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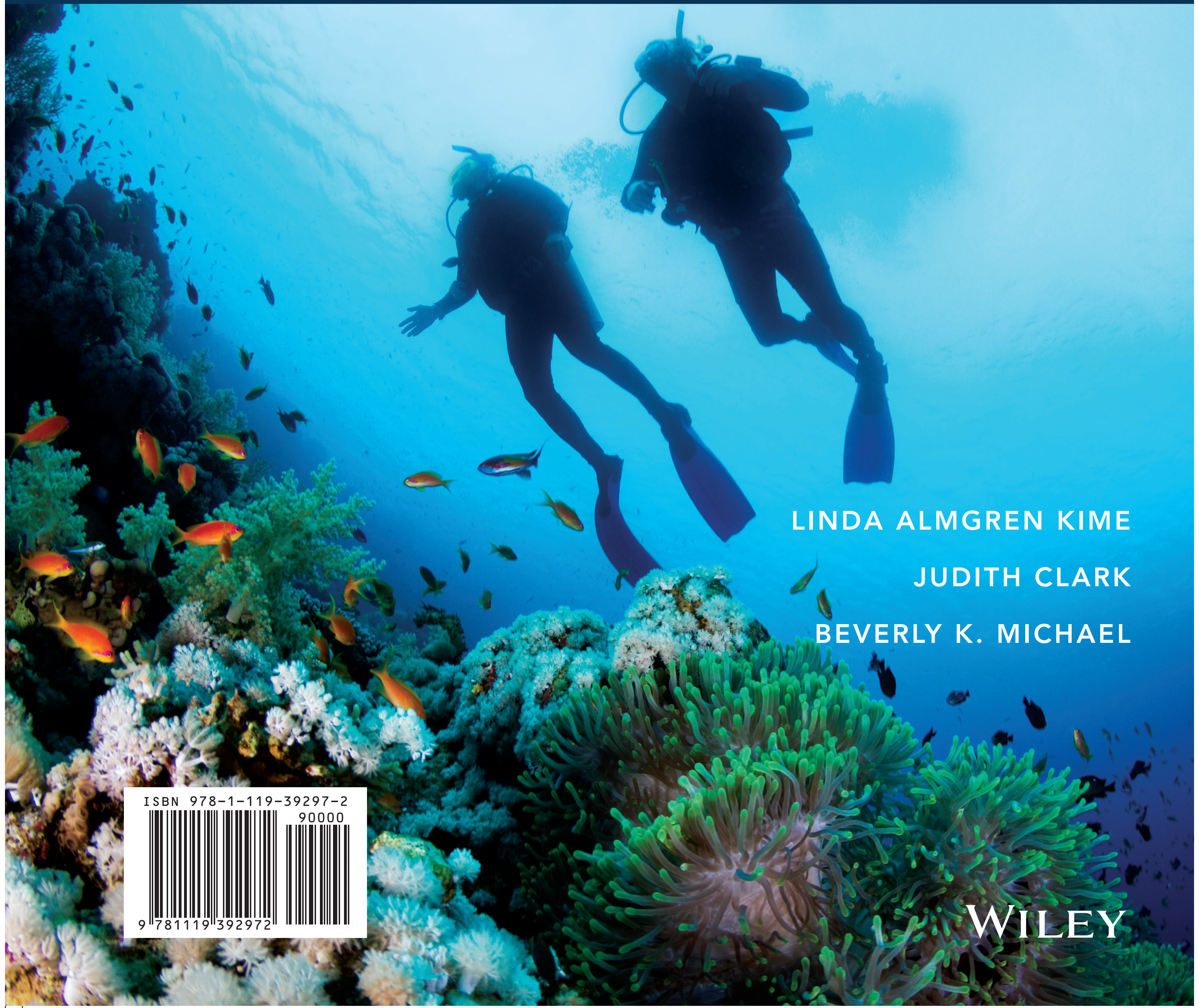
Explorations in College Algebra

SIXTH EDITION



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Sixth Edition

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Funded by a National Science Foundation Grant

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To our students, who inspired us.

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Letter from a Student

My name is Lexi Fournier and I am a freshman here at Pitt. This semester I am enrolled in the Applied Algebra course using "Explorations in College Algebra." Before coming to Pitt, I had taken numerous math courses varying from algebra to calculus, all of which produced frustration, stress, and a detestation for math as a subject. When I was told that I was required to take a math course here, I was livid. I am a pre-law and creative writing major; why do I need math? My adviser calmed me by informing me of this new math class aimed at teaching non-math/science majors the basic skills they will need in everyday life.

At first I was skeptical, but I'm writing to you now to emphatically recommend this course. What I have learned thus far in this course have been realistic math skills presented in a "left brain" method that fosters confidence and motivation. For once in my career as a student, math is relatable. The concepts are clear and realistic (as opposed to the abstract, amorphous topics addressed in my earlier math classes). I look forward to this class. I enjoy doing my homework and projects because I feel that the lessons are applicable to my life and my future and because I feel empowered by my understanding.

This course is a vital addition to the math department. It has altered my view on the subject and stimulated an appreciation for what I like to call "everyday math."

It is my belief that many students will find the class as encouraging and helpful as I have. Thank you for your attention.

Sincerely,

Lexi Fournier
Student, University of Pittsburgh

Preface

This text was born from a desire to reshape the college algebra course, to make it relevant and accessible to all of our students. Our goal is to shift the focus from learning a set of discrete mechanical rules to exploring how algebra is used in the social and physical sciences, and in the world around you. By connecting mathematics to real-life situations, we hope students come to appreciate its power and beauty.

Guiding Principles

The following principles guided our work.

- Develop mathematical concepts using real-world data.
- Pose a wide variety of problems designed to promote mathematical reasoning in different contexts.
- Make connections among the multiple representations of functions.
- Emphasize communication skills, both written and oral.
- Facilitate the use of technology.
- Provide sufficient practice in skill building to enhance problem solving.

Evolution of "Explorations in College Algebra"

The sixth edition of *Explorations* is the result of a process that started over 20 years ago. Funding by the National Science Foundation enabled us to develop and publish the first edition, and to work collaboratively with a nationwide consortium of schools. Faculty from selected schools continued to work with us on the second, third, fourth, fifth and now the sixth editions. During each stage of revision we solicited extensive feedback from our colleagues, reviewers and students.

Throughout the text, families of functions are used to model real-world phenomena. After an introductory chapter on data and functions, we first focus on linear and exponential functions, since these are the two most commonly used mathematical models. We then discuss logarithmic, power, quadratic, polynomial and rational functions. Finally we look at ways to extend and combine all these functions to create new functions and apply them in more complex situations.

The text adopts a problem-solving approach, where examples and exercises lie on a continuum from open-ended, real-life problems to skill-based problems. The materials are designed for flexibility of use and offer multiple options for a wide range of skill levels and departmental needs. The text is currently used in a variety of instructional settings including small classes, laboratory settings and large lectures, and in both two- and four-year institutions.

Special Features and Supplements

An instructor is free to choose among a number of special features. *The Instructor's Teaching and Solutions Manual* provides extensive teaching ideas and support for using these features. The manual also contains solutions to the even-numbered problems.[1]

Exploring Mathematical Ideas and Skill Building

Explore & Extend are short explorations that provide students and instructors with ideas for going deeper into topics or previewing new concepts. They can be found within the WileyPLUS course and e-book.

Exploration **Explorations** are extensive problem-solving situations available for each chapter in the WileyPLUS course and e-book that can be used for small group or individual projects.

Algebra Aerobics are collections of skill-building practice problems found in each section. All of the answers are in the back of the text.

Exercises and Examples include real-life problems with extensive data sets as well as skill-based problems.

Check Your Understanding is a set of mostly true/false questions at the end of each chapter (answers are provided in the back of the text) that offer students a chance to assess their understanding of that chapter's mathematical ideas.

[1]The manual is available free to adopters online at *www.wiley.com/college/kimeclark* or at *www.**wileyplus**.com*. You can also contact your local Wiley sales representative to obtain a printed version of the manual.

Chapter Review: Putting It All Together contains problems that apply all of the basic concepts in the chapter. The answers to the odd-numbered problems are in the back of the text.

60-Second Summaries are short writing assignments found in the exercises and the *Explore & Extend* problems that ask students to succinctly summarize their findings.

Reading **Readings** are related to topics covered in the text and are available at *www.wiley.com/college/kimeclark* and at *www.wileyplus.com*.

Using Technology

Technology is not required to teach this course. However, we provide the following online resources on the course website *www.wiley.com/college/kimeclark* and at *www.wileyplus.com*.

Applet **Interactive Course Software** provides illustrations of the properties of each function, simulations of concepts, and practice in skill building. Their modules can be used in the classroom or computer lab, or downloaded for student use.

Data **Excel and TI Connect™ Graph Link Files** contain all the major data sets used in the text and are available in Excel or TI Connect™ program formats.

GCM **Graphing Calculator Manual** is coordinated with the chapters in the text and offers step-by-step instructions for using the TI-83/TI-84 family of calculators.

NEWLY UPDATED! WileyPLUS is an online, interactive course environment for students, and course management and assessment system for instructors. See updated features in the next section.

The Sixth Edition

Changes to the Sixth Edition WileyPLUS Course

WileyPLUS online homework features a full-service, digital learning environment including additional resources for the students, such as example videos and interactive illustrations, integrated links between the online text and supplements, and ORION adaptive practice. In this sixth edition, we focused on revising the WileyPLUS course by updating and adding the following features:

- **New ORION Adaptive Practice:** this adaptive practice engine built within WileyPLUS can connect directly into the WileyPLUS gradebook or into your campus Learning Management System gradebook if you select that option. Wiley has been incorporating ORION into WileyPLUS courses for five years, and it will be included with the *Explorations in College Algebra* WileyPLUS course for the first time in this sixth edition. ORION Learning provides an adaptive, personalized learning experience that delivers easy-to-use analytics so instructors and students can see exactly where they're excelling and where they need help.
- **Other new and updated WileyPLUS features include:**
  - Assignable algorithmic questions available in each chapter
  - Newly curated student assignments that provide additional scaffolding
  - Video quiz questions for each example video
  - Closed-captioning provided for example videos

Acknowledgments

We wish to express our appreciation to all those who helped and supported us during this extensive collaborative endeavor. We are grateful for the support of the National Science Foundation, whose funding made this project possible, and for the generous help of our program officers then, Elizabeth Teles and Marjorie Enneking. Our original Advisory Board, especially Deborah Hughes-Hallett and Philip Morrison, and our original editor, Ruth Baruth, provided invaluable advice and encouragement.

For over twenty years, we worked with more faculty, students, teaching assistants, staff, and administrators than we can possibly list here. We are deeply grateful for supportive colleagues at our own universities. The generous support we received from Theresa Mortimer, Patricia Davidson, Mark Pawlak, Maura Mast, Dick Cluster, Anthony Beckwith, Bob Seeley, Randy Albelda, Art MacEwan, Rachel Skvirsky, and Brian Butler, among many others, helped to make this a successful project.

We are deeply indebted to Jennifer Blue, Celeste Hernandez, Paul Lorczak, Georgia Mederer, Ann Ostberg, and Sandra Zirkes for their dedicated search for mathematical errors in the text and solutions, and finding (we hope) all of them. A text designed around the application of real-world data would have been impossible without the time-consuming and exacting research done by Patrick Jarrett and Jie Chen. Edmond Tomastik, George Colpitts and Karl Schaffer were gracious enough to let us adapt some of their real-world examples in the text.

One of the joys of this project has been working with so many dedicated faculty who are searching for new ways to reach out to students. These faculty, their teaching assistants and students all offered incredible support, encouragement, and a

wealth of helpful suggestions. In particular, our heartfelt thanks goes to members of our original consortium: Sandi Athanassiou and all the wonderful teaching assistants at the University of Missouri, Columbia; Natalie Leone, University of Pittsburgh; Peggy Tibbs and John Watson, Arkansas Technical University; Josie Hamer, Robert Hoburg, and Bruce King, all past and present faculty at Western Connecticut State University; Judy Stubblefield, Garden City Community College; Lida McDowell, Jan Davis, and Jeff Stuart, University of Southern Mississippi; Ann Steen, Santa Fe Community College; Leah Griffith, Rio Hondo College; Mark Mills, Central College; Tina Bond, Pensacola Junior College; and Curtis Card, Black Hills State University.

The following reviewers' thoughtful comments helped shape the sixth edition: Dr. N. Paul Schembari, East Stroudsburg University of Pennsylvania; Diane Devanney, Immaculata University; Darlene Diaz, Santiago Canyon College; Dr. Amy Marolt-Alred, Northeast Misssissippi Community College; Katherine Brandl, Centenary College of Louisiana; Mark Grinshpon, Georgia State University; Dr. Ernie Solheid, California State University, Fullerton; Dr. Kevin W. Hopkins, Southwest Baptist University; Sarah Hoffman, University of Arizona; Thomas W. Crock III, University of Pittsburgh.

We are indebted to Laurie Rosatone at Wiley, whose gracious oversight helped to keep this project on track. Particular thanks goes to our editors at Wiley, Joanna Dingle and especially to Jennifer Brady who kept us on schedule. Kudos to Ashley Patterson and Valerie Zaborski in production for all their help in getting the text out. The accompanying media for *Explorations* would never have been produced without the experienced help from Tom Kulesa. Over the years many others at Wiley have been extraordinarily helpful in dealing with the myriad of endless details in producing a mathematics textbook. Our thanks goes to all of them.

Our families couldn't help but become caught up in this time-consuming endeavor. Judy's husband, Gerry, became our consortium lawyer, and her daughters, Rachel and Caroline, were there when needed for support and to mail packages. Kristin and her husband John provided editorial help and more importantly produced two grandchildren: Nola, who asks "why" and Cordelia, who sings numbers. Beverly's husband, Dan, was patient and understanding about the amount of time this edition took. Linda's husband, Milford, and her son Kristian provided invaluable scientific and, more importantly, emotional resources. Kristian and his wife Amy Mertl have produced three marvelous grandchildren. We offer our love and thanks to them.

Finally, we wish to thank all of our students. It is for them that this book was written.

Judy, Bev, and Linda

P.S. We've tried hard to write an error-free text, but we know that's impossible. You can alert us to any errors by visiting **hub.wiley.com/community/support/**. We would very much appreciate your input.

Since this text is a collaboration between authors and instructors, we encourage instructors to send new ideas and examples for the new *Explore & Extend* feature for possible future use. We'll put the best ideas on our website.

And last, but not least, we especially want to thank Dr. John Saber from Central Lake College for his kind and encouraging email: ". . . just wanted to thank you for writing this truly wonderful text." It made our day.

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- Readings
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Table of Contents

CHAPTER 1

An Introduction to Data and Functions

Overview

How can you describe patterns in data? In this chapter we explore how to use graphs to visualize the shape of single-variable data and to show changes in two-variable data. Functions, a fundamental concept in mathematics, are introduced and used to model change.

After reading this chapter, you should be able to

- describe patterns in single- and two-variable data
- construct a "60-second summary"
- define a function and represent it with words, tables, graphs, and equations
- identify properties of functions
- use the language of functions to describe and create graphs

1.1 Describing Single-Variable Data

This course starts with you. How would you describe yourself to others? Are you a 5-foot 6-inch, black, 26-year-old female studying biology? Or perhaps you are a 5-foot 10-inch, Chinese, 18-year-old male English major. In statistical terms, characteristics such as height, race, age, and major that vary from person to person are called *variables*. Information collected about a variable is called *data*.[1]

Some variables, such as age, height, or number of people in your household, can be represented by a number and a unit of measure (such as 18 years, 6 feet, or 3 people). These are called *quantitative variables*. For other variables, such as gender or college major, we use categories (such as male and female or biology and English) to classify information. These are called *categorical* (or *qualitative*) data. The dividing line between classifying a variable as categorical or quantitative is not always clear-cut. For example, you could ask individuals to list their years of education (making education a quantitative variable) or ask for their highest educational category, such as college or graduate school (making education a categorical variable).

Many of the controversies in the social sciences have centered on how particular variables are defined and measured. For nearly two centuries, the categories used by the U.S. Census Bureau to classify race and ethnicity have been the subject of debate.

GCM

See the Graphing Calculator Manual (GCM) Chapter 1, "*Histograms, Scatter Plots, and Graphs of Functions,*" on the course website for instructions on using a graphing calculator in this chapter.

Exploration

Exploration 1.1 provides an opportunity to collect your own data and to think about issues related to classifying and interpreting data.

[1]*Data* is the plural of the Latin word *datum* (meaning "something given")—hence one datum, two data.

Visualizing Single-Variable Data

Humans are visual creatures. Converting data to an image can make it much easier to recognize patterns.

Bar charts: How well educated are Americans? Categorical data are usually displayed with a bar chart. The length of the bar for a single category tells you either the *frequency count* (the number of observations that fall into that category) or the *relative frequency* (the percentage of the total observations).

Since the relative size of the bars is the same using either frequency or relative frequency counts, the two scales are often put on different vertical axes of the same chart. For example as shown in **Figure 1.1**, a bar chart of the educational attainment of Americans in 2008, approximately 60 million Americans had a high school degree but never went to college, and this represents approximately 31% of Americans 25 years or older.

Educational Attainment (2008) Americans 25 Years or Older

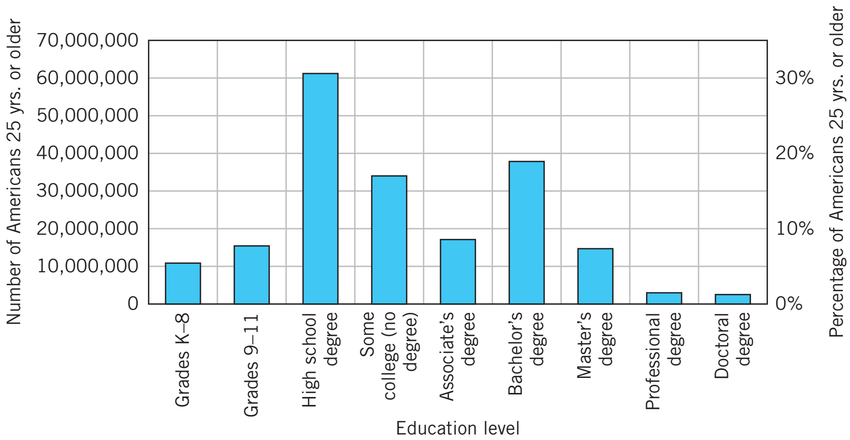

FIGURE 1.1 **Bar chart showing the education levels for Americans age 25 or older.**

Source: U.S. Bureau of the Census.

Always, remember to ask: Percentage of what? The bar chart in Figure 1.1 represents the percentage of Americans with high school degrees relative to Americans 25 years or older, not the percentage of all Americans.

EXAMPLE 1 | What Does the Bar Chart Tell Us?

a. Using Figure 1.1, estimate the number and percentage of Americans age 25 years or older who have bachelor's degrees, but no further advanced education.

b. Estimate the percentage of Americans 25 years or older who have at least a high school education.

c. What doesn't the bar chart tell us?

d. Write a brief summary of educational attainment in the United States.

Solution

a. Those with bachelor's degrees but no further education number about 38 million, or 19%.

b. Those who have completed a high school education include everyone with a high school degree up to a Ph.D. We could add up all the percentages for each of those seven categories. But it's easier to subtract from the whole those who do not meet the conditions; that is, subtract the percentage with grades K–8 and 9–11 education from 100%.

