# TRIGONOMETRY

CalcChat® and CalcView®

11e

**Ron Larson**

# TRIGONOMETRY

CalcChat® and CalcView® 11e

**Ron Larson**
The Pennsylvania State University
The Behrend College

CENGAGE

Australia • Brazil • Canada • Mexico • Singapore • United Kingdom • United States

CENGAGE

***Trigonometry***
**with CalcChat® and CalcView®**
**Eleventh Edition**
**Ron Larson**

Product Director: Mark Santee
Senior Product Manager: Gary Whalen
Product Assistant: Tim Rogers
Senior Learning Designer: Laura Gallus
Executive Marketing Manager: Tom Ziolkowski
Content Manager: Rachel Pancare
Digital Delivery Lead: Nikkita Kendrick
IP Analyst: Ashley Maynard
IP Project Manager: Nick Barrows
Manufacturing Planner: Ron Montgomery
Production Service: Larson Texts, Inc.
Compositor: Larson Texts, Inc.
Illustrator: Larson Texts, Inc.
Text and Cover Designer: Larson Texts, Inc.
Cover Image: Romolo Tavani/Shutterstock.com



For product information and technology assistance, contact us at **Cengage Customer & Sales Support, 1-800-354-9706 or support.cengage.com.**

For permission to use material from this text or product, submit all requests online at **www.cengage.com/permissions.**

Student Edition
ISBN: 978-0-357-45521-0

Loose-leaf Edition
ISBN: 978-0-357-45532-6

**Cengage**
200 Pier 4 Boulevard
Boston, MA 02210
USA

Cengage is a leading provider of customized learning solutions with employees residing in nearly 40 different countries and sales in more than 125 countries around the world. Find your local representative at **www.cengage.com.**

To learn more about Cengage platforms and services, register or access your online learning solution, or purchase materials for your course, visit **www.cengage.com.**

QR Code is a registered trademark of Denso Wave Incorporated.

Printed in the United States of America
Print Number: 01 Print Year: 2020

# Contents


## P ▶ Prerequisites 1

P.1 Review of Real Numbers and Their Properties 2
P.2 Solving Equations 14
P.3 The Cartesian Plane and Graphs of Equations 26
P.4 Linear Equations in Two Variables 40
P.5 Functions 53
P.6 Analyzing Graphs of Functions 67
P.7 A Library of Parent Functions 78
P.8 Transformations of Functions 85
P.9 Combinations of Functions: Composite Functions 94
P.10 Inverse Functions 102
**Summary and Study Strategies** 111
**Review Exercises** 114
**Chapter Test** 117
**Proofs in Mathematics** 118
**P.S. Problem Solving** 119

## 1 ▶ Trigonometry 121

1.1 Radian and Degree Measure 122
1.2 Trigonometric Functions: The Unit Circle 132
1.3 Right Triangle Trigonometry 139
1.4 Trigonometric Functions of Any Angle 150
1.5 Graphs of Sine and Cosine Functions 159
1.6 Graphs of Other Trigonometric Functions 170
1.7 Inverse Trigonometric Functions 180
1.8 Applications and Models 190
**Summary and Study Strategies** 200
**Review Exercises** 202
**Chapter Test** 205
**Proofs in Mathematics** 206
**P.S. Problem Solving** 207

## 2 ▶ Analytic Trigonometry 209

2.1 Using Fundamental Identities 210
2.2 Verifying Trigonometric Identities 217
2.3 Solving Trigonometric Equations 224
2.4 Sum and Difference Formulas 236
2.5 Multiple-Angle and Product-to-Sum Formulas 243
**Summary and Study Strategies** 252
**Review Exercises** 254
**Chapter Test** 256
**Proofs in Mathematics** 257
**P.S. Problem Solving** 259

**3 Additional Topics in Trigonometry** **261**

3.1 Law of Sines 262
3.2 Law of Cosines 271
3.3 Vectors in the Plane 278
3.4 Vectors and Dot Products 291
**Summary and Study Strategies** 300
**Review Exercises** 302
**Chapter Test** 305
**Cumulative Test for Chapters 1–3** 306
**Proofs in Mathematics** 308
**P.S. Problem Solving** 311

**4 Complex Numbers** **313**

4.1 Complex Numbers 314
4.2 Complex Solutions of Equations 321
4.3 The Complex Plane 329
4.4 Trigonometric Form of a Complex Number 336
4.5 DeMoivre's Theorem 342
**Summary and Study Strategies** 348
**Review Exercises** 350
**Chapter Test** 353
**Proofs in Mathematics** 354
**P.S. Problem Solving** 355

**5 Exponential and Logarithmic Functions** **357**

5.1 Exponential Functions and Their Graphs 358
5.2 Logarithmic Functions and Their Graphs 369
5.3 Properties of Logarithms 379
5.4 Exponential and Logarithmic Equations 386
5.5 Exponential and Logarithmic Models 396
**Summary and Study Strategies** 408
**Review Exercises** 410
**Chapter Test** 413
**Proofs in Mathematics** 414
**P.S. Problem Solving** 415

## 6 Topics in Analytic Geometry 417


6.1 Lines 418
6.2 Introduction to Conics: Parabolas 425
6.3 Ellipses 434
6.4 Hyperbolas 443
6.5 Rotation of Conics 453
6.6 Parametric Equations 461
6.7 Polar Coordinates 471
6.8 Graphs of Polar Equations 477
6.9 Polar Equations of Conics 485
**Summary and Study Strategies** 492
**Review Exercises** 494
**Chapter Test** 497
**Cumulative Test for Chapters 4–6** 498
**Proofs in Mathematics** 500
**P.S. Problem Solving** 503


## Appendix A


**Concepts in Statistics (online)***
A.1 Representing Data
A.2 Analyzing Data
A.3 Modeling Data

**Answers to Odd-Numbered Exercises and Tests** A1

**Index** A79

**Index of Applications (online)***


*Available at the text companion website *LarsonPrecalculus.com*

# Preface

Welcome to *Trigonometry* with CalcChat® & CalcView®, Eleventh Edition. I am excited to offer you a new edition with more resources than ever that will help you understand and master trigonometry. This text includes features and resources that continue to make *Trigonometry* a valuable learning tool for students and a trustworthy teaching tool for instructors.

*Trigonometry* provides the clear instruction, precise mathematics, and thorough coverage that you expect for your course. Additionally, this new edition provides you with **free** access to a variety of digital resources:

- **GO DIGITAL**—direct access to digital content on your mobile device or computer
- **CalcView.com**—video solutions to selected exercises
- **CalcChat.com**—worked-out solutions to odd-numbered exercises and access to online tutors
- **LarsonPrecalculus.com**—companion website with resources to supplement your learning

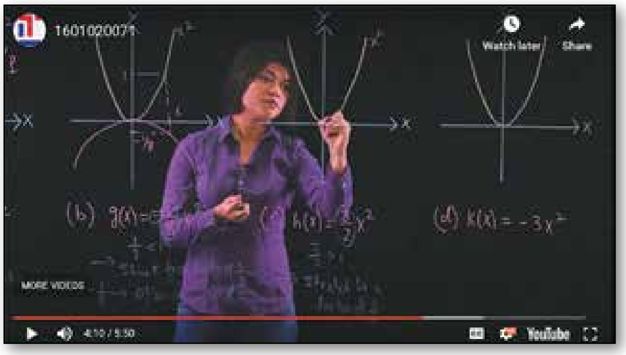
These digital resources will help enhance and reinforce your understanding of the material presented in this text and prepare you for future mathematics courses. CalcView® and CalcChat® are also available as free mobile apps.

## Features

### NEW GO DIGITAL

Scan the QR codes on the pages of this text to *GO DIGITAL* on your mobile device. This will give you easy access from anywhere to instructional videos, solutions to exercises and Checkpoint problems, Skills Refresher videos, Interactive Activities, and many other resources.

GO DIGITAL




### UPDATED CalcView®

The website *CalcView.com* provides video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google Play™ store. You can access the video solutions by scanning the QR Code® at the beginning of the Section exercises, or visiting the *CalcView.com* website.

### UPDATED CalcChat®

Solutions to all odd-numbered exercises and tests are provided for free at *CalcChat.com.* Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. For many years, millions of students have visited my site for help. The CalcChat mobile app is also available as a free download at the Apple® App Store® or Google Play™ store.

---

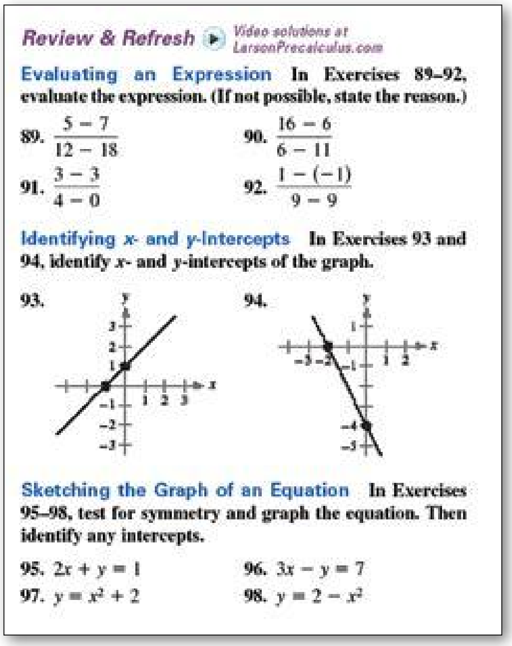
App Store is a service mark of Apple Inc. Google Play is a trademark of Google Inc.
QR Code is a registered trademark of Denso Wave Incorporated.


**REVISED LarsonPrecalculus.com**
All companion website features have been updated based on this revision, including two new features: Skills Refresher and Review & Refresh. Access to these features is free. You can view and listen to worked-out solutions of Checkpoint problems in English or Spanish, explore examples, download data sets, watch lesson videos, and much more.

**NEW Skills Refresher**
This feature directs you to an instructional video where you can review algebra skills needed to master the current topic. Scan the on-page code or go to *LarsonPrecalculus.com* to access the video.

**SKILLS REFRESHER**

For a refresher on finding the sum, difference, product, or quotient of two polynomials, watch the video at *LarsonPrecalculus.com*.


*Review & Refresh* Video solutions at LarsonPrecalculus.com

**Evaluating an Expression** In Exercises 89–92, evaluate the expression. (If not possible, state the reason.)

89. $\frac{5-7}{12-18}$
90. $\frac{16-6}{6-11}$
91. $\frac{3-3}{4-0}$
92. $\frac{1-(-1)}{9-9}$

**Identifying $x$- and $y$-Intercepts** In Exercises 93 and 94, identify $x$- and $y$-intercepts of the graph.

93.
94.

**Sketching the Graph of an Equation** In Exercises 95–98, test for symmetry and graph the equation. Then identify any intercepts.

95. $2x + y = 1$
96. $3x - y = 7$
97. $y = x^2 + 2$
98. $y = 2 - x^2$

**NEW Review and Refresh**
These exercises will help you to reinforce previously learned skills and concepts and to prepare for the next section. View and listen to worked-out solutions of the Review & Refresh exercises in English or Spanish by scanning the code on the first page of the section exercises or go to *LarsonPrecalculus.com*.

**NEW Vocabulary and Concept Check**
The Vocabulary and Concept Check appears at the beginning of the exercise set for each section. It includes fill-in-the-blank, matching, or non-computational questions designed to help you learn mathematical terminology and to test basic understanding of the concepts of the section.

**NEW Summary and Study Strategies**
The "What Did You Learn?" feature is a section-by-section overview that ties the learning objectives from the chapter to the Review Exercises for extra practice. The Study Strategies give concrete ways that you can use to help yourself with your study of mathematics.

**REVISED Algebra Help**
These notes reinforce or expand upon concepts, help you learn how to study mathematics, address special cases, or show alternative or additional steps to a solution of an example.

## REVISED Exercise Sets

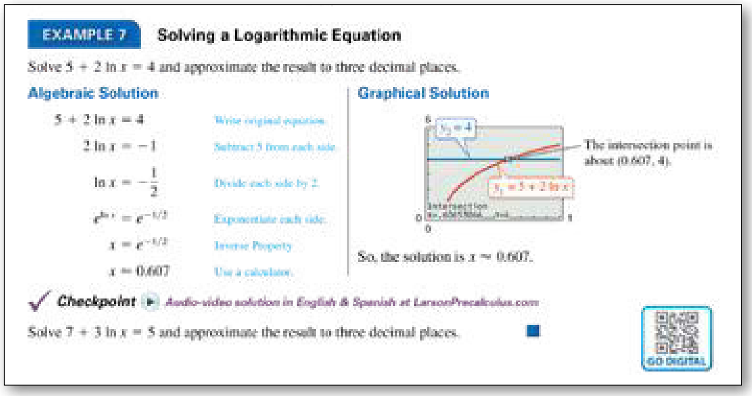
The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant, and include topics our users have suggested. The exercises have been reorganized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations. Two new sets of exercises, Vocabulary and Concept Check and Review & Refresh, have been added to help you develop and maintain your skills.

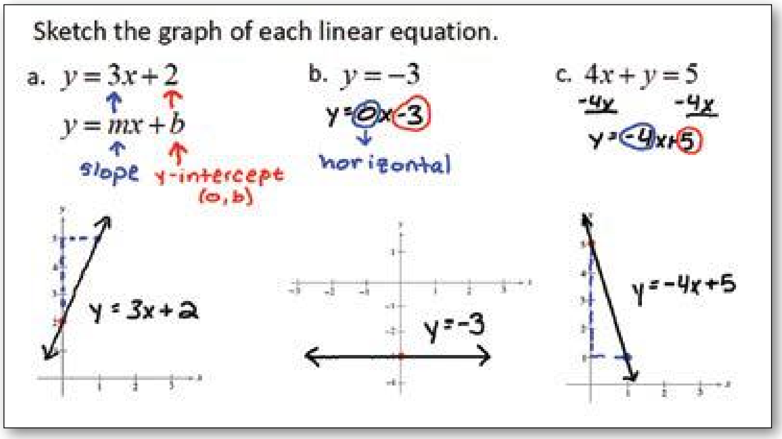
## Section Objectives

A bulleted list of learning objectives provides you the opportunity to preview what will be presented in the upcoming section.

## Side-By-Side Examples

Throughout the text, we present solutions to many examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.




## Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at *LarsonPrecalculus.com*. Scan the on-page code to access the solutions.

## Technology

The technology feature gives suggestions for effectively using tools such as calculators, graphing utilities, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

## Historical Notes

These notes provide helpful information regarding famous mathematicians and their work.

**Summarize** (Section 3.2)

1. Explain how to use transformations to sketch graphs of polynomial functions *(page 252)*. For an example of sketching transformations of monomial functions, see Example 1.
2. Explain how to apply the Leading Coefficient Test *(page 253)*. For an example of applying the Leading Coefficient Test, see Example 2.
3. Explain how to find real zeros of polynomial functions and use them as sketching aids *(page 255)*. For examples involving finding real zeros of polynomial functions, see Examples 3–5.
4. Explain how to use the Intermediate Value Theorem to help locate real zeros of polynomial functions *(page 258)*. For an example of using the Intermediate Value Theorem, see Example 6.

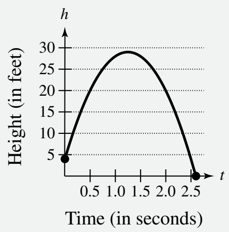
### Summarize
The Summarize feature at the end of each section helps you organize the lesson's key concepts into a concise summary, providing you with a valuable study tool. Use this feature to prepare for a homework assignment, to help you study for an exam, or as a review for previously covered sections.

### Algebra of Calculus
Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol $\int$.

### Error Analysis
This exercise presents a sample solution that contains a common error which you are asked to identify.

### How Do You See It?
The How Do You See It? feature in each section presents a real-life exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

**86. HOW DO YOU SEE IT?** The graph represents the height $h$ of a projectile after $t$ seconds.




(a) Explain why $h$ is a function of $t$.
(b) Approximate the height of the projectile after 0.5 second and after 1.25 seconds.
(c) Approximate the domain of $h$.
(d) Is $t$ a function of $h$? Explain.

### Project
The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at *LarsonPrecalculus.com*.

### Collaborative Project
You can find these extended group projects at *LarsonPrecalculus.com.* Check your understanding of the chapter concepts by solving in-depth, real-life problems. These collaborative projects provide an interesting and engaging way for you and other students to work together and investigate ideas.

## Instructor Resources

CENGAGE | WEBASSIGN

Built by educators, WebAssign from Cengage is a fully customizable online solution for STEM disciplines. WebAssign includes the flexibility, tools, and content you need to create engaging learning experiences for your students. The patented grading engine provides unparalleled answer evaluation, giving students instant feedback, and insightful analytics highlight exactly where students are struggling. For more information, visit *cengage.com/webassign*.

**Complete Solutions Manual**
This manual contains solutions to all exercises from the text, including Chapter Review Exercises and Chapter Tests, and Practice Tests with solutions. The Complete Solutions Manual is available on the Instructor Companion Site.

**Cengage Testing Powered by Cognero®**
Cengage Testing, Powered by Cognero®, is a flexible online system that allows you to author, edit, and manage test bank content online. You can create multiple versions of your test in an instant and deliver tests from your LMS or exportable PDF or Word docs you print for in-class assessment. Cengage Testing is available online via *cengage.com*.

**Instructor Companion Site**
Everything you need for your course in one place! Access and download PowerPoint® presentations, test banks, the solutions manual, and more. This collection of book-specific lecture and class tools is available online via *cengage.com*.

**Test Bank**
The test bank contains text-specific multiple-choice and free response test forms and is available online at the Instructor Companion Site.

**LarsonPrecalculus.com**
In addition to its student resources, *LarsonPrecalculus.com* also has resources to help instructors. If you wish to challenge your students with multi-step and group projects, you can assign the Section Projects and Collaborative Projects. You can assess the knowledge of your students before and after each chapter using the pre- and post-tests. You can also give your students experience using an online graphing calculator with the Interactive Activities. You can access these features by going to *LarsonPrecalculus.com* or by scanning the on-page code.

**MathGraphs.com**
For exercises that ask students to draw on the graph, I have provided **free,** printable graphs at *MathGraphs.com*. You can access these features by going to *MathGraphs.com* or by scanning the on-page code at the beginning of the section exercises, review exercises, or tests.

# Student Resources

Prepare for class with confidence using WebAssign from Cengage. This online learning platform, which includes an interactive eBook, fuels practice, so that you truly absorb what you learn and prepare better for tests. Videos and tutorials walk you through concepts and deliver instant feedback and grading, so you always know where you stand in class. Focus your study time and get extra practice where you need it most. Study smarter with WebAssign! Ask your instructor today how you can get access to WebAssign, or learn about self-study options at *cengage.com/webassign*.

