College Algebra

Thirteenth Edition

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Cengage

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About the Cover

As authors, we've always seen studying mathematics as an exciting adventure and the striking hiking cover image of this thirteenth edition continues that theme. Hiking is active and fun and sometimes challenging but as with all great adventures, you can persist and succeed. That approach is also true of mathematics and college algebra with its joys and challenges. Actively working through the mathematics will enable learning and with persistence there comes much joy in success. Too often, many people think that being successful at mathematics is out of reach for them. Perhaps they internalize thoughts like “I'm not good at math” or “I'm not a math person.” We wrote this text with that student in mind to give you confidence and as much support as possible when needed. When you think of your coursework, approach learning mathematics with the same positive growth mindset and openness you would have whenever you embark on any adventure. This textbook is here to support you and ensure that the rewards of the mathematics adventure are gratifying.

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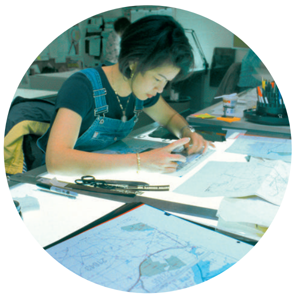
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Preface

To the Instructor

It is with great delight that we present the thirteenth edition of *College Algebra*. This revised edition maintains the same philosophy of the highly successful previous editions but is enhanced to meet the current expectations of today's students and instructors. Revised for greater clarity, this edition increases problem-solving skills and bolsters learning to prepare students for success in trigonometry, calculus, statistics, and other disciplines of study.

This textbook has six underlying goals:

1. Present solid mathematics written in way that is easy to understand for students with diverse abilities, backgrounds, and future career and course goals whether a STEM or non-STEM student.
2. Provide a strong instructional experience that will help students master algebra and understand and apply the concept of a function.
3. Motivate student learning by using popular culture and real-life applications that increase student's drive to engage with the mathematics.
4. Improve critical-thinking abilities in all students by emphasizing conceptual understanding over procedures.
5. Develop algebra skills needed for future success in mathematics courses, and where appropriate, give them a preview of future STEM topics.
6. Furnish learning support where student need it with diverse examples and multiply strategies, but not limited to prerequisite concepts and skills.

As authors, we have accomplished these goals through a successful blending of content and pedagogy, presenting thorough coverage of classic college algebra topics that can be incorporated into a contemporary framework of tested teaching strategies. With student and instructor's needs being at the center of this revision, this book emphasizes conceptual understanding, problem solving, and the appropriate and flexible use of technology.

New Features

To advance mathematics learning and increase student success the following features were included for additional scaffolded support.

- **Reframed Getting Ready Exercises** These are completely new and appear at the beginning of each of the exercise sets and are available in WebAssign. These just-in-time review problems are included to guarantee student readiness to successfully work the practice exercises. The former Getting Ready Exercises are now titled Vocabulary and Concepts and are also available in WebAssign.
- **Fix Its** These are exercises where students find and fix an error in mostly step-by-step provided solutions. The learner will first identify the step number where the first mistake occurs. Then the learner is asked to correct that mistake ensuring the error is fixed. This process is intended to emphasize problem-solving strategies students can apply to any exercise/homework they work on. The exercises are also intended to help students avoid making common mistakes that are often made in mathematics on homework and test problems. Where appropriate, there are two Fix Its per section and note that these exercises are also available in WebAssign.
- **Strategy Prominently Stated in Examples** Before the solution of an example is shown, a clearly stated strategy is written to let student know the approach that will be used to solve the problem. Students gain a

Fix It

In exercises 107 and 108, identify the step the first error is made and fix it.

107. Solve the equation: $5e^{7x} + 1 = 11$

Solution:

Step 1: $5e^{4x} = 10$

Step 2: $e^{7x} = 2$

Step 3: $\ln(e^{7x}) = \ln 2$

Step 4: $7x = \ln 2$

Step 5: $x = \ln\left(\frac{2}{7}\right)$

108. Solve the equation: $\log_6 2x + \log_6 x = \log_6 10$

Solution:

Step 1: $\log_6 2x^2 = \log_6 10$

Step 2: $2x^2 = 10$

Step 3: $x^2 = 5$

Step 4: $x = \pm\sqrt{5}$

deeper understanding of the problem-solving process and see initially why a specific procedure is used to determine the solution.

- **Looking Ahead to Calculus** These margin notes preview calculus topics explaining why they are important and how they are used. The accompanying modules in WebAssign are interactive and extend the explanation of the concepts to connect college algebra to calculus. Topics include:
  - What is Calculus?
  - Using Tables of Values to Help Understand the Definition of a Limit at a Point
  - Using Algebraic Techniques to Evaluate a Limit at a Point
  - Using the Difference Quotient to Find a Tangent Line and Define the Derivative
  - Using the Derivative to Analyze the Graph of a Function
  - Using End Behavior to Find Limits to Infinity and Infinite Limits
  - Using the Intermediate Value Theorem to Apply Newton's Method for Finding Roots
- **Group Activities** These are new group activities/projects at the end of each chapter that can be used for in-class discussions or assigned as individual or group work in the accompanying WebAssign course.

Group Activity

Installment Loans

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What Are Fixed Installment Loans?

A fixed installment loan is a loan that is repaid in equal payments. Sometimes part of the cost is paid at the time of purchase. This amount is the down payment.

Real-World Example of Installment Loans

Installment loans allow you the ability to purchase an item and use it now. This is called installment purchasing and being able to use the item is an advantage. The disadvantage is that interest is paid on the amount borrowed. A common example is purchasing an automobile.

Group Activity

1. Suppose you graduate from college, obtain an amazing job, get married, and then purchase a new vehicle. Use the given information below to answer four questions.
- Cost of automobile: $26,500
- Down payment: $2,000
- Monthly payment: $550.25
- Loan term: 48 months

Hallmark Features

We have retained and updated the pedagogical features that made the previous editions of the book so successful.

Achieving and Validating Mastery

- **Numbered Objectives** Along with clear exposition and accessible writing, numbered learning objectives are given at the beginning of each section and appear as subheadings throughout each section to keep students focused and engaged.

4.2 Polynomial Functions

In this section, we will learn to

1. Identify polynomial functions.
2. Recognize characteristics of the graphs of polynomial functions.
3. Find zeros of polynomial functions by factoring.
4. Determine end behavior.
5. Graph polynomial functions.
6. Use the Intermediate Value Theorem.

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So far, we have discussed two types of polynomial functions—first-degree (or linear) functions, and second-degree (or quadratic) functions. In this section, we will discuss polynomial functions of higher degree.

Polynomial functions can be used to model the path of a roller coaster or to model the fluctuation of gasoline prices over the past few months. The Hollywood Rip Ride Rockit is a roller coaster located at Universal Studios in Orlando, Florida. With a length of 3800 feet, a height of 167 feet, and a top speed of 65 miles per hour, it is one of the largest roller coasters ever built. Riders are video recorded for the entire ride and can choose from one of thirty songs to listen to during the experience. Portions of Rip Ride Rockit's tracks can be modeled with a polynomial function.

- **Titled Examples to Clearly Identify the Purpose of Each Example and an Example Structure to Help Students Gain a Deeper Understanding of How to Solve Each Problem** Descriptive titles serve as example identifiers. Solutions begin with a stated approach. The examples are engaging and step-by-step solutions with annotations are provided.

Example 6 **Using a Quadratic Function to Solve a Minimum Cost Problem**

A company that makes and sells backpacks for hiking has found that the total weekly cost $C(x)$, in dollars, of producing x backpacks is given by the function $C(x) = 0.5x^2 - 210x + 26{,}250$. Find the production level that minimizes the weekly cost and find that weekly minimum cost.

Strategy The weekly cost function $C(x)$ is a quadratic function whose graph is a parabola that opens upward. The minimum value of $C(x)$ occurs at the vertex of the parabola. We will use the vertex formula to find the vertex of the parabola.

Solution Since the coefficient of x^2 is 0.5 (a positive real number), the x-coordinate of the vertex is the production level that will minimize the cost, and the y-coordinate is that minimum cost. We compare the equations

$$C(x) = 0.5x^2 - 210x + 26{,}250 \quad \text{and} \quad f(x) = ax^2 + bx + c$$

to see that $a = 0.5$, $b = -210$, and $c = 26{,}250$. Using the vertex formula, we see that the vertex of the parabola is the point with coordinates

$$\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) = \left(-\frac{-210}{2(0.5)}, 26{,}250 - \frac{(-210)^2}{4(0.5)}\right) = (210, 4200)$$

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- **Self Checks That Actively Reinforce Student Understanding of Concepts and Example Solutions** Each example is followed immediately by a Self Check Exercise. The answers for students are offered at the end of each section. To assist instructors, the answers to Self Checks appear next to the problem in the AIE (Annotated Instructor's Edition), printed in blue.

- **Now Try Exercises to Provide Students an Additional Opportunity to Assess Their Understanding of the Concept Related to Each Worked Example** A reference to an exercise follows all Examples and Self Check problems. These references also show students a correspondence between the examples in the book and the exercise sets.
- **Various Boxes Including Tip Boxes, Take Note Boxes, and Caution Boxes Provide Helpful Strategies for Studying and Learning Content**. Tip boxes provide a visual representation to help students remember the tip; Take Note boxes highlight important information and often provide clarify a specific step or concept in an example; Caution Boxes alert students to common errors and misunderstandings.
- **Vocabulary and Concepts Exercises (Formerly Getting Ready) to Test Student Understanding of Concepts and Proper Use of Vocabulary** Students should be able to answer these fill-in-the blank questions before moving on to the practice exercises.
- **Chapter Reviews and Chapter Tests to Give Students the Best Opportunity for Study and Exam Preparation** Each chapter closes with a Chapter Review and Chapter Test. Chapter Reviews are comprehensive and consist of three parts: definitions and concepts, examples, and review exercises. Chapter Tests cover all the important topics and yet are brief enough to emulate a "real-time" test, so students can practice not only the math but their test-taking aptitude. As an additional quick reference, endpapers offer the important formulas and graphs developed in the book.

Tip

Although money doesn't actually grow on trees, learning about compound interest can help you as a student make wise decisions with regard to your finances.

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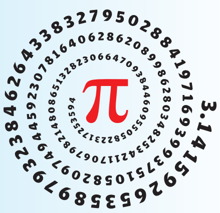
Take Note

There are two important math constants algebra students should know: π, which is approximately 3.14, and Euler's number e, which is approximately 2.718.



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Caution

When we raise the real number 2 to the 0th power, we get 1 and not 0. Also, when we raise 2 to a negative power like -1, we get $\frac{1}{2}$ and not -2. Avoid making these two common errors when completing a table of value for exponential functions.

- **Cumulative Reviews to Reinforce Student Learning and Improve Students Retention of Concepts** Cumulative Review Exercises appear after every two chapters. These comprehensive reviews revisit all the essential topics covered in prior chapters.

Emphasizing Real World Relevancy

- **Careers in Mathematics to Encourage Students to Explore Careers That Use Mathematics and to Make a Connection between Math and Real Life** Each chapter opens with "Careers in Mathematics." Exciting careers are featured in this edition. These snapshots include information on how professionals use math in their work and who employs them.

Careers and Mathematics: Information Security Analyst

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Information security analysts plan and carry out security measures to protect an organization's computer networks and systems. Their responsibilities are continually expanding as the number of cyberattacks increases. Information security analysts typically do the following:

- Monitor their organization's networks for security breaches and investigate a violation when one occurs
- Install and use software, such as firewalls and data encryption programs, to protect sensitive information
- Conduct penetration testing, which is when analysts simulate attacks to look for vulnerabilities in their systems before they can be exploited
- Research the latest information technology (IT) security trends

- **Section Openers to Peak Interest and Motivate Students to Read the Material** Each section begins with a contemporary photo and a real-life application.
- **Application Examples to Answer the Student Question: When Will I Ever Use This Math?** Applications from a wide range of disciplines demonstrate how mathematics is used to solve real problems. These applications motivate the material and help students become better problem solvers.

Deepening Problem Solving

- **Strategy Boxes to Enable Students to Build on Their Mathematical Reasoning and Approach Problems with Confidence** Strategy boxes offer problem-solving techniques and steps at appropriate points in the material.
- **Comprehensive Exercise Sets to Improve Mathematical Skills and Cement Understanding, and Interesting Applications to Emphasize Problem Solving** The exercise sets progress from routine to more challenging. The mathematics in each exercise set is sound, but not so rigorous that it will confuse students. All exercise sets include Getting Ready, Vocabulary and Concepts, Practice, Fix It, Application, Discovery and Writing, and Critical Thinking problems. The book contains more than 4000 exercises, many of them are new. New application exercises have been added and others updated. All application problems have titles.
- **Critical Thinking Exercises to Encourage Algebraic Thinking and to Assess Conceptual Understanding** Exercise sets now include Critical Thinking exercises. These exercises are primarily true–false statements. If a statement is false, students are then asked to make the necessary change to produce a true statement. Matching exercises are also used.

The Development of This Edition

We took several things into account when developing this latest edition. Guiding our work were the Four Core Principles of Learning Design: authenticity, inclusivity, intentionality, and personalization, with a focus on removing barriers to learning to offer a carefully planned and inclusive experience for all students. Our narrative, examples and exercises were also reviewed extensively throughout the revision process with the help of Dr. Dale Dawes, City University of New York, in his capacity as DEI (diversity, equity, and inclusion) advisor. The DEI review was an iterative process designed to ground and apply the learning design principles. Cengage is committed to continue this process in future editions. Lastly, we wanted to ensure the text could be used in a variety of ways. To maintain optimum flexibility, many chapters are sufficiently independent to allow instructors to pick and choose topics that are relevant to their students. Additionally, the accompanying WebAssign course is a critical extension of our approach including new features as coded problem types where possible.

Content and Organizational Changes

Each chapter and section in the text has been edited to fine-tune the presentation of topics for better flow of concepts and for clarity. There are many new exercises and applications. Key changes made to specific chapters include the following:

Chapter R: A Review of Basic Algebra

- Section R.1—A new *Tip Box* was added—Definition of a Rational Number; two *Looking Ahead to Calculus* boxes were added—The Real Number System, Absolute Value, and the Determining Limits; six *Getting Ready* Exercises were added; two new absolute value exercises added; two distance between pairs of numbers were added; and Two *Fix It* Exercises were added.
- Section R.2—Six *Getting Ready* Exercises and two *Fix It* Exercises were added.
- Section R.3—Six *Getting Ready* Exercises and two *Fix It* Exercises were added.
- Section R.4—Section title was changed from Polynomials to Polynomials and More Operations on Radicals; a *Take Note* was added on the Foil Method; a *Looking Ahead to Calculus* was added—Rationalizing Techniques; six *Getting Ready* Exercises; four new multiplication of radicals exercises; and two *Fix It* Exercises were added.
- Section R.5—A *Looking Ahead to Calculus* was added—Simplification of Derivatives by Factoring with Negative and Fractional Exponents; six *Getting Ready* Exercises were added; and eight new factoring exercises were added of which three have fractional and/or negative exponents; and two *Fix It* Exercises were added.
- Section R.6—A new objective was added—Identify the restricted numbers of a rational expression; a new Example was added—Identify the restricted numbers of a rational expression; a *Looking Ahead to Calculus* was added—Domain; six new *Getting Ready* Exercises, eight new exercises—identify the restricted numbers of a rational expression, and two *Fix It* Exercises were added.
- End of Chapter R—Four new exercises—two factoring with negative and/or fractional exponent and two identify the restricted numbers of a rational expression; and a *Group Activity*—Anesthesiology was added.

Chapter 1: Equations and Inequalities

- Section 1.1—Six new *Getting Ready* Exercises were added; eleven new linear equations were added; four new rational equations; and two *Fix It* Exercises were added.
- Section 1.2—Six new *Getting Ready* Exercises and six new applications were added.
- Section 1.3—Six new *Getting Ready* Exercises were added; four new exercises were added—simplify imaginary numbers; Six new exercises were added—identify real and imaginary parts of a complex number; Eight new exercises were added—perform operations with complex numbers along with two *Fix It* Exercises.
- Section 1.4—Two new *Caution Boxes* were added—Completing the Square Common Error; Quadratic Formula and Signs of a, b, and c; six new *Getting Ready* Exercises were added; eight new quadratic equations were added and two *Fix It* Exercises were added.
- Section 1.5—Six new *Getting Ready* Exercises and eleven new applications were added.
- Section 1.6—Six new *Getting Ready* Exercises were added; four new exercises added—solve by factoring and two *Fit It* Exercises were added.
- Section 1.7—A new example was added—Inequality Symbols; a *Looking Ahead to Calculus* was added—Signs of the Derivatives on an Interval; six new *Getting Ready* Exercises and six new Quadratic Inequality Exercises were added; six new Rational Inequality Exercises and two new *Fix It* Exercises were added.
- Section 1.8—Objectives were rewritten using words and not mathematics; a *Looking Ahead to Calculus* was added—Absolute Value and the Precise Definition of a Limit; six *Getting Ready* Exercises and two new *Fix It* Exercises were added; two Absolute Value Inequality Applications were added.
- Chapter 1 End of Chapter 1—A *Group Activity* was added—Health and Body Mass Index.

Chapter 2: Functions and Graphs

- Section 2.1—A *Looking Ahead to Calculus* was added—Difference Quotient; six *Getting Ready* Exercises were added; thirty-six new exercises added—evaluating functions with integers, fractions, and variable expressions and several types of functions were used; four new domain exercises added—rational functions and two with radicals in the denominator; two *Fix It* Exercises were added as well.
- Section 2.2—Six *Getting Ready* Exercises were added; ten new exercises on graphing lines and four new distance exercises were added; two new midpoint exercises and two new *Fix It* Exercises were added.
- Section 2.3—New objective added—Determine a function's average rate of change; a new example added—Determine a function's average rate of change on an interval; a *Looking Ahead to Calculus* was added—Instantaneous Rate of Change; six *Getting Ready* Exercises were added; one new Vocabulary and Concepts Exercises added—Average Rate of Change; ten new exercises added—average rate of change on an interval along with two new *Fix It* Exercises.

