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PRECALC with LIMITS

5e

Ron Larson

CalcChat® and CalcView®

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The Pennsylvania State University
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with CalcChat® and CalcView®
Fifth Edition
Ron Larson

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Contents

*Available at the text companion website *LarsonPrecalculus.com*

Preface

Welcome to *Precalc with Limits* with CalcChat® & CalcView®, Fifth Edition. I am excited to offer you a new edition with more resources than ever that will help you understand and master precalculus with limits. This text includes features and resources that continue to make *Precalc with Limits* a valuable learning tool for students and a trustworthy teaching tool for instructors.

Precalc with Limits provides the clear instruction, precise mathematics, and thorough coverage that you expect for your course. Additionally, this new edition provides you with **free** access to a variety of digital resources:

- **GO DIGITAL**—direct access to digital content on your mobile device or computer
- **CalcView.com**—video solutions to selected exercises
- **CalcChat.com**—worked-out solutions to odd-numbered exercises and access to online tutors
- **LarsonPrecalculus.com**—companion website with resources to supplement your learning

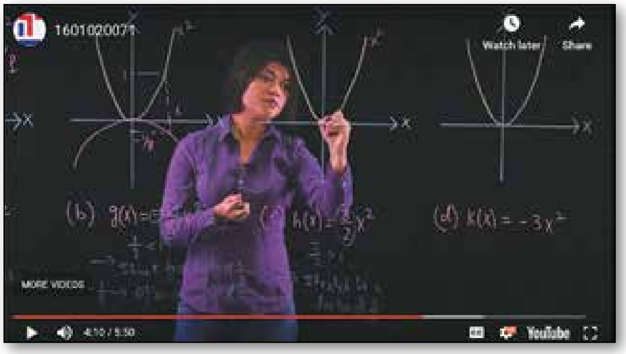
These digital resources will help enhance and reinforce your understanding of the material presented in this text and prepare you for future mathematics courses. CalcView® and CalcChat® are also available as free mobile apps.

Features

NEW GO DIGITAL

Scan the QR codes on the pages of this text to *GO DIGITAL* on your mobile device. This will give you easy access from anywhere to instructional videos, solutions to exercises and Checkpoint problems, Skills Refresher videos, Interactive Activities, and many other resources.

GO DIGITAL



UPDATED CalcView®

The website *CalcView.com* provides video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google Play™ store. You can access the video solutions by scanning the QR Code® at the beginning of the Section exercises, or visiting the *CalcView.com* website.

UPDATED CalcChat®

Solutions to all odd-numbered exercises and tests are provided for free at *CalcChat.com.* Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. For many years, millions of students have visited my site for help. The CalcChat mobile app is also available as a free download at the Apple® App Store® or Google Play™ store.

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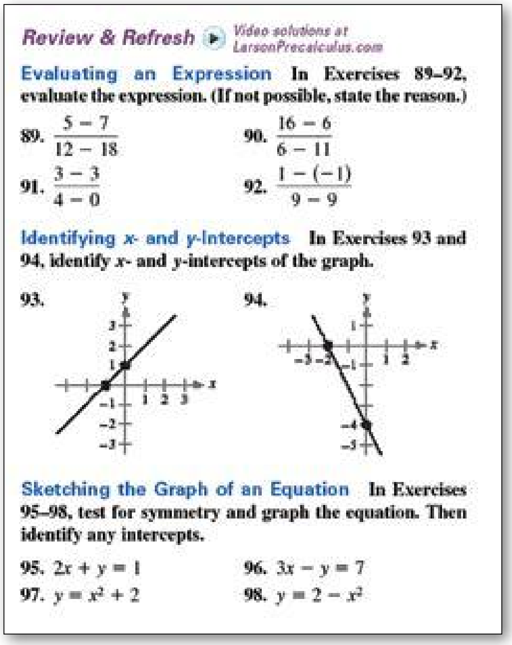
REVISED LarsonPrecalculus.com
All companion website features have been updated based on this revision, including two new features: Skills Refresher and Review & Refresh. Access to these features is free. You can view and listen to worked-out solutions of Checkpoint problems in English or Spanish, explore examples, download data sets, watch lesson videos, and much more.

NEW Skills Refresher
This feature directs you to an instructional video where you can review algebra skills needed to master the current topic. Scan the on-page code or go to *LarsonPrecalculus.com* to access the video.

SKILLS REFRESHER

For a refresher on finding the sum, difference, product, or quotient of two polynomials, watch the video at *LarsonPrecalculus.com*.



Review & Refresh *Video solutions at LarsonPrecalculus.com*

Evaluating an Expression In Exercises 89–92, evaluate the expression. (If not possible, state the reason.)

89. $\frac{5-7}{12-18}$ 90. $\frac{16-6}{6-11}$

91. $\frac{3-3}{4-0}$ 92. $\frac{1-(-1)}{9-9}$

Identifying x- and y-Intercepts In Exercises 93 and 94, identify x- and y-intercepts of the graph.

93. 94.

Sketching the Graph of an Equation In Exercises 95–98, test for symmetry and graph the equation. Then identify any intercepts.

95. $2x + y = 1$ 96. $3x - y = 7$

97. $y = x^2 + 2$ 98. $y = 2 - x^2$

NEW Review and Refresh
These exercises will help you to reinforce previously learned skills and concepts and to prepare for the next section. View and listen to worked-out solutions of the Review & Refresh exercises in English or Spanish by scanning the code on the first page of the section exercises or go to *LarsonPrecalculus.com*.

NEW Vocabulary and Concept Check
The Vocabulary and Concept Check appears at the beginning of the exercise set for each section. It includes fill-in-the-blank, matching, or non-computational questions designed to help you learn mathematical terminology and to test basic understanding of the concepts of the section.

NEW Summary and Study Strategies
The “What Did You Learn?” feature is a section-by-section overview that ties the learning objectives from the chapter to the Review Exercises for extra practice. The Study Strategies give concrete ways that you can use to help yourself with your study of mathematics.

REVISED Algebra Help
These notes reinforce or expand upon concepts, help you learn how to study mathematics, address special cases, or show alternative or additional steps to a solution of an example.

REVISED Exercise Sets

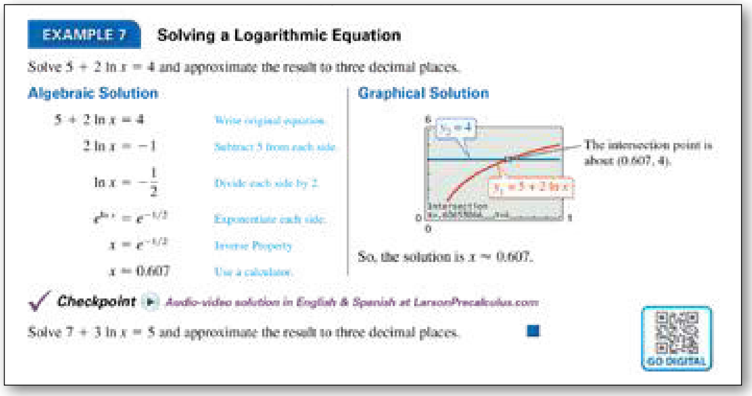
The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant, and include topics our users have suggested. The exercises have been reorganized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations. Two new sets of exercises, Vocabulary and Concept Check and Review & Refresh, have been added to help you develop and maintain your skills.

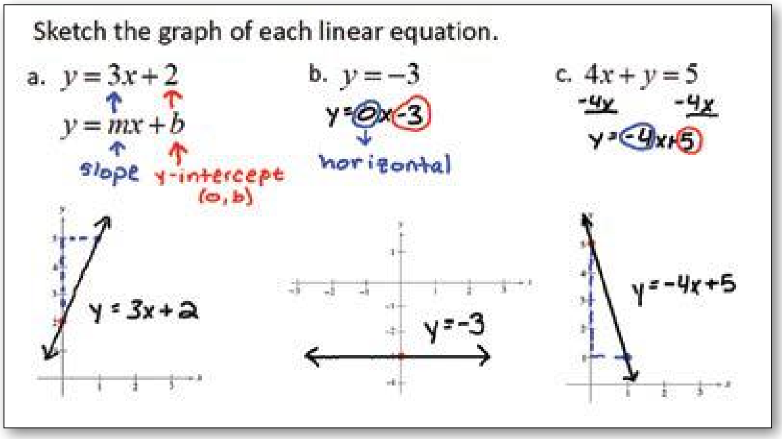
Section Objectives

A bulleted list of learning objectives provides you the opportunity to preview what will be presented in the upcoming section.

Side-By-Side Examples

Throughout the text, we present solutions to many examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.





Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at *LarsonPrecalculus.com*. Scan the on-page code to access the solutions.

Technology

The technology feature gives suggestions for effectively using tools such as calculators, graphing utilities, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

Historical Notes

These notes provide helpful information regarding famous mathematicians and their work.

Summarize (Section 2.2)

1. Explain how to use transformations to sketch graphs of polynomial functions *(page 124)*. For an example of sketching transformations of monomial functions, see Example 1.
2. Explain how to apply the Leading Coefficient Test *(page 125)*. For an example of applying the Leading Coefficient Test, see Example 2.
3. Explain how to find real zeros of polynomial functions and use them as sketching aids *(page 127)*. For examples involving finding real zeros of polynomial functions, see Examples 3–5.
4. Explain how to use the Intermediate Value Theorem to help locate real zeros of polynomial functions *(page 130)*. For an example of using the Intermediate Value Theorem, see Example 6.

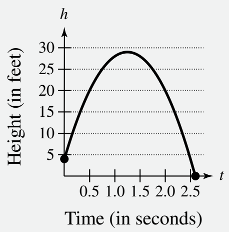
Summarize
The Summarize feature at the end of each section helps you organize the lesson's key concepts into a concise summary, providing you with a valuable study tool. Use this feature to prepare for a homework assignment, to help you study for an exam, or as a review for previously covered sections.

Algebra of Calculus
Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol ∫.

Error Analysis
This exercise presents a sample solution that contains a common error which you are asked to identify.

How Do You See It?
The How Do You See It? feature in each section presents a real-life exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

86. **HOW DO YOU SEE IT?** The graph represents the height h of a projectile after t seconds.



(a) Explain why h is a function of t.

(b) Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

(c) Approximate the domain of h.

(d) Is t a function of h? Explain.

Project
The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at *LarsonPrecalculus.com*.

Collaborative Project
You can find these extended group projects at *LarsonPrecalculus.com.* Check your understanding of the chapter concepts by solving in-depth, real-life problems. These collaborative projects provide an interesting and engaging way for you and other students to work together and investigate ideas.

Instructor Resources

CENGAGE | WEBASSIGN

Built by educators, WebAssign from Cengage is a fully customizable online solution for STEM disciplines. WebAssign includes the flexibility, tools, and content you need to create engaging learning experiences for your students. The patented grading engine provides unparalleled answer evaluation, giving students instant feedback, and insightful analytics highlight exactly where students are struggling. For more information, visit *cengage.com/webassign*.

Complete Solutions Manual
This manual contains solutions to all exercises from the text, including Chapter Review Exercises and Chapter Tests, and Practice Tests with solutions. The Complete Solutions Manual is available on the Instructor Companion Site.

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Cengage Testing, Powered by Cognero®, is a flexible online system that allows you to author, edit, and manage test bank content online. You can create multiple versions of your test in an instant and deliver tests from your LMS or exportable PDF or Word docs you print for in-class assessment. Cengage Testing is available online via *cengage.com*.

Instructor Companion Site
Everything you need for your course in one place! Access and download PowerPoint® presentations, test banks, the solutions manual, and more. This collection of book-specific lecture and class tools is available online via *cengage.com*.

Test Bank
The test bank contains text-specific multiple-choice and free response test forms and is available online at the Instructor Companion Site.

LarsonPrecalculus.com
In addition to its student resources, *LarsonPrecalculus.com* also has resources to help instructors. If you wish to challenge your students with multi-step and group projects, you can assign the Section Projects and Collaborative Projects. You can assess the knowledge of your students before and after each chapter using the pre- and post-tests. You can also give your students experience using an online graphing calculator with the Interactive Activities. You can access these features by going to *LarsonPrecalculus.com* or by scanning the on-page code.

MathGraphs.com
For exercises that ask students to draw on the graph, I have provided **free,** printable graphs at *MathGraphs.com*. You can access these features by going to *MathGraphs.com* or by scanning the on-page code at the beginning of the section exercises, review exercises, or tests.

Student Resources

CENGAGE | WEBASSIGN

Prepare for class with confidence using WebAssign from Cengage. This online learning platform, which includes an interactive eBook, fuels practice, so that you truly absorb what you learn and prepare better for tests. Videos and tutorials walk you through concepts and deliver instant feedback and grading, so you always know where you stand in class. Focus your study time and get extra practice where you need it most. Study smarter with WebAssign! Ask your instructor today how you can get access to WebAssign, or learn about self-study options at *cengage.com/webassign*.

Student Study Guide and Solutions Manual
This guide offers step-by-step solutions for all odd-numbered text exercises, Chapter Tests, and Cumulative Tests. It also contains Practice Tests. For more information on how to access this digital resource, go to cengage.com

Note-Taking Guide
This is an innovative study aid, in the form of a notebook organizer, that helps students develop a section-by-section summary of key concepts. For more information on how to access this digital resource, go to cengage.com

LarsonPrecalculus.com
Of the many features at this website, students have told me that the videos are the most helpful. You can watch lesson videos by Dana Mosely as he explains various mathematical concepts. Other helpful features are the data downloads (editable spreadsheets so you do not have to enter the data), video solutions of the Checkpoint problems in English or Spanish, and the Student Success Organizer. The Student Success Organizer will help you organize the important concepts of each section using chapter outlines. You can access these features by going to *LarsonPrecalculus.com* or by scanning the on-page code.

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Acknowledgments

I would like to thank the many people who have helped me prepare the text and the supplements package. Their encouragement, criticisms, and suggestions have been invaluable.

Thank you to all of the instructors who took the time to review the changes in this edition and to provide suggestions for improving it. Without your help, this book would not be possible.

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On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil. If you have suggestions for improving this text, please feel free to write to me. Over the past two decades, I have received many useful comments from both instructors and students, and I value these comments very highly.

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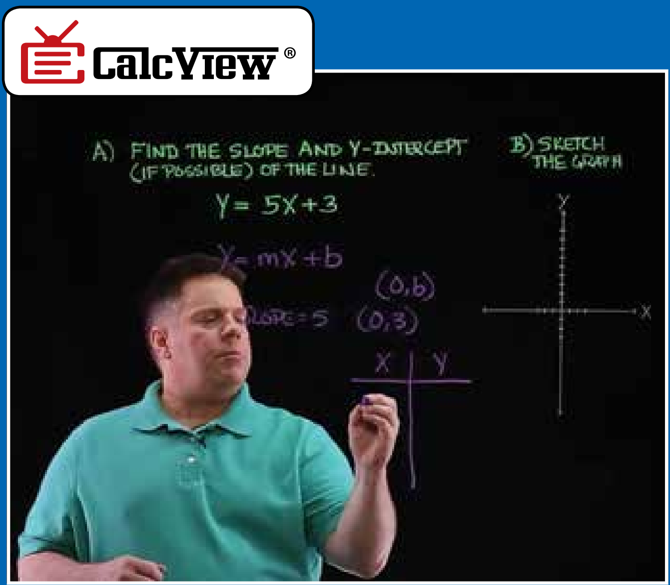
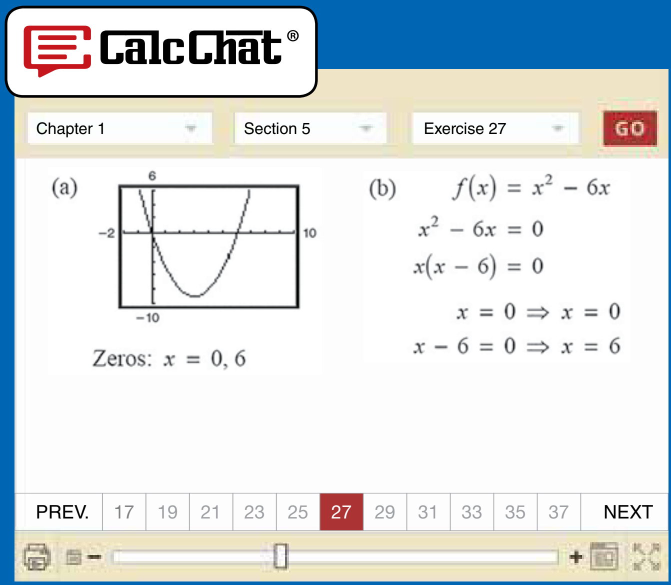
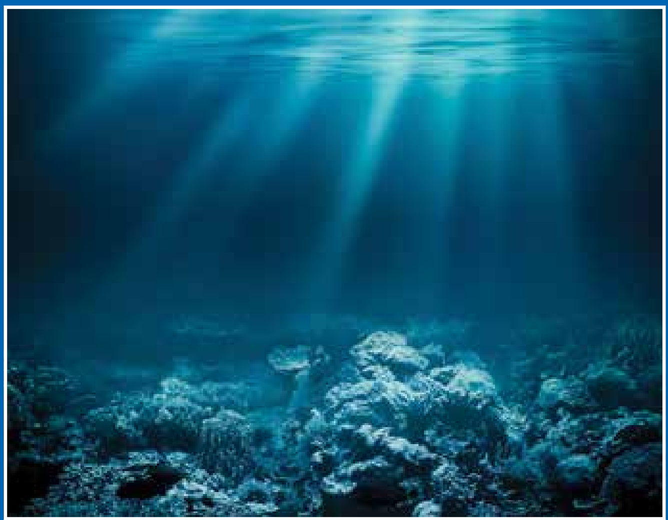
1 Functions and Their Graphs







1.10 Ocean Temperatures *(Exercise 65, p. 102)*

1.4 Force of Water *(Exercise 64, p. 46)*

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1.1 Rectangular Coordinates

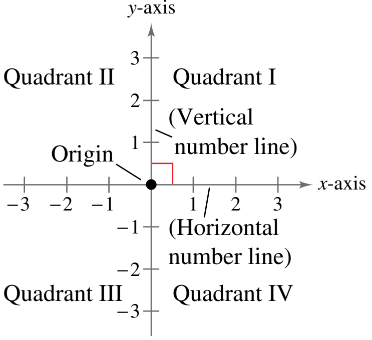
The Cartesian plane can help you visualize relationships between two variables. For example, in Exercise 44 on page 9, given how far north and west one city is from another, plotting points to represent the cities can help you visualize these distances and determine the flying distance between the cities.

