



# REINFORCED CONCRETE DESIGN

NINTH EDITION

JOSÉ A. PINCHEIRA | GUSTAVO J. PARRA-MONTESINOS  
CHU-KIA WANG | CHARLES SALMON

OXFORD  
UNIVERSITY PRESS

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*University of Wisconsin–Madison*

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# PREFACE

The ninth edition of this textbook has been substantially revised and updated to incorporate the changes introduced by the publication of the 2019 American Concrete Institute (ACI) Building Code and Commentary for Structural Concrete, as well as to reflect changes in construction and design practices that have occurred in the last few years.

## APPROACH

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This new edition follows the same philosophical approach that has gained wide acceptance among users since the first edition was published in 1965. Herein, as in past editions, considerable emphasis is placed on presenting to the student, as well as to the practicing engineer, the basic principles of reinforced concrete design and the concepts necessary to understand and properly apply the provisions of the ACI Building Code. Numerous examples are presented to illustrate the general approach to design and analysis. The material is incorporated into the chapters in a way that permits the reader to study, in detail, the concepts in logical sequence or to obtain a qualitative explanation and proceed directly to the design process using the ACI Code.

## NEW TO THIS EDITION

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The ninth edition of this book incorporates the changes arising from the publication of the 2019 American Concrete Institute Building Code and Commentary (ACI 318-19). Major changes incorporated into the 2019 ACI Building Code include new provisions for shear design of reinforced concrete members, development length of hooked bars, design of beam-column joints, and calculation of deflections in reinforced concrete members. The chapter dealing with serviceability is now Chapter 11 (formerly Chapter 12) and follows the chapter on members in compression and bending. This was done to conform to the sequence of topics that is commonly followed in either a first or second course in reinforced concrete. Most chapters in this book were thus substantially revised to accommodate these and other changes made to the ACI Building Code. Also, many sections in the book have been updated and in some cases reordered for clarity and better understanding of the material for the reader.

In addition to the content revisions indicated in the previous paragraph, all the examples and the problems at the end of each chapter have been revised and updated to conform to the current ACI Code. Some examples and problems have also been updated to reflect the increasing use of higher concrete strengths and Grade 80 ksi steel in current practice. A few examples, however, use less common values in order to emphasize specific aspects of the design process that students might otherwise overlook.

To aid instructors, a solutions manual has been prepared for the end-of-chapter problems. Many problems are solved in Mathcad®, allowing alternate solutions to be easily arrived at by modifying a few parameters, either as suggested in this textbook or at the choice of the instructor.

## COURSE SUGGESTIONS

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Depending on the proficiency required of the student, this book may provide material for two courses of three or four semester-hours each. It is suggested that the beginning course in concrete structures for undergraduate students contain all or most of the material in Chapters 1 through 6, Chapter 8, and Chapter 10. Depending on the semester-hours of the course, material related to serviceability in Chapter 11 may be incorporated.

The second course may begin with Chapter 10, using that topic (members in compression and bending) to review many of the subjects in the first course, followed by Chapter 11 on serviceability, Chapter 13 on slenderness effects on columns, and Chapter 16 on two-way floor systems. In addition, one or two of the following may be included in a second course: Chapter 15 on structural walls; Chapter 18 on torsion; Chapter 14 on strut-and-tie models, deep beams, brackets, and corbels; and Chapter 20 on prestressed concrete.

Chapters on beam-column joints (Chapter 12), yield line theory of slabs (Chapter 17), footings (Chapter 19), and composite members and connections (Chapter 21) may serve as contents for a third course.

## SI UNITS

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This edition continues the modest treatment of SI units used in previous editions. The 2019 ACI Code has an SI version (known as ACI 318-19M), and the SI versions of the ACI Code equations appear in this book as footnote equations with the same equation number. According to the ACI Code, the designer must use in its entirety either the Inch-Pound units version (ACI 318-19) or the SI version (ACI 318-19M), although the Inch-Pound units version is the official version of the Code. The authors believe that sufficient metrication should be included in a text on reinforced concrete to permit the reader to gain some familiarity with SI units, but suspect that too much would interfere with learning the basic concepts of concrete design; constant conversion back and forth between Inch-Pound and SI units is more confusing than using either one exclusively. The text provides data on reinforcing bars in accordance with American Society for Testing and Materials (ASTM) Inch-Pound units, and also ASTM SI units (the “soft” conversion of the bar sizes and strengths approved in 1996). Some design tables are provided for bars and material strengths in SI units, a few numerical examples are given in SI units, and some problems at the ends of chapters are given with an SI alternate in parentheses at the statement concluding the problem.

In all parts of this book that use metric units, force is measured in the newton (N) or kilonewton (kN) unit. The SI unit of stress is the pascal (Pa), or newton per meter squared, which because of its typically large numerical value is usually expressed in megapascals (MPa): that is,  $10^6$  pascals. A few diagrams show, along the stress axis, the kilogram force per centimeter squared ( $\text{kgf}/\text{cm}^2$ ) in addition to Inch-Pound and SI units. For the convenience of the reader, some conversion factors for forces, stresses, uniform loading, and moments are provided on a separate page following this Preface. Note that conversion factors (for forces, stresses, and dimensions) used in example problems are shown with a smaller font size to distinguish them from the values of the variables actually used in the calculations and thus facilitate understanding of the problem solution.

## ACKNOWLEDGMENTS

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The authors continue to be indebted to students, colleagues, and other users of the first eight editions of this book who have suggested improvements of wording, identified errors, and recommended items for inclusion or omission. The authors gratefully acknowledge the following reviewers, to whom they owe special thanks: Sergio F. Breña, University of Massachusetts–Amherst; Michael Kreger, University of Alabama; Rémy Lequesne, University of Kansas, and others who anonymously provided valuable feedback. Their

comments and suggestions have been carefully considered, and the results of our review are reflected in this completed revision.

Users of this ninth edition are urged to communicate with the authors regarding all aspects of this book, particularly on identification of errors and suggestions for improvement.

We are indebted to late Professors Chu-Kia (CK) Wang and Charles (Chuck) G. Salmon, who originated this textbook and entrusted us to carry on their legacy. Much of the new and expanded material presented in this ninth edition would not have been possible without their work in earlier editions of this book.

Special thanks are due to the Higher Education Group, Oxford University Press—in particular, Daniel Sayre; Theresa Stockton; Megan Carlson, and Claudia Dukeshire for their assistance in the early stages of this project, and to Petra Recter, Joan Kalkut, Joan Lewis-Milne, Arthur Pero, Brad Rau, and Wesley Morrison for helping bring this project to fruition during unparalleled and uncertain times. To all of you, our heartfelt gratitude and highest appreciation.

We acknowledge the long-time continuing patience and encouragement from our families, and especially from our respective wives, Rebeca Israel and Connie Parra, throughout the preparation of this edition of the book. Nicole and Gabriel Parra, with their frequent smiles and unbounded love, were a continuous source of inspiration to their father. We also owe a special recognition to our parents, Paulina Peña, Hernán Pincheira (deceased), Gustavo Parra Pardi (deceased), and Yolanda Montesinos Soteldo, who instilled in us from an early age the importance of learning, education, and hard work. To all of them we wholeheartedly dedicate this book.

José A. Pincheira  
Gustavo J. Parra-Montesinos



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## ABOUT THE AUTHORS

**JOSÉ A. PINCHEIRA** is Associate Professor of Civil and Environmental Engineering at the University of Wisconsin–Madison. He received his degree of Civil Engineer from the University of Chile, and holds an M.S. degree from the University of Manitoba, Canada, and a Ph.D. degree from the University of Texas at Austin. His main research interests include the behavior and design of reinforced concrete structural systems subjected to earthquakes, as well as the seismic rehabilitation of concrete structures. Dr. Pincheira is a Fellow of the American Concrete Institute (ACI), and a member of the ACI Building Code Committee 318. He is past Chair and current member of ACI Committee 369, Seismic Repair and Rehabilitation of Concrete Buildings; member of ACI Committee 374, Performance-Based Seismic Design of Concrete Buildings; and former member of the Committee on Seismic Rehabilitation of the American Society of Civil Engineers. Professor Pincheira has received several prestigious awards in recognition of his contributions to research and teaching, including the CAREER Award from the National Science Foundation; the James M. Robbins Excellence in Teaching Award from Chi Epsilon, the Civil Engineering Honor Society; the Martin P. Korn Award from the Precast/Prestressed Concrete Institute; the Wason Medal from the American Concrete Institute and the José A. Cuevas Medal from the Mexican Society of Civil Engineers.

**GUSTAVO J. PARRA-MONTESINOS** is the C. K. Wang Professor of Structural Engineering in the Department of Civil and Environmental Engineering at the University of Wisconsin–Madison. Professor Parra's main research interests include the behavior and design of reinforced concrete, fiber-reinforced concrete, and hybrid steel-concrete structures. He is a member of the ACI Building Code Committee 318 and Chair of its subcommittee 318-J (Joints and Connections). He is also Chair of ACI-ASCE Committee 352 on Joints in Monolithic Reinforced Concrete Structures. In addition, he is a member of ACI-ASCE Committee 335 on Composite and Hybrid Structures and of the Fédération Internationale du Béton (*fib*) Task Group T4.1 on fiber-reinforced concrete. Professor Parra has received several prestigious awards for his contributions to research in the field of reinforced concrete, including the Wason Medal, the Chester Paul Siess Award, and the Charles S. Whitney Medal from the American Concrete Institute. In addition, he is a recipient of the Arthur J. Boase Award from the ACI Foundation, the Walter L. Huber Research Prize from the American Society of Civil Engineers, the Shah Family Innovation Prize from the Earthquake Engineering Research Institute, and the ACI Young Member Award for Professional Achievement. Professor Parra is also a Fellow of the American Concrete Institute.

**CHU-KIA WANG\*** was Professor of Civil Engineering at the University of Wisconsin–Madison for more than 30 years. A devoted teacher throughout his career, he was the author or coauthor of many textbooks in the field of structural engineering as the outgrowth of lectures he prepared for his classes. The University of Wisconsin–Madison recognized his contribution to the education of future engineers with the College of Engineering's Benjamin Smith Reynolds Award for Excellence in Teaching. A fellow and lifetime member of the American Society of Civil Engineers, Professor Wang was also a member of the American Concrete Institute (ACI), the American Society for Engineering Education (ASCE), and other professional societies.



**CHARLES G. SALMON\*** was Professor Emeritus of Civil Engineering at the University of Wisconsin–Madison. An accomplished author, educator, researcher, and professional structural engineer, Professor Salmon received numerous honors in recognition of his contributions to the field, including the Western Electric Award for excellence in teaching from the American Society for Engineering Education, the University of Wisconsin’s Emil H. Steiger Distinguished Teaching Award, and the American Concrete Institute’s Joe W. Kelly and Delmar L. Bloem Awards. He was a long-time member of the ACI Building Code Committee for Structural Concrete (ACI 318), Committee 340 (Design Aids), and Committee 435 (Deflections of Concrete Structures). Professor Salmon was also an honorary member of ACI, an honorary member of the American Society of Civil Engineers; and a life member of the American Society for Engineering Education.

\*Deceased

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# CONVERSION FACTORS

Some Conversion Factors, between Inch-Pound and SI Units, Useful in Reinforced Concrete Design

	To Convert	To	Multiply by
Forces	kip force	kN	4.448
	lb	N	4.448
	kN	kip	0.2248
Stresses	ksi	MPa (i.e., N/mm <sup>2</sup> )	6.895
	psi	MPa	0.006895
	MPa	ksi	0.1450
	MPa	psi	145.0
Moments	ft-kip	kN · m	1.356
	kN · m	ft-kip	0.7376
Uniform Loading	kip/ft	kN/m	14.59
	kN/m	kip/ft	0.06852
	kip/ft <sup>2</sup>	kN/m <sup>2</sup>	47.88
	psf	N/m <sup>2</sup>	47.88
	kN/m <sup>2</sup>	kip/ft <sup>2</sup>	0.02089
Density	pcf	kg/m <sup>3</sup>	16.01846

Basis of Conversions: 1 in. = 25.4 mm; 1 lb force = 4.448 newtons.

