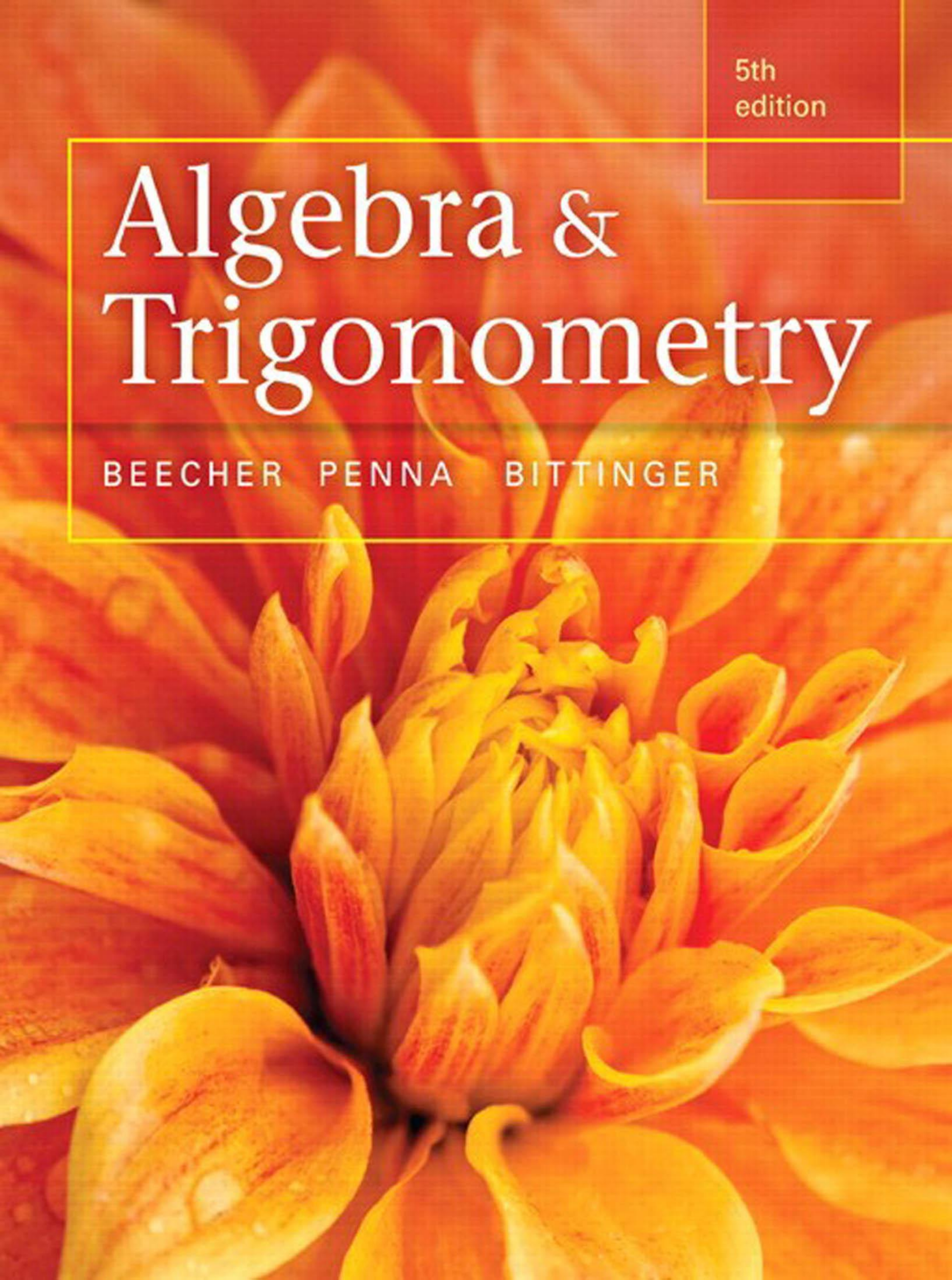
5th edition

Algebra & Trigonometry

BEECHER PENNA BITTINGER



Achieve Your Potential

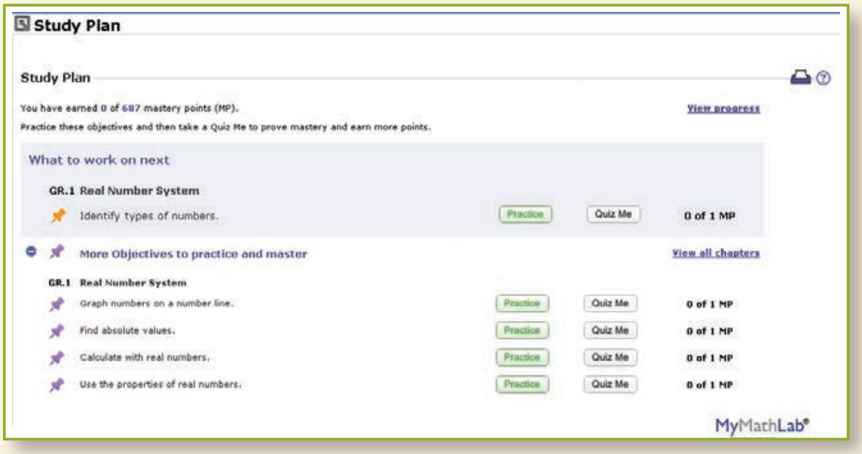
The authors have developed specific content in MyMathLab® to ensure you have many resources to help you achieve success in mathematics - and beyond! The MyMathLab features described here will help you:

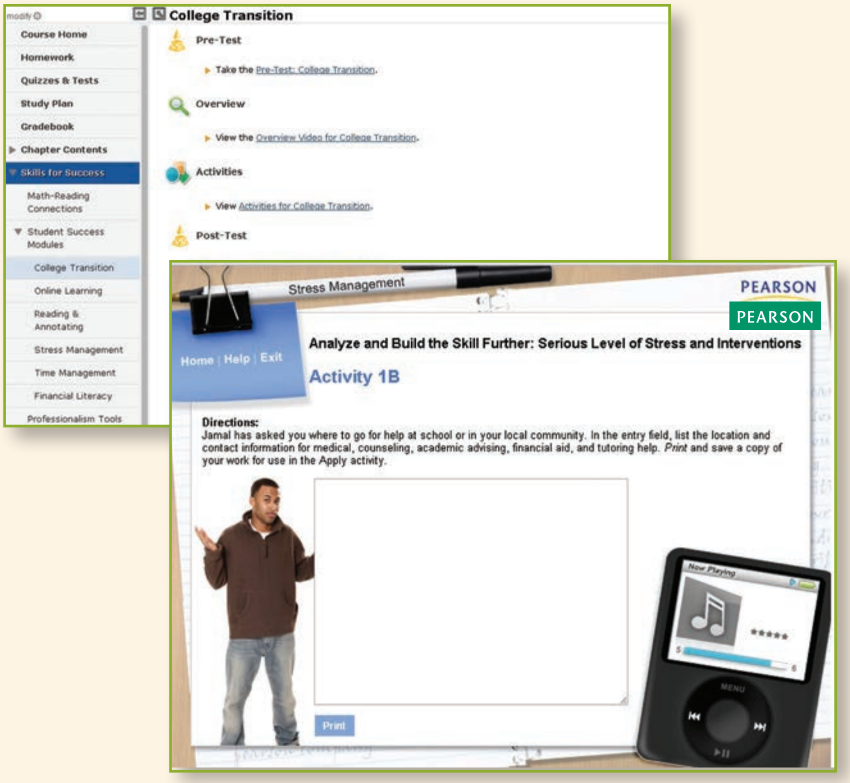
- **Review** math skills and concepts you may have forgotten
- **Retain** new concepts as you move through your math course
- **Develop** skills that will help with your transition to college

Adaptive Study Plan

The Study Plan will help you study more efficiently and effectively.

Your performance and activity are assessed continually in real time, providing a personalized experience based on your individual needs.





Skills for Success

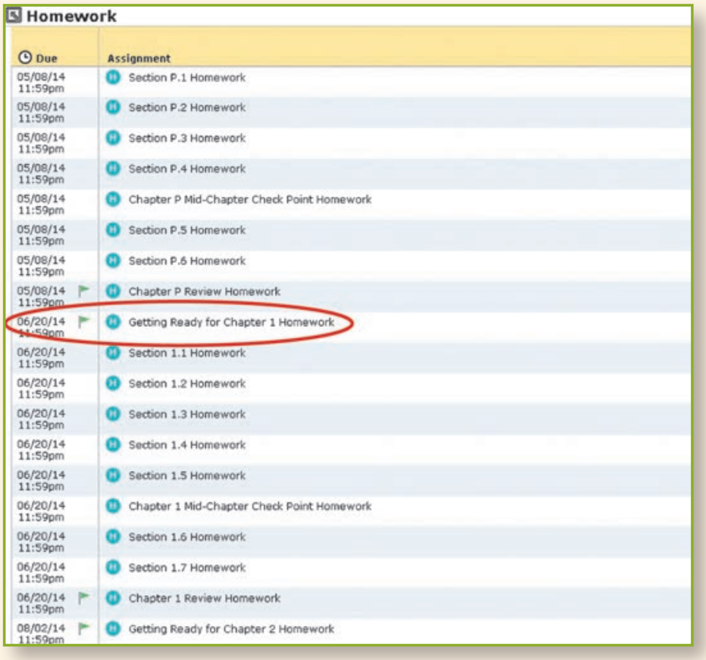
The Skills for Success Modules support your continued success in college. These modules provide tutorials and guidance on a variety of topics, including transitioning to college, online learning, time management, and more.

Additional content is provided to help with the development of professional skills such as resume writing and interview preparation.

Getting Ready

Are you frustrated when you know you learned a math concept in the past, but you can't quite remember the skill when it's time to use it? Don't worry!

The authors have included Getting Ready material so you can brush up on forgotten material efficiently by taking a quick skill review quiz to pinpoint the areas where you need help.



Then, a personalized homework assignment provides additional practice on those forgotten concepts, right when you need it.

▶ Skill Maintenance

Simplify. [3.1]

77. $(1 - 4i)(7 + 6i)$ **78.** $\dfrac{2 - i}{3 + i}$

Find the x-intercepts and the zeros of the function.

79. $f(x) = 2x^2 - 13x - 7$ [3.2]

80. $h(x) = x^3 - 3x^2 + 3x - 1$ [4.4]

81. $h(x) = x^4 - x^2$ [4.1]

82. $g(x) = x^3 + x^2 - 12x$ [4.1]

Solve.

83. $x^3 + 6x^2 - 16x = 0$ [4.1]

84. $3x^2 - 6 = 5x$ [3.2]

Skill Maintenance

As you work through your math course, these MyMathLab® assignments support ongoing review to help you maintain essential skills.

The ability to recall important math concepts as you continually acquire new mathematical skills will help you be successful in this math course and in your future math courses.

EDITION

5

Algebra & Trigonometry

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Contents

Preface

This *Algebra and Trigonometry* textbook is known for enabling students to "see the math" through its

- focus on visualization,
- early introduction of functions,
- complete, optional technology coverage, and
- connections between math concepts and the real world.

New!

With the new edition, we continue to innovate by positioning the review material as a more effective tool for teachers and students. Chapter R from the previous edition has been condensed into 25 Just-In-Time review topics that are placed at the back of the book. This new review feature is designed to give each student the opportunity to be successful in this course by providing a quick review of topics from intermediate algebra that will be built upon in new college algebra topics. The review can be used in an individualized instruction format since some students will require more review than others. Treating the review in this manner will allow more time to cover the college algebra topics in the syllabus.

On the other hand, some instructors might choose to review some or all of the topics with the entire class at the beginning of the course or in a just-in-time format as each is needed. We think instructors will appreciate the flexibility that the Just-In-Time feature offers.

Additional resources in the MyMathLab courses reflect the themes of just-in-time review and concept retention. For example, new Cumulative Review assignments allow students to synthesize and retain concepts learned throughout the course.

Our overarching goal is to provide students with a learning experience that will not only lead to success in this course, but also prepare them to be successful in the mathematics courses they take in the future.

Content Changes to the Fifth Edition

JUST IN TIME 10

- **Just-In-Time Review** Review of prerequisite algebra topics is now presented when students need it most.
  - A set of 25 numbered, short review topics creates an efficient review of intermediate algebra topics.
  - This feature is placed at the back of the text. Just-In-Time icons are positioned throughout the text next to the example where review of an intermediate algebra topic would be helpful.
- **Informed Exercises** We have analyzed the MyMathLab usage data, which has helped us revise our exercises for this new edition. The goal is to ultimately improve the quality and quantity of exercises that matter the most to instructors and students.
- **Symmetry and Transformations** These topics are now presented in two sections rather than one.

Emphasis on Functions

Functions are the core of this course and are presented as a thread that runs throughout the course rather than as an isolated topic. We introduce functions in Chapter 1, whereas many traditional college algebra textbooks cover equation-solving in Chapter 1. Our approach of introducing students to a relatively new concept at the beginning of the course, rather than requiring them to begin with a review of material that was previously covered in intermediate algebra, immediately engages them and serves to help them avoid the temptation to neglect studying early in the course because "I already know this."

The concept of a function can be challenging for students. By repeatedly exposing them to the language, notation, and use of functions, demonstrating visually how functions relate to equations and graphs, and also showing how functions can be used to model real data, we hope to ensure that students not only become comfortable with functions but also come to understand and appreciate them. You will see this emphasis on functions woven throughout the other themes that follow.

Classify the Function Exercises With a focus on conceptual understanding, students are asked periodically to identify a number of functions by their type (linear, quadratic, rational, and so on). As students progress through the text, the variety of functions with which they are familiar increases and these exercises become more challenging. The "classifying the function" exercises appear with the review exercises in the Skill Maintenance portion of an exercise set. (See pp. 266 and 356.)

Visual Emphasis

Our early introduction of functions allows graphs to be used to provide a visual aspect to solving equations and inequalities. For example, we are able to show students both algebraically and visually that the solutions of a quadratic equation $ax^2 + bx + c = 0$ are the zeros of the quadratic function $f(x) = ax^2 + bx + c$, as well as the first coordinates of the x-intercepts of the graph of that function. This makes it possible for students, particularly visual learners, to gain a quick understanding of these concepts. (See pp. 182, 185, 227, 285, and 344.)

Visualizing the Graph Appearing at least once in every chapter, this feature provides students with an opportunity to match an equation with its graph by focusing on the characteristics of the equation and the corresponding attributes of the graph. (See pp. 143, 198, and 280.) In addition to this full-page feature, many of the exercise sets include exercises in which the student is asked to match an equation with its graph or to find an equation of a function from its graph. (See pp. 145, 146, 236, and 330.) In MyMathLab, animated Visualizing the Graph features for each chapter allow students to interact with graphs on an entirely new level.

Side-by-Side Examples Many examples are presented in a side-by-side, two-column format in which the algebraic solution of an equation appears in the left column and a graphical solution appears in the right column. (See pp. 176, 290–291, 360, and 361.) This enables students to visualize and comprehend the connections among the solutions of an equation, the zeros of a function, and the x-intercepts of the graph of a function.

Technology Connections This feature appears throughout the text to demonstrate how a graphing calculator can be used to solve problems. The technology is set apart from the traditional exposition so that it does not intrude if no technology is desired. Although students might not be using graphing calculators, the graphing calculator windows that appear in the Technology Connection features enhance the visual element of the text, providing graphical interpretations of solutions of equations, zeros of functions, and x-intercepts of graphs of functions. (See pp. 21, 181, and 360.) A graphing calculator manual providing keystroke-level instruction, written by author Judy Penna, is available online.

Making Connections

Zeros, Solutions, and x-Intercepts We find that when students understand the connections among the real zeros of a function, the solutions of its associated equation, and the first coordinates of the x-intercepts of its graph, a door opens to a new level of mathematical comprehension that increases the probability of success in this course. We emphasize zeros, solutions, and x-intercepts throughout the text by using consistent, precise terminology and including exceptional graphics. Seeing this theme repeated in different contexts leads to a better understanding and retention of these concepts. (See pp. 176 and 185.)

Connecting the Concepts This feature highlights the importance of connecting concepts. When students are presented with concepts in visual form—using graphs, an outline, or a chart—rather than merely in paragraphs of text, comprehension is streamlined and retention is enhanced. The visual aspect of this feature invites students to stop and check their understanding of how concepts work together in one section or in several sections. This check in turn enhances student performance on homework assignments and exams. (See pp. 73, 185, and 253.)