- **Plot points in the Cartesian plane.**
- **Use the Distance Formula to find the distance between two points.**
- **Use the Midpoint Formula to find the midpoint of a line segment.**
- **Translate points in the plane.**

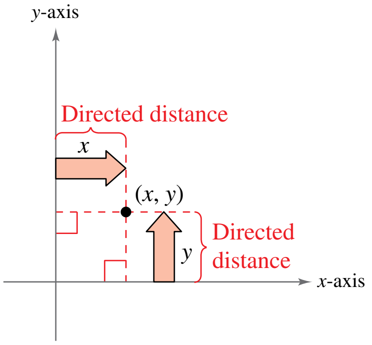
The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** named after the French mathematician René Descartes (1596–1650).

Two real number lines intersecting at right angles form the Cartesian plane, as shown in Figure 1.1. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four **quadrants.**



The Cartesian plane
Figure 1.1



Ordered pair (x, y)
Figure 1.2

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the y-axis to the point, and the ***y*-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure 1.2.

Directed distance from y-axis → (x, y) ← Directed distance from x-axis

The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

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EXAMPLE 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

Solution To plot the point $(-1, 2)$, imagine a vertical line through -1 on the x-axis and a horizontal line through 2 on the y-axis. The intersection of these two lines is the point $(-1, 2)$. Plot the other four points in a similar way, as shown in Figure 1.3.

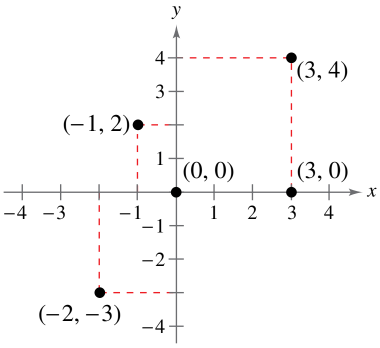

Figure 1.3

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Plot the points $(-3, 2)$, $(4, -2)$, $(3, 1)$, $(0, -2)$, and $(-1, -2)$.

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The beauty of a rectangular coordinate system is that it allows you to *see* relationships between two variables. It would be difficult to overestimate the importance of Descartes's introduction of coordinates in the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

In the next example, data are represented graphically by points plotted in a rectangular coordinate system. This type of graph is called a **scatter plot.**

EXAMPLE 2 Sketching a Scatter Plot

The table shows the numbers N (in millions) of AT&T wireless subscribers from 2013 through 2018, where t represents the year. Sketch a scatter plot of the data. *(Source: AT&T Inc.)*

DATA Year, t	2013	2014	2015	2016	2017	2018
Subscribers, N	110	121	129	134	141	153

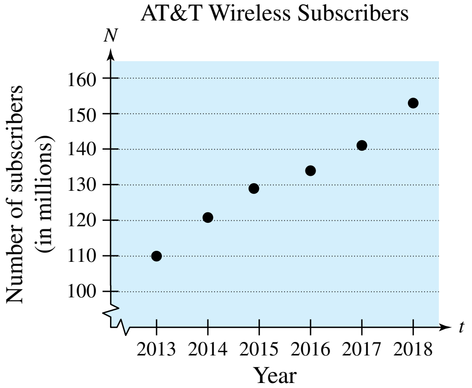
Spreadsheet at LarsonPrecalculus.com

Solution

Before sketching the scatter plot, represent each pair of values in the table by an ordered pair (t, N), as shown below.

$$(2013, 110), (2014, 121), (2015, 129), (2016, 134), (2017, 141), (2018, 153)$$

To sketch the scatter plot, first draw a vertical axis to represent the number of subscribers (in millions) and a horizontal axis to represent the year. Then plot a point for each ordered pair, as shown in the figure below. In the scatter plot, the break in the t-axis indicates omission of the numbers less than 2013, and the break in the N-axis indicates omission of the numbers less than 100 million. Also, the scatter plot shows that the number of subscribers has increased each year since 2013.



Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

The table shows the numbers N of Costco stores from 2014 through 2019, where t represents the year. Sketch a scatter plot of the data. *(Source: Costco Wholesale Corp.)*

DATA Year, t	2014	2015	2016	2017	2018	2019
Stores, N	663	686	715	741	762	782

Spreadsheet at LarsonPrecalculus.com

Another way to make the scatter plot in Example 2 is to let $t = 1$ represent the year 2013. In this scatter plot, the horizontal axis does not have a break, and the labels for the tick marks are 1 through 6 (instead of 2013 through 2018).

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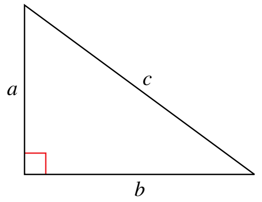
TECHNOLOGY

The scatter plot in Example 2 is only one way to represent the data graphically. You could also represent the data using a bar graph or a line graph. Use a graphing utility to represent the data given in Example 2 graphically.



The Pythagorean Theorem:
$a^2 + b^2 = c^2$
Figure 1.4

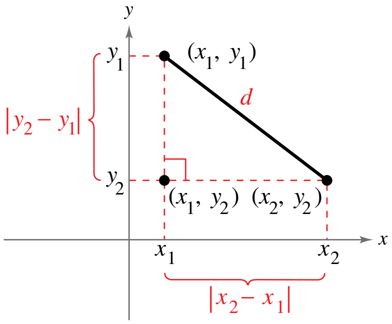


Figure 1.5

The Distance Formula

Before developing the Distance Formula, recall from the **Pythagorean Theorem** that, for a right triangle with hypotenuse of length c and sides of lengths a and b, you have

$$a^2 + b^2 = c^2 \qquad \text{Pythagorean Theorem}$$

as shown in Figure 1.4. (The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

Consider two points (x_1, y_1) and (x_2, y_2) that do not lie on the same horizontal or vertical line in the plane. With these two points, you can form a right triangle (see Figure 1.5). To determine the distance d between these two points, note that the length of the vertical side of the triangle is $|y_2 - y_1|$ and the length of the horizontal side is $|x_2 - x_1|$. By the Pythagorean Theorem,

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2 \qquad \text{Pythagorean Theorem}$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \qquad \text{Distance } d \text{ must be positive.}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \qquad \text{Property of exponents}$$

This result is the **Distance Formula.** Note that for the special case in which the two points lie on the same horizontal or vertical line, the Distance Formula still works. (See Exercise 62.)

The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 3 Finding a Distance

GO DIGITAL

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

Algebraic Solution

Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{Distance Formula}$$

$$= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} \qquad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= \sqrt{(5)^2 + (3)^2} \qquad \text{Simplify.}$$

$$= \sqrt{34} \qquad \text{Simplify.}$$

$$\approx 5.83 \qquad \text{Use a calculator.}$$

So, the distance between the points is about 5.83 units.

Check

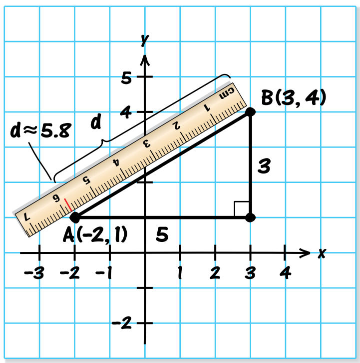
$$d^2 \stackrel{?}{=} 5^2 + 3^2 \qquad \text{Pythagorean Theorem}$$

$$(\sqrt{34})^2 \stackrel{?}{=} 5^2 + 3^2 \qquad \text{Substitute for } d.$$

$$34 = 34 \qquad \text{Distance checks. ✓}$$

Graphical Solution

Use centimeter graph paper to plot the points $A(-2, 1)$ and $B(3, 4)$. Carefully sketch the line segment from A to B. Then use a centimeter ruler to measure the length of the segment.



The line segment measures about 5.8 centimeters. So, the distance between the points is about 5.8 units.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the distance between the points $(3, 1)$ and $(-3, 0)$.

When the Distance Formula is used, it does not matter which point is (x_1, y_1) and which is (x_2, y_2), because the result will be the same. For instance, in Example 3, let $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-2, 1)$. Then

$$d = \sqrt{(-2 - 3)^2 + (1 - 4)^2} = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34} \approx 5.83.$$

EXAMPLE 4 Verifying a Right Triangle

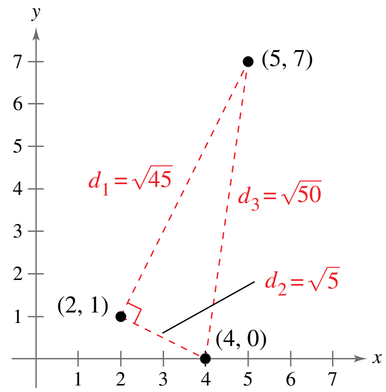


Figure 1.6

Show that the points (2, 1), (4, 0), and (5, 7) are vertices of a right triangle.

Solution

The three points are plotted in Figure 1.6. Use the Distance Formula to find the lengths of the three sides.

$$d_1 = \sqrt{(5 - 2)^2 + (7 - 1)^2} = \sqrt{9 + 36} = \sqrt{45}$$
$$d_2 = \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5}$$
$$d_3 = \sqrt{(5 - 4)^2 + (7 - 0)^2} = \sqrt{1 + 49} = \sqrt{50}$$

Using the converse of the Pythagorean Theorem and the fact that

$$(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$$

you can conclude that the triangle is a right triangle.

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Show that the points $(2, -1)$, $(5, 5)$, and $(6, -3)$ are vertices of a right triangle.

EXAMPLE 5 Finding the Length of a Pass

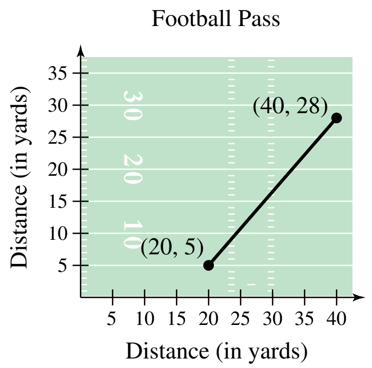

Figure 1.7

A football quarterback throws a pass from the 28-yard line, 40 yards from the sideline. A wide receiver catches the pass on the 5-yard line, 20 yards from the same sideline, as shown in Figure 1.7. How long is the pass?

Solution

The length of the pass is the distance between the points (40, 28) and (20, 5).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(40 - 20)^2 + (28 - 5)^2}$	Substitute for x_1, y_1, x_2, and y_2.
$= \sqrt{20^2 + 23^2}$	Simplify.
$= \sqrt{400 + 529}$	Simplify.
$= \sqrt{929}$	Simplify.
≈ 30	Use a calculator.

So, the pass is about 30 yards long.

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A football quarterback throws a pass from the 10-yard line, 10 yards from the sideline. A wide receiver catches the pass on the 32-yard line, 25 yards from the same sideline. How long is the pass?

In Example 5, the horizontal and vertical scales do not normally appear on a football field. However, when you use coordinate geometry to solve real-life problems, you may place the coordinate system in any way that helps you solve the problem.

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The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula.**

The Midpoint Formula

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

For a proof of the Midpoint Formula, see Proofs in Mathematics on page 110.

EXAMPLE 6 Finding the Midpoint of a Line Segment

Find the midpoint of the line segment joining the points $(-5, -3)$ and $(9, 3)$.

Solution Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula} \\ &= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= (2, 0) && \text{Simplify.} \end{aligned}$$

The midpoint of the line segment is $(2, 0)$, as shown in Figure 1.8.

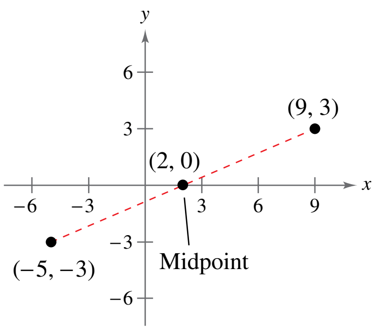

Figure 1.8

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the midpoint of the line segment joining the points $(-2, 8)$ and $(4, -10)$.

EXAMPLE 7 Estimating Annual Revenues

Microsoft Corp. had annual revenues of about \$96.7 billion in 2017 and about \$125.8 billion in 2019. Estimate the revenues in 2018. *(Source: Microsoft Corp.)*

Solution One way to solve this problem is to assume that the revenues followed a *linear* pattern. Then, to estimate the 2018 revenues, find the midpoint of the line segment connecting the points (2017, 96.7) and (2019, 125.8).

$$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula} \\ &= \left(\frac{2017 + 2019}{2}, \frac{96.7 + 125.8}{2}\right) && \text{Substitute for } x_1, x_2, y_1, \text{ and } y_2. \\ &= (2018, 111.25) && \text{Simplify.} \end{aligned}$$

So, you would estimate the 2018 revenues to have been about \$111.25 billion, as shown in Figure 1.9. (The actual 2018 revenues were about \$110.36 billion.)

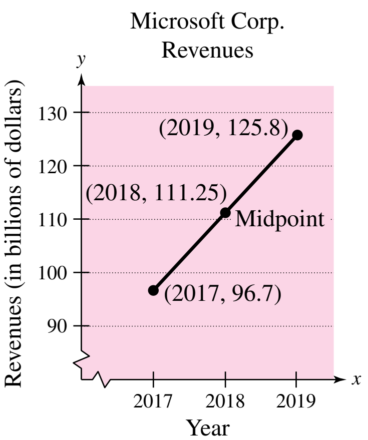

Figure 1.9

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

The Proctor & Gamble Co. had annual sales of about \$65.1 billion in 2017 and about \$67.7 billion in 2019. Estimate the sales in 2018. *(Source: Proctor & Gamble Co.)*

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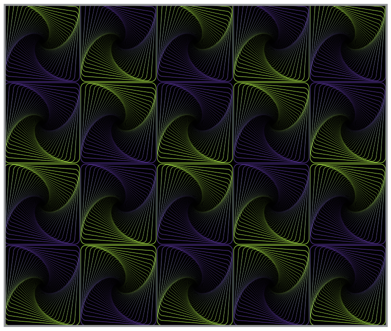


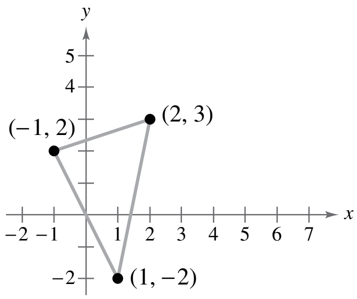
Application

Much of computer graphics, including the computer-generated tessellation shown at the left, consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 8. Other types of transformations include reflections, rotations, and stretches.

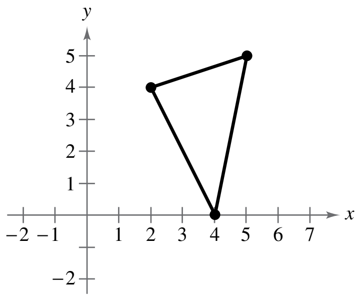
EXAMPLE 8 Translating Points in the Plane

See LarsonPrecalculus.com for an interactive version of this type of example.

The triangle in Figure 1.10 has vertices at $(-1, 2)$, $(1, -2)$, and $(2, 3)$. Shift the triangle three units to the right and two units up. What are the coordinates of the vertices of the shifted triangle (see Figure 1.11)?



Original triangle
Figure 1.10



Shifted triangle
Figure 1.11

Solution To shift the vertices three units to the right, add 3 to each of the x-coordinates. To shift the vertices two units up, add 2 to each of the y-coordinates.

Original Point	**Translated Point**
$(-1, 2)$	$(-1 + 3, 2 + 2) = (2, 4)$
$(1, -2)$	$(1 + 3, -2 + 2) = (4, 0)$
$(2, 3)$	$(2 + 3, 3 + 2) = (5, 5)$

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The parallelogram in Figure 1.12 has vertices at $(1, 4)$, $(1, 0)$, $(3, 2)$, and $(3, 6)$. Shift the parallelogram two units to the left and four units down. What are the coordinates of the vertices of the shifted parallelogram?

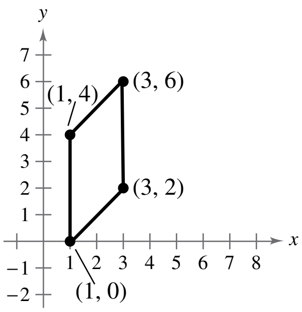

Figure 1.12

The figures in Example 8 were not really essential to the solution. Nevertheless, you should develop the habit of including sketches with your solutions because they serve as useful problem-solving tools.

Summarize (Section 1.1)

1. Describe the Cartesian plane *(page 2)*. For examples of plotting points in the Cartesian plane, see Examples 1 and 2.
2. State the Distance Formula *(page 4)*. For examples of using the Distance Formula to find the distance between two points, see Examples 3–5.
3. State the Midpoint Formula *(page 6)*. For examples of using the Midpoint Formula to find the midpoint of a line segment, see Examples 6 and 7.
4. Describe how to translate points in the plane *(page 7)*. For an example of translating points in the plane, see Example 8.

