MILLER O'NEILL HYDE

Beginning Algebra

Fifth Edition

Mc Graw Hill Education

Beginning Algebra

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Fifth Edition

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Letter from the Authors

Dear Colleagues,

Across the country, Developmental Math courses are in a state of flux and we as instructors are at the center of it all. As many of our institutions are grappling with the challenges of placement, retention, and graduation rates, we are on the front lines with our students—supporting each one of them in their educational journey.

Flexibility—No Matter Your Course Format!

The three of us each teach differently, as do many of our current users. The Miller/O'Neill/Hyde series is designed for successful use in a variety of different course formats, both traditional and modern—traditional classroom lecture settings, flipped classrooms, hybrid classes, and online-only classes.

Ease of Instructor Preparation

We've all had to fill in for a colleague, pick up a last-minute section, or find ourselves running across campus to yet a different course. The Miller/O'Neill/Hyde series is carefully designed to support instructors teaching in a variety of different settings and circumstances. Experienced, senior faculty members can draw from a massive library of static and algorithmic content found in ALEKS and Connect Hosted by ALEKS to meticulously build assignments and assessments sharply tailored to individual student needs. Newer, part-time adjunct professors, on the other hand, can lean for support on a wide range of digital resources and prebuilt assignments that are ready to go from Day 1, affording them an opportunity to facilitate successful student outcomes in spite of little or no time to prepare for class in advance.

Many instructors want to incorporate discovery-based learning and groupwork into their courses but don't have time to write or find quality materials. We have ready-made Group Activities that are found at the end of each chapter. Furthermore, each section of the text has numerous discovery-based activities that we have tested in our own classrooms. These are found in the Student Resource Manual along with other targeted worksheets for additional practice and materials for a student portfolio.

Student Success—Now and in the Future

Too often our math placement tests fail our students, which can lead to frustration, anxiety, and often withdrawal from their education journey. We encourage you to learn more about ALEKS Placement, Preparation, and Learning (ALEKS PPL), which uses adaptive learning technology to place students appropriately. No matter the skills they come in with, the Miller/O'Neill/Hyde series provides resources and support that uniquely positions them for success in that course and for their next course. Whether they need a brush-up on their basic skills, ADA supportive materials, or advanced topics to help bridge them into the next level, we've created a support system for them.

We hope you are as excited as we are about the series and the supporting resources and services that accompany it. Please reach out to any of us with any questions or comments you have about our texts.

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About the Authors

Julie Miller is from Daytona State College, where she taught developmental and upper-level mathematics courses for 20 years. Prior to her work at Daytona State College, she worked as a software engineer for General Electric in the area of flight and radar simulation. Julie earned a bachelor of science in applied mathematics from Union College in Schenectady, New York, and a master of science in mathematics from the University of Florida. In addition to this textbook, she has authored textbooks for college algebra, trigonometry, and precalculus, as well as several short works of fiction and nonfiction for young readers.

"My father is a medical researcher, and I got hooked on math and science when I was young and would visit his laboratory. I can remember using graph paper to plot data points for his experiments and doing simple calculations. He would then tell me what the peaks and features in the graph meant in the context of his experiment. I think that applications and hands-on experience made math come alive for me and I'd like to see math come alive for my students."

—Julie Miller

Molly O'Neill is also from Daytona State College, where she taught for 22 years in the School of Mathematics. She has taught a variety of courses from developmental mathematics to calculus. Before she came to Florida, Molly taught as an adjunct instructor at the University of Michigan–Dearborn, Eastern Michigan University, Wayne State University, and Oakland Community College. Molly earned a bachelor of science in mathematics and a master of arts and teaching from Western Michigan University in Kalamazoo, Michigan. Besides this textbook, she has authored several course supplements for college algebra, trigonometry, and precalculus and has reviewed texts for developmental mathematics.

"I differ from many of my colleagues in that math was not always easy for me. But in seventh grade I had a teacher who taught me that if I follow the rules of mathematics, even I could solve math problems. Once I understood this, I enjoyed math to the point of choosing it for my career. I now have the greatest job because I get to do math every day and I have the opportunity to influence my students just as I was influenced. Authoring these texts has given me another avenue to reach even more students."

—Molly O'Neill

Nancy Hyde served as a full-time faculty member of the Mathematics Department at Broward College for 24 years. During this time she taught the full spectrum of courses from developmental math through differential equations. She received a bachelor of science degree in math education from Florida State University and a master's degree in math education from Florida Atlantic University. She has conducted workshops and seminars for both students and teachers on the use of technology in the classroom. In addition to this textbook, she has authored a graphing calculator supplement for *College Algebra*.

"I grew up in Brevard County, Florida, where my father worked at Cape Canaveral. I was always excited by mathematics and physics in relation to the space program. As I studied higher levels of mathematics I became more intrigued by its abstract nature and infinite possibilities. It is enjoyable and rewarding to convey this perspective to students while helping them to understand mathematics."

—Nancy Hyde

Dedication

To Our Students

Julie Miller ⚜ Molly O'Neill ⚜ Nancy Hyde

The Miller/O'Neill/Hyde Developmental Math Series

Julie Miller, Molly O'Neill, and Nancy Hyde originally wrote their developmental math series because students were entering their College Algebra course underprepared. The students were not mathematically mature enough to understand the concepts of math, nor were they fully engaged with the material. The authors began their developmental mathematics offerings with intermediate algebra to help bridge that gap. This in turn developed into several series of textbooks from Prealgebra through Precalculus to help students at all levels before Calculus.

What sets all of the Miller/O'Neill/Hyde series apart is that they address course issues through an author-created digital package that maintains a consistent voice and notation throughout the program. This consistency—in videos, PowerPoints, Lecture Notes, and Group Activities—coupled with the power of ALEKS and Connect Hosted by ALEKS, ensures that students master the skills necessary to be successful in Developmental Math through Precalculus and prepares them for the calculus sequence.

Developmental Math Series (Hardback)

The hardback series is the more traditional in approach, yet balanced in its treatment of skills and concepts development for success in subsequent courses.

Beginning Algebra, Fifth Edition
Beginning & Intermediate Algebra, Fifth Edition
Intermediate Algebra, Fifth Edition

Developmental Math Series (Softback)

The softback series includes a stronger emphasis on conceptual learning through Skill Practice features and Concept Connections, which are intended to help students with the conceptual meaning of the problems they are solving.

Basic College Mathematics, Third Edition
Prealgebra, Second Edition
Prealgebra & Introductory Algebra, First Edition
Introductory Algebra, Third Edition
Intermediate Algebra, Third Edition

College Algebra/Precalculus Series

The Precalculus series serves as the bridge from Developmental Math coursework to setting the stage for future courses, including the skills and concepts needed for Calculus.

College Algebra, Second Edition
College Algebra and Trigonometry, First Edition
Precalculus, First Edition

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Finally, we greatly appreciate the many people behind the scenes at McGraw-Hill without whom we would still be on page 1. First and foremost, to Luke Whalen, our product developer and newest member of the team. Thanks for being our help desk. You've been a hero filling some big shoes in the day-to-day help on all things math, English, and editorial. To Amber Van Namee, our brand manager and team leader: thank you so much for leading us down this path. Your insight, creativity, and commitment to our project has made our job easier.

To the marketing team, Sally Yagan and Annie Clark: thank you for your creative ideas in making our books come to life in the market. Thank you as well to Mary Ellen Rahn for continuing to drive our long-term content vision through her market development efforts. To the digital content experts, Rob Brieler and Adam Fischer: we are most grateful for your long hours of work and innovation in a world that changes from day to day. And many thanks to the team at ALEKS for creating its spectacular adaptive technology and for overseeing the quality control in Connect Math.

To the production team: Peggy Selle, Carrie Burger, Emily Windelborn, Lora Neyens, and Lorraine Buczek—thank you for making the manuscript beautiful and for keeping the train on the track. You've been amazing. And finally, to Ryan Blankenship, Marty Lange, and Kurt Strand: thank you for supporting our projects for many years and for the confidence you've always shown in us.

Most importantly, we give special thanks to the students and instructors who use our series in their classes.

Julie Miller
Molly O'Neill
Nancy Hyde

Contents

To the Student

Take a deep breath and know that you aren't alone. Your instructor, fellow students, and we, your authors, are here to help you learn and master the material for this course and prepare you for future courses. You may feel like math just isn't your thing, or maybe it's been a long time since you've had a math class—that's okay!

We wrote the text and all the supporting materials with you in mind. Most of our students aren't really sure how to be successful in math, but we can help with that.

As you begin your class, we'd like to offer some specific suggestions:

1. **Attend class.** Arrive on time and be prepared. If your instructor has asked you to read prior to attending class—do it. How often have you sat in class and thought you understood the material, only to get home and realize you don't know how to get started? By reading and trying a couple of Skill Practice exercises, which follow each example, you will be able to ask questions and gain clarification from your instructor when needed.
2. **Be an *active* learner.** Whether you are at lecture, watching an author lecture or exercise video, or are reading the text, pick up a pencil and work out the examples given. Math is learned only by doing; we like to say, "Math is not a spectator sport." If you like a bit more guidance, we encourage you to use the Integrated Video and Study Guide. It was designed to provide structure and note-taking for lectures and while watching the accompanying videos.
3. **Schedule time to do some math every day.** Exercise, foreign language study, and math are three things that you must do every day to get the results you want. If you are used to cramming and doing all of your work in a few hours on a weekend, you should know that even mathematicians start making silly errors after an hour or so! Check your answers. Skill Practice exercises all have the answer at the bottom of that page. Odd-numbered exercises throughout the text have answers at the back of the text. If you didn't get it right, don't throw in the towel. Try again, revisit an example, or bring your questions to class for extra help.
4. **Prepare for quizzes and exams.** At the end of each chapter is a summary that highlights all the concepts and problem types you need to understand and know how to do. There are additional problem sets at the end of each chapter: a set of review exercises, a chapter test, and a cumulative review. Working through the cumulative review will help keep your skills fresh from previous chapters—one of the key ways to do well on your exams. If you use ALEKS or Connect Hosted by ALEKS, use all of the tools available within the program to test your understanding.
5. **Use your resources.** This text comes with numerous supporting resources designed to help you succeed in this class and your future classes. Additionally, your instructor can direct you to resources within your institution or community. Form a student study group. Teaching others is a great way to strengthen your own understanding and they might be able to return the favor if you get stuck.

We wish you all the best in this class and your educational journey!

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Student Guide to the Text

Clear, Precise Writing

Learning from our own students, we have written this text in simple and accessible language. Our goal is to keep you engaged and supported throughout your coursework.

Callouts

Just as your instructor will share tips and math advice in class, we provide callouts throughout the text to offer tips and warn against common mistakes.

- Tip boxes offer additional insight to a concept or procedure.
- Avoiding Mistakes help fend off common student errors.

Examples

- Each example is step-by-step, with thorough annotation to the right explaining each step.
- Following each example is a similar **Skill Practice** exercise to give you a chance to test your understanding. You will find the answer at the bottom of the page—providing a quick check.
- When you see this 

in an example, there is an online dynamic animation within your online materials. Sometimes an animation is worth a thousand words.

Exercise Sets

Each type of exercise is built for your success in learning the materials and showing your mastery on exams.

- **Study Skills Exercises** integrate your studies of math concepts with strategies for helping you grow as a student overall.
- **Vocabulary and Key Concept Exercises** check your understanding of the language and ideas presented within the section.
- **Review Exercises** keep fresh your knowledge of math content already learned by providing practice with concepts explored in previous sections.
- **Concept Exercises** assess your comprehension of the specific math concepts presented within the section.
- **Mixed Exercises** evaluate your ability to successfully complete exercises that combine multiple concepts presented within the section.
- **Expanding Your Skills** challenge you with advanced skills practice exercises around the concepts presented within the section.
- **Problem Recognition Exercises** appear in strategic locations in each chapter of the text. These will require you to distinguish between similar problem types and to determine what type of problem-solving technique to apply.

Calculator Connections

Throughout the text are materials highlighting how you can use a graphing calculator to enhance understanding through a visual approach. Your instructor will let you know if you will be using these in class.

End-of-Chapter Materials

The features at the end of each chapter are perfect for reviewing before test time.

- Section-by-section summaries provide references to key concepts, examples, and vocabulary.
- Chapter review exercises provide additional opportunities to practice material from the entire chapter.
- Chapter tests are an excellent way to test your complete understanding of the chapter concepts.
- Cumulative review exercises are the best preparation to maintain a strong foundation of skills to help you move forward into new material. These exercises cover concepts from all the material covered up to that point in the text and will help you study for your final exam.

Get Better Results

How Will Miller/O'Neill/Hyde Help Your Students *Get Better Results?*

Better Clarity, Quality, and Accuracy!

Julie Miller, Molly O'Neill, and Nancy Hyde know what students need to be successful in mathematics. Better results come from clarity in their exposition, quality of step-by-step worked examples, and accuracy of their exercises sets; but it takes more than just great authors to build a textbook series to help students achieve success in mathematics. Our authors worked with a strong mathematical team of instructors from around the country to ensure that the clarity, quality, and accuracy you expect from the Miller/O'Neill/Hyde series was included in this edition.

