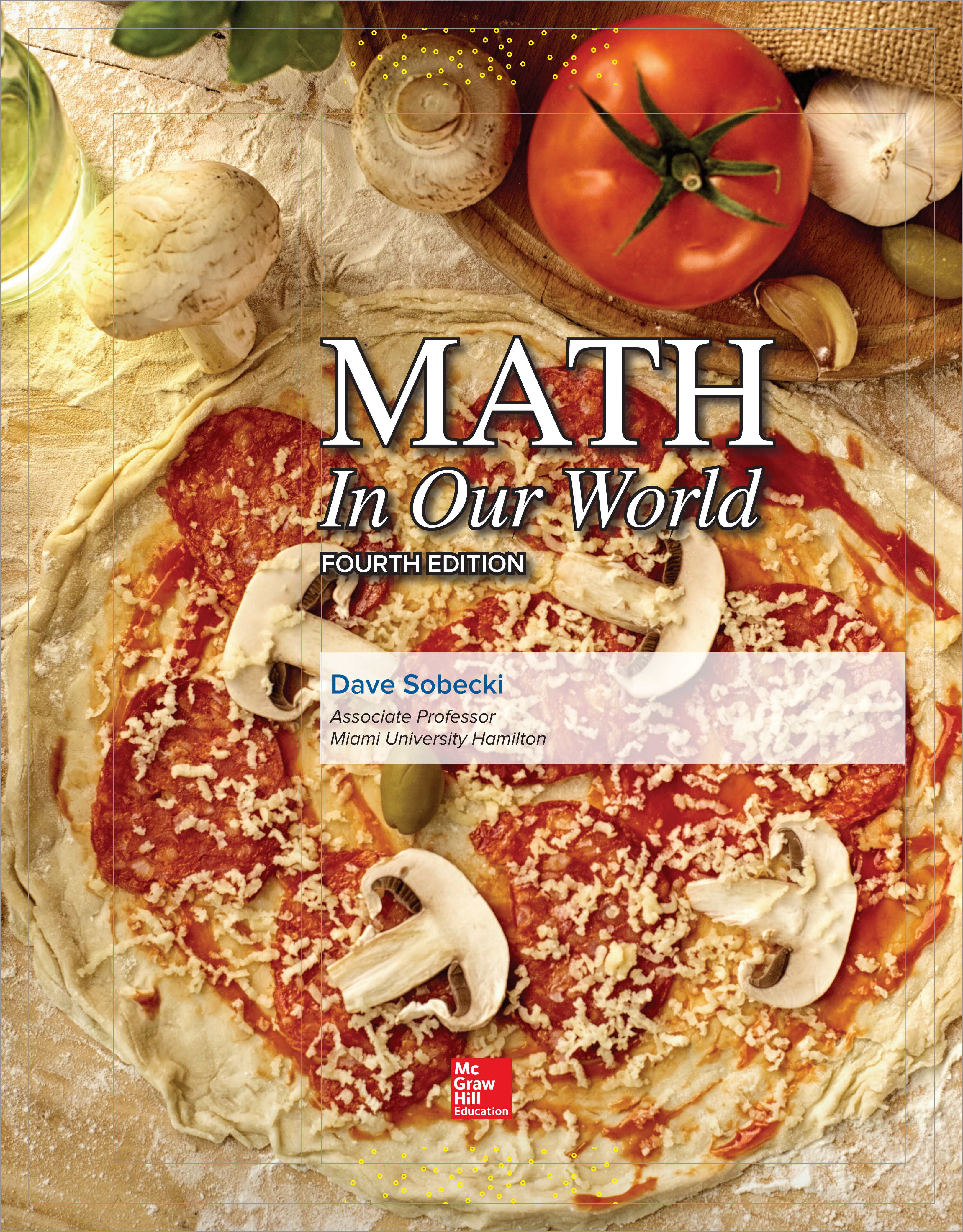
MATH IN OUR WORLD

Fourth Edition

Mc Graw Hill Education

Dave SOBECKI



MATH
In Our World

FOURTH EDITION

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McGraw Hill Education

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MATH IN OUR WORLD, FOURTH EDITION

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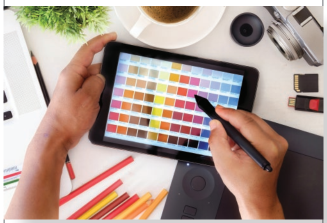
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About the Author