Basic SI units relating to structural design:

Quantity	Unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s

Derived SI units relating to structural design:

Quantity	Unit	Symbol	Formula
force	newton	N	kg · m/s <sup>2</sup>
pressure, stress	pascal	Pa	N/m <sup>2</sup>
energy, or work	joule	J	N · m



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# REINFORCED CONCRETE DESIGN



# CHAPTER 1

## INTRODUCTION, MATERIALS, AND PROPERTIES

### 1.1 REINFORCED CONCRETE STRUCTURES

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The most common materials from which most structures are built are wood, steel, reinforced (including prestressed) concrete, and masonry. Lightweight materials, such as aluminum, and advanced composite materials, such as fiber-reinforced polymers (FRPs), are also used though to a much lesser extent. Reinforced concrete, however, is unique in that two materials, reinforcing steel and concrete, are used together; thus, the principles governing structural design in reinforced concrete differ in many ways from those involving design in one material.

Many structures are built of reinforced concrete: buildings, bridges, viaducts, retaining walls, tunnels, tanks, conduits, and others. This text deals primarily with fundamental principles of behavior and design of reinforced concrete members subjected to axial force, bending moment, shear, torsion, or combinations of those. These principles are applicable to the design of any structure, as long as information is known about the variation of axial force, shear, moment along the length of each member. Although analysis and design may be treated separately, they are inseparable in practice, especially in the case of reinforced concrete structures, which usually are statically indeterminate. In such cases, reasonable sizes of members are needed in the preliminary analysis that must precede the final design, so the final conciliation between analysis and design is largely a matter of trial, judgment, and experience.

Reinforced concrete is a logical union of two materials: plain concrete, which possesses high compressive strength but little tensile strength, and steel, in the form of bars embedded in the concrete, which can provide the needed strength in tension and deformation capacity to the member. For instance, the strength and deflection capacity of the beam shown in Fig. 1.1.1 are greatly increased by placing steel bars in the tension zone. Without steel reinforcement, the beam would undergo a brittle failure once the tensile stress at the bottom of the beam reached the tensile strength of the concrete. Adding sufficient longitudinal steel reinforcement in the tension zone, however, allows the beam to sustain additional load beyond the formation of a transverse (flexural) crack. As shown in Fig. 1.1.1, several flexural cracks will likely develop as the load is increased, providing some degree of warning prior to failure. Since reinforcing steel is capable of resisting compression as well as tension, it is also used to provide part of the carrying capacity in reinforced concrete columns, and frequently in the compression zone of beams to increase ductility and to control deflections. Also, reinforcement is needed transversely to resist shear, to provide lateral support to longitudinal reinforcement, and to confine the concrete.

Steel and concrete work readily in combination for several reasons: (1) bond (interaction between bars and surrounding hardened concrete) allows transfer of forces between the two materials; (2) proper concrete mixes provide adequate impermeability of the





**Stratosphere Tower, Las Vegas;** the tallest free-standing observation tower in the United States, 1149 feet high. A three-legged concrete tower is topped by a ring beam that supports the steel dome, completed in 1996 (Photo by C. G. Salmon).

concrete against water intrusion and bar corrosion; and (3) sufficiently similar rates of thermal expansion—that is, 0.0000055 to 0.0000075 for concrete and 0.0000065 for steel per degree Fahrenheit ( $^{\circ}\text{F}$ ), or 0.000010 to 0.000013 for concrete and 0.000012 for steel per degree Celsius ( $^{\circ}\text{C}$ )—introduce negligible forces between steel and concrete under atmospheric changes of temperature.

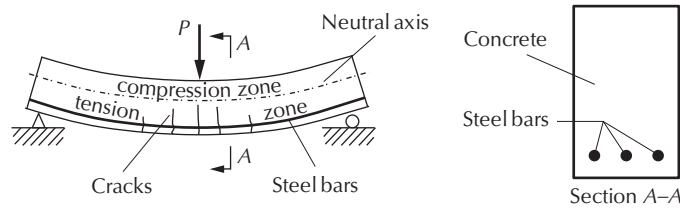
## 1.2 HISTORICAL BACKGROUND

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Joseph Monier, the owner of an important nursery in Paris, is generally given the credit for making the first practical use of reinforced concrete. In 1867, Monier recognized many of its potential uses and successfully undertook to expand the application of the new method [1.1].<sup>1</sup> Prior to his work, however, the method of reinforcing concrete with iron was

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<sup>1</sup> Numbers in brackets refer to the Selected References at the end of the chapter.



**Figure 1.1.1** Use of steel bars as tension reinforcement in a reinforced concrete beam.

known and, in some cases, was protected by patents. Ancient Grecian structures show that even much earlier builders knew something about the reinforcing of stonework for added strength [1.2].

In the mid-1800s, Joseph-Louis Lambot in France constructed and later exhibited at the Paris Exposition of 1854 a small boat, on which he received a patent in 1855. In Lambot's patent was shown a reinforced concrete beam and a column reinforced with four round iron bars. Another Frenchman, François Coignet, published a book in 1861 describing many applications and uses of reinforced concrete. In 1854, W. B. Wilkinson of England took out a patent for a reinforced concrete floor.

Monier acquired his first French patent in 1867 for iron-reinforced concrete tubs. This was followed by his many other patents, such as for pipes and tanks in 1868, flat plates in 1869, bridges in 1873, and stairways in 1875. In 1880–1881, Monier received German patents for innovations that included railroad ties, water feeding troughs, circular flower pots, flat plates, and irrigation channels. Monier's iron reinforcement was made mainly to conform to the contour of the structural element and generally strengthen it. He apparently had no quantitative knowledge regarding its behavior or any method of making design calculations [1.1].

In the United States, the pioneering efforts were made by Thaddeus Hyatt, originally a lawyer, who conducted experiments on reinforced concrete beams in the 1850s. In a perfectly correct manner, the iron bars in Hyatt's beams were located in the tension zone, bent up near the supports, and anchored in the compression zone. Additionally, transverse reinforcement (known as *vertical stirrups*) was used near the supports. However, Hyatt's experiments were unknown until 1877, when he published his work privately.

Built in 1870, the William Ward house in Port Chester, New York, is generally credited as the first cast-in-place reinforced concrete structure in the United States [1.3]. E. L. Ransome, head of the Concrete-Steel Company of San Francisco, apparently used some form of reinforced concrete in the early 1870s. He continued to increase the application of wire rope and hoop iron to many structures and was the first to use and, in 1884, patent the deformed (twisted) bar. Hurd [1.4] has provided an interesting biographical sketch of Ernest L. Ransome.

In 1890, Ransome built the Leland Stanford Jr. Museum in San Francisco, a reinforced concrete building two stories high and 312 ft (95 m) long. Since that time, development of reinforced concrete in the United States was rapid. During the period 1891–1894, various investigators in Europe published theories and test results; among them were Möller (Germany), Robert Wunsch (Hungary), Josef Melan (Austria), François Hannebique (France), and Fritz von Emperger (Hungary). Practical use, however, was less extensive than in the United States.

Throughout the entire period 1850–1900, relatively little was published, because the engineers working in the reinforced concrete field considered construction and computational methods to be trade secrets. One of the first publications that might be classified as a textbook was that of Armand Considère in 1899. By the turn of the century, there was a multiplicity of systems and methods with little uniformity in design procedures, allowable stresses, and systems of reinforcing. In 1903, with the formation in the United States of a joint committee of representatives of all organizations interested in reinforced concrete, uniform application of knowledge to design was initiated. The development of standard specifications is discussed in Chapter 2.

The earliest textbook in English was that of Turneure and Maurer [1.5], published in 1907. In the first decade of the twentieth century, progress in reinforced concrete was rapid. Extensive testing to determine beam behavior, compressive strength of concrete, and modulus of elasticity was conducted by Arthur N. Talbot at the University of Illinois, by Frederick E. Turneure and Morton O. Withey at the University of Wisconsin, and by Carl von Bach in Germany, among others. From about 1916 to the mid-1930s, research centered on axially loaded column behavior. In the late 1930s and 1940s, eccentrically loaded columns, footings, and the ultimate strength of beams received special attention.

Between the mid-1950s and 1970s, reinforced concrete design practice made the transition from one based on elastic methods to one based on strength. Prestressed concrete (Chapter 20), wherein the steel reinforcement is stressed in tension and the concrete is in compression even before external loads are applied, has advanced from an experimental technique to a major structural composite material. There has been a transition from cast-in-place reinforced concrete to elements precast at a manufacturer's plant and shipped to the job site for assembly. A summary of concrete building construction in the United States is given in Reference 1.3.

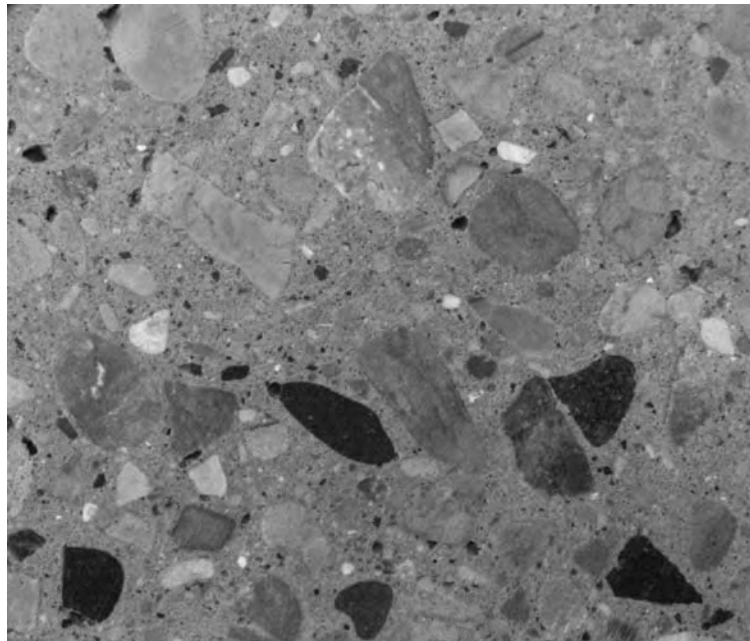
Our understanding of reinforced concrete behavior is still far from complete. Building codes and specifications that give design procedures are continually changing to reflect latest knowledge.

## 1.3 CONCRETE

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Plain concrete is made by mixing cement, fine aggregate, coarse aggregate, water, and frequently, admixtures (Fig. 1.3.1). When reinforcing steel is placed in the forms and wet concrete mix is placed around it, the final solidified mass becomes reinforced concrete. The strength of concrete depends on many factors, notably the proportion of the ingredients and the conditions of temperature and moisture under which it is placed and cured.

Subsequent sections contain brief discussions of the materials in and the properties of plain concrete. These are intended to be only introductory; an interested reader should consult standard references devoted entirely to the subject of plain concrete [1.6–1.8].



**Figure 1.3.1** Cross section of concrete. Cement-and-water paste coats each aggregate particle and fills space between particles. (Photo by José A. Pincheira.)

## 1.4 CEMENT

Cement is a material that has adhesive and cohesive properties enabling it to bond mineral fragments into a solid mass. Although this definition can apply to many materials, the cements of interest for reinforced concrete construction are those that can set and harden in the presence of water—the so-called *hydraulic cements*. These consist primarily of silicates and aluminates of lime made from limestone and clay (or shale), which is ground, blended, fused in a kiln, and crushed to a powder. Such cements chemically combine with water (hydrate) to form a hardened mass. The usual hydraulic cement used for reinforced concrete is known as *portland cement* because of its resemblance when hardened to Portland stone found near Dorset, England. The name originated in a patent obtained by Joseph Aspdin of Leeds, England, in 1824.

Concrete made with portland cement ordinarily requires several days to attain strength adequate to allow forms to be removed and construction and dead loads carried. The design or specified compressive strength of such concrete is typically assumed to be reached at about 28 days. This ordinary portland cement is identified by ASTM (American Society for Testing and Materials) C150/C150M [1.9] as Type I. Other types of portland cement and their intended uses are given in Table 1.4.1. American Concrete Institute (ACI) Committee 225 provides a guide for selection and use of hydraulic cements [1.10].

