Third Edition

Advanced Heat Transfer

Greg F. Naterer

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Dedication

In memory of my son Jordan who inspires me by his everlasting zeal of exploring and climbing to greater heights.

Contents

Chapter 1 Introduction....................1

Chapter 3 Convection

Preface

Since the publication of the Second Edition in 2018, numerous updated and emerging new fields have motivated the development of this new Third Edition of *Advanced Heat Transfer*. The Third Edition aims to provide a more comprehensive treatment of advanced topics and enrich the student learning experience with more case studies, examples, and problems. It also broadens the scope of applications of thermal engineering, including but not limited to, biomedical, micro- and nanotechnology, and machine learning. This Third Edition was updated based on input from colleagues and readers over the years since the first edition.

Typically courses on advanced topics in thermal and energy systems have specialized books on a specific aspect of thermal engineering with a focus on the selected topic in depth. Interactions among various modes of multiphase heat transfer are not fully explored or connected in a way that provides a systematic framework of analysis. Advanced methods of analysis are typically covered in depth for a single specific mode of heat transfer, such as conduction or convection, rather than an overview of methods that can be applied over a wider range of heat transfer modes. In this Third Edition, advanced solution methods are presented for a range of applications and modes of heat transfer.

Several new sections, figures, tables, example problems at the end of each chapter, and graphs were added to this latest edition. In this Third Edition, the chapter on heat conduction (Chapter 2) provides a more comprehensive treatment of solution methods including non-homogeneous systems and time-dependent boundary conditions. Also, new sections are added on emerging topics of active research such as conduction in porous media, heat transfer in living tissue, and microscale conduction. In each new section, a brief overview of the topic is outlined, along with references to other sources for more detailed analysis.

Also, the chapter on radiation (Chapter 4) was significantly updated. In particular, the transport phenomena of radiation in participating media were expanded to include the processes of attenuation by absorption, scattering from multiple directions, enhancement of intensity by emission, and the resulting radiation exchange among the participating gases. Fundamentals and the governing differential equations of energy transfer for participating media are presented. This includes the general equation of transfer, involving the radiative flux vector and overall conservation of energy. Approximate solutions of the equation of transfer are also discussed. Simplifications based on the emission and Rosseland approximations are used to find semi-analytical solutions. Coupled radiation and convection problems are considered and analyzed.

More detailed treatment of transport phenomena is also presented in the updated chapter on chemically reacting flows (Chapter 8). Diffusion processes during chemical reactions are analyzed for both heterogeneous and homogeneous reactions, as well as reactions in porous catalysts. Coupled heat and fluid flow with chemical reactions are formulated and investigated with analytical solution methods. In addition, the section on fuels and combustion is expanded to analyze burning fuel droplets and radiation exchange.

Several emerging areas of heat exchanger development are also presented in the Third Edition (Chapter 9). Further types of heat exchangers include honeycomb, moving bed, micro-scale, compact, helically coiled, and two-phase heat exchangers. Systems with more than two operating fluids are examined. Also, thermal enhancement with metal foams is analyzed. The method of entropy generation minimization is applied to the design of heat exchangers, in particular, for counterflow systems, conditions with flow imbalance, phase change, and finned tube heat exchangers.

Lastly, the chapter on computational heat transfer (Chapter 10) was also significantly updated. New sections were added on radiation heat transfer, including numerical methods of the discrete transfer model, discrete ordinates, and finite volume method. Concepts in numerical methods for two-phase flows (boiling and condensation) are presented. Also, an introduction is provided for machine learning and artificial intelligence. The fundamentals of artificial neural networks and linear regression are outlined. As an application of the fundamental principles, case studies of heat conduction and forced convection are presented and discussed.

This Third Edition aims to cover a range of advanced heat transfer topics of importance in both undergraduate and graduate-level courses. It can serve both as undergraduate and follow-up graduate courses in heat transfer, such as advanced topics courses or graduate-level heat transfer. It would normally follow a first course in fluid mechanics. The student is expected to have knowledge of vector calculus and differential equations.

This book is organized into six overall areas: (1) introduction (Chapter 1); (2) primary single-phase modes of heat transfer (Chapters 2–4); (3) multiphase heat transfer (Chapters 5–7); (4) chemically reacting flows (Chapter 8); (5) heat exchangers (Chapter 9); and (6) computational heat transfer (Chapter 10). The introduction provides the reader with fundamentals of heat transfer. The modes of single-phase heat transfer, including conduction, convection, and radiation, are covered in the second part. Then, the reader may focus on any of the specific multiphase systems (Chapters 5–7), such as gas–liquid (Chapter 5) or liquid–solid systems (Chapter 7), without a loss of continuity. The latter chapters on chemically reacting flows, heat exchangers, and numerical heat transfer, can also be studied independently of the others without a loss of continuity.

I'm grateful to faculty colleagues and students who have contributed in significant ways to the preparation in this Third Edition. My role as the Editor-in-Chief of the *AIAA Journal of Thermophysics and Heat Transfer* has provided a window into the latest and emerging developments in thermal engineering systems. Also, special thanks to Rhys Northcote at the University of British Columbia for his contributions to the machine learning section, and Brian McDonald, Nancy Chafe, Brandon Howell, Guofei Yan, Kheya Zaman, Saleh Elfasay, Mahamudul Hasan, and Logan Hornell at Memorial University, for their contributions to the problem sets.

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Dr. Naterer has made significant contributions to the fields of heat transfer, energy systems, and fluid mechanics. He led an international team that developed and constructed a copper–chlorine cycle of thermochemical hydrogen production. In addition to this Third Edition, he has co-authored two other books: *Hydrogen Production from Nuclear Energy* (with I. Dincer, C. Zamfirescu; 2013, Springer) and *Entropy-Based Analysis and Design of Fluids Engineering Systems* (with J.A. Camberos; 2008, CRC Press/Taylor & Francis).

Among his awards and honors for teaching and research, Dr. Naterer has received the EIC Julian C. Smith and K.Y. Lo Medals, CNS Innovative Achievement Award, CSME Jules Stachiewicz Medal, and Best Professor Teaching Award. He is a Fellow of the Canadian Society for Mechanical Engineering (CSME), American Society of Mechanical Engineers (ASME), Engineering Institute of Canada (EIC), Canadian Academy of Engineering (CAE), and American Institute of Aeronautics and Astronautics (AIAA). Dr. Naterer earned a PhD in mechanical engineering in 1995 at the University of Waterloo, Canada.

Lists of Abbreviations

List of Symbols

a	extinction coefficient (absorption plus scattering), m^{-1}
A	area, m^2
AF	air–fuel ratio
b	unfrozen water film thickness, m
B	Ice thickness, m
Bi	Biot number (hD/k)
B_T	Spalding number ($c_v\Delta T/h_{fg}$)
c	speed of light, m/s; capacitance matrix
c_d	drag coefficient
c_f	skin friction coefficient
c_p	specific heat at constant pressure, kJ/kgK
c_v	specific heat at constant volume, kJ/kgK
C	concentration; slip correction factor
CC	cloud cover fraction
D	diameter, m; mass diffusivity, m^2/s
e	internal (thermal) energy, kJ/kg
E	total energy, kJ; emissive power, W/m^2
Ec	Eckert number ($U^2/c_p\Delta T$)
Ei	exponential integral
Eo	Eotvos number ($\Delta\rho gD/\sigma$)
f	friction factor
F	force, N; view factor; blackbody function; correction factor
Fo	Fourier number ($\alpha t/L^2$)
Fr	Froude number ($U/g^{1/2}L^{1/2}$)
g	Gibbs free energy, kJ/kg; gravitational acceleration, m^2/s; Lamé coefficient
G	irradiation, W/m^2; mass velocity, kg/m^2s
Ga	Galileo number ($g\mu^4/\rho\sigma^3$)
Gr	Grashof number ($g\beta\Delta TL^3/\nu^3$)
h	convection coefficient, W/m^2K; enthalpy, kJ/kg; Planck's constant (6.63×10^{-34} Js)
h_{fg}	latent heat of vaporization, kJ/kg
h_{sl}	latent heat of fusion, kJ/kg
H	height, m
i, j	unit vectors in coordinate directions
I	intensity of radiation, W; turbulence intensity
j	diffusion flux; Colburn factor; drift flux, m/s
J	radiosity, W/m^2; Jacobian determinant
Ja	Jacob number ($c_p\Delta T/h_{fg}$)
k	thermal conductivity, W/mK; turbulent kinetic energy, m^2/s^2; reaction rate coefficient
K	permeability, m^2; chemical equilibrium constant
Kn	Knudsen number (λ/L)
L	length, m
Le	Lewis number (α/D)
m	mass, kg
$\dot{m}$	mass flow rate, kg/s
M	molar mass, kg/kmol; heat pipe merit figure
Mo	Morton number ($g\mu^4\Delta\rho/\rho^2\sigma^3$)

n	surface normal; order of reaction
N	shape function; number of moles; number density of particles
$\dot{N}''$	molar flux of species i, kmol/m^{2}s
nel	number of elements
NTU	number of transfer units
Nu	Nusselt number (hL/k)
Oh	Ohnesorge number ($\mu/\rho^{1/2}\sigma^{1/2}L^{1/2}$)
p	pressure, Pa; population balance
P	perimeter, m^2; porosity
Pe	Peclet number (UL/α)
Pr	Prandtl number (ν/α)
Q	heat flow, kJ
q	heat flow rate, W
q''	heat flux, W/m^2
r	radial coordinate, m; reaction rate, mol/m^{3}s
R	thermal resistance, K/W; radial phase interface position, m; residual
Ra	Rayleigh number ($g\beta\Delta TL^3/\nu\alpha$)
Re	Reynolds number (UL/ν)
s	entropy, kJ/kgK; surface element, m
S	shape factor, m; surface area, m^2
Sc	Schmidt number (ν/D)
Sh	Sherwood number (hL/D)
St	Stanton number ($h/\rho Uc_p$)
Ste	Stefan number ($c_p\Delta T/L$)
STP	standard temperature and pressure conditions (25°C, 1 atm)
t	time, s; thickness, m
T	temperature, K
TDH	transport disengaging height, m
u, v, w	x, y and z direction velocities, m/s
u', v'	turbulent fluctuating velocity components in x, y directions, m/s
U	freestream or reference velocity, m/s; total conductance, W/m^{2}K
UTS	ultimate tensile strength, N/m^2
v'''	specific volume, m^3/kg
v	velocity vector, m/s
V	velocity, m/s; volume, m^3
w	complex coordinate (u + iv)
W	work, kJ; width, m; weight function; liquid water content, kg/m^3
We	Weber number ($\rho U^2L/s$)
x	fraction of phase conversion
x, y, z	Cartesian coordinates
X	Cartesian phase interface position, m; Martinelli parameter
y_i	mole fraction
z	complex coordinate (x + iy)
Z	heat fin parameter; molar generation rate, kmol/s
∇	gradient operator ($\partial/\partial x$, $\partial/\partial y$, $\partial/\partial y$)

Greek Letters

α	thermal diffusivity ($k/\rho c_p$), m^2/s; absorptivity; solar altitude angle
β	thermal expansion coefficient, 1/K; flow excess parameter
χ_k	mass fraction of phase k
δ	boundary layer thickness, m; film thickness, m; thermal penetration depth, m

ε	emissivity; turbulent dissipation rate; heat exchanger effectiveness; perturbation parameter
ϕ	angle, rads; velocity potential; general scalar; shear parameter
Φ	viscous dissipation, 1/s^2; particle sphericity; scattering phase function
γ	shape parameter; mass flow weighting function; ratio of specific heats (c_p/c_v)
Γ	mass flow rate per unit width, kg/ms; general diffusion coefficient
η	efficiency; similarity variable
κ	mass transfer coefficient, mol/m^{3}s; Boltzmann constant (1.38×10^{-23} J/K); optical thickness, m
λ	wavelength, m; spectral; latitude; dimensionless bed parameter
Λ	mean free path, m
μ	dynamic viscosity, kg/ms; chemical potential, kJ/kmol
ν	kinematic viscosity, m^2/s; frequency, s^{-1}; stoichiometric coefficient
θ	angle, rads; dimensionless temperature
Θ	kinetic function
ρ	density, kg/m^3; reflectivity
σ	normal stress, N/m^2; Stefan–Boltzmann constant (5.67×10^{-8} W/m^{2}K); surface tension, N/m; scattering coefficient
τ	shear stress, N/m^2; transmissivity
υ	specific volume (m^3/kg)
ω	solid angle, sr
Ω	albedo function for scattering
ξ	flow alignment weighting factor; local elemental coordinate
ψ	stream function; hour angle
ζ_k	volume fraction of phase k; void fraction; porosity

Subscripts

a	air; ambient; activation; absorption
b	blood
A, B	constituents A and B in a mixture
atm	atmospheric
b	base; blackbody; boiling; Binghman slurry; body force; bubble
bndry	boundary
c	cross-section; collector; cold; critical; characteristic; combustion
civ	civic
conv	convection
crit	critical
cv	control volume
d	drift; droplet
D	diameter
dif	diffuse
dir	direct
e	east; mean beam length; eutectic
eff	effective
ev	evaporation
f	fluid; fin; fusion (phase change point); final; formation
fg	fluid–gas
g	gas; ground; glass; generation
gen	generation
h	hot; high; horizontal; hydraulic
i	inner; interfacial; initial; surface index
in	inlet

ip	integration point
j	surface index
k	phase number; kinetic energy
ko	Kolmogorov
l, L	low; laminar; liquid
le	liquid entrainment
liq	liquidus
lm	log mean
loc	local
m	mixing length; mean; melting point; mass transfer; metanbolic
min	minimum
mf	minimum fluidization
mp	melt particularization
n	north
nb	neighboring
nuc	nucleate
o	outer
opt	optimal
out	outlet
p	particle; pipe
P	product; Planck
r	reference; relative; removal
R	reactant; Rosseland
rad	radiation
ref	reflected
res	residence
s	surface; entropy; solar; solid; south; settling
sat	saturation
sg	superficial gas; solid–gas
sl	superficial liquid; solid–liquid
sol	solidus; solar time
sub	subcooled
t	thermal; turbulent
tp	two-phase
U	drift velocity
v	vapor
w	wall; wick; water; west
x, y, z	Cartesian coordinates
1, 2	node numbers
∞	ambient; freestream

Superscripts

$'$	turbulent fluctuating quantity
$''$	flux quantity (per unit area)
*	non-dimensional quantity

1 Introduction

Until the mid-nineteenth century, heat was interpreted as an invisible form of matter called a "caloric." The caloric was understood as a fluid substance that was responsible for heat transfer phenomena. This perspective was held until about 1840, when the British physicist James Joule showed that heat was not a material substance, but rather a form of energy. This led to a new interpretation of heat as a mechanism of thermal energy transfer across the boundaries of a system. This new insight led to a deeper understanding of the fundamental modes of heat transfer, namely conduction, convection, and radiation.