Courtesy of David Sobecki

Dave Sobecki

I was born and raised in Cleveland, and started college at Bowling Green State University majoring in creative writing. Eleven years later, I walked across the graduation stage to receive a PhD in math, a strange journey indeed. After two years at Franklin and Marshall College in Pennsylvania, I came home to Ohio, accepting a tenure-track job at the Hamilton campus of Miami University. I've won a number of teaching awards in my career, and while maintaining an active teaching schedule, I now spend an inordinate amount of time writing textbooks and course materials. I've written or co-authored either nine or sixteen textbooks, depending on how you count them, as well as a wide variety of solutions manuals and interactive CD-ROMS. I've also worked on an awful lot of the digital content that accompanies my texts, including Connect, LearnSmart, and Instructional videos.

I'm in a very happy place right now: my love of teaching meshes perfectly with my childhood dream of writing. (Don't tell my publisher this—they think I spend 20 hours a day working on textbooks—but I'm working on my first novel in the limited spare time that I have.) I'm also a former coordinator of Ohio Project NExT, because I believe very strongly in helping young college instructors focus on high-quality teaching as a primary career goal. I live in Fairfield, Ohio, with my lovely wife Cat and fuzzy dogs Macleod and Tessa. I'm a recovering sports fan, still rooting for Ohio State and the Cleveland teams in a saner manner. Other passions include heavy metal music, travel, golf, collecting fine art, and visiting local breweries.

This edition of MIOW is dedicated to Katie Stevens and Trina Zimmerman, whose early encouragement got me to thinking I just might be able to make something of this author gig.

Letter from the Author

This is the story behind *Math in Our World.* Liberal Arts Math is different from the other classes we typically teach to underclassmen, and I think that it requires a different approach. Many of the students have had negative experiences in algebra, and come into any math course thinking it's going to be the same old thing again—finding *x*. Liberal Arts Math provides a great opportunity to show students that math isn't just an abstract subject studied by high-level intellectuals. In this course, we have the opportunity to really teach students about reasoning and thinking, rather than train them to mimic procedures. Who wouldn't look forward to that?

Math in Our World has a different style than you'll find in most college math texts. Both the structure of the chapters and the style of writing are designed to make the students think "Wow, this isn't what I expected . . . I can actually read and understand this!" I like to call it "teaching backwards": rather than first learning the math and then studying how it can be applied, every topic is introduced from a conceptual, applied point of view. The goal is to engage students at the beginning of each topic, helping them to not fall into the old "Why do I have to know this—I'm never going to use it" trap. I've been passionate about writing as far back as I can remember, and I switched my major from creative writing to math education when I got a first taste as a tutor in college. I was captivated then by the "Aha!" moment when struggling students GET IT—and I still feel that way today. This book is special to me because it really allowed my two passions to come together into a unique product.

No one has ever become stronger by watching someone else lift weights, and our students aren't going to be any better at thinking and problem solving unless we encourage them to practice it. *Math in Our World* includes a veritable cornucopia of applications for students to hone their skills. The exercises for this edition were carefully evaluated to ensure that they are engaging for students and apply to fields of study that are common for Liberal Arts Math students. Additionally, I've tried to incorporate more key ideas from the growing quantitative reasoning movement. You'll see more open-ended questions, more discovery learning, and more critical thinking. The goal is to help students develop into problem solvers and thinkers beyond the halls of academia. This new focus is reflected in the new cover. Pizza has become our trademark, but you'll notice that the pizza is now a work in progress. To me, the essence of a QR approach is encouraging students to use the ingredients they have at hand (their background skills) to learn how to produce something great through effort, rather than expecting someone to do the preparation for them. And even when the course is over, their pizza isn't finished: learning to think quantitatively becomes an ongoing pursuit that will help in many other courses, and in life in general.

