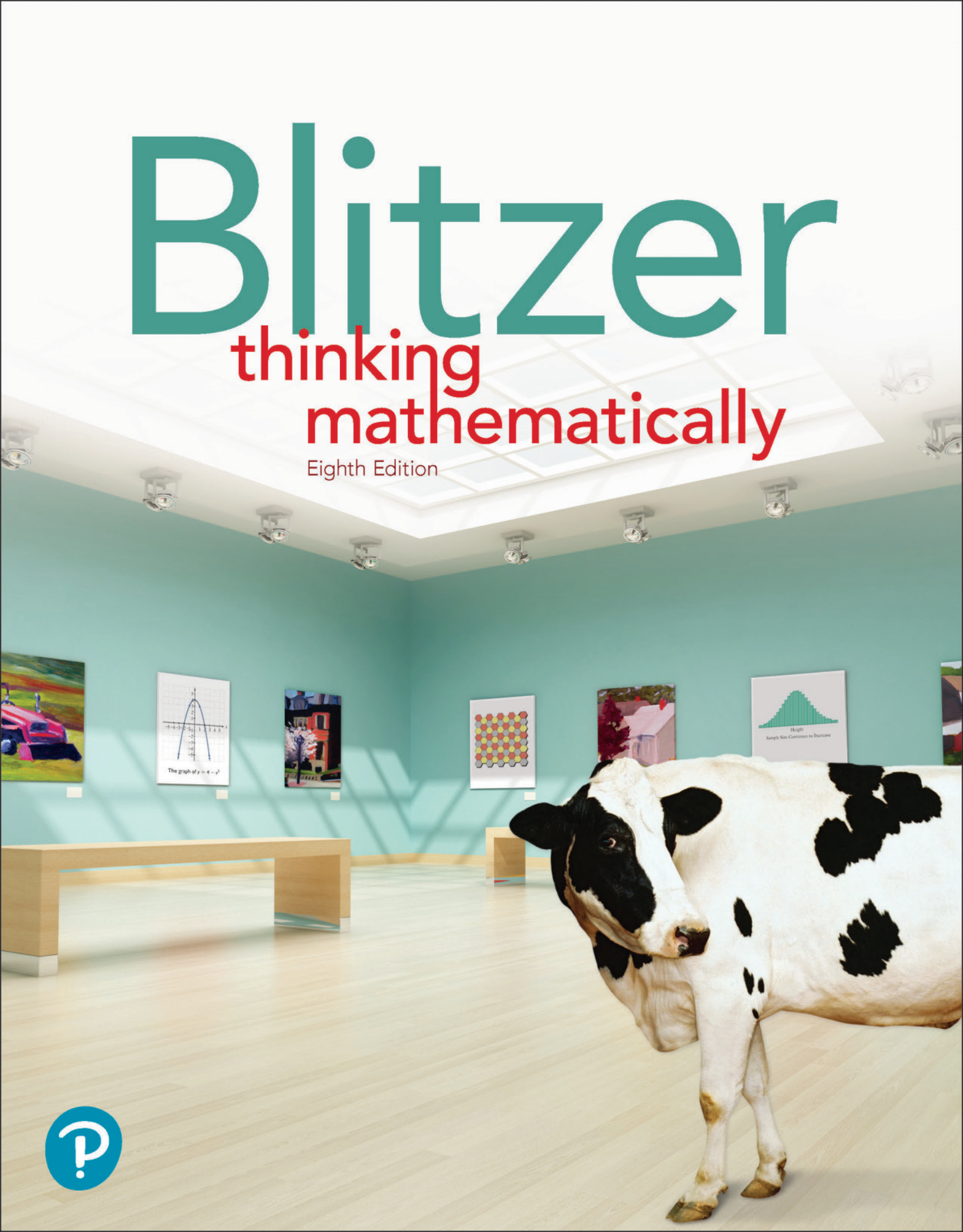
Blitzer

thinking mathematically

Eighth Edition



A Brief Guide to Getting the Most from This Book

1 Read the Book

Feature	Description	Benefit
Section-Opening Scenarios	Every section opens with a scenario presenting a unique application of mathematics in your life outside the classroom.	Realizing that mathematics is everywhere will help motivate your learning. (See page 354.)
Example	Examples are clearly written and provide step-by-step solutions. No steps are omitted, and each step is thoroughly explained to the right of the mathematics.	The blue annotations will help you understand the solutions by providing the reason why every mathematical step is true. (See pages 372–373.)
Applications Using Real-World Data	Interesting applications from nearly every discipline, supported by up-to-date real-world data, are included in every section.	Ever wondered how you'll use mathematics? This feature will show you how it can solve real problems. (See page 350.)
Great Question!	Answers to students' questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.	By seeing common mistakes, you'll be able to avoid them. This feature should help you not to feel anxious or threatened when asking questions in class. (See page 357.)
A Brief Review	Brief Reviews cover skills you already learned but may have forgotten. Some of the Brief Reviews contain exercises for you to practice.	Having these refresher boxes easily accessible will help ease anxiety about skills you may have forgotten. (See page 373.)
BLITZER BONUS	These enrichment essays provide historical, interdisciplinary, and otherwise interesting connections to the mathematics under study.	Yet even more proof that math is an interesting and dynamic discipline! (See page 794.)
Explanatory Voice Balloons	Voice balloons help to demystify mathematics. They translate math into plain English, clarify problem-solving procedures, and present alternative ways of understanding.	Does math ever look foreign to you? This feature often translates math into everyday English. (See page 346.)
What You'll Learn 1 Learning Objective	Every section begins with a list of objectives. Each objective is restated in the margin where the objective is covered.	The objectives focus your reading by emphasizing what is most important and where to find it. (See page 507.)
TECHNOLOGY	The screens displayed in the technology boxes show how to use calculators and other technology to verify and visualize mathematical results.	Even if you are not using technology in the course, this feature will help you understand different approaches to problem solving. (See page 421.)

2 Work the Problems

Feature	Description	Benefit
Check Point	Each example is followed by a matched problem, called a Check Point, that offers you the opportunity to work a similar exercise. The answers to the Check Points are provided in the answer section.	You learn best by doing. You'll solidify your understanding of worked examples if you try a similar problem right away to be sure you understand what you've just read. (See page 526.)
Animation Venn Diagram **StatCrunch** Experiment → Birthdays	Animations and StatCrunch explorations, referenced by name in the side margin (e.g., "Venn Diagram"), can be found in MyLab Math. They provide an interactive way to explore concepts.	These new tools make it easy to "play around" with the key ideas and explore them on your own. (See pages 161 and 790.)
Concept and Vocabulary Check	These short-answer questions, mainly fill-in-the-blank and true/false items, assess your understanding of the definitions and concepts presented in each section.	It is difficult to learn mathematics without knowing its special language. These exercises test your understanding of the vocabulary and concepts. (See page 873.)
Exercise Set	An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within several categories. Your instructor will usually provide guidance on which exercises to work. The exercises in the first category, Practice Exercises, follow the same order as the section's worked examples.	The parallel order of the Practice Exercises lets you refer to the worked examples and use them as models for solving these problems. (See page 174.)
Practice PLUS	This category of exercises contains more challenging problems that often require you to combine several skills or concepts.	It is important to dig in and develop your problem-solving skills. Practice PLUS Exercises provide you with ample opportunity to do so. (See page 814.)

3 Review for Quizzes and Tests

Feature	Description	Benefit
Chapter Review Chart	Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.	Review this chart and you'll know the most important material in the chapter! (See page 110.)
CHAPTER REVIEW **Exercise Set**	A comprehensive collection of review exercises for each of the chapter's sections follows the review chart.	Practice makes perfect. These exercises contain the most significant problems for each of the chapter's sections. (See page 588.)
CHAPTER TEST	Each chapter contains a practice test with approximately 25 problems that cover the important concepts in the chapter. Take the practice test, check your answers, and then watch the Chapter Test Prep Videos to see worked-out solutions for any exercises you miss.	You can use the chapter test to determine whether you have mastered the material covered in the chapter. (See page 914.)
Chapter Test Prep Videos	These videos contain worked-out solutions to every exercise in each chapter test and can be found in MyLab Math and on YouTube at **youtube.com/user/pearsonmathstats** (playlist "Blitzer Thinking Mathematically 8e").	The videos let you review any exercises you miss on the chapter test.
Objective Videos	These fresh, interactive videos walk you through the concepts from every objective of the text.	The videos provide you with active learning at your own pace.

Thinking Mathematically

Eighth Edition

Robert Blitzer

Miami Dade College

Pearson

Content Management	Jeff Weidenaar, Jonathan Krebs
Content Development	Robert Carroll
Content Production	Tamela Ambush, Shana Siegmund
Product Management	Steve Schoen
Product Marketing	Alicia Wilson, Jessica Szewczyk
Rights and Permissions	Anjali Singh
Cover Image	archideaphoto/Shutterstock

Please contact https://support.pearson.com/getsupport/s/ with any queries on this content.

Acknowledgments of third-party content appear on page C1, which constitutes an extension of this copyright page.

Library of Congress Cataloging-in-Publication Data
Names: Blitzer, Robert, author.
Title: Thinking mathematically / Robert F. Blitzer, Miami Dade College.
Description: Eighth edition. | Hoboken : Pearson, 2022. | Includes index.
Identifiers: LCCN 2021043674 (print) | LCCN 2021043675 (ebook) | ISBN 9780137551200 (hardcover) | ISBN 9780137551279 (ebook)
Subjects: LCSH: Mathematics—Textbooks.
Classification: LCC QA39.3 .B59 2022 (print) | LCC QA39.3 (ebook) | DDC 510—dc23/eng/20211006
LC record available at https://lccn.loc.gov/2021043674
LC ebook record available at https://lccn.loc.gov/2021043675

ScoutAutomatedPrintCode

Rental
ISBN-13: 978-0-13-755120-0
ISBN-10: 0-13-755120-7

Contents

11 Counting Methods and Probability Theory 709

12 Statistics 787

13 Voting and Apportionment 863

14 Graph Theory 915

About the Author

Bob Blitzer is a native of Manhattan and received a Bachelor of Arts degree with dual majors in mathematics and psychology (minor: English literature) from the City College of New York. His unusual combination of academic interests led him toward a Master of Arts in mathematics from the University of Miami and a doctorate in behavioral sciences from Nova University. Bob's love for teaching mathematics was nourished for nearly 30 years at Miami Dade College, where he received numerous teaching awards, including Innovator of the Year from the League for Innovations in the Community College and an endowed chair based on excellence in the classroom. In addition to *Thinking Mathematically*, Bob has written textbooks covering introductory algebra, intermediate algebra, college algebra, algebra and trigonometry, precalculus, and trigonometry, all published by Pearson. When not secluded in his Northern California writer's cabin, Bob can be found hiking the beaches and trails of Point Reyes National Seashore, and tending to the chores required by his beloved entourage of horses, chickens, and irritable roosters.

Preface

Thinking Mathematically, Eighth Edition provides a general survey of mathematical topics that are useful in our contemporary world. My primary purpose in writing the book was to show students how mathematics can be applied to their lives in interesting, enjoyable, and meaningful ways. The book's variety of topics and flexibility of sequence make it appropriate for a one- or two-term course in liberal arts mathematics, quantitative reasoning, finite mathematics, as well as for courses specifically designed to meet state-mandated requirements in mathematics.

I wrote the book to help diverse students, with different backgrounds and career plans, to succeed. *Thinking Mathematically, Eighth Edition*, has four major goals:

1. To help students acquire knowledge of fundamental mathematics.
2. To show students how mathematics can solve authentic problems that apply to their lives.
3. To enable students to understand and reason with quantitative issues and mathematical ideas they are likely to encounter in college, career, and life.
4. To enable students to develop problem-solving skills, while fostering critical thinking, within an interesting setting.

One major obstacle in the way of achieving these goals is the fact that very few students actually read their textbook. This has been a regular source of frustration for me and my colleagues in the classroom. Anecdotal evidence gathered over years highlights two basic reasons why students do not take advantage of their textbook:

"I'll never use this information."
"I can't follow the explanations."

I've written every page of the Eighth Edition with the intent of eliminating these two objections. The ideas and tools I've used to do so are described for the student in "A Brief Guide to Getting the Most from This Book," which appears on the endpapers of this book.

What's New in the Eighth Edition?

The Eighth Edition contains 94 worked-out examples and exercises based on new data and 258 updated examples and exercises. Many of the new and updated applications involve topics relevant to college students.

New Applications

- Mathlete John Urschel (Blitzer Bonus in Section 1.1, p. 2)
- Value of Online Classes (Section 1.1, Exercise 68)
- Probability of Divorce, by Educational Attainment (Section 1.2, pp. 21–22)
- Trust in Government and the Media (Section 1.2, Exercises 47–48)
- Hidden Figures: Katherine Johnson (Blitzer Bonus in Section 1.3, p. 31)
- Renting Movies (Section 1.3, Exercises 13–14)
- Robocalls Epidemic (Section 1.3, Exercise 58)
- Huge NFL Contract (Section 1.3, Exercise 59)
- The Companies Americans Love Most (Section 2.1, Check Point 9)
- TV Hosts Earning More Than $40 Million (Section 2.3, Great Question!, p. 78)
- Pandemic Policies and Statistics (Section 2.3, Exercises 133–134)
- A Venn Diagram for Giftedness (Blitzer Bonus in Section 2.4, p. 92)
- Popular Apps (Section 2.4, Exercises 93–98)
- LGBTQ Tolerance (Section 2.5 Discussion, p. 103; Exercises 33–38)
- Flight Insurance (Section 2.5, Exercise 40)
- Accuracy of COVID-19 Tests (Section 2.5, Exercise 63)
- The 2020 U.S. Presidential Election (Chapter 2 Review, Exercise 59)
- Living Arrangements of Young Adults (Section 3.3 Opener; Example 7)
- Student Loans and Stress (Section 3.3, Exercises 81–84)
- Noise Levels of Selected Sources (Section 3.5, Exercises 39–40)
- Percentage of Bachelor's Degrees Awarded to Men and Women (Section 3.6, Example 3)
- Are Smartphones Making Us Stupid? (Section 3.7 Opener; Example 3)
- Could/Want ∴ Is (Blitzer Bonus in Section 3.7, p. 198)
- Books on Having Better Arguments (Section 3.7, Exercise 93)
- Time per Day Spent Eating, by Country (Section 4.3, Exercises 47–52)
- Math Jokes (Section 5.4, Exercise 91)
- Value of Apple, Inc., at Various Share Prices (Section 5.7, Figure 5.16, p. 327)