Grade School +	Only Some High School	= Total without High School Degree
6% +	8%	= 14%

The percentage of Americans (age 25 or over) with a high school degree is about 100% − 14% = 86%. So more than four out of five Americans 25 years or older have completed high school.

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c. The bar chart does not tell us the total size of the population or the total number (or percentage) of Americans who have a high school degree. For example, if we include younger Americans between age 18 and age 25, we would expect the percentage with a high school degree to be higher.

d. About 86% of adult Americans (age 25 or older) have at least a high school education. The breakdown for the 86% includes 31% who completed high school but did not go on, 45% who have some college (up to a bachelor's degree), and about 10% who have graduate degrees. When compared with other countries, the United States population ranks among the most highly educated in the world.

An important aside: What a good graph should contain

When you encounter a graph in an article or you produce one for a class, there are three elements that should always be present:

1. An informative title that succinctly describes the graph
2. Clearly labeled axes (or a legend) including the units of measurement (e.g., indicating whether age is measured in months or years)
3. The source of the data cited in the data table, in the text, or on the graph

Histograms: What is the distribution of ages in the U.S. population?

A histogram is a specialized form of a bar chart that is used to visualize single-variable quantitative data. The horizontal axis on a histogram is a subset of the real numbers with the unit (representing, for example, number of years) and the size of each interval marked. The intervals are usually evenly spaced to facilitate comparisons (e.g., placed every 10 years). The size of the interval can reveal or obscure patterns in the data. As with a bar chart, the vertical axis can be labeled with a frequency or a relative frequency count. For example, the histogram in **Figure 1.2** shows the distribution of ages in the United States as projected for 2010.

Applet

See course software "F1: Histograms" on the course website.

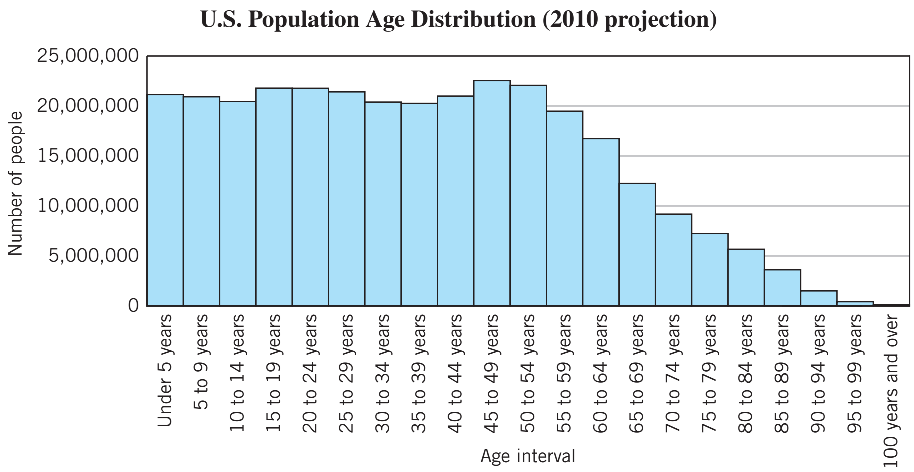


FIGURE 1.2 **Histogram of the U.S. population in 5-year intervals.**

Source: U.S. Bureau of the Census.

EXAMPLE 2 | What Does the Histogram Tell Us?

a. What 5-year age interval contains the most Americans? Roughly how many are in that interval? (Refer to Figure 1.2.)

b. Estimate the number of people under age 20.

c. Construct a topic sentence for a report about the U.S. population.

Solution

a. The interval from 45 to 49 years contains the largest number of Americans, about 23 million.

b. The sum of the frequency counts for the four intervals below age 20 is about 84 million.

c. According to estimates by the U.S. Census Bureau, in 2010 the number of Americans in each 5-year age interval will be fairly flat up to age 45, will peak between ages 45 and 54, and then fall in a gradual decline.

Numerical Descriptors: What Is "Average" Anyway?

In 2010 the U.S. Bureau of the Census reported that the mean age for Americans was 37.8 and the median age was 36.9 years.

> **The Mean and Median**
>
> The *mean* is the sum of a list of numbers divided by the number of terms in the list.
>
> The *median* is the middle value of a list of numbers ordered from the smallest to the largest; half the numbers are less than or equal to the median and half are greater than or equal to the median.
>
> If the number of observations is odd, then the median is the middle number in the list. If the number of observations is even, then the median is the mean of the two middle numbers in the list.

The mean age of 37.8 represents the sum of the ages of every American divided by the total number of Americans. The median age of 36.9 means that if you placed all the ages in order, 36.9 would lie right in the middle; that is, half of Americans are younger than or equal to 36.9 and half are 36.9 or older.

In the press you will most likely encounter the word "average" rather than the term "mean" or "median."[2] The term "average" is used very loosely. It usually represents the mean, but it could also represent the median or something much more vague, such as the "average" American household. For example, the media reported that:

> The *average* American home in 2008 had more television sets than people. . . . There were 2.86 TV sets in the typical home and 2.5 people.[3]
> The *average* American family owed more than $8,000 in credit debt in 2008 . . . and is averaging about 5.4 credit cards.[4]

Reading

See the reading "The Median Isn't the Message," on the course website, to find out how an understanding of the median gave renewed hope to the renowned scientist Stephen J. Gould when he was diagnosed with cancer.

The significance of the mean and median

The median divides the number of entries in a data set into two equal halves and is unchanged by changes in values above and below it. For example, as long as the median income is larger than the poverty level, it will remain the same even if all poor people suddenly increase their incomes up to that level and everyone else's income remains the same.

The mean is the most commonly used statistic in the news media and, unlike the median, can be affected by a few extreme values called *outliers*. For example, suppose Bill Gates, founder of Microsoft and the richest man in the world, were to move into a town of 10,000 people, all of whom earned nothing. The median income would be $0, but the mean income would be in the millions. That's why income studies usually use the median.

EXAMPLE 3 | "Million-Dollar Manhattan Apartment? Just About Average"

According to a report cited on *therealdeal.com*, in November 2009 the median price of purchasing an apartment in Manhattan was $850,000 and the mean price was $1.32 million. How could there be such a difference in price? Which value do you think better represents apartment prices in Manhattan?

[2]The word "average" has an interesting derivation according to Klein's etymological dictionary. It comes from the Arabic word *awariyan*, which means "merchandise damaged by seawater." The idea being debated was that if your ships arrived with water-damaged merchandise, should you have to bear all the losses yourself or should they be spread around, or "averaged," among all the other merchants? The words *averia* in Spanish, *avaria* in Italian, and *avarie* in French still mean "damage."
[3]Source: The Nielson Co., 2009.
[4]Source: *www.creditcards.com.*

Solution

Apartments that sold for exorbitant prices (in the millions) could raise the mean above the median. If you want to buy an apartment in Manhattan, the median price is probably more important because it tells you that half of the apartments cost $850,000 or less.

An Introduction to Explore & Extend

In the WileyPLUS course and e-book, there are *Explore & Extend* activities that provide you with ideas for going deeper or previewing what comes next. While these activities are optional, it is hoped they will be engaging, and doing them will increase your understanding. Enjoy!

An Introduction to Algebra Aerobics

In each section of the text there are "Algebra Aerobics" with answers in the back of the book. They are intended to give you practice in the algebraic skills introduced in the section and to review skills we assume you have learned in previous courses. These skills should provide a good foundation for doing the exercises at the end of each section.

Algebra Aerobics **1.1**

1. Fill in the following table. Round decimals to the nearest thousandth.

Fraction	Decimal	Percent
$\frac{7}{12}$		
	0.025	
		2%
$\frac{1}{200}$		
	0.35	
		0.8%

2. Calculate the following:

a. A survey reported that 80 people, or 16% of the group, were smokers. How many people were surveyed?

b. Of the 236 students who took a test, 16.5% received a B grade. How many students received a B grade?

c. Six of the 16 people present were from foreign countries. What percent were foreigners?

3. Given the list of numbers, find the mean and the median.

a. 9, 2, −2, 6, 5 **b.** −2, 2, 5, 6

4. The mean hourly wage of six convenience store employees is $7.50. The hourly wages of five of the employees are $4.75, $5.50, $5.75, $8.00, and $9.50. Find the hourly wage of the sixth.

5. When looking through the classified ads, you found that 16 jobs had a starting salary of $20,000, 8 had a starting salary of $32,000, and 1 had a starting salary of $50,000. Find the mean and median starting salary for these jobs.

6. a. Fill in the table. Round your answers to the nearest whole number.

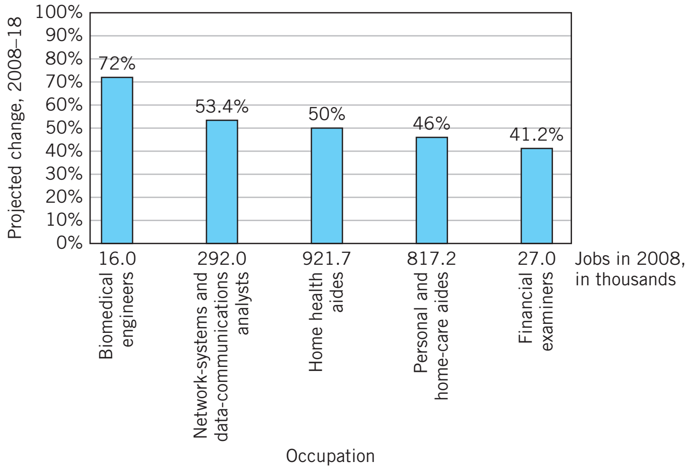
Age	Frequency Count	Relative Frequency (%)
1–20		38
21–40	35	
41–60	28	
61–80		
Total	137	

b. Calculate the percentage of the population who are over 40 years old.

7. In the chart, which job type had the largest number of jobs in 2008? The smallest? Which job type is projected to increase the most? The least? What should you consider in interpreting these percentages?

Jobs of the Future

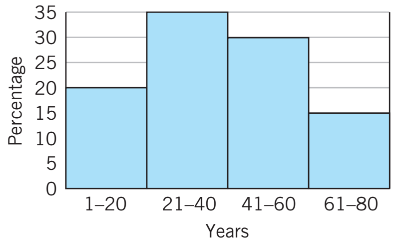
Occupations that will add jobs the fastest from 2008 to 2018, according to Labor Department projections.



Source: U.S. Dept of Labor.

8. From the histogram, create a frequency distribution table. Assume that the total number of people represented by the histogram is 1352. (*Hint:* Estimate the relative frequencies from the graph and then calculate the frequency count in each interval.)

Distribution of Ages (in years)



9. Calculate the mean and median for the following data:

a. \$475, \$250, \$300, \$450, \$275, \$300, \$6000, \$400, \$300

b. 0.4, 0.3, 0.3, 0.7, 1.2, 0.5, 0.9, 0.4

10. Explain why the mean may be a misleading numerical summary of the data in Problem 9(a).

Exercises for Section **1.1**

Course software required for Exercise 22 and recommended for Exercise 21.

1. The following graph shows the amount of venture capital investment in different types of "clean technology."

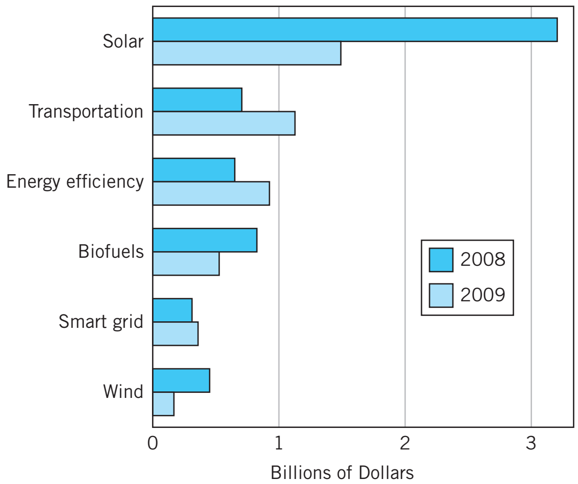
a. Which type of "clean technology" had the largest amount of venture capital investment in 2009? About how much was this?

b. Which type of "clean technology" had the greatest amount of money lost from venture capital investment from 2008 to 2009? About how much was this?

c. What types of "clean technology" gained money from venture capital investments from 2008 to 2009?

Green for Green

Venture capital investment in
selected 'clean technology' sectors

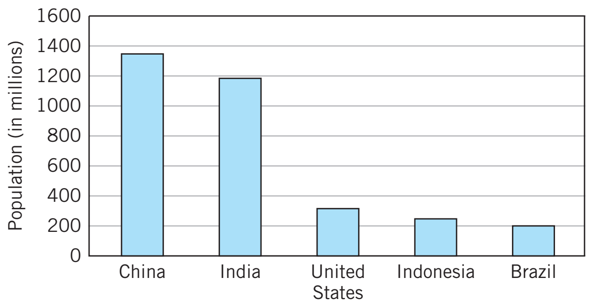


*Includes electric vehicles, advanced batteries and fuel cells

Source: Cleantech Group.

2. The accompanying bar chart shows the five countries with the largest populations in 2010 (estimates).

The Five Most Populous Countries (2010 est.)



Source: CIA Factbook, *www.cia.gov/cia*.

a. What country has the largest population, and approximately what is its population size?

b. The population of India is projected in the near future to exceed the population of China. Given the current data, what is the minimum number of additional persons needed to make India's population larger than China's?

c. What additional information would you need in order to calculate the percentage of the 2010 world population for each of these five countries?

3. The following bar graph was published in a 2009 report by the Organization for Economic Cooperation and Development (OECD).

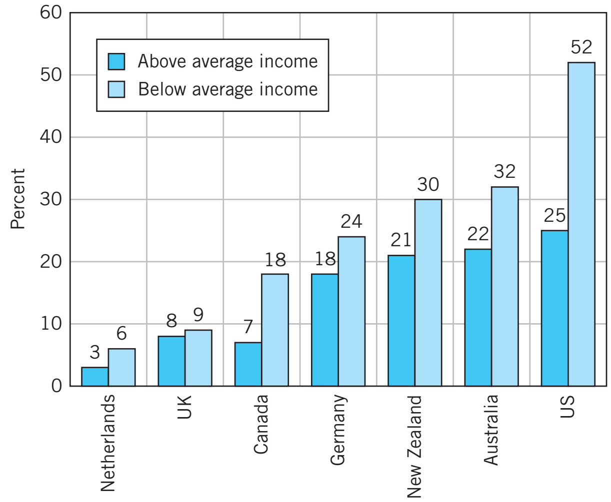
a. Summarize what you think this graph tells you about unmet health care needs.

b. For which country was the disparity between below and above average incomes the largest? The smallest?

c. What additional information would you need in order to calculate the number of people with unmet health care needs due to costs in each country?

Health Care Gap

Persons reporting an unmet health care need* due to costs in seven OECD countries, by income group, 2007



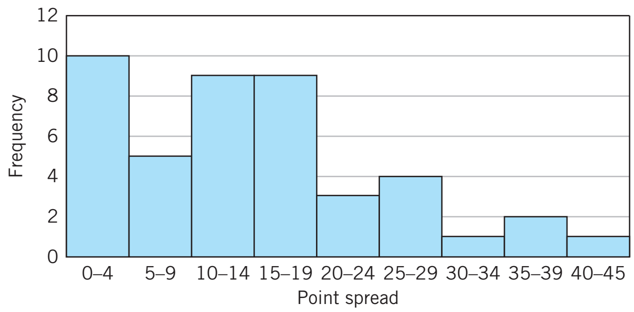
*Did not get medical care, missed medical test, treatment or follow-up, or did not fill prescription or missed doses.

Source: "Measuring disparities in health status and in access and use of health care in OECD countries", Michael de Looper and Gaetan Lafortune, OECD Health Working Paper No. 43, 2009, available at *www.oecd.org/health*.

4. The point spread in a football game is the difference between the winning team's score and the losing team's score. For example, in the 2010 Super Bowl game, the New Orleans Saints won with 31 points versus the Indianapolis Colts' 17 points. So the point spread was 14 points.

a. In the accompanying histogram, what is the interval with the most likely point spread in a Super Bowl? The least likely?

Point Spreads in 44 Super Bowls



Source: *www.docsports.com*.

b. What percentage of these 44 Super Bowl games had a point spread of 9 or less? Of 30 or more?

5. Given here is a table of salaries taken from a survey of some recent graduates (with bachelor degrees) from a well-known university in Pittsburgh.

Salary (in thousands)	Number of Graduates Receiving Salary
21–25	2
26–30	3
31–35	10
36–40	20
41–45	9
46–50	1

a. How many graduates were surveyed?

b. Is this quantitative or qualitative data? Explain.

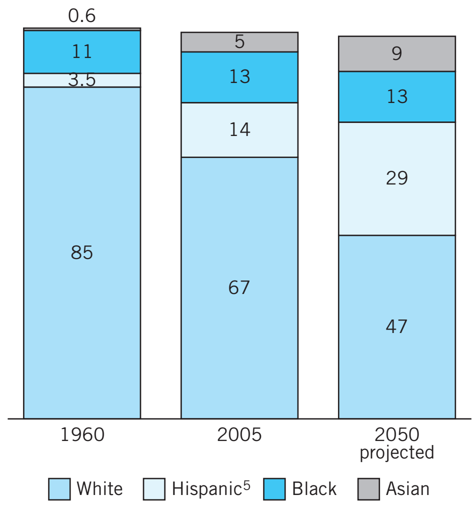
c. What is the relative frequency of people having a salary between $26,000 and $30,000?

d. Create a histogram of the data.

6. The accompanying bar chart shows the predictions of the Pew Research Center about the future racial and ethnic composition of American society. American Indian/Alaskan American comprised less than 1% of the population for each of the years on the chart and thus is not represented.

U.S. Population 1960–2050

Share of total, by racial and ethnic groups



Source: *Immigration to play lead role in future U.S. growth.* Pew Research Center, 2008.

a. The U.S. Bureau of the Census projected that there will be approximately 438 million people in the United States in the year 2050. Approximately how many people will be of Hispanic origin in 2050?

b. Describe three changes in the racial composition of the United States since 1960.

c. Write a topic sentence describing an overall trend.

7. a. Compute the mean and median for the list: 5, 18, 22, 46, 80, 105, 110.

b. Change one of the entries in the list in part (a) so that the median stays the same but the mean increases.

8. Suppose that a church congregation has 100 members, each of whom donates 10% of his or her income to the church. The church collected $250,000 last year from its members.

a. What was the mean contribution of its members?

b. What was the mean income of its members?

c. Can you predict from the given data, the median income of its members? Explain your answer.

[5] Hispanic used to be considered a racial classification. It is now considered an ethnic classification, since Hispanics can be black, white, or any other race. For this survey, Hispanics formed one category and did not overlap with other categories.

9. Suppose that annual salaries in a certain corporation are as follows:

Level I (30 employees)	\$18,000
Level II (8 employees)	\$36,000
Level III (2 employees)	\$80,000

Find the mean and median annual salary. Suppose that an advertisement is placed in the newspaper giving the mean annual salary of employees in this corporation as a way to attract applicants. Why would this be a misleading indicator of salary expectations?

10. Suppose the grades on your first four exams were 78%, 92%, 60%, and 85%. What would be the lowest possible average (using the mean) that your last two exams could have so that your grade in the class, based on the mean of the six exams, is at least 82%?

Reading **11.** Read Stephen Jay Gould's article "The Median Isn't the Message" and explain how an understanding of statistics brought hope to a cancer victim.

