EDITION 5

HESI ADMISSION ASSESSMENT

Exam Review

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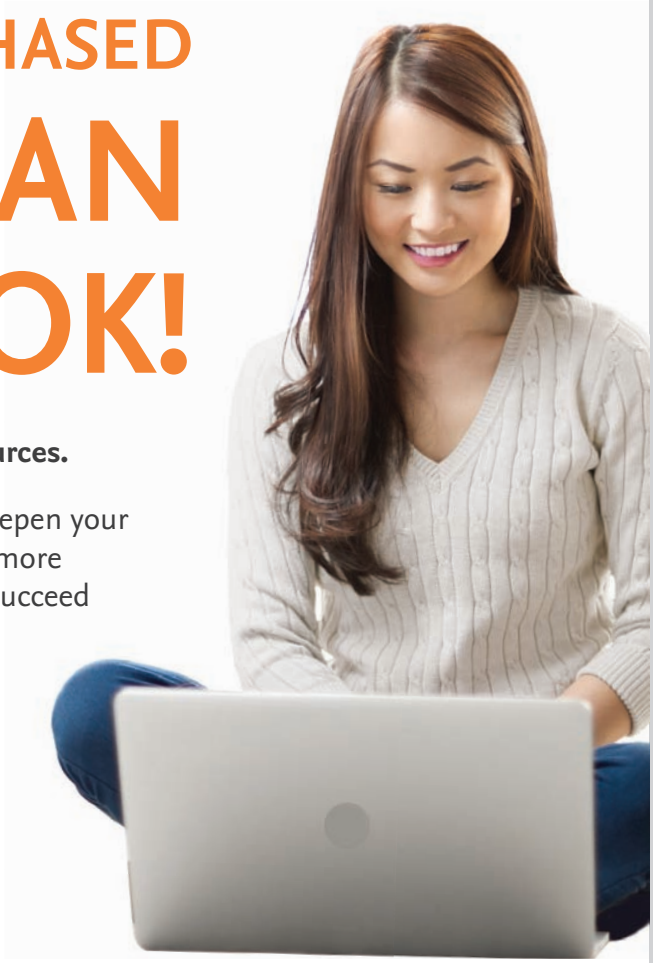

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EDITION
5

HESI
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EDITOR

E. Tina Cuellar, PhD, WHNP, PMHCNS, BC
Director of Live Review
Elsevier/HESI
Houston, Texas

Elsevier
3251 Riverport Lane
St. Louis, Missouri 63043

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Senior Content Strategist: Jamie Blum
Content Development Manager: Laurie Gower
Content Development Specialist: Elizabeth McCormac
Publishing Services Manager: Shereen Jameel
Project Manager: Aparna Venkatachalam
Designer: Bridget Hoette

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CONTRIBUTING AUTHORS

Lisa Aberle, MSRS, RT(R)(CV)
Radiography Educator
Continuing Education
Achieve RT Media
Chatsworth, Illinois

Sandra K. Anderson, BA, LMT, ABT
Instructor
The Costa Rica School of Massage Therapy
Samara, Costa Rica

Joanna Cain, BSN, BA
President & Founder
Editorial
Auctorial Pursuits, LLC
Boulder, Colorado

John Lane, RT(R)(CT), BS, DC
Technologist
Radiography, Computed Tomography
The American Registry of Radiologic Technologists (ARRT)
St. Louis, Missouri

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PREFACE

Congratulations on purchasing the *HESI Admission Assessment Exam Review*! This study guide was developed based on the HESI Admission Assessment Exam; however, test items on the HESI Admission Assessment Exam are not specifically derived from this study guide. The content in this study guide provides an overview of the subjects tested on the Admission Assessment Exam and is designed to assist students in preparation for entrance into higher education in a variety of health-related professions. The *HESI Admission Assessment Exam Review* is written at the high school and beginning college levels and offers the basic knowledge that is necessary to be successful on the Admission Assessment Exam.

The HESI Admission Assessment exam consists of 10 different exams—8 academically oriented exams and 2 personally oriented exams. The academically oriented subjects consist of:

- Mathematics
- Reading Comprehension
- Vocabulary
- Grammar
- Biology
- Chemistry
- Anatomy and Physiology
- Physics

Chapter content in the *HESI Admission Assessment Exam Review* includes conversion tables and practice problems in the Mathematics chapter; step-by-step explanations in the Reading Comprehension and Grammar chapters; a substantial list of words used in health professions in the Vocabulary chapter; rationales and sample questions in the Biology and Chemistry chapters; helpful terminology in the Anatomy and Physiology chapter; and sample problems in the Physics chapter. Also included throughout the exam review are "HESI Hint" boxes, which are designed to offer students a suggestion, an example, or a reminder pertaining to a specific topic.

The personally oriented exams consist of a Learning Style assessment and a Personality Profile. These exams are intended to offer students insights into their study habits, learning preferences, and dispositions relating to academic achievement. Students generally like to take these personally oriented exams for the purpose of personal insight and discussion. Because each of these exams takes only approximately 15 minutes to complete, the school may include them in their administration of the Admission Assessment Exam.

Schools can choose to administer any one, or all, of these exams provided by the Admission Assessment. For example, programs that do not require biology, chemistry, anatomy and physiology, or physics for entry would not administer those specific Admission Assessment science-oriented exams.

The HESI Admission Assessment Exam has been used by colleges, universities, and health-related institutions as part of the selection and placement process for applicants and newly admitted students for approximately 10 years.

Study Hints

It is always a good idea to prepare for any exam. When you begin to study for the Admission Assessment Exam, make sure you allocate adequate time and do not feel rushed. Set up a schedule that provides an hour or two each day to review material in the *HESI Admission Assessment Exam Review*. Mark the time you set aside on a calendar to remind yourself when to study each day. Before you begin, take the 25-question Pretest at the beginning of the text to help you initially assess your strengths and weaknesses of the content. For each section in the *HESI Admission Assessment Exam Review*, review the material that is relevant to your particular field of the health care professions. Complete the review questions at the end of each chapter, then complete the 50-question Posttest at the end of the text. This Posttest gives you additional practice in the text's subject areas using a more comprehensive approach. The

Posttest will help you to assess your readiness for the exam. Once you have completed your review and self-assessment of topics in the study guide, more test-taking practice is available on the text's corresponding Evolve site (http://www.elsevier.com/HESI/A2Review) with two comprehensive 82-question Practice Exams on the various subject areas that will help you prepare for the Admissions Assessment Exam. If you are having trouble with the review questions or the Practice Exams for a particular section, review that content in the *HESI Admission Assessment Exam Review* study guide again. It may also be helpful to go back to your textbook and class notes for additional review.

Test-Taking Hints

1. Read each question carefully and completely. Make sure you understand what the question is asking.
2. Identify the key words or phrases in the question. These words or phrases will provide critical information about how to answer the question.
3. Rephrase the question in your words.
   A. Ask yourself, "What is the question really asking?"
   B. Eliminate nonessential information from the question.
   C. Sometimes writers use terminology that may be unfamiliar to you. Do not be confused by a new writing style.
4. Rule out options (if they are presented).
   A. Read all of the responses completely.
   B. Rule out any options that are clearly incorrect.
   C. Mentally mark through incorrect options in your head.
   D. Differentiate between the remaining options, considering your knowledge of the subject.
5. Computer tests do not allow an option for skipping questions and returning to them later. Practice answering every question as it appears.

Do not second-guess yourself. TRUST YOUR ANSWERS.

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PRETEST

1. A pair of six-sided dice is rolled one time. What is the probability that the number 5 will appear?
A. 1/12
B. 1/6
C. 1/4
D. 1/3

2. The nurse's notation describes the patient as "febrile." Based on this information, which statement is true?
A. The patient is confused.
B. The patient had a seizure.
C. The patient feels weak.
D. The patient has a fever.

3. In the hierarchic system of classification, which is the largest?
A. Kingdom
B. Class
C. Genus
D. Species

4. Which structure is part of the respiratory system?
A. Esophagus
B. Trachea
C. Pancreas
D. Spleen

5. Write the following quantity, 1 millimeter (mm), in powers of tens: _____

6. The product of y and -25 is -100; find the value of y.
A. 4
B. -2500
C. -4
D. 2500

Use the passage below to answer questions 7-9.

Penicillin

Antibiotics are among the most commonly prescribed drugs today. Before the discovery of the first antibiotic, there was no effective treatment for bacterial infections such as pneumonia. Until antibiotics were available, even a tiny cut on the surface of the skin could result in a potentially grave infection.

The world's first true antibiotic was discovered by a professor of bacteriology named Alexander Fleming. In 1928, Fleming was performing experiments with staphylococcal bacteria. Fleming noticed that the bacteria in one of his Petri dishes was dying. Unlike the other samples, this dish had become contaminated by mold spores. Fleming was able to identify the mold as a member of the *Penicillium* genus.

When Fleming grew this mold in a pure culture, he found that it produced a substance that was effective against all Gram-positive pathogens. He would eventually name this "mold juice" penicillin. Fleming admitted that his discovery was purely accidental, but his groundbreaking work in developing the first antibiotic revolutionized medicine and saved millions of lives.

7. Which is the author's primary purpose in writing this essay?
A. To entertain the reader with stories about a famous inventor.
B. To explain how antibiotics are able to destroy bacteria.
C. To inform the reader about how penicillin was discovered.
D. To analyze the difference between types of bacteria.

8. According to the passage, which statement is **false**?
A. Penicillin is effective against Gram-positive pathogens.
B. One of Fleming's Petri dishes became contaminated with bacteria.
C. Fleming discovered penicillin by accident.
D. Fleming discovered the first antibiotic.

9. What is the meaning of the word *grave* in the first paragraph?
A. Very serious
B. Incurable
C. Highly contagious
D. Deadly

10. A paramedic treats a patient who has a deep cut in the skin caused by broken glass. Which term best describes this injury?
A. Incision
B. Laceration
C. Puncture
D. Abrasion

11. Which word in the following sentence is a verb?
Roy was afraid he would not be able to finish the race.
A. was
B. afraid
C. able
D. race

12. Twelve (12) more than a number is five (5). What is the number?
A. −7
B. 7
C. −17
D. 17

13. Select the best word for the blank in the following sentence.
I must remember to _____ my book to class today.
A. Bring
B. Take
C. Brought
D. Took

14. Which subatomic particles contain a positive charge?
A. Protons and electrons
B. Protons only
C. Protons and neutrons
D. Neutrons and electrons

15. After observing an event, you develop an explanation. This explanation is tested by performing a repeatable procedure. What is this procedure called?
A. Hypothesis
B. Experiment
C. Conclusion
D. Theory

16. Which word in the following sentence is the direct object?
Tom's dad gave him a bicycle for his birthday.
A. dad
B. him
C. bicycle
D. birthday

17. Which property of water explains why the oceans are helpful in stabilizing the climate?
A. Water molecules spread apart in freezing temperatures.
B. Water has strong cohesive and adhesive properties.
C. Water can act as both an acid and a base.
D. Water has a relatively high specific heat value.

18. Select the meaning of the underlined word in the sentence.
She felt dizzy and had difficulty <u>ambulating</u>.
A. Walking
B. Balancing
C. Seeing
D. Standing

19. Calcium is located in group IIA on the periodic table. Which is the charge on a calcium ion?
A. −1
B. +1
C. −2
D. +2

20. Which is the unit for measuring the amount of force applied to an object?
A. Joules
B. Hertz
C. Newtons
D. Watts

21. Which mineral is necessary for muscle contraction?
A. Chloride
B. Sodium
C. Calcium
D. Magnesium

22. Which biological molecules are composed of amino acids?
A. Carbohydrates
B. Lipids
C. Proteins
D. Nucleic acids

23. Which is the correct coefficient for water (H_2O) when the following equation is balanced?
$2C_2H_6 + 7O_2 \rightarrow 4CO_2 + \_H_2O$
A. 3 parts H_2O
B. 6 parts H_2O
C. 9 parts H_2O
D. 14 parts H_2O

24. A tissue examined under the microscope is from the internal surface of stomach, has layers of flat cells, and is without blood vessels. Which type of tissue is this?
A. Epithelial
B. Connective
C. Muscle
D. Nervous

25. Which physical quantity has both a magnitude and a direction?
A. Energy
B. Time
C. Velocity
D. Distance

ANSWERS TO PRETEST

1. D—There is 1 out of 6 chances on 1 die. When rolling a pair of dice, there are 12 chances in 36 possible outcomes. A 12-to-36 chance is equal to 1 out of 3.
2. D
3. A
4. B
5. 10^{-3} m
6. A—Divide: $-100 \div -25 = 4$
7. C
8. B
9. A
10. B
11. A
12. A—Add -12 to 5. The solution is -7.
13. B—In this sentence, the action is away from the speaker, who will carry the book from a near place (where the speaker is) to a far place (the classroom). Therefore, the best word is "take."
14. B
15. B
16. C—The direct object is the word that is directly affected by the action of the verb. In this sentence, the noun "bicycle" receives the action of the verb "gave."
17. D
18. A
19. D
20. C
21. C
22. C
23. B
24. A
25. C

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1 MATHEMATICS

Members of the health professions use math every day to calculate medication dosages, radiation limits, nutritional needs, mental status, intravenous drip rates, intake and output, and a host of other requirements related to their clients. Safe and effective care is the goal of all who work in health professions. Therefore, it is essential that students entering the health professions understand and make calculations using whole numbers, fractions, decimals, and percentages.

The purpose of this chapter is to review the addition, subtraction, multiplication, and division of whole numbers, fractions, decimals, and percentages. Basic algebra skills will also be reviewed, such as evaluating expressions, and solving for a specific variable. Mastery of these basic mathematic functions is an integral step toward a career in the health professions.

CHAPTER OUTLINE

Basic Addition and Subtraction
Basic Multiplication (Whole Numbers)
Basic Division (Whole Numbers)
Decimals
Fractions
Multiplication of Fractions
Division of Fractions
Changing Fractions to Decimals
Changing Decimals to Fractions
Ratios and Proportions
Percentages
12-hour Clock versus Military Time
Algebra
Helpful Information to Memorize
Answers to Sample Problems

KEY TERMS

Common Denominator
Constant
Denominator
Digit
Dividend
Divisor
Exponent
Expression
Factor
Fraction Bar
Improper Fraction
Least Common Denominator
Numerator
Percent
Place Value
Product
Proper Fraction
Proportion
Quotient
Ratio
Reciprocals
Remainder
Terminating Decimal
Variable

Basic Addition and Subtraction

Digit: Any number 1 through 9 and 0 (e.g., the number 7 is a digit)
Place Value: The value of the position of a digit in a number (e.g., in the number 321, the number 2 is in the "tens" position)

HESI Hint

1 ten = 10 ones
1 hundred = 100 ones
1 thousand = 1000 ones

Basic Addition

Example 1

382 + 212

$$\begin{array}{r} 382 \\ +\ 212 \\ \hline 594 \end{array}$$

Steps

1. Line up the **digits** according to **place value.**
2. Add the digits starting from right to left.
   - Ones: 2 + 2 = 4
   - Tens: 8 + 1 = 9
   - Hundreds: 3 + 2 = 5

Addition with Regrouping

HESI Hint

To solve an addition problem, it may be necessary to regroup by moving, or carrying over, an extra digit from one place value column to the next.

Example 2

748 + 523

$$\begin{array}{r} {\scriptstyle 1} \\ 748 \\ +\ 523 \\ \hline 1{,}271 \end{array}$$

Steps

1. Line up the digits according to place value.
2. Add:
   - Ones: 8 + 3 = 11
   - Carry the 1 to the tens place, which is one place to the left.
   - Tens: 1 + 4 + 2 = 7
   - Hundreds: 7 + 5 = 12

Basic Subtraction

Subtraction provides the difference between two numbers.

HESI Hint

It may be easier to solve a subtraction problem by first rewriting it vertically.

Example 1

3,884 − 1,671

$$\begin{array}{r} 3{,}884 \\ -\ 1{,}671 \\ \hline 2{,}213 \end{array}$$

Steps

1. Line up the digits according to place value.
2. Subtract:
   - Ones: 4 − 1 = 3
   - Tens: 8 – 7 = 1
   - Hundreds: 8 − 6 = 2
   - Thousands: 3 − 1 = 2

Subtraction with Regrouping

HESI Hint

Remember, if the number to subtract is not a positive number, you must borrow or regroup from one place value to a lower place value.

Example 1

862 − 57

$$\begin{array}{r} {\scriptstyle 5\ 12} \\ 8\not{6}\not{2} \\ -5\ 7 \\ \hline 8\,0\,5 \end{array}$$

Steps

1. Align the digits according to place value.
2. Subtract:
   - Ones: 12 – 7 = 5
   - Tens: 5 – 5 = 0
   - Hundreds: 8 – 0 = 8

SAMPLE PROBLEMS

Add or subtract each of the following problems as indicated.

1. 1,283 + 188 =
2. 273 + 47 =
3. 1,644 + 357 =
4. 73 + 211 + 22 =
5. 382 − 150 =
6. 246 − 47 =
7. 568 − 170 =
8. 17,444 − 923 =
9. Sam walks 2 blocks to Stacey's house. They both walk 5 blocks to the park. How far has Sam walked?
10. Lisa buys 12 eggs from the store so she can make pancakes. The pancake recipe calls for 3 eggs. How many eggs will Lisa have left after making the pancakes?

Basic Multiplication (Whole Numbers)

The process of multiplication is essentially repeated addition.

Product: The answer to a multiplication problem.

HESI Hint

Remember, the zero is used as a placeholder to keep the problem aligned. If you do not skip a space, the answer will be incorrect. Below is an example of a well-aligned problem.

$$\begin{array}{rl} 24571 & \\ \times\ 1233 & \\ \hline 73{,}713 & \rightarrow \text{Ones} \\ 737{,}130 & \rightarrow \text{Tens} \\ 4{,}914{,}200 & \rightarrow \text{Hundreds} \\ +\,24{,}571{,}000 & \rightarrow \text{Thousands} \\ \hline 30{,}296{,}043 & \end{array}$$

Example 1

18×4

$$\begin{array}{r} {\scriptstyle 3}\\ 18 \\ \times\ 4 \\ \hline 72 \end{array}$$

Steps

1. Multiply one digit at a time.
2. Multiply 4×18.
   - Ones: $4 \times 8 = 32$

   Carry the 1 to the tens place, and write the 2 in the ones place.
   - Tens: $4 \times 1 = 4 + 3 = 7$

Example 2

713 × 24

```
   713
 × 24
 2,852
+14,260
 17,112
```

Zero (0) represents a placeholder for the ones place

Steps

1. Multiply 713 × 24.
   - 4 × 3 = 12
   - 4 × 1 = 4 + 1 = 5
   - 4 × 7 = 28
2. Multiply 713 × 2 (remember to line up the ones digit with the 4 by using zero as a placeholder).
   - 2 × 3 = 6
   - 2 × 1 = 2
   - 2 × 7 =14
3. Add the two products together.
   - 2,852 + 14,260 = 17,112 (the final **product**)

Example 3

411 × 242

```
   411
 × 242
   822
 16,440
+82,200
 99,462
```

Steps

1. Multiply 411 × 2.
   - 2 × 1 = 2
   - 2 × 1 = 2
   - 2 × 4 = 8
2. Multiply 411 × 4.
   - 4 × 1 = 4 (remember to use a zero for a placeholder)
   - 4 × 1 = 4
   - 4 × 4 = 16
3. Multiply 411 × 2.
   - 2 × 1 = 2
   - 2 × 1 = 2
   - 2 × 4 = 8
4. Add the three products together.
   - 822 + 16,440 + 82,200 = 99,462 (the final **product**)

SAMPLE PROBLEMS

Multiply each of the following problems as indicated.

1. 246 × 4 =
2. 689 × 2 =

3. $821 \times 22 =$
4. $529 \times 47 =$
5. $954 \times 75 =$
6. $262 \times 94 =$
7. $123 \times 529 =$
8. $2{,}943 \times 293 =$
9. It takes 18 man-hours for an automobile factory to build a car. How many man-hours does it take to build 92 cars?
10. A bakery sells boxes of donuts with 13 donuts in each box. If the bakery sells 52 boxes of donuts, how many total donuts were sold?

Basic Division (Whole Numbers)

Dividend: The number being divided.
Divisor: The number by which the dividend is divided.
Quotient: The answer to a division problem.
Remainder: The portion of the dividend that is not evenly divisible by the divisor.

HESI Hint

$$5\overline{)45}\ 9$$

The 45 represents the **dividend** (the number being divided), the 5 represents the **divisor** (the number by which the dividend is divided), and the 9 represents the **quotient** (the answer to the division problem). It is best not to leave a division problem with a **remainder,** but to end it as a fraction or decimal instead. To make the problem into a decimal, add a decimal point and zeros at the end of the dividend and continue. If a remainder continues to occur, round to the hundredths place.
Example
233.547 → 233.55 (the 7 rounds the 4 to a 5)

Example 1

$42 \div 7$

$$\begin{array}{r} 6 \\ 7\overline{)42} \\ \underline{-42} \\ 0 \end{array}$$

Steps

1. Set up the problem.
2. Use a series of multiplication and subtraction problems to solve a division problem.
3. $7 \times ? = 42$
   - Multiply: $7 \times 6 = 42$
   - Subtract: $42 - 42 = 0$
   - The quotient (or answer) is 6.

Example 2

464 ÷ 4

$$\begin{array}{r} 116 \\ 4\overline{)464} \\ \underline{-4}\downarrow\downarrow \\ 06\downarrow \\ \underline{-4}\downarrow \\ 24 \\ \underline{-24} \\ 0 \end{array}$$

Steps

1. Set up the problem.
2. Begin with the hundreds place.
   - $4 \times ? = 4$. We know $4 \times 1 = 4$; therefore, place the 1 (quotient) above the 4 in the hundreds place (dividend). Place the other 4 under the hundreds place and subtract: $4 - 4 = 0$.
   - Bring down the next number, which is 6; $4 \times ? = 6$. There is no number that can be multiplied by 4 that will equal 6 exactly, so try to get as close as possible without going over 6. Use $4 \times 1 = 4$ and set it up just like the last subtraction problem: $6 - 4 = 2$.
   - Bring down the 4 from the dividend, which results in the number 24 (the 2 came from the remainder of $6 - 4 = 2$).
   - $4 \times ? = 24$; $? = 6$. The two becomes the next number in the quotient. $24 - 24 = 0$. There is no remainder left.
   - The quotient (or answer) is 116.

Example 3

163 ÷ 5

$$\begin{array}{r} 32.6 \\ 5\overline{)163.0} \\ \underline{-15}\downarrow\downarrow \\ 13\downarrow \\ \underline{-10}\downarrow \\ 30 \\ \underline{-30} \\ 0 \end{array}$$

Steps

1. Set up the problem.
2. The 5 (divisor) does not divide into 1 but does divide into 16.
3. $5 \times 3 = 15$. Write the 3 in the quotient. (It is written above the 6 in 16 because that is the last digit in the number.)
   - $5 \times 3 = 15$
   - $16 - 15 = 1$
4. Bring the 3 down. Combine the 1 (remainder from $16 - 15$) and 3 to create 13.
5. The 5 does not divide evenly into 13; therefore, try to get close without going over.
   - $5 \times 2 = 10$
   - $13 - 10 = 3$

6. There is a remainder of 3, but there is no number left in the dividend. Add a decimal point and zeros and continue to divide.
7. The quotient (or answer) is 32.6 (thirty-two and six tenths).

SAMPLE PROBLEMS

Divide in each of the following problems as indicated.

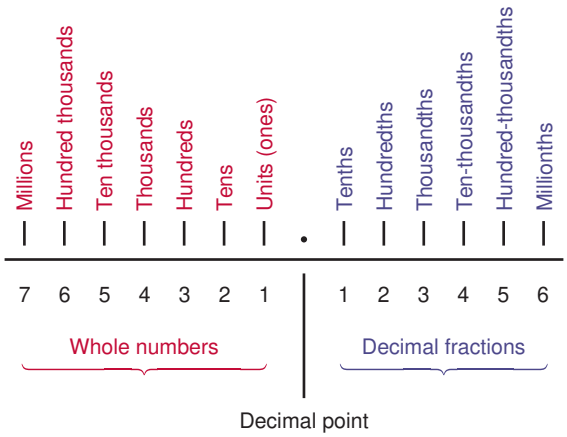
1. 135 ÷ 9 =
2. 8,400 ÷ 4 =
3. 3,732 ÷ 2 =
4. 357 ÷ 17 =
5. 4,725 ÷ 9 =
6. 2,925 ÷ 3 =
7. 787 ÷ 8 =
8. 6,281 ÷ 5 =
9. There are 72 bottles of water for a soccer team with 16 players. If each player receives the same amount of water, how many bottles of water can each player have?
10. Tim is driving 270 miles to visit his parents. Tim's car can travel 18 miles on 1 gallon of gas. How many gallons of gas does Tim need to make it to his parents' house?

Decimals

A decimal pertains to tenths or to the number 10.

Place value: Regarding decimals, numbers to the right of the decimal point have different terms from the whole numbers to the left of the decimal point. Each digit in a number occupies a position called a **place value.**

Addition and Subtraction of Decimals



HESI Hint

Remember, whole numbers are written to the left of the decimal point and place values are written to the right of the decimal point. Line the numbers up vertically before solving the problem.

HESI Hint

The word "and" stands for the decimal when writing a number in words.
Example: 5.7 (five *and* seven tenths)

Example 1

1.3 + 7.2

$$\begin{array}{r} 1.3 \\ +\ 7.2 \\ \hline 8.5 \end{array}$$

Steps

1. Align the decimal points.
2. Add the tenths together: 3 + 2 = 5
3. Add the ones together: 1 + 7 = 8
4. Final answer: 8.5 (eight and five tenths).

Example 2

6 + 22.13

$$\begin{array}{r} 22.13 \\ +\ 6.00 \\ \hline 28.13 \end{array}$$

Steps

1. Align the decimal points.
   - It might be difficult to align the 6 because it does not have a decimal point.
   - Remember that after the ones place, there is a decimal point. To help with organization, add zeros (placeholders). **Example:** 6 = 6.00
2. Add the hundredths: 3 + 0 = 3
3. Add the tenths: 1 + 0 = 1
4. Add the ones: 2 + 6 = 8
5. Add the tens: 2 + 0 = 2
6. Final answer: 28.13 (twenty-eight and thirteen hundredths).

Example 3

9.46 − 7.34

$$\begin{array}{r} 9.46 \\ -\ 7.34 \\ \hline 2.12 \end{array}$$

Steps

1. Align the decimal points.
2. Subtract the hundredths: 6 − 4 = 2
3. Subtract the tenths: 4 − 3 = 1
4. Subtract the ones: 9 − 7 = 2
5. Final answer: 2.12 (two and twelve hundredths).

Example 4

22 − 7.99

$$\begin{array}{r} {\scriptstyle 1\ \ 9\,10} \\ 2\not{2}.\not{0}\not{0} \\ -7.9\not{9} \\ \hline 14.01 \end{array}$$

Steps

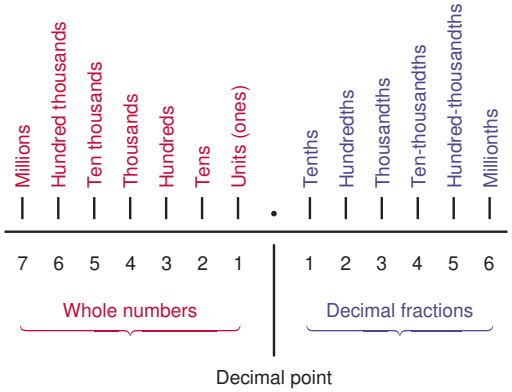
1. Align the decimal points.
2. Because 22 is a whole number, add a decimal point and zeros.
3. 0.00 − 0.99 cannot be subtracted; therefore, 1 must be borrowed from the 22 and regrouped.
4. The ones become 1, the tenths become 9, and the hundredths become 10.
5. Subtract the hundredths: 10 − 9 = 1
6. Subtract the tenths: 9 − 9 = 0
7. Subtract the ones: 21 − 7 = 14
   - 1 was borrowed from the tens in order to subtract the 7.
8. Final answer: 14.01 (fourteen and one hundredth).

SAMPLE PROBLEMS

Solve each of the following decimal problems as indicated.

1. 2.7 + 8.44 =
2. 7.125 + 38.65 =
3. 127.2 + 83 =
4. 43 + 9.57 =
5. 4.6 + 3.66 + 11 =
6. 22 – 14.18 =
7. 67.74 − 9.83 =
8. 17.56 – 9.88 =
9. Robin used the company car for 3 business trips. She traveled 3.2 miles on Monday, 4.75 miles on Wednesday, and 7.3 miles on Friday. How many total miles did Robin travel for the week?
10. A dressmaker needs 4.25 yards of fabric to make a blue dress, but only 2.5 yards of blue fabric are left in stock. How many additional yards of fabric does the dressmaker need to complete the dress?

Multiplication of Decimals



Example 1

81.2 + 4.2

81.2 × 4.2 1624 + 32480 341.04	1 decimal place + 1 decimal place 2 decimal places Move the decimal point two places to the left in the final product.

Steps

1. Multiply 812 × 42 (do not worry about the decimal point until the final product has been calculated).
2. Starting from the right, count the decimal places in both numbers and add together (two decimal places).
3. Move to the left two places, and then place the decimal point.

Example 2

0.002 + 3.4

0.002 × 3.4 0008 + 00060 0.0068	3 decimal places + 1 decimal place 4 decimal places Move four places to the left.

Steps

1. Multiply 2 × 34.
2. Starting from the right, count the decimal places in both numbers and add together (four decimal places).
3. Move to the left four places, and then place the decimal.

Example 3

8.23 × 3

8.23 × 3 24.69	2 decimal places + 0 decimal places 2 decimal places Move two places to the left.

Steps

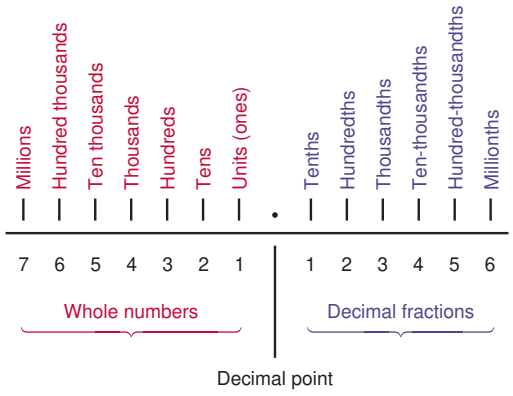
1. Multiply 823 × 3.
2. Starting from the right, count the decimal places in both numbers and add together (two decimal places).
3. Move to the left two places, and then place the decimal point.

SAMPLE PROBLEMS

Multiply the decimals in the following problems as indicated.

1. $0.012 \times 3.27 =$
2. $22.79 \times 6 =$
3. $4.04 \times 6.1 =$
4. $321.1 \times 18 =$
5. $0.9112 \times 8.13 =$
6. $21.2 \times 7 =$
7. $0.009 \times 45.2 =$
8. $245.24 \times .003 =$
9. Tina's water bottle holds 24.6 ounces of liquid. If Tina drinks 2.5 bottles of water, how much water has she drunk?
10. Dennis walks 1.2 miles to the store to buy groceries. He walks another 1.2 miles to bring the groceries home. How many miles does Dennis have to walk to complete 2 grocery trips?

Division of Decimals



HESI Hint

The number 25 is a whole number. Though this number could be written 25.0, decimals are usually not displayed after a whole number.

Example 1

$48 \div 2.5$

$$
\begin{array}{r}
19.2 \\
2.5\overline{)\,48.0.0} \\
-25\downarrow\downarrow \\
\hline
230\downarrow \\
-225\downarrow \\
\hline
50 \\
-50 \\
\hline
0
\end{array}
$$

Steps

1. Set up the division problem.
2. Move the decimal point in 2.5 one place to the right, making it a whole number.

3. What is done to one side must be done to the other side. Move the decimal point one place to the right in 48, making it 480, and then bring the decimal point up into the quotient.
4. Divide normally.
   - $25 \times 1 = 25$
   - Subtract: $48 - 25 = 23$
   - Bring down the zero to make 230.
   - $25 \times 9 = 225$. This is as close to 230 as possible without going over.
   - Subtract: $230 - 225 = 5$
   - Add a zero to the dividend and bring it down to the 5, making it 50.
   - $25 \times 2 = 50$
   - $50 - 50 = 0$
5. The quotient is 19.2.

Example 2

$2.468 \div 0.2$

Steps

1. Set up the division problem.
2. Move the decimal point in 0.2 over one place to the right, making it a whole number. 0.2 is now 2.
3. Move the same number of spaces in the dividend. 2.468 is now 24.68.
4. Bring the decimal point up to the quotient in the new position.
5. Divide normally.

Example 3

$0.854 \div 0.05$

$$
\begin{array}{r}
17.08 \\
0.05\overline{)\,0.85.40} \\
-5\downarrow\downarrow\downarrow \\
\quad 35\downarrow\downarrow \\
\quad -35\downarrow\downarrow \\
\quad\quad 040 \\
\quad\quad -40 \\
\quad\quad\quad 0
\end{array}
$$

Steps

1. Set up the division problem.
2. Move the decimal point in the divisor until it is a whole number. 0.05 is now 5.
3. Move the decimal point in the dividend the same number of spaces as was moved in the divisor. 0.854 is now 85.4.
4. Divide normally.

SAMPLE PROBLEMS

Divide the decimals in the following problems as indicated.

1. $36 \div 0.8 =$
2. $54 \div 0.3 =$
3. $65.6 \div 0.8 =$
4. $43.54 \div 0.7 =$

5. 5.202 ÷ 0.45 =
6. 912 ÷ 0.2 =
7. 0.052 ÷ 0.8 =
8. 0.624 ÷ 0.26 =
9. Dan's pie crust recipe calls for 1.25 ounces of canola oil per crust. How many pie crusts can Dan make if he has 10 ounces of canola oil?
10. A carpenter is cutting a board to make shelves for a bookcase. The board is 2.44 meters long and 0.3 meters wide. Each shelf will be 0.61 meters long and 0.3 meters wide. How many shelves will the bookcase have?

Fractions

In mathematics, a fraction is a way to express a part in relation to the total.
Numerator: The top number in a fraction.
Denominator: The bottom number in a fraction.
Fraction Bar: The line between the numerator and denominator. The bar is another symbol for division.
Factor: A number that divides evenly into another number.
Least Common Denominator (LCD): The smallest multiple that two numbers share.
Improper Fraction: A fraction where the numerator is larger than the denominator.
Proper Fraction: A fraction where the denominator is larger than the numerator.
Common Denominator: Two or more fractions having the same denominator.
Reciprocals: Pairs of numbers that equal 1 when multiplied together.
Terminating Decimal: A decimal that is not continuous.

HESI Hint

- The **numerator** is the top number of the fraction. It represents the part or pieces.
- The **denominator** is the bottom number of the fraction. It represents the total or whole amount.
- The fraction bar is the line that separates the numerator and the denominator.

$$\frac{\text{Numerator (part)}}{\text{Denominator (whole)}} \quad \text{Fraction bar}$$

Reducing Fractions Using the Greatest Common Factor

A **factor** is a number that divides evenly into another number.

Factors of 12:

- $1 \times 12 = 12$
- $2 \times 6 = 12$
- $3 \times 4 = 12$

12 {1, 2, 3, 4, 6, 12}: Listing the factors helps determine the greatest common factor between two or more numbers.

$$\frac{1}{2} = \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \frac{9}{18}, \frac{10}{20}$$

All represent one-half.

Reducing fractions can also be called reducing a fraction to its lowest terms or simplest form. A fraction is reduced to the lowest terms by finding an equivalent fraction in which the numerator and denominator are as small as possible. You may need to reduce fractions to work with them in an equation or

for solving a problem. That means that there is no number, except 1, that can be divided evenly into both the numerator and the denominator.

$$1 = \frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{7}{7}, \frac{8}{8}, \frac{9}{9}, \frac{10}{10}$$

Example 1

Reduce $\frac{4}{24}$

Factors of 4 and 24:
4 {1, 2, 4}
24 {1, 2, 4, 6, 8, 12}
The greatest common factor is 4; therefore, divide the numerator and denominator by 4.

$$\frac{4}{24} \div \frac{4}{4} = \frac{1}{6}$$

Example 2

Reduce $\frac{12}{20}$

Factors of 12 and 20:
12 {1, 2, 3, **4,** 6, 12}
20 {1, 2, **4,** 5, 10, 20}
The greatest common factor is 4 (they do have 1 and 2 in common, but the greatest factor is best).

$$\frac{12}{20} \div \frac{4}{4} = \frac{3}{5}$$

Least Common Denominator

The LCD is the smallest multiple that two numbers share. Determining the LCD is an essential step in the addition, subtraction, and ordering of fractions.

Example 1

Find the LCD for $\frac{3}{4}$ and $\frac{7}{9}$

Steps

1. List the multiples (multiplication tables) of each denominator.
   - 4: $4 \times 1 = 4$, $4 \times 2 = 8$, $4 \times 3 = 12$, $4 \times 4 = 16$, $4 \times 5 = 20$, $4 \times 6 = 24$, $4 \times 7 = 28$, $4 \times 8 = 32$, $4 \times 9 = 36$, $4 \times 10 = 40$
   - 4 {4, 8, 12, 16, 20, 24, 28, 32, 36, 40}—this will be the standard form throughout for listing multiples.
   - 9 {9, 18, 27, 36, 45, 54, 63, 72, 81, 90}
2. Compare each for the least common multiple.
   - 4 {4, 8, 12, 16, 20, 24, 28, 32, **36,** 40}
   - 9 {9, 18, 27, **36,** 45, 54, 63, 72, 81, 90}
3. The LCD between 4 and 9 is 36 ($4 \times 9 = 36$ and $9 \times 4 = 36$).

Example 2

Find the LCD for $\frac{1}{15}$ and $\frac{2}{3}$.

Steps

1. List the multiples of each denominator, and find the common multiples.
   - 15 {**15**, 30, 45, 60, 75, 90, 105, 120, 135, 150}
   - 3 {3, 6, 9, 12, **15**, 18, 21, 24, 27, 30}
2. The LCD between 15 and 3 is 15 ($15 \times 1 = 15$ and 3×5 is 15).

Changing Improper Fractions into Mixed Numbers

An **improper fraction** occurs when the numerator is larger than the denominator. An improper fraction should be reduced and made into a mixed number.

Example

$$\frac{17}{5} \rightarrow 5\overline{)17}^{3} \rightarrow 3\frac{2}{5}$$
$$\underline{15}$$
$$2$$

Steps

1. Turn an improper fraction into a mixed number through division. (The top number [numerator] goes in the box [17]; the bottom number [denominator] stays out [5].)
2. The 3 becomes the whole number.
3. The remainder (2) becomes the numerator.
4. The denominator stays the same.

Changing Mixed Numbers into Improper Fractions

A mixed number has a whole number and fraction combined.

Example

$$5\frac{2}{3} \rightarrow 5\frac{+2}{3} = (5 \times 3) + 2 = 17 \rightarrow \frac{17}{3}$$

Steps

1. To make a mixed number into an improper fraction, multiply the denominator (3) and whole number (5) together, then add the numerator (2).
2. Place this new numerator (17) over the denominator (3), which stays the same in the mixed number.

Addition of Fractions

Addition with Common Denominators

Example

$$\frac{1}{9} + \frac{7}{9} = \frac{8}{9}$$

Steps

1. Add the numerators together: $1 + 7 = 8$.
2. The denominator (9) stays the same. This makes it a **common denominator.**
3. Answer: $\frac{8}{9}$ (eight ninths).

Addition with Unlike Denominators

Example

$$\frac{1}{3}+\frac{4}{9}$$

$$\frac{1\times 3}{3\times 3}=\frac{3}{9}$$

$$\frac{4\times 1}{9\times 1}=\frac{4}{9}$$

$$\frac{3}{9}+\frac{4}{9}=\frac{7}{9}$$

Steps

1. Find the LCD by listing the multiples of each denominator.
   - 3 {3, 6, **9**, 12, 15}
   - 9 {**9**, 27, 36, 45}
   - The LCD is 9.
2. If the denominator is changed, the numerator must also be changed by the same number. Do this by multiplying the numerator and denominator by the same number.

$$\frac{1\times 3}{3\times 3}=\frac{3}{9}$$

3. Because the denominator of the second fraction is 9, no change is necessary.
4. Add the numerators together, and keep the common denominator.
5. Reduce the fraction if necessary.

Addition of Mixed Numbers

Example

$$2\frac{1}{3}+3\frac{11}{15}$$

$$2\frac{1\times 5}{3\times 5}=2\frac{5}{15}$$

$$3\frac{11\times 1}{15\times 1}=3\frac{11}{15}$$

$$2\frac{5}{15}+3\frac{11}{15}=5\frac{16}{15}=6\frac{1}{15}$$

Steps

1. Find the LCD of 3 and 15 by listing the multiples of each.
   - 3 (3, 6, 9, 12, **15**)
   - 15 (**15**, 30, 45)
2. Calculate the new numerator of each fraction to correspond to the changed denominator.

3. Add the whole numbers together, and then add the numerators together. Keep the common denominator 15.
4. The numerator is larger than the denominator (improper); change the answer to a mixed number (review vocabulary if necessary).

SAMPLE PROBLEMS

Add the fractions in the following problems as indicated (remember to reduce the fraction as needed).

1. $\frac{2}{10} + \frac{5}{10} =$
2. $\frac{4}{18} + \frac{11}{18} =$
3. $\frac{4}{6} + \frac{7}{6} =$
4. $\frac{5}{8} + \frac{1}{16} =$
5. $\frac{7}{9} + \frac{6}{11} =$
6. $6\frac{2}{3} + 4\frac{5}{12} =$
7. $2\frac{2}{7} + 3\frac{2}{9} =$
8. $5\frac{1}{6} + 3\frac{2}{18} =$
9. Ray is driving Tom to the movies. Tom lives 2½ miles from Ray's house. The movie theatre is 1¾ miles from Tom's house. How many miles must Ray drive to pick up Tom and drive to the theatre?
10. The Warrens are installing a privacy fence along two sides of their property. The length of one side is 85⅓ feet. The length of the other side is 62⅚ feet. How many feet of fencing will the Warrens install?

Subtraction of Fractions

Subtracting Fractions with Common Denominators

Example

$$\frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

Steps

1. Subtract the numerators: $(5 - 1 = 4)$
2. Keep the common denominator.
3. Reduce the fraction by dividing by the greatest common factor:

$$\frac{4}{6} \div \frac{2}{2} = \frac{2}{3}$$

Subtracting Fractions with Unlike Denominators

Example

$$\frac{5}{12} - \frac{1}{8} = ?$$

$$\frac{5 \times 2}{12 \times 2} = \frac{10}{24}$$

$$\frac{1 \times 3}{8 \times 3} = \frac{3}{24}$$

$$\frac{10}{24} - \frac{3}{24} = \frac{7}{24}$$

Steps

1. Find the LCD by listing the multiples of each denominator.
   - 12 {12, **24**, 36, 48}
   - 8 {8, 16, **24**, 32}
   - The LCD is 24.
2. Change the numerator to reflect the new denominator. (What is done to the bottom must be done to the top of a fraction.)
3. Subtract the new numerators: 10 − 3 = 7. The denominator stays the same.

Borrowing from Whole Numbers

Example

$$4\frac{2}{3} - 3\frac{5}{7}$$

$$4\frac{2 \times 7}{3 \times 7} = 4\frac{14}{21}$$

$$^{3}\not{4}\frac{14}{21} + \frac{21}{21} = 3\frac{35}{21}$$

$$3\frac{5 \times 3}{7 \times 3} = 3\frac{15}{21}$$

$$3\frac{35}{21} - 3\frac{15}{21} = \frac{20}{21}$$

Steps

1. Find the LCD.
2. Fifteen cannot be subtracted from 14; therefore, 1 must be borrowed from the whole number, making it 3, and the borrowed 1 must be added to the fraction.
3. Add the original numerator to the borrowed numerator: 14 + 21 = 35.
4. Now the whole number and the numerator can be subtracted.

HESI Hint

Fractions as a whole:

$$\frac{15}{15} = 1 \text{ (one whole)}$$

Notice in the example under "Borrowing from Whole Numbers" that we added 15 to both the numerator and the denominator. We did this because it is one whole and it is the same denominator.

SAMPLE PROBLEMS

Subtract the fractions in the following problems as indicated.

1. $\frac{5}{7} - \frac{2}{7} =$
2. $\frac{18}{24} - \frac{5}{24} =$
3. $\frac{2}{4} - \frac{3}{16} =$
4. $\frac{27}{56} - \frac{3}{7} =$
5. $1\frac{2}{5} - \frac{1}{2} =$
6. $16\frac{8}{9} - 5\frac{2}{3} =$
7. $11\frac{1}{5} - 8\frac{4}{5} =$
8. $20\frac{1}{2} - 4\frac{2}{3} =$
9. Terry has 8¼ feet of wood trim. He needs 5⅓ feet of trim to decorate one wall. After cutting the wood to size, how much trim will Terry have left?
10. Jordan has a gallon of milk, which is equal to 16 cups. She uses 1¼ cups of milk to make pancakes. How much milk does Jordan have left after making pancakes?

Multiplication of Fractions

HESI Hint

"Multiplying fractions is no problem. Top times top and bottom times bottom"; for example,

Top × Top and Bottom × Bottom.

To change an improper fraction into a mixed number, divide the numerator by the denominator.

$$\frac{20}{13} \rightarrow 13\overline{)\begin{array}{r}1\\20\\13\\\hline 07\end{array}} \rightarrow 1\frac{7}{13}$$

The quotient becomes the whole number. The remainder becomes the numerator, and the denominator stays the same.

Example 1

$$\frac{2}{3}\times\frac{5}{8}$$

$$\frac{2}{3}\times\frac{5}{8}=\frac{10}{24}=\frac{5}{12}$$

Steps

1. Multiply the numerators together: $2 \times 5 = 10$.
2. Multiply the denominators together: $3 \times 8 = 24$.
3. Reduce the product by using the greatest common factor: $\frac{10}{24}\div\frac{2}{2}=\frac{5}{12}$

Example 2

$$5\times\frac{4}{13}$$

$$\frac{5}{1}\times\frac{4}{13}=\frac{20}{13}=1\frac{7}{13}$$

Steps

1. Make the whole number 5 into a fraction by placing a 1 as the denominator.
2. Multiply the numerators: $5 \times 4 = 20$.
3. Multiply the denominators: $1 \times 13 = 13$.
4. Change the improper fraction into a mixed number.

Example 3

$$3\frac{1}{6}\times6\frac{5}{8}$$

$$\frac{19}{6}\times\frac{53}{8}=\frac{1,007}{48}$$

$$\frac{1,007}{48}=20\frac{47}{48}$$

Steps

1. Change the mixed numbers into improper fractions.

$$3\frac{+1}{\times6}=(3\times6)+1=19\rightarrow\frac{19}{6}$$

$$6\frac{+5}{\times8}=(6\times8)+5=53\rightarrow\frac{53}{8}$$

2. Multiply the numerators and denominators together.
   - $19 \times 53 = 1{,}007$ (numerator)
   - $6 \times 8 = 48$ (denominator)
   - Change the improper fraction into a mixed number.

$$48\overline{)1007}\begin{array}{l}20\\-96\\\hline 47\end{array}=20\frac{47}{48}$$

SAMPLE PROBLEMS

Multiply the following fractions and reduce the product to the lowest form and/or mixed fraction (also referred to as the common denominator).

1. $\frac{3}{4} \times \frac{3}{6} =$
2. $\frac{2}{3} \times \frac{2}{8} =$
3. $5 \times \frac{7}{6} =$
4. $2\frac{6}{7} \times 7 =$
5. $5\frac{5}{8} \times 2\frac{5}{6} =$
6. $3\frac{1}{6} \times 1\frac{4}{5} =$
7. $2\frac{3}{4} \times 3 =$
8. $1\frac{3}{9} \times 3\frac{1}{4} =$
9. A baseball pitcher throws an average of 15 pitches per inning. This pitcher throws an average of $6\frac{1}{3}$ innings per game. What is the average number of pitches per game for this pitcher?
10. Kelly's smart phone has a battery life of $10\frac{1}{6}$ hours. She buys a new phone with a battery life that is $1\frac{1}{2}$ times longer than her current phone. How many hours of battery life does her new phone have?

Division of Fractions

HESI Hint

"Dividing fractions, don't ask why, inverse the second fraction and then multiply."

Example:

$$\frac{1}{2} \div \frac{3}{8} \text{ Inverse } \frac{3}{8} \rightarrow \frac{8}{3}$$

$$\text{Then multiply } \frac{1}{2} \times \frac{8}{3}$$

$$\frac{1}{2} \times \frac{8}{3} = \frac{8}{6}$$

Write as an improper fraction: $1\frac{2}{6}$ then reduce to lowest form: $1\frac{1}{3}$

$$\frac{3}{8} \rightarrow \frac{8}{3} \; \frac{3}{8} \times \frac{8}{3} = \frac{24}{24} = 1$$

These two numbers ($\frac{3}{8}$ and $\frac{8}{3}$) are **reciprocals** of each other because when they are multiplied together, they equal 1.

Example 1

$$\frac{1}{3} \div \frac{3}{4}$$

$$\frac{1}{3} \div \frac{3}{4}$$
$$\frac{1}{3} \times \frac{4}{3} = \frac{4}{9}$$

Steps

1. Inverse (or take the reciprocal) of the second fraction: $\frac{3}{4} \rightarrow \frac{4}{3}$.
2. Rewrite the new problem and multiply.
   - $1 \times 4 = 4$ (numerator)
   - $3 \times 3 = 9$ (denominator)

Example 2

$$1\frac{5}{6} \div \frac{3}{4}$$

$$1\frac{5}{6} \div \frac{3}{4}$$
$$\frac{11}{6} \div \frac{3}{4}$$
$$\frac{11}{6} \times \frac{4}{3} = \frac{44}{18}$$
$$2\frac{8}{18} = 2\frac{4}{9}$$

Steps

1. Change the mixed number into an improper fraction: $1\frac{5}{6} = (1 \times 6) + 5 = \frac{11}{6}$.
2. Rewrite the new problem with the improper fraction.
3. Inverse the second fraction.
4. Multiply the numerators and the denominators together.
   - $11 \times 4 = 44$ (numerators)
   - $6 \times 3 = 18$ (denominators)
5. Change the improper fraction into a mixed number. Reduce the mixed number.

Example 3

$$9 \div 3\frac{2}{3}$$

$$\frac{9}{1} \div \frac{11}{3}$$
$$\frac{9}{1} \times \frac{3}{11} = \frac{27}{11}$$
$$2\frac{5}{11}$$

Steps

1. Change the whole number into a fraction and the mixed number into an improper fraction.
2. Inverse the second fraction.
3. Multiply the numerators and then denominators together.
   - $9 \times 3 = 27$
   - $1 \times 11 = 11$
4. Change the improper fraction into a mixed number.

SAMPLE PROBLEMS

Divide the fractions in the following problems and reduce to the lowest common denominator.

1. $\frac{2}{3} \div \frac{1}{5} =$
2. $\frac{5}{12} \div \frac{2}{8} =$
3. $\frac{3}{8} \div \frac{2}{6} =$
4. $2 \div \frac{1}{7} =$
5. $6 \div \frac{2}{5} =$
6. $6\frac{2}{3} \div \frac{1}{3} =$
7. $7 \div 2\frac{1}{3} =$
8. $16\frac{2}{5} \div 4 =$
9. Gary has 1½ pounds of chicken. A recipe for chicken tacos allows for ⅛ pounds of chicken for each taco. How many tacos can Gary make?
10. Patty makes decorative wreaths. She has 31¼ feet of ribbon to make bows for her wreaths. Each bow requires 6¼ feet of ribbon. How many wreaths can Patty make?

Changing Fractions to Decimals

HESI Hint

"Top goes in the box, the bottom goes out."

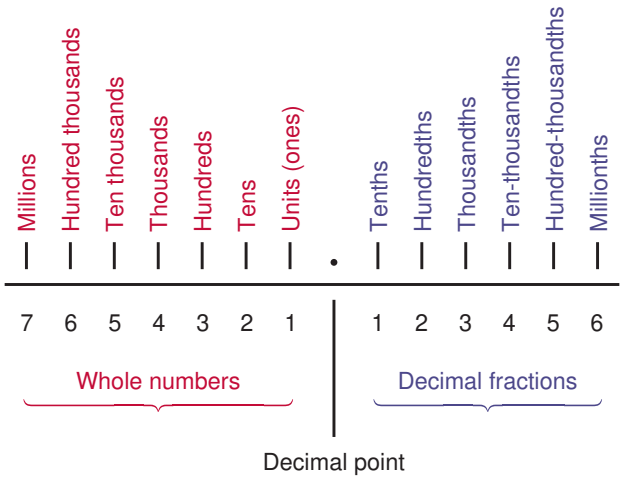
This is a helpful saying in remembering that the numerator is the dividend and the denominator is the divisor.

If the decimal does not terminate, continue to the thousandths place and then round to the hundredths place.

Example:

7.8666 → 7.87

If the number in the thousandths place is 5 or greater, round the number in the hundredths place to the next higher number. However, if the number in the thousandths place is less than 5, do not round up the number in the hundredths place.



Example 1

Change $\frac{1}{4}$ to a decimal.

$$
\begin{array}{r}
0.25 \\
4\overline{)1.00} \\
\underline{-8}\downarrow \\
20 \\
\underline{-20} \\
0
\end{array}
$$

Steps

1. Change the fraction into a division problem.
2. Add a decimal point after the 1 and add two zeros.
   - Remember to raise the decimal into the quotient area.
3. The answer is a **terminating decimal** (a decimal that is not continuous); therefore, adding additional zeros is not necessary.

Example 2

Change $\frac{5}{8}$ to a decimal.

$$
\begin{array}{r}
0.625 \\
8\overline{)5.000} \\
\underline{-48}\downarrow\downarrow \\
20\downarrow \\
\underline{-16}\downarrow \\
40 \\
\underline{-40} \\
0
\end{array}
$$

Steps

1. Change the fraction into a division problem.
2. Add a decimal point after the 5 and add two zeros.
   - Remember to raise the decimal into the quotient area.

3. If there is still a remainder, add another zero to the dividend and bring it down.
4. The decimal terminates at the thousandths place.

Example 3

Change $\frac{1}{6}$ to a decimal.

$$
\begin{array}{r}
0.1666 \\
6\overline{)1.0000} \\
\underline{-6}\downarrow\downarrow\downarrow \\
40\downarrow\downarrow \\
\underline{-36}\downarrow\downarrow \\
40\downarrow \\
\underline{-36}\downarrow \\
40
\end{array}
$$

Steps

1. Change the fraction into a division problem.
2. After the 1, add a decimal point and two zeros.
3. The decimal continues (does not terminate); therefore, round to the hundredths place: 0.1666 → 0.167. (It can also be written as $0.1\overline{6}$. The line is placed over the number that repeats.)

Example 4

Change $3\frac{3}{4}$ to a decimal.

$$
\begin{array}{r}
0.75 \\
4\overline{)3.00} \\
\underline{-28}\downarrow \\
20 \\
\underline{-20} \\
0
\end{array}
$$

Steps

1. Change the fraction into a division problem.
2. After the 3, add a decimal and two zeros.
3. Place the whole number in front of the decimal: 3.75.

SAMPLE PROBLEMS

Change the following fractions into decimals and round to the nearest thousandth.

1. $\frac{1}{9}$
2. $\frac{1}{8}$
3. $\frac{5}{8}$
4. $\frac{2}{4}$

5. $\frac{2}{5}$
6. $1\frac{1}{4}$
7. $\frac{3}{12}$
8. $3\frac{3}{5}$
9. $9\frac{3}{16}$
10. $\frac{17}{25}$

Changing Decimals to Fractions

Example 1

Change 0.9 to a fraction.

$$0.9 \rightarrow \frac{9}{10}$$

Steps

Knowing place values makes it very simple to change decimals to fractions.
1. The last digit is located in the tenths place; therefore, the 9 becomes the numerator.
2. 10 becomes the denominator.

Example 2

Change 0.02 to a fraction.

$$0.02 \rightarrow \frac{2}{100} = \frac{1}{50}$$

Steps

1. The 2 is located in the hundredths place.
2. The numerator becomes 2, and 100 becomes the denominator.
3. Reduce the fraction.

Example 3

Change 0.75 to a fraction.

$$0.75 \rightarrow \frac{75}{100} = \frac{3}{4}$$

Steps

1. Always look at the last digit in the decimal. In this example, the 5 is located in the hundredths place.
2. The numerator becomes 75, and 100 becomes the denominator.
3. Reduce the fraction.

Example 4

Change 2.045 into a fraction.

$$2.045 \rightarrow 2\frac{45}{1000} \rightarrow 2\frac{9}{200}$$

Steps

1. The 5 is located in the thousandths place.
2. The numerator becomes 45, and 1,000 becomes the denominator. The 2 is still the whole number.
3. Reduce the fraction.

SAMPLE PROBLEMS

Change the following decimals into fractions and reduce to the lowest form.

1. 0.07 =
2. 0.02 =
3. 0.175 =
4. 0.22 =
5. 0.8 =
6. 4.25 =
7. 6.4 =
8. 10.6667 =
9. 8.24 =
10. 0.075 =

Ratios and Proportions

Ratio: A relationship between two numbers.
Proportion: Two ratios that have equal values.

HESI Hint

Ratios can be written several ways.
As a fraction: $\frac{5}{12}$
Using a colon: 5:12
In words: 5 to 12
Proportions can be written two ways.

$$\frac{5}{12} = \frac{25}{60}$$

$$5:12::25:60$$

NOTE: The numerator is listed first, then the denominator (known as the foil method).

Example 1

Change the decimal to a ratio.

$$.012 \rightarrow \frac{12}{1000} \rightarrow \frac{3}{250} \rightarrow 3:250$$

Steps

1. Change the decimal to a fraction.
2. Reduce the fraction.
3. The numerator (3) is the first listed number.
4. Then write the colon.
5. Finally, place the denominator (250) after the colon.

Example 2

Change the fraction to a ratio.

$$\frac{4}{5} = 4:5$$

Steps

1. The numerator (4) is the first listed number.
2. Then write the colon.
3. Finally, place the denominator (5) after the colon.

Example 3

Solve the proportion (find the value of x).

$2:7 :: 8:x$

$$2:7 :: 8:x$$
$$\frac{2}{7} = \frac{8}{x}$$
$$\frac{2}{7} \overset{x4}{=} \frac{8}{x}$$
$$\frac{2}{7} = \frac{8}{x}$$
$$x = 28$$

Steps

1. Rewrite the proportion as a fraction (this might help to see the solution).
2. Note that $2 \times 4 = 8$; therefore, $7 \times 4 = 28$.
   - Multiply 7×8 (two diagonal numbers). The answer is 56.
   - $56 \div 2 = 28$ (Divide the remaining number.)
3. The answer is 28.

Example 4

Solve the proportion (find the value of x).

$x:28 :: 12:48$

$$\frac{x}{28} = \frac{12}{48}$$
$$\frac{x}{28} = \frac{12}{48}$$
$$12 \times 28 = 336$$
$$336 \div 48 = 7$$
$$x = 7$$

Steps

1. Rewrite the proportion as a fraction.
2. Multiply the diagonal numbers: $12 \times 28 = 336$.

3. Divide the answer (336) by the remaining number: 336 ÷ 48 = 7.
4. The value for x is 7.

Example 5

Solve the proportion (find the value of x).
240:60 :: x:12.

$$\frac{240}{60} = \frac{x}{12}$$

$$\frac{240}{60} = \frac{x}{12}$$

$$x = 48$$

HESI Hint

An example of the foil method is to remember "inside x inside and outside x outside."
240:60 :: x:12
60 times x :: 240 x 12
60x :: 2,880
Divide 60 by both sides to get x by itself
x :: 48

Steps

1. Rewrite the proportion as a fraction.
2. Multiply the diagonal numbers together: 240 × 12 = 2,880.
3. Divide the answer (2,880) by the remaining number: 2,880 ÷ 60 = 48.
4. The answer to x is 48.

SAMPLE PROBLEMS

Change the following fractions to ratios.

1. $\frac{12}{43}$
2. $\frac{7}{30}$

Solve the following for x:

3. 9:4 :: 117:x
4. 3:11 :: x:99
5. x:12 :: 29:116
6. 27:x :: 9:13
7. 126:72 :: 14:x
8. x:144 :: 12:36
9. If Barb types 130 words in 2 minutes, how long will it take her to type 325 words?
10. Steve is a painter. He needs 25 gallons of paint to cover 10 offices. Assuming all the offices are the same size, how many gallons of paint does Steve need to finish painting 3 offices?

Percentages

Percent: Per hundred (part per hundred).

Example 1

Change the decimal to a **percent:** 0.31 → 31%.

Steps

1. Move the decimal point to the right of the hundredths place (two places).
2. Put the percent sign behind the new number.

Example 2

Change the decimal to a percent: 0.007 → 0.7%.

Steps

1. Move the decimal point to the right of the hundredths place (always two places!).
2. Put the percent sign behind the new number. It is still a percent; it is just a very small percent.

Example 3

Change the percent to a decimal: 73.2% → 0.732.

Steps

1. Move the decimal two spaces away from the percent sign (to the left).
2. Drop the percent sign; it is no longer a percent, but a decimal.

Example 4

Change the percent to a decimal: 25% → 0.25.

Steps

1. The decimal point is not visible, but is always located after the last number.
2. Move the decimal two spaces away from the percent sign (toward the left).
3. Drop the percent sign; the number is no longer a percent, but a decimal.

Example 5

Change the fraction to a percent: $\frac{8}{9}$

$$
\begin{array}{r}
.888 \\
9\overline{)8.000} \\
-72\downarrow\downarrow \\
\quad 80\downarrow \\
\quad -72\downarrow \\
\quad\quad 80
\end{array}
$$

$0.888 \rightarrow 88.8\%$

Steps

1. Change the fraction into a division problem and solve.
2. Move the decimal behind the hundredths place in the quotient.
3. Place a percent sign after the new number.

SAMPLE PROBLEMS

Change the following decimals to percents.

1. 0.12 =
2. 0.04 =
3. 0.0052 =

Change the following percents to decimals.

4. 1.1% =
5. 8% =
6. 0.9% =

Change the following fractions to percents.

7. $\frac{7}{10} =$
8. $\frac{2}{5} =$
9. $\frac{5}{6} =$
10. $\frac{1}{8} =$

Using the Percent Formula

HESI Hint

The word *of* usually indicates the whole portion of the percent formula.
Percent formula:

$$\frac{\text{Part}}{\text{Whole}} = \frac{\%}{100}$$

Using this formula will help in all percent problems in which there is an unknown (solving for *x*).

Example 1

What is 7 out of 8 expressed as a percent?

$$\frac{7}{8} = \frac{\%}{100}$$
$$7 \times 100 = 700$$
$$700 \div 8 = 87.5$$
$$\% = 87.5 \text{ or } 87.5\%$$

Steps

1. Rewrite the problem using the percent formula.
2. Multiply the diagonal numbers together: $7 \times 100 = 700$.
3. Divide by the remaining number: $700 \div 8 = 87.5\%$.

Example 2

What is 32% of 75?

$$\frac{x}{75} = \frac{32}{100}$$
$$75 \times 32 = 2{,}400$$
$$2{,}400 \div 100 = 24$$
$$x = 24$$

Steps

1. Rewrite the problem using the percent formula.
2. "Of 75:" 75 is the whole.
3. Multiply the diagonal numbers together: $75 \times 32 = 2{,}400$.
4. Divide by the remaining number: $2{,}400 \div 100 = 24$.
5. $x = 24$ (this is not a percent; it is the part).

Example 3

14 is 56% of what number?

$$\frac{14}{x} = \frac{56}{100}$$
$$14 \times 100 = 1{,}400$$
$$1{,}400 \div 56 = 25$$
$$x = 25$$

Steps

1. Rewrite the problem using the percent formula.
2. We are looking for the **whole** because *of* is indicating an unknown number.
3. Multiply the diagonal numbers together: $14 \times 100 = 1{,}400$.
4. Divide by the remaining number: $1{,}400 \div 56 = 25$.

Fractions, Decimals, and Percents

Fraction	Decimal	Percent
$\frac{1}{2}$	0.50	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{5}$	0.20	20%
$\frac{2}{5}$	0.40	40%
$\frac{3}{5}$	0.60	60%
$\frac{4}{5}$	0.80	80%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{8}$	0.375	37.5%
$\frac{5}{8}$	0.625	62.5%
$\frac{7}{8}$	0.875	87.5%
$\frac{1}{3}$	0.333	33.3%
$\frac{2}{3}$	0.666	66.6%

SAMPLE PROBLEMS

Solve the following percent problems.

1. What is 14 out of 56 as a percent?
2. What is 2 out of 80 as a percent?
3. What is 45 out of 150 as a percent?
4. What is 50% of 90?
5. What is 85% of 40?
6. What is 2.5% of 400?
7. The number 5 is 20% of what number?
8. The number 30 is 25% of what number?
9. The number 9 is 30% of what number?
10. The number 51 is 25% of what number?

12-hour Clock versus Military Time

12-hour clock uses the numbers 1 through 12 with the suffixes am or pm to represent the hour in a 24-hour period. Military time uses the numbers 00 through 23 to represent the hour in a 24-hour period. The minutes and seconds in 12-hour clock and military time are expressed the same way.

HESI Hint

To convert to military time before noon, simply include a zero before the numbers 1 through 9 for AM. For example, 9:35 AM 12-hour clock time converts to 0935 military time. The zero is not needed when converting 10 AM or 11 AM. If the time is after noon, simply add 12 to the hour number. For example, 1:30 PM 12-hour clock time converts to 1330 military time (1 + 12 = 13). Midnight, or 12 AM, is converted to 0000. Noon, or 12 PM, is converted to 1200.

Table 1.1 summarizes the equivalents between military time and 12-hour clock time.

Military time is written with a colon between the minutes and seconds just as in the 12-hour clock. It can also be expressed with a colon between the hours and the minutes.

Table 1.1 Equivalents for Military Time and 12-hour Clock Time

Military Time	12-hour Clock Time	Military Time	12-hour Clock Time
0000	12:00 AM (Midnight)	1200	12:00 PM (Noon)
0100	1:00 AM	1300	1:00 PM
0200	2:00 AM	1400	2:00 PM
0300	3:00 AM	1500	3:00 PM
0400	4:00 AM	1600	4:00 PM
0500	5:00 AM	1700	5:00 PM
0600	6:00 AM	1800	6:00 PM
0700	7:00 AM	1900	7:00 PM
0800	8:00 AM	2000	8:00 PM
0900	9:00 AM	2100	9:00 PM
1000	10:00 AM	2200	10:00 PM
1100	11:00 AM	2300	11:00 PM

Military time is written as follows:

hours:minutes:seconds	OR	**hours:**minutes:seconds
0932:24 hours	OR	**09:**23:24
1926:56 hours	OR	**19:**26:56 hours

12-hour clock time is written as follows:
hours:minutes:seconds AM or PM
9:32:24 AM
7:26:56 PM

SAMPLE PROBLEMS

Convert the following 12-hour clock times to military times.

1. 12:01 AM =
2. 5:30 PM =
3. 6:10:17 AM =
4. 7:50:12 PM =
5. 2:23:15 PM =
6. 4:14:44 AM =

Convert the following military times to 12-hour clock times.

7. 1530 hours
8. 1908 hours
9. 11:10:22 hours
10. 02:08:33 hours
11. 1202:00 hours
12. 21:45:30

Algebra

Variable: A letter representing an unknown quantity (i.e., x).
Constant: A number that cannot change.
Expression: A mathematical sentence containing constants and variables (i.e., $3x - 2$).
Exponent: A number or symbol placed above and after another number or symbol (a superscript or subscript), indicating the number of times to multiply.

Algebra is a process that involves variables and constants. A **variable** is a letter that represents an unknown quantity. A **constant** is a number that cannot change. Using the operations of addition, subtraction, multiplication, and division, we can use algebra to determine the value of unknown quantities. Two algebra concepts discussed in this section will be evaluating **expressions** and solving equations for a specific variable.

HESI Hint

When working with algebra, remember to evaluate expressions by performing the "Order of Operations."

Order of Operations

1. Evaluate numbers within parentheses.	$4 \cdot (2 + 3)^2 - 5$
2. Multiply numbers based on any exponents.	$4 \cdot (5)^2 - 5$
3. Multiply and divide numbers from left to right.	$4 \cdot 25 - 5$
4. Add and subtract numbers from left to right.	$4 \cdot 25 - 5$

Continued

HESI Hint—cont'd

The variable for these expressions is 95.
Here's a mnemonic to remember the "Order of Operations":
"Please excuse my dear Aunt Sally" helps to remember the correct order of operations.
The order should be Parentheses, Exponents, Multiply, Divide, Add, Subtract.

Evaluating the Expression

- Numbers can be positive (1 or +1) or negative (–1). If a number has no sign (e.g., 1) it usually means it is a positive number.
- Adding positive numbers is similar to addition (e.g., 1 + 3 = 4).
- Subtracting positive numbers is simple subtraction (e.g., 4 – 3 = 1).
- Subtracting a negative number is the same as adding (e.g., 3 – [– 1] = 4); it is written as 3 + 1 = 4.
- Subtracting a positive number: 4 – (+3) = 4 – 3 = 1
- Adding a negative number: 3 + (–4) = 3 – 4 = – 1

Rules:

- Two like signs become positive signs: 3 + (+1) = 3(+1) = 3 + 1 = 4

$$3-(-1)=3+1=4$$

- Two unlike signs become a negative sign: 8 + (– 2) = 8 – 2 = 6

$$8-(+2)=8-2=6$$

When we substitute a specific value for each variable in the expression and then perform the operations, it's called "evaluating the expression."

Example 1

Evaluate the expression $ab + c$ if $a = 6$, $b = -4$, and $c = 12$

$$(6)(-4)+12$$
$$-24+12$$
$$-12$$

Steps

1. Substitute the numbers into the given expression. Use parentheses when inserting numbers into an expression.
2. Multiply 6 × –4 = –24
3. Add –24 + 12 = –12

Example 2

Evaluate the expression $-xy(x-y)+y$ if $x=5$ and $y=-1$

$$-(5)(-1)([5]-[-1])+(-1)$$
$$-(-5)(5+1)-1$$
$$5(6)-1$$
$$30-1$$
$$29$$

Steps

1. Substitute the numbers into the given expression.
2. Multiply $5 \times -1 = -5$.
3. Change $-(-1)$ to $+1$, and $+(-1)$ to -1.
4. Add $5 + 1 = 6$
5. Change $-(-5) + 5$
6. Multiply $5 \times 6 = 30$
7. Subtract $30 - 1 = 29$

Solving Equations for a Specific Variable

To solve equations for a specific variable, perform the operations in the reverse order in which you evaluate expressions.

Example 3

Solve: $2x + 3 = 21$

$$\frac{\overset{-3}{2x}}{2} = \frac{\overset{-3}{18}}{2}$$

$$x = 9$$

Steps

1. Subtract 3 from each side of the equation.
2. Divide both sides by 2.

Example 4

Solve: $-4k - 2 = -17$

$$\frac{\overset{+2}{-4k}}{-4} = \frac{\overset{+2}{-15}}{4}$$

$$k = \frac{15}{4}$$

Steps

1. Add 2 to both sides.
2. Divide both sides by -4.
3. Simplify. (A negative divided by a negative is a positive.)

SAMPLE PROBLEMS

Evaluate the following expressions:

1. $x + 9y$ if $x = 3$ and $y = -2$
2. $2ab - am$ if $a = 1$, $b = 2$, and $m = -2$
3. $-2x(y - 2z)$ if $x = 3$, $y = -2$, and $z = 6$
4. $-qr + r - s$ if $q = 2$, $r = 4$, and $s = 6$
5. $(a + b)(2a + bc)$ if $a = 4$, $b = -2$, and $c = 3$

Solve the following equations for the given variable:

6. $6x - 6 = 24$ solve for x.
7. $-x - 4 = 18$ solve for x.
8. $4y + 15 = 28$ solve for y.