- Electric Charging Stations in the U.S. (Chapter 5 Review, Exercise 147)
- Weekly Earnings of Workers (Section 6.1, Exercises 71–72)
- Earnings and Education (Section 6.3, Exercises 21–22)
- Debit Cards and Linear Inequalities (Section 6.4 Opener)
- Marriage and Divorce (Section 6.4, Exercises 85–86)
- Average Earnings, by Field of Study (Chapter 6 Review, Exercise 22)
- Number of Executions in the U.S. (Chapter 6 Test, Exercise 12)
- Modeling Body Temperature, Heart Rate, and Respiratory Rate (Chapter 7 Opener; Section 7.4 Opener; Example 4; Exercises 45–50)
- Political Orientation of First-Year College Students (Section 7.2, Example 7)
- Number of Threatened Species (Section 7.2, Example 8)
- Marriage Equality (Section 7.2, Check Point 8)
- Bob Moses: Civil Rights to Algebra (Blitzer Bonus in Section 7.3, p. 446)
- New Coronavirus Cases in the U.S. in 2020 (Section 7.6, Exercise 50)
- Blood-Alcohol Concentration by Number of Drinks, Weight, and Sex (Blitzer Bonus in Section 9.3, p. 620)
- Women and Extreme Temperatures at the Top of the World (Blitzer Bonus in Section 9.3, p. 622)
- A Truly Hellish Planet (Section 9.3, Exercise 70)
- Deceiving Your Brain (Blitzer Bonus in Section 10.1, p. 636)
- Happy Pythagorean Day! (Blitzer Bonus in Section 10.2, p. 646; Exercise 63)
- Hexagons in Nature: Eye-to-Eye with a Fly (Blitzer Bonus in Section 10.3, p. 653)
- Geometric Humor (Section 10.4, Exercise 60)
- Rectangular Solid Swimmer: The Yellow Boxfish (Section 10.5, Check Point 7)
- Selected Sports Balls (Section 10.5, Exercise 51)
- Jokes and Permutations (Section 11.2 Opener; Example 1)
- Celebrity Chefs (Section 11.3 Opener)
- Counting Methods and Jokes about Books (Section 11.3, Exercises 61–66)
- Rubik and His Cube (Blitzer Bonus in Section 11.5, p. 742)
- Blood Types (Section 11.6, Exercises 67–68; Exercise 108)
- Beware of Survey Results! (Blitzer Bonus in Section 12.1, p. 790)
- COVID Deaths (Blitzer Bonus in Section 12.1, p. 794)
- America's Most Misspelled Words (Blitzer Bonus in Section 12.1, p. 797; Exercise 48)
- Excuses for Not Meeting Assignment Deadlines (Section 12.1, Exercise 34)
- Marijuana Use among College Students (Section 12.1, Exercise 35)
- Federal Minimum Hourly Wage (Section 12.1, Exercise 36)
- Sports Riches: Weekly Team Pay (Section 12.1, Exercise 37)
- Educational Attainment (Section 12.1, Exercise 50)
- Price of a Movie Ticket (Section 12.2, Example 1)
- Youth Voter Turnout (Section 12.2, Check Point 1)
- Median Income, by Sex and Race (Section 12.2, Figure 12.11, p. 809)
- Top-Earning Celebrities (Section 12.2, Exercise 55)
- Most Successful Women Entrepreneurs (Section 12.2, Exercise 56)
- Gasoline Prices over Time (Section 12.3, Examples 1–3)
- Gasping at Computations! (Section 12.3, Great Question!, p. 819)
- An Inappropriate Model for New COVID-19 Cases (Blitzer Bonus in Section 12.4, p. 834)
- Cigarette Ads from the 1940s and 1950s (Section 12.6 Opener)
- Age and Blood Pressure (Section 12.6, Discussion on Scatter Plots and Correlation, pp. 844–851; Examples 2–5)
- Highest-Paid Actors (Chapter 12 Review, Exercise 7)
- Costco Paid Memberships (Chapter 12 Review, Exercise 71)
- Steve Kornacki: Crunching the Political Numbers (Blitzer Bonus in Section 13.1, p. 867)
- Gaining Momentum: Ranked-Choice Voting (Blitzer Bonus in Section 13.1, p. 870)
- Our Hodgepodge Presidential Election System (Blitzer Bonus in Section 13.4, p. 908)
- When to Use Two Letters for Vertices (Section 14.1, Great Question!, p. 919)

Updated Applications

- World Population (Section 1.2, Example 1)
- Cigarette Use by College Students (Section 1.2, Example 7)
- Average Cost of Tuition and Fees at Public and Private Four-Year U.S. Colleges (Section 1.2, Example 8 and Check Point 8)
- Populations of the Ten Most Populous U.S. States (Section 1.2, Exercises 1–2)
- Student-Loan Debt (Section 1.2, Exercise 62)
- College Graduates (Chapter 1 Review, Exercise 28)
- U.S. Population (Chapter 1 Review, Exercise 30; Section 5.7, Example 7)
- Grade Inflation (Chapter 1 Test, Exercise 16; Section 6.2, Exercises 107–108)
- Legalized Marijuana (Section 2.3, Exercises 135–140)
- Worldwide Health Indicators (Section 2.4, Exercise 105)
- Actors with the Most Oscar Nominations (Section 3.5 Opener)
- Diversity (Chapter 3 Review, Exercises 32–34; Chapter 6 Review, Exercise 4)
- Food and Health Care Spending (Chapter 3 Test, Exercise 15)

- Letters and Words in Base Two (Blitzer Bonus in Section 4.2, p. 226)
- Music in Base Two (Blitzer Bonus in Section 4.2, p. 228)
- Quantum Computers (Blitzer Bonus in Section 4.3, p. 236)
- The Curious Number 142,857 (Section 5.1, Exercise 122)
- Life Expectancy (Section 5.2, Example 5)
- Money Collected and Spent by the U.S. Government (Section 5.2, Exercises 127–130)
- Jobs and Education (Section 5.3, Exercises 123–124)
- The U.S. National Debt (Section 5.6 Opener; Example 5; Example 9)
- Growth of Our Most Populated States (Section 5.7, Exercises 133–134)
- How Long It Takes to Earn $1000 (Section 6.3 Opener)
- Attitudes of First-Year College Students (Section 6.3, Example 2)
- Options for a Toll (Section 6.3, Example 3; Exercise 32; Section 7.1, Example 3)
- Car Talk (Section 6.3, Exercises 23–24)
- Foreign-Born Americans (Section 6.5, Exercises 85–86)
- Price of a Movie Ticket (Chapter 6 Review, Exercise 23)
- Bicycle-Friendly Communities (Chapter 6 Review, Exercise 55)
- Gun Ownership (Chapter 6 Test, Exercise 8)
- Spending on Food and Health Care (Section 7.2, Exercises 61–62)
- Yearly Earnings, by Gender and Age (Section 7.2, Exercises 63–64)
- Inmates in Federal Prison (Section 7.3, Exercise 64)
- Cellphone-Only and Landline Customers (Section 7.3, Exercise 65)
- World Population (Section 7.6, Example 2)
- Income Tax (Section 8.2, Examples 1–3; Exercises 1–42)
- Reading Stock Tables (Section 8.5, Example 7; Exercises 19–20)
- Costs of Owning and Operating a Car (Section 8.6, Table 8.7, p. 560; Exercises 13–16)
- Appraising a House (Blitzer Bonus in Section 10.4, p. 662)
- The 2021 U.S. Senate (Section 11.3, Example 4)
- Marital Status of the U.S. Population (Section 11.4, Example 4; Section 11.6, Example 7; Section 11.7, Exercises 65–76)
- Probability, Permutations, and Jokes (Section 11.5, Example 1)
- U.S. Drivers, by Age (Section 11.6, Example 3)
- Gambling It Away (Blitzer Bonus in Section 11.8, p. 775)
- Employment Status of the U.S. Labor Force (Chapter 11 Review, Exercises 42–44)
- Deaths Involving Firearms (Chapter 11 Review, Exercises 100–106)
- Living below the Poverty Line (Section 12.1, Table 12.5, p. 796)
- Starting Salaries of College Graduates (Section 12.1, Exercises 22–25)
- Oscar Winners (Section 12.1, Exercise 32)
- Singers with Top Albums (Section 12.3, Exercise 37)
- U.S. Murder Rates (Section 12.4, Exercises 73–74)
- Senate Voting Power (Blitzer Bonus in Section 13.3, p. 888)
- The Electoral College (Blitzer Bonus in Section 13.3, p. 897)

Other Textbook Changes

- **Logical Fallacies —** At the suggestion of reviewers, we augmented Section 3.7 by adding content on various types of logical fallacies (Blitzer Bonus, p. 194; Exercises 63–76)
- **Spreadsheet Usage —** At the suggestion of reviewers, we added the use of spreadsheets to Chapter 8 (Personal Finance). The coverage provides enough information on how to use spreadsheets to help a non-user become familiar with the basics. Here are specific places where we integrated spreadsheets:
  - Find the Value of an Annuity (Section 8.5, Example 3)
  - Find Regular Annuity Payments (Section 8.5, Example 6)
  - Find a Car Payment (Section 8.6, Example 2)
  - Prepare a Loan Amortization Schedule (Section 8.5, Example 3; Exercises 11–12)
- **References to Free Online Technology —** In several applications where the use of technology is discussed, we mention the availability of free online resources. In the new Example 4 of Section 12.6, we demonstrate how to use one such resource to find the equation of the regression line and the correlation coefficient for a set of data.
- **Diversity, Equity, and Inclusion —** We conducted an external review of the text's content to determine how it could be improved to address issues related to diversity, equity, and inclusion. Based on the review, we revised content, including but not limited to race, ethnicity, gender, socioeconomic status, ability, age, sexual orientation, and religious or political beliefs. Please let us know if you have any concerns or needs with this product so that we can investigate and address them. See page xvii for details on how to reach us and more information about diversity, equity, and inclusion.
- **Objectives —** The list of each section's objective, previously headed "What am I supposed to learn?" (which annoyed some reviewers) has been renamed "What You'll Learn."
- **Brief Reviews —** We added a few more Brief Reviews to the sections, and we added some practice exercises to the most important Brief Reviews so that students have a chance to practice what they've just reviewed.

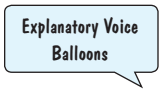
- **Learning Catalytics —** Each section of the Annotated Instructor's Edition features a keyword that can be entered into Learning Catalytics to bring you directly to questions newly written for that section. Detailed instructions for using these keywords can be found at **bit.ly/3jCeKFb**.
- **Animations and StatCrunch —** We added more animations and StatCrunch activities and now reference them at point-of-use within the textbook.
- **MyLab Resource List —** Prior to the exercises in each section, the Annotated Instructor's Edition provides a list of resources available for that section in MyLab Math.
- **Added Example —** At the suggestion of reviewers, we added a new example to Section 14.1 (Example 7, Identifying Parts of a Graph) to better prepare students for Exercises 23–48 in Exercise Set 14.1.
- **"Handy Ideas, Notations, and Formulas" —** We added six reference pages to the end of the book. They contain helpful information for each chapter.

New in MyLab Math

- **Corequisite Support Resources** provide all the content and assessment resources necessary for students and instructors. MyLab Math supports various corequisite course models, including concurrent (aka just-in-time) and consecutive (aka front-loaded) models. For more details, see page xi or the Corequisite Implementation Guide at **bit.ly/3zGzPnM**.
- **Animations —** We added more animations and now reference them at point-of-use within the textbook.
- **StatCrunch —** We added more StatCrunch activities and now reference them at point-of-use within the textbook.
- **Enhanced Assignments —** These section-level assignments have two unique properties (and are fully editable):
(1) They have learning aids strategically turned off for some exercises to ensure that students really understand how to work the exercises independently.
(2) They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.
- **Video Assignments —** These section-level assignments are especially helpful for online classes or "flipped" classes, where some or all learning takes place independently.
- **Learning Catalytics —** We added all new questions for our popular student-response system and added keywords to the Annotated Instructor's Edition to make it easy for you to use them. Detailed instructions for using these keywords can be found at **bit.ly/3jCeKFb**.
- **Guided Practice Worksheets** for each section of the text, located in the Learning Guide, now contain more examples and exercises as well as coverage for every objective.

What Familiar Features Have Been Retained in the Eighth Edition?

- **Chapter-Opening and Section-Opening Scenarios.** Every chapter and every section open with a scenario presenting a unique application of mathematics in students' lives outside the classroom. These scenarios are revisited in the course of the chapter or section in an example, discussion, or exercise. The often humorous tone of these openers is intended to help fearful and reluctant students overcome their negative perceptions about math. A feature called "Here's Where You'll Find These Applications" is included with each chapter opener.
- **Section Objectives (What You'll Learn).** Learning objectives are clearly stated at the beginning of each section. These objectives help students recognize and focus on the section's most important ideas. The objectives are restated in the margin at their point of use.
- **Detailed Worked-Out Examples.** Each example is titled, making the purpose of the example clear. Examples are clearly written and provide students with detailed step-by-step solutions. No steps are omitted and each step is thoroughly explained to the right of the mathematics.



- **Explanatory Voice Balloons.** Voice balloons are used in a variety of ways to demystify mathematics. They translate mathematical language into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect problem solving to concepts students have already learned.

Check Point

- **Check Point Examples.** Each example is followed by a similar matched problem, called a Check Point, offering students the opportunity to test for conceptual understanding by working a similar exercise. The answers to the Check Points are provided in the answer section in the back of the book. Worked-out video solutions for many Check Points are in the MyLab Math course.
- **Great Question!** This feature presents study tips in the context of students' questions. Answers to the questions offer suggestions for problem solving, point

out common errors to avoid, and provide informal hints and suggestions. As a secondary benefit, this feature should help students not to feel anxious or threatened when asking questions in class.

BLITZER BONUS

- **Blitzer Bonuses.** These enrichment vignettes provide historical, interdisciplinary, and otherwise interesting connections to the topic under study, showing students that math is an interesting and dynamic discipline.
- **Brief Reviews.** The book's Brief Review boxes summarize mathematical skills that students should have learned previously, but that many students still need to review. This feature appears whenever a particular skill is first needed and eliminates the need to reteach that skill. For the Eighth Edition, we added some practice exercises to support the content in these reviews.
- **Concept and Vocabulary Checks.** The Eighth Edition contains 653 short-answer exercises, mainly fill-in-the blank and true/false items, that assess students' understanding of the definitions and concepts presented in each section. The Concept and Vocabulary Checks appear as separate features preceding the Exercise Sets. These are assignable in the MyLab Math course.
- **Extensive and Varied Exercise Sets.** An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within seven category types: Practice Exercises, Practice PLUS Exercises, Application Exercises, Explaining the Concepts, Critical Thinking Exercises, Technology Exercises, and Group Exercises. Practice PLUS exercises contain practice problems that often require students to combine several skills or concepts, providing instructors the option of creating assignments that take Practice Exercises to a more challenging level.
- **Chapter Summaries.** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.
- **End-of-Chapter Materials.** A comprehensive collection of review exercises for each of the chapter's sections follows the Summary. This is followed by a Chapter Test that enables students to test their understanding of the material covered in the chapter. Worked-out video solutions are available for every Chapter Test Prep problem in the MyLab Math course or on YouTube.

MyLab Math Resources for Success

MyLab Math (**pearson.com/mylab/math**) is available to accompany Pearson's market-leading text options, including this text (access code required). MyLab™ is the teaching and learning platform that empowers you to reach every student. MyLab Math combines trusted author content—including full eText and assessment with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

MyLab Math supports all learners, regardless of their ability and background, to provide an equal opportunity for success. Accessible resources support learners for a more equitable experience no matter their abilities. And options to personalize learning and address individual gaps help to provide each learner with the specific resources they need to achieve success.

NEW! Corequisite Course Support

MyLab Math supports various corequisite course models, including concurrent (aka just-in-time) and consecutive (aka front-loaded) models. MyLab Math for this text contains the following learning and assessment resources to support corequisite courses:

1. **Corequisite Support eText** built from Bob Blitzer's developmental mathematics texts. This eText begins after the final page of the *Thinking Mathematically* eText.
2. **Instructional videos** for each corequisite objective.
3. **Assignable algorithmic exercises** for each corequisite objective.
4. **Worksheets** with instruction and exercises for key corequisite objective (also available in print).
5. **Mindset videos** and assignable, open-ended exercises foster a growth mindset, which is especially helpful for students who have not previously experienced success in math.
6. **Corequisite Implementation Guide** (downloadable at **bit.ly/3zGzPnM**) with specific guidelines for using the materials to teach various corequisite models.

To help target instruction on corequisite objectives, MyLab includes a **Skills Check Quiz** for each chapter. These quizzes address the prerequisite skills needed for the chapter. Based on the results of these quizzes, students can receive **personalized assignments** to address objectives that are not mastered. This way, students can focus on just the topics they need help with. You can choose either one personalized assignment per chapter (labeled "Skills Review Homework for Chapter X") or personalized Enhanced Assignments at the section level. (Either of the two options provides the same prerequisite skills support, but the Enhanced Assignments are more of a "just-in-time" approach.)

Note that the above resources can also be helpful for students in your regular (non-corequisite) courses, where we understand that students can also benefit from targeted refreshers on prerequisite skills.

Student Resources

Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student needs practice, giving all students the support they need — when and where they need it — to be successful.

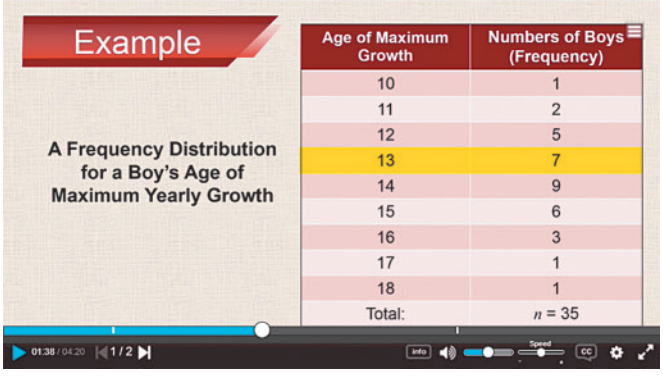
NEW! **Animations —** Located throughout the textbook you'll find Animation icons like the one pictured above. They are designed to facilitate active learning and visualization of key concepts. These Animations are housed within MyLab Math and are ideal for classroom use during lecture or by students independently. They were created in GeoGebra and are usable on any type of device. They are also editable.

StatCrunch

NEW! **StatCrunch —** In Chapters 11 and 12, you'll find StatCrunch icons like the one shown above. Like the Animation features, they are designed to facilitate active learning and exploration. They are also housed within MyLab Math. StatCrunch is Pearson's online statistical software.