Conduction heat transfer occurs from one part of a solid body or fluid to another, or between bodies in contact, without any movement on a macroscopic level. Convection occurs when heat is transferred between a solid surface and a fluid, or between different fluid regions, due to bulk fluid movement. For forced convection, external processes such as pressure-induced forces drive the fluid motion. These external processes may result from devices such as pumps, fans, or atmospheric winds. In contrast, buoyancy (rather than an external force) drives the fluid motion for natural convection (or free convection). Radiation is another fundamental mode of heat transfer. It occurs from the emission of electromagnetic waves, or photons as packets of energy, by all surfaces above absolute zero temperature (zero Kelvin, or –273.15°C). These processes will be described in detail in following chapters devoted to each mode of heat transfer.

Multiphase heat transfer, such as phase change in gas–liquid and liquid–solid flows, arises in many practical applications. For example, predicting and controlling the operation of condensers in thermal power plants in an efficient manner requires an understanding of gas–liquid transport phenomena during condensation. In material processing technologies, such as extrusion or casting, the liquid metal phase change has a significant role in the final material properties, such as tensile strength, due to the alignment of grain boundaries and solidification shrinkage. In upcoming chapters, various solution techniques and case studies dealing with such multiphase systems will be presented.

A common design issue in heat transfer engineering is finding ways to reduce (or increase) the heat transfer to a minimum (or maximum) value. For example, consider the directional solidification of Ni-based superalloy turbine blade castings by liquid metal cooling. Several complex interactions occur during the solidification process, such as shrinkage flow at the phase interface, thermal–solutal convection in the bulk liquid, and radiative heat transfer. Effective thermal control is important so that grain boundaries are aligned parallel to the blade axes during solidification. In this way, the solidified material can most effectively resist conditions of maximum stress during turbine operation.

Another common design issue in thermal engineering is how to achieve a specified heat transfer rate as efficiently and economically as possible. For example, in microelectronics assemblies, designers seek better ways of cooling the electronic circuits and more efficient alternatives to conventional cooling with a fan. Another example is deicing of aircraft, wind turbines, and other iced surfaces. Aircraft icing increases drag and weight, and it presents a serious danger to air safety. It can damage downstream components if the attached ice breaks off, and ingested ice can damage the jet engine. Several heating and cooling modes at the surface affect the ice accretion such as surface heating, convection, conduction, and incoming supercooled droplets which freeze on the surface. These combined processes are complex and involve multiple modes of heat transfer simultaneously.

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1.1 FUNDAMENTAL CONCEPTS AND DEFINITIONS

Microscopic phenomena at the molecular level affect a material's thermophysical state properties such as thermal conductivity, specific heat, viscosity, density, and phase change temperature. Differences between solids, liquids, and gases at the microscopic level affect their nature of intermolecular interactions and thermal energy exchange. Chemical bonds between atoms in a solid enable the formation of the lattice structure, or ions and molecules that form chemical compounds in a fluid. These bonds may result from electrostatic forces of attraction between oppositely charged ions in the case of ionic bonds, or the sharing of atoms in covalent bonds. The oppositely charged ions are arranged in a lattice structure. Also, there are intermolecular forces that bind a substance together. These bonds create a compact structure in the material that affects the resulting thermophysical state properties.

Unlike fluids, solids typically resist shear and compression forces, and they are self-supporting. The various types of solids can be broadly characterized as ceramics, metals, and polymers. Ceramics are compounds based predominantly on ionic bonding. Some common examples of ceramics are brick and porcelain. Ceramic phase diagrams have similar layouts as metal–metal systems. Metals usually exhibit less complex crystal structures than ceramics. Also, less energy is required to dislocate atoms in their atomic structure. Metals typically have lower yield stresses and a lower hardness than ceramics. Ceramics are harder, but usually more brittle and more difficult to plastically deform than metals.

Polymers are organic in nature and their atomic structure involves covalent bonding. Common examples of polymers are hydrocarbons, such as C_2H_4 (ethylene), plastics, rubbers, and CH_4 (methane). They are utilized in applications such as coatings, adhesives, films, foam, and many others. Polymers are neither as strong nor as stiff as metals and ceramics; they form as molecular chains. Thermophysical properties such as the melting temperature and material strength depend on their degree of crystallinity and the ability of the molecules to resist molecular chain motion. Unlike phase change at a discrete point in pure metals, a continuous phase change between liquid and solid phases is observed in polymers.

The crystal structure of polymers usually involves "spherulites." Spherulites are small semi-crystalline regions that are analogous to grain structures in metals. The extremities of spherulites impinge on one another to form linear boundaries in polymer materials. A region of high crystallinity is formed by thin layers called "lamallae" (typically of the order of 10 μm in length). These different types of regions affect the thermophysical state properties. Their varying structural forms explain why the densities of ceramic materials are normally larger than those of polymers, but less than those of metals. Metals usually have melting temperatures higher than those of polymers but less than ceramics. Also, the thermal conductivity of polymers is usually about two orders of magnitude less than that of metals and ceramics.

Unlike solids, there is a molecular freedom of movement with no fixed structure in liquids and gases. From common everyday experience, liquids need a container for storage and they cannot resist imposed shear stresses. However, they can resist compression. These characteristics indicate some key differences between solids and liquids from a microscopic point of view. Some materials, such as slurries, tar, toothpaste, and snow, exhibit multiple characteristics. For example, tar resists shear at small stresses, but it flows at high stresses. The study of these forms of hybrid materials is the subject of *rheology*.

In order to determine the macroscopic properties of a solid or fluid, such as density or thermal conductivity, it is normally assumed that the substance is a continuous medium. This approach is called the *continuum assumption*. It is an idealization that treats the substance as continuous, even though on a microscopic scale, it is composed of individual molecules. The continuum assumption is normally applicable to fluids beyond a minimum of 10^{12} molecules/mm^3. In certain circumstances, the continuum assumption cannot be used, for example, in rarefied gases at low pressure like the conditions experienced by spacecraft atmospheric re-entry at high altitudes.

The continuum assumption considers macroscopic averaging rather than microscopic properties arising from a varying spatial distribution of molecules. For example, consider the definition of

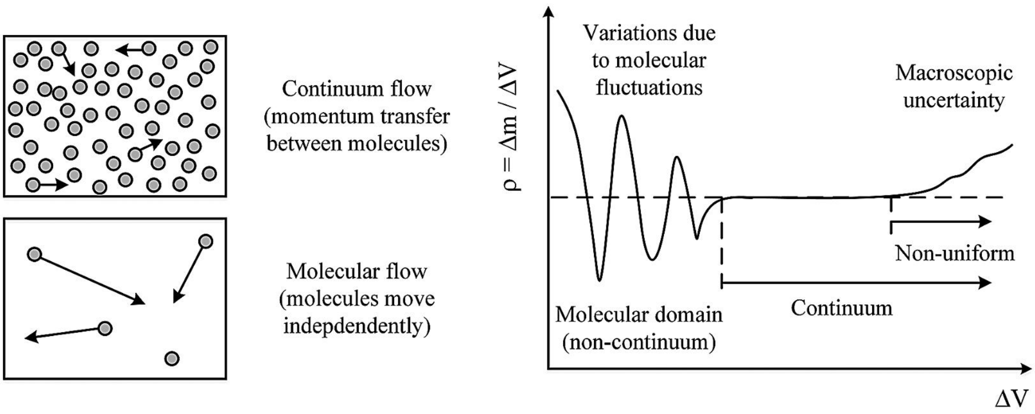

FIGURE 1.1 Continuum assumption.

density by macroscopic averaging of the mass divided by the volume (see Figure 1.1). In this definition, a volume is chosen to be large enough so that the density is properly defined. The mass of molecules is assumed to be distributed uniformly across the volume. But the number of molecules varies within the volume if the volume size approaches the scale of the mean free path of molecular motion. If the volume size is less than the mean free path, then significant variations in density can arise due to the molecular fluctuations. The molecules fluctuate randomly in and out of a selected control volume. On the other hand, if the volume is large on a macroscopic scale, then variations associated with the spatial density distribution would be observed. As a result, there is a specific limited range to be defined as the appropriate volume size for the continuum assumption to be valid. The control volume size must be larger than the scale of the mean free path, but less than the characteristic macroscopic dimensions, to properly define the local fluid density.

Different techniques may be used to describe the motion of fluids. In an *Eulerian frame of reference*, flow quantities are tracked from a fixed location in space (or a control volume), whereas in the *Lagrangian framework*, individual fluid particles are tracked along their trajectories. In general, the Eulerian approach will be adopted throughout this book. However, it should be noted that in some applications, a Lagrangian description may be more useful, such as free surface flows with tagged particles for the tracking of wave motion on a free surface.

As an example, consider a gas particle trajectory in a heated duct. If a thermocouple is placed in the duct, then the temperature varies according to the selected position, as well as time. This represents a fixed location, corresponding to the Eulerian approach. On the other hand, if individual gas particles are tracked throughout the duct, then this represents a Lagrangian approach. In this latter approach, the temperature of a specific particle is a function of time along its trajectory. The particle is tracked over a trajectory so its velocity has a functional dependence on both the trajectory and time, or in other words, spatial coordinates of the pathlines that also vary with time. It would be impractical to trace all particle trajectories within the duct so the Eulerian approach would be more suitable in this example. In the Eulerian approach, the change of temperature and velocity with time would be observed with a stationary control volume in the duct. The approaches are different but ultimately both Eulerian and Lagrangian descriptions lead to the same results.

Kinematics refers to the study of properties of motion, such as fluid velocity and acceleration. The kinematic properties involve two components: (i) a time derivative; and (ii) spatial derivatives. The total rate of change of a scalar quantity, B, denoted by the material derivative, DB/Dt, consists of both the time and spatial components:

$$\frac{DB}{Dt} = \frac{\partial B}{\partial t} + \left(u\frac{\partial B}{\partial x} + v\frac{\partial B}{\partial y} + w\frac{\partial B}{\partial z} \right) \tag{1.1}$$

where u, v, and w refer to the x-, y- and z-direction velocity components, respectively. For example, to determine the acceleration field, B represents the velocity. The notation of DB/Dt refers to the total (or material, or substantial) derivative of B. The portion of the material derivative represented by the spatial derivatives is called the convective derivative. It accounts for the variation of the fluid property, B, due to a change of position, while the temporal derivative represents the change with respect to time.

An alternative useful notation is *tensor* or *indicial notation*, especially when expressions involving vectors and/or matrices become increasingly complex or lengthy. A tensor is a variable with appropriate subscripts. When a subscript (or index) is repeated in a tensor, it denotes a summation with respect to that index over its range, for example, $i = 1, 2, 3$. Otherwise, if the index is not repeated, then it implies multiple values for each index, such as a vector of quantities (tensor of rank 1) or a matrix (tensor of rank 2). For example, the variable a_{ij} is a tensor of rank 2 that represents a 3×3 matrix, where $i = 1, 2, 3$, and $j = 1, 2, 3$.

Using tensor notation, the total derivative Equation (1.1) can be written more concisely as

$$\frac{DB}{Dt} = \frac{\partial B}{\partial t} + u_i \frac{\partial B}{\partial x_i} \tag{1.2}$$

where $i = 1, 2, 3$. Further details on tensors and indicial notation are provided in Appendix A.

1.2 CONSERVATION OF ENERGY

The conservation of energy, or first law of thermodynamics, is a fundamental basis of heat transfer engineering. Two general types of energy balances may be used – either a control mass or a control volume approach. A control mass refers to a closed system of no inflow or outflow of mass from the system. In contrast, a control volume refers to an open system consisting of a fixed region in space with inflows and/or outflows of mass across the boundary surfaces. A general energy balance for a control volume can be expressed as follows (see Figure 1.2):

$$\dot{E}_{cv} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{g} \tag{1.3}$$

From left to right, the individual terms represent: (i) the rate of energy accumulation with time in the control volume; (ii) the rate of energy inflow across the boundary surfaces; (iii) the rate of energy outflow; and (iv) the rate of internal heat generation within the control volume due to processes such as electrical resistive heating or chemical reactions. The overdot notation refers to the rate of change with respect to time. A "steady state" refers to conditions that are independent of time, or in other words, negligible changes in the problem variables with time.

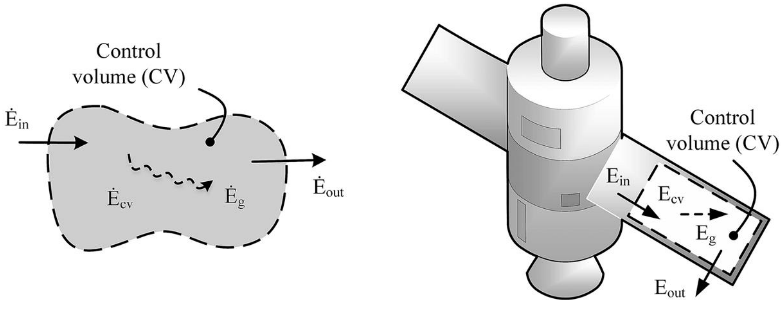

FIGURE 1.2 Schematic of energy balance for a control volume.

The energy balance states that the rate of the increase of energy over time within the control volume equals the net rate of energy inflow plus any internal heat generation. The energy inflow and outflow terms include heat and work flows across the boundary surfaces. For example, work or power input (or output) may occur due to a protruding shaft across the boundary of the control volume of a pump. A turbine shaft and blades would extract power from a control volume encompassing a steam turbine in a power plant. Although the above form of the energy balance indicates a single outlet and inlet, a more generalized expression can be written for multiple inlets and outlets by taking a summation over all inlets and outlets in the above energy balance.