Annotated Examples We have included over 1070 annotated examples designed to fully prepare the student to work the exercises. Learning is carefully guided with the use of numerous color-coded art pieces and step-by-step annotations. Substitutions and annotations are highlighted in red for emphasis. (See pp. 179 and 352.)

Now Try Exercises Now Try Exercises are found after nearly every example. This feature encourages active learning by asking students to do an exercise in the exercise set that is similar to the example the student has just read. (See pp. 182, 272, and 328.)

Synthesis Exercises These exercises appear at the end of each exercise set and encourage critical thinking by requiring students to synthesize concepts from several sections or to take a concept a step further than in the general exercises. For the Fifth Edition, these exercises are assignable in MyMathLab. (See pp. 32, 255, 333, and 380.)

Real-Data Applications We encourage students to see and interpret the mathematics that appears every day in the world around them. Throughout the writing process, we conducted an energetic search for real-data applications, and the result is a variety of examples and exercises that connect the mathematical content with everyday life. Most of these applications feature source lines and many include charts and graphs. Many are drawn from the fields of health, business and economics, life and physical sciences, social science, and areas of general interest such as sports and travel. (See pp. 39 ("Food Stamp Program"), 66 ("Words in Languages"), 133 ("Peace Corps Volunteers"), 187 ("Funding for Afghan Security"), 236 ("Vinyl Album Sales"), 331 ("Alternative-Fuel Vehicles"), 559 ("Vietnam Veterans Memorial"), 648 ("Cosmetic Surgery"), 657 ("Top Auction Art Sales"), 736 ("The Ellipse at the White House"), and 812 ("The Economic Multiplier; Super Bowl XLVII").)

Ongoing Review

The most significant change to the Fifth Edition is the new Just-in-Time Review feature, designed to provide students with efficient and effective review of basic algebra skills.

JUST IN TIME 10

New! Just-In-Time Review Chapter R has been condensed into 25 numbered, short review topics to create an efficient review of intermediate algebra topics. This feature is placed at the back of the book.

- Just-In-Time icons are placed throughout the text next to the example where review of an intermediate algebra topic would be helpful. (See pp. 35, 99, 115, 171, 232, and 319.)
- The coverage of each topic contains worked-out examples and a short exercise set. Answers to all exercises appear at the back of the book.
- Worked-out solutions to all exercises are included in the *Student Solutions Manual*.
- Students can find additional review support in the MyMathLab course for College Algebra with Integrated Review and in the Getting Ready MyMathLab.

Mid-Chapter Mixed Review This review reinforces understanding of the mathematical concepts and skills covered in the first half of the chapter before students move on to new material in the second half of the chapter. Each review begins with at least three true/false exercises that require students to consider the concepts they have studied and also contains exercises that drill the skills from all prior sections of the chapter. They are available as assignments in MyMathLab. (See pp. 125–126 and 256–257.)

Collaborative Discussion and Writing Exercises appear in the Mid-Chapter Mixed Review as well. These exercises can be discussed in small groups or by the class as a whole to encourage students to talk about the key mathematical concepts in the chapter. They can also be assigned to individual students to give them an opportunity to write about mathematics. (See pp. 202 and 257.)

A section reference is provided for each exercise in the Mid-Chapter Mixed Review. This tells the student which section to refer to if help is needed to work the exercise. Answers to all exercises in the Mid-Chapter Mixed Review are given at the back of the book.

Study Guide This feature is found at the beginning of the **Summary and Review** near the end of each chapter. Presented in a two-column format and organized by section, this feature gives key concepts and terms in the left column and a worked-out example in the right column. It provides students with a concise and effective review of the chapter that is a solid basis for studying for a test. In MyMathLab, these Study Guides are accompanied by narrated examples to reinforce the key concepts and ideas. (See pp. 214–220 and 381–387.)

Exercise Sets There are over 7060 exercises in this text. The exercise sets are enhanced with real-data applications and source lines, detailed art pieces, tables, graphs, and photographs. In addition to the exercises that provide students with concepts presented in the section, the exercise sets feature the following elements to provide ongoing review of topics presented earlier:

- **Skill Maintenance Exercises.** These exercises provide an ongoing review of concepts previously presented in the course, enhancing students' retention of these concepts. These exercises include **Vocabulary Reinforcement**, described next, and **Classifying the Function** exercises, described earlier in the section "Emphasis on Functions." A section reference is provided for each exercise. This tells the student which section to refer to if help is needed to work the exercise. Answers to all Skill Maintenance exercises appear in the answer section at the back of the book. (See pp. 133, 210, 283, and 347.)
- **Enhanced Vocabulary Reinforcement Exercises.** This feature checks and reviews students' understanding of the vocabulary introduced throughout the text. It appears once in every chapter, in the Skill Maintenance portion of an exercise set, and is intended to provide a continuing review of the terms that students must know in order to be able to communicate effectively in the language of mathematics. (See pp. 84, 154, 214, and 283.)
- **Enhanced Synthesis Exercises.** These exercises are described under the Making Connections heading and are also assignable in MyMathLab.

Review Exercises These exercises in the **Summary and Review** supplement the Study Guide by providing a thorough and comprehensive review of the skills taught in the chapter. A group of true/false exercises appears first, followed by a large number of exercises that drill the skills and concepts taught in the chapter. In addition, three multiple-choice exercises, one of which involves identifying the graph of a function, are included in the Review Exercises for every chapter. Each review exercise is accompanied by a section reference that, as in the Mid-Chapter Mixed Review, directs students to the section in which the material being reviewed can be found. Collaborative Discussion and Writing exercises are also included. These exercises are described under the Mid-Chapter Mixed Review heading on p. xiv. (See pp. 220–223 and 388–390.)

Chapter Test The test at the end of each chapter allows students to test themselves and target areas that need further study before taking the in-class test. Each Chapter Test includes a multiple-choice exercise involving identifying the graph of a function. Answers to all questions in the Chapter Tests appear in the answer section at the back of the book, along with corresponding section references. (See pp. 223–224 and 391–392.)

DOMAIN
REVIEW SECTION 1.2

Review Icons Placed next to the concept that a student is currently studying, a review icon references a section of the text in which the student can find and review topics on which the current concept is built. (See pp. 267 and 324.)

Acknowledgments

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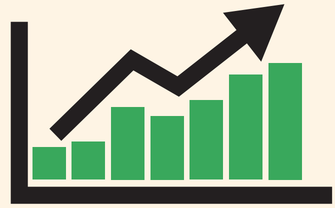
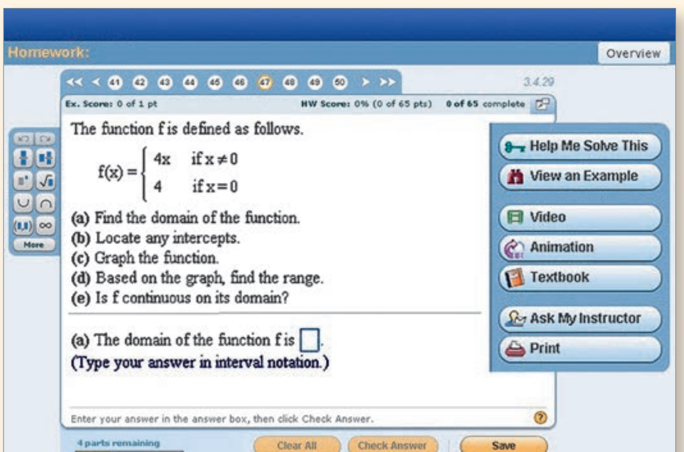
Corwin Stanford, *The University of Southern Mississippi*

J.A.B.
J.A.P.
M.L.B.

Get the most out of MyMathLab®



MyMathLab creates personalized experiences to help each student achieve success and provides powerful tools so instructors can create the perfect learning experiences for their courses.



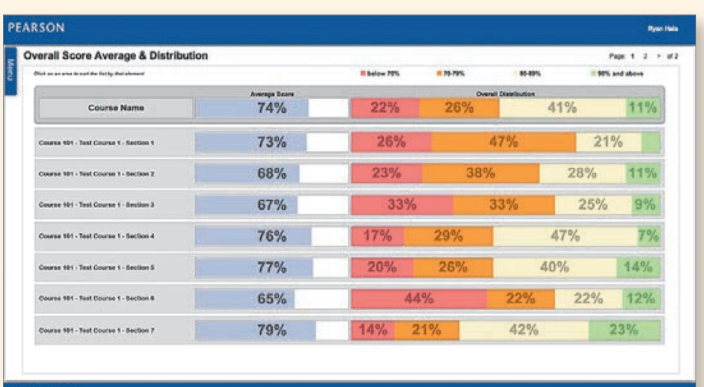
Exercise with feedback and learning aids

Personalized Support *for* Students

- MyMathLab comes with many learning resources—eText, animations, videos, and more—all designed to support you as you complete your assignments.
- Whether you're doing homework or working from the adaptive study plan, you'll receive immediate feedback, so you'll know exactly where you need help.

Data-Driven Reporting *for* Instructors

- MyMathLab's comprehensive online gradebook automatically tracks students' results on tests, quizzes, homework, and in the study plan.
- The Reporting Dashboard makes it easier than ever to identify topics where students are struggling or specific students who may need extra help.



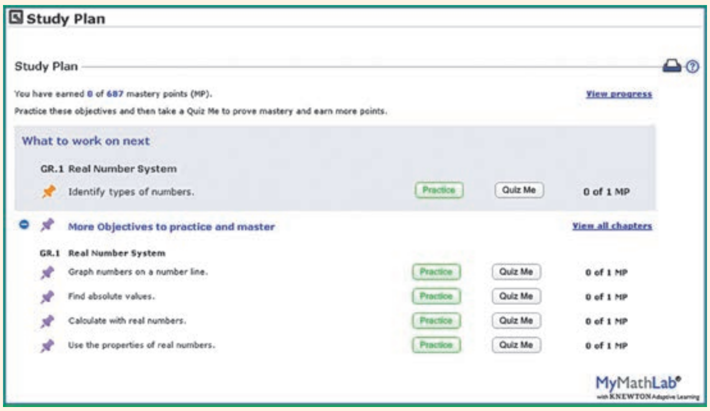
Dashboard

www.mymathlab.com

Resources for Success

MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And, it comes from an **experienced partner** with educational expertise and an eye on the future. MyMathLab helps prepare students and gets them thinking more conceptually and visually through the following features:

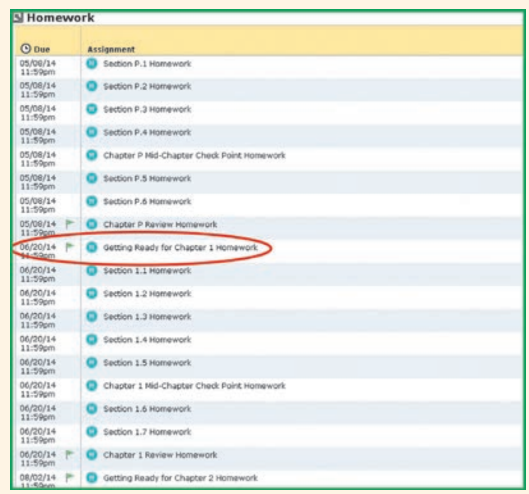


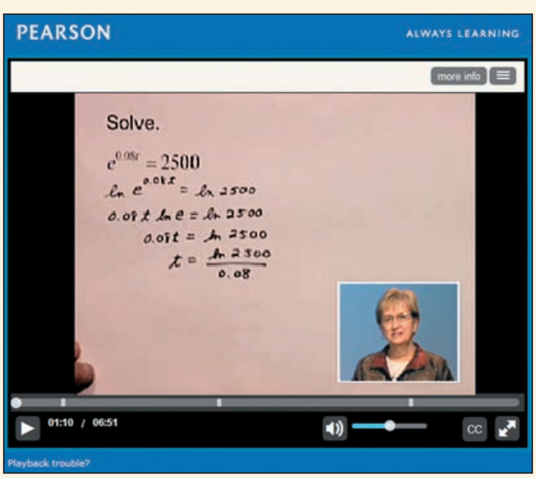
◄ Adaptive Study Plan

The Study Plan makes studying more efficient and effective for every student. Performance and activity are assessed continually in real time. The data and analytics are used to provide personalized content–reinforcing concepts that target each student's strengths and weaknesses.

Getting Ready ►

Students refresh prerequisite topics through assignable skill review quizzes and personalized homework integrated within MyMathLab.



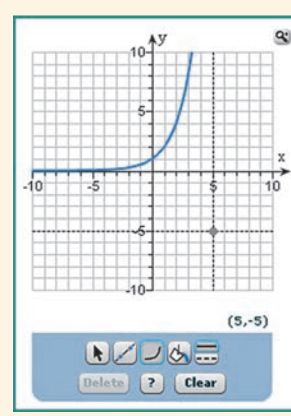


◄ Video Assessment

Video assessment is tied to key author example videos to check students' conceptual understanding of important math concepts.

Enhanced Graphing Functionality ►

New functionality within the graphing utility allows graphing of 3-point quadratic functions, 4-point cubic graphs, and transformations in exercises.



Skills for Success Modules are integrated within the MyMathLab course to help students succeed in collegiate courses and prepare for future professions.

Skill Maintenance These exercises support ongoing review at the course level and help students maintain essential skills.

Instructor Resources

Additional resources can be downloaded from **www.pearsonhighered.com** or hardcopy resources can be ordered from your sales representative.

Ready to Go MyMathLab® Course

Now it is even easier to get started with **MyMathLab**. The Ready to Go **MyMathLab** course option includes author-chosen preassigned homework, integrated review, and more.

TestGen®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides

Feature presentations written and designed specifically for this text. These lecture slides provide an outline for presenting definitions, figures, and key examples from the text.

Annotated Instructor's Edition

Includes all answers to the exercise sets, usually on the page on which the exercises appear. Sample homework assignments are indicated by a blue underline within each end-of-section exercise set and may be assigned in MyMathLab.

Instructor's Solutions Manual (Download Only)

Written by Judy Penna, this resource contains worked-out solutions to all exercises in the exercise sets, Mid-Chapter Mixed Reviews, Chapter Reviews, and Chapter Tests, as well as solutions for all the Just-In-Time exercises.

Online Test Bank (Download Only)

Contains four free-response text forms for each chapter following the same format and having the same level of difficulty as the test in the main text and two multiple-choice test forms for each chapter. It also provides six forms of the final examination, four with free-response questions and two with multiple-choice questions.

Student Resources

Additional resources to help student success.

Author Example Videos

Ideal for distance learning or supplemental instruction, these videos feature authors Judy Beecher and Judy Penna working through and explaining examples in the text. Assignable in MyMathLab with new Video Assessment questions.

New! Video Notebook

The new Video Notebook contains fill-in-the-blank worksheets to accompany the video examples presented by the authors. Key definitions, theorems, and procedures are also included. After filling in the worksheet while watching the video, the student has an excellent study guide for review and test preparation. This is available in print or as a PDF or Word document in MyMathLab.

Student's Solutions Manual

Written by author Judy Penna, this resource contains completely worked-out solutions with step-by-step annotations for all the odd-numbered exercises in the exercise sets, Mid-Chapter Mixed Reviews, and Chapter Reviews, as well as solutions for all the Chapter Test exercises and the Just-In-TIme exercises.

Graphing Calculator Manual

Contains keystroke level instruction for the Texas Instruments TI-84 Plus using MathPrint OS. Mirrors the topic order in the main text to provide a just-in-time mode of instruction.

To the Student

GUIDE TO SUCCESS

Success can be planned. Combine goals and good study habits to create a plan for success that works for you. The following list contains study tips that your authors consider most helpful.