12. a. On the first quiz (worth 25 points) given in a section of college algebra, one person received a score of 16, two people got 18, one got 21, three got 22, one got 23, and one got 25. What were the mean and median of the quiz scores for this group of students?

b. On the second quiz (again worth 25 points), the scores for eight students were 16, 17, 18, 20, 22, 23, 25, and 25.

i. If the mean of the scores for the nine students was 21, then what was the missing score?

ii. If the median of the scores was 22, then what are possible scores for the missing ninth student?

13. Why is the mean age larger than the median age in the United States? What prediction would you make for your State? What predictions would you make for other countries? You can check your predictions with data from the U.S. Census Bureau at *www.census.gov*.

14. Up to and including Barack Obama, the ages of the last 15 presidents when they first took office were 55, 51, 54, 51, 60, 62, 43, 55, 56, 52, 69, 64, 46, 54, and 48.

a. Find the mean and median ages of the past 15 presidents when they took office.

b. If the mean age of the past 16 presidents is 54.75, at what age did the missing president take office?

c. Beginning with age 40 and using 5-year intervals, find the frequency count for each age interval.

d. Create a frequency histogram using your results from part (c).

15. Herb Caen, a Pulitzer Prize–winning columnist for the *San Francisco Chronicle*, remarked that a person moving from state A to state B could raise the average IQ in both states. Is he right? Explain.

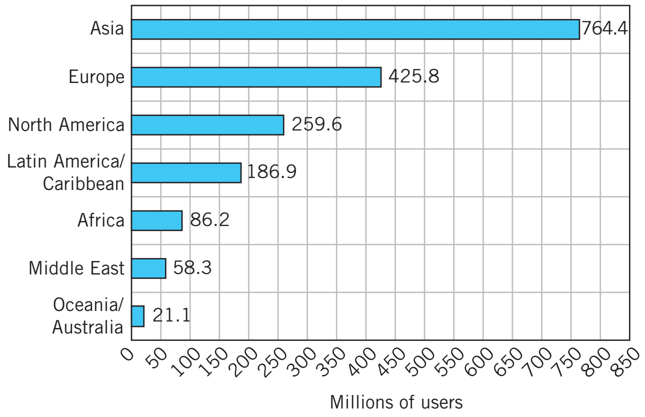
16. Why do you think most researchers use median rather than mean income when studying "typical" households?

17. According to the Federal Reserve Board, in 2007, the median net worth of American families was \$120,300 and the mean net worth was \$556,300. How could there be such a wide discrepancy?

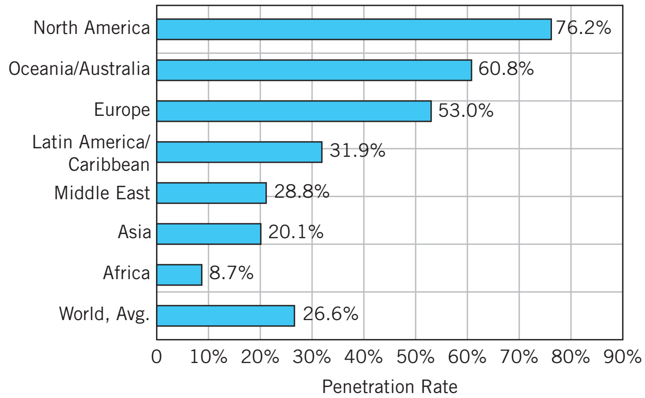
Reading **18.** Read the *CHANCE News* article and explain why the author was concerned.

19. Use the following graphs to describe three interesting facts about Internet use worldwide in 2009.

Internet Users in the World by Geographic Regions—2009



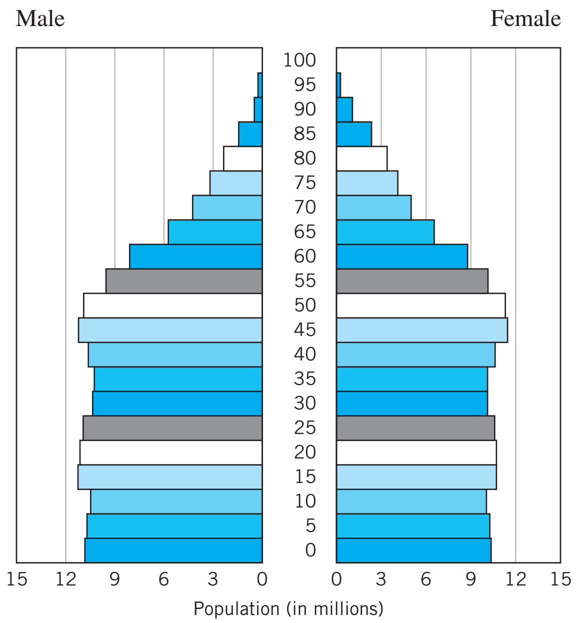
World Internet Penetration Rates by Geographic Regions—2009



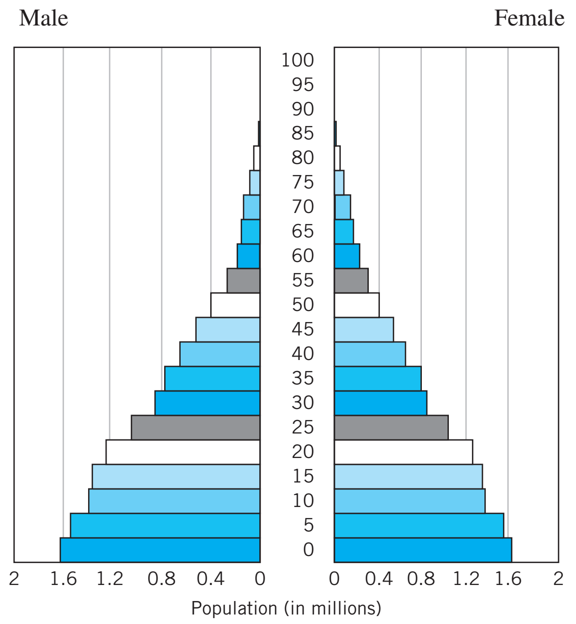
Source: Internet World Stats, *www.internetworldstats.com*. Estimated Internet users are 1,802,330,457 for December 31, 2009.

20. Population pyramid charts are used to depict the overall age structure of a society. The accompanying pyramids show the age structure in 2010 for Ghana, a developing country, and for the United States, an industrialized nation. Describe three major differences in the distribution of ages in the these two countries in 2010.

United States–2010



Ghana–2010

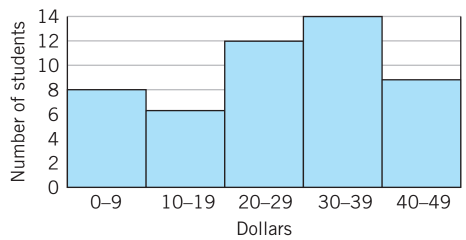


Source: U.S. Bureau of Census, International Data Base.

21. Estimate the mean and median from the given histogram.

(*Hint:* Replace each dollar interval with a dollar amount approximately in the middle of the interval.)

Monthly Allowance of Junior High School Students

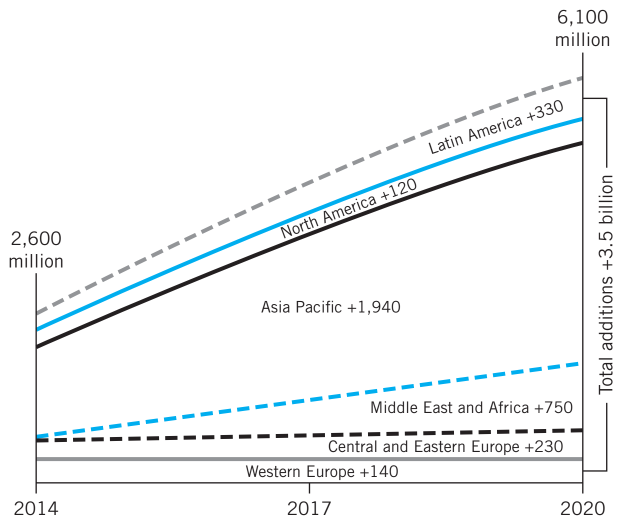


Applet **22.** (Computer and course software required.) Open up the course software "F1: Histograms" in *FAM 1000 Census Graphs* in the course software. The 2009 U.S. Census data on 1000 randomly selected U.S. individuals and their families are imbedded in this program. You can use it to create histograms for education, age, and different measures of income. Try using different interval sizes to see what patterns emerge. Decide on one variable (say education) and compare the histograms of this variable for different groups of people. For example, you could compare education histograms for men and women or for people living in two different regions of the country. Pick a comparison that you think is interesting. Create a possible headline for these data. Describe three key features that support your headline.

23. According to the Pew Research Center, roughly three quarters (77%) of Americans owned a smartphone in January 2017. The following graph shows predictions for 2020 for smartphone subscriptions in different regions of the world.

a. According to Pew Research Center, what region of the world will account for the largest increase in smartphone subscriptions between 2014 and 2020? What region will account for the smallest increase? Explain your results.

b. In 2020, the world population is predicted to be about 7.8 billion people. What percent of the world population are predicted to be using smartphones in 2020?



1.2 Describing Relationships Between Two Variables

By looking at two-variable data, we can learn how change in one variable affects change in another. How does the weight of a child determine the amount of medication prescribed by a pediatrician? How does median age or income change over time? In this section we examine how to describe these changes with graphs, data tables, words, and equations.

Exploration

Exploration 1.1 Homework provides an opportunity to collect and analyze two-variable data.

Visualizing Two-Variable Data

EXAMPLE 1 | Scatter Plots

Table 1.1 shows data for two variables, the year and the median age of the U.S. population. Plot the data in Table 1.1 and then use your graph to describe the changes in the U.S. median age over time.

TABLE 1.1 **Median Age of the U.S. Population, 1850–2050***

Year	Median Age	Year	Median Age
1850	18.9	1950	30.2
1860	19.4	1960	29.5
1870	20.2	1970	28.0
1880	20.9	1980	30.0
1890	22.0	1990	32.8
1900	22.9	2000	35.3
1910	24.1	2005	36.7
1920	25.3	2010	36.9
1930	26.4	2025	38.2
1940	29.0	2050	39.0

*Data for 2010–2050 are projected

Source: U.S., Bureau of the Census.

Data

There are Excel and graph link files for the median age data called MEDAGE on the course website.

Solution

In Table 1.1 we can think of a year and its associated median age as an *ordered pair* of the form (year, median age). For example, the first two elements correspond to the ordered pair (1850, 18.9) and the second corresponds to (1860, 19.4). **Figure 1.3** shows a scatter plot of the data. The graph is called a *time series* because it shows changes over time. In newspapers and magazines, the time series is the most frequently used form of data graphic.[6]

Median age of U.S. population over time: A time series

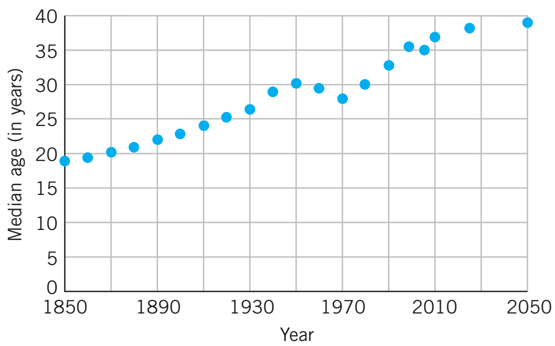


FIGURE 1.3

Our graph shows that the median age of the U.S. population grew quite steadily for one hundred years, from 1850 to 1950. Although the median age decreased between 1950 and 1970, since 1970 it has continued to increase. From 1850 to the present, the median age nearly doubled, and projections for 2025 and 2050 indicate continued increases, though at a slower pace.

Constructing a "60-Second Summary"

To communicate effectively, you need to describe your ideas succinctly and clearly. One tool for doing this is a "60-second summary"—a brief synthesis of your thoughts that could be presented in one minute. Quantitative summaries should be straightforward and concise. They often start with a topic sentence that summarizes the key idea, followed by supporting quantitative evidence.

[6]Edward Tufte in *The Visual Display of Information* (Cheshire, Conn.: Graphics Press, 2001, p. 28) reported on a study that found that more than 75% of all graphics published were time series.

After you have identified a topic you wish to write about or present orally, some recommended steps for constructing a 60-second summary are:

- Collect relevant information (possibly from multiple sources, including the Internet).
- Search for patterns, taking notes.
- Identify a key idea (out of possibly many) that could provide a topic sentence.
- Select evidence and arguments that support your key idea.
- Examine counterevidence and arguments and decide if they should be included.
- Construct a 60-second summary, starting with your topic sentence.

To help your ideas take shape, put them down on paper, then refine and modify. Quantitative reports should not be written in the first person. For example, you might say something like "The data suggest that . . ." rather than "I found that the data . . ."

EXAMPLE 2 | A 60-Second Summary

The annual federal surplus (+) or deficit (–) since World War II is shown in **Table 1.2** and **Figure 1.4** (a scatter plot where the points have been connected). Construct a 60-second summary describing the changes over time.

TABLE 1.2 Fiscal year[7] Federal Budget: Surplus (+) or Deficit (–)

Year	Billions of Dollars	Year	Billions of Dollars
1945	–48	1995	–164
1950	–3	1996	–107
1955	–3	1997	–22
1960	–0	1998	69
1965	–1	1999	126
1970	–3	2000	236
1975	–53	2001	128
1980	–74	2002	–158
1985	–212	2003	–378
1990	–221	2004	–413
1991	–269	2005	–318
1992	–290	2006	–248
1993	–255	2007	–161
1994	–203	2008	–459
		2009	–1,413

Source: U.S. Office of Management and Budget.

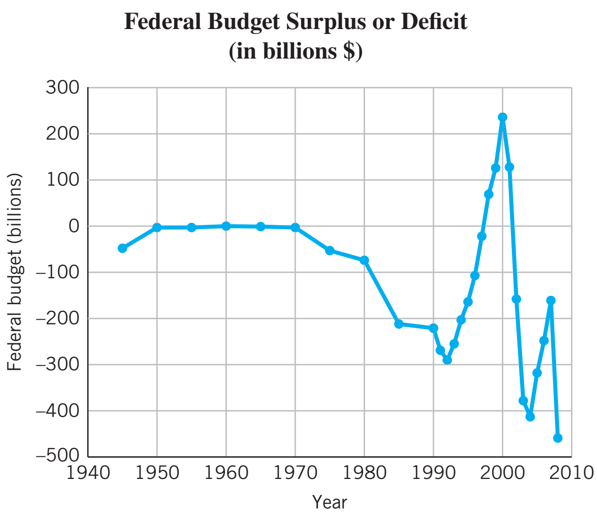


FIGURE 1.4 Note that the data point (2009, –1,413) is not included on the graph.

Solution

Between 1945 and 2009 the annual U.S. federal deficit moved from a 30-year stable period, with as little as $0 deficit, to a period of oscillations, leading in 2009 to the largest deficit ever recorded. From the 1970s to 1992, the federal budget ran an annual deficit, which generally was getting larger until it reached almost $300 billion in 1992. From 1992 to 1997, the deficit steadily decreased, and from 1998 to 2001 there were relatively large surpluses. The maximum surplus occurred in 2000, when it reached $236 billion. But by 2002 the federal government was again running large deficits. In 2009 the deficit reached $1,413 billion, the largest recorded up to that time.

[7]The government's fiscal year runs from October 1 through September 30 and is designated by the year in which it ends. For example, the fiscal year for 2010 is from October 1, 2009 to September 30, 2010.

Algebra Aerobics **1.2a**

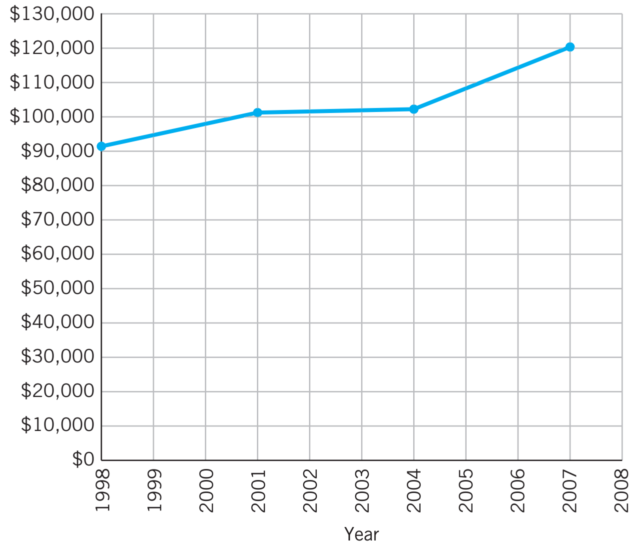
1. The net worth of a family at any given time is the difference between assets (what is *owned*) and liabilities (what is *owed*). The following table and graph show the median net worth of U.S. families, adjusted for inflation.

Median Family Net Worth (adjusted for inflation using 2007 dollars)[8]

Year	Median Net Worth ($)
1998	91,300
2001	101,200
2004	102,200
2007	120,300

Source: The Federal Reserve Board, *www.federalreserve.gov*.

Median Family Net Worth (in 2007 dollars)



a. Write a few sentences about the trend in U.S. median family net worth.

b. What additional information might be useful in describing the trend in median net worth?

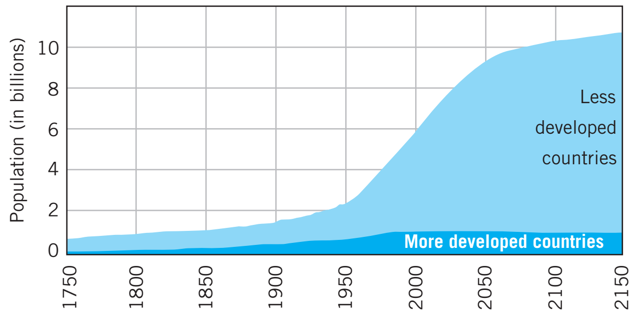
2. Use the World Population graph to estimate:

a. The year when the world population reached 4 billion.

b. The year that it is projected to reach 8 billion.

c. The number of years it will take to grow from 4 to 8 billion.

World Population with Projections to 2150



Source: Population Reference Bureau, *www.prb.org*.

3. Use the World Population graph to estimate the following projections for the year 2150.

a. The total world population.

b. The total populations of all the more developed countries.

c. The total populations of all the less developed countries.

d. Write a topic sentence about the estimated world population in 2150.

4. Use the World Population graph to answer the following:

a. The world population in 2000 was how many times greater than the world population in 1900? What was the difference in population size?

b. The world population in 2100 is projected to be how many times greater than the world population in 2000? What is the difference in population size?

c. Describe the difference in the growth in world population from 1900 to 2000 versus the projected growth from 2000 to 2100.

Using Equations to Describe Change

Sometimes the relationship between two variables can also be described with an equation. An equation gives a rule on how change in the value of one variable affects change in the value of the other. If the variable n represents the number of years of education beyond eighth grade and E represents yearly median earnings (in dollars) for people living in the United States, then the following equation and graph in **Figure 1.5** model the relationship between E and n:

$$E = 1140 + 6250\, n$$

[8] "Constant dollars" is a measure used by economists for comparing the value of money over time, eliminating the effects of inflation or deflation. It is based on the buying power of the dollar in a certain base year. For example, the net worth in 1998 of $91,300 "in constant 2007 dollars" was equal to the amount of goods and services that cost $91,300 in 2007. The actual net worth in 1998, measured in what is called "current dollars," was much lower.