For a control mass, an energy balance can be written to include both work and heat modes of energy transfer across the boundary surface of the control mass. The first law of thermodynamics over a finite period of time including work performed on the system (such as compression/expansion of a gas in the closed system), denoted by W, can be written as

$$E_i + Q + W = E_f \tag{1.4}$$

where the subscripts i and f refer to initial and final states, respectively, and Q refers to the net inflow of heat into the control mass (note: negative Q represents a heat outflow).

At the edges of a control volume, a boundary condition can be established through an energy balance for a control volume that shrinks to a zero thickness as it encompasses the boundary. In this case, the transient energy accumulation term in the energy balance becomes zero since the mass of the control volume approaches zero. The heat generation may be nonzero as a result of processes such as friction between two different phases at the interface or heat transfer due to latent heat evolved at a moving phase interface. The energy balance at the edge of the control volume can be regarded as a boundary condition that is used to solve the governing equations for variables internally within the domain.

1.3 THERMOPHYSICAL PROPERTIES

Modeling of thermal and fluid transport behavior is inherently limited by an incomplete detailed understanding of the underlying physical processes at a microscopic level. The governing equations often introduce averaged properties of the medium in order to relate dependent to independent variables. For example, in the case of heat conduction, the thermal conductivity is a proportionality constant that relates the heat flux to the temperature gradient. Transport properties reflect a further limitation in the ability to accurately measure these transport processes. Variability of experimental results introduces measurement uncertainties that affect the accuracy of models and correlations which use transport properties.

In this section, three different types of thermophysical properties of a system will be discussed: thermodynamic, transport, and other properties. Fundamentals of their associated physical processes will be described.

1.3.1 THERMODYNAMIC PROPERTIES

Thermodynamic properties or variables include pressure (p), density (ρ), enthalpy (h), specific volume (v), temperature (T), specific internal energy (e), and specific entropy (s). The fluid enthalpy, h, is defined by $h = e + p\,v$. “Specific” properties are those expressed on a per mass basis. A thermodynamic property is any property that is measurable and which describes the state of the physical system. Some thermodynamic constants, such as the ideal gas constant, R, do not describe the state of a system, and so these are not properties.

Specific or intensive properties are independent of size, whereas extensive properties (such as the total energy) are dependent on the size of the system. For example, an extensive property of a system containing two parts, A and B, is the sum of properties of both parts A and B. The *state postulate of thermodynamics* states that for a simple compressible substance, the number of intensive independent properties of a system equals the number of relevant reversible work modes plus 1. One is added because even if all properties are held constant within a system, one further property can be changed, such as temperature, through heat transfer.

Pressure, p, is the normal force per unit area acting on a fluid. It is associated with a momentum change of a fluid and represents a force applied perpendicular to the surface of an object, per unit area, over which the force is distributed. Consider the force exerted on a plate as a result of fluid impact on the plate. The impulse (or change of momentum) of a specific fluid particle near the plate is the change in momentum between an initial point upstream of the plate and its final state (a zero velocity upon impact at the wall). Summing over many molecules near the plate and taking the average normal velocity of all molecules, an expression can be obtained for the average force per unit area exerted by the molecules on the wall. For a gas at normal atmospheric conditions, the following ideal gas equation of state can be used to relate pressure to the density and temperature:

$$p = \rho R T \tag{1.5}$$

Pressure is a scalar variable that acts perpendicular to a surface and whose magnitude adjusts to conserve mass in the flow field. For example, consider an air gap in a window cell with a buoyant internal flow arising from differential heating of both sides of the cell. Due to buoyancy forces, warm air ascends near the hot wall until it reaches the top corner. Conservation of mass dictates that the fluid cannot only ascend, but there must also be a balance of a descending flow to conserve mass within the cell. Therefore, an adverse pressure gradient (i.e., increasing pressure in the flow direction) occurs as the fluid ascends toward the top corner, thereby causing the airflow to change directions and descend down along the other side to allow an overall conservation of mass within the cavity.

Another key thermodynamic property is energy. The total energy refers to the sum of internal, kinetic, and potential energies. The internal energy of a system is characterized by its temperature. Work and heat are the forms that energy takes to cross the boundaries of a system. A force alone does not change the energy of a system, but rather a force acting over a distance, leads to work and an energy change. At a visible or macroscopic scale, work is a process that changes the potential and/or kinetic energy of a system. In contrast, heat transfer leads to a change of internal energy at a microscopic scale. In other words, heat transfer corresponds to work at a microscopic or subvisible scale.

Every system above a temperature of absolute zero (zero Kelvin, or –273.15°C) has a state of microscopic disorder. Entropy represents an uncertainty about a system's microscopic state. It characterizes the disorder at the molecular level and a statistical probability or uncertainty of a particular quantum state. In a perfect crystal of a pure substance at absolute zero temperature, the molecules are motionless and stacked precisely in accordance with their crystal structure. Here there is no uncertainty about the crystal's microscopic state (called the third law of thermodynamics). The entropy at zero absolute temperature is zero. The second law of thermodynamics requires that the entropy of a system, including its surroundings (an isolated system), never decreases. So the entropy production of an isolated system is equal to zero for reversible processes, or greater than zero for irreversible processes. A process is irreversible if it is highly unlikely from a statistical probability perspective that the direction of energy conversion can be reversed. Examples of irreversible processes are dissipation of kinetic energy to frictional heating in a boundary layer and heat transfer from a higher to lower temperature.

Although entropy, s, cannot be measured directly, it can be determined indirectly from the Gibbs equation. For a simple compressible substance, the Gibbs equation is given by

$$Tds = de + pd\upsilon \tag{1.6}$$

where e and υ refer to the internal energy and specific volume, respectively. Entropy can also be expressed in terms of the specific Gibbs free energy, g. The Gibbs free energy is a thermodynamic property which represents the maximum reversible work that can be extracted from a closed system in an isothermal (constant temperature) and isobaric (constant pressure) process. It is defined by

$$g = h - Ts \tag{1.7}$$

where h refers to specific enthalpy. If a system undergoes a reversible process, the decrease in Gibbs free energy from the initial to final state equals the work performed by the system on the surroundings, minus the work of pressure forces. In multicomponent systems, the Gibbs free energy is minimized when the system reaches chemical equilibrium at constant temperature and pressure. Therefore, a decrease of Gibbs free energy is required for the spontaneity of processes to proceed at constant pressure and temperature.

For a given system, an increase in microscopic disorder (or entropy) results in a loss of ability to perform useful work. For example, consider the expansion of an ideal gas from a half-cavity into an adjacent evacuated side of the other half-cavity. A partition initially divides the two sides of the entire cavity. When the partition is removed, the total energy of the total cavity remains constant, but after the process, there was a loss of ability to perform useful work. If the first side had a piston instead of a partition, the initial state could have performed some work on the other side. But now the final state cannot perform work since the gas has expanded into both sections. In the final state, there is less certainty about a particle's location because it has moved randomly within the larger entire volume, rather than only within the half-cavity in the initial state. Thus, the system entropy has increased and there was a loss of ability to perform useful work.

1.3.2 TRANSPORT PROPERTIES

Molecular properties of a substance, such as thermal conductivity, viscosity, and diffusion coefficient (diffusivity), indicate the rate at which specific (per unit volume) heat, momentum, or mass are transferred through the substance. Examples of thermal conductivities of various solids, liquids, and gases are shown in Figure 1.3. Values are shown in SI units. Conversion factors between SI and Imperial units for a number of transport and other thermophysical properties are presented in Appendix B. An extensive source of property tables for other solids, liquids, and gases was provided by Weast (1970).

It can be observed from Figure 1.3 that the thermal conductivity varies widely, depending on the type of substance, from 0.01 W/mK for gases up to 10 to 400 W/mK for metals. Metals have much higher thermal conductivities than liquids and gases. In metallic bonds, metal atoms give up their free outer-shell valence electrons to an electron gas and take up a regular arrangement. For example, Mg^+ ions are attracted to a negative electron cloud. These loosely held electrons in an electron cloud lead to high thermal conductivities, relative to liquids and gases, since the electrons move readily through the solid.

Ionic bonds are formed between metal and nonmetal ions. A metal gives up its valence electrons to the outer shell of the nonmetal. The positive metal ions and negative nonmetal ions are attracted to each other. For example, in sodium chloride, Na (one valence electron) reacts with Cl (seven valence electrons). A Cl stable outer shell of eight valence electrons is formed. The electrons are tightly held,

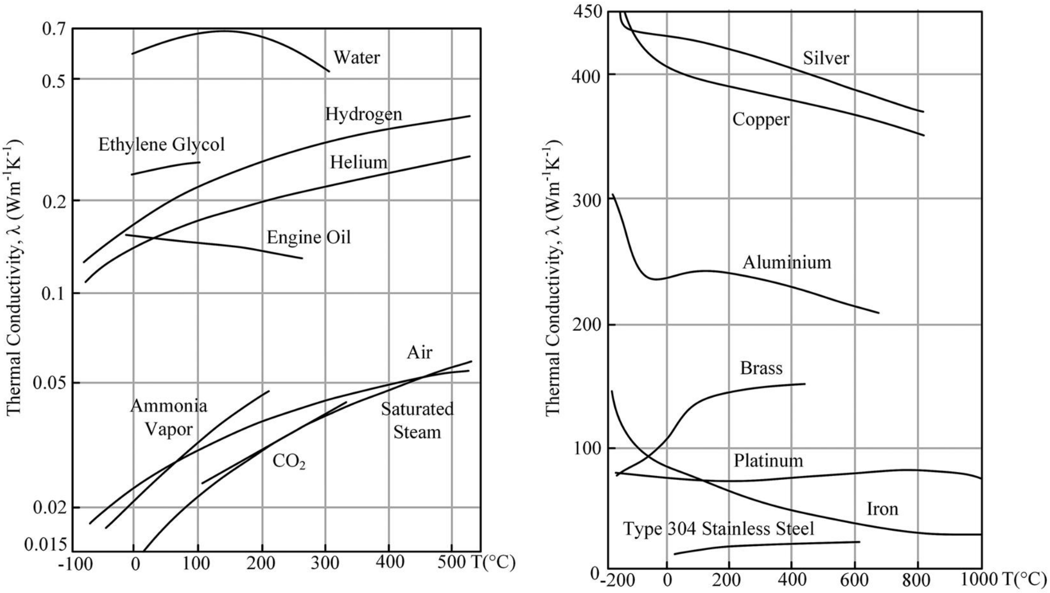

FIGURE 1.3 Thermal conductivities of fluids and solids. (adapted from Hewitt, Shires, & Polezhaev, 1997)

thereby yielding a lower thermal conductivity than solids with metallic bonds. Also, mechanical properties of the solids are affected by the nature of the ionic bonds. The electric fields of opposing ions in different planes repel each other, leading to more brittle characteristics and crystal fractures.

In covalent bonds, electrons are shared by participating atoms and held tightly together. In van der Waals bonds, secondary bonding between molecules is formed due to the charge attraction resulting from an asymmetrical charge distribution in the material. For example, water molecules are attracted to each other by negatively and positively charged sides of adjacent molecules. The type and nature of the atomic bonds within a substance have a direct impact on the magnitude of transport properties such as thermal conductivity, as well as their variations with temperature.

Fluid viscosity is a measure of the frictional resistance of a fluid element when applying a shear stress across adjacent layers of fluid that move parallel to each other at different speeds. Consider a layer of liquid between a lower, fixed plate and an upper plate, moving at a constant velocity, u. As the top plate moves, each layer of fluid will move faster than the layer below it, due to friction that is resisting their relative motion. Since the fluid applies on the top plate a force opposite to the direction of fluid motion, an external force is required to keep the top plate moving at a constant velocity. The magnitude of the force, F, divided by the plate area, A, is found to be proportional to the rate of shear deformation, or local shear velocity, $\partial u/\partial y$, according to *Newton's law of viscosity*,

$$\tau = \frac{F}{A} = \mu \frac{\partial u}{\partial y} \tag{1.8}$$

where the proportionality constant, μ, is the dynamic viscosity, τ is the shear stress, and y refers to the direction perpendicular to the plate (alternatively, $\nu = \mu/\rho$ is the kinematic viscosity). The viscosity expresses the resistance of the fluid to shear flows. For common fluids, such as oil and water, experimental measurements of the applied force confirm that the shear stress is directly proportional to the strain rate in the liquid. Fluids are called *Newtonian fluids* when the shear stress varies linearly with strain rate. For non-Newtonian fluids, such as paint films, water–sand mixtures, or liquid polymers, the applied shear stress and resulting fluid strain rates are related in a nonlinear manner.

In the evaluation of transport properties, macroscopic relationships are often used to approximate the microscopic transport phenomena. For example, heat conduction involves molecular fluctuations. Tracking of individual molecules, through their rotational, translational, and vibrational modes of motion, and assessing their energy exchange during these intermolecular motions, would be impractical. Therefore, a phenomenological law is used to relate the heat transfer rate, q, to the temperature gradient via a proportionality constant, k (thermal conductivity), similarly to Newton's law of viscosity. Phenomenological laws refer to approximations based on experimental observations that improve theoretical models but which cannot be proven rigorously from mathematical or first principles. The thermal conductivity is defined through a phenomenological law as a macroscopic approximation of microscopic intermolecular interactions which lead to the flow of heat by conduction.

Examples of other thermophysical properties are the specific heat, surface tension, coefficient of thermal expansion, and saturation pressure. Further properties are usually required when additional physical processes, such as a phase change, occur in a system. The specific heat at constant volume, c_v, and specific heat at constant pressure, c_p, for simple compressible substances as partial derivatives of the internal energy, $e(T, v)$, and enthalpy, $h(T, p)$, respectively:

$$c_v = \left.\frac{\partial e}{\partial T}\right|_{\upsilon} \tag{1.9}$$

$$c_p = \left.\frac{\partial h}{\partial T}\right|_{p} \tag{1.10}$$

where the subscripts υ and p refer to the variables, specific volume and pressure, held fixed during differentiation. The specific heats represent the amount of required thermal energy added by heat transfer to increase the temperature of the substance by 1 degree Celsius.

Surface tension, σ, refers to the tension of a surface film or droplet of liquid caused by the attraction of molecules in the surface layer with an adjoining fluid, tending to minimize surface area. The coefficient of thermal expansion, β, describes the fractional change in the size of a substance per degree of change of temperature at constant pressure. When a substance is heated, the molecular kinetic energy, vibrations, and movements increase, thereby creating a larger intermolecular separation and expansion of the fluid. Another property is the vapor saturation pressure, p_{sat}, which refers to the pressure at which liquid–vapor phase change occurs in a pure substance. The vapor pressure increases with saturation temperature since more thermal energy is required to break molecular bonds in the phase transition at higher temperatures.