Skills for Success

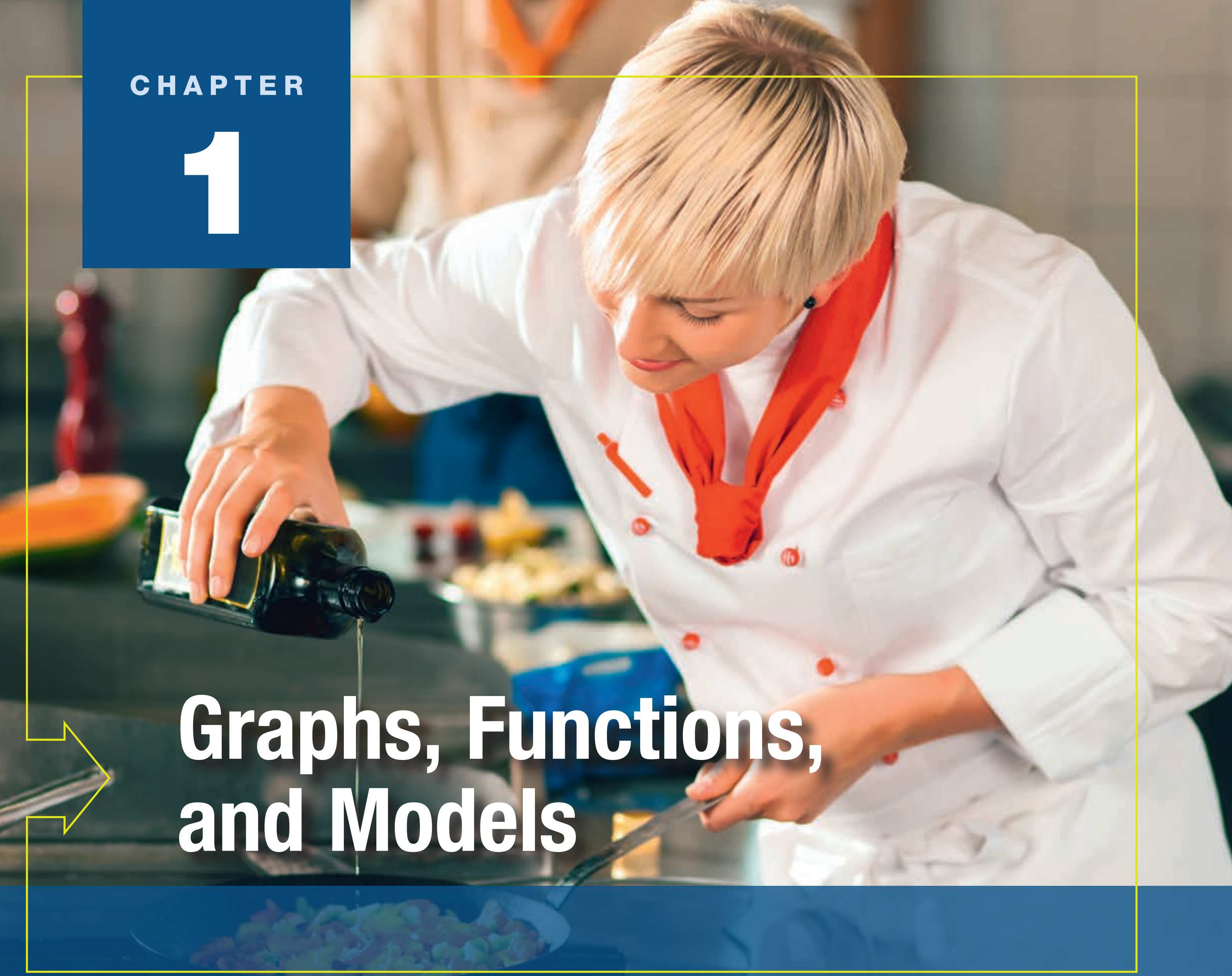
- **Set goals and expect success.** Approach this class experience with a positive attitude.
- **Communicate with your instructor** when you need extra help.
- **Take your text with you to class and lab.** Each section in the text is designed with headings and boxed information that provide an outline for easy reference.
- **Ask questions in class, lab, and tutoring sessions.** Instructors encourage them, and other students probably have the same questions.
- **Begin each homework assignment as soon as possible.** If you have difficulty, you will then have the time to access supplementary resources.
- **Carefully read the instructions** before working homework exercises **and include all steps.**
- **Form a study group** with fellow students. Verbalizing questions about topics that you do not understand can clarify the material for you.
- After each quiz or test, **write out corrected step-by step solutions** to all missed questions. They will provide a valuable study guide for the midterm exam and the final exam.
- **MyMathLab has numerous tools to help you succeed.** Use MyMathLab to create a personalized study plan and practice skills with sample quizzes and tests.
- **Knowing math vocabulary is an important step toward success.** Review vocabulary with Vocabulary Reinforcement exercises in the text and in MyMathLab.
- If you miss a lecture, **watch the video in the Multimedia Library** of MyMathLab that explains the concepts you missed.

In writing this textbook, we challenged ourselves to do everything possible to help you learn the concepts and skills contained between its covers so that you will be successful in this course and in the mathematics courses you take in the future. We realize that your time is both valuable and limited, so we communicate in a highly visual way that allows you to learn quickly and efficiently. We are confident that, if you invest an adequate amount of time in the learning process, this text will be of great value to you. We wish you a positive learning experience.

Judy Beecher
Judy Penna
Marv Bittinger

CHAPTER

1



Graphs, Functions, and Models

APPLICATION This problem appears as Exercise 67 in Exercise Set 1.5.

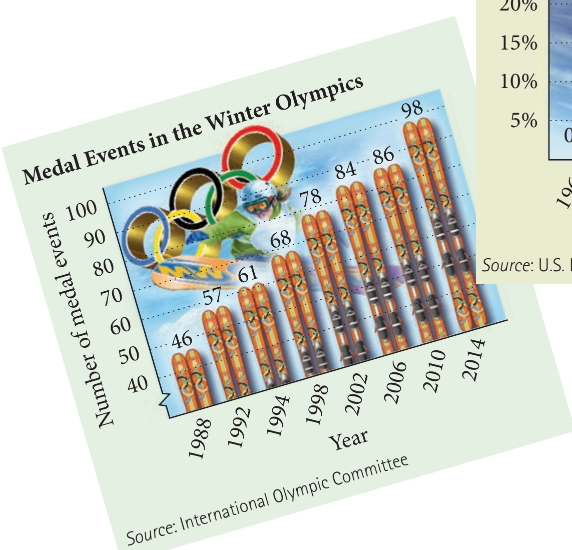
Together, Italy, Spain, and the United States consume 58% of the world's olive oil. The percentage consumed in Italy is $3\frac{3}{4}$ times the percentage consumed in the United States. The percentage consumed in Spain is $\frac{2}{3}$ of the percentage consumed in Italy. (*Source*: www.OliveOilEmporium.com) Find the percent of the world's olive oil consumed in each country.

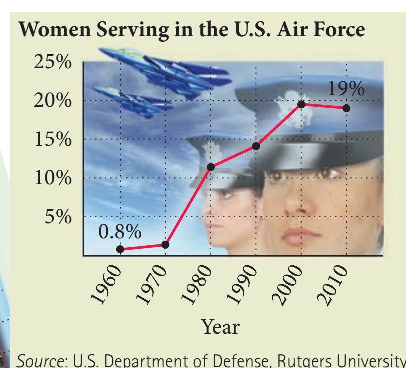
1.1 Introduction to Graphing

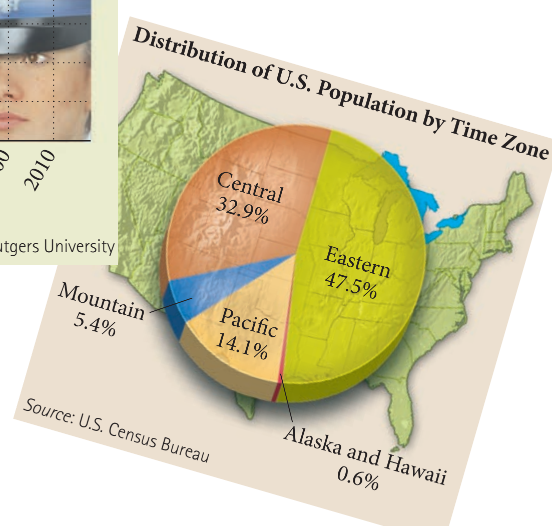
- Plot points.
- Determine whether an ordered pair is a solution of an equation.
- Find the *x*- and *y*-intercepts of an equation of the form $Ax + By = C$.
- Graph equations.
- Find the distance between two points in the plane and find the midpoint of a segment.
- Find an equation of a circle with a given center and radius, and given an equation of a circle in standard form, find the center and the radius.
- Graph equations of circles.

Graphs

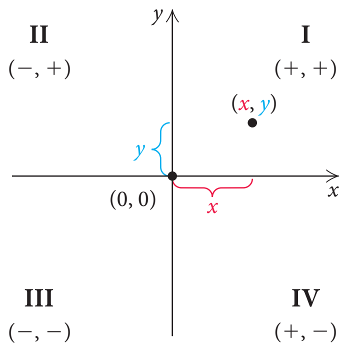
Graphs provide a means of displaying, interpreting, and analyzing data in a visual format. It is not uncommon to open a newspaper or a magazine and encounter graphs. Examples of bar, line, and circle graphs are shown below.







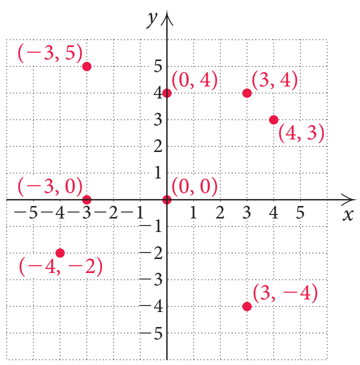
Many real-world situations can be modeled, or described mathematically, using equations in which two variables appear. We use a plane to graph a pair of numbers. To locate points on a plane, we use two perpendicular number lines, called **axes**, that intersect at (0, 0). We call this point the **origin**. The horizontal axis is called the ***x*-axis**, and the vertical axis is called the ***y*-axis**. (Other variables, such as *a* and *b*, can also be used.) The axes divide the plane into four regions,



called **quadrants**, denoted by Roman numerals and numbered counterclockwise from the upper right. Arrows show the positive direction of each axis.

Each point (x, y) in the plane is described by an **ordered pair**. The first number, x, indicates the point's horizontal location with respect to the y-axis, and the second number, y, indicates the point's vertical location with respect to the x-axis. We call x the **first coordinate,** the ***x*-coordinate**, or the **abscissa**. We call y the **second coordinate,** the ***y*-coordinate**, or the **ordinate**. Such a representation is called the **Cartesian coordinate system** in honor of the French mathematician and philosopher René Descartes (1596–1650).

In the first quadrant, both coordinates of a point are positive. In the second quadrant, the first coordinate is negative and the second is positive. In the third quadrant, both coordinates are negative, and in the fourth quadrant, the first coordinate is positive and the second is negative.



EXAMPLE 1 Graph and label the points $(-3, 5)$, $(4, 3)$, $(3, 4)$, $(-4, -2)$, $(3, -4)$, $(0, 4)$, $(-3, 0)$, and $(0, 0)$.

Solution To graph or **plot** $(-3, 5)$, we note that the x-coordinate, -3, tells us to move from the origin 3 units horizontally in the negative direction, or 3 units to the left of the y-axis. Then we move 5 units up from the x-axis.* To graph the other points, we proceed in a similar manner. (See the graph at left.) Note that the point $(4, 3)$ is different from the point $(3, 4)$.

Now Try Exercise 3.

Solutions of Equations

Equations in two variables, like $2x + 3y = 18$, have solutions (x, y) that are ordered pairs such that when the first coordinate is substituted for x and the second coordinate is substituted for y, the result is a true equation. The first coordinate in an ordered pair generally represents the variable that occurs first alphabetically.

EXAMPLE 2 Determine whether each ordered pair is a solution of the equation $2x + 3y = 18$.

a) $(-5, 7)$
b) $(3, 4)$

Solution We substitute the ordered pair into the equation and determine whether the resulting equation is true.

a)

$2x + 3y$	$= 18$	
$2(-5) + 3(7)$	$\overset{?}{=}$ 18	**We substitute −5 for *x* and 7 for *y* (alphabetical order).**
$-10 + 21$		
11	18	FALSE

The equation $11 = 18$ is false, so $(-5, 7)$ is not a solution.

*Here the notation $(-3, 5)$ represents an ordered pair. This notation can also represent an open interval. See Just-In-Time 6 review. The context in which the notation appears usually makes the meaning clear.

b)

$$\begin{array}{r|l} 2x + 3y & = 18 \\ \hline 2(3) + 3(4) & ?\ 18 \\ 6 + 12 & \\ 18 & 18 \quad \text{TRUE} \end{array}$$

We substitute 3 for x and 4 for y.

The equation $18 = 18$ is true, so $(3, 4)$ is a solution.

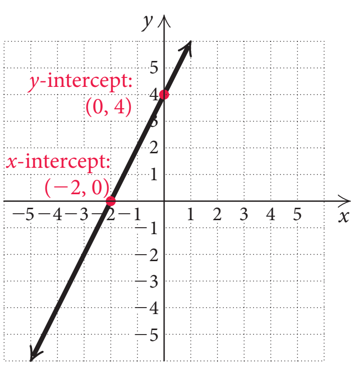
Now Try Exercise 11.

Graphs of Equations

The equation considered in Example 2 actually has an infinite number of solutions. Since we cannot list all the solutions, we will make a drawing, called a **graph**, that represents them. On the following page are some suggestions for drawing graphs.

TO GRAPH AN EQUATION

To **graph an equation** is to make a drawing that represents the solutions of that equation.



Graphs of equations of the type $Ax + By = C$ are straight lines. Many such equations can be graphed conveniently using intercepts. The ***x*-intercept** of the graph of an equation is the point at which the graph crosses the x-axis. The ***y*-intercept** is the point at which the graph crosses the y-axis. We know from geometry that only one line can be drawn through two given points. Thus, if we know the intercepts, we can graph the line. To ensure that a computational error has not been made, it is a good idea to calculate and plot a third point as a check.

***x*- AND *y*-INTERCEPTS**

An ***x*-intercept** is a point $(a, 0)$. To find a, let $y = 0$ and solve for x.

A ***y*-intercept** is a point $(0, b)$. To find b, let $x = 0$ and solve for y.

EXAMPLE 3 Graph: $2x + 3y = 18$.

Solution The graph is a line. To find ordered pairs that are solutions of this equation, we can replace either x or y with any number and then solve for the other variable. In this case, it is convenient to find the intercepts of the graph. For instance, if x is replaced with 0, then

$$\begin{aligned} 2 \cdot 0 + 3y &= 18 \\ 3y &= 18 \\ y &= 6. \end{aligned}$$

Dividing by 3 on both sides

Thus, $(0, 6)$ is a solution. It is the *y-intercept* of the graph. If y is replaced with 0, then

$$2x + 3 \cdot 0 = 18$$
$$2x = 18$$
$$x = 9. \qquad \text{Dividing by 2 on both sides}$$

Thus, $(9, 0)$ is a solution. It is the *x-intercept* of the graph. We find a third solution as a check. If x is replaced with 3, then

$$2 \cdot 3 + 3y = 18$$
$$6 + 3y = 18$$
$$3y = 12 \qquad \text{Subtracting 6 on both sides}$$
$$y = 4. \qquad \text{Dividing by 3 on both sides}$$

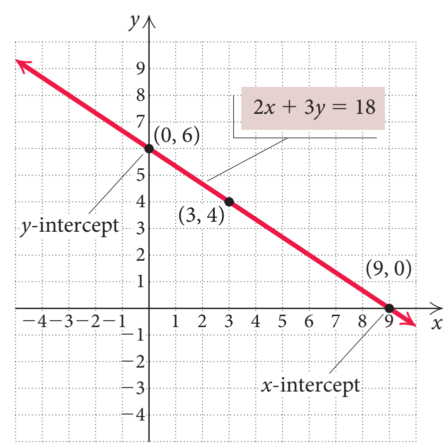
Thus, $(3, 4)$ is a solution.

We list the solutions in a table and then plot the points. Note that the points appear to lie on a straight line.

Suggestions for Drawing Graphs

1. Calculate solutions and list the ordered pairs in a table.
2. Use graph paper.
3. Draw axes and label them with the variables.
4. Use arrows on the axes to indicate positive directions.
5. Scale the axes; that is, label the tick marks on the axes. Consider the ordered pairs found in part (1) above when choosing the scale.
6. Plot the ordered pairs, look for patterns, and complete the graph. Label the graph with the equation being graphed.

x	y	(x, y)
0	6	$(0, 6)$
9	0	$(9, 0)$
3	4	$(3, 4)$



Were we to graph additional solutions of $2x + 3y = 18$, they would be on the same straight line. Thus, to complete the graph, we use a straight-edge to draw a line, as shown in the figure. This line represents all solutions of the equation. Every point on the line represents a solution; every solution is represented by a point on the line.