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1.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

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Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blanks.

1. An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the ________ plane.

2. Finding the average values of the respective coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the ________ ________.

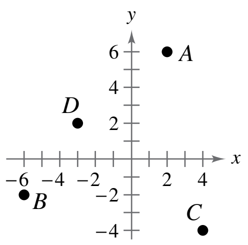
In Exercises 3–8, match each term with its definition.

3. x-axis	(a) point of intersection of vertical axis and horizontal axis
4. y-axis	(b) directed distance from the x-axis
5. origin	(c) horizontal real number line
6. quadrants	(d) four regions of the coordinate plane
7. x-coordinate	(e) directed distance from the y-axis
8. y-coordinate	(f) vertical real number line

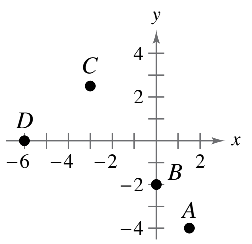
Skills and Applications

Approximating Coordinates of Points **In Exercises 9 and 10, approximate the coordinates of the points.**

9.



10.



Plotting Points in the Cartesian Plane **In Exercises 11 and 12, plot the points.**

11. $(2, 4), (3, -1), (-6, 2), (-4, 0), (-1, -8), (1.5, -3.5)$

12. $(1, -5), (-2, -7), (3, 3), (-2, 4), (0, 5), \left(\frac{2}{3}, \frac{5}{2}\right)$

Finding the Coordinates of a Point **In Exercises 13 and 14, find the coordinates of the point.**

13. The point is three units to the left of the y-axis and four units above the x-axis.

14. The point is on the x-axis and 12 units to the left of the y-axis.

Determining Quadrant(s) for a Point **In Exercises 15–20, determine the quadrant(s) in which (x, y) could be located.**

15. $x > 0$ and $y < 0$

16. $x < 0$ and $y < 0$

17. $x = -4$ and $y > 0$

18. $x < 0$ and $y = 7$

19. $x + y = 0, x \neq 0, y \neq 0$

20. $xy > 0$

Sketching a Scatter Plot **In Exercises 21 and 22, sketch a scatter plot of the data shown in the table.**

21. The table shows the number y of Dollar General stores for each year x from 2012 through 2018. *(Source: Dollar General Corporation)*

DATA

Spreadsheet at LarsonPrecalculus.com

Year, x	Number of Stores, y
2012	10,506
2013	11,132
2014	11,789
2015	12,483
2016	13,320
2017	14,609
2018	15,472

22. The table shows the annual revenues y (in billions of dollars) for Amazon.com for each year x from 2011 through 2018. *(Source: Amazon.com)*

DATA

Spreadsheet at LarsonPrecalculus.com

Month, x	Annual Revenue, y
2011	48.1
2012	61.1
2013	74.5
2014	89.0
2015	107.0
2016	136.0
2017	177.9
2018	232.9

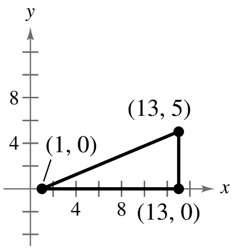
A blue exercise number indicates that a video solution can be seen at *CalcView.com.*

Finding a Distance **In Exercises 23–28, find the distance between the points.**

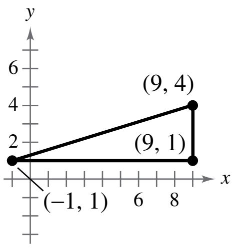
23. $(-2, 6), (3, -6)$
24. $(8, 5), (0, 20)$
25. $(1, 4), (-5, -1)$
26. $(1, 3), (3, -2)$
27. $\left(\frac{1}{2}, \frac{4}{3}\right), (2, -1)$
28. $(9.5, -2.6), (-3.9, 8.2)$

Verifying a Right Triangle **In Exercises 29 and 30, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem.**

29.



30.



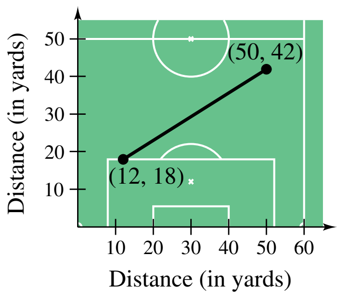
Verifying a Polygon **In Exercises 31–34, show that the points form the vertices of the polygon.**

31. Right triangle: $(4, 0), (2, 1), (-1, -5)$
32. Right triangle: $(-1, 3), (3, 5), (5, 1)$
33. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$
34. Isosceles triangle: $(2, 3), (4, 9), (-2, 7)$

Plotting, Distance, and Midpoint **In Exercises 35–42, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.**

35. $(6, -3), (6, 5)$
36. $(1, 4), (8, 4)$
37. $(1, 1), (9, 7)$
38. $(1, 12), (6, 0)$
39. $(-1, 2), (5, 4)$
40. $(2, 10), (10, 2)$
41. $(-16.8, 12.3), (5.6, 4.9)$
42. $\left(\frac{1}{2}, 1\right), \left(-\frac{5}{2}, \frac{4}{3}\right)$

43. **Sports** A soccer player passes the ball from a point that is 18 yards from the endline and 12 yards from the sideline. A teammate who is 42 yards from the same endline and 50 yards from the same sideline receives the pass. (See figure.) How long is the pass?



44. **Flying Distance**

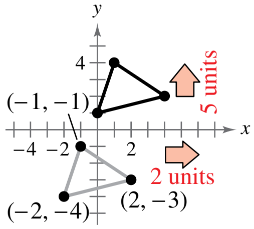
An airplane flies from Naples, Italy, in a straight line to Rome, Italy, which is about 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?

45. **Sales** Walmart had sales of \$485.9 billion in 2016 and \$514.4 billion in 2018. Use the Midpoint Formula to estimate the sales in 2017. Assume that the sales followed a linear pattern. *(Source: Walmart, Inc.)*

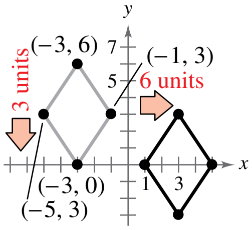
46. **Earnings per Share** The earnings per share for Facebook, Inc. were \$6.16 in 2017 and \$7.57 in 2018. Use the Midpoint Formula to estimate the earnings per share in 2019. Assume that the earnings per share followed a linear pattern. *(Source: Facebook, Inc.)*

Translating Points in the Plane **In Exercises 47–50, find the coordinates of the vertices of the polygon after the given translation to a new position in the plane.**

47.



48.



49. Original coordinates of vertices: $(-7, -2), (-2, 2), (-2, -4), (-7, -4)$

Shift: eight units up, four units to the right

50. Original coordinates of vertices: $(5, 8), (3, 6), (7, 6)$

Shift: 6 units down, 10 units to the left

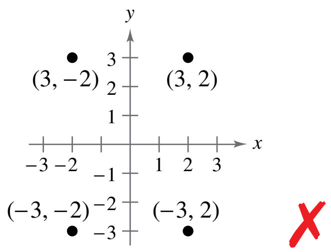
Exploring the Concepts

True or False? **In Exercises 51–54, determine whether the statement is true or false. Justify your answer.**

51. If the point (x, y) is in Quadrant II, then the point $(2x, -3y)$ is in Quadrant III.
52. To divide a line segment into 16 equal parts, you have to use the Midpoint Formula 16 times.
53. The points $(-8, 4)$, $(2, 11)$, and $(-5, 1)$ represent the vertices of an isosceles triangle.
54. If four points represent the vertices of a polygon, and the four side lengths are equal, then the polygon must be a square.

55. **Think About It** What is the y-coordinate of any point on the x-axis? What is the x-coordinate of any point on the y-axis?

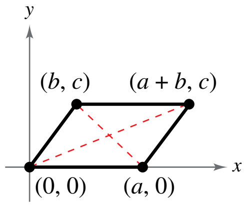
56. Error Analysis Describe the error.



57. Using the Midpoint Formula A line segment has (x_1, y_1) as one endpoint and (x_m, y_m) as its midpoint. Find the other endpoint (x_2, y_2) of the line segment in terms of x_1, y_1, x_m, and y_m.

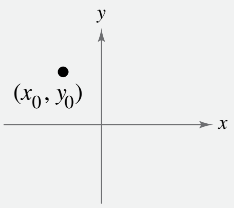
58. Using the Midpoint Formula Use the Midpoint Formula three times to find the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four equal parts.

59. Proof Prove that the diagonals of the parallelogram in the figure intersect at their midpoints.

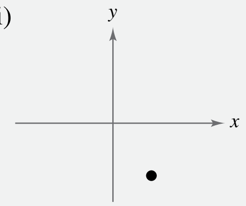


60. HOW DO YOU SEE IT?

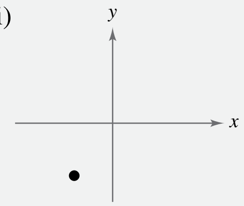
Use the plot of the point (x_0, y_0) in the figure. Match the transformation of the point with the correct plot. Explain. [The plots are labeled (i), (ii), (iii), and (iv).]



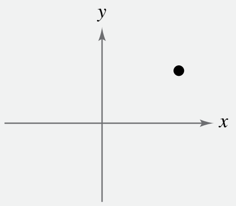
(i)



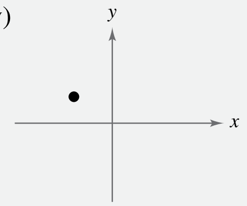
(ii)



(iii)



(iv)



(a) $(x_0, -y_0)$ (b) $(-2x_0, y_0)$

(c) $\left(x_0, \frac{1}{2}y_0\right)$ (d) $(-x_0, -y_0)$

61. Collinear Points Three or more points are collinear when they all lie on the same line. Use the steps below to determine whether the set of points $\{A(2, 3), B(2, 6), C(6, 3)\}$ and the set of points $\{A(8, 3), B(5, 2), C(2, 1)\}$ are collinear.

(a) For each set of points, use the Distance Formula to find the distances from A to B, from B to C, and from A to C. What relationship exists among these distances for each set of points?

(b) Plot each set of points in the Cartesian plane. Do all the points of either set appear to lie on the same line?

(c) Compare your conclusions from part (a) with the conclusions you made from the graphs in part (b). Make a general statement about how to use the Distance Formula to determine collinearity.

62. Points on Vertical and Horizontal Lines Use the Distance Formula to find the distance between each pair of points. Are the results what you expected? Explain.

(a) *On the same vertical line:* (x_1, y_1) and (x_1, y_2)

(b) *On the same horizontal line:* (x_1, y_1) and (x_2, y_1)

Review & Refresh

Video solutions at LarsonPrecalculus.com

Evaluating an Expression In Exercises 63–66, evaluate the expression for each value of x. (If not possible, state the reason.)

63. $4x - 6$ (a) $x = -1$ (b) $x = 0$

64. $9 - 7x$ (a) $x = -3$ (b) $x = 3$

65. $2x^3$ (a) $x = -3$ (b) $x = 0$

66. $-3x^{-4}$ (a) $x = 0$ (b) $x = -2$

Simplifying an Expression In Exercises 67–70, simplify the expression.

67. $(-x)^2 - 2$

68. $(-x)^3 - (-x)^2 + 2$

69. $-x^2 + (-x)^2$

70. $-x^3 + (-x)^3 + (-x)^4 - x^4$

71. Cost, Revenue, and Profit A manufacturer can produce and sell x electronic devices per week. The total cost C (in dollars) of producing x electronic devices is $C = 93x + 35{,}000$, and the total revenue R (in dollars) is $R = 135x$.

(a) Find the profit P in terms of x.

(b) Find the profit obtained by selling 5000 electronic devices per week.

72. Rate A copier copies at a rate of 50 pages per minute.

(a) Find the time required to copy one page.

(b) Find the time required to copy x pages.

(c) Find the time required to copy 120 pages.

(d) Find the time required to copy 10,000 pages.

1.2 Graphs of Equations

The graph of an equation can help you visualize relationships between real-life quantities. For example, in Exercise 80 on page 21, you will use a graph to analyze life expectancy.

- **Sketch graphs of equations.**
- **Identify x- and y-intercepts of graphs of equations.**
- **Use symmetry to sketch graphs of equations.**
- **Write equations of circles.**
- **Use graphs of equations to solve real-life problems.**

The Graph of an Equation

In Section 1.1, you used a coordinate system to graphically represent the relationship between two quantities as points in a coordinate plane. In this section, you will review some basic procedures for sketching the graph of an *equation in two variables*.

Frequently, a relationship between two quantities is expressed as an **equation in two variables.** For example, $y = 7 - 3x$ is an equation in x and y. An ordered pair (a, b) is a **solution** or **solution point** of an equation in x and y when the substitutions $x = a$ and $y = b$ result in a true statement. For example, $(1, 4)$ is a solution of $y = 7 - 3x$ because $4 = 7 - 3(1)$ is a true statement.

EXAMPLE 1 Determining Solution Points

Determine whether (a) $(2, 13)$ and (b) $(-1, -3)$ lie on the graph of $y = 10x - 7$.

Solution

a. $y = 10x - 7$ — Write original equation.

$13 \stackrel{?}{=} 10(2) - 7$ — Substitute 2 for x and 13 for y.

$13 = 13$ — $(2, 13)$ is a solution. ✓

The point $(2, 13)$ *does* lie on the graph of $y = 10x - 7$ because it is a solution point of the equation.

b. $y = 10x - 7$ — Write original equation.

$-3 \stackrel{?}{=} 10(-1) - 7$ — Substitute -1 for x and -3 for y.

$-3 \neq -17$ — $(-1, -3)$ is not a solution.

The point $(-1, -3)$ *does not* lie on the graph of $y = 10x - 7$ because it is *not* a solution point of the equation.

SKILLS REFRESHER

For a refresher on how to use the Basic Rules of Algebra to evaluate an expression or an equation, watch the video at *LarsonPrecalculus.com.*

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Determine whether (a) $(3, -5)$ and (b) $(-2, 26)$ lie on the graph of $y = 14 - 6x$.

The set of all solution points of an equation is the **graph of the equation.** The basic technique used for sketching the graph of an equation is the **point-plotting method.**

The Point-Plotting Method of Sketching a Graph

1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Construct a table of values showing several solution points.
3. Plot these points in a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

GO DIGITAL

Blaine Harrington III/The Image Bank/Getty Images

It is important to use negative values, zero, and positive values for x (if possible) when constructing a table. The choice of values to use in the table is somewhat arbitrary. The more values you choose, however, the easier it will be to recognize a pattern.

EXAMPLE 2 Sketching the Graph of an Equation

Sketch the graph of $3x + y = 7$.

Solution

Rewrite the equation so that y is isolated on the left.

$$3x + y = 7 \qquad \text{Write original equation.}$$

$$y = -3x + 7 \qquad \text{Subtract } 3x \text{ from each side.}$$

Next, construct a table of values that consists of several solution points of the equation. For instance, when $x = -2$,

$$y = -3(-2) + 7 = 13$$

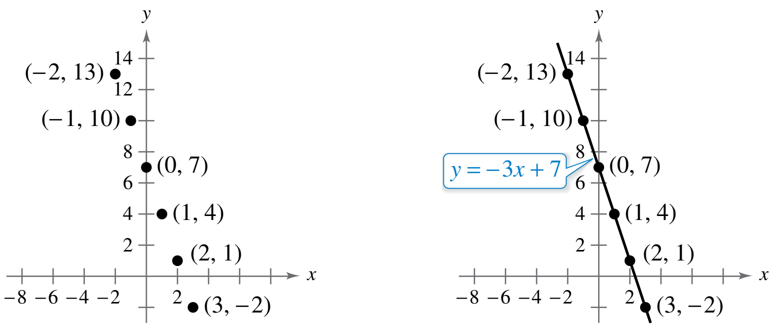
which implies that $(-2, 13)$ is a solution point of the equation.

x	-2	-1	0	1	2	3
$y = -3x + 7$	13	10	7	4	1	-2
(x, y)	$(-2, 13)$	$(-1, 10)$	$(0, 7)$	$(1, 4)$	$(2, 1)$	$(3, -2)$

From the table, it follows that

$$(-2, 13), \quad (-1, 10), \quad (0, 7), \quad (1, 4), \quad (2, 1), \quad \text{and} \quad (3, -2)$$

are solution points of the equation. Plot these points, as shown in Figure 1.13(a). It appears that all six points lie on a line, so complete the sketch by drawing a line through the points, as shown in Figure 1.13(b).



(a) **(b)**

Figure 1.13

ALGEBRA HELP

Example 2 shows three common ways to represent the relationship between two variables. The equation $y = -3x + 7$ is the *algebraic* representation, the table of values is the *numerical* representation, and the graph in Figure 1.13(b) is the *graphical* representation. You will see and use algebraic, numerical, and graphical representations throughout this course.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Sketch the graph of each equation.

a. $3x + y = 2$ **b.** $-2x + y = 1$

One of your goals in this course is to learn to classify the basic shape of a graph from its equation. For instance, you will learn that the *linear equation* in Example 2 can be written in the form $y = mx + b$ and its graph is a line. Similarly, the *quadratic equation* in Example 3 on the next page has the form $y = ax^2 + bx + c$ and its graph is a *parabola*.

GO DIGITAL

EXAMPLE 3 Sketching the Graph of an Equation

See LarsonPrecalculus.com for an interactive version of this type of example.

Sketch the graph of

$$y = x^2 - 2.$$

Solution

The equation is already solved for y, so begin by constructing a table of values.

x	-2	-1	0	1	2	3
$y = x^2 - 2$	2	-1	-2	-1	2	7
(x, y)	$(-2, 2)$	$(-1, -1)$	$(0, -2)$	$(1, -1)$	$(2, 2)$	$(3, 7)$

Next, plot the points given in the table, as shown in Figure 1.14(a). Finally, connect the points with a smooth curve, as shown in Figure 1.14(b).

