and V may take on various values, and R is fixed. This means I and V are variables and R is a constant. For a *different* resistor, the value of R may differ.

(b) The fixed cost for a calculator manufacturer to operate a certain plant is b dollars per day, and it costs a dollars to produce each calculator. The total daily cost C to produce n calculators is

$$C = an + b$$

Here, C and n are variables, and a and b are constants, and the product of a and n is shown as an. For *another* plant, the values of a and b would probably differ.

If specific numerical values of a and b are known, say $a = \$7$ per calculator and $b = \$3000$, then $C = 7n + 3000$. Thus, constants may be numerical or literal. ■

EXERCISES 1.1

In Exercises 1–4, make the given changes in the indicated examples of this section, and then answer the given questions.

1. In the first line of Example 1, change the 5 to -7 and the -19 to 12. What other changes must then be made in the first paragraph?
2. In Example 4, change the 6 to -6. What other changes must then be made in the first paragraph?
3. In the left figure of Example 5, change the 2 to -6. What other changes must then be made?
4. In Example 6, change the $\frac{2}{3}$ to $\frac{3}{2}$. What other changes must then be made?

In Exercises 5–8, designate each of the given numbers as being an integer, rational, irrational, real, or imaginary. (More than one designation may be correct.)

5. $3, \sqrt{-4}$ **6.** $\frac{\sqrt{7}}{3}, -6$ **7.** $-\frac{\pi}{6}, \frac{1}{8}$ **8.** $-\sqrt{-6}, -2.33$

In Exercises 9 and 10, find the absolute value of each real number.

9. $3, -3, \frac{\pi}{4}, \sqrt{-1}$ **10.** $-0.857, \sqrt{2}, -\frac{19}{4}, \frac{\sqrt{-5}}{-2}$

In Exercises 11–18, insert the correct sign of inequality (> or <) between the given numbers.

11. 6 8 **12.** 7 5

13. π 3.1416 **14.** -4 0

15. -4 $-|-3|$ **16.** $-\sqrt{2}$ -1.42

17. $-\frac{2}{3}$ $-\frac{3}{4}$ **18.** -0.6 0.2

In Exercises 19 and 20, find the reciprocal of each number.

19. $3, -\frac{4}{\sqrt{3}}, \frac{y}{b}$ **20.** $-\frac{1}{3}$, 0.25, $2x$

In Exercises 21 and 22, locate (approximately) each number on a number line as in Fig. 1.2.

21. $2.5, -\frac{12}{5}, \sqrt{3}, -\frac{3}{4}$ **22.** $-\frac{\sqrt{2}}{2}, 2\pi, \frac{123}{19}, -\frac{7}{3}$

In Exercises 23–46, solve the given problems. Refer to Appendix B for units of measurement and their symbols.

23. Is an absolute value always positive? Explain.

24. Is -2.17 rational? Explain.

25. What is the reciprocal of the reciprocal of any positive or negative number?

26. Is the repeating decimal $2.\overline{72}$ rational or irrational?

27. True or False: A nonterminating, nonrepeating decimal is an irrational number.

28. If $b > a$ and $a > 0$, is $|b - a| < |b| - |a|$?

29. List the following numbers in numerical order, starting with the smallest: $-1, 9, \pi, \sqrt{5}, |-8|, -|-3|, -3.1$.

30. List the following numbers in numerical order, starting with the smallest: $\frac{1}{5}, -\sqrt{10}, -|-6|, -4, 0.25, |-\pi|$.

31. If a and b are positive integers and $b > a$, what type of number is represented by the following?

(a) $b - a$ **(b)** $a - b$ **(c)** $\frac{b - a}{b + a}$

32. If a and b represent positive integers, what kind of number is represented by (a) $a + b$, (b) a/b, and (c) $a \times b$?

33. For any positive or negative integer: (a) Is its absolute value always an integer? (b) Is its reciprocal always a rational number?

34. For any positive or negative rational number: (a) Is its absolute value always a rational number? (b) Is its reciprocal always a rational number?

35. Describe the location of a number x on the number line when (a) $x > 0$ and (b) $x < -4$.

36. Describe the location of a number x on the number line when (a) $|x| < 1$ and (b) $|x| > 2$.

37. For a number $x > 1$, describe the location on the number line of the reciprocal of x.

38. For a number $x < 0$, describe the location on the number line of the number with a value of $|x|$.

39. A *complex number* is defined as $a + bj$, where a and b are real numbers and $j = \sqrt{-1}$. For what values of a and b is the complex number $a + bj$ a real number? (All real numbers and all imaginary numbers are also complex numbers.)

40. A sensitive gauge measures the total weight w of a container and the water that forms in it as vapor condenses. It is found that $w = c\sqrt{0.1t + 1}$, where c is the weight of the container and t is the time of condensation. Identify the variables and constants.

41. In an electric circuit, the reciprocal of the total capacitance of two capacitors in series is the sum of the reciprocals of the capacitances $\left(\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}\right)$. Find the total capacitance of two capacitances of 0.0040 F and 0.0010 F connected in series.

42. Alternating-current (ac) voltages change rapidly between positive and negative values. If a voltage of 100 V changes to -200 V, which is greater in absolute value?

43. The memory of a certain computer has a bits in each byte. Express the number N of bits in n kilobytes in an equation. (A *bit* is a single digit, and bits are grouped in *bytes* in order to represent special characters. Generally, there are 8 bits per byte. If necessary, see Appendix B for the meaning of *kilo*.)

44. The computer design of the base of a truss is x ft long. Later it is redesigned and shortened by y in. Give an equation for the length L, in inches, of the base in the second design.

45. In a laboratory report, a student wrote "$-20°C > -30°C$." Is this statement correct? Explain.

46. After 5 s, the pressure on a valve is less than 60 lb/in.^{2} (pounds per square inch). Using t to represent time and p to represent pressure, this statement can be written "for $t > 5$ s, $p < 60$ lb/in.^{2}." In this way, write the statement "when the current I in a circuit is less than 4 A, the resistance R is greater than 12 Ω (ohms)."

Answers to Practice Exercises

1. 4.2 **2.** $-\frac{3}{4}$ **3.** $<$ **4.** $>$ **5.** **(a)** $-\frac{1}{4}$ **(b)** $\frac{8}{3}$

1.2 Fundamental Operations of Algebra

Fundamental Laws of Algebra • Operations on Positive and Negative Numbers • Order of Operations • Operations with Zero

If two numbers are added, it does not matter in which order they are added. (For example, $5 + 3 = 8$ and $3 + 5 = 8$, or $5 + 3 = 3 + 5$.) This statement, generalized and accepted as being correct for all possible combinations of numbers being added, is called the **commutative law** for addition. It states that *the sum of two numbers is the same,* *regardless of the order in which they are added*. We make no attempt to prove this law in general, but accept that it is true.

In the same way, we have the **associative law** for addition, which states that *the sum of three or more numbers is the same, regardless of the way in which they are grouped for addition*. For example, $3 + (5 + 6) = (3 + 5) + 6$.

The laws just stated for addition are also true for multiplication. Therefore, *the product of two numbers is the same, regardless of the order in which they are multiplied,* and *the product of three or more numbers is the same, regardless of the way in which they are grouped for multiplication*. For example, $2 \times 5 = 5 \times 2$, and $5 \times (4 \times 2) = (5 \times 4) \times 2$.

Another very important law is the **distributive law.** It states that *the product of one number and the sum of two or more other numbers is equal to the sum of the products of the first number and each of the other numbers of the sum*. For example,

■ Note carefully the difference:
associative law: $5 \times (4 \times 2)$
distributive law: $5 \times (4 + 2)$

$$5(4 + 2) = 5 \times 4 + 5 \times 2$$

In this case, it can be seen that the total is 30 on each side.

In practice, we use these **fundamental laws of algebra** naturally without thinking about them, except perhaps for the distributive law.

Not all operations are commutative and associative. For example, division is not commutative, because the order of division of two numbers does matter. For instance, $\frac{6}{5} \neq \frac{5}{6}$ ($\neq$ is read "does not equal"). (Also, see Exercise 54.)

Using literal numbers, the fundamental laws of algebra are as follows:

Commutative law of addition: $a + b = b + a$

Associative law of addition: $a + (b + c) = (a + b) + c$

Commutative law of multiplication: $ab = ba$

Associative law of multiplication: $a(bc) = (ab)c$

Distributive law: $a(b + c) = ab + ac$

■ Note the meaning of *identity*.

Each of these laws is an example of an *identity,* in that the expression to the left of the $=$ sign equals the expression to the right for any value of each of *a, b,* and *c*.

OPERATIONS ON POSITIVE AND NEGATIVE NUMBERS

When using the basic operations (addition, subtraction, multiplication, division) on positive and negative numbers, we determine the result to be either positive or negative according to the following rules.

Addition of two numbers of the same sign *Add their absolute values and assign the sum their common sign.*

EXAMPLE 1 Adding numbers of the same sign

(a) $2 + 6 = 8$ — the sum of two positive numbers is positive

(b) $-2 + (-6) = -(2 + 6) = -8$ — the sum of two negative numbers is negative

■ From Section 1.1, we recall that a positive number is preceded by no sign. Therefore, in using these rules, we show the "sign" of a positive number by simply writing the number itself.

The negative number -6 is placed in parentheses because it is also preceded by a plus sign showing addition. It is not necessary to place the -2 in parentheses. ■

Addition of two numbers of different signs *Subtract the number of smaller absolute value from the number of larger absolute value and assign to the result the sign of the number of larger absolute value.*

EXAMPLE 2 Adding numbers of different signs

(a) $2 + (-6) = -(6 - 2) = -4$

(b) $-6 + 2 = -(6 - 2) = -4$ — the negative 6 has the larger absolute value

(c) $6 + (-2) = 6 - 2 = 4$

(d) $-2 + 6 = 6 - 2 = 4$ — the positive 6 has the larger absolute value

the subtraction of absolute values ■

Subtraction of one number from another *Change the sign of the number being subtracted and change the subtraction to addition. Perform the addition.*

EXAMPLE 3 Subtracting positive and negative numbers

(a) $2 - 6 = 2 + (-6) = -(6 - 2) = -4$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 2(a).

(b) $-2 - 6 = -2 + (-6) = -(2 + 6) = -8$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 1(b).

(c) $-a - (-a) = -a + a = 0$

This shows that subtracting a number from itself results in zero, even if the number is negative. [Subtracting a negative number is equivalent to adding a positive number of the same absolute value.]

NOTE ▸

(d) $-2 - (-6) = -2 + 6 = 4$

(e) The change in temperature from $-12°C$ to $-26°C$ is

$-26°C - (-12°C) = -26°C + 12°C = -14°C$ ■

Multiplication and division of two numbers *The product (or quotient) of two numbers of the same sign is positive. The product (or quotient) of two numbers of different signs is negative.*

EXAMPLE 4 Multiplying and dividing positive and negative numbers

(a) $3(12) = 3 \times 12 = 36$ $\quad \frac{12}{3} = 4$ — result is positive if both numbers are positive

(b) $-3(-12) = 3 \times 12 = 36$ $\quad \frac{-12}{-3} = 4$ — result is positive if both numbers are negative

(c) $3(-12) = -(3 \times 12) = -36$ $\quad \frac{-12}{3} = -\frac{12}{3} = -4$ — result is negative if one number is positive and the other is negative

(d) $-3(12) = -(3 \times 12) = -36$ $\quad \frac{12}{-3} = -\frac{12}{3} = -4$ ■

Practice Exercises

Evaluate: **1.** $-5 - (-8)$ **2.** $-5(-8)$

ORDER OF OPERATIONS

Often, how we are to combine numbers is clear by grouping the numbers using symbols such as **parentheses,** ( ); the **bar,** ____, between the numerator and denominator of a fraction; and **vertical lines** for absolute value. Otherwise, for an expression in which there are several operations, we use the following order of operations.

Order of Operations

1. Perform operations within grouping symbols (parentheses, brackets, or absolute value symbols).
2. Perform multiplications and divisions (from left to right).
3. Perform additions and subtractions (from left to right).

EXAMPLE 5 Order of operations

■ Note that $20 \div (2 + 3) = \frac{20}{2+3}$, whereas $20 \div 2 + 3 = \frac{20}{2} + 3$.

(a) $20 \div (2 + 3)$ is evaluated by first adding $2 + 3$ and then dividing. The grouping of $2 + 3$ is clearly shown by the parentheses. Therefore, $20 \div (2 + 3) = 20 \div 5 = 4$.

(b) $20 \div 2 + 3$ is evaluated by first dividing 20 by 2 and then adding. No specific grouping is shown, and therefore the division is done before the addition. This means $20 \div 2 + 3 = 10 + 3 = 13$.

NOTE ➧ **(c)** [$16 - 2 \times 3$ is evaluated by *first multiplying* 2 by 3 and then subtracting. We do not first subtract 2 from 16.] Therefore, $16 - 2 \times 3 = 16 - 6 = 10$.

(d) $16 \div 2 \times 4$ is evaluated by first dividing 16 by 2 and then multiplying. From left to right, the division occurs first. Therefore, $16 \div 2 \times 4 = 8 \times 4 = 32$.

(e) $|3 - 5| - |-3 - 6|$ is evaluated by first performing the subtractions within the absolute value vertical bars, then evaluating the absolute values, and then subtracting. This means that $|3 - 5| - |-3 - 6| = |-2| - |-9| = 2 - 9 = -7$. ■

Practice Exercises

Evaluate: **3.** $12 - 6 \div 2$

4. $16 \div (2 \times 4)$

When evaluating expressions, it is generally more convenient to change the operations and numbers so that the result is found by the addition and subtraction of positive numbers. When this is done, we must remember that

$$a + (-b) = a - b \quad \textbf{(1.1)}$$

$$a - (-b) = a + b \quad \textbf{(1.2)}$$

EXAMPLE 6 Evaluating numerical expressions

(a) $7 + (-3) - 6 = 7 - 3 - 6 = 4 - 6 = -2$ using Eq. (1.1)

(b) $\frac{18}{-6} + 5 - (-2)(3) = -3 + 5 - (-6) = 2 + 6 = 8$ using Eq. (1.2)

(c) $\frac{|3 - 15|}{-2} - \frac{8}{4 - 6} = \frac{12}{-2} - \frac{8}{-2} = -6 - (-4) = -6 + 4 = -2$

(d) $\frac{-12}{2 - 8} + \frac{5 - 1}{2(-1)} = \frac{-12}{-6} + \frac{4}{-2} = 2 + (-2) = 2 - 2 = 0$

In illustration (b), we see that the division and multiplication were done before the addition and subtraction. In (c) and (d), we see that the groupings were evaluated first. Then we did the divisions, and finally the subtraction and addition. ■

Practice Exercises

Evaluate: **5.** $2(-3) - \frac{4 - 8}{2}$

6. $\frac{|5 - 15|}{2} - \frac{-9}{3}$

EXAMPLE 7 Evaluating—velocity after collision

A 3000-lb van going at 40 mi/h ran head-on into a 2000-lb car going at 20 mi/h. An insurance investigator determined the velocity of the vehicles immediately after the collision from the following calculation. See Fig. 1.5.

$$\frac{3000(40) + (2000)(-20)}{3000 + 2000} = \frac{120{,}000 + (-40{,}000)}{3000 + 2000} = \frac{120{,}000 - 40{,}000}{5000}$$

$$= \frac{80{,}000}{5000} = 16 \text{ mi/h}$$

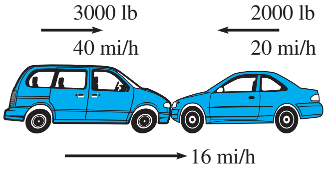


Fig. 1.5

The numerator and the denominator must be evaluated before the division is performed. The multiplications in the numerator are performed first, followed by the addition in the denominator and the subtraction in the numerator. ■

OPERATIONS WITH ZERO

Because operations with zero tend to cause some difficulty, we will show them here.

