CENGAGE

Stephen T. Thornton | Andrew Rex | Carol Hood

MODERN PHYSICS

FOR SCIENTISTS AND ENGINEERS

FIFTH EDITION

Conversion Factors

$1\,\text{y} = 3.154 \times 10^{7}\,\text{s}$

$1\text{ lightyear} = 9.461 \times 10^{15}\,\text{m}$

$1\text{ cal} = 4.184\ \text{J}$

$1\,\text{MeV}/c = 5.344 \times 10^{-22}\,\text{kg} \cdot \text{m/s}$

$1\,\text{eV} = 1.6022 \times 10^{-19}\,\text{J}$

$1\,\text{T} = 10^{4}\,\text{G}$

$1\,\text{Ci} = 3.7 \times 10^{10}\,\text{Bq}$

$1\text{ barn} = 10^{-28}\,\text{m}^2$

$1\,\text{u} = 1.66054 \times 10^{-27}\,\text{kg}$

Useful Combinations of Constants

$\hbar = h/2\pi = 1.0546 \times 10^{-34}\,\text{J}\cdot\text{s} = 6.5821 \times 10^{-16}\,\text{eV}\cdot\text{s}$

$hc = 1.9864 \times 10^{-25}\,\text{J}\cdot\text{m} = 1239.8\,\text{eV}\cdot\text{nm}$

$\hbar c = 3.1615 \times 10^{-26}\,\text{J}\cdot\text{m} = 197.33\,\text{eV}\cdot\text{nm}$

$\dfrac{1}{4\pi\varepsilon_0} = 8.9876 \times 10^{9}\,\text{N}\cdot\text{m}^2\cdot\text{C}^{-2}$

Compton wavelength $\lambda_\text{C} = \dfrac{h}{m_e c} = 2.4263 \times 10^{-12}\,\text{m}$

$\dfrac{e^2}{4\pi\varepsilon_0} = 2.3071 \times 10^{-28}\,\text{J}\cdot\text{m} = 1.4400 \times 10^{-9}\,\text{eV}\cdot\text{m}$

Fine structure constant $\alpha = \dfrac{e^2}{4\pi\varepsilon_0 \hbar c} = 0.0072974 \approx \dfrac{1}{137}$

Bohr magneton $\mu_\text{B} = \dfrac{e\hbar}{2m_e} = 9.2740 \times 10^{-24}\,\text{J/T} = 5.7884 \times 10^{-5}\,\text{eV/T}$

Nuclear magneton $\mu_\text{N} = \dfrac{e\hbar}{2m_p} = 5.0508 \times 10^{-27}\,\text{J/T} = 3.1525 \times 10^{-8}\,\text{eV/T}$

Bohr radius $a_0 = \dfrac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = 5.2918 \times 10^{-11}\,\text{m}$

Hydrogen ground state $E_0 = \dfrac{e^2}{8\pi\varepsilon_0 a_0} = 13.606\,\text{eV} = 2.1799 \times 10^{-18}\,\text{J}$

Rydberg constant $R_\infty = \dfrac{\alpha^2 m_e c}{2h} = 1.09737 \times 10^{7}\,\text{m}^{-1}$

Hydrogen Rydberg $R_\text{H} = 1.09678 \times 10^{7}\,\text{m}^{-1}$

Gas constant $R = N_\text{A} k = 8.3145\,\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$

Magnetic flux quantum $\Phi_0 = \dfrac{h}{2e} = 2.0678 \times 10^{-15}\,\text{T}\cdot\text{m}^2$

Classical electron radius $r_e = \alpha^2 a_0 = 2.8179 \times 10^{-15}\,\text{m}$

$kT = 2.5249 \times 10^{-2}\,\text{eV} \approx \dfrac{1}{40}\,\text{eV}$ at $T = 293\,\text{K}$

Note: The latest values of the fundamental constants can be found at the National Institute of Standards and Technology website at http://physics.nist.gov/cuu/Constants/index.html.

Modern Physics

For Scientists and Engineers

Fifth Edition

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CENGAGE

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CENGAGE

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Preface

Our objective in writing this book was to produce a textbook for a modern physics course of either one or two semesters for physics and engineering students. Such a course normally follows a full-year, calculus-based introductory physics course for first-year or second-year students. Before each edition we have the publisher send a questionnaire to users of modern physics books to see what needed to be changed or added. Most users like our textbook as is, especially the complete coverage of topics such as the early quantum theory, subfields of physics, general relativity, and cosmology/astrophysics. Our book continues to be useful for either a one- or two-term modern physics course. We have reordered and expanded the topics in the final two chapters, but have not made any other major changes in the order of subjects in the fifth edition.

Coverage

The first edition of our text established a trend for a contemporary approach to the exciting, thriving, and changing field of modern science. After briefly visiting the status of physics at the turn of the last century, we cover relativity and quantum theory, the basis of any study of modern physics. Almost all areas of science depend on quantum theory and the methods of experimental physics. We have included the name Quantum Mechanics in two of our chapter titles (Chapters 5 and 6) to emphasize the quantum connection. The latter part of the book is devoted to the subfields of physics (atomic, condensed matter, nuclear, and particle) and the exciting fields of cosmology and astrophysics. Our experience is that science and engineering majors particularly enjoy the study of modern physics after the sometimes-laborious study of classical mechanics, thermodynamics, electricity, magnetism, and optics. The level of mathematics is not difficult for the most part, and students feel they are finally getting to the frontiers of physics. We have brought the study of modern physics alive by presenting many current applications and challenges in physics, for example, nanoscience, high-temperature superconductors, quantum teleportation, neutrino mass and oscillations, gravitational waves, missing dark mass and energy in the universe, gamma-ray bursts, holography, quantum dots, and nuclear fusion. Modern physics texts need to be updated periodically to include recent advances. Although we have emphasized modern applications, we also provide the sound theoretical basis for quantum theory that will be needed by physics majors in their upper division and graduate courses.

Changes for the Fifth Edition

Our book continues to be the most complete and up-to-date textbook in the modern physics market for sophomores/juniors. We have made several changes for the fifth edition to aid the student in learning modern physics.

The important contributions to physics of more female scientists, including Emmy Noether, Rosalyn Yalow, Annie Jump Cannon, and Henrietta Leavitt, have been added, as well as the contributions of the female computers at the Harvard Observatory and NASA.

Additional short biographical features highlight the achievements of physicists throughout history to illustrate the importance of individual ingenuity in advancing knowledge in the field. Similarly, throughout the text, a focus on the history of physics offers a human perspective and helps students understand the context in which scientific advancements have been made.

The discussion on gravitational waves has been greatly expanded to include the recent detections and subsequent electromagnetic detections and the discussions on dark matter and dark energy have been updated to include the most up-to-date observations and theories.

The latest information on the Higgs boson brings students up-to-date on this significant research area of physics research and theory.

The latest research and updated information about the age of the universe has been added.

Chapter 15, "Modern Astrophysics and General Relativity" and Chapter 16, "Cosmology," have been rewritten to reflect the latest research and findings and expose students to this rapidly changing body of knowledge.

Special Topic boxes are up-to-date applications of interest to physicists and engineers. These features show the relevance of modern physics to the real world and allow students a more in-depth look at particularly engaging topics like exoplanets, the "age of the Earth," neutrino detection, and scanning probe microscopes.

Finally, end-of-chapter summaries will give students a quick overview of topics covered in the chapter.

Teaching Suggestions

The text has been used extensively in its first four editions in courses at our home institutions. These include a one-semester course for physics and engineering students at the University of Virginia, a two-semester course for physics and pre-engineering students at the University of Puget Sound, and a one-quarter course at California State University, San Bernardino. These are representative of the one- and two-term modern physics courses taught elsewhere. Both one- and two-term courses should cover the material through the establishment of the periodic table in Chapter 8 with few exceptions. We have eliminated the denoting of optional sections, because we believe that depends on the wishes of the instructor, but we feel Sections 2.4, 4.2, 6.4, 6.6, 7.2, 7.6, 8.2, and 8.3 from the first nine chapters may be skipped without loss of continuity. Our suggestions for the one- and two-term courses (3 or 4 credit hours per term) are then

One-term: Chapters 1–9 and selected other material as chosen by the instructor

Two-term: Chapters 1–16 with supplementary material as desired, with possible student projects

Features

End-of-Chapter Problems

Digital versions of thought-provoking end-of-chapter questions are available to assign via WebAssign in a number of formats. The large repository of problems is now further enhanced in this edition with WebAssign-only enhanced content created by Marllin Simon and Matthew Kohlmyer. The wide variety of questions allows any instructor to make different homework assignments year after year without having to repeat problems. For those users of the earlier fourth edition a correlation guide is available via online instructor resources.

Solutions Manuals

PDF files of the *Instructor's Solutions Manual* are available to the instructor on the *Instructor's Resource Center website* (or by contacting your Cengage sales representative). This manual contains the *solutions to the printed end-of-chapter problems* and has been checked by at least two physics professors. The answers to selected odd-numbered problems are given at the end of the textbook itself.

Examples

These examples are written and presented in the manner in which students are expected to work the end-of-chapter problems: that is, to develop a conceptual understanding and strategy before attempting a numerical solution. Problem solving does not come easily for most students, especially the problems requiring several steps (that is, not simply plugging numbers into one equation). We expect that the many text examples with varying degrees of difficulty will help students.

Special Topic Boxes

Users have encouraged us to keep the Special Topic boxes. We believe both students and professors find them interesting, because they add insight and detail into the excitement of physics. We have updated the material to keep them current.

History

We include historical aspects of modern physics that many students will find interesting and that others can simply ignore. We continue to include photos and biographies of scientists who have made significant contributions to modern physics. We believe this helps to enliven and humanize the material.

Acknowledgments

We acknowledge the assistance of many persons who have helped with this text. There are too many that helped us with the first four editions to list here, but the book would not have been possible without them. We acknowledge the professional staff at Cengage who helped make this fifth edition a useful, popular, and attractive textbook. They include Product Managers Nate Thibeault and Spencer Arritt, Learning Designer Michael Jacobs, Subject Matter Expert Matthew Kohlmyer, Content Manager Michael Lepera, and Product Assistants Kyra Kruger and Tim Biddick. We acknowledge the tremendous help given to us by Lori Hazzard and her colleagues from MPS Limited for their production of this fifth edition.

Prior to and during our work on this fifth edition, we conducted a survey of instructors about the modern physics course in general and our book in particular. We received many insightful comments, and we would like to thank the following for their feedback and suggestions:

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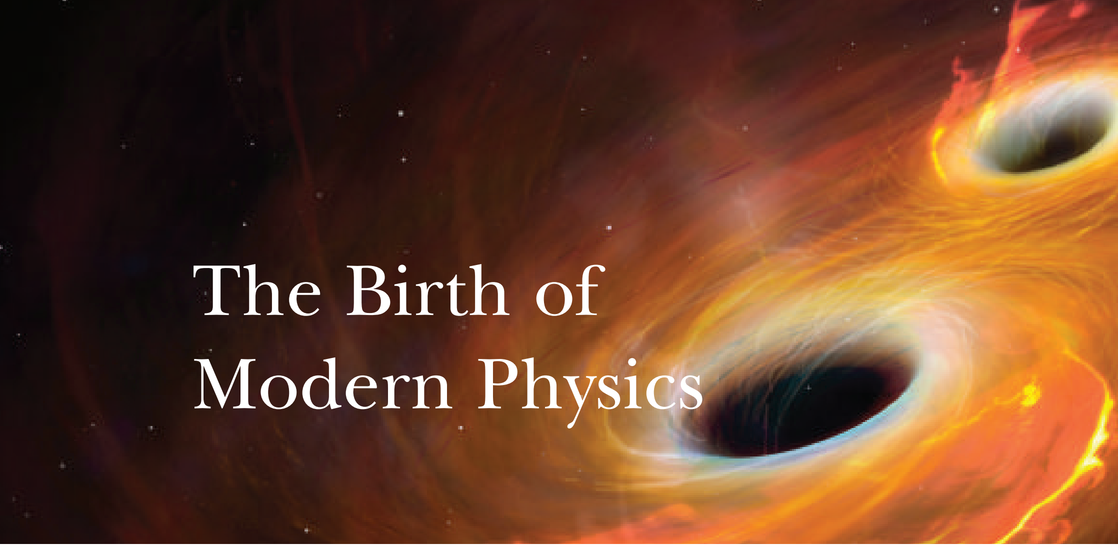

CHAPTER 1

The Birth of Modern Physics

The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote. … Our future discoveries must be looked for in the sixth place of decimals.

Albert A. Michelson, 1894

There is nothing new to be discovered in physics now. All that remains is more and more precise measurement.

William Thomson (Lord Kelvin), 1900

Although the Greek scholars Aristotle and Eratosthenes performed measurements and calculations that today we would call physics, the discipline of physics has its roots in the work of Galileo and Newton and others in the scientific revolution of the sixteenth and seventeenth centuries. The knowledge and practice of physics grew steadily for 200 to 300 years until another revolution in physics took place, which is the subject of this book. Physicists distinguish *classical physics*, which was mostly developed before 1895, from *modern physics*, which is based on discoveries made after 1895. The precise year is unimportant, but monumental changes occurred in physics around 1900.

In this chapter we briefly review the status of physics around 1895, including Newton's laws, Maxwell's equations, and the laws of thermodynamics. These results are just as important today as they were over a hundred years ago. Arguments by scientists concerning the interpretation of experimental data using wave and particle descriptions that seemed to have been resolved 200 years ago were reopened in the twentieth century. Today we look back on the evidence of the late nineteenth century and wonder how anyone could have doubted the validity of the atomic view of matter. The fundamental interactions of gravity, electricity, and magnetism were thought to be well understood in 1895. Physicists continued to be driven by the goal of understanding fundamental laws throughout the twentieth century. This is demonstrated by the fact that other fundamental forces (specifically the nuclear and weak interactions) have been added, and in some cases—curious as it may seem—various forces have even been combined. The search for the holy grail of fundamental interactions continues unabated today.

We finish this chapter with a status report on physics just before 1900. The few problems not then understood would be the basis for decades of fruitful investigations and discoveries continuing into the twenty-first century.

1.1 Classical Physics of the 1890s

Scientists and engineers of the late nineteenth century were indeed rather smug. They thought they had just about everything under control (see the quotes from Michelson and Kelvin at the beginning of the chapter). The best scientists of the day were highly recognized and rewarded. Public lectures were frequent. Some scientists had easy access to their political leaders, partly because science and engineering had benefited their war machines, but also because of the many useful technological advances. Basic research was recognized as important because of the commercial and military applications of scientific discoveries. Although there were only primitive automobiles and no airplanes in 1895, advances in these modes of transportation were soon to follow. A few people already had telephones, and plans for widespread distribution of electricity were under way.

Based on their success with what we now call macroscopic classical results, scientists felt that given enough time and resources, they could explain just about anything. They did recognize some difficult questions they still couldn't answer; for example, they didn't clearly understand the structure of matter—that was under intensive investigation. Nevertheless, on a macroscopic scale, they knew how to build efficient engines. Ships plied the lakes, seas, and oceans of the world. Travel between the countries of Europe was frequent and easy by train. Many scientists were born in one country, educated in one or two others, and eventually worked in still other countries. The most recent ideas traveled relatively quickly among the centers of research. Except for some isolated scientists, of whom Einstein is the most notable example, discoveries were quickly and easily shared. Scientific journals were becoming accessible.