Exercises with Immediate Feedback — The exercises in MyLab Math reflect the approach and learning style of this text, and regenerate algorithmically to give the student unlimited opportunity for practice and mastery. Most exercises include learning aids, such as *Help Me Solve This* and *View an Example*, and they offer helpful feedback when students enter incorrect answers.

Personalized Homework — With Personalized Homework, students take a quiz or test and receive a subsequent homework assignment that is personalized based on their performance. This way, students can focus on just the topics they have not yet mastered.



Instructional Videos — High-quality instructional videos are included for every objective in the text. Many of these feature built-in interactive quizzes.

Check Point Videos — The Check Point problems in the text mirror each worked-out Example. The Check Point Videos provide solutions to the Check Point problems.

Chapter Test Prep Videos correspond to each exercise in the Chapter Test in the text, enabling students to effectively prepare for high-stakes testing. These are available in MyLab Math and **www.youtube.com/user/pearsonmathstats** (playlist "Blitzer Thinking Mathematically 8e").

Mindset Videos and assignable, open-ended exercises foster a growth mindset in students. This material encourages them to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as learning opportunities—so often a hurdle for math students.

Learning Guide (also available in print format) consists of two parts:

1. **Guided Practice Worksheets** for each section of the text. These worksheets contain key terms, short examples, and practice exercises. Answers are provided. UPDATED! Now contains more examples and exercises as well as coverage for every objective.
2. **Integrated Review Worksheets** for every prerequisite objective. These feature both instruction and practice.

Pearson eText — The eText is "reflowable" to adapt to use on tablets and smartphones. You can insert your own highlights, notes, and bookmarks. It is also fully accessible using screen-readers. Download the Pearson+ app to access your eText on your smartphone or tablet anytime—even offline.

Study Slides — PowerPoint slides featuring key ideas and examples are available for students within the Video & Resource Library. These slides are compatible with screen readers.

Student Solutions Manual — Fully worked solutions to odd-numbered exercises. Available for download from within MyLab Math.

Instructor Resources

Your course is unique. So whether you'd like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

Pre-Built Assignments are designed to make the homework experience as effective as possible for students. All of these assignments are *fully editable*.

- NEW! **Enhanced Assignments —** These section-level assignments have two unique properties: (1) They have learning aids strategically turned off for some exercises to ensure that students really understand how to work the exercises independently. (2) They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.
- NEW! **Video Assignments —** These section-level assignments are especially helpful for online classes or "flipped" classes, where some or all learning takes place independently.

Learning Catalytics — With Learning Catalytics™, you'll hear from every student when it matters most. You pose a variety of questions (choosing from pre-loaded questions or questions of your own making) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops. NEW! Each section of the Annotated Instructor's Edition features a keyword that can be entered into Learning Catalytics to bring you directly to questions newly written for that section. Detailed instructions for using these keywords can be found at **bit.ly/3jCeKFb**.

Accessibility — Pearson works continuously to ensure our products are as accessible as possible to all students. Currently, we are working toward achieving WCAG 2.0 AA for our existing products (2.1 AA for future products) and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media (**https://www.pearson.com/us/accessibility.html**).

Other instructor resources, available for download from your MyLab Math course or Pearson's online catalog, include:

- **Annotated Instructor's Edition eText —** This page-for-page eText is available within the Instructor Resources section of MyLab Math.
- **Instructor Solution Manual,** which contains worked-out solutions for every exercise in the text.
- **PowerPoint Lecture Slides,** which are fully animated and editable, are included for each section of the text.
- **TestGen®,** which enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions.
- **Instructor's Testing Manual,** which features printable PDFs containing all of the test exercises available in TestGen.

Acknowledgments

An enormous benefit of authoring a successful textbook is the broad-based feedback I receive from students, dedicated users, and reviewers. Every change to this edition is the result of their thoughtful comments and suggestions. I would like to express my appreciation to all the reviewers, whose collective insights form the backbone of this revision. In particular, I would like to thank the following people for reviewing *Thinking Mathematically* for this Eighth Edition.

- Disa Beaty, *Rose State College*
- Ellen Carr, *Delaware State University*
- Kelly David, *Tarrant County College*
- Ryan DeFronzo, *New Hampshire Technical Institute*
- Emily Elrod, *Valencia College*
- Ryan Grossman, *Saint Mary-of-the-Woods College*
- Gail Illich, *McLennan Community College*
- Jennifer La Fleur, *Harford Community College*
- Edith Lester, *Volunteer State Community College*
- Sherry Liu, *Houston Community College*
- Avan McHenry, *Glenville State College*
- Tom Rittweger, *Georgia Northwestern Technical College*
- Camelia Salajean, *Harold Washington College*
- Linda Schott, *Ozarks Technical Community College*
- Alica Serfaty de Markus, *Miami Dade College*
- Alfredo Vaquiax-Alvarado, *Our Lady of the Lake University*

The following reviewers provided helpful feedback on improving the MyLab course for *Thinking Mathematically*. I thank them for their input.

- Cheban Acharya, *Florida SouthWestern State College*
- Gonzalo Aguerrevere, *Miami Dade College*
- Dilshad Akrayee, *Georgia Highlands College*
- Sandra Anderson, *Georgia Highlands College*
- Brandon Armstrong, *Valencia College*
- Adam Bailey, *Florida A&M University*
- Mary Ann Barber, *University of North Texas*
- Ive Barreiros, *Miami Dade College*
- Disa Beaty, *Rose State College*
- Brenda E. Brown, *University of the District of Columbia*
- Ellen Carr, *Delaware State University*
- Brandi Cline, *Lone Star College Tomball*
- Agnes Conaty, *Trinity Washington University*
- Kelly David, *Tarrant County College*
- Ryan DeFronzo, *New Hampshire Technical Institute*
- Robert Diaz, *Fullerton College*
- Ali Falahat, *Houston Community College*
- Charles Godfrey, *Arizona Western College*
- Ryan Grossman, *Saint Mary-of-the-Woods College*
- Angela Harris, *Florida State University*
- Gene Hendricks, *Central Arizona College*
- Jerry Holt, *Florida A&M University*

- Gail T. Illich, *McLennan Community College*
- Tammie Wilkins Johnson, *Spartanburg Community College*
- Gordon Jorgenson III, *Los Angeles Pacific University*
- Edith Lester, *Volunteer State Community College*
- Mike McCraith, *Cuyahoga Community College*
- Kimberly U. McIlvaine, *Camden County College*
- George Michaelides, *Hillsborough Community College*
- Daniel T. Murphree, *Great Basin College*
- Michael Murphy, *Texas State Technical College*
- Geoffrey D. Owers, *Rio Salado College*
- Tom Rittweger, *Georgia Northwestern Technical College*
- Kevin Ritzman, *Fox Valley Technical College*
- David W. Roach, *Murray State University*
- Camelia Salajean, *Harold Washington College*
- Brunilda Santiago, *Tarrant County College*
- Linda Schott, *Ozarks Technical Community College*
- Sandra D. Seifert, *Florida SouthWestern State College*
- Alicia Serfaty de Markus, *Miami Dade College*
- Shannon Solis, *San Jacinto College*
- Leslie Sterrett, *Indian River State College*
- David H. Tseng, *Miami Dade College*
- Alfredo Vaquiax-Alvarado, *Our Lady of the Lake University*
- Russ Wydeen, *Ozarks Technical Community College*

Each reviewer from every edition has contributed to the success of this product, and I would like to also continue to offer my thanks to them.

- David Allen, *Iona College*
- Deana Alexander, *Indiana University—Purdue University*
- Carl P. Anthony, *Holy Family University*
- Laurel Berry, *Bryant and Stratton College*
- Nina Bohrod, *Anoka-Ramsey Community College*
- Kris Bowers, *Florida State University*
- Gerard Buskes, *University of Mississippi*
- Fred Butler, *West Virginia University*
- Kim Caldwell, *Volunteer State Community College*
- Jimmy Chang, *St. Petersburg College*
- Kevin Charlwood, *Washburn University*
- Jerry Chen, *Suffolk County Community College*
- Ivette Chuca, *El Paso Community College*
- David Cochener, *Austin Peay State University*
- Stephanie Costa, *Rhode Island College*
- Elizabeth T. Dameron, *Tallahassee Community College*
- Tristen Denley, *University of Mississippi*
- Darlene O. Diaz, *Santiago Canyon College*
- Suzanne Feldberg, *Nassau Community College*
- Margaret Finster, *Erie Community College*
- Maryanne Frabotta, *Community Campus of Beaver County*
- Lyn Geisler III, *Randolph-Macon College*
- Patricia G. Granfield, *George Mason University*
- Cornell Grant, *Georgia Piedmont Technical College*
- Dale Grussing, *Miami Dade College*
- Cindy Gubitose, *Southern Connecticut State University*
- Virginia Harder, *College at Oneonta*
- Joseph Lloyd Harris, *Gulf Coast Community College*
- Julia Hassett, *Oakton Community College*
- Sonja Hensler, *St. Petersburg College*
- James Henson, *Edinboro University of Pennsylvania*
- Larry Hoehn, *Austin Peay State University*
- Diane R. Hollister, *Reading Area Community College*
- Kalynda Holton, *Tallahassee Community College*
- Alec Ingraham, *New Hampshire College*
- Theresa Jones, *Texas State University*
- Elizabeth Kiedaisch, *College of DuPage*
- Lauren Kieschnick, *Mineral Area College*
- Alina Klein, *University of Dubuque*
- Susan Knights, *College of Western Idaho*
- Isabelle Kumar, *Miami Dade College*
- Linda Kuroski, *Erie Community College—City Campus*
- Jamie Langille, *University of Nevada Las Vegas*
- Veronique Lanuqueitte, *St. Petersburg College*
- Dennine LaRue, *Fairmont State University*
- Julia Ledet, *Louisiana State University*
- Mitzi Logan, *Pitt Community College*
- Dmitri Logvnenko, *Phoenix College*
- Linda Lohman, *Jefferson Community College*
- Richard J. Marchand, *Slippery Rock University*
- Mike Marcozzi, *University of Nevada Las Vegas*
- Diana Martelly, *Miami Dade College*
- Jim Matovina, *Community College of Southern Nevada*
- Erik Matsuoka, *Leeward Community College*
- Marcel Maupin, *Oklahoma State University*
- Carrie McCammon, *Ivy Tech Community College*
- Diana McCammon, *Delgado Community College*
- Mex McKinley, *Florida Keys Community College*
- David Miller, *William Paterson University*
- Taranna Amani Miller, *Indian River State College*
- Carla A. Monticelli, *Camden County College*
- Paul Mosbos, *State University of New York—Cortland*
- Tammy Muhs, *University of Central Florida*
- Cornelius Nelan, *Quinnipiac University*
- Lawrence S. Orilia, *Nassau Community College*
- Richard F. Patterson, *University of North Florida*
- Frank Pecchioni, *Jefferson Community College*
- Stan Perrine, *Charleston Southern University*
- Anthony Pettofrezzo, *University of Central Florida*
- Val Pinciu, *Southern Connecticut State University*
- Evelyn Pupplo-Cody, *Marshall University*
- Virginia S. Powell, *University of Louisiana at Monroe*
- Kim Query, *Lindenwood College*
- Anne Quinn, *Edinboro University of Pennsylvania*
- Bill Quinn, *Frederick Community College*
- Sharonda Ragland, *ECPI College of Technology*
- Shawn Robinson, *Valencia Community College*
- Gary Russell, *Brevard Community College*
- Tonny Sangutei, *North Carolina Central University*
- Mary Lee Seitz, *Erie Community College*
- Laurie A. Stahl, *State University of New York—Fredonia*
- Abolhassan Taghavy, *Richard J. Daley College*
- Diane Tandy, *New Hampshire Technical Institute*
- Ann Thrower, *Kilgore College*
- Mike Tomme, *Community College of Southern Nevada*
- Sherry Tornwall, *University of Florida*
- Linda Tully, *University of Pittsburgh at Johnstown*
- Cindy Vanderlaan, *Indiana Purdue University Fort Wayne*

- Christopher Scott Vaughen, *Miami Dade College*
- Bill Vaughters, *Valencia Community College*
- Alexandra Verkhovtseva, *Anoka-Ramsey Community College*
- Karen Villareal, *University of New Orleans*
- Don Warren, *Edison Community College*
- Shirley Wilson, *North Central College*
- James Wooland, *Florida State University*
- Clifton E. Webb, *Virginia Union University*
- Cindy Zarske, *Fullerton College*
- Marilyn Zopp, *McHenry County College*

Special thanks to Brad Davis for his brilliant work on the manuscript, answer preparation, and page proof. Thanks also to:

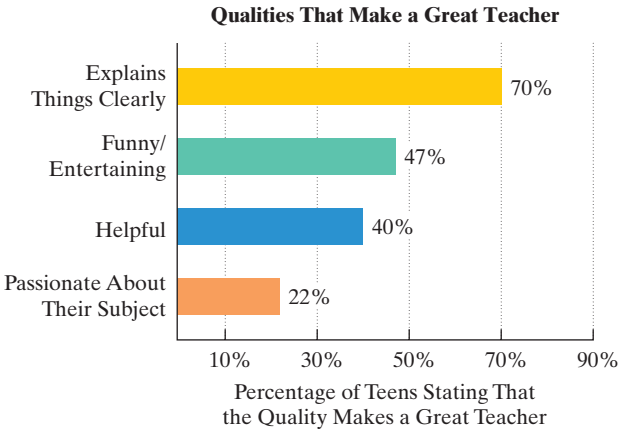
- Donna Kirk and Tim Trowbridge for their contributions to the manuscript
- Dan Miller for writing the solutions manuals
- Brian Morris at Scientific Illustrators for superbly illustrating the book
- Francesca Monaco, project manager, and Tamela Ambush, production manager, for shepherding this complex project through its many phases
- Bob Carroll and Shana Siegmund for creating the MyLab Math course for the product
- Alicia Wilson for her stellar marketing efforts
- Editor Jeff Weidenaar for his guidance and contributions
- Editorial assistant Jon Krebs for behind-the-scenes support

Final thanks go to the Pearson sales force for your confidence and enthusiasm about the book.

I hope that my love for learning, as well as my respect for the diversity of students I have taught and learned from over the years, is apparent throughout this new edition. By connecting mathematics to the whole spectrum of learning, it is my intent to show students that their world is profoundly mathematical, and indeed, π is in the sky.

Bob Blitzer

To the Student



Source: Avanta Learning System

The bar graph shows some of the qualities that students say make a great teacher. It was my goal to incorporate each of these qualities throughout the pages of this book to help you gain control over the part of your life that involves numbers and mathematical ideas.

Explains Things Clearly

I understand that your primary purpose in reading *Thinking Mathematically* is to acquire a solid understanding of the required topics in your liberal arts math course. In order to achieve this goal, I've carefully explained each topic. Important definitions and procedures are set off in boxes, and worked-out examples that present solutions in a step-by-step manner appear in every section. Each example is followed by a similar matched problem, called a Check Point, for you to try so that you can actively participate in the learning process as you read the book. (Answers to all Check Points appear in the back of the book, and video solutions are in MyLab Math.)

Funny & Entertaining

Who says that a math textbook can't be entertaining? From our engaging cover to the photos in the chapter and section openers, prepare to expect the unexpected. I hope some of the book's enrichment essays, called Blitzer Bonuses, will put a smile on your face from time to time.

Helpful

I designed the book's features to help you acquire knowledge of fundamental mathematics, as well as to show you how math can solve authentic problems that apply to your life. These helpful features include

- **Explanatory Voice Balloons:** Voice balloons are used in a variety of ways to make math less intimidating. They translate mathematical language into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect new concepts to concepts you have already learned.
- **Great Question!:** The book's Great Question! boxes are based on questions students ask in class. The answers to these questions give suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.
- **Chapter Summaries:** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples from the chapter that illustrate these key concepts are also referenced in the chart. Review these summaries and you'll know the most important material in the chapter!

Passionate about the Subject

I passionately believe that no other discipline comes close to math in offering a more extensive set of tools for application and development of your mind. I wrote the book in Point Reyes National Seashore, 40 miles north of San Francisco. The park consists of 75,000 acres with miles of pristine surf-washed beaches, forested ridges, and bays bordered by white cliffs. It was my hope to convey the beauty and excitement of mathematics using nature's unspoiled beauty as a source of inspiration and creativity. Enjoy the pages that follow as you empower yourself with the mathematics needed to succeed in college, your career, and in your life.