If a is a real number, the operations of addition, subtraction, multiplication, and division with zero are as follows:

$$a + 0 = a$$
$$a - 0 = a \qquad 0 - a = -a$$
$$a \times 0 = 0$$
$$0 \div a = \frac{0}{a} = 0 \qquad (\text{if } a \neq 0)$$

$\neq$ means "is not equal to"

EXAMPLE 8 Operations with zero

(a) $5 + 0 = 5$ **(b)** $-6 - 0 = -6$ **(c)** $0 - 4 = -4$

(d) $\frac{0}{6} = 0$ **(e)** $\frac{0}{-3} = 0$ **(f)** $\frac{5 \times 0}{7} = \frac{0}{7} = 0$ ■

Note that there is no result defined for division by zero. To understand the reason for this, consider the results for $\frac{6}{2}$ and $\frac{6}{0}$.

$$\frac{6}{2} = 3 \quad \text{since} \quad 2 \times 3 = 6$$

If $\frac{6}{0} = b$, then $0 \times b = 6$. This cannot be true because $0 \times b = 0$ for any value of b. Thus, ***division by zero is undefined***.

(The special case of $\frac{0}{0}$ is termed *indeterminate*. If $\frac{0}{0} = b$, then $0 = 0 \times b$, which is true for any value of b. Therefore, no specific value of b can be determined.)

EXAMPLE 9 Division by zero is undefined

$\frac{2}{5} \div 0$ is undefined $\frac{8}{0}$ is undefined $\frac{7 \times 0}{0 \times 6}$ is indeterminate ■

see above

CAUTION The operations with zero will not cause any difficulty if we remember to *never divide by zero.* ■

Division by zero is the only undefined basic operation. All the other operations with zero may be performed as for any other number.

EXERCISES 1.2

In Exercises 1–4, make the given changes in the indicated examples of this section, and then solve the resulting problems.

1. In Example 5(c), change 3 to (-2) and then evaluate.
2. In Example 6(b), change 18 to -18 and then evaluate.
3. In Example 6(d), interchange the 2 and 8 in the first denominator and then evaluate.
4. In the rightmost illustration in Example 9, interchange the 6 and the 0 above the 6. Is any other change needed?

In Exercises 5–38, evaluate each of the given expressions by performing the indicated operations.

5. $5 + (-8)$ **6.** $-4 + (-7)$ **7.** $-3 + 9$

8. $18 - 21$ **9.** $-19 - (-16)$ **10.** $-8 - (-10)$

11. $7(-4)$ **12.** $-9(3)$ **13.** $-7(-5)$

14. $\frac{-9}{3}$ **15.** $\frac{-6(20 - 10)}{-3}$ **16.** $\frac{-28}{-7(5 - 6)}$

17. $-2(4)(-5)$ **18.** $-3(-4)(-6)$ **19.** $2(2-7) \div 10$

20. $\dfrac{-64}{-2|4-8|}$ **21.** $16 \div 2(-4)$ **22.** $-20 \div 5(-4)$

23. $-9-|2-10|$ **24.** $(7-7) \div (5-7)$ **25.** $\dfrac{17-7}{7-7}$

26. $\dfrac{(7-7)(2)}{(7-7)(-1)}$ **27.** $8-3(-4)$ **28.** $-20+8 \div 4$

29. $-2(-6)+\left|\dfrac{8}{-2}\right|$ **30.** $\dfrac{|-2|}{-2}-(-2)(-5)$

31. $10(-8)(-3) \div (10-50)$ **32.** $\dfrac{7-|-5|}{-1(-2)}$

33. $\dfrac{24}{3+(-5)}-4(-9) \div (-3)$ **34.** $\dfrac{-18}{3}-\dfrac{4-|-6|}{-1}$

35. $-7-\dfrac{|-14|}{2(2-3)}-3|6-8|$ **36.** $-7(-3)+\dfrac{-6}{-3}-|-9|$

37. $\dfrac{3|-9-2(-3)|}{1+(-10)}$ **38.** $\dfrac{20(-12)-40(-15)}{98-|-98|}$

In Exercises 39–46, determine which of the fundamental laws of algebra is demonstrated.

39. $6(7) = 7(6)$ **40.** $6+8 = 8+6$

41. $6(3+1) = 6(3)+6(1)$ **42.** $4(5 \times \pi) = (4 \times 5)(\pi)$

43. $3+(5+9) = (3+5)+9$ **44.** $8(3-2) = 8(3)-8(2)$

45. $(\sqrt{5} \times 3) \times 9 = \sqrt{5} \times (3 \times 9)$

46. $(3 \times 6) \times 7 = 7 \times (3 \times 6)$

In Exercises 47–50, for numbers a and b, determine which of the following expressions equals the given expression.

(a) $a+b$ **(b)** $a-b$ **(c)** $b-a$ **(d)** $-a-b$

47. $-a+(-b)$ **48.** $b-(-a)$

49. $-b-(-a)$ **50.** $-a-(-b)$

In Exercises 51–66, solve the given problems. Refer to Appendix B for units of measurement and their symbols.

51. Insert the proper sign ($=, >, <$) to make the following true: $|5-(-2)| \quad |-5-|-2||$

52. Insert the proper sign ($=, >, <$) to make the following true: $|-3-|-7|| \quad ||-3|-7|$

53. (a) What is the sign of the product of an even number of negative numbers? (b) What is the sign of the product of an odd number of negative numbers?

54. Is subtraction commutative? Explain.

55. Explain why the following definition of the absolute value of a real number x is either correct or incorrect (the symbol $\geq$ means "is equal to or greater than"): If $x \geq 0$, then $|x| = x$; if $x < 0$, then $|x| = -x$.

56. Explain what is the error if the expression $24 - 6 \div 2 \cdot 3$ is evaluated as 27. What is the correct value?

57. Describe the values of x and y for which (a) $-xy = 1$ and (b) $\dfrac{x-y}{x-y} = 1$.

58. Describe the values of x and y for which (a) $|x+y| = |x| + |y|$ and (b) $|x-y| = |x| + |y|$.

59. The changes in the price of a stock (in dollars) for a given week were -0.68, $+0.42$, $+0.06$, -0.11, and $+0.02$. What was the total change in the stock's price that week?

60. Using subtraction of signed numbers, find the difference in the altitude of the bottom of the Dead Sea, 1396 m below sea level, and the bottom of Death Valley, 86 m below sea level.

61. Some solar energy systems are used to supplement the utility company power supplied to a home such that the meter runs backward if the solar energy being generated is greater than the energy being used. With such a system, if the solar power averages 1.5 kW for a 3.0-h period and only 2.1 kW · h is used during this period, what will be the change in the meter reading for this period? *Hint:* Solar power generated makes the meter run in the negative direction while power used makes it run in the positive direction.

62. A baseball player's batting average (total number of hits divided by total number of at-bats) is expressed in decimal form from 0.000 (no hits for all at-bats) to 1.000 (one hit for each at-bat). A player's batting average is often shown as 0.000 before the first at-bat of the season. Is this a correct batting average? Explain.

63. The daily high temperatures (in °C) for Edmonton, Alberta, in the first week in March were recorded as $-7, -3, 2, 3, 1, -4$, and -6. What was the average daily temperature for the week? (Divide the algebraic sum of readings by the number of readings.)

64. A flare is shot up from the top of a tower. Distances above the flare gun are positive and those below it are negative. After 5 s the vertical distance (in ft) of the flare from the flare gun is found by evaluating $(70)(5) + (-16)(25)$. Find this distance.

65. Find the sum of the voltages of the batteries shown in Fig. 1.6. Note the directions in which they are connected.

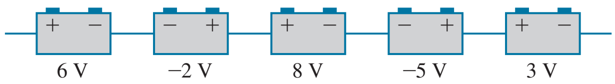


Fig. 1.6

66. A faulty gauge on a fire engine pump caused the apparent pressure in the hose to change every few seconds. The pressures (in lb/in.^{2}) above and below the set pressure were recorded as: $+7, -2, -9, -6$. What was the change between (a) the first two readings, (b) between the middle two readings, and (c) the last two readings?

67. One oil-well drilling rig drills 100 m deep the first day and 200 m deeper the second day. A second rig drills 200 m deep the first day and 100 m deeper the second day. In showing that the total depth drilled by each rig was the same, state what fundamental law of algebra is illustrated.

68. A water tank leaks 12 gal each hour for 7 h, and a second tank leaks 7 gal each hour for 12 h. In showing that the total amount leaked is the same for the two tanks, what fundamental law of algebra is illustrated?

69. On each of the 7 days of the week, a person spends 25 min on Facebook and 15 min on Twitter. Set up the expression for the total time spent on these two sites that week. What fundamental law of algebra is illustrated?

70. A jet travels 600 mi/h relative to the air. The wind is blowing at 50 mi/h. If the jet travels with the wind for 3 h, set up the expression for the distance traveled. What fundamental law of algebra is illustrated?

Answers to Practice Exercises

1. 3 **2.** 40 **3.** 9 **4.** 2 **5.** −4 **6.** 8

1.3 Calculators and Approximate Numbers

Graphing Calculators • Approximate Numbers • Significant Digits • Accuracy and Precision • Rounding Off • Operations with Approximate Numbers • Estimating Results

■ The calculator screens shown with text material are for a TI-84 Plus. They are intended only as an illustration of a calculator screen for the particular operation. Screens for other models may differ.

■ When less than half of a calculator screen is needed, a partial screen will be shown.

■ Some calculator keys on different models are labeled differently. For example, on some models, the EXE key is equivalent to the ENTER key.

■ Calculator keystrokes for various operations can be found by using the URLs given in this text. A list of all the calculator instructions is at `bit.ly/2NFYzDK`.

You will be doing many of your calculations on a calculator, and a *graphing calculator* can be used for these calculations and many other operations. In this text, we will restrict our coverage of calculator use to graphing calculators because a *scientific calculator* cannot perform many of the required operations we will cover.

A brief discussion of the graphing calculator appears in Appendix C, and sample calculator screens appear throughout the book. Since there are many models of graphing calculators, *the notation and screen appearance for many operations will differ from one model to another*. You should *practice using your calculator and* ***review its manual*** *to be sure how it is used*. Following is an example of a basic calculation done on a graphing calculator.

EXAMPLE 1 Calculating on a graphing calculator

In order to calculate the value of $38.3 - 12.9(-3.58)$, the numbers are entered as follows. The calculator will perform the multiplication first, following the order of operations shown in Section 1.2. The sign of -3.58 is entered using the (−) key, before 3.58 is entered. The display on the calculator screen is shown in Fig. 1.7.

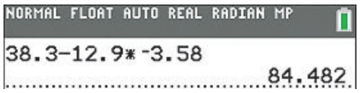


Fig. 1.7

38.3 [−] 12.9 [×] [(−)] 3.58 [ENTER] keystrokes

This means that $38.3 - 12.9(-3.58) = 84.482$.

Note in the display that the negative sign of -3.58 is smaller and a little higher to distinguish it from the minus sign for subtraction. Also note the * shown for multiplication; the asterisk is the standard computer symbol for multiplication. ■

Looking back into Section 1.2, we see that *the minus sign is used in two different ways:* (1) to indicate subtraction and (2) to designate a negative number. This is clearly shown on a graphing calculator because there is a key for each purpose. The [−] key is used for subtraction, and the [(−)] key is used before a number to make it negative.

We will first use a graphing calculator for the purpose of graphing in Section 3.5. Before then, we will show some calculational uses of a graphing calculator.

APPROXIMATE NUMBERS AND SIGNIFICANT DIGITS

Most numbers in technical and scientific work are **approximate numbers,** having been determined by some *measurement*. Certain other numbers are **exact numbers,** having been determined by a *definition* or *counting process*.

EXAMPLE 2 Approximate numbers and exact numbers

One person measures the distance between two cities on a map as 36 cm, and another person measures it as 35.7 cm. However, the distance cannot be measured *exactly*.

If a computer prints out the number of names on a list of 97, this 97 is exact. We know it is not 96 or 98. Since 97 was found from precise counting, it is exact.

By definition, 60 s = 1 min, and the 60 and the 1 are exact. ■

An approximate number may have to include some zeros to properly locate the decimal point. *Except for these zeros, all other digits are called* **significant digits.**

EXAMPLE 3 Significant digits

All numbers in this example are assumed to be approximate.

(a) 34.7 has three significant digits.

(b) 0.039 has two significant digits. The zeros properly locate the decimal point.

(c) 706.1 has four significant digits. The zero is not used for the location of the decimal point. It shows the number of tens in 706.1.

(d) 5.90 has three significant digits. ***The zero is not necessary as a placeholder*** and should not be written unless it is significant.

(e) 1400 has two significant digits, unless information is known about the number that makes either or both zeros significant. Without such information, we assume that the zeros are placeholders for proper location of the decimal point.

(f) Other approximate numbers with the number of significant digits are 0.0005 (one), 960,000 (two), 0.0709 (three), 1.070 (four), and 700.00 (five). ■

■ To show that zeros at the end of a whole number are significant, a notation that can be used is to place a bar over the last significant zero. Using this notation, $78,0\bar{0}0$ is shown to have four significant digits.

Practice Exercises

Determine the number of significant digits.

1. 1010 **2.** 0.1010

From Example 3, we see that *all nonzero digits are significant*. *Also, zeros not used as placeholders (for location of the decimal point) are significant*.

In calculations with approximate numbers, the number of significant digits and the position of the decimal point are important. *The* **accuracy** *of a number refers to the number of significant digits it has,* whereas *the* **precision** *of a number refers to the decimal position of the last significant digit*.

EXAMPLE 4 Accuracy and precision

One technician measured the thickness of a metal sheet as 3.1 cm and another technician measured it as 3.12 cm. Here, 3.12 is more precise since its last digit represents hundredths and 3.1 is expressed only to tenths. Also, 3.12 is more accurate since it has three significant digits and 3.1 has only two.

A concrete driveway is 230 ft long and 0.4 ft thick. Here, 230 is more accurate (two significant digits) and 0.4 is more precise (expressed to tenths). ■

The last significant digit of an approximate number is not exact. It has usually been determined by estimating or *rounding off*. However, it is not off by more than one-half of a unit in its place value.

EXAMPLE 5 Meaning of the last digit of an approximate number

When we write the measured distance on the map in Example 2 as 35.7 cm, we are saying that the distance is at least 35.65 cm and no more than 35.75 cm. Any value between these, rounded off to tenths, would be 35.7 cm.

In changing the fraction $\frac{2}{3}$ to the approximate decimal value 0.667, we are saying that the value is between 0.6665 and 0.6675. ■

■ On graphing calculators, it is possible to set the number of decimal places (to the right of the decimal point) to which results will be rounded off.

To **round off** *a number to a specified number of significant digits, discard all digits to the right of the last significant digit (replace them with zeros if needed to properly place the decimal point). If the first digit discarded is* 5 *or more, increase the last significant digit by* 1 *(round up). If the first digit discarded is less than* 5, *do not change the last significant digit (round down).*

EXAMPLE 6 Rounding off

(a) 70,360 rounded off to three significant digits is 70,400. Here, 3 is the third significant digit and the next digit is 6. Because $6 > 5$, the 3 is rounded up to 4 and the 6 is replaced with a zero to hold the place value.

(b) 70,430 rounded off to three significant digits, or to the nearest hundred, is 70,400. Here the 3 is replaced with a zero.

(c) 187.35 rounded off to four significant digits, or to tenths, is 187.4.

(d) 187.349 rounded off to four significant digits is 187.3. *We do not round up the* 4 *and then round up the* 3.

NOTE ▸ **(e)** 35.003 rounded off to four significant digits is 35.00. [We do not discard the zeros because they are significant and are not used only to properly place the decimal point.]

(f) 849,720 rounded off to three significant digits is $85\overline{0},000$. *The bar over the zero shows that digit is significant.* ■

Practice Exercises

Round off each number to three significant digits.

3. 2015 **4.** 0.3004

OPERATIONS WITH APPROXIMATE NUMBERS

NOTE ▸ [When performing operations on approximate numbers, we must express the result to an accuracy or precision that is valid.] Consider the following examples.

EXAMPLE 7 Precision—length of pipe

A pipe is made in two sections. One is measured as 16.3 ft long and the other as 0.927 ft long. What is the total length of the two sections together?