There are also several categories of blended hydraulic cements (ASTM C595/C595M [1.11]), such as portland blast-furnace slag cement (Type IS), portland-pozzolan cement (Type IP), portland-limestone cement (Type IL), and ternary blended cement (Type IT). Ternary blended cements are defined in ASTM C595 as those “consisting of portland cement with either a combination of two different pozzolans, slag and a pozzolan, a pozzolan and a limestone, or a slag and a limestone.” Pozzolan is a finely divided siliceous or siliceous and aluminous material that possesses little or no inherent cementitious property; in the powdery form and in the presence of moisture, however, it will chemically react with calcium hydroxide at ordinary temperatures to form compounds possessing cementitious properties.

Portland blast-furnace slag cement has lower heat of hydration than ordinary Type I cement and is useful for mass concrete structures such as dams. Because of its high sulfate resistance, it is used in seawater construction. Portland-pozzolan cement is a blended mixture of ordinary Type I cement with pozzolan. Blended cements with pozzolan gain strength more slowly than cements without pozzolan; hence, they produce less heat during hydration and are widely used in mass concrete construction.

Air-entraining portland cement contains a chemical admixture finely ground with the cement to produce intentionally air bubbles on the order of 0.002 in. (0.05 mm) diameter uniformly distributed throughout the concrete. Such air entrainment will give the concrete improved durability against frost action as well as better workability. Air-entraining portland cement for Types I, II, and III, given in Table 1.4.1, is designated IA, IIA, or IIIA. Air-entraining agents may also be added to the blended hydraulic cements in ASTM C595/C595M [1.11] at the time the concrete is mixed.

**TABLE 1.4.1** TYPES OF PORTLAND CEMENT<sup>a</sup>

Type	Uses
I	Ordinary construction where special properties are not required
II	Ordinary construction when moderate sulfate resistance is desired
II(MH)	Ordinary construction when moderate sulfate resistance and moderate heat of hydration is desired
III	When high early strength is desired; has considerably higher heat of hydration than Type I cement
IV	When low heat of hydration is desired
V	When high sulfate resistance is desired

<sup>a</sup> According to ASTM C150/C150M [1.9].

## 1.5 AGGREGATES

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Since aggregate usually occupies about 75% of the total volume of concrete, its properties have a definite influence on the behavior of hardened concrete. Not only does the strength and stiffness of the aggregate affect the strength and stiffness of the concrete, its properties also greatly affect durability (resistance to deterioration under freeze-thaw cycles). Because aggregate is less expensive than cement, it is logical to use the largest percentage feasible. In general, for maximum strength, durability, and best economy, the aggregate should be packed and cemented as densely as possible. Hence, aggregates are usually graded by size, and a proper mix specifies percentages of both *fine* and *coarse* aggregates.

Fine aggregate (sand) is any material passing through a No. 4 sieve<sup>2</sup> [i.e., less than about  $\frac{3}{16}$  in. (5 mm) diameter]. Coarse aggregate (gravel) is any material of larger size. The nominal maximum size of coarse aggregate permitted (ACI-26.4.2.1)<sup>3</sup> is governed by the clearances between sides of forms and between adjacent bars and may not exceed (a)  $\frac{1}{5}$  the narrowest dimension between sides of forms, nor (b)  $\frac{1}{3}$  the depth of slabs, nor (c)  $\frac{3}{4}$  the minimum clear spacing between individual reinforcing bars. Additional information concerning aggregate selection and use can be found in a report of ACI Committee 221 [1.13].

Natural stone aggregates conforming to ASTM C33 [1.14] are used in the majority of concrete construction, giving a unit weight for such concrete of about 145 pcf (pounds per cubic foot) or 2320 kg/m<sup>3</sup> (kilograms per cubic meter). When steel reinforcement is added, the unit weight of normal-weight reinforced concrete is taken for calculation purposes as 150 pcf or 2400 kg/m<sup>3</sup>. Actual weights for concrete and steel are rarely, if ever, computed separately. For special purposes, lightweight or extraheavy aggregates are used.

Structural lightweight concretes are usually made from aggregates conforming to ASTM C330 [1.15] that are produced artificially in a kiln, such as expanded clays and shales. The unit weight of such concretes typically ranges from 70 to 115 pcf (1120–1840 kg/m<sup>3</sup>) (see Fig. 1.5.1). Lightweight concretes ranging down to 30 pcf (480 kg/m<sup>3</sup>), often known as *cellular concretes*, are also used for insulating purposes and for masonry units. When lightweight materials are used for both coarse and fine aggregates in structural concrete, it is termed *all-lightweight concrete*. When only the coarse aggregate is of lightweight material but normal weight sand is used for the fine aggregate, it is said to be sand-lightweight concrete. Often the term *sand replacement* is used in connection with lightweight concrete. This refers to replacing all or part of the lightweight aggregate fines with natural sand. Steiger [1.16] provides historical background for the use of lightweight aggregate concrete, Mackie [1.17] has discussed uses of lightweight concrete, and ACI Committee 213 [1.18] has a guide for the use of structural lightweight aggregate concrete.

Heavyweight, high-density concrete is used for shielding against gamma and X radiation in nuclear reactor containers and other structures [1.19]. Naturally occurring iron ores, titaniferous iron ores, “hydrous iron ores” (i.e., containing bound and adsorbed water), and barites are crushed to suitable size for use as aggregates. Heavyweight concretes typically weigh from 200 to 350 pcf (3200–5600 kg/m<sup>3</sup>).

## 1.6 ADMIXTURES

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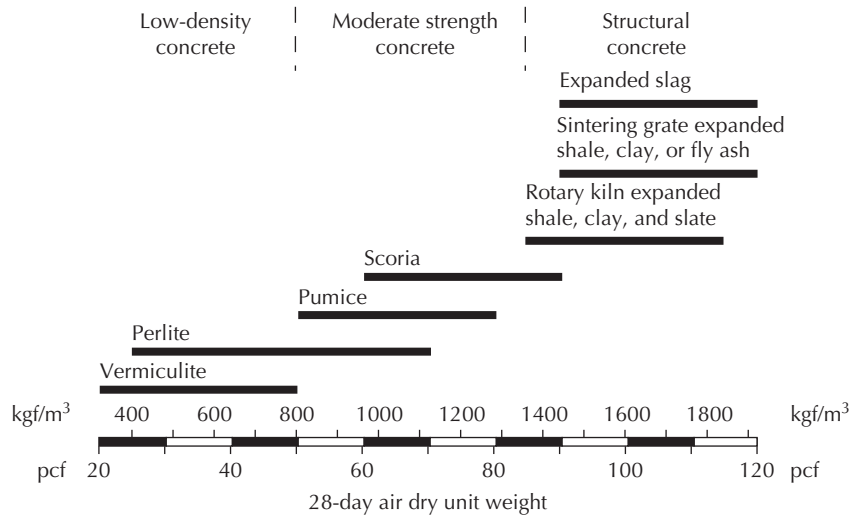
In addition to cement, coarse and fine aggregates, and water, materials known as *admixtures*, mineral or chemical, are often added to the concrete mix immediately before or during the mixing. Admixtures are used to modify the properties of the concrete to make it better serve its intended use or for better economy.

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<sup>2</sup> 4.75 mm according to ASTM Standard E11.

<sup>3</sup> Numbers refer to sections in the “ACI Code,” officially 318-19, *Building Code Requirements for Structural Concrete* [1.12].





**Figure 1.5.1** Approximate unit weight and use classification of lightweight aggregate concrete. (From Ref. 1.115.)

## Mineral Admixtures

Mineral admixtures are finely divided materials including pozzolans such as fly ash, cement slag, and silica fume. These admixtures are often used as cement replacement, but they can also be used in addition to cement, as replacement of sand. In general, mineral admixtures reduce the heat of hydration and improve workability and durability. General information about mineral admixtures is available in Mindess, Young, and Darwin [1.7] and Malhotra [1.20]. Mielenz [1.21] has given a history and background on mineral admixtures.

### Fly Ash

Fly ash is a by-product from the combustion of coal in power plants. Fly ash used in concrete shall meet ASTM C618 [1.22]. Since its cost is substantially lower than that of cement, it is typically used as cement replacement for economic reasons. The use of fly ash in concrete improves workability, reduces the heat of hydration, and increases durability. Albinger [1.23] provides general information on when to use and what to expect from fly ash concrete, and Ravina [1.24] discusses slump retention of fly ash concrete (see Section 1.7) with and without chemical admixtures.

### Cement Slag

Blast-furnace slag is a by-product of iron production. This by-product is first granulated and then ground to achieve particle sizes similar to those of cement. For use as mineral admixture in concrete, slag shall meet ASTM C989 [1.25]. Cement slag in concrete reduces the heat of hydration and provides increased durability by decreasing concrete permeability and increasing resistance to sulfate attacks. Although the strength gain of concrete with cement slag is lower in the first few days, compressive strength after that is typically greater than or comparable to that of concrete without slag cement. Additional information on the use of cement slag in concrete can be found in a report by ACI Committee 233 [1.26].

### Silica Fume

Silica fume is the finely divided solid-microsilica material collected from the fumes of electric furnaces that produce ferrosilicon or silicon metal. When used in concrete, it shall conform to ASTM C1240 [1.27]. In addition to its use as a pozzolan, silica fume in the concrete mix produces a more impermeable concrete, able to resist chloride intrusion into concrete exposed

to deicing chemicals. Silica fume is an important admixture in high-performance concrete to achieve high strength and excellent durability. Further information about silica fume is available from Cohen, Olek, and Mather [1.28] and from Durning and Hicks [1.29].

## Chemical Admixtures

A wide variety of chemical admixtures for concrete are available, most commonly air-entraining, water-reducing, and set-controlling admixtures. A history and background on chemical admixtures can be found in Durning and Hicks [1.30]. A report by ACI Committee 212 [1.31] provides an essential guidance for use of chemical admixtures. In this report, chemical admixtures are classified into categories that include air-entraining admixtures, accelerating admixtures, water-reducing and set-retarding admixtures, admixtures for flowing concrete, and admixtures for self-consolidating concrete. A brief discussion on these chemical admixtures follows.

### Air-Entraining Admixtures

These chemicals, meeting the requirements of ASTM C260 [1.32], can be added either to the hydraulic cement or as an admixture to the concrete mix. The chemical causes air in the form of minute bubbles (often 0.004 in. or 0.1 mm diameter or smaller) to be dispersed throughout the concrete mix, with the purpose of increasing workability and resistance to deterioration that results from both freeze-thaw action and ice-removal salts.

Air-entraining admixtures are probably the most widely used type of chemical admixture. In addition to resistance against freeze-thaw cycles and the corrosiveness of deicing chemicals, air entrainment improves plasticity and workability, permitting a reduction in water content. Uniformity of placement with little bleeding and segregation can be achieved. In addition, air-entrained concrete is more watertight and increases resistance to sulfate action. For exposed concrete, the possible reduction in strength (approximately 5% for each percent of entrained air) is far less important than the improved durability in terms of resistance to freeze-thaw action and deicing chemicals.

### Accelerating Admixtures

Accelerating admixtures modify the properties of concrete, particularly in cold weather, to (1) accelerate the rate of early-age strength development; (2) reduce the time required for proper curing and protection; and (3) permit earlier start of finishing operations. Accelerators must not be used as antifreeze agents for concrete. Accelerators must meet the requirements of Type C or E in ASTM C494 [1.33]; calcium chloride, the best known and most common accelerator, must also meet the requirements of ASTM D98 [1.116]. Limits on water-soluble chloride, however, are specified in ACI-19.3.2.1 to reduce potential for corrosion of reinforcement.

### Water-Reducing and Set-Retarding Admixtures

Water-reducing admixtures are used to reduce the amount of water required for a given slump or, when used without water reduction, to increase concrete workability. Some of these admixtures also increase the setting time for concrete. Water-reducing and set-retarding admixtures must meet the requirements of ASTM C494 [1.33], where they are classified as water-reducing admixtures (Type A); retarding admixtures (Type B); water-reducing and retarding admixtures (Type D); water-reducing and accelerating admixtures (Type E); water-reducing, high-range admixtures (Type F); and water-reducing, high-range, and retarding admixtures (Type G). Water-reducing, high-range admixtures are sometimes referred to as *superplasticizers*, meaning that the quantity of mixing water is reduced by 12% or more. The last two classifications (Type F and Type G) are also covered by ASTM C1017 [1.34].