**Student Study Guide and Solutions Manual**
This guide offers step-by-step solutions for all odd-numbered text exercises, Chapter Tests, and Cumulative Tests. It also contains Practice Tests. For more information on how to access this digital resource, go to cengage.com

**Note-Taking Guide**
This is an innovative study aid, in the form of a notebook organizer, that helps students develop a section-by-section summary of key concepts. For more information on how to access this digital resource, go to cengage.com

**LarsonPrecalculus.com**
Of the many features at this website, students have told me that the videos are the most helpful. You can watch lesson videos by Dana Mosely as he explains various mathematical concepts. Other helpful features are the data downloads (editable spreadsheets so you do not have to enter the data), video solutions of the Checkpoint problems in English or Spanish, and the Student Success Organizer. The Student Success Organizer will help you organize the important concepts of each section using chapter outlines. You can access these features by going to *LarsonPrecalculus.com* or by scanning the on-page code.

**CalcChat.com**
This website provides free step-by-step solutions to all odd-numbered exercises and tests. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. You can access the solutions by going to *CalcChat.com* or by scanning the on-page code on the first page of the section exercises, review exercises, or tests.

**CalcView.com**
This website has video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. You can access the videos by going to *CalcView.com* or by scanning the on-page code on the first page of the section exercises, review exercises, or tests.

**MathGraphs.com**
For exercises that ask you to draw on the graph, I have provided **free,** printable graphs at *MathGraphs.com*. You can access the printable graphs by going to *MathGraphs.com* or by scanning the on-page code on the first page of the section exercises, review exercises, or tests.

# Acknowledgments

I would like to thank the many people who have helped me prepare the text and the supplements package. Their encouragement, criticisms, and suggestions have been invaluable.

Thank you to all of the instructors who took the time to review the changes in this edition and to provide suggestions for improving it. Without your help, this book would not be possible.

**Reviewers of the Eleventh Edition**
Ivette Chuca, *El Paso Community College*
Russell Murray, *St. Louis Community College-Meramec*
My Linh Nguyen, *The University of Texas at Dallas*
Michael Wallace, *Northern Virginia Community College*

**Reviewers of the Previous Editions**
Gurdial Arora, *Xavier University of Louisiana;* Darin Bauguess, *Surry Community College;* Timothy Andrew Brown, *South Georgia College;* Blair E. Caboot, *Keystone College;* Russell C. Chappell, *Twinsburg High School, Ohio;* Shannon Cornell, *Amarillo College;* Gayla Dance, *Millsaps College;* John Elias, *Glenda Dawson High School;* John Fellers, *North Allegheny School District;* Paul Finster, *El Paso Community College;* Paul A. Flasch, *Pima Community College West Campus;* Vadas Gintautas, *Chatham University;* Lorraine A. Hughes, *Mississippi State University;* Shu-Jen Huang, *University of Florida;* Renyetta Johnson, *East Mississippi Community College;* George Keihany, *Fort Valley State University;* Brianna Kurtz, *Daytona State College;* Mulatu Lemma, *Savannah State University;* Darlene Martin, *Lawson State Community College;* William Mays Jr., *Salem Community College;* Marcella Melby, *University of Minnesota;* Jonathan Prewett, *University of Wyoming;* Denise Reid, *Valdosta State University;* Professor Steven Sikes, *Collin College;* Ann Slate, *Surry Community College;* David L. Sonnier, *Lyon College;* David H. Tseng, *Miami Dade College—Kendall Campus;* Kimberly Walters, *Mississippi State University;* Richard Weil, *Brown College;* Solomon Willis, *Cleveland Community College;* Kathy Wood, *Lansing Catholic High School;* Bradley R. Young, *Darton College*

My thanks to Robert Hostetler, The Behrend College, The Pennsylvania State University, David Heyd, The Behrend College, The Pennsylvania State University, and David C. Falvo, The Behrend College, The Pennsylvania State University, for their significant contributions to previous editions of this text.

I would also like to thank the staff at Larson Texts, Inc. who assisted with proofreading the manuscript, preparing and proofreading the art package, checking and typesetting the supplements, and developing the websites *LarsonPrecalculus.com*, *CalcView.com*, *CalcChat.com*, and *MathGraphs.com*.

On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil. If you have suggestions for improving this text, please feel free to write to me. Over the past two decades, I have received many useful comments from both instructors and students, and I value these comments very highly.

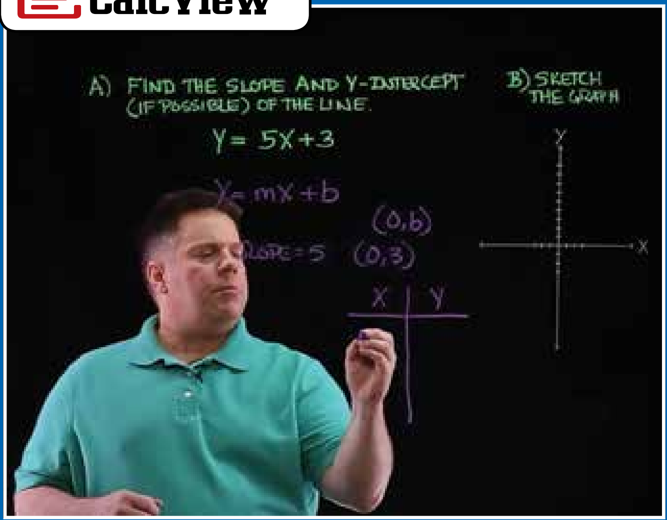
Ron Larson, Ph.D.
Professor of Mathematics
Penn State University
*www.RonLarson.com*

# P Prerequisites

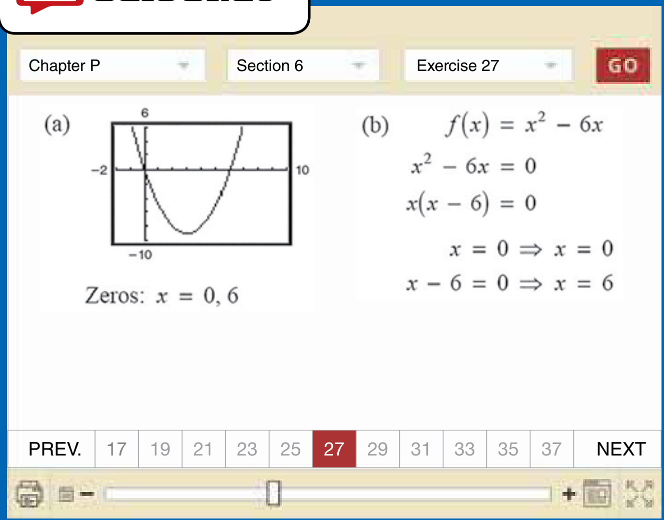
GO DIGITAL


P.1 Review of Real Numbers and Their Properties
P.2 Solving Equations
P.3 The Cartesian Plane and Graphs of Equations
P.4 Linear Equations in Two Variables
P.5 Functions
P.6 Analyzing Graphs of Functions
P.7 A Library of Parent Functions
P.8 Transformations of Functions
P.9 Combinations of Functions: Composite Functions
P.10 Inverse Functions


CalcView®




CalcChat®




**P.6** Temperature *(Exercise 87, p. 76)*

**P.10** Diesel Mechanics *(Exercise 70, p. 109)*



left, © iStockPhoto.com/Dsafanda; right, © Shaun/Getty Images

# P.1 Review of Real Numbers and Their Properties

Real numbers can represent many real-life quantities. For example, in Exercises 47–50 on page 13, you will use real numbers to represent the federal surplus or deficit.

- **Represent and classify real numbers.**
- **Order real numbers and use inequalities.**
- **Find the absolute values of real numbers and find the distance between two real numbers.**
- **Evaluate algebraic expressions.**
- **Use the basic rules and properties of algebra.**

## Real Numbers

**Real numbers** can describe quantities in everyday life such as age, miles per gallon, and population. Real numbers are represented by symbols such as

$$-5, 9, 0, \frac{4}{3}, 0.666\ldots, 28.21, \quad \sqrt{2}, \quad \pi, \quad \text{and} \quad \sqrt[3]{-32}.$$

Three commonly used **subsets** of real numbers are listed below. Each member in these subsets is also a member of the set of real numbers. (The three dots, called an *ellipsis*, indicate that the pattern continues indefinitely.)

$\{1, 2, 3, 4, \ldots\}$ Set of natural numbers

$\{0, 1, 2, 3, 4, \ldots\}$ Set of whole numbers

$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ Set of integers

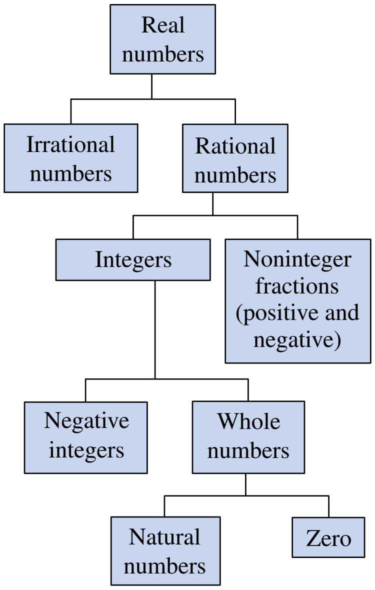
A real number is **rational** when it can be written as the ratio $p/q$ of two integers, where $q \neq 0$. For example, the numbers

$$\frac{1}{3} = 0.3333\ldots = 0.\overline{3}, \quad \frac{1}{8} = 0.125, \quad \text{and} \quad \frac{125}{111} = 1.126126\ldots = 1.\overline{126}$$

are rational. The decimal form of a rational number either repeats $\left(\text{as in } \frac{173}{55} = 3.1\overline{45}\right)$ or terminates $\left(\text{as in } \frac{1}{2} = 0.5\right)$. A real number that cannot be written as the ratio of two integers is **irrational.** The decimal form of an irrational number neither terminates nor repeats. For example, the numbers

$$\sqrt{2} = 1.4142135\ldots \approx 1.41 \quad \text{and} \quad \pi = 3.1415926\ldots \approx 3.14$$

are irrational. (The symbol $\approx$ means "is approximately equal to.") Figure P.1 shows several common subsets of the real numbers and their relationships to each other.




Common subsets of the real numbers
**Figure P.1**

### EXAMPLE 1 Classifying Real Numbers

Determine which numbers in the set $\left\{-13, -\sqrt{5}, -1, -\frac{1}{3}, 0, \frac{5}{8}, \sqrt{2}, \pi, 7\right\}$ are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.

### Solution

**a.** Natural numbers: $\{7\}$

**b.** Whole numbers: $\{0, 7\}$

**c.** Integers: $\{-13, -1, 0, 7\}$

**d.** Rational numbers: $\left\{-13, -1, -\frac{1}{3}, 0, \frac{5}{8}, 7\right\}$

**e.** Irrational numbers: $\left\{-\sqrt{5}, \sqrt{2}, \pi\right\}$

***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

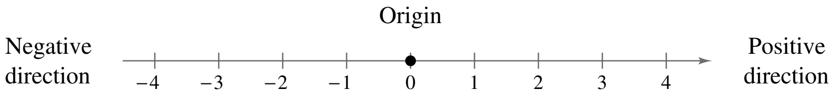
Repeat Example 1 for the set $\left\{-\pi, -\frac{1}{4}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -1, 8, -22\right\}$.

GO DIGITAL

Scan the QR code to access digital content available for this page.

© Gary718/Shutterstock.com

Real numbers are represented graphically on the **real number line.** When you draw a point on the real number line that corresponds to a real number, you are **plotting** the real number. The point representing 0 on the real number line is the **origin.** Numbers to the right of 0 are positive, and numbers to the left of 0 are negative, as shown in Figure P.2. The term **nonnegative** describes a number that is either positive or zero.




The real number line
**Figure P.2**

As illustrated in Figure P.3, there is a *one-to-one correspondence* between real numbers and points on the real number line.


Every real number corresponds to exactly one point on the real number line.
**Figure P.3**

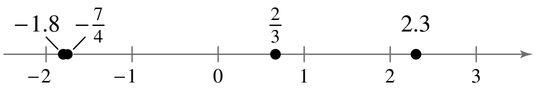
Every point on the real number line corresponds to exactly one real number.

## EXAMPLE 2 Plotting Points on the Real Number Line

Plot the real numbers on the real number line.

**a.** $-\frac{7}{4}$ **b.** 2.3 **c.** $\frac{2}{3}$ **d.** $-1.8$

**Solution** The figure below shows all four points.




**a.** The point representing the real number

$$-\frac{7}{4} = -1.75 \qquad \text{Write in decimal form.}$$

lies between $-2$ and $-1$, but closer to $-2$, on the real number line.

**b.** The point representing the real number 2.3 lies between 2 and 3, but closer to 2, on the real number line.

**c.** The point representing the real number

$$\frac{2}{3} = 0.666\ldots \qquad \text{Write in decimal form.}$$

lies between 0 and 1, but closer to 1, on the real number line.

**d.** The point representing the real number $-1.8$ lies between $-2$ and $-1$, but closer to $-2$, on the real number line. Note that the point representing $-1.8$ lies slightly to the left of the point representing $-\frac{7}{4}$.

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Plot the real numbers on the real number line.

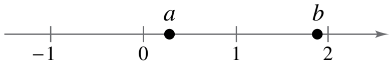
**a.** $\frac{5}{2}$ **b.** $-1.6$ **c.** $-\frac{3}{4}$ **d.** 0.7

GO DIGITAL

## Ordering Real Numbers

One important property of real numbers is that they are *ordered.* If $a$ and $b$ are real numbers, then $a$ is *less than* $b$ when $b - a$ is positive. The **inequality** $a < b$ denotes the **order** of $a$ and $b$. This relationship can also be described by saying that $b$ is *greater than* $a$ and writing $b > a$. The inequality $a \le b$ means that $a$ is *less than or equal to* $b$, and the inequality $b \ge a$ means that $b$ is *greater than or equal to* $a$. The symbols $<$, $>$, $\le$, and $\ge$ are *inequality symbols.*

Geometrically, this implies that $a < b$ if and only if $a$ lies to the *left* of $b$ on the real number line, as shown in Figure P.4.




$a < b$ if and only if $a$ lies to the left of $b$.

**Figure P.4**

### EXAMPLE 3 Ordering Real Numbers

Place the appropriate inequality symbol ($<$ or $>$) between the pair of real numbers.

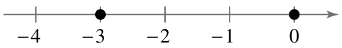
**a.** $-3, 0$ **b.** $-2, -4$ **c.** $\frac{1}{4}, \frac{1}{3}$

#### Solution

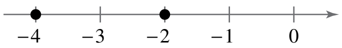
**a.** On the real number line, $-3$ lies to the left of 0, as shown in Figure P.5(a). So, you can say that $-3$ is *less than* 0, and write $-3 < 0$.

**b.** On the real number line, $-2$ lies to the right of $-4$, as shown in Figure P.5(b). So, you can say that $-2$ is *greater than* $-4$, and write $-2 > -4$.

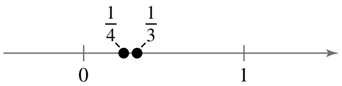
**c.** On the real number line, $\frac{1}{4}$ lies to the left of $\frac{1}{3}$, as shown in Figure P.5(c). So, you can say that $\frac{1}{4}$ is *less than* $\frac{1}{3}$, and write $\frac{1}{4} < \frac{1}{3}$.




**(a)**




**(b)**




**(c)**

**Figure P.5**

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Place the appropriate inequality symbol ($<$ or $>$) between the pair of real numbers.

**a.** $1, -5$ **b.** $\frac{3}{2}, 7$ **c.** $-\frac{2}{3}, -\frac{3}{4}$

### EXAMPLE 4 Interpreting Inequalities

*See LarsonPrecalculus.com for an interactive version of this type of example.*

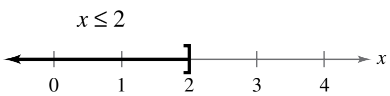
Describe the subset of real numbers that the inequality represents.

**a.** $x \le 2$ **b.** $-2 \le x < 3$

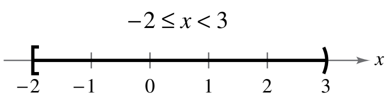
#### Solution

**a.** The inequality $x \le 2$ denotes all real numbers less than or equal to 2, as shown in Figure P.6(a). In the figure, the bracket at 2 indicates 2 is *included* in the interval.

**b.** The inequality $-2 \le x < 3$ means that $x \ge -2$ *and* $x < 3$. This "double inequality" denotes all real numbers between $-2$ and 3, including $-2$ but not including 3, as shown in Figure P.6(b). In the figure, the bracket at $-2$ indicates $-2$ is *included* in the interval, and the parenthesis at 3 indicates that 3 is *not* included in the interval.




**(a)**




**(b)**

**Figure P.6**

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Describe the subset of real numbers that the inequality represents.

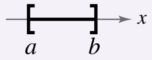
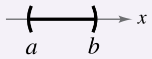
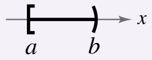
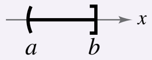
**a.** $x > -3$ **b.** $0 < x \le 4$

Inequalities can describe subsets of real numbers called **intervals.** In the bounded intervals on the next page, the real numbers $a$ and $b$ are the **endpoints** of each interval. The endpoints of a closed interval are included in the interval, whereas the endpoints of an open interval are not included in the interval.

GO DIGITAL

### Bounded Intervals on the Real Number Line

Let $a$ and $b$ be real numbers such that $a < b$.

| Notation | Interval Type | Inequality | Graph |
|---|---|---|---|
| $[a, b]$ | Closed | $a \leq x \leq b$ |  |
| $(a, b)$ | Open | $a < x < b$ |  |
| $[a, b)$ | | $a \leq x < b$ |  |
| $(a, b]$ | | $a < x \leq b$ |  |

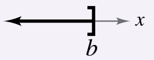
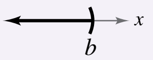
The reason that the four types of intervals above are called **bounded** is that each has a finite length. An interval that does not have a finite length is **unbounded.** Note in the unbounded intervals below that the symbols $\infty$, **positive infinity,** and $-\infty$, **negative infinity,** do not represent real numbers. They are convenient symbols used to describe the unboundedness of intervals such as $(1, \infty)$ or $(-\infty, 3]$.

**ALGEBRA HELP**

Whenever you write an interval containing $\infty$ or $-\infty$, always use a parenthesis and never a bracket next to these symbols. This is because $\infty$ and $-\infty$ are never included in the interval.