1.4 HEAT CONDUCTION

Conduction is a primary mode of heat transfer. Thermal energy is transported by intermolecular motion during microscopic particle to particle interactions. The carrier motions are the translational, rotational, and vibrational movements of molecules. In this way, heat is transferred even though no net bulk movement of the substance occurs.

Consider heat conduction through a solid or liquid layer contained between a heated left wall and a cooled right wall in Figure 1.4. Molecular energy is transferred between the hotter side of the layer and the colder side. Also, consider an imaginary plane dividing the left and right sides of the layer into equal portions. On average, an equal number of molecules crosses the imaginary plane in both directions. However, the molecules coming from the hotter section possess more energy than the right side. As a result, there is a net transfer of thermal energy from the hotter side to the colder side, without any net transport of mass across the dividing plane.

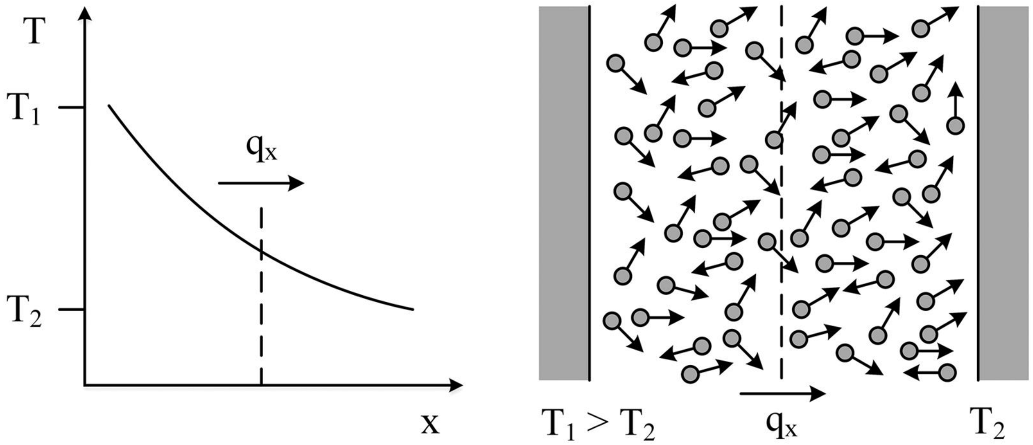

FIGURE 1.4 Schematic of heat conduction by molecular motion.

As discussed earlier, it is impractical and difficult to track the energy exchange among individual molecules during all of the intermolecular interactions. Therefore, the conduction heat flow is typically approximated by macroscopic averaged quantities, such as temperature and thermal conductivity, *k*. In a one-dimensional layer in Figure 1.4, the conduction heat transfer rate is governed by *Fourier's law*, named after Jean–Baptiste Fourier (1768–1830):

$$q''_{cond} = \frac{q_{cond}}{A} = -k\frac{\partial T}{\partial x} \approx -k\frac{\Delta T}{\Delta x} \tag{1.11}$$

where A, q''_{cond} and q_{cond} refer to the heat flow area, conduction heat flux (in units of W/m^2), and heat flow rate (in units of W), respectively. For steady-state one-dimensional problems, the temperature gradient in Equation (1.11) can be approximated by the temperature difference divided by the thickness of the one-dimensional layer, Δx. In multidimensional problems, the partial derivative in Equation (1.11) becomes a gradient of temperature and q becomes a vector of heat flow components in the x, y, and z directions. Further detailed analysis of heat conduction will be presented in Chapter 2.

The negative sign in Fourier's law indicates that a negative temperature gradient (i.e., decreasing temperature in the positive x-direction) multiplied by a minus sign yields a positive heat flow in the positive x-direction. This sign convention is illustrated in Figure 1.5. It can be observed that heat flows in the direction of decreasing temperature (as expected).

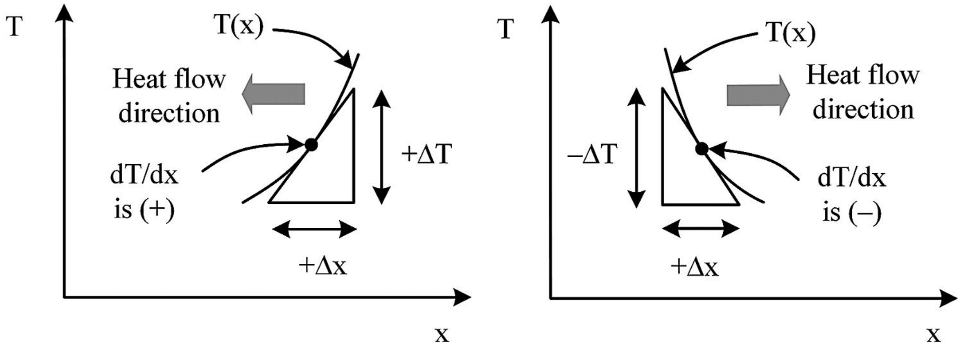

FIGURE 1.5 Sign convention for conduction heat flow.

From a one-dimensional energy balance under steady-state conditions, the rate of heat flow into the control volume balances the rate of outflow. Then based on Fourier's law for a homogenous material of constant thermal conductivity, the slope of the temperature profile at the inlet boundary matches the temperature slope (or gradient) at the outlet surface. The energy balance requires that both of these temperature gradients are constant and equal to one another, assuming zero sources/sinks of energy within the control volume. Therefore, the temperature profile must decrease linearly within the control volume to match the temperature slopes at both the inlet and outlet. This confirms the prior approximation of linearization of the temperature gradient in Equation (1.11) for steady-state one-dimensional problems.

In Fourier's law, the thermal conductivity is a transport property that characterizes the rate at which a substance can conduct heat. As discussed earlier, the variations of thermal conductivity with temperature are influenced by the molecular structure and intermolecular interactions. For example, the thermal conductivity of pure metals is often higher at low temperatures, but then increases to 100–400 W/mK at room temperature. Less uniform molecular structures in alloys often lead to lower thermal conductivities than pure metals. The thermal conductivity of water rises to about 0.7 W/mK at 200 K, but then gradually falls afterward. In gases, an opposite trend is usually observed where the thermal conductivity continually rises with temperature.

1.5 CONVECTION

Convective heat transfer refers to the combination of molecular diffusion (conduction) and bulk fluid motion (or advection). *Newton's law of cooling* states that the rate of convective heat flow from an object is proportional to the surface area and difference between its temperature and the surrounding ambient temperature. The proportionality constant is called the convective heat transfer coefficient, h. The name of this law is attributed to Sir Isaac Newton (1642–1727), largely due to the associated *Newton's laws of motion* in governing the motion of the fluid which drives the process of convective heat transfer.

Consider a hot surface at a constant temperature, T_s, placed into contact with a uniform fluid stream flowing at a temperature of T_f above the surface (see Figure 1.6). Then from Newton's law of cooling, the convective heat flux to the fluid stream from the surface of area A, is given by

$$q''_{conv} = \frac{q_{conv}}{A} = h\left(T_s - T_f\right) \tag{1.12}$$

where the convective heat transfer coefficient is denoted by h. Also, q''_{conv} and q_{conv} refer to the convective heat flux (in units of W/m^2) and convective heat flow rate (in units of W), respectively. This coefficient depends on many complex factors, including the fluid properties, geometrical

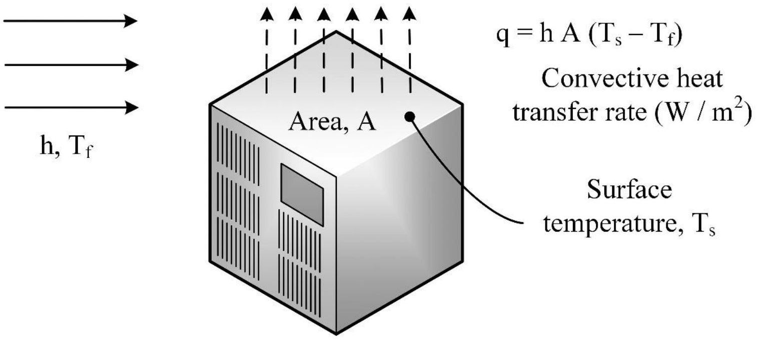

FIGURE 1.6 Convective heat transfer from a heated surface.

configuration, fluid velocity, surface roughness, and other factors to be discussed in forthcoming chapters of this book.

Heat transfer near the surface occurs through a *boundary layer*. The boundary layer is a thin layer of fluid close to the solid surface of the wall in contact with the moving fluid stream. The fluid velocity varies from zero at the wall (called a *no-slip condition*) up to the freestream velocity at the edge of the boundary layer, which approximately within 1% corresponds to the freestream velocity. The boundary layer thickness is the distance from the surface up to the point at which the velocity is 99% of the freestream velocity. To determine how the velocity and temperature fields change through a boundary layer, and more generally the entire flow field, the governing differential equations of mass, momentum, and energy conservation must be solved. These three-dimensional equations in Cartesian, cylindrical and spherical coordinates are shown in Appendix C and will be discussed further in Chapter 3.

Convection can be induced by external means (e.g., fan, pump, atmospheric winds), called *forced convection*, or else buoyancy induced motion in *free convection* (or *natural convection*). Mixed convection includes both forced and free convection. The magnitude of the convective heat transfer coefficient usually indicates the type of problem. Examples of typical values of convective heat transfer coefficients are listed below:

- 5–30 W/m^{2}K: free convection in air;
- 100–500 W/m^{2}K: forced convection in air;
- 100–15,000 W/m^{2}K: forced convection in water;
- 5,000–10,000 W/m^{2}K: convection with water condensation.

Thermophysical properties, phase change and turbulence are common factors leading to higher convective heat transfer coefficients. Thermodynamic and transport property tables which are commonly used for convection problems involving solids, liquids and gases are presented in Appendices D–F, respectively.

In convection problems, the fluid temperature in Newton's law of cooling must be carefully specified. The selection of an appropriate temperature depends on the type of problem under consideration. In the case of external flow, such as external flow over a circular cylinder, the fluid temperature is given by the freestream temperature. However, for internal flow, such as liquid flow in a pipe, the fluid temperature in Newton's law of cooling becomes the mean temperature of the fluid, since a freestream ambient temperature is not applicable. The mean temperature is obtained by spatial averaging of the velocity multiplied by temperature over the cross-sectional area of the pipe. It represents the mass flow weighted average temperature of the fluid in the pipe at a particular axial location within the pipe.

The convective heat transfer coefficient varies with position along a surface. The local convection coefficient is the value at a specific point on the surface, whereas the average or total heat transfer coefficient refers to an integrated value across the surface. For example, consider external flow over the top surface of the object in Figure 1.6. Define an x-coordinate as the position from the leading edge of the surface. As the gas flows over the surface, a thin boundary layer forms and grows. This affects the convective heat transfer process and leads to a variation of the "local" convection coefficient with position, x. Averaging of the local coefficient by integrating $h(x)$ over the surface area yields an average convection coefficient, which is more commonly used in convective heat transfer analysis. In Chapter 3, a further detailed analysis of convective heat transfer will be presented.

1.6 THERMAL RADIATION

Thermal radiation is a form of electromagnetic energy emitted by all matter above absolute zero temperature. These emissions arise from changes in the electron configurations of the constituent atoms and molecules at an atomic scale. Energy is transported through space as a result of this electron activity in the form of electromagnetic waves (or photons). Thermal radiation does not require the presence of a material medium between the objects exchanging energy by thermal radiation. It occurs most efficiently in a vacuum.

Consider a surface that exchanges heat by radiation and convection with the surroundings (see Figure 1.7). Incident radiation arrives on the surface from the surroundings. Some of the incident radiation is reflected from the surface, while other incoming energy is absorbed into the surface and potentially transmitted through the object. Some of the electromagnetic waves emitted from the surface may not be absorbed by other surrounding objects if they are not in the direct line of sight of another surface.

In heat conduction problems, the main governing equation was Fourier's law, whereas Newton's law of cooling was adopted for convective heat transfer problems. In radiative heat transfer, a primary governing equation is *Stefan–Boltzmann's law*. For a surface at a temperature of T_s (in absolute units, i.e., Kelvin or Rankine units) and a surface emissivity of ε, the rate of radiative heat transfer from the surface is given by

$$q''_{rad} = \frac{q_{rad}}{A} = \varepsilon\sigma T_s^4 \tag{1.13}$$

where A is the surface area and $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4 is the Stefan–Boltzmann constant, named after Jozef Stefan (1835–1893) and Ludwig Boltzmann (1844–1906). The radiative heat flux (in units of W/m^2) and radiation heat flow rate (in units of W) are represented by q''_{rad} and q_{rad}, respectively. The surface emissivity, ε, represents the surface's ability to emit radiation in comparison to an ideal emitter (i.e., $\varepsilon = 1$ for a *blackbody*). Radiative properties of various surfaces and gases are presented in Appendix G.

Radiation heat transfer between two surfaces or objects, at temperatures T_1 and T_2, respectively, can be obtained based on the net radiation exchange between the surfaces,

$$q''_{rad} = \varepsilon\sigma\left(T_1^4 - T_2^4\right) \tag{1.14}$$

Alternatively, linearizing this expression to follow a similar form as Newton's law of cooling,

$$q''_{rad} = h_r\left(T_1 - T_2\right) \tag{1.15}$$

where the effective radiation heat transfer coefficient, h_r, is given by

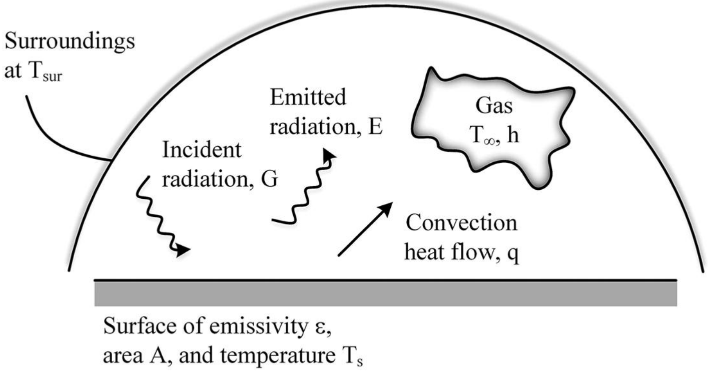

FIGURE 1.7 Radiative heat transfer between and a surface and its surroundings.

$$h_r = \varepsilon\sigma\left(T_1 + T_2\right)\left(T_1^2 + T_2^2\right) \tag{1.16}$$

In this way, radiation and convection coefficients can be combined together in a thermal analysis involving both modes of heat transfer.