A report by ACI Committee 212 [1.31] lists seven general categories for materials used as water-reducing admixtures, including lignosulfonic acids and their salts, hydroxylated carboxylic acids and their salts, carbohydrate-based compounds and polysaccharides,

and polycarboxylates. Materials used for water-reducing, high-range admixtures, on the other hand, include sulfonated naphthalene condensates, sulfonated melamine condensates, modified lignosulfonates, and polycarboxylates. Ramezaniapour, Sivasundaram, and Malhotra [1.35] have discussed superplasticizers and their effect on strength properties of concrete.

Set-retarding admixtures are used primarily to offset the accelerating and damaging effect of high temperature, to keep concrete workable during placement, and to minimize form-deflection cracks. A variety of water-reducing admixtures also serve as set-retarding admixtures.

#### Admixtures for Flowing Concrete

Flowing concrete is defined as “concrete that is characterized as having a slump greater than 7½ in. [190 mm] while maintaining a cohesive nature” [1.34]. These admixtures are classified by ASTM C1017 [1.34] into two types: Type I—Plasticizing, and Type II—Plasticizing and Retarding. Flowing concrete is commonly used where high rates of casting are required or in highly reinforcement-congested members [1.31].

#### Admixtures for Self-Consolidating Concrete

Self-consolidating concrete is highly flowable concrete that requires no vibration. Given its high flowability, a flow slump, rather than a slump, is measured in self-consolidating concretes. In general, flow slumps between 22 and 30 in. (550–750 mm) are associated with these concretes. Two primary types of chemical admixtures are used in self-consolidating concrete: high-range, water-reducing admixtures (typically polycarboxylate based) and viscosity-modifying admixtures (polymer based or in the form of fine particles). High-range, water-reducing admixtures are used to lower the yield stress of the material, while viscosity-modifying admixtures serve to increase cohesion and plastic viscosity when a concrete with low yield stress and high plastic viscosity is desired [1.31]. More information about self-consolidating concrete can be found in an ACI Committee 237 report [1.36].

The chemical admixtures discussed above are only a few of those covered in ACI Committee 212 [1.31]. The reader is referred to this document for a complete list and description of chemical admixtures for concrete.

## 1.7 COMPRESSIVE STRENGTH

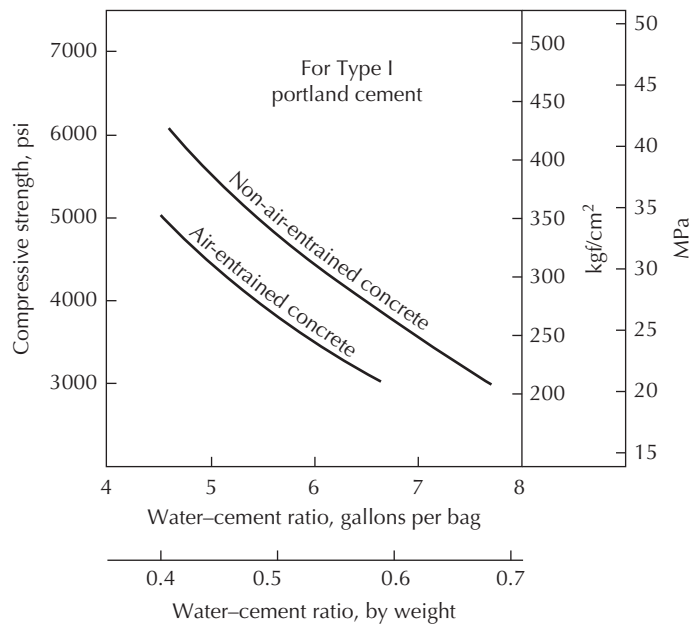
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The strength of concrete is primarily controlled by the proportioning of cement, coarse and fine aggregates, water, and various admixtures. In reinforced concrete design, “concrete strength” means uniaxial compressive strength measured by a compression test, typically of a standard test cylinder. The most important variable in determining concrete strength is the water-to-cement (w/c) ratio, as shown in Fig. 1.7.1. The lower the w/c ratio, the higher the compressive strength. This relationship has been recognized since the 1920s.

In the past decades, with the increasing use of admixtures, many of which contain cementitious materials, researchers have confirmed that any cementitious admixtures should be included with the cement in determining the proper mix to obtain a specified strength. This is recognized in ACI 318, where a water-to-cementitious (w/cm) ratio shall be calculated, including the weight of fly ash and other pozzolans (ASTM C618 [1.22]), slag cement (ASTM C989 [1.25]), and silica fume (ASTM C1240 [1.27]). Popovics [1.37] as well as Popovics and Popovics [1.38, 1.39] have reviewed the validity of strength based on the w/cm ratio.

A certain minimum amount of water is necessary for the proper chemical action in the hardening of concrete; extra water increases the workability (the ease of concrete flow) but reduces strength. A measure of the workability is obtained by a slump test. A truncated cone-shaped metal mold 12 in. (300 mm) high is filled with fresh concrete. The mold is then lifted off, and a measurement is made of the distance to the top of the wet mass “slump” from its position before the mold was removed. The smaller the slump, the stiffer and less workable the mix. In building construction, a 3 to 4 in. (75–100 mm) slump is





**Figure 1.7.1** Effect of water-to-cement ratio on 28-day compressive strength. Average values for concrete containing 1.5 to 2% trapped air for non-air-entrained concrete and no more than 5 to 6% air for air-entrained concrete. (Curves drawn from data in Ref. 1.42, Table 6.3.4a.)

common. Vibration of the concrete mix will greatly improve workability, and even very stiff no-slump concrete can be placed [1.40].

Proportioning of concrete mixes can be done in accordance with Design and Control of Concrete Mixtures [1.41], as well as ACI Standard 211.1 for normal-weight, heavyweight, and mass concrete [1.42], ACI Standard 211.2 for structural lightweight concrete [1.43], and ACI Standard 211.3 for no-slump concrete [1.40]. Strength of concrete in place is also greatly affected by quality-control procedures for placement and inspection. Details regarding good practice are available in ACI Standard 304 [1.44] and in the ACI Manual of Concrete Inspection [1.45].

Durability, long recognized as an important quality of concrete, is related to the  $w/cm$  ratio and compressive strength, among other factors. Durability requirements for concrete can be found in Chapter 19 of the ACI Code (Concrete: Design and Durability Requirements). A source for obtaining durable concrete is the Committee 201 Guide to Durable Concrete [1.46].

The strength of concrete is denoted in the United States by  $f'_c$ , which is the compressive strength in psi of test cylinders with diameter and height, respectively, of either 6 in. (150 mm) and 12 in. (300 mm) or 4 in. (100 mm) and 8 in. (200 mm), typically measured at 28 days after casting. In many parts of the world, the standard test unit is the cube, frequently measuring 8 in. (200 mm) to a side.

Since nearly all reinforced concrete behavior is related to the standard 28-day compressive strength,  $f'_c$ , it is important to note that such strength depends on the size and shape of the test specimen and the manner of testing [1.47]. Properties such as tensile strength of concrete and size of contact area of the testing machine have more effect on cube strength than on cylinder strength. As an average, the 6 × 12 in. (150 × 300 mm) cylinder strength is 80% of the 6 in. (150 mm) cube strength and 83% of the 8 in. (200 mm) cube strength [1.48]. For lightweight concrete, cylinder strength and cube strength are nearly equal.

Given the effect on cylinder compressive strength of numerous test variables (e.g., load rate, specimen dimensions, casting and curing conditions), it is clear that such strength will differ from the in-place concrete strength in a structure. Results from a thorough investigation of in-place versus molded cylinder concrete compressive strengths were reported by Bloem [1.49]. Compressive strengths obtained from tests of drilled cores were less than those of cylinders. Compared with strengths of field-cured cylinders, the compressive strength of drilled cores averaged between 11 and 21% less, depending on curing conditions.

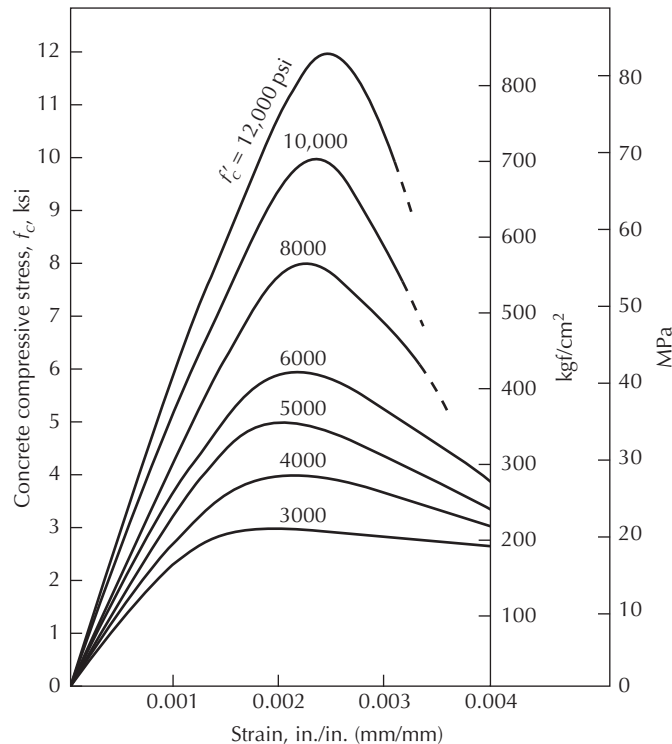
An interesting discussion of the cylinder test is given by Shilstone [1.50], and Tait [1.51] has discussed the use of test results. When an assessment of the strength of in-place concrete is desired, procedures ranging from tests of cylindrical cores cut from the structure to the use of nondestructive tests [1.52–1.58] are available. ACI Committee 214 [1.59] has a recommended practice for evaluating strength test results from concrete cores.

## Stress-Strain Relationship

The stress-strain relationship for concrete depends on its strength, age at loading, rate of loading, aggregates and cement properties, and type and size of specimens [1.60, 1.61]. Typical curves for specimens ( $6 \times 12$  in. cylinders) loaded in compression at 28 days using normal testing speeds are shown in Fig. 1.7.2. The rate of applied strain during testing influences the shape of the stress-strain curve, as shown in Fig. 1.7.3, particularly the portion after the maximum stress has been reached.

Note from Fig. 1.7.2 that lower-strength concrete has greater deformability (ductility) than higher-strength concrete, as evidenced by the length and smaller slope of the descending portion of the curve after the maximum stress has been reached at a strain between 0.002 and 0.0025. Ultimate strain at crushing of concrete often varies from 0.003 to as high as 0.008.

In usual reinforced concrete design, specified concrete strengths  $f'_c$  of 3500 to 5000 psi (24–35 MPa) are used for nonprestressed structures, and strengths of 5000 to 8000 psi (35–56 MPa) are used for prestressed concrete. For special situations, particularly in columns of tall buildings, concretes ranging from 6000 to 14,000 psi (42–97 MPa) have been used [1.66–1.68]. On the Pacific First Center in Seattle, the specified strength was 14,000 psi (97 MPa) at 56 days [1.66]. The average strength obtained throughout the project was about 18,000 psi (124 MPa). Research continues on high-strength concrete (often referred to as *high-performance concrete*) because in addition to high strength, the concrete must have other excellent characteristics [1.69, 1.70].



**Figure 1.7.2** Typical stress-strain curves for concrete in compression under short-time loading. (Curves represent a compromise adapted from curves and results given by Wang, Shah, and Naaman [1.61], Bertero [1.62], Naaman [1.63], and Nilson [1.64].)