### Unbounded Intervals on the Real Number Line

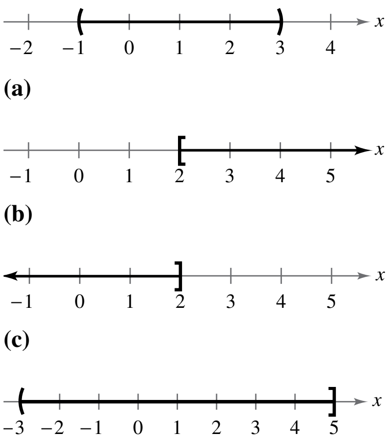
Let $a$ and $b$ be real numbers.

| Notation | Interval Type | Inequality | Graph |
|---|---|---|---|
| $[a, \infty)$ | | $x \geq a$ |  |
| $(a, \infty)$ | Open | $x > a$ |  |
| $(-\infty, b]$ | | $x \leq b$ |  |
| $(-\infty, b)$ | Open | $x < b$ |  |
| $(-\infty, \infty)$ | Entire real line | $-\infty < x < \infty$ |  |

GO DIGITAL

## EXAMPLE 5 Representing Intervals

| | Verbal | Algebraic | Graphical |
|---|---|---|---|
| **a.** | All real numbers greater than $-1$ and less than 3 | $(-1, 3)$ or $-1 < x < 3$ | See Figure P.7(a). |
| **b.** | All real numbers greater than or equal to 2 | $[2, \infty)$ or $x \geq 2$ | See Figure P.7(b). |
| **c.** | All real numbers less than or equal to 2 | $(-\infty, 2]$ or $x \leq 2$ | See Figure P.7(c). |
| **d.** | All real numbers greater than $-3$ and less than or equal to 5 | $(-3, 5]$ or $-3 < x \leq 5$ | See Figure P.7(d). |



(a) (b) (c) (d)

**Figure P.7**

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

**a.** Represent the interval $[-2, 5)$ verbally, as an inequality, and as a graph.

**b.** Represent the statement "$x$ is less than 4 and at least $-2$" as an interval, an inequality, and a graph.

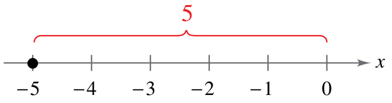
## Absolute Value and Distance

The **absolute value** of a real number is its *magnitude,* or the distance between the origin and the point representing the real number on the real number line.

### Definition of Absolute Value

If $a$ is a real number, then the **absolute value** of $a$ is

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}.$$




Absolute value as the distance from the origin
**Figure P.8**

Notice in this definition that the absolute value of a real number is never negative. For example, if $a = -5$, then $|-5| = -(-5) = 5$, as shown in Figure P.8. The absolute value of a real number is either positive or zero. Moreover, 0 is the only real number whose absolute value is 0. So, $|0| = 0$.

### Properties of Absolute Values

**1.** $|a| \geq 0$

**2.** $|-a| = |a|$

**3.** $|ab| = |a||b|$

**4.** $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \quad b \neq 0$

### EXAMPLE 6 Finding Absolute Values

**a.** $|-15| = 15$ **b.** $\left|\dfrac{2}{3}\right| = \dfrac{2}{3}$

**c.** $|-4.3| = 4.3$ **d.** $-|-6| = -(6) = -6$

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Evaluate each expression.

**a.** $|1|$ **b.** $-\left|\dfrac{3}{4}\right|$ **c.** $\dfrac{2}{|-3|}$ **d.** $-|0.7|$

### EXAMPLE 7 Evaluating an Absolute Value Expression

Evaluate $\dfrac{|x|}{x}$ for (a) $x > 0$ and (b) $x < 0$.

**Solution**

**a.** If $x > 0$, then $x$ is positive and $|x| = x$. So, $\dfrac{|x|}{x} = \dfrac{x}{x} = 1$.

**b.** If $x < 0$, then $x$ is negative and $|x| = -x$. So, $\dfrac{|x|}{x} = \dfrac{-x}{x} = -1$.

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Evaluate $\dfrac{|x + 3|}{x + 3}$ for (a) $x > -3$ and (b) $x < -3$.

GO DIGITAL

The **Law of Trichotomy** states that for any two real numbers $a$ and $b$, *precisely* one of three relationships is possible:

$a = b, \quad a < b, \quad \text{or} \quad a > b.$ Law of Trichotomy

In words, this property tells you that if $a$ and $b$ are any two real numbers, then $a$ is equal to $b$, $a$ is less than $b$, or $a$ is greater than $b$.

## EXAMPLE 8 Comparing Real Numbers

Place the appropriate symbol ($<$, $>$, or $=$) between the pair of real numbers.

**a.** $|-4| \; \square \; |3|$ **b.** $|-10| \; \square \; |10|$ **c.** $-|-7| \; \square \; |-7|$

### Solution

**a.** $|-4| > |3|$ because $|-4| = 4$ and $|3| = 3$, and 4 is greater than 3.

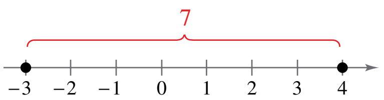
**b.** $|-10| = |10|$ because $|-10| = 10$ and $|10| = 10$.

**c.** $-|-7| < |-7|$ because $-|-7| = -7$ and $|-7| = 7$, and $-7$ is less than 7.

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Place the appropriate symbol ($<$, $>$, or $=$) between the pair of real numbers.

**a.** $|-3| \; \square \; |4|$ **b.** $-|-4| \; \square \; -|4|$ **c.** $|-3| \; \square \; -|-3|$




The distance between $-3$ and 4 is 7.
**Figure P.9**

Absolute value can be used to find the distance between two points on the real number line. For example, the distance between $-3$ and 4 is

$$|-3 - 4| = |-7|$$
$$= 7 \qquad \text{Distance between } -3 \text{ and } 4$$

as shown in Figure P.9.

### Distance Between Two Points on the Real Number Line

Let $a$ and $b$ be real numbers. The **distance between $a$ and $b$** is

$$d(a, b) = |b - a| = |a - b|.$$

One application of finding the distance between two points on the real number line is finding a change in temperature.

## EXAMPLE 9 Finding a Distance

Find the distance between $-25$ and 13.

### Solution

The distance between $-25$ and 13 is

$|-25 - 13| = |-38| = 38.$ Distance between $-25$ and 13

The distance can also be found as follows.

$|13 - (-25)| = |38| = 38$ Distance between $-25$ and 13

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the distance between each pair of real numbers.

**a.** 35 and $-23$ **b.** $-35$ and $-23$ **c.** 35 and 23

GO DIGITAL

© Alexander Uhrin/Shutterstock.com

## Algebraic Expressions

One characteristic of algebra is the use of letters to represent numbers. The letters are **variables,** and combinations of letters and numbers are **algebraic expressions.** Here are a few examples of algebraic expressions.

$$5x, \quad 2x - 3, \quad \frac{4}{x^2 + 2}, \quad 7x + y$$

**Definition of an Algebraic Expression**

An **algebraic expression** is a collection of letters **(variables)** and real numbers **(constants)** combined using the operations of addition, subtraction, multiplication, division, and exponentiation.

The **terms** of an algebraic expression are those parts that are separated by *addition.* For example, $x^2 - 5x + 8 = x^2 + (-5x) + 8$ has three terms: $x^2$ and $-5x$ are the **variable terms** and 8 is the **constant term.** For terms such as $x^2$, $-5x$, and 8, the numerical factor is the **coefficient.** Here, the coefficients are 1, $-5$, and 8.

### EXAMPLE 10 Identifying Terms and Coefficients

| | Algebraic Expression | Terms | Coefficients |
|---|---|---|---|
| **a.** | $5x - \frac{1}{7}$ | $5x, -\frac{1}{7}$ | $5, -\frac{1}{7}$ |
| **b.** | $2x^2 - 6x + 9$ | $2x^2, -6x, 9$ | $2, -6, 9$ |
| **c.** | $\frac{3}{x} + \frac{1}{2}x^4 - y$ | $\frac{3}{x}, \frac{1}{2}x^4, -y$ | $3, \frac{1}{2}, -1$ |

✓ ***Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Identify the terms and coefficients of $-2x + 4$. ■

The **Substitution Principle** states, "If $a = b$, then $b$ can replace $a$ in any expression involving $a$." Use the Substitution Principle to **evaluate** an algebraic expression by substituting values for each of the variables in the expression. The next example illustrates this.

### EXAMPLE 11 Evaluating Algebraic Expressions

| | Expression | Value of Variable | Substitution | Value of Expression |
|---|---|---|---|---|
| **a.** | $-3x + 5$ | $x = 3$ | $-3(3) + 5$ | $-9 + 5 = -4$ |
| **b.** | $3x^2 + 2x - 1$ | $x = -1$ | $3(-1)^2 + 2(-1) - 1$ | $3 - 2 - 1 = 0$ |
| **c.** | $\frac{2x}{x + 1}$ | $x = -3$ | $\frac{2(-3)}{-3 + 1}$ | $\frac{-6}{-2} = 3$ |

Note that you must substitute the value for *each* occurrence of the variable.

✓ ***Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Evaluate $4x - 5$ when $x = 0$. ■

GO DIGITAL

## Basic Rules of Algebra

There are four arithmetic operations with real numbers: *addition, multiplication, subtraction,* and *division,* denoted by the symbols $+$, $\times$ or $\cdot$, $-$, and $\div$ or $/$, respectively. Of these, addition and multiplication are the two primary operations. Subtraction and division are the inverse operations of addition and multiplication, respectively.

### Definitions of Subtraction and Division

**Subtraction:** Add the opposite.

$$a - b = a + (-b)$$

**Division:** Multiply by the reciprocal.

If $b \neq 0$, then $a/b = a\left(\frac{1}{b}\right) = \frac{a}{b}$.

In these definitions, $-b$ is the **additive inverse** (or opposite) of $b$, and $1/b$ is the **multiplicative inverse** (or reciprocal) of $b$. In the fractional form $a/b$, $a$ is the **numerator** of the fraction and $b$ is the **denominator.**

The properties of real numbers below are true for variables and algebraic expressions as well as for real numbers, so they are often called the **Basic Rules of Algebra.** Formulate a verbal description of each of these properties. For example, the first property states that *the order in which two real numbers are added does not affect their sum.*

### Basic Rules of Algebra

Let $a$, $b$, and $c$ be real numbers, variables, or algebraic expressions.

| **Property** | | **Example** |
|---|---|---|
| Commutative Property of Addition: | $a + b = b + a$ | $4x + x^2 = x^2 + 4x$ |
| Commutative Property of Multiplication: | $ab = ba$ | $(4 - x)x^2 = x^2(4 - x)$ |
| Associative Property of Addition: | $(a + b) + c = a + (b + c)$ | $(x + 5) + x^2 = x + (5 + x^2)$ |
| Associative Property of Multiplication: | $(ab)c = a(bc)$ | $(2x \cdot 3y)(8) = (2x)(3y \cdot 8)$ |
| Distributive Properties: | $a(b + c) = ab + ac$ | $3x(5 + 2x) = 3x \cdot 5 + 3x \cdot 2x$ |
| | $(a + b)c = ac + bc$ | $(y + 8)y = y \cdot y + 8 \cdot y$ |
| Additive Identity Property: | $a + 0 = a$ | $5y^2 + 0 = 5y^2$ |
| Multiplicative Identity Property: | $a \cdot 1 = a$ | $(4x^2)(1) = 4x^2$ |
| Additive Inverse Property: | $a + (-a) = 0$ | $5x^3 + (-5x^3) = 0$ |
| Multiplicative Inverse Property: | $a \cdot \frac{1}{a} = 1, \quad a \neq 0$ | $(x^2 + 4)\left(\frac{1}{x^2 + 4}\right) = 1$ |

Subtraction is defined as "adding the opposite," so the Distributive Properties are also true for subtraction. For example, the "subtraction form" of $a(b + c) = ab + ac$ is $a(b - c) = ab - ac$. Note that the operations of subtraction and division are neither commutative nor associative. The examples

$$7 - 3 \neq 3 - 7 \quad \text{and} \quad 20 \div 4 \neq 4 \div 20$$

show that subtraction and division are not commutative. Similarly

$$5 - (3 - 2) \neq (5 - 3) - 2 \quad \text{and} \quad 16 \div (4 \div 2) \neq (16 \div 4) \div 2$$

demonstrate that subtraction and division are not associative.

## EXAMPLE 12 Identifying Rules of Algebra

Identify the rule of algebra illustrated by the statement.

**a.** $(5x^3)2 = 2(5x^3)$ **b.** $(4x + 3) - (4x + 3) = 0$

**c.** $7x \cdot \dfrac{1}{7x} = 1, \quad x \neq 0$ **d.** $(2 + 5x^2) + x^2 = 2 + (5x^2 + x^2)$

### Solution

**a.** This statement illustrates the Commutative Property of Multiplication. In other words, you obtain the same result whether you multiply $5x^3$ by 2, or 2 by $5x^3$.

**b.** This statement illustrates the Additive Inverse Property. In terms of subtraction, this property states that when any expression is subtracted from itself, the result is 0.

**c.** This statement illustrates the Multiplicative Inverse Property. Note that $x$ must be a nonzero number. The reciprocal of $x$ is undefined when $x$ is 0.

**d.** This statement illustrates the Associative Property of Addition. In other words, to form the sum $2 + 5x^2 + x^2$, it does not matter whether 2 and $5x^2$, or $5x^2$ and $x^2$ are added first.

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Identify the rule of algebra illustrated by the statement.

**a.** $x + 9 = 9 + x$ **b.** $5(x^3 \cdot 2) = (5x^3)2$ **c.** $(2 + 5x^2)y^2 = 2 \cdot y^2 + 5x^2 \cdot y^2$

### ALGEBRA HELP

Notice the difference between the *opposite of a number* and a *negative number.* If $a$ is already negative, then its opposite, $-a$, is positive. For example, if $a = -5$, then

$$-a = -(-5) = 5.$$

### Properties of Negation and Equality

Let $a$, $b$, and $c$ be real numbers, variables, or algebraic expressions.

| | Property | Example |
|---|---|---|
| **1.** | $(-1)a = -a$ | $(-1)7 = -7$ |
| **2.** | $-(-a) = a$ | $-(-6) = 6$ |
| **3.** | $(-a)b = -(ab) = a(-b)$ | $(-5)3 = -(5 \cdot 3) = 5(-3)$ |
| **4.** | $(-a)(-b) = ab$ | $(-2)(-x) = 2x$ |
| **5.** | $-(a + b) = (-a) + (-b)$ | $-(x + 8) = (-x) + (-8)$ $= -x - 8$ |
| **6.** | If $a = b$, then $a \pm c = b \pm c$. | $\frac{1}{2} + 3 = 0.5 + 3$ |
| **7.** | If $a = b$, then $ac = bc$. | $4^2 \cdot 2 = 16 \cdot 2$ |
| **8.** | If $a \pm c = b \pm c$, then $a = b$. | $1.4 - 1 = \frac{7}{5} - 1 \Rightarrow 1.4 = \frac{7}{5}$ |
| **9.** | If $ac = bc$ and $c \neq 0$, then $a = b$. | $3x = 3 \cdot 4 \Rightarrow x = 4$ |

### ALGEBRA HELP

The "or" in the Zero-Factor Property includes the possibility that either or both factors may be zero. This is an *inclusive or,* and it is generally the way the word "or" is used in mathematics.

### Properties of Zero

Let $a$ and $b$ be real numbers, variables, or algebraic expressions.

**1.** $a + 0 = a$ and $a - 0 = a$ **2.** $a \cdot 0 = 0$

**3.** $\dfrac{0}{a} = 0, \quad a \neq 0$ **4.** $\dfrac{a}{0}$ is undefined.

**5. Zero-Factor Property:** If $ab = 0$, then $a = 0$ or $b = 0$.

GO DIGITAL

### Properties and Operations of Fractions

Let $a$, $b$, $c$, and $d$ be real numbers, variables, or algebraic expressions such that $b \neq 0$ and $d \neq 0$.

1. **Equivalent Fractions:** $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.
2. **Rules of Signs:** $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ and $\frac{-a}{-b} = \frac{a}{b}$
3. **Generate Equivalent Fractions:** $\frac{a}{b} = \frac{ac}{bc}, \quad c \neq 0$
4. **Add or Subtract with Like Denominators:** $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$
5. **Add or Subtract with Unlike Denominators:** $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$
6. **Multiply Fractions:** $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
7. **Divide Fractions:** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, \quad c \neq 0$

**ALGEBRA HELP**

In Property 1, the phrase "if and only if" implies two statements. One statement is: If $a/b = c/d$, then $ad = bc$. The other statement is: If $ad = bc$, where $b \neq 0$ and $d \neq 0$, then $a/b = c/d$.

### EXAMPLE 13 Properties and Operations of Fractions

**a.** $\frac{x}{5} = \frac{3 \cdot x}{3 \cdot 5} = \frac{3x}{15}$ Property 3

**b.** $\frac{7}{x} \div \frac{3}{2} = \frac{7}{x} \cdot \frac{2}{3} = \frac{14}{3x}$ Property 7

**✓ Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

**a.** Multiply fractions: $\frac{3}{5} \cdot \frac{x}{6}$ **b.** Add fractions: $\frac{x}{10} + \frac{2x}{5}$

If $a$, $b$, and $c$ are integers such that $ab = c$, then $a$ and $b$ are **factors** or **divisors** of $c$. A **prime number** is an integer that has exactly two positive factors—itself and 1—such as 2, 3, 5, 7, and 11. The numbers 4, 6, 8, 9, and 10 are **composite** because each can be written as the product of two or more prime numbers. The number 1 is neither prime nor composite. The **Fundamental Theorem of Arithmetic** states that every positive integer greater than 1 is prime or can be written as the product of prime numbers in precisely one way (disregarding order). For example, the **prime factorization** of 24 is $24 = 2 \cdot 2 \cdot 2 \cdot 3$.

### Summarize (Section P.1)

1. Explain how to represent and classify real numbers *(pages 2 and 3)*. For examples of representing and classifying real numbers, see Examples 1 and 2.
2. Explain how to order real numbers and use inequalities *(pages 4 and 5)*. For examples of ordering real numbers and using inequalities, see Examples 3–5.
3. State the definition of the absolute value of a real number *(page 6)*. For examples of using absolute value, see Examples 6–9.
4. Explain how to evaluate an algebraic expression *(page 8)*. For examples involving algebraic expressions, see Examples 10 and 11.
5. State the basic rules and properties of algebra *(pages 9–11)*. For examples involving the basic rules and properties of algebra, see Examples 12 and 13.