"The most complete text at this level in its thoroughness, accuracy, and pedagogical soundness. The best developmental mathematics text I have seen."

—Frederick Bakenhus, *Saint Phillips College*

Better Exercise Sets!

Comprehensive sets of exercises are available for every student level. Julie Miller, Molly O'Neill, and Nancy Hyde worked with a board of advisors from across the country to offer the appropriate depth and breadth of exercises for your students. **Problem Recognition Exercises** were created to improve student performance while testing.

Practice exercise sets help students progress from skill development to conceptual understanding. Student tested and instructor approved, the Miller/O'Neill/Hyde exercise sets will help your students *get better results*.

- **Problem Recognition Exercises**
- **Skill Practice Exercises**
- **Study Skills Exercises**
- **Mixed Exercises**
- **Expanding Your Skills Exercises**
- **Vocabulary and Key Concepts Exercises**

"This series was thoughtfully constructed with students' needs in mind. The Problem Recognition section was extremely well designed to focus on concepts that students often misinterpret."

—Christine V. Wetzel-Ulrich, *Northampton Community College*

Better Step-By-Step Pedagogy!

Beginning Algebra provides enhanced step-by-step learning tools to help students *get better results*.

- **Worked Examples** provide an "easy-to-understand" approach, clearly guiding each student through a step-by-step approach to master each practice exercise for better comprehension.
- **TIPs** offer students extra cautious direction to help improve understanding through hints and further insight.
- **Avoiding Mistakes** boxes alert students to common errors and provide practical ways to avoid them. All three learning aids will help students get better results by showing how to work through a problem using a clearly defined step-by-step methodology that has been class tested and student approved.

"The book is designed with both instructors and students in mind. I appreciate that great care was used in the placement of 'Tips' and 'Avoiding Mistakes' as it creates a lot of teachable moments in the classroom."

—Shannon Vinson, *Wake Tech Community College*

Get Better Results

Formula for Student Success

Step-by-Step Worked Examples

- Do you get the feeling that there is a disconnection between your students' class work and homework?
- Do your students have trouble finding worked examples that match the practice exercises?
- Do you prefer that your students see examples in the textbook that match the ones you use in class?

Miller/O'Neill/Hyde's *Worked Examples* offer a clear, concise methodology that replicates the mathematical processes used in the authors' classroom lectures!

Example 5 Solving a Linear Equation

Solve the equation. $2.2y - 8.3 = 6.2y + 12.1$

Solution:

$2.2y - 8.3 = 6.2y + 12.1$

Step 1: The right- and left-hand sides are already simplified.

$8.3 = 6.2y - 2.2y + 12.1$
$8.3 = 4y + 12.1$

Step 2: Subtract $2.2y$ from both sides to collect the variable terms on one side of the equation.

$-4.2 = 4y + 12.1 - 12.1$
$-0.4 = 4y$

Step 3: Subtract 12.1 from both sides to collect the constant terms on the other side.

$\frac{-0.4}{4} = \frac{4y}{4}$

Step 4:

$-0.1 = y$

$y = -0.1$

Step 5:

$2.2y$
$2.2(-0.1)$
-0.22

The solution set is $\{-0.1\}$.

Skill Practice Solve the equation.

5. $1.5t + 2.3 = 3.5t - 1.9$

"As always, MOH's Worked Examples are so clear and useful for the students. All steps have wonderfully detailed explanations written with wording that the students can understand. MOH is also excellent with arrows and labels making the Worked Examples extremely clear and understandable."

—Kelli Hammer, *Broward College–South*

"Easy to read step-by-step solutions to sample textbook problems. The 'why' is provided for students, which is invaluable when working exercises without available teacher/tutor assistance."

—Arcola Sullivan, *Copiah-Lincoln Community College*

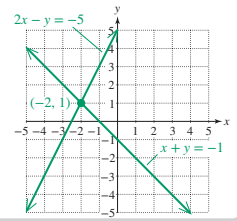
Classroom Example: p. 282, Exercise 30

Example 3 Solving a System of Linear Equations by Graphing

Solve the system by the graphing method.

$x - 2y = -2$
$-3x + 2y = 6$

30. $x + y = -1$
$2x - y = -5$ $\{(-2, 1)\}$



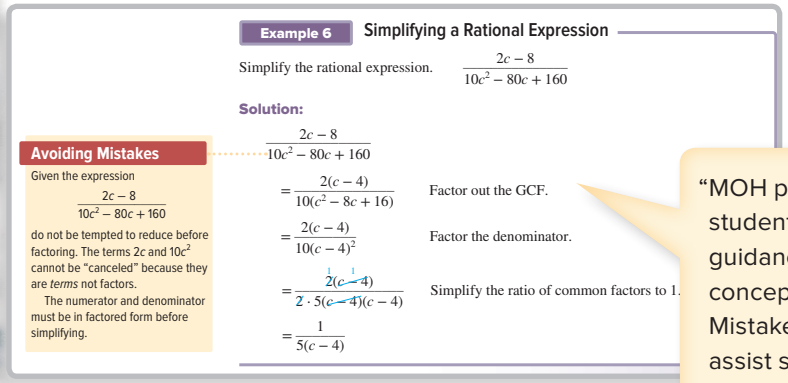
Classroom Examples

To ensure that the classroom experience also matches the examples in the text and the practice exercises, we have included references to even-numbered exercises to be used as Classroom Examples. These exercises are highlighted in the Practice Exercises at the end of each section.

Better Learning Tools

TIP and Avoiding Mistakes Boxes

TIP and **Avoiding Mistakes** boxes have been created based on the authors' classroom experiences—they have also been integrated into the **Worked Examples.** These pedagogical tools will help students get better results by learning how to work through a problem using a clearly defined step-by-step methodology.



Example 6 Simplifying a Rational Expression

Simplify the rational expression. $\frac{2c-8}{10c^2-80c+160}$

Solution:

$$\frac{2c-8}{10c^2-80c+160}$$

$$=\frac{2(c-4)}{10(c^2-8c+16)}$$ Factor out the GCF.

$$=\frac{2(c-4)}{10(c-4)^2}$$ Factor the denominator.

$$=\frac{\overset{1}{\cancel{2}}(\overset{1}{\cancel{c-4}})}{\underset{1}{\cancel{2}}\cdot 5(\underset{1}{\cancel{c-4}})(c-4)}$$ Simplify the ratio of common factors to 1.

$$=\frac{1}{5(c-4)}$$

Avoiding Mistakes

Given the expression $\frac{2c-8}{10c^2-80c+160}$ do not be tempted to reduce before factoring. The terms $2c$ and $10c^2$ cannot be "canceled" because they are *terms* not factors.

The numerator and denominator must be in factored form before simplifying.

Avoiding Mistakes Boxes

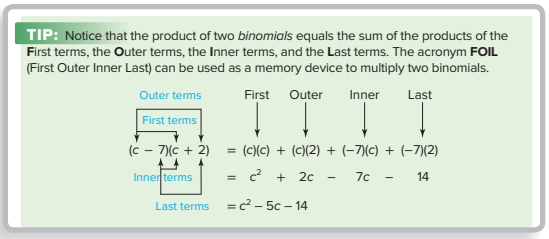
Avoiding Mistakes boxes are integrated throughout the textbook to alert students to common errors and how to avoid them.

"MOH presentation of reinforcement concepts builds students' confidence and provides easy to read guidance in developing basic skills and understanding concepts. I love the visual clue boxes 'Avoiding Mistakes.' Visual clue boxes provide tips and advice to assist students in avoiding common mistakes."

—Arcola Sullivan, *Copiah-Lincoln Community College*

TIP Boxes

Teaching tips are usually revealed only in the classroom. Not anymore! TIP boxes offer students helpful hints and extra direction to help improve understanding and provide further insight.



TIP: Notice that the product of two *binomials* equals the sum of the products of the **F**irst terms, the **O**uter terms, the **I**nner terms, and the **L**ast terms. The acronym **FOIL** (First Outer Inner Last) can be used as a memory device to multiply two binomials.

Outer terms / First terms / Inner terms / Last terms

$(c-7)(c+2)$ = First Outer Inner Last

$= (c)(c) + (c)(2) + (-7)(c) + (-7)(2)$

$= c^2 + 2c - 7c - 14$

$= c^2 - 5c - 14$

Better Exercise Sets! Better Practice! Better Results!

- Do your students have trouble with problem solving?
- Do you want to help students overcome math anxiety?
- Do you want to help your students improve performance on math assessments?

Get Better Results

Problem Recognition Exercises

Problem Recognition Exercises present a collection of problems that look similar to a student upon first glance, but are actually quite different in the manner of their individual solutions. Students sharpen critical thinking skills and better develop their "solution recall" to help them distinguish the method needed to solve an exercise—an essential skill in developmental mathematics.

Problem Recognition Exercises were tested in the authors' developmental mathematics classes and were created to improve student performance on tests.

"The PREs are an excellent source of additional mixed problem sets. Frequently students have questions/comments like 'Where do I start?' or 'I know what to do once I get started, but I have trouble getting started.' Perhaps with these PREs, students will be able to overcome this obstacle."

—Erika Blanken, *Daytona State College*

Problem Recognition Exercises

Operations on Polynomials

For Exercises 1–40, perform the indicated operations and simplify.

1. **a.** $6x^2 + 2x^2$ **b.** $(6x^2)(2x^2)$
2. **a.** $8y^3 + y^3$ **b.** $(8y^3)(y^3)$
3. **a.** $(4x + y)^2$ **b.** $(4xy)^2$
4. **a.** $(2a + b)^2$ **b.** $(2ab)^2$
5. **a.** $(2x + 3) + (4x - 2)$ **b.** $(2x + 3)(4x - 2)$
6. **a.** $(5m^2 + 1) + (m^2 + m)$ **b.** $(5m^2 + 1)(m^2 + m)$
7. **a.** $(3z + 2)^2$ **b.** $(3z + 2)(3z - 2)$
8. **a.** $(6y - 7)^2$ **b.** $(6y - 7)(6y + 7)$
9. **a.** $(2x - 4)(x^2 - 2x + 3)$ **b.** $(2x - 4) + (x^2 - 2x + 3)$
10. **a.** $(3y^2 + 8)(-y^2 - 4)$ **b.** $(3y^2 + 8) - (-y^2 - 4)$
11. **a.** $x + x$ **b.** $x \cdot x$
12. **a.** $2c + 2c$ **b.** $2c \cdot 2c$
13. $(7mn)^2$
14. $(8pq)^2$
15. $(-2x^4 - 6x^3 + 8x^2) \div (2x^2)$
16. $(-15m^3 + 12m^2 - 3m) \div (-3m)$
17. $(m^3 - 4m^2 - 6) - (3m^2 + 7m) + (-m^3 - 9m + 6)$
18. $(n^4 + 2n^2 - 3n) + (4n^2 + 2n - 1) - (4n^5 + 6n - 3)$
19. $(8x^3 + 2x + 6) \div (x - 2)$
20. $(-4x^3 + 2x^2 - 5) \div (x - 3)$
21. $(2x - y)(3x^2 + 4xy - y^2)$
22. $(3a + b)(2a^2 - ab + 2b^2)$
24. $(m^2 + 1)(m^4 - m^2 + 1)$
27. $(a^3 + 2b)(a^3 - 2b)$
28. $(y^3 - 6z)(y^3 + 6z)$
31. $\dfrac{12x^3y^7}{3xy^5}$
32. $\dfrac{-18p^2q^4}{2pq^3}$

"These are so important to test whether a student can recognize different types of problems and the method of solving each. They seem very unique—I have not noticed this feature in many other texts or at least your presentation of the problems is very organized and unique."

—Linda Kuroski, *Erie Community College*

Student-Centered Applications!

The Miller/O'Neill/Hyde Board of Advisors partnered with our authors to bring the *best applications* from every region in the country! These applications include real data and topics that are more relevant and interesting to today's student.

11. A bicyclist rides 24 mi against a wind and returns 24 mi with the same wind. His average speed for the return trip traveling with the wind is 8 mph faster than his speed going out against the wind. If x represents the bicyclist's speed going out against the wind, then the total time, t, required for the round trip is given by

$$t = \frac{24}{x} + \frac{24}{x + 8}$$ where t is measured in hours.

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a. Find the time required for the round trip if the cyclist rides 12 mph against the wind.

b. Find the time required for the round trip if the cyclist rides 24 mph against the wind.

Group Activities!

Each chapter concludes with a Group Activity to promote classroom discussion and collaboration—helping students not only to solve problems but to explain their solutions for better mathematical mastery. Group Activities are great for both full-time and adjunct instructors—bringing a more interactive approach to teaching mathematics! All required materials, activity time, and suggested group sizes are provided in the end-of-chapter material.

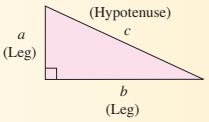
Group Activity

The Pythagorean Theorem and a Geometric "Proof"

Estimated Time: 25–30 minutes

Group Size: 2

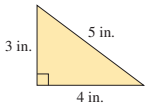
Right triangles occur in many applications of mathematics. By definition, a right triangle is a triangle that contains a 90° angle. The two shorter sides in a right triangle are referred to as the "legs," and the longest side is called the "hypotenuse." In the triangle shown, the legs are labeled as a and b, and the hypotenuse is labeled as c.