Regards,

Bob

Pearson's Commitment to Diversity, Equity, and Inclusion

Pearson is dedicated to creating bias-free content that reflects the diversity, depth, and breadth of all learners' lived experiences.

We embrace the many dimensions of diversity, including but not limited to race, ethnicity, gender, sex, sexual orientation, socioeconomic status, ability, age, and religious or political beliefs.

Education is a powerful force for equity and change in our world. It has the potential to deliver opportunities that improve lives and enable economic mobility. As we work with authors to create content for every product and service, we acknowledge our responsibility to demonstrate inclusivity and incorporate diverse scholarship so that everyone can achieve their potential through learning. As the world's leading learning company, we have a duty to help drive change and live up to our purpose to help more people create a better life for themselves and to create a better world.

Our ambition is to purposefully contribute to a world where:

- Everyone has an equitable and lifelong opportunity to succeed through learning.
- Our educational content accurately reflects the histories and lived experiences of the learners we serve.
- Our educational products and services are inclusive and represent the rich diversity of learners.
- Our educational content prompts deeper discussions with students and motivates them to expand their own learning (and worldview).

Accessibility

We are also committed to providing products that are fully accessible to all learners. As per Pearson's guidelines for accessible educational Web media, we test and retest the capabilities of our products against the highest standards for every release, following the WCAG guidelines in developing new products for copyright year 2022 and beyond.

You can learn more about Pearson's commitment to accessibility at **https://www.pearson.com/us/accessibility.html**

Contact Us

While we work hard to present unbiased, fully accessible content, we want to hear from you about any concerns or needs with this Pearson product so that we can investigate and address them.

Please contact us with concerns about any potential bias at **https://www.pearson.com/report-bias.html**

For accessibility-related issues, such as using assistive technology with Pearson products, alternative text requests, or accessibility documentation, email the Pearson Disability Support team at **disability.support@pearson.com**

Pearson

Index of Applications

D

E

F

S

T

This page intentionally left blank

Problem Solving and Critical Thinking

1

How would your lifestyle change if a gallon of gas cost $9.15? Or if the price of a staple such as milk was $15? That's how much those products would cost if their prices had increased at the same rate college tuition has increased since 1980.

If these trends continue, what can we expect in the 2020s and beyond? We can answer this question by using estimation techniques that allow us to represent the data mathematically. With such representations, called *mathematical models*, we can gain insights and predict what might occur in the future on a variety of issues, ranging from college costs to global warming.

Tuition and Fees at Four-Year Colleges

	School Year Ending 2000	School Year Ending 2020
Public	$3349	$9439
Private	$14,616	$32,679

Source: National Center for Education Statistics

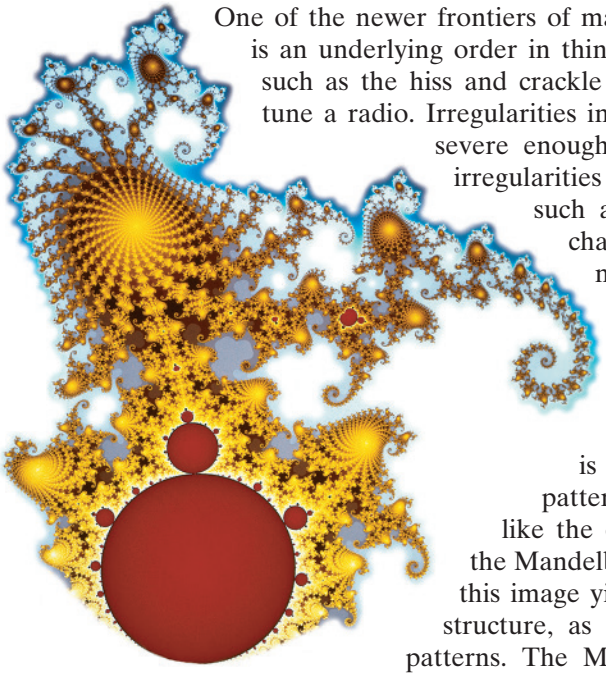
Here's where you'll find these applications:

Mathematical models involving college costs are developed in Example 8 and Check Point 8 of Section 1.2. In Exercises 51 and 52 in Exercise Set 1.2, you will approach our climate crisis mathematically by developing models for data related to global warming.

1.1 Inductive and Deductive Reasoning

What You'll Learn

1. Understand and use inductive reasoning.
2. Understand and use deductive reasoning.



A magnification of the Mandelbrot set
Richard F. Voss

One of the newer frontiers of mathematics suggests that there is an underlying order in things that appear to be random, such as the hiss and crackle of background noises as you tune a radio. Irregularities in the heartbeat, some of them severe enough to cause a heart attack, or irregularities in our sleeping patterns, such as insomnia, are examples of chaotic behavior. Chaos in the mathematical sense does not mean a complete lack of form or arrangement. In mathematics, chaos is used to describe something that appears to be random but is not actually random. The patterns of chaos appear in images like the one shown on the left, called the Mandelbrot set. Magnified portions of this image yield repetitions of the original structure, as well as new and unexpected patterns. The Mandelbrot set transforms the hidden structure of chaotic events into a source of wonder and inspiration.

Many people associate mathematics with tedious computation, meaningless algebraic procedures, and intimidating sets of equations. The truth is that mathematics is the most powerful means we have of exploring our world and describing how it works. The word *mathematics* comes from the Greek word *mathematikos*, which means "inclined to learn." To be mathematical literally means to be inquisitive, open-minded, and interested in a lifetime of pursuing knowledge!

Mathematics and Your Life

A major goal of this book is to show you how mathematics can be applied to your life in interesting, enjoyable, and meaningful ways. The ability to think mathematically and reason with quantitative issues will help you so that you can:

- order and arrange your world by using sets to sort and classify information (Chapter 2, Set Theory);
- use logic to evaluate the arguments of others and become a more effective advocate for your own beliefs (Chapter 3, Logic);
- understand the relationship between cutting-edge technology and ancient systems of number representation (Chapter 4, Number Representation and Calculation);
- put the numbers you encounter in the news, from contemplating the national debt to grasping just how colossal $1 trillion actually is, into perspective (Chapter 5, Number Theory and the Real Number System);
- use mathematical models to gain insights into a variety of issues, including the positive benefits that humor and laughter can have on your life (Chapter 6, Algebra: Equations and Inequalities);
- use basic ideas about savings, loans, and investments to achieve your financial goals (Chapter 8, Personal Finance);
- use geometry to study the shape of your world, enhancing your appreciation of nature's patterns and beauty (Chapter 10, Geometry);
- develop an understanding of the fundamentals of statistics and how these numbers are used to make decisions (Chapter 12, Statistics);

BLITZER BONUS

Mathlete

John Urschel (1991–) pursued dual dreams of pro football and math research. At age 26, after three seasons with the NFL (National Football League) as an offensive guard for the Baltimore Ravens, Urschel left the sport to become a mathematician. He's now at the Massachusetts Institute of Technology working on his Ph.D. Urschel plans to become a professor so he can teach students how to use math to think critically about issues that affect their daily lives.

- understand the mathematical paradoxes of voting in a democracy, increasing your ability to function as a more fully aware citizen (Chapter 13, Voting and Apportionment);
- use graph theory to examine how mathematics is used to solve problems in the business world (Chapter 14, Graph Theory).

Mathematics and Your Career

"It is better to take what may seem to be too much math rather than too little. Career plans change, and one of the biggest roadblocks in undertaking new educational or training goals is poor preparation in mathematics. Furthermore, not only do people qualify for more jobs with more math, they are also better able to perform their jobs."

—Occupational Outlook Quarterly

Generally speaking, the income of an occupation is related to the amount of education required. This, in turn, is usually related to the skill level required in language and mathematics. With our increasing reliance on technology, the more mathematics you know, the more career choices you will have.

Mathematics and Your World

Mathematics is a science that helps us recognize, classify, and explore the hidden patterns of our universe. Focusing on areas as different as planetary motion, animal markings, shapes of viruses, aerodynamics of figure skaters, and the very origin of the universe, mathematics is the most powerful tool available for revealing the underlying structure of our world. Within the last 40 years, mathematicians have even found order in chaotic events such as the uncontrolled storm of noise in the nerve cells of the brain during an epileptic seizure.

Inductive Reasoning

1 Understand and use inductive reasoning.

Mathematics involves the study of patterns. In everyday life, we frequently rely on patterns and routines to draw conclusions. Here is an example:

> The last six times I went to the beach, the traffic was light on Wednesdays and heavy on Sundays. My conclusion is that weekdays have lighter traffic than weekends.

This type of reasoning process is referred to as *inductive reasoning*, or *induction*.

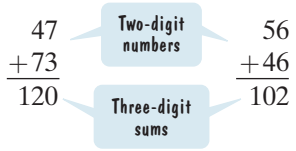
Inductive Reasoning

Inductive reasoning is the process of arriving at a general conclusion based on observations of specific examples.

Although inductive reasoning is a powerful method of drawing conclusions, we can never be absolutely certain that these conclusions are true. For this reason, the conclusions are called **conjectures**, **hypotheses**, or educated guesses. A strong inductive argument does not guarantee the truth of the conclusion, but rather provides strong support for the conclusion. If there is just one case for which the conjecture does not hold, then the conjecture is false. Such a case is called a **counterexample**.

Example 1 Finding a Counterexample

The ten symbols that we use to write numbers, namely, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, are called **digits**. In each example shown below, the sum of two two-digit numbers is a three-digit number.



Is the sum of two two-digit numbers always a three-digit number? Find a counterexample to show that the statement

> The sum of two two-digit numbers is a three-digit number

is false.

Great Question!

Why is it so important to work each of the book's Check Points?

You learn best by doing. Do not simply look at the worked examples and conclude that you know how to solve them. To be sure you understand the worked examples, try each Check Point. Check your answer in the answer section before continuing your reading. Expect to read this book with pencil and paper handy to work the Check Points.

Solution

There are many counterexamples, but we need to find only one. Here is an example that makes the statement false:

Two-digit numbers

$$\begin{array}{r} 56 \\ +43 \\ \hline 99 \end{array}$$

This is a two-digit sum, not a three-digit sum.

This example is a counterexample that shows the statement

The sum of two two-digit numbers is a three-digit number

is false.

Check Point 1. Find a counterexample to show that the statement

The product of two two-digit numbers is a three-digit number

is false.

Here are two examples of inductive reasoning:

- **Strong Inductive Argument** In a random sample of 380,000 freshmen at 722 four-year colleges, 25% said they frequently came to class without completing readings or assignments (*Source:* National Survey of Student Engagement). We can conclude that there is a 95% probability that between 24.84% and 25.15% of all college freshmen frequently come to class unprepared.
- **Weak Inductive Argument** Neither my dad nor my boyfriend has ever cried in front of me. Therefore, men have difficulty expressing their feelings.

In Chapter 12, you will learn how observations from a randomly selected group, one in which each member of the population has an equal chance of being selected, can provide probabilities of what is true about an entire population.

When generalizing from observations about your own circumstances and experiences, avoid jumping to hasty conclusions based on a few observations. Psychologists theorize that we do this—that is, place everyone in a neat category—to feel more secure about ourselves and our relationships to others.

Inductive reasoning is extremely important to mathematicians. Discovery in mathematics often begins with an examination of individual cases to reveal patterns about numbers.

Example 2 Using Inductive Reasoning

Identify a pattern in each list of numbers. Then use this pattern to find the next number.

a. 3, 12, 21, 30, 39, _____

b. 3, 12, 48, 192, 768, _____

c. 3, 4, 6, 9, 13, 18, _____

d. 3, 6, 18, 36, 108, 216, _____

Solution

a. Because 3, 12, 21, 30, 39, _____ is increasing relatively slowly, let's use addition as the basis for our individual observations.

3, 12, 21, 30, 39, _____

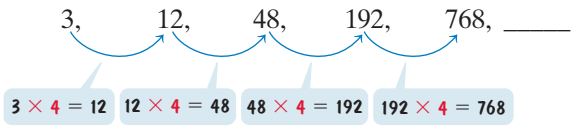
$3 + 9 = 12$ $12 + 9 = 21$ $21 + 9 = 30$ $30 + 9 = 39$

"For thousands of years, people have loved numbers and found patterns and structures among them. The allure of numbers is not limited to or driven by a desire to change the world in a practical way. When we observe how numbers are connected to one another, we are seeing the inner workings of a fundamental concept."

—Edward B. Burger and Michael Starbird, *Coincidences, Chaos, and All That Math Jazz*, W. W. Norton and Company, 2005

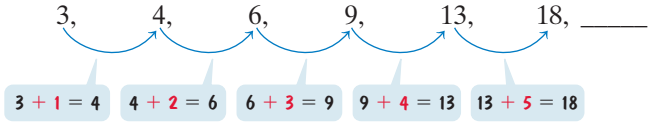
Generalizing from these observations, we conclude that each number after the first is obtained by adding 9 to the previous number. Using this pattern, the next number is $39 + 9$, or 48.

b. Because 3, 12, 48, 192, 768, _____ is increasing relatively rapidly, let's use multiplication as the basis for our individual observations.



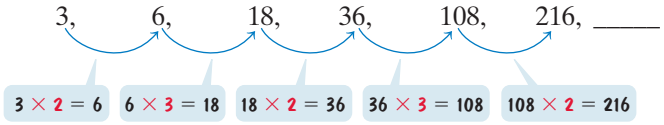
Generalizing from these observations, we conclude that each number after the first is obtained by multiplying the previous number by 4. Using this pattern, the next number is 768×4, or 3072.

c. Because 3, 4, 6, 9, 13, 18, _____ is increasing relatively slowly, let's use addition as the basis for our individual observations.



Generalizing from these observations, we conclude that each number after the first is obtained by adding a counting number to the previous number. The additions begin with 1 and continue through each successive counting number. Using this pattern, the next number is $18 + 6$, or 24.

d. Because 3, 6, 18, 36, 108, 216, _____ is increasing relatively rapidly, let's use multiplication as the basis for our individual observations.



Generalizing from these observations, we conclude that each number after the first is obtained by multiplying the previous number by 2 or by 3. The multiplications begin with 2 and then alternate, multiplying by 2, then 3, then 2, then 3, and so on. Using this pattern, the next number is 216×3, or 648.

Check Point 2. Identify a pattern in each list of numbers. Then use this pattern to find the next number.

a. 3, 9, 15, 21, 27, _____

b. 2, 10, 50, 250, _____

c. 3, 6, 18, 72, 144, 432, 1728, _____

d. 1, 9, 17, 3, 11, 19, 5, 13, 21, _____

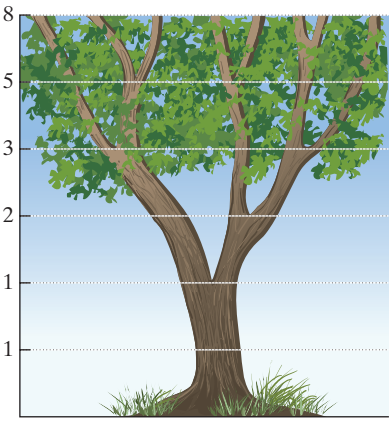
In our next example, the patterns are a bit more complex than the additions and multiplications we encountered in Example 2.

Example 3 Using Inductive Reasoning

Identify a pattern in each list of numbers. Then use this pattern to find the next number.

a. 1, 1, 2, 3, 5, 8, 13, 21, _____

b. 23, 54, 95, 146, 117, 98, _____



As this tree branches, the number of branches forms the Fibonacci sequence.

Solution

a. We begin with 1, 1, 2, 3, 5, 8, 13, 21. Starting with the third number in the list, let's form our observations by comparing each number with the two numbers that immediately precede it.

1, 1, 2, 3, 5, 8, 13, 21, _____

2	3	5	8	13	21
preceded by 1 and 1: $1 + 1 = 2$	preceded by 1 and 2: $1 + 2 = 3$	preceded by 2 and 3: $2 + 3 = 5$	preceded by 3 and 5: $3 + 5 = 8$	preceded by 5 and 8: $5 + 8 = 13$	preceded by 8 and 13: $8 + 13 = 21$

The first two numbers are 1. Generalizing from these observations, we conclude that each number thereafter is the sum of the two preceding numbers. Using this pattern, the next number is $13 + 21$, or 34. (The numbers 1, 1, 2, 3, 5, 8, 13, 21, and 34 are the first nine terms of the *Fibonacci sequence*, discussed in Chapter 5, Section 5.7.)

b. Now, we consider 23, 54, 95, 146, 117, 98. Let's use the digits that form each number as the basis for our individual observations. Focus on the sum of the digits, as well as the final digit increased by 1.