The ideas of classical physics are just as important and useful today as they were at the end of the nineteenth century. For example, they allow us to build automobiles and produce electricity. The conservation laws of energy, linear momentum, angular momentum, and charge can be stated as follows:

Classical conservation laws

Conservation of energy: The total sum of energy (in all its forms) is conserved in all interactions.

Conservation of linear momentum: In the absence of external forces, linear momentum is conserved in all interactions (vector relation).

Conservation of angular momentum: In the absence of external torque, angular momentum is conserved in all interactions (vector relation).

Conservation of charge: Electric charge is conserved in all interactions.

A nineteenth-century scientist might have added the **conservation of mass** to this list, but we know it not to be valid today (you will find out why in Chapter 2). These conservation laws are reflected in the laws of mechanics, electromagnetism, and thermodynamics. Electricity and magnetism, separate subjects for hundreds of years, were combined by James Clerk Maxwell (1831–1879) in his four equations. Maxwell showed optics to be a special case of electromagnetism. Waves, which permeated mechanics and optics, were known to be an important component of nature. Many natural phenomena could be explained by wave motion using the laws of physics.

Mechanics

The laws of mechanics were developed over hundreds of years by many researchers. Important contributions were made by astronomers because of the great interest in the heavenly bodies. Galileo (1564–1642) may rightfully be called the first great experimenter. His experiments and observations laid the groundwork for the important discoveries to follow during the next 200 years.

Isaac Newton (1642–1727) was certainly the greatest scientist of his time and one of the best the world has ever seen. His discoveries were in the fields of mathematics, astronomy, and physics and include gravitation, optics, motion, and forces.

We owe to Newton our present understanding of motion. He understood clearly the relationships among position, displacement, velocity, and acceleration. He understood how motion was possible and that a body at rest was just a special case of a body having constant velocity. It may not be so apparent to us today, but we should not forget the tremendous unification that Newton made when he pointed out that the motions of the planets about our sun can be understood by the same laws that explain motion on Earth, like apples falling from trees or a soccer ball being kicked toward a goal. Newton was able to elucidate carefully the relationship between net force and acceleration, and his concepts were stated in three laws that bear his name today:

Newton's laws

Newton's first law: *An object in motion with a constant velocity will continue in motion unless acted upon by some net external force.* A body at rest is just a special case of Newton's first law with zero velocity. Newton's first law is often called the *law of inertia* and is also used to describe inertial reference frames.

Newton's second law: *The acceleration $\vec{a}$ of a body is proportional to the net external force $\vec{F}$ and inversely proportional to the mass m of the body. It is stated mathematically as*

$$\vec{F} = m\vec{a} \tag{1.1a}$$


Science History Images/Alamy Stock Photo

Galileo Galilei (1564–1642) was born, educated, and worked in Italy. Often said to be the "father of physics" because of his careful experimentation, he is shown here performing experiments by rolling balls on an inclined plane. He is perhaps best known for his experiments on motion, the development of the telescope, and his many astronomical discoveries. He came into disfavor with the Catholic Church for his belief in the Copernican theory. He was finally cleared of heresy by Pope John Paul II in 1992, 350 years after his death.

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Isaac Newton (1642–1727), the great English physicist and mathematician, did most of his work at Cambridge where he was educated and became the Lucasian Professor of Mathematics. He was known not only for his work on the laws of motion but also as a founder of optics. His useful works are too numerous to list here, but it should be mentioned that he spent a considerable amount of his time on alchemy, theology, and the spiritual universe. His writings on these subjects, which were dear to him, were quite unorthodox. This painting shows him performing experiments with light.

A more general statement* relates force to the time rate of change of the linear momentum $\vec{p}$.

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{1.1b}$$

Newton's third law: *The force exerted by body 1 on body 2 is equal in magnitude and opposite in direction to the force that body 2 exerts on body 1.* If the force on body 2 by body 1 is denoted by $\vec{F}_{21}$, then Newton's third law is written as

$$\vec{F}_{21} = -\vec{F}_{12} \tag{1.2}$$

It is often called the *law of action and reaction.*

These three laws develop the concept of force. Using that concept together with the concepts of velocity $\vec{v}$, acceleration $\vec{a}$, linear momentum $\vec{p}$, rotation (angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$), and angular momentum $\vec{L}$, we can describe the complex motion of bodies.

Electromagnetism

Electromagnetism developed over a long period of time. Important contributions were made by Charles Coulomb (1736–1806), Hans Christian Oersted (1777–1851), Thomas Young (1773–1829), André Ampère (1775–1836), Michael Faraday (1791–1867), Joseph Henry (1797–1878), James Clerk Maxwell (1831–1879), and Heinrich Hertz (1857–1894). Maxwell showed that electricity and magnetism were intimately connected and were related by a change in the inertial frame of reference. His work also led to the understanding of electromagnetic radiation, of which light and optics are special cases. Maxwell's four equations [Equations (1.3–1.6)], together with the Lorentz force law [Equation (1.7)], explain much of electromagnetism.

Maxwell's equations

Gauss's law for electricity
$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} \tag{1.3}$$

Gauss's law for magnetism
$$\oint \vec{B} \cdot d\vec{A} = 0 \tag{1.4}$$

Faraday's law
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \tag{1.5}$$

Generalized Ampere's law
$$\oint \vec{B} \cdot d\vec{s} = \mu_0\epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I \tag{1.6}$$

Lorentz force law

Lorentz force law
$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{1.7}$$

Maxwell's equations indicate that charges and currents create electric and magnetic fields, and in turn, these fields can create other fields, both electric and magnetic.

*It is a remarkable fact that Newton wrote his second law not as $\vec{F} = m\vec{a}$, but as $\vec{F} = d(m\vec{v})/dt$, focusing on what we now call momentum. This has applications in both fluid mechanics and rocket propulsion.

Thermodynamics

Thermodynamics deals with temperature T, heat Q, work W, and the internal energy of systems U. The understanding of the concepts used in thermodynamics—such as pressure P, volume V, temperature, thermal equilibrium, heat, entropy, and especially energy—was slow in coming. We can understand the concepts of pressure and volume as mechanical properties, but the concept of temperature must be carefully considered. The internal energy of a system of noninteracting point masses depends only on the temperature.

Important contributions to thermodynamics were made by Benjamin Thompson (Count Rumford, 1753–1814), Sadi Carnot (1796–1832), James Joule (1818–1889), Rudolf Clausius (1822–1888), and William Thomson (Lord Kelvin, 1824–1907). The primary results of thermodynamics can be described in two laws:

Laws of thermodynamics

First law of thermodynamics: *The change in the internal energy ΔU of a system is equal to the heat Q added to the system plus the work W done on the system.*

$$\Delta U = Q + W \tag{1.8}$$

The first law of thermodynamics generalizes the conservation of energy by including heat.

Second law of thermodynamics: *It is not possible to convert heat completely into work without some other change taking place.* Equivalent forms of the second law may appear different, but instead describe what kinds of energy processes can or cannot take place. For example, it is not possible to build a perfect engine or a perfect refrigerator. It is not possible to build a perpetual motion machine. Heat does not spontaneously flow from a colder body to a hotter body without some other change taking place. The second law forbids all these from happening.

Two other laws of thermodynamics are sometimes expressed. One is called the zeroth law, and it is useful in understanding temperature. It states that *if two thermal systems are in thermodynamic equilibrium with a third system, they are in equilibrium with each other.* We can state it more simply by saying that *two systems at the same temperature as a third system have the same temperature as each other.* This concept was not explicitly stated until the twentieth century. The third law of thermodynamics expresses that *it is not possible to achieve an absolute zero temperature.*

1.2 The Kinetic Theory of Gases

We understand now that gases are composed of atoms and molecules in rapid motion, bouncing off each other and the container walls, but in the 1890s this had just gained acceptance. The kinetic theory of gases is related to thermodynamics and to the atomic theory of matter, which we discuss in Section 1.5. Experiments were relatively easy to perform on gases, and the Irish chemist Robert Boyle (1627–1691) showed around 1662 that the pressure times the volume of a gas was constant for a constant temperature. The relation PV = constant (for constant T) is now referred to as *Boyle's law*. The French physicist Jacques Charles (1746–1823) found that V/T = constant (at constant pressure), referred to as *Charles's law.* Joseph Louis Gay-Lussac (1778–1850) later produced the same result, and the law is sometimes associated with his name. If we combine these two laws, we obtain the ideal gas equation,

Ideal gas equation

$$PV = nRT \tag{1.9}$$

where n is the number of moles and R is the ideal gas constant, 8.31 J/mol·K. The ideal gas equation is also written as $PV = NkT$, where N is the number of molecules and k is Boltzmann's constant.

In 1811 the Italian physicist Amedeo Avogadro (1776–1856) proposed that equal volumes of gases at the same temperature and pressure contained equal numbers of molecules. This hypothesis was so far ahead of its time that it was not accepted for many years. The famous English chemist John Dalton opposed the idea because he apparently misunderstood the difference between atoms and molecules. Considering the rudimentary nature of the atomic theory of matter at the time, this was not surprising.

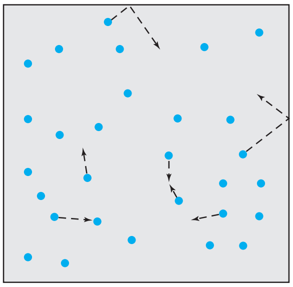


Figure 1.1 Molecules inside a closed container are shown colliding with the walls and with each other. The motions of a few molecules are indicated by the arrows.

Daniel Bernoulli (1700–1782) apparently originated the kinetic theory of gases in 1738, but his results were generally ignored. Many scientists, including Newton, Laplace, Davy, Herapath, and Waterston, had contributed to the development of kinetic theory by 1850. Theoretical calculations were being compared with experiments, and by 1895 the kinetic theory of gases was widely accepted. The statistical interpretation of thermodynamics was made in the latter half of the nineteenth century by Maxwell, the Austrian physicist Ludwig Boltzmann (1844–1906), and the American physicist J. Willard Gibbs (1839–1903).

In introductory physics classes, the kinetic theory of gases is usually taught by applying Newton's laws to the collisions that a molecule makes with other molecules and with the walls. A representation of a few molecules colliding is shown in Figure 1.1. In the simple model of an ideal gas, only elastic collisions are considered. By taking averages over the collisions of many molecules, the ideal gas law, Equation (1.9), is revealed. The average kinetic energy of the molecules is shown to be linearly proportional to the temperature, and the internal energy U is

Statistical thermodynamics

$$U = nN_A\overline{K} = \frac{3}{2}nRT \tag{1.10}$$

where n is the number of moles of gas, N_A is Avogadro's number, $\overline{K}$ is the average kinetic energy of a molecule, and R is the ideal gas constant. This relation ignores any nontranslational contributions to the molecular energy, such as rotations and vibrations.

Equipartition theorem

However, energy is not represented only by translational motion. It became clear that all *degrees of freedom,* including rotational and vibrational, were also capable of carrying energy. The *equipartition theorem* states that each degree of freedom of a molecule has an average energy of $kT/2$, where k is the Boltzmann constant ($k = R/N_A$). Translational motion has three degrees of freedom, and rotational and vibrational modes can also be excited at higher temperatures. If there are f degrees of freedom, then Equation (1.10) becomes

Internal energy

$$U = \frac{f}{2}nRT \tag{1.11}$$

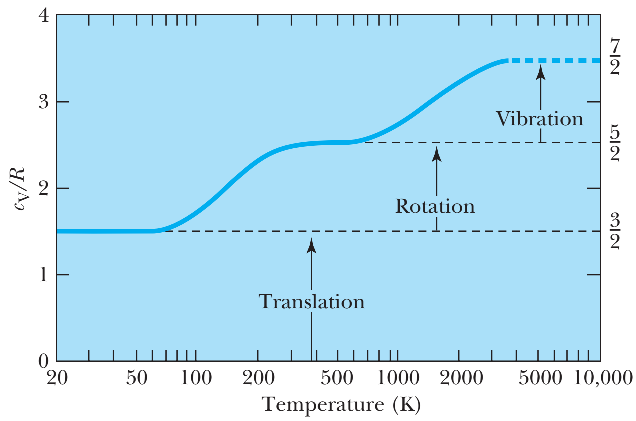


Figure 1.2 The molar heat capacity at constant volume (c_V) divided by R (c_V/R is dimensionless) is displayed as a function of temperature for molecular hydrogen gas. Note that as the temperature increases, the rotational and vibrational modes become important. This experimental result is consistent with the equipartition theorem, which adds $kT/2$ of energy per molecule ($RT/2$ per mole) for each degree of freedom.

The molar ($n = 1$) heat capacity c_V at constant volume for an ideal gas is the rate of change in internal energy with respect to change in temperature and is given by

$$c_V = \frac{fR}{2} \tag{1.12}$$

Heat capacity

The experimental quantity c_V/R is plotted versus temperature for molecular hydrogen in Figure 1.2. The ratio c_V/R is equal to 3/2 for low temperatures, where only translational kinetic energy is important, but it rises to 5/2 at 300 K, where rotations occur for H_2, and finally reaches 7/2, because of vibrations at still higher temperatures, before the H_2 molecule dissociates.

In the 1850s Maxwell derived a relation for the distribution of speeds of the molecules in gases. The distribution of speeds $f(v)$ is given as a function of the speed and the temperature by the equation

$$f(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} \tag{1.13}$$

Maxwell's speed distribution

where m is the mass of a molecule and T is the temperature. This result is plotted for nitrogen in Figure 1.3 for temperatures of 300 K, 1000 K, and 4000 K. The

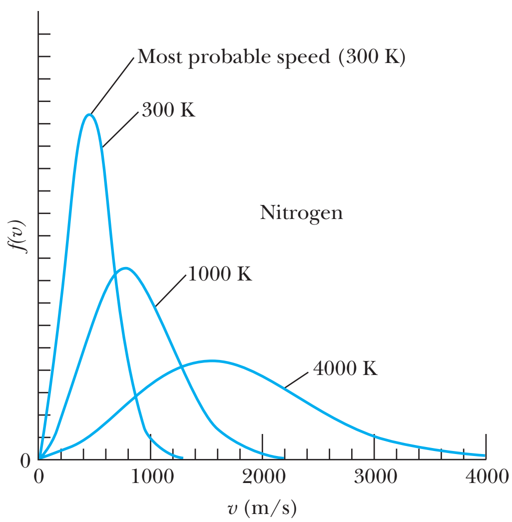


Figure 1.3 The Maxwell distribution of molecular speeds (for nitrogen), $f(v)$, is shown as a function of speed for three temperatures.

peak of each distribution is the most probable speed of a gas molecule for the given temperature. In 1895 measurement was not precise enough to confirm Maxwell's distribution, and it was not confirmed experimentally until 1921.

By 1895 Boltzmann had made Maxwell's calculation more rigorous, and the general relation is called the *Maxwell–Boltzmann distribution.* The distribution can be used to find the *root-mean-square* speed v_{rms},

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} \tag{1.14}$$

which shows the relationship of the energy to the temperature for a monatomic ideal gas:

$$U = nN_A\overline{K} = nN_A\frac{m\overline{v^2}}{2} = nN_A\frac{3mkT}{2m} = \frac{3}{2}nRT \tag{1.15}$$

This was the result of Equation (1.10).

1.3 Waves and Particles

We first learned the concepts of velocity, acceleration, force, momentum, and energy in introductory physics by using a single particle with its mass concentrated in one small point. In order to adequately describe nature, we add two- and three-dimensional bodies and rotations and vibrations. However, many aspects of physics can still be treated as if the bodies are simple particles. In particular, the kinetic energy of a moving particle is one way that energy can be transported from one place to another.

But we have found that many natural phenomena can be explained only in terms of *waves*, which are traveling disturbances that carry energy. This description includes standing waves, which are superpositions of traveling waves. Most waves, like water waves and sound waves, need an elastic medium in which to move. Curiously enough, matter is not transported in waves—but energy is. Mass may oscillate, but it doesn't actually propagate along with the wave. Two examples are a cork and a boat on water. As a water wave passes, the cork gains energy as it moves up and down, and after the wave passes, the cork remains. The boat also reacts to the wave, but it primarily rocks back and forth, throwing around things that are not fixed on the boat. The boat obtains considerable kinetic energy from the wave.

Waves and particles were the subject of disagreement as early as the seventeenth century, when there were two competing theories of the nature of light. Newton supported the idea that light consisted of corpuscles (or particles). He performed extensive experiments on light for many years and finally published his book *Opticks* in 1704. *Geometrical optics* uses straight-line, particle-like trajectories called *rays* to explain familiar phenomena such as reflection and refraction. Geometrical optics was also able to explain the apparent observation of sharp shadows. The competing theory considered light as a wave phenomenon. Its strongest proponent was the Dutch physicist Christian Huygens (1629–1695), who presented his theory in 1678. The wave theory could also explain reflection and refraction, but it could not explain the sharp shadows observed. Experimental physics of the 1600s and 1700s was not able to discern between the two competing theories. Huygens's poor health and other duties kept him from working on optics much after 1678. Although Newton did not feel strongly about his corpuscular theory, the magnitude of his reputation caused it to be almost universally accepted for more than a hundred years and throughout most of the eighteenth century.

Finally, in 1802, the English physician Thomas Young (1773–1829) announced the results of his two-slit interference experiment, indicating that light behaved as a wave. Even after this singular event, the corpuscular theory had its supporters. During the next few years Young and, independently, Augustin Fresnel (1788–1827) performed several experiments that clearly showed that light behaved as a wave. By 1830 most physicists believed in the wave theory—some 150 years after Newton performed his first experiments on light.

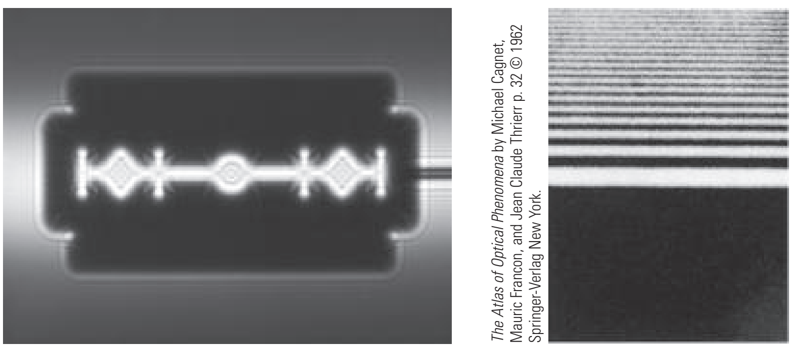
One final experiment indicated that the corpuscular theory was difficult to accept. Let c be the speed of light in vacuum and v be the speed of light in another medium. If light behaves as a particle, then to explain refraction, light must speed up when going through denser material ($v > c$). The wave theory of Huygens predicts just the opposite ($v < c$). The measurements of the speed of light in various media were slowly improving, and finally, in 1850, Léon Foucault showed that *light traveled more slowly in water than in air.* The corpuscular theory seemed incorrect. Newton would probably have been surprised that his weakly held beliefs lasted as long as they did. Now we realize that geometrical optics is correct only if the wavelength of light is much smaller than the size of the obstacles and apertures that the light encounters.

Figure 1.4 shows the "shadows" or *diffraction patterns* from light falling on sharp edges. In Figure 1.4a the alternating black and white lines can be seen all around the razor blade's edges. Figure 1.4b is a highly magnified photo of the diffraction from a sharp edge. The bright and dark regions can be understood only if light is a wave and not a particle.

In the 1860s Maxwell showed that electromagnetic waves consist of oscillating electric and magnetic fields. Visible light covers just a narrow range of the total electromagnetic spectrum, and all electromagnetic radiation travels at the speed of light c in free space, given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \lambda f \tag{1.16}$$

where λ is the wavelength and f is the frequency. The fundamental constants μ_0 and ϵ_0 are defined in electricity and magnetism and reveal the connection to the speed of light. In 1887 the German physicist Heinrich Hertz (1857–1894) succeeded in generating and detecting electromagnetic waves having wavelengths



Courtesy Ken Kay/Fundamental Photographs New York

The Atlas of Optical Phenomena by Michael Cagnet, Mauric Francon, and Jean Claude Thrierr p. 32 © 1962 Springer-Verlag New York.

(a) (b)

Figure 1.4 In contradiction to what scientists thought in the seventeenth century, shadows are not sharp, but show dramatic diffraction patterns—as seen here (a) for a razor blade and (b) for a highly magnified sharp edge.

far outside the visible range ($\lambda \approx 5$ m). The properties of these waves were just as Maxwell had predicted. His results continue to have far-reaching effects in modern telecommunications: cable TV, cell phones, lasers, fiber optics, wireless Internet, and so on.

Some unresolved issues about electromagnetic waves in the 1890s eventually led to one of the two great modern theories, the *theory of relativity* (see Section 1.6 and Chapter 2). Waves play a central and essential role in the other great modern physics theory, *quantum mechanics*, which is sometimes called *wave mechanics*. Because waves play such a central role in modern physics, we review their properties in Chapter 5.

1.4 Conservation Laws and Fundamental Forces

Conservation laws are the guiding principles of physics. The application of a few laws explains a vast quantity of physical phenomena. We listed the conservation laws of classical physics in Section 1.1. They include energy, linear momentum, angular momentum, and charge. Each of these is extremely useful in introductory physics. We use linear momentum when studying collisions, and the conservation laws when examining dynamics. We have seen the concept of the conservation of energy change. At first we had only the conservation of kinetic energy in a force-free region. Then we added potential energy and formed the conservation of mechanical energy. In our study of thermodynamics, we added internal energy, and so on. The study of electrical circuits was made easier by the conservation of charge flow at each junction and the conservation of energy throughout all the circuit elements.

In our study of modern physics we will find that mass is added to the conservation of energy, and the result is sometimes called the *conservation of mass–energy*, although the term conservation *of energy* is still sufficient and generally used. When we study fundamental particles we will add the conservation of baryons and the conservation of leptons. Closely related to conservation laws are invariance principles. Some parameters are invariant in some interactions or in specific systems but not in others. Examples include time reversal, parity, and distance. We will study the Newtonian or Galilean invariance and find it lacking in our study of relativity; a new invariance principle will be needed. In our study of nuclear and elementary particles, conservation laws and invariance principles will often be used (see Figure 1.5).

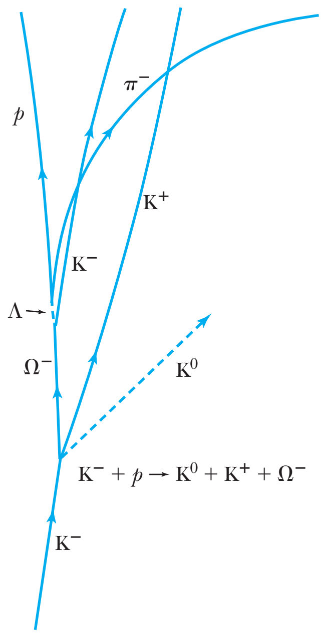


Figure 1.5 The conservation laws of momentum and energy are invaluable in untangling complex particle reactions like the one shown here, where a 5-GeV K^- meson interacts with a proton at rest to produce an Ω^- in a bubble chamber. The uncharged K^0 is not observed. Notice the curved paths of the charged particles in the magnetic field. Such reactions are explained in Chapter 14.

Fundamental Forces

In introductory physics, we often begin our study of forces by examining the reaction of a mass at the end of a spring, because the spring force can be easily calibrated. We subsequently learn about tension, friction, gravity, surface, electrical, and magnetic forces. Despite the seemingly complex array of forces, we presently believe there are only three fundamental forces. All the other forces can be derived from them. These three forces are the **gravitational**, **electroweak**, and **strong** forces. Some physicists refer to the electroweak interaction as separate electromagnetic and weak forces because the unification occurs only at very high energies. The approximate strengths and ranges of the three fundamental forces are listed in Table 1.1. Physicists sometimes use the term *interaction* when

Table 1.1 Fundamental Forces

Interaction		Relative Strength*	Range
Strong		1	Short, $\sim 10^{-15}$ m
Electroweak	Electromagnetic	10^{-2}	Long, $1/r^2$
	Weak	10^{-9}	Short, $\sim 10^{-15}$ m
Gravitational		10^{-39}	Long, $1/r^2$

*These strengths are quoted for neutrons and/or protons in close proximity.

referring to the fundamental forces because it is the overall interaction among the constituents of a system that is of interest.

The gravitational force is the weakest. It is the force of mutual attraction between masses and, according to Newton, is given by

Gravitational interaction

$$\vec{F}_g = -G\frac{m_1 m_2}{r^2}\hat{r} \tag{1.17}$$

where m_1 and m_2 are two point masses, G is the gravitational constant, r is the distance between the masses, and $\hat{r}$ is a unit vector directed along the line between the two point masses (attractive force). The gravitational force is noticeably effective only on a macroscopic scale, but it has tremendous importance: it is the force that keeps Earth in orbit about our source of life energy—the sun—and that keeps us and our atmosphere anchored to the ground. Gravity is a long-range force that diminishes as $1/r^2$.

Weak interaction

The primary component of the electroweak force is *electromagnetic*. The other component is the *weak* interaction, which is responsible for beta decay in nuclei, among other processes. In the 1970s Sheldon Glashow, Steven Weinberg, and Abdus Salam predicted that the electromagnetic and weak forces were in fact facets of the same force. Their theory predicted the existence of new particles, called W and Z bosons, which were discovered in 1983. We discuss bosons and the experiment in Chapter 14. For all practical purposes, the weak interaction is effective in the nucleus only over distances the size of 10^{-15} m. Except when dealing with very high energies, physicists mostly treat nature as if the electromagnetic and weak forces were separate. Therefore, you will sometimes see references to the *four* fundamental forces (gravity, strong, electromagnetic, and weak).

Electromagnetic interaction

The electromagnetic force is responsible for holding atoms together, for friction, for contact forces, for tension, and for electrical and optical signals. It is responsible for all chemical and biological processes, including cellular structure and nerve processes. The list is long because the electromagnetic force is responsible for practically all nongravitational forces that we experience. The electrostatic, or Coulomb, force between two point charges q_1 and q_2, separated by a distance r, is given by

Coulomb force

$$\vec{F}_C = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \tag{1.18}$$

The easiest way to remember the vector direction is that like charges repel and unlike charges attract. Moving charges also create and react to magnetic fields [see Equation (1.7)].

Strong interaction

The third fundamental force, the strong force, is the one holding the nucleus together. It is the strongest of all the forces, but it is effective only over short distances—on the order of 10^{-15} m. The strong force is so strong that it easily binds two protons inside a nucleus even though the electrical force of repulsion over the tiny confined space is huge. The strong force is able to contain dozens of protons inside the nucleus before the electrical force of repulsion becomes strong enough to cause nuclear decay. We study the strong force extensively in this book, learning that neutrons and protons are composed of *quarks*, and that the part of the strong force acting between quarks has the unusual name of *color* force.

Physicists strive to combine forces into more fundamental ones. Centuries ago the forces responsible for friction, contact, and tension were all believed to be different. Today we know they are all part of the electroweak force. Two hundred years ago scientists thought the electrical and magnetic forces were independent, but after a series of experiments, physicists slowly began to see their connection. This culminated in the 1860s in Maxwell's work, which clearly showed they were but part of one force and at the same time explained light and other radiation. Figure 1.6 is a diagram of the unification of forces over time. Newton certainly had an inspiration when he was able to unify the planetary motions with the apple falling from the tree. We will see in Chapter 15 that Einstein was even able to link gravity with space and time.

Unification of forces

The further unification of forces currently remains one of the most active research fields. Considerable efforts have been made to unify the electroweak and strong forces through the *grand unified theories*, or GUTs. A leading GUT is the mathematically complex *string theory*. Several predictions of these theories have not yet been verified experimentally (for example, the instability of the

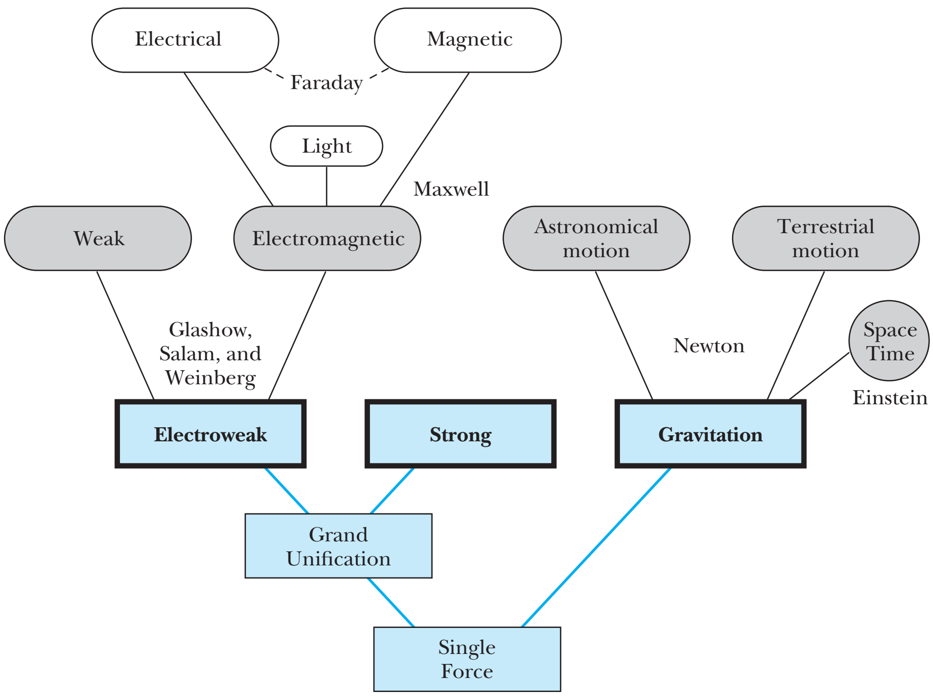


Figure 1.6 The three fundamental forces (shown in the heavy boxes) are themselves unifications of forces that were once believed to be fundamental. Present research is underway (see blue lines) to further unify the fundamental forces into a single force sometimes called the Theory of Everything.

proton and the existence of magnetic monopoles). A Theory of Everything would combine quantum theory and gravity, called quantum gravity, with the Standard Model of particle physics (see Chapter 14) and the Standard Model of cosmology (see Chapter 16). We present some of the exciting research areas in present-day physics throughout this book, because these topics are at the forefront of current research.

1.5 The Atomic Theory of Matter

Today the idea that matter is composed of tiny particles called *atoms* is taught in grade school and expounded throughout later schooling. We are told that the Greek philosophers Democritus and Leucippus proposed the concept of atoms as early as 450 BC. The smallest piece of matter, which could not be subdivided further, was called an *atom*, after the Greek word *atomos*, meaning "indivisible."

Not many new ideas were proposed about atoms until the seventeenth century, when scientists started trying to understand the properties and laws of gases. The work of Boyle, Charles, and Gay-Lussac presupposed the interactions of tiny particles in gases. Chemists and physical chemists made many important advances. In 1799 the French chemist Joseph Proust (1754–1826) proposed the *law of definite proportions*, which states that when two or more elements combine to form a compound, the proportions by weight (or mass) of the elements are always the same. Water (H_2O) is always formed of one part hydrogen and eight parts oxygen by mass.

The English chemist John Dalton (1766–1844) is given most of the credit for originating the modern atomic theory of matter. In 1803 he proposed that the atomic theory of matter could explain the law of definite proportions if the elements are composed of atoms. Each element has atoms that are physically and chemically characteristic. The concept of atomic weights (or masses) was the key to the atomic theory.

In 1811 the Italian physicist Amedeo Avogadro proposed the existence of molecules, consisting of individual or combined atoms. He stated without proof that *all gases contain the same number of molecules in equal volumes at the same temperature and pressure*. Avogadro's ideas were ridiculed by Dalton and others who could not imagine that atoms of the same element could combine. If this could happen, they argued, then all the atoms of a gas would combine to form a liquid. The concept of molecules and atoms was indeed difficult to imagine, but finally, in 1858, the Italian chemist Stanislao Cannizzaro (1826–1910) solved the problem and showed how Avogadro's ideas could be used to find atomic masses. Today we think of an atom as the smallest unit of matter that can be identified with a particular element. A molecule is a combination of two or more atoms of either like or dissimilar elements. Molecules can consist of thousands of atoms.

Avogadro's number

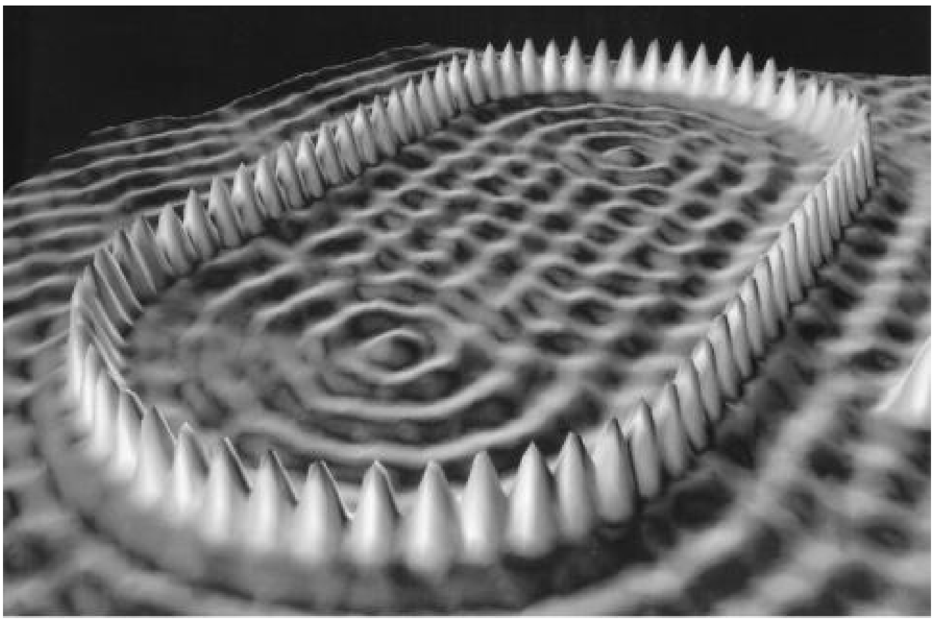
The number of constituent particles, usually atoms or molecules, that are contained in one gram-molecular weight of a particular substance (6.023×10^{23} molecules/mol) is called Avogadro's number (N_A). For example, one mole of hydrogen (H_2) has a mass of about 2 g and one mole of carbon has a mass of about 12 g; one mole of each substance consists of 6.023×10^{23} atoms. Avogadro's number was not even estimated until 1865, and it was finally accurately measured by Perrin, as we discuss at the end of this section.

During the mid-1800s the kinetic theory of gases was being developed, and because it was based on the concept of atoms, its successes gave validity to the atomic theory. The experimental results of specific heats, Maxwell speed distribution, and transport phenomena (see the discussion in Section 1.2) all supported the concept of the atomic theory.

In 1827 the English botanist Robert Brown (1773–1858) observed with a microscope the motion of tiny pollen grains suspended in water. The pollen appeared to dance around in random motion, while the water was still. At first the motion (now called *Brownian motion*) was ascribed to convection or organic matter, but eventually it was observed to occur for any tiny particle suspended in liquid. The explanation according to the atomic theory is that the molecules in the liquid are constantly bombarding the tiny grains. A satisfactory explanation was not given until the twentieth century (by Einstein).

Although it may appear, according to the preceding discussion, that the atomic theory of matter was universally accepted by the end of the nineteenth century, that was not the case. Certainly most physicists believed in it, but there was still opposition. A principal leader in the antiatomic movement was the renowned Austrian physicist Ernst Mach. Mach was an absolute positivist, believing in the reality of nothing but our own sensations. A simplified version of his line of reasoning would be that because we have never *seen* an atom, we cannot say anything about its reality. The Nobel Prize–winning German physical chemist Wilhelm Ostwald supported Mach philosophically but also had more practical arguments on his side. In 1900 there were difficulties in understanding radioactivity, x rays, discrete spectral lines, and how atoms formed molecules and solids. Ostwald contended that we should therefore think of atoms as hypothetical constructs, useful for bookkeeping in chemical reactions.

On the other hand, there were many believers in the atomic theory. Max Planck, the originator of quantum theory, grudgingly accepted the atomic theory of matter because his radiation law supported the existence of submicroscopic quanta. Boltzmann was convinced that atoms must exist, mainly because they were necessary in his statistical mechanics. Today we have pictures of the atom (see Figure 1.7) that would undoubtedly have convinced even Mach, who died in 1916 still unconvinced of the validity of the atomic theory.

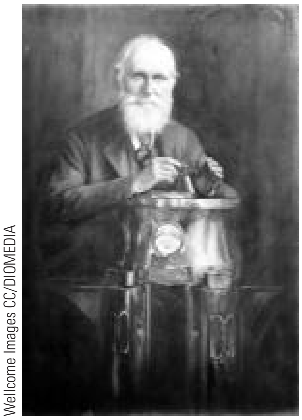


Courtesy of International Business Machines.

Figure 1.7 This scanning tunneling microscope photo, called the "stadium corral," shows 76 individually placed iron atoms on a copper surface. The IBM researchers were trying to contain and modify electron density, observed by the wave patterns, by surrounding the electrons inside the quantum "corral." Researchers are thus able to study the quantum behavior of electrons. See also the Special Topic on Scanning Probe Microscopes in Chapter 6.

Overwhelming evidence for the existence of atoms was finally presented in the first decade of the twentieth century. First, Einstein, in one of his three famous papers published in 1905 (the others were about special relativity and the photoelectric effect), provided an explanation of the Brownian motion observed almost 80 years earlier by Robert Brown. Einstein explained the motion in terms of molecular motion and presented theoretical calculations for the *random walk* problem. A random walk is a statistical process that determines how far from its initial position a tiny grain may be after many random molecular collisions. Einstein was able to determine the approximate masses and sizes of atoms and molecules from experimental data.

Finally, in 1908, the French physicist Jean Perrin (1870–1942) presented data from an experiment designed using kinetic theory that agreed with Einstein's predictions. Perrin's experimental method of observing many particles of different sizes is a classic work, for which he received the Nobel Prize for Physics in 1926. His experiment utilized four types of measurements. Each was consistent with the atomic theory, and each gave a quantitative determination of Avogadro's number—the first accurate measurements that had been made. By 1908 the atomic theory was well accepted.


Wellcome Images CC/DIOMEDIA

William Thomson (Lord Kelvin, 1824–1907) was born in Belfast, Ireland, and at age 10 entered the University of Glasgow in Scotland where his father was a professor of mathematics. He graduated from the University of Cambridge and, at age 22, accepted the chair of natural philosophy (later called physics) at the University of Glasgow, where he finished his illustrious 53-year career, finally resigning in 1899 at age 75. Lord Kelvin's contributions to nineteenth-century science were far reaching, and he made contributions in electricity, magnetism, thermodynamics, hydrodynamics, and geophysics. He was involved in the successful laying of the transatlantic cable. He was arguably the preeminent scientist of the latter part of the nineteenth century. He was particularly well known for his prediction of the Earth's age, which would later turn out to be inaccurate (see Chapter 12).

1.6 Unresolved Questions of 1895 and New Horizons

We choose 1895 as a convenient time to separate the periods of classical and modern physics, although this is an arbitrary choice based on discoveries made in 1895–1897. The thousand or so physicists living in 1895 were rightfully proud of the status of their profession. The precise experimental method was firmly established. Theories were available that could explain many observed phenomena. In large part, scientists were busy measuring and understanding such physical parameters as specific heats, densities, compressibility, resistivity, indices of refraction, and permeabilities. The pervasive feeling was that, given enough time, everything in nature could be understood by applying the careful thinking and experimental techniques of physics. The field of mechanics was in particularly good shape, and its application had led to the stunning successes of the kinetic theory of gases and statistical thermodynamics.

In hindsight we can see now that this euphoria of success applied only to the macroscopic world. Objects of human dimensions such as automobiles, steam engines, airplanes, telephones, and electric lights either existed or were soon to appear and were triumphs of science and technology. However, the atomic theory of matter was not universally accepted, and what made up an atom was purely conjecture. The structure of matter was unknown.

Clouds on the horizon

There were certainly problems that physicists could not resolve. Only a few of the deepest thinkers seemed to be concerned with them. Lord Kelvin, in a speech in 1900 to the Royal Institution, referred to "two clouds on the horizon." These were the electromagnetic medium and the failure of classical physics to explain blackbody radiation. We mention these and other problems here. Their solutions were soon to lead to two of the greatest breakthroughs in human thought ever recorded—the theories of quantum physics and of relativity.

Electromagnetic Medium. The waves that were well known and understood by physicists all had media in which the waves propagated. Water waves traveled in water, and sound waves traveled in any material. It was natural for nineteenth-century physicists to assume that electromagnetic waves also traveled in a medium, and this medium was called the *ether*. Several experiments, the most notable of which were done by Albert Michelson, had sought to detect the ether without success. An extremely careful experiment by Michelson and Morley in 1887 was so sensitive, it should have revealed the effects of the ether. Subsequent experiments to check other possibilities were also negative. In 1895 some physicists were concerned that the elusive ether could not be detected. Was there an alternative explanation?

Electrodynamics. The other difficulty with Maxwell's electromagnetic theory had to do with the electric and magnetic fields as seen and felt by moving bodies. What appears as an electric field in one reference system may appear as a magnetic field in another system moving with respect to the first. Although the relationship between electric and magnetic fields seemed to be understood by using Maxwell's equations, the equations do not keep the same form under a Galilean transformation [see Equations (2.1) and (2.2)], a situation that concerned both Hertz and Lorentz. Hertz unfortunately died in 1894 at the young age of 36 and never experienced the modern physics revolution. The Dutch physicist Hendrik Lorentz (1853–1928), on the other hand, proposed a radical idea that solved the electrodynamics problem: space was contracted along the direction of motion of the body. George FitzGerald in Ireland independently proposed the same concept. The Lorentz–FitzGerald hypothesis, proposed in 1892, was a precursor to Einstein's theory advanced in 1905 (see Chapter 2).

Blackbody Radiation. In 1895 thermodynamics was on a strong footing; it had achieved much success. One of the interesting experiments in thermodynamics concerns an object, called a *blackbody*, that absorbs the entire spectrum of electromagnetic radiation incident on it. An enclosure with a small hole serves as a blackbody, because all the radiation entering the hole is absorbed. A blackbody also emits radiation, and the emission spectrum shows the electromagnetic power emitted per unit area. The radiation emitted covers all frequencies, each with its own intensity. Precise measurements were carried out to determine the spectrum of blackbody radiation, shown in Figure 1.8. Blackbody radiation was a fundamental issue, because the emission spectrum is independent of the body itself—it is characteristic of all blackbodies.

Many physicists of the period—including Kirchhoff, Stefan, Boltzmann, Rubens, Pringsheim, Lummer, Wien, Lord Rayleigh, Jeans, and Planck—had worked on the problem. It was possible to understand the spectrum either at the low-frequency end or at the high-frequency end, but no single theory could account for the entire spectrum. When the most modern theory of the day (the equipartition of energy applied to standing waves in a cavity) was applied to the problem, the result led to an *infinite* emissivity (or energy density) for high frequencies. The failure of the theory was known as the "ultraviolet catastrophe." The solution of the problem by Max Planck in 1900 would shake the very foundations of physics.

Ultraviolet catastrophe

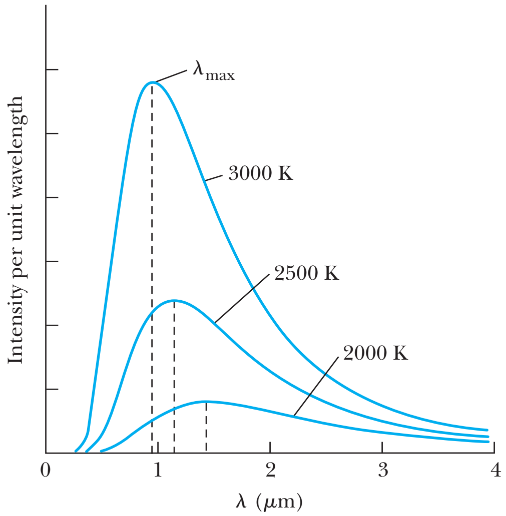


Figure 1.8 The blackbody spectrum, showing the emission spectrum of radiation emitted from a blackbody as a function of the radiation wavelength. Different curves are produced for different temperatures, but they are independent of the type of blackbody cavity. The intensity peaks at λ_{max}.

On the Horizon

During the years 1895–1897 there were four discoveries that were all going to require deeper understanding of the atom. The first was the discovery of x rays by the German physicist Wilhelm Röntgen (1845–1923) in November 1895. Next came the accidental discovery of radioactivity by the French physicist Henri Becquerel (1852–1908), who in February 1896 placed uranium salt next to a carefully wrapped photographic plate. When the plate was developed, a silhouette of the uranium salt was evident—indicating the presence of a very penetrating ray.

Discovery of x rays

Discovery of radioactivity

The third discovery, that of the electron, was actually the work of several physicists over a period of years. Michael Faraday, as early as 1833, observed a gas discharge glow—evidence of electrons. Over the next few years, several scientists detected evidence of particles, called *cathode rays*, being emitted from charged cathodes. In 1896 Perrin proved that cathode rays were negatively charged. The discovery of the electron, however, is generally credited to the British physicist J. J. Thomson (1856–1940), who in 1897 isolated the electron (cathode ray) and measured its velocity and its ratio of charge to mass.

Discovery of the electron

The final important discovery of the period was made by the Dutch physicist Pieter Zeeman (1865–1943), who in 1896 found that a single spectral line was sometimes separated into two or three lines when the sample was placed in a magnetic field. The (normal) *Zeeman effect* was quickly explained by Lorentz as the result of light being emitted by the motion of electrons inside the atom. Zeeman and Lorentz showed that the frequency of the light was affected by the magnetic field according to the classical laws of electromagnetism.

Discovery of the Zeeman effect

The unresolved issues of 1895 and the important discoveries of 1895–1897 bring us to the subject of this book, *Modern Physics*. In 1900 Max Planck completed his radiation law, which solved the blackbody problem but required that energy be quantized. In 1905 Einstein presented his three important papers on Brownian motion, the photoelectric effect, and special relativity. While the work of Planck and Einstein may have solved the problems of the nineteenth-century physicists, they broadened the horizons of physics and have kept physicists active ever since.

Summary

Physicists of the 1890s felt that almost anything in nature could be explained by the application of careful experimental methods and intellectual thought. The application of mechanics to the kinetic theory of gases and statistical thermodynamics, for example, was a great success.

The particle viewpoint of light had prevailed for over a hundred years, mostly because of the weakly held belief of the great Newton, but in the early 1800s the nature of light was resolved in favor of waves. In the 1860s Maxwell showed that his electromagnetic theory predicted a much wider frequency range of electromagnetic radiation than the visible optical phenomena. In the twentieth century, the question of waves versus particles was to reappear.

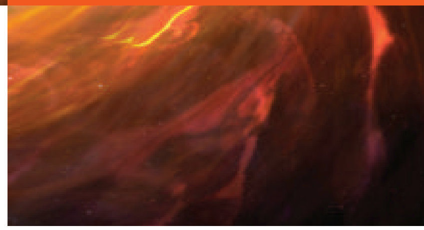
The conservation laws of energy, momentum, angular momentum, and charge are well established. The three fundamental forces are gravitational, electroweak, and strong. Over the years many forces have been unified into these three. Physicists are actively pursuing attempts to unify these three forces into only two or even just one single fundamental force.

The atomic theory of matter assumes atoms are the smallest unit of matter that is identified with a characteristic element. Molecules are composed of atoms, which can be from different elements. The kinetic theory of gases assumes the atomic theory is correct, and the development of the two theories proceeded together. The atomic theory of matter was not fully accepted until around 1910, by which time Einstein had explained Brownian motion and Perrin had published overwhelming experimental evidence.

The year 1895 saw several outstanding problems that seemed to worry only a few physicists. These problems included the inability to detect an electromagnetic medium, the difficulty in understanding the electrodynamics of moving bodies, and blackbody radiation. Four important discoveries during the period 1895–1897 were to signal the atomic age: x rays, radioactivity, the electron, and the splitting of spectral lines (Zeeman effect). The understanding of these problems and discoveries (among many others) is the object of this book on modern physics.

Special Theory of Relativity

CHAPTER 2



Relativity challenges your basic intuitions that you've built up from everyday experience. It says your experience of time is not what you think it is, that time is malleable. Your experience of space is not what you think it is; it can stretch and shrink.

Brian Greene, The Elegant Universe, 1999

One of the great theories of physics appeared early in the twentieth century when Albert Einstein presented his special theory of relativity in 1905. We learned in introductory physics that Newton's laws of motion must be measured relative to some reference frame. A reference frame is called an **inertial frame** if Newton's laws are valid in that frame. If a body subject to no net external force moves with constant velocity, then the coordinate system attached to that body defines an inertial frame. If Newton's laws are valid in one reference frame, then they are also valid in a reference frame moving at a uniform velocity relative to the first system. This is known as the **Newtonian principle of relativity** or **Galilean invariance**.

Inertial frame

Galilean invariance

Newton showed that it was not possible to determine absolute motion in space by any experiment, so he decided to use relative motion. In addition, the Newtonian concepts of time and space are completely separable. Consider two inertial reference frames, K and K′, that move along their x and x' axes, respectively, with uniform relative velocity $\vec{v}$ as shown in Figure 2.1. We show system K′ moving to the right with velocity $\vec{v}$ with respect to system K, which is fixed or

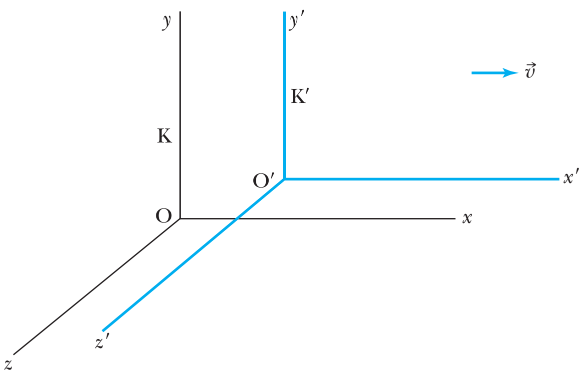


Figure 2.1 Two inertial systems are moving with relative speed v along their x axes. We show the system K at rest and the system K′ moving with speed v relative to the system K.

stationary. One result of the relativity theory is that there are no fixed, absolute frames of reference. We use the term *fixed* to refer to a system that is fixed on a particular object, such as a planet, star, or spaceship that itself is moving in space. The transformation of the coordinates of a point in one system to the other system is given by

$$x' = x - vt$$
$$y' = y \qquad (2.1)$$
$$z' = z$$

Similarly, the inverse transformation is given by

$$x = x' + vt$$
$$y = y' \qquad (2.2)$$
$$z = z'$$

Galilean transformation

where we have set $t = t'$ because Newton considered time to be absolute. Equations (2.1) and (2.2) are known as the **Galilean transformation**. Newton's laws of motion are invariant under a Galilean transformation; that is, they have the same form in both systems K and K′.

In the late nineteenth century Albert Einstein was concerned that although Newton's laws of motion had the same form under a Galilean transformation, Maxwell's equations did not. Einstein believed so strongly in Maxwell's equations that he showed there was a significant problem in our understanding of the Newtonian principle of relativity. In 1905 he published ideas that rocked the very foundations of physics and science. He proposed that space and time are not separate and that Newton's laws are only an approximation. This special theory of relativity and its ramifications are the subject of this chapter. We begin by presenting the experimental situation historically—showing why a problem existed and what was done to try to rectify the situation. Then we discuss Einstein's two postulates on which the special theory is based. The interrelation of space and time is discussed, and several amazing and remarkable predictions based on the new theory are presented.

As the concepts of relativity became used more often in everyday research and development, it became essential to understand the transformation of momentum, force, and energy. Here we study relativistic dynamics and the relationship between mass and energy, which leads to one of the most famous equations in physics and a new conservation law of mass–energy. Finally, we return to electromagnetism to investigate the effects of relativity. We learn that Maxwell's equations don't require change, and electric and magnetic effects are relative, depending on the observer. We leave until Chapter 15 our discussion of Einstein's general theory of relativity.

2.1 The Apparent Need for Ether

Thomas Young, an English physicist and physician, performed his famous experiments on the interference of light in 1802. A decade later, the French physicist and engineer Augustin Fresnel published his calculations showing the detailed understanding of interference, diffraction, and polarization. Because all known waves (other than light) require a medium in which to propagate (water waves have water, sound waves have, for example, air, and so on), it was naturally assumed that light also required a medium, even though light was apparently able to travel in vacuum through outer space. This medium was called the *luminiferous ether*, or just **ether** for short, and it must have some amazing properties. The ether had to have such a low density that planets could pass through it, seemingly for eternity, with no apparent loss of orbit position. Its elasticity must be strong enough to pass waves of incredibly high speeds!

The concept of ether

The electromagnetic theory of light (1860s) of the Scottish mathematical physicist James Clerk Maxwell shows that the speed of light in different media depends only on the electric and magnetic properties of matter. In vacuum, the speed of light is given by $v = c = 1/\sqrt{\mu_0\epsilon_0}$, where μ_0 and ϵ_0 are the permeability and permittivity of free space, respectively. The properties of the ether, as proposed by Maxwell in 1873, must be consistent with electromagnetic theory, and it was thought that to be able to discern the ether's various properties required only a sensitive enough experiment. The concept of ether was well accepted by 1880.

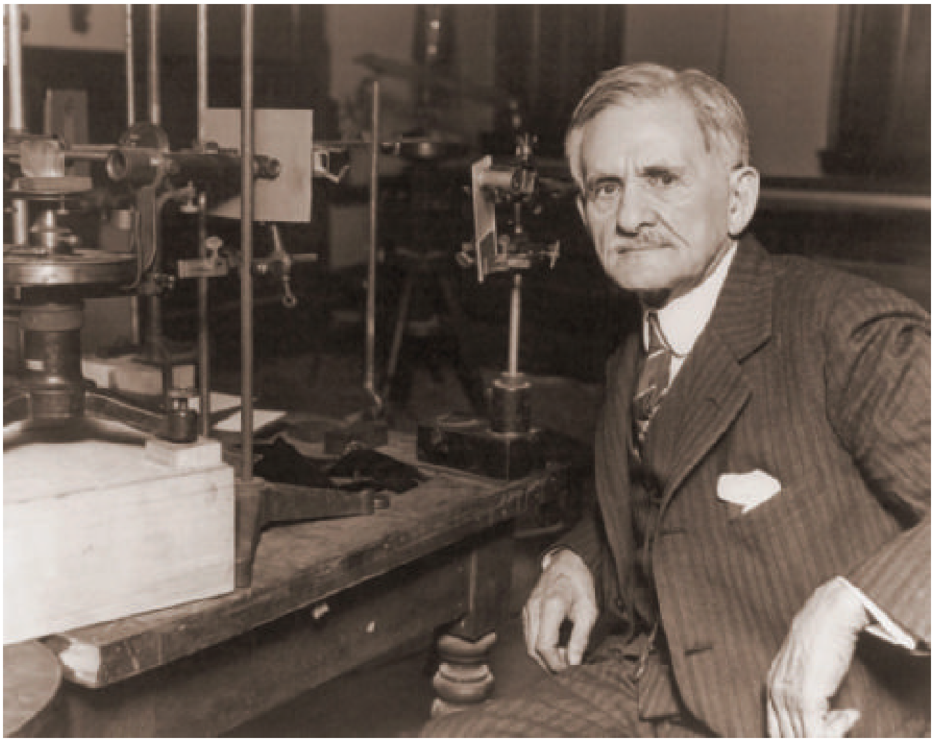
When Maxwell presented his electromagnetic theory, scientists were so confident in the laws of classical physics that they immediately pursued the aspects of Maxwell's theory that were in contradiction with those laws. As it turned out, this investigation led to a new, deeper understanding of nature. Maxwell's equations predict the speed of light in a vacuum to be c. If we have a flashbulb go off in the moving system K′, an observer in system K′ measures the speed of the light pulse to be c. However, if we make use of Equation (2.1) to find the relation between speeds, we find the speed measured in system K to be $c + v$, where v is the relative speed of the two systems. However, Maxwell's equations don't differentiate between these two systems. Physicists of the late nineteenth century proposed that there must be one preferred inertial reference frame in which the ether was stationary and that in this system the speed of light was c. In the other systems, the speed of light would indeed be affected by the relative speed of the reference system. Because the speed of light was known to be so enormous, 3×10^8 m/s, no experiment had as yet been able to discern an effect due to the relative speed v. The ether frame would in fact be an absolute standard, from which other measurements could be made. Scientists set out to find the effects of the ether.

2.2 The Michelson–Morley Experiment

The Earth orbits the sun at a high orbital speed, about $10^{-4}c$, so an obvious experiment is to try to find the effects of the Earth's motion through the ether. Even though we don't know how fast the sun might be moving through the ether, the Earth's orbital *velocity* changes significantly throughout the year because of its change in direction, even if its orbital *speed* is nearly constant. That is, the Earth's velocity through the ether would appear opposite in the summer and winter.

Albert Michelson (1852–1931) performed perhaps the most significant American physics experiment of the 1800s. Michelson, who was the first U.S. citizen to receive the Nobel Prize in Physics (1907), was an ingenious scientist who built an extremely precise device called an *interferometer*, which measures the phase difference between two light waves. Michelson used his interferometer to detect the difference in the speed of light passing through the ether in different

Albert A. Michelson (1852–1931) shown in his lab at the University of Chicago. He was born in Prussia but came to the United States when he was two years old. He was educated at the U.S. Naval Academy and later returned on the faculty. Michelson had appointments at several American universities including the Case School of Applied Science, Cleveland, in 1883; Clark University, Worcester, Massachusetts, in 1890; and the University of Chicago in 1892 until his retirement in 1929. During World War I he returned to the U.S. Navy, where he developed a rangefinder for ships. He spent his retirement years in Pasadena, California, where he continued to measure the speed of light at Mount Wilson.



Everett Collection Historical/Alamy Stock Photo

directions. The basic technique is shown in Figure 2.2. Initially, it is assumed that one of the interferometer arms (AC) is parallel to the motion of the Earth through the ether. Light leaves the source S and passes through the glass plate at A. Because the back of A is partially silvered, part of the light is reflected, eventually going to the mirror at D, and part of the light travels through A on to the mirror at C. The light is reflected at the mirrors C and D and comes back to

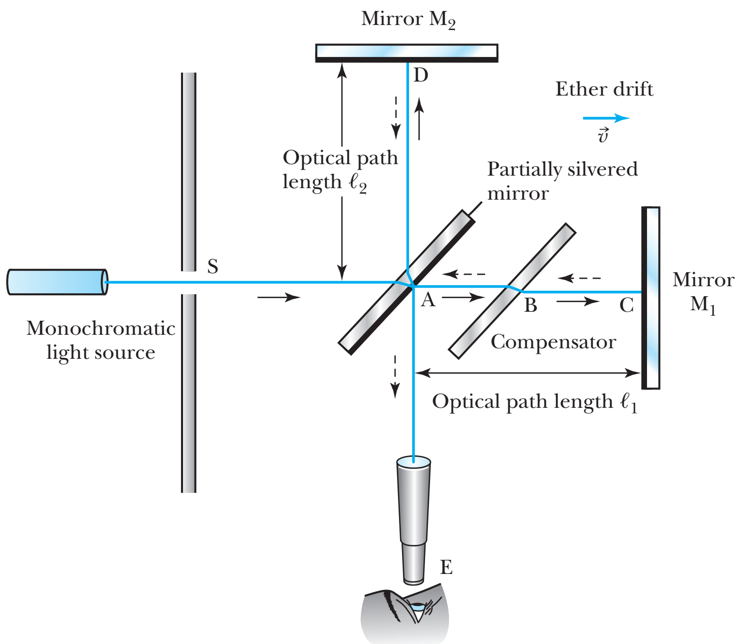


Figure 2.2 A schematic diagram of Michelson's interferometer experiment. Light of a single wavelength is partially reflected and partially transmitted by the glass at A. The light is subsequently reflected by mirrors at C and D, and, after reflection or transmission again at A, enters the telescope at E. Interference fringes are visible to the observer at E.

the partially silvered mirror A, where part of the light from each path passes on to the telescope and eye at E. The compensator is added at B to make sure both light paths pass through equal thicknesses of glass. Interference fringes can be found by using a bright light source such as sodium, with the light filtered to make it monochromatic, and the apparatus is adjusted for maximum intensity of the light at E. We will show that the fringe pattern should shift if the apparatus is rotated through 90° such that arm AD becomes parallel to the motion of the Earth through the ether and arm AC is perpendicular to the motion.

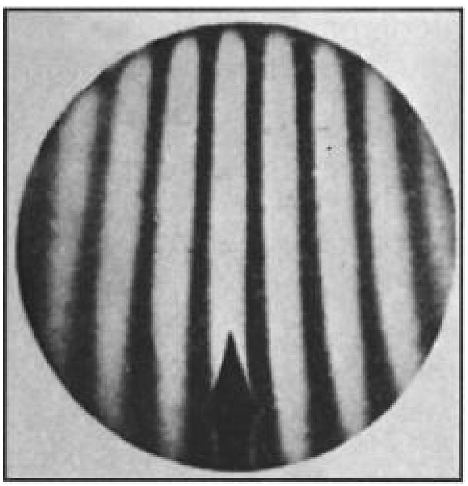
We let the optical path lengths of AC and AD be denoted by ℓ_1 and ℓ_2, respectively. The observed interference pattern consists of alternating bright and dark bands, corresponding to constructive and destructive interference, respectively (Figure 2.3). For constructive interference, the difference between the two path lengths (to and from the mirrors) is given by some number of wavelengths, $2(\ell_1 - \ell_2) = n\lambda$, where λ is the wavelength of the light and n is an integer.