- Section 2.4—Six *Getting Ready* Exercises were added; thirteen new exercises on finding an equation for a line and four new graphing line exercises were added; two new *Fix It* Exercises were added.
- Section 2.5—Six *Getting Ready* Exercises were added; two new graphing exercises were added—absolute value equations along with two *Fix It* Exercises.
- Section 2.6—Six *Getting Ready* Exercises and two *Fix It* Exercises were added.
- End of Chapter 2—Four new exercises—evaluate a function; average rate of change added to review concepts as well as two new average rate of change exercises added; a *Group Activity* was added—Average Velocity Roller Coaster.

Chapter 3: Functions

- Section 3.1—Six *Getting Ready* exercises were added; two new graphing exercises added—simple rational functions; four new exercises added—determine whether the given graph represents a Function; two *Fix It* Exercises were added.
- Section 3.2—Six new *Getting Ready* exercises were added; eight new graphing exercises added—use transformations; two *Fix It* Exercises were added.
- Section 3.3—Two *Looking Ahead to Calculus* were added—First Derivative (Increasing and Decreasing) and Local Extreme Values; six new *Getting Ready* Exercises were added; two new graphing exercises—piecewise-defined functions; and four new greatest-integer function exercises added—two to evaluate and two to graph; two *Fix It* Exercises were added.
- Section 3.4—A *Looking Ahead to Calculus* was added—Chair Rule and Composition of Functions; six new *Getting Ready* Exercises were added; eight new exercises added—operations of functions on functions and specify domain; six new composition of function exercises (specify domains) added; two *Fix It* Exercises and four Applications were added.
- Section 3.5—Six new *Getting Ready* Exercises were added; twenty exercises added—find the inverse of a function; eight new exercises added—graph a function and its inverse on the same axes; four new exercises—given the graph of a function graph its inverse; two *Fix It* Exercises were added.
- End of Chapter 3—A *Group Activity* was added—Cryptography and Cybersecurity.

Chapter 4: Polynomial and Rational Functions

- Section 4.1—A *Looking Ahead to Calculus* was added—Optimization; six new *Getting Ready* Exercises were added; two new exercises added—given the graph of a quadratic function identify intercepts, domain and range, axis of symmetry, and max/min point; two *Fix It* Exercises and five new applications were added.
- Section 4.2—Two *Looking Ahead to Calculus* were added—Curve Sketching and Finding Zeros; four *Getting Ready* Exercises were added; four new exercises added—Given the graph of a polynomial function identify zeros, leading coefficient positive or negative, and even or odd degree; two *Fix It* Exercises and two new applications were added.
- Section 4.3—Six *Getting Ready* Exercises and two *Fix It* Exercises were added.
- Section 4.4—A *Looking Ahead to Calculus* was added—Boundness; six *Getting Ready* Exercises and two *Fix It* Exercises were added.
- Section 4.5—Four *Getting Ready* Exercises and two *Fix It* Exercises were added; four new applications were added, and six answers were rewritten to list all zeros and not use plus or minus notation.
- Section 4.6—Two *Looking Ahead to Calculus* were added—Vertical Asymptote and Infinite Limits; Applications Involving Cost, Revenue, and Profit; six new *Getting Ready* Exercises were added; two *Fix It* Exercises and four new applications were added.
- End of Chapter 4—A *Group Activity* was added—Polynomial Function Trending.

Chapter 5: Exponential and Logarithmic Functions

- Section 5.1—A *Looking Ahead to Calculus* was added—Exponential Functions and e^x; six *Getting Ready* Exercises were added; eight new exercises were added—students use calculator to approximate exponential functions; sixteen new exercises were added—students evaluate exponential functions, finding exact values; eight new exercises were added—the graph of an exponential function is given and students are asked to identify the domain, range, horizontal asymptote, and increasing or decreasing; four new exercises—graph an exponential function; two new *Fix It* Exercises were added.
- Section 5.2—Six *Getting Ready* Exercises were added.
- Section 5.3—Six new *Getting Ready* Exercises were added; sixteen new exercises were added—evaluate a logarithmic expression; six new exercises were added—find the domain of a logarithmic function; four new exercises were added—evaluate an exponential function; four new exercises were added—the graph of an exponential function is given and students are asked to find the domain, range, and vertical asymptote; two new exercises were added—graph the logarithmic function along with two *Fix It* Exercises.
- Section 5.4—Four *Getting Ready* Exercises were added.

- Section 5.5—A *Looking Ahead to Calculus* was added—Logarithmic Differentiation; six new *Getting Ready* Exercises and two *Fix It* Exercises were added.
- Section 5.6—Six new *Getting Ready* Exercises were added; exact answers are now asked for when solving exponential and logarithmic equations, as well as decimals; two new exponential equations with unlike bases and four new exponential equations with bases of e were added; four new logarithmic equations involving natural log were added as well as two *Fix It* Exercises.
- End of Chapter 5—Two new exercises—evaluate an exponential function were added; two new exercises added—given the graph of an exponential function identify the domain, range, horizontal asymptote, and increasing or decreasing; two new exercises added—find the domain of a logarithmic function; two new exercises added—evaluate a logarithmic function; two new exercises added—given the graph of a logarithmic function identify the domain, range, and vertical asymptote; a *Group Activity* was added—Installment Loans.

Chapter 6: System of Linear Equations, Matrices, and Inequalities Title changed to System of Linear Equations, Matrices, and Inequalities and chapter opener changed to Information Security Analyst.

- Section 6.1—Title changed to Systems of Linear Equations in Two and Three Variables; six new *Getting Ready* Exercises were added; four new exercises—two equations with two unknowns; two *Fix It* Exercises and six new applications were added.
- Section 6.2—Title changed to Using Matrices to Solve Systems of Linear Equations; six new *Getting Ready* Exercises were added; six new exercises added—write the augmented matrix; six new exercises added—write the system represented by the augmented matrix; four new exercises added—students are asked to perform row operations on a matrix; twelve new exercises added—use matrices to solve a system of linear equations along with two *Fix It* Exercises.
- Section 6.3—Title changed to Matrix Operations; use multiplicative identity vocabulary instead of simple identity; six new *Getting Ready* Exercises were added; four new exercises added—identify the size of a matrix; six new exercises—identify a specific entry in a matrix; two *Fix It* Exercises were added.
- Section 6.4—Title changed to Multiplicative Inverses of Matrices and Matrix Equations; use multiplicative inverse vocabulary instead of simply inverse; four new *Getting Ready* Exercises added; four new exercises added—verify that two matrices are multiplicative inverses; four new exercises added—find the multiplicative inverse of 2 by 2 matrices; four new exercises added—use the "shorter" method to find the inverse of a 2 by 2 matrix; four new exercises added—write a liner system in the for m $AX = B$; four new exercises added—write the matrix equation as a linear system; four new exercises added—solve the linear system and the multiplicative inverse is given along with two *Fix It* Exercises.
- Section 6.5—Title changed to Determinants and Cramer's Rule; a *Looking Ahead to Calculus* was added—Cross Product; six new *Getting Ready* Exercises were added; six new exercises added—find the determinant of a 2 by 2 matrix; eight new exercises added—find a specific minor or cofactor of a matrix; one new exercise added—find the determinant of a 4 by 4 matrix; five new exercises added—find the determinant using row or column operations to help; six new exercises added—solve the system using Cramer's Rule; two new exercises added—find the equation of a line passing through two points; two new exercises added—determine the area of each triangle along with two *Fix It* Exercises.
- Section 6.6—A *Looking Ahead to Calculus* was added—Partial Fraction Decomposition and Integration; six new *Getting Ready* Exercises and two *Fix It* Exercises were added.
- Section 6.7—Title changed to Graphs of Inequalities in Two Variables; six new *Getting Ready* Exercises were added; four new exercises added—graphing an inequality in two variables; four new exercises added—solve the system of inequalities in two variables along with two *Fix It* Exercises.
- Section 6.8—Four new *Getting Ready* Exercises and two *Fix It* Exercises were added.
- End of Chapter 6—A *Group Activity* for Chapter 6 was added—Vacation Budget.

Chapter 7: Conic Sections and Systems of Nonlinear Equations Title changed to Conic Sections and Systems of Nonlinear Equations and chapter opener changed to Civil Engineer.

- Section 7.1—A *Looking Ahead to Calculus* was added—Conics; six new *Getting Ready* Exercises were added; six new exercises added—Find the vertex, focus, and directrix of the parabola; eight new exercises added—Write the equation of the parabola in standard form; eight new exercises added—Find the vertex, focus, directrix, and graph the parabola; two *Fix It* Exercises and six new applications were added.
- Section 7.2—Six new *Getting Ready* Exercises were added; four new exercises added—Given the graph of the ellipse, write its equation in standard form; eight new exercises added—Graph the ellipse given in standard form and identify the foci; two new exercises added—Write the ellipse in standard form and graph it; six new exercises added—Write the ellipse in standard form, graph it, and identify the Foci along with two *Fix It* Exercises.

- Section 7.3—Six new *Getting Ready* Exercises were added; two new exercises added—Given the graph of the hyperbola, write its equation in standard form; four new exercises added—Write the hyperbola's equation in standard form; six new exercises added—Graph each hyperbola and identify the center, vertices, and foci; two *Fix It* Exercises were added.
- Section 7.4—Title changed to Solving Nonlinear Systems of Equations in Two Variables; six new *Getting Ready* Exercises and two *Fix It* Exercises were added.
- End of Chapter 7—Two new exercises were added to Chapter Review—Graph each parabola and identify the focus and directrix; two new exercises added to Chapter Review—Graph each ellipse and identify the foci; two new exercises were added to Chapter Review—Graph each hyperbola and identify the foci; a *Group Activity* for Chapter 7 was added—Suspension Bridges.

Chapter 8: Sequences, Series, Induction, and Probability Title changed to Sequences, Series, Induction, and Probability and a Looking Ahead to Calculus was added—Expanding a Binomial.

- Section 8.1—Six new *Getting Ready* Exercises were added; two new exercises added—Use Pascal's Triangle to expand the binomial along with two *Fix It* Exercises.
- Section 8.2—A *Looking Ahead to Calculus* added—Sequences; six new *Getting Ready* Exercises were added; four new exercises added—Write the first six terms of the sequence represented by the Function along with two *Fix It* Exercises.
- Section 8.3—Six new *Getting Ready* Exercises were added; two new exercises added—Write the first six terms of the arithmetic sequence with the given properties along with two *Fix It* Exercises.
- Section 8.4—A *Looking Ahead to Calculus* was added—Infinite Series; six new *Getting Ready* Exercises were added; four new exercises added—Write the first four terms of the geometric sequence with the given properties along with two *Fix It* Exercises.
- Section 8.5—Six new *Getting Ready* Exercises and two *Fix It* Exercises were added.
- Section 8.6—Title changed to Fundamental Counting Principle, Permutations, and Combinations; six new *Getting Ready* Exercises were added. 12 new exercises added—Evaluate each permutation or combination along with two *Fix It* Exercises.
- Section 8.7—Title changed to Introduction to Probability; six new *Getting Ready* Exercises were added; eight new exercises added—Determine the probability roulette wheel problems along with two *Fix It* Exercises.
- End of Chapter 8—A *Group Activity* was added—Lottery Mathematics.

Ancillaries for the Instructor

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Go beyond the answers and see what it takes to understand the concepts and improve your grade! This manual provides worked-out, step-by-step solutions to the odd-numbered problems in the text, giving you the information needed to understand how these problems are solved. Available in WebAssign and on the companion website.

Student Notetaking Guide
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To the Student


Dmitry Molchanov/Shutterstock.com

Congratulations! You now own a state-of-art textbook that has been written especially for you. You are about to travel some beautiful college algebra terrain and I'm thrilled to see you on this mathematics hiking trail. You will discover and explore many breathtaking math concepts as you reach the summit of success in mathematics. So, put on your hiking boots, grab your backpack, and let the hiking begin. An exhilarating adventure in mathematics awaits you! Happy hiking!

Just as a hiker needs navigation tools and skills for hiking, we provide the same for you with the content in this textbook. Whether you use this text in a print of eBook form, you can still maximize its impact. The textbook can serve as your hiking backpack and it is full of the support needed to succeed no matter what your initial math-skill level. Easy-to-follow examples are provided, and Tips, Take Notes, and Cautions are shared to help you traverse whatever algebra terrain or section in the textbook you are trekking. To maximize your experience on the hike, there are Definition, Property and Theorem, and Strategy Boxes along the way to guide you. Getting Ready Exercises are also provided to serve as stretching before the big hike. Our Looking Ahead to Calculus feature provides you with a glimpse of how math is applied in future courses and keeps you moving forward on the trail. Are you ready to set out on the journey and discover and learn new things? If yes, grab your favorite pencil, some papers, and your textbook or eBookand, and lets hit the trail. You'll soon experience the thrill of reaching mountain peak in college algebra and completing mathematics successfully. We wish you the very best on your mathematics hike. Stay positive, keep focused, and enjoy this math hike of a lifetime!

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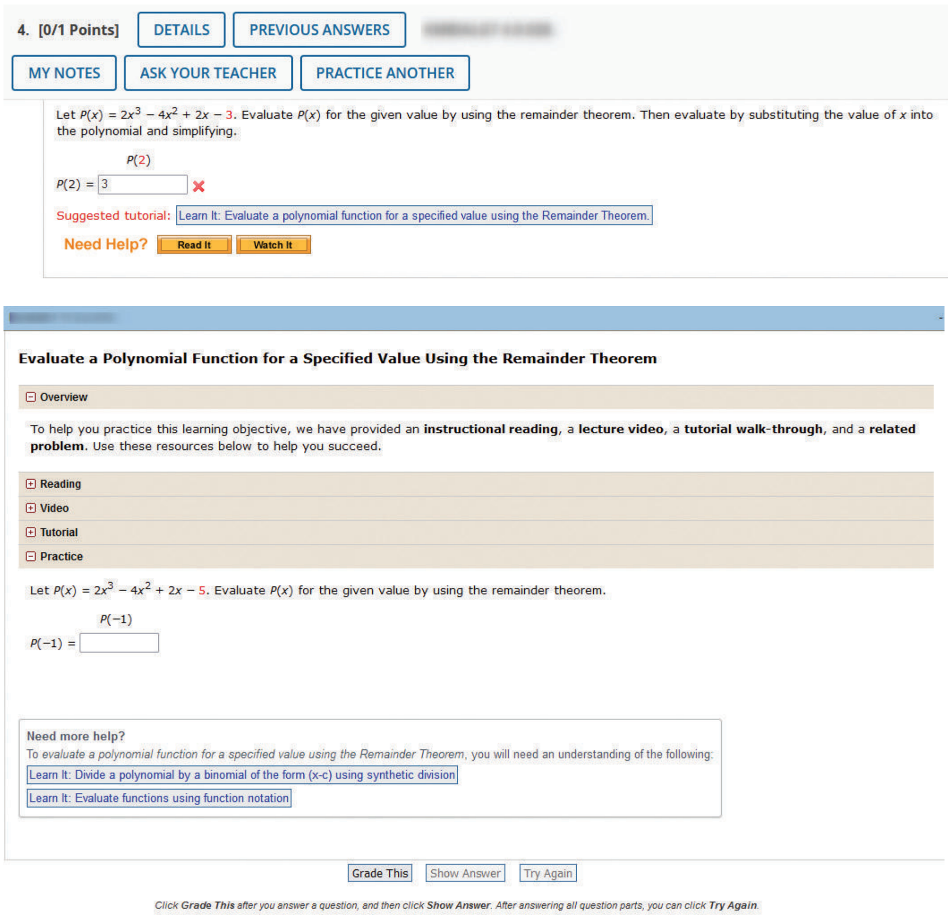
R. David Gustafson

Jeffrey D. Hughes

The WebAssign course accompanying this thirteenth edition is very much an extension of the textbook's approach. In addition to offering students the opportunity to learn and practice and get immediate feedback, this edition focuses strongly on providing scaffolded help to students when need it, deepening their problem-solving abilities and providing engaging ways to apply course concepts.

Scaffolded Help

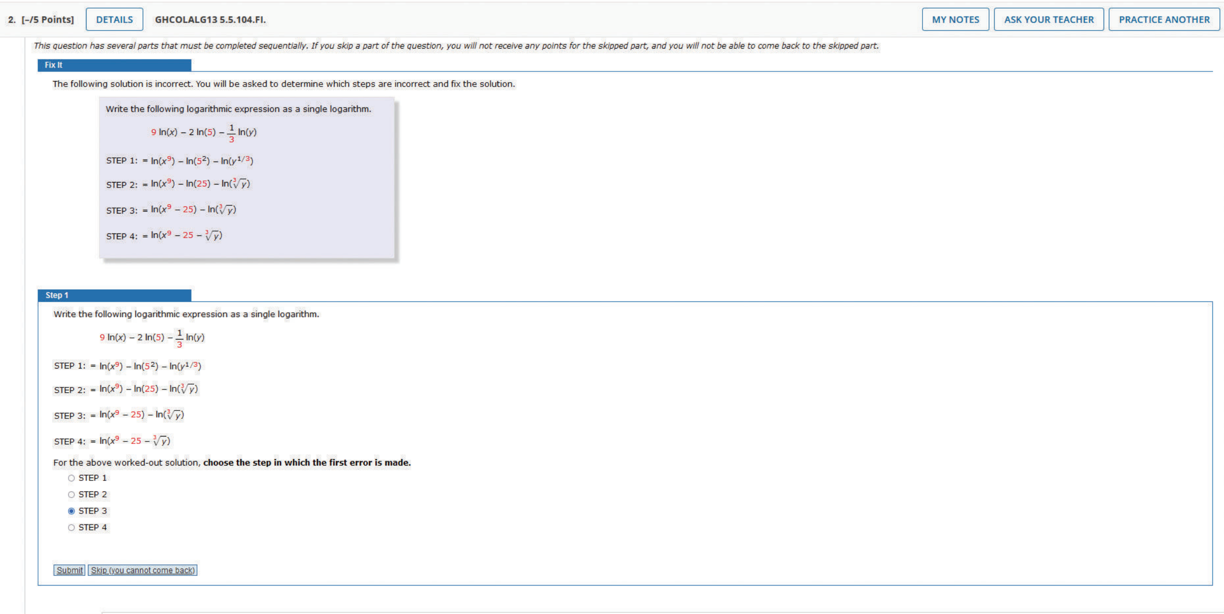
Learn Its help students become confident, self-sufficient learners by offering just-in-time instruction/help that meets their diverse learning styles. Embedded within the majority of coded problems from the textbook, students are presented with a Learn It when they get a question incorrect by default. The Learn It then provides immediate targeted instruction and practice on the topic using narrative, videos and tutorials—all in one place. If a topic is still too challenging, students can choose to continue learning through associated prerequisite Learn Its until they feel confident in their knowledge and preparedness.