In addition to radiative heat transfer between surfaces in view of each other, Stefan–Boltzmann's law can also be used to predict radiation exchange among objects or groups of surfaces. For example, consider the net radiation exchange between a small object at a temperature of T_1 and a surrounding cavity at T_2, where $T_1 < T_2$. Since all of the radiation emitted from the small object (object 1) is absorbed by the surrounding cavity (object 2), the surroundings behave like a blackbody at T_2. Then the net radiative heat gain by the small object is given by

$$q_{rad} = \varepsilon\sigma A\left(T_2^4 - T_1^4\right) \tag{1.17}$$

where A refers to the surface area of object 1 and ε denotes the emissivity of the object.

The thermal physics of radiation is based on the random movements of atoms and molecules, composed of charged particles (protons and electrons), whose movement leads to the emission of electromagnetic waves that carry energy away from the surface. In Chapter 4, more detailed analysis, physical processes, governing equations, and methods of analysis of radiation heat transfer will be presented and discussed.

1.7 PHASE CHANGE HEAT TRANSFER

There are four states of matter – solid, liquid, gas, and plasma – although matter on Earth exists mostly in the former three phases (solid, liquid, and gas). Plasma is a gaseous mixture of negatively charged electrons and highly charged positive ions. Unlike the other three states of matter, plasma does not exist naturally on Earth under normal conditions. It can be artificially generated by heating neutral gases to very high temperatures or subjecting a gas to a strong electromagnetic field.

A phase is a distinctive state of a substance. All matter can change among any of the phases although in some cases this may require extreme temperatures or pressures. There are six distinct forms of phase change which occur at different temperatures for different substances. These changes of phase are given as follows: freezing or solidification (from liquid to solid); melting (solid to liquid); boiling or vaporization (liquid to gas); condensation (gas to liquid), sublimation (directly from a solid to a gas without passing through the liquid phase); and deposition (gas to solid without passing through the liquid phase). The heat released or absorbed at the phase interface during a phase change process is called the *latent heat*. For example, the latent heat of vaporization is absorbed by the liquid during a boiling process, and the latent heat of fusion is released by a liquid during a solidification process.

A phase diagram is often used in the analysis of phase change heat transfer. It is a type of chart used to illustrate the thermodynamic properties (pressure, temperature, volume, etc.) at which distinct phases occur and co-exist at equilibrium. Consider a typical phase diagram for solid, liquid and gas phases in Figure 1.8. The figure depicts a process pathline from region 1 to 5 in which an initially solid substance is heated at a constant pressure until it eventually becomes entirely gas. In the solid phase (region 1), heat is added and the temperature increases until it eventually reaches the melting temperature. In region 2, further heat input leads to solid–liquid phase change at a constant temperature until eventually all of the solid has melted entirely. Then continued heating causes the temperature to increase in region 3 through sensible heating (i.e., change of temperature) until the liquid temperature reaches the saturation point (onset of boiling). At this next onset of phase change, heat is added to sustain the boiling process (region 4) at constant temperature until the saturated vapor point is reached, beyond which further heating is transferred as sensible heating and

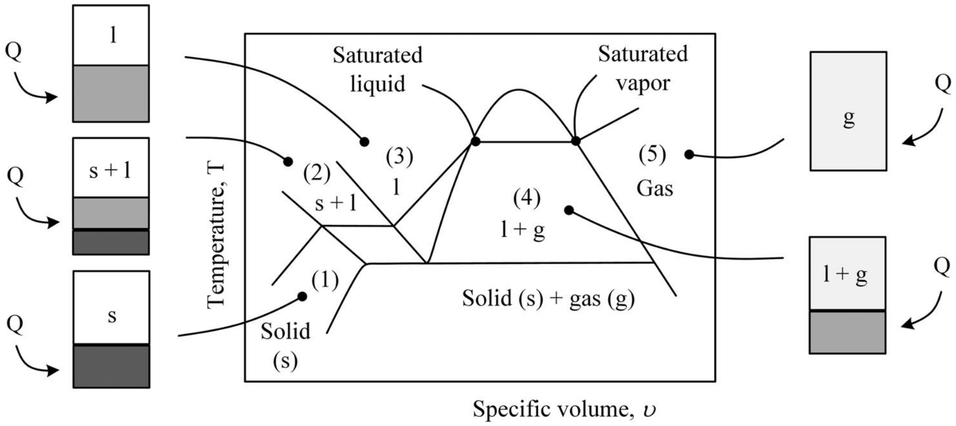

FIGURE 1.8 Phase diagram for solid–liquid–gas pure material.

temperature increase of the gas (region 5). If these steps are conducted at several different pressures, then the resulting measurements and state points from each experiment (i.e., temperature, pressure, specific volume, etc.) can be joined together. These resulting data points would provide the entire phase diagram for phase change of a pure material from a solid to a gas.

In region 4 of Figure 1.8, the boiling process begins when individual bubbles form and grow along the heated surface. The thin liquid layer beneath a growing bubble is vaporized after which the bubble detaches from the wall and ascends by buoyancy through the liquid. During this process, heat is transferred by conduction from the heated surface to the liquid layer in contact with the surface. The heat transfer process at the wall may be represented by a combination of liquid and vapor periods in contact with the wall which are characterized by the frequency and sizes of detaching bubbles. The average heat flux over both periods depends on the frequency of bubble departure. The changing liquid layer thickness beneath a bubble is related to the heat conduction process in the liquid and latent heat absorbed by the adjacent vapor phase. Further detailed analysis of boiling and condensation processes will be presented in Chapter 5.

Melting occurs in region 2 of Figure 1.8. The importance of solidification and melting in our everyday lives is evident in various ways. For example, consider water and its density difference between the solid and liquid phases. Unlike many materials, its density is higher in the liquid phase. This property arises from a unique angular arrangement of hydrogen and oxygen atoms in a water molecule. As a result, in the natural environment, ice freezes and floats on the top of oceans and lakes. Otherwise, if solidification of water occurred with a higher density in the solid phase (like most metals), freezing ice would descend. This could cause oceans, lakes, and rivers to potentially completely freeze, so that life on Earth may not exist. So this relatively basic anomaly of the freezing process illustrates the importance of solidification and melting in our everyday experiences. In Chapter 7, further detailed investigation of solid–liquid phase change processes will be presented.

1.8 MASS TRANSFER

Similar to the flow of heat by conduction due to a change of temperature, the driving force for mass transfer by diffusion is a difference of species concentration in a mixture. In a manner similar to Fourier's law for the conduction of heat, Fick's law of mass transfer states that the species diffusion flux from regions of high concentration to regions of low concentration has a magnitude that is proportional to the concentration gradient. For example, in a two-component mixture, the solute diffuses from a region of high concentration to a region of low concentration down the concentration gradient, where the constant of proportionality is called the *mass diffusivity*.

There is a close analogy between heat and mass transfer by diffusion. When the term "mass transfer" is used in this context, it refers to the relative movement of species in a mixture due to the presence of concentration gradients, not the movement of mass due to bulk fluid motion. As with the transfer of heat and momentum, the extent of mass transfer is affected by flow patterns within the system and diffusivities of the species in each phase. Also, mass transfer coefficients are calculated and applied similarly as the convective heat transfer and skin friction coefficients.

Recall that intermolecular energy exchange by heat conduction from a hot to cold side of a layer was discussed earlier. Consider instead an analogous process of mass diffusion of interstitial atoms (component A) through a solvent (component B), across an imaginary plane over a distance of Δx. A layer of fluid with molecular motion and interstitial atoms migrate across the imaginary plane (Figure 1.9).

Let Γ_A, N, N_1, and N_2 refer to the average number of interstitial atom jumps per second in random directions, number of adjacent sites (i.e., $n = 6$ in three dimensions), number of A atoms per unit area in plane 1 (immediately to the left of the imaginary plane), and the number of A atoms per unit area in plane 2 (immediately to the right of the imaginary plane), respectively. Then the net flux of interstitial atoms across the imaginary control surface can be written as

$$j''_A = \frac{j_A}{A} = j''_{A,1\to 2} - j''_{A,2\to 1} = \frac{1}{N}\Gamma_A N_1 - \frac{1}{N}\Gamma_A N_2 \tag{1.18}$$

where the double prime notation (″) represents a flux quantity (per unit area). A positive expression for j_A indicates that there is a net species mass flux from the left side to the right side of the imaginary plane.

The concentration of A atoms at planes 1 and 2, $C_{A,1}$ and $C_{A,2}$, respectively, is

$$C_{A,1} = \frac{N_1}{\Delta x};\ C_{A,2} = \frac{N_2}{\Delta x} \tag{1.19}$$

Performing a Taylor series expansion of the concentration of A atoms at plane 2, in terms of the concentration nearby to the left at plane 1,

$$C_{A,2} = C_{A,1} + \Delta x \frac{\partial C_A}{\partial x} + \cdots \tag{1.20}$$

Combining these equations, neglecting higher order terms, and multiplying by the surface area, A, yields the following total mass diffusion flow of interstitial atoms across the imaginary plane,

$$j_A = -\left(\frac{\Gamma_A}{6}\Delta x^2\right) A \frac{\partial C_A}{\partial x} \tag{1.21}$$

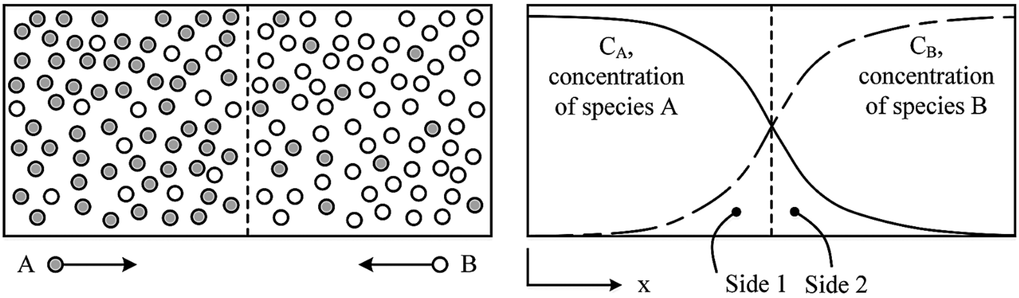

FIGURE 1.9 Schematic of mass diffusion of interstitial atoms.

where the coefficient in parentheses is the mass diffusivity, or mass diffusion coefficient (denoted by D_{AB}) of constituents A and B. This equation can be generalized to the following one-dimensional form of *Fick's law*, named after Adolf Fick (1829–1901):

$$j_A'' = \frac{j_A}{A} = -D_{AB}\frac{dC}{dx} \tag{1.22}$$

This result is analogous to Fourier's law of heat conduction earlier in this chapter. Fick's law describes the diffusion of mass down a species concentration gradient in a manner analogous to Fourier's law, which prescribes the diffusion of heat down a temperature gradient. A more detailed analysis of mass transfer processes is presented in a classic book by Bird, Stewart, and Lightfoot (2007) and will be described in forthcoming chapters, particularly for chemically reacting flows in Chapter 8.

REFERENCES

Bird, R.B., W.E. Stewart and E.N. Lightfoot. 2007. *Transport Phenomena*, 2nd Edition, New York: John Wiley & Sons.

Hewitt, G.F., G.L. Shires and Y.V. Polezhaev, Eds. 1997. *International Encyclopedia of Heat and Mass Transfer*, Boca Raton: CRC Press/Taylor & Francis Group.

Weast, R.C., Ed., 1970. *CRC Handbook of Tables for Applied Engineering Science*, Boca Raton: CRC Press/Taylor & Francis Group.

PROBLEMS

1. Consider touching and holding two different materials, such as a piece of steel and a piece of wood, which were both initially at the same subzero temperature. Would either material "feel" colder than the other? Explain your response.
2. The heat loss through a common brick wall in a building is 1,800 W. The height, width, and thickness of the wall are 3, 8, and 0.22 m, respectively. If the inside temperature of the wall is 22°C, find the outside surface temperature of the wall.
3. Heat flows in a one-dimensional direction through a layer of material of unknown thermal conductivity. Explain a method to estimate the material's conductivity based on temperature measurements recorded by thermocouples at various positions within the layer of material.
4. The heat loss through a brick wall with an outside temperature of 16°C is 800 W. The height, width, and thickness of the wall are 4, 10, and 0.3 m, respectively. What is the inside surface temperature of the wall?
5. The walls of a house are made from a half-inch sheet of gypsum sheetrock, 4 inches of foam insulation, and a half inch of light fiberboard. Heat loss from the house during a winter day occurs with an average convective heat transfer coefficient of 200 W/m^2. The outside air temperature is –5°C.
   a. If the inside temperature is to be maintained at 20°C, how much internal heating on average is required per square meter of wall area?
   b. If the house has 5,000 ft^2 of wall area and the floor and ceiling are assumed adiabatic, how much energy would the house require per day in kWh to stay heated at 20°C?
6. A steel plate in a manufacturing plant is removed from a furnace at a temperature of 650°C. Hoss loss from the plate to the surrounding air at 20°C occurs at a rate of 36 kW/m^2. Find the average plate temperature at a location where the convection coefficient is 30% higher than the initial value where the plate was initially removed from the furnace.
7. A heat loss of 1,600 W is experienced through a glass window (1.4 m width × 2.6 m height) at the outer wall of a building. The convection coefficient is 30 W/m^{2}K. Estimate the window temperature when the outside air temperature reaches –10°C.