Now Try Exercise 17.

When graphing some equations, it is convenient to first solve for y and then find ordered pairs. We can use the addition and multiplication principles to solve for y.

JUST IN TIME 14

EXAMPLE 4 Graph: $3x - 5y = -10$.

Solution We first solve for y:

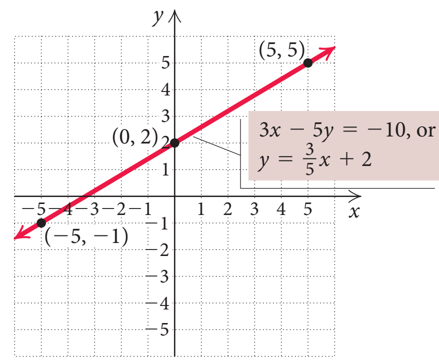
$$3x - 5y = -10$$
$$-5y = -3x - 10 \quad \text{Subtracting } 3x \text{ on both sides}$$
$$y = \tfrac{3}{5}x + 2. \quad \text{Multiplying by } -\tfrac{1}{5} \text{ on both sides}$$

By choosing multiples of 5 for x, we can avoid adding and subtracting fraction values when calculating y. For example, if we choose -5 for x, we get

$$y = \tfrac{3}{5}x + 2 = \tfrac{3}{5}(-5) + 2 = -3 + 2 = -1.$$

The following table lists a few points. We plot the points and draw the graph.

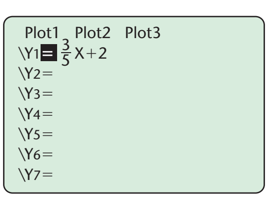
x	y	(x, y)
-5	-1	$(-5, -1)$
0	2	$(0, 2)$
5	5	$(5, 5)$

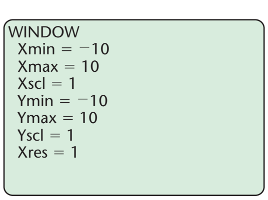


Now Try Exercise 29.

In the equation $y = \frac{3}{5}x + 2$ in Example 4, the value of y *depends* on the value chosen for x, so x is said to be the **independent variable** and y the **dependent variable**.

Technology Connection

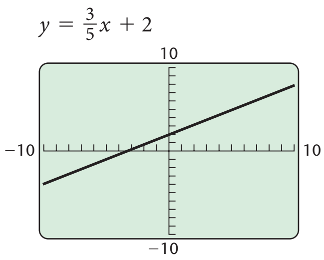




We can graph an equation on a graphing calculator. Many calculators require an equation to be entered in the form "$y =$." In such a case, if the equation is not initially given in this form, it must be solved for y before it is entered in the calculator. For the equation $3x - 5y = -10$ in Example 4, we enter $y = \frac{3}{5}x + 2$ on the equation-editor, or $y =$, screen in the form $y = (3/5)x + 2$, as shown in the window at left.

Next, we determine the portion of the xy-plane that will appear on the calculator's screen. That portion of the plane is called the **viewing window**.

The notation used in this text to denote a window setting consists of four numbers [L, R, B, T], which represent the **L**eft and **R**ight endpoints of the x-axis and the **B**ottom and **T**op endpoints of the y-axis, respectively. The window with the settings $[-10, 10, -10, 10]$ is the **standard viewing window**. On some graphing calculators, the standard window can be selected quickly using the ZSTANDARD feature from the ZOOM menu.



Xmin and Xmax are used to set the left and right endpoints of the x-axis, respectively; Ymin and Ymax are used to set the bottom and top endpoints of the y-axis, respectively. The settings Xscl and Yscl give the scales for the axes. For example, Xscl = 1 and Yscl = 1 means that there is 1 unit between tick marks on each of the axes. In this text, scaling factors other than 1 will be listed by the window unless they are readily apparent.

After entering the equation $y = (3/5)x + 2$ and choosing a viewing window, we can then draw the graph shown at left.

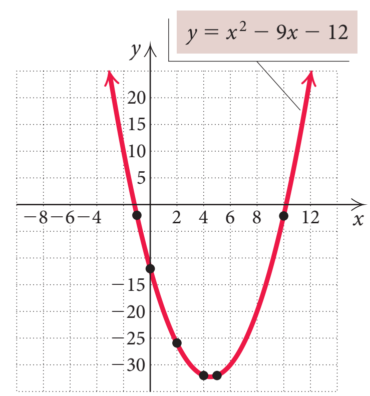
JUST IN TIME 7

EXAMPLE 5 Graph: $y = x^2 - 9x - 12$.

Solution Note that since this equation is not of the form $Ax + By = C$, its graph is not a straight line. We make a table of values, plot enough points to obtain an idea of the shape of the curve, and connect the points with a smooth curve. It is important to scale the axes to include most of the ordered pairs listed in the table. Here it is appropriate to use a larger scale on the y-axis than on the x-axis.

x	y	(x, y)
−3	24	(−3, 24)
−1	−2	(−1, −2)
0	−12	(0, −12)
2	−26	(2, −26)
4	−32	(4, −32)
5	−32	(5, −32)
10	−2	(10, −2)
12	24	(12, 24)

① **Select values for x.**
② **Compute values for y.**



Now Try Exercise 39.

Technology Connection

TABLE SETUP
TblStart = −3
ΔTbl = 1
Indpnt: Auto Ask
Depend: Auto Ask

X	Y1
−3	24
−2	10
−1	−2
0	−12
1	−20
2	−26
3	−30

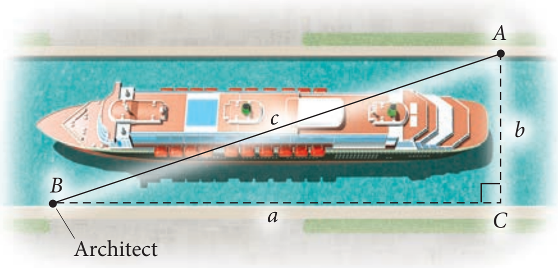
X = −3

A graphing calculator can be used to create a table of ordered pairs that are solutions of an equation. For the equation in Example 5, $y = x^2 - 9x - 12$, we first enter the equation on the equation-editor screen. Then we set up a table in AUTO mode by designating a value for TBLSTART and a value for ΔTBL. The calculator will produce a table starting with the value of TBLSTART and continuing by adding ΔTBL to supply succeeding x-values. For the equation $y = x^2 - 9x - 12$, we let TBLSTART = −3 and ΔTBL = 1. We can scroll up and down in the table to find values other than those shown here.

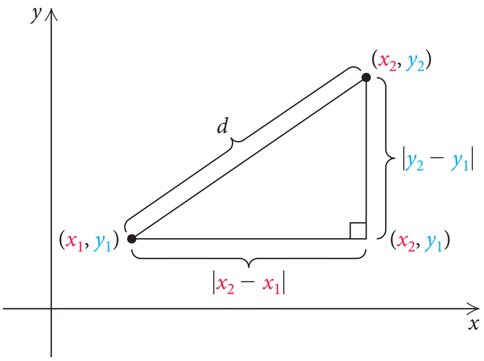
The Distance Formula

Suppose that an architect must determine the distance between two points, A and B, on opposite sides of a lane of the Panama Canal. One way in which he or she might proceed is to measure two legs of a right triangle that is situated as shown in the following figure. The Pythagorean equation, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the lengths of the legs, can then be used to find the length of the hypotenuse, which is the distance from A to B.

The \$5.25 billion expansion of the Panama Canal will soon double its capacity. A third canal lane is scheduled to open in 2015. (Source: *Panama Canal Authority*)



A similar strategy is used to find the distance between two points in a plane. For two points (x_1, y_1) and (x_2, y_2), we can draw a right triangle in which the legs have lengths $|x_2 - x_1|$ and $|y_2 - y_1|$.



JUST IN TIME 3, 25

Using the Pythagorean equation $c^2 = a^2 + b^2$, we have

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2.$$

Substituting d for c, $|x_2 - x_1|$ for a, and $|y_2 - y_1|$ for b in the Pythagorean equation

Because we are squaring, we can use parentheses to replace the absolute-value symbols:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

Taking the principal square root, we obtain the distance formula.

THE DISTANCE FORMULA

The **distance d** between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The subtraction of the x-coordinates can be done in any order, as can the subtraction of the y-coordinates. Although we derived the distance formula by considering two points not on a horizontal line or a vertical line, the distance formula holds for *any* two points.

JUST IN TIME 4, 22

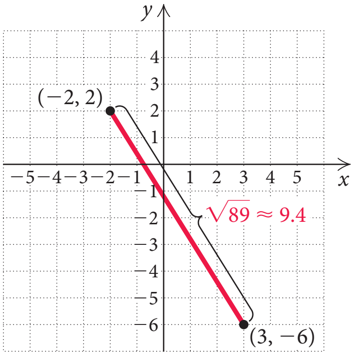
EXAMPLE 6 Find the distance between each pair of points.

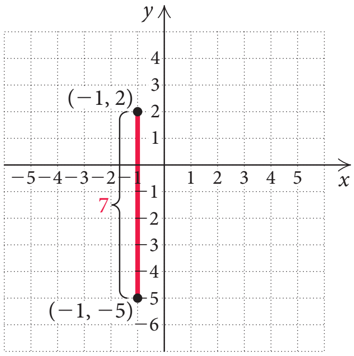
a) $(-2, 2)$ and $(3, -6)$ **b)** $(-1, -5)$ and $(-1, 2)$

Solution We substitute into the distance formula.

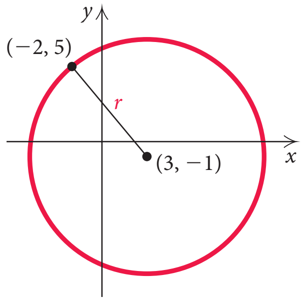
a)
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[3 - (-2)]^2 + (-6 - 2)^2} \\ &= \sqrt{5^2 + (-8)^2} = \sqrt{25 + 64} \\ &= \sqrt{89} \approx 9.4 \end{aligned}$$

b)
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-1 - (-1)]^2 + (-5 - 2)^2} \\ &= \sqrt{0^2 + (-7)^2} = \sqrt{0 + 49} \\ &= \sqrt{49} = 7 \end{aligned}$$





Now Try Exercises 41 and 49.



EXAMPLE 7 The point $(-2, 5)$ is on a circle that has $(3, -1)$ as its center. Find the length of the radius of the circle.

Solution Since the length of the radius is the distance from the center to a point on the circle, we substitute into the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{[3 - (-2)]^2 + (-1 - 5)^2}$$

Substituting r for d, $(3, -1)$ for (x_2, y_2), and $(-2, 5)$ for (x_1, y_1). Either point can serve as (x_1, y_1).

$$= \sqrt{5^2 + (-6)^2} = \sqrt{25 + 36}$$

$$= \sqrt{61} \approx 7.8.$$

Rounded to the nearest tenth

The radius of the circle is approximately 7.8.

Now Try Exercise 55.

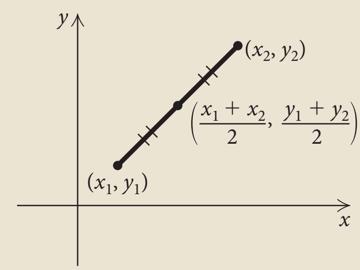
Midpoints of Segments

The distance formula can be used to develop a method of determining the *midpoint* of a segment when the endpoints are known. We state the formula and leave its proof to the exercises.

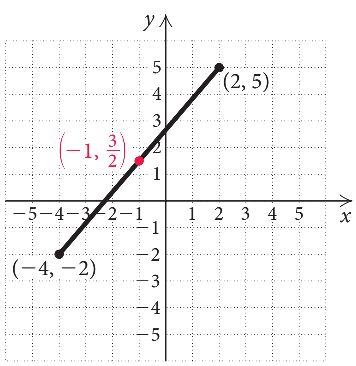
THE MIDPOINT FORMULA

If the endpoints of a segment are (x_1, y_1) and (x_2, y_2), then the coordinates of the **midpoint** of the segment are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



Note that we obtain the coordinates of the midpoint by averaging the coordinates of the endpoints. This is a good way to remember the midpoint formula.



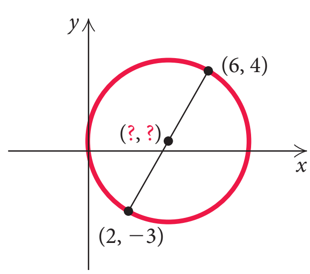
EXAMPLE 8 Find the midpoint of the segment whose endpoints are $(-4, -2)$ and $(2, 5)$.

Solution Using the midpoint formula, we obtain

$$\left(\frac{-4 + 2}{2}, \frac{-2 + 5}{2}\right) = \left(\frac{-2}{2}, \frac{3}{2}\right) = \left(-1, \frac{3}{2}\right).$$

Now Try Exercise 61.

EXAMPLE 9 The diameter of a circle connects the points $(2, -3)$ and $(6, 4)$ on the circle. Find the coordinates of the center of the circle.



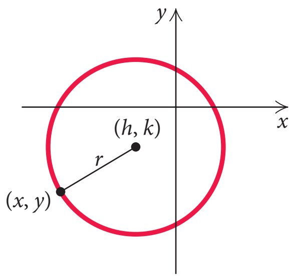
Solution Since the center of the circle is the midpoint of the diameter, we use the midpoint formula:

$$\left(\frac{2 + 6}{2}, \frac{-3 + 4}{2}\right), \quad \text{or} \quad \left(\frac{8}{2}, \frac{1}{2}\right), \quad \text{or} \quad \left(4, \frac{1}{2}\right).$$

The coordinates of the center are $\left(4, \frac{1}{2}\right)$.

Now Try Exercise 73.

Circles



A **circle** is the set of all points in a plane that are a fixed distance r from a *center* (h, k). Thus if a point (x, y) is to be r units from the center, we must have

$$r = \sqrt{(x - h)^2 + (y - k)^2}.$$

Using the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Squaring both sides gives an equation of a circle. The distance r is the length of a *radius* of the circle.

> **THE EQUATION OF A CIRCLE**
>
> The standard form of the equation of a circle with center (h, k) and radius r is
>
> $$(x - h)^2 + (y - k)^2 = r^2.$$

EXAMPLE 10 Find an equation of the circle having radius 5 and center $(3, -7)$.

Solution Using the standard form, we have

$$[x - 3]^2 + [y - (-7)]^2 = 5^2 \qquad \text{Substituting}$$
$$(x - 3)^2 + (y + 7)^2 = 25.$$

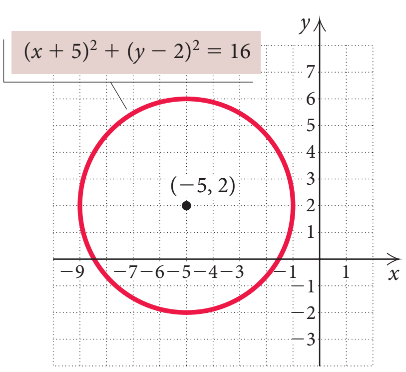
Now Try Exercise 75.

EXAMPLE 11 Graph the circle $(x + 5)^2 + (y - 2)^2 = 16$.

Solution We write the equation in standard form to determine the center and the radius:

$$[x - (-5)]^2 + [y - 2]^2 = 4^2.$$

The center is $(-5, 2)$ and the radius is 4. We locate the center and draw the circle using a compass.



Now Try Exercise 87.

Technology Connection

When we graph a circle, we select a viewing window in which the distance between units is visually the same on both axes. This procedure is called **squaring the viewing window**. We do this so that the graph will not be distorted. A graph of the circle $x^2 + y^2 = 36$ in a nonsquared window is shown in Fig. 1.

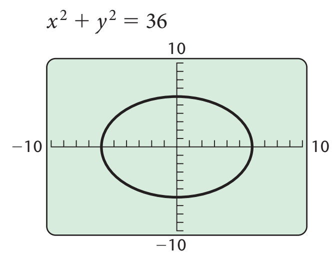


Figure 1.

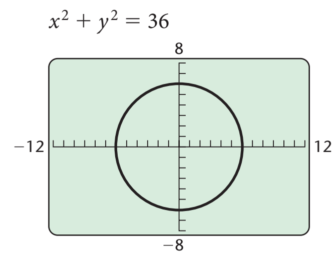
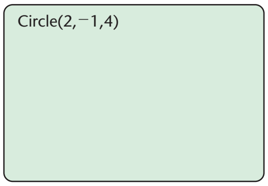
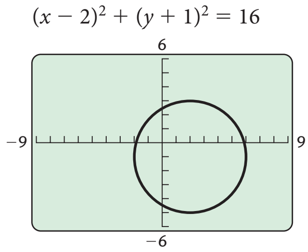


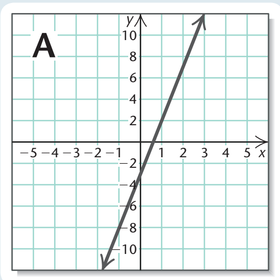
Figure 2.

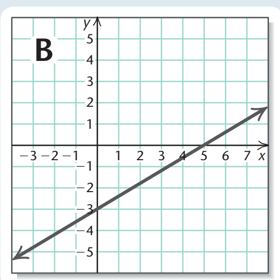
On many graphing calculators, the ratio of the height to the width of the viewing screen is $\frac{2}{3}$. When we choose a window in which Xscl = Yscl and the length of the y-axis is $\frac{2}{3}$ the length of the x-axis, the window will be squared. The windows with dimensions $[-6, 6, -4, 4]$, $[-9, 9, -6, 6]$, and $[-12, 12, -8, 8]$ are examples of squared windows. A graph of the circle $x^2 + y^2 = 36$ in a squared window is shown in Fig. 2. Many graphing calculators have an option on the ZOOM menu that squares the window automatically.

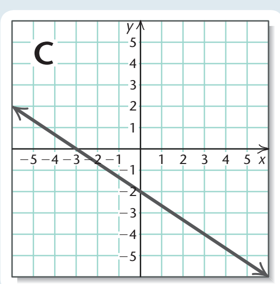
To graph a circle, we select the CIRCLE feature from the DRAW menu and enter the coordinates of the center and the length of the radius. The graph of the circle $(x - 2)^2 + (y + 1)^2 = 16$ is shown here. For more on graphing circles with a graphing calculator, see Section 7.2.

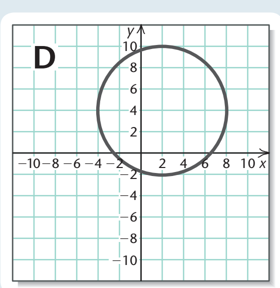


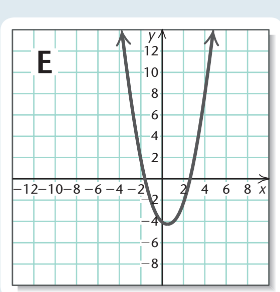












Visualizing the Graph

Match the equation with its graph.

1. $y = -x^2 + 5x - 3$

2. $3x - 5y = 15$

3. $(x - 2)^2 + (y - 4)^2 = 36$

4. $y - 5x = -3$

5. $x^2 + y^2 = \frac{25}{4}$

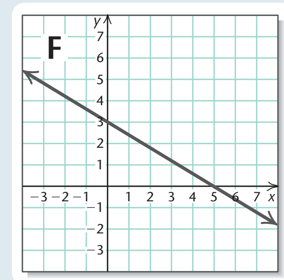
6. $15y - 6x = 90$

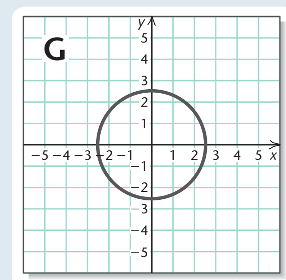
7. $y = -\frac{2}{3}x - 2$

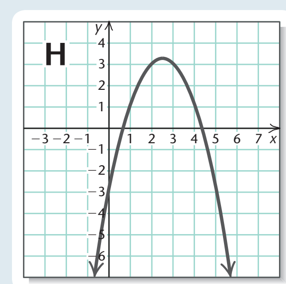
8. $(x + 3)^2 + (y - 1)^2 = 16$

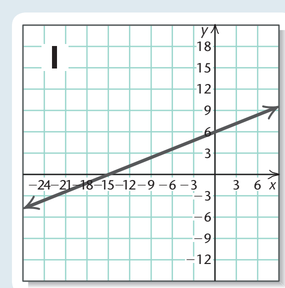
9. $3x + 5y = 15$

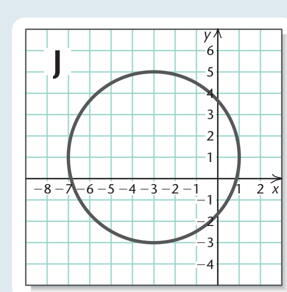
10. $y = x^2 - x - 4$







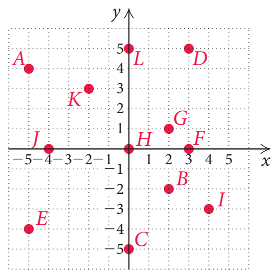




Answers on page A-1

1.1 Exercise Set

Use this graph for Exercises 1 and 2.



1. Find the coordinates of points A, B, C, D, E, and F.

2. Find the coordinates of points G, H, I, J, K, and L.

Graph and label the given points.

3. $(4, 0)$, $(-3, -5)$, $(-1, 4)$, $(0, 2)$, $(2, -2)$

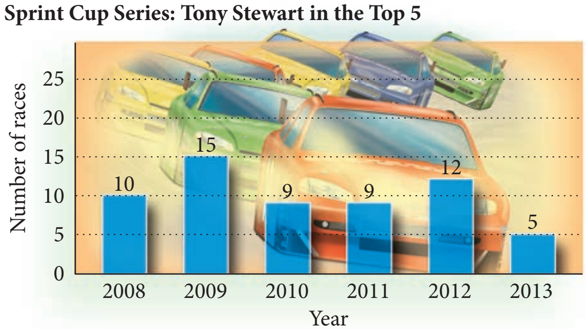
4. $(1, 4)$, $(-4, -2)$, $(-5, 0)$, $(2, -4)$, $(4, 0)$

5. $(-5, 1)$, $(5, 1)$, $(2, 3)$, $(2, -1)$, $(0, 1)$

6. $(4, 0)$, $(4, -3)$, $(-5, 2)$, $(-5, 0)$, $(-1, -5)$

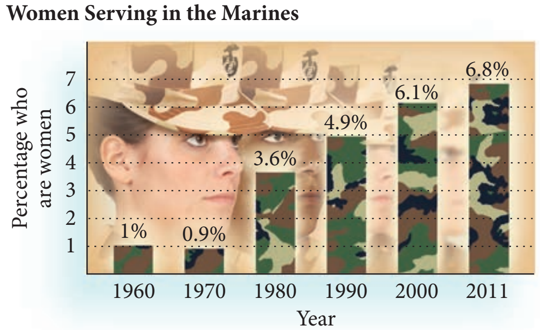
Express the data pictured in the graph as ordered pairs, letting the first coordinate represent the year and the second coordinate the amount or percent.

7. **Sprint Cup Series: Tony Stewart in the Top 5**



Source: ESPN NASCAR

8. **Women Serving in the Marines**



Source: U. S. Department of Veterans Affairs, Rutgers University

Use substitution to determine whether the given ordered pairs are solutions of the given equation.

9. $(-1, -9)$, $(0, 2)$; $y = 7x - 2$

10. $\left(\frac{1}{2}, 8\right)$, $(-1, 6)$; $y = -4x + 10$

11. $\left(\frac{2}{3}, \frac{3}{4}\right)$, $\left(1, \frac{3}{2}\right)$; $6x - 4y = 1$

12. $(1.5, 2.6)$, $(-3, 0)$; $x^2 + y^2 = 9$

13. $\left(-\frac{1}{2}, -\frac{4}{5}\right)$, $\left(0, \frac{3}{5}\right)$; $2a + 5b = 3$

14. $\left(0, \frac{3}{2}\right)$, $\left(\frac{2}{3}, 1\right)$; $3m + 4n = 6$

15. $(-0.75, 2.75)$, $(2, -1)$; $x^2 - y^2 = 3$

16. $(2, -4)$, $(4, -5)$; $5x + 2y^2 = 70$

Find the intercepts and then graph the line.

17. $5x - 3y = -15$

18. $2x - 4y = 8$

19. $2x + y = 4$

20. $3x + y = 6$

21. $4y - 3x = 12$

22. $3y + 2x = -6$

Graph the equation.

23. $y = 3x + 5$

24. $y = -2x - 1$

25. $x - y = 3$

26. $x + y = 4$

27. $y = -\frac{3}{4}x + 3$

28. $3y - 2x = 3$

29. $5x - 2y = 8$

30. $y = 2 - \frac{4}{3}x$

31. $x - 4y = 5$

32. $6x - y = 4$

33. $2x + 5y = -10$

34. $4x - 3y = 12$

35. $y = -x^2$

36. $y = x^2$

37. $y = x^2 - 3$

38. $y = 4 - x^2$

39. $y = -x^2 + 2x + 3$

40. $y = x^2 + 2x - 1$

Find the distance between the pair of points. Give an exact answer and, where appropriate, an approximation to three decimal places.

41. $(4, 6)$ and $(5, 9)$

42. $(-3, 7)$ and $(2, 11)$

43. $(-11, -8)$ and $(1, -13)$

44. $(-60, 5)$ and $(-20, 35)$

45. $(6, -1)$ and $(9, 5)$

46. $(-4, -7)$ and $(-1, 3)$

47. $\left(-8, \frac{7}{11}\right)$ and $\left(8, \frac{7}{11}\right)$

48. $\left(\frac{1}{2}, -\frac{4}{25}\right)$ and $\left(\frac{1}{2}, -\frac{13}{25}\right)$

49. $\left(-\frac{3}{5}, -4\right)$ and $\left(-\frac{3}{5}, \frac{2}{3}\right)$

50. $\left(-\frac{11}{3}, -\frac{1}{2}\right)$ and $\left(\frac{1}{3}, \frac{5}{2}\right)$

51. $(-4.2, 3)$ and $(2.1, -6.4)$

52. $(0.6, -1.5)$ and $(-8.1, -1.5)$

53. $(0, 0)$ and (a, b)

54. (r, s) and $(-r, -s)$

55. The points $(-3, -1)$ and $(9, 4)$ are the endpoints of the diameter of a circle. Find the length of the radius of the circle.

56. The point $(0, 1)$ is on a circle that has center $(-3, 5)$. Find the length of the diameter of the circle.

The converse of the Pythagorean theorem is also a true statement: If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle. Use the distance formula and the Pythagorean theorem to determine whether the set of points could be vertices of a right triangle.

57. $(-4, 5)$, $(6, 1)$, and $(-8, -5)$

58. $(-3, 1)$, $(2, -1)$, and $(6, 9)$

59. $(-4, 3)$, $(0, 5)$, and $(3, -4)$

60. The points $(-3, 4)$, $(2, -1)$, $(5, 2)$, and $(0, 7)$ are vertices of a quadrilateral. Show that the quadrilateral is a rectangle. (*Hint*: Show that the quadrilateral's opposite sides are the same length and that the two diagonals are the same length.)

Find the midpoint of the segment having the given endpoints.

61. $(4, -9)$ and $(-12, -3)$

62. $(7, -2)$ and $(9, 5)$

63. $\left(0, \frac{1}{2}\right)$ and $\left(-\frac{2}{5}, 0\right)$

64. $(0, 0)$ and $\left(-\frac{7}{13}, \frac{2}{7}\right)$

65. $(6.1, -3.8)$ and $(3.8, -6.1)$

66. $(-0.5, -2.7)$ and $(4.8, -0.3)$

67. $(-6, 5)$ and $(-6, 8)$

68. $(1, -2)$ and $(-1, 2)$

69. $\left(-\frac{1}{6}, -\frac{3}{5}\right)$ and $\left(-\frac{2}{3}, \frac{5}{4}\right)$

70. $\left(\frac{2}{9}, \frac{1}{3}\right)$ and $\left(-\frac{2}{5}, \frac{4}{5}\right)$

71. Graph the rectangle described in Exercise 60. Then determine the coordinates of the midpoint of each of the four sides. Are the midpoints vertices of a rectangle?

72. Graph the square with vertices $(-5, -1)$, $(7, -6)$, $(12, 6)$, and $(0, 11)$. Then determine the midpoint of each of the four sides. Are the midpoints vertices of a square?

73. The points $(\sqrt{7}, -4)$ and $(\sqrt{2}, 3)$ are endpoints of the diameter of a circle. Determine the center of the circle.

74. The points $(-3, \sqrt{5})$ and $(1, \sqrt{2})$ are endpoints of the diagonal of a square. Determine the center of the square.

Find an equation for a circle satisfying the given conditions.

75. Center $(2, 3)$, radius of length $\frac{5}{3}$

76. Center $(4, 5)$, diameter of length 8.2

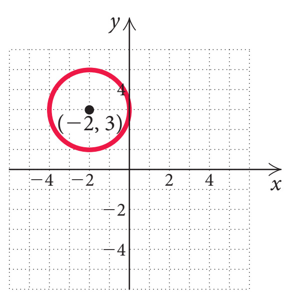
77. Center $(-1, 4)$, passes through $(3, 7)$

78. Center $(6, -5)$, passes through $(1, 7)$

79. The points $(7, 13)$ and $(-3, -11)$ are at the ends of a diameter.

80. The points $(-9, 4)$, $(-2, 5)$, $(-8, -3)$, and $(-1, -2)$ are vertices of an inscribed square.

81. Center $(-2, 3)$, tangent (touching at one point) to the y-axis



82. Center $(4, -5)$, tangent to the x-axis

Find the center and the radius of the circle. Then graph the circle.

83. $x^2 + y^2 = 4$

84. $x^2 + y^2 = 81$

85. $x^2 + (y - 3)^2 = 16$

86. $(x + 2)^2 + y^2 = 100$

87. $(x - 1)^2 + (y - 5)^2 = 36$

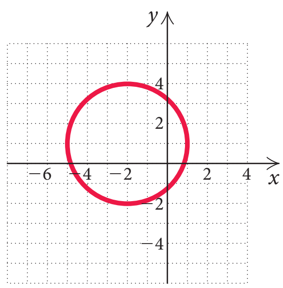
88. $(x - 7)^2 + (y + 2)^2 = 25$

89. $(x + 4)^2 + (y + 5)^2 = 9$

90. $(x + 1)^2 + (y - 2)^2 = 64$

Find the equation of the circle. Express the equation in standard form.

91.



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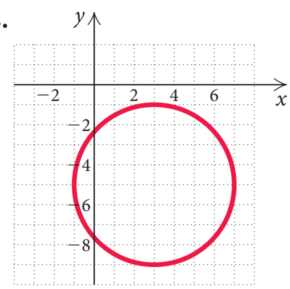

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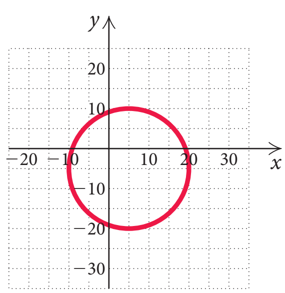

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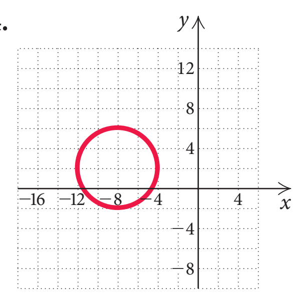


▶ Synthesis

To the student and the instructor: *The Synthesis exercises found at the end of every exercise set challenge students to combine concepts or skills studied in that section or in preceding parts of the text.*

95. If the point (p, q) is in the fourth quadrant, in which quadrant is the point $(q, -p)$?

Find the distance between the pair of points and find the midpoint of the segment having the given points as endpoints.

96. $\left(a, \frac{1}{a}\right)$ and $\left(a + h, \frac{1}{a + h}\right)$

97. $(a, \sqrt{a})$ and $(a + h, \sqrt{a + h})$

Find an equation of a circle satisfying the given conditions.

98. Center $(-5, 8)$ with a circumference of 10π units

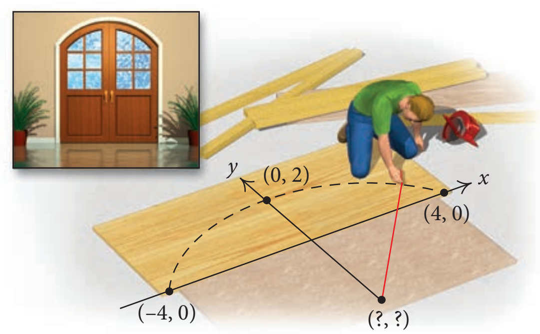
99. Center $(2, -7)$ with an area of 36π square units

100. Find the point on the x-axis that is equidistant from the points $(-4, -3)$ and $(-1, 5)$.

101. Find the point on the y-axis that is equidistant from the points $(-2, 0)$ and $(4, 6)$.

102. Determine whether the points $(-1, -3)$, $(-4, -9)$, and $(2, 3)$ are collinear.

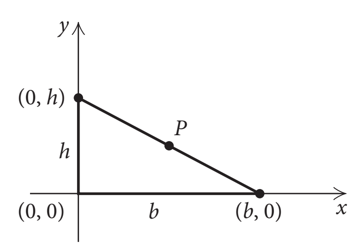
103. *An Arch of a Circle in Carpentry.* Matt is remodeling the front entrance to his home and needs to cut an arch for the top of an entranceway. The arch must be 8 ft wide and 2 ft high. To draw the arch, he will use a stretched string with chalk attached at an end as a compass.



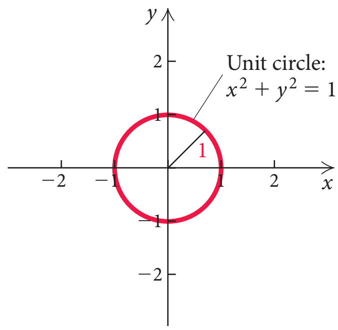
a) Using a coordinate system, locate the center of the circle.

b) What radius should Matt use to draw the arch?

104. Consider any right triangle with base b and height h, situated as shown. Show that the midpoint of the hypotenuse P is equidistant from the three vertices of the triangle.



Determine whether each of the following points lies on the ***unit circle***, $x^2 + y^2 = 1$.



105. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

106. $(0, -1)$

107. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

108. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

109. Prove the midpoint formula by showing that $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is equidistant from the points (x_1, y_1) and (x_2, y_2).

1.2 Functions and Graphs

- Determine whether a correspondence or a relation is a function.
- Find function values, or outputs, using a formula or a graph.
- Graph functions.
- Determine whether a graph is that of a function.
- Find the domain and the range of a function.
- Solve applied problems using functions.

We now focus our attention on a concept that is fundamental to many areas of mathematics—the idea of a *function*.

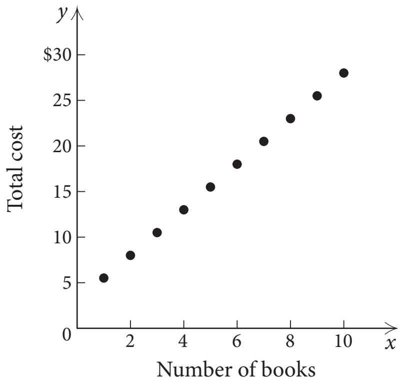
Functions

Used-Book Co-op. A community center operates a used-book co-op, and the proceeds are donated to summer youth programs. The total cost of a purchase is \$2.50 per book plus a flat-rate surcharge of \$3. If a customer selects 6 books, the total cost of the purchase is

$$\$2.50(6) + \$3, \quad \text{or} \quad \$18.$$

We can express this relationship with a set of ordered pairs, a graph, and an equation. A few ordered pairs are listed in the following table.

x	y	Ordered Pairs: (x, y)	Correspondence
1	5.50	(1, 5.50)	$1 \to 5.50$
2	8.00	(2, 8)	$2 \to 8$
4	13.00	(4, 13)	$4 \to 13$
7	20.50	(7, 20.50)	$7 \to 20.50$
10	28.00	(10, 28)	$10 \to 28$



The ordered pairs express a relationship, or a correspondence, between the first coordinate and the second coordinate. We can see this relationship in the graph as well. The equation that describes the correspondence is

$$y = 2.50x + 3, \quad \text{where } x \text{ is a natural number.}$$

This is an example of a *function*. In this case, the total cost of the purchase y is a function of the number of books purchased x; that is, y is a function of x, where x is the independent variable and y is the dependent variable.

Let's consider some other correspondences before giving the definition of a function.

First Set	Correspondence	Second Set
To each person	there corresponds	that person's DNA.
To each blue spruce sold	there corresponds	its price.
To each real number	there corresponds	the square of that number.

In each correspondence, the first set is called the **domain** and the second set is called the **range**. For each member, or **element**, in the domain, there is *exactly one* member in the range to which it corresponds. Thus each person has exactly *one* DNA, each blue spruce has exactly *one* price, and each real number has exactly *one* square. Each correspondence is a *function*.

FUNCTION

A **function** is a correspondence between a first set, called the **domain**, and a second set, called the **range**, such that each member of the domain corresponds to *exactly one* member of the range.

It is important to note that not every correspondence between two sets is a function.

EXAMPLE 1 Determine whether each of the following correspondences is a function.

a)
$-6 \to 36$
$6 \to 36$
$-3 \to 9$
$3 \to 9$
$0 \to 0$

b) **APPOINTING PRESIDENT** → **SUPREME COURT JUSTICE**

George H. W. Bush → Clarence Thomas
William Jefferson Clinton → Stephen G. Breyer
William Jefferson Clinton → Ruth Bader Ginsburg
George W. Bush → Samuel A. Alito, Jr.
George W. Bush → John G. Roberts, Jr.
Barack H. Obama → Elena Kagan
Barack H. Obama → Sonia M. Sotomayor

Solution

a) This correspondence *is* a function because each member of the domain corresponds to exactly one member of the range. Note that the definition of a function allows more than one member of the domain to correspond to the same member of the range.

b) This correspondence *is not* a function because there is at least one member of the domain who is paired with more than one member of the range (William Jefferson Clinton with Stephen G. Breyer and Ruth Bader Ginsburg; George W. Bush with Samuel A. Alito, Jr., and John G. Roberts, Jr.; Barack H. Obama with Elena Kagan and Sonia M. Sotomayor). Now Try Exercises 5 and 7.

EXAMPLE 2 Determine whether each of the following correspondences is a function.

	DOMAIN	CORRESPONDENCE	RANGE
a)	Years in which a presidential election occurs	The person elected	A set of presidents
b)	All automobiles produced in 2014	Each automobile's VIN (Vehicle Identification Number)	A set of VINs
c)	The set of all professional golfers who won a PGA tournament in 2013	The tournament won	The set of all PGA tournaments in 2013
d)	The set of all PGA tournaments in 2013	The winner of the tournament	The set of all golfers who won a PGA tournament in 2013

Solution

a) This correspondence *is* a function because in each presidential election *exactly one* president is elected.

b) This correspondence *is* a function because each automobile has *exactly one* VIN.

c) This correspondence *is not* a function because a winning golfer could be paired with more than one tournament.

d) This correspondence *is* a function because each tournament has only one winning golfer. Now Try Exercises 11 and 13.

When a correspondence between two sets is not a function, it may still be an example of a **relation**.

> **RELATION**
>
> A **relation** is a correspondence between a first set, called the **domain**, and a second set, called the **range**, such that each member of the domain corresponds to *at least one* member of the range.

All the correspondences in Examples 1 and 2 are relations, but, as we have seen, not all are functions. Relations are sometimes written as sets of ordered pairs (as we saw earlier in the example on the total cost of a purchase of used books) in which elements of the domain are the first coordinates of the ordered pairs and elements of the range are the second coordinates. For example, instead of writing $-3 \longrightarrow 9$, as we did in Example 1(a), we could write the ordered pair $(-3, 9)$.

EXAMPLE 3 Determine whether each of the following relations is a function. Identify the domain and the range.

a) $\{(9, -5), (9, 5), (2, 4)\}$
b) $\{(-2, 5), (5, 7), (0, 1), (4, -2)\}$
c) $\{(-5, 3), (0, 3), (6, 3)\}$

Solution

a) The relation *is not* a function because the ordered pairs $(9, -5)$ and $(9, 5)$ have the same first coordinate and different second coordinates. (See Fig. 1.)

The domain is the set of all first coordinates: $\{9, 2\}$.
The range is the set of all second coordinates: $\{-5, 5, 4\}$.

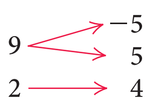


Figure 1.

b) The relation *is* a function because *no* two ordered pairs have the same first coordinate and different second coordinates. (See Fig. 2.)

The domain is the set of all first coordinates: $\{-2, 5, 0, 4\}$.
The range is the set of all second coordinates: $\{5, 7, 1, -2\}$.

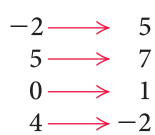


Figure 2.

c) The relation *is* a function because *no* two ordered pairs have the same first coordinate and different second coordinates. (See Fig. 3.)

The domain is $\{-5, 0, 6\}$.
The range is $\{3\}$.

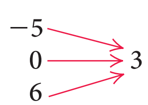


Figure 3.

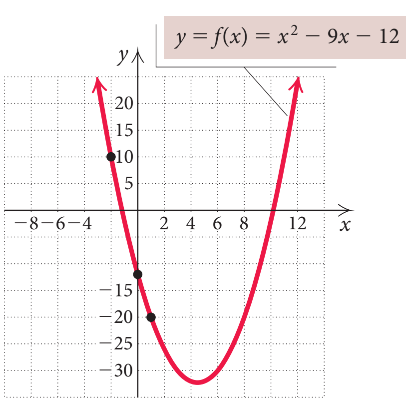
Now Try Exercises 15 and 17.

Notation for Functions

JUST IN TIME 9

Functions used in mathematics are often given by equations. They generally require that certain calculations be performed in order to determine which member of the range is paired with each member of the domain. For example, in Section 1.1 we graphed the function $y = x^2 - 9x - 12$ by doing calculations like the following:

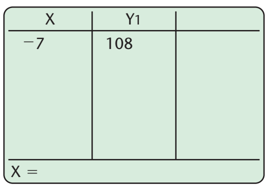
for $x = -2$, $y = (-2)^2 - 9(-2) - 12 = 10$,
for $x = 0$, $y = 0^2 - 9 \cdot 0 - 12 = -12$, and
for $x = 1$, $y = 1^2 - 9 \cdot 1 - 12 = -20$.

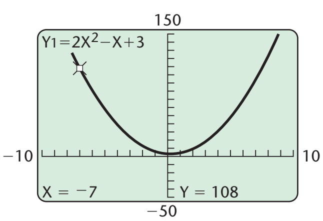


A more concise notation is often used. For $y = x^2 - 9x - 12$, the **inputs** (members of the domain) are values of x substituted into the equation. The **outputs** (members of the range) are the resulting values of y. If we call the function f, we can use x to represent an arbitrary *input* and $f(x)$—read "f of x," or "f at x," or "the value of f at x"—to represent the corresponding *output*. In this notation, the

Technology Connection

We can find function values with a graphing calculator. Below, we illustrate finding $f(-7)$ from Example 4(b), first with the TABLE feature set in ASK mode and then with the VALUE feature from the CALC menu. On both screens, we see that $f(-7) = 108$.





function given by $y = x^2 - 9x - 12$ is written as $f(x) = x^2 - 9x - 12$ and the above calculations would be

$$f(-2) = (-2)^2 - 9(-2) - 12 = 10,$$
$$f(0) = 0^2 - 9 \cdot 0 - 12 = -12, \quad \text{and}$$
$$f(1) = 1^2 - 9 \cdot 1 - 12 = -20.$$

Keep in mind that $f(x)$ *does not* mean $f \cdot x$.

Thus, instead of writing "when $x = -2$, the value of y is 10," we can simply write "$f(-2) = 10$," which can be read as "f of -2 is 10" or "for the input -2, the output of f is 10." The letters g and h are also often used to name functions.

EXAMPLE 4 A function f is given by $f(x) = 2x^2 - x + 3$. Find each of the following.

a) $f(0)$ **b)** $f(-7)$
c) $f(5a)$ **d)** $f(a - 4)$

Solution We can think of this formula as follows:

$$f(\square) = 2(\square)^2 - (\square) + 3.$$

Then to find an output for a given input, we think: "Whatever goes in the blank on the left goes in the blank(s) on the right." This gives us a "recipe" for finding outputs.

a) $f(0) = 2(0)^2 - 0 + 3$
$\quad = 0 - 0 + 3 = 3$

b) $f(-7) = 2(-7)^2 - (-7) + 3$
$\quad = 2 \cdot 49 + 7 + 3 = 108$

c) $f(5a) = 2(5a)^2 - 5a + 3$
$\quad = 2 \cdot 25a^2 - 5a + 3$
$\quad = 50a^2 - 5a + 3$

JUST IN TIME 12

d) $f(a - 4) = 2(a - 4)^2 - (a - 4) + 3$
$\quad = 2(a^2 - 8a + 16) - (a - 4) + 3$
$\quad = 2a^2 - 16a + 32 - a + 4 + 3$
$\quad = 2a^2 - 17a + 39$

Now Try Exercise 21.

Graphs of Functions

We graph functions in the same way that we graph equations. We find ordered pairs (x, y), or $(x, f(x))$, plot points, and complete the graph.

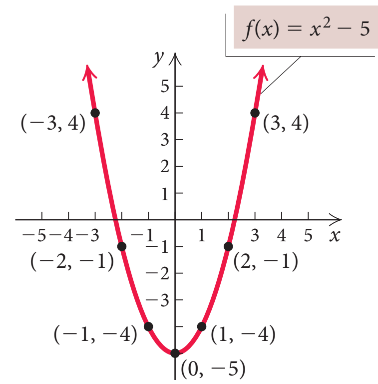
EXAMPLE 5 Graph each of the following functions.

a) $f(x) = x^2 - 5$
b) $f(x) = x^3 - x$
c) $f(x) = \sqrt{x + 4}$

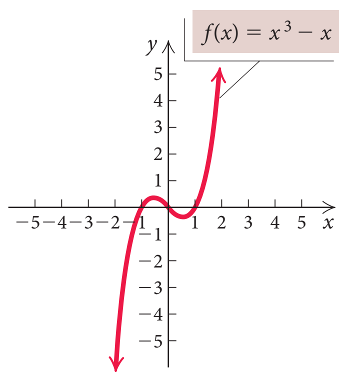
Solution We select values for x and find the corresponding values of $f(x)$. Then we plot the points and connect them with a smooth curve.

a) $f(x) = x^2 - 5$

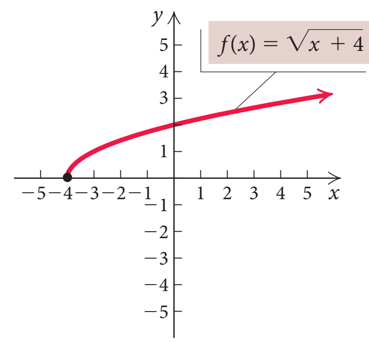
x	$f(x)$	$(x, f(x))$
-3	4	$(-3, 4)$
-2	-1	$(-2, -1)$
-1	-4	$(-1, -4)$
0	-5	$(0, -5)$
1	-4	$(1, -4)$
2	-1	$(2, -1)$
3	4	$(3, 4)$



b) $f(x) = x^3 - x$

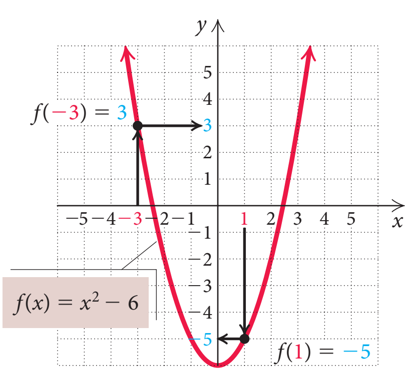


c) $f(x) = \sqrt{x + 4}$



Now Try Exercise 31.

Function values can also be determined from a graph.



EXAMPLE 6 For the function $f(x) = x^2 - 6$, use the graph at left to find each of the following function values.

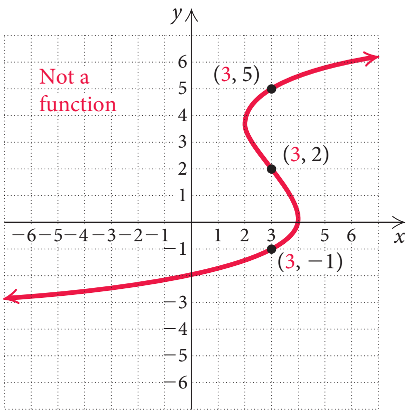
a) $f(-3)$ **b)** $f(1)$

Solution

a) To find the function value $f(-3)$ from the graph, we locate the input -3 on the horizontal axis, move vertically to the graph of the function, and then move horizontally to find the output on the vertical axis. We see that $f(-3) = 3$.

b) To find the function value $f(1)$, we locate the input 1 on the horizontal axis, move vertically to the graph, and then move horizontally to find the output on the vertical axis. We see that $f(1) = -5$.