Right triangles have an important property that the sum of the squares of the two legs of a right triangle equals the square of the hypotenuse. This fact is referred to as the Pythagorean theorem. In symbols, the Pythagorean theorem is stated as:

$$a^2 + b^2 = c^2$$

1. The following triangles are right triangles. Verify that $a^2 + b^2 = c^2$. (The units may be left off when performing these calculations.)

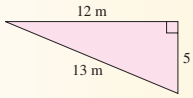


$a = 3$
$b = 4$
$c = 5$

$a^2 + b^2 = c^2$

$(3)^2 + (4)^2 \stackrel{?}{=} (5)^2$

$9 + 16 = 25$ ✓



$a =$ _____
$b =$ _____
$c =$ _____

$a^2 + b^2 = c^2$

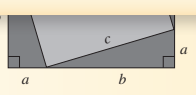
$(\_\_\_\_)^2 + (\_\_\_\_)^2 \stackrel{?}{=} (\_\_\_\_)^2$

$\_\_\_\_ + \_\_\_\_ = \_\_\_\_$ ✓

an theorem uses addition, subtraction, an
e figure. The length of each side of the
the large outer square is $(a + b)^2$.
ound by adding the area of the inner
four right triangles (pictured in dark gray

ht triangles: $4 \cdot (\frac{1}{2} a\, b)$

$\frac{1}{2}$ Base · Height



rea of the large outer square:

"This is one part of the book that would have me adopt the MOH book. I am very big on group work for *Beginning Algebra* and many times it is difficult to think of an activity. I would conclude the chapter doing the group activity in the class. Many books just have problems for this, but the MOH book provides an actual activity."

—Sharon Giles, *Grossmont College*

"MOH's group activity involves true participation and interaction; fun with fractions!"

—Monika Bender, *Central Texas College*

Get Better Results

Dynamic Math Animations

The Miller/O'Neill/Hyde author team has developed a series of animations to illustrate difficult concepts where static images and text fall short. The animations leverage the use of on-screen movement and morphing shapes to enhance conceptual learning.

Through their classroom experience, the authors recognize that such media assets are great teaching tools for the classroom and excellent for online learning. The Miller/O'Neill/Hyde animations are interactive and quite diverse in their use. Some provide a virtual laboratory for which an application is simulated and where students can collect data points for analysis and modeling. Others provide interactive question-and-answer sessions to test conceptual learning. For word problem applications, the animations ask students to estimate answers and practice "number sense."

The animations were created by the authors based on over 75 years of combined teaching experience! To facilitate the use of the animations, the authors have placed icons in the text to indicate where animations are available. Students and instructors can access these assets online in either the ALEKS 360 Course product or Connect Math Hosted by ALEKS.

Additional Supplements

SmartBook. . . NOW with Learning Resources!

SmartBook is the first and only adaptive reading experience available for the world of higher education, and facilitates the reading process by identifying what content a student knows and doesn't know. As a student reads, the material continuously adapts to ensure the student is focused on the content he or she needs the most to close specific knowledge gaps. Additionally, new interactive Learning Resources now allow students to explore connections between different representations of problems, and also serve as an added resource right at the moment when a student answers a probe incorrectly and needs help. These Learning Resources—such as videos, interactive activities, and kaleidoscopes—are available at all times to provide support for students, even when they are working late at night or over the weekend and therefore do not have access to an instructor.

NEW Integrated Video and Study Workbooks

The Integrated Video and Study Workbooks were built to be used in conjunction with the Miller/O'Neill/Hyde Developmental Math series online lecture videos. These new video guides allow students to consolidate their notes as they work through the material in the book, and provide students with an opportunity to focus their studies on particular topics that they are struggling with rather than entire chapters at a time. Each video guide contains written examples to reinforce the content students are watching in the corresponding lecture video, along with additional written exercises for extra practice. There is also space provided for students to take their own notes alongside the guided notes already provided. By the end of the academic term, the video guides will not only be a robust study resource for exams, but will serve as a portfolio showcasing the hard work of students throughout the term.

Student Resource Manual

The *Student Resource Manual (SRM),* created by the authors, is a printable, electronic supplement available to students through Connect Math Hosted by ALEKS Corp. Instructors can also choose to customize this manual and package with their course materials. With increasing demands on faculty schedules, this resource offers a convenient means for both full-time and adjunct faculty to promote active learning and success strategies in the classroom.

This manual supports the series in a variety of different ways:

- Discovery-based classroom activities written by the authors for each section
- Worksheets for extra practice written by the authors

- Excel activities that not only provide students with numerical insights into algebraic concepts, but also teach simple computer skills to manipulate data in a spreadsheet
- Additional fun group activities
- Lecture Notes designed to help students organize and take notes on key concepts
- Materials for a student portfolio

Lecture Videos Created by the Authors (Available in ALEKS and Connect Math Hosted by ALEKS Corp.)

Julie Miller began creating these lecture videos for her own students to use when they were absent from class. The student response was overwhelmingly positive, prompting the author team to create the lecture videos for their entire developmental math book series. In these new videos, the authors walk students through the learning objectives using the same language and procedures outlined in the book. Students learn and review right alongside the author! Students can also access the written notes that accompany the videos.

All videos are closed-captioned for the hearing-impaired, and meet the Americans with Disabilities Act Standards for Accessible Design. These videos are available online through Connect Math Hosted by ALEKS Corp. as well as in ALEKS 360.

Exercise Videos (Available in ALEKS and Connect Math Hosted by ALEKS Corp.)

The authors, along with a team of faculty who have used the Miller/O'Neill/Hyde textbooks for many years, have created new exercise videos for designated exercises in the textbook. These videos cover a representative sample of the main objectives in each section of the text. Each presenter works through selected problems, following the solution methodology employed in the text.

Annotated Instructor's Edition

In the *Annotated Instructor's Edition* (*AIE*), answers to all exercises appear adjacent to each exercise in a color used *only* for annotations. The *AIE* also contains Instructor Notes that appear in the margin. These notes offer instructors assistance with lecture preparation. In addition, there are Classroom Examples referenced in the text that are highlighted in the Practice Exercises. Also found in the *AIE* are icons within the Practice Exercises that serve to guide instructors in their preparation of homework assignments and lessons.

Powerpoints (Available in the Resources of Connect Math Hosted by ALEKS Corp.)

The Powerpoints present key concepts and definitions with fully editable slides that follow the textbook. An instructor may project the slides in class or post to a website in an online course.

McGraw-Hill Connect Math Hosted by ALEKS Corp.

Connect Math Hosted by ALEKS Corp. is an exciting new assignment and assessment ehomework platform. Instructors can assign an AI-driven ALEKS Asssessment to identify the strengths and weaknesses of each student at the beginning of the term rather than after the first exam. Assignment creation and navigation is efficient and intuitive. The gradebook, based on instructor feedback, has a straightforward design and allows flexibility to import and export additional grades.

ALEKS Prep for Developmental Mathematics

ALEKS Prep for Beginning Algebra focuses on prerequisite and introductory material for this text. The prep products can be used during the first 3 weeks of a traditional course or in a corequisite course where students need to quickly narrow the gap in their skill and concept base.

ALEKS Prep Course Products Feature:

- Artificial Intelligence Targeting Gaps in Individual Student's Knowledge
- Assessment and Learning Directed Toward Individual Student's Needs
- Open Response Environment with Realistic Input Tools
- Unlimited Online Access—PC and Mac Compatible

Free trial at www.aleks.com/free_trial/instructor

Get Better Results

Instructor's and Student's Solutions Manuals (Available in the Resources of Connect Math Hosted by ALEKS Corp., the Online Learning Center, and at www.mcgrawhillcreate.com, our print-on-demand book-building website)

The *Instructor's Solutions Manual* provides comprehensive, worked-out solutions to all exercises in the Chapter Openers, the Practice Exercises, the Problem Recognition Exercises, the end-of-chapter Review Exercises, the Chapter Tests, and the Cumulative Review Exercises. The *Student's Solutions Manual* provides answers to the odd-numbered exercises in the text.

Instructor's Test Bank (Available in the Resources of Connect Math Hosted by ALEKS Corp.)

Among the supplements is a computerized test bank utilizing algorithm-based testing software to create customized exams quickly. Hundreds of text-specific, open-ended, and multiple-choice questions are included in the question bank. Sample chapter tests are also provided.

Loose-Leaf Text

This three-hole punched version of the traditional printed text allows students to carry it lightly and comfortably in a binder, integrated with notes and workbook pages as desired.

Acknowledgments and Reviewers

Paramount to the development of *Beginning Algebra* was the invaluable feedback provided by the instructors from around the country that reviewed the manuscript or attended a market development event over the course of the several years the text was in development.

Reviewers of Miller/O'Neill/Hyde Developmental Mathematics Series

Maryann Faller, *Adirondack Community College*
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Debra Pearson, *Ball State University*
Patricia Parkison, *Ball State University*
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Sounny Slitine, *Saint Philips College*
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Natalie Johnson, *Tarrant County College South*
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Shannon Vinson, *Wake Tech Community College*
Kim Walaski, *Wake Tech Community College*
Lisa Rombes, *Washtenaw Community College*
Maziar Ouliaeinia, *Western Iowa Tech Community College*
Keith McCoy, *Wilbur Wright College*

Also, a special thanks to all instructors who have reviewed previous editions of this series.

Get Better Results

Our Commitment to Market Development and Accuracy

McGraw-Hill's Development Process is an ongoing, never-ending, market-oriented approach to building accurate and innovative print and digital products. We begin developing a series by partnering with authors that desire to make an impact within their discipline to help students succeed. Next, we share these ideas and manuscript with instructors for review for feedback and to ensure that the authors' ideas represent the needs within that discipline. Throughout multiple drafts, we help our authors adapt to incorporate ideas and suggestions from reviewers to ensure that the series carries the same pulse as today's classrooms. With any new series, we commit to accuracy across the series and its supplements. In addition to involving instructors as we develop our content, we also utilize accuracy checks through our various stages of development and production. With our commitment to this process, we are confident that our series has the most developed content the industry has to offer, thus pushing our desire for quality and accurate content that meets the needs of today's students and instructors.

New and Updated Content for Miller/O'Neill/Hyde Beginning Algebra, Fifth Edition:

- New Chapter Openers focused on contextualized learning that introduce the main idea of each chapter in an applied setting
- New SmartBook with Learning Resources digital resource added for students that includes over 1,400 student learning resources, over 500 learning objectives, and over 1,300 student activity probes
- New Integrated Video & Study Guide workbook to accompany the online lecture video series created by the Miller/O'Neill/Hyde author team
- Updated Applications to be timely in all instances where appropriate
- Modularized content for easier course customization and flexibility in a digital or traditional classroom environment
- Over 200 new algorithmic exercises added to Connect Math to better cover developmental math content for students
- New Introduction to Modeling online appendix added to provide students with additional targeted instruction on linear, quadratic, and exponential models

Mc Graw Hill Education

connect® MATH
HOSTED BY ALEKS
+SMARTBOOK®

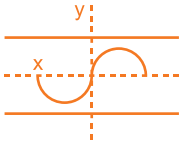
Students lacking confidence in math? Looking for a consistent voice between text and digital?

Problem Solved!

Connect Math Hosted by ALEKS + SmartBook is a complete system that offers everything students and instructors need in one, intuitive platform. ConnectMath is an online homework engine where the problems and solutions are consistent with the textbook authors' approach. SmartBook is an assignable, adaptive eBook and study tool that directs students to the content they don't know and helps them study more efficiently. This combination gives you tools you need to be the teacher you want to be.

"I like that ConnectMath reaches students with different learning styles … our students are engaged, attend class, and ask excellent questions."
— Kelly Davis, South Texas College

Developed by instructors for instructors to create a seamless transition from text to digital



SmartBook integration offers an interactive reading and learning experience

Access to author-developed, text-specific assignments, learning resources, and videos

How can ConnectMath + SmartBook help solve your students' math challenges?

I like to learn by _____.

Whether it's reading, watching, discovering, or doing, ConnectMath has something for everyone. Instructors can create assignments that accommodate different learning styles, and students aren't stuck with boring multiple-choice problems. Instead they have a myriad of motivational learning and media resources at their fingertips. SmartBook delivers an interactive reading and learning experience that provides personalized guidance and just-in-time remediation. This helps students to focus on what they need, right when they need it.

I still don't get it. Can you do that problem again?

Because the content in ConnectMath is author-developed and goes through a rigorous quality control process, students hear one voice, one style, and don't get lost moving from text to digital. The high-quality, author-developed videos provide students ample opportunities to master concepts and practice skills that they need extra help with . . . all of which are integrated in the ConnectMath platform and the eBook.

Let's Talk!

Ready to take the next step?