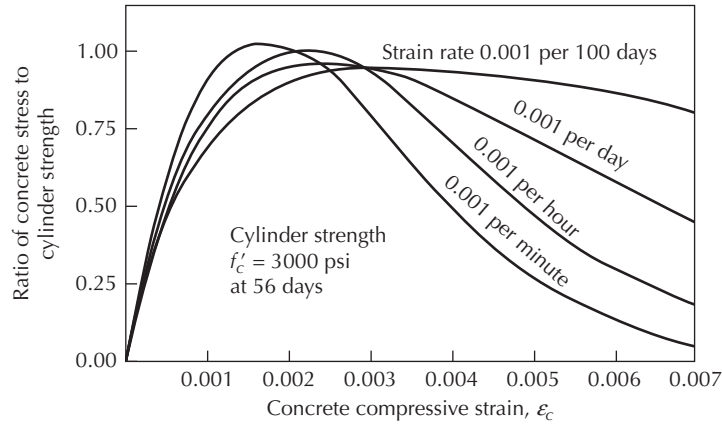


Figure 1.7.3 Stress-strain curves for various strain rates of concentric loading. (From Rüsch [1.65].)

## 1.8 TENSILE STRENGTH

The strength of concrete in tension is an important property that greatly affects the extent and size of cracking in structures. Tensile strength is usually determined by using the *split-cylinder* test in accordance with ASTM C496/C496M [1.71]. In this test, the same size cylinder used for the compression test is placed on its side in the testing machine so that the compression load  $P$  is applied uniformly along the length of the cylinder in the direction of the diameter. The cylinder will split in half when the tensile strength is reached. The stress is computed by  $2P/[\pi(\text{diameter})(\text{length})]$  based on the theory of elasticity for a homogeneous material in a biaxial state of stress.<sup>4</sup> Tensile strength is a more variable property than compressive strength and is about 10 to 15% of it. The split-cylinder tensile strength  $f_{ct}$  has been found to be proportional to  $\sqrt{f'_c}$ ,<sup>5</sup> such that

$$f_{ct} = 6\sqrt{f'_c} \quad \text{to} \quad 7\sqrt{f'_c} \text{ psi} \quad \text{for normal-weight concrete}^6$$

$$f_{ct} = 5\sqrt{f'_c} \quad \text{to} \quad 6\sqrt{f'_c} \text{ psi} \quad \text{for lightweight concrete}^6$$

Tensile strength in flexure, known as *modulus of rupture* and measured in accordance with ASTM C78 [1.72], is also important when considering cracking and deflection of beams. The modulus of rupture  $f_r$ , computed from the flexure formula  $f = Mc/I$ , gives higher values for tensile strength than the split-cylinder test, primarily because the concrete stress distribution is not linear when tensile failure is imminent, as is assumed in the computation of the nominal  $Mc/I$  stress. It is generally accepted (ACI-19.2.3.1) that an average value for the modulus of rupture  $f_r$  may be taken as  $7.5\lambda\sqrt{f'_c}$  ( $0.62\lambda\sqrt{f'_c}$  MPa), where  $\lambda$  is meant to account for the lower tensile properties of lightweight concrete compared to normal-weight concrete. Because of the large variability in modulus of rupture, as shown in Fig. 1.8.1, the selection of the coefficient 7.5, or even the entire expression  $7.5\lambda\sqrt{f'_c}$ , should be viewed as a practical choice for design purposes.

ACI Table 19.2.4.1(a) specifies  $\lambda$  values based on the equilibrium density of the concrete  $w_c$ . Based on the equilibrium density  $\lambda = 1$  for concrete with  $w_c > 135$  lb/ft<sup>3</sup>,  $\lambda = 0.75$  for concrete with  $w_c \leq 100$  lb/ft<sup>3</sup>, and  $\lambda = 0.0075w_c$  for  $100 \text{ lb/ft}^3 < w_c \leq 135 \text{ lb/ft}^3$ . Alternatively, the value of  $\lambda$  can be obtained from ACI Table 19.2.4.1(b) based on the aggregate composition, where  $\lambda = 0.75$  for *all-lightweight* concrete and  $\lambda = 0.85$  for

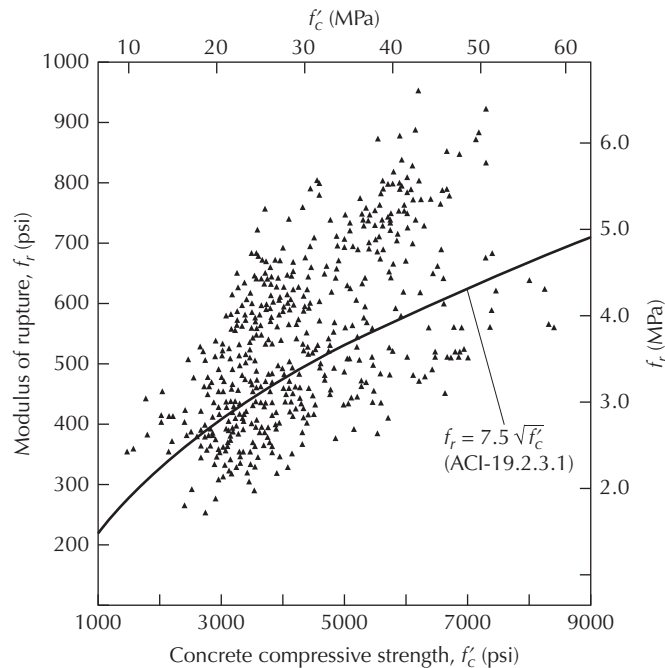
<sup>4</sup> See, for example, S. Timoshenko and J. N. Goodier, *Theory of Elasticity*, 3rd ed., McGraw-Hill, 1970, pp. 122–123.

<sup>5</sup>  $\sqrt{f'_c}$  is in psi units; thus,  $f'_c = 3000$  psi and  $\sqrt{f'_c} = 54.8$  psi. When  $f'_c$  is in newtons per square millimeter, that is, megapascals (MPa), the constant in front of  $\sqrt{f'_c}$  is to be multiplied by 0.083.

<sup>6</sup> In SI units, with  $f'_c$  and  $f_{ct}$  in MPa,

$$f_{ct} = 0.5\sqrt{f'_c} \quad \text{to} \quad 0.6\sqrt{f'_c} \quad \text{for normal-weight concrete}$$

$$f_{ct} = 0.4\sqrt{f'_c} \quad \text{to} \quad 0.5\sqrt{f'_c} \quad \text{for lightweight concrete}$$



**Figure 1.8.1** Comparison of test results for modulus of rupture of normal-weight concrete with ACI Code expression. (Adapted from Mirza, Hatzinikolas, and MacGregor [1.73].)

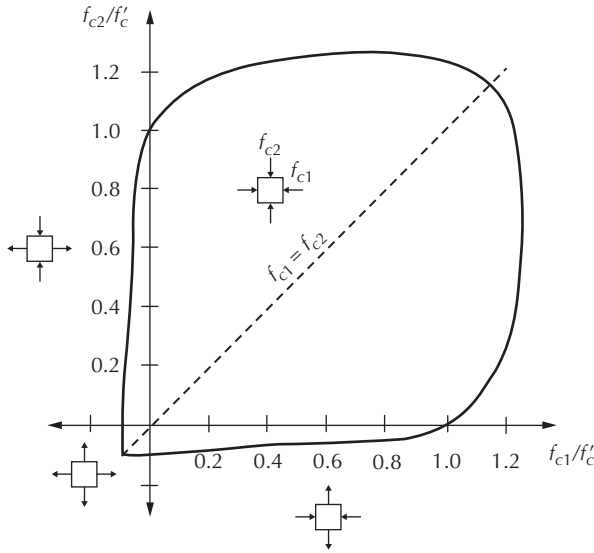
*sand-lightweight* concrete. For other lightweight blends,  $\lambda$  varies depending on the volumetric ratios of lightweight and normal-weight aggregate. Nonetheless, ACI-19.2.4.2 permits the use of  $\lambda = 0.75$  for all types of lightweight concrete. For normal-weight concrete,  $\lambda$  is taken as 1.0 (ACI-19.2.4.3).

One may note that neither the split-cylinder nor the modulus of rupture tensile strength is correctly a measure of the strength under axial tension. However, axial tensile strength is difficult to measure accurately, and when compared with the modulus of rupture or split-cylinder strength, it does *not* give better correlation with tension-related failure behavior such as inclined cracking from shear and torsion or splitting from interaction of reinforcing bars with surrounding concrete.

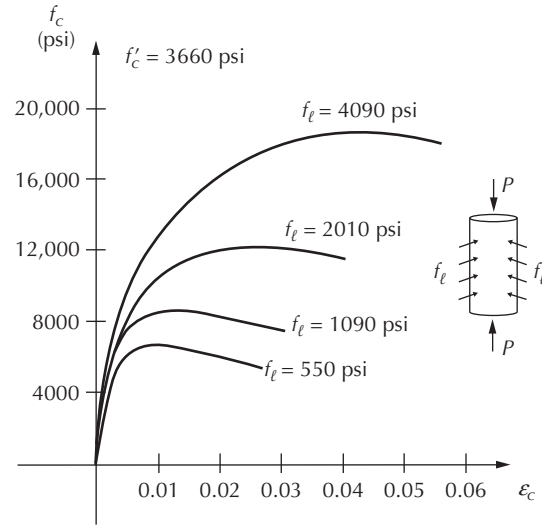
## 1.9 BIAXIAL AND TRIAXIAL STRENGTH

Concrete is seldom subjected to only uniaxial compressive or tensile stress. For example, the presence of shear in flexural members generates biaxial stresses in the concrete, and the restraint against lateral expansion provided by transverse reinforcement in members under axial compression leads to triaxial compressive stresses. Fig. 1.9.1 shows a biaxial stress interaction diagram from Kupfer, Hilsdorf, and Rusch [1.74]. Under biaxial compression, concrete compressive strength is greater than the uniaxial compressive strength, being approximately 30% higher on average for a 1:0.5 ratio of biaxial compressive stresses. Under combined tension and compression, the strength interaction is approximately linear with strengths lower than both the uniaxial compressive and tensile strengths. Under equal biaxial tension, concrete strength is approximately equal to the uniaxial tensile strength.

Compressive strength and deformation capacity are greatly increased by the presence of lateral compressive (confining) stresses. From tests of cylinders subjected to triaxial compression with the largest stress applied in the longitudinal direction, Richart, Brandtzaeg, and Brown [1.75] showed that concrete strength increases at a rate of approximately 4.1 times the magnitude of the lateral (confining) stress. Strain capacity also increased with an increase in lateral pressure, with strain at peak stress ranging between 0.5 and 7% for the range of lateral pressures considered (Fig. 1.9.2).



**Figure 1.9.1** Strength of concrete under biaxial stress. (Adapted from Fig. 6 in Ref. 1.74.)



**Figure 1.9.2** Stress-strain response of concrete under triaxial compression. (Adapted from Fig. 23 in Ref. 1.75.)

## 1.10 MODULUS OF ELASTICITY

The modulus of elasticity of concrete, unlike that of steel, varies with strength. It also depends, though to a much lesser extent, on the age of the concrete, the properties of the aggregates and cement, the rate of loading, and the type and size of the specimen. Furthermore, since concrete exhibits some permanent set even under small loads, there are various definitions of the modulus of elasticity.