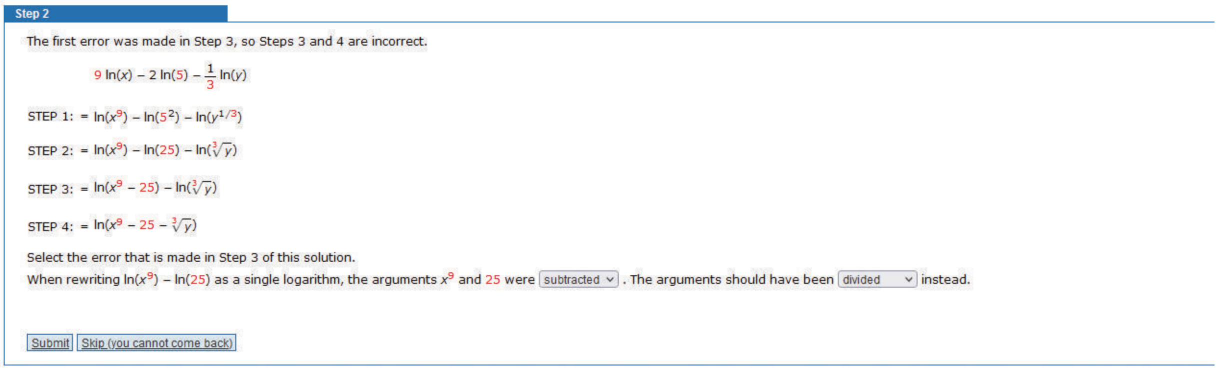


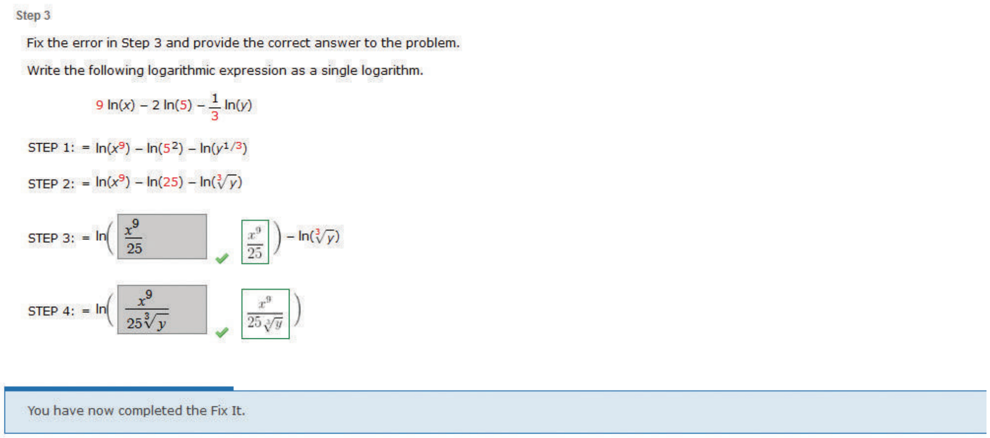
Deepening Problem-Solving Abilities

Fix Its

These are exercises where students find and fix an error in a step-by-step provided solution. The learner will first identify the step number where the first mistake occurs. Then the learner is asked to correct that mistake ensuring the error is fixed. This process is intended to emphasize problem-solving strategies students can apply to any exercise or homework assignment they work on. These exercises are included in WebAssign. The exercises are also intended to help students avoid making common mistakes that are often made in mathematics on homework and test problems.







Looking Ahead to Calculus Topics

These margin notes preview calculus topics explaining why they are important and how they are used. The accompanying modules in WebAssign are interactive and extend the explanation of the concepts to connect college algebra to calculus. Previewing these topics sets the stage for strong mastery and understanding of these topics in future courses. Module topics include:

- What is Calculus?
- Using Tables of Values to Help Understand the Definition of a Limit at a Point
- Using Algebraic Techniques to Evaluate a Limit at a Point
- Using the Difference Quotient to Find a Tangent Line and Define the Derivative
- Using the Derivative to Analyze the Graph of a Function
- Using End Behavior to Find Limits to Infinity and Infinite Limits
- Using the Intermediate Value Theorem to Apply Newton's Method for Finding Roots

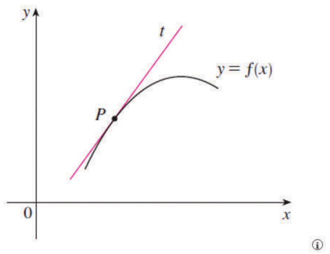
1. [–/72 Points] DETAILS GHCOLALG13 4.LAC.001. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Looking Ahead to Calculus: Using the Derivative to Analyze the Graph of a Function

⊟ Introduction

You have already learned how to sketch the graph of a polynomial function using information about the end behavior, symmetry, the intercepts, and a sign chart for where the graph is above and below the x-axis. In calculus, the **derivative** of a function provides us with even more information to accurately sketch the graph of a function.

The **derivative** of a function represents the slope of the tangent line to the curve $y = f(x)$ at the point $(x, f(x))$. The derivative of a function is denoted $f'(x)$ (read as f prime of x).



In the figure above, the slope of the tangent line t is the derivative of $f(x)$ at the point P.

So, the derivative will tell us the direction in which the curve proceeds at each point. In other words, it tells us where the curve is increasing or decreasing. The derivative will also tell us when the curve changes directions, and thus the location of any local maximum or minimum values.

We focus on polynomial functions in this lab, but, in calculus, the concepts extend to other types of functions as well.

⊞ Interactive Reading
⊞ Interactive Example
⊞ Practice 1
⊞ Practice 2

1. [–/72 Points] DETAILS GHCOLALG13 4.LAC.001. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Looking Ahead to Calculus: Using the Derivative to Analyze the Graph of a Function

⊞ Introduction
⊞ Interactive Reading
⊟ Interactive Example

For the polynomial function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$, the derivative is $f'(x) = 12x^3 - 12x^2 - 24x$.

Use a sign chart for $f'(x)$ to determine the open intervals where $f(x)$ is increasing and where it is decreasing. Also, identify any local maximum and minimum values of $f(x)$. Then, sketch the graph of $f(x)$.

Solution

First, we must set the derivative equal to zero and solve for any critical numbers. These values will help us construct our sign chart.

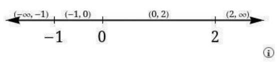
We factor $f'(x)$ and then solve for x.

$$12x^3 - 12x^2 - 24x = 0$$

$$12x(x + 1)\left(\boxed{}\right) = 0$$

$$x = 0,\ x = -1,\ \text{or } x = \boxed{}$$

Now, we create a sign chart. The three critical numbers divide the domain of $f(x)$ into four intervals.



In each interval, we choose a test point and evaluate the derivative at the test point. If the sign of $f'(x)$ is positive, we know that $f(x)$ is increasing, and if the sign of $f'(x)$ is negative, we know that $f(x)$ is decreasing.

Interval	Test Point	Sign of $f'(x)$	Behavior of $f(x)$
$(-\infty, -1)$	$f'(-2) = -96$	−	decreasing
$(-1, 0)$	$f'\left(-\frac{1}{2}\right) = \frac{15}{2}$	?	---Select---
$(0, 2)$	$f'(1) =$ ☐	?	---Select---
$(2, \infty)$	$f'(3) =$ ☐	+	increasing

From the sign chart, we see that $f(x)$ is increasing on the intervals ☐ and $(2, \infty)$. We see that $f(x)$ is decreasing on the intervals $(-\infty, -1)$ and ☐ .

Applying the Content

Group Activities based on projects at the end of the chapter are now available in WebAssign and can be assigned individually or as group work.

3. [–/9 Points] DETAILS GHCOLALG13 5.GA.001. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Group Activity: Installment Loans

What Are Fixed Installment Loans?

A fixed installment loan is a loan that is repaid in equal payments. Sometimes part of the cost is paid at the time of purchase. This amount is the down payment.

Real-World Example of Installment Loans

Installment loans allow you the ability to purchase an item and use it now. This is called installment purchasing, and being able to use the item is an advantage. The disadvantage is that interest is paid on the amount borrowed. A common example is purchasing an automobile.

Group Activity

1. Suppose you graduate from college, obtain an amazing job, get married, and then purchase a new vehicle. Use the given information below to answer four questions.
   - Cost of automobile: $22,500
   - Down payment: $2,000
   - Monthly payment: $450.25
   - Loan term: 48 months

   a. What is the amount financed (in dollars)? Note that the amount financed is the price of the car minus the down payment.

   $ ____

   b. What is the total amount (in dollars) of all monthly payments?

   $ ____

   c. What is the total installment price (in dollars)? Note that the total installment price is the sum of all the monthly payments plus the down payment.

   $ ____

   d. What was the financial charge (in dollars)? Note that the financial charge is the total installment price minus the purchase price of the automobile.

   $ ____

   Explain what it represents.

   The financial charge represents the ---Select--- .

2. Prior to making an automobile purchase, it is often helpful to know in advance what the monthly payment will be. The monthly payment can be determined using the formula shown below, where M is the monthly payment, P is the principal value of the loan, r is the APR (annual percentage rate) in decimal form, and n is the total number of payments.

$$M = \frac{P\left(\frac{r}{12}\right)}{1 - \left(1 + \frac{r}{12}\right)^{-n}}$$

   a. Using the internet, identify the MSRP (Manufacturer's Suggested Retail Price) of an automobile you would be interested in purchasing (in dollars). A suggested website would be www.edmunds.com.

   $ ____

   b. Use the MSRP from part (2a) and the given information below to determine the monthly payment (in dollars) for the automobile. Compare your calculated monthly payment with an online loan calculator. A suggested loan calculator is found at www.bankrate.com. (Round your answer to two decimal places.)
   - APR is 3.39%
   - 60 months
   - Trade-in value of your current automobile is $4,950

   M = $ ____

   c. Use the rounded value you found in part (2b) to find the total installment price (in dollars). (Round your answer to two decimal places.)

   $ ____

   d. Use the rounded value you found in part (2b) to find the financial charge (in dollars). (Round your answer to two decimal places.)

   $ ____

R

A Review of Basic Algebra

Careers and Mathematics: Pharmacist

Pharmacists distribute prescription drugs to individuals. They also advise patients, physicians, and other healthcare workers on the selection, dosages, interactions, and side effects of medications. They also monitor patients to ensure that they are using their medications safely and effectively. Some pharmacists specialize in oncology, nuclear pharmacy, geriatric pharmacy, or psychiatric pharmacy.

Education and Mathematics Required

- Pharmacists are required to possess a Pharm.D. degree from an accredited college or school of pharmacy. This degree generally takes four years to complete. To be admitted to a Pharm.D. program, at least two years of college must be completed, which includes courses in the natural sciences, mathematics, humanities, and the social sciences. A series of examinations must also be passed to obtain a license to practice pharmacy.
- College Algebra, Trigonometry, Statistics, and Calculus I are courses required for admission to a Pharm.D. program.

How Pharmacists Use Math and Who Employs Them

- Pharmacists use math throughout their work to calculate dosages of various drugs. These dosages are based on weight and whether the medication is given in pill form, by infusion, or intravenously.
- Most pharmacists work in a community setting, such as a retail drugstore, or in a healthcare facility, such as a hospital.

Career Outlook and Salary

- Employment of pharmacists is expected to decrease by 3% between 2019 and 2029, about as fast as the average for all occupations.
- Median annual wages of wage and salary pharmacists is approximately $128,710.

For more information see: www.bls.gov/ooh.

In this chapter, we review many concepts and skills learned in previous algebra courses. Be sure to master this material now, because it is the basis for the rest of this course.

R.1 Sets of Real Numbers

R.2 Integer Exponents and Scientific Notation

R.3 Rational Exponents and Radicals

R.4 Polynomials and More Operations on Radicals

R.5 Factoring

R.6 Rational Expressions

Chapter Review

Chapter Test

Group Activity

R.1 Sets of Real Numbers

In this section, we will learn to

1. Identify sets of real numbers.
2. Identify properties of real numbers.
3. Graph subsets of real numbers on the number line.
4. Graph intervals on the number line.
5. Simplify absolute value expressions.
6. Find distances on the number line.

Sudoku, a game that involves number placement, is very popular. The objective is to fill a 9 by 9 grid so that each column, each row, and each of the 3 by 3 blocks contains the numbers from 1 to 9. A man playing Sudoku on his tablet is shown in the margin.

To solve Sudoku puzzles, logic and the set of numbers, {1, 2, 3, 4, 5, 6, 7, 8, 9}, are used. Sets of numbers are important in mathematics, and we begin our study of algebra with this topic.

A **set** is a collection of objects, such as a set of dishes or a set of golf clubs. The set of vowels in the English language can be denoted as {a, e, i, o, u}, where the braces { } are read as "the set of."

If every member of one set B is also a member of another set A, we say that B is a **subset** of A. We can denote this by writing $B \subset A$, where the symbol $\subset$ is read as "is a subset of." (See Figure R-1.) If set B equals set A, we can write $B \subseteq A$.

If A and B are two sets, we can form a new set consisting of all members that are in set A or set B or both. This set is called the **union** of A and B. We can denote this set by writing $A \cup B$, where the symbol $\cup$ is read as "union." (See Figure R-1.)

We can also form the set consisting of all members that are in both set A and set B. This set is called the **intersection** of A and B. We can denote this set by writing $A \cap B$, where the symbol $\cap$ is read as "intersection." (See Figure R-1.)

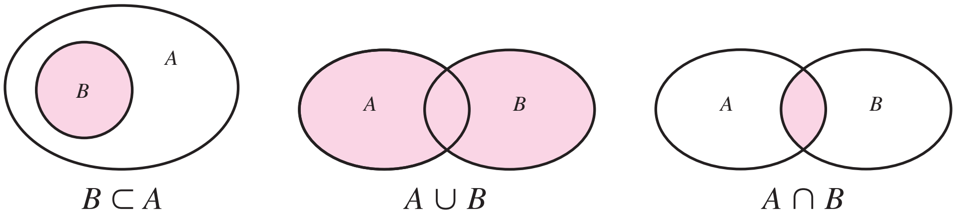


Figure R-1

Example 1 **Understanding Subsets and Finding the Union and Intersection of Two Sets**

Let $A = \{a, e, i\}$, $B = \{c, d, e\}$, and $V = \{a, e, i, o, u\}$.

a. Is $A \subset V$? **b.** Find $A \cup B$. **c.** Find $A \cap B$.

Strategy We will apply the definitions of subset, union, and intersection.

Solution

a. Since each member of set A is also a member of set V, $A \subset V$.

b. The union of set A and set B contains the members of set A, set B, or both. Thus, $A \cup B = \{a, c, d, e, i\}$.

c. The intersection of set A and set B contains the members that are in both set A and set B. Thus, $A \cap B = \{e\}$.

Self Check 1 **a.** Is $B \subset V$? **b.** Find $B \cup V$ **c.** Find $A \cap V$

Now Try Exercise 39.

If a set has no elements, it is called the **empty set**, or the **null set**, and is represented by either { } or ∅. The symbol ∅ is the Greek letter phi. Here is an example of how the empty set can result when finding the intersection of two sets.

$$\{5, 10, 15, 20\} \cap \{25, 30, 35, 40\} = \varnothing$$

Because the sets have no common elements, their intersection has no elements and is the empty set.

1. Identify Sets of Real Numbers

There are several sets of numbers that we use in everyday life.

Natural Numbers, Whole Numbers, and Integers

Natural numbers
The numbers that we use for counting: $\{1, 2, 3, 4, 5, 6, \ldots\}$

Whole numbers
The set of natural numbers including 0: $\{0, 1, 2, 3, 4, 5, 6, \ldots\}$

Integers
The set of whole numbers and their negatives:
$\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

In the definitions above, each group of three dots (called an *ellipsis*) indicates that the numbers continue forever in the indicated direction.

Two important subsets of the natural numbers are the *prime* and *composite* numbers. A **prime number** is a natural number greater than 1 that is divisible only by itself and 1. A **composite number** is a natural number greater than 1 that is not prime.

- **The set of prime numbers:** $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, \ldots\}$
- **The set of composite numbers:** $\{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, \ldots\}$

Two important subsets of the set of integers are the *even* and *odd integers*. The **even integers** are the integers that are exactly divisible by 2. The **odd integers** are the integers that are not exactly divisible by 2.

- **The set of even integers:** $\{\ldots, -10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, \ldots\}$
- **The set of odd integers:** $\{\ldots, -9, -7, -5, -3, -1, 1, 3, 5, 7, 9, \ldots\}$

So far, we have listed numbers inside braces to specify sets. This method is called the **roster method**. When we give a rule to determine which numbers are in a set, we are using **set-builder notation**. To use set-builder notation to denote the set of prime numbers, we write

$$\{x \mid x \text{ is a prime number}\}$$

variable — such that — rule that determines membership in the set

Read as "the set of all numbers x such that x is a prime number." Recall that when a letter stands for a number, it is called a variable.