23	54	95	146	117	98	_____
$2 + 3 = 5$	$5 + 4 = 9$	$9 + 5 = 14$	$1 + 4 + 6 = 11$	$1 + 1 + 7 = 9$		
$3 + 1 = 4$	$4 + 1 = 5$	$5 + 1 = 6$	$6 + 1 = 7$	$7 + 1 = 8$		

Generalizing from these observations, we conclude that for each number after the first, we obtain the first digit or the first two digits by adding the digits of the previous number. We obtain the last digit by adding 1 to the final digit of the preceding number. Applying this pattern to find the number that follows 98, the first two digits are $9 + 8$, or 17. The last digit is $8 + 1$, or 9. Thus, the next number in the list is 179.

Great Question!

Can a list of numbers have more than one pattern?

Yes. Consider the illusion in **Figure 1.1**. This ambiguous figure contains two patterns, where it is not clear which pattern should predominate. Do you see a wine goblet or two faces looking at each other? Like this ambiguous figure, some lists of numbers can display more than one pattern, particularly if only a few numbers are given. Inductive reasoning can result in more than one probable next number in a list.

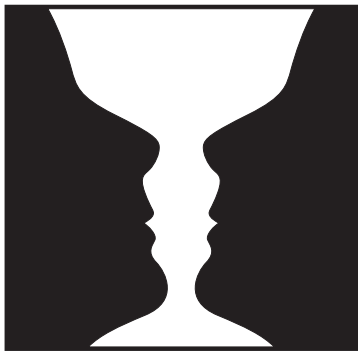


Figure 1.1

Example: 1, 2, 4, _____

Pattern: Each number after the first is obtained by multiplying the previous number by 2. The missing number is 4×2, or 8.

Pattern: Each number after the first is obtained by adding successive counting numbers, starting with 1, to the previous number. The second number is $1 + 1$, or 2. The third number is $2 + 2$, or 4. The missing number is $4 + 3$, or 7.

Inductive reasoning can also result in different patterns that produce the same probable next number in a list.

Example: 1, 4, 9, 16, 25, _____

Pattern: Start by adding 3 to the first number. Then add successive odd numbers, 5, 7, 9, and so on. The missing number is $25 + 11$, or 36.

Pattern: Each number is obtained by squaring its position in the list: The first number is $1^2 = 1 \times 1 = 1$, the second number is $2^2 = 2 \times 2 = 4$, the third number is $3^2 = 3 \times 3 = 9$, and so on. The missing sixth number is $6^2 = 6 \times 6$, or 36.

The numbers that we found in Examples 2 and 3 are probable numbers. Perhaps you found patterns other than the ones we pointed out that might have resulted in different answers.

Check Point 3. Identify a pattern in each list of numbers. Then use this pattern to find the next number.

a. 1, 3, 4, 7, 11, 18, 29, 47, _____

b. 2, 3, 5, 9, 17, 33, 65, 129, _____

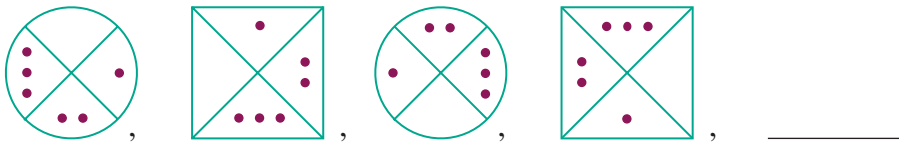
This electron microscope photograph shows the knotty shape of the Ebola virus.

Mathematics is more than recognizing number patterns. It is about the patterns that arise in the world around us. For example, by describing patterns formed by various kinds of knots, mathematicians are helping scientists investigate the knotty shapes and patterns of viruses. One of the weapons used against viruses is based on recognizing visual patterns in the possible ways that knots can be tied.

Our next example deals with recognizing visual patterns.

Example 4 Finding the Next Figure in a Visual Sequence

Describe two patterns in this sequence of figures. Use the patterns to draw the next figure in the sequence.



Solution

The more obvious pattern is that the figures alternate between circles and squares. We conclude that the next figure will be a circle. We can identify the second pattern in the four regions containing no dots, one dot, two dots, and three dots. The dots are placed in order (no dots, one dot, two dots, three dots) in a clockwise direction. However, the entire pattern of the dots rotates counterclockwise as we follow the figures from left to right. This means that the next figure should be a circle with a single dot in the right-hand region, two dots in the bottom region, three dots in the left-hand region, and no dots in the top region.

The missing figure in the visual sequence, a circle with a single dot in the right-hand region, two dots in the bottom region, three dots in the left-hand region, and no dots in the top region, is drawn in **Figure 1.2**.

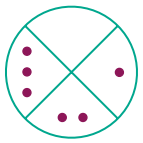
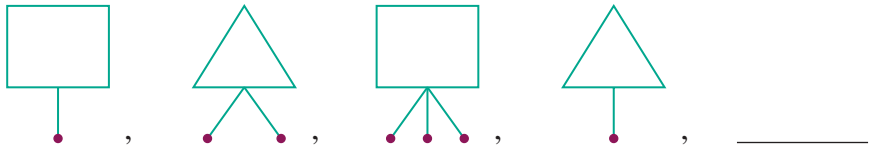


Figure 1.2

"What you really need to do is show students how imperfect people can be and still succeed.

—Karen Uhlenbeck (1942–), the first woman to win the Abel Prize in mathematics

Check Point 4. Describe two patterns in this sequence of figures. Use the patterns to draw the next figure in the sequence.



BLITZER BONUS *Are You Smart Enough to Work at Google?*

In *Are You Smart Enough to Work at Google?* (Little, Brown, and Company, 2012), author William Poundstone guides readers through the surprising solutions to challenging job-interview questions. The book covers the importance of creative thinking in inductive reasoning, estimation, and problem solving. Best of all, Poundstone explains the answers.

Whether you're preparing for a job interview or simply want to increase your critical thinking skills, we highly recommend tackling the puzzles in *Are You Smart Enough to Work at Google?* Here is a sample of two of the book's problems that involve inductive reasoning. We've provided hints to help you recognize the pattern in each sequence. The answers appear in the answer section.

1. Determine the next entry in the sequence.
SSS, SCC, C, SC, ___?___

Hint: Think of the capital letters in the English alphabet. A is made up of three straight lines. B consists of one straight line and two curved lines. C is made up of one curved line.

2. Determine the next line in this sequence of digits.

1 — The first row contains one 1.
1 1 — The second row contains two 1s.
2 1 — The third row contains one 2 and one 1.
1 2 1 1
1 1 1 2 2 1
? ? ? ? ? ?

2 Understand and use deductive reasoning.

Deductive Reasoning

We use inductive reasoning in everyday life. Many of the conjectures that come from this kind of thinking seem highly likely, although we can never be absolutely certain that they are true. Another method of reasoning, called *deductive reasoning*, or *deduction*, can be used to prove that some conjectures are true.

Deductive Reasoning

Deductive reasoning is the process of proving a specific conclusion from one or more general statements. A conclusion that is proved to be true by deductive reasoning is called a **theorem**.

Deductive reasoning allows us to draw a specific conclusion from one or more general statements. Two examples of deductive reasoning are shown below. Notice that in both everyday situations, the general statement from which the conclusion is drawn is implied rather than directly stated.

Everyday Situation	Deductive Reasoning
One player to another in a Scrabble game: "You have to remove those five letters. You can't use TEXAS as a word."	• All proper names are prohibited in Scrabble. — general statement TEXAS is a proper name. Therefore, TEXAS is prohibited in Scrabble. — conclusion
Advice to college freshmen on choosing classes: "Never sign up for a 7 A.M. class. Yes, you did it in high school, but someone was always there to keep waking you up, and if by some miracle you do make it to an early class, you will sleep through the lecture when you get there." (*Source: How to Survive Your Freshman Year*, Hundreds of Heads Books, 2004)	• All people need to sleep at 7 A.M. — general statement You sign up for a class at 7 A.M. Therefore, you'll sleep through the lecture or not even make it to class. — conclusion In Chapter 3, you'll learn how to prove this conclusion from the general statement in the first line. But is the general statement really true? Can we make assumptions about the sleeping patterns of all people, or are we using deductive reasoning to reinforce an untrue reality assumption?

Our next example illustrates the difference between inductive and deductive reasoning. The first part of the example involves reasoning that moves from specific examples to a general statement, illustrating inductive reasoning. The second part of the example begins with the general case rather than specific examples and illustrates deductive reasoning. To begin the general case, we use a letter to represent any one of various numbers. A letter used to represent any number in a collection of numbers is called a **variable**. Variables and other mathematical symbols allow us to work with the general case in a very concise manner.

A Brief Review

In case you have forgotten some basic terms of arithmetic, the following list should be helpful.

Sum:	the result of addition
Difference:	the result of subtraction
Product:	the result of multiplication
Quotient:	the result of division

Example 5 Using Inductive and Deductive Reasoning

Consider the following procedure:

Select a number. Multiply the number by 6. Add 8 to the product. Divide this sum by 2. Subtract 4 from the quotient.

a. Repeat this procedure for at least four different numbers. Write a conjecture that relates the result of this process to the original number selected.

b. Use the variable n to represent the original number and use deductive reasoning to prove the conjecture in part (a).

Solution

a. First, let us pick our starting numbers. We will use 4, 7, 11, and 100, but we could pick any four numbers. Next we will apply the procedure given in this example to 4, 7, 11, and 100, four individual cases, in **Table 1.1**.

Table 1.1 Applying a Procedure to Four Individual Cases

Select a number.	4	7	11	100
Multiply the number by 6.	$4 \times 6 = 24$	$7 \times 6 = 42$	$11 \times 6 = 66$	$100 \times 6 = 600$
Add 8 to the product.	$24 + 8 = 32$	$42 + 8 = 50$	$66 + 8 = 74$	$600 + 8 = 608$
Divide this sum by 2.	$\frac{32}{2} = 16$	$\frac{50}{2} = 25$	$\frac{74}{2} = 37$	$\frac{608}{2} = 304$
Subtract 4 from the quotient.	$16 - 4 = 12$	$25 - 4 = 21$	$37 - 4 = 33$	$304 - 4 = 300$

Because we are asked to write a conjecture that relates the result of this process to the original number selected, let us focus on the result of each case.

Original number selected	4	7	11	100
Result of the process	12	21	33	300

Do you see a pattern? Our conjecture is that the result of the process is three times the original number selected. We have used inductive reasoning.

b. Now we begin with the general case rather than specific examples. We use the variable n to represent any number.

Select a number.	n
Multiply the number by 6.	$6n$ (This means 6 times n.)
Add 8 to the product.	$6n + 8$
Divide this sum by 2.	$\frac{6n + 8}{2} = \frac{6n}{2} + \frac{8}{2} = 3n + 4$
Subtract 4 from the quotient.	$3n + 4 - 4 = 3n$

Using the variable n to represent any number, the result is $3n$, or three times the number n. This proves that the result of the procedure is three times the original number selected for any number. We have used deductive reasoning. Observe how algebraic notation allows us to work with the general case quite efficiently through the use of a variable.

Number Tricks Using Deductive Reasoning

Check Point 5. Consider the following procedure:

Select a number. Multiply the number by 4. Add 6 to the product. Divide this sum by 2. Subtract 3 from the quotient.

a. Repeat this procedure for at least four different numbers. Write a conjecture that relates the result of this process to the original number selected.

b. Use the variable n to represent the original number and use deductive reasoning to prove the conjecture in part (a).

BLITZER BONUS *Surprising Friends with Induction*

Ask a few friends to follow this procedure:

Write down a whole number from 2 to 10. Multiply the number by 9. Add the digits. Subtract 3. Assign a letter to this result using $A = 1, B = 2, C = 3$, and so on. Write down the name of a state that begins with this letter. Select the name of an insect that begins with the last letter of the state. Name a fruit or vegetable that begins with the last letter of the insect.

After following this procedure, surprise your friend by asking, "Are you thinking of an ant in Florida eating a tomato?" (Try using inductive reasoning to determine how you came up with this "astounding" question. Are other less-probable "astounding" questions possible using inductive reasoning?)

Concept and Vocabulary Check

Great Question!

What am I supposed to do with the exercises in the Concept and Vocabulary Check?

An important component of thinking mathematically involves knowing the special language and notation used in mathematics. The exercises in the Concept and Vocabulary Check, mainly fill-in-the-blank and true/false items, test your understanding of the definitions and concepts presented in each section. **Work all of the exercises in the Concept and Vocabulary Check** regardless of which exercises your professor assigns in the Exercise Set that follows.

Fill in each blank so that the resulting statement is true.

C1. The statement $3 + 3 = 6$ serves as a/an _____ to the conjecture that the sum of two odd numbers is an odd number.

C2. Arriving at a specific conclusion from one or more general statements is called _____ reasoning.

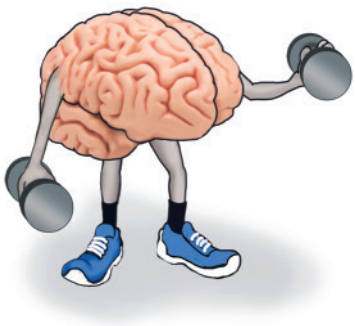
C3. Arriving at a general conclusion based on observations of specific examples is called _____ reasoning.

C4. True or False: A theorem cannot have counterexamples. _____

Exercise Set 1.1

Great Question!

Any way that I can perk up my brain before working the book's Exercise Sets?



Researchers say the mind can be strengthened, just like your muscles, with regular training and rigorous practice. Think of the book's Exercise Sets as brain calisthenics. If you're feeling a bit sluggish before any of your mental workouts, try this warmup:

In the list below, say the color the word is printed in, not the word itself. Once you can do this in 15 seconds without an error, the warmup is over and it's time to move on to the assigned exercises.

Blue Yellow Red Green Yellow Green Blue Red Yellow Red

Practice Exercises

In Exercises 1–8, find a counterexample to show that each of the statements is false.

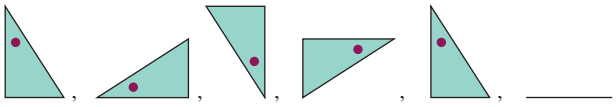
1. No U.S. president has been younger than 65 at the time of inauguration.
2. No singers appear in movies.
3. If a number is multiplied by itself, the result is even.
4. The sum of two three-digit numbers is a four-digit number.
5. Adding the same number to both the numerator and the denominator (top and bottom) of a fraction does not change the fraction's value.
6. If the difference between two numbers is odd, then the two numbers are both odd.
7. If a number is added to itself, the sum is greater than the original number.
8. If 1 is divided by a number, the quotient is less than that number.

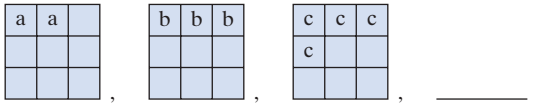
In Exercises 9–38, identify a pattern in each list of numbers. Then use this pattern to find the next number. (More than one pattern might exist, so it is possible that there is more than one correct answer.)

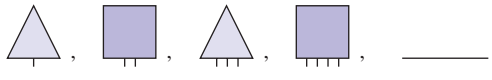
9. $8, 12, 16, 20, 24, \_\_\_\_\_$
10. $19, 24, 29, 34, 39, \_\_\_\_\_$
11. $37, 32, 27, 22, 17, \_\_\_\_\_$
12. $33, 29, 25, 21, 17, \_\_\_\_\_$
13. $3, 9, 27, 81, 243, \_\_\_\_\_$
14. $2, 8, 32, 128, 512, \_\_\_\_\_$
15. $1, 2, 4, 8, 16, \_\_\_\_\_$
16. $1, 5, 25, 125, \_\_\_\_\_$
17. $1, 4, 1, 8, 1, 16, 1, \_\_\_\_\_$
18. $1, 4, 1, 7, 1, 10, 1, \_\_\_\_\_$
19. $4, 2, 0, -2, -4, \_\_\_\_\_$
20. $6, 3, 0, -3, -6, \_\_\_\_\_$
21. $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \frac{1}{18}, \_\_\_\_\_$
22. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \_\_\_\_\_$
23. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \_\_\_\_\_$
24. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \_\_\_\_\_$
25. $3, 7, 12, 18, 25, 33, \_\_\_\_\_$
26. $2, 5, 9, 14, 20, 27, \_\_\_\_\_$
27. $3, 6, 11, 18, 27, 38, \_\_\_\_\_$
28. $2, 5, 10, 17, 26, 37, \_\_\_\_\_$
29. $3, 7, 10, 17, 27, 44, \_\_\_\_\_$
30. $2, 5, 7, 12, 19, 31, \_\_\_\_\_$
31. $2, 7, 12, 5, 10, 15, 8, 13, \_\_\_\_\_$
32. $3, 9, 15, 5, 11, 17, 7, 13, \_\_\_\_\_$
33. $3, 6, 5, 10, 9, 18, 17, 34, \_\_\_\_\_$
34. $2, 6, 5, 15, 14, 42, 41, 123, \_\_\_\_\_$
35. $64, -16, 4, -1, \_\_\_\_\_$
36. $125, -25, 5, -1, \_\_\_\_\_$
37. $(6, 2), (0, -4), (7\frac{1}{2}, 3\frac{1}{2}), (2, -2), (3, \_\_\_\_\_)$
38. $(\frac{2}{3}, \frac{4}{9}), (\frac{1}{5}, \frac{1}{25}), (7, 49), (-\frac{5}{6}, \frac{25}{36}), (-\frac{4}{7}, \_\_\_\_\_)$

In Exercises 39–42, identify a pattern in each sequence of figures. Then use the pattern to find the next figure in the sequence.