Figure 1.10.1 represents a typical stress-strain curve for concrete in compression. In the figure, the initial modulus (tangent at origin), the tangent modulus (at  $0.5f'_c$ ), and the secant modulus (also at  $0.5f'_c$ ) are noted. Usually, the secant modulus at 25 to 50% of the compressive strength is considered to be the modulus of elasticity. For many years the modulus was approximated adequately as  $1000f'_c$  by the ACI Code, but with the rapidly increasing use of lightweight concrete, the variable of density needed to be included. As a result of a statistical analysis of available data, the empirical formula given by ACI-19.2.2.1a,

$$E_c = 33w_c^{1.5} \sqrt{f'_c} \quad (1.10.1)^7$$

was developed [1.76] for values of  $w_c$  between 90 and 155 pcf, though the ACI Code allows Eq. (1.10.1) to be used for  $w_c$  values up to 160 pcf. Equation (1.10.1) is representative of the secant modulus at a compressive stress of  $0.45f'_c$ . Reviews of the applicability of Eq. (1.10.1) have been made by Shih, Lee, and Chang [1.77] and also Oluokun, Burdette, and Deatherage [1.78]. For normal-weight concrete weighing 145 pcf, Eq. (1.10.1) gives  $E_c = 57,600\sqrt{f'_c}$ . For normal-weight concrete, ACI-19.2.2.1b allows

$$E_c = 57,000\sqrt{f'_c} \quad (1.10.2)^8$$

<sup>7</sup> For SI units, with  $w_c$  in kg/m<sup>3</sup> and  $E_c$  and  $f'_c$  in MPa,

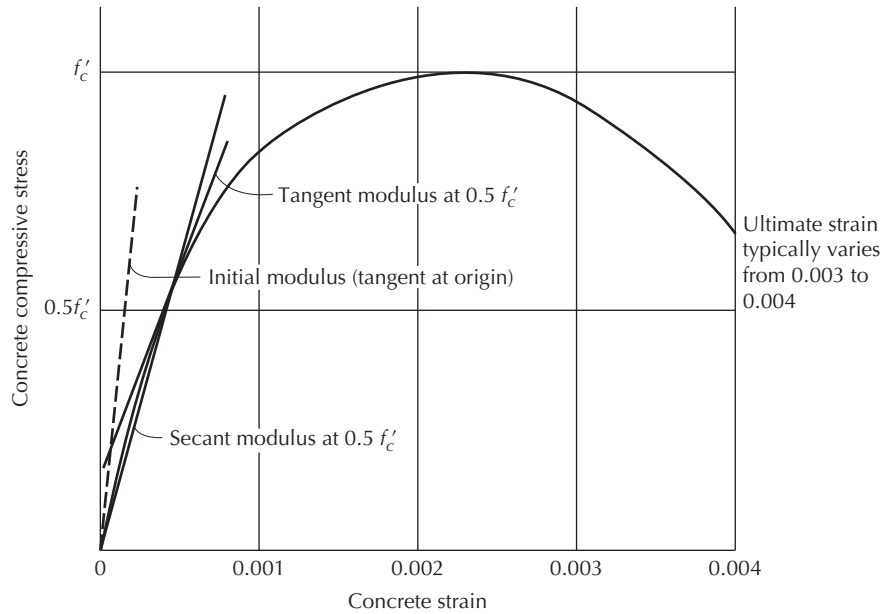
$$E_c = 0.043w_c^{1.5} \sqrt{f'_c} \quad (\text{ACI 318-19M}) \quad (1.10.1)$$

<sup>8</sup> For SI units, with  $E_c$  and  $f'_c$  in MPa,

$$E_c = 4700\sqrt{f'_c} \quad (\text{ACI 318-19M}) \quad (1.10.2)$$

and with  $E_c$  and  $f'_c$  in kgf/cm<sup>2</sup>,

$$E_c = 15,000\sqrt{f'_c} \quad (\text{approximate})$$



**Figure 1.10.1** Definitions of the modulus of elasticity for concrete in compression. Values of the modulus of elasticity for various concrete strengths appear in Table 1.10.1.

**TABLE 1.10.1** VALUES OF MODULUS OF ELASTICITY (USING  $E_c = 33w_c^{1.5}\sqrt{f'_c}$  FOR NORMAL-WEIGHT CONCRETE WEIGHING 145 PCF)

Inch-Pound Units		SI Units <sup>b</sup>	
$f'_c$ (psi)	$E_c$ (psi)	$f'_c$ (MPa)	$E_c^\dagger$ (MPa)
3000	3,150,000	21 <sup>a</sup>	21,500
3500	3,400,000	24	23,000
4000	3,640,000	28	24,900
4500	3,860,000	31	26,200
5000	4,070,000	35	27,800
6000	4,460,000	41	30,100
8000	5,150,000	55	34,900

<sup>a</sup> These metric values are rounded values approximating concrete strengths in Inch-Pound units; actual equivalents for 3000, 3500, 4000, 4500, 5000, 6000, and 8000 psi are 20.7, 24.1, 27.6, 31.0, 34.5, 41.3, and 55.1 MPa, respectively.

<sup>b</sup> Multiply MPa values by 10.2 to obtain kgf/cm<sup>2</sup>.

<sup>†</sup> Using  $E_c = 4700\sqrt{f'_c}$  as per ACI 318-19M.

## 1.11 CREEP AND SHRINKAGE

Creep and shrinkage are time-dependent deformations that, along with cracking, cause a great concern for the designer because of the inaccuracies and unknowns that surround them. Concrete may behave as essentially elastic only under loads of short duration, and because of additional deformation with time, the effective behavior is that of an inelastic material. Deflection after a long period of time is therefore difficult to predict, but its control is needed to assure serviceability during the life of the structure.

## Creep

Creep is the property of concrete (and other materials) by which it continues to deform with time under sustained loads at unit stresses within the accepted elastic range (say, below  $0.5f'_c$ ). This inelastic deformation increases at a decreasing rate during the time of loading, and its total magnitude may be several times as large as the short-time elastic deformation. Frequently, creep is associated with shrinkage, since the two occur simultaneously and often provide the same net effect: increased deformation with time. As may be noted by the general relationship of deformation versus time in Fig. 1.11.1, the “true elastic strain” decreases, since the modulus of elasticity  $E_c$  is a function of concrete strength, which increases with time.

Although creep is separate from shrinkage, it is related to it. Detailed information is available for estimating creep [1.79, 1.80]. The internal mechanism of creep, or *plastic flow* as it is sometimes called, may be due to any one or a combination of the following: (1) crystalline flow in the aggregate and hardened cement paste, (2) plastic flow of the cement paste surrounding the aggregate, (3) closing of internal voids, and (4) the flow of water out of the cement gel due to external load and drying.

Factors affecting the magnitude of creep are (1) the constituents, such as the composition and fineness of the cement, the admixtures, and the size, grading, and mineral content of the aggregates; (2) proportions, such as water content and the w/c ratio; (3) curing temperature and humidity; (4) relative humidity during period of use; (5) age at loading; (6) duration of loading; (7) magnitude of stress; (8) surface-to-volume ratio of the member; and (9) slump.

Accurate prediction of creep is complicated because of the variables involved; however, a general prediction method developed by Branson [1.80] gives a standard creep coefficient equation (for 4 in. or less slump, 40% relative humidity, moist cured, and loading at 7 days or more)

$$C_t = \frac{\text{creep strain}}{\text{initial elastic strain}} \quad (1.11.1)$$

$$= \frac{t^{0.60}}{10 + t^{0.60}} C_u$$

shown in Fig. 1.11.2, where  $t$  is the duration of loading (days) and  $C_u$  is the ultimate creep coefficient. (Branson [1.80] suggests using an average of 2.35 for  $C_u$  under standard conditions, but the range is shown to be from 1.3 to 4.15.) Correction factors are given for relative humidity, loading age, minimum thickness of member, slump, percent fines, and air content. For practical purposes, the only factors significant enough to require correction are humidity and age at loading.

The effect of unloading may be seen from Fig. 1.11.3, where at a certain time the load is removed. There is an immediate elastic recovery and a long-time creep recovery, but a residual deformation remains.

Creep of concrete will often cause an increase in the long-term deflection of members. Unlike concrete, steel is not susceptible to creep. For this reason, steel reinforcement is often provided in the compression zone of beams to reduce their long-term deflection.

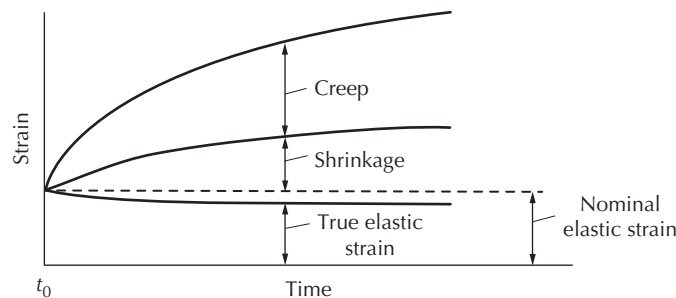
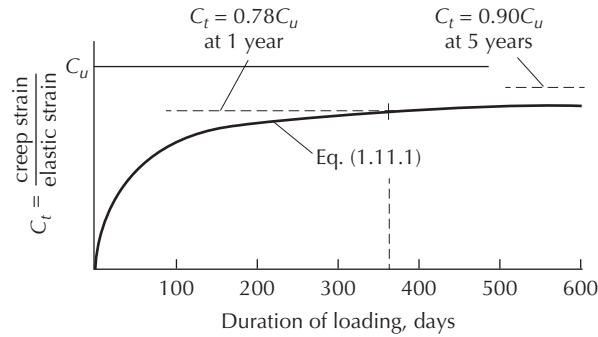
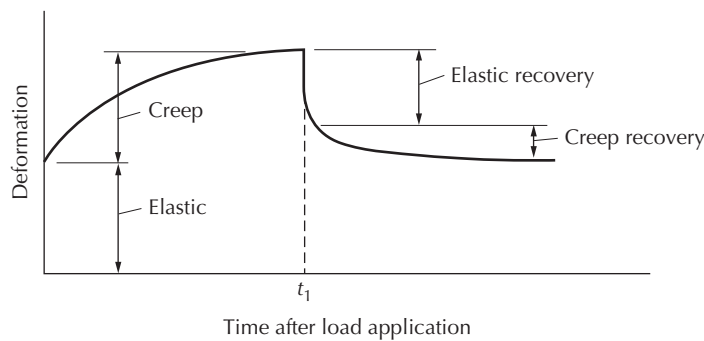


Figure 1.11.1 Change in strain of a loaded and drying specimen;  $t_0$  is the time at application of load.





**Figure 1.11.2** Standard creep coefficient variation with duration of loading (for 4 in. or less slump, 40% relative humidity, moist cured, and loading at 7 days or more).



**Figure 1.11.3** Typical relationship between creep and recovery with time.

## Shrinkage

Shrinkage, broadly defined, is the volume change during hardening and curing of the concrete. It is unrelated to load application. The main cause of shrinkage is the loss of water as the concrete dries and hardens. It is possible for concrete cured continuously under water to increase in volume; however, the usual concern is with a decrease in volume. A discussion of the mechanisms of shrinkage may be found in Mindess, Young, and Darwin [1.7]. In general, the same factors have been found to influence shrinkage strain as those that influence creep—primarily those factors related to moisture loss.

The Branson general prediction method [1.80] gives a standard shrinkage strain equation (for 4 in. or less slump, 40% ambient relative humidity, and minimum member dimension of 6 in. or less, after 7 days of moist curing)

$$\epsilon_{sh} = \left( \frac{t}{35 + t} \right) (\epsilon_{sh})_u \quad (1.11.2)$$

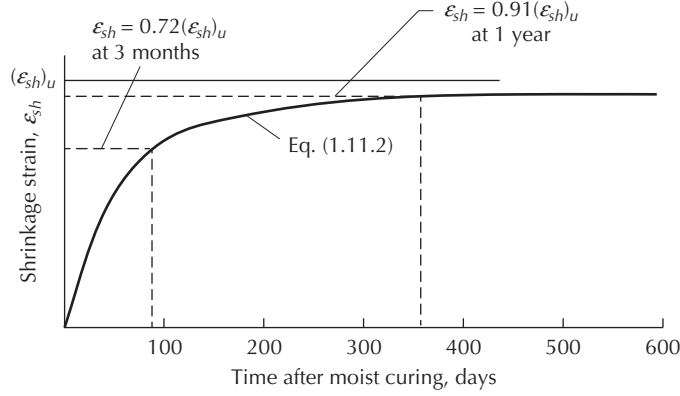
shown in Fig. 1.11.4, where  $t$  is time (days) after moist curing and  $(\epsilon_{sh})_u$  is the ultimate shrinkage strain. (Branson [1.80] suggests using  $800 \times 10^{-6}$  for average conditions, but the range is from approximately 400 to more than  $1000 \times 10^{-6}$ .) Correction factors are given with the primary one relating to humidity  $H$ :

$$\text{correction factor} = 1.40 - 0.01H \quad \text{for } 40\% \leq H \leq 80\%$$

$$\text{correction factor} = 3.00 - 0.03H \quad \text{for } 80\% \leq H \leq 100\%$$

Shrinkage, particularly when restrained unsymmetrically by reinforcement, causes deformations that are generally additive to those of creep. For proper serviceability, it is desirable to estimate or compensate for shrinkage in the structure.