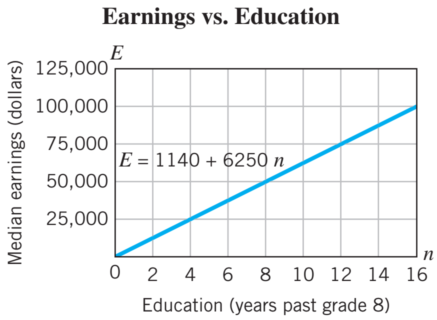


FIGURE 1.5

This equation provides a useful tool for describing how earnings and education are linked and for making predictions.[9] For example, to predict the median earnings, E, for those with a high school education, we replace n with 4 (representing 4 years beyond eighth grade, or a high school education) in our equation to get

$$\begin{aligned} E &= 1140 + 6250 \cdot 4 \\ &= \$26{,}140 \end{aligned}$$

Our equation predicts that for those with a high school education, median earnings will be about \$26,140.

An equation that is used to describe a real-world situation is called a *mathematical model*. Such models offer compact, often simplified descriptions of what may be a complex situation. The accuracy of the predictions made with such models can be questioned and disciplines outside of mathematics may be needed to help answer such questions. Yet these models are valuable guides in our quest to understand social and physical phenomena in our world.

Describing the relationship between abstract variables Variables can represent quantities that are not associated with real objects or events. The following equation or mathematical sentence defines a relationship between two quantities, which are named by the abstract variables x and y:

$$y = x^2 + 2x - 3$$

Solutions to the equation are pairs of values for x and y that make the sentence true. By convention, we express these solutions as ordered pairs of the form (x, y).

EXAMPLE 3 | Identifying Solutions to an Equation

Given $y = x^2 + 2x - 3$

a. Are (1, 0) and (0, 1) solutions to $y = x^2 + 2x - 3$?

b. How many solutions are there?

c. Create a table with several solutions for this equation.

d. Use technology to graph the equation. State the relationship between points on the graph and solutions to the equation. Label one solution on your graph.

Solution

a. (1, 0) would be a solution to $y = x^2 + 2x - 3$, since $0 = 1^2 + 2(1) - 3$, whereas (0, 1) would not be a solution, since $1 \neq 0^2 + 2(0) - 3$.

b. There are infinitely many possible solutions to the equation $y = x^2 + 2x - 3$, since we could substitute any real number for x and find a corresponding y.

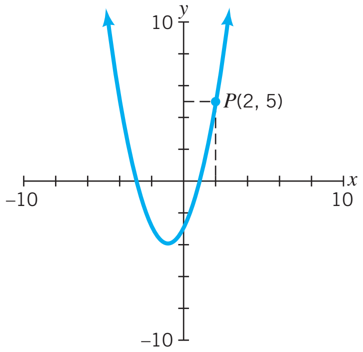
c. Table 1.3 lists a few solutions.

d. The graph of the equation is shown in Figure 1.6. All the points on the graph represent solutions to the equation, and every solution is a point on the graph of the equation.

[9] In "Looking for Links Between Education and Earnings," in Section 2.11, we show how such equations are derived and how they are used to analyze the relationship between education and earnings.

TABLE 1.3

x	y
–4	5
–3	0
–2	–3
–1	–4
0	–3
1	0
2	5
3	12



Coordinates of point P are:
(*horizontal* coordinate, *vertical* coordinate)
(x, y)
$(2, 5)$

FIGURE 1.6 **Graph of $y = x^2 + 2x - 3$ where one solution of infinitely many solutions is labeled.**

Note that sometimes an arrow is used to show that a graph extends indefinitely in the indicated direction. In Figure 1.6, the arrows show that both arms of the graph extend indefinitely upward.

> ### Solutions of an Equation
> The *solutions* of an equation in two variables x and y are the ordered pairs (x, y) that make the equation a true statement.
>
> ### Graph of an Equation
> The *graph* of an equation in two variables displays the set of points that are solutions to the equation.

EXAMPLE 4 | Solutions for Equations in One or Two Variables

Describe how the solutions for the following equations are similar and how they differ.

a. $3x + 5 = 11$

b. $x + 2 = x + 2$

c. $3 + x = y + 5$

Solution

The solutions are similar in the sense that each solution for each particular statement makes the statement true. They are different because:

a. There is only one solution $(x = 2)$ of the single-variable equation $3x + 5 = 11$.

b. There are an infinite number of solutions for x of the single-variable equation $x + 2 = x + 2$, since the left-side expression is the same as the right-side expression, any real number will make the statement a true statement.

c. There are infinitely many solutions, in the form of ordered pairs (x, y), of the two-variable equation $3 + x = y + 5$.

EXAMPLE 5 | Estimating Solutions From a Graph

The graph of the equation $x^2 + 4y^2 = 4$ is shown in **Figure 1.7**.

a. From the graph, estimate three solutions of the equation.

b. Check your solutions using the equation.

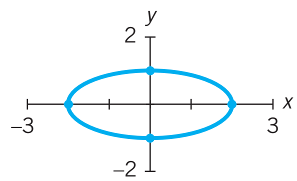


FIGURE 1.7

Solution

a. The coordinates (0, 1), (–2, 0), and (1, 0.8) appear to lie on the ellipse, which is the graph of the equation $x^2 + 4y^2 = 4$.

b. If substituting the ordered pair (0, 1) into the equation makes it a true statement, then (0, 1) is a solution.

Given	$x^2 + 4y^2 = 4$
substitute $x = 0$ and $y = 1$	$(0)^2 + 4(1)^2 = 4$
evaluate	$4 = 4$

We get a true statement, so (0, 1) is a solution to the equation.
For the ordered pair (–2, 0):

Given	$x^2 + 4y^2 = 4$
substitute $x = -2$ and $y = 0$	$(-2)^2 + 4(0)^2 = 4$
evaluate	$4 + 0 = 4$
	$4 = 4$

Again we get a true statement, so (–2, 0) is a solution to the equation.
For the ordered pair (1, 0.8):

Given	$x^2 + 4y^2 = 4$
substitute $x = 1$ and $y = 0.8$	$(1)^2 + 4(0.8)^2 = 4$
evaluate	$1 + 4(0.64) = 4$
	$3.56 \neq 4$

We get a false statement, so (1, 0.8) is not a solution, although it is close to a solution.

Algebra Aerobics **1.2b**

1. **a.** Describe in your own words how to compute the value for y, given a value for x, using the following equation:

$$y = 3x^2 - x + 1$$

b. Which of the following ordered pairs represent solutions to the equation?

(0, 0), (0, 1), (1, 0), (–1, 2), (–2, 3), (–1, 0)

c. Use $x = 0, \pm1, \pm2, \pm3$ to generate a small table of values that represent solutions to the equation.

2. Repeat the directions in Problem 1(a), (b), and (c) using the equation $y = (x - 1)^2$.

3. Given the equations $y_1 = 4 - 3x$ and $y_2 = -2x^2 - 3x + 5$, fill in the following table.

x	–4	–2	–1	0	1	2	4
y_1							
y_2							

a. Use the table to create two scatter plots, one for the ordered pairs (x, y_1) and the other for (x, y_2).

b. Draw a smooth curve through the points on each graph.

c. Is (1, 1) a solution for equation y_1? For y_2?

d. Is (–1, 6) a solution for equation y_1? For y_2?

e. Look at the graphs. Is the ordered pair (–3, 2) a solution for either equation? Verify your answer by substituting the values into each equation.

4. Given the equation $y = x^2 - 3x + 2$,

a. If $x = 3/2$, find y.

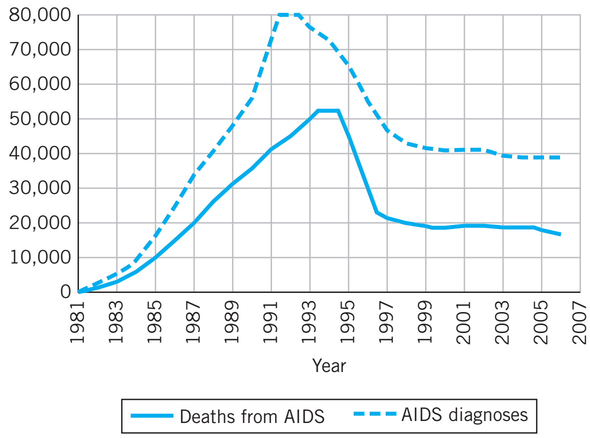
b. Find two points that are *not* solutions to this equation.

c. If available, use technology to graph the equation and then confirm your results for parts (a) and (b).

Exercises for Section **1.2**

1. In June 1981, the first cases of AIDS were reported in the United States. In 2008, it was estimated that more than one million people were living with HIV in the United States and that more than half a million died after developing AIDS. The following graph shows AIDS diagnoses and deaths from 1981 to 2007. Assume you work for a newspaper and are asked to report on the following data.

AIDS Diagnoses and Deaths



Source: *www.avert.org*. Avert is an international AIDS charity.

a. What are three important facts that emerge from this graph?

b. Construct a 60-second summary that could accompany the graph in the newspaper article.

2. The following graph shows changes in the Dow Jones Industrial Average, which is based on 30 stocks that trade on the New York Stock Exchange and is the best-known index of U.S. stocks.

Dow Jones Industrial Average

Note that the symbol on the lower left corner of the graph indicates that the vertical axis does not start at 0.

Source: Yahoo Finance. *http://finance.yahoo.com*.

a. What time period does the graph cover?

b. Estimate the lowest Dow Jones Industrial Average. During what month and year did it occur?

c. Estimate the highest value for the Dow Jones Industrial Average during that period. When did it occur?

d. Write a topic sentence describing the change in the Dow Jones Industrial Average over the given time period.

3. The accompanying table shows the number of personal and property crimes in the United States from 1980 to 2007.

Year	Personal Crimes (in thousands)	Property Crimes (in thousands)
1980	1,345	12,064
1985	1,328	11,103
1990	1,820	12,655
1995	1,799	12,064
2000	1,425	10,183
2005	1,391	10,175
2006	1,418	9,984
2007	1,408	9,843

Source: U.S. Bureau of the Census.

a. Create a scatter plot of the personal crimes over time. Connect the points with line segments.

b. Approximately how many *times* more property crimes than personal crimes were committed in 1980? In 2007?

c. Write a topic sentence about property and personal crime from 1980 to 2007.

4. The following graph shows the amount of revenues from newspaper ad sales over time.

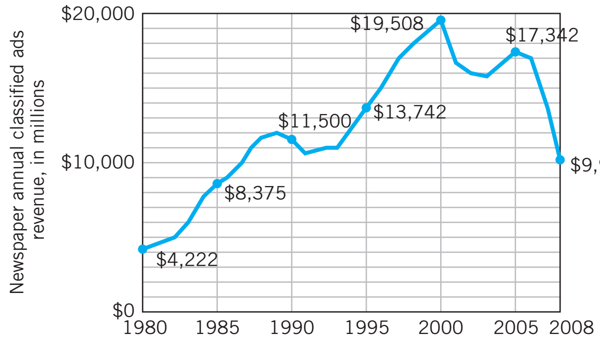
a. What are the ordered pairs associated with the highest and lowest revenues?

b. What are the units for the ordered pair (1995, 13742)? Interpret the meaning of this ordered pair.

c. Write a 60-second summary about these changes. Can you think of any reasons to explain changes in revenue over time?

Newspaper Classified Ads Revenue Has Plummeted in the Past Few Years

Newspaper revenue from classied ads, 1980–2008



Source: Business Analysis and Research Newspaper Association of America, "Advertising Expenditures." Available at *http://www.naa.org*.

5. The National Center for Chronic Disease Prevention and Health Promotion published the following data on the chances that a man has had prostate cancer at different ages.

Lifetime Risk of Developing Prostate Cancer

Age	Risk	Percent Risk
45	1 in 25,000	0.004%
50	1 in 476	0.21%
55	1 in 120	0.83%
60	1 in 43	2.3%
65	1 in 21	4.8%
70	1 in 13	7.7%
75	1 in 9	11.1%
80	1 in 6	16.7%

a. What is the relationship between age and getting prostate cancer?

b. Make a scatter plot of the percent risk for men of the ages given.

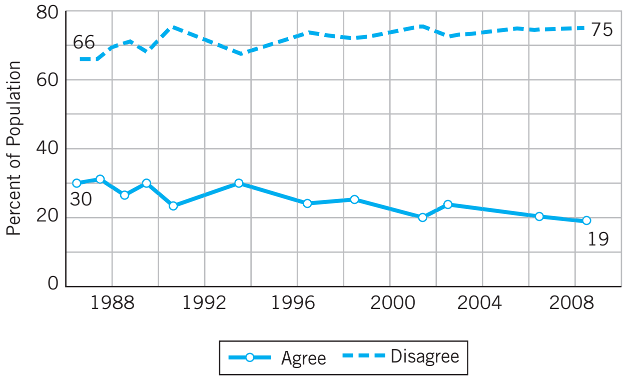
c. Use the "percent risk" data to find how much more likely men 50 years old will develop prostate cancer than men who are 45. How much more likely are men 55 years old to develop prostate cancer than men who are 50?

d. Looking at these data, when would you recommend annual prostate checkups to begin for men? Explain your answer in terms of the interests of the patient and of the insurance company.

6. Use the following graph to write a 60-second summary of the attitudes of Americans about the role of women in society.

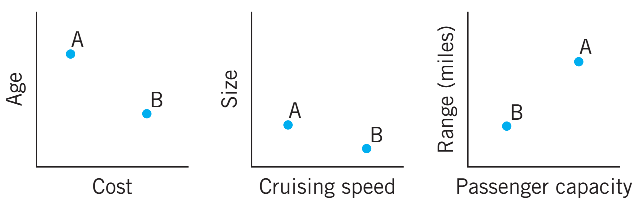
The Changing Role of Women

Women should return to their traditional role in society



Source: Pew Research Center for the People & the Press May 21, 2009 survey report "Trends in Political Values and Core Attitudes: 1987–2009."

7. The following three graphs describe two cars, A and B.



For parts (a)–(d), decide whether the statement is true or false. Explain your reasoning.

a. The newer car is more expensive.

b. The slower car is larger.

c. The larger car is newer.

d. The less expensive car carries more passengers.

e. State two other facts you can derive from the graphs.

f. Which car would you buy? Why?

8. **a.** Which (if any) of the following ordered pairs (x, y) is a solution to the equation $y = x^2 - 2x + 1$? Show how you came to your conclusion.

$$(-2, 7) \quad (1, 0) \quad (2, 1)$$

b. Find one additional ordered pair that is a solution to the preceding equation. Show how you found your solution.

9. Consider the equation $R = 2 - 5T$.

a. Determine which, if any, of the following points (T, R) satisfy this equation.

$$(0, 4) \quad (1, -3) \quad (2, 0)$$

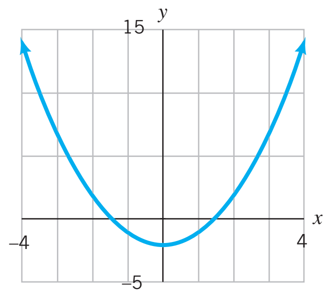
b. Find two additional ordered pairs that are solutions to the equation.

c. Make a scatter plot of the solution points found.

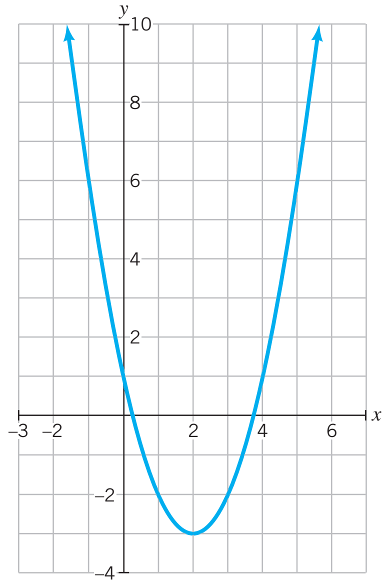
d. What does the scatter plot suggest about where more solutions could be found? Check your predictions.

10. Use the accompanying graph to estimate the missing values for x or y in the table.

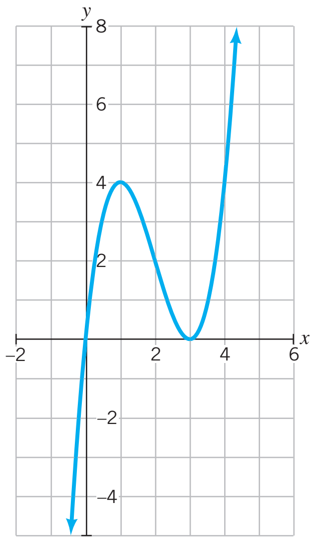
x	y
−3	
	2
−1	
	−2
	−1
2	
	7



11. The graph of the equation $(x - 2)^2 - 3 = y$ is shown. Estimate three solutions from the graph and then check your solutions using the equation.



12. The graph of the equation $x(x - 3)^2 = y$ is shown. Estimate three solutions from the graph and then check your solutions using the equation.



13. For parts (a)–(d) use the following equation: $y = -2x^2$.

a. If $x = 0$, find the value of y.

b. If x is greater than zero, what can you say about the value of y?

c. If x is a negative number, what can you say about the value of y?

d. Can you find an ordered pair that represents a solution to the equation when y is greater than zero? If so, find it; if not, explain why.

14. Find the ordered pairs that represent solutions to each of the following equations when $x = 0$, when $x = 3$, and when $x = -2$.

a. $y = 2x^2 + 5x$ **c.** $y = x^3 + x^2$

b. $y = -x^2 + 1$ **d.** $y = 3(x - 2)(x - 1)$

15. Given the four ordered pairs $(-1, 3)$, $(1, 0)$, $(2, 3)$, and $(1, 2)$, for each of the following equations, identify which points (if any) are solutions for that equation.

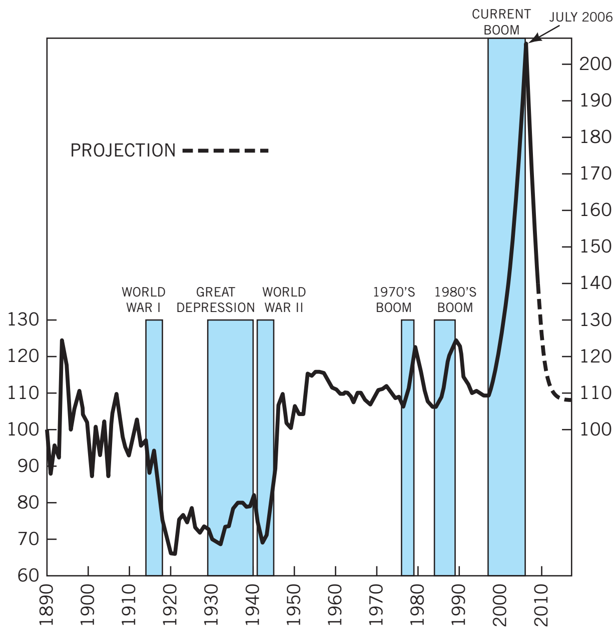
a. $y = 2x + 5$ **c.** $y = x^2 - x + 1$

b. $y = x^2 - 1$ **d.** $y = \frac{4}{x+1}$

16. Yale economist Robert J. Shiller and Wellesley economist Karl Case created the Case-Shiller index of American housing prices that tracks the value of housing back to 1890 in consistent terms, factoring out the effects of inflation. The 1890 benchmark is 100 on the chart. If a standard house sold in 1890 for $100,000 (inflation-adjusted to today's dollars), an equivalent house would have sold for $66,000 in 1920 (66 on the index scale) and $220,000 in 2006 (220 on the index scale).