It may appear that we simply add the numbers as shown at the left. However, both numbers are approximate, and adding the smallest possible values and the largest possible values, the result differs by 0.1 (17.2 and 17.3) when rounded off to tenths. Rounded off to hundredths (17.18 and 17.28), they do not agree at all because the tenths digit is different. Thus, we get a good approximation for the total length if it is rounded off to *tenths,* the precision of the least precise length, and it is written as 17.2 ft. ■

	smallest values	largest values
16.3 ft	16.25 ft	16.35 ft
0.927 ft	0.9265 ft	0.9275 ft
17.227 ft	17.1765 ft	17.2775 ft

EXAMPLE 8 Accuracy—area of land plot

We find the area of the rectangular piece of land in Fig. 1.8 by multiplying the length, 207.54 ft, by the width, 81.4 ft. Using a calculator, we find that $(207.54)(81.4) = 16{,}893.756$. This apparently means the area is $16{,}893.756\ \text{ft}^2$.

However, the area should not be expressed with this accuracy. Because the length and width are both approximate, we have

$$(207.535\ \text{ft})(81.35\ \text{ft}) = 16{,}882.97225\ \text{ft}^2 \quad \text{least possible area}$$

$$(207.545\ \text{ft})(81.45\ \text{ft}) = 16{,}904.54025\ \text{ft}^2 \quad \text{greatest possible area}$$

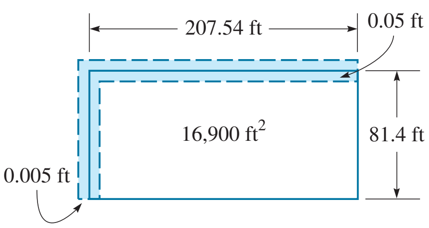


Fig. 1.8

These values agree when rounded off to three significant digits ($16{,}900\ \text{ft}^2$) but do not agree when rounded off to a greater accuracy. Thus, we conclude that the result is accurate only to *three* significant digits, the accuracy of the least accurate measurement, and that the area is written as $16{,}900\ \text{ft}^2$. ■

■ The results of operations on approximate numbers shown at the right are based on reasoning that is similar to that shown in Examples 7 and 8.

The Result of Operations on Approximate Numbers

1. When approximate numbers are added or subtracted, the result is expressed with the precision of the least precise number.
2. When approximate numbers are multiplied or divided, the result is expressed with the accuracy of the least accurate number.
3. When the root of an approximate number is found, the result is expressed with the accuracy of the number.
4. When approximate numbers and exact numbers are involved, the accuracy of the result is limited only by the approximate numbers.

CAUTION Always express the result of a calculation with the proper accuracy or precision. When using a calculator, if additional digits are displayed, round off the *final* result (do not round off in any of the intermediate steps). ■

■ When rounding off a number, it may seem difficult to discard the extra digits. However, if you keep those digits, you show a number with too great an accuracy, and it is incorrect to do so.

EXAMPLE 9 Adding approximate numbers

Find the sum of the approximate numbers 73.2, 8.0627, and 93.57.

Showing the addition in the standard way and using a calculator, we have

$$\begin{array}{r} 73.2 \\ 8.0627 \\ 93.57 \\ \hline 174.8327 \end{array}$$

73.2 ⟵ least precise number (expressed to tenths)

174.8327 ⟵ final display must be rounded to tenths

Therefore, the sum of these approximate numbers is 174.8. ■

EXAMPLE 10 Multiplying approximate numbers

In finding the product of the approximate numbers 2.4832 and 30.5 on a calculator, the final display shows 75.7376. However, since 30.5 has only three significant digits, the product is 75.7. ■

EXAMPLE 11 Combined operations

For problems with multiple operations that require using more than one of the rules given on the previous page, follow the correct order of operations as given in Section 1.2. Keep all the digits in the intermediate steps, but keep track of (perhaps by underlining) the significant digits that would be retained according to the appropriate rounding rule for each step. Then round off the final answer according to the last operation that is performed. For example,

$$(4.265 \times 2.60) \div (3.7 + 5.14) = \underline{11.0}89 \div \underline{8.8}4 = 1.3$$

Note that three significant digits are retained from the multiplication and one decimal place precision is retained from the addition. The final answer is rounded off to two significant digits, which is the accuracy of the least accurate number in the final division (based on the underlined significant digits). ■

Practice Exercise

Evaluate using a calculator.

5. $40.5 + \dfrac{3275}{-60.041}$ (Numbers are approximate.)

EXAMPLE 12 Operations with exact numbers and approximate numbers

Using the exact number 600 and the approximate number 2.7, we express the result to tenths if the numbers are added or subtracted. If they are multiplied or divided, we express the result to two significant digits. Since 600 is exact, the accuracy of the result depends only on the approximate number 2.7.

$$600 + 2.7 = 602.7 \qquad 600 - 2.7 = 597.3$$
$$600 \times 2.7 = 1600 \qquad 600 \div 2.7 = 220$$

There are 16 pieces in a pile of lumber and the average length of a piece is 482 mm. Here 16 is exact, but 482 is approximate. To get the total length of the pieces in the pile, the product $16 \times 482 = 7712$ must be rounded off to three significant digits, the accuracy of 482. Therefore, we can state that the total length is about 7710 mm. ■

NOTE [A note regarding the equal sign (=) is in order. We will use it for its defined meaning of "equals exactly" and when the result is an approximate number that has been properly rounded off.] Although $\sqrt{27.8} \approx 5.27$, where $\approx$ means "equals approximately," we write $\sqrt{27.8} = 5.27$, since 5.27 has been properly rounded off.

You should *make a rough estimate* of the result when using a calculator. An estimation may prevent accepting an incorrect result after using an incorrect calculator sequence, particularly if the calculator result is far from the estimated value.

EXAMPLE 13 Estimating results

In Example 1, we found that

$$38.3 - 12.9(-3.58) = 84.482 \quad \text{using exact numbers}$$

When using the calculator, if we forgot to make 3.58 negative, the display would be −7.882, or if we incorrectly entered 38.3 as 83.3, the display would be 129.482.

However, if we estimate the result as

$$40 - 10(-4) = 80$$

we know that a result of −7.882 or 129.482 cannot be correct.

When estimating, we can often use one-significant-digit approximations. If the calculator result is far from the estimate, we should do the calculation again. ■

EXERCISES 1.3

In Exercises 1–4, make the given changes in the indicated examples of this section, and then solve the given problems.

1. In Example 3(b), change 0.039 to 0.390. Is there any change in the conclusion?
2. In Example 6(e), change 35.003 to 35.303 and then find the result.
3. In Example 10, change 2.4832 to 2.5 and then find the result.
4. In Example 13, change 12.9 to 21.9 and then find the estimated value.

In Exercises 5–10, determine whether the given numbers are approximate or exact.

5. A car with 8 cylinders travels at 55 mi/h.
6. A computer chip 0.002 mm thick is priced at $7.50.
7. In 24 h there are 1440 min.
8. A calculator has 50 keys, and its battery lasted for 50 h of use.
9. A cube of copper 1 cm on an edge has a mass of 9 g.
10. Of a building's 90 windows, 75 were replaced 15 years ago.

In Exercises 11–18, determine the number of significant digits in each of the given approximate numbers.

11. 107; 3004; 1040 **12.** 3600; 730; 2055

13. 6.80; 6.08; 0.068 **14.** 0.8730; 0.0075; 0.0305

15. 3000; 3000.1; 3000.10 **16.** 1.00; 0.01; 0.0100

17. 5000; 5000.0; $500\bar{0}$ **18.** 200; $20\bar{0}$; 200.00

In Exercises 19–24, determine which of the pair of approximate numbers is (a) more precise and (b) more accurate.

19. 30.8; 0.010 **20.** 0.041; 7.673

21. 0.1; 78.0 **22.** 7040; 0.004

23. 7000; 0.004 **24.** 50.060; $|-8.914|$

In Exercises 25–32, round off the given approximate numbers (a) to three significant digits and (b) to two significant digits.

25. 4.936 **26.** 80.53 **27.** −50.893 **28.** 7.004

29. 5968 **30.** 30.96 **31.** 0.9449 **32.** 0.9999

In Exercises 33–42, perform the indicated operations assuming all numbers are approximate and round the answer appropriately. For Exercises 37–42, use the procedure shown in Example 11.

33. $12.78 + 1.0495 - 1.633$ **34.** $35.78 - 22.944 + 5.1$

35. $0.6572(3.94)$ **36.** $6.3(4.18) \div 3.7$

37. $8.75 + (1.2)(3.84)$ **38.** $28 - \frac{20.955}{2.2}$

39. $\frac{8.75(15.32)}{8.75 + 15.32}$ **40.** $\frac{8.97(4.003)}{2.0 + 4.78}$

41. $4.52 - \frac{2.056(309.6)}{395.2}$ **42.** $8.195 + \frac{14.9}{1.7 + 2.1}$

In Exercises 43–46, perform the indicated operations. The first number is approximate, and the second number is exact.

43. $0.9788 + 14.9$ **44.** $17.311 - 22.98$

45. $-3.142(65)$ **46.** $8.62 \div 1728$

In Exercises 47–50, answer the given questions. Refer to Appendix B for units of measurement and their symbols.

47. The manual for a heart monitor lists the frequency of the ultrasound wave as 2.75 MHz. What are the least possible and the greatest possible frequencies?
48. A car manufacturer states that the engine displacement for a certain model is 2400 cm^3. What should be the least possible and greatest possible displacements?
49. A flash of lightning struck a tower 3.25 mi from a person. The thunder was heard 15 s later. The person calculated the speed of sound and reported it as 1144 ft/s. What is wrong with this conclusion?
50. A technician records 4.4 s as the time for a robot arm to swing from the extreme left to the extreme right, 2.72 s as the time for the return swing, and 1.68 s as the difference in these times. What is wrong with this conclusion?

In Exercises 51–58, perform the calculations on a calculator without rounding.

51. Evaluate: (a) $2.2 + 3.8 \times 4.5$ (b) $(2.2 + 3.8) \times 4.5$

52. Evaluate: (a) $6.03 \div 2.25 + 1.77$ (b) $6.03 \div (2.25 + 1.77)$

53. Evaluate: (a) $2 + 0$ (b) $2 - 0$ (c) $0 - 2$ (d) 2×0 (e) $2 \div 0$ Compare with operations with zero on page 10.

54. Evaluate: (a) $2 \div 0.0001$ and $2 \div 0$ (b) $0.0001 \div 0.0001$ and $0 \div 0$ (c) Explain why the displays differ.

55. Show that π is not equal exactly to (a) 3.1416, or (b) 22/7.

56. At some point in the decimal equivalent of a rational number, some sequence of digits will start repeating endlessly. An irrational number never has an endlessly repeating sequence of digits. Find the decimal equivalents of (a) 8/33 and (b) π. Note the repetition for 8/33 and that no such repetition occurs for π.

57. Following Exercise 56, show that the decimal equivalents of the following fractions indicate they are rational: (a) 1/3 (b) 5/11 (c) 2/5. What is the repeating part of the decimal in (c)?

58. Following Exercise 56, show that the decimal equivalent of the fraction 124/990 indicates that it is rational. Why is the last digit different?

In Exercises 59–64, assume that all numbers are approximate unless stated otherwise.

59. In 3 successive days, a home solar system produced 32.4 MJ, 26.704 MJ, and 36.23 MJ of energy. What was the total energy produced in these 3 days?

60. A shipment contains eight LED televisions, each weighing 24.6 lb, and five video game consoles, each weighing 15.3 lb. What is the total weight of the shipment?

61. Certain types of iPhones and iPads weigh approximately 129 g and 298.8 g, respectively. What is the total weight of 12 iPhones and 16 iPads of these types? (*Source:* Apple.com.)

62. Find the voltage in a certain electric circuit by multiplying the sum of the resistances 15.2 Ω, 5.64 Ω, and 101.23 Ω by the current 3.55 A.

63. The percent of alcohol in a certain car engine coolant is found by performing the calculation $\dfrac{100(40.63 + 52.96)}{105.30 + 52.96}$. Find this percent of alcohol. The number 100 is exact.

64. The tension (in N) in a cable lifting a crate at a construction site was found by calculating the value of $\dfrac{50.45(9.80)}{1 + 100.9 \div 23}$, where the 1 is exact. Calculate the tension.

In Exercises 65 and 66, all numbers are approximate. (a) Estimate the result mentally using one-significant-digit approximations of all the numbers, and (b) compute the result using the appropriate rounding rules and compare with the estimate.

65. $7.84 \times 4.932 - 11.317$ **66.** $21.6 - 53.14 \div 9.64$

Answers to Practice Exercises

1. 3 **2.** 4 **3.** 2020 **4.** 0.300 **5.** -14.0

1.4 Exponents and Unit Conversions

Positive Integer Exponents • Zero and Negative Exponents • Order of Operations • Evaluating Algebraic Expressions • Converting Units

In mathematics and its applications, we often have a number multiplied by itself several times. To show this type of product, we use the notation a^n, where a is the number and n is the number of times it appears. *In the expression a^n, the number a is called the* **base,** *and n is called the* **exponent;** in words, a^n *is read as "the* **nth power of *a*.**"

EXAMPLE 1 Meaning of exponents

(a) $4 \times 4 \times 4 \times 4 \times 4 = 4^5$ — the fifth power of 4

(b) $(-2)(-2)(-2)(-2) = (-2)^4$ — the fourth power of -2

(c) $a \times a = a^2$ — the second power of a, called "a squared"

(d) $\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right) = \left(\frac{1}{5}\right)^3$ — the third power of $\frac{1}{5}$, called "$\frac{1}{5}$ cubed" ■

We now state the basic operations with exponents using positive integers as exponents. Therefore, with m and n as positive integers, we have the following operations:

■ Two forms are shown for Eq. (1.4) in order that the resulting exponent is a positive integer. We consider negative and zero exponents after the next three examples.

$$a^m \times a^n = a^{m+n} \tag{1.3}$$

$$\frac{a^m}{a^n} = a^{m-n} \quad (m > n, a \neq 0) \qquad \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \quad (m < n, a \neq 0) \tag{1.4}$$

$$(a^m)^n = a^{mn} \tag{1.5}$$

$$(ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0) \tag{1.6}$$

EXAMPLE 2 Illustrating Eqs. (1.3) and (1.4)

■ In a^3, which equals $a \times a \times a$, each a is called a factor. A more general definition of factor is given in Section 1.7.

Using Eq. (1.3): (add exponents)

$$a^3 \times a^5 = a^{3+5} = a^8$$

Using the meaning of exponents:

$$a^3 \times a^5 = \underbrace{(a \times a \times a)}_{3 \text{ factors of } a}\underbrace{(a \times a \times a \times a \times a)}_{5 \text{ factors of } a} = a^8 \quad (8 \text{ factors of } a)$$

■ Here we are using the fact that a (not zero) divided by itself equals 1, or $a/a = 1$.

Using first form Eq. (1.4): ($5 > 3$)

$$\frac{a^5}{a^3} = a^{5-3} = a^2$$

Using the meaning of exponents:

$$\frac{a^5}{a^3} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times a \times a}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}}} = a^2$$

■ Note that Eq. (1.3) can be verified numerically, for example, by

$2^3 \times 2^5 = 8 \times 32 = 256$

$2^3 \times 2^5 = 2^{3+5} = 2^8 = 256$

Using second form Eq. (1.4): ($5 > 3$)

$$\frac{a^3}{a^5} = \frac{1}{a^{5-3}} = \frac{1}{a^2}$$

Using the meaning of exponents:

$$\frac{a^3}{a^5} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}}}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times a \times a} = \frac{1}{a^2}$$ ■

EXAMPLE 3 Illustrating Eqs. (1.5) and (1.6)

Using Eq. (1.5): (multiply exponents)

$$(a^5)^3 = a^{5(3)} = a^{15}$$

Using the meaning of exponents:

$$(a^5)^3 = (a^5)(a^5)(a^5) = a^{5+5+5} = a^{15}$$

Using first form Eq. (1.6):

$$(ab)^3 = a^3b^3$$

Using the meaning of exponents:

$$(ab)^3 = (ab)(ab)(ab) = a^3b^3$$

Using second form Eq. (1.6):

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

Using the meaning of exponents:

$$\left(\frac{a}{b}\right)^3 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) = \frac{a^3}{b^3}$$ ■

CAUTION When an expression involves a product or a quotient of different bases, only exponents of the same base may be combined. ■ Consider the following example.