Let's talk. Whether you're looking to redesign your course, change technologies, or just explore, our team of dedicated faculty consultants, reps, and digital specialists are standing by.

http://shop.mheducation.com/store/paris/user/findltr.html

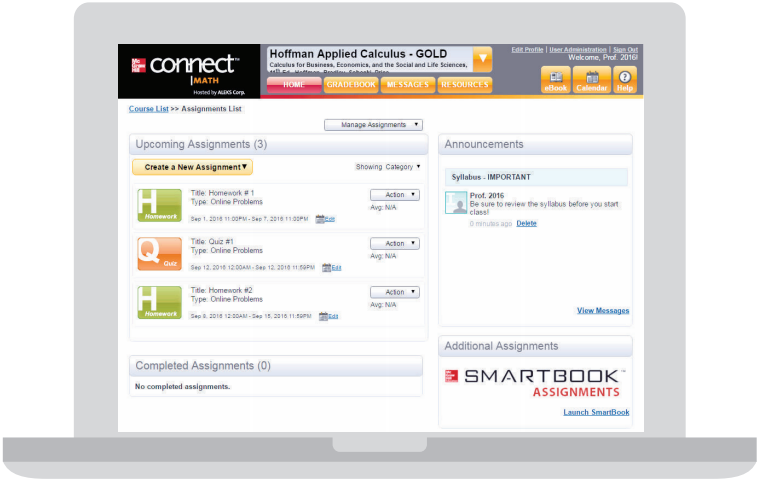
How can ConnectMath + SmartBook help solve your classroom challenges?

I need meaningful data to measure student success!

From helping the student in the back row to tracking learning trends for your entire course, ConnectMath + SmartBook delivers the data you need to make an impactful, meaningful learning experience for students. With easy-to-interpret, downloadable reports, you can analyze learning rates for each assignment, monitor time on task, and learn where students' strengths and weaknesses are in each course area.

We're going with the _____ (flipped classroom, corequisite model, hybrid, etc.) implementation.

ConnectMath + SmartBook is an intuitive digital solution that can be used in any course setup. Each course in ConnectMath comes complete with its own set of text-specific assignments, author-developed videos and learning resources, and an integrated eBook that cater to the needs of your specific course. The easy-to-navigate home page keeps the learning curve at a minimum, but offers an abundance of tutorials and videos to help get you and your colleagues started.



Looking for stable technology? Connect Math Hosted by ALEKS has an uptime of 99.97%.

Problem Solved!

Mc Graw Hill Education

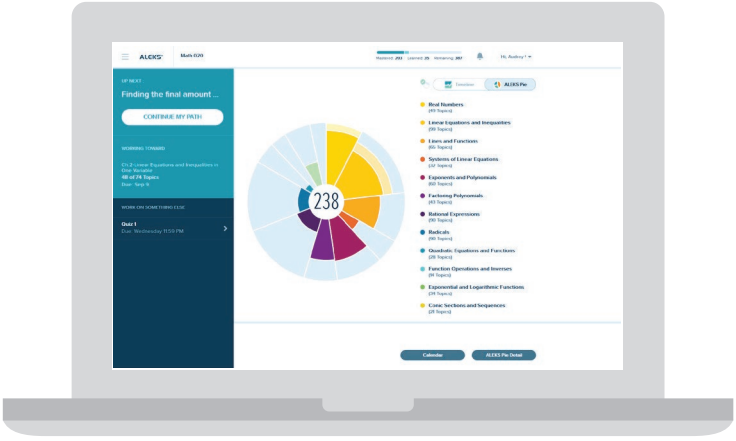
ALEKS®

Students unprepared, unmotivated, or unsure about their math abilities?

Problem Solved!

As the leading Learning Science Company, McGraw-Hill Education delivers the most effective, efficient, and engaging educational technology today. ALEKS is that technology.

ALEKS uses artificial intelligence to precisely map what each student knows, doesn't know, and is most ready to learn in a given course area. The system interacts with each student like a skilled human tutor, delivering a cycle of highly individualized learning and assessment that ensures mastery. Students are shown an optimal path to success, and instructors have the analytics they need to deliver a data-informed, impactful learning experience.



"ALEKS has helped to create the best classroom experience that I've had in 36 years. **Pass rates significantly increased by 40%.** I can reach each student, in each class, with ALEKS."

– Tommy Thompson, Cedar Valley College, TX

How can ALEKS help solve your students' challenges?

I've never been good at math, so why should I try?

The perceived struggle with math is often too real for many students. ALEKS offers a chance to break from that struggle through its cycle of individualized assessment and learning. Students only work on topics they are ready to learn, which have a proven learning success rate of 93% or higher. Periodic assessments reinforce content mastery and provide targeted remediation. As students watch their progress in the ALEKS Pie grow, their confidence grows with it.

I did all my homework, so why am I failing my exams?

The purpose of homework is to ensure mastery and prepare students for exams. Yet how well do homework scores correlate to exam scores? ALEKS is the only adaptive learning system that ensures mastery through periodic reassessments and delivers just-in-time remediation to efficiently prepare students. Because of how ALEKS presents lessons and practice, students learn by understanding the core principle of a concept rather than just memorizing a process.

I'm too far behind to catch up. - OR - I've already done this, I'm bored.

No two students are alike. So why start everyone on the same page? ALEKS diagnoses what each student knows and doesn't know, and prescribes an optimized learning path through the curriculum you put forth. Students are only working on what they need, when they need it, rather than focusing on topics they already know or aren't ready for. The frustration of falling behind and the boredom from redundant review is virtually eliminated.

"ALEKS improved my math skills tremendously. What I love the most about it is that it makes sure you understand the material ... Thanks to ALEKS I got the job that I have now."
– Student, Broward College, FL

How can ALEKS help solve your classroom challenges?

I need something that solves the problem of cost, time to completion, and student preparedness.

ALEKS is the perfect solution to the trifecta of these problems. It provides an efficient path to mastery through its individualized cycle of learning and assessment that targets prerequisite gaps and focuses students on what they are ready to learn. Students move through their math requirements more quickly and are better prepared for subsequent courses. This saves both the institution and the student money.

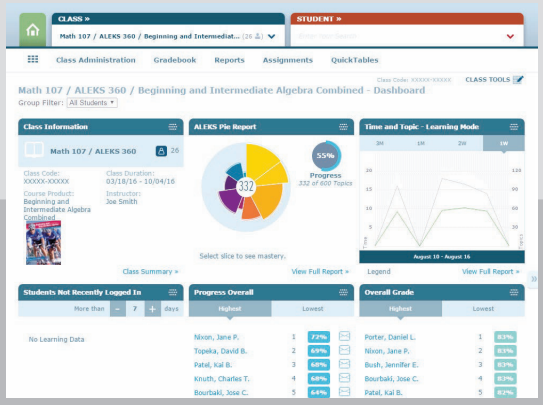
We're going with the _____ (flipped classroom, corequisite, accelerated, etc.) implementation.

No matter your course setup, ALEKS can handle it. ALEKS courses cover a broad curriculum, so that you can easily tailor it to cover just the topics that you need. ALEKS Objectives gives you control over when and in what order students move through the content so that the pacing matches that of the textbook and/or curriculum. The flexibility of ALEKS allows you to use it in conjunction with any textbook or set of resources that you want to use.

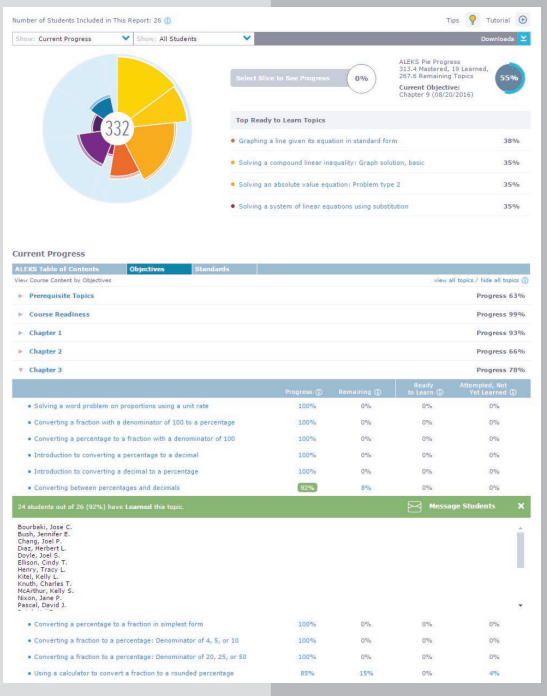
My administration and department measure success differently. How can we compare notes?

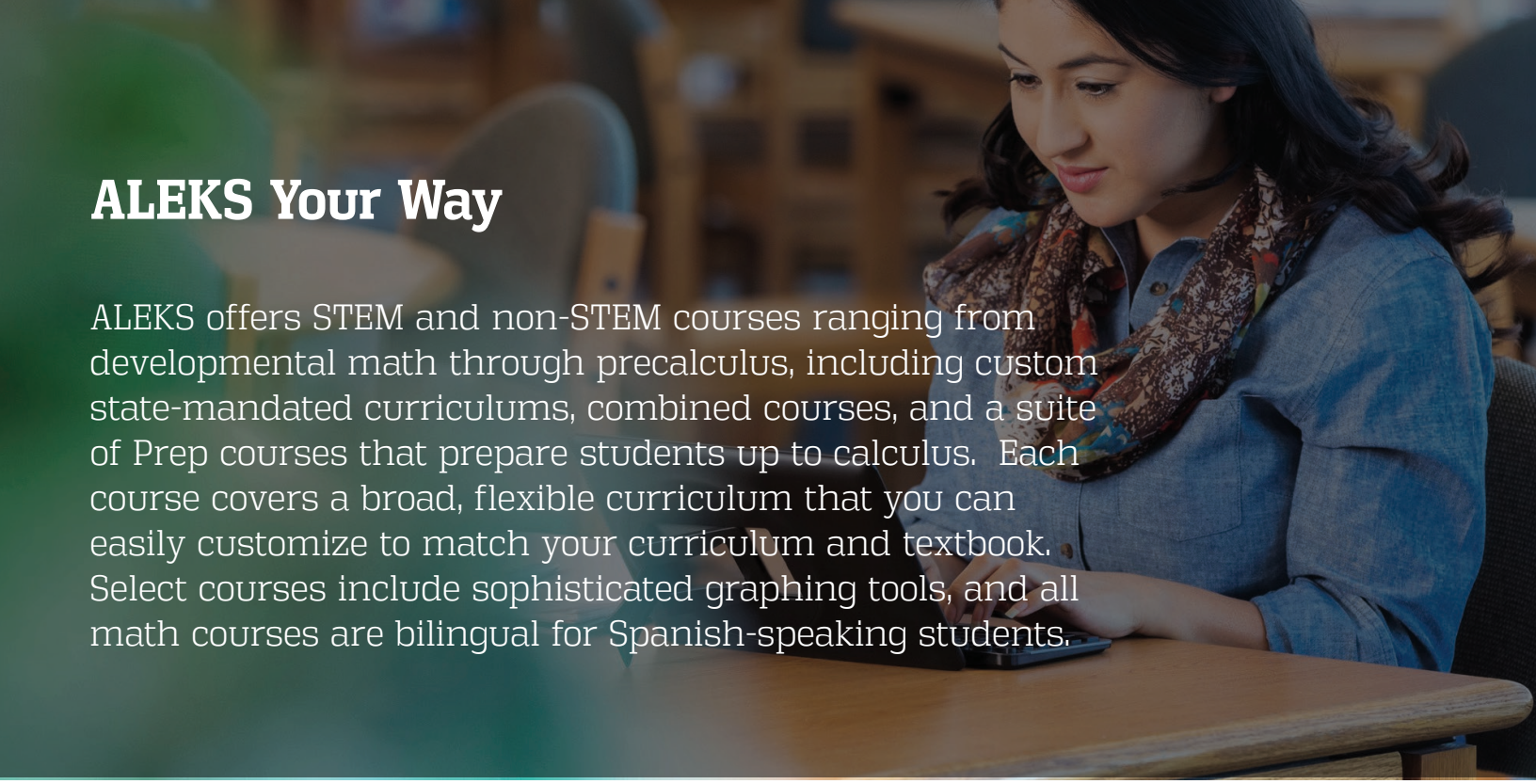
ALEKS offers the most comprehensive and detailed data analytics on the market. From helping the student in the back row to monitoring pass rates across the department and institution, ALEKS delivers the data needed at all levels. The customizable and intuitive reporting features allow you and your colleagues to easily gather, interpret, and share the data you need, when you need it.

"ALEKS is the only product that diagnoses and prescribes an individualized learning path. The ALEKS management system is intuitive and easy to use. The reports are easy to construct, read and interpret, allowing us to help students focus and succeed."
–Yoshi Yamato and Marie McClendon, Pasadena City College, CA



The ALEKS Instructor Module is an intuitive interface where you can easily manage courses and track student progress.





ALEKS Your Way

ALEKS offers STEM and non-STEM courses ranging from developmental math through precalculus, including custom state-mandated curriculums, combined courses, and a suite of Prep courses that prepare students up to calculus. Each course covers a broad, flexible curriculum that you can easily customize to match your curriculum and textbook. Select courses include sophisticated graphing tools, and all math courses are bilingual for Spanish-speaking students.

ALEKS® 360

ALEKS 360 combines the power of ALEKS with an integrated, interactive McGraw-Hill eBook. Students have direct access to the text, lecture videos, supplementary resources, and practice examples right when they need it as they work each topic. ALEKS 360 is available with most ALEKS courses, and at an affordable price. For those students who still prefer to hold a page in their hand, they can order a loose-leaf version of the eBook for a significant discount.