GO DIGITAL

# P.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

GO DIGITAL

## Vocabulary and Concept Check

**In Exercises 1–4, fill in the blanks.**

**1.** The decimal form of an ________ number neither terminates nor repeats.

**2.** The point representing 0 on the real number line is the ________.

**3.** The ________ of an algebraic expression are those parts that are separated by addition.

**4.** The ________ ________ states that if $ab = 0$, then $a = 0$ or $b = 0$.

**5.** Is $|3 - 10|$ equal to $|10 - 3|$? Explain.

**6.** Match each property with its name.

(a) Commutative Property of Addition — (i) $a \cdot 1 = a$

(b) Additive Inverse Property — (ii) $a(b + c) = ab + ac$

(c) Distributive Property — (iii) $a + b = b + a$

(d) Associative Property of Addition — (iv) $a + (-a) = 0$

(e) Multiplicative Identity Property — (v) $(a + b) + c = a + (b + c)$

## Skills and Applications

**Classifying Real Numbers** **In Exercises 7–10, determine which numbers in the set are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.**

**7.** $\left\{-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{3}, 0, 8, -4, 2, -11\right\}$

**8.** $\left\{\sqrt{5}, -7, -\frac{7}{3}, 0, 3.14, \frac{5}{4}, -3, 12, 5\right\}$

**9.** $\{2.01, 0.\overline{6}, -13, 0.010110111\ldots, 1, -6\}$

**10.** $\left\{25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 18, -11.1, 13\right\}$

**Plotting and Ordering Real Numbers** **In Exercises 11–16, plot the two real numbers on the real number line. Then place the appropriate inequality symbol (< or >) between them.**

**11.** $-4, -8$ **12.** $1, \frac{16}{3}$

**13.** $\frac{5}{6}, \frac{2}{3}$ **14.** $-\frac{8}{7}, -\frac{3}{7}$

**15.** $-5.2, -8.5$ **16.** $-\frac{4}{3}, -4.75$

**Interpreting an Inequality** **In Exercises 17–20, describe the subset of real numbers that the inequality represents.**

**17.** $x \le 5$ **18.** $x < 0$

**19.** $-2 < x < 2$ **20.** $0 < x \le 6$

**Representing an Interval** **In Exercises 21–24, represent the interval verbally, as an inequality, and as a graph.**

**21.** $[4, \infty)$ **22.** $(-\infty, 2)$

**23.** $[-5, 2)$ **24.** $(-1, 2]$

**Representing an Interval** **In Exercises 25–28, represent the statement as an interval, an inequality, and a graph.**

**25.** $y$ is nonpositive.

**26.** $y$ is no more than 25.

**27.** $t$ is at least 10 and at most 22.

**28.** $k$ is less than 5 but no less than $-3$.

**Evaluating an Absolute Value Expression** **In Exercises 29–38, evaluate the expression.**

**29.** $|-10|$ **30.** $|0|$

**31.** $|3 - 8|$ **32.** $|6 - 2|$

**33.** $|-1| - |-2|$ **34.** $-3 - |-3|$

**35.** $5|-5|$ **36.** $-4|-4|$

**37.** $\dfrac{|x + 2|}{x + 2}, \quad x < -2$ **38.** $\dfrac{|x - 1|}{x - 1}, \quad x > 1$

**Comparing Real Numbers** **In Exercises 39–42, place the appropriate symbol (<, >, or =) between the pair of real numbers.**

**39.** $|-4|$ ▢ $|4|$ **40.** $-5$ ▢ $-|5|$

**41.** $-|-6|$ ▢ $|-6|$ **42.** $-|-2|$ ▢ $-|2|$

**Finding a Distance** **In Exercises 43–46, find the distance between $a$ and $b$.**

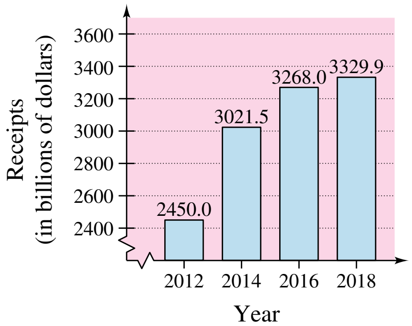
**43.** $a = 126, b = 75$ **44.** $a = -20, b = 30$

**45.** $a = -\frac{5}{2}, b = 0$ **46.** $a = -\frac{1}{4}, b = -\frac{11}{4}$

A blue exercise number indicates that a video solution can be seen at *CalcView.com*.

## Federal Deficit

**In Exercises 47–50, use the bar graph, which shows the receipts of the federal government (in billions of dollars) for selected years from 2012 through 2018. In each exercise, you are given the expenditures of the federal government. Find the magnitude of the surplus or deficit for the year.** *(Source: U.S. Office of Management and Budget)*




| | Year | Receipts, $R$ | Expenditures, $E$ | $\lvert R - E \rvert$ |
|---|---|---|---|---|
| **47.** | 2012 | | \$3526.6 billion | |
| **48.** | 2014 | | \$3506.3 billion | |
| **49.** | 2016 | | \$3852.6 billion | |
| **50.** | 2018 | | \$4109.0 billion | |

**Identifying Terms and Coefficients** **In Exercises 51–54, identify the terms. Then identify the coefficients of the variable terms of the expression.**

**51.** $7x + 4$

**52.** $6x^3 - 5x$

**53.** $4x^3 + 0.5x - 5$

**54.** $3\sqrt{3}x^2 + 1$

**Evaluating an Algebraic Expression** **In Exercises 55 and 56, evaluate the expression for each value of $x$. (If not possible, state the reason.)**

**55.** $x^2 - 3x + 2$ (a) $x = 0$ (b) $x = -1$

**56.** $\dfrac{x - 2}{x + 2}$ (a) $x = 2$ (b) $x = -2$

**Operations with Fractions** **In Exercises 57–60, perform the operation. (Write fractional answers in simplest form.)**

**57.** $\dfrac{2x}{3} - \dfrac{x}{4}$

**58.** $\dfrac{3x}{4} + \dfrac{x}{5}$

**59.** $\dfrac{3x}{10} \cdot \dfrac{5}{6}$

**60.** $\dfrac{2x}{3} \div \dfrac{6}{7}$

© Gary718/Shutterstock.com

## *Exploring the Concepts*

**True or False?** **In Exercises 61 and 62, determine whether the statement is true or false. Justify your answer.**

**61.** Every nonnegative number is positive.

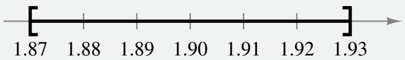
**62.** If $a < 0$ and $b < 0$, then $ab > 0$.

**63. Error Analysis** Describe the error.

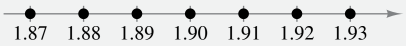
$5(2x + 3) = 5 \cdot 2x + 3 = 10x + 3$ ✗

**64. HOW DO YOU SEE IT?** Match each description with its graph. Explain.

(i)




(ii)




(a) The price of an item is within \$0.03 of \$1.90.

(b) The distance between the prongs of an electric plug may not differ from 1.9 centimeters by more than 0.03 centimeter.

**65. Conjecture** Make a conjecture about the value of the expression $5/n$ as $n$ approaches 0. Explain.

**66. Conjecture** Make a conjecture about the value of the expression $5/n$ as $n$ increases without bound. Explain.

## *Review & Refresh*

*Video solutions at LarsonPrecalculus.com*

**Finding the Least Common Denominator** **In Exercises 67–70, find the least common denominator.**

**67.** $\dfrac{x}{3}, \dfrac{3x}{4}$

**68.** $\dfrac{4x}{9}, \dfrac{1}{3}, x, \dfrac{5}{3}$

**69.** $\dfrac{3x}{x - 4}, 5, \dfrac{12}{x - 4}$

**70.** $\dfrac{1}{x - 2}, \dfrac{3}{x + 2}, \dfrac{6x}{x^2 - 4}$

**Factoring Completely** **In Exercises 71–80, completely factor the expression.**

**71.** $3x^4 - 48x^2$

**72.** $9x^4 - 12x^2$

**73.** $x^3 - 3x^2 + 3x - 9$

**74.** $x^3 - 5x^2 - 2x + 10$

**75.** $6x^3 - 27x^2 - 54x$

**76.** $12x^3 - 16x^2 - 60x$

**77.** $x^4 - 3x^2 + 2$

**78.** $x^4 - 7x^2 + 12$

**79.** $9x^4 - 37x^2 + 4$

**80.** $4x^4 - 37x^2 + 9$

**Evaluating an Expression** **In Exercises 81–84, evaluate the expression for each value of $x$.**

**81.** $\sqrt{2x + 7} - x$ (a) $x = -3$ (b) $x = 1$

**82.** $x + \sqrt{40 - 9x}$ (a) $x = 4$ (b) $x = -9$

**83.** $\lvert x^2 - 3x \rvert + 4x - 6$ (a) $x = -3$ (b) $x = 2$

**84.** $\lvert x^2 + 4x \rvert - 7x - 18$ (a) $x = -3$ (b) $x = -9$

# P.2 Solving Equations

Linear equations have many real-life applications, such as in forensics. For example, in Exercises 101 and 102 on page 25, you will use linear equations to determine height from femur length.

- **Identify different types of equations.**
- **Solve linear equations in one variable and rational equations.**
- **Solve quadratic equations by factoring, extracting square roots, completing the square, and using the Quadratic Formula.**
- **Solve polynomial equations of degree three or greater.**
- **Solve radical equations.**
- **Solve absolute value equations.**

## Equations and Solutions of Equations

An **equation** in $x$ is a statement that two algebraic expressions are equal. For example, $3x - 5 = 7$, $x^2 - x - 6 = 0$, and $\sqrt{2x} = 4$ are equations in $x$. To **solve** an equation in $x$ means to find all values of $x$ for which the equation is true. Such values are **solutions.** For example, $x = 4$ is a solution of the equation $3x - 5 = 7$ because $3(4) - 5 = 7$ is a true statement.

The solutions of an equation depend on the kinds of numbers being considered. For example, in the set of rational numbers, $x^2 = 10$ has no solution because there is no rational number whose square is 10. In the set of real numbers, however, the equation has the two solutions $x = \sqrt{10}$ and $x = -\sqrt{10}$. The **domain** is the set of all real numbers for which the equation is defined.

An equation that is true for *every* real number in the domain of the variable is an **identity.** For example,

$$x^2 - 9 = (x + 3)(x - 3) \qquad \text{Identity}$$

is an identity because it is a true statement for any real value of $x$. The equation

$$\frac{x}{3x^2} = \frac{1}{3x} \qquad \text{Identity}$$

is an identity because it is true for any nonzero real value of $x$.

An equation that is true for just *some* (but not all) of the real numbers in the domain of the variable is a **conditional equation.** For example, the equation

$$x^2 - 9 = 0 \qquad \text{Conditional equation}$$

is conditional because $x = 3$ and $x = -3$ are the only values in the domain that satisfy the equation.

A **contradiction** is an equation that is false for *every* real number in the domain of the variable. For example, the equation

$$2x - 4 = 2x + 1 \qquad \text{Contradiction}$$

is a contradiction because there are no real values of $x$ for which the equation is true.

## Linear and Rational Equations

**Definition of a Linear Equation in One Variable**

A **linear equation in one variable** $x$ is an equation that can be written in the standard form $ax + b = 0$, where $a$ and $b$ are real numbers with $a \neq 0$.

Some examples of linear equations in one variable that are written in the standard form $ax + b = 0$ are $3x + 2 = 0$ and $5x - 9 = 0$.

GO DIGITAL

© iStockPhoto.com/Nastasic

**HISTORICAL NOTE**

This ancient Egyptian papyrus, discovered in 1858, contains one of the earliest examples of mathematical writing in existence. The papyrus itself dates back to around 1650 B.C., but it is actually a copy of writings from two centuries earlier. The algebraic equations on the papyrus were written in words. Diophantus, a Greek who lived around A.D. 250, is often called the Father of Algebra. He was the first to use abbreviated word forms in equations.

A linear equation in one variable has exactly one solution. To see this, consider the steps below. (Remember that $a \neq 0$.)

| | |
|---|---|
| $ax + b = 0$ | Original equation |
| $ax = -b$ | Subtract $b$ from each side. |
| $x = -\dfrac{b}{a}$ | Divide each side by $a$. |

It is clear that the last equation has only one solution, $x = -b/a$, and that this equation is equivalent to the original equation. So, you can conclude that every linear equation in one variable, written in standard form, has exactly one solution.

To solve a conditional equation in $x$, isolate $x$ on one side of the equation using a sequence of **equivalent equations,** each having the same solution as the original equation. The operations that yield equivalent equations come from the properties of equality reviewed in Section P.1.

### Generating Equivalent Equations

An equation can be transformed into an *equivalent equation* by one or more of the steps listed below.

| | **Given Equation** | **Equivalent Equation** |
|---|---|---|
| **1.** Remove symbols of grouping, combine like terms, or simplify fractions on one or both sides of the equation. | $2x - x = 4$ | $x = 4$ |
| **2.** Add (or subtract) the same quantity to (or from) *each* side of the equation. | $x + 1 = 6$ | $x = 5$ |
| **3.** Multiply (or divide) *each* side of the equation by the same *nonzero* quantity. | $2x = 6$ | $x = 3$ |
| **4.** Interchange the two sides of the equation. | $2 = x$ | $x = 2$ |

GO DIGITAL

### EXAMPLE 1 Solving Linear Equations

| | |
|---|---|
| **a.** $3x - 6 = 0$ | Original equation |
| $3x = 6$ | Add 6 to each side. |
| $x = 2$ | Divide each side by 3. |
| **b.** $5x + 4 = 3x - 8$ | Original equation |
| $2x + 4 = -8$ | Subtract $3x$ from each side. |
| $2x = -12$ | Subtract 4 from each side. |
| $x = -6$ | Divide each side by 2. |

**ALGEBRA HELP**

After solving an equation, you should check each solution in the original equation. For instance, to check the solution to Example 1(a), substitute 2 for $x$ in the original equation and simplify.

| | |
|---|---|
| $3x - 6 = 0$ | Write original equation. |
| $3(2) - 6 \stackrel{?}{=} 0$ | Substitute 2 for $x$. |
| $0 = 0$ | Solution checks. ✓ |

Check the solution to Example 1(b) on your own.

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve each equation.

**a.** $7 - 2x = 15$ **b.** $7x - 9 = 5x + 7$

© The Print Collector/Alamy

**SKILLS REFRESHER**

For a refresher on how to find the least common denominator (LCD) of two or more rational expressions, watch the video at *LarsonPrecalculus.com*.

A **rational equation** involves one or more rational expressions. To solve a rational equation, multiply every term by the least common denominator (LCD) of all the terms. This clears the original equation of fractions and produces a simpler equation.

## EXAMPLE 2 Solving a Rational Equation

Solve $\frac{x}{3} + \frac{3x}{4} = 2$.

### Solution

The LCD is 12, so multiply each term by 12.

$$\frac{x}{3} + \frac{3x}{4} = 2 \qquad \text{Write original equation.}$$

$$(12)\frac{x}{3} + (12)\frac{3x}{4} = (12)2 \qquad \text{Multiply each term by the LCD.}$$

$$4x + 9x = 24 \qquad \text{Simplify.}$$

$$13x = 24 \qquad \text{Combine like terms.}$$

$$x = \frac{24}{13} \qquad \text{Divide each side by 13.}$$

The solution is $x = \frac{24}{13}$. Check this in the original equation.

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $\frac{4x}{9} - \frac{1}{3} = x + \frac{5}{3}$.

When multiplying or dividing an equation by a *variable expression,* it is possible to introduce an **extraneous solution,** which is a solution that does not satisfy the original equation. So, it is essential to check your solutions.

## EXAMPLE 3 An Equation with an Extraneous Solution

*See LarsonPrecalculus.com for an interactive version of this type of example.*

GO DIGITAL

Solve $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$.

### Solution

The LCD is $(x + 2)(x - 2)$. Multiply each term by the LCD.

$$\frac{1}{x-2}(x+2)(x-2) = \frac{3}{x+2}(x+2)(x-2) - \frac{6x}{x^2-4}(x+2)(x-2)$$

$$x + 2 = 3(x - 2) - 6x, \quad x \neq \pm 2$$

$$x + 2 = 3x - 6 - 6x$$

$$x + 2 = -3x - 6$$

$$4x = -8$$

$$x = -2 \qquad \text{Extraneous solution}$$

In the original equation, $x = -2$ yields a denominator of zero. So, $x = -2$ is an extraneous solution, and the original equation has *no solution.*

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $\frac{3x}{x-4} = 5 + \frac{12}{x-4}$.

**ALGEBRA HELP**

Recall that the least common denominator of two or more fractions consists of the product of all prime factors in the denominators, with each factor given the highest power of its occurrence in any denominator. For instance, in Example 3, the factored forms of the denominators are $x - 2$, $x + 2$, and $(x + 2)(x - 2)$. The factors $x - 2$ and $x + 2$ each appear once, so the LCD is $(x + 2)(x - 2)$.

## Quadratic Equations

A **quadratic equation** in $x$ is an equation that can be written in the general form

$$ax^2 + bx + c = 0 \qquad \text{General form}$$

where $a$, $b$, and $c$ are real numbers with $a \neq 0$. A quadratic equation in $x$ is also called a **second-degree polynomial equation** in $x$.

You should be familiar with the four methods for solving quadratic equations listed below.

### Solving a Quadratic Equation

**Factoring**

If $ab = 0$, then $a = 0$ or $b = 0$. Zero-Factor Property

*Example:*

$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 2 = 0 \Rightarrow x = -2$$

**Extracting Square Roots**

If $u^2 = c$, where $c > 0$, then $u = \pm\sqrt{c}$. Square Root Principle

*Example:* $(x + 3)^2 = 16$

$$x + 3 = \pm 4$$
$$x = -3 \pm 4$$
$$x = 1 \quad \text{or} \quad x = -7$$

**Completing the Square**

If $x^2 + bx = c$, then

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2 \qquad \text{Add } \left(\frac{b}{2}\right)^2 \text{ to each side.}$$
$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}.$$

*Example:*

$$x^2 + 6x = 5$$
$$x^2 + 6x + 3^2 = 5 + 3^2 \qquad \text{Add } \left(\frac{6}{2}\right)^2 \text{ to each side.}$$
$$(x + 3)^2 = 14$$
$$x + 3 = \pm\sqrt{14}$$
$$x = -3 \pm \sqrt{14}$$

**Quadratic Formula**

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

*Example:* $2x^2 + 3x - 1 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{17}}{4}$$

**ALGEBRA HELP**

It is possible to solve every quadratic equation by completing the square or using the Quadratic Formula.

GO DIGITAL

**SKILLS REFRESHER**

For a refresher on factoring quadratic polynomials, watch the video at *LarsonPrecalculus.com.*

## EXAMPLE 4 Solving Quadratic Equations by Factoring

**a.** $2x^2 + 9x + 7 = 3$ — Original equation

$2x^2 + 9x + 4 = 0$ — Write in general form.

$(2x + 1)(x + 4) = 0$ — Factor.

$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$ — Set 1st factor equal to 0.

$x + 4 = 0 \Rightarrow x = -4$ — Set 2nd factor equal to 0.

The solutions are $x = -\frac{1}{2}$ and $x = -4$. Check these in the original equation.