EXAMPLE 4 Other illustrations of exponents

CAUTION In illustration (b), note that ax^2 means a times the square of x and does not mean a^2x^2, whereas $(ax)^3$ does mean a^3x^3. ■

(a) $(-x^2)^3 = [(-1)x^2]^3 = (-1)^3(x^2)^3 = -x^6$

(b) $ax^2(ax)^3 = ax^2(a^3x^3) = a^4x^5$ (exponent of 1; add exponents of a; add exponents of x)

(c) $\dfrac{(3 \times 2)^4}{(3 \times 5)^3} = \dfrac{3^4 2^4}{3^3 5^3} = \dfrac{3 \times 2^4}{5^3}$

(d) $\dfrac{(ry^3)^2}{r(y^2)^4} = \dfrac{r^2y^6}{ry^8} = \dfrac{r}{y^2}$ ■

Practice Exercises

Use Eqs. (1.3)–(1.6) to simplify the given expressions.

1. $ax^3(-ax)^2$ **2.** $\dfrac{(2c)^5}{(3cd)^2}$

EXAMPLE 5 Exponents—beam deflection

In analyzing the amount a beam bends, the following simplification may be used. (P is the force applied to a beam of length L; E and I are constants related to the beam.)

$$\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{2}{3}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{2}{3}\right)\left(\frac{L^2}{2^2}\right)$$

$$= \frac{\overset{1}{\cancel{2}}\,PL(L^2)}{\underset{1}{\cancel{2}}(3)(4)(4)EI} = \frac{PL^3}{48EI}$$

In *simplifying* this expression, we combined exponents of L and divided out the 2 that was a factor common to the numerator and the denominator. ■

ZERO AND NEGATIVE EXPONENTS

If $n = m$ in Eq. (1.4), we have $a^m/a^m = a^{m-m} = a^0$. Also, $a^m/a^m = 1$, since any nonzero quantity divided by itself equals 1. Therefore, for Eq. (1.4) to hold for $m = n$,

$$a^0 = 1 \quad (a \neq 0) \tag{1.7}$$

Equation (1.7) states that *any nonzero expression raised to the zero power is* 1. Zero exponents can be used with any of the operations for exponents.

EXAMPLE 6 Zero as an exponent

(a) $5^0 = 1$ **(b)** $(-3)^0 = 1$ **(c)** $-(-3)^0 = -1$ **(d)** $(2x)^0 = 1$

(e) $(ax + b)^0 = 1$ **(b)** $(a^2b^0c)^2 = a^4c^2$ ($b^0 = 1$) **(g)** $2t^0 = 2(1) = 2$

We note in illustration (*g*) that *only t is raised to the zero power*. If the quantity $2t$ were raised to the zero power, it would be written as $(2t)^0$, as in part (d). ■

Practice Exercise

3. Evaluate: $-(3x)^0$

Applying both forms of Eq. (1.4) to the case where $n > m$ leads to the definition of a negative exponent. For example, applying both forms to a^2/a^7, we have

$$\frac{a^2}{a^7} = a^{2-7} = a^{-5} \quad \text{and} \quad \frac{a^2}{a^7} = \frac{1}{a^{7-2}} = \frac{1}{a^5}$$

■ Although positive exponents are generally preferred in a final result, there are some cases in which zero or negative exponents are to be used. Also, negative exponents are very useful in some operations that we will use later.

For these results to be equal, then $a^{-5} = 1/a^5$. Thus, if we define

$$a^{-n} = \frac{1}{a^n} \quad (a \neq 0) \tag{1.8}$$

then all the laws of exponents will hold for negative integers.

■ Note carefully the difference in parts (d) and (e) of Example 7.

Practice Exercises

Simplify: **4.** $\frac{-7^0}{c^{-3}}$ **5.** $\frac{(3x)^{-1}}{2a^{-2}}$

EXAMPLE 7 Negative exponents

(a) $3^{-1} = \frac{1}{3}$ **(b)** $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ **(c)** $\frac{1}{a^{-3}} = a^3$ change signs of exponents

(d) $(3x)^{-1} = \frac{1}{3x}$ **(e)** $3x^{-1} = 3\left(\frac{1}{x}\right) = \frac{3}{x}$ **(f)** $\left(\frac{a^3}{x}\right)^{-2} = \frac{(a^3)^{-2}}{x^{-2}} = \frac{a^{-6}}{x^{-2}} = \frac{x^2}{a^6}$ ■

ORDER OF OPERATIONS

We have seen that the basic operations on numbers must be performed in a particular order. Since raising a number to a power is actually multiplication, it is performed before additions and subtractions, and in fact, before multiplications and divisions.

■ The use of exponents is taken up in more detail in Chapter 11.

Order of Operations

1. Operations within grouping symbols
2. Exponents
3. Multiplications and divisions (from left to right)
4. Additions and subtractions (from left to right)

EXAMPLE 8 Using order of operations

(a) $8 - (-1)^2 - 2(-3)^2 = 8 - 1 - 2(9)$

$= 8 - 1 - 18 = -11$

apply exponents first, then multiply, and then subtract

(b) $806 \div (26.1 - 9.09)^2 = 806 \div (\underline{17.01})^2$

$= 806 \div \underline{289}.3401 = 2.79$

subtract inside parentheses first, then square the answer, and then divide

NOTE ▸ [In part (b), the significant digits retained from each intermediate step are underlined.] ■

EXAMPLE 9 Even and odd powers

Using the meaning of a power of a number, we have

$$(-2)^2 = (-2)(-2) = 4 \quad (-2)^3 = (-2)(-2)(-2) = -8$$

$$(-2)^4 = 16 \quad (-2)^5 = -32 \quad (-2)^6 = 64 \quad (-2)^7 = -128$$

NOTE ▸ [Note that a negative number raised to an even power gives a positive value, and a negative number raised to an odd power gives a negative value.] ■

EVALUATING ALGEBRAIC EXPRESSIONS

An algebraic expression is **evaluated** *by* **substituting** *given values of the literal numbers in the expression and calculating the result.* On a calculator, the $\boxed{x^2}$ key is used to square numbers, and the $\boxed{\wedge}$ or $\boxed{x^y}$ key is used for other powers.

To calculate the value of $20 \times 6 + 200/5 - 3^4$, we use the key sequence

$$20 \boxed{\times} 6 \boxed{+} 200 \boxed{\div} 5 \boxed{-} 3 \boxed{\wedge} 4$$

Fig. 1.9

with the result of 79 shown in the display of Fig. 1.9. **Note that calculators are programmed to follow the correct order of operations.**

EXAMPLE 10 Evaluating an expression—free-fall distance

The distance (in ft) that an object falls in 4.2 s is found by substituting 4.2 for t in the expression $16.0t^2$ as shown below:

$$16.0(4.2)^2 = 280 \text{ ft}$$

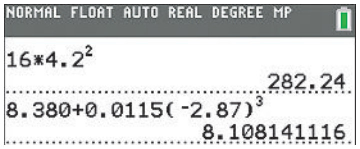


Fig. 1.10

The calculator result from the first line of Fig. 1.10 has been rounded off to two significant digits, the accuracy of 4.2. ■

EXAMPLE 11 Evaluating an expression—length of a wire

A wire made of a special alloy has a length L (in m) given by $L = a + 0.0115T^3$, where T (in °C) is the temperature (between $-4°C$ and $4°C$). To find the wire length for L for $a = 8.380$ m and $T = -2.87°C$, we substitute these values to get

$$L = 8.380 + 0.0115(-2.87)^3 = 8.108 \text{ m}$$

The calculator result from the second line of Fig. 1.10 has been rounded to the nearest thousandth. ■

OPERATIONS WITH UNITS AND UNIT CONVERSIONS

Many problems in science and technology require us to perform operations on numbers with units. *For multiplication, division, powers, or roots, whatever operation is performed on the numbers also is performed on the units. For addition and subtraction, only numbers with the same units can be combined, and the answer will have the same units as the numbers in the problem*. Essentially, units are treated the same as any other algebraic symbol.

EXAMPLE 12 Algebraic operations with units

(a) $(2 \text{ ft})(4 \text{ lb}) = 8 \text{ ft} \cdot \text{lb}$ — the dot symbol represents multiplication

(b) $255 \text{ m} + 121 \text{ m} = 376 \text{ m}$ — note that the units are not added

(c) $(3.45 \text{ in.})^2 = 11.9 \text{ in.}^2$ — the unit is squared as well as the number

(d) $\left(65.0\frac{\text{mi}}{\text{h}}\right)(3.52 \text{ h}) = 229 \text{ mi}$ — note that $\frac{\text{mi}}{\not{\text{h}}} \times \frac{\not{\text{h}}}{1} = \frac{\text{mi}}{1} = \text{mi}$

(e) $\frac{8.48 \text{g}}{(1.69\text{m})^3} = \frac{8.48\text{g}}{(1.69)^3\text{m}^3} = 1.76 \text{ g/m}^3$ — the units are divided

(f) $\left(\frac{8.75 \text{ mi}}{1.32 \text{ min}^2}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)^2\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) = \frac{(8.75 \text{ mi})(1 \text{ min}^2)(5280 \text{ ft})}{(1.32 \text{ min}^2)(3600 \text{ s}^2)(1 \text{ mi})}$ — the units min^2 and mi both cancel

$$= 9.72 \text{ ft/s}^2$$ ■

Often, it is necessary to **convert from one set of units to another.** This can be accomplished by using **conversion factors** (for example, 1 in. = 2.54 cm). Several useful conversion factors are shown in Table 1.1.

Metric prefixes are sometimes attached to units to indicate they are multiplied by a given power of ten. Table 1.2 (on the next page) shows some commonly used prefixes.

Table 1.1 Conversion Factors

Length	*Volume/Capacity*	*Weight/Mass*	*Energy/Power*
1 in. = 2.54 cm (exact)	1 ft^3 = 28.32 L	1 lb = 453.6 g	1 Btu = 778.2 ft · lb
1 ft = 0.3048 m (exact)	1 L = 1.057 qt	1 kg = 2.205 lb	1 ft · lb = 1.356 J
1 km = 0.6214 mi	1 gal = 3.785 L	1 lb = 4.448 N	1 hp = 550 ft · lb/s (exact)
1 mi = 5280 ft (exact)			1 hp = 746.0 W

Table 1.2 Metric Prefixes

Prefix	*Factor*	*Symbol*
exa	10^{18}	E
peta	10^{15}	P
tera	10^{12}	T
giga	10^{9}	G
mega	10^{6}	M
kilo	10^{3}	k
hecto	10^{2}	h
deca	10^{1}	da
deci	10^{-1}	d
centi	10^{-2}	c
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f
atto	10^{-18}	a

■ See Appendix B for a description of the U.S. Customary and SI units, as well as a list of all the units used in this text and their symbols.

When a conversion factor is written in fractional form, *the fraction has a value of 1 since the numerator and denominator represent the same quantity*. For example, $\frac{1\text{ in.}}{2.54\text{ cm}} = 1$ or $\frac{1\text{ km}}{1000\text{ m}} = 1$. To convert units, we *multiply* the given number (including its units) by one or more of these fractions placed in such a way that the units we wish to eliminate will cancel and the units we wish to retain will remain in the answer. Since we are multiplying the given number by fractions that have a value of 1, the original quantity remains unchanged even though it will be expressed in different units.

EXAMPLE 13 Converting units

(a) The length of a certain smartphone is 13.8 cm. Convert this to inches.

$$13.8\text{ cm} = \underbrace{\frac{13.8\text{ cm}}{1}}_{\text{original number}} \times \underbrace{\frac{1\text{ in.}}{2.54\text{ cm}}}_{1} = \frac{(13.8)(1)\text{ in.}}{(1)(2.54)} = 5.43\text{ in.}$$

Notice that the unit cm appears in both the numerator and denominator and therefore cancels, leaving only the unit inches in the final answer.

(b) A car is traveling at 65.0 mi/h. Convert this speed to km/min (kilometers per minute).

From Table 1.1, we note that $1\text{ km} = 0.6214\text{ mi}$, and we know that $1\text{ h} = 60\text{ min}$. Using these values, we have

$$65.0\frac{\text{mi}}{\text{h}} = \frac{65.0\text{ mi}}{1\text{ h}} \times \frac{1\text{ km}}{0.6214\text{ mi}} \times \frac{1\text{ h}}{60\text{ min}} = \frac{(65.0)(1)(1)\text{ km}}{(1)(0.6214)(60)\text{ min}}$$
$$= 1.74\text{ km/min}$$

We note that the units mi and h appear in both numerator and denominator and therefore cancel out, leaving the units km and min. Also note that the 1's and 60 are exact.

(c) The density of iron is 7.86 g/cm^3 (grams per cubic centimeter). Express this density in kg/m^3 (kilograms per cubic meter).

From Table 1.2, $1\text{ kg} = 1000\text{ g}$ exactly, and $1\text{ cm} = 0.01\text{ m}$ exactly. Therefore,

$$7.86\frac{\text{g}}{\text{cm}^3} = \frac{7.86\text{ g}}{1\text{ cm}^3} \times \frac{1\text{ kg}}{1000\text{ g}} \times \left(\frac{1\text{ cm}}{0.01\text{ m}}\right)^3 = \frac{(7.86)(1)(1^3)\text{ kg}}{(1)(1000)(0.01^3)\text{ m}^3}$$
$$= \frac{(7.86)\text{ kg}}{0.001\text{ m}^3} = 7860\text{ kg/m}^3$$

Here, the units g and cm^3 are in both numerator and denominator and therefore cancel out, leaving units of kg and m^3. Also, all numbers are exact, except 7.86. ■

Practice Exercise

6. Convert 725 g/cm^2 to lb/in.^2

EXERCISES 1.4

In Exercises 1 and 2, make the given changes in the indicated examples of this section, and then simplify the resulting expression.

1. In Example 4(a), change $(-x^2)^3$ to $(-x^3)^2$.

2. In Example 6(d), change $(2x)^0$ to $2x^0$.

In Exercises 3–42, simplify the given expressions. Express results with positive exponents only.

3. x^3x^4 **4.** y^2y^7 **5.** $2b^4b^2$ **6.** $3k^5k$

7. $\frac{m^5}{m^3}$ **8.** $\frac{2x^6}{-x}$ **9.** $\frac{-n^5}{7n^9}$ **10.** $\frac{3s}{s^4}$

11. $(P^2)^4$ **12.** $(x^8)^3$ **13.** $(aT^2)^{30}$ **14.** $(3r^2)^3$

15. $\left(\frac{2}{b}\right)^3$ **16.** $\left(\frac{F}{t}\right)^{20}$ **17.** $\left(\frac{x^2}{-2}\right)^4$ **18.** $\left(\frac{3}{n^3}\right)^3$

19. $(8a)^0$ **20.** $-v^0$ **21.** $-3x^0$ **22.** $-(-2)^0$

23. 6^{-1} **24.** $-w^{-5}$ **25.** $\frac{1}{R^{-2}}$ **26.** $\frac{1}{-t^{-48}}$

27. $(-t^2)^7$ **28.** $(-y^3)^5$ **29.** $-\dfrac{L^{-3}}{L^{-5}}$ **30.** $2i^{40}i^{-70}$

31. $\dfrac{2v^4}{(2v)^4}$ **32.** $\dfrac{x^2x^3}{(x^2)^3}$ **33.** $\dfrac{(n^2)^4}{(n^4)^2}$ **34.** $\dfrac{(3t)^{-1}}{-3t^{-1}}$

35. $(\pi^0 x^2 a^{-1})^{-1}$ **36.** $(3m^{-2}n^4)^{-2}$ **37.** $(-8g^{-1}s^3)^2$

38. $ax^{-2}(-a^2x)^3$ **39.** $\left(\dfrac{4x^{-1}}{a^{-1}}\right)^{-3}$ **40.** $\left(\dfrac{2b^2}{-y^5}\right)^{-2}$

41. $\dfrac{15n^2T^5}{3n^{-1}T^6}$ **42.** $\dfrac{(nRT^{-2})^{32}}{R^{-2}T^{32}}$

In Exercises 43–50, evaluate the given expressions. In Exercises 45–50, all numbers are approximate.