39. 

40. 

41. 

42. 

Exercises 43–46 describe procedures that are to be applied to numbers. In each exercise,

a. *Repeat the procedure for four numbers of your choice. Write a conjecture that relates the result of the process to the original number selected.*

b. *Use the variable n to represent the original number and use deductive reasoning to prove the conjecture in part (a).*

43. Select a number. Multiply the number by 4. Add 8 to the product. Divide this sum by 2. Subtract 4 from the quotient.
44. Select a number. Multiply the number by 3. Add 6 to the product. Divide this sum by 3. Subtract the original selected number from the quotient.
45. Select a number. Add 5. Double the result. Subtract 4. Divide by 2. Subtract the original selected number.
46. Select a number. Add 3. Double the result. Add 4. Divide by 2. Subtract the original selected number.

In Exercises 47–52, use inductive reasoning to predict the next line in each sequence of computations. Then use a calculator or perform the arithmetic by hand to determine whether your conjecture is correct.

47.
$$1 + 2 = \frac{2 \times 3}{2}$$
$$1 + 2 + 3 = \frac{3 \times 4}{2}$$
$$1 + 2 + 3 + 4 = \frac{4 \times 5}{2}$$
$$1 + 2 + 3 + 4 + 5 = \frac{5 \times 6}{2}$$

48.
$$3 + 6 = \frac{6 \times 3}{2}$$
$$3 + 6 + 9 = \frac{9 \times 4}{2}$$
$$3 + 6 + 9 + 12 = \frac{12 \times 5}{2}$$
$$3 + 6 + 9 + 12 + 15 = \frac{15 \times 6}{2}$$

49.
$$1 + 3 = 2 \times 2$$
$$1 + 3 + 5 = 3 \times 3$$
$$1 + 3 + 5 + 7 = 4 \times 4$$
$$1 + 3 + 5 + 7 + 9 = 5 \times 5$$

50.
$$1 \times 9 + (1 + 9) = 19$$
$$2 \times 9 + (2 + 9) = 29$$
$$3 \times 9 + (3 + 9) = 39$$
$$4 \times 9 + (4 + 9) = 49$$

51.
$$9 \times 9 + 7 = 88$$
$$98 \times 9 + 6 = 888$$
$$987 \times 9 + 5 = 8888$$
$$9876 \times 9 + 4 = 88{,}888$$

52.
$$1 \times 9 - 1 = 8$$
$$21 \times 9 - 1 = 188$$
$$321 \times 9 - 1 = 2888$$
$$4321 \times 9 - 1 = 38{,}888$$

Practice PLUS

In Exercises 53–54, use inductive reasoning to predict the next line in each sequence of computations. Then use a calculator or perform the arithmetic by hand to determine whether your conjecture is correct.

53.
$$33 \times 3367 = 111{,}111$$
$$66 \times 3367 = 222{,}222$$
$$99 \times 3367 = 333{,}333$$
$$132 \times 3367 = 444{,}444$$

54.
$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12{,}345 \times 8 + 5 = 98{,}765$$

55. Study the pattern in these examples:

$$a^2 \# a^4 = a^{10} \quad a^3 \# a^2 = a^7 \quad a^5 \# a^3 = a^{11}.$$

Select the equation that describes the pattern.

a. $a^x \# a^y = a^{2x+y}$ **b.** $a^x \# a^y = a^{x+2y}$

c. $a^x \# a^y = a^{x+y+4}$ **d.** $a^x \# a^y = a^{xy+2}$

56. Study the pattern in these examples:

$$a^5 * a^3 * a^2 = a^5 \quad a^3 * a^7 * a^2 = a^6 \quad a^2 * a^4 * a^8 = a^7.$$

Select the equation that describes the pattern.

a. $a^x * a^y * a^z = a^{x+y+z}$ **b.** $a^x * a^y * a^z = a^{\frac{xyz}{2}}$

c. $a^x * a^y * a^z = a^{\frac{x+y+z}{2}}$ **d.** $a^x * a^y * a^z = a^{\frac{xy}{2}+z}$

Application Exercises

In Exercises 57–60, identify the reasoning process, induction or deduction, in each example. Explain your answer.

57. It can be shown that

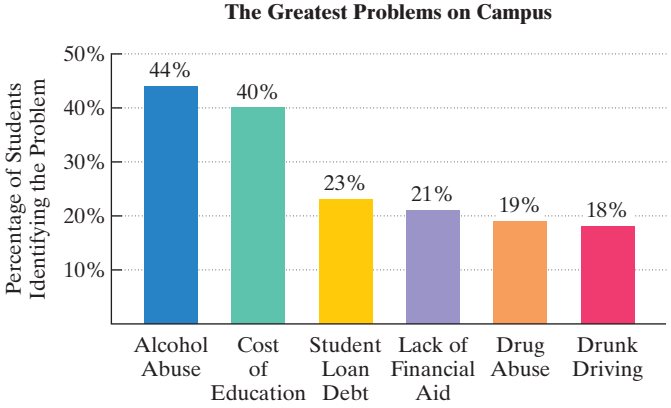
$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

I can use this formula to conclude that the sum of the first 100 counting numbers, $1 + 2 + 3 + \cdots + 100$, is

$$\frac{100(100+1)}{2} = \frac{100(101)}{2} = 50(101), \text{ or } 5050.$$

58. An HMO does a follow-up study on 200 randomly selected patients given a flu shot. None of these people became seriously ill with the flu. The study concludes that all HMO patients should be urged to get a flu shot in order to prevent a serious case of the flu.

59. Problems on Campus The data in the graph are from a random sample of 1200 full-time four-year undergraduate college students on 100 U.S. campuses.



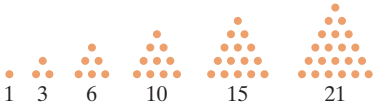
Source: Student Monitor LLC

Using the graph, we can conclude that there is a high probability that approximately 44% of all full-time four-year college students in the United States believe that alcohol abuse is one of the greatest problem on campus.

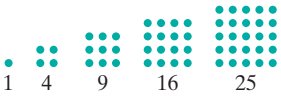
60. The course policy states that work turned in late will be marked down a grade. I turned in my report a day late, so it was marked down from B to C.

61. The ancient Greeks studied **figurate numbers**, so named because of their representations as geometric arrangements of points.

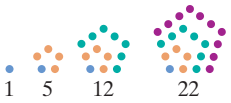
Triangular Numbers



Square Numbers



Pentagonal Numbers



a. Use inductive reasoning to write the five triangular numbers that follow 21.

b. Use inductive reasoning to write the five square numbers that follow 25.

c. Use inductive reasoning to write the five pentagonal numbers that follow 22.

d. Use inductive reasoning to complete this statement: If a triangular number is multiplied by 8 and then 1 is added to the product, a _____ number is obtained.

62. The triangular arrangement of numbers shown below is known as **Pascal's triangle**, credited to French mathematician Blaise Pascal (1623–1662). Use inductive reasoning to find the six numbers designated by question marks.

$$1$$
$$1 \quad 1$$
$$1 \quad 2 \quad 1$$
$$1 \quad 3 \quad 3 \quad 1$$
$$1 \quad 4 \quad 6 \quad 4 \quad 1$$
$$? \quad ? \quad ? \quad ? \quad ? \quad ?$$

Explaining the Concepts

An effective way to understand something is to explain it to someone else. *You can do this by using the Explaining the Concepts exercises that ask you to respond with verbal or written explanations. Speaking about a new concept uses a different part of your brain than thinking about the concept. Explaining new ideas verbally will quickly reveal any gaps in your understanding. It will also help you to remember new concepts for longer periods of time.*

63. The word *induce* comes from a Latin term meaning *to lead*. Explain what leading has to do with inductive reasoning.

64. Describe what is meant by deductive reasoning. Give an example.

65. Give an example of a decision that you made recently in which the method of reasoning you used to reach the decision was induction. Describe your reasoning process.

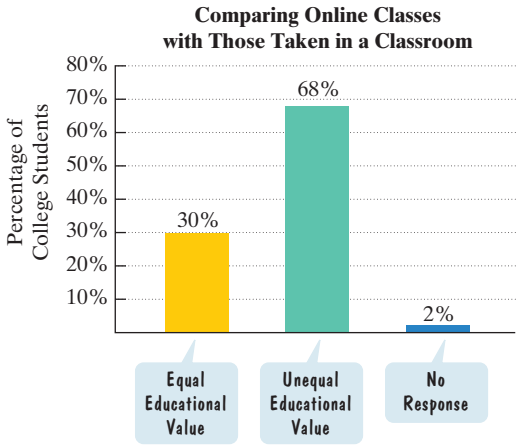
Critical Thinking Exercises

Make Sense? *In Exercises 66–69, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

66. I use deductive reasoning to draw conclusions that are not certain but likely.

67. Additional information may strengthen or weaken the probability of my inductive arguments.

68. Value of Online Classes I used the data shown in the bar graph, which summarizes a random sample of college students taken in 2000, to conclude that in 2000 online classes were not perceived as equal in educational value to those taken in a classroom.



Source: Pew Research Center Survey, 2000

69. I used the 2000 data shown in the bar graph for Exercise 68 to conclude inductively that by 2030 online classes will not be perceived as equal in value to those taken in a classroom.

70. If $(6-2)^2 = 36-24+4$ and $(8-5)^2 = 64-80+25$, use inductive reasoning to write a compatible expression for $(11-7)^2$.

71. The rectangle shows an array of nine numbers represented by combinations of the variables a, b, and c.

$a+b$	$a-b-c$	$a+c$
$a-b+c$	a	$a+b-c$
$a-c$	$a+b+c$	$a-b$

a. Determine the nine numbers in the array for $a = 10$, $b = 6$, and $c = 1$. What do you observe about the sum of the numbers in all rows, all columns, and the two diagonals?

b. Repeat part (a) for $a = 12, b = 5$, and $c = 2$.

c. Repeat part (a) for values of a, b, and c of your choice.

d. Use the results of parts (a) through (c) to make an inductive conjecture about the rectangular array of nine numbers represented by a, b, and c.

e. Use deductive reasoning to prove your conjecture in part (d).

72. Write a list of numbers that has two patterns so that the next number in the list can be 15 or 20.

73. a. Repeat the following procedure with at least five people. Write a conjecture that relates the result of the procedure to each person's birthday.

Take the number of the month of your birthday (January $= 1$, February $= 2, \ldots,$ December $= 12$), multiply by 5, add 6, multiply this sum by 4, add 9, multiply this new sum by 5, and add the number of the day on which you were born. Finally, subtract 165.

b. Let M represent the month number and let D represent the day number of any person's birthday. Use deductive reasoning to prove your conjecture in part (a).

Technology Exercises

74. a. Use a calculator to find 6×6, 66×66, 666×666, and 6666×6666.

b. Describe a pattern in the numbers being multiplied and the resulting products.

c. Use the pattern to write the next two multiplications and their products. Then use your calculator to verify these results.

d. Is this process an example of inductive or deductive reasoning? Explain your answer.

75. a. Use a calculator to find 3367×3, 3367×6, 3367×9, and 3367×12.

b. Describe a pattern in the numbers being multiplied and the resulting products.

c. Use the pattern to write the next two multiplications and their products. Then use your calculator to verify these results.

d. Is this process an example of inductive or deductive reasoning? Explain your answer.

Group Exercise

76. Stereotyping refers to classifying people, places, or things according to common traits. Prejudices and stereotypes can function as assumptions in our thinking, appearing in inductive and deductive reasoning. For example, it is not difficult to find inductive reasoning that results in generalizations such as these, as well as deductive reasoning in which these stereotypes serve as assumptions:

School has nothing to do with life.

Intellectuals are nerds.

People on welfare are lazy.

Each group member should find one example of inductive reasoning and one example of deductive reasoning in which stereotyping occurs. Upon returning to the group, present each example and then describe how the stereotyping results in faulty conjectures or prejudging situations and people.

1.2 Estimation, Graphs, and Mathematical Models

What You'll Learn

1. Use estimation techniques to arrive at an approximate answer to a problem.
2. Apply estimation techniques to information given by graphs.
3. Develop mathematical models that estimate relationships between variables.

Octogenarian hobby? Why not?

If present trends continue, is it possible that our descendants could live to be 200 years of age? To answer this question, we need to examine data for life expectancy and develop estimation techniques for representing the data mathematically. In this section, you will learn estimation methods that will enable you to obtain mathematical representations of data displayed by graphs, using these representations to predict what might occur in the future.

Estimation

1. Use estimation techniques to arrive at an approximate answer to a problem.

Estimation is the process of arriving at an approximate answer to a question. For example, companies estimate the amount of their products consumers are likely to use, and economists estimate financial trends. If you are about to cross a street, you may estimate the speed of oncoming cars so that you know whether or not to wait before crossing. Rounding numbers is also an estimation method. You might round a number without even being aware that you are doing so. You may say that you are 20 years old, rather than 20 years 5 months, or that you will be home in about a half-hour, rather than 25 minutes.

You will find estimation to be equally valuable in your work for this class. Making mistakes with a calculator or a computer is easy. Estimation can tell us whether the answer displayed for a computation makes sense.

In this section, we demonstrate several estimation methods. In the second part of the section, we apply these techniques to information given by graphs.

Rounding Whole Numbers

The numbers that we use for counting, 1, 2, 3, 4, 5, 6, 7, and so on, are called **natural numbers**. When we combine 0 with the natural numbers, we obtain the whole numbers.

Whole Numbers

The **whole numbers** are

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \ldots .$$

The smallest whole number is 0.

The three dots mean that the list continues without end. There is no largest whole number.

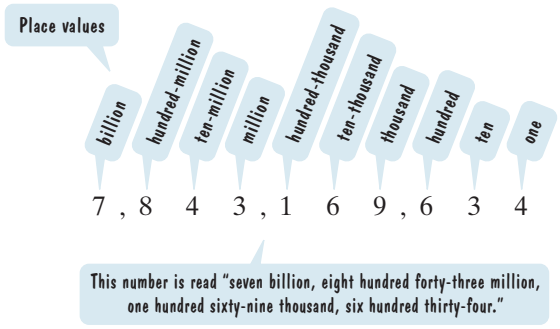
The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are called **digits**, from the Latin word for fingers. Digits are used to write whole numbers.

Great Question!

When do I need to use hyphens to write the names of numbers?

Hyphenate the names for the numbers 21 (twenty-one) through 99 (ninety-nine), except 30, 40, 50, 60, 70, 80, and 90.

The position of each digit in a whole number tells us the value of that digit. Here is an example using world population at 6:30 A.M. Eastern Time on January 2, 2021.



Rounding Whole Numbers

1. Look at the digit to the right of the digit where rounding is to occur.
2. **a.** If the digit to the right is 5 or greater, add 1 to the digit to be rounded. Replace all digits to the right with zeros.
   b. If the digit to the right is less than 5, do not change the digit to be rounded. Replace all digits to the right with zeros.

The symbol $\approx$ means *is approximately equal to*. We will use this symbol when rounding numbers.

Example 1 Rounding a Whole Number

Round world population (7,843,169,634) as follows:

a. to the nearest hundred-million
b. to the nearest million
c. to the nearest hundred-thousand.

Solution

a. $7{,}843{,}169{,}634 \approx 7{,}800{,}000{,}000$

Hundred-millions digit, where rounding is to occur. Digit to the right is less than 5. Do not change the digit to be rounded. Replace all digits to the right with zeros.

World population to the nearest hundred-million is seven billion, eight hundred-million.

b. $7{,}843{,}169{,}634 \approx 7{,}843{,}000{,}000$

Millions digit, where rounding is to occur. Digit to the right is less than 5. Do not change the digit to be rounded. Replace all digits to the right with zeros.