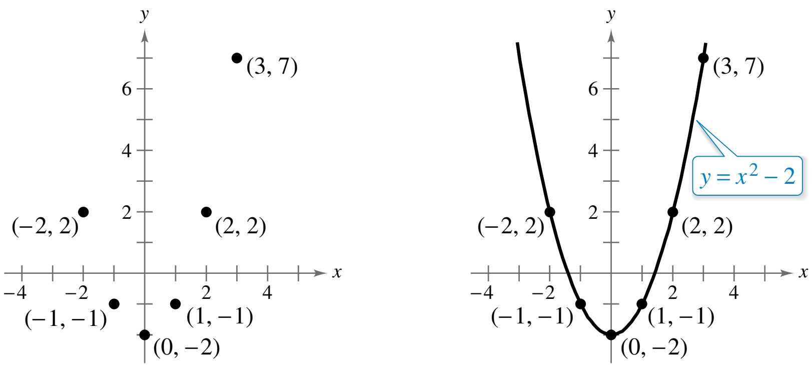


Figure 1.14

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

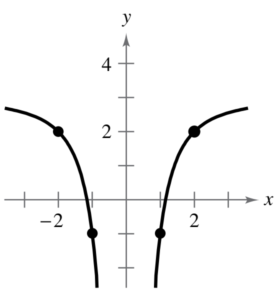
Sketch the graph of each equation.

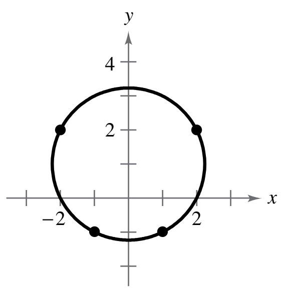
a. $y = x^2 + 3$ **b.** $y = 1 - x^2$

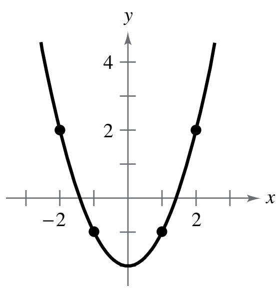
The point-plotting method demonstrated in Examples 2 and 3 is straightforward, but it has shortcomings. For instance, with too few solution points, it is possible to misrepresent the graph of an equation. To illustrate, when you only plot the four points

$$(-2, 2), \quad (-1, -1), \quad (1, -1), \quad \text{and} \quad (2, 2)$$

in Example 3, any one of the three graphs below is reasonable.







TECHNOLOGY

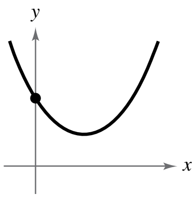
To graph an equation involving x and y on a graphing utility, use the procedure below.

1. If necessary, rewrite the equation so that y is isolated on the left side.
2. Enter the equation in the graphing utility.
3. Determine a *viewing window* that shows all important features of the graph.
4. Graph the equation.

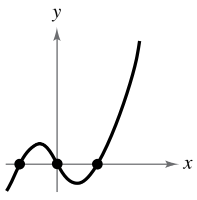
GO DIGITAL

Intercepts of a Graph

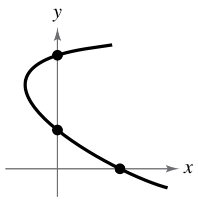
Solution points of an equation that have zero as either the x-coordinate or the y-coordinate are called **intercepts.** They are the points at which the graph intersects or touches the x- or y-axis. It is possible for a graph to have no intercepts, one intercept, or several intercepts, as shown in the graphs below.



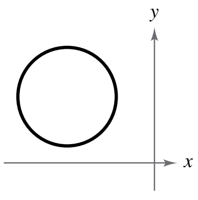
No x-intercepts
One y-intercept



Three x-intercepts
One y-intercept



One x-intercept
Two y-intercepts



No intercepts

Note that an x-intercept can be written as the ordered pair $(a, 0)$ and a y-intercept can be written as the ordered pair $(0, b)$. Sometimes it is convenient to denote the x-intercept as the x-coordinate a of the point $(a, 0)$, or the y-intercept as the y-coordinate b of the point $(0, b)$. Unless it is necessary to make a distinction, the term *intercept* will refer to either the point or the coordinate.

Finding Intercepts

1. To find x-intercepts, let y be zero and solve the equation for x.
2. To find y-intercepts, let x be zero and solve the equation for y.

EXAMPLE 4 Finding *x*- and *y*-Intercepts

Find the x- and y-intercepts of the graph of $y = x^3 - 4x$.

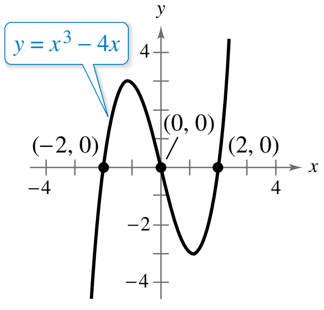
Solution To find the x-intercepts of the graph of $y = x^3 - 4x$, let $y = 0$ and solve for x.

$y = x^3 - 4x$	Write original equation.
$0 = x^3 - 4x$	Substitute 0 for y.
$0 = x(x^2 - 4)$	Factor out common factor.
$0 = x(x + 2)(x - 2)$	Factor difference of two squares.

The solutions of this equations are $x = 0, -2$, and 2. So, the x-intercepts are $(0, 0)$, $(-2, 0)$, and $(2, 0)$. To find the y-intercept, let $x = 0$ and solve for y.

$y = x^3 - 4x$	Write original equation.
$y = (0)^3 - 4(0)$	Substitute 0 for x.
$y = 0$	Simplify.

This equation has one solution, $y = 0$. So, the y-intercept is $(0, 0)$. Check each intercept by sketching a graph, as shown at the right.



Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the x- and y-intercepts of the graph of $y = -x^2 - 5x$.

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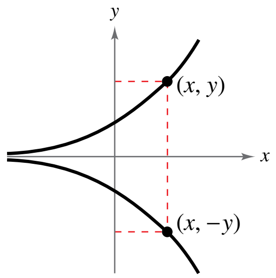
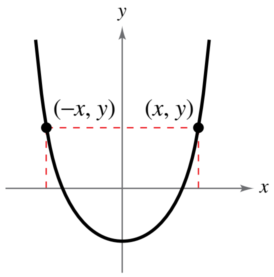
Symmetry

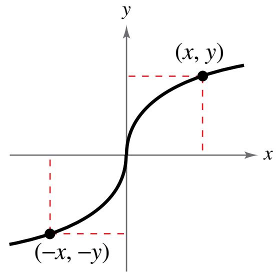
Graphs of equations can have **symmetry** with respect to one of the coordinate axes or with respect to the origin. Symmetry with respect to the x-axis means that when you fold the Cartesian plane along the x-axis, the portion of the graph above the x-axis coincides with the portion below the x-axis. Symmetry with respect to the y-axis or the origin can be described in a similar manner. The graphs below show these three types of symmetry.



x-Axis symmetry



y-Axis symmetry



Origin symmetry

Knowing the symmetry of a graph *before* attempting to sketch it is helpful because you need only half as many solution points to sketch the graph. Graphical and algebraic tests for these three basic types of symmetry are described below.

Tests for Symmetry

Graphical

1. A graph is **symmetric with respect to the x-axis** if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph.
2. A graph is **symmetric with respect to the y-axis** if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
3. A graph is **symmetric with respect to the origin** if, whenever (x, y) is on the graph, $(-x, -y)$ is also on the graph.

Algebraic

1. The graph of an equation is **symmetric with respect to the x-axis** when replacing y with $-y$ yields an equivalent equation.
2. The graph of an equation is **symmetric with respect to the y-axis** when replacing x with $-x$ yields an equivalent equation.
3. The graph of an equation is **symmetric with respect to the origin** when replacing x with $-x$ and y with $-y$ yields an equivalent equation.

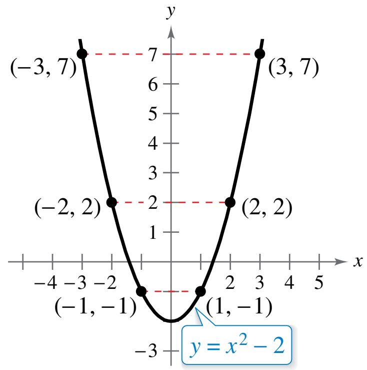
GO DIGITAL

Using the graphical tests for symmetry, the graph of $y = x^2 - 2$ is symmetric with respect to the y-axis because (x, y) and $(-x, y)$ are on its graph, as shown in Figure 1.15. To verify this algebraically, replace x with $-x$ in $y = x^2 - 2$

$$y = (-x)^2 - 2 = x^2 - 2 \qquad \text{Replace } x \text{ with } -x \text{ in } y = x^2 - 2.$$

and note that the result is an equivalent equation. To support this result numerically, create a table of values (see below).

x	-3	-2	-1	1	2	3
y	7	2	-1	-1	2	7
(x, y)	$(-3, 7)$	$(-2, 2)$	$(-1, -1)$	$(1, -1)$	$(2, 2)$	$(3, 7)$



y-Axis symmetry
Figure 1.15

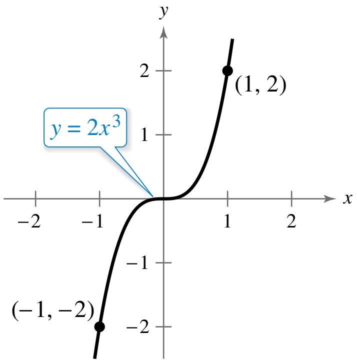
EXAMPLE 5 Testing for Symmetry

Test $y = 2x^3$ for symmetry with respect to both axes and the origin.

Solution

x-Axis:	$y = 2x^3$	Write original equation.
	$-y = 2x^3$	Replace y with $-y$. The result is *not* an equivalent equation.
y-Axis:	$y = 2x^3$	Write original equation.
	$y = 2(-x)^3$	Replace x with $-x$.
	$y = -2x^3$	Simplify. The result is *not* an equivalent equation.
Origin:	$y = 2x^3$	Write original equation.
	$-y = 2(-x)^3$	Replace y with $-y$ and x with $-x$.
	$-y = -2x^3$	Simplify.
	$y = 2x^3$	Simplify. The result is an equivalent equation.

Of the three tests for symmetry, the test for origin symmetry is the only one satisfied. So, the graph of $y = 2x^3$ is symmetric with respect to the origin (see Figure 1.16).



Origin symmetry
Figure 1.16

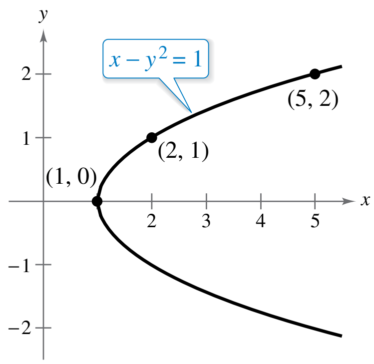
Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Test $y^2 = 6 - x$ for symmetry with respect to both axes and the origin.

EXAMPLE 6 Using Symmetry as a Sketching Aid

Use symmetry to sketch the graph of $x - y^2 = 1$.

Solution Of the three tests for symmetry, the test for x-axis symmetry is the only one satisfied, because $x - (-y)^2 = 1$ is equivalent to $x - y^2 = 1$. So, the graph is symmetric with respect to the x-axis. Find solution points above (or below) the x-axis and then use symmetry to obtain the graph, as shown in Figure 1.17.



x-Axis symmetry
Figure 1.17

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Use symmetry to sketch the graph of $y = x^2 - 4$.

EXAMPLE 7 Sketching the Graph of an Equation

Sketch the graph of $y = |x - 1|$.

Solution This equation fails all three tests for symmetry, so its graph is not symmetric with respect to either axis or to the origin. The absolute value bars tell you that y is always nonnegative. Construct a table of values. Then plot and connect the points, as shown in Figure 1.18. Notice from the table that $x = 0$ when $y = 1$. So, the y-intercept is $(0, 1)$. Similarly, $y = 0$ when $x = 1$. So, the x-intercept is $(1, 0)$.

x	-2	-1	0	1	2	3	4
$y = \|x - 1\|$	3	2	1	0	1	2	3
(x, y)	$(-2, 3)$	$(-1, 2)$	$(0, 1)$	$(1, 0)$	$(2, 1)$	$(3, 2)$	$(4, 3)$

GO DIGITAL

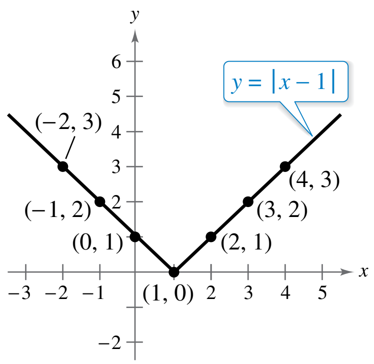


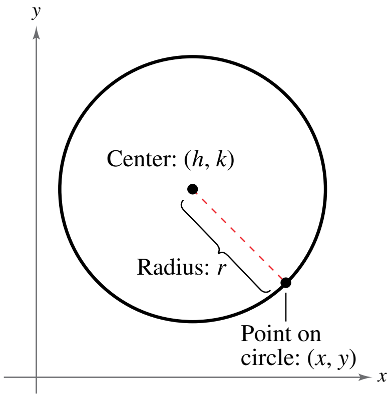
Figure 1.18

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Sketch the graph of $y = |x - 2|$.

Circles

A **circle** is the set of all points in a plane that are the same distance from a fixed point. The fixed point is the **center** of the circle, and the distance between the center and a point on the circle is the **radius,** as shown in the figure.



Definition of a circle

You can use the Distance Formula to write an equation for the circle with center (h, k) and radius r. Let (x, y) be any point on the circle. Then the distance between (x, y) and the center (h, k) is

$$\sqrt{(x-h)^2+(y-k)^2}=r.$$

By squaring each side of this equation, you obtain the **standard form of the equation of a circle.**

Standard Form of the Equation of a Circle

A point (x, y) lies on the circle of **radius** r and **center** (h, k) if and only if

$$(x-h)^2+(y-k)^2=r^2.$$

From this result, the standard form of the equation of a circle with radius r *and center at the origin,* $(h, k) = (0, 0)$, is

$$x^2+y^2=r^2.$$ Circle with radius r and center at origin

When $r = 1$, the circle is called the **unit circle.**

EXAMPLE 8 Writing the Equation of a Circle

The point $(3, 4)$ lies on a circle whose center is at $(-1, 2)$, as shown in Figure 1.19. Write the standard form of the equation of this circle.

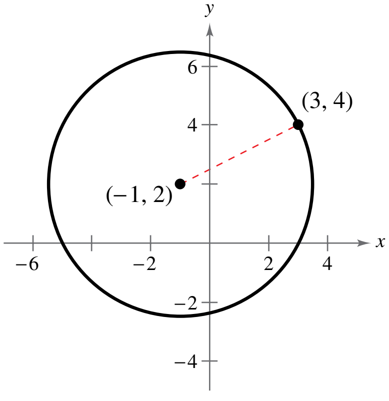


Figure 1.19

Solution

The radius of the circle is the distance between $(-1, 2)$ and $(3, 4)$.

$r=\sqrt{(x-h)^2+(y-k)^2}$	Distance Formula
$=\sqrt{[3-(-1)]^2+(4-2)^2}$	Substitute for x, y, h, and k.
$=\sqrt{4^2+2^2}$	Simplify.
$=\sqrt{16+4}$	Simplify.
$=\sqrt{20}$	Radius

Using $(h, k) = (-1, 2)$ and $r = \sqrt{20}$, the equation of the circle is

$(x-h)^2+(y-k)^2=r^2$	Equation of circle
$[x-(-1)]^2+(y-2)^2=(\sqrt{20})^2$	Substitute for h, k, and r.
$(x+1)^2+(y-2)^2=20.$	Standard form

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The point $(1, -2)$ lies on a circle whose center is at $(-3, -5)$. Write the standard form of the equation of this circle.

GO DIGITAL

Application

In this course, you will learn that there are many ways to approach a problem. Three common approaches are listed below.

A numerical approach: Construct and use a table.

A graphical approach: Draw and use a graph.

An algebraic approach: Use the rules of algebra.

Note how Example 9 uses these approaches.

ALGEBRA HELP

You should develop the habit of using at least two approaches to solve every problem. This helps build your intuition and helps you check that your answers are reasonable.

EXAMPLE 9 Maximum Weight

The maximum allowable weight y (in pounds) for a male in the United States Marine Corps can be approximated by the mathematical model

$$y = 0.040x^2 - 0.11x + 3.9, \quad 58 \le x \le 80$$

where x is the male's height (in inches). *(Source: U.S. Department of Defense)*

a. Construct a table of values that shows the maximum allowable weights for males with heights of 62, 64, 66, 68, 70, 72, 74, and 76 inches.

b. Use the table of values to sketch a graph of the model. Then use the graph to estimate *graphically* the maximum allowable weight for a male whose height is 71 inches.

c. Use the model to estimate the weight in part (b) *algebraically*.

Solution

a. Use a calculator to construct a table, as shown at the left.

DATA

Spreadsheet at LarsonPrecalculus.com

Height, x	Weight, y
62	150.8
64	160.7
66	170.9
68	181.4
70	192.2
72	203.3
74	214.8
76	226.6

b. Use the table of values to sketch the graph of the equation, as shown in Figure 1.20. From the graph, you can estimate that a height of 71 inches corresponds to a maximum allowable weight of about 198 pounds.