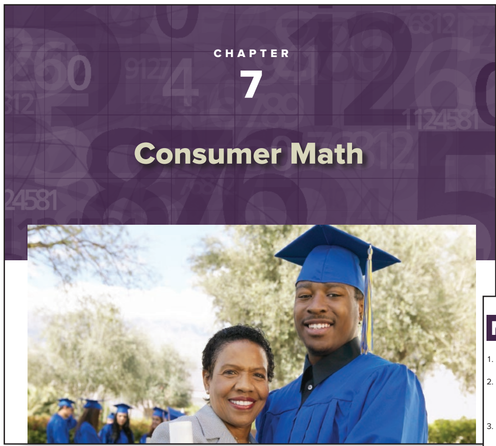
While no book can prevent the lack of preparedness of students, we believe the digital component of the *Math in Our World* program (Connect and SmartBook) will help engage students and encourage them to develop their own questions about the world. Both programs can play an important role in your class by providing opportunities to practice and master the computational as well as conceptual aspects of this course. In addition, we've added a corequisite guide to our supplements, recognizing that many schools are working on creative ways to help students fulfill their math requirements expediently.

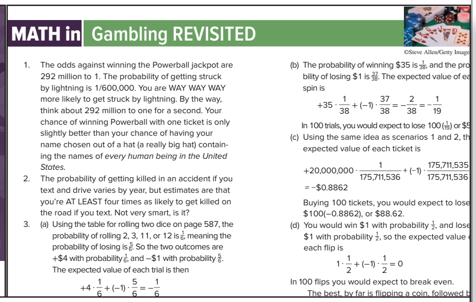
I'm confident that this book offers a fantastic vehicle to drive your classes to higher pass rates because of the pedagogical elements, writing style, interesting problem sets, and digital components. I hope you and your students enjoy using *Math in Our World* as much as our team enjoyed creating this program together. Good luck, and please don't hesitate to reach out with comments, questions or suggestions. I love hearing from instructors and students at davesobecki@gmail.com.

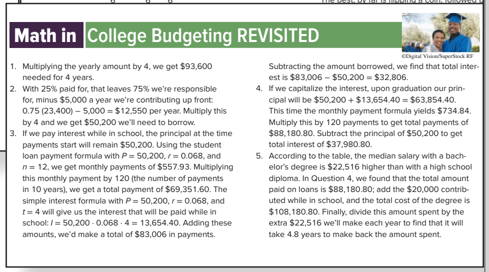
—Dave

Highly relevant **Application Exercises and Examples** drawn from the experiences and research of the author further emphasize the importance that *Math in Our World* places upon students' ability to form a distinct connection with the mathematical content. The new edition brings many brand new and updated application exercises to students in each chapter, ranging in topic from credit card usage, college degree majors, elections, and relevant business decisions to scenarios involving popular statistics.

- **Chapter Openers** engage student interest by immediately tying mathematical concepts to their everyday lives. These vignettes introduce concepts by referencing popular topics familiar to a wide variety of students—travel, demographics, the economy, television, and even college football.
- Used to clarify concepts and emphasize particularly important points, **Math Notes** provide suggestions for students to keep in mind as they progress through the chapter.
- **Sidelights** highlight relevant interdisciplinary connections within math to encourage and motivate students who have a wide variety of interests. These include biographic vignettes as well as other interesting facts that emphasize the importance of math in areas like weather, photography, music, and health.

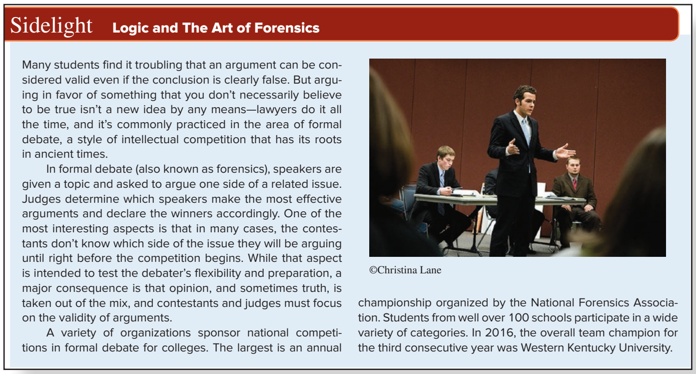






Math Note

When finding percent increase or decrease, make sure you divide by the *original* amount, not the ending amount.



Practice

Carefully chosen questions help students to form a connection between relevant examples and the mathematical concepts of the chapter. Using the engaging writing style characteristic of the text, the author supports concepts through abundant examples, helpful practice problems, and rich exercise sets.