World population to the nearest million is seven billion, eight hundred forty-three million.

c. $7{,}843{,}169{,}634 \approx 7{,}843{,}200{,}000$

Hundred-thousands digit, where rounding is to occur. Digit to the right is greater than 5. Add 1 to the digit to be rounded. Replace all digits to the right with zeros.

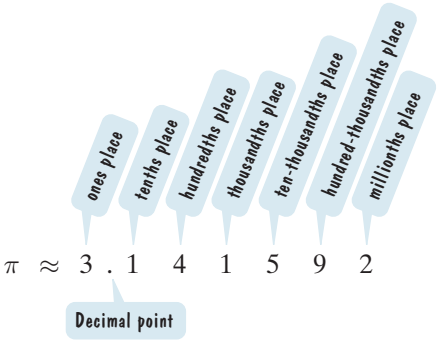
World population to the nearest hundred-thousand is seven billion, eight hundred forty-three million, two hundred-thousand.

Check Point 1. Round world population (7,843,169,634) as follows:

a. to the nearest billion

b. to the nearest ten-million.

Rounding can also be applied to decimal notation, used to denote a part of a whole. Once again, the place that a digit occupies tells us its value. Here's an example using the first seven digits of the number π (pi). (We'll have more to say about π, whose digits extend endlessly with no repeating pattern, in Chapter 5.)



We round the decimal part of a decimal number in nearly the same way that we round whole numbers. The only difference is that we drop the digits to the right of the rounding place rather than replacing these digits with zeros.

Animation

Rounding Decimals

Example 2 Rounding the Decimal Part of a Number

Round 3.141592, the first seven digits of π, as follows:

a. to the nearest hundredth

b. to the nearest thousandth.

Solution

a. 3.141592 $\approx$ 3.14

Hundredths digit, where rounding is to occur. Digit to the right is less than 5. Do not change the digit to be rounded. Drop all digits to the right.

The number π to the nearest hundredth is three and fourteen hundredths.

b. 3.141592 $\approx$ 3.142

Thousandths digit, where rounding is to occur. Digit to the right is 5. Add 1 to the digit to be rounded. Drop all digits to the right.

The number π to the nearest thousandth is three and one hundred forty-two thousandths.

Check Point 2. Round 3.141592, the first seven digits of π, as follows:

a. to the nearest tenth

b. to the nearest ten-thousandth.

Great Question!

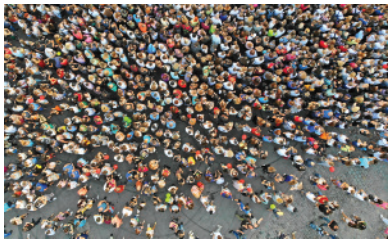
Could you please explain how the decimal numbers in Example 2 are read?

Of course! The whole-number part to the left of the decimal point is read like any whole number, which is *three* in both parts of Example 2. The decimal point is read as *and*. The decimal part to the right of the decimal point is read like a whole number followed by the place value of the rightmost digit. In 3.14, the 4 is in the hundredths place, so there are *fourteen hundredths*. In 3.142, the 2 is in the thousandths place, so there are *one hundred forty-two thousandths*.

BLITZER BONUS

Estimating Support for a Cause

Police often need to estimate the size of a crowd at a political demonstration. One way to do this is to select a reasonably sized rectangle within the crowd and estimate (or count) the number of people within the rectangle. Police then estimate the number of such rectangles it would take to completely fill the area occupied by the crowd. The police estimate is obtained by multiplying the number of such rectangles by the number of demonstrators in the representative rectangle. The organizers of the demonstration might give a larger estimate than the police to emphasize the strength of their support.



Example 3 Estimation by Rounding

You purchased bread for \$2.59, detergent for \$5.17, a sandwich for \$3.65, an apple for \$0.47, and coffee for \$8.79. The total bill was given as \$24.67. Is this amount reasonable?

Solution

If you are in the habit of carrying your smartphone to the store, you can answer the question by finding the exact cost of the purchase. However, estimation can be used to determine if the bill is reasonable even if you do not have a smartphone. We will round the cost of each item to the nearest dollar.

Round to the nearest dollar. Use digits in the tenths place to do the rounding.

Bread	\$2.59	$\approx$	\$3.00
Detergent	\$5.17	$\approx$	\$5.00
Sandwich	\$3.65	$\approx$	\$4.00
Apple	\$0.47	$\approx$	\$0.00
Coffee	\$8.79	$\approx$	\$9.00
			\$21.00

The total bill that you were given, \$24.67, seems a bit high compared to the \$21.00 estimate. You should check the bill before paying it. Adding the prices of all five items gives the true total bill of \$20.67.

Check Point 3. You and a friend ate lunch at Ye Olde Cafe. The check for the meal showed soup for \$3.40, tomato juice for \$2.25, a roast beef sandwich for \$5.60, a chicken salad sandwich for \$5.40, two coffees totaling \$3.40, apple pie for \$2.85, and chocolate cake for \$3.95.

a. Round the cost of each item to the nearest dollar and obtain an estimate for the food bill.

b. The total bill before tax was given as \$29.85. Is this amount reasonable?

Example 4 Estimation by Rounding

A carpenter who works full time earns \$28 per hour.

a. Estimate the carpenter's weekly salary.

b. Estimate the carpenter's annual salary.

Solution

a. In order to simplify the calculation, we can round the hourly rate of \$28 to \$30. Be sure to write out the units for each number in the calculation. The work week is 40 hours per week, and the rounded salary is \$30 per hour. We express this as

$$\frac{40 \text{ hours}}{\text{week}} \quad \text{and} \quad \frac{\$30}{\text{hour}}.$$

The word *per* is represented by the division bar. We multiply these two numbers to estimate the carpenter's weekly salary. We cancel out units that are identical if they are above and below the division bar.

$$\frac{40 \cancel{\text{ hours}}}{\text{week}} \times \frac{\$30}{\cancel{\text{hour}}} = \frac{\$1200}{\text{week}}$$

Thus, the carpenter earns approximately \$1200 per week, written $\approx$ \$1200.

Great Question!

Is it OK to cancel identical units if one unit is singular and the other is plural?

Yes. It does not matter whether a unit is singular, such as *week*, or plural, such as *weeks*. *Week* and *weeks* are identical units and can be canceled out, as shown on the right.

b. For the estimate of annual salary, we may round 52 weeks to 50 weeks. The annual salary is approximately the product of \$1200 per week and 50 weeks per year:

$$\frac{\$1200}{\text{week}} \times \frac{50 \text{ weeks}}{\text{year}} = \frac{\$60{,}000}{\text{year}}.$$

Thus, the carpenter earns approximately \$60,000 per year, or \$60,000 annually, written $\approx$ \$60,000.

Check Point 4. A landscape architect who works full time earns \$52 per hour.

a. Estimate the landscape architect's weekly salary.

b. Estimate the landscape architect's annual salary.

2 Apply estimation techniques to information given by graphs.

Estimation with Graphs

Magazines, newspapers, and websites often display information using circle, bar, and line graphs. The following examples illustrate how rounding and other estimation techniques can be applied to data displayed in each of these types of graphs.

Circle graphs, also called **pie charts**, show how a whole quantity is divided into parts. Circle graphs are divided into pieces, called **sectors**. **Figure 1.3** shows a circle graph that indicates how Americans disagree as to when "old age" begins.

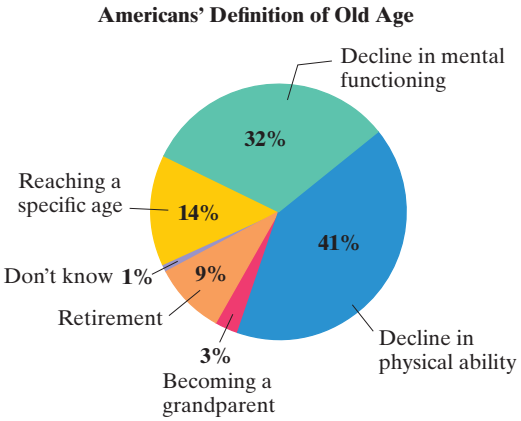


Figure 1.3

Source: American Demographics

A Brief Review *Percents*

- **Percents** are the result of expressing numbers as part of 100. The word *percent* means *per hundred*. For example, the circle graph in **Figure 1.3** shows that 41% of Americans define old age by a decline in physical ability. Thus, 41 out of every 100 Americans define old age in this manner: $41\% = \frac{41}{100}$.
- To convert a number from percent form to decimal form, move the decimal point two places to the left and drop the percent sign. Example:

$$41\% = 41.\% = 0.41\%$$

Thus, $41\% = 0.41$.

- Many applications involving percent are based on the following formula:

$$\underbrace{A}_{A} \ \underbrace{\text{is}}_{=} \ \underbrace{P \text{ percent}}_{P \cdot} \ \underbrace{\text{of}}_{\cdot} \ \underbrace{B.}_{B.}$$

Note that the word *of* implies multiplication.

Exercises

Convert each percent to decimal form.

1. 99% **2.** 7% **3.** 12.4% **4.** 0.3% **5.** 123%

Translate each sentence into a percent equation. Do not solve the equation.

6. A number A is 5% of 1000.

7. 45 is 10% of a number B.

8. 12 is P% of 60.

In our next example, we will use the information in the circle graph on page 18 to estimate a quantity. Although different rounding results in different estimates, the whole idea behind the rounding process is to make calculations simple.

Example 5 Applying Estimation Techniques to a Circle Graph

According to the U.S. Census Bureau, in 2020, there were 249,019,818 Americans 25 years and older. Assuming the circle graph in **Figure 1.3** is representative of this age group,

a. Use the appropriate information displayed by the graph to determine a calculation that shows the number of Americans 25 years and older who define old age by a decline in physical ability.

b. Use rounding to find a reasonable estimate for this calculation.

Solution

a. The circle graph in **Figure 1.3** indicates that 41% of Americans define old age by a decline in physical ability. Among the 249,019,818 Americans 25 years and older, the number who define old age in this manner is determined by finding 41% of 249,019,818.

The number of Americans 25 and older who define old age by a decline in physical ability | is | 41% | of | the number of Americans 25 and older.

$$= \quad 0.41 \quad \times \quad 249{,}019{,}818$$

b. We can use rounding to obtain a reasonable estimate of $0.41 \times 249{,}019{,}818$.

Round to the nearest ten-million.

$$0.41 \times 249{,}019{,}818 \approx 0.4 \times 250{,}000{,}000 = 100{,}000{,}000$$

Round to the nearest tenth.

$$\begin{array}{r} 250{,}000{,}000 \\ \times \quad 0.4 \\ \hline 100{,}000{,}000.0 \end{array}$$

Our answer indicates that approximately 100,000,000 (100 million) Americans 25 years and older define old age by a decline in physical ability.

Check Point 5. Being aware of which appliances and activities in your home use the most energy can help you make sound decisions that allow you to decrease energy consumption and increase savings. The circle graph in **Figure 1.4** shows how energy consumption is distributed throughout a typical home.

Suppose that last year your family spent $2148.72 on natural gas and electricity. Assuming the circle graph in **Figure 1.4** is representative of your family's energy consumption,

a. Use the appropriate information displayed by the graph to determine a calculation that shows the amount your family spent on heating and cooling for the year.

b. Use rounding to find a reasonable estimate for this calculation.

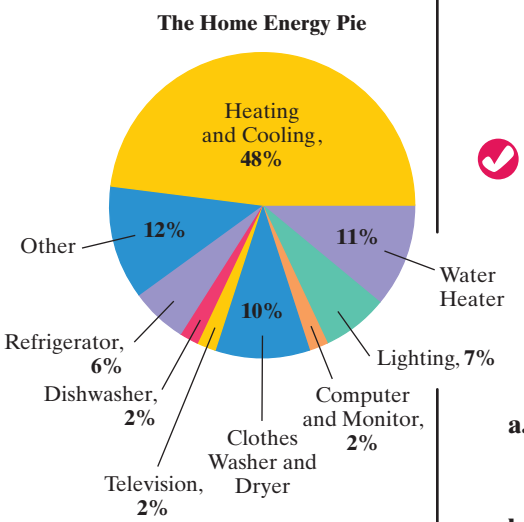


Figure 1.4

Source: Natural Home and Garden

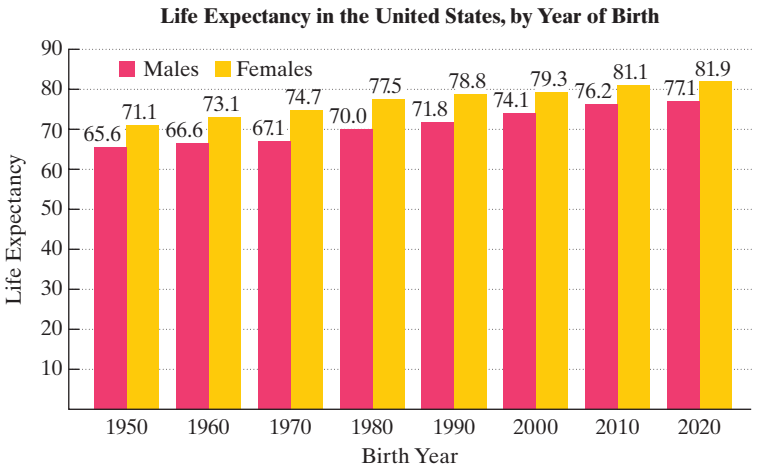


Figure 1.5
Source: National Center for Health Statistics

Bar graphs are convenient for comparing some measurable attribute of various items. The bars may be either horizontal or vertical, and their heights or lengths are used to show the amounts of different items. **Figure 1.5** is an example of a typical bar graph. The graph shows life expectancy for American males and American females born in various years from 1950 through 2020.

Example 6 Applying Estimation and Inductive Reasoning to Data in a Bar Graph

Use the data for males in **Figure 1.5** to estimate each of the following:

a. a male's increased life expectancy, rounded to the nearest hundredth of a year, for each subsequent birth year

b. the life expectancy of a male born in 2030.

Solution

a. One way to estimate increased life expectancy for each subsequent birth year is to generalize from the information given for 1950 (male life expectancy: 65.6 years) and for 2020 (male life expectancy: 77.1 years). The average yearly increase in life expectancy is the change in life expectancy from 1950 to 2020 divided by the change in time from 1950 to 2020.

Yearly increase in life expectancy is approximately $\frac{\text{change in life expectancy from 1950 to 2020}}{\text{change in time from 1950 to 2020}}$.

$$\approx \frac{77.1 - 65.6}{2020 - 1950}$$

life expectancy in 2020 minus life expectancy in 1950

Change in time is 2020 − 1950, or 70 years.

$$\approx 0.16$$

Use a calculator. See the Technology feature below.

For each subsequent birth year, a male's life expectancy is increasing by approximately 0.16 year.

TECHNOLOGY

Here is the calculator keystroke sequence needed to perform the computation in Example 6(a).

$$(\,77.1\,-\,65.6\,)\,\div\,(\,2020\,-\,1950\,)$$

Press $=$ on a scientific calculator or ENTER on a graphing calculator to display the answer. As specified, we round to the nearest hundredth.

The computation shown on a graphing calculator screen

```
(77.1-65.6)/(2020-1950)
                .1642857143
```

≈ 0.16

Hundredths digit, where rounding is to occur

Digit to the right is 4, so do not change the digit to be rounded.

b. We can use our computation in part (a) to estimate the life expectancy of an American male born in 2030. The bar graph indicates that males born in 1950 had a life expectancy of 65.6 years. The year 2030 is 80 years after 1950, and life expectancy is increasing by approximately 0.16 year for each subsequent birth year.

Life expectancy for a male born in 2030	is approximately	life expectancy for a male born in 1950	plus	yearly increase in life expectancy	times	the number of years from 1950 to 2030.

$$\approx 65.6 + 0.16 \times 80$$
$$= 65.6 + 12.8 = 78.4$$

An American male born in 2030 will have a life expectancy of approximately 78.4 years.

Check Point 6. Use the data for females in **Figure 1.5** on page 20 to estimate each of the following:

a. a female's increased life expectancy, rounded to the nearest hundredth of a year, for each subsequent birth year

b. the life expectancy, to the nearest tenth of a year, of a female born in 2050.