43. $7(-4) - (-5)^2$ **44.** $6 - |-2|^5 - (-2)(8)$

45. $-(-26.5)^2 - (-9.85)^3$ **46.** $-0.711^2 - (-|-0.809|)^6$

47. $\dfrac{3.07(-|-1.86|)}{(-1.86)^4 + 1.596}$ **48.** $\dfrac{15.66^2 - (-4.017)^4}{1.044(-3.68)}$

49. $2.38(-10.7)^2 - \dfrac{254}{1.17^3}$ **50.** $4.2(2.6) + \dfrac{0.889}{1.89 - 1.09^2}$

In Exercises 51–62, perform the indicated operations.

51. Does $\left(\dfrac{1}{x^{-1}}\right)^{-1}$ represent the reciprocal of x?

52. Does $\left(\dfrac{0.2 - 5^{-1}}{10^{-2}}\right)^0$ equal 1? Explain.

53. If $a^3 = 5$, then what does a^{12} equal?

54. Is $a^{-2} < a^{-1}$ for any negative value of a? Explain.

55. If a is a positive integer, simplify $(x^a \cdot x^{-a})^5$.

56. If a and b are positive integers, simplify $(-y^{a-b} \cdot y^{a+b})^2$.

57. In developing the "big bang" theory of the origin of the universe, the expression $(kT/(hc))^3(GkThc)^2c$ arises. Simplify this expression.

58. In studying planetary motion, the expression $(GmM)(mr)^{-1}(r^{-2})$ arises. Simplify this expression.

59. In designing a cam for a fire engine pump, the expression $\pi\left(\dfrac{r}{2}\right)^3\left(\dfrac{4}{3\pi r^2}\right)$ is used. Simplify this expression.

60. For a certain integrated electric circuit, it is necessary to simplify the expression $\dfrac{gM(2\pi fM)^{-2}}{2\pi fC}$. Perform this simplification.

61. If \$2500 is invested at 2.1% interest, compounded quarterly, the amount in the account after 6 years is $2500(1 + 0.021/4)^{24}$. Round the answer to the nearest penny.

62. In designing a building, it was determined that the forces acting on an I beam would deflect the beam an amount (in cm), given by $\dfrac{x(1000 - 20x^2 + x^3)}{1850}$, where x is the distance (in m) from one end of the beam. Find the deflection for $x = 6.85$ m. (The 1000 and 20 are exact.)

63. Calculate the value of $\left(1 + \dfrac{1}{n}\right)^n$ for $n = 1, 10, 100, 1000$ on a calculator. Round to four decimal places. (For even larger values of n, the value will never exceed 2.7183. The limiting value is a number called e, which will be important in future chapters.)

64. For computer memory, the metric prefixes have an unusual meaning: 1 KB = 2^{10} bytes, 1 MB = 2^{10} KB, 1 GB = 2^{10} MB, and 1 TB = 2^{10} GB. How many bytes are there in 1 TB? (KB is kilobyte, MB is megabyte, GB is gigabyte, TB is terabyte.)

In Exercises 65–68, perform the indicated operations and attach the correct units to your answers.

65. $\left(28.2\dfrac{\text{ft}}{\text{s}}\right)(9.81\text{ s})$

66. $\left(40.5\dfrac{\text{mi}}{\text{gal}}\right)(3.7\text{ gal})$

67. $\left(7.25\dfrac{\text{m}}{\text{s}^2}\right)\left(\dfrac{1\text{ ft}}{0.3048\text{ m}}\right)\left(\dfrac{60\text{ s}}{1\text{ min}}\right)^2$

68. $\left(238\dfrac{\text{kg}}{\text{m}^3}\right)\left(\dfrac{1000\text{ g}}{1\text{ kg}}\right)\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)^3$

In Exercises 69–74, make the indicated conversions.

69. 15.7 qt to L **70.** 7.50 W to hp

71. 245 cm^2 to in.^2 **72.** 85.7 mi^2 to km^2

73. 65.2 $\dfrac{\text{m}}{\text{s}}$ to $\dfrac{\text{ft}}{\text{min}}$ **74.** 25.0 $\dfrac{\text{mi}}{\text{gal}}$ to $\dfrac{\text{km}}{\text{L}}$

In Exercises 75–82, solve the given problems.

75. A laptop computer has a screen that measures 15.6 in. across its diagonal. Convert this to centimeters.

76. GPS satellites orbit the Earth at an altitude of about 12,500 mi. Convert this to kilometers.

77. A wastewater treatment plant processes 575,000 gal/day. Convert this to liters per hour.

78. Water flows from a fire hose at a rate of 85 gal/min. Convert this to liters per second.

79. The speed of sound is about 1130 ft/s. Change this to kilometers per hour.

80. A military jet flew at a rate of 7200 km/h. What is this speed in meters per second?

81. At sea level, atmospheric pressure is about 14.7 lb/in.^{2}. Express this in pascals (Pa). *Hint:* A pascal is a N/m^2 (see Appendix B).

82. The density of water is about 62.4 lb/ft^3. Convert this to kilograms per cubic meter.

Answers to Practice Exercises

1. a^3x^5 **2.** $\dfrac{2^5c^3}{3^2d^2} = \dfrac{32c^3}{9d^2}$ **3.** -1 **4.** $-c^3$ **5.** $\dfrac{a^2}{6x}$ **6.** 10.3 lb/in.^{2}

1.5 Scientific Notation

Meaning of Scientific Notation • Changing Numbers to and from Scientific Notation • Scientific Notation on a Calculator • Engineering Notation

■ Television was invented in the 1920s and first used commercially in the 1940s.

The use of fiber optics was developed in the 1950s.

X-rays were discovered by Roentgen in 1895.

In technical and scientific work, we encounter numbers that are inconvenient to use in calculations. Examples are: radio and television signals travel at 30,000,000,000 cm/s; the mass of Earth is 6,600,000,000,000,000,000,000 tons; a fiber in a fiber-optic cable has a diameter of 0.000005 m; some X-rays have a wavelength of 0.000000095 cm. Although calculators and computers can handle such numbers, a convenient and useful notation, called *scientific notation*, is used to represent these or any other numbers.

A number in **scientific notation** *is expressed as the product of a number greater than or equal to* 1 *and less than* 10, *and a power of* 10, *and is written as*

$$P \times 10^k$$

where $1 \leq P < 10$ and k is an integer. (The symbol $\leq$ means "is less than or equal to.")

EXAMPLE 1 Scientific notation

(a) $34,000 = 3.4 \times 10,000 = 3.4 \times 10^4$ **(b)** $6.82 = 6.82 \times 1 = 6.82 \times 10^0$

between 1 and 10

(c) $0.00503 = \frac{5.03}{1000} = \frac{5.03}{10^3} = 5.03 \times 10^{-3}$ ■

From Example 1, we see how a number is changed from ordinary notation to scientific notation. NOTE ▸ [The decimal point is moved so that only one nonzero digit is to its left. The number of places moved is the power of 10 (k), which is positive if the decimal point is moved to the left and negative if moved to the right.] To change a number from scientific notation to ordinary notation, this procedure is reversed. The next two examples illustrate these procedures.

EXAMPLE 2 Changing numbers to scientific notation

(a) $34,000 = 3.4 \times 10^4$ **(b)** $6.82 = 6.82 \times 10^0$ **(c)** $0.00503 = 5.03 \times 10^{-3}$

4 places to left　　0 places　　3 places to right ■

EXAMPLE 3 Changing numbers to ordinary notation

(a) To change 5.83×10^6 to ordinary notation, we move the decimal point six places to the right, including additional zeros to properly locate the decimal point.

$$5.83 \times 10^6 = 5,830,000$$

6 places to right

(b) To change 8.06×10^{-3} to ordinary notation, we must move the decimal point three places to the left, again including additional zeros to properly locate the decimal point.

$$8.06 \times 10^{-3} = 0.00806$$

3 places to left ■

Practice Exercises

1. Change 2.35×10^{-3} to ordinary notation.

2. Change 235 to scientific notation.

Scientific notation provides a practical way of handling very large or very small numbers. First, all numbers are expressed in scientific notation. Then the calculation can be done with numbers between 1 and 10, using the laws of exponents to find the power of ten of the result. Thus, scientific notation gives an important use of exponents.

■ See Exercise 43 of Exercises 1.1 for a brief note on computer data.

EXAMPLE 4 Scientific notation in calculations—processing rate

A 4.25 GHz computer processor can process 4.25×10^9 instructions in one second. The number of instructions it can process in 55.0 seconds is found by multiplying:

$$(4.25 \times 10^9)(55.0) = 2.34 \times 10^{11} \text{ instructions}$$

As shown, it is proper to leave the result (*rounded off*) in scientific notation. ■

Another advantage of scientific notation is that it clearly shows the precise number of significant digits when the final significant digit is 0, making it unnecessary to use the "bar" notation introduced in Section 1.3.

EXAMPLE 5 Scientific notation and significant digits—gravity

In determining the gravitational force between two stars 750,000,000,000 km apart, it is necessary to evaluate $750{,}000{,}000{,}000^2$. If 750,000,000,000 has *three* significant digits, we can show this by writing

$$75\overline{0},000,000,000^2 = (7.50 \times 10^{11})^2 = 7.50^2 \times 10^{2\times 11} = 56.3 \times 10^{22}$$

Since 56.3 is not between 1 and 10, we can write this result in scientific notation as

$$56.3 \times 10^{22} = (5.63 \times 10)(10^{22}) = 5.63 \times 10^{23}$$ ■

We can enter numbers in scientific notation on a calculator, as well as have the calculator give results automatically in scientific notation. See the next example.

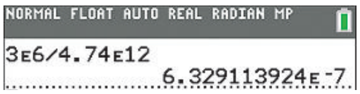


Fig. 1.11

EXAMPLE 6 Scientific notation on a calculator—wavelength

The wavelength λ (in m) of the light in a red laser beam can be found from the following calculation. Note the significant digits in the numerator.

$$\lambda = \frac{3{,}000{,}000}{4{,}740{,}000{,}000{,}000} = \frac{3.00 \times 10^6}{4.74 \times 10^{12}} = 6.33 \times 10^{-7} \text{ m}$$

The key sequence is 3 [EE] 6 [÷] 4.74 [EE] 12 [ENTER]. See Fig. 1.11. ■

Another commonly used notation, which is similar to scientific notation, is **engineering notation.** A number expressed in engineering notation is of the form

$$P \times 10^k$$

where $\mathbf{1 \leq P < 1000}$ and ***k* is an integral multiple of 3.** Since the exponent k is a multiple of 3, the metric prefixes in Table 1.2 (Section 1.4) can be used to replace the power of ten. For example, an infrared wave that has a frequency of 850×10^9 Hz written in engineering notation can also be expressed as 850 GHz. The prefix *giga* (G) replaces the factor of 10^9.

EXAMPLE 7 Engineering notation and metric prefixes

Express each of the following quantities using engineering notation, and then replace the power of ten with the appropriate metric prefix.

less than 1000 → ; ← multiple of 3

(a) $48{,}000{,}000\ \Omega = 48 \times 10^6\ \Omega = 48\ \text{M}\Omega$

(b) $0.00000036\ \text{m} = 360 \times 10^{-9}\ \text{m} = 360\ \text{nm}$

(c) $1.3 \times 10^{-4}\ \text{A} = 0.00013\ \text{A} = 130 \times 10^{-6}\ \text{A} = 130\ \mu\text{A}$ ■

Practice Exercise

3. Write 0.0000728 s in engineering notation and using the appropriate metric prefix.

EXERCISES 1.5

In Exercises 1 and 2, make the given changes in the indicated examples of this section, and then rewrite the number as directed.

1. In Example 3(b), change the exponent -3 to 3 and then write the number in ordinary notation.
2. In Example 5, change the exponent 2 to -1 and then write the result in scientific notation.

In Exercises 3–10, change the numbers from scientific notation to ordinary notation.

3. 4.5×10^4 **4.** 6.8×10^7 **5.** 2.01×10^{-3} **6.** 9.61×10^{-5}

7. 3.23×10^0 **8.** 8×10^0 **9.** 1.86×10 **10.** 1×10^{-1}

In Exercises 11–18, change the numbers from ordinary notation to scientific notation.

11. 4000 **12.** 56,000 **13.** 0.0087 **14.** 0.00074

15. 609,000,000 **16.** 10 **17.** 0.0528 **18.** 0.0000908

In Exercises 19–22, perform the indicated calculations using a calculator and by first expressing all numbers in scientific notation. Assume that all numbers are exact.

19. 28,000(2,000,000,000) **20.** 50,000(0.006)

21. $\frac{88{,}000}{0.0004}$ **22.** $\frac{0.00003}{6{,}000{,}000}$

In Exercises 23–28, change the number from ordinary notation to engineering notation.

23. 35,600,000 **24.** 0.0000056 **25.** 0.0973

26. 925,000,000,000 **27.** 0.000000475 **28.** 370,000

In Exercises 29–32, perform the indicated calculations and then check the result using a calculator. Assume that all numbers are exact.

29. $2 \times 10^{-35} + 3 \times 10^{-34}$ **30.** $5.3 \times 10^{12} - 3.7 \times 10^{10}$

31. $(1.2 \times 10^{29})^3$ **32.** $(2 \times 10^{-16})^{-5}$

In Exercises 33–40, perform the indicated calculations using a calculator. All numbers are approximate.

33. 1320(649,000)(85.3) **34.** 0.0000569(3,190,000)

35. $\frac{0.0732(6710)}{0.00134(0.0231)}$ **36.** $\frac{0.00452}{2430(97{,}100)}$

37. $(3.642 \times 10^{-8})(2.736 \times 10^5)$ **38.** $\frac{(7.309 \times 10^{-1})^2}{5.9843(2.5036 \times 10^{-20})}$

39. $\frac{(3.69 \times 10^{-7})(4.61 \times 10^{21})}{0.0504}$ **40.** $\frac{(9.9 \times 10^7)(1.08 \times 10^{12})^2}{(3.603 \times 10^{-5})(2054)}$

In Exercises 41–50, change numbers in ordinary notation to scientific notation or change numbers in scientific notation to ordinary notation. See Appendix B for an explanation of symbols used.

41. The average number of tweets per day on Twitter in 2020 was 500,000,000.

42. A certain laptop computer has 2,048,000,000,000 bytes of memory.

43. A fiber-optic system requires 0.000003 W of power.

44. A red blood cell measures 0.0075 mm across.

45. The frequency of a certain cell phone signal is 1,200,000,000 Hz.

46. The PlayStation 5 game console has a graphic processing unit that can perform 1.028×10^{13} floating point operations per second (10.28 teraflops). (*Source*: playstation.com.)

47. The Gulf of Mexico oil spill in 2010 covered more than 12,000,000,000 m^2 of ocean surface.

48. A *parsec*, a unit used in astronomy, is about 3.086×10^{16} m.

49. The power of the signal of a laser beam probe is 1.6×10^{-12} W.

50. The electrical force between two electrons is about 2.4×10^{-43} times the gravitational force between them.

In Exercises 51–56, solve the given problems.

51. Write the following numbers in engineering notation and then replace the power of 10 with the appropriate metric prefix.
(a) 2300 W (b) 0.23 W (c) 2,300,000 W (d) 0.00023 W

52. Write the following numbers in engineering notation and then replace the power of 10 with the appropriate metric prefix.
(a) 8,090,000 Ω (b) 809,000 Ω (c) 0.0809 Ω

53. A *googol* is defined as 1 followed by 100 zeros. (a) Write this number in scientific notaion. (b) A *googolplex* is defined as 10 to the googol power. Write this number using powers of 10, and not the word *googol*. (Note the name of the Internet company.)

54. The number of electrons in the universe has been estimated at 10^{79}. How many times greater is a googol than the estimated number of electrons in the universe? (See Exercise 53.)

55. The diameter of the sun, 1.4×10^9 m, is about 109 times the diameter of Earth. Express the diameter of Earth in scientific notation.

56. GB means *gigabyte* where *giga* means *billion*, or 10^9. Actually, 1 GB $= 2^{30}$ bytes. Use a calculator to show that the use of *giga* is a reasonable choice of terminology.

In Exercises 57–60, perform the indicated calculations.