ALEKS® PREP

ALEKS Prep courses target students' prerequisite knowledge gaps and provide individualized learning and remediation to ensure students are ready to succeed in their math course. It is the perfect solution for the accelerated "bootcamp" program, instructors that need to get students up to speed during the first few weeks of a course, or as the remedial component of a corequisite course.

ALEKS® PPL

PLACEMENT, PREPARATION, AND LEARNING

ALEKS Placement, Preparation, and Learning (ALEKS PPL) is the first and only institution-wide placement solution to both diagnose and remediate, all in one completely personalized program. The adaptive placement assessment places students from Basic Math through Calculus I. Then students get six months of personalized learning and remediation. ALEKS PPL gives institutions the tools to improve college preparedness, course performance, and retention.

"My students are much more prepared because of ALEKS Prep. I saw noticeably fewer errors on exams and fewer Algebra-related questions during class. I can now focus on Calculus material without having to review."
–Jeanette Martin, University of Washington

Study Tips

In taking a course in algebra, you are making a commitment to yourself, your instructor, and your classmates. Following some or all of the study tips presented here can help you be successful in this endeavor. The features of this text that will assist you are printed in blue.

Concepts

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1. Before the Course

1. Purchase the necessary materials for the course before the course begins or on the first day.
2. Obtain a three-ring binder to keep and organize your notes, homework, tests, and any other materials acquired in the class. We call this type of notebook a portfolio.
3. Arrange your schedule so that you have enough time to attend class and to do homework. A common rule is to set aside at least 2 hours for homework for every hour spent in class. That is, if you are taking a 4-credit-hour course, plan on at least 8 hours a week for homework. A 6-credit-hour course will then take *at least* 12 hours each week—about the same as a part-time job. If you experience difficulty in mathematics, plan for more time.
4. Communicate with your employer and family members the importance of your success in this course so that they can support you.
5. Be sure to find out the type of calculator (if any) that your instructor requires.

2. During the Course

1. Read the section in the text *before* the lecture to familiarize yourself with the material and terminology. It is recommended that you read your math book with paper and pencil in hand. Write a one-sentence preview of what the section is about.
2. Attend every class and be on time. Be sure to bring any materials that are needed for class such as graph paper, a ruler, or a calculator.
3. Take notes in class. Write down all of the examples that the instructor presents. Read the notes after class, and add any comments to make your notes clearer to you. Use an audio recorder to record the lecture if the instructor permits the recording of lectures.
4. Ask questions in class.
5. Read the section in the text *after* the lecture, and pay special attention to the Tip boxes and Avoiding Mistakes boxes.
6. After you read an example, try the accompanying Skill Practice problem. The skill practice problem mirrors the example and tests your understanding of what you have read.
7. Do homework every day. Even if your class does not meet every day, you should still do some work every day to keep the material fresh in your mind.
8. Check your homework with the answers that are supplied in the back of this text. Correct the exercises that do not match, and circle or star those that you cannot correct yourself. This way you can easily find them and ask your instructor, tutor, online tutor, or math lab staff the next day.
9. Be sure to do the Vocabulary and Key Concepts exercises found at the beginning of the Practice Exercises.

10. The Problem Recognition Exercises are located in all chapters. These provide additional practice distinguishing among a variety of problem types. Sometimes the most difficult part of learning mathematics is retaining all that you learn. These exercises are excellent tools for retention of material.
11. Form a study group with fellow students in your class, and exchange phone numbers. You will be surprised by how much you can learn by talking about mathematics with other students.
12. If you use a calculator in your class, read the Calculator Connections boxes to learn how and when to use your calculator.
13. Ask your instructor where you might obtain extra help if necessary.

3. Preparation for Exams

1. Look over your homework. Pay special attention to the exercises you have circled or starred to be sure that you have learned that concept.
2. Begin preparations for exams on the first day of class. As you do each homework assignment, think about how you would recognize similar problems when they appear on a test.
3. Read through the Summary at the end of the chapter. Be sure that you understand each concept and example. If not, go to the section in the text and reread that section.
4. Give yourself enough time to take the Chapter Test uninterrupted. Then check the answers. For each problem you answered incorrectly, go to the Review Exercises and do all of the problems that are similar.
5. To prepare for the final exam, complete the Cumulative Review Exercises at the end of each chapter. If you complete the cumulative reviews after finishing each chapter, then you will be preparing for the final exam throughout the course. The Cumulative Review Exercises are another excellent tool for helping you retain material.

4. Where to Go for Help

1. At the first sign of trouble, see your instructor. Most instructors have specific office hours set aside to help students. Don't wait until after you have failed an exam to seek assistance.
2. Get a tutor. Most colleges and universities have free tutoring available. There may also be an online tutor available.
3. When your instructor and tutor are unavailable, use the Student Solutions Manual for step-by-step solutions to the odd-numbered problems in the exercise sets.
4. Work with another student from your class.
5. Work on the computer. Many mathematics tutorial programs and websites are available on the Internet, including the website that accompanies this text.

Group Activity

Becoming a Successful Student

Materials: Computer with Internet access

Estimated Time: 15 minutes

Group Size: 4

Good time management, good study skills, and good organization will help you be successful in this course. Answer the following questions and compare your answers with your group members.

1. To motivate yourself to complete a course, it is helpful to have clear reasons for taking the course. List your goals for taking this course and discuss them with your group.

2. For the following week, write down the times each day that you plan to study math.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday

3. Write down the date of your next math test. _____

4. Taking 12 credit-hours is the equivalent of a full-time job. Often students try to work too many hours while taking classes at school.

a. Write down the number of hours you work per week and the number of credit-hours you are taking this term.

Number of hours worked per week _____

Number of credit-hours this term _____

b. The table gives a recommended limit to the number of hours you should work for the number of credit-hours you are taking at school. (Keep in mind that other responsibilities in your life such as your family might also make it necessary to limit your hours at work even more.) How do your numbers from part (a) compare to those in the table? Are you working too many hours?

Number of Credit-Hours	Maximum Number of Hours of Work per Week
3	40
6	30
9	20
12	10
15	0

5. Discuss with your group members where you can go for extra help in math. Then write down three of the suggestions.

_____

_____

_____

6. Do you keep an organized notebook for this class? Can you think of any suggestions that you can share with your group members to help them keep their materials organized?

7. Look through a chapter and find the page number corresponding to each feature in that chapter. Discuss with your group members how you might use each feature.

Problem Recognition Exercises: page _____

Chapter Summary: page _____

Chapter Review Exercises: page _____

Chapter Test: page _____

Cumulative Review Exercises: page _____

8. Look at the Skill Practice exercises. For example, find Skill Practice exercises 1 and 2 in the first section. Where are the answers to these exercises located? Discuss with your group members how you might use the Skill Practice exercises.

9. Do you think that you have math anxiety? Read the following list for some possible solutions. Check the activities that you can realistically try to help you overcome this problem.

_____ Read a book on math anxiety.

_____ Search the Web for tips on handling math anxiety.

_____ See a counselor to discuss your anxiety.

_____ Talk with your instructor to discuss strategies to manage math anxiety.

_____ Evaluate your time management to see if you are trying to do too much. Then adjust your schedule accordingly.

10. Some students favor different methods of learning over others. For example, you might prefer:

- Learning through listening and hearing.
- Learning through seeing images, watching demonstrations, and visualizing diagrams and charts.
- Learning by experience through a hands-on approach.
- Learning through reading and writing.

Most experts believe that the most effective learning comes when a student engages in *all* of these activities. However, each individual is different and may benefit from one activity more than another. You can visit a number of different websites to determine your "learning style." Try doing a search on the Internet with the key words "*learning styles assessment*." Once you have found a suitable website, answer the questionnaire and the site will give you feedback on what method of learning works best for you.

11. As you read through Chapter 1, try to become familiar with the features of this textbook. Then match the feature in column B with its description in column A.

Column A

1. Allows you to check your work as you do your homework
2. Shows you how to avoid common errors
3. Provides an online tutorial and exercise supplement
4. Outlines key concepts for each section in the chapter
5. Provides exercises that help you distinguish between different types of problems
6. Offers helpful hints and insight
7. Offers practice exercises that go along with each example

Column B

a. Tips
b. ConnectMath
c. Skill Practice exercises
d. Problem Recognition Exercises
e. Answers to odd-numbered exercises
f. Chapter Summary
g. Avoiding Mistakes

The Set of Real Numbers

1

CHAPTER OUTLINE

Numbers in Our World

Imagine a world where the only numbers known are the counting or natural numbers (1, 2, 3, 4, . . .). Now imagine that you want to sell only a fraction of your land to another person, or that you owe twenty dollars and fifteen cents to your bank. How could these values be written without formal numerical symbols? Living in such a world would deter the growth of a complex society like ours.

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It is difficult to fathom that the use of zero, fractions, and negative numbers was formally accepted only about a thousand years ago! Before that time, communicating about parts of items, the absence of value, and owing money to a lender was likely done in creative but arduous ways. When we talk about a *third* of a parcel of land, temperatures below *zero* such as *negative* 20 degrees, and the number π, we make use of the many subsets of the **set of real numbers**.

Real numbers enable us to talk about parts of things, to expand our thinking to explain phenomena in precise ways, and to operate with numbers in a consistent and predicable manner. In this chapter we will explore how real numbers are used, and the way they open the door to algebra.

Section 1.1 Fractions

Concepts

1. Basic Definitions

The study of algebra involves many of the operations and procedures used in arithmetic. Therefore, we begin this text by reviewing the basic operations of addition, subtraction, multiplication, and division on fractions and mixed numbers.

We begin with the numbers used for counting:

the **natural numbers:** 1, 2, 3, 4, . . .

and

the **whole numbers:** 0, 1, 2, 3, . . .

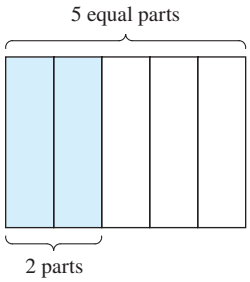
Whole numbers are used to count the number of whole units in a quantity. A fraction is used to express part of a whole unit. If a child gains $2\frac{1}{2}$ lb, the child has gained two whole pounds plus a portion of a pound. To express the additional half pound mathematically, we may use the fraction, $\frac{1}{2}$.

A Fraction and Its Parts

Fractions are numbers of the form $\frac{a}{b}$, where $\frac{a}{b} = a \div b$ and b does not equal zero.

In the fraction $\frac{a}{b}$, the **numerator** is a, and the **denominator** is b.

The denominator of a fraction indicates how many equal parts divide the whole. The numerator indicates how many parts are being represented. For instance, suppose Jack wants to plant carrots in $\frac{2}{5}$ of a rectangular garden. He can divide the garden into five equal parts and use two of the parts for carrots (Figure 1-1).



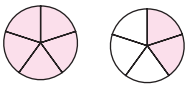
The shaded region represents $\frac{2}{5}$ of the garden.

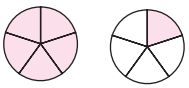
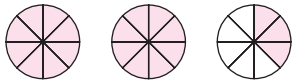
Figure 1-1

Proper Fractions, Improper Fractions, and Mixed Numbers

1. If the numerator of a fraction is less than the denominator, the fraction is a **proper fraction**. A proper fraction represents a quantity that is less than a whole unit.
2. If the numerator of a fraction is greater than or equal to the denominator, then the fraction is an **improper fraction**. An improper fraction represents a quantity greater than or equal to a whole unit.
3. A **mixed number** is a whole number added to a proper fraction.

Proper Fractions: $\frac{3}{5}$  $\frac{1}{8}$ 

Improper Fractions: $\frac{7}{5}$  $\frac{8}{8}$ 

Mixed Numbers: $1\frac{1}{5}$  $2\frac{3}{8}$ 

2. Prime Factorization

To perform operations on fractions it is important to understand the concept of a factor. For example, when the numbers 2 and 6 are multiplied, the result (called the **product**) is 12.

$$\underset{\text{factors}}{2 \times 6} = \underset{\text{product}}{12}$$

The numbers 2 and 6 are said to be **factors** of 12. (In this context, we refer only to natural number factors.) The number 12 is said to be factored when it is written as the product of two or more natural numbers. For example, 12 can be factored in several ways:

$$12 = 1 \times 12 \qquad 12 = 2 \times 6 \qquad 12 = 3 \times 4 \qquad 12 = 2 \times 2 \times 3$$

A natural number greater than 1 that has only two factors, 1 and itself, is called a **prime number**. The first several prime numbers are 2, 3, 5, 7, 11, and 13. A natural number greater than 1 that is not prime is called a **composite number**. That is, a composite number has factors other than itself and 1. The first several composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, and 16.

Avoiding Mistakes

The number 1 is neither prime nor composite.

Example 1 Writing a Natural Number as a Product of Prime Factors

Write each number as a product of prime factors.

a. 12 **b.** 30

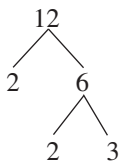
Solution:

a. $12 = 2 \times 2 \times 3$

Divide 12 by prime numbers until the result is also a prime number.