A Brief Review

Order of Operations

When performing calculations like the one shown at the right, recall that operations need to be performed in this order:

1. Evaluate expressions in grouping symbols such as parentheses.
2. Evaluate exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

You may use the acronym PEMDAS to help you remember the Order of Operations, which are described in full detail on page 273:

- **P**arentheses
- **E**xponents
- **M**ultiplication
- **D**ivision
- **A**ddition
- **S**ubtraction

Line graphs are often used to illustrate trends over time. Some measure of time, such as months or years, frequently appears on the horizontal axis. Points are drawn to represent the given information. The graph is formed by connecting the points with line segments.

Figure 1.6 is an example of two line graphs. The graphs show the percentage of marriages ending in divorce after 5, 10, and 15 years of marriage for two levels of educational attainment.

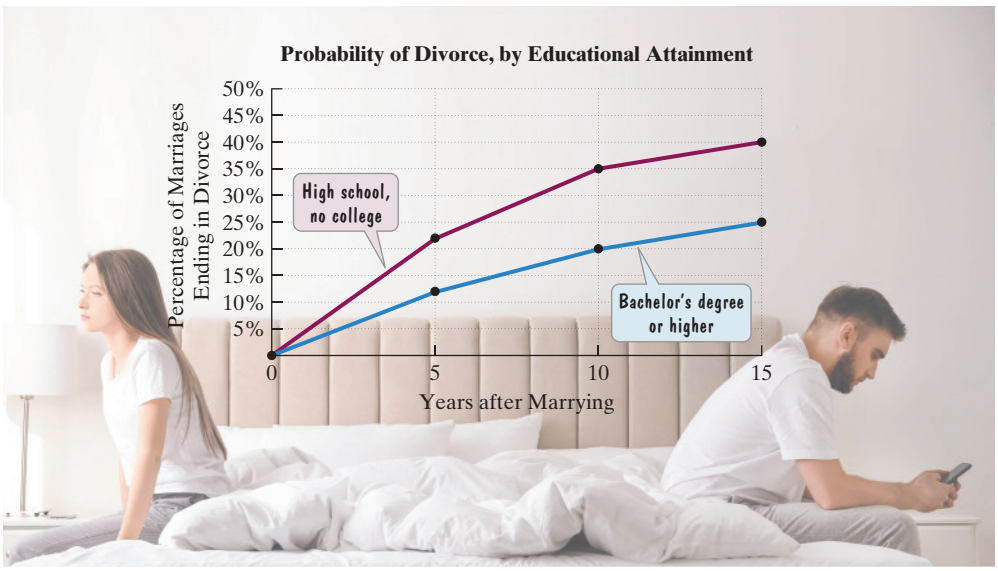


Figure 1.6

Source: U.S. Bureau of Labor Statistics

Great Question!

The data presented in Figure 1.6 indicate longer-lasting marriages for those with more education. Does this mean that a college education prepares you for marriage?

Not necessarily. Although the academic and social interactions in college may broaden a person's perspective of relationships, there are underlying issues at play here. One such issue is finances: More education typically means higher earnings, which may lead to greater financial stability. Financial stress in a marriage is a major contributor to divorce.

Throughout this text, you will encounter graphs based on real-world data. We want you to be aware of the existence of underlying issues that may offer an alternative explanation for the trends observed in these graphs. While our goal is that you acquire a greater appreciation of graphs and their applications, we also want you to be aware of the limitations of the conclusions you may draw from these graphs.

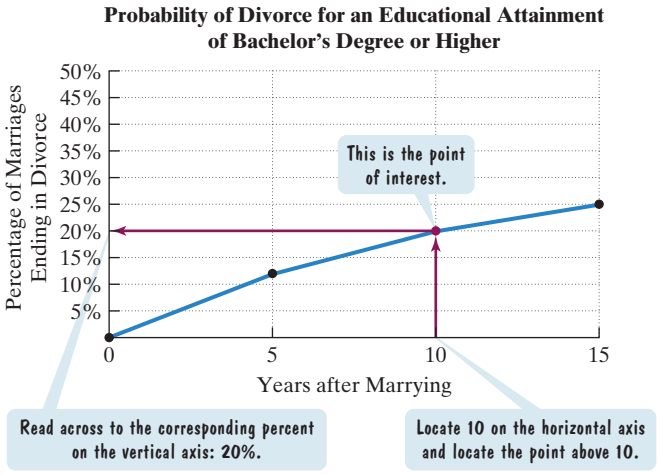


Figure 1.7

Figure 1.7 shows how to find the percentage of marriages for a bachelor's degree or higher ending in divorce after 10 years.

Step 1. Locate 10 on the horizontal axis.

Step 2. Locate the point on the blue graph above 10.

Step 3. Read across to the corresponding percent on the vertical axis.

The percent is 20%. Thus, for a bachelor's degree or higher, after 10 years, 20% of marriages end in divorce.

Example 7 Using a Line Graph

The line graph in **Figure 1.8** shows the percentage of U.S. college students who smoked cigarettes from 1982 through 2020.

a. Find an estimate for the percentage of college students who smoked cigarettes in 2010.

b. In which four-year period did the percentage of college students who smoked cigarettes decrease at the greatest rate?

c. In which year did 30% of college students smoke cigarettes?

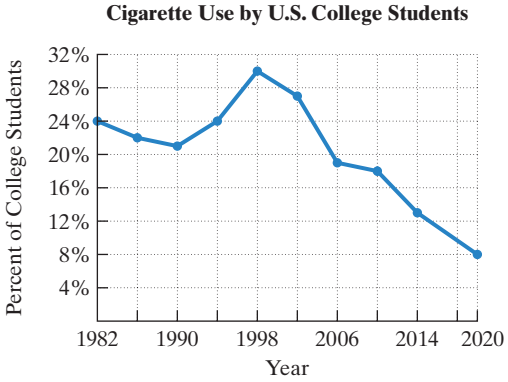
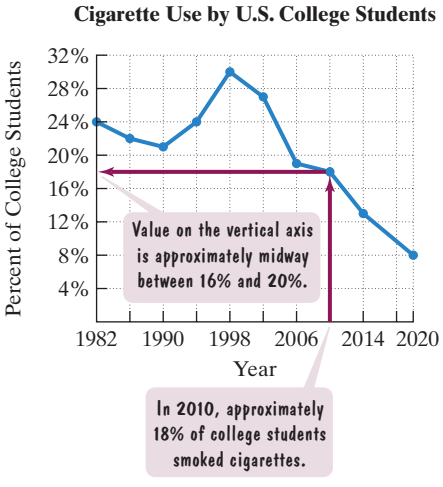


Figure 1.8

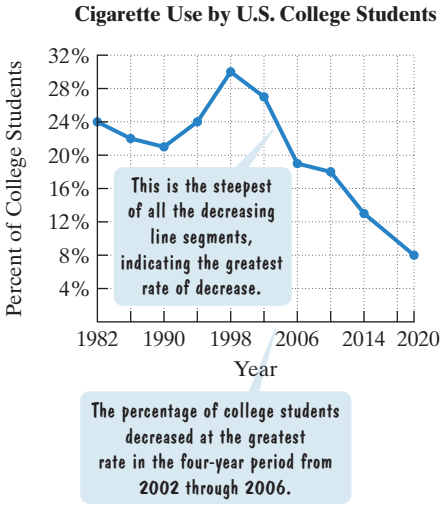
Source: Rebecca Donatelle, *Health The Basics*, 10th Edition, Pearson; *Monitoring the Future Study*, University of Michigan.

Solution

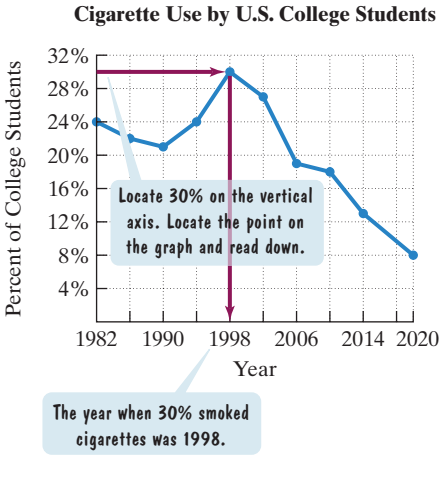
a. Estimating the Percentage Smoking Cigarettes in 2010



b. Identifying the Period of the Greatest Rate of Decreasing Cigarette Smoking



c. Identifying the Year When 30% of College Students Smoked Cigarettes



Check Point 7. Use the line graph in **Figure 1.8** on the previous page to solve this exercise.

a. Find an estimate for the percentage of college students who smoked cigarettes in 1986.

b. In which four-year period did the percentage of college students who smoked cigarettes increase at the greatest rate?

c. In which years corresponding to a tick mark on the horizontal axis did 24% of college students smoke cigarettes?

d. In which year did the least percentage of college students smoke cigarettes? What percentage of students smoked in that year?

3 Develop mathematical models that estimate relationships between variables.

Mathematical Models

We have seen in Example 6 that American males born in 1950 have a life expectancy of 65.6 years, increasing by approximately 0.16 year for each subsequent birth year. We can use variables to express the life expectancy, E, for American males born x years after 1950.

Life expectancy for American males | is | life expectancy for a male born in 1950 | plus | yearly increase in life expectancy | times the number of birth years after 1950.

$$E = 65.6 + 0.16x$$

A **formula** is a statement of equality that uses letters to express a relationship between two or more variables. Thus, $E = 65.6 + 0.16x$ is a formula describing life expectancy, E, for American males born x years after 1950. Be aware that this formula provides *estimates* of life expectancy, as shown in **Table 1.2**.

Table 1.2 Comparing Given Data with Estimates Determined by a Formula

Birth Year	Male Life Expectancy: Given Data	Male Life Expectancy: Formula Estimate $E = 65.6 + 0.16x$
1950	65.6	$E = 65.6 + 0.16(0) = 65.6 + 0 = 65.6$
1960	66.6	$E = 65.6 + 0.16(10) = 65.6 + 1.6 = 67.2$
1970	67.1	$E = 65.6 + 0.16(20) = 65.6 + 3.2 = 68.8$
1980	70.0	$E = 65.6 + 0.16(30) = 65.6 + 4.8 = 70.4$
1990	71.8	$E = 65.6 + 0.16(40) = 65.6 + 6.4 = 72.0$
2000	74.1	$E = 65.6 + 0.16(50) = 65.6 + 8.0 = 73.6$
2010	76.2	$E = 65.6 + 0.16(60) = 65.6 + 9.6 = 75.2$
2020	77.1	$E = 65.6 + 0.16(70) = 65.6 + 11.2 = 76.8$

In each row, we substitute the number of years after 1950 for x. The better estimates occur in 1950, 1990, and 2020.

The process of finding formulas to describe real-world phenomena is called **mathematical modeling**. Such formulas, together with the meaning assigned to the variables, are called **mathematical models**. We often say that these formulas model, or describe, the relationships among the variables.

BLITZER BONUS

Predicting Your Own Life Expectancy

The formula in **Table 1.2** does not take into account your current health, lifestyle, and family history, all of which could increase or decrease your life expectancy. Thomas Perls at Boston University Medical School, who studies centenarians, developed a much more detailed formula for life expectancy at livingto100.com. The model takes into account everything from your stress level to your sleep habits and gives you the exact age it predicts you will live to.

Example 8 Modeling the Cost of Attending a Public College

The bar graph in **Figure 1.9** shows the average cost of tuition and fees for public four-year colleges. (Note that the symbol ⫰ on the vertical axis shows that there is a break in values between 0 and 3000.)

a. Estimate the yearly increase in tuition and fees.

b. Write a mathematical model that estimates the average cost of tuition and fees, T, at public four-year colleges for the school year ending x years after 2000.

c. Use the mathematical model from part (b) to project the average cost of tuition and fees at public four-year colleges for the school year ending in 2030.

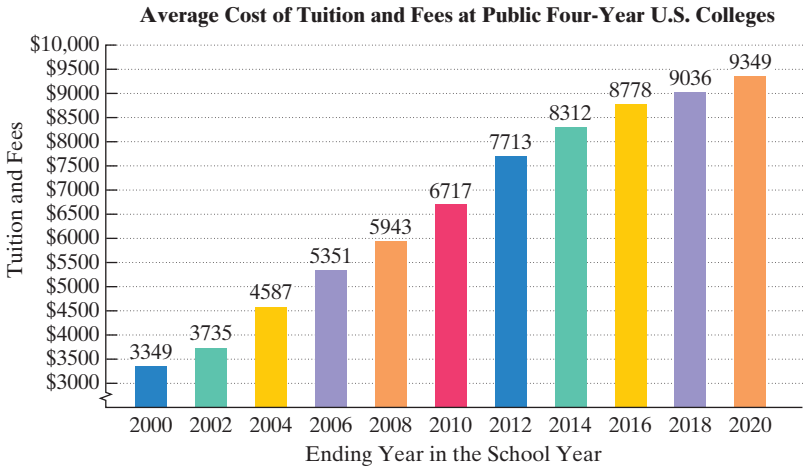


Figure 1.9

Source: National Center for Education Statistics

Solution

a. We can use the data in **Figure 1.9** from 2000 and 2020 to estimate the yearly increase in tuition and fees.

$$\text{Yearly increase in tuition and fees} \text{ is approximately } \frac{\text{change in tuition and fees from 2000 to 2020}}{\text{change in time from 2000 to 2020}}.$$

$$\approx \frac{9349 - 3349}{2020 - 2000}$$

$$= \frac{6000}{20} = 300$$

Each year the average cost of tuition and fees for public four-year colleges is increasing by approximately \$300.

b. Now we can use variables to obtain a mathematical model that estimates the average cost of tuition and fees, T, for the school year ending x years after 2000.

The average cost of tuition and fees is tuition and fees in 2000 plus yearly increase in tuition and fees times the number of years after 2000.

$$T = 3349 + 300x$$

The mathematical model $T = 3349 + 300x$ estimates the average cost of tuition and fees, T, at public four-year colleges for the school year ending x years after 2000.

c. Now let's use the mathematical model to project the average cost of tuition and fees for the school year ending in 2030. Because 2030 is 30 years after 2000, we substitute 30 for x.

$$\begin{aligned} T &= 3349 + 300x && \text{This is the mathematical model from part (b).} \\ T &= 3349 + 300(30) && \text{Substitute 30 for } x. \\ &= 3349 + 9000 && \text{Multiply: } 300(30) = 9000. \\ &= 12{,}349 && \text{Add. On a calculator, enter } 3349 \boxed{+} 300 \boxed{\times} 30 \text{ and press } \boxed{=} \text{ or } \boxed{\text{ENTER}}. \end{aligned}$$

Our model projects that the average cost of tuition and fees at public four-year colleges for the school year ending in 2030 will be \$12,349.

Check Point 8. The bar graph in **Figure 1.10** shows the average cost of tuition and fees for private four-year colleges.

a. Estimate the yearly increase in tuition and fees. Round to the nearest dollar.

b. Write a mathematical model that estimates the average cost of tuition and fees, T, at private four-year colleges for the school year ending x years after 2000.

c. Use the mathematical model from part (b) to project the average cost of tuition and fees at private four-year colleges for the school year ending in 2030.

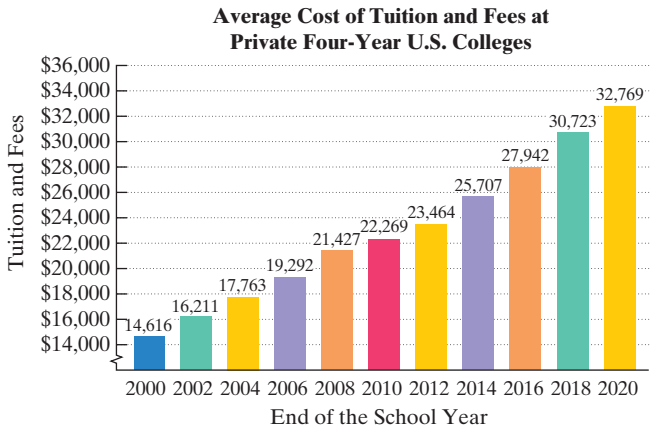


Figure 1.10

Source: National Center for Education Statistics

BLITZER BONUS *Is College Worthwhile?*

"Questions have intensified about whether going to college is worthwhile," says *Education Pays*, released by the College Board Advocacy & Policy Center. "For the typical student, the investment pays off very well over the course of a lifetime, even considering the expense."

Among the findings in *Education Pays*:

- In 2018, mean (average) full-time earnings with a bachelor's degree were approximately \$71,000, which was \$45,000 more than earnings with a high school diploma.
- Compared with a high school graduate, a four-year college graduate who enrolled in a public university at age 18 will break even by age 33. The college graduate will have earned enough by then to compensate for being out of the labor force for four years and for borrowing enough to pay tuition and fees, shown in **Figure 1.9**.