8. An aluminum sheet leaves a hot-roll section (called the entry point) of a mill at a temperature of 620°C. The surrounding air temperature is 20°C. For a desired uniform cooling rate of 38 kW/m^2 across the sheet, estimate the aluminum surface temperature at a position where the convection coefficient is 40% higher than the value at the entry point.
9. Heat loss occurs by convection through the surface of an object. The surface temperature is 1,000°C while the surrounding fluid is at 100°C. The convective heat transfer coefficient is 2,000 W/m^2 K and the surface heat flux is 1,800 kW/m^2.
   a. Determine the heat flux by first converting both the surface temperature and surrounding fluid temperatures to Kelvin units prior to finding ΔT and the heat flux. Then find the heat flux without converting any temperatures to Kelvin. Explain why the results are the same and correct even though the units are wrong.
   b. Find the ΔT value in Celsius degrees and then convert it to Kelvin units prior to finding the heat flux. Explain why this result is incorrect.
   c. Then repeat the above questions but consider the same object in the same surroundings while emitting radiation as a blackbody. Find the resulting radiative heat flux.
10. Air flows across a tube surface (emissivity of $\varepsilon = 0.9$) in a heat exchanger. The ambient air temperature and convection coefficient are 20°C and 120 W/m^2K, respectively. Compare the heat transfer by convection and radiation at surface temperatures of (a) 60°C, (b) 400°C, and (c) 1,300°C.
11. A metal block (0.8 m × 1.2 m × 0.4 m) in a furnace is heated by radiation exchange with the walls at 1,100°C. The block temperature is 420°C. The walls are approximated as blackbodies. Find the net rate of heat transfer to the block due to radiation exchange with the walls. How would the result be altered if the block was placed in a corner of the furnace?
12. Superheated steam at 240°C flows through an uninsulated pipe in a basement hallway at 20°C. The pipe emissivity is 0.7 and the coefficient of free convection is 20 W/m^2K. Find an expression for the total heat loss from the pipe (per unit length of pipe) in terms of the pipe diameter.
13. Water is vaporized in a distillation unit and superheated steam exits from the heating tank through a single tube. Once cooled, this steam is condensed and collected in a second storage tank of purified water. If the electrical element provides a net heat input of 400 W to the water, how many liters of distilled water are produced after 6 h of operation? Assume that external heat losses from the heating tank are negligible and that heat input is directed entirely into phase change (i.e., 6 h refers to the time taken once the water reaches the saturation temperature).
14. An ice-making machine uses a refrigeration system to freeze water with a net heat removal rate of 11 kW. How much time is required to produce 1,200 kg of ice? It may be assumed that water is cooled to slightly above 0°C before it enters the refrigeration unit.
15. An industrial heat exchanger is used to vaporize water by electrical heating of 3,900 W of heat input to the storage tank of saturated water. How much vapor is generated after 5 hours of operation, assuming that the storage tank is well insulated, and all heat input is directed to the vaporization process?
16. The diameter of the sun is approximately 1.39×10^9 m. A heat flux of about 1,353 W/m^2 from the sun's radiant heat reaches the outer atmosphere of the earth. Estimate the radius of the orbit of Earth's trajectory around the sun (i.e., Earth's distance from the sun). The sun can be approximated as a blackbody at 5,800 K.
17. A cylindrical spacecraft 7.5 m long and 2.5 m in diameter is orbiting the earth where the incoming solar radiation is 1,300 W/m^2. The surface of the spacecraft has an absorptivity and emissivity of 0.5 and 0.5, respectively. It can be assumed that the ship's outer surface has a uniform temperature.

   a. What is the steady-state temperature of the spacecraft if its longer side is exposed continuously to the sun?
   b. What additional heat generation/removal is needed to maintain an average temperature of 15°C when the longer side is exposed to the sun?
   c. Repeat the analysis if the small circular end is exposed to the sun instead of the longer side.
18. The wall of a walk-in freezer in a meat processing plant contains insulation with a thickness of 10 cm and a thermal conductivity of 0.02 W/mK. The inside wall temperature is –10°C. The outside air temperature and convection coefficient (outside) are 20°C and 5 W/m^{2}K, respectively. What additional thickness of insulation is required to reduce the heat gain (into the freezer) by 10%?
19. The temperature of the indoor side of a stone concrete wall in a building is 20°C. The wall thickness is 20 cm. The convection coefficient and ambient air temperature for the outdoor air are 50 W/m^{2}K and –5°C, respectively. Determine the outer wall temperature of the concrete wall.
20. A thin plate is exposed to an incident radiation flux of 2 kW/m^2 on its top surface. The bottom surface is well insulated. The exposed surface absorbs 80% of the incident radiation and exchanges heat by convection and radiation to the ambient air at 290 K. If the plate's emissivity is 0.8 and the wall temperature is 350 K, estimate the convection coefficient, *h*, under steady-state conditions.
21. The gas temperature in an autoclave is measured with a thermocouple (a wire of 1-mm outer diameter). Composite material components for an aircraft are cured in the autoclave. The junction of the thermocouple ($\varepsilon = 0.6$) is located at the end of the thermocouple wire, which protrudes into the autoclave and exchanges heat by radiation with the walls at 410°C. The freestream temperature and convection coefficient of gas flow past the thermocouple are 180°C and 30 W/m^{2}K, respectively. What error, arising from the difference between the thermocouple reading and gas temperature, is expected from the measurement of the gas temperature? How can this temperature measurement error be reduced? Conduction losses through the thermocouple wire can be neglected.
22. Resistive heat generation occurs within an overhead power transmission cable. Air flows past the cable with a freestream temperature and convection coefficient of –6°C and 80 W/m^{2}K, respectively. For a specified copper cable (3-cm diameter), estimate the required heat generation rate (per unit length of cable) to maintain the surface temperature above 0°C.
23. The top cover of a solar collector plate is exposed to ambient air at 20°C with a convection coefficient of 6 W/m^{2}K. A surface coating is developed and applied to modify the radiative properties of this absorbing cover plate. The incident solar radiation is 860 W/m^2 and the outer surface temperature of the absorber plate is 70°C.
    a. What plate emissivity is required to provide a conduction heat flux of 500 W/m^2 through the absorber plate?
    b. What is the proportion of heat exchange by radiation between the cover plate and surroundings (ambient air) relative to the convective heat loss from the plate?
24. Heating, ventilating, and air conditioning (HVAC) equipment maintains a room in a building at desired temperature conditions. Identify the heat transfer processes and appropriate energy balance(s) to find the steady-state temperature of the room in the building.
25. Aircraft icing may occur when an aircraft passes through a cloud containing supercooled droplets. Describe the heat flows to/from an iced aircraft surface and write an energy balance for the surface based on these heat flows.
26. Particles of pulverized coal are injected and burned in a boiler of a thermal power plant. Identify and briefly describe the relevant heat transfer modes that contribute to an energy balance for a pulverized coal particle.

27. Identify the relevant heat transfer processes that arise in a thermal balance of the human body. Explain how these processes are combined in an overall energy balance to find the total heat losses from the body.
28. Various technical challenges are encountered by thermal engineers in the design of more efficient systems for heating and cooling of buildings. Perform a technical literature (including journals and magazines) to describe the main technological advances and challenges facing thermal engineers in the development of next-generation HVAC systems.

2 Heat Conduction

2.1 INTRODUCTION

In this chapter, the governing equations and methods of analysis of conduction heat transfer will be presented. One-dimensional and two-dimensional problems will be analyzed, as well as steady-state and transient problems. Fundamental transport processes will be investigated. Also, advanced solution methods such as separation of variables, conformal mapping, and solutions in orthogonal curvilinear coordinates, will be outlined.

As discussed in the prior chapter, conduction heat transfer occurs at a molecular level when thermal energy is exchanged among intermolecular interactions by rotational, vibrational, and translational energies at a molecular scale. The temperature of the substance characterizes the thermal energy of this atomic motion. When collisions occur among molecules, energy is passed from more energetic to less energetic neighboring atoms, thereby allowing thermal energy to travel through the substance. Since the atomic structure is more closely packed in solids, the atoms are closer together and unable to move compared to liquids and gases, where atoms can more easily move past each other. As a result, there are fewer collisions and a lower resulting thermal conductivity in liquids and gases.

As introduced in the previous chapter, heat conduction is governed by Fourier's law, which relates the magnitude and direction of the heat flux to the temperature gradient. A negative sign is placed in front of the temperature gradient to indicate that heat flows in the direction of decreasing temperature. Or alternatively, a positive temperature gradient suggests that temperature is increasing, and heat must then flow in the opposite (negative) direction. The proportionality constant between the heat flux and temperature gradient is called the thermal conductivity. This chapter will examine various formulations and solutions of Fourier's Law and the heat equation for a range of coordinate systems.

The flow of heat by conduction depends on three key factors: the temperature gradient, cross-sectional area of heat flow, and thermal conductivity. The temperature gradient prescribes the rate and direction of temperature change at a particular location. The transfer of heat continues until there is no longer any temperature difference within the domain at which point a state of thermal equilibrium is reached. The second factor, cross-sectional area, directly affects the amount of heat that can flow across the surface. Also, the properties of the material affect the heat flow, particularly the thermal conductivity. As discussed in the previous chapter, solids are better conductors than liquids and gases, while materials such as metals are better conductors than solids such as wood, paper, or cloth. Materials that are poor heat conductors are called insulators. Air is an effective insulator because it can be readily contained within any space so it is often used as an insulator in enclosed spaces, such as double-pane glass windows.

When graphically displaying temperature results, *isotherms* (lines of constant temperature) are often used. Plotting the isotherms within a domain shows the regions of high and low temperatures. Heat flows by conduction in a direction perpendicular to a local isotherm. In other words, heat flows in a direction of steepest temperature descent. Thus, the heat flow lines and isotherms are mutually perpendicular to each other. The heat flow pathlines may be constructed by joining perpendicular crossing points between successive isotherms.

Topics to be covered in this chapter include the formulation and analysis of the heat equation, thermal resistance networks, fins and extended surfaces, advanced solution methods for multidimensional problems, and transient heat conduction. The reader is also referred to other classic books on conduction heat transfer by Carslaw and Jaeger (1959), Yener and Kakac (2008), and Han (2011).

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2.2 ONE-DIMENSIONAL HEAT CONDUCTION

2.2.1 HEAT CONDUCTION EQUATION

The heat conduction equation may be derived from the general form of an energy balance for a control volume,

$$\dot{E}_{cv} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \tag{2.1}$$

From left to right, the terms represent the transient accumulation of energy within the control volume, energy inflow and outflow from the control volume, and internal energy generation. A sample control volume of the energy balance is illustrated in Figure 2.1.

The transient energy change, $\dot{E}_{cv}$, may be written in terms of a temperature change using the definition of the specific heat (Chapter 1), multiplied by the volume size, *Adx*. The energy inflow, $\dot{E}_{in}$, can be expressed by the conduction heat flux, multiplied by the surface area. The energy outflow can be written similarly, along with a Taylor series expansion about the influx value. Also, the total internal energy generation rate is expressed by the volumetric generation rate multiplied by the size of the volume. Substituting these individual terms into the energy balance,

$$\rho c_p A dx \frac{\partial T}{\partial t} = q_x'' A - \left[q_x'' A + \frac{\partial}{\partial x}\left(q_x'' A\right) dx + \cdots \right] + \dot{q} A dx \tag{2.2}$$

where refers to $\dot{q}$ refers to the volumetric heat generation rate.

Furthermore, Fourier's law may be used, as defined earlier in Chapter 1. In this case for the control volume at location *x*,

$$q_x'' = -k \frac{\partial T}{\partial x} \tag{2.3}$$

Substituting this expression into the energy balance, canceling terms, and neglecting higher order terms,

$$\rho c_p A dx \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} A \right) dx + \dot{q} A dx \tag{2.4}$$

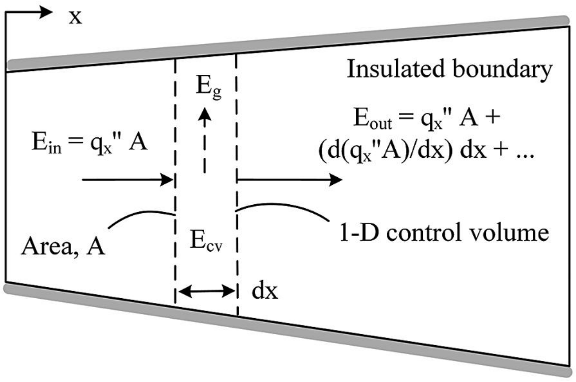


FIGURE 2.1 Schematic of one-dimensional control volume energy balance.

Then, assuming a constant thermal conductivity and cross-sectional area, and dividing by Adx, yields the following form of the one-dimensional heat conduction equation:

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + \dot{q} \tag{2.5}$$

In addition to the above governing heat equation, a problem definition also requires boundary conditions for completion of the problem specification. For heat conduction problems, there are three main types of boundary conditions: Dirichlet, Neumann, and Robin boundary conditions (see Figure 2.2). For a *Dirichlet condition*, a surface temperature is specified at the boundary of the domain. For example, a Dirichlet boundary condition at a wall ($x = 0$) can be represented by $T(0, t) = T_w$ where T_w is the known or specified wall temperature.

The second type of boundary condition is a heat flux or temperature gradient specified condition (called a *Neumann condition*). For example, at a solid boundary ($x = 0$), a constant specified wall heat flux balances the rate of heat flow into the solid. Applying Fourier's law, the Neumann condition becomes

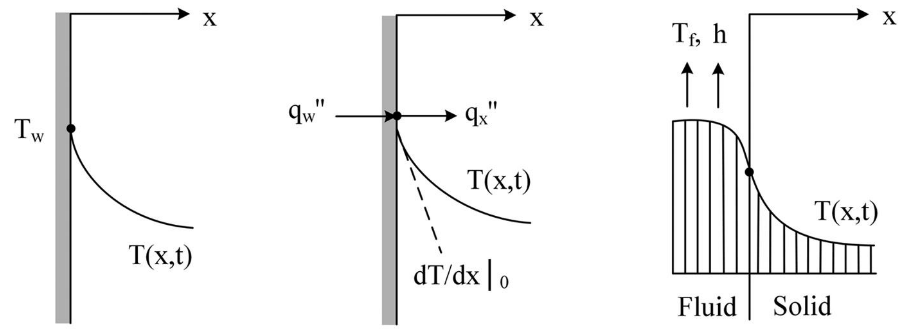
$$\left.\frac{\partial T}{\partial x}\right|_0 = -\frac{q''_w}{k} \tag{2.6}$$

where q'''_w refers to the known or specified wall heat flux. If the temperature gradient at the boundary is zero, this represents an adiabatic (or insulated) boundary condition, as there is no heat flow across the boundary. An insulated boundary is a special case of a Neumann boundary condition.