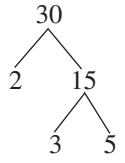
$$\begin{array}{r} 2\overline{)12} \\ 2\overline{)6} \\ 3 \end{array}$$

Or use a factor tree



b. $30 = 2 \times 3 \times 5$

$$\begin{array}{r} 2\overline{)30} \\ 3\overline{)15} \\ 5 \end{array}$$



Skill Practice Write the number as a product of prime factors.

1. 40 **2.** 72

Answers

1. $2 \times 2 \times 2 \times 5$
2. $2 \times 2 \times 2 \times 3 \times 3$

3. Simplifying Fractions to Lowest Terms

The process of factoring numbers can be used to reduce or simplify fractions to lowest terms. A fractional portion of a whole can be represented by infinitely many fractions. For example, Figure 1-2 shows that $\frac{1}{2}$ is equivalent to $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, and so on.

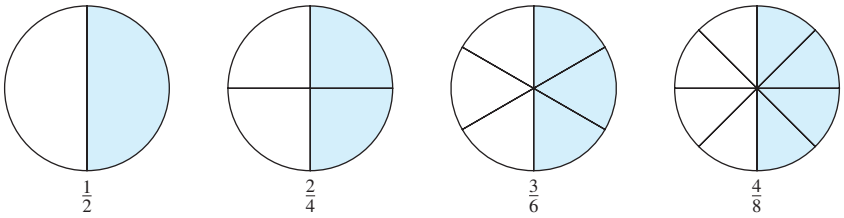


Figure 1-2

The fraction $\frac{1}{2}$ is said to be in **lowest terms** because the numerator and denominator share no common factor other than 1.

To simplify a fraction to lowest terms, we use the following important principle.

Fundamental Principle of Fractions

Suppose that a number, c, is a common factor in the numerator and denominator of a fraction. Then

$$\frac{a \times c}{b \times c} = \frac{a}{b} \times \frac{c}{c} = \frac{a}{b} \times 1 = \frac{a}{b}$$

To simplify a fraction, we begin by factoring the numerator and denominator into prime factors. This will help identify the common factors.

Example 2 Simplifying a Fraction to Lowest Terms

Simplify $\frac{45}{30}$ to lowest terms.

Solution:

$$\frac{45}{30} = \frac{3 \times 3 \times 5}{2 \times 3 \times 5}$$ Factor the numerator and denominator.

$$= \frac{3}{2} \times \frac{3}{3} \times \frac{5}{5}$$ Apply the fundamental principle of fractions.

$$= \frac{3}{2} \times 1 \times 1$$ Any nonzero number divided by itself is 1.

$$= \frac{3}{2}$$ Any number multiplied by 1 is itself.

Skill Practice Simplify to lowest terms.

3. $\frac{20}{50}$

Answer

3. $\frac{2}{5}$

In Example 2, we showed numerous steps to reduce fractions to lowest terms. However, the process is often simplified. Notice that the same result can be obtained by dividing out the greatest common factor from the numerator and denominator. (The **greatest common factor** is the largest factor that is common to both numerator and denominator.)

$$\frac{45}{30} = \frac{3 \times 15}{2 \times 15}$$ The greatest common factor of 45 and 30 is 15.

$$= \frac{3 \times \overset{1}{\cancel{15}}}{2 \times \underset{1}{\cancel{15}}}$$ The symbol / is often used to show that a common factor has been divided out.

$$= \frac{3}{2}$$ Notice that "dividing out" the common factor of 15 has the same effect as dividing the numerator and denominator by 15. This is often done mentally.

$\frac{\overset{3}{\cancel{45}}}{\underset{2}{\cancel{30}}} = \frac{3}{2}$ ← 45 divided by 15 equals 3. ← 30 divided by 15 equals 2.

Example 3 **Simplifying a Fraction to Lowest Terms**

Simplify $\frac{14}{42}$ to lowest terms.

Solution:

$$\frac{14}{42} = \frac{1 \times 14}{3 \times 14}$$ The greatest common factor of 14 and 42 is 14.

$$= \frac{1 \times \overset{1}{\cancel{14}}}{3 \times \underset{1}{\cancel{14}}}$$

$$= \frac{1}{3}$$

$\frac{\overset{1}{\cancel{14}}}{\underset{3}{\cancel{42}}} = \frac{1}{3}$ ← 14 divided by 14 equals 1. ← 42 divided by 14 equals 3.

Avoiding Mistakes

In Example 3, the common factor 14 in the numerator and denominator simplifies to 1. It is important to remember to write the factor of 1 in the numerator. The simplified form of the fraction is $\frac{1}{3}$.

Skill Practice Simplify to lowest terms.

4. $\frac{32}{12}$

4. Multiplying Fractions

Multiplying Fractions

If b is not zero and d is not zero, then

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

To multiply fractions, multiply the numerators and multiply the denominators.

Answer

4. $\frac{8}{3}$

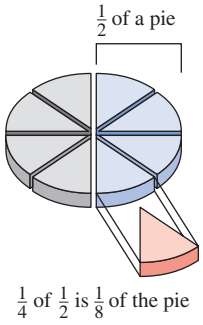


Figure 1-3

Example 4 Multiplying Fractions

Multiply the fractions: $\frac{1}{4}\times\frac{1}{2}$

Solution:

$$\frac{1}{4}\times\frac{1}{2}=\frac{1\times 1}{4\times 2}=\frac{1}{8}$$ Multiply the numerators. Multiply the denominators.

Notice that the product $\frac{1}{4}\times\frac{1}{2}$ represents a quantity that is $\frac{1}{4}$ of $\frac{1}{2}$. Taking $\frac{1}{4}$ of a quantity is equivalent to dividing the quantity by 4. One-half of a pie divided into four pieces leaves pieces that each represent $\frac{1}{8}$ of the pie (Figure 1-3).

Skill Practice Multiply.

5. $\frac{2}{7}\times\frac{3}{5}$

Example 5 Multiplying Fractions

Multiply the fractions.

a. $\frac{7}{10}\times\frac{15}{14}$ **b.** $\frac{2}{13}\times\frac{13}{2}$ **c.** $5\times\frac{1}{5}$

Solution:

a. $\frac{7}{10}\times\frac{15}{14}=\frac{7\times 15}{10\times 14}$ Multiply the numerators. Multiply the denominators.

$=\frac{\overset{1}{\cancel{7}}\times\overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}}\times\underset{2}{\cancel{14}}}$ Divide out the common factors.

$=\frac{3}{4}$ Multiply.

b. $\frac{2}{13}\times\frac{13}{2}=\frac{2\times 13}{13\times 2}=\frac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{13}}}{\underset{1}{\cancel{13}}\times\underset{1}{\cancel{2}}}=\frac{1}{1}=1$

Multiply $1\times 1=1$.

Multiply $1\times 1=1$.

c. $5\times\frac{1}{5}=\frac{5}{1}\times\frac{1}{5}$ The whole number 5 can be written as $\frac{5}{1}$.

$=\frac{\overset{1}{\cancel{5}}\times 1}{1\times\underset{1}{\cancel{5}}}=\frac{1}{1}=1$ Divide out the common factors and multiply.

TIP: The same result can be obtained by dividing out common factors *before* multiplying.

$$\frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{10}}}\times\frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{14}}}=\frac{3}{4}$$

Skill Practice Multiply.

6. $\frac{8}{9}\times\frac{3}{4}$ **7.** $\frac{4}{5}\times\frac{5}{4}$ **8.** $10\times\frac{1}{10}$

Answers

5. $\frac{6}{35}$ **6.** $\frac{2}{3}$ **7.** 1 **8.** 1

5. Dividing Fractions

Before we divide fractions, we need to know how to find the reciprocal of a fraction. Notice from Example 5 that $\frac{2}{13} \times \frac{13}{2} = 1$ and $5 \times \frac{1}{5} = 1$. The numbers $\frac{2}{13}$ and $\frac{13}{2}$ are said to be reciprocals because their product is 1. Likewise the numbers 5 and $\frac{1}{5}$ are reciprocals.

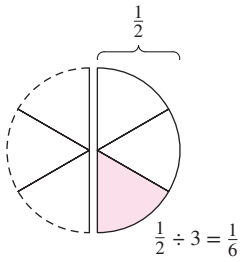
The Reciprocal of a Number

Two nonzero numbers are **reciprocals** of each other if their product is 1. Therefore, the reciprocal of the fraction

$$\frac{a}{b} \text{ is } \frac{b}{a} \qquad \text{because} \qquad \frac{a}{b} \times \frac{b}{a} = 1$$

Number	Reciprocal	Product
$\frac{2}{15}$	$\frac{15}{2}$	$\frac{2}{15} \times \frac{15}{2} = 1$
$\frac{1}{8}$	$\frac{8}{1}$ (or equivalently 8)	$\frac{1}{8} \times 8 = 1$
$6 \left(\text{or equivalently } \frac{6}{1}\right)$	$\frac{1}{6}$	$6 \times \frac{1}{6} = 1$

To understand the concept of dividing fractions, consider a pie that is half-eaten. Suppose the remaining half must be divided among three people, that is, $\frac{1}{2} \div 3$. However, dividing by 3 is equivalent to taking $\frac{1}{3}$ of the remaining $\frac{1}{2}$ of the pie (Figure 1-4).



$$\frac{1}{2} \div 3 = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

Figure 1-4

This example illustrates that dividing two numbers is equivalent to multiplying the first number by the reciprocal of the second number.

Dividing Fractions

Let a, b, c, and d be numbers such that b, c, and d are not zero. Then,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

multiply

reciprocal

To divide fractions, multiply the first fraction by the reciprocal of the second fraction.

Example 6 Dividing Fractions

Divide the fractions.

a. $\frac{8}{5} \div \frac{3}{10}$ **b.** $\frac{12}{13} \div 6$

Solution:

a. $\frac{8}{5} \div \frac{3}{10} = \frac{8}{5} \times \frac{10}{3}$ Multiply by the reciprocal of $\frac{3}{10}$, which is $\frac{10}{3}$.

$= \frac{8 \times \overset{2}{\cancel{10}}}{\underset{1}{\cancel{5}} \times 3} = \frac{16}{3}$ Divide out the common factors and multiply.

b. $\frac{12}{13} \div 6 = \frac{12}{13} \div \frac{6}{1}$ Write the whole number 6 as $\frac{6}{1}$.

$= \frac{12}{13} \times \frac{1}{6}$ Multiply by the reciprocal of $\frac{6}{1}$, which is $\frac{1}{6}$.

$= \frac{\overset{2}{\cancel{12}} \times 1}{13 \times \underset{1}{\cancel{6}}} = \frac{2}{13}$ Divide out the common factors and multiply.

Avoiding Mistakes

Always check that the final answer is in lowest terms.

Skill Practice Divide.

9. $\frac{12}{25} \div \frac{8}{15}$ **10.** $\frac{1}{4} \div 2$

6. Adding and Subtracting Fractions

Adding and Subtracting Fractions

Two fractions can be added or subtracted if they have a common denominator. Let a, b, and c be numbers such that b does not equal zero. Then,

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

To add or subtract fractions with the same denominator, add or subtract the numerators and write the result over the common denominator.

Example 7 Adding and Subtracting Fractions with the Same Denominator

Add or subtract as indicated.

a. $\frac{1}{12} + \frac{7}{12}$ **b.** $\frac{13}{5} - \frac{3}{5}$

Answers

9. $\frac{9}{10}$ **10.** $\frac{1}{8}$

Solution:

a. $\frac{1}{12}+\frac{7}{12}=\frac{1+7}{12}$ Add the numerators.

$=\frac{8}{12}$

$=\frac{2}{3}$ Simplify to lowest terms.

b. $\frac{13}{5}-\frac{3}{5}=\frac{13-3}{5}$ Subtract the numerators.

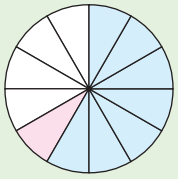
$=\frac{10}{5}$ Simplify.

$=2$ Simplify to lowest terms.

Skill Practice Add or subtract as indicated.

11. $\frac{2}{3}+\frac{5}{3}$ **12.** $\frac{5}{8}-\frac{1}{8}$

TIP: The sum $\frac{1}{12}+\frac{7}{12}$ can be visualized as the sum of the pink and blue sections of the figure.



In Example 7, we added and subtracted fractions with the same denominators. To add or subtract fractions with different denominators, we must first become familiar with the idea of the least common multiple between two or more numbers. The **least common multiple (LCM)** of two numbers is the smallest whole number that is a multiple of each number. For example, the LCM of 6 and 9 is 18.

multiples of 6: 6, 12, 18, 24, 30, 36, . . .
multiples of 9: 9, 18, 27, 36, 45, 54, . . .

Listing the multiples of two or more given numbers can be a cumbersome way to find the LCM. Therefore, we offer the following method to find the LCM of two numbers.

Finding the LCM of Two Numbers

Step 1 Write each number as a product of prime factors.

Step 2 The LCM is the product of unique prime factors from *both* numbers. Use repeated factors the maximum number of times they appear in *either* factorization.

Animation

Example 8 Finding the LCM of Two Numbers

Find the LCM of 9 and 15.

Solution:

	3's	5's
9 =	(3 × 3)	
15 =	3 ×	(5)

LCM = 3 × 3 × 5 = 45

For the factors of 3 and 5, we circle the greatest number of times each occurs. The LCM is the product.