**b.** $6x^2 - 3x = 0$ — Original equation

$3x(2x - 1) = 0$ — Factor.

$3x = 0 \Rightarrow x = 0$ — Set 1st factor equal to 0.

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$ — Set 2nd factor equal to 0.

The solutions are $x = 0$ and $x = \frac{1}{2}$. Check these in the original equation.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $2x^2 - 3x + 1 = 6$ by factoring.

Note that the method of solution in Example 4 is based on the Zero-Factor Property from Section P.1. This property applies *only* to equations written in general form (in which the right side of the equation is zero). So, collect all terms on one side *before* factoring. For example, in the equation $(x - 5)(x + 2) = 8$, it is *incorrect* to set each factor equal to 8. Solve this equation correctly on your own. Then check the solutions in the original equation.

## EXAMPLE 5 Extracting Square Roots

**a.** $4x^2 = 12$ — Original equation

$x^2 = 3$ — Divide each side by 4.

$x = \pm\sqrt{3}$ — Extract square roots.

The solutions are $x = \sqrt{3}$ and $x = -\sqrt{3}$. Check these in the original equation.

**b.** $(x - 3)^2 = 7$ — Original equation

$x - 3 = \pm\sqrt{7}$ — Extract square roots.

$x = 3 \pm \sqrt{7}$ — Add 3 to each side.

The solutions are $x = 3 \pm \sqrt{7}$. Check these in the original equation.

**c.** $(3x - 6)^2 - 18 = 0$ — Original equation

$(3x - 6)^2 = 18$ — Add 18 to each side.

$3x - 6 = \pm 3\sqrt{2}$ — Extract square roots.

$3x = 6 \pm 3\sqrt{2}$ — Add 6 to each side.

$x = 2 \pm \sqrt{2}$ — Divide each side by 3.

The solutions are $x = 2 \pm \sqrt{2}$. Check these in the original equation.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve (a) $3x^2 = 36$ and (b) $(x - 1)^2 = 10$ by extracting square roots.

GO DIGITAL

**ALGEBRA HELP**

When extracting square roots in Example 5(c), note that $\pm\sqrt{18} = \pm\sqrt{3^2 \cdot 2} = \pm 3\sqrt{2}$.

When solving quadratic equations by completing the square, you must add $(b/2)^2$ to *each side* in order to maintain equality. When the leading coefficient is *not* 1, divide each side of the equation by the leading coefficient *before* completing the square, as shown in Example 7.

## EXAMPLE 6 Completing the Square: Leading Coefficient Is 1

Solve $x^2 + 2x - 6 = 0$ by completing the square.

**ALGEBRA HELP**

Note that when you complete the square to solve a quadratic equation, you are rewriting the equation so it can be solved by extracting square roots, as shown in Example 6.

**Solution**

$$x^2 + 2x - 6 = 0 \qquad \text{Write original equation.}$$
$$x^2 + 2x = 6 \qquad \text{Add 6 to each side.}$$
$$x^2 + 2x + 1^2 = 6 + 1^2 \qquad \text{Add } 1^2 \text{ to each side.}$$

(Half of 2)$^2$

$$(x + 1)^2 = 7 \qquad \text{Simplify.}$$
$$x + 1 = \pm\sqrt{7} \qquad \text{Extract square roots.}$$
$$x = -1 \pm \sqrt{7} \qquad \text{Subtract 1 from each side.}$$

The solutions are

$$x = -1 \pm \sqrt{7}.$$

Check these in the original equation.

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $x^2 - 4x - 1 = 0$ by completing the square.

## EXAMPLE 7 Completing the Square: Leading Coefficient Is Not 1

Solve $3x^2 - 4x - 5 = 0$ by completing the square.

**Solution** Note that the leading coefficient is 3.

$$3x^2 - 4x - 5 = 0 \qquad \text{Write original equation.}$$
$$3x^2 - 4x = 5 \qquad \text{Add 5 to each side.}$$
$$x^2 - \frac{4}{3}x = \frac{5}{3} \qquad \text{Divide each side by 3.}$$
$$x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 = \frac{5}{3} + \left(-\frac{2}{3}\right)^2 \qquad \text{Add } \left(-\frac{2}{3}\right)^2 \text{ to each side.}$$

$\left(\text{Half of } -\frac{4}{3}\right)^2$

$$\left(x - \frac{2}{3}\right)^2 = \frac{19}{9} \qquad \text{Simplify.}$$
$$x - \frac{2}{3} = \pm\frac{\sqrt{19}}{3} \qquad \text{Extract square roots.}$$
$$x = \frac{2}{3} \pm \frac{\sqrt{19}}{3} \qquad \text{Add } \tfrac{2}{3} \text{ to each side.}$$

Check these in the original equation.

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $3x^2 - 10x - 2 = 0$ by completing the square.

GO DIGITAL

## EXAMPLE 8 The Quadratic Formula: Two Distinct Solutions

Use the Quadratic Formula to solve $x^2 + 3x = 9$.

### Solution

$$x^2 + 3x = 9$$ Write original equation.

$$x^2 + 3x - 9 = 0$$ Write in general form.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-9)}}{2(1)}$$ Substitute 1 for $a$, 3 for $b$, and $-9$ for $c$.

$$x = \frac{-3 \pm \sqrt{45}}{2}$$ Simplify.

$$x = \frac{-3 \pm 3\sqrt{5}}{2}$$ Simplify.

The two solutions are

$$x = \frac{-3 + 3\sqrt{5}}{2} \quad \text{and} \quad x = \frac{-3 - 3\sqrt{5}}{2}.$$

Check these in the original equation.

**ALGEBRA HELP**

When you use the Quadratic Formula, remember that *before* applying the formula, you must first write the quadratic equation in general form.

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use the Quadratic Formula to solve $3x^2 + 2x = 10$.

## EXAMPLE 9 The Quadratic Formula: One Solution

Use the Quadratic Formula to solve $8x^2 - 24x + 18 = 0$.

### Solution

$$8x^2 - 24x + 18 = 0$$ Write original equation.

$$4x^2 - 12x + 9 = 0$$ Divide out common factor of 2.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$$ Substitute 4 for $a$, $-12$ for $b$, and 9 for $c$.

$$x = \frac{12 \pm \sqrt{0}}{8}$$ Simplify.

$$x = \frac{3}{2}$$ Simplify.

This quadratic equation has only one solution: $x = \frac{3}{2}$. Check this in the original equation.

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use the Quadratic Formula to solve $18x^2 - 48x + 32 = 0$. ■

Note that you could have solved Example 9 without first dividing out a common factor of 2. Substituting $a = 8$, $b = -24$, and $c = 18$ into the Quadratic Formula produces the same result.

GO DIGITAL

## Polynomial Equations of Higher Degree

Sometimes, the methods used to solve quadratic equations can be extended to solve polynomial equations of higher degrees.

**ALGEBRA HELP**

A common mistake when solving an equation such as that in Example 10 is to divide each side of the equation by the variable factor $x^2$. This loses the solution $x = 0$. When solving a polynomial equation, always write the equation in general form, then factor the polynomial and set each factor equal to zero. Do not divide each side of an equation by a variable factor in an attempt to simplify the equation.

### EXAMPLE 10 Solving a Polynomial Equation by Factoring

Solve $3x^4 = 48x^2$.

**Solution** First write the polynomial equation in general form. Then factor the polynomial, set each factor equal to zero, and solve.

$$3x^4 = 48x^2 \quad \text{Write original equation.}$$
$$3x^4 - 48x^2 = 0 \quad \text{Write in general form.}$$
$$3x^2(x^2 - 16) = 0 \quad \text{Factor out common factor.}$$
$$3x^2(x + 4)(x - 4) = 0 \quad \text{Factor difference of two squares.}$$
$$3x^2 = 0 \Rightarrow x = 0 \quad \text{Set 1st factor equal to 0.}$$
$$x + 4 = 0 \Rightarrow x = -4 \quad \text{Set 2nd factor equal to 0.}$$
$$x - 4 = 0 \Rightarrow x = 4 \quad \text{Set 3rd factor equal to 0.}$$

Check these solutions by substituting in the original equation.

**Check**

$$3(0)^4 \stackrel{?}{=} 48(0)^2 \Rightarrow 0 = 0 \quad \text{0 checks. ✓}$$
$$3(-4)^4 \stackrel{?}{=} 48(-4)^2 \Rightarrow 768 = 768 \quad -4 \text{ checks. ✓}$$
$$3(4)^4 \stackrel{?}{=} 48(4)^2 \Rightarrow 768 = 768 \quad \text{4 checks. ✓}$$

So, the solutions are

$x = 0, \quad x = -4, \quad$ and $\quad x = 4.$

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $9x^4 - 12x^2 = 0$.

### EXAMPLE 11 Solving a Polynomial Equation by Factoring

Solve $x^3 - 3x^2 - 3x + 9 = 0$.

**Solution**

$$x^3 - 3x^2 - 3x + 9 = 0 \quad \text{Write original equation.}$$
$$x^2(x - 3) - 3(x - 3) = 0 \quad \text{Group terms and factor.}$$
$$(x - 3)(x^2 - 3) = 0 \quad (x - 3) \text{ is a common factor.}$$
$$x - 3 = 0 \Rightarrow x = 3 \quad \text{Set 1st factor equal to 0.}$$
$$x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3} \quad \text{Set 2nd factor equal to 0.}$$

The solutions are $x = 3$, $x = \sqrt{3}$, and $x = -\sqrt{3}$. Check these in the original equation.

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve each equation.

**a.** $x^3 - 5x^2 - 2x + 10 = 0$

**b.** $6x^3 - 27x^2 - 54x = 0$

GO DIGITAL

## Radical Equations

A **radical equation** is an equation that involves one or more radical expressions. A radical equation can often be cleared of radicals by raising each side of the equation to an appropriate power. This procedure may introduce extraneous solutions, so checking your solutions is crucial.

### EXAMPLE 12 Solving Radical Equations

**a.**

| | |
|---|---|
| $\sqrt{2x+7} - x = 2$ | Original equation |
| $\sqrt{2x+7} = x + 2$ | Isolate radical. |
| $2x + 7 = x^2 + 4x + 4$ | Square each side. |
| $0 = x^2 + 2x - 3$ | Write in general form. |
| $0 = (x+3)(x-1)$ | Factor. |
| $x + 3 = 0 \Rightarrow x = -3$ | Set 1st factor equal to 0. |
| $x - 1 = 0 \Rightarrow x = 1$ | Set 2nd factor equal to 0. |

Checking these values shows that the only solution is $x = 1$.

**b.**

| | |
|---|---|
| $\sqrt{2x-5} - \sqrt{x-3} = 1$ | Original equation |
| $\sqrt{2x-5} = \sqrt{x-3} + 1$ | Isolate $\sqrt{2x-5}$. |
| $2x - 5 = x - 3 + 2\sqrt{x-3} + 1$ | Square each side. |
| $x - 3 = 2\sqrt{x-3}$ | Isolate $2\sqrt{x-3}$. |
| $x^2 - 6x + 9 = 4(x-3)$ | Square each side. |
| $x^2 - 10x + 21 = 0$ | Write in general form. |
| $(x-3)(x-7) = 0$ | Factor. |
| $x - 3 = 0 \Rightarrow x = 3$ | Set 1st factor equal to 0. |
| $x - 7 = 0 \Rightarrow x = 7$ | Set 2nd factor equal to 0. |

The solutions are $x = 3$ and $x = 7$. Check these in the original equation.

**ALGEBRA HELP**

When an equation contains two radical expressions, it may not be possible to isolate both of them in the first step. In such cases, you may have to isolate radical expressions at *two* different stages in the solution, as shown in Example 12(b).

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $-\sqrt{40 - 9x} + 2 = x$.

**SKILLS REFRESHER**

For a refresher on how to simplify radical expressions with real numbers, watch the video at *LarsonPrecalculus.com.*

### EXAMPLE 13 Solving an Equation Involving a Rational Exponent

| | |
|---|---|
| $(x-4)^{2/3} = 25$ | Original equation |
| $\sqrt[3]{(x-4)^2} = 25$ | Rewrite in radical form. |
| $(x-4)^2 = 15{,}625$ | Cube each side. |
| $x - 4 = \pm 125$ | Extract square roots. |
| $x = 129, \ x = -121$ | Add 4 to each side. |

The solutions are $x = 129$ and $x = -121$. Check these in the original equation.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $(x-5)^{2/3} = 16$.

GO DIGITAL

## Absolute Value Equations

An **absolute value equation** is an equation that involves one or more absolute value expressions. To solve an absolute value equation, remember that the expression inside the absolute value bars can be positive or negative. This results in *two* separate equations, each of which must be solved. For example, the equation $|x - 2| = 3$ results in the two equations $x - 2 = 3$ and $-(x - 2) = 3$, which implies that the original equation has two solutions: $x = 5$ and $x = -1$.

### EXAMPLE 14 Solving an Absolute Value Equation

Solve $|x^2 - 3x| = -4x + 6$.

**Solution**

The variable expression inside the absolute value signs can be positive or negative, so you must solve the two *quadratic* equations

$$x^2 - 3x = -4x + 6 \quad \text{and} \quad -(x^2 - 3x) = -4x + 6.$$

*First Equation*

$$\begin{aligned} x^2 - 3x &= -4x + 6 && \text{Use positive expression.} \\ x^2 + x - 6 &= 0 && \text{Write in general form.} \\ (x + 3)(x - 2) &= 0 && \text{Factor.} \\ x + 3 &= 0 \implies x = -3 && \text{Set 1st factor equal to 0.} \\ x - 2 &= 0 \implies x = 2 && \text{Set 2nd factor equal to 0.} \end{aligned}$$

*Second Equation*

$$\begin{aligned} -(x^2 - 3x) &= -4x + 6 && \text{Use negative expression.} \\ x^2 - 7x + 6 &= 0 && \text{Write in general form.} \\ (x - 1)(x - 6) &= 0 && \text{Factor.} \\ x - 1 &= 0 \implies x = 1 && \text{Set 1st factor equal to 0.} \\ x - 6 &= 0 \implies x = 6 && \text{Set 2nd factor equal to 0.} \end{aligned}$$

Check the values in the original equation to determine that the only solutions are $x = -3$ and $x = 1$.

**✓ Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $|x^2 + 4x| = 7x + 18$. ■

### Summarize (Section P.2)

1. State the definitions of an identity, a conditional equation, and a contradiction *(page 14)*.
2. State the definition of a linear equation in one variable *(page 14)*. For examples of solving linear equations and rational equations, see Examples 1–3.
3. List the four methods for solving quadratic equations discussed in this section *(page 17)*. For examples of solving quadratic equations, see Examples 4–9.
4. Explain how to solve a polynomial equation of degree three or greater *(page 21)*, a radical equation *(page 22)*, and an absolute value equation *(page 23)*. For examples of solving these types of equations, see Examples 10–14.

GO DIGITAL

# P.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

GO DIGITAL

## Vocabulary and Concept Check

**In Exercises 1 and 2, fill in the blank.**

1. An ________ is a statement that equates two algebraic expressions.
2. A linear equation in one variable $x$ is an equation that can be written in the standard form ________.
3. List four methods that can be used to solve a quadratic equation.
4. Describe the step needed to remove the radical from the equation $\sqrt{x+2} = x$.

## Skills and Applications

**Solving a Linear Equation** **In Exercises 5–12, solve the equation and check your solution. (If not possible, explain why.)**

5. $2x + 11 = 15$
6. $7x + 2 = 23$
7. $7 - 2x = 25$
8. $7 - x = 19$
9. $3x - 5 = 2x + 7$
10. $4y + 2 - 5y = 7 - 6y$
11. $x - 3(2x + 3) = 8 - 5x$
12. $9x - 10 = 5x + 2(2x - 5)$

**Solving a Rational Equation** **In Exercises 13–24, solve the equation and check your solution. (If not possible, explain why.)**

13. $\dfrac{3x}{8} - \dfrac{4x}{3} = 4$
14. $\dfrac{x}{5} - \dfrac{x}{2} = 3 + \dfrac{3x}{10}$
15. $\dfrac{5x-4}{5x+4} = \dfrac{2}{3}$
16. $\dfrac{10x+3}{5x+6} = \dfrac{1}{2}$
17. $10 - \dfrac{13}{x} = 4 + \dfrac{5}{x}$
18. $\dfrac{1}{x} + \dfrac{2}{x-5} = 0$
19. $\dfrac{x}{x+4} + \dfrac{4}{x+4} = -2$
20. $\dfrac{7}{2x+1} - \dfrac{8x}{2x-1} = -4$
21. $\dfrac{2}{(x-4)(x-2)} = \dfrac{1}{x-4} + \dfrac{2}{x-2}$
22. $\dfrac{12}{(x-1)(x+3)} = \dfrac{3}{x-1} + \dfrac{2}{x+3}$
23. $\dfrac{1}{x-3} + \dfrac{1}{x+3} = \dfrac{10}{x^2-9}$
24. $\dfrac{1}{x-2} + \dfrac{3}{x+3} = \dfrac{4}{x^2+x-6}$

**Solving a Quadratic Equation by Factoring** **In Exercises 25–34, solve the quadratic equation by factoring.**

25. $6x^2 + 3x = 0$
26. $8x^2 - 2x = 0$
27. $x^2 + 10x + 25 = 0$
28. $x^2 - 2x - 8 = 0$
29. $3 + 5x - 2x^2 = 0$
30. $4x^2 + 12x + 9 = 0$
31. $16x^2 - 9 = 0$
32. $-x^2 + 4x = 3$
33. $\frac{3}{4}x^2 + 8x + 20 = 0$
34. $\frac{1}{8}x^2 - x - 16 = 0$

**Extracting Square Roots** **In Exercises 35–44, solve the equation by extracting square roots. When a solution is irrational, list both the exact solution *and* its approximation rounded to two decimal places.**

35. $x^2 = 49$
36. $x^2 = 43$
37. $3x^2 = 81$
38. $9x^2 = 36$
39. $(x - 4)^2 = 49$
40. $(x + 9)^2 = 24$
41. $(2x - 1)^2 = 18$
42. $(4x + 7)^2 = 44$
43. $(x - 7)^2 = (x + 3)^2$
44. $(x + 5)^2 = (x + 4)^2$

**Completing the Square** **In Exercises 45–54, solve the quadratic equation by completing the square.**

45. $x^2 + 4x - 32 = 0$
46. $x^2 - 2x - 3 = 0$
47. $x^2 + 4x + 2 = 0$
48. $x^2 + 8x + 14 = 0$
49. $6x^2 - 12x = -3$
50. $4x^2 - 4x = 1$
51. $7 + 2x - x^2 = 0$
52. $-x^2 + x - 1 = 0$
53. $2x^2 + 5x - 8 = 0$
54. $3x^2 - 4x - 7 = 0$

**Using the Quadratic Formula** **In Exercises 55–68, use the Quadratic Formula to solve the equation.**

55. $2x^2 + x - 1 = 0$
56. $2x^2 - x - 1 = 0$
57. $9x^2 + 30x + 25 = 0$
58. $28x - 49x^2 = 4$
59. $2x^2 - 7x + 1 = 0$
60. $3x + x^2 - 1 = 0$
61. $4x^2 + 6x = 8$
62. $16x^2 + 5 = 40x$
63. $2 + 2x - x^2 = 0$
64. $x^2 + 10 + 8x = 0$
65. $8t = 5 + 2t^2$
66. $25h^2 + 80h = -61$
67. $(y - 5)^2 = 2y$
68. $(z + 6)^2 = -2z$

**Choosing a Method** **In Exercises 69–78, solve the equation using any convenient method.**

69. $x^2 - 2x - 1 = 0$
70. $14x^2 + 42x = 0$
71. $(x + 2)^2 = 64$
72. $x^2 - 14x + 49 = 0$
73. $x^2 - x - \frac{11}{4} = 0$
74. $x^2 + 3x - \frac{3}{4} = 0$
75. $3x + 4 = 2x^2 - 7$
76. $(x + 1)^2 = x^2$
77. $4x^2 + 2x + 4 = 2x + 8$
78. $a^2x^2 - b^2 = 0$, $a$ and $b$ are real numbers

### Solving a Polynomial Equation

**In Exercises 79–84, solve the equation. Check your solutions.**

**79.** $6x^4 - 54x^2 = 0$

**80.** $5x^3 + 30x^2 + 45x = 0$

**81.** $x^3 + 2x^2 - 8x = 16$

**82.** $x^3 - 3x^2 - x = -3$

**83.** $x^4 - 4x^2 + 3 = 0$

**84.** $x^4 - 13x^2 + 36 = 0$

### Solving a Radical Equation

**In Exercises 85–92, solve the equation, if possible. Check your solutions.**

**85.** $\sqrt{5x} - 10 = 0$

**86.** $\sqrt{3x + 1} = 7$

**87.** $4 + \sqrt[3]{2x - 9} = 0$

**88.** $\sqrt[3]{12 - x} - 3 = 0$

**89.** $\sqrt{x + 8} = 2 + x$

**90.** $2x = \sqrt{-5x + 24} - 3$

**91.** $\sqrt{x - 3} + 1 = \sqrt{x}$

**92.** $2\sqrt{x + 1} - \sqrt{2x + 3} = 1$

### Solving an Equation Involving a Rational Exponent

**In Exercises 93–96, solve the equation. Check your solutions.**

**93.** $(x - 5)^{3/2} = 8$

**94.** $(x^2 - x - 22)^{3/2} = 27$

**95.** $3x(x - 1)^{1/2} + 2(x - 1)^{3/2} = 0$

**96.** $4x^2(x - 1)^{1/3} + 6x(x - 1)^{4/3} = 0$

### Solving an Absolute Value Equation

**In Exercises 97–100, solve the equation. Check your solutions.**

**97.** $|2x - 5| = 11$

**98.** $|3x + 2| = 7$

**99.** $|x + 1| = x^2 - 5$

**100.** $|x - 15| = x^2 - 15x$

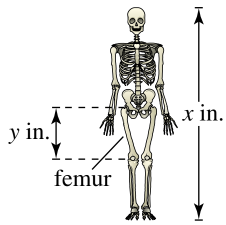
### Forensics

**In Exercises 101 and 102, use the following information. The relationship between the length of an adult's femur (thigh bone) and the height of the adult can be approximated by the linear equations**

$y = 0.514x - 14.75$ Female

$y = 0.532x - 17.03$ Male

**where $y$ is the length of the femur in inches and $x$ is the height of the adult in inches (see figure).**




**101.** A crime scene investigator discovers a femur belonging to an adult human female. The bone is 18 inches long. Estimate the height of the female.

**102.** Officials search a forest for a missing man who is 6 feet 3 inches tall. They find an adult male femur that is 23 inches long. Is it possible that the femur belongs to the missing man?



## Exploring the Concepts

### True or False?

**In Exercises 103 and 104, determine whether the statement is true or false. Justify your answer.**

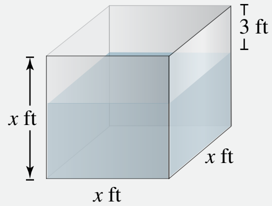
**103.** The equation $2(x + 3) = 3x + 3$ has no solution.

**104.** If $(2x - 3)(x + 5) = 8$, then either $2x - 3 = 8$ or $x + 5 = 8$.

**105. Error Analysis** Describe the error.

A quadratic equation that has solutions $x = 2$ and $x = 4$ is $(x + 2)(x + 4) = 0$ ⇨ $x^2 + 6x + 8 = 0$. ✗

**106. HOW DO YOU SEE IT?** The figure shows a glass cube partially filled with water.




(a) What does the expression $x^2(x - 3)$ represent?

(b) Given $x^2(x - 3) = 320$, explain how to find the volume of the cube.

**107. Think About It** Are $(3x + 2)/5 = 7$ and $x + 9 = 20$ equivalent equations? Explain.

**108. Think About It** To solve the equation $3x^2 = 3 - x$ using the Quadratic Formula, what are the values of $a$, $b$, and $c$?

## Review & Refresh

Video solutions at LarsonPrecalculus.com

### Plotting Points on the Real Number Line

**In Exercises 109 and 110, plot the real numbers on the real number line.**

**109.** (a) $3$ (b) $\frac{7}{2}$ (c) $-\frac{5}{2}$ (d) $-5.2$

**110.** (a) $8.5$ (b) $\frac{4}{3}$ (c) $-4.75$ (d) $-\frac{8}{3}$

### Simplifying Radical Expressions

**In Exercises 111 and 112, simplify the radical expression.**

**111.** $\sqrt{25 + 20}$

**112.** $\sqrt{284 + 321}$

### Evaluating an Expression

**In Exercises 113–116, evaluate the expression for each value of $x$. (If not possible, state the reason.)**

**113.** $4x - 6$ (a) $x = -1$ (b) $x = 0$

**114.** $9 - 7x$ (a) $x = -3$ (b) $x = 3$

**115.** $2x^3$ (a) $x = -3$ (b) $x = 0$

**116.** $-3x^{-4}$ (a) $x = 0$ (b) $x = -2$

### Simplifying an Expression

**In Exercises 117 and 118, simplify the expression.**

**117.** $-x^2 + (-x)^2$

**118.** $-x^3 + (-x)^3 + (-x)^4$

# P.3 The Cartesian Plane and Graphs of Equations

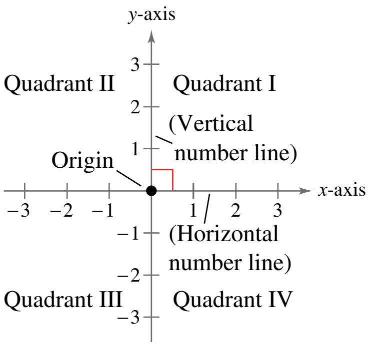
The Cartesian plane can help you visualize relationships between two variables. For example, in Exercise 44 on page 37, given how far north and west one city is from another, plotting points to represent the cities can help you visualize these distances and determine the flying distance between the cities.

- **Plot points in the Cartesian plane.**
- **Use the Distance Formula to find the distance between two points.**
- **Use the Midpoint Formula to find the midpoint of a line segment.**
- **Sketch graphs of equations.**
- **Find *x*- and *y*-intercepts of graphs of equations.**
- **Use symmetry to sketch graphs of equations.**
- **Write equations of circles.**

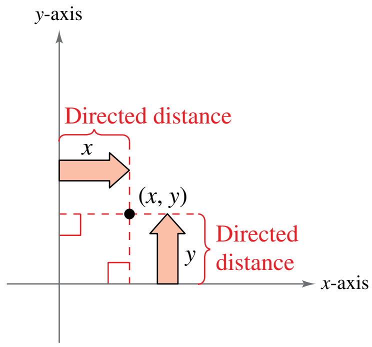
## The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** named after the French mathematician René Descartes (1596–1650).

Two real number lines intersecting at right angles form the Cartesian plane, as shown in Figure P.10. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four **quadrants.**




The Cartesian plane
**Figure P.10**




Ordered pair $(x, y)$
**Figure P.11**

GO DIGITAL

Each point in the plane corresponds to an **ordered pair** $(x, y)$ of real numbers $x$ and $y$, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the $y$-axis to the point, and the ***y*-coordinate** represents the directed distance from the $x$-axis to the point, as shown in Figure P.11.

The notation $(x, y)$ denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

### EXAMPLE 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

**Solution** To plot the point $(-1, 2)$, imagine a vertical line through $-1$ on the $x$-axis and a horizontal line through 2 on the $y$-axis. The intersection of these two lines is the point $(-1, 2)$. Plot the other four points in a similar way, as shown in Figure P.12.

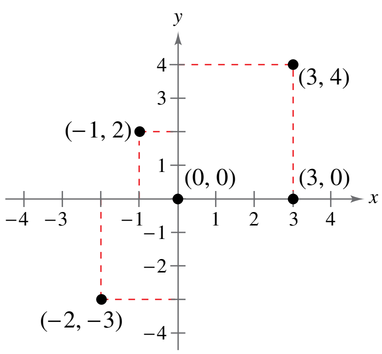



**Figure P.12**

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Plot the points $(-3, 2)$, $(4, -2)$, $(3, 1)$, $(0, -2)$, and $(-1, -2)$.

© Song_about_summer/Shutterstock.com

The beauty of a rectangular coordinate system is that it allows you to *see* relationships between two variables. It would be difficult to overestimate the importance of Descartes's introduction of coordinates in the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

In the next example, data are represented graphically by points plotted in a rectangular coordinate system. This type of graph is called a **scatter plot.**

## EXAMPLE 2 Sketching a Scatter Plot

The table shows the numbers $N$ (in millions) of AT&T wireless subscribers from 2013 through 2018, where $t$ represents the year. Sketch a scatter plot of the data. *(Source: AT&T Inc.)*

| DATA Year, $t$ | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|
| Subscribers, $N$ | 110 | 121 | 129 | 134 | 141 | 153 |

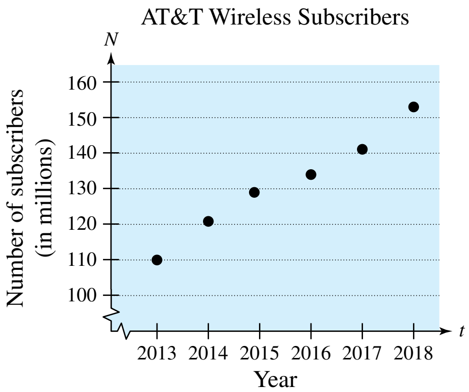
Spreadsheet at LarsonPrecalculus.com

### Solution

Before sketching the scatter plot, represent each pair of values in the table by an ordered pair $(t, N)$, as shown below.

(2013, 110), (2014, 121), (2015, 129), (2016, 134), (2017, 141), (2018, 153)

To sketch the scatter plot, first draw a vertical axis to represent the number of subscribers (in millions) and a horizontal axis to represent the year. Then plot a point for each ordered pair, as shown in the figure below. In the scatter plot, the break in the $t$-axis indicates omission of the numbers less than 2013, and the break in the $N$-axis indicates omission of the numbers less than 100 million. Also, the scatter plot shows that the number of subscribers has increased each year since 2013.




**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

The table shows the numbers $N$ of Costco stores from 2014 through 2019, where $t$ represents the year. Sketch a scatter plot of the data. *(Source: Costco Wholesale Corp.)*

| DATA Year, $t$ | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
|---|---|---|---|---|---|---|
| Stores, $N$ | 663 | 686 | 715 | 741 | 762 | 782 |

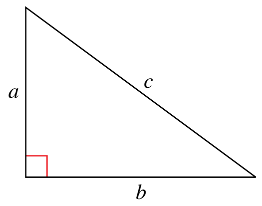
Spreadsheet at LarsonPrecalculus.com

Another way to make the scatter plot in Example 2 is to let $t = 1$ represent the year 2013. In this scatter plot, the horizontal axis does not have a break, and the labels for the tick marks are 1 through 6 (instead of 2013 through 2018).

GO DIGITAL

### TECHNOLOGY

The scatter plot in Example 2 is only one way to represent the data graphically. You could also represent the data using a bar graph or a line graph. Use a graphing utility to represent the data given in Example 2 graphically.




The Pythagorean Theorem:
$a^2 + b^2 = c^2$
**Figure P.13**

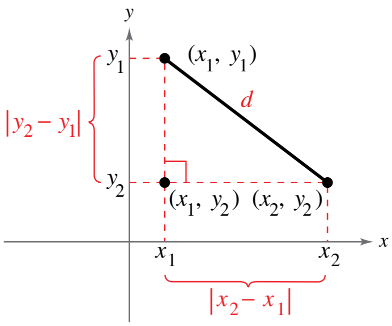



**Figure P.14**

## The Distance Formula

Before developing the Distance Formula, recall from the **Pythagorean Theorem** that, for a right triangle with hypotenuse of length $c$ and sides of lengths $a$ and $b$, you have

$$a^2 + b^2 = c^2 \qquad \text{Pythagorean Theorem}$$

as shown in Figure P.13. (The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

Consider two points $(x_1, y_1)$ and $(x_2, y_2)$ that do not lie on the same horizontal or vertical line in the plane. With these two points, you can form a right triangle (see Figure P.14). To determine the distance $d$ between these two points, note that the length of the vertical side of the triangle is $|y_2 - y_1|$ and the length of the horizontal side is $|x_2 - x_1|$. By the Pythagorean Theorem,

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2 \qquad \text{Pythagorean Theorem}$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \qquad \text{Distance } d \text{ must be positive.}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \qquad \text{Property of exponents}$$

This result is the **Distance Formula.** Note that for the special case in which the two points lie on the same horizontal or vertical line, the Distance Formula still works.

### The Distance Formula

The distance $d$ between the points $(x_1, y_1)$ and $(x_2, y_2)$ in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

## EXAMPLE 3 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

### Algebraic Solution

Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{Distance Formula}$$

$$= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} \qquad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= \sqrt{(5)^2 + (3)^2} \qquad \text{Simplify.}$$

$$= \sqrt{34} \qquad \text{Simplify.}$$

$$\approx 5.83 \qquad \text{Use a calculator.}$$

So, the distance between the points is about 5.83 units.

### Check

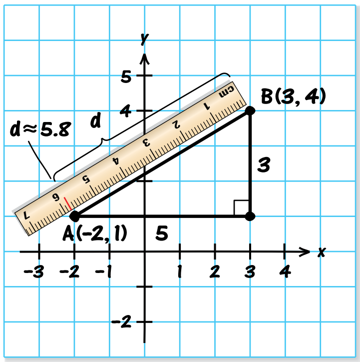
$$d^2 \stackrel{?}{=} 5^2 + 3^2 \qquad \text{Pythagorean Theorem}$$

$$(\sqrt{34})^2 \stackrel{?}{=} 5^2 + 3^2 \qquad \text{Substitute for } d.$$

$$34 = 34 \qquad \text{Distance checks. ✓}$$

### Graphical Solution

Use centimeter graph paper to plot the points $A(-2, 1)$ and $B(3, 4)$. Carefully sketch the line segment from $A$ to $B$. Then use a centimeter ruler to measure the length of the segment.




The line segment measures about 5.8 centimeters. So, the distance between the points is about 5.8 units.

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the distance between the points $(3, 1)$ and $(-3, 0)$.

GO DIGITAL

When the Distance Formula is used, it does not matter which point is $(x_1, y_1)$ and which is $(x_2, y_2)$, because the result will be the same. For instance, in Example 3, let $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-2, 1)$. Then

$$d = \sqrt{(-2-3)^2 + (1-4)^2} = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34} \approx 5.83.$$

## EXAMPLE 4 Verifying a Right Triangle

Show that the points (2, 1), (4, 0), and (5, 7) are vertices of a right triangle.

### Solution

The three points are plotted in Figure P.15. Use the Distance Formula to find the lengths of the three sides.

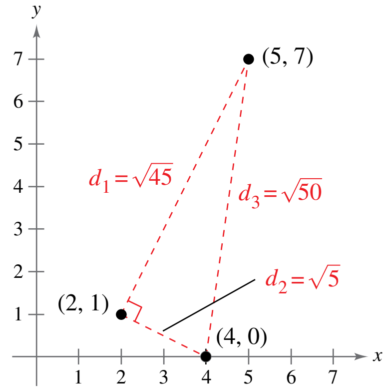


**Figure P.15**

$$d_1 = \sqrt{(5-2)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45}$$
$$d_2 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$$
$$d_3 = \sqrt{(5-4)^2 + (7-0)^2} = \sqrt{1+49} = \sqrt{50}$$

Using the converse of the Pythagorean Theorem and the fact that

$$(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$$

you can conclude that the triangle is a right triangle.

**✓ Checkpoint** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Show that the points $(2, -1)$, $(5, 5)$, and $(6, -3)$ are vertices of a right triangle.

## EXAMPLE 5 Finding the Length of a Pass

A football quarterback throws a pass from the 28-yard line, 40 yards from the sideline. A wide receiver catches the pass on the 5-yard line, 20 yards from the same sideline, as shown in Figure P.16. How long is the pass?

### Solution

The length of the pass is the distance between the points (40, 28) and (20, 5).

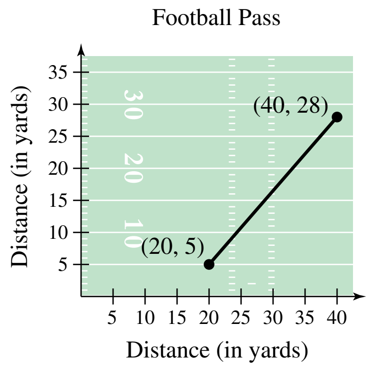


**Figure P.16**

| | |
|---|---|
| $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | Distance Formula |
| $= \sqrt{(40 - 20)^2 + (28 - 5)^2}$ | Substitute for $x_1$, $y_1$, $x_2$, and $y_2$. |
| $= \sqrt{20^2 + 23^2}$ | Simplify. |
| $= \sqrt{400 + 529}$ | Simplify. |
| $= \sqrt{929}$ | Simplify. |
| $\approx 30$ | Use a calculator. |

So, the pass is about 30 yards long.

**✓ Checkpoint** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

A football quarterback throws a pass from the 10-yard line, 10 yards from the sideline. A wide receiver catches the pass on the 32-yard line, 25 yards from the same sideline. How long is the pass? ■

In Example 5, the horizontal and vertical scales do not normally appear on a football field. When you use coordinate geometry to solve real-life problems, however, you may place the coordinate system in any way that helps you solve the problem.

GO DIGITAL

## The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula.**

### The Midpoint Formula

The midpoint of the line segment joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

For a proof of the Midpoint Formula, see Proofs in Mathematics on page 118.

### EXAMPLE 6 Finding the Midpoint of a Line Segment

Find the midpoint of the line segment joining the points

$(-5, -3)$ and $(9, 3)$.

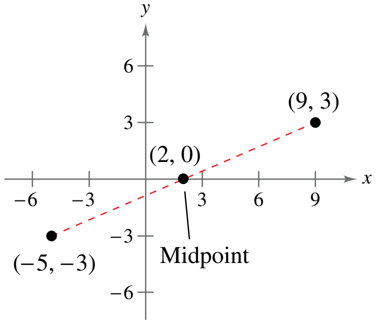



**Figure P.17**

**Solution** Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula}\\ &= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.\\ &= (2, 0) && \text{Simplify.}\end{aligned}$$

The midpoint of the line segment is $(2, 0)$, as shown in Figure P.17.

**✓ *Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

Find the midpoint of the line segment joining the points $(-2, 8)$ and $(4, -10)$.

### EXAMPLE 7 Estimating Annual Revenues

Microsoft Corp. had annual revenues of about \$96.7 billion in 2017 and about \$125.8 billion in 2019. Estimate the revenues in 2018. *(Source: Microsoft Corp.)*

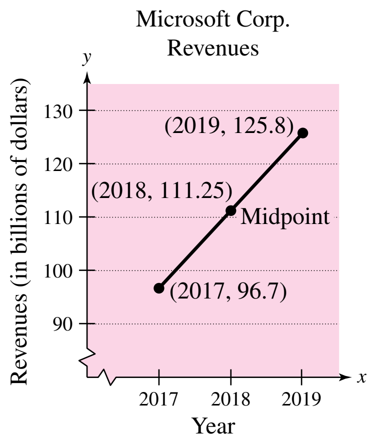



**Figure P.18**

**Solution** One way to solve this problem is to assume that the revenues followed a *linear* pattern. Then, to estimate the 2018 revenues, find the midpoint of the line segment connecting the points (2017, 96.7) and (2019, 125.8).

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula}\\ &= \left(\frac{2017 + 2019}{2}, \frac{96.7 + 125.8}{2}\right) && \text{Substitute for } x_1, x_2, y_1, \text{ and } y_2.\\ &= (2018, 111.25) && \text{Simplify.}\end{aligned}$$

So, you would estimate the 2018 revenues to have been about \$111.25 billion, as shown in Figure P.18. (The actual 2018 revenues were about \$110.36 billion.)

**✓ *Checkpoint*** ▶ ***Audio-video solution in English & Spanish at LarsonPrecalculus.com***

The Proctor & Gamble Co. had annual sales of about \$65.1 billion in 2017 and about \$67.7 billion in 2019. Estimate the sales in 2018. *(Source: Proctor & Gamble Co.)*

GO DIGITAL

## The Graph of an Equation

Earlier in this section, you used a coordinate system to graphically represent the relationship between two quantities as points in a coordinate plane. Next, you will review some basic procedures for sketching the graph of an *equation in two variables*.

Frequently, a relationship between two quantities is expressed as an **equation in two variables.** For example, $y = 7 - 3x$ is an equation in $x$ and $y$. An ordered pair $(a, b)$ is a **solution** or **solution point** of an equation in $x$ and $y$ when the substitutions $x = a$ and $y = b$ result in a true statement. For example, $(1, 4)$ is a solution of $y = 7 - 3x$ because $4 = 7 - 3(1)$ is a true statement.

The **graph of an equation** is the set of all points that are solutions of the equation. The basic technique used for sketching the graph of an equation is the **point-plotting method.** To sketch a graph using the point-plotting method, first, when possible, rewrite the equation so that one of the variables is isolated on one side of the equation. Next, construct a table of values showing several solution points. Then, plot these points in a rectangular coordinate system. Finally, connect the points with a smooth curve or line.

### TECHNOLOGY

To graph an equation involving $x$ and $y$ on a graphing utility, use the procedure below.

1. If necessary, rewrite the equation so that $y$ is isolated on the left side.
2. Enter the equation in the graphing utility.
3. Determine a *viewing window* that shows all important features of the graph.
4. Graph the equation.

### EXAMPLE 8 Sketching the Graph of an Equation

*See LarsonPrecalculus.com for an interactive version of this type of example.*

Sketch the graph of $y = x^2 - 2$.

**Solution**

The equation is already solved for $y$, so begin by constructing a table of values.

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $y = x^2 - 2$ | $2$ | $-1$ | $-2$ | $-1$ | $2$ | $7$ |
| $(x, y)$ | $(-2, 2)$ | $(-1, -1)$ | $(0, -2)$ | $(1, -1)$ | $(2, 2)$ | $(3, 7)$ |

Next, plot the points given in the table, as shown in Figure P.19(a). Finally, connect the points with a smooth curve, as shown in Figure P.19(b).

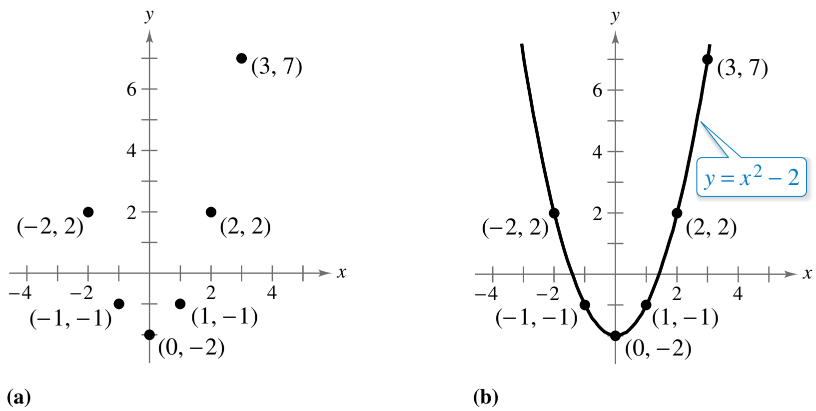



**Figure P.19**

### ALGEBRA HELP

One of your goals in this course is to learn to classify the basic shape of a graph from its equation. For instance, you will learn that a *linear equation* can be written in the form

$$y = mx + b$$

and its graph is a line. Similarly, the *quadratic equation* in Example 8 has the form

$$y = ax^2 + bx + c$$

and its graph is a *parabola*.

**Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Sketch the graph of each equation.

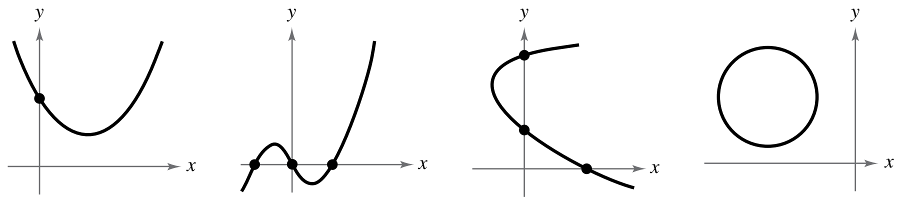
**a.** $y = x^2 + 3$

**b.** $y = 1 - x^2$

GO DIGITAL

## Intercepts of a Graph

Solution points of an equation that have zero as either the $x$-coordinate or the $y$-coordinate are called **intercepts.** They are the points at which the graph intersects or touches the $x$- or $y$-axis. It is possible for a graph to have no intercepts, one intercept, or several intercepts, as shown in the graphs below.




No $x$-intercepts
One $y$-intercept

Three $x$-intercepts
One $y$-intercept

One $x$-intercept
Two $y$-intercepts

No intercepts

Note that an $x$-intercept can be written as the ordered pair $(a, 0)$ and a $y$-intercept can be written as the ordered pair $(0, b)$. Sometimes it is convenient to denote the $x$-intercept as the $x$-coordinate $a$ of the point $(a, 0)$ or the $y$-intercept as the $y$-coordinate $b$ of the point $(0, b)$. Unless it is necessary to make a distinction, the term *intercept* will refer to either the point or the coordinate.

### Finding Intercepts

1. To find $x$-intercepts, let $y$ be zero and solve the equation for $x$.
2. To find $y$-intercepts, let $x$ be zero and solve the equation for $y$.

### EXAMPLE 9 Finding $x$- and $y$-Intercepts

Find the $x$- and $y$-intercepts of the graph of $y = x^3 - 4x$.

**Solution** To find the $x$-intercepts of the graph of $y = x^3 - 4x$, let $y = 0$ and solve for $x$.

$y = x^3 - 4x$ Write original equation.

$0 = x^3 - 4x$ Substitute 0 for $y$.

$0 = x(x^2 - 4)$ Factor out common factor.

$0 = x(x + 2)(x - 2)$ Factor difference of two squares.

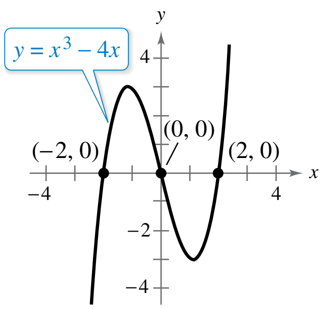
The solutions of this equation are $x = 0, -2$, and 2. So, the $x$-intercepts are $(0, 0)$, $(-2, 0)$, and $(2, 0)$. To find the $y$-intercept, let $x = 0$ and solve for $y$.

$y = x^3 - 4x$ Write original equation.

$y = (0)^3 - 4(0)$ Substitute 0 for $x$.

$y = 0$ Simplify.

This equation has one solution, $y = 0$. So, the $y$-intercept is $(0, 0)$. Check each intercept by sketching a graph, as shown at the right.



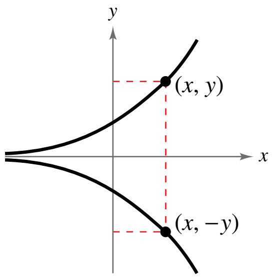

**Checkpoint** Audio-video solution in English & Spanish at LarsonPrecalculus.com

Find the $x$- and $y$-intercepts of the graph of $y = -x^2 - 5x$.

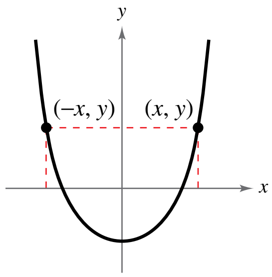
GO DIGITAL

## Symmetry

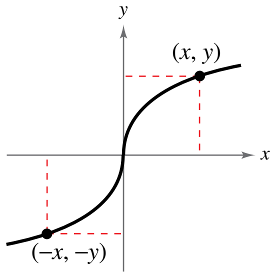
Graphs of equations can have **symmetry** with respect to one of the coordinate axes or with respect to the origin. Symmetry with respect to the $x$-axis means that when you fold the Cartesian plane along the $x$-axis, the portion of the graph above the $x$-axis coincides with the portion below the $x$-axis. Symmetry with respect to the $y$-axis or the origin can be described in a similar manner. The graphs below show these three types of symmetry.



$x$-Axis symmetry



$y$-Axis symmetry



Origin symmetry

Knowing the symmetry of a graph *before* attempting to sketch it is helpful because you need only half as many solution points to sketch the graph. Graphical and algebraic tests for these three basic types of symmetry are described below.

### Tests for Symmetry

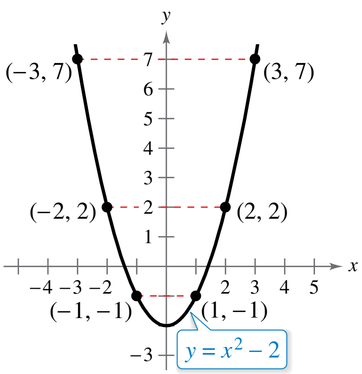
**Graphical**

1. A graph is **symmetric with respect to the $x$-axis** if, whenever $(x, y)$ is on the graph, $(x, -y)$ is also on the graph.
2. A graph is **symmetric with respect to the $y$-axis** if, whenever $(x, y)$ is on the graph, $(-x, y)$ is also on the graph.
3. A graph is **symmetric with respect to the origin** if, whenever $(x, y)$ is on the graph, $(-x, -y)$ is also on the graph.

**Algebraic**

1. The graph of an equation is **symmetric with respect to the $x$-axis** when replacing $y$ with $-y$ yields an equivalent equation.
2. The graph of an equation is **symmetric with respect to the $y$-axis** when replacing $x$ with $-x$ yields an equivalent equation.
3. The graph of an equation is **symmetric with respect to the origin** when replacing $x$ with $-x$ and $y$ with $-y$ yields an equivalent equation.

GO DIGITAL



$y$-Axis symmetry
**Figure P.20**

Using the graphical tests for symmetry, the graph of $y = x^2 - 2$ is symmetric with respect to the $y$-axis because $(x, y)$ and $(-x, y)$ are on its graph, as shown in Figure P.20. To verify this algebraically, replace $x$ with $-x$ in $y = x^2 - 2$

$$y = (-x)^2 - 2 = x^2 - 2$$ Replace $x$ with $-x$ in $y = x^2 - 2$.

and note that the result is an equivalent equation. To support this result numerically, create a table of values (see below).

| $x$ | $-3$ | $-2$ | $-1$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $y$ | $7$ | $2$ | $-1$ | $-1$ | $2$ | $7$ |
| $(x, y)$ | $(-3, 7)$ | $(-2, 2)$ | $(-1, -1)$ | $(1, -1)$ | $(2, 2)$ | $(3, 7)$ |

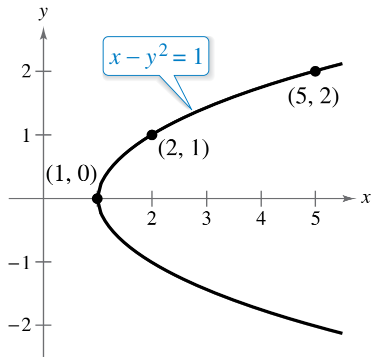



**Figure P.21**

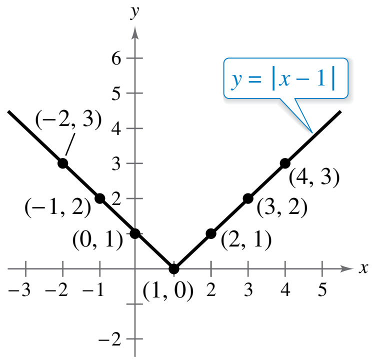



**Figure P.22**

## EXAMPLE 10 Using Symmetry as a Sketching Aid

Use symmetry to sketch the graph of $x - y^2 = 1$.

**Solution** Of the three tests for symmetry, the test for $x$-axis symmetry is the only one satisfied, because $x - (-y)^2 = 1$ is equivalent to $x - y^2 = 1$. So, the graph is symmetric with respect to the $x$-axis. Find solution points above (or below) the $x$-axis and then use symmetry to obtain the graph, as shown in Figure P.21.

***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use symmetry to sketch the graph of $y = x^2 - 4$.

## EXAMPLE 11 Sketching the Graph of an Equation

Sketch the graph of $y = |x - 1|$.

**Solution** This equation fails all three tests for symmetry, so its graph is not symmetric with respect to either axis or to the origin. The absolute value bars tell you that $y$ is always nonnegative. Construct a table of values. Then plot and connect the points, as shown in Figure P.22. Notice from the table that $x = 0$ when $y = 1$. So, the $y$-intercept is $(0, 1)$. Similarly, $y = 0$ when $x = 1$. So, the $x$-intercept is $(1, 0)$.

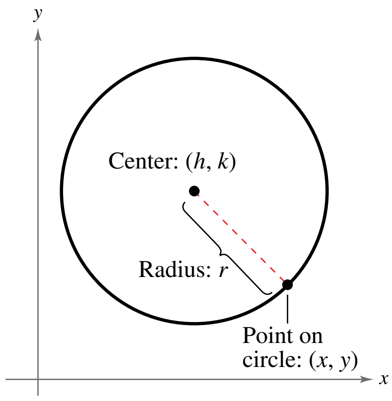
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|
| $y = \|x - 1\|$ | 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| $(x, y)$ | $(-2, 3)$ | $(-1, 2)$ | $(0, 1)$ | $(1, 0)$ | $(2, 1)$ | $(3, 2)$ | $(4, 3)$ |

***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Sketch the graph of $y = |x - 2|$.

## Circles

A **circle** is the set of all points in a plane that are the same distance from a fixed point. The fixed point is the **center** of the circle, and the distance between the center and a point on the circle is the **radius,** as shown in the figure.




Definition of a circle

You can use the Distance Formula to write an equation for the circle with center $(h, k)$ and radius $r$. Let $(x, y)$ be any point on the circle. Then the distance between $(x, y)$ and the center $(h, k)$ is $\sqrt{(x - h)^2 + (y - k)^2} = r$. By squaring each side of this equation, you obtain the **standard form of the equation of a circle.**

GO DIGITAL

### Standard Form of the Equation of a Circle

A point $(x, y)$ lies on the circle of **radius** $r$ and **center** $(h, k)$ if and only if

$$(x - h)^2 + (y - k)^2 = r^2.$$

**ALGEBRA HELP**

To find $h$ and $k$ from the standard form of the equation of a circle, you may want to rewrite one or both of the quantities in parentheses. For example, $x + 1 = x - (-1)$.

From this result, the standard form of the equation of a circle with radius $r$ *and center at the origin,* $(h, k) = (0, 0)$, is $x^2 + y^2 = r^2$. When $r = 1$, the circle is called the **unit circle.**

## EXAMPLE 12 Writing the Equation of a Circle

The point $(3, 4)$ lies on a circle whose center is at $(-1, 2)$, as shown in Figure P.23. Write the standard form of the equation of this circle.

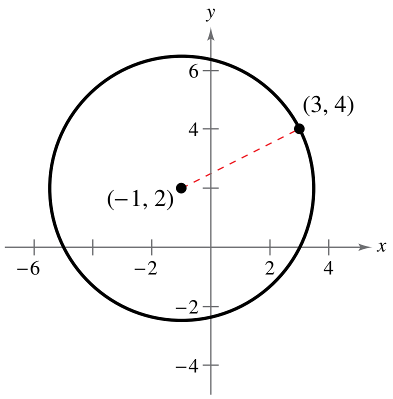



**Figure P.23**

### Solution

The radius of the circle is the distance between $(-1, 2)$ and $(3, 4)$.

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$ Distance Formula

$$= \sqrt{[3 - (-1)]^2 + (4 - 2)^2}$$ Substitute for $x$, $y$, $h$, and $k$.

$$= \sqrt{4^2 + 2^2}$$ Simplify.

$$= \sqrt{16 + 4}$$ Simplify.

$$= \sqrt{20}$$ Radius

Using $(h, k) = (-1, 2)$ and $r = \sqrt{20}$, the equation of the circle is

$$(x - h)^2 + (y - k)^2 = r^2$$ Equation of circle

$$[x - (-1)]^2 + (y - 2)^2 = \left(\sqrt{20}\right)^2$$ Substitute for $h$, $k$, and $r$.

$$(x + 1)^2 + (y - 2)^2 = 20.$$ Standard form

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

The point $(1, -2)$ lies on a circle whose center is at $(-3, -5)$. Write the standard form of the equation of this circle.

### Summarize (Section P.3)

1. Describe the Cartesian plane *(page 26)*. For examples of plotting points in the Cartesian plane, see Examples 1 and 2.
2. State the Distance Formula *(page 28)*. For examples of using the Distance Formula to find the distance between two points, see Examples 3–5.
3. State the Midpoint Formula *(page 30)*. For examples of using the Midpoint Formula to find the midpoint of a line segment, see Examples 6 and 7.
4. Explain how to sketch the graph of an equation *(page 31)*. For an example of sketching the graph of an equation, see Example 8.
5. Explain how to find the $x$- and $y$-intercepts of a graph *(page 32)*. For an example of finding $x$- and $y$-intercepts, see Example 9.
6. Explain how to use symmetry to graph an equation *(pages 33 and 34)*. For an example of using symmetry to graph an equation, see Example 10.
7. State the standard form of the equation of a circle *(page 35)*. For an example of writing the standard form of the equation of a circle, see Example 12.

GO DIGITAL

# P.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

GO DIGITAL

## Vocabulary and Concept Check

**In Exercises 1–6, fill in the blanks.**

**1.** An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the ________ plane.

**2.** Finding the average values of the respective coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the ________ ________.

**3.** An ordered pair $(a, b)$ is a ________ of an equation in $x$ and $y$ when the substitutions $x = a$ and $y = b$ result in a true statement.

**4.** The set of all solution points of an equation is the ________ of the equation.

**5.** The points at which a graph intersects or touches an axis are the ________ of the graph.

**6.** A graph is symmetric with respect to the ________ if, whenever $(x, y)$ is on the graph, $(-x, y)$ is also on the graph.

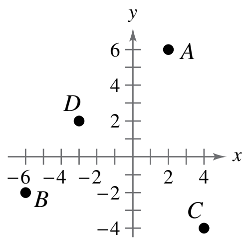
**7.** Define each of the following terms: *x-axis, y-axis, origin, quadrants, x-coordinate, y-coordinate*.

**8.** Explain how to use the Distance Formula to write an equation for the circle with center $(h, k)$ and radius $r$.

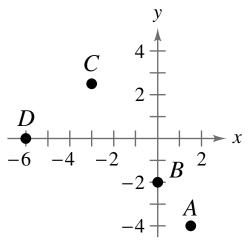
## Skills and Applications

**Approximating Coordinates of Points** **In Exercises 9 and 10, approximate the coordinates of the points.**

**9.**




**10.**




**Plotting Points in the Cartesian Plane** **In Exercises 11 and 12, plot the points.**

**11.** $(2, 4), (3, -1), (-6, 2), (-4, 0), (-1, -8), (1.5, -3.5)$

**12.** $(1, -5), (-2, -7), (3, 3), (-2, 4), (0, 5), \left(\frac{2}{3}, \frac{5}{2}\right)$

**Finding the Coordinates of a Point** **In Exercises 13 and 14, find the coordinates of the point.**

**13.** The point is three units to the left of the $y$-axis and four units above the $x$-axis.

**14.** The point is on the $x$-axis and 12 units to the left of the $y$-axis.

**Determining Quadrant(s) for a Point** **In Exercises 15–20, determine the quadrant(s) in which $(x, y)$ could be located.**

**15.** $x > 0$ and $y < 0$

**16.** $x < 0$ and $y < 0$

**17.** $x = -4$ and $y > 0$

**18.** $x < 0$ and $y = 7$

**19.** $x + y = 0, x \neq 0, y \neq 0$

**20.** $xy > 0$

**Sketching a Scatter Plot** **In Exercises 21 and 22, sketch a scatter plot of the data shown in the table.**

**21.** The table shows the numbers $y$ of Dollar General stores for each year $x$ from 2013 through 2019. *(Source: Dollar General Corporation)*

DATA — Spreadsheet at LarsonPrecalculus.com

| Year, $x$ | Number of Stores, $y$ |
|---|---|
| 2013 | 11,132 |
| 2014 | 11,789 |
| 2015 | 12,483 |
| 2016 | 13,320 |
| 2017 | 14,609 |
| 2018 | 15,472 |
| 2019 | 16,368 |

**22.** The table shows the annual revenues $y$ (in billions of dollars) for Amazon.com for each year $x$ from 2012 through 2019. *(Source: Amazon.com)*

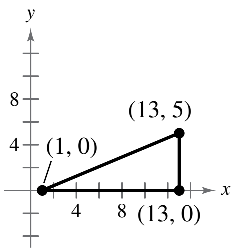
DATA — Spreadsheet at LarsonPrecalculus.com

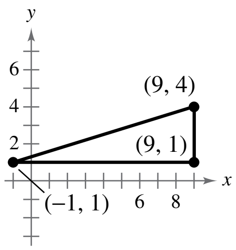
| Month, $x$ | Annual Revenue, $y$ |
|---|---|
| 2012 | 61.1 |
| 2013 | 74.5 |
| 2014 | 89.0 |
| 2015 | 107.0 |
| 2016 | 136.0 |
| 2017 | 177.9 |
| 2018 | 232.9 |
| 2019 | 280.5 |

**Finding a Distance** **In Exercises 23–28, find the distance between the points.**

23. $(-2, 6), (3, -6)$
24. $(8, 5), (0, 20)$
25. $(1, 4), (-5, -1)$
26. $(1, 3), (3, -2)$
27. $\left(\frac{1}{2}, \frac{4}{3}\right), (2, -1)$
28. $(9.5, -2.6), (-3.9, 8.2)$

**Verifying a Right Triangle** **In Exercises 29 and 30, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem.**

29.



30.


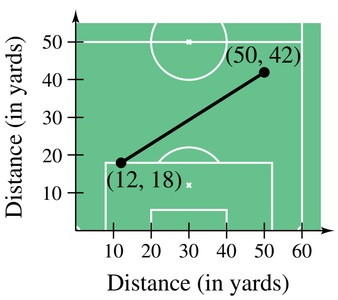

**Verifying a Polygon** **In Exercises 31–34, show that the points form the vertices of the polygon.**

31. Right triangle: $(4, 0), (2, 1), (-1, -5)$
32. Right triangle: $(-1, 3), (3, 5), (5, 1)$
33. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$
34. Isosceles triangle: $(2, 3), (4, 9), (-2, 7)$

**Plotting, Distance, and Midpoint** **In Exercises 35–42, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.**

35. $(6, -3), (6, 5)$
36. $(1, 4), (8, 4)$
37. $(1, 1), (9, 7)$
38. $(1, 12), (6, 0)$
39. $(-1, 2), (5, 4)$
40. $(2, 10), (10, 2)$
41. $(-16.8, 12.3), (5.6, 4.9)$
42. $\left(\frac{1}{2}, 1\right), \left(-\frac{5}{2}, \frac{4}{3}\right)$

43. **Sports** A soccer player passes the ball from a point that is 18 yards from the endline and 12 yards from the sideline. A teammate who is 42 yards from the same endline and 50 yards from the same sideline receives the pass. (See figure.) How long is the pass?






44. **Flying Distance**

An airplane flies from Naples, Italy, in a straight line to Rome, Italy, which is about 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?

45. **Sales** Walmart had sales of \$500.3 billion in 2017 and \$524.0 billion in 2019. Use the Midpoint Formula to estimate the sales in 2018. Assume that the sales followed a linear pattern. *(Source: Walmart, Inc.)*

46. **Earnings per Share** The earnings per share for Facebook, Inc. were \$7.57 in 2018 and \$8.56 in 2019. Use the Midpoint Formula to estimate the earnings per share in 2020. Assume that the earnings per share followed a linear pattern. *(Source: Facebook, Inc.)*

**Determining Solution Points** **In Exercises 47–50, determine whether each point lies on the graph of the equation.**

| | **Equation** | **Points** | |
|---|---|---|---|
| 47. | $y = x^2 - 3x + 2$ | (a) $(2, 0)$ | (b) $(-2, 8)$ |
| 48. | $y = \sqrt{x + 4}$ | (a) $(0, 2)$ | (b) $(5, 3)$ |
| 49. | $y = 4 - \lvert x - 2 \rvert$ | (a) $(1, 5)$ | (b) $(6, 0)$ |
| 50. | $2x^2 + 5y^2 = 8$ | (a) $(6, 0)$ | (b) $(0, 4)$ |

**Sketching the Graph of an Equation** **In Exercises 51–54, complete the table. Use the resulting solution points to sketch the graph of the equation.**

51. $y = -2x + 5$

| $x$ | $-1$ | $0$ | $1$ | $2$ | $\frac{5}{2}$ |
|---|---|---|---|---|---|
| $y$ | | | | | |
| $(x, y)$ | | | | | |

52. $y + 1 = \frac{3}{4}x$

| $x$ | $-2$ | $0$ | $1$ | $\frac{4}{3}$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |
| $(x, y)$ | | | | | |

53. $y + 3x = x^2$

| $x$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|
| $y$ | | | | | |
| $(x, y)$ | | | | | |

54. $y = 5 - x^2$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |
| $(x, y)$ | | | | | |

**Finding $x$- and $y$-Intercepts** In Exercises 55–64, find the $x$- and $y$-intercepts of the graph of the equation.

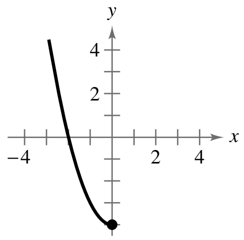
**55.** $y = 5x - 6$
**56.** $y = 8 - 3x$
**57.** $y = \sqrt{x + 4}$
**58.** $y = \sqrt{2x - 1}$
**59.** $y = |3x - 7|$
**60.** $y = -|x + 10|$
**61.** $y = 2x^3 - 4x^2$
**62.** $y = x^4 - 25$
**63.** $y^2 = 6 - x$
**64.** $y^2 = x + 1$

**Testing for Symmetry** In Exercises 65–72, use the algebraic tests to check for symmetry with respect to both axes and the origin.

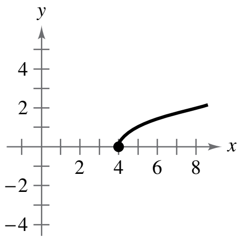
**65.** $x^2 - y = 0$
**66.** $x - y^2 = 0$
**67.** $y = x^3$
**68.** $y = x^4 - x^2 + 3$
**69.** $y = \dfrac{x}{x^2 + 1}$
**70.** $y = \dfrac{1}{x^2 + 1}$
**71.** $xy^2 + 10 = 0$
**72.** $xy = 4$

**Using Symmetry as a Sketching Aid** In Exercises 73–76, assume that the graph has the given type of symmetry. Complete the graph of the equation. To print an enlarged copy of the graph, go to *MathGraphs.com*.

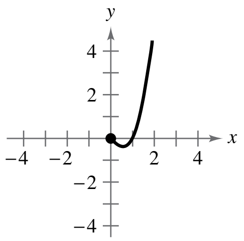
**73.** $y$-Axis symmetry



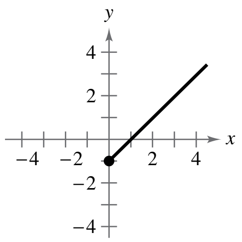

**74.** $x$-Axis symmetry




**75.** Origin symmetry




**76.** $y$-Axis symmetry




**Sketching the Graph of an Equation** In Exercises 77–88, test for symmetry and graph the equation. Then identify any intercepts.

**77.** $y = -3x + 1$
**78.** $y = 2x - 3$
**79.** $y = x^2 - 2x$
**80.** $y = -x^2 - 2x$
**81.** $y = x^3 + 3$
**82.** $y = x^3 - 1$
**83.** $y = \sqrt{x - 3}$
**84.** $y = \sqrt{1 - x}$
**85.** $y = |x - 6|$
**86.** $y = 1 - |x|$
**87.** $x = y^2 - 1$
**88.** $x = y^2 - 5$

**Writing the Equation of a Circle** In Exercises 89–96, write the standard form of the equation of the circle with the given characteristics.

**89.** Center: $(0, 0)$; Radius: 3
**90.** Center: $(0, 0)$; Radius: 7
**91.** Center: $(-4, 5)$; Radius: 2
**92.** Center: $(1, -3)$; Radius: $\sqrt{11}$
**93.** Center: $(3, 8)$; Solution point: $(-9, 13)$
**94.** Center: $(-2, -6)$; Solution point: $(1, -10)$
**95.** Endpoints of a diameter: $(3, 2)$, $(-9, -8)$
**96.** Endpoints of a diameter: $(11, -5)$, $(3, 15)$

**Sketching a Circle** In Exercises 97–102, find the center and radius of the circle with the given equation. Then sketch the circle.

**97.** $x^2 + y^2 = 25$
**98.** $x^2 + y^2 = 36$
**99.** $(x - 1)^2 + (y + 3)^2 = 9$
**100.** $x^2 + (y - 1)^2 = 1$
**101.** $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$
**102.** $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

**103. Depreciation** A hospital purchases a new magnetic resonance imaging (MRI) machine for \$1.2 million. The depreciated value $y$ (reduced value) after $t$ years is given by $y = 1{,}200{,}000 - 80{,}000t$, $0 \le t \le 10$. Sketch the graph of the equation.

**104. Depreciation** You purchase an all-terrain vehicle (ATV) for \$9500. The depreciated value $y$ (reduced value) after $t$ years is given by $y = 9500 - 1000t$, $0 \le t \le 6$. Sketch the graph of the equation.

**105. Electronics** The resistance $y$ (in ohms) of 1000 feet of solid copper wire at 68 degrees Fahrenheit is given by

$$y = \frac{10{,}370}{x^2}$$

where $x$ is the diameter of the wire in mils (0.001 inch).

(a) Complete the table.

| $x$ | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | |

(b) Use the table of values in part (a) to sketch a graph of the model. Then use your graph to estimate the resistance when $x = 85.5$.

(c) Use the model to confirm algebraically the estimate you found in part (b).

(d) What can you conclude about the relationship between the diameter of the copper wire and the resistance?

**106. Population Statistics** The table shows the life expectancies of a child (at birth) in the United States for selected years from 1950 through 2020. *(Source: Macrotrends LLC)*

| Year | Life Expectancy, $y$ |
|---|---|
| 1950 | 68.14 |
| 1960 | 69.84 |
| 1970 | 70.78 |
| 1980 | 73.70 |
| 1990 | 75.19 |
| 2000 | 76.75 |
| 2010 | 78.49 |
| 2020 | 78.93 |

Spreadsheet at LarsonPrecalculus.com

A model for the life expectancy during this period is $y = (68.0 + 0.33t)/(1 + 0.002t)$, $0 \le t \le 70$, where $y$ represents the life expectancy and $t$ is the time in years, with $t = 0$ corresponding to 1950.

(a) Use a graphing utility to graph the data from the table and the model in the same viewing window. How well does the model fit the data? Explain.

(b) Determine the life expectancy in 1990 both graphically and algebraically.

(c) Use the graph to determine the year when life expectancy was approximately 70.1. Verify your answer algebraically.

(d) Identify the $y$-intercept of the graph of the model. What does it represent in the context of the problem?

(e) Do you think this model can be used to predict the life expectancy of a child 50 years from now? Explain.

## Exploring the Concepts

**True or False?** **In Exercises 107 and 108, determine whether the statement is true or false. Justify your answer.**

**107.** The graph of a linear equation cannot be symmetric with respect to the origin.

**108.** If the point $(x, y)$ is in Quadrant II, then the point $(2x, -3y)$ is in Quadrant III.

**109. Think About It** When plotting points on the rectangular coordinate system, when should you use different scales for the $x$- and $y$-axes? Explain.

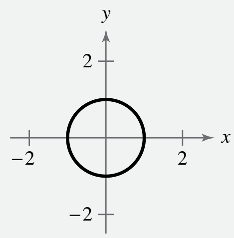
**110. Think About It** What is the $y$-coordinate of any point on the $x$-axis? What is the $x$-coordinate of any point on the $y$-axis?

**111. Error Analysis** Describe the error.

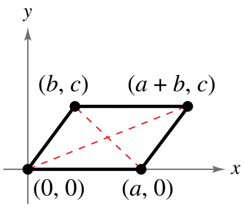
The graph of $x = 3y^2$ is symmetric with respect to the $y$-axis because $x = 3(-y)^2 = 3y^2$. ✗

The symbol indicates an exercise or a part of an exercise in which you are instructed to use a graphing utility.

**112. HOW DO YOU SEE IT?** The graph shows the circle with the equation $x^2 + y^2 = 1$. Describe the types of symmetry that you observe.




**113. Proof** Prove that the diagonals of the parallelogram in the figure intersect at their midpoints.




**114. Using the Midpoint Formula** A line segment has $(x_1, y_1)$ as one endpoint and $(x_m, y_m)$ as its midpoint. Find the other endpoint $(x_2, y_2)$ of the line segment in terms of $x_1$, $y_1$, $x_m$, and $y_m$. Then use the result to find the missing point.

(a) $(x_1, y_1) = (1, -2)$
$(x_m, y_m) = (4, -1)$
$(x_2, y_2) = $ ▭

(b) $(x_1, y_1) = (-5, 11)$
$(x_m, y_m) = (2, 4)$
$(x_2, y_2) = $ ▭

## Review & Refresh

Video solutions at LarsonPrecalculus.com

**Evaluating an Expression** **In Exercises 115–118, evaluate the expression. (If not possible, state the reason.)**

**115.** $\dfrac{5 - 7}{12 - 18}$ **116.** $\dfrac{16 - 6}{6 - 11}$

**117.** $\dfrac{3 - 3}{4 - 0}$ **118.** $\dfrac{1 - (-1)}{9 - 9}$

**Solving for $y$** **In Exercises 119–122, solve the equation for $y$.**

**119.** $3y + 5 = 0$ **120.** $2x - 3y = 5$

**121.** $5x + 2y = 8$ **122.** $x + 3y + 8 = 0$

**123. Cost, Revenue, and Profit** A manufacturer can produce and sell $x$ electronic devices per week. The total cost $C$ (in dollars) of producing $x$ electronic devices is $C = 93x + 35{,}000$, and the total revenue $R$ (in dollars) is $R = 135x$.

(a) Find the profit $P$ in terms of $x$.

(b) Find the profit obtained by selling 5000 electronic devices per week.

**124. Rate** A copier copies at a rate of 50 pages per minute.

(a) Find the time required to copy one page.

(b) Find the time required to copy $x$ pages.

(c) Find the time required to copy 10,000 pages.