Use the following graph, based on the Case-Shiller index, to write a 60-second summary about home values in the United States since 1890. You can use the Internet to find the current value of homes and see if these projections are accurate.

A History of Home Values



1.3 An Introduction to Functions

What Is a Function?

When we speak informally of one quantity being a function of some other quantity, we mean that one depends on the other. For example, someone may say that what they wear is a function of where they are going, or what they weigh is a function of what they eat, or how well a car runs is a function of how well it is maintained.

In mathematics, the word "function" has a precise meaning. A function is a special type of relationship between two quantities. If the value of one quantity uniquely determines the value of a second quantity, then the second quantity is a *function* of the first.

For example, median age (as shown in Example 1) is a function of time, since each input of a year determines a unique (one and only one) output value of a median age.

> A *function* is a rule that assigns to each input quantity exactly one output quantity. The output is a function of the input.

Representing Functions: Words, Tables, Graphs, and Equations

A function rule can be described using words, data tables, graphs, or equations. The median age example uses data tables and graphs. When a function is described in these various ways, each description provides the same information but with a different emphasis.

EXAMPLE 1 | Sales Tax

Twelve states have a sales tax of 6%; that is, for each dollar spent in a store in these states, the law says that you must pay a tax of 6 cents, or $0.06. Represent the sales tax as a function of purchase price using words, an equation, table, and graph.[10]

[10]In 2009, a sales tax of 6% was the most common rate for a sales tax in the United States. See *taxadmin.org* for a listing of the sales tax rates for all of the states.

Solution

Using Words

The sales tax is 6% of the purchase price, which means you multiply the purchase price by six hundredths to get the sales tax.

Using an Equation

We can write this relationship as an equation where T represents the amount of sales tax and P represents the price of the purchase (both measured in dollars):

$$\text{amount of sales tax} = 0.06 \cdot \text{price of purchase}$$
$$T = 0.06P$$

Our function rule says: "Take the given value of P and multiply it by 0.06; the result is the corresponding value of T." The equation represents T as a function of P, since for each value of P the equation determines a unique (one and only one) value of T. The purchase price, P, is restricted to dollar amounts greater than or equal to zero.

Using a Table

We can use this formula to make a table of values for T determined by the different values of P (see **Table 1.4**). Such tables are sometimes seen beside cash registers.

TABLE 1.4

P (purchase price in $)	0	1	2	3	4	5	6	7	8	9	10
T (sales tax in $)	0.00	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60

Using a Graph

The points in Table 1.4 were used to create a graph of the function (**Figure 1.8**). The table shows the sales tax only for selected purchase prices, but we could have used any positive dollar amount for P. We connected the points on the scatter plot to suggest the many possible intermediate values for price. For example, if $P = \$2.50$, then $T = \$0.15$.

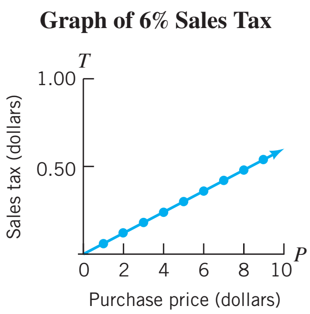


FIGURE 1.8

Input and Output: Independent and Dependent Variables

Since a function is a rule that assigns to each input a unique output, we think of the output as being dependent on the input. We call the input of a function the *independent variable* and the output the *dependent variable*. When a set of ordered pairs represents a function, then each ordered pair is written in the form

(input, output)

or equivalently, (independent variable, dependent variable)

If x is the independent and y the dependent variable, then the ordered pairs would be of the form

$$(x, y)$$

The mathematical convention is for the first variable, or input of a function, to be represented on the horizontal axis and the second variable, or output, on the vertical axis (see **Figure 1.9**).

Sometimes the choice of the independent variable is arbitrary or not obvious. For example, economists argue as to whether wealth is a function of education or education is a function of wealth. As seen in the next example, there may be more than one correct choice.

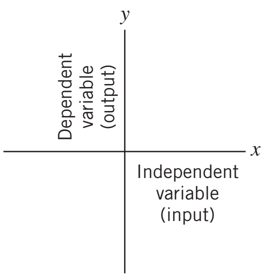


FIGURE 1.9

EXAMPLE 2 | Identifying Independent and Dependent Variables

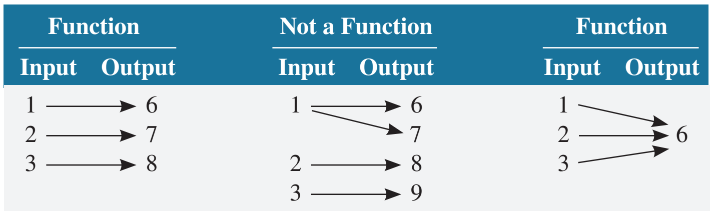
In the sales tax example, the equation $T = 0.06P$ gives the sales tax, T, as a function of purchase price, P. In this case T is the dependent variable, or output, and P is the independent variable, or input. But for this equation we can see that P is also a function of T; that is, each value of T corresponds to one and only one value of P. It is easier to see the relationship if we solve for P in terms of T, to get

$$P = \frac{T}{0.06}$$

Now we are thinking of the purchase price, P, as the dependent variable, or output, and the sales tax, T, as the independent variable, or input. So, if you tell me how much tax you paid, I can find the purchase price.

When Is a Relationship Not a Function?

Not all relationships define functions. A function is a special type of relationship, one where for each input, the rule specifies one and only one output. Examine the following examples.



Each input has only one output. | The input of 1 gives *two different outputs*, 6 and 7, so this relationship is *not* a function. | Each input has only one output. Note that a function may have identical outputs for different inputs.

EXAMPLE 3 | Does the Table Represent a Function?

Consider the data in **Table 1.5** on the rate of unemployment in the United States. Which variable/s could be used as the input of a function? What about the output?

TABLE 1.5

Unemployment in the United States

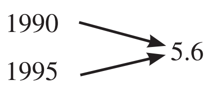
Year, Y	Unemployment Rate, U
1975	8.5
1980	7.1
1985	7.2
1990	5.6
1995	5.6
2000	4.0
2005	5.1
2008	5.8
2009	9.3
2010	10.0 (est)

Source: Bureau of Labor Statistics, 2010 *http://www.bls.gov*.

Solution

U is a Function of *Y*

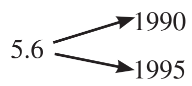
The year, Y, can be used as the input of a function where the unemployment rate, U, is the output. For each year there is one and only one output; therefore, U is a function of Y.



Note that different inputs (such as 1990 and 1995) can have the same output (5.6) and still satisfy the conditions of a function.

Y is NOT a Function of *U*

The unemployment rate, *U*, can NOT be used as the input of a function where the year, *Y*, is the output. When $U = 5.6$, there are two corresponding values for *Y*, 1990 and 1995, and this would violate the condition for a function of a unique (one and only one) output for each input. Therefore *Y* is NOT a function of *U*.



Remember that each input can have only one output to satisfy the conditions of a function.

EXAMPLE 4 | The Input and Output of a Function on a Graph

How would the axes be labeled for each graph of the following functions?

a. Density of water is a function of temperature.

b. Radiation intensity is a function of wavelength.

c. A quantity *Q* is a function of time *t*.

Solution

Figure 1.10 shows how to label the input and output of these functions on a graph.

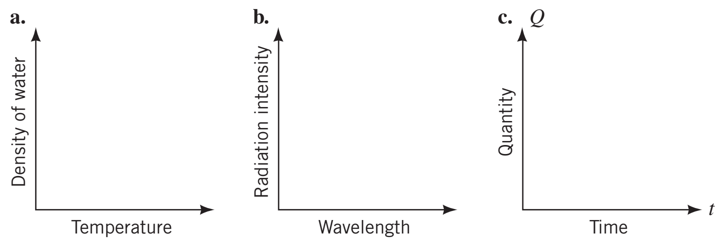


FIGURE 1.10 **Various labels for axes, depending on the context.**

How to tell if a graph represents a function: The vertical line test For a graph to represent a function, each value of the input on the horizontal axis must be associated with one and only one value of the output on the vertical axis. If you can draw a vertical line that intersects a graph in more than one point, then at least one input is associated with two or more outputs, and the graph does not represent a function.

> **Vertical Line Test**
>
> If there is a vertical line that intersects a graph more than once, the graph does not represent a function.

The graph in **Figure 1.11** represents *y* as a function of *x*. For each value of *x*, there is only one corresponding value of *y*. No vertical line intersects the curve in more than one point. The graph in **Figure 1.12** does *not* represent a function. One can draw a vertical line (an infinite number, in fact) that intersects the graph in more than one point. Figure 1.12 shows a vertical line that intersects the graph at both (4, 2) and (4, –2). That means that the value $x = 4$ does not determine one and only one value of *y*. It corresponds to *y* values of both 2 and –2.

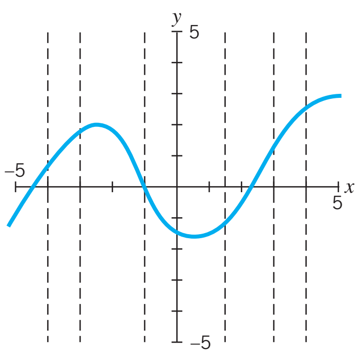


FIGURE 1.11 **The graph represents *y* as a function of *x* since there is no vertical line that intersects the curve at more than one point.**

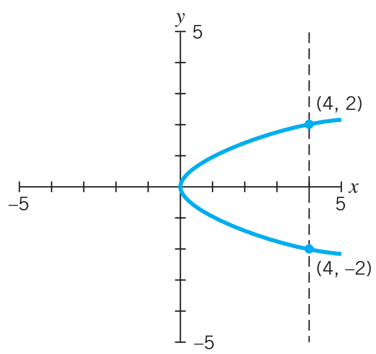


FIGURE 1.12 **The graph does not represent *y* as a function of *x* since there is at least one vertical line that intersects this curve at more than one point.**

Algebra Aerobics 1.3

1. Which of the following tables represent functions? Justify your answer.

TABLE A

Input	Output
1	5
2	8
3	8
4	10

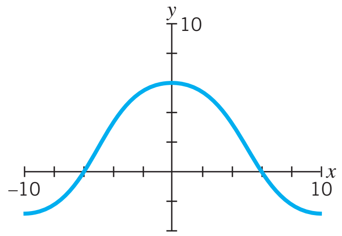
TABLE B

Input	Output
1	5
2	7
2	8
4	10

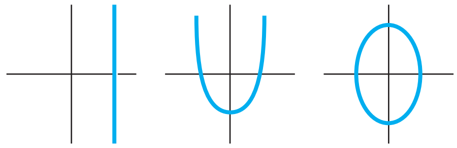
2. Does the following table represent a function? If so, why? If not, how could you change the values in the table so that it represents a function?

Input	Output
1	5
1	7
3	8
4	10

3. Refer to the following graph. Is y a function of x?



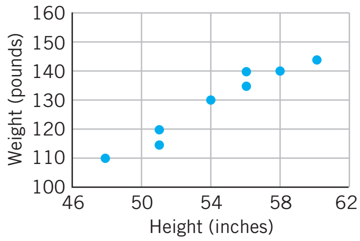
4. Which of the graphs represent functions and which do not? Why?



Graph *A* Graph *B* Graph *C*

5. Consider the scatter plot in the graph. Is weight a function of height? Is height a function of weight? Explain your answer.

Graph of Weight versus Height



6. Consider the following table.

a. Is D a function of Y?

b. Is Y a function of D?

Y	1992	1993	1994	1995	1996	1997
D	\$2.50	\$2.70	\$2.40	–\$0.50	\$0.70	\$2.70

7. a. Write an equation for computing a 15% tip in a restaurant. Does your equation represent a function? If so, what are your choices for the independent and dependent variables?

b. How much would the equation suggest you tip for an \$8 meal?

c. Compute a 15% tip on a total check of \$26.42.

Exercises for Section 1.3

A graphing program is required for Exercise 12.

1. The following table gives the high temperature in Santa Cruz, California, for each of five days in January, 2010.

a. Is the temperature a function of the date?

b. Is the date a function of the temperature?

Date	High Temperature Santa Cruz, California
Jan. 1	62°F
Jan. 2	65°F
Jan. 3	64°F
Jan. 4	66°F
Jan. 5	64°F

Source: *weather.com*.

2. Which of the following tables describe functions? Explain your answer.

a.

Input value	–2	–1	0	1	2
Output value	–8	–1	0	1	8

b.

Input value	0	1	2	1	0
Output value	–4	–2	0	2	4

c.

Input value	10	7	4	7	10
Output value	3	6	9	12	15

d.

Input value	0	3	9	12	15
Output value	3	3	3	3	3

3. Determine whether each set of points represents a function. (*Hint:* It may be helpful to plot the points.)

a. (2, 6), (−4, 6), (1, −3), (4, −3)

b. (2, −2), (3, −2), (4, −2), (6, −2)

c. (2, −3), (2, 3), (−2, −3), (−2, 3)

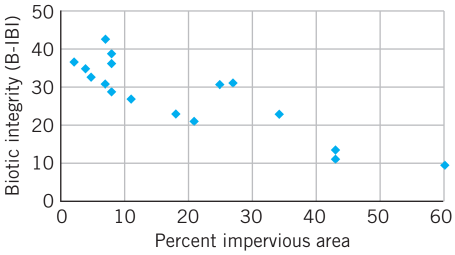
d. (−1, 2), (−1, 0), (−1, −1), (−1, −2)

4. Consider the accompanying table, listing the weights (*W*) and heights (*H*) of five individuals. Based on this table, is height a function of weight? Is weight a function of height? Justify your answers.

Weight *W* (lb)	Height *H* (in)
120	54
120	55
125	58
130	60
135	56

5. The accompanying graph uses a measure of urbanization as the percent of impervious area (area covered by materials such as asphalt, concrete, and so on that prevent the absorption of water) and shows the relationship to biotic integrity (the capability of supporting and maintaining a natural habitat).

Urbanization Effects on Streams



Source: Seattle Central College, Quantitative Environment Learning Project.

Use the graph to determine if the level of biotic integrity is a function of the percent of imperious area. Explain your answer.

6. Consider the accompanying graph describing the sales of organic products from 2005 to 2008. Use the vertical line test to determine if the percent change in sales of organic products is a function of time. Explain your answer.

Organic Product Sales

Source: The Nielsen Company.

7. **a.** Find an equation that represents the relationship between *x* and *y* in each of the accompanying tables.

i.

x	*y*
0	5
1	6
2	7
3	8
4	9

ii.

x	*y*
0	1
1	2
2	5
3	10
4	17

iii.

x	*y*
0	3
1	3
2	3
3	3
4	3

b. Which of your equations represents *y* as function of *x*? Justify your answer.

8. For each of the accompanying tables find a function that takes the *x* values and produces the given *y* values.

a.

x	*y*
0	0
1	3
2	6
3	9
4	12

b.

x	*y*
0	−2
1	1
2	4
3	7
4	10

c.

x	*y*
0	0
1	−1
2	−4
3	−9
4	−16

9. The basement of a large department store features discounted merchandise. Their policy is to reduce the previous month's price of the item by 10% each month for 5 months, and then give the unsold items to charity.

a. Let S_1 be the sale price for the first month and *P* the original price. Express S_1 as a function of *P*. What is the price of a \$100 garment on sale for the first month?

b. Let S_2 be the sale price for the second month and *P* the original price. Express S_2 as a function of *P*. What is the price of a \$100 garment on sale for the second month?

c. Let S_3 be the sale price for the third month and *P* the original price. Express S_3 as a function of *P*. What is the price of a \$100 garment on sale for the third month?

d. Let S_5 be the sale price for the fifth month and *P* the original price. Express S_5 as a function of *P*. What is the final price of a \$100 garment on sale for the fifth month? By what total percentage has the garment now been reduced from its original price?

10. Write a formula to express each of the following sentences:

a. The sale price is 20% off the original price. Use *S* for sale price and *P* for original price to express *S* as a function of *P*.

b. The time in Paris is 6 hours ahead of New York. Use *P* for Paris time and *N* for New York time to express *P* as a function of *N*. (Represent your answer in terms of a 12-hour clock.) How would you adjust your formula if *P* comes out greater than 12?

c. For temperatures above 0°F the wind chill effect can be estimated by subtracting two-thirds of the wind speed (in miles per hour) from the outdoor temperature. Use *C* for the effective wind chill temperature, *W* for wind speed, and *T* for the actual outdoor temperature to write an equation expressing *C* in terms of *W* and *T*.

11. Determine whether y is a function of x in each of the following equations. If the equation does not define a function, find a value of x that is associated with two different y values.

a. $y = x^2 + 1$

b. $y = 3x - 2$

c. $y = 5$

d. $y^2 = x$

12. (Graphing program required.) For each equation, write an equivalent equation that expresses z in terms of t. Use technology to sketch the graph of each equation. Is z a function of t? Why or why not?

a. $3t - 5z = 10$

b. $12t^2 - 4z = 0$

c. $2(t - 4) - (z + 1) = 0$

13. If we let D stand for ampicillin dosage expressed in milligrams and W stand for a child's weight in kilograms, then the equation

$$D = 50W$$

gives a rule for finding the safe maximum daily drug dosage of ampicillin (used to treat respiratory infections) for children who weigh less than 10 kilograms (about 22 pounds).[11]

a. What are logical choices for the independent and dependent variables?

b. Does the equation represent a function? Why?

c. Generate a small table and graph of the function.

1.4 The Language of Functions

Not all equations represent functions. (See Figure 1.12 where the graph of the equation does not pass the vertical line test.) But functions have important qualities, so it is useful to have a way to indicate when a relationship is a function.

Function Notation

When a quantity y is a function of x, we can write

$$y \text{ is a function of } x$$

or in abbreviated form, we say:

$$y \text{ equals "} f \text{ of } x\text{"}$$

or using function notation,

$$y = f(x)$$

The expression $y = f(x)$ means that the rule f is applied to the input value x to give the output value, $f(x)$:

$$\textbf{output} = f(\textbf{input})$$

or

$$\textbf{dependent variable} = f(\textbf{independent variable})$$

The letter f is often used to denote the function, but we could use any letter, not just f.

Understanding the symbols Suppose we create a function where the input is multiplied by three and 1 is added to the product. We could write this function as

$$y = 3x + 1 \tag{1}$$

or with function notation as

$$R(x) = 3x + 1 \quad \text{where } y = R(x) \tag{2}$$

Equations (1) and (2) represent the same function, but with function notation we name the function—in this case R—and identify the input, x, and output, $3x + 1$.

Function notation can provide considerable economy in writing and reading, by using $R(x)$ instead of the full expression to represent the function.

[11]Information extracted from Anna M. Curren and Laurie D. Muntlay, *Math for Meds: Dosages and Solutions*, 6th ed., San Diego: W. I. Publications, 1990.

The Language of Functions

If y is a function, f, of x, then

$$y = f(x)$$

where f is the name of the function,
y is the output or *dependent* variable,
x is the input or *independent* variable.

f(input) = output
f(independent) = dependent

Finding Output Values: Evaluating a Function

Function notation is particularly useful when a function is being evaluated at a specific input value. Suppose we want to find the value of the previous function $R(x)$ when our input value is 10. Using Equation (1) we would say, "find the value of y when $x = 10$." With function notation, we simply write $R(10)$. To evaluate the function R at 10 means calculating the value of the output when the value of the input is 10:

$$R(\text{input}) = \text{output}$$
$$R(x) = 3x + 1$$

Substitute 10 for x

$$R(10) = 3(10) + 1 = 31$$

So, applying the function rule R to the input value of 10 gives an output value of 31.