Take Note
Remember that the denominator of a fraction can **never** be 0.

The fractions of arithmetic are called *rational numbers*.

Rational Numbers

Rational numbers are fractions that have an integer numerator and a nonzero integer denominator. Using set-builder notation, the rational numbers are

$$\left\{\frac{a}{b} \,\middle|\, a \text{ is an integer and } b \text{ is a nonzero integer}\right\}$$

Tip
To easily recall the definition of a rational number, consider the first five letters of the word **ratio**nal. Those letters spell the word **ratio**. A rational number is the ratio of two integers.

Rational numbers can be written as fractions or decimals. Some examples of rational numbers are

$$5 = \frac{5}{1}, \quad \frac{3}{4} = 0.75, \quad -\frac{1}{3} = -0.333\ldots, \quad \text{and} \quad -\frac{5}{11} = -0.454545\ldots$$

The = sign indicates that two quantities are equal.

These examples suggest that the decimal forms of all rational numbers are either *terminating decimals* or *repeating decimals*.

Example 2 **Determining whether the Decimal Form of a Fraction Terminates or Repeats**

Determine whether the decimal form of each fraction terminates or repeats:

a. $\frac{7}{16}$ **b.** $\frac{65}{99}$

Strategy In each case, we perform a long division and write the quotient as a decimal.

Solution **a**. To change $\frac{7}{16}$ to a decimal, we perform a long division to get $\frac{7}{16} = 0.4375$. Since 0.4375 terminates, we can write $\frac{7}{16}$ as a terminating decimal.

b. To change $\frac{65}{99}$ to a decimal, we perform a long division to get $\frac{65}{99} = 0.656565\ldots$. Since $0.656565\ldots$ repeats, we can write $\frac{65}{99}$ as a repeating decimal.

We can write repeating decimals in compact form by using an overbar. For example, $0.656565\ldots = 0.\overline{65}$.

Self Check 2 Determine whether the decimal form of each fraction terminates or repeats:

a. $\frac{38}{99}$ **b.** $\frac{7}{8}$

Now Try Exercise 41.

Some numbers have decimal forms that neither terminate nor repeat. These nonterminating, nonrepeating decimals are called **irrational numbers**. Three examples of irrational numbers are

$$1.010010001000010\ldots, \qquad \sqrt{2} = 1.414213562\ldots, \qquad \text{and} \qquad \pi = 3.141592654\ldots$$

The union of the set of rational numbers (the terminating and repeating decimals) and the set of irrational numbers (the nonterminating, nonrepeating decimals) is the set of *real numbers* (the set of all decimals).

Real Numbers A **real number** is any number that is rational or irrational. Using set-builder notation, the set of real numbers is

$$\{x \mid x \text{ is a rational or an irrational number}\}$$

Example 3 **Classifying Real Numbers**

In the set $\{-3, -2, 0, \frac{1}{2}, 1, \sqrt{5}, 2, 4, 5, 6\}$, list all

a. even integers **b.** prime numbers **c.** rational numbers

Strategy We will check to see whether each number is a member of the set of even integers, the set of prime numbers, and the set of rational numbers.

Solution **a.** even integers: $-2, 0, 2, 4, 6$

b. prime numbers: 2, 5

c. rational numbers: $-3, -2, 0, \frac{1}{2}, 1, 2, 4, 5, 6$

Self Check 3 In the set in Example 3, list all **a.** odd integers **b.** composite numbers **c.** irrational numbers.

Now Try Exercise 49.

Tip

Learning several sets of numbers at one time can initially cause your head to spin. To help, examine closely the figure shown. You will easily grasp connections between the sets of numbers in algebra and not be overwhelmed.

Looking Ahead to Calculus

Concept: **The Real Number System**

1. Why is it important?
Numerical answers in Calculus and Business Calculus are real numbers.

2. How is it used?
Examples such as rates of change of volume functions, revenue functions, or acceleration functions will always be real number valued functions.

Figure R-2 shows how the previous sets of numbers are related.

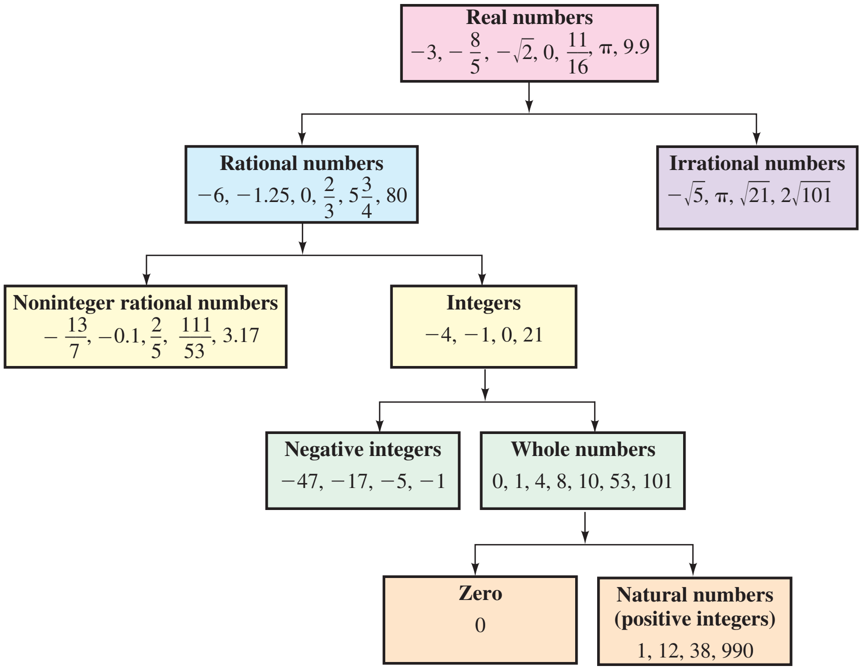


Figure R-2

2. Identify Properties of Real Numbers

When we work with real numbers, we will use the following properties.

Properties of Real Numbers

If a, b, and c are real numbers,

The Commutative Properties for Addition and Multiplication

$$a + b = b + a \qquad ab = ba$$

The Associative Properties for Addition and Multiplication

$$(a + b) + c = a + (b + c) \qquad (ab)c = a(bc)$$

The Distributive Property of Multiplication over Addition or Subtraction

$$a(b + c) = ab + ac \quad \text{or} \quad a(b - c) = ab - ac$$

The Double Negative Rule

$$-(-a) = a$$

Take Note

When the Associative Property is used, the order of the real numbers does not change. The real numbers that occur within the parentheses change.

The Distributive Property also applies when more than two terms are within parentheses.

Example 4 **Identifying Properties of Real Numbers**

Determine which property of real numbers justifies each statement.

a. $(9 + 2) + 3 = 9 + (2 + 3)$ **b.** $3(x + y + 2) = 3x + 3y + 3 \cdot 2$

Strategy We will compare the form of each statement to the forms listed in the Properties of Real Numbers box.

Solution **a.** This form matches the Associative Property of Addition.

b. This form matches the Distributive Property.

Self Check 4 Determine which property of real numbers justifies each statement:

a. $mn = nm$ **b.** $(xy)z = x(yz)$ **c.** $p + q = q + p$

Now Try Exercise 23.

3. Graph Subsets of Real Numbers on the Number Line

We can graph subsets of real numbers on the **number line**. The number line shown in Figure R-3 continues forever in both directions. The **positive numbers** are represented by the points to the right of 0, and the **negative numbers** are represented by the points to the left of 0.

Take Note

Zero is neither positive nor negative.

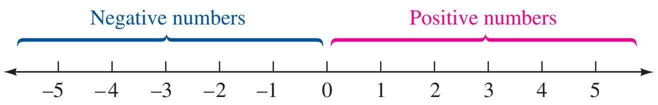


Figure R-3

Figure R-4(a) shows the graph of the natural numbers from 1 to 5. The point associated with each number is called the *graph* of the number, and the number is called the *coordinate* of its point.

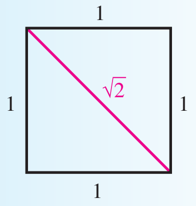
Figure R-4(b) shows the graph of the prime numbers that are less than 10.

Figure R-4(c) shows the graph of the integers from -4 to 3.

Figure R-4(d) shows the graph of the real numbers $-\frac{7}{3}$, $-\frac{3}{4}$, $0.\overline{3}$, and $\sqrt{2}$.

Take Note

$\sqrt{2}$ can be shown as the diagonal of a square with sides of length 1.



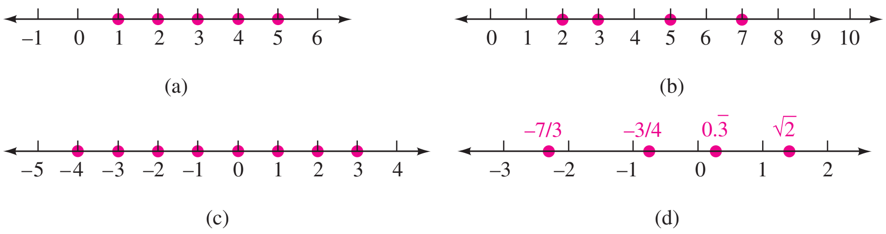


Figure R-4

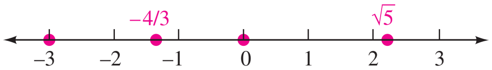
The graphs in Figure R-4 suggest that there is a **one-to-one correspondence** between the set of real numbers and the points on a number line. This means that to each real number there corresponds exactly one point on the number line, and to each point on the number line there corresponds exactly one real-number coordinate.

Example 5 **Graphing a Set of Numbers on a Number Line**

Graph the set $\left\{-3, -\frac{4}{3}, 0, \sqrt{5}\right\}$.

Strategy We will mark (plot) each number on the number line. To the nearest tenth, $\sqrt{5} = 2.2$.

Solution

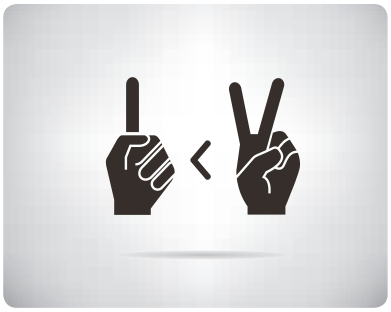


Self Check 5 Graph the set $\left\{-2, \frac{3}{4}, \sqrt{3}\right\}$. (*Hint:* To the nearest tenth, $\sqrt{3} = 1.7$.)

Now Try Exercise 57.

4. Graph Intervals on the Number Line

To show that two quantities are not equal, we can use an **inequality symbol**.



Phipatbig/Shutterstock.com

Symbol	Read as	Examples		
$\neq$	"is not equal to"	$5 \neq 8$	and	$0.25 \neq \frac{1}{3}$
$<$	"is less than"	$12 < 20$	and	$0.17 < 1.1$
$>$	"is greater than"	$15 > 9$	and	$\frac{1}{2} > 0.2$
$\leq$	"is less than or equal to"	$25 \leq 25$	and	$1.7 \leq 2.3$
$\geq$	"is greater than or equal to"	$19 \geq 19$	and	$15.2 \geq 13.7$
$\approx$	"is approximately equal to"	$\sqrt{2} \approx 1.414$	and	$\sqrt{3} \approx 1.732$

It is possible to write an inequality with the inequality symbol pointing in the opposite direction. For example,

- $12 < 20$ is equivalent to $20 > 12$
- $2.3 \geq -1.7$ is equivalent to $-1.7 \leq 2.3$

In Figure R-3, the coordinates of points get larger as we move from left to right on a number line. Thus, if a and b are the coordinates of two points, the one to the right is the greater. This suggests the following facts:

- If $a > b$, point a lies to the right of point b on a number line.
- If $a < b$, point a lies to the left of point b on a number line.

Take Note

Parentheses indicate that endpoints are not included in an interval. Square brackets indicate that endpoints are included in an interval.

Figure R-5(a) shows the graph of the *inequality* $x > -2$ (or $-2 < x$). This graph includes all real numbers x that are greater than -2. The parenthesis at -2 indicates that -2 is not included in the graph. Figure R-5(b) shows the graph of $x \leq 5$ (or $5 \geq x$). The bracket at 5 indicates that 5 is included in the graph.

(a) (b)

Figure R-5

Sometimes two inequalities can be written as a single expression called a **compound inequality**. For example, the compound inequality

$5 < x < 12$

is a combination of the inequalities $5 < x$ and $x < 12$. It is read as "5 is less than x, and x is less than 12," and it means that x is between 5 and 12. Its graph is shown in Figure R-6.

Figure R-6

The graphs shown in Figures R-5 and R-6 are portions of a number line called **intervals**. The interval shown in Figure R-7(a) is denoted by the inequality $-2 < x < 4$, or in **interval notation** as $(-2, 4)$. The parentheses indicate that the endpoints are not included. The interval shown in Figure R-7(b) is denoted by the inequality $x > 1$, or as $(1, \infty)$ in interval notation. The symbol ∞ (infinity) is not a real number. It is used to indicate that the graph in Figure R-7(b) extends infinitely far to the right.

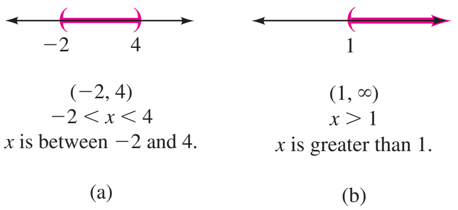


Figure R-7

A compound inequality such as $-2 < x < 4$ can be written as two separate inequalities:

$x > -2$ and $x < 4$

This expression represents the **intersection** of two intervals. In interval notation, this expression can be written as

Tip
Parentheses are always used with ∞ or $-\infty$.

$(-2, \infty) \cap (-\infty, 4)$ Read the symbol $\cap$ as "intersection."

Since the graph of $-2 < x < 4$ will include all points whose coordinates satisfy both $x > -2$ and $x < 4$ at the same time, its graph will include all points that are larger than -2 but less than 4. This is the interval $(-2, 4)$, whose graph is shown in Figure R-7(a).

Example 6 Writing an Inequality in Interval Notation and Graphing the Inequality

Write the inequality $-3 < x < 5$ in interval notation and graph it.

Strategy This is the interval $(-3, 5)$. Its graph includes all real numbers between -3 and 5, as shown in Figure R-8.

Solution

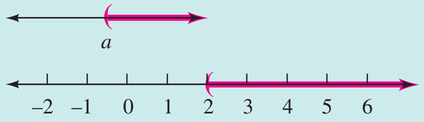
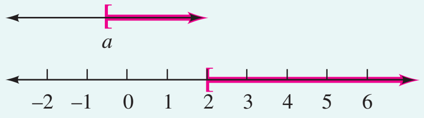
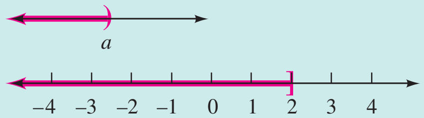
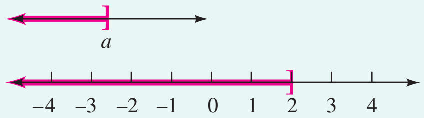
Figure R-8

Self Check 6 Write the inequality $-2 < x \le 5$ in interval notation and graph it.

Now Try Exercise 69.

If an interval extends forever in one direction, it is called an **unbounded interval**.

Unbounded Intervals

Interval	Inequality	Graph
(a, ∞) $(2, \infty)$	$x > a$ $x > 2$	
$[a, \infty)$ $[2, \infty)$	$x \geq a$ $x \geq 2$	
$(-\infty, a)$ $(-\infty, 2)$	$x < a$ $x < 2$	
$(-\infty, a]$ $(-\infty, 2]$	$x \leq a$ $x \leq 2$	
$(-\infty, \infty)$	$-\infty < x < \infty$	

$(-3, 2)$
$-3 < x < 2$

(a)

$[-2, 3)$
$-2 \leq x < 3$

(b)

Figure R-9

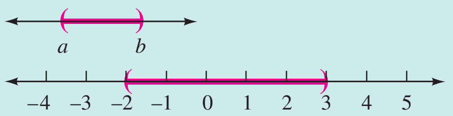
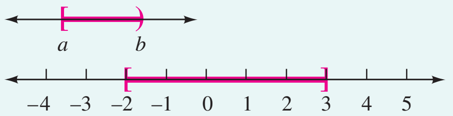
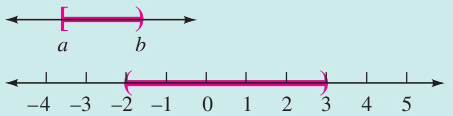
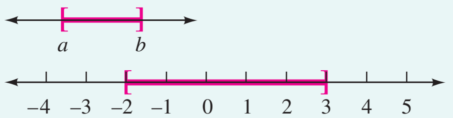
A bounded interval with no endpoints is called an **open interval**. Figure R-9(a) shows the open interval between −3 and 2. A bounded interval with one endpoint is called a **half-open interval**. Figure R-9(b) shows the half-open interval between −2 and 3, including −2.

Intervals that include two endpoints are called **closed intervals**. Figure R-10 shows the graph of a closed interval from −2 to 4.

$[-2, 4]$
$-2 \leq x \leq 4$

Figure R-10

Open Intervals, Half-Open Intervals, and Closed Intervals

Interval	Inequality	Graph
Open		
(a, b) $(-2, 3)$	$a < x < b$ $-2 < x < 3$	
Half-Open		
$[a, b)$ $[-2, 3)$	$a \leq x < b$ $-2 \leq x < 3$	
Half-Open		
$(a, b]$ $(-2, 3]$	$a < x \leq b$ $-2 < x \leq 3$	
Closed		
$[a, b]$ $[-2, 3]$	$a \leq x \leq b$ $-2 \leq x \leq 3$	

Example 7 **Writing an Inequality in Interval Notation and Graphing the Inequality**

Write the inequality $3 \le x$ in interval notation and graph it.