**Figure 1.11.4** Standard shrinkage strain variation with time after 7 days of moist curing (for 4 in. or less slump, 40% ambient relative humidity, and minimum member dimension of 6 in. or less).

## 1.12 CONCRETE QUALITY CONTROL

In reinforced concrete design, concrete sections are proportioned and reinforced using a *specified compressive strength*  $f'_c$ . The strength  $f'_c$  for which each part of a structure has been designed should be clearly indicated on the design drawings. In the United States, as indicated in Section 1.7,  $f'_c$  is based on cylinder strength ( $6 \times 12$  or  $4 \times 8$  in. cylinders).

Because concrete is a material whose strength and other properties are not precisely predictable, test cylinders from a mix designed to provide, say, 4000 psi (roughly 28 MPa) concrete will show considerable variability. Therefore, mixes must be designed to provide an average compressive strength greater than the specified value.

ACI-26.4.3.1(b) allows concrete to be proportioned to achieve the specified compressive strength following ACI 301 [1.81], based on either *field test data* or *trial mixes*. When the ready-mix plant or other concrete production facility has a field test record based on at least 15 consecutive strength tests, or two groups of consecutive strength tests with a total of no less than 30 tests and at least 10 tests in a group, for materials and conditions similar to those expected, the *standard deviation*  $s_s$  can be computed based on those tests to establish how variable the concrete strength is. These records shall correspond to a concrete with compressive strength within 1000 psi (6.9 MPa) from the specified concrete strength and obtained within the past 24 months, from a period no less than 45 calendar days.

ACI 301 indicates that the *required average compressive strength*  $f'_{cr}$  used for proportioning the mix must be taken as the *larger* of Eq. (1.12.1) and the appropriate one of either Eq. (1.12.2) or Eq. (1.12.3):

$$f'_{cr} = f'_c + 1.34k s_s \quad (1.12.1)$$

and when  $f'_c \leq 5000$  psi,

$$f'_{cr} = f'_c + 2.33k s_s - 500 \quad (1.12.2)$$

or when  $f'_c > 5000$  psi,

$$f'_{cr} = 0.90f'_c + 2.33k s_s \quad (1.12.3)$$

The factor  $k$  in Eqs. (1.12.1) through (1.12.3) depends on the number of consecutive tests considered. When at least 30 consecutive strength tests are the basis for computing the standard deviation  $s_s$ ,  $k = 1.0$ . For cases where the number of tests considered is 25, 20, and 15,  $k$  should be taken as 1.03, 1.08, and 1.16, respectively. For example, if the designer has used a specified strength  $f'_c$  of 4000 psi, and if the concrete producer has shown field test

data with a standard deviation of 450 psi based on 30 consecutive tests or more, the mix should be designed for an average strength of 4600 psi [i.e., the larger of  $4000 + 1.34(1)(450)$  and  $4000 + 2.33(1)(450) - 500$ ].

When data are not available to establish a standard deviation, the required average compressive strength  $f'_{cr}$  is calculated as follows:

when,  $f'_c \leq 3000$  psi,

$$f'_{cr} = f'_c + 1000$$

when  $3000 \text{ psi} \leq f'_c \leq 5000 \text{ psi}$ ,

$$f'_{cr} = f'_c + 1200$$

when  $f'_c > 5000$  psi,

$$f'_{cr} = 1.1f'_c + 700$$

According to ACI-26.12.3.1(a), concrete compressive strength is considered acceptable if both of the following conditions are satisfied:

1. Every arithmetic average of any three consecutive strength tests<sup>9</sup> equals or exceeds  $f'_c$ .
2. No individual strength test falls below  $f'_c$  by more than 500 psi when  $f'_c$  is 5000 psi or less or by more than  $0.10f'_c$  when  $f'_c$  exceeds 5000 psi.

Statistical variations are to be expected, and strength tests failing to meet these criteria will occur perhaps once in 100 tests even though all proper procedures have been followed. A discussion on the risks inherent in the consideration of limited test data is provided by Tait [1.82]. ACI-26.12.6 specifies steps to be taken in case low-strength test results are obtained.

The foregoing discussion of concrete strength variation should merely give an awareness that concrete with a specified compressive strength cannot be expected to provide precisely known actual strength and other properties.

Quality control in the broader sense for reinforced concrete construction is a subject of great importance but generally lies outside the scope of this text. The ACI Committee 121 Report [1.83] and the papers by Tuthill [1.84], Mather [1.85], Newman [1.86], and Scanlon [1.87] provide an excellent overall treatment of this subject.

## 1.13 STEEL REINFORCEMENT

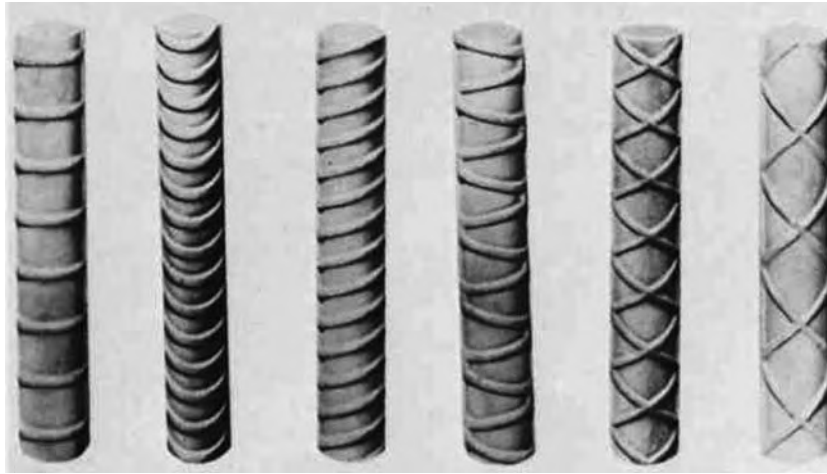
Steel reinforcement may consist of bars, welded wire reinforcement, wires, or discrete fibers.

### Deformed Bars

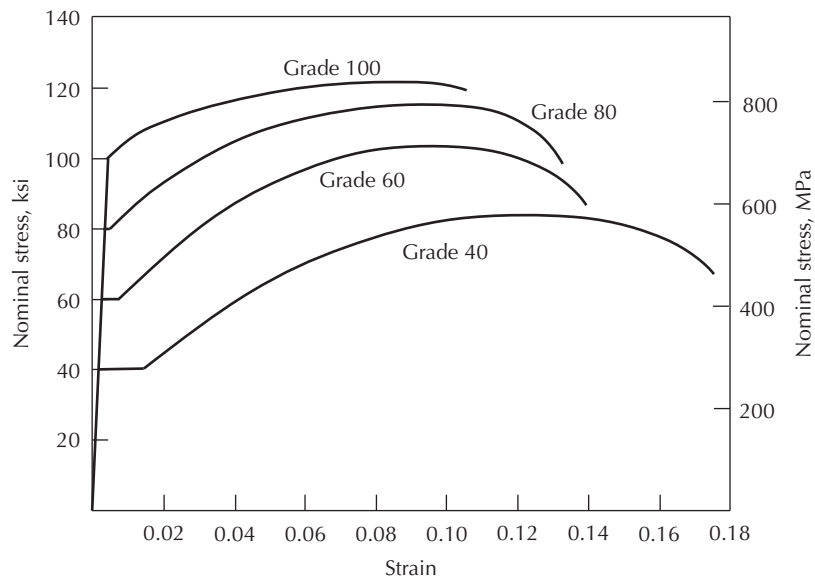
For usual construction, bars (called *deformed bars*) having lugs or protrusions (*deformations*) are used (Fig. 1.13.1). Such deformations serve to deter slip of the bar relative to the concrete that surrounds it resulting from tension or compression in the bar. These deformations can have different patterns depending on the bar producer (see Fig. 1.13.1), but they all have to meet minimum requirements of spacing, height, and gap according to ASTM standards. These deformed bars are available in the United States in sizes of  $\frac{3}{8}$  to  $2\frac{1}{4}$  in. (9.5–57 mm) nominal diameter.

Sizes of ASTM bars (in Inch-Pound units) are indicated by numbers (see Table 1.13.1). For sizes #3 through #8, they are based on the number of eighths of an inch included in the nominal diameter of the bars. Bars designated #9 through #11 are round bars corresponding

<sup>9</sup> According to ACI 301-4.2.2.7a, a strength test is the "average of at least two  $6 \times 12$  in. cylinders or the average of at least three  $4 \times 8$  in. cylinders made from the same concrete sample."



**Figure 1.13.1** Deformed reinforcing bars. (Courtesy of Concrete Reinforcing Steel Institute.)



**Figure 1.13.2** Typical stress-strain curves for Grade 40, 60, 80, and 100 reinforcing bars in tension.

to the former 1 in. square,  $1\frac{1}{8}$  in. square, and  $1\frac{1}{4}$  in. square sizes, and bars designated #14 and #18 are round bars having cross-sectional areas equal to those of  $1\frac{1}{2}$  and 2 in. square sizes, respectively. The nominal diameter of a deformed bar is equivalent to the diameter of a plain bar having the same weight per foot as the deformed bar. For metric units, ASTM standards use “soft” conversion, as given in Table 1.13.2.

Reinforcing bar steel in the United States is covered under ASTM designations as shown in Table 1.13.3 [1.88–1.92]. The “Grade” of steel is the minimum specified yield stress<sup>10</sup> expressed in ksi for Inch-Pound reinforcing bar Grades 40, 50, 60, 75, 80, 100, and 120 and in MPa for SI reinforcing bar Grades 280, 350, 420, 520, 550, 690, and 830. Both Grades 40 and 60 exhibit the well-defined yield point and elastic-plastic strain behavior shown in Fig. 1.13.2. Higher grade steels, on the other hand, often exhibit little or no yield plateau and lower ductility compared to Grade 40 and 60 steels.

<sup>10</sup> The term *yield stress* refers either to *yield point*, the well-defined deviation from perfect elasticity, or to *yield strength*, the value obtained by a 0.2% offset strain for material having no well-defined yield point (ACI-20.2.1.2).

**TABLE 1.13.1** ASTM STANDARD REINFORCING BAR DIMENSIONS AND WEIGHTS (BARS IN INCH-POUND UNITS)

Bar Number	Nominal Dimensions				Weight	
	Diameter		Area			
	(in.)	(mm)	(sq in.)	(cm²)	(lb/ft)	(kg/m)
3	0.375	9.5	0.11	0.71	0.376	0.559
4	0.500	12.7	0.20	1.29	0.668	0.994
5	0.625	15.9	0.31	2.00	1.043	1.552
6	0.750	19.1	0.44	2.84	1.502	2.235
7	0.875	22.2	0.60	3.87	2.044	3.041
8	1.000	25.4	0.79	5.10	2.670	3.973
9	1.128	28.7	1.00	6.45	3.400	5.059
10	1.270	32.3	1.27	8.19	4.303	6.403
11	1.410	35.8	1.56	10.06	5.313	7.906
14	1.693	43.0	2.25	14.52	7.65	11.38
18	2.257	57.3	4.00	25.81	13.60	20.24

**TABLE 1.13.2** 1996 ASTM ("SOFT" METRIC) REINFORCING BAR DIMENSIONS AND WEIGHTS IN SI UNITS

Metric Bar Number	Inch-Pound Bar Number	Diameter (mm)	Mass (kg/m)	Area (mm <sup>2</sup> )
10	3	9.5	0.560	71
13	4	12.7	0.994	129
16	5	15.9	1.552	199
19	6	19.1	2.235	284
22	7	22.2	3.042	387
25	8	25.4	3.973	510
29	9	28.7	5.060	645
32	10	32.3	6.404	819
36	11	35.8	7.907	1006
43	14	43.0	11.38	1452
57	18	57.3	20.24	2581

To ensure sufficient ductility for use in earthquake-resistant structures as well as adequate weldability, ASTM A706/A706M [1.91] has more restrictive mechanical and chemical properties than the other types of steel. Minimum elongation measured over a length of 8 in. (200 mm) ranges between 10 and 14% depending on the grade of steel and the bar size. Also, restrictions apply to the ratio between tensile and actual yield strength, as well as to the actual yield strength (see footnote *d* of Table 1.13.3). Deformation requirements for ASTM A615/A615M steel [1.88] are lower than those for ASTM A706/A706M steel, ranging between 6 and 9% for Grades 60 and 80 steel. For Grade 100 steel, ASTM A615/A615M requires a minimum elongation of either 6 or 7% depending on the bar size. Higher deformation requirements apply to smaller bar sizes. No special requirements are specified for enhanced weldability. Axle and rail steel bars, both of which are rarely used now, are rerolled from old axles and rails and are generally less ductile than bars satisfying ASTM A615/A615M.