Common Error

The expresssion $f(x)$ does not mean "f times x." It means the function f evaluated at x.

EXAMPLE 1 | Using Function Notation with Equations

Evaluate $f(5)$, $f(0)$, and $f(-2)$ for the function $f(x) = 2x^2 + 3$.

Solution

To evaluate $f(5)$, we replace every x in the formula with 5.

Given $\quad f(x) = 2x^2 + 3$

substitute 5 for x $\quad f(5) = 2(5)^2 + 3 = 2(5)(5) + 3 = 53$

Similarly,

if $x = 0$, $\quad f(0) = 2(0)^2 + 3 = 3$

if $x = -2$, $\quad f(-2) = 2(-2)^2 + 3 = (2 \cdot 4) + 3 = 11$

Finding Input Values: Solving Equations

In Example 1, we found output values when we knew the input. Sometimes the situation is reversed and we know the output and want to find the corresponding input.

EXAMPLE 2 | What is the Value of the Input?

a. Use the sales tax function $T = 0.06P$ to find the purchase price P when the sales tax T on a purchase is \$8.40.

b. Given $f(x) = 5x - 2.5$, find x when $f(x) = 21$.

Solution

a. We want to find P when $T = \$8.40$, so substitute \$8.40 for T and then solve the equation:

$$
\begin{array}{ll}
 & T = 0.06P \\
\text{substitute \$8.40 for } T & \$8.40 = 0.06P \\
\text{divide both sides by } 0.06 & \$140 = P
\end{array}
$$

The purchase price is \$140 when the sales tax is \$8.40.

b. To find an input value for x that results in $f(x) = 21$, solve the equation

$$
\begin{array}{ll}
 & 21 = 5x - 2.5 \\
\text{add 2.5 to each side} & 23.5 = 5x \\
\text{divide by 5} & 4.7 = x
\end{array}
$$

When the input is $x = 4.7$, then $f(x) = 21$ or equivalently $f(4.7) = 21$.

Finding Input and Output Values From Tables and Graphs

EXAMPLE 3 | Using Function Notation with Data Tables

Use **Table 1.6** to fill in the missing values:

a. $S(0) = ?$

b. $S(-1) = ?$

c. $S(?) = 4$

TABLE 1.6

Input, x	–1	–2	0	1	2
Output, $S(x)$	1	4	0	1	4

Solution

a. $S(0)$ means to evaluate S when the input $x = 0$. The table says that the corresponding output is also 0, so $S(0) = 0$.

b. $S(-1) = 1$.

c. $S(?) = 4$ means to find the input when the output is 4. When the output is 4, the input is –2 or 2, so $S(-2) = 4$ and $S(2) = 4$.

EXAMPLE 4 | Using Function Notation with Graphs

Use the graph in **Figure 1.13** to estimate the missing values:

a. $f(0) = ?$

b. $f(-5) = ?$

c. $f(?) = 0$

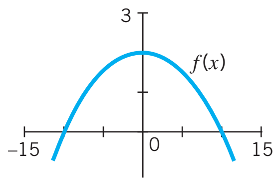


FIGURE 1.13 Graph of a function.

Solution

Remember that by convention the horizontal axis represents the input (or independent variable) and the vertical axis represents the output (or dependent variable).

a. $f(0) = 2$

b. $f(-5) = 1.5$

c. $f(10) = 0$ and $f(-10) = 0$

Rewriting Equations Using Function Notation

In order to use function notation, an equation needs to be in the form

$$\text{output} = \text{some rule applied to input}$$

or equivalently

$$\text{dependent variable} = \text{some rule applied to independent variable}$$

Translating an equation into this format is called putting the equation in *function form*. Many graphing calculators and computer graphing programs accept only equations in function form.

To put an equation into function form, we first need to identify the independent and the dependent variables. The choice is sometimes obvious, at other times arbitrary. If we use the mathematical convention that x represents the input or independent variable and y the output or dependent variable, when we put equations into function form, we want to solve for y or

$$y = \text{some rule applied to } x$$

EXAMPLE 5 | Does the Equation Represent a Function?

Analyze the equation $4x - 3y = 6$. Decide whether or not the equation represents y as a function of x. If it does, write the relationship using function notation.

Solution

First, put the equation into function form. Since y is the output, then

given the equation	$4x - 3y = 6$
subtract $4x$ from both sides	$-3y = 6 - 4x$
divide both sides by -3	$\dfrac{-3y}{-3} = \dfrac{6 - 4x}{-3}$
simplify	$y = \dfrac{6}{-3} + \dfrac{-4x}{-3}$
simplify and rearrange terms	$y = \dfrac{4}{3}x - 2$

We now have an expression for y in terms of x.

Using technology or by hand, we can generate a table and graph of the equation (see **Table 1.7** and **Figure 1.14**). Since the graph passes the vertical line test, y is a function of x.

TABLE 1.7

x	y
−3	−6
0	−2
3	2
6	6

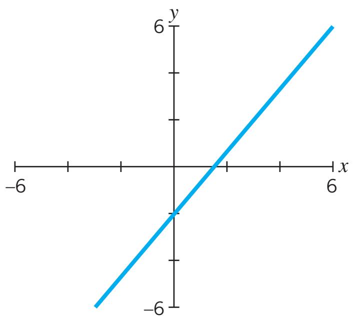


FIGURE 1.14 Graph of $y = \frac{4}{3}x - 2$.

If we name our function f, then using function notation, we have

$$y = f(x) \qquad \text{where } f(x) = \tfrac{4}{3}x - 2$$

EXAMPLE 6 | An Equation Where the Input Gives More Than One Output

Analyze the equation $y^2 - x = 0$. Generate a graph of the equation. Decide whether or not the equation represents a function. If the equation represents a function, write the relationship using function notation. Assume y is the output.

Solution

Put the equation in function form:

Given the equation	$y^2 - x = 0$
add x to both sides	$y^2 = x$

To solve this equation, we take the *square root* of both sides of the equation and we get

$$y = \pm\sqrt{x}$$

This gives us two solutions for any value of $x > 0$ as shown in **Table 1.8**. For example, if $x = 4$, then y can either be 2 or -2 since both $2^2 = 4$ and $(-2)^2 = 4$.

TABLE 1.8

x	y
0	0
1	1 or -1
2	$\sqrt{2}$ or $-\sqrt{2}$
4	2 or -2
9	3 or -3

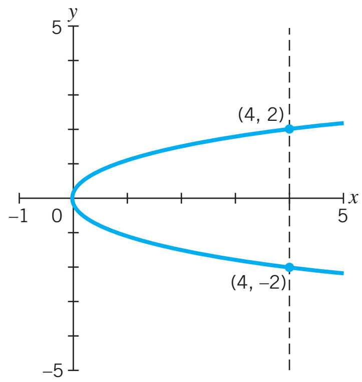


FIGURE 1.15 **Graph of the equation $y^2 = x$ is not a function.**

The graph of the equation in **Figure 1.15** does not pass the vertical line test. In particular, the solutions $(4, -2)$ and $(4, 2)$ lie on the same vertical line. So y is not a function of x and we cannot use function notation to represent this relationship.

Algebra Aerobics **1.4a**

1. Given $g(x) = 3x$, evaluate $g(0)$, $g(-1)$, $g(1)$, $g(20)$, and $g(100)$.

2. Consider the function $f(x) = x^2 - 5x + 6$. Find $f(0)$, $f(1)$, and $f(-3)$.

3. Given the function $f(x) = \frac{2}{x-1}$, evaluate $f(0)$, $f(-1)$, and $f(-3)$.

4. Determine the value of t for which each of the functions has a value of 3.

$$r(t) = 5 - 2t \qquad p(t) = 3t - 9 \qquad m(t) = 5t - 12$$

In Problems 5–7 solve for y in terms of x. Determine if y is a function of x. If it is, rewrite using $f(x)$ notation.

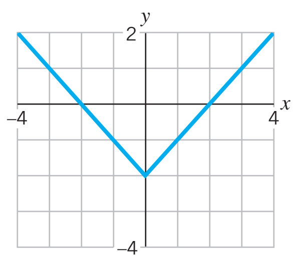
5. $2(x - 1) - 3(y + 5) = 10$

6. $x^2 + 2x - y + 4 = 0$

7. $7x - 2y = 5$

8. From the accompanying graph, estimate $f(-4)$, $f(-1)$, $f(0)$, and $f(3)$. Find two approximate values for x such that $f(x) = 0$.

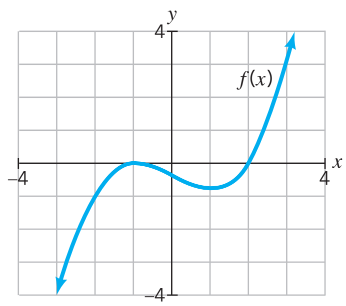
Graph of $f(x)$.



9. From the table, find $f(0)$ and $f(20)$. Find two values of x for which $f(x) = 10$. Explain why $f(x)$ is a function.

x	$f(x)$
0	20
10	10
20	0
30	10
40	20

10. From the accompanying graph, estimate $f(-3)$, $f(0)$, $f(1)$, and $f(3)$. Find two approximate values of x for which $f(x) = 0$ and $f(x) = 1.5$.



Domain and Range

A function is often defined only for certain values of the input (or independent variable) and corresponding values of the output (or dependent variable).

> **Domain and Range of a Function**
> The *domain* of a function is the set of possible values of the input.
> The *range* is the set of corresponding values of the output.

Many times, especially under real-world conditions, there are restrictions on the domain and the range. The restrictions are often implied, as in Example 7. As we study each family of functions, we return to the question of whether there are restrictions on the domain and range.

EXAMPLE 7 | Finding a Reasonable Domain and Range

In the sales tax example at the beginning of this section, we used the equation

$$T = 0.06P$$

to represent the sales tax, T, as a function of the purchase price, P (where all units are in dollars). What are the domain and range of this function?

Solution

Since negative values for P are meaningless, P is restricted to dollar amounts greater than or equal to zero. In theory there is no upper limit on prices, so we assume P has no maximum amount. In this example,

the domain is all dollar values of P greater than or equal to 0

We can express this more compactly as

$$\text{domain} = \text{dollar values of } P \geq 0$$

What are the corresponding values for the tax T? The values for T in our model cannot be negative. As long as there is no maximum value for P, there will be no maximum value for T. So,

the range is all dollar values of T greater than or equal to 0

or

$$\text{range} = \text{dollar values of } T \geq 0$$

(Note: The symbol ≥ means "greater than or equal to and the symbol ≤ means "less than or equal to")

Representing the domain and range with interval notation Interval notation is often used to represent the domain and range of a function.

Interval Notation

A *closed interval* $[a, b]$ indicates all real numbers x for which $a \le x \le b$. Closed intervals include their endpoints.

An *open interval* (a, b) indicates all real numbers x for which $a < x < b$. Open intervals exclude their endpoints.

Half-open (or equivalently half-closed) intervals are represented by $[a, b)$ which indicates all real numbers x for which $a \le x < b$ or $(a, b]$ which indicates all real numbers x for which $a < x \le b$.

For example, if the domain is values of x greater than or equal to 50 and less than or equal to 100, then

$$\begin{aligned} \text{domain} &= \text{all } x \text{ values with } 50 \le x \le 100 \\ &= [50, 100] \end{aligned}$$

If the domain is values of x greater than 50 and less than 100, then

$$\begin{aligned} \text{domain} &= \text{all } x \text{ values with } 50 < x < 100 \\ &= \text{interval } (50, 100) \end{aligned}$$

If we want to exclude 50 but include 100 as part of the domain, we would represent the interval as (50, 100]. The interval can be displayed on the real number line as:

In general, a hollow dot indicates exclusion and a solid dot inclusion.
Note: Since the notation (a, b) can also mean the coordinates of a point, we will say the *interval* (a, b) when we want to refer to an interval.

Visualizing the domain and range

Recall that by convention, the input of a function is represented on the horizontal axis and the output on the vertical axis. **Figure 1.16** illustrates the set of inputs or domain = [–4, 7] and the outputs or range of $f(x) = [-2, 6]$.

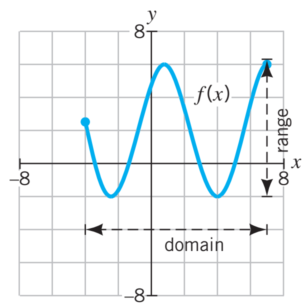


FIGURE 1.16

EXAMPLE 8 | Finding the Domain and Range From a Graph

The graph in **Figure 1.17** shows the water level of the tides in Pensacola, Florida, over a 24-hour period. Are the Pensacola tides a function of the time of day? If so, identify the independent and dependent variables. Use interval notation to describe the domain and range of this function.

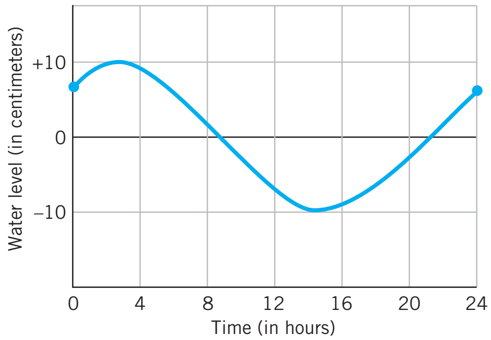


FIGURE 1.17 **Diurnal tides in a 24-hour period in Pensacola, Florida.**

Source: Adapted from Fig. 8.2 in *Oceanography: An Introduction to the Planet Oceanus*, by Paul R. Pinet.

Solution

The Pensacola tides are a function of the time of day since the graph passes the vertical line test. The independent variable is time, and the dependent variable is water level. The domain is from 0 to 24 hours, and the range is from about −10 to +10 centimeters. Using interval notation:

$$\text{domain} = [0, 24]$$
$$\text{range} = [-10, 10]$$

EXAMPLE 9 | Summarizing Music Cassette Sales

The following graph in **Figure 1.18** shows music cassette tape sales, $S(t)$ over time, t.

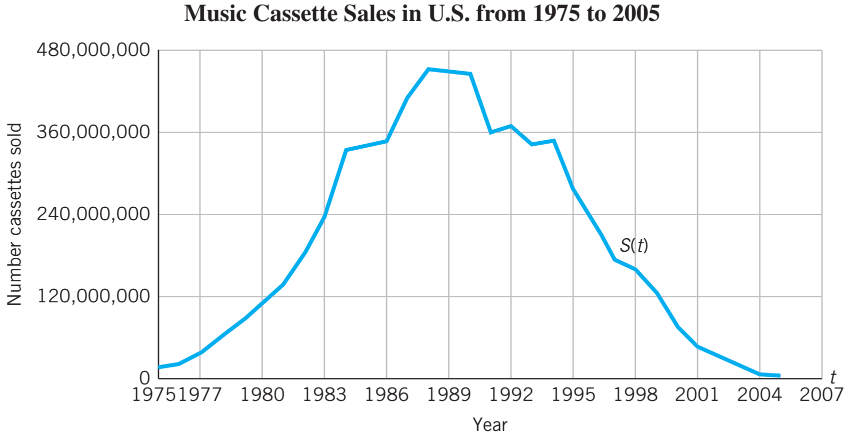


FIGURE 1.18

Find the following:

a. __________ is a function of __________.

b. Input (with units): __________ Output (with units): __________

c. Estimate: Domain: __________ Range: __________

d. Estimate: $S(1983) \approx$ __________

t such that $S(t) \approx 120{,}000{,}000$

What are the practical implications of these statements?

e. Estimate the highest and lowest values of $S(t)$.

What are the practical implications of these statements?

Solution

a. Cassette sales $S(t)$ is a function of time, t. For each input t, there is one and only one output, $S(t)$.

b. Input: time in years Output: number of cassettes sold

c. Domain: 1975 to 2005 Range: approx. 1 million to 450 million

d. $S(1983) \approx 240{,}000{,}000$, which means that in 1983 there were approximately 240 million cassettes sold.

$S(1980) \approx 120{,}000{,}000$ and $S(1999) \approx 120{,}000{,}000$, which means that in both 1980 and in 1999 there were approximately 120 million cassettes sold.

e. The highest value of $S(t)$ is approximately 450 million in 1988. The lowest value is approximately 1 million in 2005. Cassette sales soared from the mid-1970s up until 1988, reaching 450 million. But with CD sales and the Internet market, cassette sales declined rapidly to about 1 million in 2005.

Algebra Aerobics 1.4b

1. Express each of the following using interval notation.

a. $x > 2$

b. $4 \leq x < 20$

2. Express the given interval as an inequality.

a. $[-3, 10)$

b. $(-2.5, 6.8]$

3. Express each of the following statements in interval notation.

a. Harry's GPA is at least 2.5 but at most 3.6.

b. A good hitter has a batting average of at least 0.333.

c. Starting annual salary at a position is anything from \$35,000 to \$50,000 depending upon experience.

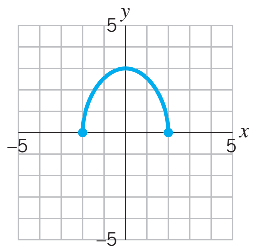
4. You invest \$1,500 in a 10-year bond that gives you 4% interest on \$1,500 each year for 10 years.

a. Construct a function that represents the amount of interest, $I(n)$, earned after n years. What is the domain and range in this context? What is the input and output?

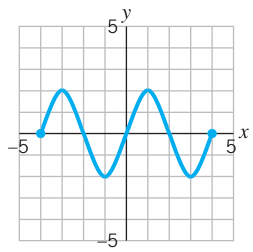
b. Find $I(6)$.

c. Find n if $I(n) = 240$.

5. Determine the domain and the range of the functions whose graphs are given.



GRAPH A



GRAPH B

Exercises for Section 1.4

1. Given $T(x) = x^2 - 3x + 2$, evaluate $T(0)$, $T(-1)$, $T(1)$, and $T(-5)$.

2. Given $f(x) = \frac{x}{x-1}$, evaluate $f(0)$, $f(-1)$, $f(1)$, $f(20)$, and $f(100)$.

3. Assume that for persons who earn less than \$20,000 a year, income tax is 16% of their income.

a. Generate a formula that describes income tax in terms of income for people earning less than \$20,000 a year.

b. What are you treating as the independent variable? The dependent variable?

c. Does your formula represent a function? Explain.

d. If it is a function, what is the domain? The range?

4. Suppose that the price of gasoline is \$3.09 per gallon.

a. Generate a formula that describes the cost, C, of buying gas as a function of the number of gallons of gasoline, G, purchased.

b. What is the independent variable? The dependent variable?

c. Does your formula represent a function? Explain.

d. If it is a function, what is the domain? The range?

e. Generate a small table of values and a graph.

5. The cost of driving a car to work is estimated to be \$2.00 in tolls plus 32 cents per mile. Write an equation for computing the total cost C of driving M miles to work. Does your equation represent a function? What is the independent variable? What is the dependent variable? Generate a table of values and then graph the equation.