Strategy The inequality $3 \le x$ can be written in the form $x \ge 3$. This is the interval $[3, \infty)$. Its graph includes all real numbers greater than or equal to 3, as shown in Figure R-11.

Solution

Figure R-11

Self Check 7 Write the inequality $5 > x$ in interval notation and graph it.

Now Try Exercise 71.

Example 8 **Writing an Inequality in Interval Notation and Graphing the Inequality**

Write the inequality $5 \ge x \ge -1$ in interval notation and graph it.

Strategy The inequality $5 \ge x \ge -1$ can be written in the form

$$-1 \le x \le 5$$

Solution This is the interval $[-1, 5]$. Its graph includes all real numbers from -1 to 5. The graph is shown in Figure R-12.

Figure R-12

Self Check 8 Write the inequality $0 \le x \le 3$ in interval notation and graph it.

Now Try Exercise 75.

The expression

$x < -2$ or $x \ge 3$ Read as "x is less than -2 or x is greater than or equal to 3."

represents the *union* of two intervals. In interval notation, it is written as

$(-\infty, -2) \cup [3, \infty)$ Read the symbol $\cup$ as "union."

Its graph is shown in Figure R-13.

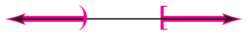


Figure R-13

5. Simplify Absolute Value Expressions

The **absolute value** of a real number x (denoted as $|x|$) is the distance on a number line between 0 and the point with a coordinate of x. For example, points with coordinates of 4 and -4 both lie four units from 0, as shown in Figure R-14. Therefore, it follows that

$$|-4| = |4| = 4$$

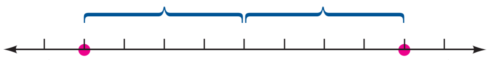


Figure R-14

In general, for any real number x,

$$|-x| = |x|$$

We can define absolute value algebraically as follows.

Absolute Value If x is a real number, then

$$|x| = x \qquad \text{when } x \geq 0$$

$$|x| = -x \qquad \text{when } x < 0$$

Take Note

Remember that x is **not** always positive and $-x$ is **not** always negative.

This definition indicates that when x is positive or 0, then x is its own absolute value. However, when x is negative, then $-x$ (which is positive) is its absolute value. Thus, $|x|$ is always nonnegative.

$$|x| \geq 0 \qquad \text{for all real numbers } x$$

Example 9 Using the Definition of Absolute Value

Write each number without using absolute value symbols:

a. $|3|$ **b.** $|-4|$ **c.** $|0|$ **d.** $-|-7|$

Strategy In each case, we will use the definition of absolute value.

Solution **a.** $|3| = 3$ **b.** $|-4| = 4$ **c.** $|0| = 0$ **d.** $-|-7| = -(7) = -7$

Self Check 9 Write each number without using absolute value symbols:

a. $|-10|$ **b.** $|12|$ **c.** $-|6|$

Now Try Exercise 91.

In Example 10, we must determine whether the number inside the absolute value is positive or negative.

Example 10 Simplifying an Expression with Absolute Value Symbols

Write each number without using absolute value symbols:

a. $|\pi - 1|$ **b.** $|2 - \pi|$ **c.** $|2 - x|$ if $x \geq 5$

Strategy In each case, we will use the definition of absolute value.

Solution

a. Since $\pi \approx 3.1416$, $\pi - 1$ is positive, and $\pi - 1$ is its own absolute value.

$$|\pi - 1| = \pi - 1$$

b. Since $2 - \pi$ is negative, its absolute value is $-(2 - \pi)$.

$$|2 - \pi| = -(2 - \pi) = -2 - (-\pi) = -2 + \pi = \pi - 2$$

c. Since $x \geq 5$, the expression $2 - x$ is negative, and its absolute value is $-(2 - x)$.

$$|2 - x| = -(2 - x) = -2 + x = x - 2 \text{ provided } x \geq 5$$

Looking Ahead to Calculus

Concept: **Absolute Value and Determining a Limit**

1. Why is it important?
Absolute value occurs in the precise definition of limit.

2. How is it used?
A knowledge of absolute value is required to prove limits in calculus.

Self Check 10 Write each number without using absolute value symbols. (*Hint:* $\sqrt{5} \approx 2.236$.)

a. $|2 - \sqrt{5}|$ **b.** $|2 - x|$ if $x \leq 1$

Now Try Exercise 95.

6. Find Distances on the Number Line

On the number line shown in Figure R-15, the distance between the points with coordinates of 1 and 4 is $4 - 1$, or 3 units. However, if the subtraction were done in the other order, the result would be $1 - 4$, or -3 units. To guarantee that the distance between two points is always positive, we can use absolute value symbols. Thus, the distance d between two points with coordinates of 1 and 4 is

$$d = |4 - 1| = |1 - 4| = 3$$

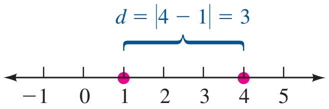


Figure R-15

In general, we have the following definition for the distance between two points on the number line.

Distance between Two Points If a and b are the coordinates of two points on the number line, the distance d between the points is given by the formula

$$d = |b - a|$$

Example 11 **Finding the Distance between Two Points on a Number Line**

Find the distance on a number line between the points with the given coordinates.

a. 3 and 5 **b.** -2 and 3 **c.** -5 and -1

Strategy We will use the formula for finding the distance between two points.

Solution

a. $d = |5 - 3| = |2| = 2$

b. $d = |3 - (-2)| = |3 + 2| = |5| = 5$

c. $d = |-1 - (-5)| = |-1 + 5| = |4| = 4$

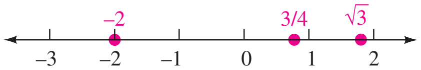
Self Check 11 Find the distance on a number line between the points with the given coordinates.

a. 4 and 10 **b.** -2 and -7

Now Try Exercise 107.

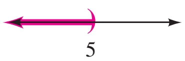
Self Check Answers

1. a. no **b.** {a, c, d, e, i, o, u} **c.** {a, e, i} **2. a.** repeats **b.** terminates **3. a.** $-3, 1, 5$ **b.** 4, 6 **c.** $\sqrt{5}$ **4. a.** Commutative Property of Multiplication **b.** Associative Property of Multiplication **c.** Commutative Property of Addition **5.**



6. $(-2, 5]$

7. $(-\infty, 5)$



8. $[0, 3]$

9. a. 10 **b.** 12 **c.** -6

10. a. $\sqrt{5} - 2$ **b.** $2 - x$ **11. a.** 6 **b.** 5

Exercises R.1

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. Simplify. $10 - 20$
2. Simplify. $5 - (-10)$
3. Division by what number is undefined?
4. Given $\frac{5}{0}$, $\frac{0}{5}$, and $\frac{0}{0}$. Which one is equivalent to 0?
5. Write the inequality symbol for greater than.
6. Write the math symbol for infinity.

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. A ____ is a collection of objects.
8. If every member of one set B is also a member of a second set A, then B is called a ______ of A.
9. If A and B are two sets, the set that contains all members that are in sets A and B or both is called the ______ of A and B.
10. If A and B are two sets, the set that contains all members that are in both sets is called the ________ of A and B.
11. A real number is any number that can be expressed as a ______ .
12. A ______ is a letter that is used to represent a number.
13. The smallest prime number is ___ .
14. All integers that are exactly divisible by 2 are called _____ integers.
15. Natural numbers greater than 1 that are not prime are called _______ numbers.
16. Fractions such as $\frac{2}{3}$, $\frac{8}{2}$, and $-\frac{7}{9}$ are called ______ numbers.
17. Irrational numbers are _______ that don't terminate and don't repeat.
18. The symbol ____ is read as "is less than or equal to."
19. On a number line, the _______ numbers are to the left of 0.
20. The only integer that is neither positive nor negative is ___ .
21. The Associative Property of Addition states that $(x + y) + z =$ ________ .
22. The Commutative Property of Multiplication states that $xy =$ ____ .
23. Use the Distributive Property to complete the statement: $5(m + 2) =$ ________ .
24. The statement $(m + n)p = p(m + n)$ illustrates the ________ Property of ________ .
25. The graph of an ______ is a portion of a number line.
26. The graph of an open interval has ____ endpoints.
27. The graph of a closed interval has ____ endpoints.
28. The graph of a _______ interval has one endpoint.
29. Except for 0, the absolute value of every number is ______ .
30. The ______ between two distinct points on a number line is always positive.

Let

N = the set of natural numbers
W = the set of whole numbers
Z = the set of integers
Q = the set of rational numbers
R = the set of real numbers

Determine whether each statement is true or false. Read the symbol ⊂ as "is a subset of."

31. N ⊂ W
32. Q ⊂ R
33. Q ⊂ N
34. Z ⊂ Q
35. W ⊂ Z
36. R ⊂ Z

Practice

Let A = {a, b, c, d, e}, B = (d, e, f, g}, and C = {a, c, e, f}. Find each set.

37. $A \cup B$
38. $A \cap B$
39. $A \cap C$
40. $B \cup C$

Determine whether the decimal form of each fraction terminates or repeats.

41. $\frac{7}{16}$
42. $\frac{5}{8}$
43. $\frac{5}{11}$
44. $\frac{7}{12}$

Consider the following set:

$\{-5, -4, -\frac{2}{3}, 0, 1, \sqrt{2}, 2, 2.75, 6, 7\}$

45. Which numbers are natural numbers?

46. Which numbers are whole numbers?

47. Which numbers are integers?

48. Which numbers are rational numbers?

49. Which numbers are irrational numbers?

50. Which numbers are prime numbers?

51. Which numbers are composite numbers?

52. Which numbers are even integers?

53. Which numbers are odd integers?

54. Which numbers are negative numbers?

Graph each subset of the real numbers on a number line.

55. The natural numbers between 1 and 5

56. The composite numbers less than 10

57. The prime numbers between 10 and 20

58. The integers from -2 to 4

59. The integers between -5 and 0

60. The even integers between -9 and -1

61. The odd integers between -6 and 4

62. -0.7, 1.75, and $3\frac{7}{8}$

Write each inequality in interval notation and graph the interval.

63. $x > 2$

64. $x < 4$

65. $0 < x < 5$

66. $-2 < x < 3$

67. $x > -4$

68. $x < 3$

69. $-2 \leq x < 2$

70. $-4 < x \leq 1$

71. $x \leq 5$

72. $x \geq -1$

73. $-5 < x \leq 0$

74. $-3 \leq x < 4$

75. $-2 \leq x \leq 3$

76. $-4 \leq x \leq 4$

77. $6 \geq x \geq 2$

78. $3 \geq x \geq -2$

Write each pair of inequalities as the intersection of two intervals and graph the result.

79. $x > -5$ and $x < 4$

80. $x \geq -3$ and $x < 6$

81. $x \geq -8$ and $x \leq -3$

82. $x > 1$ and $x \leq 7$

Write each inequality as the union of two intervals and graph the result.

83. $x < -2$ or $x > 2$

84. $x \leq -5$ or $x > 0$

85. $x \leq -1$ or $x \geq 3$

86. $x < -3$ or $x \geq 2$

Write each expression without using absolute value symbols.

87. $|13|$

88. $|-17|$

89. $|0|$

90. $-|63|$

91. $-|-8|$

92. $|-25|$

93. $-|32|$

94. $-|-6|$

95. $|\pi - 5|$

96. $|8 - \pi|$

97. $|\pi - \pi|$

98. $|2\pi|$

99. $|x + 1|$ and $x \geq 2$

100. $|x + 1|$ and $x \leq -2$

101. $|x - 4|$ and $x < 0$

102. $|x - 7|$ and $x > 10$

103. $\dfrac{|x - 7|}{x - 7}$ and $x > 7$

104. $\dfrac{x - 8}{|x - 8|}$ and $x < 8$

Find the distance between each pair of points on the number line.

105. 3 and 8

106. -5 and 12

107. -8 and -3

108. 6 and -20

109. -100 and 50

110. -200 and -50

Fix It

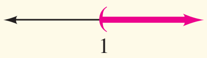
In exercises 111 and 112, identify the error made and fix it.

111. Let $A = \{s,n,i,c,k,e,r,s\}$ and $B = \{t,w,i,x\}$, find $A \cup B$

Solution: $A \cup B = \{i\}$

112. Graph the inequality $x < 1$ on a number line.

Solution:



Applications

113. What subset of the real numbers would you use to describe the populations of Memphis and Miami?

114. What subset of the real numbers would you use to describe the subdivisions of an inch on a ruler?

115. What subset of the real numbers would you use to report temperatures in London and Lisbon?

116. What subset of the real numbers would you use to describe the prices of hoodies at Old Navy?

117. Temperature The average low temperature in International Falls, Minnesota, in January is $-7°F$. The average high temperature is $15°F$. Determine the degrees difference between the average high and the average low.

118. Temperature Harbin, China, is one of the world's coldest cities and known for its ice and snow festivals. In February, the average nightly low temperature is $-20°C$ and the average daily high temperature is $-7°C$. What is the temperature drop from day to night?

The Curious Travelers/Shutterstock.com

Discovery and Writing

119. Explain why $-x$ could be positive.

120. Explain why every integer is a rational number.

121. Is the statement $|ab| = |a| \cdot |b|$ always true? Explain.

122. Is the statement $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$ $(b \neq 0)$ always true? Explain.

123. Is the statement $|a + b| = |a| + |b|$ always true? Explain.

124. Explain why it is incorrect to write $a < b > c$ if $a < b$ and $b > c$.

Critical Thinking

Determine if the statement is true or false. If the statement is false, then correct it and make it true.

125. There are six integers between -3 and 3.

126. $\frac{725}{0}$ is a rational number because 725 and 0 are integers.

127. ∞ is a real number.

128. $|a - b| = |b - a|$

129. $\varnothing \subset \left\{5, \pi, \sqrt{3}, \dfrac{13}{4}\right\}$

130. $\varnothing \subset \varnothing$

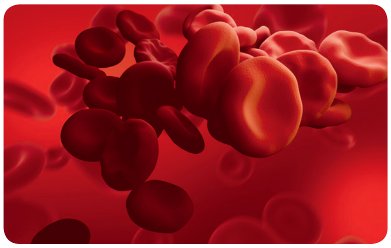
131. There are six subsets of $\{11, 22, 33\}$.

132. A set is always a subset of itself.

R.2 Integer Exponents and Scientific Notation

In this section, we will learn to

1. Simplify expressions with natural-number exponents.
2. Apply the rules of exponents.
3. Apply the rules for order of operations to evaluate expressions.
4. Express numbers in scientific notation.
5. Use scientific notation to simplify computations.



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The number of cells in the human body is approximated to be one hundred trillion or 100,000,000,000,000. One hundred trillion is $(10)(10)(10) \cdots (10)$, where ten occurs fourteen times. Fourteen factors of ten can be written as 10^{14}.

In this section, we will use integer exponents to represent repeated multiplication of numbers.

1. Simplify Expressions with Natural-Number Exponents

When two or more quantities are multiplied together, each quantity is called a *factor* of the product. The exponential expression x^4 indicates that x is to be used as a factor four times.

$$x^4 = \overbrace{x \cdot x \cdot x \cdot x}^{\text{4 factors of } x}$$

In general, the following is true.

Natural-Number Exponents

For any natural number n,

$$x^n = \overbrace{x \cdot x \cdot x \cdot \cdots \cdot x}^{n \text{ factors of } x}$$

In the **exponential expression** x^n, x is called the **base**, and n is called the **exponent** or the **power** to which the base is raised. The expression x^n is called a **power of** $\boldsymbol{x}$. From the definition, we see that a natural-number exponent indicates how many times the base of an exponential expression is to be used as a factor in a product. If an exponent is 1, the 1 is usually not written:

$$x^1 = x$$

Example 1 **Using the Definition of Natural-Number Exponents**

Write each expression without using exponents:

a. 4^2 **b.** $(-4)^2$ **c.** -5^3 **d.** $(-5)^3$ **e.** $3x^4$ **f.** $(3x)^4$

Strategy In each case, we apply the definition of natural-number exponents.

Solution

a. $4^2 = 4 \cdot 4 = 16$ Read 4^2 as "four squared."

b. $(-4)^2 = (-4)(-4) = 16$ Read $(-4)^2$ as "negative four squared."

c. $-5^3 = -5(5)(5) = -125$ Read -5^3 as "the negative of five cubed."

d. $(-5)^3 = (-5)(-5)(-5) = -125$ Read $(-5)^3$ as "negative five cubed."

e. $3x^4 = 3 \cdot x \cdot x \cdot x \cdot x$ Read $3x^4$ as "3 times x to the fourth power."

f. $(3x)^4 = (3x)(3x)(3x)(3x) = 81 \cdot x \cdot x \cdot x \cdot x$ Read $(3x)^4$ as "$3x$ to the fourth power."

Take Note

To find powers on a scientific calculator use the $\boxed{y^x}$ key.

Self Check 1 Write each expression without using exponents:

a. 7^3 **b.** $(-3)^2$ **c.** $5a^3$ **d.** $(5a)^4$

Now Try Exercise 19.

Take Note

Note the distinction between ax^n and $(ax)^n$:

$$ax^n = a \cdot \overbrace{x \cdot x \cdot x \cdot \cdots \cdot x}^{n \text{ factors of } x} \qquad (ax)^n = \overbrace{(ax)(ax)(ax) \cdot \cdots \cdot (ax)}^{n \text{ factors of } ax}$$

Also note the distinction between $-x^n$ and $(-x)^n$:

$$-x^n = -\overbrace{(x \cdot x \cdot x \cdot \cdots \cdot x)}^{n \text{ factors of } x} \qquad (-x)^n = \overbrace{(-x)(-x)(-x) \cdot \cdots \cdot (-x)}^{n \text{ factors of } -x}$$

2. Apply the Rules of Exponents

We begin the review of the rules of exponents by considering the product $x^m x^n$. Since x^m indicates that x is to be used as a factor m times, and since x^n indicates that x is to be used as a factor n times, there are $m + n$ factors of x in the product $x^m x^n$.

$$x^m x^n = \overbrace{\overbrace{x \cdot x \cdot x \cdot \cdots \cdot x}^{m \text{ factors of } x} \cdot \overbrace{x \cdot x \cdot x \cdot \cdots \cdot x}^{n \text{ factors of } x}}^{m + n \text{ factors of } x} = x^{m+n}$$

This suggests that to multiply exponential expressions with the same base, we *keep the base and add the exponents*.

Product Rule for Exponents If m and n are natural numbers, then

$$x^m x^n = x^{m+n}$$

Take Note

The Product Rule applies to exponential expressions with the same base. A product of two powers with different bases, such as $x^4 y^3$, cannot be simplified.

To find another property of exponents, we consider the exponential expression $(x^m)^n$. In this expression, the exponent n indicates that x^m is to be used as a factor n times. This implies that x is to be used as a factor mn times.

$$(x^m)^n = \overbrace{\overbrace{(x^m)(x^m)(x^m) \cdot \cdots \cdot (x^m)}^{n \text{ factors of } x^m}}^{mn \text{ factors of } x} = x^{mn}$$

This suggests that to raise an exponential expression to a power, we *keep the base and multiply the exponents*.

To raise a product to a power, we raise each factor to that power.

$$(xy)^n = \overbrace{(xy)(xy)(xy) \cdot \cdots \cdot (xy)}^{n \text{ factors of } xy} = \overbrace{(x \cdot x \cdot x \cdot \cdots \cdot x)}^{n \text{ factors of } x}\overbrace{(y \cdot y \cdot y \cdot \cdots \cdot y)}^{n \text{ factors of } y} = x^n y^n$$

To raise a fraction to a power, we raise both the numerator and the denominator to that power. If $y \neq 0$, then

$$\left(\frac{x}{y}\right)^n = \overbrace{\left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) \cdot \cdots \cdot \left(\frac{x}{y}\right)}^{n \text{ factors of } \frac{x}{y}}$$

$$= \frac{\overbrace{xxx \cdot \cdots \cdot x}^{n \text{ factors of } x}}{\underbrace{yyy \cdot \cdots \cdot y}_{n \text{ factors of } y}}$$

$$= \frac{x^n}{y^n}$$

The previous three results are called the *Power Rules of Exponents*.

Power Rules of Exponents If m and n are natural numbers, then

$$(x^m)^n = x^{mn} \qquad (xy)^n = x^n y^n \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \quad (y \neq 0)$$

Example 2 Using Exponent Rules to Simplify Expressions with Natural-Number Exponents

Simplify: **a.** x^5x^7 **b.** $x^2y^3x^5y$ **c.** $(x^4)^9$ **d.** $(x^2x^5)^3$

e. $\left(\frac{x}{y^2}\right)^5$ **f.** $\left(\frac{5x^2y}{z^3}\right)^2$

Strategy In each case, we will apply the appropriate rule of exponents.

Solution

a. $x^5x^7 = x^{5+7} = x^{12}$ **b.** $x^2y^3x^5y = x^{2+5}y^{3+1} = x^7y^4$

c. $(x^4)^9 = x^{4\cdot 9} = x^{36}$ **d.** $(x^2x^5)^3 = (x^7)^3 = x^{21}$

e. $\left(\frac{x}{y^2}\right)^5 = \frac{x^5}{(y^2)^5} = \frac{x^5}{y^{10}} \quad (y \neq 0)$

f. $\left(\frac{5x^2y}{z^3}\right)^2 = \frac{5^2(x^2)^2y^2}{(z^3)^2} = \frac{25x^4y^2}{z^6} \quad (z \neq 0)$

Self Check 2 Simplify:

a. $(y^3)^2$ **b.** $(a^2a^4)^3$ **c.** $(x^2)^3(x^3)^2$ **d.** $\left(\frac{3a^3b^2}{c^3}\right)^3 \quad (c \neq 0)$

Now Try Exercise 49.

If we assume that the rules for natural-number exponents hold for exponents of 0, we can write

$$x^0x^n = x^{0+n} = x^n = 1x^n$$

Since $x^0x^n = 1x^n$, it follows that if $x \neq 0$, then $x^0 = 1$.

Zero Exponent $x^0 = 1 \quad (x \neq 0)$

Take Note

0 raised to the power of 0 is not defined.

If we assume that the rules for natural-number exponents hold for exponents that are negative integers, we can write

$$x^{-n}x^n = x^{-n+n} = x^0 = 1 \quad (x \neq 0)$$

However, we know that

$$\frac{1}{x^n} \cdot x^n = 1 \quad (x \neq 0)$$

$\frac{1}{x^n} \cdot x^n = \frac{x^n}{x^n}$, and any nonzero number divided by itself is 1.

Since $x^{-n}x^n = \frac{1}{x^n} \cdot x^n$, it follows that $x^{-n} = \frac{1}{x^n}$ $(x \neq 0)$.

Negative Exponents If n is an integer and $x \neq 0$, then

$$x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \frac{1}{x^{-n}} = x^n$$

Because of the previous definitions, all of the rules for natural-number exponents will hold for integer exponents.

Example 3 Simplifying Expressions with Integer Exponents

Simplify and write all answers without using negative exponents:

a. $(3x)^0$ **b.** $3(x^0)$ **c.** x^{-4} **d.** $\frac{1}{x^{-6}}$ **e.** $x^{-3}x$ **f.** $(x^{-4}x^8)^{-5}$

Strategy We will use the definitions of zero exponent and negative exponents to simplify each expression.

Caution

A common mistake is to give an answer of 0 when a number is raised to the power of 0. This is incorrect! Avoid making this error.

$17^0 \neq 0, \left(-\frac{3}{5}\right)^0 \neq 0, \text{ and } (8x)^0 \neq 0$

These expressions equal 1 and not 0.

Solution

a. $(3x)^0 = 1$ **b.** $3(x^0) = 3(1) = 3$ **c.** $x^{-4} = \frac{1}{x^4}$ **d.** $\frac{1}{x^{-6}} = x^6$

e. $x^{-3}x = x^{-3+1} = x^{-2} = \frac{1}{x^2}$

f. $(x^{-4}x^8)^{-5} = (x^4)^{-5} = x^{-20} = \frac{1}{x^{20}}$

Self Check 3 Simplify and write all answers without using negative exponents:

a. $7a^0$ **b.** $3a^{-2}$ **c.** $a^{-4}a^2$ **d.** $(a^3a^{-7})^3$

Now Try Exercise 59.

To develop the Quotient Rule for Exponents, we proceed as follows:

$$\frac{x^m}{x^n} = x^m\left(\frac{1}{x^n}\right) = x^mx^{-n} = x^{m+(-n)} = x^{m-n} \quad (x \neq 0)$$

This suggests that to divide two exponential expressions with the same nonzero base, we *keep the base and subtract the exponent in the denominator from the exponent in the numerator*.

Quotient Rule for Exponents If m and n are integers, then

$$\frac{x^m}{x^n} = x^{m-n} \quad (x \neq 0)$$

Example 4 Simplifying Expressions with Integer Exponents

Simplify and write all answers without using negative exponents:

a. $\frac{x^8}{x^5}$ **b.** $\frac{x^2x^4}{x^{-5}}$

Strategy We will apply the Product and Quotient Rules of Exponents.

Solution

a. $\frac{x^8}{x^5} = x^{8-5} = x^3$

b. $\frac{x^2x^4}{x^{-5}} = \frac{x^6}{x^{-5}} = x^{6-(-5)} = x^{11}$

Self Check 4 Simplify and write all answers without using negative exponents:

a. $\frac{x^{-6}}{x^2}$ **b.** $\frac{x^4x^{-3}}{x^2}$

Now Try Exercise 69.

Example 5 Simplifying Expressions with Integer Exponents

Simplify and write all answers without using negative exponents:

a. $\left(\dfrac{x^3y^{-2}}{x^{-2}y^3}\right)^{-2}$ **b.** $\left(\dfrac{x}{y}\right)^{-n}$

Strategy We will apply the appropriate rules of exponents.

Solution **a.**

$$\left(\frac{x^3y^{-2}}{x^{-2}y^3}\right)^{-2} = (x^{3-(-2)}y^{-2-3})^{-2}$$
$$= (x^5y^{-5})^{-2}$$
$$= x^{-10}y^{10}$$
$$= \frac{y^{10}}{x^{10}}$$

b.

$$\left(\frac{x}{y}\right)^{-n} = \frac{x^{-n}}{y^{-n}}$$
$$= \frac{x^{-n}x^ny^n}{y^{-n}x^ny^n}$$ Multiply numerator and denominator by 1 in the form $\frac{x^ny^n}{x^ny^n}$
$$= \frac{x^0y^n}{y^0x^n}$$ $x^{-n}x^n = x^0$ and $y^{-n}y^n = y^0$
$$= \frac{y^n}{x^n}$$ $x^0 = 1$ and $y^0 = 1$
$$= \left(\frac{y}{x}\right)^n$$

Self Check 5 Simplify and write all answers without using negative exponents:

a. $\left(\dfrac{x^4y^{-3}}{x^{-3}y^2}\right)^2$ **b.** $\left(\dfrac{2a}{3b}\right)^{-3}$

Now Try Exercise 75.

Part b of Example 5 establishes the following rule.

A Fraction to a Negative Power If n is a natural number, then

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n \qquad (x \neq 0 \quad \text{and} \quad y \neq 0)$$

Tip

Learning and applying several exponent rules at one time can often be confusing. This a common feeling experienced. Some students admit to being intimidated by them because the formal statements of the rules involve x's, y's, m's, and n's. It can be very helpful to rewrite the rules in your own words. When you do, your understanding will be clearer. Here are some examples.

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- When I multiply exponential expressions with the same bases, then I keep the common base and add the exponents.
- When I raise a power to a power, then I multiply the exponents.
- When I raise a product to a power, then I raise each factor to the power.
- When I raise a quotient to a power, then I raise both the numerator and the denominator to the power.
- When I divide exponential expressions with the same bases, then I keep the common base and subtract the exponents.
- When I raise an exponential expression to a negative power, I can invert the exponential expression and raise it to a positive power.

3. Apply the Rules for Order of Operations to Evaluate Expressions

When several operations occur in an expression, we must perform the operations in the following order to get the correct result.

Strategy for Evaluating Expressions Using Order of Operations

If an expression does not contain grouping symbols such as parentheses or brackets, follow these steps:

1. Find the values of any exponential expressions.
2. Perform all multiplications and/or divisions, working from left to right.
3. Perform all additions and/or subtractions, working from left to right.

- If an expression contains grouping symbols such as parentheses, brackets, or braces, use the rules above to perform the calculations within each pair of grouping symbols, working from the innermost pair to the outermost pair.
- In a fraction, simplify the numerator and the denominator of the fraction separately. Then simplify the fraction, if possible.

Take Note

Many students remember the Order of Operations Rule with the acronym **PEMDAS:**

- **P**arentheses
- **E**xponents
- **M**ultiplication
- **D**ivision
- **A**ddition
- **S**ubtraction

For example, to simplify $\frac{3[4-(6+10)]}{2^2-(6+7)}$, we proceed as follows:

$$\frac{3[4-(6+10)]}{2^2-(6+7)} = \frac{3(4-16)}{2^2-(6+7)}$$ Simplify within the inner parentheses: $6+10=16$.

$$= \frac{3(-12)}{2^2-13}$$ Simplify within each parentheses: $4-16=-12$ and $6+7=13$.

$$= \frac{3(-12)}{4-13}$$ Evaluate the power: $2^2=4$.

$$= \frac{-36}{-9}$$ $3(-12)=-36$; $4-13=-9$

$$= 4$$

Example 6 **Evaluating Algebraic Expressions**

If $x = -2$, $y = 3$, and $z = -4$, evaluate

a. $-x^2 + y^2z$ **b.** $\dfrac{2z^3 - 3y^2}{5x^2}$

Strategy In each part, we will substitute the numbers for the variables, apply the rules of order of operations, and simplify.

Solution **a.**

$$\begin{aligned} -x^2 + y^2z &= -(-2)^2 + 3^2(-4) \\ &= -(4) + 9(-4) && \text{Evaluate the powers.} \\ &= -4 + (-36) && \text{Do the multiplication.} \\ &= -40 && \text{Do the addition.} \end{aligned}$$

b.

$$\begin{aligned} \frac{2z^3 - 3y^2}{5x^2} &= \frac{2(-4)^3 - 3(3)^2}{5(-2)^2} \\ &= \frac{2(-64) - 3(9)}{5(4)} && \text{Evaluate the powers.} \\ &= \frac{-128 - 27}{20} && \text{Do the multiplications.} \\ &= \frac{-155}{20} && \text{Do the subtraction.} \\ &= -\frac{31}{4} && \text{Simplify the fraction.} \end{aligned}$$

Self Check 6 If $x = 3$ and $y = -2$, evaluate $\dfrac{2x^2 - 3y^2}{x - y}$.

Now Try Exercise 91.

4. Express Numbers in Scientific Notation

Scientists often work with numbers that are very large or very small. These numbers can be written compactly by expressing them in *scientific notation*.

Scientific Notation A number is written in *scientific notation* when it is written in the form

$$N \times 10^n$$

where $1 \le |N| < 10$ and n is an integer.

Fztommy/Shutterstock.com

Light travels 29,980,000,000 centimeters per second. To express this number in scientific notation, we must write it as the product of a number between 1 and 10 and some integer power of 10. The number 2.998 lies between 1 and 10. To get 29,980,000,000, the decimal point in 2.998 must be moved ten places to the right. This is accomplished by multiplying 2.998 by 10^{10}.

Standard notation ⟶ $29{,}980{,}000{,}000 = 2.998 \times 10^{10}$ ⟵ Scientific notation

One meter is approximately 0.0006214 mile. To express this number in scientific notation, we must write it as the product of a number between 1 and 10 and some integer power of 10. The number 6.214 lies between 1 and 10. To get 0.0006214, the decimal point in 6.214 must be moved four places to the left. This is accomplished by multiplying 6.214 by $\frac{1}{10^4}$ or by multiplying 6.214 by 10^{-4}.

Standard notation ⟶ $0.0006214 = 6.214 \times 10^{-4}$ ⟵ Scientific notation

To write each of the following numbers in scientific notation, we start to the right of the first nonzero digit and count to the decimal point. The exponent gives the number of places the decimal point moves, and the sign of the exponent indicates the direction in which it moves.

a. $372000 = 3.72 \times 10^5$ 5 places to the right

b. $0.000537 = 5.37 \times 10^{-4}$ 4 places to the left

c. $7.36 = 7.36 \times 10^0$ No movement of the decimal point

Example 7 Writing Numbers in Scientific Notation

Write each number in scientific notation: **a.** 62,000 **b.** -0.0027

Strategy **a.** We must express 62,000 as a product of a number between 1 and 10 and some integer power of 10. This is accomplished by multiplying 6.2 by 10^4.

Solution $62{,}000 = 6.2 \times 10^4$

Strategy **b.** We must express -0.0027 as a product of a number whose absolute value is between 1 and 10 and some integer power of 10. This is accomplished by multiplying -2.7 by 10^{-3}.

Solution $-0.0027 = -2.7 \times 10^{-3}$

Self Check 7 Write each number in scientific notation:

a. $-93{,}000{,}000$ **b.** 0.0000087

Now Try Exercise 103.

Example 8 Writing Numbers in Standard Notation

Write each number in standard notation:

a. 7.35×10^2 **b.** 3.27×10^{-5}

Strategy **a.** The factor of 10^2 indicates that 7.35 must be multiplied by 2 factors of 10. Because each multiplication by 10 moves the decimal point one place to the right, we are required to move the decimal point two places to the right.

Solution $7.35 \times 10^2 = 735$

Strategy **b.** The factor of 10^{-5} indicates that 3.27 must be divided by 5 factors of 10. Because each division by 10 moves the decimal point one place to the left, we are required to move the decimal point five places to the left.

Solution $3.27 \times 10^{-5} = 0.0000327$

Self Check 8 Write each number in standard notation:

a. 6.3×10^3 **b.** 9.1×10^{-4}

Now Try Exercise 111.

5. Use Scientific Notation to Simplify Computations

Another advantage of scientific notation becomes evident when we multiply and divide very large and very small numbers.

Example 9 **Using Scientific Notation to Simplify Computations**

Use scientific notation to calculate $\dfrac{(3{,}400{,}000)(0.00002)}{170{,}000{,}000}$.

Strategy After changing each number to scientific notation, we can do the arithmetic on the numbers and the exponential expressions separately.

Solution

$$\frac{(3{,}400{,}000)(0.00002)}{170{,}000{,}000} = \frac{(3.4 \times 10^{6})(2.0 \times 10^{-5})}{1.7 \times 10^{8}}$$

$$= \frac{6.8}{1.7} \times 10^{6+(-5)-8}$$

$$= 4.0 \times 10^{-7}$$

$$= 0.0000004$$

Self Check 9 Use scientific notation to simplify $\dfrac{(192{,}000)(0.0015)}{(0.0032)(4500)}$.

Now Try Exercise 119.

Take Note

To calculate an expression like $\dfrac{21^{8}}{0.000000000061}$ on a scientific calculator, it is necessary to convert the denominator to scientific notation because the number has too many digits to fit on the screen.

- For scientific notation, we enter these numbers and press these keys: 6.1 [EXP] 11 [+/−].
- To evaluate the expression above, we enter these numbers and press these keys:

  21 [y^x] 8 [=] [÷] 6.1 [EXP] 11 [+/−] [=]

The display will read [6.200468748^{20}]. In standard notation, the answer is approximately 620,046,874,800,000,000,000.

Self Check Answers

1. a. $7 \cdot 7 \cdot 7 = 343$ **b.** $(-3)(-3) = 9$ **c.** $5 \cdot a \cdot a \cdot a$ **d.** $(5a)(5a)(5a)(5a) = 625 \cdot a \cdot a \cdot a \cdot a$ **2. a.** y^6 **b.** a^{18} **c.** x^{12} **d.** $\dfrac{27a^9b^6}{c^9}$ **3. a.** 7 **b.** $\dfrac{3}{a^2}$ **c.** $\dfrac{1}{a^2}$ **d.** $\dfrac{1}{a^{12}}$ **4. a.** $\dfrac{1}{x^8}$ **b.** $\dfrac{1}{x}$ **5. a.** $\dfrac{x^{14}}{y^{10}}$ **b.** $\dfrac{27b^3}{8a^3}$ **6.** $\dfrac{6}{5}$ **7. a.** -9.3×10^7 **b.** 8.7×10^{-6} **8. a.** 6300 **b.** 0.00091 **9.** 20

Exercises R.2

Getting Ready

Complete these just-in-time review problems to prepare you to successfully work the practice exercises.

1. Match each expression with the proper description given below.

   $\frac{x^7}{x^2}$ $(x^3y^4)^5$ $(x^2)^7$ $\left(\frac{x^4}{x^2}\right)^5$ $x^3 . x^8$

   a. Product of exponential expressions with the same base

   b. Quotient of exponential expressions with the same base

   c. Power of an exponential expression

   d. Power of a product

   e. Power of a quotient

2. Simplify the expression. $x \cdot x \cdot x \cdot x \cdot x \cdot x$
3. Simplify the expression. $\frac{y \cdot y \cdot y \cdot y \cdot y \cdot y}{y \cdot y \cdot y}$
4. Fill in the box. $(x^2)^3 = (x^2)(x^2)(x^2) = x^{\square}$
5. If $x = -5$ what is $x^3 + x$?
6. These look alike. Match each with its correct simplification $x^5 \cdot x^5$ $(x^5)^5$ $x^5 + x^5$

   a. $2x^5$

   b. x^{10}

   c. x^{25}

Vocabulary and Concepts

You should be able to complete these vocabulary and concept statements before you proceed to the practice exercises.

Fill in the blanks.

7. Each quantity in a product is called a _______ of the product.
8. A ________ number exponent tells how many times a base is used as a factor.
9. In the expression $(2x)^3$, ____ is the exponent and ____ is the base.
10. The expression x^n is called an __________ expression.
11. A number is in _________ notation when it is written in the form $N \times 10^n$, where $1 \leq |N| < 10$ and n is an _______.
12. Unless __________ indicate otherwise, _____________ are performed before additions.

Complete each exponent rule. Assume $x \neq 0$.

13. $x^m x^n =$ ____
14. $(x^m)^n =$ ____
15. $(xy)^n =$ ____
16. $\frac{x^m}{x^n} =$ ____
17. $x^0 =$ _
18. $x^{-n} =$ ____

Practice

Write each number or expression without using exponents.

19. 13^2
20. 10^3
21. -5^2
22. $(-5)^2$
23. $4x^3$
24. $(4x)^3$
25. $(-5x)^4$
26. $-6x^2$
27. $-8x^4$
28. $(-8x)^4$

Write each expression using exponents.

29. $7xxx$
30. $-8yyyy$
31. $(-x)(-x)$
32. $(2a)(2a)(2a)$
33. $(3t)(3t)(-3t)$
34. $-(2b)(2b)(2b)(2b)$
35. $xxxyy$
36. $aaabbbb$

Use a calculator to simplify each expression.

37. 2.2^3
38. 7.1^4
39. -0.5^4
40. $(-0.2)^4$

Simplify each expression. Write all answers without using negative exponents. Assume that all variables are restricted to those numbers for which the expression is defined.

41. x^2x^3
42. y^3y^4
43. $(z^2)^3$
44. $(t^6)^7$
45. $(y^5y^2)^3$
46. $(a^3a^6)a^4$
47. $(z^2)^3(z^4)^5$
48. $(t^3)^4(t^5)^2$
49. $(a^2)^3(a^4)^2$
50. $(a^2)^4(a^3)^3$
51. $(3x)^3$
52. $(-2y)^4$
53. $(x^2y)^3$
54. $(x^3z^4)^6$
55. $\left(\frac{a^2}{b}\right)^3$
56. $\left(\frac{x}{y^3}\right)^4$
57. $(-x)^0$
58. $4x^0$

59. $(4x)^0$ **60.** $-2x^0$

61. z^{-4} **62.** $\dfrac{1}{t^{-2}}$

63. $y^{-2}y^{-3}$ **64.** $-m^{-2}m^3$

65. $(x^3x^{-4})^{-2}$ **66.** $(y^{-2}y^3)^{-4}$

67. $\dfrac{x^7}{x^3}$ **68.** $\dfrac{r^5}{r^2}$

69. $\dfrac{a^{21}}{a^{17}}$ **70.** $\dfrac{t^{13}}{t^4}$

71. $\dfrac{(x^2)^2}{x^2x}$ **72.** $\dfrac{s^9s^3}{(s^2)^2}$

73. $\left(\dfrac{m^3}{n^2}\right)^3$ **74.** $\left(\dfrac{t^4}{t^3}\right)^3$

75. $\dfrac{(a^3)^{-2}}{aa^2}$ **76.** $\dfrac{r^9r^{-3}}{(r^{-2})^3}$

77. $\left(\dfrac{a^{-3}}{b^{-1}}\right)^{-4}$ **78.** $\left(\dfrac{t^{-4}}{t^{-3}}\right)^{-2}$

79. $\left(\dfrac{r^4r^{-6}}{r^3r^{-3}}\right)^2$ **80.** $\dfrac{(x^{-3}x^2)^2}{(x^2x^{-5})^{-3}}$

81. $\left(\dfrac{x^5y^{-2}}{x^{-3}y^2}\right)^4$ **82.** $\left(\dfrac{x^{-7}y^5}{x^7y^{-4}}\right)^3$

83. $\left(\dfrac{5x^{-3}y^{-2}}{3x^2y^{-3}}\right)^{-2}$ **84.** $\left(\dfrac{3x^2y^{-5}}{2x^{-2}y^{-6}}\right)^{-3}$

85. $\left(\dfrac{3x^5y^{-3}}{6x^{-5}y^3}\right)^{-2}$ **86.** $\left(\dfrac{12x^{-4}y^3z^{-5}}{4x^4y^{-3}z^5}\right)^3$

87. $\dfrac{(8^{-2}z^{-3}y)^{-1}}{(5y^2z^{-2})^3(5yz^{-2})^{-1}}$

88. $\dfrac{(m^{-2}n^3p^4)^{-2}(mn^{-2}p^3)^4}{(mn^{-2}p^3)^{-4}(mn^2p)^{-1}}$

Simplify each expression.

89. $-\dfrac{5[6^2 + (9-5)]}{4(2-3)^2}$ **90.** $\dfrac{6[3-(4-7)^2]}{-5(2-4^2)}$

Let $x = -2$, $y = 0$, and $z = 3$ and evaluate each expression.

91. x^2 **92.** $-x^2$

93. x^3 **94.** $-x^3$

95. $(-xz)^3$ **96.** $-xz^3$

97. $\dfrac{-(x^2z^3)}{z^2-y^2}$ **98.** $\dfrac{z^2(x^2-y^2)}{x^3z}$

99. $5x^2 - 3y^3z$ **100.** $3(x-z)^2 + 2(y-z)^3$

101. $\dfrac{-3x^{-3}z^{-2}}{6x^2z^{-3}}$ **102.** $\dfrac{(-5x^2z^{-3})^2}{5xz^{-2}}$

Express each number in scientific notation.

103. 372,000 **104.** 89,500

105. −177,000,000 **106.** −23,470,000,000

107. 0.007 **108.** 0.00052

109. −0.000000693 **110.** −0.000000089

111. one trillion **112.** one millionth

Express each number in standard notation.

113. 9.37×10^5 **114.** 4.26×10^9

115. 2.21×10^{-5} **116.** 2.774×10^{-2}

117. 0.00032×10^4 **118.** $9{,}300.0 \times 10^{-4}$

119. -3.2×10^{-3} **120.** -7.25×10^3

Use the method of Example 9 to do each calculation. Write all answers in scientific notation.

121. $\dfrac{(65{,}000)(45{,}000)}{250{,}000}$

122. $\dfrac{(0.000000045)(0.00000012)}{45{,}000{,}000}$

123. $\dfrac{(0.00000035)(170{,}000)}{0.00000085}$

124. $\dfrac{(0.0000000144)(12{,}000)}{600{,}000}$

125. $\dfrac{(45{,}000{,}000{,}000)(212{,}000)}{0.00018}$

126. $\dfrac{(0.00000000275)(4750)}{500{,}000{,}000{,}000}$

Fix It

In exercises 127, identify the step the first error is made and fix it.

127. Use the properties of exponents to simplify the expression $\left(\dfrac{3x^3y^{-2}}{x^2y}\right)^{-3}$.

Solution:

Step 1: $\left(\dfrac{x^2y}{3x^3y^{-2}}\right)^3$

Step 2: $\dfrac{(x^2y)^3}{(3x^3y^{-2})^3}$

Step 3: $\dfrac{x^6y^3}{3x^9y^{-6}}$

Step 4: $\dfrac{y^9}{3x^3}$

In exercises 128, identify the error made and fix it.

128. 123,456,789 written in scientific notation is $1.23456789 \times 10^{-8}$.

Applications

Use scientific notation to compute each answer. Write all answers in scientific notation.

129. Speed of sound The speed of sound in air is 3.31×10^4 centimeters per second. Compute the speed of sound in meters per minute.

130. Volume of a box Calculate the volume of a box that has dimensions of 6000 by 9700 by 4700 millimeters.

131. Mass of a proton The mass of one proton is 0.00000000000000000000000167248 gram. Find the mass of one billion protons.

132. Speed of light The speed of light in a vacuum is approximately 30,000,000,000 centimeters per second. Find the speed of light in miles per hour. (160,934.4 cm = 1 mile.)

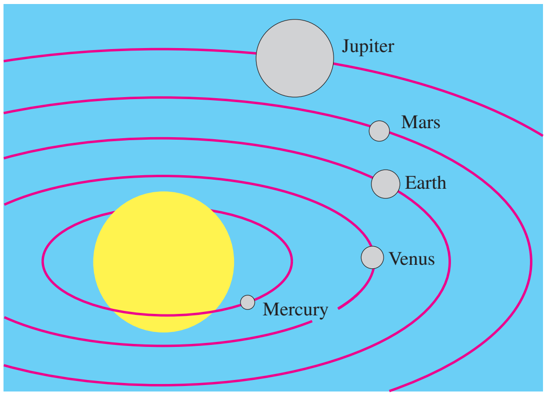
133. License plates License plates come in various forms. The number of different license plates of the form three digits followed by three letters, is $10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26$. Write this expression using exponents. Then evaluate it and express the result in scientific notation.

Leonard Zhukovsky/Shutterstock.com

134. Astronomy The distance d, in miles, of the nth planet from the sun is given by the formula

$$d = 9{,}275{,}200[3(2^{n-2}) + 4]$$

To the nearest million miles, find the distance of Earth and the distance of Mars from the sun. Give each answer in scientific notation.



135. New way to the center of the Earth The spectacular "blue marble" image is the most detailed true-color image of the entire Earth to date. A new NASA-developed technique estimates Earth's center of mass within 1 millimeter (0.04 inch) a year by using a combination of four space-based techniques.

NASA Images

The distance from the Earth's center to the North Pole (the **polar radius**) measures approximately 6356.750 km, and the distance from the center to the equator (the **equatorial radius**) measures approximately 6378.135 km. Express each distance using scientific notation.

136. Refer to Exercise 127. Given that 1 km is approximately equal to 0.62 miles, use scientific notation to express each distance in miles.

Discovery and Writing

Write each expression with a single base.

137. $x^n x^2$

138. $\dfrac{x^m}{x^3}$

139. $\dfrac{x^m x^2}{x^3}$

140. $\dfrac{x^{3m+5}}{x^2}$

141. $x^{m+1}x^3$

142. $a^{n-3}a^3$

143. Explain why $-x^4$ and $(-x)^4$ represent different numbers.

144. Explain why $-x^{55}$ and $(-x)^{55}$ represent equal numbers.

145. Explain how to write a number in scientific notation.

146. Explain why 32×10^2 is not in scientific notation.

147. Explain why $x^{11} \cdot x^{11} \neq x^{121}$.

148. Explain why $11^2 \cdot 11^3 \neq 121^5$.

149. Explain why $\frac{y^{50}}{y^{10}} \neq y^5$.

150. Explain why $(6xyz)^6 \neq 6x^6y^6z^6$.

Critical Thinking

In Exercises 143–148, determine if the statement is true or false. If the statement is false, then correct it and make it true.

151. $0^0 = 1$

152. $\pi^0 = 1$

153. $x^{-n} = -\frac{1}{x^n}$

154. $(x + y)^{-n} = \frac{1}{x^n} + \frac{1}{y^n}$

155. $2^{-1} > 2^{-2}$

156. $(-2)^{-1} < (-2)^{-2}$

157. Young adults between the ages of 18 and 24 send an average of 110 text messages per day. If there are approximately 31.5 million young adults in the USA in this age group, how many text messages are sent in one year? Write the answer using scientific notation.

158. Health authorities recommend that we drink eight 8-ounce glasses of water each day. How many glasses of water would you drink over a lifetime of 80 years? Write the answer using scientific notation.

R.3 Rational Exponents and Radicals

In this section, we will learn to

1. Simplify expression of the form $a^{1/n}$.
2. Simplify expressions of the form $a^{m/n}$.
3. Use radical notation and determine *n*th roots.
4. Simplify and combine radicals.
5. Rationalize denominators and numerators.

Love Silhouette/Shutterstock.com

"Dead Man's Curve" is a 1964 hit song by the rock and roll duo Jan Berry and Dean Torrence. The song details a teenage drag race that ends in an accident. Today, dead man's curve is a commonly used expression given to dangerous curves on our roads. Every curve has a "critical speed." If we exceed this speed, regardless of how skilled a driver we are, we will lose control of the vehicle.

The radical expression $3.9\sqrt{r}$ gives the critical speed in miles per hour when we travel a curved road with a radius of r feet. A knowledge of square roots and radicals is important and used in the construction of safe highways and roads. We will study the topic of radicals in this section.

1. Simplify Expressions of the Form $a^{1/n}$

If we apply the rule $(x^m)^n = x^{mn}$ to $(25^{1/2})^2$, we obtain

$$(25^{1/2})^2 = 25^{(1/2)2} \quad \text{Keep the base and multiply the exponents.}$$
$$= 25^1 \quad \tfrac{1}{2} \cdot 2 = 1$$
$$= 25$$

Thus, $25^{1/2}$ is a real number whose square is 25. Although both $5^2 = 25$ and $(-5)^2 = 25$, we define $25^{1/2}$ to be the positive real number whose square is 25:

$$25^{1/2} = 5 \quad \text{Read } 25^{1/2} \text{ as "the square root of 25."}$$

In general, we have the following definition.

Take Note

In the expression $a^{1/n}$, there is **no** real-number *n*th root of *a* when *n* is even and $a < 0$. For example, $(-64)^{1/2}$ is **not** a real number, because the square of **no** real number is -64.

Rational Exponents If $a \geq 0$ and n is a natural number, then $a^{1/n}$ (read as "the nth root of a") is the nonnegative real number b such that

$$b^n = a$$

Since $b = a^{1/n}$, we have $b^n = (a^{1/n})^n = a$.

Example 1 **Simplifying Expressions of the Form $a^{1/n}$**

Simplify each expression.

Strategy In each case we will apply the definition of $a^{1/n}$.

Solution

a. $16^{1/2} = 4$ — Because $4^2 = 16$. Read $16^{1/2}$ as "the square root of 16."

b. $27^{1/3} = 3$ — Because $3^3 = 27$. Read $27^{1/3}$ as "the cube root of 27."

c. $\left(\frac{1}{81}\right)^{1/4} = \frac{1}{3}$ — Because $\left(\frac{1}{3}\right)^4 = \frac{1}{81}$. Read $\left(\frac{1}{81}\right)^{1/4}$ as "the fourth root of $\frac{1}{81}$."

d. $-32^{1/5} = -(32^{1/5})$ — Read $32^{1/5}$ as "the fifth root of 32."

$= -(2)$ — Because $2^5 = 32$.

$= -2$

Self Check 1 Simplify: **a.** $100^{1/2}$ **b.** $243^{1/5}$

Now Try Exercise 21.

If n is even in the expression $a^{1/n}$ and the base contains variables, we often use absolute value symbols to guarantee that an even root is nonnegative.

$(49x^2)^{1/2} = 7|x|$ — Because $(7|x|)^2 = 49x^2$. Since x could be negative, absolute value symbols are necessary to guarantee that the square root is nonnegative.

$(16x^4)^{1/4} = 2|x|$ — Because $(2|x|)^4 = 16x^4$. Since x could be negative, absolute value symbols are necessary to guarantee that the fourth root is nonnegative.

$(729x^{12})^{1/6} = 3x^2$ — Because $(3x^2)^6 = 729x^{12}$. Since x^2 is always nonnegative, no absolute value symbols are necessary. Read $(729x^{12})^{1/6}$ as "the sixth root of $729x^{12}$."

If n is an odd number in the expression $a^{1/n}$, the base a can be negative.

Example 2 **Simplifying Expressions with Rational Exponents**

Simplify each expression.

Strategy In each case, we will use the definition of $a^{1/n}$.

Solution

a. $(-8)^{1/3} = -2$ — Because $(-2)^3 = -8$.

b. $(-3125)^{1/5} = -5$ — Because $(-5)^5 = -3125$.

c. $\left(-\frac{1}{1000}\right)^{1/3} = -\frac{1}{10}$ — Because $\left(-\frac{1}{10}\right)^3 = -\frac{1}{1000}$.