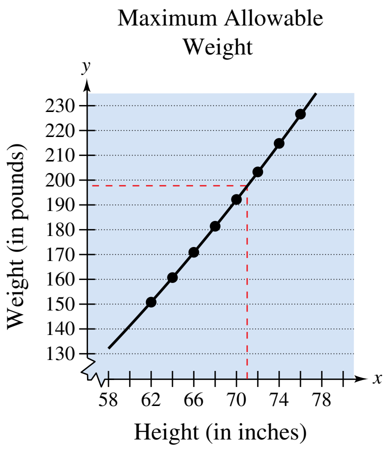

Figure 1.20

c. To estimate the weight in part (b) *algebraically*, substitute 71 for x in the model.

$$y = 0.040(71)^2 - 0.11(71) + 3.9 \approx 197.7$$

The estimate is about 197.7 pounds, which is similar to the estimate in part (b).

✓ ***Checkpoint*** ▶ *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use Figure 1.20 to estimate *graphically* the maximum allowable weight for a man whose height is 75 inches. Then estimate the weight *algebraically*. ■

Summarize (Section 1.2)

1. Explain how to sketch the graph of an equation *(page 11)*. For examples of sketching graphs of equations, see Examples 2 and 3.
2. Explain how to identify the x- and y-intercepts of a graph *(page 14)*. For an example of identifying x- and y-intercepts, see Example 4.
3. Explain how to use symmetry to graph an equation *(page 15)*. For an example of using symmetry to graph an equation, see Example 6.
4. State the standard form of the equation of a circle *(page 17)*. For an example of writing the standard form of the equation of a circle, see Example 8.
5. Describe an example of how to use the graph of an equation to solve a real-life problem *(page 18, Example 9)*.

GO DIGITAL

1.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

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Vocabulary and Concept Check

In Exercises 1–6, fill in the blanks.

1. An ordered pair (a, b) is a ________ of an equation in x and y when the substitutions $x = a$ and $y = b$ result in a true statement.
2. The set of all solution points of an equation is the ________ of the equation.
3. The points at which a graph intersects or touches an axis are the ________ of the graph.
4. A graph is symmetric with respect to the ________ if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
5. A graph is symmetric with respect to the ________ if, whenever (x, y) is on the graph, $(-x, -y)$ is also on the graph.
6. When you construct and use a table to solve a problem, you are using a ________ approach.

7. Besides your answer for Exercise 6, name two other approaches you can use to solve problems mathematically.
8. Explain how to use the Distance Formula to write an equation for the circle with center (h, k) and radius r.

Skills and Applications

Determining Solution Points **In Exercises 9–12, determine whether each point lies on the graph of the equation.**

	Equation	Points	
9.	$y = x^2 - 3x + 2$	(a) $(2, 0)$	(b) $(-2, 8)$
10.	$y = \sqrt{x + 4}$	(a) $(0, 2)$	(b) $(5, 3)$
11.	$y = 4 - \lvert x - 2\rvert$	(a) $(1, 5)$	(b) $(6, 0)$
12.	$2x^2 + 5y^2 = 8$	(a) $(6, 0)$	(b) $(0, 4)$

Sketching the Graph of an Equation **In Exercises 13–16, complete the table. Use the resulting solution points to sketch the graph of the equation.**

13. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y					
(x, y)					

14. $y + 1 = \frac{3}{4}x$

x	-2	0	1	$\frac{4}{3}$	2
y					
(x, y)					

15. $y + 3x = x^2$

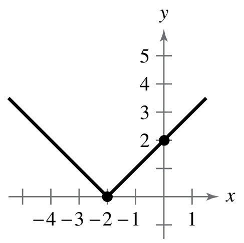
x	-1	0	1	2	3
y					
(x, y)					

16. $y = 5 - x^2$

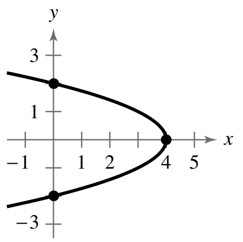
x	-2	-1	0	1	2
y					
(x, y)					

Identifying x- and y-Intercepts **In Exercises 17–20, identify the x- and y-intercepts of the graph.**

17.



18.



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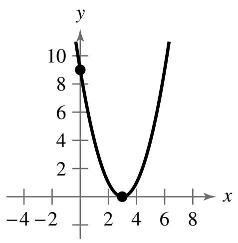

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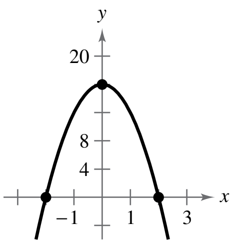


Finding x- and y-Intercepts **In Exercises 21–28 find the x- and y-intercepts of the graph of the equation.**

21. $y = 5x - 6$
22. $y = 8 - 3x$
23. $y = \sqrt{x + 4}$
24. $y = \sqrt{2x - 1}$
25. $y = \lvert 3x - 7\rvert$
26. $y = -\lvert x + 10\rvert$
27. $y = 2x^3 - 4x^2$
28. $y^2 = x + 1$

Testing for Symmetry **In Exercises 29–36, use the algebraic tests to check for symmetry with respect to both axes and the origin.**

29. $x^2 - y = 0$ **30.** $x - y^2 = 0$

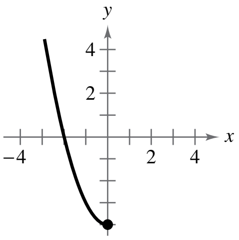
31. $y = x^3$ **32.** $y = x^4 - x^2 + 3$

33. $y = \dfrac{x}{x^2 + 1}$ **34.** $y = \dfrac{1}{x^2 + 1}$

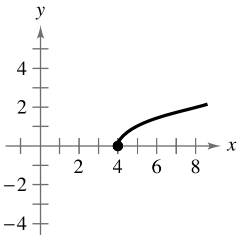
35. $xy^2 + 10 = 0$ **36.** $xy = 4$

Using Symmetry as a Sketching Aid **In Exercises 37–40, assume that the graph has the given type of symmetry. Complete the graph of the equation.**

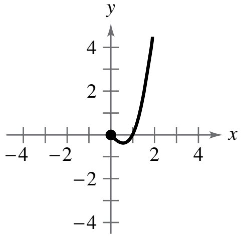
37. y-Axis symmetry



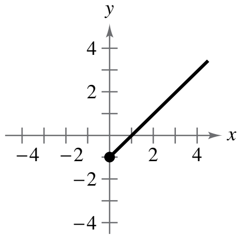
38. x-Axis symmetry



39. Origin symmetry



40. y-Axis symmetry



Sketching the Graph of an Equation **In Exercises 41–52, test for symmetry and graph the equation. Then identify any intercepts.**

41. $y = -3x + 1$ **42.** $y = 2x - 3$

43. $y = x^2 - 2x$ **44.** $y = -x^2 - 2x$

45. $y = x^3 + 3$ **46.** $y = x^3 - 1$

47. $y = \sqrt{x - 3}$ **48.** $y = \sqrt{1 - x}$

49. $y = |x - 6|$ **50.** $y = 1 - |x|$

51. $x = y^2 - 1$ **52.** $x = y^2 - 5$

Approximating Intercepts **In Exercises 53–62, use a graphing utility to graph the equation. Use a standard setting. Approximate any intercepts.**

53. $y = 5 - \frac{1}{2}x$ **54.** $y = \frac{2}{3}x - 1$

55. $y = x^2 - 4x + 3$ **56.** $y = x^2 + x - 2$

57. $y = \dfrac{2x}{x - 1}$ **58.** $y = \dfrac{4}{x^2 + 1}$

59. $y = \sqrt[3]{x + 1}$ **60.** $y = x\sqrt{x + 6}$

61. $y = |x + 3|$ **62.** $y = 2 - |x|$

Writing the Equation of a Circle **In Exercises 63–70, write the standard form of the equation of the circle with the given characteristics.**

63. Center: $(0, 0)$; Radius: 3

64. Center: $(0, 0)$; Radius: 7

65. Center: $(-4, 5)$; Radius: 2

66. Center: $(1, -3)$; Radius: $\sqrt{11}$

67. Center: $(3, 8)$; Solution point: $(-9, 13)$

68. Center: $(-2, -6)$; Solution point: $(1, -10)$

69. Endpoints of a diameter: $(3, 2), (-9, -8)$

70. Endpoints of a diameter: $(11, -5), (3, 15)$

Sketching a Circle **In Exercises 71–76, find the center and radius of the circle with the given equation. Then sketch the circle.**

71. $x^2 + y^2 = 25$ **72.** $x^2 + y^2 = 36$

73. $(x - 1)^2 + (y + 3)^2 = 9$

74. $x^2 + (y - 1)^2 = 1$

75. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

76. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

77. Depreciation A hospital purchases a new magnetic resonance imaging (MRI) machine for \$1.2 million. The depreciated value y (reduced value) after t years is given by

$$y = 1{,}200{,}000 - 80{,}000t,\ 0 \le t \le 10.$$

Sketch the graph of the equation.

78. Depreciation You purchase an all-terrain vehicle (ATV) for \$9500. The depreciated value y (reduced value) after t years is given by

$$y = 9500 - 1000t,\ 0 \le t \le 6.$$

Sketch the graph of the equation.

79. Geometry A regulation NFL playing field of length x and width y has a perimeter of $346\frac{2}{3}$ or $\frac{1040}{3}$ yards.

(a) Draw a rectangle that gives a visual representation of the problem. Use the specified variables to label the sides of the rectangle.

(b) Show that the width of the rectangle is $y = \frac{520}{3} - x$ and its area is $A = x\left(\frac{520}{3} - x\right)$.

(c) Use a graphing utility to graph the area equation. Be sure to adjust your window settings.

(d) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.

(e) Use an appropriate research source to determine the actual dimensions and area of a regulation NFL playing field and compare your findings with the results of part (d).

The symbol indicates an exercise or a part of an exercise in which you are instructed to use a graphing utility.

80. **Population Statistics** The table shows the life expectancies of a child (at birth) in the United States for selected years from 1950 through 2020. *(Source: Macrotrends LLC)*

Spreadsheet at LarsonPrecalculus.com

Year	Life Expectancy, y
1950	68.14
1960	69.84
1970	70.78
1980	73.70
1990	75.19
2000	76.75
2010	78.49
2020	78.93

A model for the life expectancy during this period is

$$y = \frac{68.0 + 0.33t}{1 + 0.002t}, \quad 0 \le t \le 70$$

where y represents the life expectancy and t is the time in years, with $t = 0$ corresponding to 1950.

(a) Use a graphing utility to graph the data from the table and the model in the same viewing window. How well does the model fit the data? Explain.

(b) Determine the life expectancy in 1990 both graphically and algebraically.

(c) Use the graph to determine the year when life expectancy was approximately 70.1. Verify your answer algebraically.

(d) Identify the y-intercept of the graph of the model. What does it represent in the context of the problem?

(e) Do you think this model can be used to predict the life expectancy of a child 50 years from now? Explain.

Exploring the Concepts

True or False? **In Exercises 81 and 82, determine whether the statement is true or false. Justify your answer.**

81. The graph of a linear equation cannot be symmetric with respect to the origin.

82. The graph of a linear equation can have either no x-intercepts or only one x-intercept.

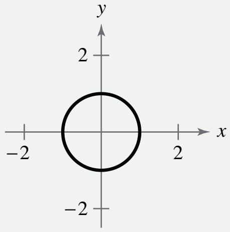
Blaine Harrington III/The Image Bank/Getty Images

83. **Error Analysis** Describe the error.

The graph of $x = 3y^2$ is symmetric with respect to the y-axis because

$$\begin{aligned} x &= 3(-y)^2 \\ &= 3y^2. \end{aligned}$$

84. **HOW DO YOU SEE IT?** The graph shows the circle with the equation $x^2 + y^2 = 1$. Describe the types of symmetry that you observe.



85. **Think About It** Find a and b when the graph of $y = ax^2 + bx^3$ is symmetric with respect to (a) the y-axis and (b) the origin. (There are many correct answers.)

86. **Graph of a Linear Equation** When the graph of a linear equation in two variables has a negative x-intercept and a positive y-intercept, does the line rise or fall from left to right? Through which quadrant(s) does the line pass? Use a graph to illustrate your answer.

Review & Refresh

Video solutions at LarsonPrecalculus.com

Using the Distributive Property **In Exercises 87–90, use the Distributive Property to rewrite the expression. Simplify your results.**

87. $3(7x + 1)$ **88.** $5(x - 6)$

89. $6(x - 1) + 4$ **90.** $4(x + 2) - 12$

Finding the Least Common Denominator **In Exercises 91–94, find the least common denominator.**

91. $\frac{x}{3}, \frac{3x}{4}$ **92.** $\frac{4x}{9}, \frac{1}{3}, x, \frac{5}{3}$

93. $\frac{3x}{x - 4}, 5, \frac{12}{x - 4}$ **94.** $\frac{1}{x - 2}, \frac{3}{x + 2}, \frac{6x}{x^2 - 4}$

Simplifying an Expression **In Exercises 95–98, simplify the expression.**

95. $7\sqrt{72} - 5\sqrt{18}$ **96.** $-10\sqrt{25y} - \sqrt{y}$

97. $7^{3/2} \cdot 7^{11/2}$ **98.** $\frac{10^{17/4}}{10^{5/4}}$

Operations with Polynomials **In Exercises 99–102, perform the operation and write the result in standard form.**

99. $(9x - 4) + (2x^2 - x + 15)$

100. $4x(11 - x + 3x^2)$ **101.** $(2x + 9)(x - 7)$

102. $(3x^2 - 5)(-x^2 + 1)$

1.3 Linear Equations in Two Variables

Linear equations in two variables can help you model and solve real-life problems. For example, in Exercise 90 on page 33, you will use a surveyor's measurements to find a linear equation that models a mountain road.

- **Use slope to graph linear equations in two variables.**
- **Find the slope of a line given two points on the line.**
- **Write linear equations in two variables.**
- **Use slope to identify parallel and perpendicular lines.**
- **Use slope and linear equations in two variables to model and solve real-life problems.**

Using Slope

The simplest mathematical model relating two variables x and y is the **linear equation**

$$y = mx + b \qquad \text{Linear equation in two variables } x \text{ and } y$$

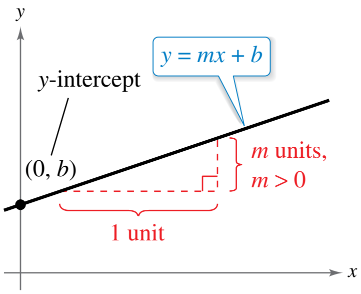
where m and b are constants. The equation is called *linear* because its graph is a line. (In mathematics, the term *line* means *straight line*.) By letting $x = 0$, you obtain

$$y = m(0) + b = b.$$

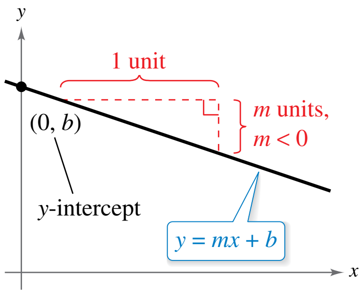
So, the line crosses the y-axis at $y = b$, as shown in Figure 1.21. In other words, the y-intercept is $(0, b)$. The steepness, or *slope,* of the line is m.

$$y = \underbrace{m}_{\text{Slope}}x + \underbrace{b}_{y\text{-Intercept}}$$

The **slope** of a nonvertical line is the number of units the line rises (or falls) vertically for each unit of horizontal change from left to right. When the line rises from left to right, the slope is positive (see Figure 1.21). When the line falls from left to right, the slope is negative (see Figure 1.22).



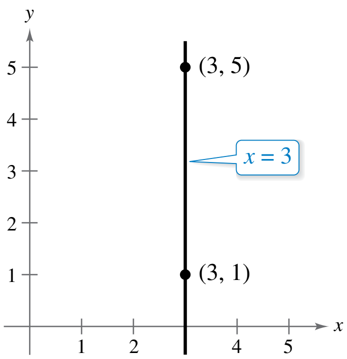
Positive slope, line rises
Figure 1.21



Negative slope, line falls
Figure 1.22

The Slope-Intercept Form of the Equation of a Line

The linear equation $y = mx + b$ is in **slope-intercept form.** The graph of the equation $y = mx + b$ is a line whose slope is m and whose y-intercept is $(0, b)$.



Slope is undefined.
Figure 1.23

Once you determine the slope and the y-intercept of a line, it is relatively simple to sketch its graph. In the next example, note that none of the lines is vertical. A vertical line has an equation of the form

$$x = a \qquad \text{Vertical line}$$

where a is a real number. The equation of a vertical line cannot be written in the form $y = mx + b$ because the slope of a vertical line is undefined (see Figure 1.23).

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EXAMPLE 1 Graphing Linear Equations

See LarsonPrecalculus.com for an interactive version of this type of example.