6. For each equation, write the equivalent equation that expresses y in terms of x. If the equation represents a function, use function notation to express the relationship.

a. $3x + 5x - y = 3y$ **c.** $x(x - 1) + y = 2x - 5$

b. $3x(5 - x) = x - y$ **d.** $2(y - 1) = y + 5x(x + 1)$

7. If $f(x) = x^2 - x + 2$, find:

a. $f(2)$ **b.** $f(-1)$ **c.** $f(0)$ **d.** $f(-5)$

8. If $g(x) = 2x + 3$, evaluate $g(0)$, $g(1)$, and $g(-1)$.

9. Look at the accompanying table.

a. Find $p(-4)$, $p(5)$, and $p(1)$.

b. For what value(s) of n does $p(n) = 2$?

n	–4	–3	–2	–1	0	1	2	3	4	5
$p(n)$	0.063	0.125	0.25	0.5	1	2	4	8	16	32

10. The data in the table show the median weight for various ages of Caucasian men. (1 kg ≈ 2.2 lbs).

Age (years)	20	25	30	35	40	45	50	55	60	65	70	75
Weight (kg)	72	76	78	80	82	83	83.5	83.5	82.5	82	80	78

Source: Steven Halls, MD.

a. Which variable(s) could be used as the input of a function? Explain your answer.

b. What is the input (with units)? Output (with units)?

c. What is the domain? The range?

d. If f represents this function, complete the following for one pair of values:

f(_______) = _______.

11. The data in the table show the depth and corresponding velocity of the Columbia River in the state of Washington.

Depth (ft)	Velocity (ft/sec)	Depth (ft)	Velocity (ft/sec)
0.7	1.55	7.3	0.91
2.0	1.11	8.6	0.59
2.6	1.42	9.9	0.59
3.3	1.39	10.6	0.41
4.6	1.39	11.2	0.22
5.9	1.14		

Source: Savini, J., and Bodhaine, G. L. (1971), Analysis of current meter data at Columbia River gauging stations, Washington and Oregon; USGS Water Supply Paper 1869-F.

a. Which variable(s) could be used as the input of a function? Explain your answer.

b. What is the input (with units)? Output (with units)?

c. What is the domain? The range?

d. If g represents this function, complete the following for one pair of values:

g(_______) = _______.

12. Pennsylvania has a flat-rate personal income tax. It can be represented as $T(I) = 0.0307I$, where I is the income in dollars and $T(I)$ is the tax owed in dollars.

a. What is the income tax rate in Pennsylvania?

b. What is the input? The output?

c. Estimate $T(35{,}000)$. What is its meaning?

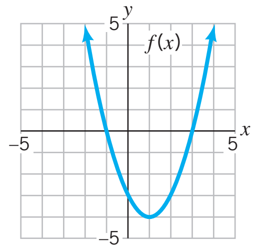
d. Estimate I if $T(I) = 1535$. What is its meaning?

13. From the accompanying graph of $y = f(x)$:

a. Find $f(-2)$, $f(-1)$, $f(0)$, and $f(1)$.

b. Find two values of x for which $f(x) = -3$.

c. Estimate the range of f. Assume that the arms of the graph extend upward indefinitely.



14. Given $f(x) = 1 - 0.5x$ and $g(x) = x^2 + 1$, evaluate:

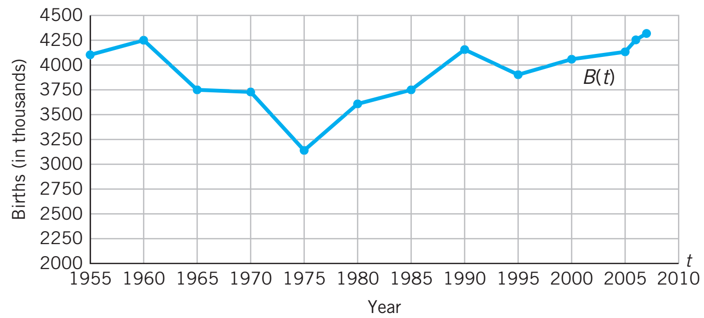
a. $f(0)$, $g(0)$ **c.** $f(2)$, $g(1)$

b. $f(-2)$, $g(-3)$ **d.** $f(3)$, $g(3)$

15. If $f(x) = (2x - 1)^2$, evaluate $f(0)$, $f(1)$, and $f(-2)$.

16. The following graph shows live births, $B(t)$, over time, t, for the years 1955–2007.

Live Births in the USA 1955–2007



Source: U.S. Statistical Abstract, 2010.

Determine the following:

a. _______ is a function of _______.

b. Input (with units): _______ Output (with units): _______

c. Estimate Domain: _______ Range: _______

What is the practical meaning of these values?

d. Estimate $B(2000) \approx$ _______

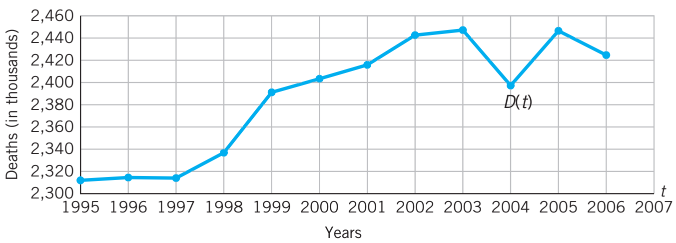
What is the practical meaning of this value?

e. Estimate t if $B(t) = 3750$

What is the practical meaning of these values?

17. The following graph shows U.S. deaths, $D(t)$, over time, t, for the years 1995–2006 in 1-year intervals.

Deaths in USA 1995–2006



Source: U.S. Statistical Abstract, 2010.

Determine the following:

a. _______ is a function of year _______.

b. Input (with units): _______
Output (with units): _______

c. Domain: _______ Range: _______
What is the practical meaning of these values?

d. Estimate $D(2000) \approx$ _______
What is the practical meaning of this value?

e. Estimate t if $D(t) = 2390$
What is the practical meaning of this value?

f. Using Exercise 16, find $B(2000) - D(2000)$ and interpret its meaning.

1.5 Visualizing Functions

In this section we return to the question: How does change in one variable affect change in another variable? Graphs are one of the easiest ways to recognize change. We start with four basic questions:

Is There a Maximum or Minimum Value?

If a function has a *maximum* (or *minimum*) value, then it appears as the highest point (or lowest point) on its graph.

EXAMPLE 1 | Estimating the Maximum and Minimum From a Graph

Determine if each function in **Figure 1.19** has a maximum or minimum, then estimate its value.

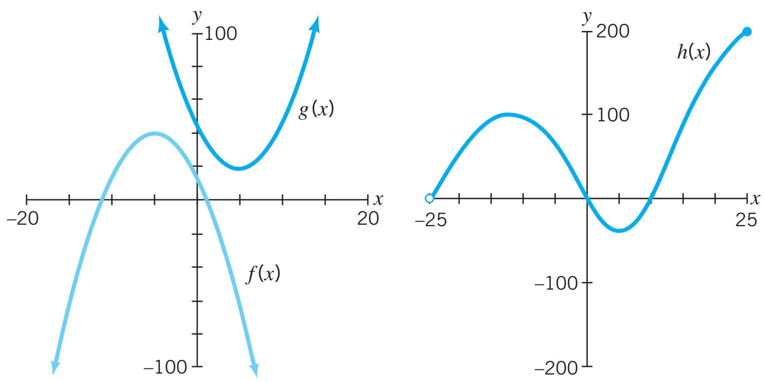


FIGURE 1.19 **Graphs of $f(x)$, $g(x)$, and $h(x)$.**

Solution

The function $f(x)$ in Figure 1.19 appears to have a maximum value of 40 when $x = -5$ but has no minimum value since both arms of the function extend indefinitely downward.

The function $g(x)$ appears to have a minimum value of 20 when $x = 5$, but no maximum value since both arms of the function extend indefinitely upward.

The function $h(x)$ appears to have a maximum value of 200, which occurs when $x = 25$, and a minimum value of -50, when $x = 5$.

When Is the Output of the Function Positive, Negative, or Zero?

From a graph we can estimate over what intervals a function $f(x) > 0$, $f(x) < 0$ and when $f(x) = 0$.

EXAMPLE 2 | Estimating the Output of a Function From a Graph

In Figure 1.19, estimate the x interval when $h(x)$ is positive, negative, or zero. Use interval notation when appropriate.

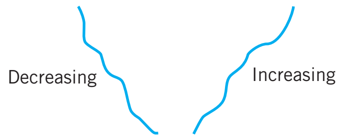
Solution

$h(x) > 0$ over x intervals $(-25, 0)$ and $(10, 25]$
$h(x) < 0$ over x interval $(0, 10)$
$h(x) = 0$ when $x = 0$ and 10

Is the Function Increasing or Decreasing?

A function f is *decreasing* over a specified interval if the values of $f(x)$ decrease as x increases over the interval. A function f is *increasing* over a specified interval if the values of $f(x)$ increase as x increases over the interval.

The graph of an increasing function rises as we move from left to right. The graph of a decreasing function falls as we move from left to right.



EXAMPLE 3 | Increasing and Decreasing Production

Figure 1.20 shows over 100 years of annual natural gas production in the United States. Create a 60-second summary about gas production from 1900 to 2009 in the U.S.

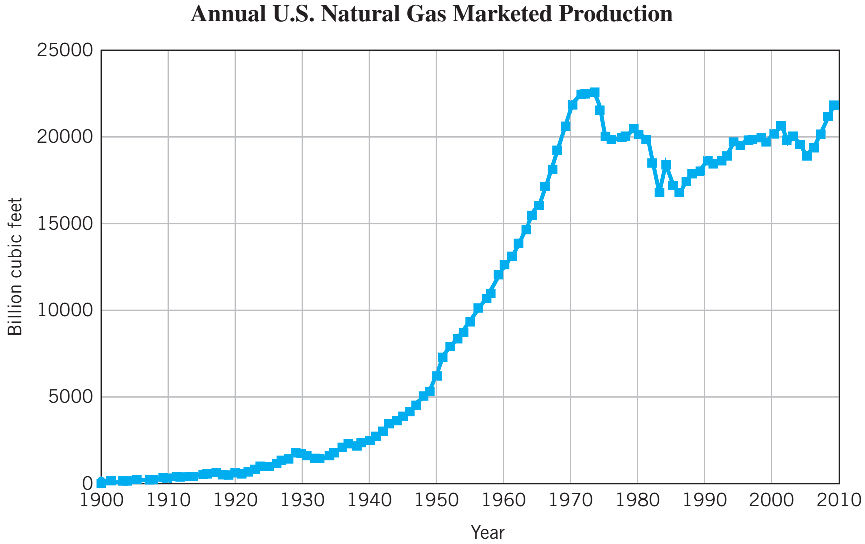


FIGURE 1.20

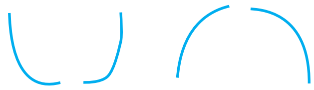
Source: U.S. Energy Information Administration.

Solution

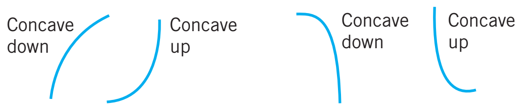
Natural gas production in the United States overall showed a steady increase from 1900 to the early 1970s, increasing from a relatively small amount in 1900 to a peak of over 22,000 billion cubic feet in 1973. For the next 10 years production generally decreased, with a few exceptions, to about 16,000 billion cubic feet in the early 1980s, reaching a low not seen since the mid-1960s. From the mid-1980s to the beginning of the twenty-first century there again was a steady increase in production. In the twenty-first century production at first decreased and then from 2006 to 2009 increased steadily, reaching 21,000 billion cubic feet, coming close to its all-time high in 1973.

Is the Graph Concave Up or Concave Down?

What does the concavity of a graph mean? The graph of a function is *concave up* if it bends upward and it is *concave down* if it bends downward.



Concavity is independent of whether the function is increasing or decreasing.



EXAMPLE 4 | Graphs are Not Necessarily Pictures of Events[12]

The graph in **Figure 1.21** shows the speed of a roller coaster car as a function of time.

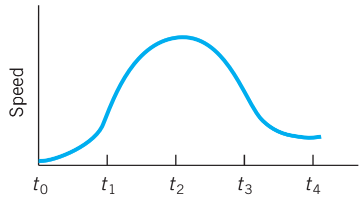


FIGURE 1.21

a. Describe how the speed of the roller coaster car changes over time. Describe the changes in the graph as the speed changes over time.

b. Draw a picture of a possible track for this roller coaster.

Solution

a. The speed of the roller coaster car increases from t_0 to t_2, reaching a maximum for this part of the ride at t_2. The speed decreases from t_2 to t_4.

The graph of speed versus time is concave up and increasing from t_0 to t_1 and then concave down and increasing from t_1 to t_2. From t_2 to t_3 the graph is concave down and decreasing, and from t_3 to t_4 it is concave up and decreasing.

b. A picture for a possible track of the roller coaster is shown in **Figure 1.22**. Notice how the track is an upside-down picture of the graph of speed versus time. (When the roller coaster car goes down the speed increases, and when the roller coaster car goes up, the speed decreases.)

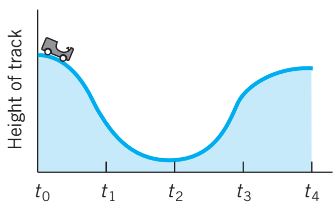


FIGURE 1.22

[12]Example 4 is adapted from Shell Centre for Mathematical Education, *The Language of Functions and Graphs*. Manchester, England: University of Nottingham, 1985.

Getting the Big Idea

We now have the basic vocabulary for describing a function's behavior. Think of a function and its graph as telling a story. We want to decipher not just the individual words, but the basic plot. In each case we should ask: Is there an overall pattern? Are there significant deviations from that pattern?

EXAMPLE 5 | Generating a Rough Graph

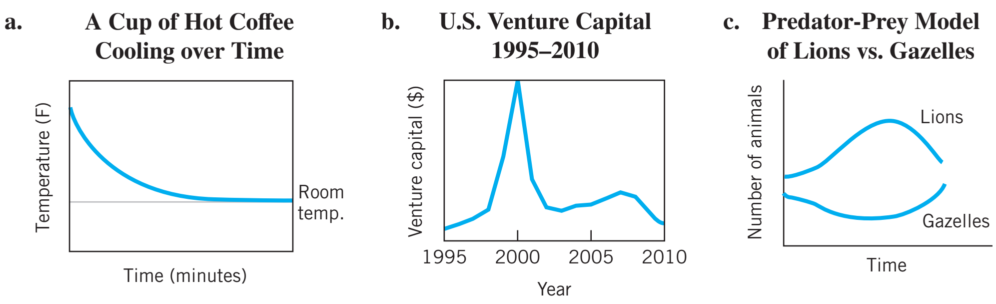
Generate a rough graph of each of the following situations.

a. A cup of hot coffee cooling.

b. U.S. venture capital (money provided by investment companies to business start-ups) increased modestly but steadily in the mid-1990s, soared during the "dotcom bubble" (in the late 1990s), with a high in 2000, and then suffered a drastic decrease in 2001 back to pre-dotcom levels. After a few years of growth another sharp decline started in 2008.

c. A simple predator-prey model: initially as the number of lions (the predators) increases, the number of gazelles (their prey) decreases. When there are not enough gazelles to feed all the lions, the number of lions decreases and the number of gazelles starts to increase.

Solution



EXAMPLE 6 | Looking for Patterns

You are a TV journalist. Summarize for your viewers the essence of the graph in **Figure 1.23**. The poverty rate is the percentage of the population living in poverty.

Note: The vertical scale is used in two different ways on this graph.

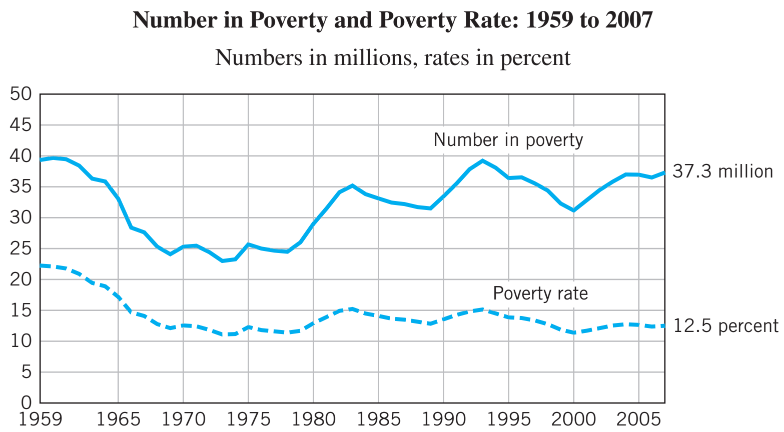


FIGURE 1.23

Source: U.S. Census Bureau, *Current Population Survey*, 1960 to 2008. Annual Social and Economic Supplements.

Solution

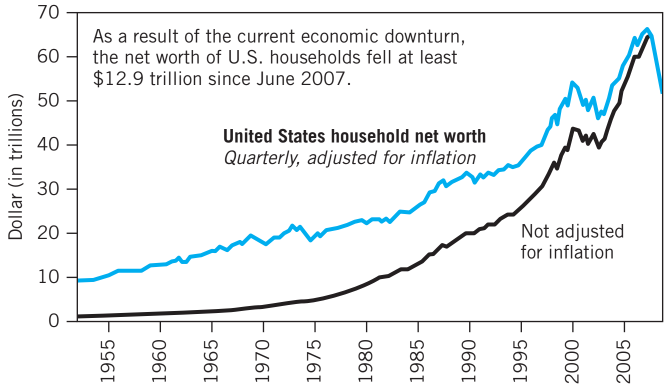
Not surprisingly there is a general relationship between the number of people in poverty and the poverty rate, mirrored in the shapes of their graphs. In 1959 both hit a record high over the 48-year span from 1959 to 2007. Both poverty measures declined after that until about 1967. The poverty rate then continued to hover around 12.5%, close to a record low. However the number of people in poverty fluctuated, reaching 37.3 million in 2007, almost the 1959 high.

EXAMPLE 7 | Looking for Deviations From a Pattern

The following graph illustrates household net worth in the United States. Household net worth encompasses all of a household's assets (such as housing, stocks, personal property) minus their total debts (such as mortgages, other unpaid loans, credit card debt).

a. Describe how household net worth has changed since 1955.

b. What title would you give to the graph for a newspaper article about household net worth in the United States?



Source: *Federal Reserve Board.*

Solution

a. From 1955 to 2000, household net worth in the United States (both adjusted and not adjusted for inflation) generally increased, with only a few relatively slight decreases at the end of the 1960s and from 1973 to 1975. The more dramatic decreases were in the twenty-first century. After falling for a few years at the beginning of the twenty-first century, household net worth began a steep climb to an all-time high of about $65 trillion dollars in 2007, which was followed by its steepest decline, losing almost $13 trillion. At end of the first decade of the twenty-first century, household net worth had declined to slightly over $50 trillion, and if adjusted for inflation, was close to where it started at the beginning of the century.

b. A similar graph appeared in *The New York Times* entitled "A Sudden Decline." Another title could be "A Dramatic Loss after Growth for Years."

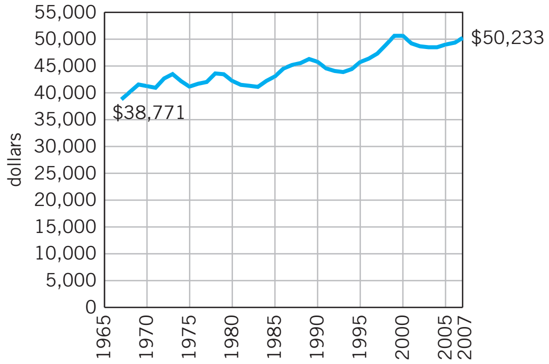
Algebra Aerobics **1.5**

1. Create a topic sentence for each of the following graphs for a newspaper article.

a.