Now Try Exercise 35.



Since 3 is paired with more than one member of the range, the graph does not represent a function.

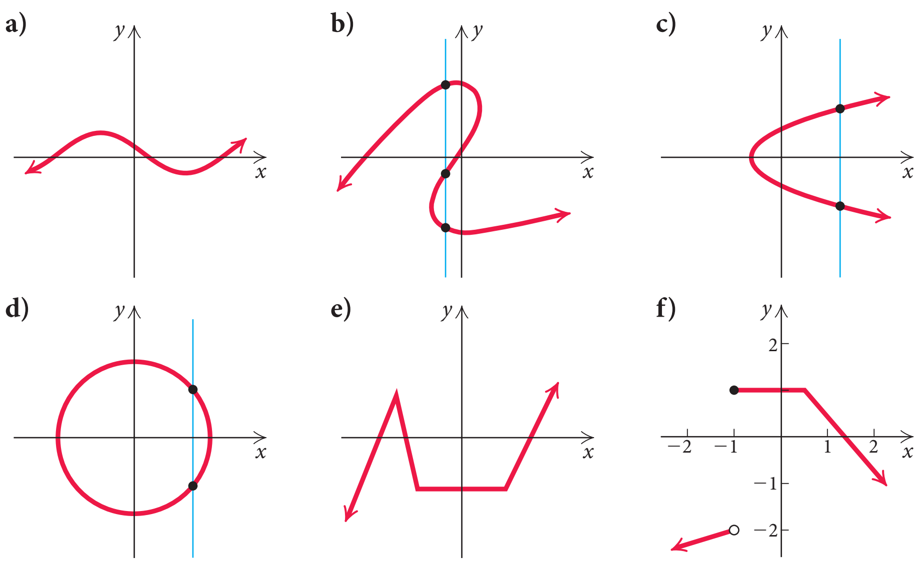
We know that when one member of the domain is paired with two or more different members of the range, the correspondence *is not* a function. Thus, when a graph contains two or more different points with the same first coordinate, the graph cannot represent a function. (See the graph at left. Note that 3 is paired with -1, 2, and 5.) Points sharing a common first coordinate are vertically above or below each other. This leads us to the *vertical-line test*.

THE VERTICAL-LINE TEST

If it is possible for a vertical line to cross a graph more than once, then the graph *is not* the graph of a function.

To apply the vertical-line test, we try to find a vertical line that crosses the graph more than once. If we succeed, then the graph is not that of a function. If we do not, then the graph is that of a function.

EXAMPLE 7 Which of graphs (a)–(f) (in red) are graphs of functions? In graph (f), the solid dot shows that $(-1, 1)$ belongs to the graph. The open circle shows that $(-1, -2)$ does *not* belong to the graph.



Solution Graphs (a), (e), and (f) are graphs of functions because we cannot find a vertical line that crosses any of them more than once. In (b), the vertical line drawn crosses the graph at three points, so graph (b) is not that of a function. Also, in (c) and (d), we can find a vertical line that crosses the graph more than once, so these are not graphs of functions.

Now Try Exercises 43 and 47.

Finding Domains of Functions

When a function *f* whose inputs and outputs are real numbers is given by a formula, the *domain* is understood to be the set of all inputs for which the expression is defined as a real number. When an input results in an expression that is not defined as a real number, we say that the function value *does not exist* and that the number being substituted *is not* in the domain of the function.

Technology Connection

When we use a graphing calculator to find function values and a function value does not exist, the calculator indicates this with an ERROR message.

In the following tables, we see in Example 8 that $f(3)$ for $f(x) = 1/(x - 3)$ and $g(-7)$ for $g(x) = \sqrt{x} + 5$ do not exist. Thus, 3 and -7 are *not* in the domains of the corresponding functions.

$y = \dfrac{1}{x - 3}$

X	Y1
1	−.5
3	ERROR

X =

$y = \sqrt{x} + 5$

X	Y1
16	9
−7	ERROR

X =

EXAMPLE 8 Find the indicated function values, if possible, and determine whether the given values are in the domain of the function.

a) $f(1)$ and $f(3)$, for $f(x) = \dfrac{1}{x - 3}$

b) $g(16)$ and $g(-7)$, for $g(x) = \sqrt{x} + 5$

Solution

a) $f(1) = \dfrac{1}{1 - 3} = \dfrac{1}{-2} = -\dfrac{1}{2}$

Since $f(1)$ is defined, 1 is in the domain of f.

$f(3) = \dfrac{1}{3 - 3} = \dfrac{1}{0}$

Since division by 0 is not defined, $f(3)$ does not exist and the number 3 is not in the domain of f.

b) $g(16) = \sqrt{16} + 5 = 4 + 5 = 9$

Since $g(16)$ is defined, 16 is in the domain of g.

$g(-7) = \sqrt{-7} + 5$

Since $\sqrt{-7}$ is not defined as a real number, $g(-7)$ does not exist and the number -7 is not in the domain of g. ■

As we see in Example 8, inputs that make a denominator 0 or that yield a negative radicand in an even root are not in the domain of a function.

EXAMPLE 9 Find the domain of each of the following functions.

a) $f(x) = \dfrac{1}{x - 7}$ **b)** $h(x) = \dfrac{3x^2 - x + 7}{x^2 + 2x - 3}$

c) $f(x) = x^3 + |x|$ **d)** $g(x) = \sqrt[3]{x - 1}$

JUST IN TIME
6, 13, 16

Solution

a) Because $x - 7 = 0$ when $x = 7$, the only input that results in a denominator of 0 is 7. The domain is $\{x \mid x \neq 7\}$. We can also write the solution using interval notation and the symbol $\cup$ for the **union**, or inclusion, of both sets: $(-\infty, 7) \cup (7, \infty)$.

b) We can substitute any real number in the numerator, but we must avoid inputs that make the denominator 0. To find those inputs, we solve $x^2 + 2x - 3 = 0$, or $(x + 3)(x - 1) = 0$. Since $x^2 + 2x - 3$ is 0 for -3 and 1, the domain consists of the set of all real numbers except -3 and 1, or $\{x \mid x \neq -3 \textit{ and } x \neq 1\}$, or $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$.

c) We can substitute any real number for x. Thus the domain is the set of all real numbers, $\mathbb{R}$, or $(-\infty, \infty)$.

d) Because the index is odd, the radicand, $x - 1$, can be any real number. Thus x can be any real number. The domain is all real numbers, $\mathbb{R}$, or $(-\infty, \infty)$.

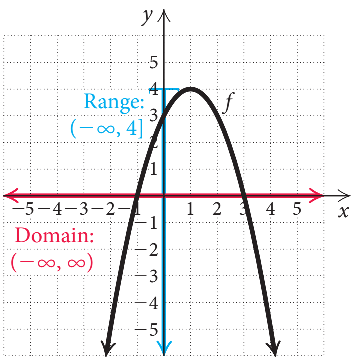
Now Try Exercises 55, 57, and 61.

Visualizing Domain and Range

Keep the following in mind regarding the *graph* of a function:

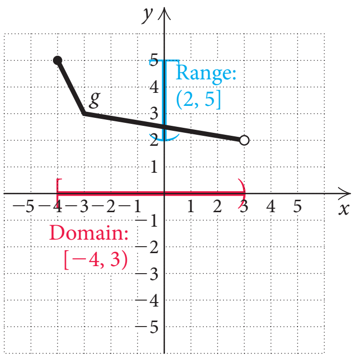
Domain = the set of a function's inputs, found on the horizontal axis (x-axis);

Range = the set of a function's outputs, found on the vertical axis (y-axis).



Consider the graph of function f, shown at left. To determine the domain of f, we look for the inputs on the x-axis that correspond to a point on the graph. We see that they include the entire set of real numbers, illustrated in red on the x-axis. Thus the domain is $(-\infty, \infty)$. To find the range, we look for the outputs on the y-axis that correspond to a point on the graph. We see that they include 4 and all real numbers less than 4, illustrated in blue on the y-axis. The bracket at 4 indicates that 4 is included in the interval. The range is $\{y \mid y \le 4\}$, or $(-\infty, 4]$.

Let's now consider the following graph of function g. The solid dot shows that $(-4, 5)$ belongs to the graph. The open circle shows that $(3, 2)$ does *not* belong to the graph.



We see that the inputs of the function include -4 and all real numbers between -4 and 3, illustrated in red on the x-axis. The bracket at -4 indicates that -4 is included in the interval. The parenthesis at 3 indicates that 3 is not included in the interval. The domain is $\{x \mid -4 \le x < 3\}$, or $[-4, 3)$. The outputs of the function include 5 and all real numbers between 2 and 5, illustrated in blue on the y-axis. The parenthesis at 2 indicates that 2 is not included in the interval. The bracket at 5 indicates that 5 is included in the interval. The range is $\{y \mid 2 < y \le 5\}$, or $(2, 5]$.

EXAMPLE 10 Using the graph of the function, find the domain and the range of the function.

a) $f(x) = \frac{1}{2}x + 1$

b) $f(x) = \sqrt{x + 4}$

c) $f(x) = x^3 - x$

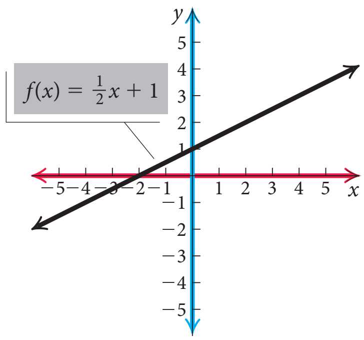
d) $f(x) = \frac{1}{x - 2}$

e) $f(x) = x^4 - 2x^2 - 3$

f) $f(x) = \sqrt{4 - (x - 3)^2}$

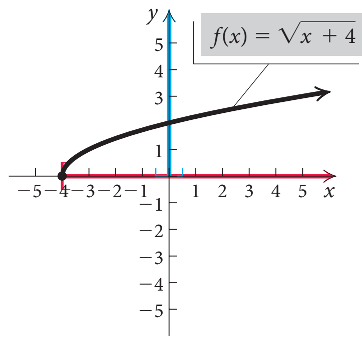
Solution

a)



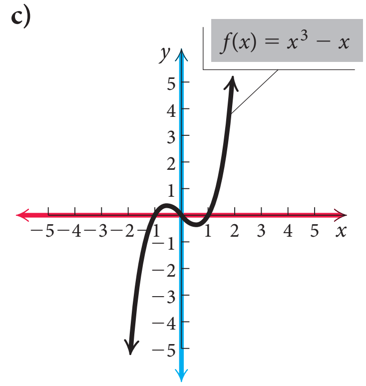
Domain = all real numbers, $(-\infty, \infty)$; range = all real numbers, $(-\infty, \infty)$

b)



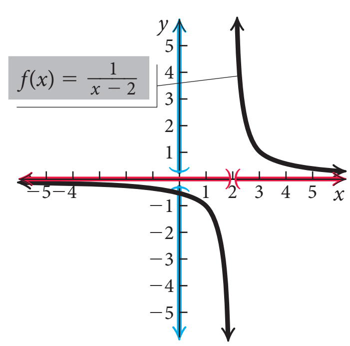
Domain = $[-4, \infty)$; range = $[0, \infty)$

c)



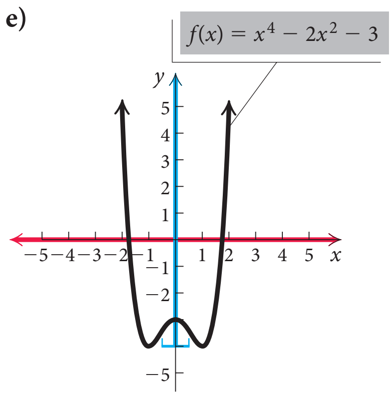
Domain = all real numbers, $(-\infty, \infty)$; range = all real numbers, $(-\infty, \infty)$

d)



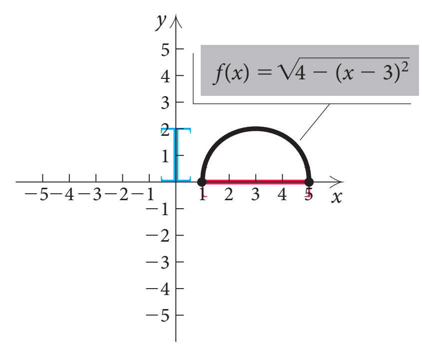
Since the graph does not touch or cross either the vertical line $x = 2$ or the x-axis, $y = 0$, 2 is excluded from the domain and 0 is excluded from the range. Domain = $(-\infty, 2) \cup (2, \infty)$; range = $(-\infty, 0) \cup (0, \infty)$

e)



Domain = all real numbers, $(-\infty, \infty)$; range = $[-4, \infty)$

f)



Domain = $[1, 5]$; range = $[0, 2]$

Now Try Exercises 71 and 77.

Always consider adding the reasoning of Example 9 to a graphical analysis. Think, "What can I input?" to find the domain. Think, "What do I get out?" to find the range. Thus, in Examples 10(c) and 10(e), it might not appear as though the domain is all real numbers because the graph rises steeply, but by examining the equation we see that we can indeed substitute any real number for x.

Applications of Functions

EXAMPLE 11 ***Linear Expansion of a Bridge.*** The linear expansion L of the steel center span of a suspension bridge that is 1420 m long is a function of the change in temperature t, in degrees Celsius, from winter to summer and is given by

$$L(t) = 0.000013 \cdot 1420 \cdot t,$$

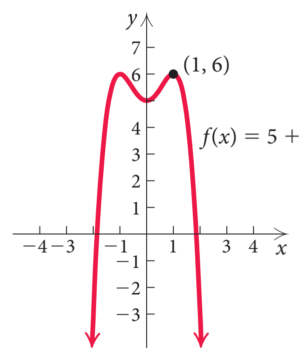
where 0.000013 is the coefficient of linear expansion for steel and L is in meters. Find the linear expansion of the steel center span when the change in temperature from winter to summer is 30°, 42°, 50°, and 56° Celsius.

Solution Using a calculator, we compute function values. We find that

$$L(30) = 0.5538 \text{ m},$$
$$L(42) = 0.77532 \text{ m},$$
$$L(50) = 0.923 \text{ m}, \quad \text{and}$$
$$L(56) = 1.03376 \text{ m}.$$

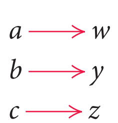
Now Try Exercise 85.

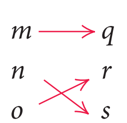
CONNECTING THE CONCEPTS

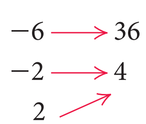
FUNCTION CONCEPTS	GRAPH
Formula for f: $f(x) = 5 + 2x^2 - x^4$. For every input, there is exactly one output. $(1, 6)$ is on the graph. For the input 1, the output is 6. $f(1) = 6$ Domain: set of all inputs $= (-\infty, \infty)$ Range: set of all outputs $= (-\infty, 6]$	

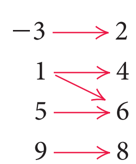
1.2 Exercise Set

In Exercises 1–14, determine whether the correspondence is a function.

1.


2.


3.


4.


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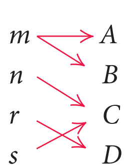

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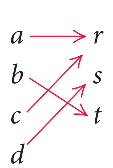
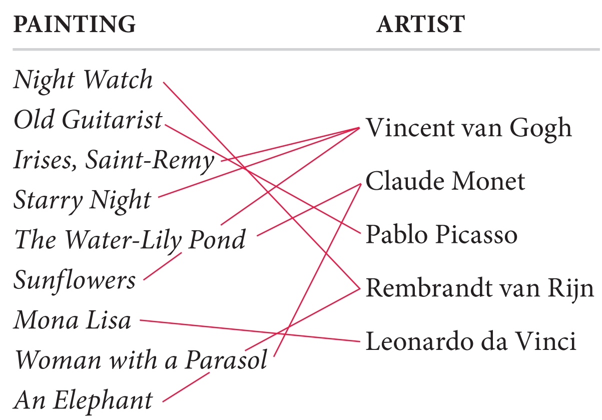


7.


8.