- **Worked Example Problems** with detailed solutions help students master and in some cases discover key concepts. Solutions demonstrate a logical, orderly approach to solving problems. Each example is titled to help you clearly identify the relevant learning objective.

- One of our hallmark features, **Try This One** practice exercises provide immediate reinforcement for students. Designed to follow each example, these practice exercises ask students to solve a similar problem, actively involving them in the learning process. All answers to Try This One exercises can be found just prior to the end-of-section exercise sets for students to complete the problem-solving process by confirming their solutions.

Example 3 Computing Simple Interest for a Term in Months

To meet payroll during a down period, United Ceramics Inc. needed to borrow $2,000 at 4% simple interest for 3 months. Find the interest.

SOLUTION

Change 3 months to years by dividing by 12, and change the rate to a decimal. Substitute in the formula $I = Prt$.

$$I = (\$2{,}000)(0.04)\left(\frac{3}{12}\right) \qquad 4\% = 0.04$$

$$= \$20$$

The interest is $20.

1. Compute simple interest and future value.

Try This One 3

Marta needs some quick cash for books at the beginning of spring semester, so she borrows $600 at 11% simple interest for 2 months. How much interest will she pay?

- With an increased emphasis on using technology appropriately to allow more focus on interpretation and understanding, **Using Technology** boxes and **Calculator Guides** show students how to perform important calculations using spreadsheets and calculators (both graphing and scientific).

- The rich variety of problem material found in the **End-of-Section Exercise Sets** helps check student knowledge in a variety of different ways to cater to the varying interests and educational backgrounds of students. In each end-of-section exercise set, there are **Writing Exercises, Computational Exercises, Applications in Our World, and Critical Thinking Exercises.**

- Exercises and activities located in the **End-of-Chapter Material** provide opportunities for students to prepare for success on quizzes or tests. At the conclusion of each chapter, students find critical summary information that helps them pull together each concept learned while moving through the chapter. In each end-of-chapter segment, students and instructors will find an end-of-chapter summary, Review Exercises, and Chapter Test.

- Also featured are the **Chapter Projects,** which encourage more in-depth investigation for students working to summarize key concepts from the entire chapter. These projects are valuable assets for instructors looking for ways that students can work collaboratively.

Calculator Guide

Computing log base 10 of 1.25 with a calculator is really simple:

Standard Scientific Calculator:
1.25 LOG

Standard Graphing Calculator:
LOG 1.25 ENTER

- With an increased emphasis on using technology appropriately to allow more focus on interpretation and understanding, **Using Technology** boxes and **Calculator Guides** show students how to perform important calculations using spreadsheets and calculators (both graphing and scientific).

Using Technology

Using a Spreadsheet for Trial and Error

One of the things that spreadsheets do best is carry out repetitive calculations, which is a pretty good description of what we did in Example 3. Here's a screenshot of a spreadsheet I put together when writing that problem. I knew that the number of items had to add to 12,

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Looking for a consistent voice between text and digital? Problem solved!

McGraw-Hill Connect® Math Hosted by ALEKS® offers everything students and instructors need in one intuitive platform. ConnectMath is an online homework engine where the problems and solutions are consistent with the textbook authors' approach. It also offers integration with SmartBook, an assignable, adaptive eBook and study tool that directs students to the content they don't know and helps them study more efficiently. With ConnectMath, you get the tools you need to be the teacher you want to be.

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> "I like that ConnectMath reaches students with different learning styles . . . our students are engaged, attend class, and ask excellent questions."
> – *Kelly Davis, South Texas College*

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I like to learn by ___________.

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I still don't get it. Can you do that problem again?

Because the content in ConnectMath is author-developed and goes through a rigorous quality control process, students hear one voice, one style, and don't get lost moving from text to digital. The high-quality, author-developed videos provide students ample opportunities to master concepts and practice skills that they need extra help with . . . all of which are integrated in the ConnectMath platform and the eBook.

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From helping the student in the back row to tracking learning trends for your entire course, ConnectMath delivers the data you need to make an impactful, meaningful learning experience for students. With easy-to-interpret, downloadable reports, you can analyze learning rates for each assignment, monitor time on task, and learn where students' strengths and weaknesses are in each course area.

We're going with the _________ (flipped classroom, corequisite model, etc.) implementation.