Real Median Household Income: 1967 to 2007

(in 2007 dollars)

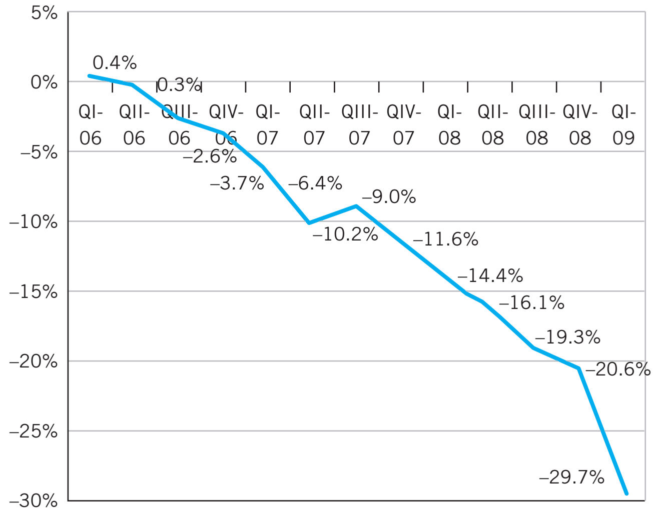


Note: Median household income data are not available before 1967.
Source: U.S. Census Bureau, Current Population Survey, 1968 to 2008 Annual Social and Economic Supplements.

b.

Accelerating Slide

Quarterly newspaper print ad sales



Source: Newspaper Association of America.

2. Use the accompanying graph showing mean contract mortgage rates, $D(t)$, over time, t, to estimate the following:

a. Domain: Range:

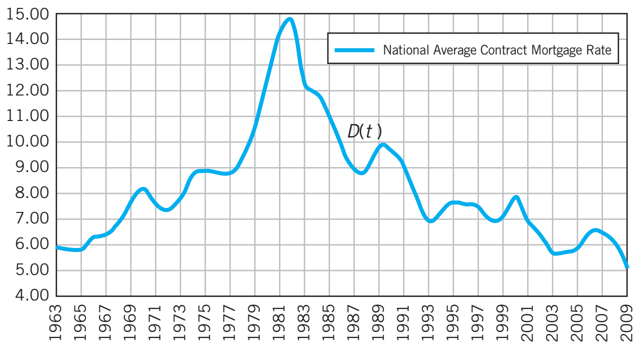
b. $D(2008) \approx$

c. t if $D(t) \approx 7.00$

d. The maximum value for mortgage rates during the time period represented on the graph. In what year does the maximum occur? What are the approximate coordinates at the maximum point?

e. The minimum value for mortgage rates. In what year does this occur? What are the coordinates of this point?

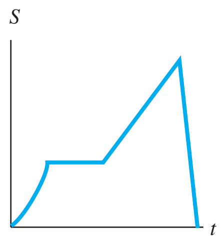
Mean Contract Mortgage Rates in United States 1963 to 2009



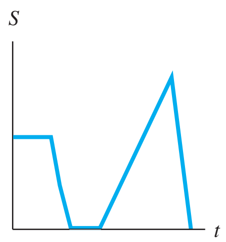
Source: *www.Mortgage-X.com*.
Note: The contract mortgage rate is based on conventional fixed and adjustable rate mortgages for a given time period.

3. Choose which of the graphs is the "best" graph to describe the following situation. Speed (S) is on the vertical axis and time (t) is on the horizontal axis.

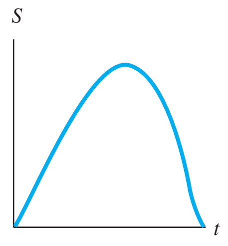
A child in a playground tentatively climbs the steps of a large slide, first at a steady pace, then gradually slowing down until she reaches the top, where she stops to rest before sliding down.



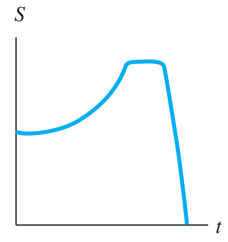
Graph *A*



Graph *B*



Graph *C*



Graph *D*

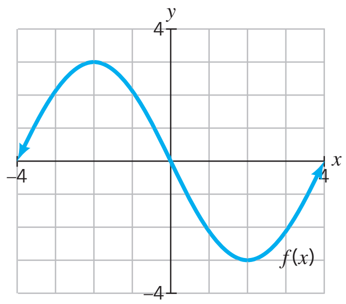
4. Generate a rough sketch of the following situation. U.S. AIDS cases increased dramatically, reaching an all-time high for a relatively short period, and then consistently decreased, until a recent small increase.

5. Sketch a graph for each of the following characteristics, and then indicate with arrows where the functions are increasing and where they are decreasing.

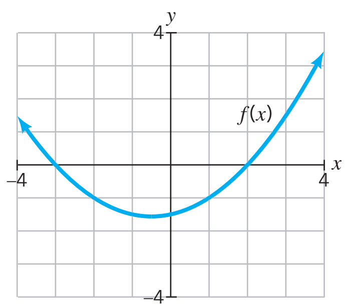
a. Concave up with a minimum point at (–2, 1).

b. Concave down with a maximum point at (3, –2).

6. Estimate the interval for x when $f(x) = 0$, when $f(x) < 0$, and when $f(x) > 0$ for each function in Graphs *A* and *B*.



Graph *A*



Graph *B*

Exercises for Section 1.5

Graphing program required for Exercise 27.

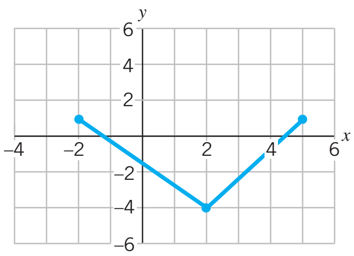
1. Identify the graph (*A* or *B*) that

a. Increases for $1 < x < 3$

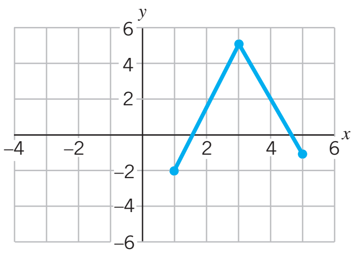
b. Increases for $2 < x < 5$

c. Decreases for $-2 < x < 2$

d. Decreases for $3 < x < 5$



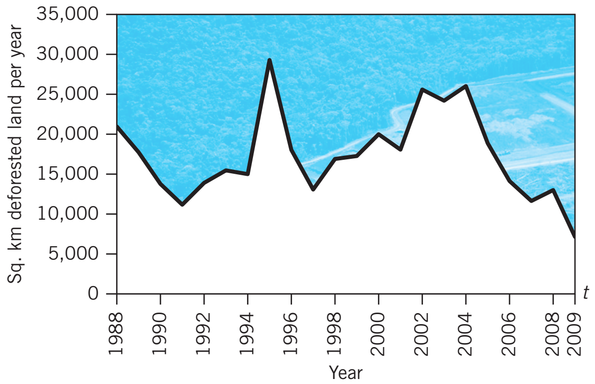
Graph *A*



Graph *B*

2. The following graph shows the number of square kilometers of deforestation, $D(t)$, in the Brazilian Amazon during the years, t, from 1988 to 2009.

Deforestation in the Brazilian Amazon, 1988–2009



Source: ©luoman/iStockphoto

Find the following:

a. _______ is a function of _______.

b. Input (with units): _______
Output (with units): _______

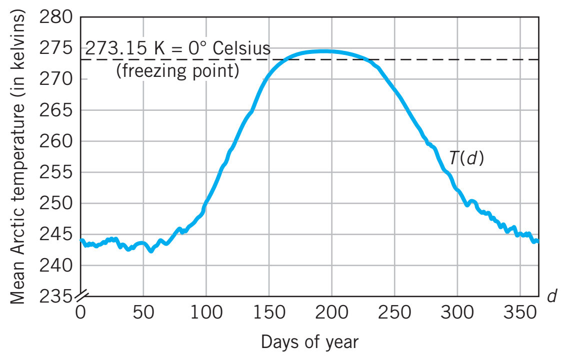
c. Estimate the Domain: _______ Range: _______

d. Estimate $D(1990) \approx$ $D(2008) \approx$
Estimate t so that $D(t) \approx 22{,}000$

e. From the graph, estimate the maximum amount of land deforested in a particular year. In what year did that occur? In what year was the least amount of land deforested?

3. The following graph shows the daily mean Arctic temperature: $T(d)$, in kelvins, over d days of the year. (*Note:* Kelvin is a unit of temperature; 273.15 K = 0° Celsius and 1 kelvin = −272.15° Celsius.)

Mean Arctic Temperature



Source: Danish Meteorological Institute.

Find the following:

a. _______ is a function of _______.

b. Input (with units): _______
Output (with units): _______

c. Estimate Domain: _______ Range: _______

d. Estimate $T(100) \approx$ _______

Estimate the interval for d over which $T(d) > 273.15$ (the melting point of ice).

What are the practical implications of these statements?

e. Estimate the maximum and minimum values of $T(d)$.

What are the practical implications of these statements?

f. Describe the overall shape of the graph of $T(d)$.

4. For each of the following functions,

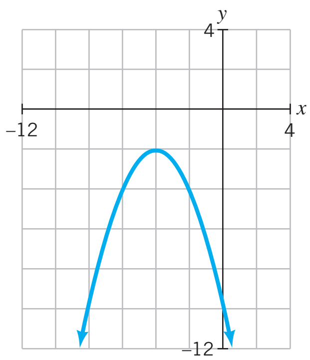
a. Over which interval is the function decreasing?

b. Over which interval is the function increasing?

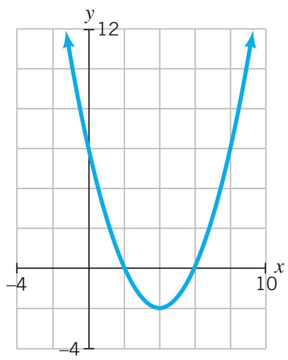
c. Does the function appear to have a minimum? If so, where?

d. Does the function appear to have a maximum? If so, where?

e. Describe the concavity.

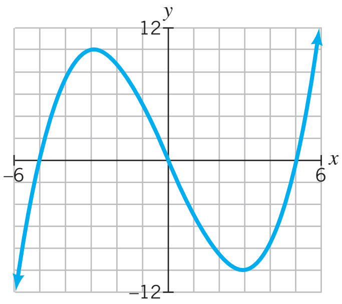


Graph *A*



Graph *B*

5. For the following function,



a. Over which interval(s) is the function positive?

b. Over which interval(s) is the function negative?

c. Over which interval(s) is the function decreasing?

d. Over which interval(s) is the function increasing?

e. Does the function appear to have a minimum? If so, where?

f. Does the function appear to have a maximum? If so, where?

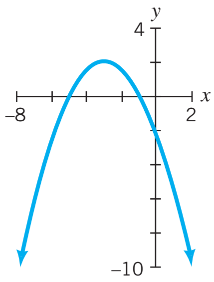
6. Choose which graph(s) A, B, C, D, match the description: As x increases, the graph is:

a. Increasing and concave up

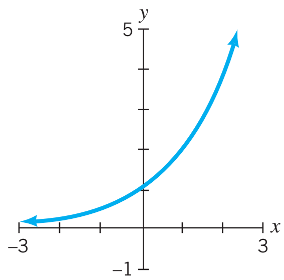
b. Increasing and concave down

c. Concave up and appears to have a minimum value at (−3, 2)

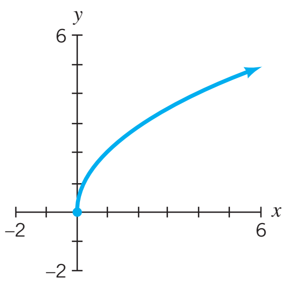
d. Concave down and appears to have a maximum value at (−3, 2)



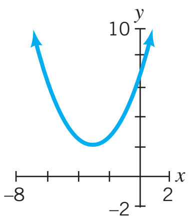
Graph *A*



Graph *C*



Graph *B*



Graph *D*

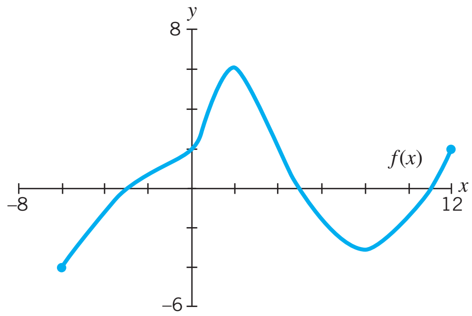
7. Examine each of the graphs in Exercise 6. Assume each graph describes a function $f(x)$. The arrows indicate that the graph extends indefinitely in the direction shown.

a. For each function estimate the domain and range.

b. For each function estimate the x interval(s) where $f(x) > 0$.

c. For each function estimate the x interval(s) where $f(x) < 0$.

8. Look at the graph of $y = f(x)$ in the accompanying figure.



a. Find $f(-6)$, $f(2)$, and $f(12)$.

b. Find $f(0)$.

c. For what values of x is $f(x) = 0$?

d. Is $f(8) > 0$ or is $f(8) < 0$?

e. How many times would the line $y = 1$ intersect the graph of $f(x)$?

f. What are the domain and range of $f(x)$?

g. What is the maximum? The minimum?

9. Use the graph of Exercise 8 to answer the following questions about $f(x)$.

a. Over which interval(s) is $f(x) < 0$?

b. Over which interval(s) is $f(x) > 0$?

c. Over which interval(s) is $f(x)$ increasing?

d. Over which interval(s) is $f(x)$ decreasing?

e. How would you describe the concavity of $f(x)$ over the interval (0, 5) for x? Over (5, 8) for x?

f. Find a value for x when $f(x) = 4$.

g. $f(-8) = ?$

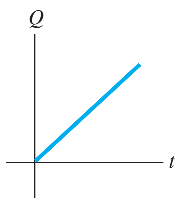
10. Match each graph with the best description of the function. Assume that the horizontal axis represents time, t.

i. The height of a ball thrown straight up is a function of time.

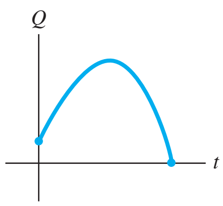
ii. The distance a truck travels at a constant speed is a function of time.

iii. The number of daylight hours is a function of the day of the year.

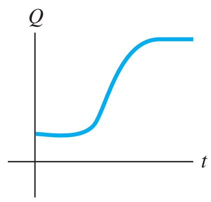
iv. The temperature of a pie baking in an oven is a function of time.



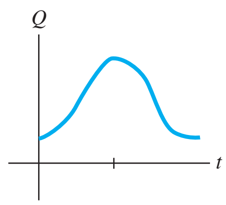
Graph *A*



Graph *C*



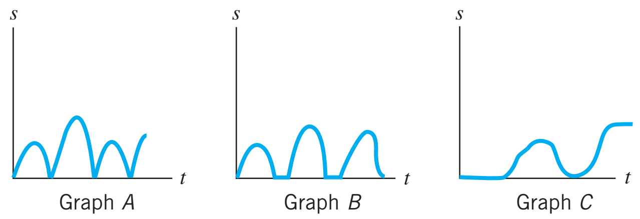
Graph *B*



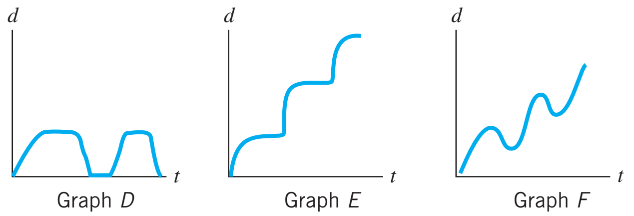
Graph *D*

11. Choose the best graph to describe the situation.

a. A student in a large urban area takes a local bus whose route ends at the college. Time, *t*, is on the horizontal axis and speed, *s*, is on the vertical axis.

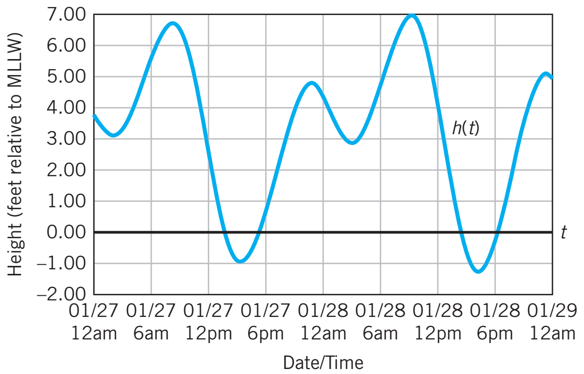


b. What graph depicts the total distance the student traveled in the bus? Time, *t*, is on the horizontal axis and distance, *d*, is on the vertical axis.



12. The accompanying graph shows the changes in the height of tides in San Francisco Bay, $h(t)$, over time, t, between January 27, 2010, and January 29, 2010.

Height of Tides in San Francisco



Source: National Oceanic and Atmospheric Association. *tidesandcurrents.hoaa.gov*.
Note: MLLW (mean lower low water level) is the mean lowest tide.

Estimate the following:

a. The t interval(s) when $h(t) > 0$. Explain what this means about the tide level.

b. The t interval(s) when $h(t) < 0$. Explain what this means about the tide level.

c. The value(s) of t when $h(t) = 0$.

d. One interval when $h(t)$ is decreasing. Explain what this means about the tide level.

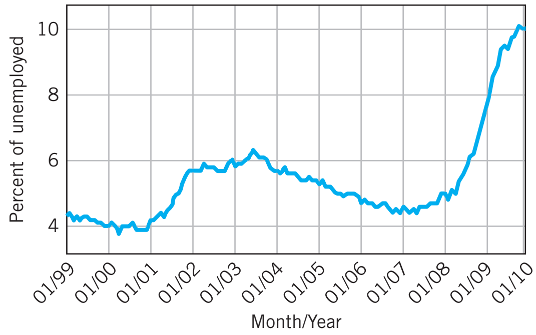
e. The value of $h(t)$ when t is 6 p.m., 1/27.

f. The t interval(s) when the graph is concave down.

g. The maximum and minimum values of $h(t)$. Explain these results in terms of tide levels.

13. The accompanying graph shows the unemployment rate for those people 16 years and older from January 1999 to January 2010.

Unemployment Rate in the United States



Source: Bureau of Labor Statistics, *www.bls.gov*.

a. Estimate the maximum and the minimum unemployment rate between January 1999 and January 2010.

b. Estimate the time frame when the unemployment rate was experiencing an overall decrease.

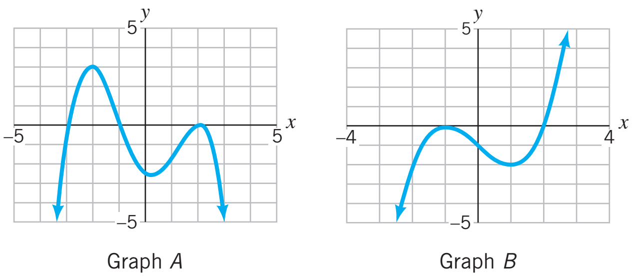
c. When does the unemployment rate seem to be rising the fastest?

d. Write a 60-second summary about the unemployment rate from January 1999 to January 2010.

14. a. In the accompanying graphs, estimate the coordinates of the maximum and minimum points (if any) of the function.

b. Specify the interval(s) over which each function is increasing.

c. Estimate when $f(x) = 0$, $f(x) > 0$, and $f(x) < 0$.



15. Use the graph to estimate:

a. When $f(x) = 0$, when $f(x) > 0$, and when $f(x) < 0$

b. The coordinates of the maximum and minimum for $f(x)$

c. When $f(x)$ is increasing and decreasing

d. The domain and range of $f(x)$