57. A computer can do an addition in 7.5×10^{-15} s. How long does it take to perform 5.6×10^6 additions?

58. Uranium is used in nuclear reactors to generate electricity. About 0.000000039% of the uranium disintegrates each day. How much of 0.085 mg of uranium disintegrates in a day?

59. If it takes 0.078 s for a GPS signal traveling at 2.998×10^8 m/s to reach the receiver in a car, find the distance from the receiver to the satellite.

60. (a) Determine the number of seconds in a day in scientific notation. (b) Using the result of part (a), determine the number of seconds in a century (assume 365.24 days/year).

In Exercises 61–64, perform the indicated calculations by first expressing all numbers in scientific notation.

61. One *atomic mass unit* (amu) is 1.66×10^{-27} kg. If one oxygen atom has 16 amu (an exact number), what is the mass of 125,000,000 oxygen atoms?

62. The rate of energy radiation (in W) from an object is found by evaluating the expression kT^4, where T is the thermodynamic temperature. Find this value for the human body, for which $k = 0.000000057$ W/K^4 and $T = 303$ K.

63. In a microwave receiver circuit, the resistance R of a wire 1 m long is given by $R = k/d^2$, where d is the diameter of the wire. Find R if $k = 0.00000002196\ \Omega \cdot \text{m}^2$ and $d = 0.00007998$ m.

64. The average distance between the sun and Earth, 149,600,000 km, is called an *astronomical unit* (AU). If it takes light 499.0 s to travel 1 AU, what is the speed of light? Compare this with the speed of the GPS signal in Exercise 59.

Answers to Practice Exercises

1. 0.00235 **2.** 2.35×10^2 **3.** 72.8×10^{-6} s, 72.8 μs

1.6 Roots and Radicals

Principal *n*th Root • Simplifying Radicals • Using a Calculator • Imaginary Numbers

At times, we have to find the *square root* of a number, or maybe some other root of a number, such as a *cube root*. This means we must find a number that when squared, or cubed, and so on equals some given number. For example, to find the square root of 9, we must find a number that when squared equals 9. In this case, either 3 or −3 is an answer. Therefore, *either* 3 *or* −3 *is a square root of* 9 *since* $3^2 = 9$ and $(-3)^2 = 9$.

To have a general notation for the square root and have it represent *one* number, *we define the* **principal square root** *of a to be positive if a is positive and represent it by* $\sqrt{a}$. This means $\sqrt{9} = 3$ and not −3.

■ Unless we state otherwise, when we refer to the root of a number, it is the principal root.

The general notation for the **principal *n*th root** *of a is* $\sqrt[n]{a}$. (When $n = 2$, do not write the 2 for n.) *The* $\sqrt{\ }$ *sign is called a* **radical sign**.

EXAMPLE 1 Roots of numbers

(a) $\sqrt{2}$ (the square root of 2) **(b)** $\sqrt[3]{2}$ (the cube root of 2)

(c) $\sqrt[4]{2}$ (the fourth root of 2) **(d)** $\sqrt[7]{6}$ (the seventh root of 6) ■

NOTE [To have a single defined value for all roots (not just square roots) and to consider only real-number roots, we define the **principal *n*th root** of a to be positive if a is positive and to be negative if a is negative and n is odd.] (If a is negative and n is even, the roots are not real.)

EXAMPLE 2 Principal *n*th root

(a) $\sqrt{169} = 13$ $(\sqrt{169} \neq -13)$ **(b)** $-\sqrt{64} = -8$

(c) $\sqrt[3]{27} = 3$ since $3^3 = 27$ **(d)** $\sqrt{0.04} = 0.2$ since $0.2^2 = 0.04$

odd

(e) $-\sqrt[4]{256} = -4$ **(f)** $\sqrt[3]{-27} = -3$ **(g)** $-\sqrt[3]{27} = -(+3) = -3$ ■

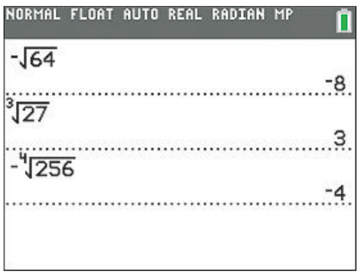


Fig. 1.12

Graphing calculator keystrokes: bit.ly/2OV0mtn

The calculator evaluations of (b), (c), and (e) are shown in Fig. 1.12. The [√] key is used for square roots and other roots are listed under the [MATH] key.

Another property of square roots is developed by noting illustrations such as $\sqrt{36} = \sqrt{4 \times 9} = \sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$. In general, this property states that *the square root of a product of positive numbers is the product of their square roots*.

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (a \text{ and } b \text{ positive real numbers}) \tag{1.9}$$

■ Try this one on your calculator: $\sqrt{12345678987654321}$

This property is used in simplifying radicals. It is most useful if either a or b is a **perfect square,** *which is the square of a rational number*.

EXAMPLE 3 Simplifying square roots

(a) $\sqrt{8} = \sqrt{(4)(2)} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

perfect squares — simplest form

(b) $\sqrt{75} = \sqrt{(25)(3)} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

(c) $\sqrt{4 \times 10^2} = \sqrt{4}\sqrt{10^2} = 2(10) = 20$

(Note that the square root of the square of a positive number is that number.) ■

In order to represent the square root of a number *exactly*, use Eq. (1.9) to write it in simplest form. However, in many applied problems, a decimal *approximation* obtained from a calculator is acceptable.

EXAMPLE 4 Approximating a square root—rocket descent

After reaching its greatest height, the time (in s) for a rocket to fall h ft is found by evaluating $0.25\sqrt{h}$. Find the time for the rocket to fall 1260 ft.

Using a calculator,

$$0.25\sqrt{1260} = 8.9 \text{ s}$$

The rocket takes 8.9 s to fall 1260 ft. The result from the calculator is rounded off to two significant digits, the accuracy of 0.25 (an approximate number). ■

In simplifying a radical, *all operations under a radical sign must be done before finding the root*.

EXAMPLE 5 More on simplifying square roots

(a) $\sqrt{16 + 9} = \sqrt{25}$ first perform the addition 16 + 9

$= 5$

NOTE ▸ [However, $\sqrt{16 + 9}$ is *not* $\sqrt{16} + \sqrt{9} = 4 + 3 = 7$.]

(b) $\sqrt{2^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$,

NOTE ▸ [However, $\sqrt{2^2 + 6^2}$ is *not* $\sqrt{2^2} + \sqrt{6^2} = 2 + 6 = 8$.] ■

Practice Exercises

Simplify:

1. $\sqrt{12}$ **2.** $\sqrt{36 + 144}$

In defining the principal square root, we did not define the square root of a negative number. However, in Section 1.1, we defined the square root of a negative number to be an **imaginary number.** More generally, *the even root of a negative number is an imaginary number, and the odd root of a negative number is a negative real number*.

EXAMPLE 6 Imaginary roots and real roots

even: $\sqrt{-64}$ is imaginary

even: $\sqrt[4]{-243}$ is imaginary

odd: $\sqrt[3]{-64} = -4$ (a real number) ■

Practice Exercise

3. Is $\sqrt[3]{-8}$ real or imaginary? If it is real, evaluate it.

A much more detailed coverage of roots, radicals, and imaginary numbers is taken up in Chapters 11 and 12.

EXERCISES 1.6

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the given problems.

1. In Example 2(b), change the square root to a cube root and then evaluate.

2. In Example 3(b), change $\sqrt{(25)(3)}$ to $\sqrt{(15)(5)}$ and explain whether or not this would be a better expression to use.

3. In Example 5(a), change the + to × and then evaluate.

4. In the first illustration of Example 6, place a $-$ sign before the radical. Is there any other change in the statement?

In Exercises 5–38, simplify the given expressions. In each of 5–9 and 12–21, the result is an integer.

5. $\sqrt{49}$ **6.** $\sqrt{225}$ **7.** $-\sqrt{121}$ **8.** $-\sqrt{36}$

9. $-\sqrt{64}$ **10.** $\sqrt{0.25}$ **11.** $\sqrt{0.09}$ **12.** $-\sqrt{900}$

13. $\sqrt[3]{125}$ **14.** $\sqrt[4]{16}$ **15.** $\sqrt[4]{81}$ **16.** $-\sqrt[5]{-32}$

17. $(\sqrt{5})^2$ **18.** $(\sqrt[3]{31})^3$ **19.** $(-\sqrt[3]{-47})^3$ **20.** $(\sqrt[5]{-23})^5$

21. $(-\sqrt[4]{53})^4$ **22.** $\sqrt{75}$ **23.** $\sqrt{18}$ **24.** $-\sqrt{32}$

25. $\sqrt{1200}$ **26.** $\sqrt{50}$ **27.** $2\sqrt{84}$ **28.** $\frac{\sqrt{108}}{2}$

29. $\sqrt{\frac{80}{|3-7|}}$ **30.** $\sqrt{81 \times 10^2}$ **31.** $\sqrt[3]{-8^2}$ **32.** $\sqrt[4]{9^2}$

33. $\frac{7^2\sqrt{81}}{(-3)^2\sqrt{49}}$ **34.** $\frac{2^5\sqrt[5]{-243}}{-3\sqrt{144}}$ **35.** $\sqrt{36+64}$

36. $\sqrt{25+144}$ **37.** $\sqrt{3^2+9^2}$ **38.** $\sqrt{8^2-4^2}$

In Exercises 39–46, find the value of each square root by use of a calculator. Each number is approximate.

39. $\sqrt{85.4}$ **40.** $\sqrt{3762}$ **41.** $\sqrt{0.8152}$ **42.** $\sqrt{0.0627}$

43. **(a)** $\sqrt{1296+2304}$ **(b)** $\sqrt{1296}+\sqrt{2304}$

44. **(a)** $\sqrt{10.6276+2.1609}$ **(b)** $\sqrt{10.6276}+\sqrt{2.1609}$

45. **(a)** $\sqrt{0.0429^2-0.0183^2}$ **(b)** $\sqrt{0.0429^2}-\sqrt{0.0183^2}$

46. **(a)** $\sqrt{3.625^2+0.614^2}$ **(b)** $\sqrt{3.625^2}+\sqrt{0.614^2}$

In Exercises 47–58, solve the given problems.

47. The speed (in mi/h) of a car that skids to a stop on dry pavement is often estimated by $\sqrt{24s}$, where s is the length (in ft) of the skid marks. Estimate the speed if $s = 150$ ft.

48. The resistance in an amplifier circuit is found by evaluating $\sqrt{Z^2 - X^2}$. Find the resistance for $Z = 5.362\ \Omega$ and $X = 2.875\ \Omega$.

49. The speed (in m/s) of sound in seawater is found by evaluating $\sqrt{B/d}$ for $B = 2.18 \times 10^9$ Pa and $d = 1.03 \times 10^3$ kg/m^3. Find this speed, which is important in locating underwater objects using sonar.

50. The terminal speed (in m/s) of a skydiver can be approximated by $\sqrt{40m}$, where m is the mass (in kg) of the skydiver. Calculate the terminal speed (after reaching this speed, the skydiver's speed remains fairly constant before opening the parachute) of a 75-kg skydiver.

51. A TV screen is 52.3 in. wide and 29.3 in. high. The length of a diagonal (the dimension used to describe it—from one corner to the opposite corner) is found by evaluating $\sqrt{w^2 + h^2}$, where w is the width and h is the height. Find the diagonal.

52. A car costs \$38,000 new and is worth \$24,000 2 years later. The annual rate of depreciation is found by evaluating $100(1 - \sqrt{V/C})$, where C is the cost and V is the value after 2 years. At what rate did the car depreciate? (100 and 1 are exact.)

53. A *tsunami* is a very high ocean tidal wave (or series of waves) often caused by an earthquake. An Alaskan tsunami in 1958 measured over 500 m high; an Asian tsunami in 2004 killed over 230,000 people; a tsunami in Japan in 2011 killed over 10,000 people. An equation that approximates the speed v (in m/s) of a tsunami is $v = \sqrt{gd}$, where $g = 9.8$ m/s^2 and d is the average depth (in m) of the ocean floor. Find v (in km/h) for $d = 3500$ m (valid for many parts of the Indian Ocean and Pacific Ocean).

54. The greatest distance (in km) a person can see from a height h (in m) above the ground is $\sqrt{1.27 \times 10^4 h + h^2}$. What is this distance for the pilot of a plane 9500 m above the ground?

55. Is it always true that $\sqrt{a^2} = a$? Explain.

56. Evaluate $\sqrt{x^2 + y^2}$ if $x = 12.7$ and $y = -18.2$.

57. A graphing calculator has a specific key sequence to find cube roots. Using a calculator, find $\sqrt[3]{2140}$ and $\sqrt[3]{-0.214}$.

58. A graphing calculator has a specific key sequence to find nth roots. Using a calculator, find $\sqrt[7]{0.382}$ and $\sqrt[7]{-382}$.

59. The resonance frequency f (in Hz) in an electronic circuit containing inductance L (in H) and capacitance C (in F) is given by $f = \frac{1}{2\pi\sqrt{LC}}$. Find the resonance frequency if $L = 0.250$ H and $C = 40.52 \times 10^{-6}$ F.

60. In statistics, the standard deviation is the square root of the variance. Find the standard deviation if the variance is 80.5 kg^2.

Answers to Practice Exercises

1. $2\sqrt{3}$ **2.** $6\sqrt{5}$ **3.** real, -2

1.7 Addition and Subtraction of Algebraic Expressions

Algebraic Expressions • Terms • Factors • Polynomials • Similar Terms • Simplifying • Symbols of Grouping

Because we use letters to represent numbers, we can see that all operations that can be used on numbers can also be used on literal numbers. In this section, we discuss the methods for adding and subtracting literal numbers.

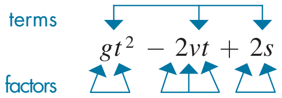
Addition, subtraction, multiplication, division, and taking powers or roots are known as **algebraic operations.** *Any combination of numbers and literal symbols that results from algebraic operations is known as an* **algebraic expression.**

When an algebraic expression consists of several parts connected by plus signs and minus signs, each part (along with its sign) is known as a **term** *of the expression. If a given expression is made up of the product of a number of quantities, each of these quantities, or any product of them, is called a* **factor** *of the expression.*

CAUTION It is very important to distinguish clearly between *terms* and *factors*, because some operations that are valid for terms are not valid for factors, and conversely. Some of the common errors in handling algebraic expressions occur because these operations are not handled properly. ■

EXAMPLE 1 Terms and factors

In the study of the motion of a rocket, the following algebraic expression may be used.



This expression has three terms: $gt^2, -2vt$, and $2s$. The first term, gt^2, has a factor of g and two factors of t. Any product of these factors is also a factor of gt^2. This means other factors are gt, t^2, and gt^2 itself. ■

EXAMPLE 2 Terms and factors

$7a(x^2 + 2y) - 6x(5 + x - 3y)$ is an expression with terms $7a(x^2 + 2y)$ and $-6x(5 + x - 3y)$.

The term $7a(x^2 + 2y)$ has individual factors of 7, a, and $(x^2 + 2y)$, as well as products of these factors. The factor $x^2 + 2y$ has two terms, x^2 and $2y$.

The term $-6x(5 + x - 3y)$ has factors 2, 3, x, and $(5 + x - 3y)$. The negative sign in front can be treated as a factor of -1. The factor $5 + x - 3y$ has three terms, 5, x, and $-3y$. ■

■ In Chapter 11, we will see that roots are equivalent to noninteger exponents.

A **polynomial** *is an algebraic expression with only nonnegative integer exponents on one or more variables, and has no variable in a denominator. The* **degree of a term** *is the sum of the exponents of the variables of that term, and the* **degree of the polynomial** *is the degree of the term of highest degree.*

A **multinomial** *is any algebraic expression of more than one term.* Terms like $1/x$ and $\sqrt{x}$ can be included in a multinomial, but not in a polynomial. (Since $1/x = x^{-1}$, the exponent is negative.)

EXAMPLE 3 Polynomials

Some examples of polynomials are as follows:

(a) $4x^2 - 5x + 3$ (degree 2) **(b)** $2x^6 - x$ (degree 6) **(c)** $3x$ (degree 1)

(d) $xy^3 + 7x - 3$ (degree 4) (add exponents of x and y)

(e) -6 (degree 0) $(-6 = -6x^0)$

From (c), note that a single term can be a polynomial, and from (e), note that a constant can be a polynomial. The expressions in (a), (b), and (d) are also multinomials.

The expression $x^2 + \sqrt{y + 2} - 8$ is a multinomial, but not a polynomial because of the square root term. ■

A polynomial with one term is called a **monomial.** *A polynomial with two terms is called a* **binomial,** *and one with three terms is called a* **trinomial.** *The numerical factor is called the* **numerical coefficient** (or simply **coefficient**) *of the term. All terms that differ at most in their numerical coefficients are known as* **similar** *or* **like** *terms.* That is, similar terms have the same variables with the same exponents.

EXAMPLE 4 Monomial, binomial, trinomial

(a) $7x^4$ is a monomial. The numerical coefficient is 7.

(b) $3ab - 6a$ is a binomial. The numerical coefficient of the first term is 3, and the numerical coefficient of the second term is -6. Note that the sign is attached to the coefficient.

(c) $8cx^3 - x + 2$ is a trinomial. The coefficients of the first two terms are 8 and -1.

(d) $x^2y^2 - 2x + 3y - 9$ is a polynomial with four terms (no special name). ■

EXAMPLE 5 Similar terms

(a) $8b - 6ab + 81b$ is a trinomial. The first and third terms are similar because they differ only in their numerical coefficients. The middle term is not similar to the others because it has a factor of a.

(b) $4x^2 - 3x$ is a binomial. The terms are not similar since the first term has two factors of x, and the second term has only one factor of x.

(c) $3x^2y^3 - 5y^3x^2 + x^2 - 2y^3$ is a polynomial. The commutative law tells us that $x^2y^3 = y^3x^2$, which means the first two terms are similar. ■

In adding and subtracting algebraic expressions, we combine similar terms into a single term. The ***simplified*** expression will contain only terms that are not similar.

EXAMPLE 6 Simplifying expressions

(a) $3x + 2x - 5y = 5x - 5y$ add similar terms—result has unlike terms

(b) $6a^2 - 7a + 8ax$ cannot be simplified since there are no like terms.

■ CAS (computer algebra system) calculators can display algebraic expressions and perform algebraic operations. The TI-89 graphing calculator is an example of such a calculator.

(c)

$$6a + 5c + 2a - c = 6a + 2a + 5c - c \quad \text{commutative law}$$
$$= 8a + 4c \quad \text{add like terms}$$ ■

To group terms in an algebraic expression, we use **symbols of grouping.** In this text, we use **parentheses,** ( ); **brackets,** [ ]; and **braces,** { }. All operations that occur in the numerator or denominator of a fraction are implied to be inside grouping symbols, as well as all operations under a radical symbol.

CAUTION In simplifying an expression using the distributive law, to remove the symbols of grouping if a MINUS sign precedes the grouping, change the sign of EVERY term in the grouping, or if a plus sign precedes the grouping retain the sign of every term. ■

EXAMPLE 7 Symbols of grouping

(a)

$$2(a + 2x) = 2a + 2(2x) \quad \text{use distributive law}$$
$$= 2a + 4x$$

(b)

$$-(+a - 3c) = (-1)(+a - 3c) \quad \text{treat } - \text{ sign as } -1$$
$$= (-1)(+a) + (-1)(-3c)$$
$$= -a + 3c \quad \text{note change of signs}$$

Normally, $+a$ would be written simply as a. ■

Practice Exercises

Use the distributive law.

1. $3(2a + y)$ **2.** $-3(-2r + s)$

EXAMPLE 8 Simplifying: signs before parentheses

+ sign before parentheses

(a) $3c + (2b - c) = 3c + 2b - c = 2b + 2c$ use distributive law

$2b = +2b$ signs retained

− sign before parentheses

(b) $3c - (2b - c) = 3c - 2b + c = -2b + 4c$ use distributive law

$2b = +2b$ signs retained

■ Note in each case that the parentheses are removed and the sign before the parentheses is also removed.

(c) $3c - (-2b + c) = 3c + 2b - c = 2b + 2c$ use distributive law

signs retained

(d) $y(3 - y) - 2(y - 3) = 3y - y^2 - 2y + 6$ note the $-2(-3) = +6$

$= -y^2 + y + 6$ ■

Practice Exercise

3. Simplify $2x - 3(4y - x)$

EXAMPLE 9 Simplifying—machine part design

In designing a certain machine part, it is necessary to perform the following simplification.

$$16(8 - x) - 2(8x - x^2) - (64 - 16x + x^2) = 128 - 16x - 16x + 2x^2 - 64 + 16x - x^2$$
$$= 64 - 16x + x^2$$ ■

At times, we have expressions in which more than one symbol of grouping is to be removed in the simplification. [Normally, when several symbols of grouping are to be removed, it is more convenient to remove the innermost symbols first.]

NOTE ▸

CAUTION One of the most common errors made is changing the sign of only the first term when removing symbols of grouping preceded by a minus sign. Remember, if the symbols are preceded by a minus sign, we must change the sign of *every* term. ■

EXAMPLE 10 Several symbols of grouping

(a) $3ax - [ax - (5s - 2ax)] = 3ax - [ax - 5s + 2ax]$ remove parentheses

$= 3ax - ax + 5s - 2ax$ remove brackets

$= 5s$

(b) $3a^2b - \{[a - (2a^2b - a)] + 2b\} = 3a^2b - \{[a - 2a^2b + a] + 2b\}$ remove parentheses

$= 3a^2b - \{a - 2a^2b + a + 2b\}$ remove brackets

$= 3a^2b - a + 2a^2b - a - 2b$ remove braces

$= 5a^2b - 2a - 2b$ ■

Calculators use only parentheses for grouping symbols, and we often need to use one set of parentheses within another set. These are called **nested parentheses.** In the next example, note that the innermost parentheses are removed first.

EXAMPLE 11 Nested parentheses

TI-89 graphing calculator keystrokes for Example 11: `bit.ly/2yAEedB`

$$2 - (3x - 2(5 - (7 - x))) = 2 - (3x - 2(5 - 7 + x))$$
$$= 2 - (3x - 10 + 14 - 2x)$$
$$= 2 - 3x + 10 - 14 + 2x = -x - 2$$ ■

EXERCISES 1.7

In Exercises 1–4, make the given changes in the indicated examples of this section, and then solve the resulting problems.

1. In Example 6(a), change $2x$ to $2y$.
2. In Example 8(a), change the sign before $(2b - c)$ from $+$ to $-$.
3. In Example 10(a), change $[ax - (5s - 2ax)]$ to $[(ax - 5s) - 2ax]$.
4. In Example 10(b), change $\{[a - (2a^2b - a)] + 2b\}$ to $\{a - [2a^2b - (a + 2b)]\}$.

In Exercises 5–51, simplify the given algebraic expressions.

5. $5x + 7x - 4x$
6. $6t - 3t - 4t$
7. $2y - y + 4x$
8. $-4C + L - 6C$
9. $3t - 4s - 3t - s$
10. $-8a - b + 12a + b$
11. $2F - 2T - 2 + 3F - T$
12. $x - 2y - 3x - y + z$
13. $a^2b - a^2b^2 - 2a^2b$
14. $-xy^2 - 3x^2y^2 + 2xy^2$
15. $2p + (p - 6 - 2p)$
16. $5 + (3 - 4n + p)$
17. $v - (7 - 9x + 2v)$
18. $-2a - \frac{1}{2}(b - a)$
19. $2 - 3 - (4 - 5a)$
20. $\sqrt{A} + (h - 2\sqrt{A}) - 3\sqrt{A}$
21. $(a - 3) + (5 - 6a)$
22. $(4x - y) - (-2x - 4y)$
23. $-(t - 2u) + (3u - t)$
24. $-2(6x - 3y) - (5y - 4x)$
25. $3(2r + s) - (-5s - r)$
26. $3(a - b) - 2(a - 2b)$
27. $-7(6 - 3j) - 2(j + 4)$
28. $-(5t + a^2) - 2(3a^2 - 2st)$
29. $-[(4 - 6n) - (n - 3)]$
30. $-[(A - B) - (B - A)]$
31. $2[4 - (t^2 - 5)]$
32. $-3\left[-3 - \frac{2}{3}(-a - 4)\right]$
33. $-2[-x - 2a - (a - x)]$
34. $-2[-3(x - 2y) + 4y]$
35. $aZ - [3 - (aZ + 4)]$
36. $9v - [6 - (-v - 4) + 4v]$
37. $5z - \{8 - [4 - (2z + 1)]\}$
38. $7y - \{y - [2y - (x - y)]\}$
39. $5p - (q - 2p) - [3q - (p - q)]$
40. $-(4 - \sqrt{LC}) - [(5\sqrt{LC} - 7) - (6\sqrt{LC} + 2)]$
41. $-2\{-(4 - x^2) - [3 + (4 - x^2)]\}$
42. $-\{-[-(x - 2a) - b] - (a - x)\}$
43. $5V^2 - (6 - (2V^2 + 3))$
44. $-2F + 2((2F - 1) - 5)$
45. $-(3t - (7 + 2t - (5t - 6)))$
46. $a^2 - 2(x - 5 - (7 - 2(a^2 - 2x) - 3x))$
47. $-4[4R - 2.5(Z - 2R) - 1.5(2R - Z)]$
48. $-3\{2.1e - 1.3[-f - 2(e - 5f)]\}$
49. In determining the size of a V belt to be used with an engine, the expression $3D - (D - d)$ is used. Simplify this expression.
50. When finding the current in a transistor circuit, the expression $i_1 - (2 - 3i_2) + i_2$ is used. Simplify this expression. (The numbers below the i's are *subscripts*. Different subscripts denote different variables.)
51. Research on a plastic building material leads to $[(B + \frac{4}{3}\alpha) + 2(B - \frac{2}{3}\alpha)] - [(B + \frac{4}{3}\alpha) - (B - \frac{2}{3}\alpha)]$. Simplify this expression.
52. One car goes 30 km/h for $t - 1$ hours, and a second car goes 40 km/h for $t + 2$ hours. Find the expression for the sum of the distances traveled by the two cars.
53. A shipment contains x hard drives with 4 terabytes of memory and $x + 25$ hard drives with 8 terabytes. Express the total number of terabytes of memory in the shipment as a variable expression and simplify.
54. Each of two suppliers has $2n + 1$ bundles of shingles costing \$30 each and $n - 2$ bundles costing \$20 each. How much more is the total value of the \$30 bundles than the \$20 bundles?
55. For the expressions $2x^2 - y + 2a$ and $3y - x^2 - b$ find (a) the sum, and (b) the difference if the second is subtracted from the first.
56. For the following expressions, subtract the third from the sum of the first two: $3a^2 + b - c^3$, $2c^3 - 2b - a^2$, $4c^3 - 4b + 3$.

In Exercises 57–60, answer the given questions.

57. Is the following simplification correct? Explain.

$$2x - 3y + 5 - (4x - y + 3) = 2x - 3y + 5 - 4x - y + 3$$
$$= -2x - 4y + 8$$

58. Is the following simplification correct? Explain.

$$2a - 3b - 4c - (-5a + 3b - 2c)$$
$$= 2a - 3b - 4c + 5a - 3b - 2c$$
$$= 7a - 6b - 6c$$

59. For any real numbers a and b, is it true that $|a - b| = |b - a|$? Explain.
60. Is subtraction associative? That is, in general, does $(a - b) - c$ equal $a - (b - c)$? Explain.

Answers to Practice Exercises

1. $6a + 3y$ 2. $6r - 3s$ 3. $5x - 12y$

1.8 Multiplication of Algebraic Expressions

Multiplying Monomials • Products of Monomials and Polynomials • Powers of Polynomials

To find the product of two or more monomials, we multiply the numerical coefficients to find the numerical coefficient of the product, and multiply the literal numbers, remembering that *the exponents may be combined only if the base is the same*.

EXAMPLE 1 Multiplying monomials

(a) $3c^5(-4c^2) = -12c^7$ multiply numerical coefficients and add exponents of c

(b) $(-2b^2y^3)(-9aby^5) = 18ab^3y^8$ add exponents of same base

(c) $2xy(-6cx^2)(3xcy^2) = -36c^2x^4y^3$ ■

NOTE ▸ [If a product contains a monomial that is raised to a power, we must first raise it to the indicated power before proceeding with the multiplication.]

EXAMPLE 2 Product containing power of a monomial

(a) $-3a(2a^2x)^3 = -3a(8a^6x^3) = -24a^7x^3$

(b) $2s^3(-st^4)^2(4s^2t) = 2s^3(s^2t^8)(4s^2t) = 8s^7t^9$ ■

We find the product of a monomial and a polynomial by using the distributive law, which states that we *multiply each term of the polynomial by the monomial.* In doing so, we must be careful to give the correct sign to each term of the product.

EXAMPLE 3 Product of monomial and polynomial

(a) $2ax(3ax^3 - 4yz) = 2ax(3ax^2) + (2ax)(-4yz) = 6a^2x^3 - 8axyz$

(b) $5cy^2(-7cx - ac) = (5cy^2)(-7cx) + (5cy^2)(-ac) = -35c^2xy^2 - 5ac^2y^2$ ■

Practice Exercises

Perform the indicated multiplications.

1. $2a^3b(-6ab^2)$ **2.** $-5x^2y^3(-2xy - y^4)$

It is generally not necessary to write out the middle step as it appears in the preceding example. We write the answer directly. For instance, Example 3(a) would appear as $2ax(3ax^2 - 4yz) = 6a^2x^3 - 8axyz$.

We find the product of two polynomials by using the distributive law. The result is that *we multiply each term of one polynomial by each term of the other and add the results.* Again we must be careful to give each term of the product its correct sign.

EXAMPLE 4 Product of polynomials

$$(x - 2)(x + 3) = x(x) + x(3) + (-2)(x) + (-2)(3)$$
$$= x^2 + 3x - 2xw - 6 = x^2 + x - 6$$ ■

■ Note that, using the distributive law, $(x - 2)(x + 3) = (x - 2)(x) + (x - 2)(3)$ leads to the same result.

Finding the power of a polynomial is equivalent to using the polynomial as a factor the number of times indicated by the exponent. It is sometimes convenient to write the power of a polynomial in this form before multiplying.

EXAMPLE 5 Power of a polynomial

two factors

(a) $(x + 5)^2 = (x + 5)(x + 5) = x^2 + 5x + 5x + 25$
$= x^2 + 10x + 25$

(b) $(2a - b)^3 = (2a - b)(2a - b)(2a - b)$ the exponent 3 indicates three factors

$$= (2a - b)(4a^2 - 2ab - 2ab + b^2)$$
$$= (2a - b)(4a^2 - 4ab + b^2)$$
$$= 8a^3 - 8a^2b + 2ab^2 - 4a^2b + 4ab^2 - b^3$$
$$= 8a^3 - 12a^2b + 6ab^2 - b^3$$ ■

TI-89 graphing calculator keystrokes for Example 5: `bit.ly/2Ee0A93`

CAUTION We should note that in Example 5(a) $(x + 5)^2$ is *not* equal to $x^2 + 25$ because the term $10x$ is not included. We must follow the proper procedure and not simply square each of the terms within the parentheses. ■

Practice Exercises

Perform the indicated multiplications.

3. $(2s - 5t)(s + 4t)$ **4.** $(3u + 2v)^2$

EXAMPLE 6 Simplifying products—telescope lens

An expression used with a lens of a certain telescope is simplified as shown.

$$a(a + b)^2 + a^3 - (a + b)(2a^2 - s^2)$$
$$= a(a + b)(a + b) + a^3 - (2a^3 - as^2 + 2a^2b - bs^2)$$
$$= a(a^2 + ab + ab + b^2) + a^3 - 2a^3 + as^2 - 2a^2b + bs^2$$
$$= a^3 + a^2b + a^2b + ab^2 - a^3 + as^2 - 2a^2b + bs^2$$
$$= ab^2 + as^2 + bs^2$$ ■

EXERCISES 1.8

In Exercises 1–4, make the given changes in the indicated examples of this section, and then solve the resulting problems.