The third type of boundary condition is a convection condition (called a *Robin condition*). This type is a combination of the previous two types, where both the temperature and the temperature gradient appear in the boundary condition. It is a commonly encountered condition at a convective boundary. At the boundary between a solid and a fluid, heat conduction within the solid (characterized by the temperature gradient in the solid) balances the convective heat flow which is represented by Newton's law of cooling. For example, at a solid boundary ($x = 0$), a heat balance between the conduction heat flux in the solid from Fourier's law and the convective heat flux in the fluid yields

$$-k\left.\frac{\partial T}{\partial x}\right|_0 = h\left(T(0,t) - T_f\right) \tag{2.7}$$

where T_f refers to the fluid temperature.



$T(0,t) = T_w$
Dirichlet condition

$dT/dx|_0 = -q''_w / k$
Neumann condition

$-k(dT/dx)|_0 = h(T(0,t) - T_f)$
Robin condition

FIGURE 2.2 Types of boundary conditions.

In a similar manner, a linearized radiation condition at a boundary, coupled with heat conduction in the solid, would lead to a Robin-type boundary condition. As expected, problems involving a Robin condition are often more difficult since both the temperature and its gradient are unknown at the boundary. Robin conditions often lead to a *conjugate problem.* A conjugate problem refers to classes of problems where convection in the fluid and conduction in the adjacent solid are coupled and must be solved simultaneously, typically through an iterative procedure involving the Robin condition at the boundary. Both heat transfer in the solid and fluid must be solved separately and then coupled together iteratively by the Robin condition.

2.2.2 STEADY CONDUCTION IN A PLANE LAYER

Consider steady-state heat conduction through a plane layer such as the wall of a building. If the wall surface temperatures are uniform, then the heat transfer will occur primarily in the direction between the outer and inner surfaces. In this case, the temperature in the wall depends on one direction only (x-direction) and therefore can be written as $T(x)$.

Performing an energy balance for the wall,

$$\dot{E}_{wall} = \dot{q}_{in} - \dot{q}_{out} \tag{2.8}$$

where the terms represent, from left to right, the rate of energy change of the wall with time, and rates of heat inflow and outflow from the wall, respectively. Under steady-state conditions, the transient term becomes zero, and therefore the rate of heat inflow equals the outflow, or in other words, the heat transfer rate remains constant.

For a wall of thickness, L, and thermal conductivity, k, Fourier's Law is expressed as

$$q''_{cond} = -k\frac{dT}{dx} \tag{2.9}$$

Since this heat transfer rate is constant, it implies that dT/dx is also constant, and so the temperature varies linearly with x. For steady-state heat conduction, the temperature distribution is linear (see Figure 2.3).

Separating variables in Fourier's Law and integrating across the wall,

$$\int_0^L q''_{cond}dx = -\int_{T_1}^{T_2} kAdT \tag{2.10}$$

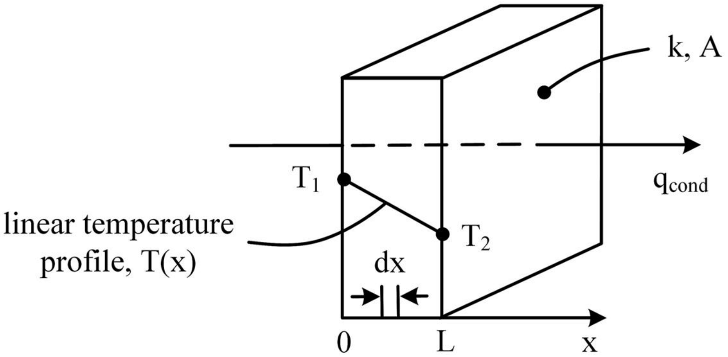


FIGURE 2.3 Heat conduction and temperature distribution through a plane wall.

Since the integrands are constant,

$$q_{cond} = -kA\left(\frac{T_2 - T_1}{L}\right) \tag{2.11}$$

This indicates that the heat transfer rate is proportional to the thermal conductivity, wall area, and temperature difference between the surfaces, but inversely proportional to the wall thickness, for steady-state heat conduction.

2.3 THERMAL RESISTANCE AND SHAPE FACTOR

The concept of a *thermal resistance* is useful for the analysis of one-dimensional steady-state heat transfer problems. The thermal resistance is analogous to the concept of electrical resistance. In electrical systems, a widely used equation is *Ohm's law*,

$$I = \frac{V_1 - V_2}{R_e} \tag{2.12}$$

where I, $V_1 - V_2$, and R_e refer to the electrical current, potential (voltage) difference, and electrical resistance, respectively. From Ohm's law, the current flow is proportional to the driving potential (a difference between the high and low applied voltages) and inversely proportional to the resistance through the electrically conducting medium.

The thermal analogy of Ohm's law may be written as follows:

$$q = \frac{T_1 - T_2}{R_t} \tag{2.13}$$

In this case, the analog of the electrical current is the heat transfer rate, q. Rather than a voltage difference, a temperature difference, $T_1 - T_2$, represents the driving potential for the heat flow. Also, the value R_t refers to the *thermal resistance*, instead of electrical resistance. Then this above thermal analogy and associated concept of a resistance network can be used to analyze steady-state heat transfer problems. Like electrical circuits, a thermal circuit can be formulated with resistances combined together in series or parallel to analyze each stage of heat transfer. Using this approach, thermal circuit diagrams involving several heat flows may be constructed to analyze heat transfer problems in a similar way to how electrical circuits are used in electrical problems.

For one-dimensional heat conduction in Cartesian coordinates, Fourier's law and (2.13) imply that the thermal resistance of heat conduction through a plane layer of thickness L can be written as

$$R_{t,cond} = \frac{L}{k} \tag{2.14}$$

For convective heat transfer, the thermal resistance of a boundary layer is obtained from Newton's law of cooling. In this case, the convective heat transfer rate can be written as a temperature difference (between the surface and fluid) divided by the convection thermal resistance, $R_{t,conv}$, where

$$R_{t,conv} = \frac{1}{h_{conv}A} \tag{2.15}$$

Similarly, for radiation problems where the heat transfer coefficient is linearized (as described previously in Chapter 1), the radiative thermal resistance becomes

$$R_{t,rad} = \frac{1}{h_{rad}A} \tag{2.16}$$

Similar expressions can be obtained for other geometrical configurations and combined systems with multiple resistances in series and/or parallel. The following example demonstrates how the thermal resistance can be calculated for a cylindrical geometry.

Example: Radial Conduction in a Circular Tube

A fluid of temperature T_f flows through a tube of radius r_1 (see Figure 2.4). The ambient air temperature around the tube is $T_\infty > T_f$. An insulation layer is added on the outside surface of the tube. As the insulation later increases, the conduction resistance also increases, but the convection resistance decreases due to a larger outer surface area. Is there an optimal thickness of insulation that minimizes the total thermal resistance?

The steady-state heat conduction equation in cylindrical coordinates is given by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \tag{2.17}$$

where r refers to the radial position. Integrating this equation twice yields the following general solution,

$$T(r) = C_1 \ln(r) + C_2 \tag{2.18}$$

The coefficients of integration may be obtained by applying specified temperatures at the inner and outer radii, $T(r_1) = T_1$ and $T(r_2) = T_2$, respectively, thereby yielding

$$T(r) = \frac{T_1 - T_2}{\ln(r_1/r_2)}\ln\left(\frac{r}{r_2}\right) + T_2 \tag{2.19}$$

Differentiating this equation with respect to r and using Fourier's law,

$$q_r = -kA\frac{dT}{dr} = \frac{T_1 - T_2}{\ln(r_2/r_1)/(2\pi Lk)} = \frac{T_1 - T_2}{R_{t,cond}} \tag{2.20}$$

where L is the tube length. Adding this conduction resistance and the convection resistance yields the following total thermal resistance per unit length of tube,

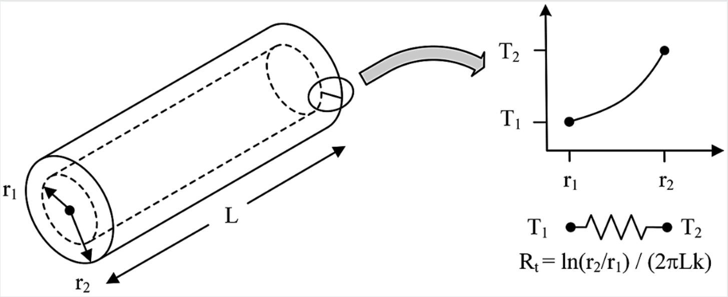


FIGURE 2.4 Schematic of cylindrical heat transfer.

$$R'_{t,tot} = \frac{\ln(r/r_1)}{2\pi k} + \frac{1}{2\pi rh} \tag{2.21}$$

An optimal thickness can be obtained by minimizing this thermal resistance with respect to the outer insulation radius. Differentiating the expression for the total thermal resistance and setting the result to zero,

$$\frac{dR'_{t,tot}}{dr} = \frac{1}{2\pi kr} - \frac{1}{2\pi r^2 h} = 0 \tag{2.22}$$

Solving this equation yields a radius of $r = k/h$ for which the total thermal resistance is a minimum. The result may be also interpreted as a *critical insulation radius*, below which q' increases with increasing radius, and above which q' decreases with increasing radius.

This previous example presented a method to find the total thermal resistance in a cylindrical geometry. The approach solved the heat equation and derived the temperature distribution and then the heat flux by Fourier's law. Then, this result was rewritten as a temperature difference divided by the heat flux, which is defined as the thermal resistance. Using this procedure, thermal resistances for various other geometrical configurations can be readily obtained.

More complex systems such as composite walls can be represented by equivalent thermal circuits. Composite walls of multiple types of thermal resistances and materials can be assembled as a number of thermal resistances in series and/or parallel. Generalizing the heat transfer rate and thermal resistance in Equation (2.13) to include multiple thermal resistances,

$$q = \frac{\Delta T}{\sum R_t}; \; U = \frac{1}{R_{tot} A} \tag{2.23}$$

where ΔT, R_{tot}, and U refer to the overall temperature difference across the entire thermal circuit, total thermal resistance (summation of all thermal resistances), and *overall heat transfer coefficient*, defined by an expression analogous to Newton's law of cooling,

$$q = UA\Delta T \tag{2.24}$$

Consider heat flow through a composite wall consisting of multiple sections in both series and parallel as illustrated in Figure 2.5. The total resistance to heat flow through the composite wall, R_{tot}, can be expressed by a combination of resistances in series and parallel as follows:

$$R_{tot} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} + R_4 \tag{2.25}$$

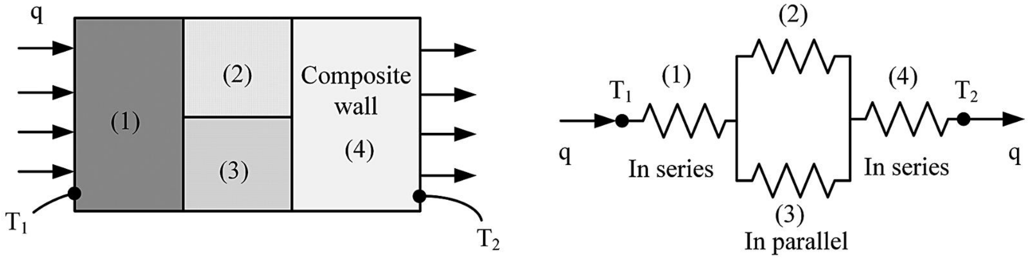


FIGURE 2.5 Thermal circuit for a series-parallel composite wall.

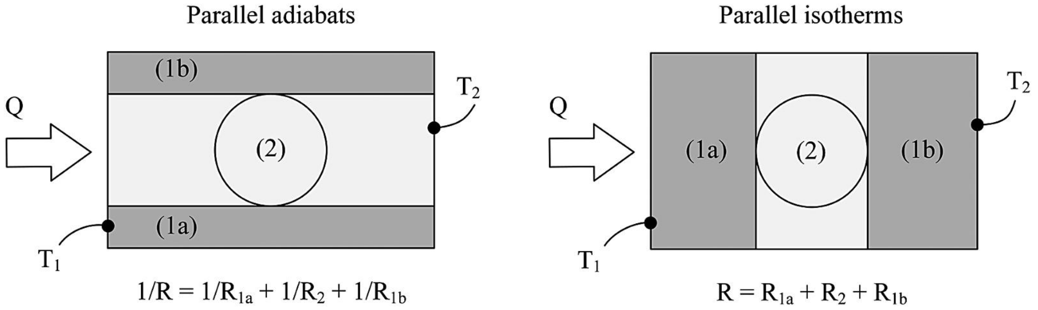


FIGURE 2.6 Parallel adiabats and isotherms.

This thermal circuit approach to approximate the heat flow is quasi one-dimensional in the sense that isotherms are approximately perpendicular to the wall at the interface between sections of the composite wall in parallel (sections 2 and 3). The boundary between sections 2 and 3 is modelled as an adiabatic interface in this approach since heat is assumed to flow only in the x-direction, leading to a temperature variation in the x-direction only.

The *method of parallel adiabats* and *parallel isotherms* can be used to estimate the upper and lower bounds of the overall thermal resistance of a composite material. In Figure 2.6, if the heat flow was directionally unrestrained (a uniform conductivity throughout the element), such that no cross-diffusion occurred against the principal heat flow direction, then the thermal circuit would represent a series of one-dimensional resistances. In this case, a series of adiabats parallel to the adiabatic boundaries would approximate the heat flow lines and yield the maximum thermal resistance (an upper bound) of the composite material. On the other hand, if the resistances between different material sections are aligned in parallel, then this circuit yields the minimum thermal resistance. Thus, a parallel isotherm construction leads to a lower bound on the material's thermal resistance. Since the upper and lower bounds on the thermal resistance, R_{UB} and R_{LB}, represent asymptotes, the actual thermal resistance lies between these two limit cases. An arithmetic mean, $R = (R_{UB} + R_{LB})/2$, is an approximation of the actual thermal resistance. If the upper and lower bounds are far apart, then a geometric mean is a more accurate approximation of the actual resistance.

The *conduction shape factor* can be obtained from the reciprocal of the thermal resistance multiplied by the thermal conductivity. The shape factor and resulting heat flow rate are expressed by

$$S = \frac{1}{R_t k} \tag{2.26}$$

$$q = Sk\left(T_1 - T_2\right) \tag{2.27}$$

The previous solution method can be applied to different geometrical configurations to determine their conduction shape factors. Various shape factors of common geometrical configurations are summarized below in Table 2.1 and Figure 2.7.