Sketch the graph of each linear equation.

a. $y = 2x + 1$

b. $y = 2$

c. $x + y = 2$

Solution

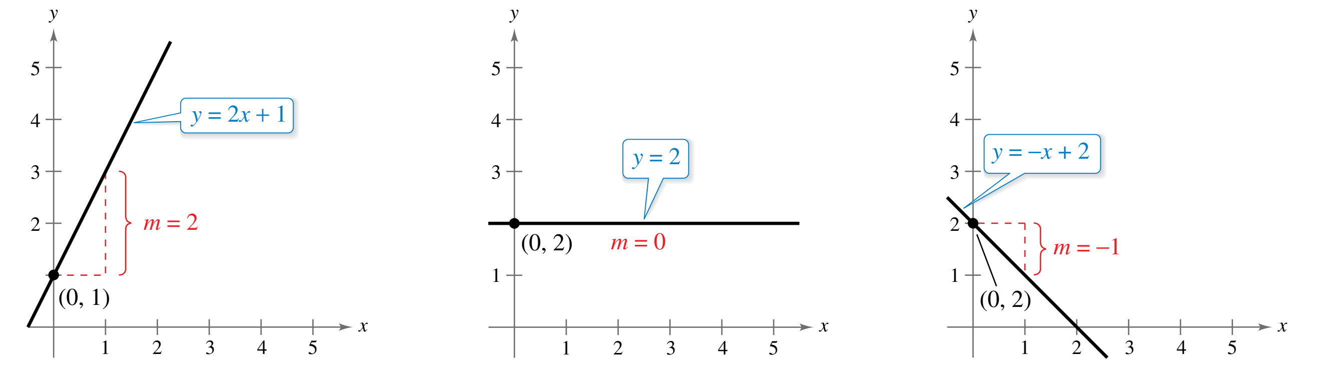
a. Because $b = 1$, the y-intercept is $(0, 1)$. Moreover, the slope is $m = 2$, so the line *rises* two units for each unit the line moves to the right, as shown in Figure 1.24(a).

b. By writing this equation in the form $y = (0)x + 2$, you find that the y-intercept is $(0, 2)$ and the slope is $m = 0$. A slope of 0 implies that the line is horizontal—that is, it does not rise *or* fall, as shown in Figure 1.24(b).

c. By writing this equation in slope-intercept form

$$x + y = 2 \qquad \text{Write original equation.}$$
$$y = -x + 2 \qquad \text{Subtract } x \text{ from each side.}$$
$$y = (-1)x + 2 \qquad \text{Write in slope-intercept form.}$$

you find that the y-intercept is $(0, 2)$. Moreover, the slope is $m = -1$, so the line *falls* one unit for each unit the line moves to the right, as shown in Figure 1.24(c).



(a) When m is positive, the line rises. **(b)** When m is 0, the line is horizontal. **(c)** When m is negative, the line falls.

Figure 1.24

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Sketch the graph of each linear equation.

a. $y = 3x + 2$ **b.** $y = -3$ **c.** $4x + y = 5$

From the lines shown in Figures 1.23 and 1.24, you can make several generalizations about the slope of a line.

1. A line with positive slope $(m > 0)$ *rises* from left to right. [See Figure 1.24(a).]
2. A line with negative slope $(m < 0)$ *falls* from left to right. [See Figure 1.24(c).]
3. A line with zero slope $(m = 0)$ is *horizontal*. [See Figure 1.24(b).]
4. A line with undefined slope is *vertical*. (See Figure 1.23.)

From the slope-intercept form of the equation of a line, you can see that a horizontal line $(m = 0)$ has an equation of the form

$$y = (0)x + b \quad \text{or} \quad y = b. \qquad \text{Horizontal line}$$

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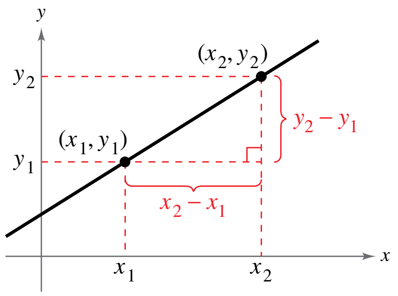
Finding the Slope of a Line

Given an equation of a line, you can find its slope by writing the equation in slope-intercept form. When you are not given an equation, you can still find the slope by using two points on the line. For example, consider the line passing through the points (x_1, y_1) and (x_2, y_2) in the figure below.



As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction. That is,

$$y_2 - y_1 = \text{change in } y = \text{rise}$$

and

$$x_2 - x_1 = \text{change in } x = \text{run}.$$

The ratio of $(y_2 - y_1)$ to $(x_2 - x_1)$ represents the slope of the line that passes through the points (x_1, y_1) and (x_2, y_2).

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

ALGEBRA HELP

Be sure you understand that the definition of slope does not apply to *vertical* lines. For instance, consider the points $(3, 5)$ and $(3, 1)$ on the vertical line shown in Figure 1.23. Applying the formula for slope, you obtain

$$m = \frac{5 - 1}{3 - 3} = \frac{4}{0}. \quad ✗$$

Because division by zero is undefined, the slope of a vertical line is undefined.

Definition of the Slope of a Line

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where $x_1 \neq x_2$.

When using the formula for slope, the *order of subtraction* is important. Given two points on a line, you are free to label either one of them as (x_1, y_1) and the other as (x_2, y_2). However, once you do this, you must form the numerator and denominator using the same order of subtraction.

$$\underbrace{m = \frac{y_2 - y_1}{x_2 - x_1}}_{\text{Correct}} \qquad \underbrace{m = \frac{y_1 - y_2}{x_1 - x_2}}_{\text{Correct}} \qquad \underbrace{m = \frac{y_2 - y_1}{x_1 - x_2}}_{\text{Incorrect}} \quad ✗$$

For example, the slope of the line passing through the points $(3, 4)$ and $(5, 7)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{5 - 3} = \frac{3}{2}$$

or

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{4 - 7}{3 - 5} = \frac{-3}{-2} = \frac{3}{2}.$$

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SKILLS REFRESHER

For a refresher on simplifying rational expressions, watch the video at *LarsonPrecalculus.com*.

EXAMPLE 2 Finding the Slope of a Line Through Two Points

Find the slope of the line passing through each pair of points.

a. $(-2, 0)$ and $(3, 1)$ **b.** $(-1, 2)$ and $(2, 2)$

c. $(0, 4)$ and $(1, -1)$ **d.** $(3, 4)$ and $(3, 1)$

Solution

a. Letting $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (3, 1)$, you find that the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{3 - (-2)} = \frac{1}{5}.$$ See Figure 1.25(a).

b. The slope of the line passing through $(-1, 2)$ and $(2, 2)$ is

$$m = \frac{2 - 2}{2 - (-1)} = \frac{0}{3} = 0.$$ See Figure 1.25(b).

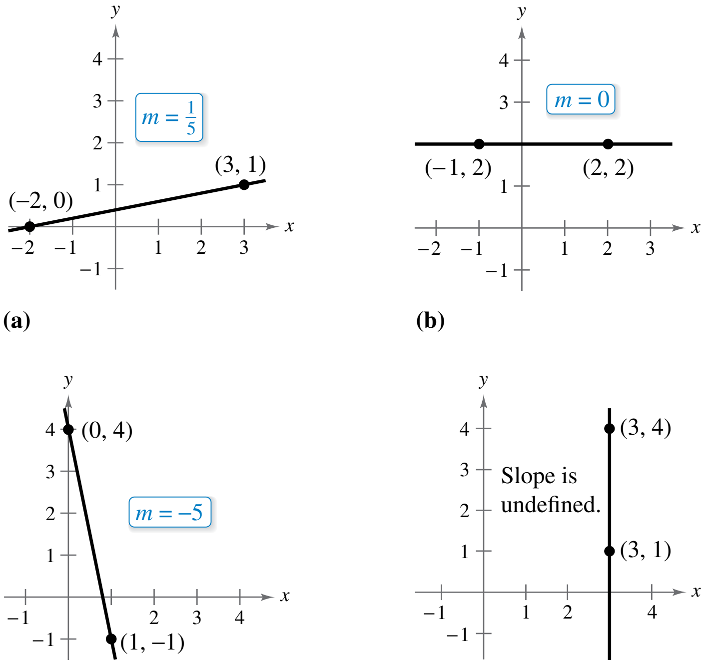
c. The slope of the line passing through $(0, 4)$ and $(1, -1)$ is

$$m = \frac{-1 - 4}{1 - 0} = \frac{-5}{1} = -5.$$ See Figure 1.25(c).

d. The slope of the line passing through $(3, 4)$ and $(3, 1)$ is

$$m = \frac{1 - 4}{3 - 3} = \frac{-3}{0}.$$ See Figure 1.25(d).

Division by 0 is undefined, so the slope is undefined and the line is vertical.



(a) (b) (c) (d)

Figure 1.25

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the slope of the line passing through each pair of points.

a. $(-5, -6)$ and $(2, 8)$ **b.** $(4, 2)$ and $(2, 5)$

c. $(0, 0)$ and $(0, -6)$ **d.** $(0, -1)$ and $(3, -1)$

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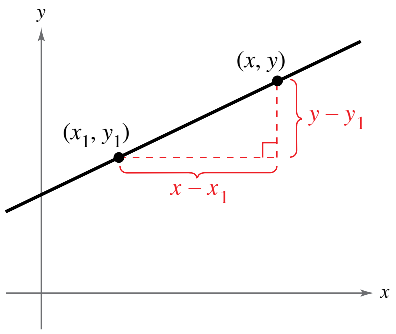
Writing Linear Equations in Two Variables

When you know the slope of a line *and* you also know the coordinates of one point on the line, you can find an equation of the line. For example, in the figure at the right, let (x_1, y_1) be a point on the line whose slope is m. When (x, y) is any *other* point on the line, it follows that

$$\frac{y - y_1}{x - x_1} = m.$$



This equation in the variables x and y can be rewritten in the **point-slope form** of the equation of a line.

Point-Slope Form of the Equation of a Line

The equation of the line with slope m passing through the point (x_1, y_1) is

$y - y_1 = m(x - x_1)$. Point-slope form

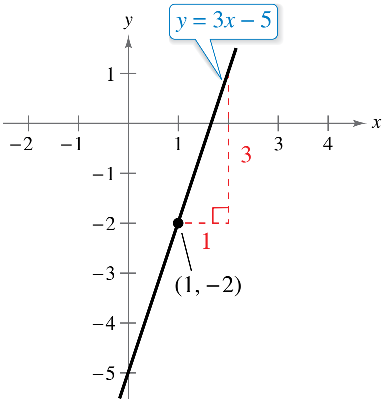


Figure 1.26

EXAMPLE 3 Using the Point-Slope Form

Find the slope-intercept form of the equation of the line that has a slope of 3 and passes through the point $(1, -2)$.

Solution Use the point-slope form with $m = 3$ and $(x_1, y_1) = (1, -2)$.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = 3(x - 1)$	Substitute for m, x_1, and y_1.
$y + 2 = 3x - 3$	Simplify.
$y = 3x - 5$	Write in slope-intercept form.

The slope-intercept form of the equation of the line is $y = 3x - 5$. Figure 1.26 shows the graph of this equation.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the slope-intercept form of the equation of the line that has the given slope and passes through the given point.

a. $m = 2$, $(3, -7)$ **b.** $m = -\frac{2}{3}$, $(1, 1)$ **c.** $m = 0$, $(1, 1)$

ALGEBRA HELP

When you find an equation of the line that passes through two given points, you only need to substitute the coordinates of one of the points in the point-slope form. It does not matter which point you choose because both points will yield the same result.

The point-slope form can be used to find an equation of the line passing through two points (x_1, y_1) and (x_2, y_2). To do this, first find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Then use the point-slope form to obtain the equation.

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \qquad \text{Two-point form}$$

This is sometimes called the **two-point form** of the equation of a line.

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Parallel and Perpendicular Lines

The slope of a line is a convenient way for determining whether two lines are parallel or perpendicular. Specifically, nonvertical lines with the same slope are parallel, and nonvertical lines whose slopes are negative reciprocals are perpendicular.

Parallel and Perpendicular Lines

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is,

$$m_1 = m_2. \qquad \text{Parallel} \Leftrightarrow \text{Slopes are equal.}$$

2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = -\frac{1}{m_2}. \qquad \text{Perpendicular} \Leftrightarrow \text{Slopes are negative reciprocals.}$$

Note that $m_1 m_2 = -1$.

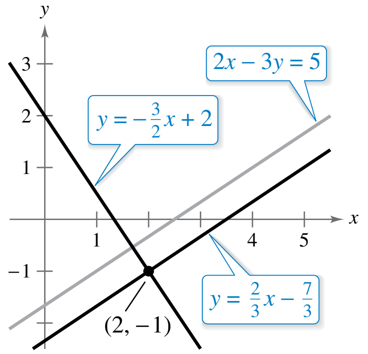

Figure 1.27

TECHNOLOGY

The standard viewing window on some graphing utilities does not give a true geometric perspective because the screen is rectangular, which distorts the image. So, perpendicular lines will not appear to be perpendicular, and circles will not appear to be circular. To overcome this, use a square setting, in which the horizontal and vertical tick marks have equal spacing. On many graphing utilities, a square setting can be obtained when the ratio of the range of y to the range of x is 2 to 3.

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EXAMPLE 4 Finding Parallel and Perpendicular Lines

Find the slope-intercept form of the equations of the lines that pass through the point $(2, -1)$ and are (a) parallel to and (b) perpendicular to the line $2x - 3y = 5$.

Solution Write the equation $2x - 3y = 5$ in slope-intercept form.

$$2x - 3y = 5 \qquad \text{Write original equation.}$$
$$-3y = -2x + 5 \qquad \text{Subtract } 2x \text{ from each side.}$$
$$y = \tfrac{2}{3}x - \tfrac{5}{3} \qquad \text{Write in slope-intercept form.}$$

Notice that the line has a slope of $m = \frac{2}{3}$.

a. Any line parallel to the given line must also have a slope of $\frac{2}{3}$. Use the point-slope form with $m = \frac{2}{3}$ and $(x_1, y_1) = (2, -1)$.

$$y - (-1) = \tfrac{2}{3}(x - 2) \qquad \text{Write in point-slope form.}$$
$$y + 1 = \tfrac{2}{3}x - \tfrac{4}{3} \qquad \text{Simplify.}$$
$$y = \tfrac{2}{3}x - \tfrac{7}{3} \qquad \text{Write in slope-intercept form.}$$

Notice the similarity between the slope-intercept form of this equation and the slope-intercept form of the given equation, $y = \frac{2}{3}x - \frac{5}{3}$.

b. Any line perpendicular to the given line must have a slope of $-\frac{3}{2}$ (because $-\frac{3}{2}$ is the negative reciprocal of $\frac{2}{3}$). Use the point-slope form with $m = -\frac{3}{2}$ and $(x_1, y_1) = (2, -1)$.

$$y - (-1) = -\tfrac{3}{2}(x - 2) \qquad \text{Write in point-slope form.}$$
$$y + 1 = -\tfrac{3}{2}x + 3 \qquad \text{Simplify.}$$
$$y = -\tfrac{3}{2}x + 2 \qquad \text{Write in slope-intercept form.}$$

The graphs of all three equations are shown in Figure 1.27.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

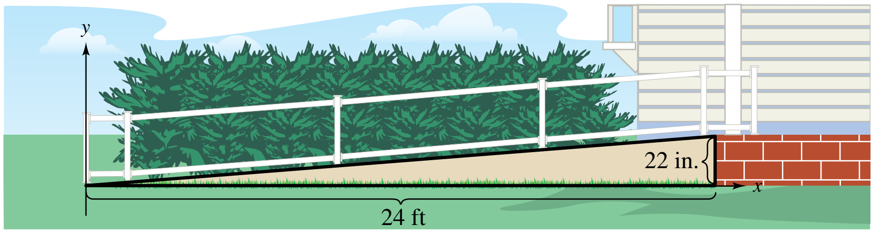
Find the slope-intercept form of the equations of the lines that pass through the point $(-4, 1)$ and are (a) parallel to and (b) perpendicular to the line $5x - 3y = 8$.

Applications

In real-life problems, the slope of a line can be interpreted as either a *ratio* or a *rate*. When the x-axis and y-axis have the same unit of measure, the slope has no units and is a **ratio.** When the x-axis and y-axis have different units of measure, the slope is a **rate** or **rate of change.**

EXAMPLE 5 Using Slope as a Ratio

The maximum recommended slope of a wheelchair ramp is $\frac{1}{12}$. A business installs a wheelchair ramp that rises 22 inches over a horizontal length of 24 feet (see figure). Is the ramp steeper than recommended? *(Source: ADA Standards for Accessible Design)*



Solution The horizontal length of the ramp is 24 feet or $12(24) = 288$ inches. The slope of the ramp is the ratio of its height (the rise) to its length (the run).

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{22 \text{ in.}}{288 \text{ in.}} \approx 0.076$$

The slope of the ramp is about 0.076, which is less than $\frac{1}{12} \approx 0.083$. So, the ramp is not steeper than recommended. Note that the slope of the ramp is a ratio and has no units.

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The business in Example 5 installs a second ramp that rises 36 inches over a horizontal length of 32 feet. Is the ramp steeper than recommended?

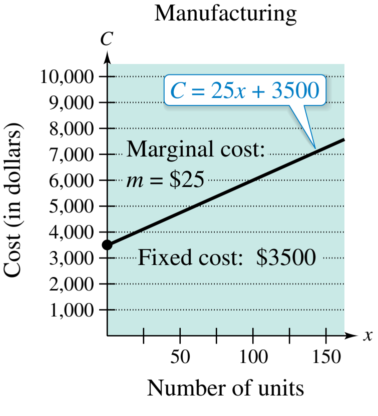
EXAMPLE 6 Using Slope as a Rate of Change

A kitchen appliance manufacturing company determines that the total cost C (in dollars) of producing x units of a blender is given by

$$C = 25x + 3500. \qquad \text{Cost equation}$$

Interpret the y-intercept and slope of this line.

Solution The y-intercept $(0, 3500)$ tells you that the cost of producing 0 units is \$3500. This is the *fixed cost* of production—it includes costs that must be paid regardless of the number of units produced. The slope of $m = 25$ tells you that the cost of producing each unit is \$25, as shown in Figure 1.28. Economists call the cost per unit the *marginal cost*. When the production increases by one unit, the "margin," or extra amount of cost, is \$25. So, the cost increases at a rate of \$25 per unit.



Production cost
Figure 1.28

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An accounting firm determines that the value V (in dollars) of a copier t years after its purchase is given by

$$V = -300t + 1500.$$

Interpret the y-intercept and slope of this line.