1. In Example 2(b), change the factor $(-st^4)^2$ to $(-st^4)^3$.
2. In Example 3(a), change the factor $2ax$ to $-2ax$.
3. In Example 4, change the factor $(x + 3)$ to $(x - 3)$.
4. In Example 5(b), change the exponent 3 to 2.

In Exercises 5–66, perform the indicated multiplications.

5. $(a^2)(ax)$ **6.** $(2xy)(x^2y^3)$ **7.** $-a^2c^2(a^2cx^3)$

8. $-2cs^2(-4cs)^2$ **9.** $(2ax^2)^2(-2ax)$ **10.** $6pq^3(3pq^2)^2$

11. $i^2(Ri + 2i)$ **12.** $2x(-p - q)$ **13.** $-3s(s^2 - 5t)$

14. $-3b(2b^2 - b)$ **15.** $5m(m^2n + 3mn)$ **16.** $a^2bc(2ac - 3b^2c)$

17. $3M(-M - N + 2)$ **18.** $-4c^2(-9gc - 2c + g^2)$

19. $xy(tx^2)(x + y^3)$ **20.** $-2(-3st^3)(3s - 4t)$

21. $(x - 3)(x + 5)$ **22.** $(a + 7)(a + 1)$

23. $(x + 5)(2x - 1)$ **24.** $(4t_1 + t_2)(2t_1 - 3t_2)$

25. $(y + 8)(y - 8)$ **26.** $(z - 4)(z + 4)$

27. $(2a - b)(-2b + 3a)$ **28.** $(-3 + 4w^2)(3w^2 - 1)$

29. $(2s + 7t)(3s - 5t)$ **30.** $(5p - 2q)(p + 8q)$

31. $(x^2 - 1)(2x + 5)$ **32.** $(3y^2 + 2)(2y - 9)$

33. $(x - 2y - 4)(x - 2y + 4)$

34. $(2a + 3b + 1)(2a + 3b - 1)$

35. $2(a + 1)(a - 9)$ **36.** $-5(y - 3)(y + 6)$

37. $-3(3 - 2T)(3T + 2)$ **38.** $2n(-n + 5)(6n + 5)$

39. $2L(L + 1)(4 - L)$ **40.** $ax(x + 4)(7 - x^2)$

41. $(3x - 7)^2$ **42.** $(x - 3y)^2$ **43.** $(x_1 + 3x_2)^2$

44. $(-7m - 1)^2$ **45.** $(xyz - 2)^2$ **46.** $(-6x^2 + b)^2$

47. $2(x + 8)^2$ **48.** $3(3R - 4)^2$

49. $(2 + x)(3 - x)(x - 1)$ **50.** $(-c^2 + 3x)^3$

51. $3T(T + 2)(2T - 1)$ **52.** $[(x - 2)^2(x + 2)]^2$

53. Let $x = 3$ and $y = 4$ to show that (a) $(x + y)^2 \neq x^2 + y^2$ and (b) $(x - y)^2 \neq x^2 - y^2$. ($\neq$ means "does not equal")

54. *Explain how* you would perform $(x + 3)^5$. Do not actually do the operations.

55. By multiplication, show that $(x + y)^3$ is not equal to $x^3 + y^3$.

56. By multiplication, show that $(x + y)(x^2 - xy + y^2) = x^3 + y^3$.

57. In finding the value of a certain savings account, the expression $P(1 + 0.01r)^2$ is used. Multiply out this expression.

58. A savings account of \$1000 that earns r% annual interest, compounded quarterly, has a value of $1000(1 + 0.0025r)^2$ after 6 months. Perform the indicated multiplication.

59. A contractor is designing a rectangular room that will have a pool table. The length of the pool table is twice its width. The contractor wishes to have 5 ft of open space between each wall and the pool table. See Fig. 1.13. Express the area of the room in terms of the width w of the pool table. Then perform the indicated operations.

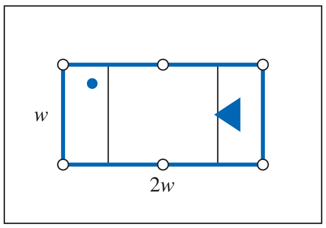


Fig. 1.13

60. The weekly revenue R (in dollars) of a flash drive manufacturer is given by $R = xp$, where x is the number of flash drives sold each week and p is the price (in dollars). If the price is given by the demand equation $p = 30 - 0.01x$, express the revenue in terms of x and simplify.

61. In using aircraft radar, the expression $(2R - X)^2 - (R^2 + X^2)$ arises. Simplify this expression.

62. In calculating the temperature variation of an industrial area, the expression $(2T^3 + 3)(T^2 - T - 3)$ arises. Perform the indicated multiplication.

63. In a particular computer design containing n circuit elements, n^2 switches are needed. Find the expression for the number of switches needed for $n + 100$ circuit elements.

64. Simplify the expression $(T^2 - 100)(T - 10)(T + 10)$, which arises when analyzing the energy radiation from an object.

65. In finding the maximum power in part of a microwave transmiter circuit, the expression $(R_1 + R_2)^2 - 2R_2(R_1 + R_2)$ is used. Multiply and simplify.

66. In determining the deflection of a certain steel beam, the expression $27x^2 - 24(x - 6)^2 - (x - 12)^3$ is used. Multiply and simplify.

Answers to Practice Exercises

1. $-12a^4b^3$ **2.** $-10x^3y^4 + 5x^2y^7$
3. $2s^2 + 3st - 20t^2$ **4.** $9u^2 + 12uv + 4v^2$

1.9 Division of Algebraic Expressions

Dividing Monomials • Dividing by a Monomial • Dividing One Polynomial by Another

To find the quotient of one monomial divided by another, we use the laws of exponents and the laws for dividing signed numbers. Again, the *exponents may be combined only if the base is the same*.

EXAMPLE 1 Dividing monomials

(a) $\dfrac{3c^7}{c^2} = 3c^{7-2} = 3c^5$

(b) $\dfrac{16x^3y^5}{4xy^2} = \dfrac{16}{4}(x^{3-1})(y^{5-2}) = 4x^2y^3$ (divide coefficients; subtract exponents)

(c) $\dfrac{-6a^2xy^2}{2axy^4} = -\left(\dfrac{6}{2}\right)\dfrac{a^{2-1}x^{1-1}}{y^{4-2}} = \dfrac{-3a}{y^2}$

As shown in illustration (c), we use only positive exponents in the final result unless there are specific instructions otherwise. ■

From arithmetic, we may show how a multinomial is to be divided by a monomial. When adding fractions (say $\frac{2}{7}$ and $\frac{3}{7}$), we have $\dfrac{2}{7} + \dfrac{3}{7} = \dfrac{2+3}{7}$.

Looking at this from *right to left,* we see that *the quotient of a multinomial divided by a monomial is found by dividing each term of the multinomial by the monomial and adding the results*. This can be shown as

■ This is an identity and is valid for all values of a and b, and all values of c except zero (which would make it undefined).

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

CAUTION Be careful: Although $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$, we must note that $\dfrac{c}{a+b}$ is *not* $\dfrac{c}{a} + \dfrac{c}{b}$. ■

EXAMPLE 2 Dividing by a monomial

(a) $\dfrac{4a^2 + 8a}{2a} = \dfrac{4a^2}{2a} + \dfrac{8a}{2a} = 2a + 4$

(b) $\dfrac{4x^3y - 8x^3y^2 + 2x^2y}{2x^2y} = \dfrac{4x^3y}{2x^2y} - \dfrac{8x^3y^2}{2x^2y} + \dfrac{2x^2y}{2x^2y}$ (each term of numerator divided by denominator)

$= 2x - 4xy + 1$

■ Until you are familiar with the method, it is recommended that you do write out the middle steps.

Practice Exercise

1. Divide: $\dfrac{4ax^2 - 6a^2x}{2ax}$

We usually do not write out the middle step as shown in these illustrations. The divisions of the terms of the numerator by the denominator are usually done by inspection (mentally), and the result is shown as it appears in the next example.

NOTE ▸ [Note carefully the last term 1 of the result. When all factors of the numerator are the same as those in the denominator, we are dividing a number by itself, which gives a result of 1.] ■

EXAMPLE 3 Dividing by a monomial—irrigation pump

The expression $\dfrac{2p + v^2d + 2ydg}{2dg}$ is used when analyzing the operation of an irrigation pump. Performing the indicated division, we have

$$\frac{2p + v^2d + 2ydg}{2dg} = \frac{p}{dg} + \frac{v^2}{2g} + y$$ ■

DIVISION OF ONE POLYNOMIAL BY ANOTHER

To divide one polynomial by another, use the following steps.

1. Arrange the dividend (the polynomial to be divided) and the divisor in descending powers of the variable.
2. Divide the first term of the dividend by the first term of the divisor. The result is the first term of the quotient.
3. Multiply the entire divisor by the first term of the quotient and *subtract* the product from the dividend.
4. Divide the first term of this difference by the first term of the divisor. This gives the second term of the quotient.
5. Multiply this term by the entire divisor and *subtract* the product from the first difference.
6. Repeat this process until the remainder is zero or of lower degree than the divisor.
7. Express the answer in the form quotient + $\dfrac{\text{remainder}}{\text{divisor}}$.

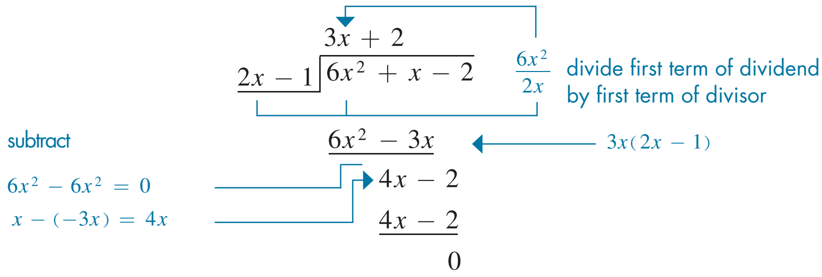
■ This is similar to long division of numbers.

EXAMPLE 4 Dividing one polynomial by another

Perform the division $(6x^2 + x - 2) \div (2x - 1)$.

$\left(\text{This division can also be indicated in the fractional form } \dfrac{6x^2 + x - 2}{2x - 1}.\right)$

We set up the division as we would for long division in arithmetic. Then, following the procedure outlined above, we have the following:



■ The answer to Example 4 can be checked by showing $(2x - 1)(3x + 2) = 6x^2 + x - 2$.

The remainder is zero and the quotient is $3x + 2$. Therefore, the answer is

$$3x + 2 + \frac{0}{2x - 1}, \text{ or simply } 3x + 2$$ ■

Practice Exercise

2. Divide: $(6x^2 + 7x - 3) \div (3x - 1)$

EXAMPLE 5 Quotient with a remainder

Perform the division $\frac{8x^3 + 4x^2 + 3}{4x^2 - 1}$. Because there is no x-term in the dividend, we should leave space for any x-terms that might arise (which we will show as $0x$).

$$
\begin{array}{rl}
 & 2x + 1 \\
4x^2 - 1 \,\big) & 8x^3 + 4x^2 + 0x + 3 \\
 & 8x^3 \qquad\quad - 2x \\
\hline
 & \qquad 4x^2 + 2x + 3 \\
 & \qquad 4x^2 \qquad\; - 1 \\
\hline
 & \qquad\qquad 2x + 4
\end{array}
$$

divisor: $4x^2 - 1$; dividend: $8x^3 + 4x^2 + 0x + 3$; $\frac{8x^3}{4x^2} = 2x$; subtract; $0x - (-2x) = 2x$; $\frac{4x^2}{4x^2} = 1$; subtract; remainder: $2x + 4$

TI-89 graphing calculator keystrokes for Example 5: **bit.ly/2CF8Q27**

■ The answer to Example 5 can be checked by showing $(4x^2 - 1)(2x + 1) + (2x + 4) = 8x^3 + 4x^2 + 3$

Because the degree of the remainder $2x + 4$ is less than that of the divisor, the long-division process is complete and the answer is $2x + 1 + \frac{2x + 4}{4x^2 - 1}$. ■

EXERCISES 1.9

In Exercises 1–4, make the given changes in the indicated examples of this section and then perform the indicated divisions.

1. In Example 1(c), change the denominator to $-2a^2xy^5$.
2. In Example 2(b), change the denominator to $2xy^2$.
3. In Example 4, change the dividend to $6x^2 - 7x + 2$.
4. In Example 5, change the sign of the middle term of the numerator from + to −.

In Exercises 5–24, perform the indicated divisions.

5. $\frac{8x^3y^2}{-2xy}$
6. $\frac{-18b^7c^3}{bc^2}$
7. $\frac{-16r^3t^5}{-4r^5t}$
8. $\frac{51mn^5}{17m^2n^2}$
9. $\frac{(15x^2y)(2xz)}{10xy}$
10. $\frac{(5sT)(8s^2T^3)}{10s^3T^2}$
11. $\frac{(4a^3)(2x)^2}{(4ax)^2}$
12. $\frac{12a^2b}{(3ab^2)^2}$
13. $\frac{3a^2x + 6xy}{3x}$
14. $\frac{2m^2n - 6mn}{-2m}$
15. $\frac{3rst - 6r^2st^2}{3rs}$
16. $\frac{-5a^2n - 10an^2}{5an}$
17. $\frac{4pq^3 + 8p^2q^2 - 16pq^5}{4pq^2}$
18. $\frac{a^2x_1x_2^2 + ax_1^3 - ax_1}{ax_1}$
19. $\frac{2\pi fL - \pi fR^2}{\pi fR}$
20. $\frac{9(aB)^4 - 6aB^4}{-3aB^3}$
21. $\frac{-7a^2b + 14ab^2 - 21a^3}{14a^2b^2}$
22. $\frac{2x^{n+2} + 4ax^n}{2x^n}$
23. $\frac{6y^{2n} - 4ay^{n+1}}{2y^n}$
24. $\frac{3a(F + T)b^2 - (F + T)}{a(F + T)}$

In Exercises 25–44, perform the indicated divisions. Express the answer as shown in Example 5 when applicable.

25. $(x^2 + 9x + 20) \div (x + 4)$
26. $(x^2 + 7x - 18) \div (x - 2)$
27. $(2x^2 + 7x + 3) \div (x + 3)$
28. $(3t^2 - 7t + 4) \div (t - 1)$
29. $(x^2 - 3x + 2) \div (x - 2)$
30. $(2x^2 - 5x - 7) \div (x + 1)$
31. $(x - 14x^2 + 8x^3) \div (2x - 3)$
32. $(6 + 7y + 6y^2) \div (2y + 1)$
33. $(4Z^2 - 5Z - 7) \div (4Z + 3)$
34. $(6x^2 - 5x - 9) \div (-4 + 3x)$
35. $\frac{x^3 + 3x^2 - 4x - 12}{x + 2}$
36. $\frac{3x^3 + 19x^2 + 13x - 20}{3x - 2}$
37. $\frac{2a^4 + 4a^2 - 16}{a^2 - 2}$
38. $\frac{6T^3 + T^2 + 2}{3T^2 - T + 2}$
39. $\frac{y^3 + 27}{y + 3}$
40. $\frac{D^3 - 1}{D - 1}$
41. $\frac{x^2 - 2xy + y^2}{x - y}$
42. $\frac{3r^2 - 5rR + 2R^2}{r - 3R}$
43. $\frac{t^3 - 8}{t^2 + 2t + 4}$
44. $\frac{a^4 + b^4}{a^2 - 2ab + 2b^2}$

In Exercises 45–56, solve the given problems.

45. When $2x^2 - 9x - 5$ is divided by $x + c$, the quotient is $2x + 1$. Find c.
46. When $6x^2 - x + k$ is divided by $3x + 4$, the remainder is zero. Find k.
47. By division show that $\frac{x^4 + 1}{x + 1}$ is not equal to x^3.
48. By division show that $\frac{x^3 + y^3}{x + y}$ is not equal to $x^2 + y^2$.
49. If a gas under constant pressure has volume V_1 at temperature T_1 (in kelvin), then the new volume V_2 when the temperature changes from T_1 to T_2 is given by $V_2 = V_1\left(1 + \frac{T_2 - T_1}{T_1}\right)$. Simplify the right-hand side of this equation.