Skill Practice Find the LCM.

13. 10 and 25

Answers

11. $\frac{7}{3}$ or $2\frac{1}{3}$ **12.** $\frac{1}{2}$ **13.** 50

To add or subtract fractions with *different* denominators, we must first write each fraction as an equivalent fraction with a common denominator. A common denominator may be *any* common multiple of the denominators. However, we will use the least common denominator. The **least common denominator (LCD)** of two or more fractions is the LCM of the denominators of the fractions. The following example uses the fundamental principle of fractions to rewrite fractions with the desired denominator. *Note:* Multiplying the numerator and denominator by the *same* nonzero quantity will not change the value of the fraction.

Example 9 **Writing Equivalent Fractions and Subtracting Fractions**

a. Write each of the fractions $\frac{1}{9}$ and $\frac{1}{15}$ as an equivalent fraction with the LCD as its denominator.

b. Subtract $\frac{1}{9} - \frac{1}{15}$.

Solution:

From Example 8, we know that the LCM for 9 and 15 is 45. Therefore, the LCD of $\frac{1}{9}$ and $\frac{1}{15}$ is 45.

a. $\frac{1}{19} = \frac{\quad}{45}$ $\qquad$ $\frac{1 \times 5}{9 \times 5} = \frac{5}{45}$ $\qquad$ So, $\frac{1}{9}$ is equivalent to $\frac{5}{45}$.

What number must we multiply 9 by to get 45?

Multiply numerator and denominator by 5.

$\frac{1}{15} = \frac{\quad}{45}$ $\qquad$ $\frac{1 \times 3}{15 \times 3} = \frac{3}{45}$ $\qquad$ So, $\frac{1}{15}$ is equivalent to $\frac{3}{45}$.

What number must we multiply 15 by to get 45?

Multiply numerator and denominator by 3.

b. $\frac{1}{9} - \frac{1}{15}$

$= \frac{5}{45} - \frac{3}{45}$ Write $\frac{1}{9}$ and $\frac{1}{15}$ as equivalent fractions with the same denominator.

$= \frac{2}{45}$ Subtract.

Skill Practice

14. Write each of the fractions $\frac{5}{8}$ and $\frac{5}{12}$ as an equivalent fraction with the LCD as its denominator.

15. Subtract. $\frac{5}{8} - \frac{5}{12}$

Answers

14. $\frac{5}{8} = \frac{15}{24}$ and $\frac{5}{12} = \frac{10}{24}$ **15.** $\frac{5}{24}$

Example 10 **Adding and Subtracting Fractions**

Simplify. $\frac{5}{12}+\frac{3}{4}-\frac{1}{2}$

Solution:

$$\frac{5}{12}+\frac{3}{4}-\frac{1}{2}$$

To find the LCD, we have: LCD $= 2\times2\times3 = 12$

	2's	3's
12 =	2×2	3
4 =	2×2	
2 =	2	

$$=\frac{5}{12}+\frac{3\times3}{4\times3}-\frac{1\times6}{2\times6}$$

Write each fraction as an equivalent fraction with the LCD as its denominator.

$$=\frac{5}{12}+\frac{9}{12}-\frac{6}{12}$$

$$=\frac{5+9-6}{12}$$

Add and subtract the numerators.

$$=\frac{8}{12}$$

Simplify to lowest terms.

$$=\frac{2}{3}$$

Skill Practice Add.

16. $\frac{2}{3}+\frac{1}{2}+\frac{5}{6}$

7. Operations on Mixed Numbers

Recall that a mixed number is a whole number added to a fraction. The number $3\frac{1}{2}$ represents the sum of three wholes plus a half, that is, $3\frac{1}{2} = 3+\frac{1}{2}$. For this reason, any mixed number can be converted to an improper fraction by using addition.

$$3\tfrac{1}{2} = 3+\frac{1}{2}=\frac{6}{2}+\frac{1}{2}=\frac{7}{2}$$

TIP: A shortcut to writing a mixed number as an improper fraction is to multiply the whole number by the denominator of the fraction. Then add this value to the numerator of the fraction, and write the result over the denominator.

$3\frac{1}{2}$ ⟶ Multiply the whole number by the denominator: $3\times2=6$
Add the numerator: $6+1=7$
Write the result over the denominator: $\frac{7}{2}$

To add, subtract, multiply, or divide mixed numbers, we will first write the mixed number as an improper fraction.

Answer
16. 2

Example 11 Operations on Mixed Numbers

Subtract. $5\frac{1}{3} - 2\frac{1}{4}$

Solution:

$$5\frac{1}{3} - 2\frac{1}{4}$$

$$= \frac{16}{3} - \frac{9}{4}$$ Write the mixed numbers as improper fractions.

$$= \frac{16 \times 4}{3 \times 4} - \frac{9 \times 3}{4 \times 3}$$ The LCD is 12. Multiply numerators and denominators by the missing factors from the denominators.

$$= \frac{64}{12} - \frac{27}{12}$$

$$= \frac{37}{12} \text{ or } 3\frac{1}{12}$$ Subtract the fractions.

Skill Practice Subtract.

17. $2\frac{3}{4} - 1\frac{1}{3}$

TIP: An improper fraction can also be written as a mixed number. Both answers are acceptable. Note that

$$\frac{37}{12} = \frac{36}{12} + \frac{1}{12} = 3 + \frac{1}{12}, \text{ or } 3\frac{1}{12}$$

This can easily be found by dividing.

$$\frac{37}{12} \longrightarrow 12\overline{)37}\ \begin{matrix} 3 \\ -36 \\ \hline 1 \end{matrix}$$

quotient → 3, remainder → 1, divisor → 12

$$3\frac{1}{12}$$

Example 12 Operations on Mixed Numbers

Divide. $7\frac{1}{2} \div 3$

Solution:

$$7\frac{1}{2} \div 3$$

$$= \frac{15}{2} \div \frac{3}{1}$$ Write the mixed number and whole number as fractions.

$$= \frac{\overset{5}{\cancel{15}}}{2} \times \frac{1}{\underset{1}{\cancel{3}}}$$ Multiply by the reciprocal of $\frac{3}{1}$, which is $\frac{1}{3}$.

$$= \frac{5}{2} \text{ or } 2\frac{1}{2}$$ The answer may be written as an improper fraction or as a mixed number.

Avoiding Mistakes

Remember that when dividing (or multiplying) fractions, a common denominator is not necessary.

Answer

17. $\frac{17}{12}$ or $1\frac{5}{12}$

Skill Practice Divide.

18. $5\frac{5}{6} \div 3\frac{2}{3}$

Answer

18. $\frac{35}{22}$ or $1\frac{13}{22}$

Section 1.1 Practice Exercises

Study Skills Exercise

To enhance your learning experience, we provide study skills that focus on eight areas: learning about your course, using your text, taking notes, doing homework, taking an exam (test and math anxiety), managing your time, recognizing your learning style, and studying for the final exam.

Each activity requires only a few minutes and will help you pass this course and become a better math student. Many of these skills can be carried over to other disciplines and help you become a model college student. To begin, write down the following information:

a. Instructor's name

b. Instructor's office number

c. Instructor's telephone number

d. Instructor's e-mail address

e. Instructor's office hours

f. Days of the week that the class meets

g. The room number in which the class meets

h. Is there a lab requirement for this course? How often must you attend lab and where is it located?

Vocabulary and Key Concepts

1. a. A _____ is the result of multiplying two or more numbers.

b. The numbers being multiplied in a product are called _____.

c. Given a fraction $\frac{a}{b}$ with $b \neq 0$, the value a is the _____ and _____ is the denominator.

d. A fraction is said to be in _____ terms if the numerator and denominator share no common factor other than 1.

e. The fraction $\frac{4}{4}$ can also be written as the whole number _____, and the fraction $\frac{4}{1}$ can be written as the whole number _____.

f. Two nonzero numbers $\frac{a}{b}$ and $\frac{b}{a}$ are _____ because their product is 1.

g. The least common multiple (LCM) of two numbers is the smallest whole number that is a _____ of both numbers.

h. The _____ common denominator of two or more fractions is the LCM of their denominators.

Concept 1: Basic Definitions

For Exercises 2–10, identify the numerator and denominator of each fraction. Then determine if the fraction is a proper fraction or an improper fraction.

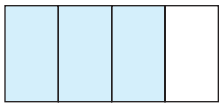
2. $\frac{7}{8}$ **3.** $\frac{2}{3}$ **4.** $\frac{9}{5}$

5. $\frac{5}{2}$ **6.** $\frac{6}{6}$ **7.** $\frac{4}{4}$

8. $\frac{12}{1}$ **9.** $\frac{5}{1}$ **10.** $\frac{6}{7}$

For Exercises 11–18, write a proper or improper fraction associated with the shaded region of each figure.

11.



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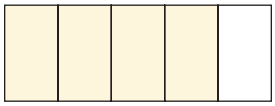

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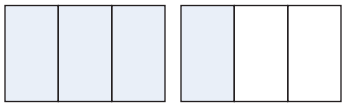

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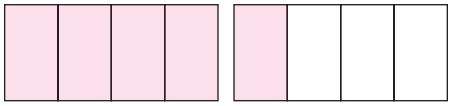

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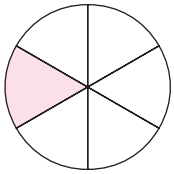

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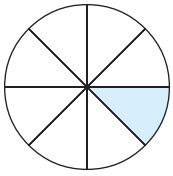

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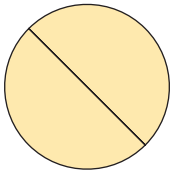

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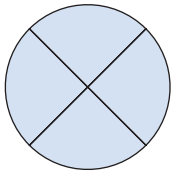
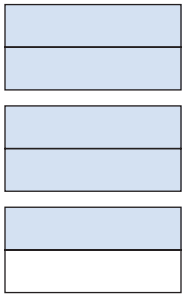


For Exercises 19–22, write both an improper fraction and a mixed number associated with the shaded region of each figure.

19.



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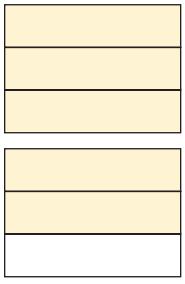

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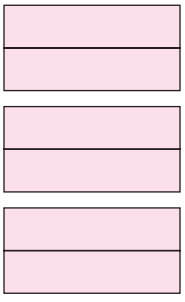


22.

23. Explain the difference between the set of whole numbers and the set of natural numbers.

24. Explain the difference between a proper fraction and an improper fraction.

25. Write a fraction that simplifies to $\frac{1}{2}$. (Answers may vary.)

26. Write a fraction that simplifies to $\frac{1}{3}$. (Answers may vary.)

Concept 2: Prime Factorization

For Exercises 27–34, identify each number as either a prime number or a composite number.

27. 5

28. 9

29. 4

30. 2

31. 39

32. 23

33. 53

34. 51

For Exercises 35–42, write each number as a product of prime factors. **(See Example 1.)**

35. 36

36. 70

37. 42

38. 35

39. 110

40. 136

41. 135

42. 105

Concept 3: Simplifying Fractions to Lowest Terms

For Exercises 43–54, simplify each fraction to lowest terms. **(See Examples 2–3.)**

43. $\frac{3}{15}$ **44.** $\frac{8}{12}$ **45.** $\frac{16}{6}$ **46.** $\frac{20}{12}$

47. $\frac{42}{48}$ **48.** $\frac{35}{80}$ **49.** $\frac{48}{64}$ **50.** $\frac{32}{48}$

51. $\frac{110}{176}$ **52.** $\frac{70}{120}$ **53.** $\frac{200}{150}$ **54.** $\frac{210}{119}$

Concepts 4–5: Multiplying and Dividing Fractions

For Exercises 55–56, determine if the statement is true or false. If it is false, rewrite as a true statement.

55. When multiplying or dividing fractions, it is necessary to have a common denominator.

56. When dividing two fractions, it is necessary to multiply the first fraction by the reciprocal of the second fraction.

For Exercises 57–68, multiply or divide as indicated. **(See Examples 4–6.)** Animation

57. $\frac{10}{13} \times \frac{26}{15}$ **58.** $\frac{15}{28} \times \frac{7}{9}$ **59.** $\frac{3}{7} \div \frac{9}{14}$ **60.** $\frac{7}{25} \div \frac{1}{5}$

61. $\frac{9}{10} \times 5$ **62.** $\frac{3}{7} \times 14$ **63.** $\frac{12}{5} \div 4$ **64.** $\frac{20}{6} \div 5$

65. $\frac{5}{2} \times \frac{10}{21} \times \frac{7}{5}$ **66.** $\frac{55}{9} \times \frac{18}{32} \times \frac{24}{11}$ **67.** $\frac{9}{100} \div \frac{13}{1000}$ **68.** $\frac{1000}{17} \div \frac{10}{3}$

69. Gus decides to save $\frac{1}{3}$ of his pay each month. If his monthly pay is \$2112, how much will he save each month?

70. Stephen's take-home pay is \$4200 a month. If he budgeted $\frac{1}{4}$ of his pay for rent, how much is his rent?

71. In Professor Foley's Beginning Algebra class, $\frac{5}{6}$ of the students passed the first test. If there are 42 students in the class, how many passed the test?