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Businesses can deduct most of their expenses in the same year they occur. One exception is the cost of property that has a useful life of more than 1 year. Such costs must be *depreciated* (decreased in value) over the useful life of the property. Depreciating the *same amount* each year is called *linear* or *straight-line depreciation*. The *book value* is the difference between the original value and the total amount of depreciation accumulated to date.

EXAMPLE 7 Straight-Line Depreciation

A college purchases exercise equipment worth \$12,000 for the new campus fitness center. The equipment has a useful life of 8 years. The salvage value at the end of 8 years is \$2000. Write a linear equation that describes the book value of the equipment each year.

Solution Let V represent the value of the equipment at the end of year t. Represent the initial value of the equipment by the data point $(0, 12{,}000)$ and the salvage value of the equipment by the data point $(8, 2000)$. The slope of the line is

$$m = \frac{2000 - 12{,}000}{8 - 0} = -\$1250 \text{ per year}$$

which represents the annual depreciation in *dollars per year*. So, the value of the equipment decreases \$1250 per year. Using the point-slope form, write an equation of the line.

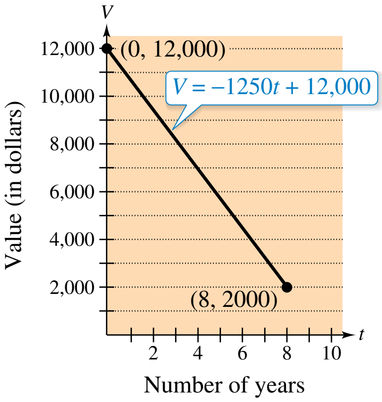
$$V - 12{,}000 = -1250(t - 0) \qquad \text{Write in point-slope form.}$$

$$V = -1250t + 12{,}000 \qquad \text{Write in slope-intercept form.}$$

Note that the domain of the equation is $0 \le t \le 8$. The table shows the book value at the end of each year, and Figure 1.29 shows the graph of the equation.

Year, t	Value, V
0	12,000
1	10,750
2	9500
3	8250
4	7000
5	5750
6	4500
7	3250
8	2000

Useful Life of Equipment



Straight-line depreciation
Figure 1.29

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A manufacturing firm purchases a machine worth \$24,750. The machine has a useful life of 6 years. After 6 years, the machine will have to be discarded and replaced, because it will have no salvage value. Write a linear equation that describes the book value of the machine each year.

ALGEBRA HELP
Some real-life applications may refer to an ordered pair as a *data point*.

In many real-life applications, the two data points that determine the line are often given in a disguised form. Note how the data points are described in Example 7.

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EXAMPLE 8 Predicting a Population

The population of Texas was about 27.9 million in 2016 and about 28.7 million in 2018. Use this information to write a linear equation that gives the population (in millions) in terms of the year. Predict the population of Texas in 2024. *(Source: U.S. Census Bureau)*

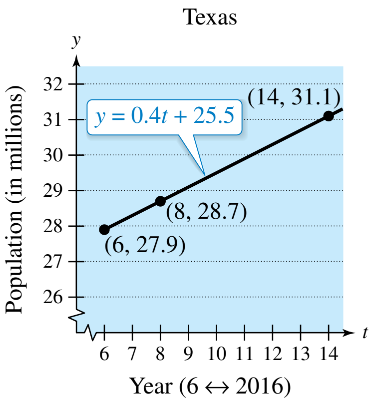


Figure 1.30

Solution Let $t = 6$ represent 2016. Then the two given values are represented by the data points $(6, 27.9)$ and $(8, 28.7)$. The slope of the line through these points is

$$m = \frac{28.7 - 27.9}{8 - 6} = \frac{0.8}{2} = 0.4 \text{ million people per year.}$$

Use the point-slope form to write an equation that relates the population y and the year t.

$$y - 27.9 = 0.4(t - 6) \quad \Longrightarrow \quad y = 0.4t + 25.5$$

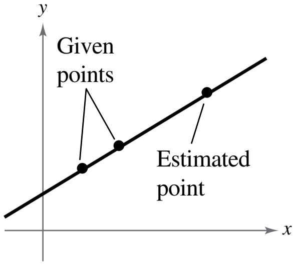
According to this equation, the population in 2024 will be

$$y = 0.4(14) + 25.5 = 5.6 + 25.5 = 31.1 \text{ million people.}$$

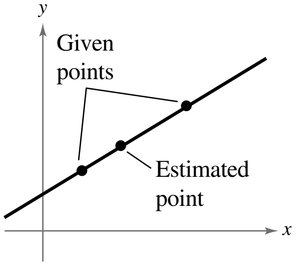
See Figure 1.30.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

The population of Oregon was about 3.9 million in 2013 and about 4.2 million in 2018. Repeat Example 8 using this information. *(Source: U.S. Census Bureau)*



Linear extrapolation
Figure 1.31



Linear interpolation
Figure 1.32

The prediction method illustrated in Example 8 is called **linear extrapolation.** Note in Figure 1.31 that an extrapolated point does not lie between the given points. When the estimated point lies between two given points, as shown in Figure 1.32, the procedure is called **linear interpolation.**

The slope of a vertical line is undefined, so its equation cannot be written in slope-intercept form. However, every line has an equation that can be written in the **general form** $Ax + By + C = 0$, where A and B are not both zero.

Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$
6. Two-point form: $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Summarize (Section 1.3)

1. Explain how to use slope to graph a linear equation in two variables *(page 22)* and how to find the slope of a line passing through two points *(page 24)*. For examples of using and finding slopes, see Examples 1 and 2.
2. State the point-slope form of the equation of a line *(page 26)*. For an example of using point-slope form, see Example 3.
3. Explain how to use slope to identify parallel and perpendicular lines *(page 27)*. For an example of finding parallel and perpendicular lines, see Example 4.
4. Describe examples of how to use slope and linear equations in two variables to model and solve real-life problems *(pages 28–30, Examples 5–8)*.

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1.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

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Vocabulary and Concept Check

In Exercises 1–6, fill in the blanks.

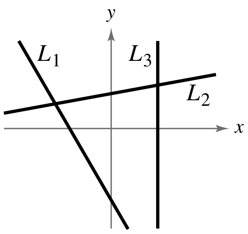
1. The simplest mathematical model for relating two variables is the ________ equation in two variables $y = mx + b$.
2. For a line, the ratio of the change in y to the change in x is the ________ of the line.
3. The ________-________ form of the equation of a line with slope m passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$.
4. Two distinct nonvertical lines are ________ if and only if their slopes are equal.
5. When the x-axis and y-axis have different units of measure, the slope can be interpreted as a ________.
6. ________ ________ is the prediction method used to estimate a point on a line when the point does not lie between the given points.

7. What is the relationship between two lines whose slopes are -3 and $\frac{1}{3}$?
8. Write the point-slope form equation $y - y_1 = m(x - x_1)$ in general form.

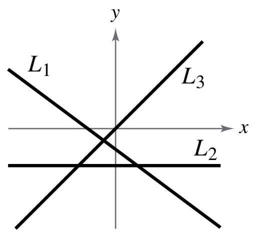
Skills and Applications

Identifying Lines **In Exercises 9 and 10, identify the line that has each slope.**

9. (a) $m = \frac{2}{3}$
(b) m is undefined.
(c) $m = -2$



10. (a) $m = 0$
(b) $m = -\frac{3}{4}$
(c) $m = 1$

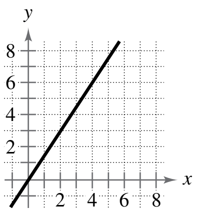


Sketching Lines **In Exercises 11 and 12, sketch the lines through the point with the given slopes on the same set of coordinate axes.**

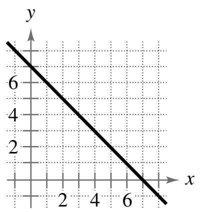
	Point	Slopes	
11.	(2, 3)	(a) 0	(b) 1
		(c) 2	(d) -3
12.	$(-4, 1)$	(a) 3	(b) -3
		(c) $\frac{1}{2}$	(d) Undefined

Estimating the Slope of a Line **In Exercises 13 and 14, estimate the slope of the line.**

13.



14.



Graphing a Linear Equation **In Exercises 15–24, find the slope and y-intercept (if possible) of the line. Sketch the line.**

15. $y = 5x + 3$
16. $y = -x - 10$
17. $y = -\frac{3}{4}x - 1$
18. $y = \frac{2}{3}x + 2$
19. $y - 5 = 0$
20. $x + 4 = 0$
21. $5x - 2 = 0$
22. $3y + 5 = 0$
23. $7x - 6y = 30$
24. $2x + 3y = 9$

Finding the Slope of a Line Through Two Points **In Exercises 25–34, find the slope of the line passing through the pair of points.**

25. $(0, 9), (6, 0)$
26. $(10, 0), (0, -5)$
27. $(-3, -2), (1, 6)$
28. $(2, -1), (-2, 1)$
29. $(5, -7), (8, -7)$
30. $(-2, 1), (-4, -5)$
31. $(-6, -1), (-6, 4)$
32. $(0, -10), (-4, 0)$
33. $(4.8, 3.1), (-5.2, 1.6)$
34. $\left(\frac{11}{2}, -\frac{4}{3}\right), \left(-\frac{3}{2}, -\frac{1}{3}\right)$

Using the Slope and a Point **In Exercises 35–42, use the slope of the line and the point on the line to find three additional points through which the line passes. (There are many correct answers.)**

35. $m = 0$, $(5, 7)$
36. $m = 0$, $(3, -2)$
37. $m = 2$, $(-5, 4)$
38. $m = -2$, $(0, -9)$
39. $m = -\frac{1}{3}$, $(4, 5)$
40. $m = \frac{1}{4}$, $(3, -4)$
41. m is undefined, $(-4, 3)$
42. m is undefined, $(2, 14)$

Using the Point-Slope Form **In Exercises 43–54, find the slope-intercept form of the equation of the line that has the given slope and passes through the given point. Sketch the line.**

43. $m = 3, \quad (0, -2)$
44. $m = -1, \quad (0, 10)$
45. $m = -2, \quad (-3, 6)$
46. $m = 4, \quad (0, 0)$
47. $m = -\frac{1}{3}, \quad (4, 0)$
48. $m = \frac{1}{4}, \quad (8, 2)$
49. $m = -\frac{1}{2}, \quad (2, -3)$
50. $m = \frac{3}{4}, \quad (-2, -5)$
51. $m = 0, \quad \left(4, \frac{5}{2}\right)$
52. $m = 6, \quad \left(2, \frac{3}{2}\right)$
53. $m = 5, \quad (-5.1, 1.8)$
54. $m = 0, \quad (-2.5, 3.25)$

Finding an Equation of a Line **In Exercises 55–64, find an equation of the line passing through the pair of points. Sketch the line.**

55. $(5, -1), (-5, 5)$
56. $(4, 3), (-4, -4)$
57. $(-7, 2), (-7, 5)$
58. $(-6, -3), (2, -3)$
59. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$
60. $(1, 1), \left(6, -\frac{2}{3}\right)$
61. $(1, 0.6), (-2, -0.6)$
62. $(-8, 0.6), (2, -2.4)$
63. $(2, -1), \left(\frac{1}{3}, -1\right)$
64. $\left(\frac{7}{3}, -8\right), \left(\frac{7}{3}, 1\right)$

Parallel and Perpendicular Lines **In Exercises 65–68, determine whether the lines are parallel, perpendicular, or neither.**

65. $L_1\colon\ y = -\frac{2}{3}x - 3$
$L_2\colon\ y = -\frac{2}{3}x + 4$
66. $L_1\colon\ y = \frac{1}{4}x - 1$
$L_2\colon\ y = 4x + 7$
67. $L_1\colon\ y = \frac{1}{2}x - 3$
$L_2\colon\ y = -\frac{1}{2}x + 1$
68. $L_1\colon\ y = -\frac{4}{5}x - 5$
$L_2\colon\ y = \frac{5}{4}x + 1$

Parallel and Perpendicular Lines **In Exercises 69–72, determine whether the lines L_1 and L_2 passing through the pairs of points are parallel, perpendicular, or neither.**

69. $L_1\colon\ (0, -1), (5, 9)$
$L_2\colon\ (0, 3), (4, 1)$
70. $L_1\colon\ (-2, -1), (1, 5)$
$L_2\colon\ (1, 3), (5, -5)$
71. $L_1\colon\ (-6, -3), (2, -3)$
$L_2\colon\ \left(3, -\frac{1}{2}\right), \left(6, -\frac{1}{2}\right)$
72. $L_1\colon\ (4, 8), (-4, 2)$
$L_2\colon\ (3, -5), \left(-1, \frac{1}{3}\right)$

Finding Parallel and Perpendicular Lines **In Exercises 73–80, find equations of the lines that pass through the given point and are (a) parallel to and (b) perpendicular to the given line.**

73. $4x - 2y = 3, \quad (2, 1)$
74. $x + y = 7, \quad (-3, 2)$
75. $3x + 4y = 7, \quad \left(-\frac{2}{3}, \frac{7}{8}\right)$
76. $5x + 3y = 0, \quad \left(\frac{7}{8}, \frac{3}{4}\right)$
77. $y + 5 = 0, \quad (-2, 4)$
78. $x - 4 = 0, \quad (3, -2)$
79. $x - y = 4, \quad (2.5, 6.8)$
80. $6x + 2y = 9, \quad (-3.9, -1.4)$

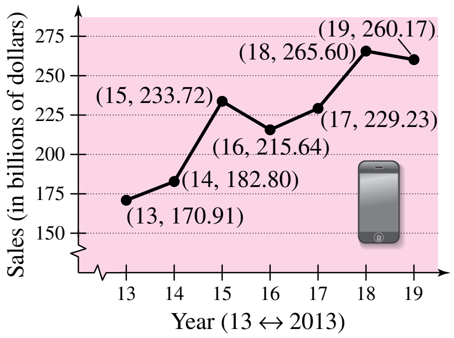
Using Intercept Form **In Exercises 81–86, use the *intercept form* to find the general form of the equation of the line with the given intercepts. The intercept form of the equation of a line with intercepts $(a, 0)$ and $(0, b)$ is**

$$\frac{x}{a} + \frac{y}{b} = 1, \quad a \neq 0, \quad b \neq 0.$$

81. x-intercept: $(3, 0)$; y-intercept: $(0, 5)$
82. x-intercept: $(-3, 0)$; y-intercept: $(0, 4)$
83. x-intercept: $\left(-\frac{1}{6}, 0\right)$; y-intercept: $\left(0, -\frac{2}{3}\right)$
84. x-intercept: $\left(\frac{2}{3}, 0\right)$; y-intercept: $(0, -2)$
85. Point on line: $(1, 2)$
x-intercept: $(c, 0), \quad c \neq 0$
y-intercept: $(0, c), \quad c \neq 0$
86. Point on line: $(-3, 4)$
x-intercept: $(d, 0), \quad d \neq 0$
y-intercept: $(0, d), \quad d \neq 0$

87. Sales The slopes of lines representing annual sales y in terms of time x in years are given below. Use the slopes to interpret any change in annual sales for a one-year increase in time.
(a) The line has a slope of $m = 135$.
(b) The line has a slope of $m = 0$.
(c) The line has a slope of $m = -40$.

88. Sales The graph shows the sales (in billions of dollars) for Apple Inc. in the years 2013 through 2019. *(Source: Apple Inc.)*



(a) Use the slopes of the line segments to determine the years in which the sales showed the greatest increase and the greatest decrease.
(b) Find the slope of the line segment connecting the points for the years 2013 and 2019.
(c) Interpret the meaning of the slope in part (b) in the context of the problem.

89. Road Grade You are driving on a road that has a 6% uphill grade. This means that the slope of the road is $\frac{6}{100}$. Approximate the amount of vertical change in your position when you drive 200 feet.

90. **Road Grade**

From the top of a mountain road, a surveyor takes several horizontal measurements x and several vertical measurements y, as shown in the table (x and y are measured in feet).

x	300	600	900	1200
y	-25	-50	-75	-100

x	1500	1800	2100
y	-125	-150	-175

(a) Sketch a scatter plot of the data.

(b) Use a straightedge to sketch the line that you think best fits the data.

(c) Find an equation for the line you sketched in part (b).

(d) Interpret the meaning of the slope of the line in part (c) in the context of the problem.

(e) The surveyor needs to put up a road sign that indicates the steepness of the road. For example, a surveyor would put up a sign that states "8% grade" on a road with a downhill grade that has a slope of $-\frac{8}{100}$. What should the sign state for the road in this problem?

91. **Temperature Conversion** Write a linear equation that expresses the relationship between the temperature in degrees Celsius C and degrees Fahrenheit F. Use the fact that water freezes at 0°C (32°F) and boils at 100°C (212°F).

92. **Neurology** The average weight of a male child's brain is 970 grams at age 1 and 1270 grams at age 3. *(Source: American Neurological Association)*

(a) Assuming that the relationship between brain weight y and age t is linear, write a linear model for the data.

(b) What is the slope and what does it tell you about brain weight?

(c) Use your model to estimate the average brain weight at age 2.

(d) Use your school's library, the Internet, or some other reference source to find the actual average brain weight at age 2. How close was your estimate?

(e) Do you think your model could be used to determine the average brain weight of an adult? Explain.

93. **Depreciation** A sandwich shop purchases a used pizza oven for \$830. After 5 years, the oven will have to be discarded and replaced. Write a linear equation giving the value V of the equipment during the 5 years it will be in use.

94. **Depreciation** A school district purchases a high-volume printer, copier, and scanner for \$24,000. After 10 years, the equipment will have to be replaced. Its value at that time is expected to be \$2000. Write a linear equation giving the value V of the equipment during the 10 years it will be in use.

95. **Cost, Revenue, and Profit** A roofing contractor purchases a shingle delivery truck with a shingle elevator for \$42,000. The vehicle requires an average expenditure of \$9.50 per hour for fuel and maintenance, and the operator is paid \$11.50 per hour.