ACTOR PORTRAYING JAMES BOND		**MOVIE TITLE**
Sean Connery	→	*Goldfinger*, 1964
Sean Connery	→	*Diamonds Are Forever*, 1971
George Lazenby	→	*On Her Majesty's Secret Service*, 1969
Roger Moore	→	*Moonraker*, 1979
Roger Moore	→	*For Your Eyes Only*, 1981
Timothy Dalton	→	*The Living Daylights*, 1987
Pierce Brosnan	→	*GoldenEye*, 1995
Pierce Brosnan	→	*The World Is Not Enough*, 1999
Daniel Craig	→	*Quantum of Solace*, 2008

	DOMAIN	**CORRESPONDENCE**	**RANGE**
9.	A set of cars in a parking lot	Each car's license number	A set of letters and numbers
10.	A set of people in a town	A doctor a person uses	A set of doctors
11.	The integers less than 9	Five times the integer	A subset of integers
12.	A set of members of a rock band	An instrument each person plays	A set of instruments
13.	A set of students in a class	A student sitting in a neighboring seat	A set of students
14.	A set of bags of chips on a shelf	Each bag's weight	A set of weights

Determine whether the relation is a function. Identify the domain and the range.

15. $\{(2, 10), (3, 15), (4, 20)\}$

16. $\{(3, 1), (5, 1), (7, 1)\}$

17. $\{(-7, 3), (-2, 1), (-2, 4), (0, 7)\}$

18. $\{(1, 3), (1, 5), (1, 7), (1, 9)\}$

19. $\{(-2, 1), (0, 1), (2, 1), (4, 1), (-3, 1)\}$

20. $\{(5, 0), (3, -1), (0, 0), (5, -1), (3, -2)\}$

21. Given that $g(x) = 3x^2 - 2x + 1$, find each of the following.

a) $g(0)$ **b)** $g(-1)$
c) $g(3)$ **d)** $g(-x)$
e) $g(1 - t)$

22. Given that $f(x) = 5x^2 + 4x$, find each of the following.

a) $f(0)$ **b)** $f(-1)$
c) $f(3)$ **d)** $f(t)$
e) $f(t - 1)$

23. Given that $g(x) = x^3$, find each of the following.

a) $g(2)$ **b)** $g(-2)$
c) $g(-x)$ **d)** $g(3y)$
e) $g(2 + h)$

24. Given that $f(x) = 2|x| + 3x$, find each of the following.

a) $f(1)$ **b)** $f(-2)$
c) $f(-x)$ **d)** $f(2y)$
e) $f(2 - h)$

25. Given that

$$g(x) = \frac{x - 4}{x + 3},$$

find each of the following.

a) $g(5)$ **b)** $g(4)$
c) $g(-3)$ **d)** $g(-16.25)$
e) $g(x + h)$

26. Given that

$$f(x) = \frac{x}{2 - x},$$

find each of the following.

a) $f(2)$ **b)** $f(1)$
c) $f(-16)$ **d)** $f(-x)$
e) $f\left(-\frac{2}{3}\right)$

27. Find $g(0)$, $g(-1)$, $g(5)$, and $g\left(\frac{1}{2}\right)$ for

$$g(x) = \frac{x}{\sqrt{1 - x^2}}.$$

28. Find $h(0)$, $h(2)$, and $h(-x)$ for

$$h(x) = x + \sqrt{x^2 - 1}.$$

Graph the function.

29. $f(x) = \frac{1}{2}x + 3$

30. $f(x) = \sqrt{x} - 1$

31. $f(x) = -x^2 + 4$

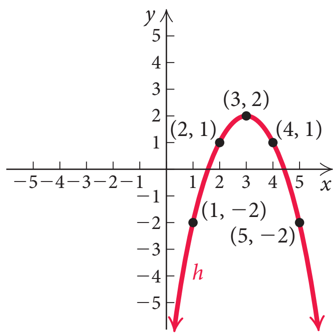
32. $f(x) = x^2 + 1$

33. $f(x) = \sqrt{x - 1}$

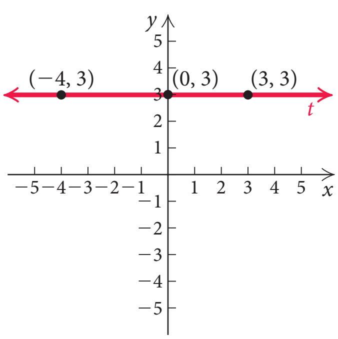
34. $f(x) = x - \frac{1}{2}x^3$

In each of Exercises 35–40, a graph of a function is shown. Using the graph, find the indicated function values; that is, given the inputs, find the outputs.

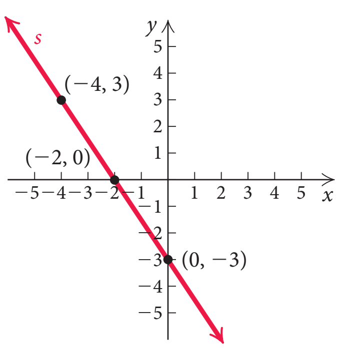
35. $h(1)$, $h(3)$, and $h(4)$



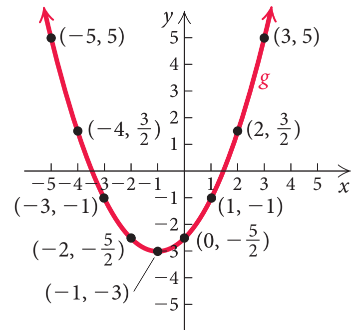
36. $t(-4)$, $t(0)$, and $t(3)$



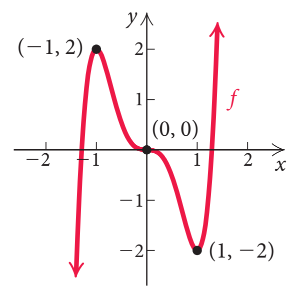
37. $s(-4)$, $s(-2)$, and $s(0)$



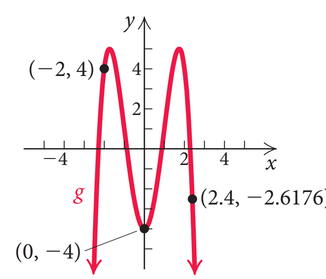
38. $g(-4)$, $g(-1)$, and $g(0)$



39. $f(-1)$, $f(0)$, and $f(1)$

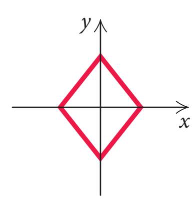


40. $g(-2)$, $g(0)$, and $g(2.4)$



In Exercises 41–48, determine whether the graph is that of a function. An open circle indicates that the point does not belong to the graph.

41.



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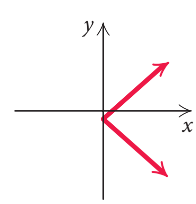

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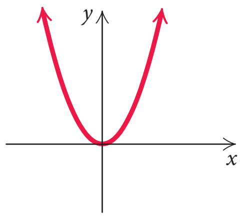

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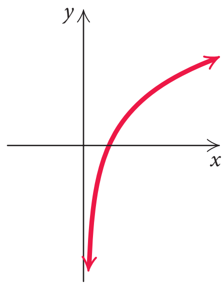

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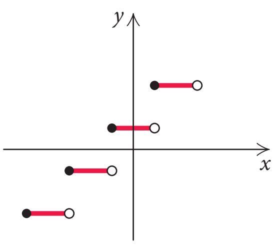

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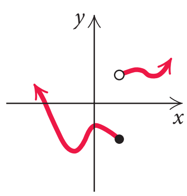

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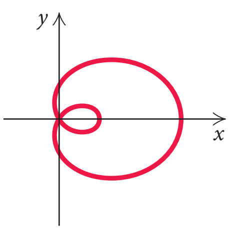

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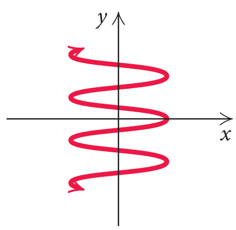


Find the domain of the function.

49. $f(x) = 7x + 4$

50. $f(x) = |3x - 2|$

51. $f(x) = |6 - x|$

52. $f(x) = \dfrac{1}{x^4}$

53. $f(x) = 4 - \dfrac{2}{x}$

54. $f(x) = \dfrac{1}{5}x^2 - 5$

55. $f(x) = \dfrac{x + 5}{2 - x}$

56. $f(x) = \dfrac{8}{x + 4}$

57. $f(x) = \dfrac{1}{x^2 - 4x - 5}$

58. $f(x) = \dfrac{(x - 2)(x + 9)}{x^3}$

59. $f(x) = \sqrt[3]{x + 10} - 1$

60. $f(x) = \sqrt[3]{4 - x}$

61. $f(x) = \dfrac{8 - x}{x^2 - 7x}$

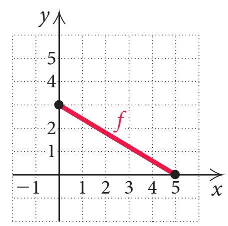
62. $f(x) = \dfrac{x^4 - 2x^3 + 7}{3x^2 - 10x - 8}$

63. $f(x) = \dfrac{1}{10}|x|$

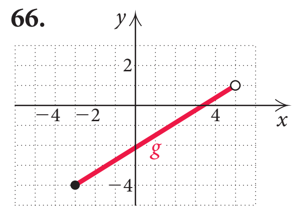
64. $f(x) = x^2 - 2x$

In Exercises 65–72, determine the domain and the range of the function.

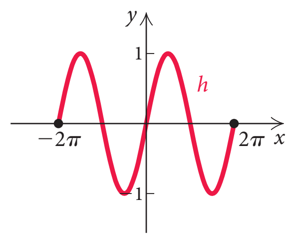
65.



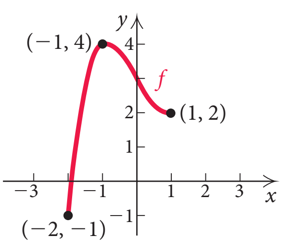
66.



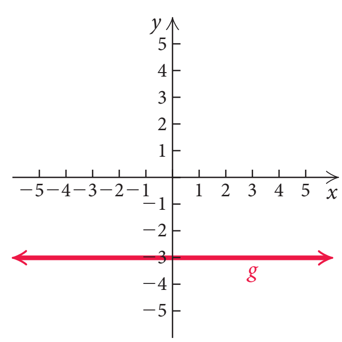
67.



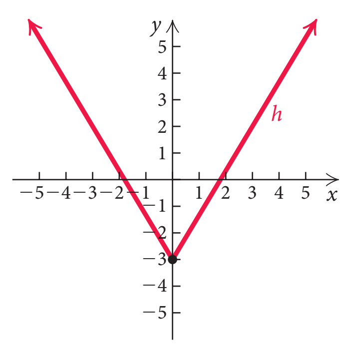
68.



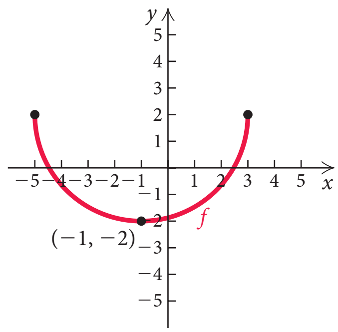
69.



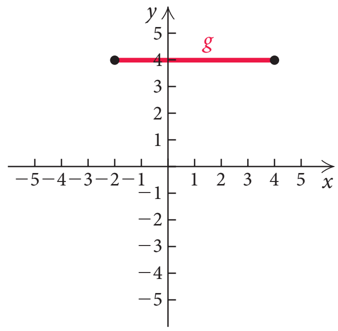
70.



71.



72.



In Exercises 73–84, graph the given function. Then visually estimate the domain and the range.

73. $f(x) = |x|$

74. $f(x) = |x| - 2$

75. $f(x) = 3x - 2$

76. $f(x) = 5 - 3x$

77. $f(x) = \dfrac{1}{x - 3}$

78. $f(x) = \dfrac{1}{x + 1}$

79. $f(x) = (x - 1)^3 + 2$

80. $f(x) = (x - 2)^4 + 1$

81. $f(x) = \sqrt{7 - x}$

82. $f(x) = \sqrt{x + 8}$

83. $f(x) = -x^2 + 4x - 1$

84. $f(x) = 2x^2 - x^4 + 5$

85. *Decreasing Value of the Dollar.* In 2014, it took \$23.63 to equal the value of \$1 in 1913. In 2000, it took only \$17.39 to equal the value of \$1 in 1913. The amount that it takes to equal the value of \$1 in 1913 can be estimated by the linear function V given by

$$V(x) = 0.4306x + 11.0043,$$

where x is the number of years since 1985. Thus, $V(10)$ gives the amount that it took in 1995 to equal the value of \$1 in 1913.

Source: usinflationcalculator.com

a) Use this function to predict the amount that it will take in 2018 and in 2025 to equal the value of \$1 in 1913.

b) When will it take approximately \$32 to equal the value of \$1 in 1913?

86. *Population of the United States.* The population P of the United States in 1960 was 179,323,175. In 2010, the population was 308,745,538. The population of the United States can be estimated by the linear function P given by

$$P(x) = 2{,}578{,}409x + 151{,}116{,}864,$$

where x is the number of years after 1950. Thus, $P(20)$ gives the population in 1970.

a) Use this function to estimate the population in 1980 and in 2018.

b) When will the population be approximately 400,000,000?

87. *Boiling Point and Elevation.* The elevation E, in meters, above sea level at which the boiling point of water is t degrees Celsius is given by the function

$$E(t) = 1000(100 - t) + 580(100 - t)^2.$$

At what elevation is the boiling point 99.5°? 100°?

88. *Windmill Power.* Under certain conditions, the power P, in watts per hour, generated by a windmill with winds blowing v miles per hour is given by

$$P(v) = 0.015v^3.$$

Find the power generated by 15-mph winds and 35-mph winds.

▶ Skill Maintenance

To the student and the instructor: *The Skill Maintenance exercises review skills covered previously in the text. You can expect such exercises in every exercise set. They provide excellent review for a final examination. Answers to all skill maintenance exercises, along with section references, appear in the answer section at the back of the book.*

Use substitution to determine whether the given ordered pairs are solutions of the given equation.

89. $(-3, -2), (2, -3)$; $y^2 - x^2 = -5$ **[1.1]**

90. $(0, -7), (8, 11)$; $y = 0.5x + 7$ **[1.1]**

91. $\left(\frac{4}{5}, -2\right), \left(\frac{11}{5}, \frac{1}{10}\right)$; $15x - 10y = 32$ **[1.1]**

Graph the equation. **[1.1]**

92. $y = (x - 1)^2$

93. $y = \frac{1}{3}x - 6$

94. $-2x - 5y = 10$

95. $(x - 3)^2 + y^2 = 4$

▶ Synthesis

Find the domain of the function.

96. $f(x) = \sqrt[4]{2x + 5} + 3$

97. $f(x) = \dfrac{\sqrt{x + 1}}{x}$

98. $f(x) = \dfrac{\sqrt{x + 6}}{(x + 2)(x - 3)}$

99. $f(x) = \sqrt{x} - \sqrt{4 - x}$

100. Give an example of two different functions that have the same domain and the same range, but have no pairs in common. Answers may vary.

101. Draw a graph of a function for which the domain is $[-4, 4]$ and the range is $[1, 2] \cup [3, 5]$. Answers may vary.

102. Suppose that for some function g, $g(x + 3) = 2x + 1$. Find $g(-1)$.

103. Suppose $f(x) = |x + 3| - |x - 4|$. Write $f(x)$ without using absolute-value notation if x is in each of the following intervals.

a) $(-\infty, -3)$
b) $[-3, 4)$
c) $[4, \infty)$

1.3 Linear Functions, Slope, and Applications

- ▶ Determine the slope of a line given two points on the line.
- ▶ Solve applied problems involving slope, or average rate of change.
- ▶ Find the slope and the y-intercept of a line given the equation $y = mx + b$, or $f(x) = mx + b$.
- ▶ Graph a linear equation using the slope and the y-intercept.
- ▶ Solve applied problems involving linear functions.

In real-life situations, we often need to make decisions on the basis of limited information. When the given information is used to formulate an equation or an inequality that at least approximates the situation mathematically, we have created a **model**. One of the most frequently used mathematical models is *linear*. The graph of a linear model is a straight line.

▶ Linear Functions

Let's examine the connections among equations, functions, and graphs that are *straight lines*. First, examine the graphs of linear functions and nonlinear functions shown here. Note that the graphs of the two types of functions are quite different.