Thermal contact between adjoining materials is usually imperfect as some *contact resistance* is typically encountered at the interface between the materials. The contact resistance is generally due to machined surfaces in contact that are not perfectly smooth. Some roughness elements are formed at each interface. When rough surfaces come into contact, gaps between the contact points and roughness elements lead to additional thermal resistances of convection and radiation across the gaps which exceed the thermal resistance of pure conduction alone for perfectly smooth surfaces in contact.

The thermal contact resistance is influenced by several factors, including contact pressure, interstitial materials, surface roughness, cleanliness, and surface deformations. As the contact pressure

TABLE 2.1
Conduction Shape Factors for Various Geometrical Configurations

Geometrical Configuration	Shape Factor
(a) Plane wall with a width of *L* and cross-sectional area of *A*	$S = \frac{A}{L}$
(b) Cylindrical shell with inner and outer radii of r_1 and r_2, respectively	$S = \frac{2\pi L}{ln\left(r_2/r_1\right)}$
(c) Spherical shell with inner and outer radii of r_1 and r_2, respectively	$S = \frac{4\pi}{1/r_1 - 1/r_2}$
(d) Isothermal horizontal cylinder of length *L* buried with its axis a distance *z* below the surface ($z/L \ll 1$ and $D/L \ll 1$)	$S = \frac{2\pi L}{\cos\ h^{-1}\left(2z/D\right)}$
(e) Isothermal sphere of diameter *D* buried a distance *z* below an isothermal surface ($z > D/2$)	$S = \frac{2\pi D}{1 - D/\left(4z\right)}$
(f) Isothermal sphere of diameter *D* buried a distance *z* below an insulated surface ($z > D/2$)	$S = \frac{2\pi D}{1 + D/\left(4z\right)}$
(g) Vertical cylinder of diameter *D* buried in a semi-infinite medium to a depth of *z*	$S = \frac{2\pi z}{\ln\left(4z/D\right)}$
(h) Isothermal rectangular parallelepiped of length *L*, width *b*, and height *a*, buried in a semi-infinite medium at a depth of *z*	$S = 1.685L\left[\log\left(1 + \frac{z}{b}\right)\right]^{-0.59}\left(\frac{z}{a}\right)^{-0.078}$
(i) Cylindrical hole of diameter *D* through the center of a square bar of height *H* and length *L*	$S = \frac{2\pi L}{\ln\left(1.08H/D\right)}$
(j) Eccentric cylinder of length *L* and diameter D_1 in a cylinder of equal length and larger diameter D_2	$S = \frac{2\pi L}{\cosh^{-1}\left[\left(D_2^2 - D_1^2 - 4z^2\right)/\left(2D_1D_2\right)\right]}$
(k) Horizontal cylinder of length *L* midway between parallel planes of equal length and infinite width	$S = \frac{2\pi L}{ln\left(8z/\pi D\right)}$
(l) Two isothermal cylinders of length *L* spaced a distance of *W* apart and buried in an infinite medium	$S = \frac{2\pi L}{\cosh^{-1}\left[\left(4w^2 - D_1^2 - D_2^2\right)/\left(2D_1D_2\right)\right]}$
(m) Two isothermal spheres spaced a distance of *W* apart and buried in an infinite medium	$S = \frac{2\pi D_2}{\frac{D_2}{D_1}\left[1 - \frac{\left(D_1/2W\right)^4}{1 - \left(D_2/2W\right)^2}\right] - \frac{D_2}{W}}$
(n) Row of horizontal cylinders of length *L* and diameter *D* spaced *W* apart in a semi-infinite medium buried at a depth of *z* below the surface	$S = \frac{2\pi L}{\ln\left[\left(2W/\pi D\right)\sinh\left(2\pi z/W\right)\right]}$
(o) Thin disk of diameter *D* on a semi-infinite medium	$S = 2D$
(p) Thin disk of diameter *D* buried in a semi-infinite medium with an adiabatic surface	$S = \frac{2\pi D}{\pi/2 + \tan^{-1}\left(D/4z\right)}$

increases, the thermal resistance becomes smaller since the contact surface area between the adjoining materials is larger. Most thermal contact resistance correlations are made as a function of contact pressure since this is the most significant factor. Another factor is interstitial gaps, due to rough surfaces in contact, which influence heat flows through the gases/fluids filling these gaps. Also, surface features such as roughness, waviness and flatness may be significant factors. Furthermore, surface cleanliness and the presence of dust particles can also affect the thermal resistance.

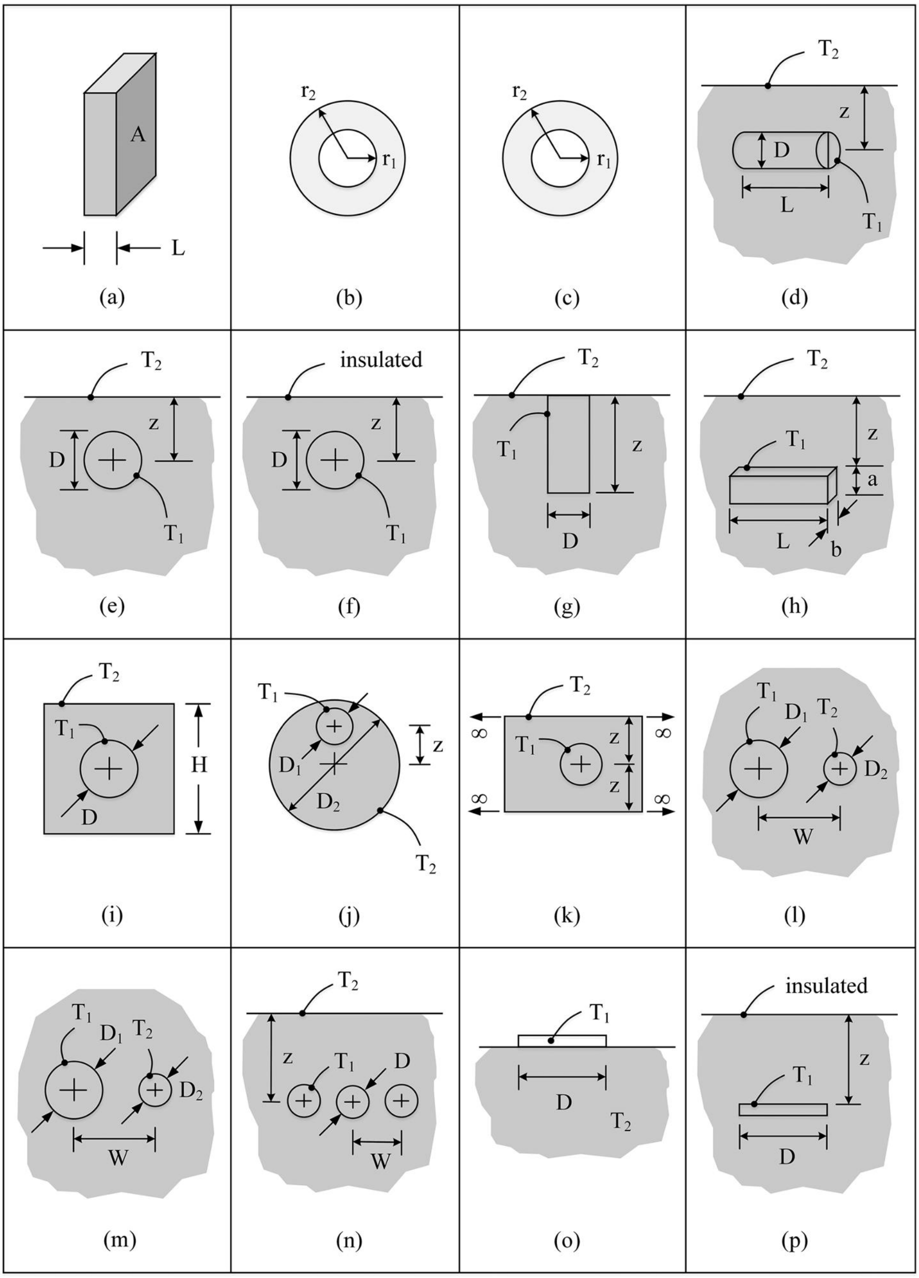


FIGURE 2.7 Conduction shape factors for selected two-dimensional geometries [$q = Sk(T_1 - T_2)$].

Surface deformations, either plastic or elastic, also affect the contact resistance since the deformation causes the contact area between adjoining surfaces to increase. In some cases, the surface roughness at the contact interface may be quantified in terms of the mean roughness height and waviness to determine an approximate surface profile. However, in many cases it is difficult to accurately characterize the microscopic behavior of roughness elements in contact with each other. Therefore, the thermal contact resistance is often best determined by experimental measurements. Once an estimate of the thermal contact resistance is obtained, it can be included in the thermal circuit as a component in series at the location of the interface between the two materials. A comprehensive review of thermal contact resistance modeling and experimental data was presented by Yovanovich (2005).

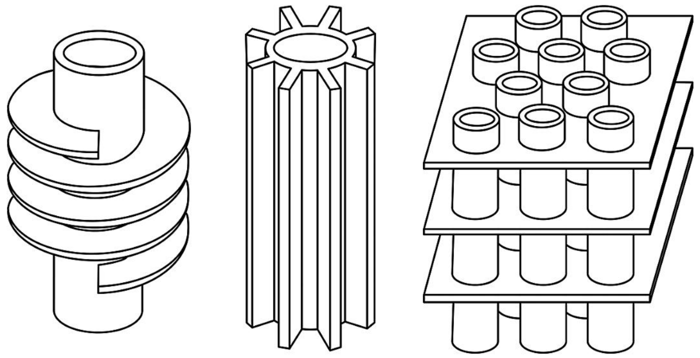
2.4 FINS AND EXTENDED SURFACES

Enhancing the rate of heat transfer from a surface is a common design objective in many thermal engineering systems. Fins, or extended surfaces from an object, are common techniques for heat transfer enhancement. Examples of common finned surfaces are illustrated in Figure 2.8. Along a fin, heat is transferred by conduction through the wall of the tube, as well as convection and/or radiation from the extended surfaces to the surrounding fluid. This section will investigate heat transfer through fins and extended surfaces.

In fin analysis, a thin fin is generally assumed so that a one-dimensional idealization for heat transfer within the fin can be used. In the thin fin approximation, it will be assumed that isotherms are one-dimensional and heat transfer occurs only in the axial direction of the fin. Then the fin heat transfer equation can be obtained in a manner similar to the procedure used previously for the general heat conduction equation.

Consider a variable cross-section fin with the base at a temperature of T_b (see Figure 2.9). Using Equation (2.1) and performing a steady-state heat balance over the control volume,

$$q''_x A_c = q''_x A_c + \frac{d}{dx}\left(q''_x A_c\right)dx + dA_s h\left(T - T_\infty\right) \tag{2.28}$$



(a) Spiral (b) Axial (c) Plate

FIGURE 2.8 Finned surfaces.

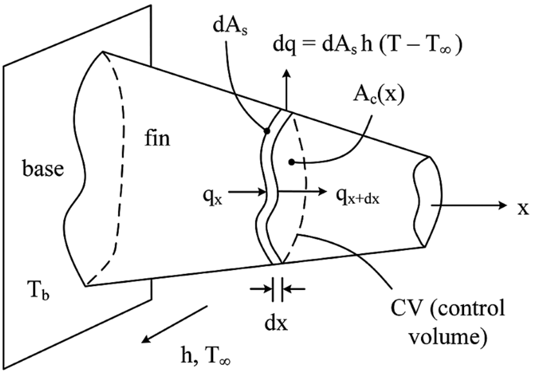


FIGURE 2.9 Schematic of a fin with a variable cross-sectional area.

where A_c and A_s refer to the cross-sectional and outer surface areas, respectively, for a differential control volume of thickness dx. The left side represents the heat inflow into the control volume, while the heat outflows are shown on the right side of the equation. Using Fourier's law, and general fin equation becomes:

$$\frac{d^2T}{dx^2}+\left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx}-\left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)\left(T-T_\infty\right)=0 \tag{2.29}$$

In one-dimensional problems, analytical solutions of this fin equation can often be obtained. Once the geometrical profile of the fin is known, the derivatives of the cross-sectional and surface areas can be determined, after which the fin equation can be solved analytically subject to appropriate boundary conditions. The following example presents an exact solution for a rectangular fin with a specified base temperature.

Example: Heat Transfer in a Uniform Fin

Rectangular fins are used to enhance the thermal effectiveness of tubes in an industrial heat exchanger. Find the temperature and heat flux distributions in a rectangular fin with a uniform cross-sectional area and length of L. The fin base temperature is specified. For the fin tip, consider both an adiabatic condition and a specified temperature at the tip of the fin.

For a fin of uniform cross-sectional area, the previous general fin equation can be reduced to:

$$\frac{d^2\theta}{dx^2}-m^2\theta=0 \tag{2.30}$$

where $\theta = T(x) - T_\infty$, $m^2 = hP/(kA_c)$, and P, T_∞, and A_c refer to the fin perimeter, ambient fluid temperature, and cross-sectional area of the fin, respectively. The solution of this equation is:

$$\theta\left(x\right)=D_1\sin h\left(mx\right)+D_2\cos h\left(mx\right) \tag{2.31}$$

where $x = 0$ at the base of the fin and the coefficients D_1 and D_2 are obtained through specification of the boundary conditions.

Applying the boundary conditions of an adiabatic fin tip with a specified base temperature, the following results are obtained for the temperature excess, θ, and heat flow rate, q_f, respectively:

$$\frac{\theta}{\theta_b}=\frac{\cos h\left(m\left(L-x\right)\right)}{\cos h\left(mL\right)} \tag{2.32}$$

$$q_f=-kA_c\left.\frac{d\theta}{dx}\right|_0=M\tanh\left(mL\right) \tag{2.33}$$

where $M = \theta_b = (hPkA_c)^{1/2}$. It can be shown that as the fin length becomes very long (i.e., $L \rightarrow \infty$), the following results are obtained:

$$\frac{\theta}{\theta_b}=e^{-mx} \tag{2.34}$$

$$q_f=M \tag{2.35}$$

Another type of boundary condition is a specified temperature at the tip of the fin, $\theta(L) = \theta_L$. In this case, the results for the temperature and heat flux become: