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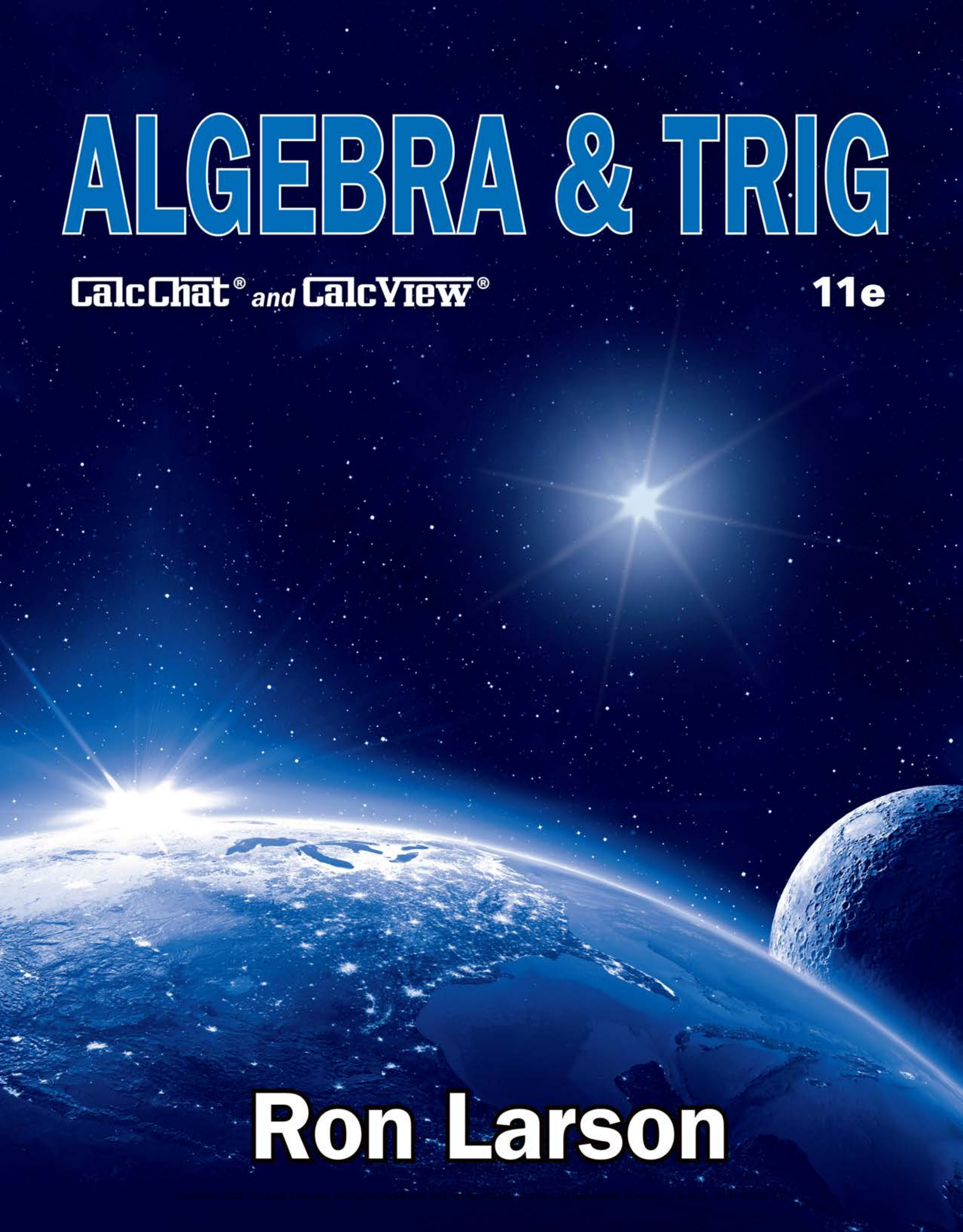

Ron Larson

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with CalcChat® and CalcView®
Eleventh Edition
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*Available at the text companion website *LarsonPrecalculus.com*

Preface

Welcome to *Algebra & Trig* with CalcChat® & CalcView®, Eleventh Edition. I am excited to offer you a new edition with more resources than ever that will help you understand and master algebra and trigonometry. This text includes features and resources that continue to make *Algebra & Trig* a valuable learning tool for students and a trustworthy teaching tool for instructors.

Algebra & Trig provides the clear instruction, precise mathematics, and thorough coverage that you expect for your course. Additionally, this new edition provides you with **free** access to a variety of digital resources:

- **GO DIGITAL**—direct access to digital content on your mobile device or computer
- **CalcView.com**—video solutions to selected exercises
- **CalcChat.com**—worked-out solutions to odd-numbered exercises and access to online tutors
- **LarsonPrecalculus.com**—companion website with resources to supplement your learning

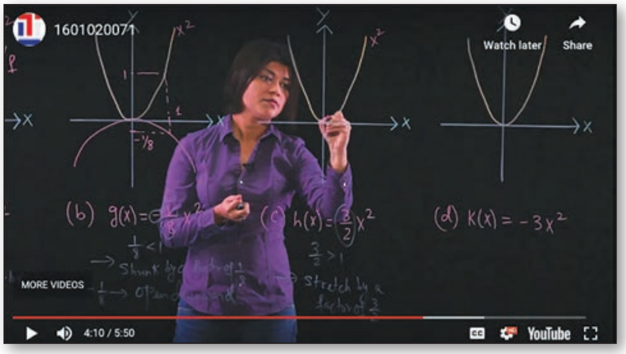
These digital resources will help enhance and reinforce your understanding of the material presented in this text and prepare you for future mathematics courses. CalcView® and CalcChat® are also available as free mobile apps.

Features

NEW GO DIGITAL

Scan the QR codes on the pages of this text to *GO DIGITAL* on your mobile device. This will give you easy access from anywhere to instructional videos, solutions to exercises and Checkpoint problems, Skills Refresher videos, Interactive Activities, and many other resources.

GO DIGITAL



UPDATED CalcView®

The website *CalcView.com* provides video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google Play™ store. You can access the video solutions by scanning the QR Code® at the beginning of the Section exercises, or visiting the *CalcView.com* website.

UPDATED CalcChat®

Solutions to all odd-numbered exercises and tests are provided for free at *CalcChat.com.* Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. For many years, millions of students have visited my site for help. The CalcChat mobile app is also available as a free download at the Apple® App Store® or Google Play™ store.

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REVISED LarsonPrecalculus.com

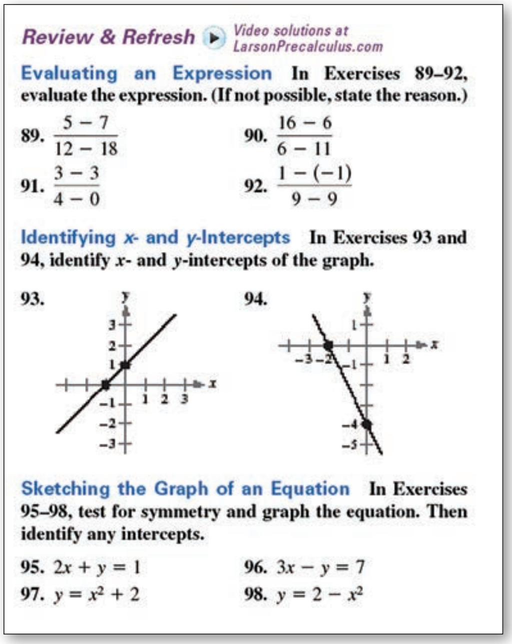
All companion website features have been updated based on this revision, including two new features: Skills Refresher and Review & Refresh. Access to these features is free. You can view and listen to worked-out solutions of Checkpoint problems in English or Spanish, explore examples, download data sets, watch lesson videos, and much more.

NEW Skills Refresher

This feature directs you to an instructional video where you can review algebra skills needed to master the current topic. Scan the on-page code or go to *LarsonPrecalculus.com* to access the video.

SKILLS REFRESHER

For a refresher on finding the sum, difference, product, or quotient of two polynomials, watch the video at *LarsonPrecalculus.com*.



Review & Refresh *Video solutions at LarsonPrecalculus.com*

Evaluating an Expression In Exercises 89–92, evaluate the expression. (If not possible, state the reason.)

89. $\frac{5-7}{12-18}$
90. $\frac{16-6}{6-11}$
91. $\frac{3-3}{4-0}$
92. $\frac{1-(-1)}{9-9}$

Identifying x- and y-Intercepts In Exercises 93 and 94, identify x- and y-intercepts of the graph.

93.
94.

Sketching the Graph of an Equation In Exercises 95–98, test for symmetry and graph the equation. Then identify any intercepts.

95. $2x + y = 1$
96. $3x - y = 7$
97. $y = x^2 + 2$
98. $y = 2 - x^2$

NEW Review and Refresh

These exercises will help you to reinforce previously learned skills and concepts and to prepare for the next section. View and listen to worked-out solutions of the Review & Refresh exercises in English or Spanish by scanning the code on the first page of the section exercises or go to *LarsonPrecalculus.com*.

NEW Vocabulary and Concept Check

The Vocabulary and Concept Check appears at the beginning of the exercise set for each section. It includes fill-in-the-blank, matching, or non-computational questions designed to help you learn mathematical terminology and to test basic understanding of the concepts of the section.

NEW Summary and Study Strategies

The "What Did You Learn?" feature is a section-by-section overview that ties the learning objectives from the chapter to the Review Exercises for extra practice. The Study Strategies give concrete ways that you can use to help yourself with your study of mathematics.

REVISED Algebra Help

These notes reinforce or expand upon concepts, help you learn how to study mathematics, address special cases, or show alternative or additional steps to a solution of an example.

REVISED Exercise Sets

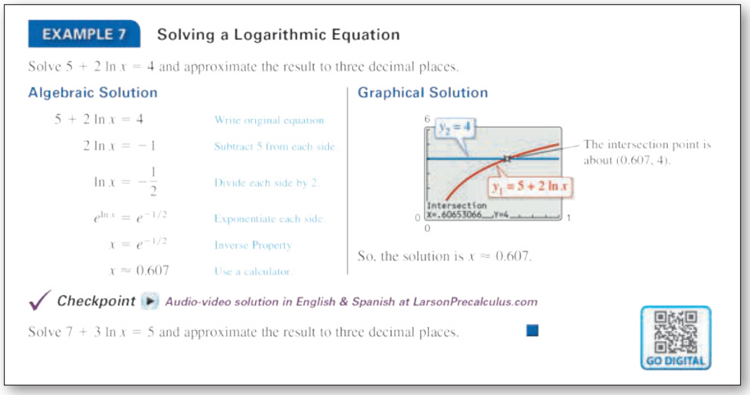
The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant, and include topics our users have suggested. The exercises have been reorganized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations. Two new sets of exercises, Vocabulary and Concept Check and Review & Refresh, have been added to help you develop and maintain your skills.

Section Objectives

A bulleted list of learning objectives provides you the opportunity to preview what will be presented in the upcoming section.

Side-By-Side Examples

Throughout the text, we present solutions to many examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.



EXAMPLE 7 **Solving a Logarithmic Equation**

Solve $5 + 2 \ln x = 4$ and approximate the result to three decimal places.

Algebraic Solution

$5 + 2 \ln x = 4$ Write original equation.

$2 \ln x = -1$ Subtract 5 from each side.

$\ln x = -\frac{1}{2}$ Divide each side by 2.

$e^{\ln x} = e^{-1/2}$ Exponentiate each side.

$x = e^{-1/2}$ Inverse Property

$x \approx 0.607$ Use a calculator.

Graphical Solution

The intersection point is about (0.607, 4).

So, the solution is $x \approx 0.607$.

✓ *Checkpoint* ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $7 + 3 \ln x = 5$ and approximate the result to three decimal places. ■

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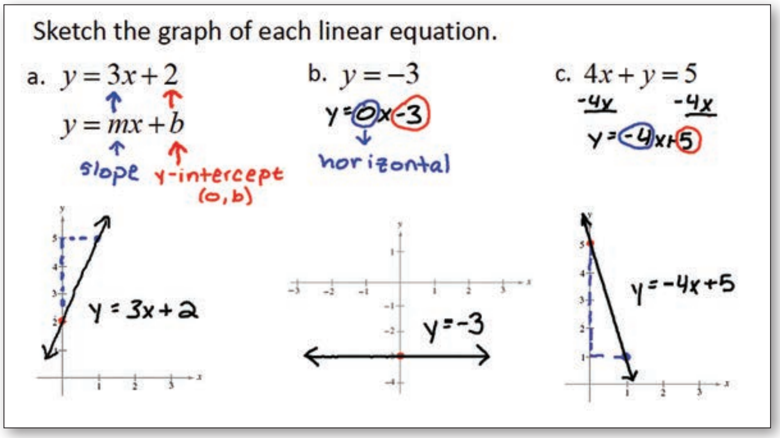


Sketch the graph of each linear equation.

a. $y = 3x + 2$

b. $y = -3$

c. $4x + y = 5$

Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at *LarsonPrecalculus.com*. Scan the on-page code to access the solutions.

Technology

The technology feature gives suggestions for effectively using tools such as calculators, graphing utilities, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

Historical Notes

These notes provide helpful information regarding famous mathematicians and their work.

Summarize (Section 3.2)

1. Explain how to use transformations to sketch graphs of polynomial functions *(page 252)*. For an example of sketching transformations of monomial functions, see Example 1.
2. Explain how to apply the Leading Coefficient Test *(page 253)*. For an example of applying the Leading Coefficient Test, see Example 2.
3. Explain how to find real zeros of polynomial functions and use them as sketching aids *(page 255)*. For examples involving finding real zeros of polynomial functions, see Examples 3–5.
4. Explain how to use the Intermediate Value Theorem to help locate real zeros of polynomial functions *(page 258)*. For an example of using the Intermediate Value Theorem, see Example 6.

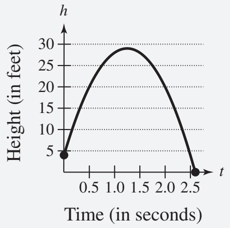
Summarize
The Summarize feature at the end of each section helps you organize the lesson's key concepts into a concise summary, providing you with a valuable study tool. Use this feature to prepare for a homework assignment, to help you study for an exam, or as a review for previously covered sections.

Algebra of Calculus
Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol ∫.

Error Analysis
This exercise presents a sample solution that contains a common error which you are asked to identify.

How Do You See It?
The How Do You See It? feature in each section presents a real-life exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

86. **HOW DO YOU SEE IT?** The graph represents the height h of a projectile after t seconds.



(a) Explain why h is a function of t.

(b) Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

(c) Approximate the domain of h.

(d) Is t a function of h? Explain.

Project
The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at *LarsonPrecalculus.com*.

Collaborative Project
You can find these extended group projects at *LarsonPrecalculus.com.* Check your understanding of the chapter concepts by solving in-depth, real-life problems. These collaborative projects provide an interesting and engaging way for you and other students to work together and investigate ideas.

Instructor Resources

CENGAGE | WEBASSIGN

Built by educators, WebAssign from Cengage is a fully customizable online solution for STEM disciplines. WebAssign includes the flexibility, tools, and content you need to create engaging learning experiences for your students. The patented grading engine provides unparalleled answer evaluation, giving students instant feedback, and insightful analytics highlight exactly where students are struggling. For more information, visit *cengage.com/webassign*.

Complete Solutions Manual

This manual contains solutions to all exercises from the text, including Chapter Review Exercises and Chapter Tests, and Practice Tests with solutions. The Complete Solutions Manual is available on the Instructor Companion Site.

Cengage Testing Powered by Cognero®

Cengage Testing, Powered by Cognero®, is a flexible online system that allows you to author, edit, and manage test bank content online. You can create multiple versions of your test in an instant and deliver tests from your LMS or exportable PDF or Word docs you print for in-class assessment. Cengage Testing is available online via *cengage.com*.

Instructor Companion Site

Everything you need for your course in one place! Access and download PowerPoint® presentations, test banks, the solutions manual, and more. This collection of book-specific lecture and class tools is available online via *cengage.com*.

Test Bank

The test bank contains text-specific multiple-choice and free response test forms and is available online at the Instructor Companion Site.

LarsonPrecalculus.com

In addition to its student resources, *LarsonPrecalculus.com* also has resources to help instructors. If you wish to challenge your students with multi-step and group projects, you can assign the Section Projects and Collaborative Projects. You can assess the knowledge of your students before and after each chapter using the pre- and post-tests. You can also give your students experience using an online graphing calculator with the Interactive Activities. You can access these features by going to *LarsonPrecalculus.com* or by scanning the on-page code.

MathGraphs.com

For exercises that ask students to draw on the graph, I have provided **free,** printable graphs at *MathGraphs.com*. You can access these features by going to *MathGraphs.com* or by scanning the on-page code at the beginning of the section exercises, review exercises, or tests.

Student Resources

CENGAGE | WEBASSIGN

Prepare for class with confidence using WebAssign from Cengage. This online learning platform, which includes an interactive eBook, fuels practice, so that you truly absorb what you learn and prepare better for tests. Videos and tutorials walk you through concepts and deliver instant feedback and grading, so you always know where you stand in class. Focus your study time and get extra practice where you need it most. Study smarter with WebAssign! Ask your instructor today how you can get access to WebAssign, or learn about self-study options at *cengage.com/webassign*.

Student Study Guide and Solutions Manual

This guide offers step-by-step solutions for all odd-numbered text exercises, Chapter Tests, and Cumulative Tests. It also contains Practice Tests. For more information on how to access this digital resource, go to cengage.com

Note-Taking Guide

This is an innovative study aid, in the form of a notebook organizer, that helps students develop a section-by-section summary of key concepts. For more information on how to access this digital resource, go to cengage.com

LarsonPrecalculus.com

Of the many features at this website, students have told me that the videos are the most helpful. You can watch lesson videos by Dana Mosely as he explains various mathematical concepts. Other helpful features are the data downloads (editable spreadsheets so you do not have to enter the data), video solutions of the Checkpoint problems in English or Spanish, and the Student Success Organizer. The Student Success Organizer will help you organize the important concepts of each section using chapter outlines. You can access these features by going to *LarsonPrecalculus.com* or by scanning the on-page code.

CalcChat.com

This website provides free step-by-step solutions to all odd-numbered exercises and tests. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. You can access the solutions by going to *CalcChat.com* or by scanning the on-page code on the first page of the section exercises, review exercises, or tests.

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Thank you to all of the instructors who took the time to review the changes in this edition and to provide suggestions for improving it. Without your help, this book would not be possible.

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On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil. If you have suggestions for improving this text, please feel free to write to me. Over the past two decades, I have received many useful comments from both instructors and students, and I value these comments very highly.

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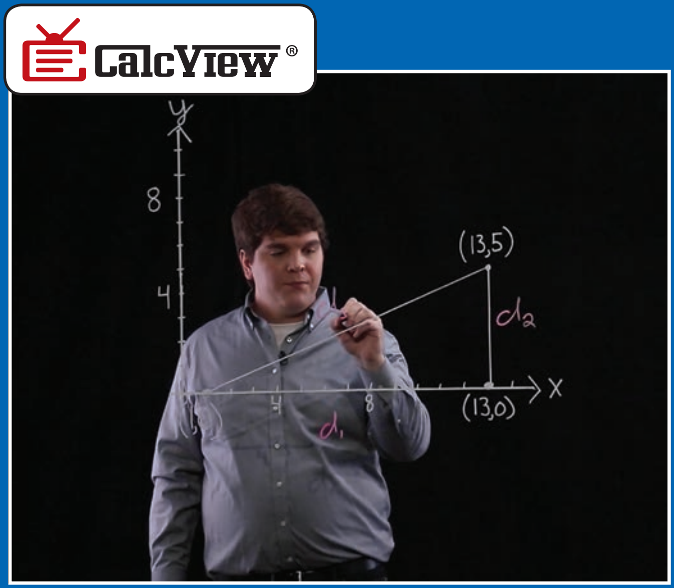
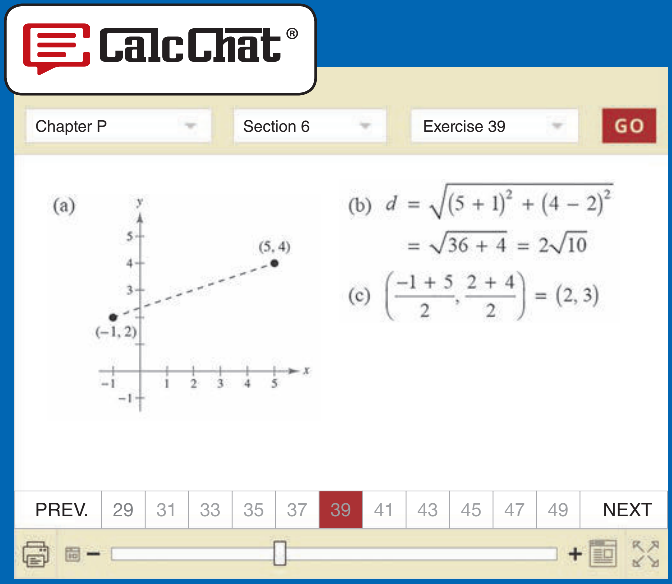
P Prerequisites





(b) $d = \sqrt{(5+1)^2 + (4-2)^2}$

$= \sqrt{36 + 4} = 2\sqrt{10}$

(c) $\left(\frac{-1+5}{2}, \frac{2+4}{2}\right) = (2, 3)$

P.1 Federal Deficit *(Exercises 47–50, p. 13)*

P.6 Flying Distance *(Exercise 44, p. 58)*

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P.1 Review of Real Numbers and Their Properties

Real numbers can represent many real-life quantities. For example, in Exercises 47–50 on page 13, you will use real numbers to represent the federal surplus or deficit.

- **Represent and classify real numbers.**
- **Order real numbers and use inequalities.**
- **Find the absolute values of real numbers and find the distance between two real numbers.**
- **Evaluate algebraic expressions.**
- **Use the basic rules and properties of algebra.**

Real Numbers

Real numbers can describe quantities in everyday life such as age, miles per gallon, and population. Real numbers are represented by symbols such as

$$-5,\ 9,\ 0,\ \frac{4}{3},\ 0.666\ .\ .\ .\ ,\ 28.21,\ \sqrt{2},\ \pi,\ \text{and}\ \sqrt[3]{-32}.$$

Three commonly used **subsets** of real numbers are listed below. Each member in these subsets is also a member of the set of real numbers. (The three dots, called an *ellipsis*, indicate that the pattern continues indefinitely.)

$\{1, 2, 3, 4, .\ .\ .\}$ Set of natural numbers

$\{0, 1, 2, 3, 4, .\ .\ .\}$ Set of whole numbers

$\{.\ .\ .\ , -3, -2, -1, 0, 1, 2, 3, .\ .\ .\}$ Set of integers

A real number is **rational** when it can be written as the ratio p/q of two integers, where $q \neq 0$. For example, the numbers

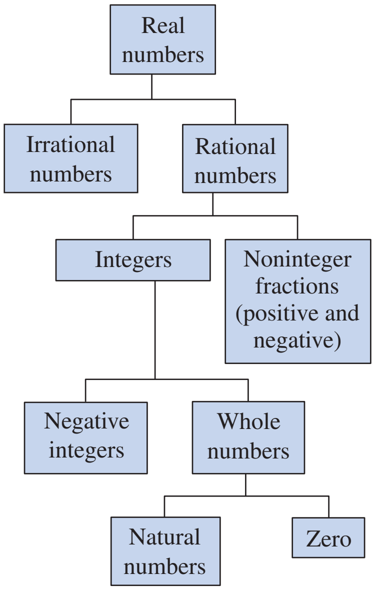
$$\frac{1}{3} = 0.3333\ .\ .\ . = 0.\overline{3},\quad \frac{1}{8} = 0.125,\quad \text{and}\quad \frac{125}{111} = 1.126126\ .\ .\ . = 1.\overline{126}$$

are rational. The decimal form of a rational number either repeats $\left(\text{as in } \frac{173}{55} = 3.1\overline{45}\right)$ or terminates $\left(\text{as in } \frac{1}{2} = 0.5\right)$. A real number that cannot be written as the ratio of two integers is **irrational.** The decimal form of an irrational number neither terminates nor repeats. For example, the numbers

$$\sqrt{2} = 1.4142135\ .\ .\ . \approx 1.41 \quad \text{and} \quad \pi = 3.1415926\ .\ .\ . \approx 3.14$$

are irrational. (The symbol $\approx$ means "is approximately equal to.")

Several common subsets of the real numbers and their relationships to each other are shown in Figure P.1.



Common subsets of the real numbers
Figure P.1

EXAMPLE 1 Classifying Real Numbers

Determine which numbers in the set $\left\{-13, -\sqrt{5}, -1, -\frac{1}{3}, 0, \frac{5}{8}, \sqrt{2}, \pi, 7\right\}$ are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.

Solution

a. Natural numbers: $\{7\}$

b. Whole numbers: $\{0, 7\}$

c. Integers: $\{-13, -1, 0, 7\}$

d. Rational numbers: $\left\{-13, -1, -\frac{1}{3}, 0, \frac{5}{8}, 7\right\}$

e. Irrational numbers: $\left\{-\sqrt{5}, \sqrt{2}, \pi\right\}$

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Repeat Example 1 for the set $\left\{-\pi, -\frac{1}{4}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -1, 8, -22\right\}$. ■

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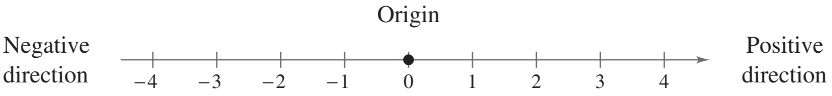
Real numbers are represented graphically on the **real number line.** When you draw a point on the real number line that corresponds to a real number, you are **plotting** the real number. The point representing 0 on the real number line is the **origin.** Numbers to the right of 0 are positive, and numbers to the left of 0 are negative, as shown in Figure P.2. The term **nonnegative** describes a number that is either positive or zero.



The real number line
Figure P.2

As illustrated in Figure P.3, there is a *one-to-one correspondence* between real numbers and points on the real number line.

Every real number corresponds to exactly one point on the real number line.

Every point on the real number line corresponds to exactly one real number.

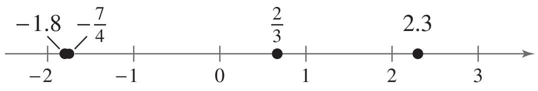
Figure P.3

EXAMPLE 2 Plotting Points on the Real Number Line

Plot the real numbers on the real number line.

a. $-\frac{7}{4}$ **b.** 2.3 **c.** $\frac{2}{3}$ **d.** -1.8

Solution The figure below shows all four points.



a. The point representing the real number

$$-\frac{7}{4} = -1.75 \qquad \text{Write in decimal form.}$$

lies between -2 and -1, but closer to -2, on the real number line.

b. The point representing the real number 2.3 lies between 2 and 3, but closer to 2, on the real number line.

c. The point representing the real number

$$\frac{2}{3} = 0.666\ldots \qquad \text{Write in decimal form.}$$

lies between 0 and 1, but closer to 1, on the real number line.

d. The point representing the real number -1.8 lies between -2 and -1, but closer to -2, on the real number line. Note that the point representing -1.8 lies slightly to the left of the point representing $-\frac{7}{4}$.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Plot the real numbers on the real number line.

a. $\frac{5}{2}$ **b.** -1.6 **c.** $-\frac{3}{4}$ **d.** 0.7

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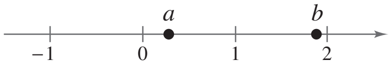
Ordering Real Numbers

One important property of real numbers is that they are *ordered.* If a and b are real numbers, then a is *less than* b when $b - a$ is positive. The **inequality** $a < b$ denotes the **order** of a and b. This relationship can also be described by saying that b is *greater than* a and writing $b > a$. The inequality $a \le b$ means that a is *less than or equal to* b, and the inequality $b \ge a$ means that b is *greater than or equal to* a. The symbols $<$, $>$, $\le$, and $\ge$ are *inequality symbols.*

Geometrically, this implies that $a < b$ if and only if a lies to the *left* of b on the real number line, as shown in Figure P.4.



$a < b$ if and only if a lies to the left of b.

Figure P.4

EXAMPLE 3 Ordering Real Numbers

Place the appropriate inequality symbol ($<$ or $>$) between the pair of real numbers.

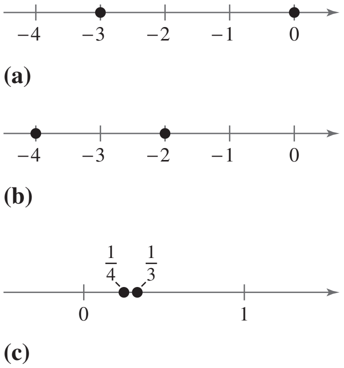
a. $-3, 0$ **b.** $-2, -4$ **c.** $\frac{1}{4}, \frac{1}{3}$

Solution

a. On the real number line, -3 lies to the left of 0, as shown in Figure P.5(a). So, you can say that -3 is *less than* 0, and write $-3 < 0$.

b. On the real number line, -2 lies to the right of -4, as shown in Figure P.5(b). So, you can say that -2 is *greater than* -4, and write $-2 > -4$.

c. On the real number line, $\frac{1}{4}$ lies to the left of $\frac{1}{3}$, as shown in Figure P.5(c). So, you can say that $\frac{1}{4}$ is *less than* $\frac{1}{3}$, and write $\frac{1}{4} < \frac{1}{3}$.



(a)

(b)

(c)

Figure P.5

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Place the appropriate inequality symbol ($<$ or $>$) between the pair of real numbers.

a. $1, -5$ **b.** $\frac{3}{2}, 7$ **c.** $-\frac{2}{3}, -\frac{3}{4}$

EXAMPLE 4 Interpreting Inequalities

See LarsonPrecalculus.com for an interactive version of this type of example.

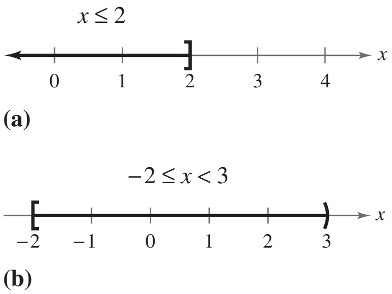
Describe the subset of real numbers that the inequality represents.

a. $x \le 2$ **b.** $-2 \le x < 3$

Solution

a. The inequality $x \le 2$ denotes all real numbers less than or equal to 2, as shown in Figure P.6(a). In the figure, the bracket at 2 indicates 2 is *included* in the interval.

b. The inequality $-2 \le x < 3$ means that $x \ge -2$ *and* $x < 3$. This "double inequality" denotes all real numbers between -2 and 3, including -2 but not including 3, as shown in Figure P.6(b). In the figure, the bracket at -2 indicates -2 is *included* in the interval, and the parenthesis at 3 indicates that 3 is *not* included in the interval.



(a)

(b)

Figure P.6

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Describe the subset of real numbers that the inequality represents.

a. $x > -3$ **b.** $0 < x \le 4$

Inequalities can describe subsets of real numbers called **intervals.** In the bounded intervals on the next page, the real numbers a and b are the **endpoints** of each interval. The endpoints of a closed interval are included in the interval, whereas the endpoints of an open interval are not included in the interval.

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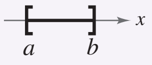
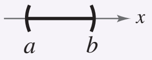
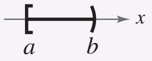
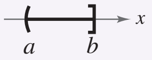
Bounded Intervals on the Real Number Line

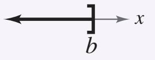
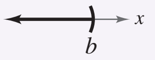
Let a and b be real numbers such that $a < b$.

Notation	Interval Type	Inequality	Graph
$[a, b]$	Closed	$a \le x \le b$	
(a, b)	Open	$a < x < b$	
$[a, b)$		$a \le x < b$	
$(a, b]$		$a < x \le b$	

The reason that the four types of intervals above are called **bounded** is that each has a finite length. An interval that does not have a finite length is **unbounded.** Note in the unbounded intervals below that the symbols ∞, **positive infinity,** and $-\infty$, **negative infinity,** do not represent real numbers. They are convenient symbols used to describe the unboundedness of intervals such as $(1, \infty)$ or $(-\infty, 3]$.

Unbounded Intervals on the Real Number Line

Let a and b be real numbers.

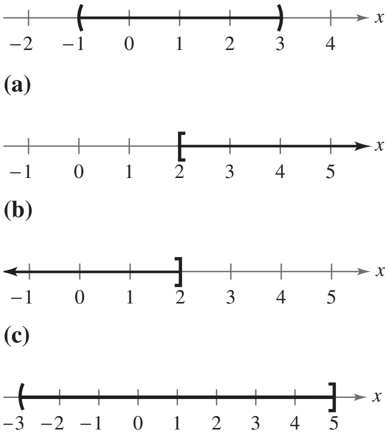
Notation	Interval Type	Inequality	Graph
$[a, \infty)$		$x \ge a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$		$x \le b$	
$(-\infty, b)$	Open	$x < b$	
$(-\infty, \infty)$	Entire real line	$-\infty < x < \infty$	

ALGEBRA HELP

Whenever you write an interval containing ∞ or $-\infty$, always use a parenthesis and never a bracket next to these symbols. This is because ∞ and $-\infty$ are never included in the interval.

EXAMPLE 5 Representing Intervals

	Verbal	Algebraic	Graphical
a.	All real numbers greater than -1 and less than 3	$(-1, 3)$ or $-1 < x < 3$	See Figure P.7(a).
b.	All real numbers greater than or equal to 2	$[2, \infty)$ or $x \ge 2$	See Figure P.7(b).
c.	All real numbers less than or equal to 2	$(-\infty, 2]$ or $x \le 2$	See Figure P.7(c).
d.	All real numbers greater than -3 and less than or equal to 5	$(-3, 5]$ or $-3 < x \le 5$	See Figure P.7(d).



(a)

(b)

(c)

(d)

Figure P.7

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a. Represent the interval $[-2, 5)$ verbally, as an inequality, and as a graph.

b. Represent the statement "x is less than 4 and at least -2" as an interval, an inequality, and a graph.

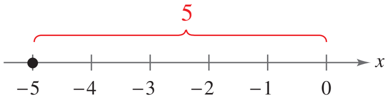
Absolute Value and Distance

The **absolute value** of a real number is its *magnitude,* or the distance between the origin and the point representing the real number on the real number line.

Definition of Absolute Value

If a is a real number, then the **absolute value** of a is

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}.$$



Absolute value as the distance from the origin

Figure P.8

Notice in this definition that the absolute value of a real number is never negative. For example, if $a = -5$, then $|-5| = -(-5) = 5$, as shown in Figure P.8. The absolute value of a real number is either positive or zero. Moreover, 0 is the only real number whose absolute value is 0. So, $|0| = 0$.

Properties of Absolute Values

1. $|a| \geq 0$

2. $|-a| = |a|$

3. $|ab| = |a||b|$

4. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \quad b \neq 0$

EXAMPLE 6 Finding Absolute Values

a. $|-15| = 15$

b. $\left|\dfrac{2}{3}\right| = \dfrac{2}{3}$

c. $|-4.3| = 4.3$

d. $-|-6| = -(6) = -6$

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Evaluate each expression.

a. $|1|$

b. $-\left|\dfrac{3}{4}\right|$

c. $\dfrac{2}{|-3|}$

d. $-|0.7|$

EXAMPLE 7 Evaluating an Absolute Value Expression

Evaluate $\dfrac{|x|}{x}$ for (a) $x > 0$ and (b) $x < 0$.

Solution

a. If $x > 0$, then x is positive and $|x| = x$. So, $\dfrac{|x|}{x} = \dfrac{x}{x} = 1$.

b. If $x < 0$, then x is negative and $|x| = -x$. So, $\dfrac{|x|}{x} = \dfrac{-x}{x} = -1$.

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Evaluate $\dfrac{|x + 3|}{x + 3}$ for (a) $x > -3$ and (b) $x < -3$. ■

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The **Law of Trichotomy** states that for any two real numbers a and b, *precisely* one of three relationships is possible:

$$a = b, \quad a < b, \quad \text{or} \quad a > b. \qquad \text{Law of Trichotomy}$$

In words, this property tells you that if a and b are any two real numbers, then a is equal to b, a is less than b, or a is greater than b.

EXAMPLE 8 Comparing Real Numbers

Place the appropriate symbol (<, >, or =) between the pair of real numbers.

a. $|-4| \ \square\ |3|$ **b.** $|-10| \ \square\ |10|$ **c.** $-|-7| \ \square\ |-7|$

Solution

a. $|-4| > |3|$ because $|-4| = 4$ and $|3| = 3$, and 4 is greater than 3.

b. $|-10| = |10|$ because $|-10| = 10$ and $|10| = 10$.

c. $-|-7| < |-7|$ because $-|-7| = -7$ and $|-7| = 7$, and -7 is less than 7.

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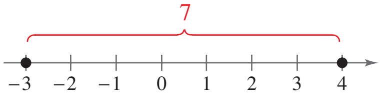
Place the appropriate symbol (<, >, or =) between the pair of real numbers.

a. $|-3| \ \square\ |4|$ **b.** $-|-4| \ \square\ -|4|$ **c.** $|-3| \ \square\ -|-3|$ ■



The distance between −3 and 4 is 7.

Figure P.9

Absolute value can be used to find the distance between two points on the real number line. For example, the distance between −3 and 4 is

$$|-3-4| = |-7|$$
$$= 7 \qquad \text{Distance between } -3 \text{ and } 4$$

as shown in Figure P.9.

Distance Between Two Points on the Real Number Line

Let a and b be real numbers. The **distance between a and b** is

$$d(a, b) = |b - a| = |a - b|.$$

One application of finding the distance between two points on the real number line is finding a change in temperature.

EXAMPLE 9 Finding a Distance

Find the distance between −25 and 13.

Solution

The distance between −25 and 13 is

$$|-25 - 13| = |-38| = 38. \qquad \text{Distance between } -25 \text{ and } 13$$

The distance can also be found as follows.

$$|13 - (-25)| = |38| = 38 \qquad \text{Distance between } -25 \text{ and } 13$$

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Find the distance between each pair of real numbers.

a. 35 and −23 **b.** −35 and −23 **c.** 35 and 23 ■

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Algebraic Expressions

One characteristic of algebra is the use of letters to represent numbers. The letters are **variables,** and combinations of letters and numbers are **algebraic expressions.** Here are a few examples of algebraic expressions.

$$5x, \qquad 2x - 3, \qquad \frac{4}{x^2 + 2}, \qquad 7x + y$$

Definition of an Algebraic Expression

An **algebraic expression** is a collection of letters **(variables)** and real numbers **(constants)** combined using the operations of addition, subtraction, multiplication, division, and exponentiation.

The **terms** of an algebraic expression are those parts that are separated by *addition.* For example, $x^2 - 5x + 8 = x^2 + (-5x) + 8$ has three terms: x^2 and $-5x$ are the **variable terms** and 8 is the **constant term.** For terms such as x^2, $-5x$, and 8, the numerical factor is the **coefficient.** Here, the coefficients are 1, -5, and 8.

EXAMPLE 10 Identifying Terms and Coefficients

	Algebraic Expression	Terms	Coefficients
a.	$5x - \frac{1}{7}$	$5x, -\frac{1}{7}$	$5, -\frac{1}{7}$
b.	$2x^2 - 6x + 9$	$2x^2, -6x, 9$	$2, -6, 9$
c.	$\frac{3}{x} + \frac{1}{2}x^4 - y$	$\frac{3}{x}, \frac{1}{2}x^4, -y$	$3, \frac{1}{2}, -1$

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Identify the terms and coefficients of $-2x + 4$. ■

The **Substitution Principle** states, "If $a = b$, then b can replace a in any expression involving a." Use the Substitution Principle to **evaluate** an algebraic expression by substituting values for each of the variables in the expression. The next example illustrates this.

EXAMPLE 11 Evaluating Algebraic Expressions

	Expression	Value of Variable	Substitution	Value of Expression
a.	$-3x + 5$	$x = 3$	$-3(3) + 5$	$-9 + 5 = -4$
b.	$3x^2 + 2x - 1$	$x = -1$	$3(-1)^2 + 2(-1) - 1$	$3 - 2 - 1 = 0$
c.	$\frac{2x}{x + 1}$	$x = -3$	$\frac{2(-3)}{-3 + 1}$	$\frac{-6}{-2} = 3$

Note that you must substitute the value for *each* occurrence of the variable.

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Evaluate $4x - 5$ when $x = 0$. ■

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Basic Rules of Algebra

There are four arithmetic operations with real numbers: *addition, multiplication, subtraction,* and *division,* denoted by the symbols $+$, $\times$ or $\cdot$, $-$, and $\div$ or $/$, respectively. Of these, addition and multiplication are the two primary operations. Subtraction and division are the inverse operations of addition and multiplication, respectively.

Definitions of Subtraction and Division

Subtraction: Add the opposite.

$$a - b = a + (-b)$$

Division: Multiply by the reciprocal.

If $b \neq 0$, then $a/b = a\left(\frac{1}{b}\right) = \frac{a}{b}$.

In these definitions, $-b$ is the **additive inverse** (or opposite) of b, and $1/b$ is the **multiplicative inverse** (or reciprocal) of b. In the fractional form a/b, a is the **numerator** of the fraction and b is the **denominator.**

The properties of real numbers below are true for variables and algebraic expressions as well as for real numbers, so they are often called the **Basic Rules of Algebra.** Formulate a verbal description of each of these properties. For example, the first property states that *the order in which two real numbers are added does not affect their sum.*

Basic Rules of Algebra

Let a, b, and c be real numbers, variables, or algebraic expressions.

Property		**Example**
Commutative Property of Addition:	$a + b = b + a$	$4x + x^2 = x^2 + 4x$
Commutative Property of Multiplication:	$ab = ba$	$(4 - x)x^2 = x^2(4 - x)$
Associative Property of Addition:	$(a + b) + c = a + (b + c)$	$(x + 5) + x^2 = x + (5 + x^2)$
Associative Property of Multiplication:	$(ab)c = a(bc)$	$(2x \cdot 3y)(8) = (2x)(3y \cdot 8)$
Distributive Properties:	$a(b + c) = ab + ac$	$3x(5 + 2x) = 3x \cdot 5 + 3x \cdot 2x$
	$(a + b)c = ac + bc$	$(y + 8)y = y \cdot y + 8 \cdot y$
Additive Identity Property:	$a + 0 = a$	$5y^2 + 0 = 5y^2$
Multiplicative Identity Property:	$a \cdot 1 = a$	$(4x^2)(1) = 4x^2$
Additive Inverse Property:	$a + (-a) = 0$	$5x^3 + (-5x^3) = 0$
Multiplicative Inverse Property:	$a \cdot \frac{1}{a} = 1, \quad a \neq 0$	$(x^2 + 4)\left(\frac{1}{x^2 + 4}\right) = 1$

Subtraction is defined as "adding the opposite," so the Distributive Properties are also true for subtraction. For example, the "subtraction form" of $a(b + c) = ab + ac$ is $a(b - c) = ab - ac$. Note that the operations of subtraction and division are neither commutative nor associative. The examples

$$7 - 3 \neq 3 - 7 \quad \text{and} \quad 20 \div 4 \neq 4 \div 20$$

show that subtraction and division are not commutative. Similarly

$$5 - (3 - 2) \neq (5 - 3) - 2 \quad \text{and} \quad 16 \div (4 \div 2) \neq (16 \div 4) \div 2$$

demonstrate that subtraction and division are not associative.

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EXAMPLE 12 Identifying Rules of Algebra

Identify the rule of algebra illustrated by the statement.

a. $(5x^3)2 = 2(5x^3)$ **b.** $(4x + 3) - (4x + 3) = 0$

c. $7x \cdot \frac{1}{7x} = 1, \quad x \neq 0$ **d.** $(2 + 5x^2) + x^2 = 2 + (5x^2 + x^2)$

Solution

a. This statement illustrates the Commutative Property of Multiplication. In other words, you obtain the same result whether you multiply $5x^3$ by 2, or 2 by $5x^3$.

b. This statement illustrates the Additive Inverse Property. In terms of subtraction, this property states that when any expression is subtracted from itself, the result is 0.

c. This statement illustrates the Multiplicative Inverse Property. Note that x must be a nonzero number. The reciprocal of x is undefined when x is 0.

d. This statement illustrates the Associative Property of Addition. In other words, to form the sum $2 + 5x^2 + x^2$, it does not matter whether 2 and $5x^2$, or $5x^2$ and x^2 are added first.

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Identify the rule of algebra illustrated by the statement.

a. $x + 9 = 9 + x$ **b.** $5(x^3 \cdot 2) = (5x^3)2$ **c.** $(2 + 5x^2)y^2 = 2 \cdot y^2 + 5x^2 \cdot y^2$ ■

ALGEBRA HELP

Notice the difference between the *opposite of a number* and a *negative number.* If a is already negative, then its opposite, $-a$, is positive. For example, if $a = -5$, then

$$-a = -(-5) = 5.$$

Properties of Negation and Equality

Let a, b, and c be real numbers, variables, or algebraic expressions.

	Property	**Example**
1.	$(-1)a = -a$	$(-1)7 = -7$
2.	$-(-a) = a$	$-(-6) = 6$
3.	$(-a)b = -(ab) = a(-b)$	$(-5)3 = -(5 \cdot 3) = 5(-3)$
4.	$(-a)(-b) = ab$	$(-2)(-x) = 2x$
5.	$-(a + b) = (-a) + (-b)$	$-(x + 8) = (-x) + (-8)$ $= -x - 8$
6.	If $a = b$, then $a \pm c = b \pm c$.	$\frac{1}{2} + 3 = 0.5 + 3$
7.	If $a = b$, then $ac = bc$.	$4^2 \cdot 2 = 16 \cdot 2$
8.	If $a \pm c = b \pm c$, then $a = b$.	$1.4 - 1 = \frac{7}{5} - 1$ ⇨ $1.4 = \frac{7}{5}$
9.	If $ac = bc$ and $c \neq 0$, then $a = b$.	$3x = 3 \cdot 4$ ⇨ $x = 4$

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ALGEBRA HELP

The "or" in the Zero-Factor Property includes the possibility that either or both factors may be zero. This is an *inclusive or,* and it is generally the way the word "or" is used in mathematics.

Properties of Zero

Let a and b be real numbers, variables, or algebraic expressions.

1. $a + 0 = a$ and $a - 0 = a$

2. $a \cdot 0 = 0$

3. $\frac{0}{a} = 0, \quad a \neq 0$

4. $\frac{a}{0}$ is undefined.

5. Zero-Factor Property: If $ab = 0$, then $a = 0$ or $b = 0$.

Properties and Operations of Fractions

Let a, b, c, and d be real numbers, variables, or algebraic expressions such that $b \neq 0$ and $d \neq 0$.

1. **Equivalent Fractions:** $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.
2. **Rules of Signs:** $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ and $\frac{-a}{-b} = \frac{a}{b}$
3. **Generate Equivalent Fractions:** $\frac{a}{b} = \frac{ac}{bc}, \quad c \neq 0$
4. **Add or Subtract with Like Denominators:** $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$
5. **Add or Subtract with Unlike Denominators:** $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$
6. **Multiply Fractions:** $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
7. **Divide Fractions:** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, \quad c \neq 0$

ALGEBRA HELP

In Property 1, the phrase "if and only if" implies two statements. One statement is: If $a/b = c/d$, then $ad = bc$. The other statement is: If $ad = bc$, where $b \neq 0$ and $d \neq 0$, then $a/b = c/d$.

EXAMPLE 13 Properties and Operations of Fractions

a. $\frac{x}{5} = \frac{3 \cdot x}{3 \cdot 5} = \frac{3x}{15}$ Property 3

b. $\frac{7}{x} \div \frac{3}{2} = \frac{7}{x} \cdot \frac{2}{3} = \frac{14}{3x}$ Property 7

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a. Multiply fractions: $\frac{3}{5} \cdot \frac{x}{6}$ **b.** Add fractions: $\frac{x}{10} + \frac{2x}{5}$ ■

If a, b, and c are integers such that $ab = c$, then a and b are **factors** or **divisors** of c. A **prime number** is an integer that has exactly two positive factors—itself and 1—such as 2, 3, 5, 7, and 11. The numbers 4, 6, 8, 9, and 10 are **composite** because each can be written as the product of two or more prime numbers. The number 1 is neither prime nor composite. The **Fundamental Theorem of Arithmetic** states that every positive integer greater than 1 is prime or can be written as the product of prime numbers in precisely one way (disregarding order). For example, the **prime factorization** of 24 is $24 = 2 \cdot 2 \cdot 2 \cdot 3$.

Summarize (Section P.1)

1. Explain how to represent and classify real numbers *(pages 2 and 3)*. For examples of representing and classifying real numbers, see Examples 1 and 2.
2. Explain how to order real numbers and use inequalities *(pages 4 and 5)*. For examples of ordering real numbers and using inequalities, see Examples 3–5.
3. State the definition of the absolute value of a real number *(page 6)*. For examples of using absolute value, see Examples 6–9.
4. Explain how to evaluate an algebraic expression *(page 8)*. For examples involving algebraic expressions, see Examples 10 and 11.
5. State the basic rules and properties of algebra *(pages 9–11)*. For examples involving the basic rules and properties of algebra, see Examples 12 and 13.

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P.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary and Concept Check

In Exercises 1–4, fill in the blanks.

1. The decimal form of an ________ number neither terminates nor repeats.

2. The point representing 0 on the real number line is the ________.

3. The ________ of an algebraic expression are those parts that are separated by addition.

4. The ________ ________ states that if $ab = 0$, then $a = 0$ or $b = 0$.

5. Is $|3 - 10|$ equal to $|10 - 3|$? Explain.

6. Match each property with its name.

(a) Commutative Property of Addition	(i) $a \cdot 1 = a$
(b) Additive Inverse Property	(ii) $a(b + c) = ab + ac$
(c) Distributive Property	(iii) $a + b = b + a$
(d) Associative Property of Addition	(iv) $a + (-a) = 0$
(e) Multiplicative Identity Property	(v) $(a + b) + c = a + (b + c)$

Skills and Applications

Classifying Real Numbers **In Exercises 7–10, determine which numbers in the set are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.**

7. $\left\{-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{3}, 0, 8, -4, 2, -11\right\}$

8. $\left\{\sqrt{5}, -7, -\frac{7}{3}, 0, 3.14, \frac{5}{4}, -3, 12, 5\right\}$

9. $\{2.01, 0.\overline{6}, -13, 0.010110111 . . . , 1, -6\}$

10. $\left\{25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 18, -11.1, 13\right\}$

Plotting and Ordering Real Numbers **In Exercises 11–16, plot the two real numbers on the real number line. Then place the appropriate inequality symbol ($<$ or $>$) between them.**

11. $-4, -8$

12. $1, \frac{16}{3}$

13. $\frac{5}{6}, \frac{2}{3}$

14. $-\frac{8}{7}, -\frac{3}{7}$

15. $-5.2, -8.5$

16. $-\frac{4}{3}, -4.75$

Interpreting an Inequality **In Exercises 17–20, describe the subset of real numbers that the inequality represents.**

17. $x \le 5$

18. $x < 0$

19. $-2 < x < 2$

20. $0 < x \le 6$

Representing an Interval **In Exercises 21–24, represent the interval verbally, as an inequality, and as a graph.**

21. $[4, \infty)$

22. $(-\infty, 2)$

23. $[-5, 2)$

24. $(-1, 2]$

Representing an Interval **In Exercises 25–28, represent the statement as an interval, an inequality, and a graph.**

25. y is nonpositive.

26. y is no more than 25.

27. t is at least 10 and at most 22.

28. k is less than 5 but no less than -3.

Evaluating an Absolute Value Expression **In Exercises 29–38, evaluate the expression.**

29. $|-10|$

30. $|0|$

31. $|3 - 8|$

32. $|6 - 2|$

33. $|-1| - |-2|$

34. $-3 - |-3|$

35. $5|-5|$

36. $-4|-4|$

37. $\dfrac{|x + 2|}{x + 2}, \quad x < -2$

38. $\dfrac{|x - 1|}{x - 1}, \quad x > 1$

Comparing Real Numbers **In Exercises 39–42, place the appropriate symbol ($<$, $>$, or $=$) between the pair of real numbers.**

39. $|-4|$ ☐ $|4|$

40. -5 ☐ $-|5|$

41. $-|-6|$ ☐ $|-6|$

42. $-|-2|$ ☐ $-|2|$

Finding a Distance **In Exercises 43–46, find the distance between a and b.**

43. $a = 126, b = 75$

44. $a = -20, b = 30$

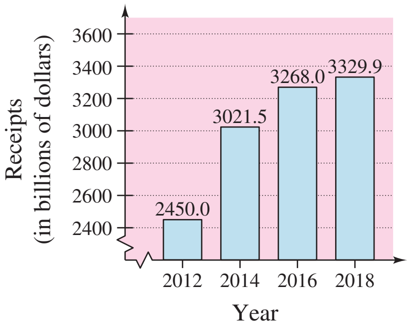
45. $a = -\frac{5}{2}, b = 0$

46. $a = -\frac{1}{4}, b = -\frac{11}{4}$

A blue exercise number indicates that a video solution can be seen at *CalcView.com*.

Federal Deficit

In Exercises 47–50, use the bar graph, which shows the receipts of the federal government (in billions of dollars) for selected years from 2012 through 2018. In each exercise, you are given the expenditures of the federal government. Find the magnitude of the surplus or deficit for the year. *(Source: U.S. Office of Management and Budget)*



	Year	Receipts, R	Expenditures, E	$\lvert R - E \rvert$
47.	2012		\$3526.6 billion	
48.	2014		\$3506.3 billion	
49.	2016		\$3852.6 billion	
50.	2018		\$4109.0 billion	

Identifying Terms and Coefficients

In Exercises 51–54, identify the terms. Then identify the coefficients of the variable terms of the expression.

51. $7x + 4$

52. $6x^3 - 5x$

53. $4x^3 + 0.5x - 5$

54. $3\sqrt{3}x^2 + 1$

Evaluating an Algebraic Expression

In Exercises 55 and 56, evaluate the expression for each value of x. (If not possible, state the reason.)

55. $x^2 - 3x + 2$ (a) $x = 0$ (b) $x = -1$

56. $\dfrac{x - 2}{x + 2}$ (a) $x = 2$ (b) $x = -2$

Operations with Fractions

In Exercises 57–60, perform the operation. (Write fractional answers in simplest form.)

57. $\dfrac{2x}{3} - \dfrac{x}{4}$

58. $\dfrac{3x}{4} + \dfrac{x}{5}$

59. $\dfrac{3x}{10} \cdot \dfrac{5}{6}$

60. $\dfrac{2x}{3} \div \dfrac{6}{7}$

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Exploring the Concepts

True or False?

In Exercises 61 and 62, determine whether the statement is true or false. Justify your answer.

61. Every nonnegative number is positive.

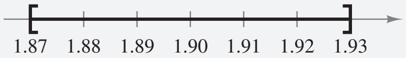
62. If $a < 0$ and $b < 0$, then $ab > 0$.

63. Error Analysis Describe the error.

$5(2x + 3) = 5 \cdot 2x + 3 = 10x + 3$ ✗

64. HOW DO YOU SEE IT? Match each description with its graph. Explain.

(i)



(ii)

(a) The price of an item is within \$0.03 of \$1.90.

(b) The distance between the prongs of an electric plug may not differ from 1.9 centimeters by more than 0.03 centimeter.

65. Conjecture Make a conjecture about the value of the expression $5/n$ as n approaches 0. Explain.

66. Conjecture Make a conjecture about the value of the expression $5/n$ as n increases without bound. Explain.

Review & Refresh

Video solutions at LarsonPrecalculus.com

Finding Greatest Common Factor and Least Common Multiple

In Exercises 67–70, find (a) the greatest common factor and (b) the least common multiple of the numbers.

67. 6, 8

68. 10, 25

69. 27, 36, 54

70. 49, 98, 112

Evaluating an Expression

In Exercises 71–74, evaluate the expression.

71. $-(3 \cdot 3)$

72. $(-3)(-3)$

73. $\dfrac{(-5)(-5)}{-5}$

74. $\dfrac{-(5 \cdot 5)}{(-5)(-5)(-5)}$

Finding a Prime Factorization

In Exercises 75–78, find the prime factorization of the number.

75. 48

76. 250

77. 792

78. 4802

Evaluating an Expression

In Exercises 79–82, evaluate the expression.

79. 3.785(10,000)

80. 1.42(1,000,000)

81. 6.09/1000

82. 8.603/100,000

P.2 Exponents and Radicals

Real numbers and algebraic expressions are often written with exponents and radicals. For example, in Exercises 85 and 86 on page 25, you will use an expression involving rational exponents to find the times required for a funnel to empty for different water heights.

- **Use properties of exponents.**
- **Use scientific notation to represent real numbers.**
- **Use properties of radicals.**
- **Simplify and combine radical expressions.**
- **Use properties of rational exponents.**

Integer Exponents and Their Properties

Repeated *multiplication* can be written in **exponential form.**

Repeated Multiplication	**Exponential Form**
$a \cdot a \cdot a \cdot a \cdot a$	a^5
$(-4)(-4)(-4)$	$(-4)^3$
$(2x)(2x)(2x)(2x)$	$(2x)^4$

Exponential Notation

If a is a real number and n is a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdot \cdot a}_{n \text{ factors}}$$

where n is the **exponent** and a is the **base.** You read a^n as "a to the nth **power.**"

An exponent can also be negative or zero. Properties 3 and 4 below show how to use negative and zero exponents.

Properties of Exponents

Let a and b be real numbers, variables, or algebraic expressions, and let m and n be integers. (All denominators and bases are nonzero.)

	Property	**Example**
1.	$a^m a^n = a^{m+n}$	$3^2 \cdot 3^4 = 3^{2+4} = 3^6 = 729$
2.	$\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{x^7}{x^4} = x^{7-4} = x^3$
3.	$a^{-n} = \dfrac{1}{a^n} = \left(\dfrac{1}{a}\right)^n$	$y^{-4} = \dfrac{1}{y^4} = \left(\dfrac{1}{y}\right)^4$
4.	$a^0 = 1$	$(x^2 + 1)^0 = 1$
5.	$(ab)^m = a^m b^m$	$(5x)^3 = 5^3 x^3 = 125x^3$
6.	$(a^m)^n = a^{mn}$	$(y^3)^{-4} = y^{3(-4)} = y^{-12} = \dfrac{1}{y^{12}}$
7.	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$\left(\dfrac{2}{x}\right)^3 = \dfrac{2^3}{x^3} = \dfrac{8}{x^3}$
8.	$\lvert a^2 \rvert = \lvert a \rvert^2 = a^2$	$\lvert(-2)^2\rvert = \lvert -2 \rvert^2 = 2^2 = 4 = (-2)^2$

GO DIGITAL

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The properties of exponents listed on the preceding page apply to *all* integers m and n, not just to positive integers. For instance, by Property 2, you can write

$$\frac{2^4}{2^{-5}} = 2^{4-(-5)} = 2^{4+5} = 2^9.$$

Note how the properties of exponents are used in Examples 1–4.

EXAMPLE 1 Evaluating Exponential Expressions

Evaluate each expression.

a. $(-5)^2$ **b.** -5^2 **c.** $2 \cdot 2^4$ **d.** $\frac{4^4}{4^6}$ **e.** $\left(\frac{7}{2}\right)^2$

Solution

a. $(-5)^2 = (-5)(-5) = 25$ Negative sign is part of the base.

b. $-5^2 = -(5)(5) = -25$ Negative sign is *not* part of the base.

c. $2 \cdot 2^4 = 2^{1+4} = 2^5 = 32$ Property 1

d. $\frac{4^4}{4^6} = 4^{4-6} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ Properties 2 and 3

e. $\left(\frac{7}{2}\right)^2 = \frac{7^2}{2^2} = \frac{49}{4}$ Property 7

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Evaluate each expression.

a. -3^4 **b.** $(-3)^4$

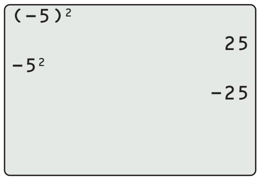
c. $3^2 \cdot 3$ **d.** $\frac{3^5}{3^8}$

ALGEBRA HELP

It is important to recognize the difference between expressions such as $(-5)^2$ and -5^2. In $(-5)^2$, the parentheses tell you that the exponent applies to the negative sign as well as to the 5, but in $-5^2 = -(5^2)$, the exponent applies only to the 5. So, $(-5)^2 = 25$ whereas $-5^2 = -25$.

TECHNOLOGY

Be sure you know how to use parentheses when evaluating exponential expressions using a graphing utility. The figure below shows that a graphing utility follows the order of operations, so $(-5)^2 = 25$. Without the parentheses, the result is -25.



```
(-5)²
                 25
-5²
                -25
```

GO DIGITAL

EXAMPLE 2 Evaluating Algebraic Expressions

Evaluate each algebraic expression when $x = 3$.

a. $5x^{-2}$ **b.** $\frac{1}{3}(-x)^3$

Solution

a. When $x = 3$, the expression $5x^{-2}$ has a value of

$$5x^{-2} = 5(3)^{-2} = \frac{5}{3^2} = \frac{5}{9}.$$

b. When $x = 3$, the expression $\frac{1}{3}(-x)^3$ has a value of

$$\frac{1}{3}(-x)^3 = \frac{1}{3}(-3)^3 = \frac{1}{3}(-27) = -9.$$

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Evaluate each algebraic expression when $x = 4$.

a. $-x^{-2}$ **b.** $\frac{1}{4}(-x)^4$ ■

EXAMPLE 3 Using Properties of Exponents

Use the properties of exponents to simplify each expression.

a. $(-3ab^4)(4ab^{-3})$ **b.** $(2xy^2)^3$ **c.** $3a(-4a^2)^0$ **d.** $\left(\frac{5x^3}{y}\right)^2$

Solution

a. $(-3ab^4)(4ab^{-3}) = (-3)(4)(a)(a)(b^4)(b^{-3}) = -12a^2b$

b. $(2xy^2)^3 = 2^3(x)^3(y^2)^3 = 8x^3y^6$

c. $3a(-4a^2)^0 = 3a(1) = 3a$

d. $\left(\frac{5x^3}{y}\right)^2 = \frac{5^2(x^3)^2}{y^2} = \frac{25x^6}{y^2}$

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Use the properties of exponents to simplify each expression.

a. $(2x^{-2}y^3)(-x^4y)$ **b.** $(4a^2b^3)^0$ **c.** $(-5z)^3(z^2)$ **d.** $\left(\frac{3x^4}{x^2y^2}\right)^2$

ALGEBRA HELP

Rarely in algebra is there only one way to solve a problem. Do not be concerned when the steps you use to solve a problem are not exactly the same as the steps presented in this text. It is important to use steps that you understand and, of course, steps that are justified by the rules of algebra. For example, the fractional form of Property 3 is

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m.$$

So, you might prefer the steps below for Example 4(d).

$$\left(\frac{3x^2}{y}\right)^{-2} = \left(\frac{y}{3x^2}\right)^2 = \frac{y^2}{9x^4}$$

EXAMPLE 4 Rewriting with Positive Exponents

a. $x^{-1} = \frac{1}{x}$ Property 3

b. $\frac{1}{3x^{-2}} = \frac{1(x^2)}{3}$ Property 3 (The exponent -2 does not apply to 3.)

$= \frac{x^2}{3}$ Simplify.

c. $\frac{12a^3b^{-4}}{4a^{-2}b} = \frac{12a^3 \cdot a^2}{4b \cdot b^4}$ Property 3

$= \frac{3a^5}{b^5}$ Property 1

d. $\left(\frac{3x^2}{y}\right)^{-2} = \frac{3^{-2}(x^2)^{-2}}{y^{-2}}$ Properties 5 and 7

$= \frac{3^{-2}x^{-4}}{y^{-2}}$ Property 6

$= \frac{y^2}{3^2x^4}$ Property 3

$= \frac{y^2}{9x^4}$ Simplify.

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Rewrite each expression with positive exponents. Simplify, if possible.

a. $2a^{-2}$ **b.** $\frac{3a^{-3}b^4}{15ab^{-1}}$

c. $\left(\frac{x}{10}\right)^{-1}$ **d.** $(-2x^2)^3(4x^3)^{-1}$ ■

GO DIGITAL

Scientific Notation

Exponents provide an efficient way of writing and computing with very large (or very small) numbers. For example, there are about 366 billion billion gallons of water on Earth—that is, 366 followed by 18 zeros.

366,000,000,000,000,000,000 Decimal form

It is convenient to write such numbers in **scientific notation.** This notation has the form $\pm c \times 10^n$, where $1 \le c < 10$ and n is an integer. So, the number of gallons of water on Earth, written in scientific notation, is

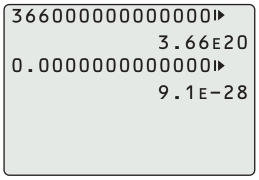
$$3.66 \times 100{,}000{,}000{,}000{,}000{,}000{,}000 = 3.66 \times 10^{20}.$$

The *positive* exponent 20 tells you that the number is *large* (10 or greater) and that the decimal point has been moved 20 places. A *negative* exponent tells you that the number is *small* (less than 1). For example, the mass (in grams) of one electron is approximately

$$9.1 \times 10^{-28} = 0.\underbrace{0000000000000000000000000091}_{\text{28 decimal places}}.$$

TECHNOLOGY

Most graphing utilities automatically switch to scientific notation when showing large (or small) numbers that exceed the display range, as shown below. Note that the figure uses 3.66E20 to represent 3.66×10^{20} and 9.1E−28 to represent 9.1×10^{-28}.



```
366000000000000▸
            3.66E20
0.0000000000000▸
            9.1E-28
```

EXAMPLE 5 Scientific Notation

a. $0.0000782 = 7.82 \times 10^{-5}$ Small number ⇨ negative exponent

b. $836{,}100{,}000 = 8.361 \times 10^{8}$ Large number ⇨ positive exponent

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Write 45,850 in scientific notation.

EXAMPLE 6 Decimal Form

a. $-9.36 \times 10^{-6} = -0.00000936$ Negative exponent ⇨ small number

b. $1.345 \times 10^{2} = 134.5$ Positive exponent ⇨ large number

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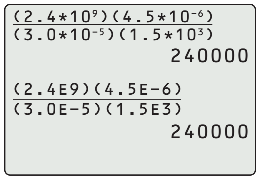
Write -2.718×10^{-3} in decimal form.

TECHNOLOGY

The figure below shows two ways to enter numbers in scientific notation on a graphing utility.



```
(2.4*10⁹)(4.5*10⁻⁶)
(3.0*10⁻⁵)(1.5*10³)
             240000
(2.4E9)(4.5E-6)
(3.0E-5)(1.5E3)
             240000
```

In the second way, the numbers were entered using an exponential key such as

EE or EXP.

EXAMPLE 7 Using Scientific Notation

Evaluate $\dfrac{(2{,}400{,}000{,}000)(0.0000045)}{(0.00003)(1500)}$.

Solution Begin by rewriting each number in scientific notation. Then simplify.

$$\begin{aligned}\frac{(2{,}400{,}000{,}000)(0.0000045)}{(0.00003)(1500)} &= \frac{(2.4 \times 10^9)(4.5 \times 10^{-6})}{(3.0 \times 10^{-5})(1.5 \times 10^3)}\\ &= \frac{(2.4)(4.5)(10^3)}{(4.5)(10^{-2})}\\ &= (2.4)(10^5)\\ &= 240{,}000\end{aligned}$$

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Evaluate (24,000,000,000)(0.00000012)(300,000). ■

GO DIGITAL

Radicals and Their Properties

A **square root** of a number is one of its two equal factors. For example, 5 is a square root of 25 because 5 is one of the two equal factors of 25. In a similar way, a **cube root** of a number is one of its three equal factors, as in $125 = 5^3$.

Definition of *n*th Root of a Number

Let a and b be real numbers, and let n be a positive integer, where $n \geq 2$. If

$$a = b^n$$

then b is an ***n*th root of *a*.** If $n = 2$, then the root is a **square root.** If $n = 3$, then the root is a **cube root.**

Some numbers have more than one nth root. For example, both 5 and -5 are square roots of 25. The *principal square* root of 25, written as $\sqrt{25}$, is the positive root, 5.

Principal *n*th Root of a Number

Let a be a real number that has at least one nth root. The **principal *n*th root of *a*** is the nth root that has the same sign as a. It is denoted by a **radical symbol**

$$\sqrt[n]{a}. \qquad \text{Principal } n\text{th root}$$

The number n is the **index** of the radical, and the number a is the **radicand.** When $n = 2$, omit the index and write $\sqrt{a}$ rather than $\sqrt[2]{a}$. (The plural of index is *indices.*)

A common misunderstanding is that the square root sign implies both negative and positive roots. This is not correct. The square root sign implies only a positive root. When a negative root is needed, you must use the negative sign with the square root sign.

Incorrect: $\sqrt{4} = \pm 2$ ✗ *Correct:* $-\sqrt{4} = -2$ *and* $\sqrt{4} = 2$

EXAMPLE 8 Evaluating Radical Expressions

a. $\sqrt{36} = 6$ because $6^2 = 36$.

b. $-\sqrt{36} = -6$ because $-\left(\sqrt{36}\right) = -\left(\sqrt{6^2}\right) = -(6) = -6$.

c. $\sqrt[3]{\dfrac{125}{64}} = \dfrac{5}{4}$ because $\left(\dfrac{5}{4}\right)^3 = \dfrac{5^3}{4^3} = \dfrac{125}{64}$.

d. $\sqrt[5]{-32} = -2$ because $(-2)^5 = -32$.

e. $\sqrt[4]{-81}$ is not a real number because no real number raised to the fourth power produces -81.

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Evaluate each expression, if possible.

a. $-\sqrt{144}$ **b.** $\sqrt{-144}$

c. $\sqrt{\dfrac{25}{64}}$ **d.** $-\sqrt[3]{\dfrac{8}{27}}$ ■

GO DIGITAL

Here are some generalizations about the *n*th roots of real numbers.

Generalizations About *n*th Roots of Real Numbers			
Real Number a	Index n	Root(s) of a	Example
$a > 0$	n is even.	$\sqrt[n]{a},\ -\sqrt[n]{a}$	$\sqrt[4]{81} = 3,\ -\sqrt[4]{81} = -3$
$a > 0$ or $a < 0$	n is odd.	$\sqrt[n]{a}$	$\sqrt[3]{-8} = -2$
$a < 0$	n is even.	No real roots	$\sqrt{-4}$ is not a real number.
$a = 0$	n is even or odd.	$\sqrt[n]{0} = 0$	$\sqrt[5]{0} = 0$

Integers such as 1, 4, 9, 16, 25, and 36 are **perfect squares** because they have integer square roots. Similarly, integers such as 1, 8, 27, 64, and 125 are **perfect cubes** because they have integer cube roots.

Properties of Radicals

Let a and b be real numbers, variables, or algebraic expressions such that the indicated roots are real numbers, and let m and n be positive integers.

	Property	**Example**
1.	$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$	$\sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$
2.	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt{5} \cdot \sqrt{7} = \sqrt{5 \cdot 7} = \sqrt{35}$
3.	$\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}},\quad b \neq 0$	$\dfrac{\sqrt[4]{27}}{\sqrt[4]{9}} = \sqrt[4]{\dfrac{27}{9}} = \sqrt[4]{3}$
4.	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt[3]{\sqrt{10}} = \sqrt[6]{10}$
5.	$\left(\sqrt[n]{a}\right)^n = a$	$\left(\sqrt{3}\right)^2 = 3$
6.	For n even, $\sqrt[n]{a^n} = \lvert a \rvert$.	$\sqrt{(-12)^2} = \lvert -12 \rvert = 12$
	For n odd, $\sqrt[n]{a^n} = a$.	$\sqrt[3]{(-12)^3} = -12$

ALGEBRA HELP

A common special case of Property 6 is

$$\sqrt{a^2} = \lvert a \rvert.$$

EXAMPLE 9 Using Properties of Radicals

Use the properties of radicals to simplify each expression.

a. $\sqrt{8} \cdot \sqrt{2}$ **b.** $\left(\sqrt[3]{5}\right)^3$ **c.** $\sqrt[3]{x^3}$ **d.** $\sqrt[6]{y^6}$

Solution

a. $\sqrt{8} \cdot \sqrt{2} = \sqrt{8 \cdot 2} = \sqrt{16} = 4$	Property 2
b. $\left(\sqrt[3]{5}\right)^3 = 5$	Property 5
c. $\sqrt[3]{x^3} = x$	Property 6
d. $\sqrt[6]{y^6} = \lvert y \rvert$	Property 6

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Use the properties of radicals to simplify each expression.

a. $\dfrac{\sqrt{125}}{\sqrt{5}}$ **b.** $\sqrt[3]{125^2}$ **c.** $\sqrt[3]{x^2} \cdot \sqrt[3]{x}$ **d.** $\sqrt{\sqrt{x}}$ ■

GO DIGITAL

Simplifying Radical Expressions

An expression involving radicals is in **simplest form** when the three conditions below are satisfied.

1. All possible factors are removed from the radical.
2. All fractions have radical-free denominators (a process called *rationalizing the denominator* accomplishes this).
3. The index of the radical is reduced.

To simplify a radical, factor the radicand into factors whose exponents are multiples of the index. Write the roots of these factors outside the radical. The "leftover" factors make up the new radicand.

ALGEBRA HELP

When you simplify a radical, it is important that both the original and the simplified expressions are defined for the same values of the variable. For instance, in Example 10(c), $\sqrt{75x^3}$ and $5x\sqrt{3x}$ are both defined only for nonnegative values of x. Similarly, in Example 10(e), $\sqrt[4]{(5x)^4}$ and $5|x|$ are both defined for all real values of x.

EXAMPLE 10 Simplifying Radical Expressions

a. $\sqrt[3]{24} = \sqrt[3]{\overset{\text{Perfect cube}}{\overset{\downarrow}{8}} \cdot \overset{\text{Leftover factor}}{\overset{\downarrow}{3}}} = \sqrt[3]{2^3 \cdot 3} = 2\sqrt[3]{3}$

b. $\sqrt[4]{48} = \sqrt[4]{\overset{\text{Perfect 4th power}}{\overset{\downarrow}{16}} \cdot \overset{\text{Leftover factor}}{\overset{\downarrow}{3}}} = \sqrt[4]{2^4 \cdot 3} = 2\sqrt[4]{3}$

c. $\sqrt{75x^3} = \sqrt{25x^2 \cdot 3x} = \sqrt{(5x)^2 \cdot 3x} = 5x\sqrt{3x}$

d. $\sqrt[3]{24a^4} = \sqrt[3]{8a^3 \cdot 3a} = \sqrt[3]{(2a)^3 \cdot 3a} = 2a\sqrt[3]{3a}$

e. $\sqrt[4]{(5x)^4} = |5x| = 5|x|$

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Simplify each radical expression.

a. $\sqrt{32}$ **b.** $\sqrt[3]{250}$ **c.** $\sqrt{24a^5}$ **d.** $\sqrt[3]{-135x^3}$ ■

Radical expressions can be combined (added or subtracted) when they are **like radicals**—that is, when they have the same index and radicand. For example, $\sqrt{2}$, $3\sqrt{2}$, and $\frac{1}{2}\sqrt{2}$ are like radicals, but $\sqrt{3}$ and $\sqrt{2}$ are unlike radicals. To determine whether two radicals can be combined, first simplify each radical.

EXAMPLE 11 Combining Radical Expressions

a. $2\sqrt{48} - 3\sqrt{27} = 2\sqrt{16 \cdot 3} - 3\sqrt{9 \cdot 3}$ Find square factors.

$= 8\sqrt{3} - 9\sqrt{3}$ Find square roots and multiply by coefficients.

$= (8 - 9)\sqrt{3}$ Combine like radicals.

$= -\sqrt{3}$ Simplify.

b. $\sqrt[3]{16x} - \sqrt[3]{54x^4} = \sqrt[3]{8 \cdot 2x} - \sqrt[3]{27x^3 \cdot 2x}$ Find cube factors.

$= 2\sqrt[3]{2x} - 3x\sqrt[3]{2x}$ Find cube roots.

$= (2 - 3x)\sqrt[3]{2x}$ Combine like radicals.

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Simplify each radical expression.

a. $3\sqrt{8} + \sqrt{18}$ **b.** $\sqrt[3]{81x^5} - \sqrt[3]{24x^2}$ ■

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To remove radicals from a denominator or a numerator, use a process called **rationalizing the denominator** or **rationalizing the numerator.** This involves multiplying by an appropriate form of 1 to obtain a perfect *n*th power (see Example 12). Note that pairs of expressions of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ are **conjugates.** The product of these two expressions contains no radicals. You can use this fact to rationalize a denominator or a numerator (see Examples 13 and 14).

EXAMPLE 12 Rationalizing Single-Term Denominators

a.
$$\begin{aligned} \frac{5}{2\sqrt{3}} &= \frac{5}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} && \sqrt{3} \text{ is rationalizing factor.} \\ &= \frac{5\sqrt{3}}{2(3)} && \text{Multiply.} \\ &= \frac{5\sqrt{3}}{6} && \text{Simplify.} \end{aligned}$$

b.
$$\begin{aligned} \frac{2}{\sqrt[3]{5}} &= \frac{2}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}} && \sqrt[3]{5^2} \text{ is rationalizing factor.} \\ &= \frac{2\sqrt[3]{5^2}}{\sqrt[3]{5^3}} && \text{Multiply.} \\ &= \frac{2\sqrt[3]{25}}{5} && \text{Simplify.} \end{aligned}$$

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Rationalize the denominators of (a) $\dfrac{5}{3\sqrt{2}}$ and (b) $\dfrac{1}{\sqrt[3]{25}}$.

EXAMPLE 13 Rationalizing a Denominator with Two Terms

$$\begin{aligned} \frac{2}{3 + \sqrt{7}} &= \frac{2}{3 + \sqrt{7}} \cdot \frac{3 - \sqrt{7}}{3 - \sqrt{7}} && \text{Multiply numerator and denominator by conjugate of denominator.} \\ &= \frac{2\left(3 - \sqrt{7}\right)}{3\left(3 - \sqrt{7}\right) + \sqrt{7}\left(3 - \sqrt{7}\right)} && \text{Distributive Property} \\ &= \frac{2\left(3 - \sqrt{7}\right)}{3(3) - 3\left(\sqrt{7}\right) + \sqrt{7}(3) - \sqrt{7}\left(\sqrt{7}\right)} && \text{Distributive Property} \\ &= \frac{2\left(3 - \sqrt{7}\right)}{(3)^2 - \left(\sqrt{7}\right)^2} && \text{Simplify.} \\ &= \frac{2\left(3 - \sqrt{7}\right)}{2} && \text{Simplify.} \\ &= 3 - \sqrt{7} && \text{Divide out common factor.} \end{aligned}$$

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Rationalize the denominator: $\dfrac{8}{\sqrt{6} - \sqrt{2}}$. ■

Sometimes it is necessary to rationalize the numerator of an expression. For instance, in Section P.5 you will use the technique shown in Example 14 on the next page to rationalize the numerator of an expression from calculus.

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EXAMPLE 14 Rationalizing a Numerator

$$\begin{aligned}
\frac{\sqrt{5}-\sqrt{7}}{2} &= \frac{\sqrt{5}-\sqrt{7}}{2} \cdot \frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} && \text{Multiply numerator and denominator by conjugate of numerator.}\\
&= \frac{(\sqrt{5})^2-(\sqrt{7})^2}{2(\sqrt{5}+\sqrt{7})} && \text{Simplify.}\\
&= \frac{5-7}{2(\sqrt{5}+\sqrt{7})} && \text{Property 5 of radicals}\\
&= \frac{-2}{2(\sqrt{5}+\sqrt{7})} && \text{Simplify.}\\
&= \frac{-1}{\sqrt{5}+\sqrt{7}} && \text{Divide out common factor.}
\end{aligned}$$

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Rationalize the numerator: $\frac{2-\sqrt{2}}{3}$. ■

ALGEBRA HELP

Do not confuse the expression $\sqrt{5}+\sqrt{7}$ with the expression $\sqrt{5+7}$. In general, $\sqrt{x+y}$ does not equal $\sqrt{x}+\sqrt{y}$. Similarly, $\sqrt{x^2+y^2}$ does not equal $x+y$.

Rational Exponents and Their Properties

Definition of Rational Exponents

If a is a real number and n is a positive integer $(n \geq 2)$ such that the principal nth root of a exists, then $a^{1/n}$ is defined as

$$a^{1/n} = \sqrt[n]{a}.$$

Moreover, if m is a positive integer that has no common factor with n, then

$$a^{m/n} = (a^{1/n})^m = \left(\sqrt[n]{a}\right)^m \quad \text{and} \quad a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}.$$

ALGEBRA HELP

Be sure you understand that the expression $a^{m/n}$ is not defined unless $\sqrt[n]{a}$ is a real number. For instance, $(-8)^{1/3}$ is defined because $\sqrt[3]{-8} = -2$ but $(-8)^{2/6}$ is undefined because $\sqrt[6]{-8}$ is not a real number.

The numerator of a rational exponent denotes the *power* to which the base is raised, and the denominator denotes the *index* or the *root* to be taken.

Power

Index

$$b^{m/n} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

When you are working with rational exponents, the properties of integer exponents still apply. For example, $2^{1/2}2^{1/3} = 2^{(1/2)+(1/3)} = 2^{5/6}$.

EXAMPLE 15 Changing From Radical to Exponential Form

a. $\sqrt{3} = 3^{1/2}$

b. $\sqrt{(3xy)^5} = \sqrt[2]{(3xy)^5} = (3xy)^{5/2}$

c. $2x\sqrt[4]{x^3} = (2x)(x^{3/4}) = 2x^{1+(3/4)} = 2x^{7/4}$

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Write (a) $\sqrt[3]{27}$, (b) $\sqrt{x^3y^5z}$, and (c) $3x\sqrt[3]{x^2}$ in exponential form. ■

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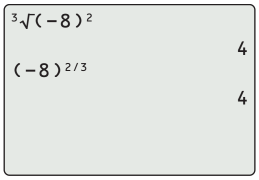
TECHNOLOGY

There are several ways to use a graphing utility to evaluate radicals and rational exponents, as shown below. Consult the user's guide for your graphing utility for specific keystrokes.



```
³√(-8)²
                4
(-8)²ᐟ³
                4
```

EXAMPLE 16 Changing From Exponential to Radical Form

See LarsonPrecalculus.com for an interactive version of this type of example.

a. $(x^2 + y^2)^{3/2} = \left(\sqrt{x^2 + y^2}\right)^3 = \sqrt{(x^2 + y^2)^3}$

b. $2y^{3/4}z^{1/4} = 2(y^3z)^{1/4} = 2\sqrt[4]{y^3z}$

c. $a^{-3/2} = \dfrac{1}{a^{3/2}} = \dfrac{1}{\sqrt{a^3}}$

d. $x^{0.2} = x^{1/5} = \sqrt[5]{x}$

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Write each expression in radical form.

a. $(x^2 - 7)^{-1/2}$ **b.** $-3b^{1/3}c^{2/3}$

c. $a^{0.75}$ **d.** $(x^2)^{2/5}$ ■

Rational exponents are useful for evaluating roots of numbers on a calculator, for reducing the index of a radical, and for simplifying expressions in calculus.

EXAMPLE 17 Simplifying with Rational Exponents

a. $(-32)^{-4/5} = \left(\sqrt[5]{-32}\right)^{-4} = (-2)^{-4} = \dfrac{1}{(-2)^4} = \dfrac{1}{16}$

b. $(-5x^{5/3})(3x^{-3/4}) = -15x^{(5/3)-(3/4)} = -15x^{11/12}, \quad x \neq 0$

c. $\sqrt[9]{a^3} = a^{3/9} = a^{1/3} = \sqrt[3]{a}$ Reduce index.

d. $\sqrt[3]{\sqrt{125}} = \sqrt[6]{125} = \sqrt[6]{(5)^3} = 5^{3/6} = 5^{1/2} = \sqrt{5}$

e. $(2x - 1)^{4/3}(2x - 1)^{-1/3} = (2x - 1)^{(4/3)-(1/3)} = 2x - 1, \quad x \neq \frac{1}{2}$

ALGEBRA HELP

The expression in Example 17(b) is not defined when $x = 0$ because $0^{-3/4}$ is not a real number. Similarly, the expression in Example 17(e) is not defined when $x = \frac{1}{2}$ because

$$\left(2 \cdot \tfrac{1}{2} - 1\right)^{-1/3} = (0)^{-1/3}$$

is not a real number.

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Simplify each expression.

a. $(-125)^{-2/3}$ **b.** $(4x^2y^{3/2})(-3x^{-1/3}y^{-3/5})$

c. $\sqrt[3]{\sqrt[4]{27}}$ **d.** $(3x + 2)^{5/2}(3x + 2)^{-1/2}$ ■

Summarize (Section P.2)

1. Make a list of the properties of exponents *(page 14)*. For examples that use these properties, see Examples 1–4.
2. Explain how to write a number in scientific notation *(page 17)*. For examples involving scientific notation, see Examples 5–7.
3. Make a list of the properties of radicals *(page 19)*. For examples involving radicals, see Examples 8 and 9.
4. Explain how to simplify a radical expression *(page 20)*. For examples of simplifying radical expressions, see Examples 10 and 11.
5. Explain how to rationalize a denominator or a numerator *(page 21)*. For examples of rationalizing denominators and numerators, see Examples 12–14.
6. State the definition of a rational exponent *(page 22)*. For examples involving rational exponents, see Examples 15–17.

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P.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

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Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blanks.

1. In the exponential form a^n, n is the ________ and a is the ________.
2. In the radical form $\sqrt[n]{a}$, the number n is the ________ of the radical and the number a is the ________.

3. When is an expression involving radicals in simplest form?
4. Is 64 a perfect square, a perfect cube, or both?

Skills and Applications

Evaluating an Exponential Expression **In Exercises 5–16, evaluate the expression.**

5. $5 \cdot 5^3$
6. $(2^3 \cdot 3^2)^2$
7. $(3^3)^2$
8. $(-2)^0$
9. $\dfrac{5^2}{5^4}$
10. $\left(-\dfrac{3}{5}\right)^3\left(\dfrac{5}{3}\right)^2$
11. -3^2
12. $(-4)^{-3}$
13. $\dfrac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}}$
14. $\dfrac{3}{3^{-4}}$
15. $3^2 + 2^3$
16. $(3^{-2})^2$

Evaluating an Algebraic Expression **In Exercises 17–20, evaluate the expression for the given value of x.**

17. $-3x^3, \quad x = 2$
18. $7x^{-2}, \quad x = 4$
19. $6x^2, \quad x = 0.1$
20. $12(-x)^3, \quad x = -\frac{1}{3}$

Using Properties of Exponents **In Exercises 21–30, simplify the expression.**

21. $(5z)^3$
22. $(4x^3)^0$
23. $6y^2(2y^0)^2$
24. $(-z)^3(3z^4)$
25. $\dfrac{7x^2}{x^3}$
26. $\dfrac{12(x+y)^3}{9(x+y)}$
27. $\left(\dfrac{4}{y}\right)^3\left(\dfrac{3}{y}\right)^4$
28. $\left(\dfrac{b^{-2}}{a^{-2}}\right)\left(\dfrac{b}{a}\right)^2$
29. $[(x^2y^{-2})^{-1}]^{-1}$
30. $(5x^2z^6)^3(5x^2z^6)^{-3}$

Rewriting with Positive Exponents **In Exercises 31–36, rewrite the expression with positive exponents. Simplify, if possible.**

31. $(2x^2)^{-2}$
32. $(4y^{-2})(8y^{-4})$
33. $\left(\dfrac{x^{-3}y^4}{5}\right)^{-3}$
34. $\left(\dfrac{a^{-2}}{b^{-2}}\right)\left(\dfrac{b}{a}\right)^{-3}$
35. $\dfrac{3^n \cdot 3^{2n}}{3^{3n} \cdot 3^2}$
36. $\dfrac{x^2 \cdot x^n}{x^3 \cdot x^n}$

Scientific Notation **In Exercises 37 and 38, write the number in scientific notation.**

37. 10,250.4
38. −0.000125

Decimal Form **In Exercises 39 and 40, write the number in decimal form.**

39. 3.14×10^{-4}
40. -2.058×10^6

Using Scientific Notation **In Exercises 41–44, evaluate the expression without using a calculator.**

41. $(2.0 \times 10^9)(3.4 \times 10^{-4})$
42. $(1.2 \times 10^7)(5.0 \times 10^{-3})$
43. $\dfrac{6.0 \times 10^8}{3.0 \times 10^{-3}}$
44. $\dfrac{2.5 \times 10^{-3}}{5.0 \times 10^2}$

Evaluating Radical Expressions **In Exercises 45 and 46, evaluate each expression without using a calculator.**

45. (a) $\sqrt{9}$ (b) $\sqrt[3]{\dfrac{27}{8}}$
46. (a) $\sqrt[3]{27}$ (b) $\left(\sqrt{36}\right)^3$

Using Properties of Radicals **In Exercises 47 and 48, use the properties of radicals to simplify each expression.**

47. (a) $(\sqrt[5]{2})^5$ (b) $\sqrt[5]{32x^5}$
48. (a) $\sqrt{12} \cdot \sqrt{3}$ (b) $\sqrt[4]{(3x^2)^4}$

Simplifying a Radical Expression **In Exercises 49–62, simplify the radical expression.**

49. $\sqrt{20}$
50. $\sqrt[3]{128}$
51. $\sqrt[3]{\dfrac{16}{27}}$
52. $\sqrt{\dfrac{75}{4}}$
53. $\sqrt{72x^3}$
54. $\sqrt{54xy^4}$
55. $\sqrt{\dfrac{18^2}{z^4}}$
56. $\sqrt{\dfrac{32a^4}{b^2}}$
57. $\sqrt{75x^2y^{-4}}$
58. $\sqrt[4]{3x^4y^2}$
59. $2\sqrt{20x^2} + 5\sqrt{125x^2}$
60. $8\sqrt{147x} - 3\sqrt{48x}$
61. $3\sqrt[3]{54x^3} + \sqrt[3]{16x^3}$
62. $\sqrt[3]{64x} - \sqrt[3]{27x^4}$

Rationalizing a Denominator **In Exercises 63–66, rationalize the denominator of the expression. Then simplify your answer.**

63. $\frac{1}{\sqrt{3}}$ **64.** $\frac{8}{\sqrt[3]{2}}$

65. $\frac{5}{\sqrt{14} - 2}$ **66.** $\frac{3}{\sqrt{5} + \sqrt{6}}$

Rationalizing a Numerator **In Exercises 67 and 68, rationalize the numerator of the expression. Then simplify your answer.**

67. $\frac{\sqrt{5} + \sqrt{3}}{3}$ **68.** $\frac{\sqrt{7} - 3}{4}$

Writing Exponential and Radical Forms **In Exercises 69–72, fill in the missing form of the expression.**

	Radical Form	Rational Exponent Form
69.	$\sqrt[3]{64}$	
70.	$x^2\sqrt{x}$	
71.		$3x^{-2/3}$
72.		$a^{0.4}$

Simplifying an Expression **In Exercises 73–84, simplify the expression.**

73. $32^{-3/5}$ **74.** $\left(\frac{16}{81}\right)^{-3/4}$

75. $\left(\frac{9}{4}\right)^{-1/2}$ **76.** $100^{-3/2}$

77. $\sqrt[4]{3^2}$ **78.** $\sqrt[4]{(3x^2)^4}$

79. $\sqrt{\sqrt{32}}$ **80.** $\sqrt{\sqrt[4]{2x}}$

81. $(x-1)^{1/3}(x-1)^{2/3}$

82. $(x-1)^{1/3}(x-1)^{-4/3}$

83. $(4x+3)^{5/2}(4x+3)^{-5/3}$

84. $(4x+3)^{-5/2}(4x+3)^{2/3}$

Mathematical Modeling

In Exercises 85 and 86, use the following information. A funnel is filled with water to a height of h centimeters. The formula $t = 0.03[12^{5/2} - (12 - h)^{5/2}]$, $0 \le h \le 12$, represents the amount of time t (in seconds) that it will take for the funnel to empty.

85. Use a graphing utility to find the times required for the funnel to empty for integer-valued water heights from 0 to 12 centimeters.

86. Use the graphing utility to find the water height corresponding to an emptying time of 10 seconds.

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Exploring the Concepts

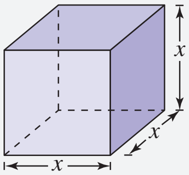
True or False? **In Exercises 87 and 88, determine whether the statement is true or false. Justify your answer.**

87. $\frac{x^{k+1}}{x} = x^k$ **88.** $\frac{a}{\sqrt{b}} = \frac{a^2}{(\sqrt{b})^2} = \frac{a^2}{b}$

89. **Error Analysis** Describe the error.

$$\left(\frac{a^4}{a^6}\right)^{-3} = (a^{-2})^{-3} = \left(\frac{1}{a^2}\right)^{-3} = \frac{-1}{a^{-6}} = -a^6 \quad ✗$$

90. **HOW DO YOU SEE IT?** Package A is a cube with a volume of 500 cubic inches. Package B is a cube with a volume of 250 cubic inches. Is the length x of a side of package A greater than, less than, or equal to twice the length of a side of package B? Explain.



91. **Think About It** Verify that $a^0 = 1$, $a \neq 0$. (*Hint:* Use the property of exponents $a^m/a^n = a^{m-n}$.)

92. **Exploration** List all possible digits that occur in the units place of the square of a positive integer. Use that list to determine whether $\sqrt{5233}$ is an integer.

Review & Refresh

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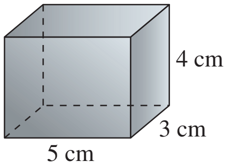
Finding Surface Area and Volume **In Exercises 93–96, find the (a) surface area and (b) volume of the rectangular solid.**

93.



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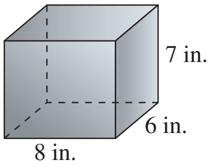

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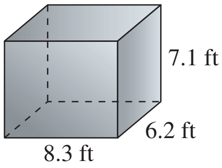

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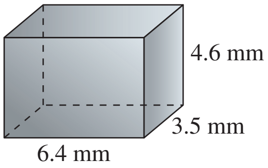

Evaluating an Algebraic Expression **In Exercises 97 and 98, evaluate the algebraic expression when (a) $x = 2$ and (b) $x = -3$.**

97. $(x+4)(x-4)$ **98.** $(9x+5)(3x-1)$

Identifying Terms and Coefficients **In Exercises 99 and 100, identify the terms. Then identify the coefficients of the variable terms of the expression.**

99. $2x - 3$ **100.** $4x^3 - x^2 + 5x + 1$

P.3 Polynomials and Special Products

Polynomials have many real-life applications. For example, in Exercise 75 on page 32, you will work with polynomials that model uniformly distributed safe loads for steel beams.

- **Write polynomials in standard form.**
- **Add, subtract, and multiply polynomials, and use special products.**
- **Use polynomials to solve real-life problems.**

Polynomials

One of the most common types of algebraic expressions is the **polynomial.** Some examples are $2x + 5$, $3x^4 - 7x^2 + 2x + 4$, and $5x^2y^2 - xy + 3$. The first two are *polynomials in x* and the third is a *polynomial in x and y*. The terms of a polynomial in x have the form ax^k, where a is the **coefficient** and k is the **degree** of the term. For example, the polynomial $2x^3 - 5x^2 + 1 = 2x^3 + (-5)x^2 + (0)x + 1$ has coefficients 2, -5, 0, and 1.

Definition of a Polynomial in *x*

Let $a_0, a_1, a_2, \ldots, a_n$ be real numbers and let n be a nonnegative integer. A **polynomial in *x*** is an expression of the form

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

where $a_n \neq 0$. The polynomial is of **degree** n, a_n is the **leading coefficient,** and a_0 is the **constant term.**

In **standard form**, a polynomial in x is written with descending powers of x. Polynomials with one, two, and three terms are **monomials, binomials,** and **trinomials,** respectively.

EXAMPLE 1 Writing Polynomials in Standard Form

	Polynomial	**Standard Form**	**Degree**	**Leading Coefficient**
a.	$4x^2 - 5x^7 - 2 + 3x$	$-5x^7 + 4x^2 + 3x - 2$	7	-5
b.	$4 - 9x^2$	$-9x^2 + 4$	2	-9
c.	8	8 or $8x^0$	0	8

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Write the polynomial $6 - 7x^3 + 2x$ in standard form. Then identify the degree and leading coefficient of the polynomial. ■

A polynomial that has all zero coefficients is called the **zero polynomial,** denoted by 0. No degree is assigned to the zero polynomial. For polynomials in more than one variable, the degree of a *term* is the sum of the exponents of the variables in the term. The degree of the *polynomial* is the highest degree of its terms. For example, the degree of the polynomial $-2x^3y^6 + 4xy - x^7y^4$ is 11 because the sum of the exponents in the last term is the greatest. The leading coefficient of the polynomial is the coefficient of the highest-degree term. Expressions are not polynomials when a variable is underneath a radical or when a polynomial expression (with degree greater than 0) is in the denominator of a term. For example, the expressions $x^3 - \sqrt{3x} = x^3 - (3x)^{1/2}$ and $x^2 + (5/x) = x^2 + 5x^{-1}$ are not polynomials.

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Operations with Polynomials and Special Products

You can add and subtract polynomials in much the same way you add and subtract real numbers. Add or subtract the *like terms* (terms having the same variables to the same powers) by adding or subtracting their coefficients. For example, $-3xy^2$ and $5xy^2$ are like terms and their sum is

$$-3xy^2 + 5xy^2 = (-3 + 5)xy^2 = 2xy^2.$$

EXAMPLE 2 Adding or Subtracting Polynomials

a. $(5x^3 - 7x^2 - 3) + (x^3 + 2x^2 - x + 8)$

$= (5x^3 + x^3) + (-7x^2 + 2x^2) + (-x) + (-3 + 8)$ Group like terms.

$= 6x^3 - 5x^2 - x + 5$ Combine like terms.

b. $(7x^4 - x^2 - 4x + 2) - (3x^4 - 4x^2 + 3x)$

$= 7x^4 - x^2 - 4x + 2 - 3x^4 + 4x^2 - 3x$ Distributive Property

$= (7x^4 - 3x^4) + (-x^2 + 4x^2) + (-4x - 3x) + 2$ Group like terms.

$= 4x^4 + 3x^2 - 7x + 2$ Combine like terms.

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Find the difference $(2x^3 - x + 3) - (x^2 - 2x - 3)$ and write the resulting polynomial in standard form. ■

ALGEBRA HELP

When a negative sign precedes an expression inside parentheses, remember to distribute the negative sign to each term inside the parentheses. In other words, multiply each term by -1.

$$-(3x^4 - 4x^2 + 3x)$$
$$= -3x^4 + 4x^2 - 3x$$

To find the *product* of two polynomials, use the right and left Distributive Properties. For example, you can find the product of $3x - 2$ and $5x + 7$ by first treating $5x + 7$ as a single quantity.

$$(3x - 2)(5x + 7) = 3x(5x + 7) - 2(5x + 7)$$
$$= (3x)(5x) + (3x)(7) - (2)(5x) - (2)(7)$$
$$= \underbrace{15x^2}_{\substack{\text{Product of}\\ \text{First terms}}} + \underbrace{21x}_{\substack{\text{Product of}\\ \text{Outer terms}}} - \underbrace{10x}_{\substack{\text{Product of}\\ \text{Inner terms}}} - \underbrace{14}_{\substack{\text{Product of}\\ \text{Last terms}}}$$
$$= 15x^2 + 11x - 14$$

Note that when using the **FOIL Method** to multiply two binomials, some of the terms in the product may be like terms that can be combined into one term.

EXAMPLE 3 Finding a Product by the FOIL Method

Use the FOIL Method to find the product of $2x - 4$ and $x + 5$.

Solution

$$(2x - 4)(x + 5) = \overset{\text{F}}{2x^2} + \overset{\text{O}}{10x} - \overset{\text{I}}{4x} - \overset{\text{L}}{20} = 2x^2 + 6x - 20$$

✓ ***Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Use the FOIL Method to find the product of $3x - 1$ and $x - 5$. ■

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When multiplying two polynomials, be sure to multiply *each* term of one polynomial by *each* term of the other. A vertical arrangement can be helpful, as shown in the next example.

EXAMPLE 4 A Vertical Arrangement for Multiplication

Multiply $-2x + 2 + x^2$ by $x^2 + 2x + 2$ using a vertical arrangement.

Solution First, write $-2x + 2 + x^2$ in standard form, $x^2 - 2x + 2$.

$$\begin{array}{rrrrrcl}
 & & x^2 & -\ 2x & +\ 2 & & \text{Write in standard form.} \\
 & \times & x^2 & +\ 2x & +\ 2 & & \text{Write in standard form.} \\
\hline
 & & 2x^2 & -\ 4x & +\ 4 & \Leftarrow & 2(x^2 - 2x + 2) \\
 & 2x^3 & -\ 4x^2 & +\ 4x & & \Leftarrow & 2x(x^2 - 2x + 2) \\
x^4 & -\ 2x^3 & +\ 2x^2 & & & \Leftarrow & x^2(x^2 - 2x + 2) \\
\hline
x^4 & +\ 0x^3 & +\ 0x^2 & +\ 0x & +\ 4 = x^4 + 4 & & \text{Combine like terms.}
\end{array}$$

So, $(x^2 - 2x + 2)(x^2 + 2x + 2) = x^4 + 4$.

✓ **Checkpoint** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Multiply $x^2 + 2x + 3$ by $x^2 - 2x + 3$ using a vertical arrangement. ■

ALGEBRA HELP

When multiplying two polynomials, it is best to write each in standard form before using either the horizontal or the vertical format.

Some binomial products have special forms that occur frequently in algebra. You do not need to memorize these formulas because you can use the Distributive Property to multiply. However, becoming familiar with these formulas will enable you to manipulate the algebra more quickly.

Special Products

Let u and v be real numbers, variables, or algebraic expressions.

Special Product	**Example**
Sum and Difference of Same Terms	
$(u + v)(u - v) = u^2 - v^2$	$(x + 4)(x - 4) = x^2 - 4^2$ $= x^2 - 16$
Square of a Binomial	
$(u + v)^2 = u^2 + 2uv + v^2$	$(x + 3)^2 = x^2 + 2(x)(3) + 3^2$ $= x^2 + 6x + 9$
$(u - v)^2 = u^2 - 2uv + v^2$	$(3x - 2)^2 = (3x)^2 - 2(3x)(2) + 2^2$ $= 9x^2 - 12x + 4$
Cube of a Binomial	
$(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$	$(x + 2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3$ $= x^3 + 6x^2 + 12x + 8$
$(u - v)^3 = u^3 - 3u^2v + 3uv^2 - v^3$	$(x - 1)^3 = x^3 - 3x^2(1) + 3x(1^2) - 1^3$ $= x^3 - 3x^2 + 3x - 1$

ALGEBRA HELP

Note that $u + v$ and $u - v$ are conjugates. In words, you can say that the product of conjugates equals the square of the first term minus the square of the second term.

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EXAMPLE 5 Sum and Difference of Same Terms

Find the product of $5x + 9$ and $5x - 9$.

Solution

The product of a sum and a difference of the *same* two terms has no middle term and takes the form $(u + v)(u - v) = u^2 - v^2$.

$$(5x + 9)(5x - 9) = (5x)^2 - 9^2 = 25x^2 - 81$$

✓ ***Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Find the product of $3x - 2$ and $3x + 2$.

ALGEBRA HELP

When squaring a binomial, note that the resulting middle term is always *twice* the product of the two terms of the binomial.

EXAMPLE 6 Square of a Binomial

Find $(6x - 5)^2$.

Solution

The square of the binomial $u - v$ is $(u - v)^2 = u^2 - 2uv + v^2$.

$$(6x - 5)^2 = (6x)^2 - 2(6x)(5) + 5^2 = 36x^2 - 60x + 25$$

✓ ***Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Find $(x + 10)^2$.

EXAMPLE 7 Cube of a Binomial

Find $(3x + 2)^3$.

Solution

The cube of the binomial $u + v$ is $(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$. Note the *decreasing* powers of u and the *increasing* powers of v. Letting $u = 3x$ and $v = 2$,

$$(3x + 2)^3 = (3x)^3 + 3(3x)^2(2) + 3(3x)(2^2) + 2^3 = 27x^3 + 54x^2 + 36x + 8.$$

✓ ***Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Find $(4x - 1)^3$.

EXAMPLE 8 Multiplying Two Trinomials

See LarsonPrecalculus.com for an interactive version of this type of example.

Find the product of $x + y - 2$ and $x + y + 2$.

Solution

One way to find this product is to group $x + y$ and form a special product.

$$\begin{aligned}(x + y - 2)(x + y + 2) &= \underset{\text{Difference}}{[(x + y) - 2]}\underset{\text{Sum}}{[(x + y) + 2]}\\ &= (x + y)^2 - 2^2 && \text{Sum and difference of same terms}\\ &= x^2 + 2xy + y^2 - 4 && \text{Square of a binomial}\end{aligned}$$

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Find the product of $x - 2 + 3y$ and $x - 2 - 3y$. ■

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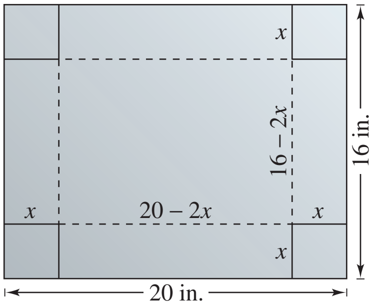
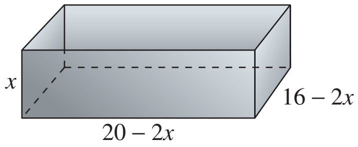
Application

EXAMPLE 9 Finding the Volume of a Box





An open box is made by cutting squares from the corners of a piece of metal that is 20 inches by 16 inches, as shown in the figure. The edge of each cut-out square is x inches. Find the volume of the box in terms of x. Then find the volume of the box when $x = 1$, $x = 2$, and $x = 3$.

Solution

The volume of a rectangular box is equal to the product of its length, width, and height. From the figure, the length is $20 - 2x$, the width is $16 - 2x$, and the height is x. So, the volume of the box is

$$\begin{aligned}\text{Volume} &= (20 - 2x)(16 - 2x)(x)\\ &= (320 - 72x + 4x^2)(x)\\ &= 320x - 72x^2 + 4x^3.\end{aligned}$$

When $x = 1$ inch, the volume of the box is

$$\begin{aligned}\text{Volume} &= 320(1) - 72(1)^2 + 4(1)^3\\ &= 252 \text{ cubic inches.}\end{aligned}$$

When $x = 2$ inches, the volume of the box is

$$\begin{aligned}\text{Volume} &= 320(2) - 72(2)^2 + 4(2)^3\\ &= 384 \text{ cubic inches.}\end{aligned}$$

When $x = 3$ inches, the volume of the box is

$$\begin{aligned}\text{Volume} &= 320(3) - 72(3)^2 + 4(3)^3\\ &= 420 \text{ cubic inches.}\end{aligned}$$

✓ ***Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

In Example 9, find the volume of the box in terms of x when the piece of metal is 12 inches by 10 inches. Then find the volume when $x = 2$ and $x = 3$.

Summarize (Section P.3)

1. State the definition of a polynomial in x and explain what is meant by the standard form of a polynomial *(page 26)*. For an example of writing polynomials in standard form, see Example 1.
2. Explain how to add and subtract polynomials *(page 27)*. For an example of adding and subtracting polynomials, see Example 2.
3. Explain the FOIL Method *(page 27)*. For an example of finding a product using the FOIL Method, see Example 3.
4. Explain how to find binomial products that have special forms *(page 28)*. For examples of binomial products that have special forms, see Examples 5–8.
5. Describe an example of how to use polynomials to model and solve a real-life problem *(page 30, Example 9)*.

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P.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

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Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blanks.

1. For the polynomial $a_nx^n + a_{n-1}x^{n-1} + \cdot\cdot\cdot + a_1x + a_0$, $a_n \neq 0$, the degree is ________, the leading coefficient is ________, and the constant term is ________.

2. The letters in "FOIL" stand for F ________, O ________, I ________, and L ________.

3. Is it possible for a binomial and a trinomial to have the same degree? If so, give examples. If not, explain why.

4. Match each special product with its equivalent form.

(a) $(u + v)(u - v)$

(b) $(u + v)^2$

(c) $(u - v)^2$

(d) $(u + v)^3$

(e) $(u - v)^3$

(i) $u^3 - 3u^2v + 3uv^2 - v^3$

(ii) $u^3 + 3u^2v + 3uv^2 + v^3$

(iii) $u^2 + 2uv + v^2$

(iv) $u^2 - v^2$

(v) $u^2 - 2uv + v^2$

Skills and Applications

Writing a Polynomial in Standard Form **In Exercises 5–10, (a) write the polynomial in standard form, (b) identify the degree and leading coefficient of the polynomial, and (c) state whether the polynomial is a monomial, a binomial, or a trinomial.**

5. $7x$

6. 3

7. $14x - \frac{1}{2}x^5$

8. $3 + 2x$

9. $1 + 6x^4 - 4x^5$

10. $-y + 25y^2 + 1$

Identifying Polynomials **In Exercises 11–16, determine whether the expression is a polynomial. If so, write the polynomial in standard form.**

11. $2x - 3x^3 + 8$

12. $5x^4 - 2x^2 + x^{-2}$

13. $\dfrac{3x + 4}{x}$

14. $\dfrac{x^2 + 2x - 3}{2}$

15. $y^2 - y^4 + y^3$

16. $y^4 - \sqrt{y}$

Adding or Subtracting Polynomials **In Exercises 17–24, add or subtract and write the result in standard form.**

17. $(6x + 5) - (8x + 15)$

18. $(t^3 - 1) + (6t^3 - 5t)$

19. $(4y^2 - 3) + (-7y^2 + 9)$

20. $(2x^2 + 1) - (x^2 - 2x + 1)$

21. $(15x^2 - 6) + (-8.3x^3 - 14.7x^2 - 17)$

22. $(15.6w^4 - 14w - 17.4) + (16.9w^4 - 9.2w + 13)$

23. $5z - [3z - (10z + 8)]$

24. $(y^3 + 1) - [(y^2 + 1) + (3y - 7)]$

Multiplying Polynomials **In Exercises 25–36, multiply the polynomials.**

25. $3x(x^2 - 2x + 1)$

26. $y^2(4y^2 + 2y - 3)$

27. $-5z(3z - 1)$

28. $-3x(5x + 2)$

29. $(1.5t^2 + 5)(-3t)$

30. $(2 - 3.5y)(2y^3)$

31. $(3x - 5)(2x + 1)$

32. $(7x - 2)(4x - 3)$

33. $(x + 7)(x^2 + 2x + 5)$

34. $(x - 8)(2x^2 + x + 4)$

35. $(x^2 - x + 2)(x^2 + x + 1)$

36. $(2x^2 - x + 4)(x^2 + 3x + 2)$

Finding Special Products **In Exercises 37–60, find the special product.**

37. $(x + 10)(x - 10)$

38. $(2x + 3)(2x - 3)$

39. $(x + 2y)(x - 2y)$

40. $(4a + 5b)(4a - 5b)$

41. $(2x + 3)^2$

42. $(5 - 8x)^2$

43. $(4x^3 - 3)^2$

44. $(8x + 3)^2$

45. $(x + 3)^3$

46. $(x - 2)^3$

47. $(2x - y)^3$

48. $(3x + 2y)^3$

49. $\left(\frac{1}{5}x - 3\right)\left(\frac{1}{5}x + 3\right)$

50. $(1.5x - 4)(1.5x + 4)$

51. $\left(\frac{1}{4}x - 5\right)^2$

52. $(2.4x + 3)^2$

53. $[(x - 3) + y]^2$

54. $[(x + 1) - y]^2$

55. $(3y - 6x)(-3y - 6x)$

56. $(3a^3 - 4b^2)(3a^3 + 4b^2)$

57. $[(m - 3) + n][(m - 3) - n]$

58. $[(x - 3y) + z][(x - 3y) - z]$

59. $(u + 2)(u - 2)(u^2 + 4)$

60. $(x + y)(x - y)(x^2 + y^2)$

Operations with Polynomials **In Exercises 61–64, perform the operation.**

61. Subtract $4x^2 - 5$ from $-3x^3 + x^2 + 9$.

62. Subtract $-7t^4 + 5t^2 - 1$ from $2t^4 - 10t^3 - 4t$.

63. Multiply $y^2 + 3y - 5$ by $y^2 - 6y + 4$.

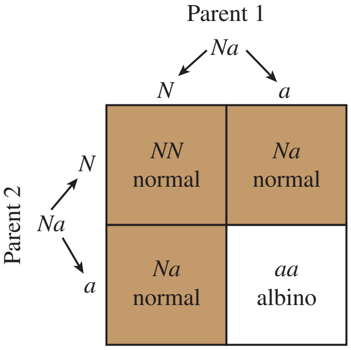
64. Multiply $x^2 + 4x - 1$ by $x^2 - x + 3$.

Finding a Product **In Exercises 65–68, find the product. (The expressions are not polynomials, but the special products formulas can still be used.)**

65. $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$ **66.** $(5 + \sqrt{x})(5 - \sqrt{x})$

67. $(x - \sqrt{y})^2$ **68.** $(x + \sqrt{y})^2$

69. Genetics In deer, the gene N is for normal coloring and the gene a is for albino. Any gene combination with an N results in normal coloring. The Punnett square shows the possible gene combinations of an offspring and the resulting colors when both parents have the gene combination Na.



	N	a
N	NN normal	Na normal
a	Na normal	aa albino

(a) What percent of the possible gene combinations result in albino coloring?

(b) Each parent's gene combination is represented by the polynomial $0.5N + 0.5a$. The product $(0.5N + 0.5a)^2$ represents the possible gene combinations of an offspring. Find this product.

(c) The coefficient of each term of the polynomial you wrote in part (b) is the probability (in decimal form) of the offspring having that gene combination. Use this polynomial to confirm your answer in part (a). Explain.

70. Construction Management A square-shaped foundation for a building with 100-foot sides is reduced by x feet on one side and extended by x feet on an adjacent side.

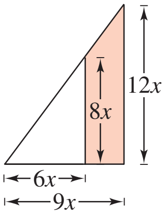
(a) The area of the new foundation is represented by $(100 - x)(100 + x)$. Find this product.

(b) Does the area of the foundation increase, decrease, or stay the same? Explain.

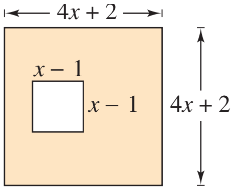
(c) Use the polynomial in part (a) to find the area of the new foundation when $x = 21$.

Geometry **In Exercises 71 and 72, find the area of the shaded region in terms of x. Write your result as a polynomial in standard form.**

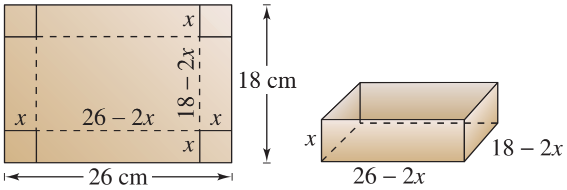
71.



72.



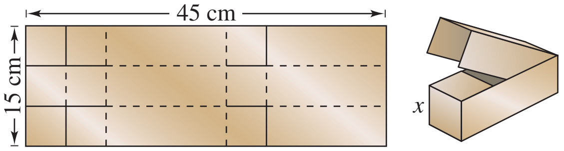
73. Volume of a Box A take-out fast-food restaurant is constructing an open box by cutting squares from the corners of the piece of cardboard shown in the figure. The edge of each cut-out square is x centimeters.



(a) Find the volume of the box in terms of x.

(b) Find the volume when $x = 1$, $x = 2$, and $x = 3$.

74. Volume of a Box An overnight shipping company designs a closed box by cutting along the solid lines and folding along the broken lines on the rectangular piece of corrugated cardboard shown in the figure.



(a) Find the volume of the shipping box in terms of x.

(b) Find the volume when $x = 3$, $x = 5$, and $x = 7$.

75. Engineering

A one-inch-wide steel beam has a uniformly distributed load. When the span of the beam is x feet and its depth is 6 inches, the safe load S (in pounds) is approximately $S_6 = (0.06x^2 - 2.42x + 38.71)^2$. When the depth is 8 inches, the safe load is approximately $S_8 = (0.08x^2 - 3.30x + 51.93)^2$.

(a) Approximate the difference of the safe loads for these two beams when the span is 12 feet.

(b) How does the difference of the safe loads change as the span increases?

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76. **Stopping Distance** The stopping distance of an automobile is the distance traveled during the driver's reaction time plus the distance traveled after the driver applies the brakes. In an experiment, researchers measured these distances (in feet) when the automobile was traveling at a speed of x miles per hour on dry, level pavement, as shown in the bar graph. The distance traveled during the reaction time R was

$$R = 1.1x$$

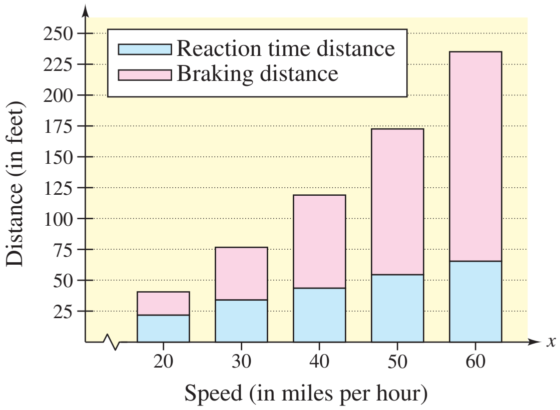
and the braking distance B was

$$B = 0.0475x^2 - 0.001x + 0.23.$$

(a) Determine the polynomial that represents the total stopping distance T.

(b) Use the result of part (a) to estimate the total stopping distance when $x = 30$, $x = 40$, and $x = 55$ miles per hour.

(c) Use the bar graph to make a statement about the total stopping distance required for increasing speeds.



Exploring the Concepts

True or False? **In Exercises 77–80, determine whether the statement is true or false. Justify your answer.**

77. The product of two binomials is always a second-degree polynomial.

78. The sum of two second-degree polynomials is always a second-degree polynomial.

79. The sum of two binomials is always a binomial.

80. The leading coefficient of the product of two polynomials is always the product of the leading coefficients of the two polynomials.

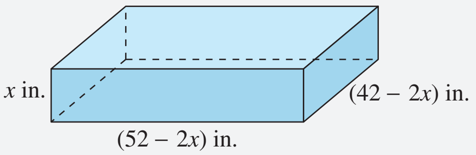
81. **Degree of a Product** Find the degree of the product of two polynomials of degrees m and n.

82. **Degree of a Sum** Find the degree of the sum of two polynomials of degrees m and n, where $m < n$.

83. **Error Analysis** Describe the error.

$(x - 3)^2 = x^2 + 9$ ✗

84. **HOW DO YOU SEE IT?** An open box has a length of $(52 - 2x)$ inches, a width of $(42 - 2x)$ inches, and a height of x inches, as shown.



(a) Describe a way that you could make the box from a rectangular piece of cardboard. Give the original dimensions of the cardboard.

(b) What degree is the polynomial that represents the volume of the box? Explain.

(c) Describe a procedure for finding the value of x (to the nearest tenth of an inch) that yields the maximum possible volume of the box.

85. **Think About It** When the polynomial $2x - 1$ is subtracted from an unknown polynomial, the difference is $5x^2 + 8$. Find the unknown polynomial.

86. **Reasoning** Verify that $(x + y)^2$ is not equal to $x^2 + y^2$ by letting $x = 3$ and $y = 4$ and evaluating both expressions. Are there any values of x and y for which $(x + y)^2$ and $x^2 + y^2$ are equal? Explain.

Review & Refresh

Video solutions at LarsonPrecalculus.com

Multiplying Radicals **In Exercises 87–90, find each product.**

87. $\sqrt{3}(\sqrt{3})$

88. $\sqrt{6}(-\sqrt{6})$

89. $-\sqrt{15}(\sqrt{15})$

90. $-\sqrt{21}(-\sqrt{21})$

Finding a Greatest Common Factor **In Exercises 91–94, find the greatest common factor of the expressions.**

91. $2x^2, x^3, 4x$

92. $3x^3, 12x^2, 42x^3$

93. x^{10}, x^{20}, x^{30}

94. $45x^5, 9x^3, 15x^2$

Using Properties of Exponents **In Exercises 95 and 96, simplify the expression.**

95. $(3x^2)^3$

96. $(4x^4)^2$

Identifying Rules of Algebra **In Exercises 97–102, identify the rule(s) of algebra illustrated by the statement.**

97. $\dfrac{1}{h + 6}(h + 6) = 1, \; h \neq -6$

98. $(x + 3) - (x + 3) = 0$

99. $x(3y) = (x \cdot 3)y = (3x)y$

100. $\frac{1}{7}(7 \cdot 12) = \left(\frac{1}{7} \cdot 7\right)12 = 1 \cdot 12 = 12$

101. $x(2x^2 + 3x - 1) = 2x^3 + 3x^2 - x$

102. $(x - 2)(2x + 3) = (x - 2)(2x) + (x - 2)(3)$

P.4 Factoring Polynomials

Polynomial factoring has many real-life applications. For example, in Exercise 80 on page 40, you will use polynomial factoring to write an alternative form of an expression that models the rate of change of an autocatalytic chemical reaction.

- **Factor out common factors from polynomials.**
- **Factor special polynomial forms.**
- **Factor trinomials as the product of two binomials.**
- **Factor polynomials by grouping.**

Polynomials with Common Factors

The process of writing a polynomial as a product is called **factoring.** It is an important tool for solving equations and for simplifying rational expressions.

Unless noted otherwise, when you are asked to factor a polynomial, assume that you are looking for factors that have integer coefficients. If a polynomial does not factor using integer coefficients, then it is **prime** or **irreducible over the integers.** For example, the polynomial $x^2 - 3$ is irreducible over the integers. Over the *real numbers,* this polynomial factors as

$$x^2 - 3 = \left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right).$$

A polynomial is **completely factored** when each of its factors is prime. For example,

$$x^3 - x^2 + 4x - 4 = (x - 1)(x^2 + 4) \qquad \text{Completely factored}$$

is completely factored, but

$$x^3 - x^2 - 4x + 4 = (x - 1)(x^2 - 4) \qquad \text{Not completely factored}$$

is not completely factored. Its complete factorization is

$$x^3 - x^2 - 4x + 4 = (x - 1)(x + 2)(x - 2).$$

The simplest type of factoring involves a polynomial that can be written as the product of a monomial and another polynomial. The technique used here is the Distributive Property, $a(b + c) = ab + ac$, in the *reverse* direction.

$$ab + ac = a(b + c) \qquad a \text{ is a common factor.}$$

Factoring out any common factors is the first step in completely factoring a polynomial.

EXAMPLE 1 Factoring Out Common Factors

Factor each expression.

a. $6x^3 - 4x$ **b.** $-4x^2 + 12x - 16$ **c.** $(x - 2)(2x) + (x - 2)(3)$

Solution

a. $6x^3 - 4x = 2x(3x^2) - 2x(2)$ $\qquad$ $2x$ is a common factor.

$\qquad = 2x(3x^2 - 2)$

b. $-4x^2 + 12x - 16 = -4(x^2) + (-4)(-3x) + (-4)4$ $\qquad$ -4 is a common factor.

$\qquad = -4(x^2 - 3x + 4)$

c. $(x - 2)(2x) + (x - 2)(3) = (x - 2)(2x + 3)$ $\qquad$ $(x - 2)$ is a common factor.

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Factor each expression.

a. $5x^3 - 15x^2$ **b.** $-3 + 6x - 12x^3$ **c.** $(x + 1)(x^2) - (x + 1)(2)$ ■

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Factoring Special Polynomial Forms

Some polynomials have special forms that arise from the special product forms on page 28. You should learn to recognize these forms so that you can factor such polynomials efficiently.

Factoring Special Polynomial Forms

Factored Form	**Example**
Difference of Two Squares	
$u^2 - v^2 = (u + v)(u - v)$	$9x^2 - 4 = (3x)^2 - 2^2 = (3x + 2)(3x - 2)$
Perfect Square Trinomial	
$u^2 + 2uv + v^2 = (u + v)^2$	$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2 = (x + 3)^2$
$u^2 - 2uv + v^2 = (u - v)^2$	$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2 = (x - 3)^2$
Sum or Difference of Two Cubes	
$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$	$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$
$u^3 - v^3 = (u - v)(u^2 + uv + v^2)$	$27x^3 - 1 = (3x)^3 - 1^3 = (3x - 1)(9x^2 + 3x + 1)$

For the difference of two squares, you can think of this form as

$$u^2 \underset{\substack{\uparrow \\ \text{Difference}}}{-} v^2 = (u \underbrace{+ v)(u -}_{\text{Opposite signs}} v). \qquad \text{Factors are conjugates.}$$

To recognize perfect square terms, look for coefficients that are squares of integers and variables raised to *even powers.*

EXAMPLE 2 Factoring Out a Common Factor First

$$\begin{aligned} 3 - 12x^2 &= 3(1 - 4x^2) && \text{3 is a common factor.} \\ &= 3[1^2 - (2x)^2] && \text{Rewrite } 1 - 4x^2 \text{ as the difference of two squares.} \\ &= 3(1 + 2x)(1 - 2x) && \text{Factor.} \end{aligned}$$

✓ ***Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Factor $100 - 4y^2$.

ALGEBRA HELP

In Example 2, note that the first step in factoring a polynomial is to check for any common factors. Once you have removed any common factors, it is often possible to recognize patterns that were not immediately obvious.

EXAMPLE 3 Factoring the Difference of Two Squares

a. $$\begin{aligned} (x + 2)^2 - y^2 &= [(x + 2) + y][(x + 2) - y] \\ &= (x + 2 + y)(x + 2 - y) \end{aligned}$$

b. $$\begin{aligned} 16x^4 - 81 &= (4x^2)^2 - 9^2 && \text{Rewrite as the difference of two squares.} \\ &= (4x^2 + 9)(4x^2 - 9) && \text{Factor.} \\ &= (4x^2 + 9)[(2x)^2 - 3^2] && \text{Rewrite } 4x^2 - 9 \text{ as the difference of two squares.} \\ &= (4x^2 + 9)(2x + 3)(2x - 3) && \text{Factor.} \end{aligned}$$

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Factor $(x - 1)^2 - 9y^4$. ■

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A perfect square trinomial is the square of a binomial, and it has the form

$$u^2 + 2uv + v^2 = (u + v)^2 \quad \text{or} \quad u^2 - 2uv + v^2 = (u - v)^2.$$

Like signs Like signs

Note that the first and last terms are squares and the middle term is twice the product of u and v.

EXAMPLE 4 Factoring Perfect Square Trinomials

Factor each trinomial.

a. $x^2 - 10x + 25$ **b.** $16x^2 + 24x + 9$

Solution

a. $x^2 - 10x + 25 = x^2 - 2(x)(5) + 5^2 = (x - 5)^2$

b. $16x^2 + 24x + 9 = (4x)^2 + 2(4x)(3) + 3^2 = (4x + 3)^2$

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Factor $9x^2 - 30x + 25$.

The next two formulas show the sum and difference of two cubes. Pay special attention to the signs of the terms.

Like signs Like signs

$$u^3 + v^3 = (u + v)(u^2 - uv + v^2) \qquad u^3 - v^3 = (u - v)(u^2 + uv + v^2)$$

Unlike signs Unlike signs

EXAMPLE 5 Factoring the Difference of Two Cubes

$$x^3 - 27 = x^3 - 3^3 \qquad \text{Rewrite 27 as } 3^3.$$

$$= (x - 3)(x^2 + 3x + 9) \qquad \text{Factor.}$$

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Factor $64x^3 - 1$.

EXAMPLE 6 Factoring the Sum of Two Cubes

a. $y^3 + 8 = y^3 + 2^3$ Rewrite 8 as 2^3.

$= (y + 2)(y^2 - 2y + 4)$ Factor.

b. $3x^3 + 192 = 3(x^3 + 64)$ 3 is a common factor.

$= 3(x^3 + 4^3)$ Rewrite 64 as 4^3.

$= 3(x + 4)(x^2 - 4x + 16)$ Factor.

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Factor each expression.

a. $x^3 + 216$ **b.** $5y^3 + 135$

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Trinomials with Binomial Factors

To factor a trinomial of the form $ax^2 + bx + c$, use the pattern below.

Factors of *a*

$$ax^2 + bx + c = (\square x + \square)(\square x + \square)$$

Factors of *c*

The goal is to find a combination of factors of a and c such that the sum of the outer and inner products is the middle term bx. For example, for the trinomial $6x^2 + 17x + 5$, you can write all possible factorizations and determine which one has outer and inner products whose sum is $17x$.

$$(6x + 5)(x + 1),\ (6x + 1)(x + 5),\ (2x + 1)(3x + 5),\ (2x + 5)(3x + 1)$$

The correct factorization is $(2x + 5)(3x + 1)$ because the sum of the outer (O) and inner (I) products is $17x$.

$$(2x + 5)(3x + 1) = \overset{\text{F}}{6x^2} + \overset{\text{O}}{2x} + \overset{\text{I}}{15x} + \overset{\text{L}}{5} = 6x^2 + \overset{\text{O + I}}{17x} + 5$$

ALGEBRA HELP

Factoring a trinomial can involve trial and error. However, you can check your answer by multiplying the factors. The product should be the original trinomial. For instance, in Example 7, verify that $(x - 3)(x - 4) = x^2 - 7x + 12$.

EXAMPLE 7 Factoring a Trinomial: Leading Coefficient Is 1

Factor $x^2 - 7x + 12$.

Solution For this trinomial, $a = 1$, $b = -7$, and $c = 12$. Because b is negative and c is positive, both factors of 12 must be negative. So, the possible factorizations of $x^2 - 7x + 12$ are

$$(x - 1)(x - 12), \quad (x - 2)(x - 6), \quad \text{and} \quad (x - 3)(x - 4).$$

Testing the middle term, you will find the correct factorization to be

$$x^2 - 7x + 12 = (x - 3)(x - 4). \qquad \text{O + I} = -4x + (-3x) = -7x$$

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Factor $x^2 + x - 6$.

EXAMPLE 8 Factoring a Trinomial: Leading Coefficient Is Not 1

See LarsonPrecalculus.com for an interactive version of this type of example.

Factor $2x^2 + x - 15$.

Solution For this trinomial, $a = 2$, $b = 1$, and $c = -15$. Because c is negative, its factors must have unlike signs. The eight possible factorizations are below.

$$(2x - 1)(x + 15) \quad (2x + 1)(x - 15) \quad (2x - 3)(x + 5) \quad (2x + 3)(x - 5)$$

$$(2x - 5)(x + 3) \quad (2x + 5)(x - 3) \quad (2x - 15)(x + 1) \quad (2x + 15)(x - 1)$$

Testing the middle term, you will find the correct factorization to be

$$2x^2 + x - 15 = (2x - 5)(x + 3). \qquad \text{O + I} = 6x + (-5x) = x$$

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Factor $2x^2 - 5x + 3$.

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Factoring by Grouping

Sometimes, polynomials with more than three terms can be **factored by grouping.**

EXAMPLE 9 Factoring by Grouping

$$
\begin{aligned}
x^3 - 2x^2 - 3x + 6 &= (x^3 - 2x^2) - (3x - 6) && \text{Group terms.}\\
&= x^2(x - 2) - 3(x - 2) && \text{Factor each group.}\\
&= (x - 2)(x^2 - 3) && (x - 2) \text{ is a common factor.}
\end{aligned}
$$

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Factor $x^3 + x^2 - 5x - 5$. ■

ALGEBRA HELP

Sometimes, more than one grouping will work. For instance, another way to factor the polynomial in Example 9 is

$$
\begin{aligned}
&x^3 - 2x^2 - 3x + 6\\
&= (x^3 - 3x) - (2x^2 - 6)\\
&= x(x^2 - 3) - 2(x^2 - 3)\\
&= (x^2 - 3)(x - 2).
\end{aligned}
$$

Notice that this is the same result as in Example 9.

Factoring by grouping can eliminate some of the trial and error involved in factoring a trinomial. To factor a trinomial of the form $ax^2 + bx + c$ by grouping, choose factors of the product ac that sum to b and use these factors to rewrite the middle term. Example 10 illustrates this technique.

EXAMPLE 10 Factoring a Trinomial by Grouping

In the trinomial $2x^2 + 5x - 3$, $a = 2$ and $c = -3$, so the product ac is -6. Now, -6 factors as $(6)(-1)$ and $6 + (-1) = 5 = b$. So, rewrite the middle term as $5x = 6x - x$ and factor by grouping.

$$
\begin{aligned}
2x^2 + 5x - 3 &= 2x^2 + 6x - x - 3 && \text{Rewrite middle term.}\\
&= (2x^2 + 6x) - (x + 3) && \text{Group terms.}\\
&= 2x(x + 3) - (x + 3) && \text{Factor } 2x^2 + 6x.\\
&= (x + 3)(2x - 1) && (x + 3) \text{ is a common factor.}
\end{aligned}
$$

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Use factoring by grouping to factor $2x^2 + 5x - 12$. ■

Guidelines for Factoring Polynomials

1. Factor out any common factors using the Distributive Property.
2. Factor according to one of the special polynomial forms.
3. Factor as $ax^2 + bx + c = (mx + r)(nx + s)$.
4. Factor by grouping.

Summarize (Section P.4)

1. Explain what it means to completely factor a polynomial *(page 34)*. For an example of factoring out common factors, see Example 1.
2. Make a list of the special polynomial forms of factoring *(page 35)*. For examples of factoring these special forms, see Examples 2–6.
3. Explain how to factor a trinomial of the form $ax^2 + bx + c$ *(page 37)*. For examples of factoring trinomials of this form, see Examples 7 and 8.
4. Explain how to factor a polynomial by grouping *(page 38)*. For examples of factoring by grouping, see Examples 9 and 10.

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P.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

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Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blanks.

1. The process of writing a polynomial as a product is called ________.

2. A ________ ________ ________ is the square of a binomial, and it has the form $u^2 + 2uv + v^2$ or $u^2 - 2uv + v^2$.

3. When is a polynomial completely factored?

4. List four guidelines for factoring polynomials.

Skills and Applications

Factoring Out a Common Factor

In Exercises 5–8, factor out the common factor.

5. $2x^3 - 6x$
6. $3z^3 - 6z^2 + 9z$
7. $3x(x - 5) + 8(x - 5)$
8. $(x + 3)^2 - 4(x + 3)$

Factoring the Difference of Two Squares

In Exercises 9–18, completely factor the difference of two squares.

9. $x^2 - 81$
10. $x^2 - 64$
11. $25y^2 - 4$
12. $4y^2 - 49$
13. $64 - 9z^2$
14. $81 - 36z^2$
15. $(x - 1)^2 - 4$
16. $25 - (z + 5)^2$
17. $81u^4 - 1$
18. $x^4 - 16y^4$

Factoring a Perfect Square Trinomial

In Exercises 19–24, factor the perfect square trinomial.

19. $x^2 - 4x + 4$
20. $4t^2 + 4t + 1$
21. $25z^2 - 30z + 9$
22. $36y^2 + 84y + 49$
23. $4y^2 - 12y + 9$
24. $9u^2 + 24uv + 16v^2$

Factoring the Sum or Difference of Two Cubes

In Exercises 25–30, factor the sum or difference of two cubes.

25. $x^3 - 8$
26. $x^3 + 125$
27. $8t^3 - 1$
28. $27z^3 + 1$
29. $27x^3 + 8$
30. $64y^3 - 125$

Factoring a Trinomial

In Exercises 31–40, factor the trinomial.

31. $x^2 + x - 2$
32. $x^2 + 5x + 6$
33. $s^2 - 5s + 6$
34. $t^2 - t - 6$
35. $3x^2 + 10x - 8$
36. $2x^2 - 3x - 27$
37. $5x^2 + 31x + 6$
38. $8x^2 + 51x + 18$
39. $-5y^2 - 8y + 4$
40. $-6z^2 + 17z + 3$

Factoring by Grouping

In Exercises 41–48, factor by grouping.

41. $x^3 - x^2 + 2x - 2$
42. $x^3 + 5x^2 - 5x - 25$
43. $2x^3 - x^2 - 6x + 3$
44. $3x^3 + x^2 - 15x - 5$
45. $6 + 2x - 3x^3 - x^4$
46. $x^5 + 2x^3 + x^2 + 2$
47. $3x^5 + 6x^3 - 2x^2 - 4$
48. $8x^5 - 6x^2 + 12x^3 - 9$

Factoring a Trinomial by Grouping

In Exercises 49–52, factor the trinomial by grouping.

49. $2x^2 + 9x + 9$
50. $6x^2 + x - 2$
51. $6x^2 - x - 15$
52. $12x^2 - 13x + 1$

Factoring Completely

In Exercises 53–70, completely factor the expression.

53. $6x^2 - 54$
54. $12x^2 - 48$
55. $x^3 - x^2$
56. $x^3 - 16x$
57. $1 - 4x + 4x^2$
58. $-9x^2 + 6x - 1$
59. $2x^2 + 4x - 2x^3$
60. $9x^2 + 12x - 3x^3$
61. $(x^2 + 3)^2 - 16x^2$
62. $(x^2 + 8)^2 - 36x^2$
63. $2x^3 + x^2 - 8x - 4$
64. $3x^3 + x^2 - 27x - 9$
65. $2x(3x + 1) + (3x + 1)^2$
66. $4x(2x - 1) + (2x - 1)^2$
67. $2(x - 2)(x + 1)^2 - 3(x - 2)^2(x + 1)$
68. $2(x + 1)(x - 3)^2 - 3(x + 1)^2(x - 3)$
69. $5(2x + 1)^2(x + 1)^2 + (2x + 1)(x + 1)^3$
70. $7(3x + 2)^2(1 - x)^2 + (3x + 2)(1 - x)^3$

Fractional Coefficients

In Exercises 71–76, completely factor the expression. (*Hint:* The factors will contain fractional coefficients.)

71. $16x^2 - \frac{1}{9}$
72. $\frac{4}{25}y^2 - 64$
73. $z^2 + z + \frac{1}{4}$
74. $9y^2 - \frac{3}{2}y + \frac{1}{16}$
75. $y^3 + \frac{8}{27}$
76. $x^3 - \frac{27}{64}$