TABLE 1.13.3 REINFORCING BAR STEELS

ASTM Designation	Grade	Bar Sizes	Minimum Yield Stress, <sup>a</sup> $f_y$		Minimum Tensile Strength, $f_u$	
			ksi	MPa	ksi	MPa
A615/A615M <sup>b</sup> (Carbon steel)	40	#3–#6	40		60	
	60	#3–#20	60		80	
	80	#3–#20	80		100	
	100	#3–#20	100		115	
	280	10–19		280		420
	420	10–64		420		550
	550	10–64		550		690
	690	10–64		690		790
A955/A995M (Stainless steel)	60	#3–#18	60		90	
	75	#3–#18	75		100	
	420	10–57		420		620
	520	10–57		520		690
A996/A996M (Rail steel, <sup>c</sup> axle steel)	40	#3–#8	40		70	
	50	#3–#11	50		80	
	60	#3–#11	60		90	
	280	10–25		280		500
	350	10–36		350		550
	420	10–36		420		620
A706/A706M <sup>d</sup> (Low-alloy steel)	60	#3–#18	60		80	
	80	#3–#18	80		100	
	420	10–57		420		550
	550	10–57		550		690
A1035/A1035M (Low-carbon, chromium steel)	100	#3–#18	100		150	
	120	#3–#18	120		150	
	690	10–57		690		1030
	830	10–57		830		1030

<sup>a</sup> The term *yield stress* refers either to *yield point*, the well-defined deviation from perfect elasticity, or to *yield strength*, the value obtained by a 0.2% offset strain for material having no well-defined yield point (ACI-20.2.1.2).

<sup>b</sup> Metric (SI) specification applies to bars designated numbers 10 through 64.

<sup>c</sup> Although rail steel is no longer considered a practical source of bar reinforcement, its use is still permitted.

<sup>d</sup> In addition to the yield and tensile strength limits given in the table, the tensile strength shall be at least 1.25 times the actual yield strength, where actual yield strength shall not exceed 78 and 98 ksi (540 and 675 MPa) for Grade 60 and Grade 80 steel, respectively.

Grade 60 is the most widely used grade for reinforcing bars. However, higher strength steels are gaining popularity, since the ACI Code allows the use of a design yield strength of up to 100 ksi (690 MPa) for longitudinal reinforcement in structural members other than those of “special, intermediate, and ordinary moment frames resisting earthquake demands.” [ACI Table 20.2.2.4(a)]. Steel with yield strength of 100 ksi (690 MPa), however, is permitted as confinement reinforcement in earthquake-resistant members and spirals in columns, and as longitudinal reinforcement in special structural walls.

Besides the ASTM requirements, ACI Table 20.2.1.3(a) requires a minimum ratio of actual tensile strength to actual yield strength for ASTM A615/A615M steel of 1.10. At the time of the publication of the ACI 318-19 Code, ASTM A706/A706M did not include Grade 100 reinforcement. However, it was the intention of the ACI Code to allow the use of ASTM A706/A706M Grade 100 steel once it is included in the ASTM standard, with the following requirements [ACI Table 20.2.1.3(b) and (c)]: minimum and maximum yield strength of 100 and 118 ksi, respectively; minimum ratio of actual tensile strength to actual yield strength of 1.17; minimum fracture elongation in 8 in. of 10%; and minimum uniform elongation of 6%.

## Wire Reinforcement

*Welded wire reinforcement* is used in thin slabs, thin shells, thin webs of T-beams, and other locations where available space would not permit the placement of deformed bars with proper cover and clearance. Welded wire reinforcement shall conform to either ASTM A1064 [1.93] for carbon steel or ASTM A1022 [1.94] for stainless steel. Welded wire reinforcement consists of cold-worked wire, cold-rolled or hot-rolled from steel rod, in orthogonal patterns, square or rectangular, and resistance welded at all intersections. The wires may be smooth or deformed. The wire is specified by the symbol W (for smooth wires) or D (for deformed wires), followed by a number representing the cross-sectional area in hundredths of a square inch, varying from 1.5 to 45. On design drawings such reinforcement usually is indicated by the spacings of wires in the two orthogonal directions, followed by the type and wire sizes. Thus,  $6 \times 8$ —W5  $\times$  W5 indicates welded wire reinforcement with 6 in. longitudinal wire spacing, 8 in. transverse wire spacing, and both sets of wires smooth and having a cross-sectional area of 0.05 sq in. Unlike most hot-rolled steel bars, the wire used in welded wire reinforcement *does not* generally have a well-defined yield point and is less ductile. Figure 1.13.3 shows typical stress-strain curves for welded wire reinforcement. Additional information about welded wire reinforcement is available from the Wire Reinforcement Institute [1.95].

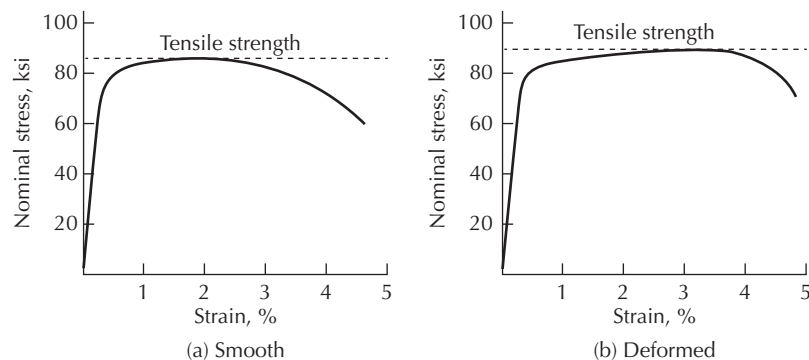
*Wires* in the form of individual wires conforming to either ASTM A1064/A1064M or A1022/A1022M can also be used as reinforcement for certain purposes. When deformed, wires can be used for confinement and as spiral reinforcement, among other uses. Plain wires, on the other hand, can be used only in the form of spirals.

## Prestressing Reinforcement

Wire reinforcement in the form of groups of wires forming *strands* or as individual wires are used for prestressing concrete. Wire and strands are available in great variety. The most prevalent strand is the 7-wire, low-relaxation strand conforming to ASTM A416/A416M [1.96]. These strands have a center wire enclosed by six helically wound outside wires (see Fig. 1.13.4). Usual nominal diameters for 7-wire strand are  $\frac{1}{4}$ ,  $\frac{3}{8}$ , and  $\frac{1}{2}$  in. The minimum tensile strength for strands of Grade 250 is 250,000 psi (1725 MPa), and that of Grade 270 is 270,000 psi (1860 MPa). There is no well-defined yield point. A typical stress-strain curve for stress-relieved strands is shown in Fig. 1.13.5. ASTM A416 requires that the yield strength, measured at a 1% extension under load, should be at least 90% of the tensile strength. Typically, under service conditions, these prestressed strands have a stress of 150,000 to 160,000 psi (1030–1100 MPa).

Additionally, for prestressing, stress-relieved and low-relaxation wire under ASTM A421 [1.97] and high-strength steel bars under ASTM A722 [1.98] are used.

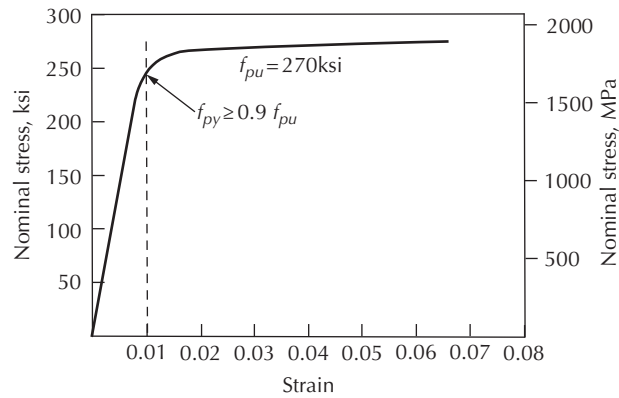
The modulus of elasticity,  $E_s$ , for all nonprestressed steel is permitted to be taken (ACI-20.2.2.2) as 29,000,000 psi (200,000 MPa). For prestressing steel, the modulus of elasticity is lower and more variable; therefore, it must be obtained from the manufacturer or from tests. A value of 27,000,000 psi (186,000 MPa) is often used for 7-wire strands conforming to ASTM A416 [1.96].



**Figure 1.13.3** Typical stress-strain curves for welded wire reinforcement.



**Figure 1.13.4** Typical 7-wire strand used in prestressed concrete construction (Photo by José A. Pincheira.)



**Figure 1.13.5** Typical stress-strain curve for 270-ksi, low-relaxation strands.

## Coated Reinforcement

Corrosion of the steel reinforcement can occur when the structure is subjected to severe environmental conditions, such as in structures exposed to marine environments or in bridge decks or parking garages subjected to deicing salts. The corrosion of a reinforcing bar embedded in concrete is a slow process that eventually will lead to cracking and spalling of the concrete cover. The repair of corrosion-induced damage is often expensive and difficult. A proper concrete cover can effectively delay the corrosion of steel bars, and it is generally agreed that larger covers will provide better protection. In severe environments, however, large concrete covers alone will not be effective.

A common method to prevent or ameliorate corrosion of the reinforcement in concrete structures is the use of epoxy-coated reinforcing bars. The surface of these bars is protected with a thin coat of epoxy (between 7 and 12 mils) to isolate the steel from the oxygen, moisture, and chlorides that will induce corrosion. Although the performance of epoxy-coated bars has been the subject of some controversy in the past, many studies have shown that epoxy-coated bars can effectively reduce corrosion of the reinforcement and extend the service life of concrete structures. Care must be exercised during transportation, handling, storage, and placing of the bars to prevent damage to the coating. Current practice requires that any damage to the coating (cracking, nicks, and cuts) be repaired before bar placement in the forms. The manufacturing requirements of these bars are presently covered by ASTM A775, *Standard Specification for Epoxy-Coated Reinforcing Steel Bars* [1.99], and ASTM A934, *Standard Specification for Epoxy-Coated Prefabricated Steel Reinforcing Bars* [1.100]. It must be noted that epoxy coating of bars will result in reduced bond strength. The ACI Code contains specific provisions that account for the reduced ability of an epoxy-coated bar to transfer the force in the reinforcement to the surrounding concrete. These provisions are discussed in detail in Chapter 6.

Zinc-coated (galvanized) bars are sometimes specified to reduce corrosion of steel reinforcement. Similar to epoxy-coated bars, galvanized bars are protected with a thin layer of zinc on the surface. Although zinc coating will protect the steel bar, zinc will corrode in concrete [1.101], and eventually, corrosion of the steel bar will also occur. While the use of galvanized bars can delay the onset of concrete spalling, it will not significantly extend service life in a severe chloride environment [1.102]. In this case, the use of zinc and epoxy dual-coated bars would be more advantageous. A layer of zinc alloy is first applied, followed by a layer of epoxy. Requirements for the manufacture of galvanized bars are given in ASTM A767, *Specification for Zinc-Coated (Galvanized) Steel Bars for Concrete Reinforcement* [1.102] and in ASTM A1055/A1055M, *Standard Specification for Zinc and Epoxy Dual-Coated Steel Reinforcing Bars* [1.103] for zinc and epoxy dual-coated bars.