Self Check 2 Simplify: **a.** $(-125)^{1/3}$ **b.** $(-100{,}000)^{1/5}$

Now Try Exercise 25.

If n is odd in the expression $a^{1/n}$, and the base contains variables, we do not use absolute value symbols, because odd roots can be negative.

$(-27x^3)^{1/3} = -3x$ Because $(-3x)^3 = -27x^3$.

$(-128a^7)^{1/7} = -2a$ Because $(-2a)^7 = -128a^7$.

We summarize the definitions concerning $a^{1/n}$ as follows.

Summary of Definitions of $a^{1/n}$

If n is a natural number and a is a real number in the expression $a^{1/n}$, then

If $a \geq 0$, then $a^{1/n}$ is the nonnegative real number b such that $b^n = a$.

If $a < 0$ $\begin{cases} \text{and } n \text{ is odd, then } a^{1/n} \text{ is the real number } b \text{ such that } b^n = a. \\ \text{and } n \text{ is even, then } a^{1/n} \text{ is not a real number.} \end{cases}$

The following chart also shows the possibilities that can occur when simplifying $a^{1/n}$.

Strategy for Simplifying Expressions of the Form $a^{1/n}$

a	n	$a^{1/n}$	Examples
$a = 0$	n is a natural number.	$0^{1/n}$ is the real number 0 because $0^n = 0$.	$0^{1/2} = 0$ because $0^2 = 0$. $0^{1/5} = 0$ because $0^5 = 0$.
$a > 0$	n is a natural number.	$a^{1/n}$ is the nonnegative real number such that $(a^{1/n})^n = a$.	$16^{1/2} = 4$ because $4^2 = 16$. $27^{1/3} = 3$ because $3^3 = 27$.
$a < 0$	n is an odd natural number.	$a^{1/n}$ is the real number such that $(a^{1/n})^n = a$.	$(-32)^{1/5} = -2$ because $(-2)^5 = -32$. $(-125)^{1/3} = -5$ because $(-5)^3 = -125$.
$a < 0$	n is an even natural number.	$a^{1/n}$ is not a real number.	$(-9)^{1/2}$ is not a real number. $(-81)^{1/4}$ is not a real number.

2. Simplify Expressions of the Form $a^{m/n}$

The definition of $a^{1/n}$ can be extended to include rational exponents whose numerators are not 1. For example, $4^{3/2}$ can be written as either

$(4^{1/2})^3$ or $(4^3)^{1/2}$ Because of the Power Rule, $(x^m)^n = x^{mn}$.

This suggests the following rule.

Rule for Rational Exponents

If m and n are positive integers, the fraction $\frac{m}{n}$ is in lowest terms, and $a^{1/n}$ is a real number, then

$$a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$$

Tip

Keep a positive attitude when approaching problems with fractions as exponents. This can help you master them.

Karen Roach/Shutterstock.com

In the previous rule, we can view the expression $a^{m/n}$ in two ways:

1. $(a^{1/n})^m$: the mth power of the nth root of a
2. $(a^m)^{1/n}$: the nth root of the mth power of a

For example, $(-27)^{2/3}$ can be simplified in two ways:

$$\begin{aligned}(-27)^{2/3} &= [(-27)^{1/3}]^2 \\ &= (-3)^2 \\ &= 9\end{aligned} \qquad \text{or} \qquad \begin{aligned}(-27)^{2/3} &= [(-27)^2]^{1/3} \\ &= (729)^{1/3} \\ &= 9\end{aligned}$$

As this example suggests, it is usually easier to take the root of the base first to avoid large numbers.

Tip

It is helpful to think of the phrase *power over root* when we see a rational exponent. The numerator of the fraction represents the *power* and the denominator represents the *root*. Begin with the *root* when simplifying to avoid large numbers.

Negative Rational Exponents

If m and n are positive integers, the fraction $\frac{m}{n}$ is in lowest terms and $a^{1/n}$ is a real number, then

$$a^{-m/n} = \frac{1}{a^{m/n}} \qquad \text{and} \qquad \frac{1}{a^{-m/n}} = a^{m/n} \qquad (a \neq 0)$$

Example 3 **Simplifying Expressions of the Form $a^{m/n}$**

Simplify each expression.

Strategy We will apply the definition of $a^{m/n}$.

Solution

a. $$\begin{aligned}25^{3/2} &= (25^{1/2})^3 \\ &= 5^3 \\ &= 125\end{aligned}$$

b. $$\begin{aligned}\left(-\frac{x^6}{1000}\right)^{2/3} &= \left[\left(-\frac{x^6}{1000}\right)^{1/3}\right]^2 \\ &= \left(-\frac{x^2}{10}\right)^2 \\ &= \frac{x^4}{100}\end{aligned}$$

c. $$\begin{aligned}32^{-2/5} &= \frac{1}{32^{2/5}} \\ &= \frac{1}{(32^{1/5})^2} \\ &= \frac{1}{2^2} \\ &= \frac{1}{4}\end{aligned}$$

d. $$\begin{aligned}\frac{1}{81^{-3/4}} &= 81^{3/4} \\ &= (81^{1/4})^3 \\ &= 3^3 \\ &= 27\end{aligned}$$

Self Check 3 Simplify: **a.** $49^{3/2}$ **b.** $16^{-3/4}$ **c.** $\dfrac{1}{(27x^3)^{-2/3}}$

Now Try Exercise 49.

Because of the definition, rational exponents follow the same rules as integer exponents.

Example 4 **Using Exponent Rules to Simplify Expressions with Rational Exponents**

Simplify each expression. Assume that all variables represent positive numbers, and write answers without using negative exponents.

Strategy We will use exponent rules to simplify each expression.

Solution

a. $(36x)^{1/2} = 36^{1/2}x^{1/2}$

$= 6x^{1/2}$

b. $\dfrac{(a^{1/3}b^{2/3})^6}{(y^3)^2} = \dfrac{a^{6/3}b^{12/3}}{y^6}$

$= \dfrac{a^2b^4}{y^6}$

c. $\dfrac{a^{x/2}a^{x/4}}{a^{x/6}} = a^{x/2+x/4-x/6}$

$= a^{6x/12+3x/12-2x/12}$

$= a^{7x/12}$

d. $\left[\dfrac{-c^{-2/5}}{c^{4/5}}\right]^{5/3} = (-c^{-2/5-4/5})^{5/3}$

$= [(-1)(c^{-6/5})]^{5/3}$

$= (-1)^{5/3}(c^{-6/5})^{5/3}$

$= -1c^{-30/15}$

$= -c^{-2}$

$= -\dfrac{1}{c^2}$

Self Check 4 Use the directions for Example 4:

a. $\left(\dfrac{y^2}{49}\right)^{1/2}$ **b.** $\dfrac{b^{3/7}b^{2/7}}{b^{4/7}}$ **c.** $\dfrac{(9r^2s)^{1/2}}{rs^{-3/2}}$

Now Try Exercise 65.

3. Use Radical Notation and Determine *n*th Roots

Radical signs can also be used to express roots of numbers.

Definition of $\sqrt[n]{a}$

If n is a natural number greater than 1 and if $a^{1/n}$ is a real number, then

$$\sqrt[n]{a} = a^{1/n}$$

In the **radical expression** $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is the **radical sign**, a is the **radicand**, and n is the **index** (or the **order**) of the radical expression. If the order is 2, the expression is a **square root**, and we do not write the index.

$$\sqrt{a} = \sqrt[2]{a}$$

If the index of a radical is 3, we call the radical a **cube root**.

***n*th Root of a Nonnegative Number**

If n is a natural number greater than 1 and $a \geq 0$, then $\sqrt[n]{a}$ is the nonnegative number whose nth power is a.

$$\left(\sqrt[n]{a}\right)^n = a$$

Take Note

In the expression $\sqrt[n]{a}$, there is **no** real-number nth root of a when n is even and $a < 0$. For example, $\sqrt{-64}$ is **not** a real number, because the square of **no** real number is -64.

If 2 is substituted for n in the equation $\left(\sqrt[n]{a}\right)^n = a$, we have

$$\left(\sqrt[2]{a}\right)^2 = \left(\sqrt{a}\right)^2 = \sqrt{a}\sqrt{a} = a \text{ for } a \geq 0$$

This shows that if a number a can be factored into two equal factors, either of those factors is a square root of a. Furthermore, if a can be factored into n equal factors, any one of those factors is an nth root of a.

If n is an odd number greater than 1 in the expression $\sqrt[n]{a}$, the radicand can be negative.

Example 5 Finding *n*th Roots of Real Numbers

Find each nth root.

Strategy We will apply the definitions of cube root and fifth root.

Solution

a. $\sqrt[3]{-27} = -3$ Because $(-3)^3 = -27$.

b. $\sqrt[3]{-8} = -2$ Because $(-2)^3 = -8$.

c. $\sqrt[3]{-\frac{27}{1000}} = -\frac{3}{10}$ Because $\left(-\frac{3}{10}\right)^3 = -\frac{27}{1000}$.

d.
$$\begin{aligned} -\sqrt[5]{-243} &= -\left(\sqrt[5]{-243}\right) \\ &= -(-3) \\ &= 3 \end{aligned}$$

Self Check 5 Find each root: **a.** $\sqrt[3]{216}$ **b.** $\sqrt[5]{-\frac{1}{32}}$

Now Try Exercise 75.

We summarize the definitions concerning $\sqrt[n]{a}$ as follows.

Summary of Definitions of $\sqrt[n]{a}$

If n is a natural number greater than 1 and a is a real number, then

If $a \geq 0$, then $\sqrt[n]{a}$ is the nonnegative real number such that $\left(\sqrt[n]{a}\right)^n = a$.

If $a < 0$ $\begin{cases} \text{and } n \text{ is odd, then } \sqrt[n]{a} \text{ is the real number such that } \left(\sqrt[n]{a}\right)^n = a. \\ \text{and } n \text{ is even, then } \sqrt[n]{a} \text{ is not a real number.} \end{cases}$

The following chart also shows the possibilities that can occur when simplifying $\sqrt[n]{a}$.

Strategy for Simplifying Expressions of the Form $\sqrt[n]{a}$

a	n	$\sqrt[n]{a}$	Examples
$a = 0$	n is a natural number greater than 1.	$\sqrt[n]{0}$ is the real number 0 because $0^n = 0$.	$\sqrt[3]{0} = 0$ because $0^3 = 0$. $\sqrt[5]{0} = 0$ because $0^5 = 0$.
$a > 0$	n is a natural number greater than 1.	$\sqrt[n]{a}$ is the nonnegative real number such that $\left(\sqrt[n]{a}\right)^n = a$.	$\sqrt{16} = 4$ because $4^2 = 16$. $\sqrt[3]{27} = 3$ because $3^3 = 27$.
$a < 0$	n is an odd natural number greater than 1.	$\sqrt[n]{a}$ is the real number such that $\left(\sqrt[n]{a}\right)^n = a$.	$\sqrt[5]{-32} = -2$ because $(-2)^5 = -32$. $\sqrt[3]{-125} = -5$ because $(-5)^3 = -125$.
$a < 0$	n is an even natural number.	$\sqrt[n]{a}$ is not a real number.	$\sqrt{-9}$ is not a real number. $\sqrt[4]{-81}$ is not a real number.

We have seen that if $a^{1/n}$ is real, then $a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$. This same fact can be stated in radical notation.

$$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

Thus, *the mth power of the nth root of a* is the same as *the nth root of the mth power of a*. For example, to find $\sqrt[3]{27^2}$, we can proceed in either of two ways:

$$\sqrt[3]{27^2} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9 \text{ or } \sqrt[3]{27^2} = \sqrt[3]{729} = 9$$

By definition, $\sqrt{a^2}$ represents a nonnegative number. If a could be negative, we must use absolute value symbols to guarantee that $\sqrt{a^2}$ will be nonnegative. Thus, if a is unrestricted,

$$\sqrt{a^2} = |a|$$

A similar argument holds when the index is any even natural number. The symbol $\sqrt[4]{a^4}$, for example, means the *positive* fourth root of a^4. Thus, if a is unrestricted,

$$\sqrt[4]{a^4} = |a|$$

Example 6 **Simplifying Radical Expressions**

If x is unrestricted, simplify **a.** $\sqrt[6]{64x^6}$ **b.** $\sqrt[3]{x^3}$ **c.** $\sqrt{9x^8}$

Strategy We apply the definitions of sixth roots, cube roots, and square roots.

Solution

a. $\sqrt[6]{64x^6} = 2|x|$ Use absolute value symbols to guarantee that the result will be nonnegative.

b. $\sqrt[3]{x^3} = x$ Because the index is odd, no absolute value symbols are needed.

c. $\sqrt{9x^8} = 3x^4$ Because $3x^4$ is always nonnegative, no absolute value symbols are needed.

Self Check 6 Use the directions for Example 6:

a. $\sqrt[4]{16x^4}$ **b.** $\sqrt[3]{27y^3}$ **c.** $\sqrt[4]{x^8}$

Now Try Exercise 79.

Tip

If you do not know whether or not to include absolute value in your answers when simplifying expressions with rational exponents or radical expressions, these two tips should help.

1. If n is **even** in the expression $a^{m/n}$ and your answer involves a variable raised to the **odd** power, then include the variable part of your answer in absolute value.
2. If the index of a radical is **even** and your answer involves a variable raised to the **odd** power, then include the variable part of your answer in absolute value.

4. Simplify and Combine Radicals

Many properties of exponents have counterparts in radical notation. For example, since $a^{1/n}b^{1/n} = (ab)^{1/n}$ and $\frac{a^{1/n}}{b^{1/n}} = (\frac{a}{b})^{1/n}$ and $(b \neq 0)$, we have the following.

Multiplication and Division Properties of Radicals

If all expressions represent real numbers,

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab} \qquad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \quad (b \neq 0)$$

In words, we say

The product of two nth roots is equal to the nth root of their product.

The quotient of two nth roots is equal to the nth root of their quotient.

These properties involve the nth root of the product of two numbers or the nth root of the quotient of two numbers. There is no such property for sums or differences.

Caution

Some students mistakenly think there are sum and difference properties for radicals and use them to simplify radical expressions. Avoid this common error. For example,

$\sqrt{9+4} \neq \sqrt{9} + \sqrt{4}$, because

$$\sqrt{9+4} = \sqrt{13} \quad \text{but} \quad \sqrt{9} + \sqrt{4} = 3 + 2 = 5$$

and $\sqrt{13} \neq 5$. In general,

~~$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$~~ and ~~$\sqrt{a-b} = \sqrt{a} - \sqrt{b}$~~

Numbers that are squares of positive integers, such as

1, 4, 9, 16, 25, and 36

are called **perfect squares**. Expressions such as $4x^2$ and $\frac{1}{9}x^6$ are also perfect squares, because each one is the square of another expression with integer exponents and rational coefficients.

$$4x^2 = (2x)^2 \text{ and } \frac{1}{9}x^6 = \left(\frac{1}{3}x^3\right)^2$$

Numbers that are cubes of positive integers, such as

1, 8, 27, 64, 125, and 216

are called **perfect cubes**. Expressions such as $64x^3$ and $\frac{1}{27}x^9$ are also perfect cubes, because each one is the cube of another expression with integer exponents and rational coefficients.

$$64x^3 = (4x)^3 \text{ and } \frac{1}{27}x^9 = \left(\frac{1}{3}x^3\right)^3$$

There are also perfect fourth powers, perfect fifth powers, and so on.

We can use perfect powers and the Multiplication Property of Radicals to simplify many radical expressions. For example, to simplify $\sqrt{12x^5}$, we factor $12x^5$ so that one factor is the largest perfect square that divides $12x^5$. In this case, it is $4x^4$. We then rewrite $12x^5$ as $4x^4 \cdot 3x$ and simplify.

$$\begin{aligned} \sqrt{12x^5} &= \sqrt{4x^4 \cdot 3x} && \text{Factor } 12x^5 \text{ as } 4x^4 \cdot 3x. \\ &= \sqrt{4x^4}\sqrt{3x} && \text{Use the Multiplication Property of Radicals: } \sqrt{ab} = \sqrt{a}\sqrt{b}. \\ &= 2x^2\sqrt{3x} && \sqrt{4x^4} = 2x^2 \end{aligned}$$

To simplify $\sqrt[3]{432x^9y}$, we find the largest perfect-cube factor of $432x^9y$ (which is $216x^9$) and proceed as follows:

$$\begin{aligned} \sqrt[3]{432x^9y} &= \sqrt[3]{216x^9 \cdot 2y} && \text{Factor } 432x^9y \text{ as } 216x^9 \cdot 2y. \\ &= \sqrt[3]{216x^9}\,\sqrt[3]{2y} && \text{Use the Multiplication Property of Radicals: } \sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}. \\ &= 6x^3\sqrt[3]{2y} && \sqrt[3]{216x^9} = 6x^3 \end{aligned}$$

Tip

There are commonly occurring fourth and fifth roots. Memorizing the ones listed in the following table can be very helpful when simplifying radicals. Assume that the variables represent positive numbers.

Fourth Roots	Fifth Roots
$\sqrt[4]{1} = 1$	$\sqrt[5]{1} = 1$
$\sqrt[4]{16} = 2$	$\sqrt[5]{32} = 2$
$\sqrt[4]{81} = 3$	$\sqrt[5]{243} = 3$
$\sqrt[4]{256} = 4$	$\sqrt[5]{1024} = 4$
$\sqrt[4]{625} = 5$	$\sqrt[5]{x^5} = 1$
$\sqrt[4]{x^4} = x$	$\sqrt[5]{x^{10}} = x^2$
$\sqrt[4]{x^8} = x^2$	$\sqrt[5]{x^{15}} = x^3$
$\sqrt[4]{x^{12}} = x^3$	$\sqrt[5]{x^{20}} = x^4$