72. Shontell had only enough paper to print out $\frac{3}{5}$ of her book report before school. If the report is 10 pages long, how many pages did she print out?

73. Marty will reinforce a concrete walkway by cutting a steel rod (called rebar) that is 4 yd long. How many pieces can he cut if each piece must be $\frac{1}{2}$ yd in length?

74. There are 4 cups of oatmeal in a box. If each serving is $\frac{1}{3}$ of a cup, how many servings are contained in the box?

75. Anita buys 6 lb of mixed nuts to be divided into decorative jars that will each hold $\frac{3}{4}$ lb of nuts. How many jars will she be able to fill?

76. Beth has a $\frac{7}{8}$-in. nail that she must hammer into a board. Each strike of the hammer moves the nail $\frac{1}{16}$ in. into the board. How many strikes of the hammer must she make to drive the nail completely into the board?

Concept 6: Adding and Subtracting Fractions

For Exercises 77–80, add or subtract as indicated. **(See Example 7.)**

77. $\frac{5}{14} + \frac{1}{14}$ **78.** $\frac{9}{5} + \frac{1}{5}$ **79.** $\frac{17}{24} - \frac{5}{24}$ **80.** $\frac{11}{18} - \frac{5}{18}$

For Exercises 81–84, find the least common multiple for each list of numbers. **(See Example 8.)**

81. 6, 15 **82.** 12, 30 **83.** 20, 8, 4 **84.** 24, 40, 30

For Exercises 85–100, add or subtract as indicated. **(See Examples 9–10.)**

85. $\frac{1}{8}+\frac{3}{4}$ **86.** $\frac{3}{16}+\frac{1}{2}$ **87.** $\frac{11}{8}-\frac{3}{10}$ **88.** $\frac{12}{35}-\frac{1}{10}$

89. $\frac{7}{26}-\frac{2}{13}$ **90.** $\frac{25}{24}-\frac{5}{16}$ **91.** $\frac{7}{18}+\frac{5}{12}$ **92.** $\frac{3}{16}+\frac{9}{20}$

93. $\frac{5}{4}-\frac{1}{20}$ **94.** $\frac{7}{6}-\frac{1}{24}$ **95.** $\frac{5}{12}+\frac{5}{16}$ **96.** $\frac{3}{25}+\frac{8}{35}$

97. $\frac{1}{6}+\frac{3}{4}-\frac{5}{8}$ **98.** $\frac{1}{2}+\frac{2}{3}-\frac{5}{12}$ **99.** $\frac{4}{7}+\frac{1}{2}+\frac{3}{4}$ **100.** $\frac{9}{10}+\frac{4}{5}+\frac{3}{4}$

101. For his famous brownie recipe, Chef Alfonso combines $\frac{2}{3}$ cup granulated sugar with $\frac{1}{4}$ cup brown sugar. What is the total amount of sugar in his recipe?

102. Chef Alfonso eats too many of his brownies and his waistline increased by $\frac{3}{4}$ in. during one month and $\frac{3}{8}$ in. the next month. What was his total increase for the 2-month period?

103. Currently the most popular smartphone has a thickness of $\frac{9}{25}$ in. The second most popular is $\frac{1}{2}$ in. thick. How much thicker is the second most popular smartphone?

104. The diameter of a penny is $\frac{3}{4}$ in. while the dime is $\frac{7}{10}$ in. How much larger is the penny than the dime?

Concept 7: Operations on Mixed Numbers

For Exercises 105–118, perform the indicated operations. **(See Examples 11–12.)**

105. $3\frac{1}{5}\times 2\frac{7}{8}$ **106.** $2\frac{1}{2}\times 1\frac{4}{5}$

107. $1\frac{2}{9}\div 7\frac{1}{3}$ **108.** $2\frac{2}{5}\div 1\frac{2}{7}$

109. $1\frac{2}{9}\div 6$ **110.** $2\frac{2}{5}\div 2$

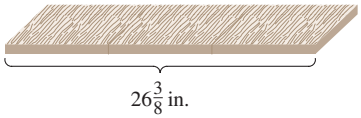
111. $2\frac{1}{8}+1\frac{3}{8}$ **112.** $1\frac{3}{14}+1\frac{1}{14}$

113. $3\frac{1}{2}-1\frac{7}{8}$ **114.** $5\frac{1}{3}-2\frac{3}{4}$

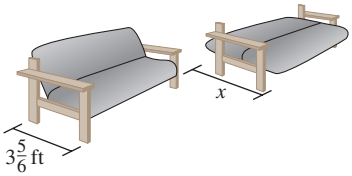
115. $1\frac{1}{6}+3\frac{3}{4}$ **116.** $4\frac{1}{2}+2\frac{2}{3}$

117. $1-\frac{7}{8}$ **118.** $2-\frac{3}{7}$

119. A board $26\frac{3}{8}$ in. long must be cut into three pieces of equal length. Find the length of each piece.



120. A futon, when set up as a sofa, measures $3\frac{5}{6}$ ft wide. When it is opened to be used as a bed, the width is increased by $1\frac{3}{4}$ ft. What is the total width of this bed?



121. A plane trip from Orlando to Detroit takes $2\frac{3}{4}$ hr. If the plane traveled for $1\frac{1}{6}$ hr, how much time remains for the flight?

122. Silvia manages a sub shop and needs to prepare smoked turkey sandwiches. She has $3\frac{3}{4}$ lb of turkey in the cooler, and each sandwich requires $\frac{3}{8}$ lb of turkey. How many sandwiches can she make?

123. José's catering company plans to prepare two different shrimp dishes for an upcoming event. One dish requires $1\frac{1}{2}$ lb of shrimp and the other requires $\frac{3}{4}$ lb of shrimp. How much shrimp should José order for the two dishes?

124. Ayako took a trip to the store $5\frac{1}{2}$ mi away. If she rode the bus for $4\frac{5}{6}$ mi and walked the rest of the way, how far did she have to walk?

125. If Tampa, Florida, averages $6\frac{1}{4}$ in. of rain during each summer month, how much total rain would be expected in June, July, August, and September?

126. Pete started working out and found that he lost approximately $\frac{3}{4}$ in. off his waistline every month. How much would he lose around his waist in 6 months?

Introduction to Algebra and the Set of Real Numbers — Section 1.2

1. Variables and Expressions

Concepts

Doctors promote daily exercise as part of a healthy lifestyle. Aerobic exercise is exercise for the heart. During aerobic exercise, the goal is to maintain a heart rate level between 65% and 85% of an individual's maximum recommended heart rate. The maximum recommended heart rate, in beats per minute, for an adult of age a is given by:

$$\text{Maximum recommended heart rate} = 220 - a$$

In this example, value a is called a **variable**. This is a symbol or letter, such as x, y, z, a, and the like, that is used to represent an unknown number that is subject to change. The number 220 is called a **constant**, because it does not vary. The quantity $220 - a$ is called an algebraic expression. An algebraic **expression** is a collection of variables and constants under algebraic operations. For example, $\frac{3}{x}$, $y + 7$, and $t - 1.4$ are algebraic expressions.

The symbols used in algebraic expressions to show the four basic operations are shown here:

Addition $a + b$

Subtraction $a - b$

Multiplication $a \times b$, $a \cdot b$, $(a)b$, $a(b)$, $(a)(b)$, ab
(*Note:* We rarely use the notation $a \times b$ because the symbol $\times$ may be confused with the variable x.)

Division $a \div b$, $\frac{a}{b}$, a/b, $b\overline{)a}$

The value of an algebraic expression depends on the values of the variables within the expression.

Example 1 Evaluating an Algebraic Expression

The expression $220 - a$ represents the maximum recommended heart rate for an adult of age a. Determine the maximum heart rate for:

a. A 20-year-old **b.** A 45-year-old

Solution:

a. In the expression $220 - a$, the variable, a, represents the age of the individual. To calculate the maximum recommended heart rate for a 20-year-old, we substitute 20 for a in the expression.

$220 - a$

$220 - (\quad)$	When substituting a number for a variable, use parentheses.
$= 220 - (20)$	Substitute $a = 20$.
$= 200$	Subtract.

The maximum recommended heart rate for a 20-year-old is 200 beats per minute.

b. $220 - a$

$220 - (\quad)$	When substituting a number for a variable, use parentheses.
$= 220 - (45)$	Substitute $a = 45$.
$= 175$	Subtract.

The maximum recommended heart rate for a 45-year-old is 175 beats per minute.

Skill Practice

1. After dining out at a restaurant, the recommended minimum amount for tipping the server is 15% of the cost of the meal. This can be represented by the expression $0.15c$, where c is the cost of the meal. Compute the tip for a meal that costs:

a. \$18 **b.** \$46

Example 2 Evaluating Algebraic Expressions

Evaluate the algebraic expression when $p = 4$ and $q = \frac{3}{4}$.

a. $100 - p$ **b.** pq

Solution:

a. $100 - p$

$100 - (\quad)$	When substituting a number for a variable, use parentheses.
$= 100 - (4)$	Substitute $p = 4$ in the parentheses.
$= 96$	Subtract.

Answers

1. a. \$2.70 **b.** \$6.90

b. pq

$= (\quad)(\quad)$ When substituting a number for a variable, use parentheses.

$= (4)\left(\frac{3}{4}\right)$ Substitute $p = 4$ and $q = \frac{3}{4}$.

$= \frac{\cancel{4}^{1}}{1} \cdot \frac{3}{\cancel{4}_{1}}$ Write the whole number as a fraction.

$= \frac{3}{1}$ Multiply fractions.

$= 3$ Simplify.

Skill Practice Evaluate the algebraic expressions when $x = 5$ and $y = 2$.

2. $20 - y$ **3.** xy

2. The Set of Real Numbers

Typically, the numbers represented by variables in an algebraic expression are all part of the set of **real numbers**. These are the numbers that we work with on a day-to-day basis. The real numbers encompass zero, all positive, and all negative numbers, including those represented by fractions and decimal numbers. The set of real numbers can be represented graphically on a horizontal number line with a point labeled as 0. Positive real numbers are graphed to the right of 0, and negative real numbers are graphed to the left of 0. Zero is neither positive nor negative. Each point on the number line corresponds to exactly one real number. For this reason, this number line is called the *real number line* (Figure 1-5).

Figure 1-5

Example 3 Plotting Points on the Real Number Line

Plot the numbers on the real number line.

a. -3 **b.** $\frac{3}{2}$ **c.** -4.7 **d.** $\frac{16}{5}$

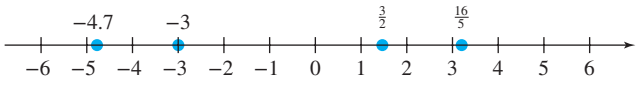
Solution:

a. Because -3 is negative, it lies three units to the left of 0.

b. The fraction $\frac{3}{2}$ can be expressed as the mixed number $1\frac{1}{2}$ which lies halfway between 1 and 2 on the number line.

c. The negative number -4.7 lies $\frac{7}{10}$ unit to the left of -4 on the number line.

d. The fraction $\frac{16}{5}$ can be expressed as the mixed number $3\frac{1}{5}$, which lies $\frac{1}{5}$ unit to the right of 3 on the number line.

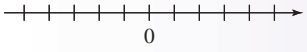


Answers

2. 18 **3.** 10

Skill Practice Plot the numbers on the real number line.

4. $\{-1, \frac{3}{4}, -2.5, \frac{10}{3}\}$



TIP: The natural numbers are used for counting. For this reason, they are sometimes called the "counting numbers."

In mathematics, a well-defined collection of elements is called a **set**. "Well-defined" means the set is described in such a way that it is clear whether an element is in the set. The symbols { } are used to enclose the elements of the set. For example, the set {A, B, C, D, E} represents the set of the first five letters of the alphabet.

Several sets of numbers are used extensively in algebra and are *subsets* (or part) of the set of real numbers.

Natural Numbers, Whole Numbers, and Integers

The set of **natural numbers** is $\{1, 2, 3, \ldots\}$

The set of **whole numbers** is $\{0, 1, 2, 3, \ldots\}$

The set of **integers** is $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Notice that the set of whole numbers includes the natural numbers. Therefore, every natural number is also a whole number. The set of integers includes the set of whole numbers. Therefore, every whole number is also an integer.

Fractions are also among the numbers we use frequently. A number that can be written as a fraction whose numerator is an integer and whose denominator is a nonzero integer is called a *rational number*.

Rational Numbers

The set of **rational numbers** is the set of numbers that can be expressed in the form $\frac{p}{q}$, where both p and q are integers and q does not equal 0.

We also say that a rational number $\frac{p}{q}$ is a *ratio* of two integers, p and q, where q is not equal to zero.

Example 4 **Identifying Rational Numbers**

Show that the following numbers are rational numbers by finding an equivalent ratio of two integers.

a. $\frac{-2}{3}$ **b.** -12 **c.** 0.5 **d.** $0.\overline{6}$

Solution:

a. The fraction $\frac{-2}{3}$ is a rational number because it can be expressed as the ratio of -2 and 3.

b. The number -12 is a rational number because it can be expressed as the ratio of -12 and 1, that is, $-12 = \frac{-12}{1}$. In this example, we see that an integer is also a rational number.

Answer

4.