The expected shift in the interference pattern can be calculated by determining the time difference between the two paths. When the light travels from A to C, the velocity of light according to the Galilean transformation is $c + v$, because the ether carries the light along with it. On the return journey from C to A the velocity is $c - v$, because the light travels opposite to the path of the ether. The total time for the round-trip journey to mirror M_1 is t_1:

$$t_1 = \frac{\ell_1}{c+v} + \frac{\ell_1}{c-v} = \frac{2c\ell_1}{c^2 - v^2} = \frac{2\ell_1}{c}\left(\frac{1}{1 - v^2/c^2}\right)$$

Now imagine what happens to the light that is reflected from mirror M_2. If the light is pointed directly at point D, the ether will carry the light with it, and the light misses the mirror, much as the wind can affect the flight of an arrow. If a swimmer (who can swim with speed v_2 in still water) wants to swim across a swiftly moving river (speed v_1), the swimmer must start heading upstream, so that when the current carries her downstream, she will move directly across the river. Careful reasoning shows that the swimmer's velocity is $\sqrt{v_2^2 - v_1^2}$ throughout her journey (Problem 4). Thus the time t_2 for the light to pass to mirror M_2 at D and back is

$$t_2 = \frac{2\ell_2}{\sqrt{c^2 - v^2}} = \frac{2\ell_2}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$



From L. S. Swenson, Jr., Invention and Discovery 43 (Fall 1987).

Figure 2.3 Interference fringes as they would appear in the eyepiece of the Michelson–Morley experiment.

The time difference between the two journeys Δt is

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{\ell_2}{\sqrt{1 - v^2/c^2}} - \frac{\ell_1}{1 - v^2/c^2}\right) \tag{2.3}$$

We now rotate the apparatus by 90° so that the ether passes along the length ℓ_2 toward the mirror M_2. We denote the new quantities by primes and carry out an analysis similar to that just done. The time difference $\Delta t'$ is now

$$\Delta t' = t_2' - t_1' = \frac{2}{c}\left(\frac{\ell_2}{1 - v^2/c^2} - \frac{\ell_1}{\sqrt{1 - v^2/c^2}}\right) \tag{2.4}$$

Michelson looked for a shift in the interference pattern when his apparatus was rotated by 90°. The time difference is

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{\ell_1 + \ell_2}{1 - v^2/c^2} - \frac{\ell_1 + \ell_2}{\sqrt{1 - v^2/c^2}}\right)$$

Because we know $c \gg v$, we can use the binomial expansion* to expand the terms involving v^2/c^2, keeping only the lowest terms.

$$\Delta t' - \Delta t = \frac{2}{c}(\ell_1 + \ell_2)\left[\left(1 + \frac{v^2}{c^2} + \cdots\right) - \left(1 + \frac{v^2}{2c^2} + \cdots\right)\right]$$

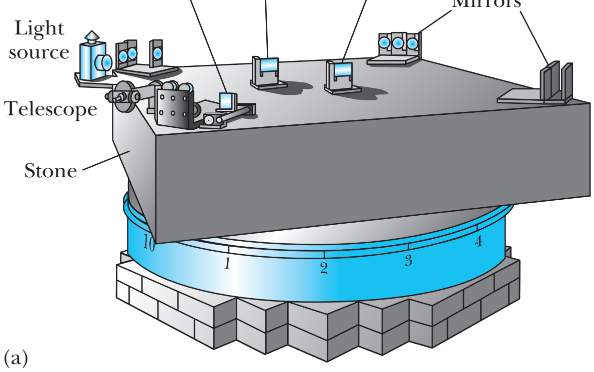
$$\approx \frac{v^2(\ell_1 + \ell_2)}{c^3} \tag{2.5}$$

Michelson in Europe

Michelson left his position at the U.S. Naval Academy in 1880 and took his interferometer to Europe for postgraduate studies with some of Europe's best physicists, particularly Hermann Helmholtz in Berlin. After a few false starts he finally was able to perform a measurement in Potsdam (near Berlin) in 1881. In order to use Equation (2.5) for an estimate of the expected time difference, the value of the Earth's orbital speed around the sun, 3×10^4 m/s, was used. Michelson's apparatus had $\ell_1 \approx \ell_2 \approx \ell = 1.2$ m. Thus Equation (2.5) predicts a time difference of 8×10^{-17} s. This is an exceedingly small time, but for a visible wavelength of 6×10^{-7} m, the period of one wavelength amounts to $T = 1/f = \lambda/c = 2 \times 10^{-15}$ s. Thus the time period of 8×10^{-17} s represents 0.04 fringes in the interference pattern. Michelson reasoned that he should be able to detect a shift of at least half this value but found none. Michelson concluded that the hypothesis of the stationary ether must be incorrect.

The result of Michelson's experiment was so surprising that he was asked by several well-known physicists to repeat it. In 1882 Michelson accepted a position at the then-new Case School of Applied Science in Cleveland. Together with Edward Morley (1838–1923), a professor of chemistry at nearby Western Reserve College who had become interested in Michelson's work, he put together the more sophisticated experiment shown in Figure 2.4. The new experiment had an optical path length of 11 m, created by reflecting the light for eight round trips. The new apparatus was mounted on soapstone that floated on mercury to eliminate vibrations and was so effective that Michelson and Morley believed they could detect a fraction of a fringe shift as small as 0.005. With their new

*See Appendix 3 for the binomial expansion.



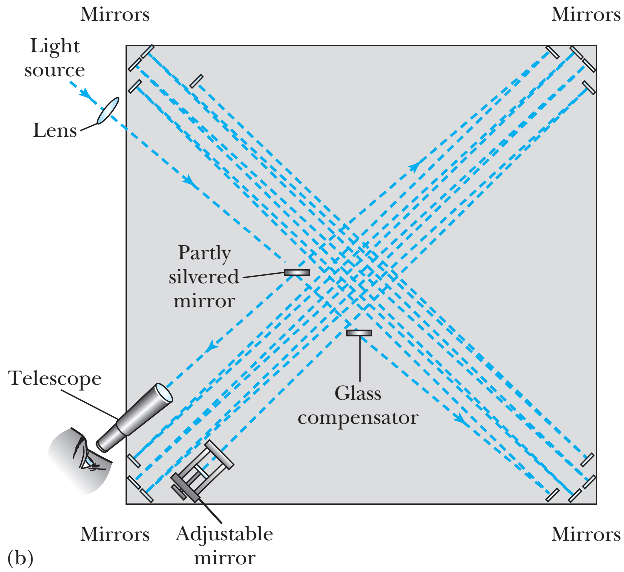


Figure 2.4 An adaptation of the Michelson and Morley 1887 experiment taken from their publication [A. A. Michelson and E. M. Morley, *Philosophical Magazine* **190**, 449 (1887)]. (a) A perspective view of the apparatus. To reduce vibration, the experiment was done on a massive soapstone, 1.5 m square and 0.3 m thick. This stone was placed on a wooden float that rested on mercury inside the annular piece shown underneath the stone. The entire apparatus rested on a brick pier. (b) The incoming light is focused by the lens and is both transmitted and reflected by the partly silvered mirror. The adjustable mirror allows fine adjustments in the interference fringes. The stone was rotated slowly and uniformly on the mercury to look for the interference effects of the ether.

apparatus they expected the ether to produce a shift as large as 0.4 of a fringe. They reported in 1887 a *null result*—no effect whatsoever! The ether does not seem to exist. It is this famous experiment that has become known as the *Michelson–Morley experiment*.

Null result of Michelson–Morley experiment

The measurement so shattered a widely held belief that many suggestions were made to explain it. What if the Earth just happened to have a zero motion through the ether at the time of the experiment? Michelson and Morley repeated their experiment during night and day and for different seasons throughout the year. It is unlikely that at least sometime during these many experiments, the Earth would not be moving through the ether. Michelson and Morley even took their experiment to a mountaintop to see if the effects of the ether might be different. There was no change.

Ether drag

Stellar aberration

Of the many possible explanations of the null ether measurement, the one taken most seriously was the *ether drag* hypothesis. Some scientists proposed that the Earth somehow dragged the ether with it as the Earth rotates on its own axis and revolves around the sun. However, the ether drag hypothesis contradicts results from several experiments, including that of *stellar aberration* noted by the British astronomer James Bradley in 1728. Bradley noticed that the apparent position of the stars seems to rotate in a circular motion with a period of one year. The angular diameter of this circular motion with respect to the Earth is 41 seconds of arc. This effect can be understood by an analogy. From the viewpoint of a person sitting in a car during a rainstorm, the raindrops appear to fall vertically when the car is at rest but appear to be slanted toward the windshield

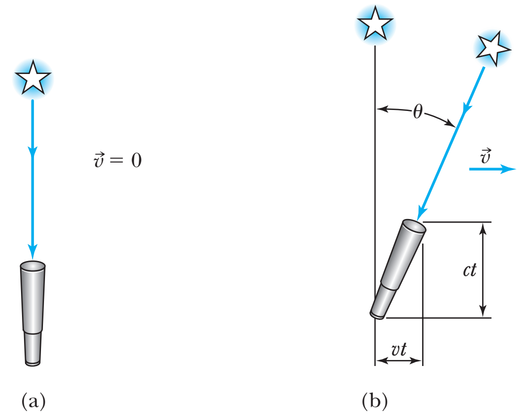


Figure 2.5 The effect of stellar aberration. (a) If a telescope is at rest, light from a distant star will pass directly into the telescope. (b) However, if the telescope is traveling at speed v (because it is fixed on the Earth, which has a motion about the sun), it must be slanted slightly to allow the starlight to enter the telescope. This leads to an apparent circular motion of the star as seen by the telescope, as the motion of the Earth about the sun changes throughout the solar year. The angle of the telescope is greatly exaggerated in the figure.

when the car is moving forward. The same effect occurs for light coming from stars directly above the Earth's orbital plane. If the telescope and star are at rest with respect to the ether, the light enters the telescope as shown in Figure 2.5a. However, because the Earth is moving in its orbital motion, the apparent position of the star is at an angle θ as shown in Figure 2.5b. The telescope must actually be slanted at an angle θ to observe the light from the overhead star. During a time period t the starlight moves a vertical distance ct while the telescope moves a horizontal distance vt, so that the tangent of the angle θ is

$$\tan\theta = \frac{vt}{ct} = \frac{v}{c}$$

The orbital speed of the Earth is about 3×10^4 m/s; therefore, the angle θ is 10^{-4} rad, or 20.6 seconds of arc, with a total opening of $2\theta = 41$ arcsec as the Earth rotates—in agreement with Bradley's observation. The aberration reverses itself over the course of six months as the Earth orbits about the sun, in effect giving a circular motion to the star's position. This observation is in disagreement with the hypothesis of the Earth dragging the ether. If the ether were dragged with the Earth, there would be no need to tilt the telescope! The experimental observation of stellar aberration together with the null result of the Michelson and Morley experiment is enough evidence to refute the suggestions that the ether exists. Many other experimental observations have now been made that also confirm this conclusion.

The inability to detect the ether was a serious blow to reconciling the invariant form of the electromagnetic equations of Maxwell. There seems to be no single reference inertial system in which the speed of light is actually c. H. A. Lorentz and G. F. FitzGerald suggested, apparently independently, that the results of the Michelson–Morley experiment could be understood if length is contracted by the factor $\sqrt{1 - v^2/c^2}$ in the direction of motion, where v is the speed in the direction of travel. For this situation, the length ℓ_1, in the direction of motion, will be contracted by the factor $\sqrt{1 - v^2/c^2}$, whereas the length ℓ_2, perpendicular to v, will not. The result in Equation (2.3) is that t_1 will have the extra factor $\sqrt{1 - v^2/c^2}$, making Δt precisely zero as determined experimentally by Michelson. This contraction postulate, which became known as the *Lorentz–FitzGerald contraction*, was not proven from first principles using Maxwell's equations, and its true significance was not understood for several years until Einstein presented his explanation. An obvious problem with the Lorentz–FitzGerald contraction is that it is an ad hoc assumption that cannot be directly tested. Any measuring device would presumably be shortened by the same factor.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Albert Einstein (1879–1955), shown here sailing on the Long Island Sound, was born in Germany and studied in Munich and Zurich. After having difficulty finding an academic position, he served seven years in the Swiss Patent Office in Bern (1902–1909), where he did some of his best work. He obtained his doctorate at the University of Zurich in 1905. His fame quickly led to appointments in Zurich, Prague, back to Zurich, and then to Berlin in 1914. In 1933, after Hitler came to power, Einstein left for the Institute for Advanced Study at Princeton University, where he became a U.S. citizen in 1940 and remained until his death in 1955. Einstein's total contributions to physics are arguably rivaled only by those of Isaac Newton.

2.3 Einstein's Postulates

At the turn of the twentieth century, the Michelson–Morley experiment had laid to rest the idea of finding a preferred inertial system for Maxwell's equations, yet the Galilean transformation, which worked for the laws of mechanics, was invalid for Maxwell's equations. This quandary represented a turning point for physics.

Albert Einstein (1879–1955) was only two years old when Michelson reported his first null measurement for the existence of the ether. Einstein said that he began thinking at age 16 about the form of Maxwell's equations in moving inertial systems, and in 1905, when he was 26 years old, he published his startling proposal* about the principle of relativity, which he believed to be fundamental. Working without the benefit of discussions with colleagues outside his small circle of friends, Einstein was apparently unaware of the interest concerning the null result of Michelson and Morley.† Einstein instead looked at the problem in a more formal manner and believed that Maxwell's equations must be valid in all inertial frames. With piercing insight and genius, Einstein was able to bring together seemingly inconsistent results concerning the laws of mechanics and electromagnetism with two postulates (as he called them; today we would call them laws). These postulates are

Einstein's two postulates

1. **The principle of relativity:** The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.
2. **The constancy of the speed of light:** Observers in all inertial systems measure the same value for the speed of light in a vacuum.

The first postulate indicates that the laws of physics are the same in all coordinate systems moving with uniform relative motion to each other. Einstein showed that postulate 2 actually follows from the first one. He returned to the principle of relativity as espoused by Newton. Although Newton's principle referred only to the laws of mechanics, Einstein expanded it to include all laws of

*In one issue of the German journal *Annalen der Physik* 17, No. 4 (1905), Einstein published three remarkable papers. The first, on the quantum properties of light, explained the photoelectric effect; the second, on the statistical properties of molecules, included an explanation of Brownian motion; and the third was on special relativity. All three papers contained predictions that were subsequently confirmed experimentally.

†The question of whether Einstein knew of Michelson and Morley's null result before he produced his special theory of relativity is somewhat uncertain. For example, see J. Stachel, "Einstein and Ether Drift Experiments," *Physics Today* (May 1987), p. 45.