(a) Write a linear equation giving the total cost C of operating this equipment for t hours. (Include the purchase cost of the equipment.)

(b) Assuming that customers are charged \$45 per hour of machine use, write an equation for the revenue R obtained from t hours of use.

(c) Use the formula for profit $P = R - C$ to write an equation for the profit obtained from t hours of use.

(d) Use the result of part (c) to find the break-even point—that is, the number of hours this equipment must be used to yield a profit of 0 dollars.

96. **Geometry** The length and width of a rectangular garden are 15 meters and 10 meters, respectively. A walkway of width x surrounds the garden.

(a) Draw a diagram that gives a visual representation of the problem.

(b) Write the equation for the perimeter y of the walkway in terms of x. Explain what the slope of the equation represents.

(c) Use a graphing utility to graph the equation for the perimeter.

Exploring the Concepts

True or False? **In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.**

97. A line with a slope of $-\frac{5}{7}$ is steeper than a line with a slope of $-\frac{6}{7}$.

98. The line through $(-8, 2)$ and $(-1, 4)$ and the line through $(0, -4)$ and $(-7, 7)$ are parallel.

99. **Right Triangle** Explain how you can use slope to show that the points $A(-1, 5)$, $B(3, 7)$, and $C(5, 3)$ are the vertices of a right triangle.

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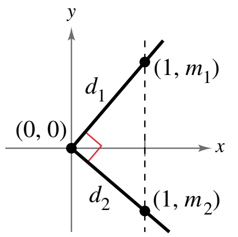
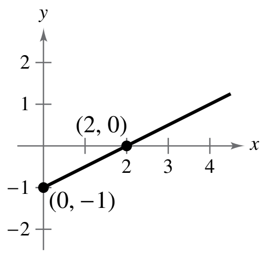
100. Perpendicular Segments Find d_1 and d_2 in terms of m_1 and m_2, respectively (see figure). Then use the Pythagorean Theorem to find a relationship between m_1 and m_2.

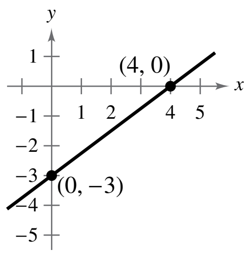


101. Error Analysis Describe the error in finding the equation of each graph.

(a) $y = 2x - 1$ ✗

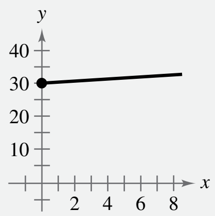


(b) $y = \frac{3}{4}x + 4$ ✗

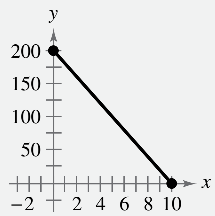


102. HOW DO YOU SEE IT? Match the description of the situation with its graph. Also determine the slope and y-intercept of each graph and interpret the slope and y-intercept in the context of the situation. [The graphs are labeled (i), (ii), (iii), and (iv).]

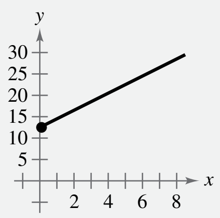
(i)



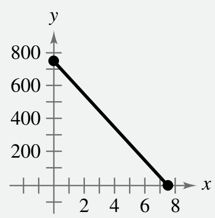
(ii)



(iii)



(iv)



(a) A person is paying \$20 per week to a friend to repay a \$200 loan.

(b) An employee receives \$12.50 per hour plus \$2 for each unit produced per hour.

(c) A sales representative receives \$30 per day for food plus \$0.32 for each mile traveled.

(d) A computer that was purchased for \$750 depreciates \$100 per year.

103. Comparing Slopes Use a graphing utility to compare the slopes of the lines $y = mx$, where $m = 0.5$, 1, 2, and 4. Which line rises most quickly? Now, let $m = -0.5$, -1, -2, and -4. Which line falls most quickly? Use a square setting to obtain a true geometric perspective. What can you conclude about the slope and the "rate" at which the line rises or falls?

104. Slope and Steepness The slopes of two lines are -4 and $\frac{5}{2}$. Which is steeper? Explain.

Review & Refresh

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Solving for a Variable **In Exercises 105–108, evaluate the equation when $x = -2, 0, 3,$ and 6.**

105. $y = x^2 - x$

106. $y = x + 5y$

107. $2f = \dfrac{7 - x^3}{5}$

108. $\dfrac{g}{4} = \dfrac{\sqrt{2 + x}}{3}$

Solving an Equation **In Exercises 109–118, solve the equation, if possible. Check your solutions.**

109. $2x^3 - 5x^2 - 2x = -5$

110. $x^4 - 3x^3 - x - 2 = 1 - 2x$

111. $x^6 + 4x^3 - 5 = 0$

112. $x^4 - 5x^2 + 4 = 0$

113. $3\sqrt{x} - 5\sqrt{18} = 0$

114. $(x + 1)^{3/5} - 8 = 0$

115. $x = \dfrac{2}{x} + 1$

116. $\dfrac{3}{x} - \dfrac{1}{2} = \dfrac{x}{4}$

117. $x + |x - 9| = 7$

118. $|3x - 6| = x^2 - 4$

Simplifying an Expression **In Exercises 119–122, simplify the expression.**

119. $\dfrac{[(x - 1)^2 + 1] - (x^2 + 1)}{x}$

120. $\dfrac{[(t + 1)^2 - (t + 1) - 2] - (t^2 - t - 2)}{t}$

121. $\dfrac{4 - x^2 - 3}{x - 1} + x^2 + x + 4$

122. $\dfrac{x^3 + x}{(x^2 - 4)} - \dfrac{5}{2(x + 2)} + \dfrac{5}{2(x - 2)} + 2$

Project: Bachelor's Degrees To work an extended application analyzing the numbers of bachelor's degrees earned by women in the United States from 2006 through 2017, visit this text's website at *LarsonPrecalculus.com*. *(Source: National Center for Education Statistics)*

1.4 Functions

Functions are used to model and solve real-life problems. For example, in Exercise 64 on page 46, you will use a function that models the force of water against the face of a dam.

- **Determine whether relations between two variables are functions, and use function notation.**
- **Find the domains of functions.**
- **Use functions to model and solve real-life problems, and evaluate difference quotients.**

Introduction to Functions and Function Notation

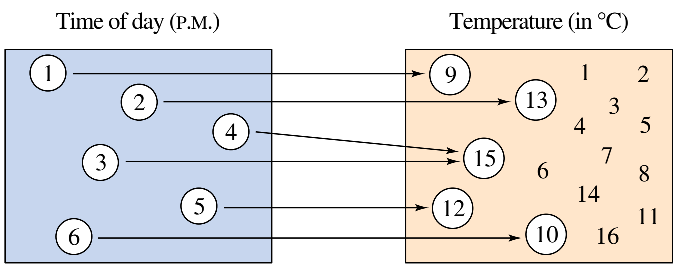
Many everyday phenomena involve two quantities that are related to each other by some rule of correspondence. The mathematical term for such a rule of correspondence is a **relation.** In mathematics, equations and formulas often represent relations. For example, the simple interest I earned on \$1000 for 1 year is related to the annual interest rate r by the formula $I = 1000r$.

The formula $I = 1000r$ represents a special kind of relation that matches each item from one set with *exactly one* item from a different set. Such a relation is a **function.**

Definition of Function

A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

To help understand this definition, look at the figure below, which shows a function that relates the time of day to the temperature.



Set A is the domain.
Inputs: 1, 2, 3, 4, 5, 6

Set B contains the range.
Outputs: 9, 10, 12, 13, 15

This function can be represented by the ordered pairs

$$\{(1, 9), (2, 13), (3, 15), (4, 15), (5, 12), (6, 10)\}.$$

In each ordered pair, the first coordinate (x-value) is the **input** and the second coordinate (y-value) is the **output**.

Characteristics of a Function from Set *A* to Set *B*

1. Each element in A must be matched with an element in B.
2. Some elements in B may not be matched with any element in A.
3. Two or more elements in A may be matched with the same element in B.
4. An element in A (the domain) cannot be matched with two different elements in B.

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There are four common ways to represent a function—verbally, numerically, graphically, and algebraically.

Four Ways to Represent a Function

1. *Verbally* by a sentence that describes how the input variable is related to the output variable
2. *Numerically* by a table or a list of ordered pairs that matches input values with output values
3. *Graphically* by points in a coordinate plane in which the horizontal positions represent the input values and the vertical positions represent the output values
4. *Algebraically* by an equation in two variables

To determine whether a relation is a function, you must decide whether each input value is matched with exactly one output value. When any input value is matched with two or more output values, the relation is not a function.

EXAMPLE 1 Testing for Functions

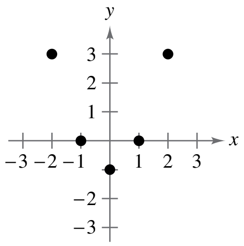
Determine whether the relation represents y as a function of x.

a. The input value x is the number of representatives from a state, and the output value y is the number of senators.

b.

Input, x	Output, y
2	11
2	10
3	8
4	5
5	1

c.



Solution

a. This verbal description *does* describe y as a function of x. Regardless of the value of x, the value of y is always 2. This is an example of a *constant function*.

b. This table *does not* describe y as a function of x. The input value 2 is matched with two different y-values.

c. The graph *does* describe y as a function of x. Each input value is matched with exactly one output value.

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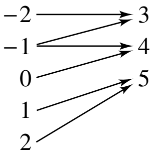
Determine whether the relation represents y as a function of x.

a. *Domain, x* *Range, y*



b.

Input, x	0	1	2	3	4
Output, y	−4	−2	0	2	4

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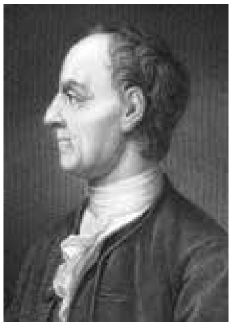
HISTORICAL NOTE



Many consider Leonhard Euler (1707–1783), a Swiss mathematician, to be the most prolific and productive mathematician in history. One of his greatest influences on mathematics was his use of symbols, or notation. Euler introduced the function notation $y = f(x)$.

Representing functions by sets of ordered pairs is common in *discrete mathematics*. In algebra, however, it is more common to represent functions by equations or formulas involving two variables. For example, the equation

$$y = x^2 \qquad \text{y is a function of x.}$$

represents the variable y as a function of the variable x. In this equation, x is the **independent variable** (the input) and y is the **dependent variable** (the output). The domain of the function is the set of all values taken on by the independent variable x, and the range of the function is the set of all values taken on by the dependent variable y.

EXAMPLE 2 Testing for Functions Represented Algebraically

See LarsonPrecalculus.com for an interactive version of this type of example.

Determine whether each equation represents y as a function of x.

a. $x^2 + y = 1$ **b.** $-x + y^2 = 1$

Solution To determine whether y is a function of x, solve for y in terms of x.

a. Solving for y yields

$$x^2 + y = 1 \qquad \text{Write original equation.}$$
$$y = 1 - x^2. \qquad \text{Solve for } y.$$

To each value of x there corresponds exactly one value of y. So, y is a function of x.

b. Solving for y yields

$$-x + y^2 = 1 \qquad \text{Write original equation.}$$
$$y^2 = 1 + x \qquad \text{Add } x \text{ to each side.}$$
$$y = \pm\sqrt{1 + x}. \qquad \text{Solve for } y.$$

The $\pm$ indicates that to a given value of x there correspond two values of y. For instance, when $x = 3$, $y = 2$ or $y = -2$. So, y is not a function of x.

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Determine whether each equation represents y as a function of x.

a. $x^2 + y^2 = 8$ **b.** $y - 4x^2 = 36$

When using an equation to represent a function, it is convenient to name the function for easy reference. For instance, you know from Example 2(a) that the equation $y = 1 - x^2$ describes y as a function of x. By naming this function "f," you can write the input, output, and equation using **function notation,** as shown below.

Input	**Output**	**Equation**
x	$f(x)$	$f(x) = 1 - x^2$

The symbol $f(x)$ is read as *the value of f at x* or simply *f of x*. The symbol $f(x)$ corresponds to the y-value for a given x. So, $y = f(x)$. Keep in mind that f is the *name* of the function, whereas $f(x)$ is the *output value* of the function at the *input value* x. For example, the function $f(x) = 3 - 2x$ has *function values* denoted by $f(-1)$, $f(0)$, $f(2)$, and so on. To find these values, substitute the specified input values into f.

For $x = -1$, $\quad f(-1) = 3 - 2(-1) = 3 + 2 = 5.$

For $x = 0$, $\quad f(0) = 3 - 2(0) = 3 - 0 = 3.$

For $x = 2$, $\quad f(2) = 3 - 2(2) = 3 - 4 = -1.$

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Although it is often convenient to use f as a function name and x as the independent variable, other letters may be used as well. For example,

$$f(x) = x^2 - 4x + 7, \quad f(t) = t^2 - 4t + 7, \quad \text{and} \quad g(s) = s^2 - 4s + 7$$

all define the same function. In fact, the role of the independent variable is that of a "placeholder." Consequently, the function can be described by

$$f(\square) = (\square)^2 - 4(\square) + 7.$$

EXAMPLE 3 Evaluating a Function

Let $g(x) = -x^2 + 4x + 1$. Find $g(2)$, $g(t)$, and $g(x + 2)$.

Solution

To find $g(2)$, replace x with 2 in $g(x) = -x^2 + 4x + 1$ and simplify.

$$\begin{aligned} g(2) &= -(2)^2 + 4(2) + 1 \\ &= -4 + 8 + 1 \\ &= 5 \end{aligned}$$

To find $g(t)$, replace x with t and simplify.

$$\begin{aligned} g(t) &= -(t)^2 + 4(t) + 1 \\ &= -t^2 + 4t + 1 \end{aligned}$$

To find $g(x + 2)$, replace x with $x + 2$ and simplify.

$$\begin{aligned} g(x + 2) &= -(x + 2)^2 + 4(x + 2) + 1 && \text{Substitute } x + 2 \text{ for } x. \\ &= -(x^2 + 4x + 4) + 4x + 8 + 1 && \text{Multiply.} \\ &= -x^2 - 4x - 4 + 4x + 8 + 1 && \text{Distributive Property} \\ &= -x^2 + 5 && \text{Simplify.} \end{aligned}$$

ALGEBRA HELP

In Example 3, note that $g(x + 2)$ is not equal to $g(x) + g(2)$. In general, $g(u + v) \neq g(u) + g(v)$.

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Let $f(x) = 10 - 3x^2$. Find $f(2)$, $f(-4)$, and $f(x - 1)$.

A function defined by two or more equations over a specified domain is called a **piecewise-defined function.**

EXAMPLE 4 A Piecewise-Defined Function

Evaluate the function f when $x = -1$, 0, and 1.

$$f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

Solution Because $x = -1$ is less than 0, use $f(x) = x^2 + 1$ to obtain

$$f(-1) = (-1)^2 + 1 = 2.$$

For $x = 0$, use $f(x) = x - 1$ to obtain

$$f(0) = (0) - 1 = -1.$$

For $x = 1$, use $f(x) = x - 1$ to obtain

$$f(1) = (1) - 1 = 0.$$

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Evaluate the function f given in Example 4 when $x = -2$, 2, and 3.

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SKILLS REFRESHER

For a refresher on solving equations, watch the video at *LarsonPrecalculus.com.*

EXAMPLE 5 Finding Values for Which $f(x) = 0$

Find all real values of x for which $f(x) = 0$.

a. $f(x) = -2x + 10$ **b.** $f(x) = x^2 - 5x + 6$

Solution For each function, set $f(x) = 0$ and solve for x.

a. $-2x + 10 = 0$ Set $f(x)$ equal to 0.

$-2x = -10$ Subtract 10 from each side.

$x = 5$ Divide each side by -2.

So, $f(x) = 0$ when $x = 5$.

b. $x^2 - 5x + 6 = 0$ Set $f(x)$ equal to 0.

$(x - 2)(x - 3) = 0$ Factor.

$x - 2 = 0 \Rightarrow x = 2$ Set 1st factor equal to 0.

$x - 3 = 0 \Rightarrow x = 3$ Set 2nd factor equal to 0.

So, $f(x) = 0$ when $x = 2$ or $x = 3$.

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Find all real values of x for which $f(x) = 0$, where $f(x) = x^2 - 16$.

EXAMPLE 6 Finding Values for Which $f(x) = g(x)$

Find the values of x for which $f(x) = g(x)$.

a. $f(x) = x^2 + 1$ and $g(x) = 3x - x^2$

b. $f(x) = x^2 - 1$ and $g(x) = -x^2 + x + 2$

Solution

a. $x^2 + 1 = 3x - x^2$ Set $f(x)$ equal to $g(x)$.

$2x^2 - 3x + 1 = 0$ Write in general form.

$(2x - 1)(x - 1) = 0$ Factor.

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$ Set 1st factor equal to 0.

$x - 1 = 0 \Rightarrow x = 1$ Set 2nd factor equal to 0.

So, $f(x) = g(x)$ when $x = 1/2$ or $x = 1$.

b. $x^2 - 1 = -x^2 + x + 2$ Set $f(x)$ equal to $g(x)$.

$2x^2 - x - 3 = 0$ Write in general form.

$(2x - 3)(x + 1) = 0$ Factor.

$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$ Set 1st factor equal to 0.

$x + 1 = 0 \Rightarrow x = -1$ Set 2nd factor equal to 0.

So, $f(x) = g(x)$ when $x = 3/2$ or $x = -1$.

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Find the values of x for which $f(x) = g(x)$, where $f(x) = x^2 + 6x - 24$ and $g(x) = 4x - x^2$. ■

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