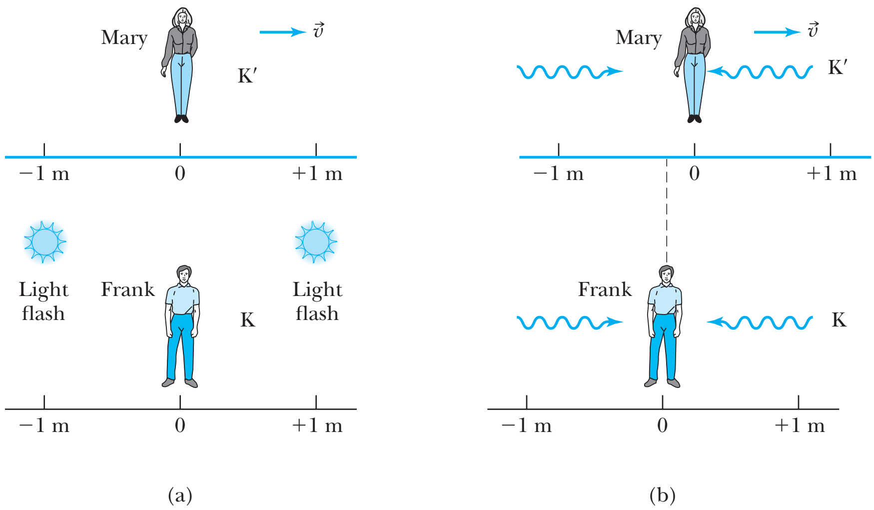

Figure 2.6 The problem of simultaneity. Flashes of light positioned in system K at one meter on either side of Frank go off simultaneously in (a). Frank indeed sees both flashes simultaneously in (b). However, Mary, at rest in system K′ moving to the right with speed v, does not see the flashes simultaneously despite the fact that she was alongside Frank when the flashes of light occurred. During the finite time it took light to travel the one meter, Mary has moved slightly, as shown in exaggerated form in (b).

Inertial frames of reference revisited

physics—including those of electromagnetism. We can now modify our previous definition of *inertial frames of reference* to be those frames of reference in which *all the laws of physics* are valid.

Einstein's solution requires us to take a careful look at time. Return to the two systems of Figure 2.1 and remember that we had previously assumed that $t = t'$. We assumed that events occurring in system K′ and in system K could easily be synchronized. Einstein realized that each system must have its own observers with their own clocks and metersticks. *An event in a given system must be specified by stating both its space and time coordinates.* Consider two light flashes fixed in system K as shown in Figure 2.6a. **M**ary, in system K′ (the **M**oving system) is beside **F**rank, who is in system K (the **F**ixed system), when the lights flash. As seen in Figure 2.6b the light pulses travel the same distance in system K and arrive at Frank *simultaneously*. Frank sees the two flashes at the same time. However, the two light pulses do not reach Mary simultaneously, because system K′ is moving to the right, and she has moved closer to the origin of the light flash on the right by the time the flash reaches her. The light flash coming from the left will reach her at some later time. Mary thus determines that the light on the right flashed before the one on the left, because she is at rest in her frame and both flashes approach her at speed c. We make the following conclusion:

Simultaneity

Two events that are simultaneous in one reference frame (K) are not necessarily simultaneous in another reference frame (K′) moving with respect to the first frame.

We must be careful when comparing the same event in two systems moving with respect to one another. Time comparison can be accomplished by sending light signals from one observer to another, but this information can travel only as fast as the finite speed of light. It is best if each system has its own observers with clocks that are synchronized. How can we do this? We place observers with clocks throughout a given system. If, when we bring all the clocks together at one spot at rest, all the clocks agree, then the clocks are said to be **synchronized**. However, we have to move the clocks relative to each other to reposition them, and this might affect the synchronization. A better way would be to have a light flash half-way between each pair of clocks at rest and make sure the pulses arrive

Synchronization of clocks

simultaneously at each clock. This will require many measurements, but it is a safe way to synchronize the clocks. We can determine the time of an event occurring far away from us by having a colleague at the event, with a clock fixed at rest, measure the time of the particular event, and send us the results, for example, by text, email, telephone, or even by mail. If we need to check our clocks, we can always send light signals to each other over known distances at some predetermined time.

In the next section we derive the correct transformation, called the **Lorentz transformation**, that makes the laws of physics invariant between inertial frames of reference. We use the coordinate systems described by Figure 2.1. At $t = t' = 0$, the origins of the two coordinate systems are coincident, and the system K′ is traveling along the x and x' axes. For this special case, the Lorentz transformation equations are

Lorentz transformation equations

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - (vx/c^2)}{\sqrt{1 - v^2/c^2}} \end{aligned} \tag{2.6}$$

Relativistic factor

We commonly use the symbols β and the *relativistic factor* γ to represent two longer expressions:

$$\beta = \frac{v}{c} \tag{2.7}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{2.8}$$

which allows the Lorentz transformation equations to be rewritten in compact form as

$$\begin{aligned} x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \beta x/c) \end{aligned} \tag{2.6}$$

Note that $\gamma \geq 1$ ($\gamma = 1$ when $v = 0$).

2.4 The Lorentz Transformation

In this section we use Einstein's two postulates to find a transformation between inertial frames of reference such that all the physical laws, including Newton's laws of mechanics and Maxwell's electrodynamics equations, will have the same form. We use the fixed system K and moving system K′ of Figure 2.1. At $t = t' = 0$ the origins and axes of both systems are coincident, and system K′ is moving to the right along the x axis. A light flash occurs at the origins when $t = t' = 0$.

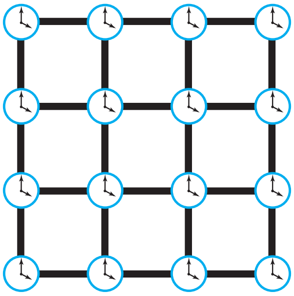

Figure 2.7 In order to make sure accurate event measurements can be obtained, synchronized clocks and uniform measuring sticks are placed throughout a system.

According to postulate 2, the speed of light is c in both systems, and the wavefronts observed in both systems must be spherical and described by

$$x^2 + y^2 + z^2 = c^2t^2 \tag{2.9a}$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2 \tag{2.9b}$$

These two equations are inconsistent with a Galilean transformation because a wavefront can be spherical in only one system when the second is moving at speed v with respect to the first. The Lorentz transformation *requires* both systems to have a spherical wavefront centered on each system's origin.

Another clear break with Galilean and Newtonian physics is that we do not assume that $t = t'$. Each system must have its own clocks and metersticks as indicated in a two-dimensional system in Figure 2.7. Because the systems move only along their x axes, observers in both systems agree by direct observation that

$$y' = y$$

$$z' = z$$

We know that the Galilean transformation $x' = x - vt$ is incorrect, but what is the correct transformation? We require a linear transformation so that each event in system K corresponds to one, and only one, event in system K′. The simplest *linear* transformation is of the form

$$x' = \gamma(x - vt) \tag{2.10}$$

We will see if such a transformation suffices. The parameter γ cannot depend on x or t because the transformation must be linear. The parameter γ must be close to 1 for $v \ll c$ in order for Newton's laws of mechanics to be valid for most of our measurements. We can use similar arguments from the standpoint of an observer stationed in system K′ to obtain an equation similar to Equation (2.10).

$$x = \gamma'(x' + vt') \tag{2.11}$$

Because postulate 1 requires that the laws of physics be the same in both reference systems, we demand that $\gamma' = \gamma$. Notice that the only difference between Equations (2.10) and (2.11) other than the primed and unprimed quantities being switched is that $v \rightarrow -v$, which is reasonable because according to the observer in each system, the other observer is moving either forward or backward.

According to postulate 2, the speed of light is c in both systems. Therefore, in each system the wavefront of the light pulse along the respective x axes must be described by $x = ct$ and $x' = ct'$, which we substitute into Equations (2.10) and (2.11) to obtain

$$ct' = \gamma(ct - vt) \tag{2.12a}$$

and

$$ct = \gamma(ct' + vt') \tag{2.12b}$$

We divide each of these equations by c and obtain

$$t' = \gamma t\left(1 - \frac{v}{c}\right) \tag{2.13}$$

and

$$t = \gamma t'\left(1 + \frac{v}{c}\right) \tag{2.14}$$

We substitute the value of t from Equation (2.14) into Equation (2.13).

$$t' = \gamma^2 t'\left(1 - \frac{v}{c}\right)\left(1 + \frac{v}{c}\right) \tag{2.15}$$

We solve this equation for γ^2 and obtain

$$\gamma^2 = \frac{1}{1 - v^2/c^2}$$

or

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{2.16}$$

In order to find a transformation for time t', we rewrite Equation (2.13) as

$$t' = \gamma\left(t - \frac{vt}{c}\right)$$

We substitute $t = x/c$ for the light pulse and find

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = \frac{t - vx/c^2}{\sqrt{1 - \beta^2}}$$

We are now able to write the complete Lorentz transformations as

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - \beta^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - (vx/c^2)}{\sqrt{1 - \beta^2}} \end{aligned} \tag{2.17}$$

These equations are the same as Equations (2.6). The inverse transformation equations are obtained by replacing v by $-v$ as discussed previously and by exchanging the primed and unprimed quantities.

Inverse Lorentz transformation equations

$$\begin{aligned} x &= \frac{x' + vt'}{\sqrt{1 - \beta^2}} \\ y &= y' \\ z &= z' \\ t &= \frac{t' + (vx'/c^2)}{\sqrt{1 - \beta^2}} \end{aligned} \tag{2.18}$$

Notice that Equations (2.17) and (2.18) both reduce to the Galilean transformation when $v \ll c$. It is only for speeds that are a significant fraction of the

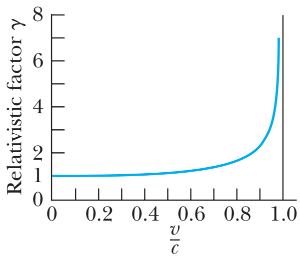

Figure 2.8 A plot of the relativistic factor γ as a function of speed v/c, showing that γ becomes large quickly as v approaches c.

speed of light that the Lorentz transformation equations become significantly different from the Galilean equations. In our studies of mechanics we normally do not consider such high speeds, and our previous results probably require no corrections. The laws of mechanics credited to Newton are still valid over the region of their applicability. Even for a speed as high as the Earth orbiting about the sun, 30 km/s, the value of the relativistic factor γ is 1.000000005. We show a plot of the relativistic parameter γ versus speed in Figure 2.8. Note that a variation of 0.5% (or $\gamma \approx 1.005$) occurs when the speed reaches $0.1c$. While the choice is arbitrary, we will use the relativistic equations at speeds greater than $0.1c$.

Finally, consider the implications of the Lorentz transformation. The linear transformation equations ensure that a single event in one system is described by a single event in another inertial system. However, space and time are not separate. In order to express the position of x in system K′, we must use both x' and t'. We have also found that the Lorentz transformation does not allow a speed greater than c; the relativistic factor γ becomes imaginary in this case. We show later in this chapter that no object of nonzero mass can have a speed greater than c.

2.5 Time Dilation and Length Contraction

The Lorentz transformations have immediate consequences with respect to time and length measurements made by observers in different inertial frames. We shall consider time and length measurements separately and then see how they are related to one another.

Time Dilation

Consider again our two systems K and K′ with system K fixed and system K′ moving along the x axis with velocity $\vec{v}$ as shown in Figure 2.9a. Frank lights a sparkler at position x_1 in system K. A clock placed beside the sparkler indicates the time to be t_1 when the sparkler is lit and t_2 when the sparkler goes out (Figure 2.9b). The sparkler burns for time T_0, where $T_0 = t_2 - t_1$. The time interval between two events occurring at the same position in a system as measured by a clock at rest in the system is called the **proper time**. We use the subscript zero on the time interval T_0 to denote the proper time.

Proper time

Now what is the time interval as determined by Mary who is passing by (but at rest in her own system K′)? All the clocks in both systems have been synchronized when the systems are at rest with respect to one another. The two events (sparkler lit and then going out) do not occur at the same place according to Mary. She is beside the sparkler when it is lit, but she has moved far away from the sparkler when it goes out (Figure 2.9b). Her friend Melinda, also at rest in system K′, is beside the sparkler when it goes out. Mary and Melinda measure the two times for the sparkler to be lit and to go out in system K′ as times t_1' and t_2'. The Lorentz transformation relates these times to those measured in system K as

$$t_2' - t_1' = \frac{(t_2 - t_1) - (v/c^2)(x_2 - x_1)}{\sqrt{1 - v^2/c^2}}$$

In system K the clock is fixed at x_1, so $x_2 - x_1 = 0$; that is, the two events occur at the same position. The time $t_2 - t_1$ is the proper time T_0, and we denote the time interval $t_2' - t_1' = T'$ as measured in the moving system K′:

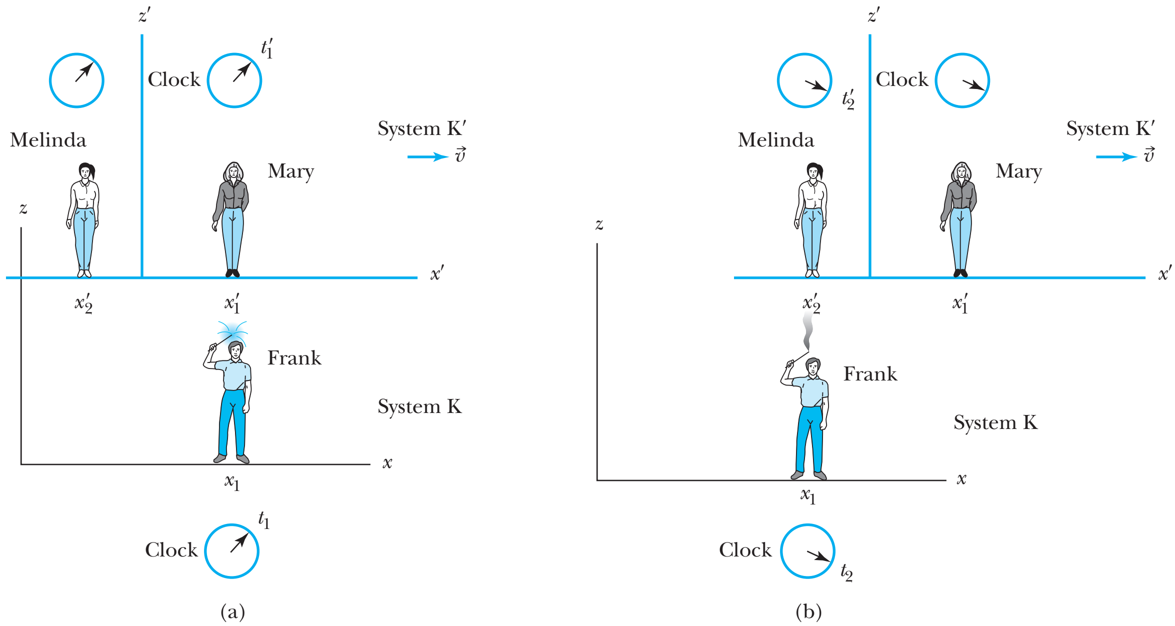

Figure 2.9 Frank measures the proper time for the time interval that a sparkler stays lit. His clock is at the same position in system K when the sparkler is lit in (a) and when it goes out in (b). Mary, in the moving system K′, is beside the sparkler at position x_1' when it is lit in (a), but by the time it goes out in (b), she has moved away. Melinda, at position x_2', measures the time in system K′ when the sparkler goes out in (b).

$$T' = \frac{T_0}{\sqrt{1 - v^2/c^2}} = \gamma T_0 \tag{2.19}$$

Time dilation

Thus the time interval measured in the moving system K′ is greater than the time interval measured in system K where the sparkler is at rest. This effect is known as **time dilation** and is a direct result of Einstein's two postulates. The time measured by Mary and Melinda in their system K′ for the time interval was greater than T_0 by the relativistic factor γ (where $\gamma > 1$). The two events, sparkler being lit and then going out, did not occur at the same position ($x_2' \neq x_1'$) in system K′ (see Figure 2.9b). This result occurs because of the absence of simultaneity. The events do not occur at the same space and time coordinates in the two systems. It requires three clocks to perform the measurement: one in system K and two in system K′.

Moving clocks run slow

The time dilation result is often interpreted by saying that *moving clocks run slow* by the factor γ^{-1}, and sometimes this is a useful way to remember the effect. The moving clock in this case can be any kind of clock. It can be the time that sand takes to pass through an hourglass, the time a sparkler stays lit, the time between heartbeats, the time between ticks of a clock, or the time spent in a class lecture. In all cases, the actual time interval on a moving clock is greater than the proper time as measured on a clock at rest. The proper time is always the smallest possible time interval between two events.

Each person will claim the clock in the other (moving) system is running slow. If Mary had a sparkler in her system K′ at rest, Frank (fixed in system K) would also measure a longer time interval on his clock in system K because the sparkler would be moving with respect to his system.

EXAMPLE 2.1

Show that Frank in the fixed system will also determine the time dilation result by having the sparkler be at rest in the system K′.

Strategy We should be able to proceed similarly to the derivation we did before when the sparkler was at rest in system K. In this case Mary lights the sparkler in the moving system K′. The time interval over which the sparkler is lit is given by $T'_0 = t'_2 - t'_1$, and the sparkler is placed at the position $x'_1 = x'_2$ so that $x'_2 - x'_1 = 0$. In this case T'_0 is the proper time. We use the Lorentz transformation from Equation (2.18) to determine the time interval $T = t_2 - t_1$ as measured by the clocks of Frank and his colleagues.

Solution We use Equation (2.18) to find $t_2 - t_1$:

$$T = t_2 - t_1 = \frac{(t'_2 - t'_1) + (v/c^2)(x'_2 - x'_1)}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{T'_0}{\sqrt{1 - v^2/c^2}} = \gamma T'_0$$

The time interval is still smaller in the system where the sparkler is at rest.

***Gedanken* experiments**

The preceding results naturally seem a little strange to us. In relativity we often carry out thought (or *gedanken* from the German word) experiments, because the actual experiments would be somewhat impractical. Consider the following *gedanken* experiment. Mary, in the moving system K′, flashes a light at her origin along her y' axis (Figure 2.10). The light travels a distance L, reflects off a mirror, and returns. Mary says that the total time for the journey is $T'_0 = t'_2 - t'_1 = 2L/c$, and this is indeed the proper time, because the clock in K′ beside Mary is at rest.

What do Frank and other observers in system K measure? Let T be the round-trip time interval measured in system K for the light to return to the x axis. The light is flashed when the origins are coincident, as Mary passes by Frank with relative velocity v. When the light reaches the mirror in the system K′ at time $T/2$, the system K′ will have moved a distance $vT/2$ down the x axis. When the light is reflected back to the x axis, Frank will not even see the light return, because it will return a distance vT away, where another observer, Fred, is positioned. Because observers Frank and Fred have previously synchronized their clocks, they can still measure the total elapsed time for the light to be reflected from the mirror and return. According to observers in the K system, the total distance the light travels (as shown in Figure 2.10) is $2\sqrt{(vT/2)^2 + L^2}$. And according to postulate 2, the light must travel at the speed of light, so the total time interval T measured in system K is

$$T = \frac{\text{distance}}{\text{speed}} = \frac{2\sqrt{(vT/2)^2 + L^2}}{c}$$

As can be determined from above, $L = cT'_0/2$, so we have

$$T = \frac{2\sqrt{(vT/2)^2 + (cT'_0/2)^2}}{c}$$

which reduces to

$$T = \frac{T'_0}{\sqrt{1 - v^2/c^2}} = \gamma T'_0$$

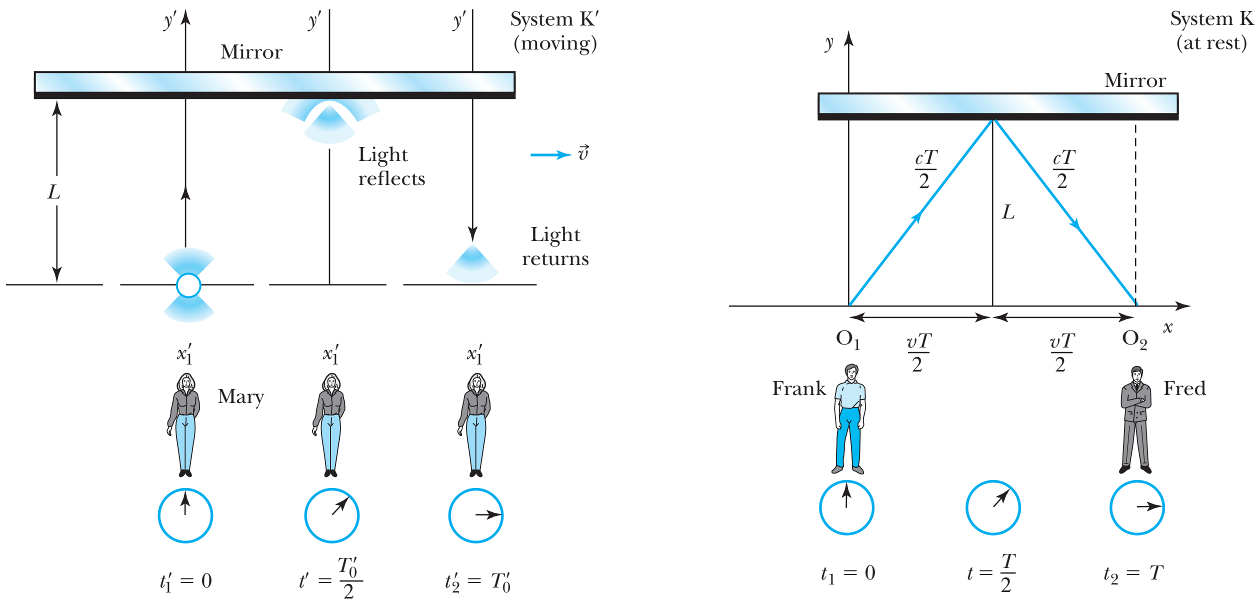

Figure 2.10 Mary, in system K′, flashes a light along her y' axis and measures the proper time $T'_0 = 2L/c$ for the light to return. In system K Frank will see the light travel partially down his x axis, because system K′ is moving. Fred times the arrival of the light in system K. The time interval T that Frank and Fred measure is related to the proper time by $T = \gamma T'_0$.

This is consistent with the earlier result. In this case $T > T'_0$. The proper time is always the shortest time interval, and we find that the clock in Mary's system K′ is "running slow."

EXAMPLE 2.2

It is the year 2150 and the United Nations Space Federation has finally perfected the storage of antiprotons for use as fuel in a spaceship. (Antiprotons are the antiparticles of protons. We discuss antiprotons in Chapter 3.) Preparations are under way for a manned spacecraft visit to possible planets orbiting one of the three stars in the star system Alpha Centauri, some 4.30 lightyears away. Provisions are placed on board to allow a trip of 16 years' total duration. How fast must the spacecraft travel if the provisions are to last? Neglect the period of acceleration, turnaround, and visiting times, because they are negligible compared with the actual travel time.

Strategy The time interval as measured by the astronauts on the spacecraft can be no longer than 16 years, because that is how long the provisions will last. However, from Earth we realize that the spacecraft will be moving at a high relative speed v to us, and that according to our clock in the stationary system K, the trip will last $T = 2L/v$, where L is the distance to the star.

Because provisions on board the spaceship will last for only 16 years, we let the proper time T'_0 in system K′ be 16 years. Using the time dilation result, we determine the relationship between T, the time measured on Earth, and the proper time T'_0 to be

$$T = \frac{2L}{v} = \frac{T'_0}{\sqrt{1 - v^2/c^2}} \tag{2.20}$$

We then solve this equation for the required speed v.

Solution A lightyear is a convenient way to measure large distances. It is the distance light travels in one year and is denoted by ly:

$$1\text{ ly} = \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)(1\text{ year})\left(365 \frac{\text{days}}{\text{year}}\right)\left(24 \frac{\text{h}}{\text{day}}\right)\left(3600 \frac{\text{s}}{\text{h}}\right)$$

$$= 9.46 \times 10^{15}\text{ m}$$

Note that the distance of one lightyear is the speed of light, c, multiplied by the time of one year. The dimension of a lightyear works out to be length. In this case, the result is $4.30\text{ ly} = c(4.30\text{ y}) = 4.07 \times 10^{16}\text{ m}$.

We insert the appropriate numbers into Equation (2.20) and obtain

$$\frac{2(4.30\text{ ly})(9.46 \times 10^{15}\text{ m/ly})}{v} = \frac{16\text{ y}}{\sqrt{1 - v^2/c^2}}$$

The solution to this equation is $v = 0.473c = 1.42 \times 10^8$ m/s. The time interval as measured on Earth will be $\gamma T'_0 = 18.2$ y. Notice that the astronauts will age only 16 years (their clocks run slow), whereas their friends remaining on Earth will age 18.2 years. Can this really be true? We shall discuss this question again in Section 2.8.

Length Contraction

Now consider what might happen to the length of objects in relativity. Let an observer in each system K and K′ have a meterstick at rest in his or her own respective system. Each observer lays the stick down along his or her respective x axis, putting the left end at x_ℓ (or x'_ℓ) and the right end at x_r (or x'_r). Thus, Frank in system K measures his stick to be $L_0 = x_r - x_\ell$. Similarly, in system K′, Mary measures her stick to be $L'_0 = x'_r - x'_\ell$. Because every observer measures a meterstick at rest in his or her own system to have the same length, $L_0 = L'_0$. Every observer measures a meterstick at rest in his or her own system to have the same length, namely one meter. The length as measured at rest is called the **proper length**.

Proper length

Let system K be at rest and system K′ move along the x axis with speed v. Frank, who is at rest in system K, measures the length of the stick moving in K′. The difficulty is to measure the ends of the stick simultaneously. We insist that Frank measure the ends of the stick at the same time so that $t = t_r = t_\ell$. The events denoted by (x, t) are (x_ℓ, t) and (x_r, t). We use Equation (2.17) and find

$$x'_r - x'_\ell = \frac{(x_r - x_\ell) - v(t_r - t_\ell)}{\sqrt{1 - v^2/c^2}}$$

The meterstick is at rest in system K′, so the length $x'_r - x'_\ell$ must be the proper length L'_0. Denote the length measured by Frank as $L = x_r - x_\ell$. The times t_r and t_ℓ are identical, as we insisted, so $t_r - t_\ell = 0$. Notice that the times of measurement by Mary in her system, t'_ℓ and t'_r, are *not* identical. It makes no difference when Mary makes the measurements in her own system, because the stick is at rest. However, it makes a big difference when Frank makes his measurements, because the stick is moving with speed v with respect to him. The measurements must be done simultaneously! With these results, the previous equation becomes

$$L'_0 = \frac{L}{\sqrt{1 - v^2/c^2}} = \gamma L$$

or, because $L'_0 = L_0$,

Length contraction

$$L = L_0\sqrt{1 - v^2/c^2} = \frac{L_0}{\gamma} \tag{2.21}$$

Notice that $L_0 > L$, so the moving meterstick shrinks according to Frank. This effect is known as **length** or **space contraction** and is characteristic of relative motion. This effect is also sometimes called the *Lorentz–FitzGerald contraction* because Lorentz and FitzGerald independently suggested the contraction as a way to solve the electrodynamics problem. This effect, like time dilation, is also reciprocal. Each observer will say that the other moving stick is shorter. There is no length contraction perpendicular to the relative motion, however, because $y' = y$ and $z' = z$. Observers in both systems can check the length of the other meterstick placed perpendicular to the direction of motion as the metersticks pass each other. They will agree that both metersticks are one meter long.

We can perform another *gedanken* experiment to arrive at the same result. This time we lay the meterstick along the x' axis in the moving system K′ (Figure 2.11a). The two systems K and K′ are aligned at $t = t' = 0$. A mirror is placed at the end of the meterstick, and a light flash goes off at the origin at $t = t' = 0$, sending a light pulse down the x' axis, where it is reflected and returned. Mary sees the stick at rest in system K′ and measures the proper length L_0 (which should of course be one meter). Mary uses the same clock fixed at $x' = 0$ for the time measurements. The stick is moving at speed v with respect to Frank in the fixed system K. The clocks at $x = x' = 0$ both read zero when the origins are

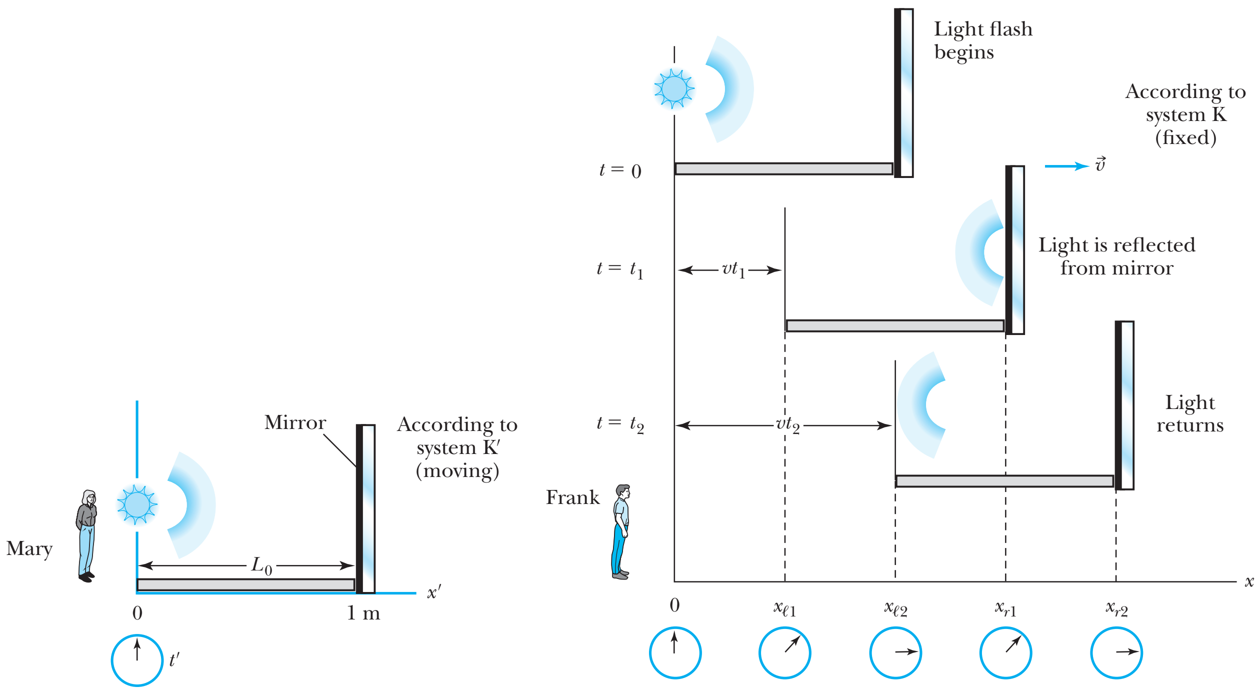

Figure 2.11 (a) Mary, in system K′, flashes a light down her x' axis along a stick at rest in her system of length L_0, which is the proper length. The time interval for the light to travel down the stick and back is $2L_0/c$. (b) Frank, in system K, sees the stick moving, and the mirror has moved a distance vt_1 by the time the light is reflected. By the time the light returns to the beginning of the stick, the stick has moved a total distance of vt_2. The times can be compared to show that the moving stick has been length contracted by $L = L_0\sqrt{1 - v^2/c^2}$.