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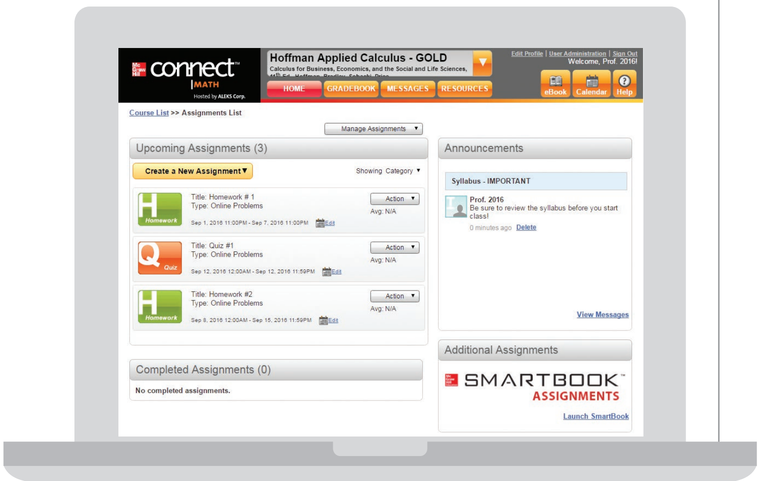

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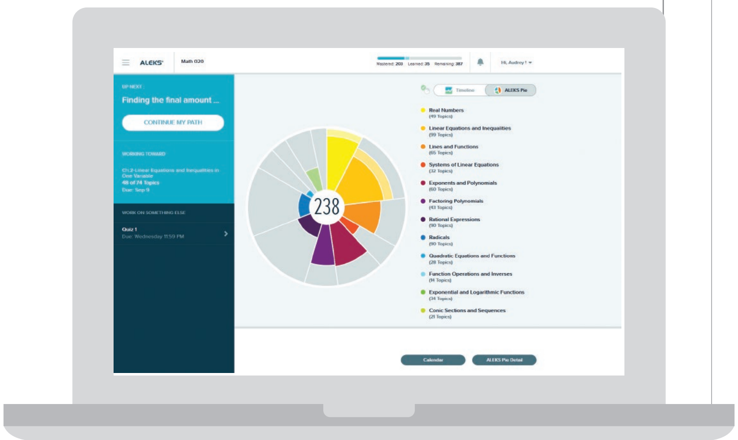
ALEKS® uses artificial intelligence to precisely map what each student knows, doesn't know, and is most ready to learn in a given course area. The system interacts with each student like a skilled human tutor, delivering a cycle of highly individualized learning and assessment that ensures mastery. Students are shown an optimal path to success, and instructors have the analytics they need to deliver a data-informed, impactful learning experience.



> "ALEKS has helped to create the best classroom experience I've had in 36 years. I can reach each student with ALEKS."
> – *Tommy Thompson, Cedar Valley College, TX*

How can ALEKS help solve your students' challenges?

I did all my homework, so why am I failing my exams?

The purpose of homework is to ensure mastery and prepare students for exams. ALEKS is the only adaptive learning system that ensures mastery through periodic assessments and delivers just-in-time remediation to efficiently prepare students. Because of how ALEKS presents lessons and practice, students learn by understanding the core principle of a concept rather than just memorizing a process.

I'm too far behind to catch up. - OR - I've already done this, I'm bored.

No two students are alike. So why start everyone on the same page? ALEKS diagnoses what each student knows and doesn't know, and prescribes an optimized learning path through your curriculum. Students only work on topics they are ready to learn, and they have a proven learning success rate of 93% or higher. As students watch their progress in the ALEKS Pie grow, their confidence grows with it.

How can ALEKS help solve your classroom challenges?

I need something that solves the problem of cost, time to completion, and student preparedness.

ALEKS is the perfect solution to these common problems. It provides an efficient path to mastery through its individualized cycle of learning and assessment. Students move through the course content more efficiently and are better prepared for subsequent courses. This saves both the institution and the student money. Increased student success means more students graduate.

My administration and department measure success differently. How can we compare notes?

ALEKS offers the most comprehensive and detailed data analytics on the market. From helping the student in the back row to monitoring pass rates across the department and institution, ALEKS delivers the data needed at all levels.

The customizable and intuitive reporting features allow you and your colleagues to easily gather, interpret, and share the data you need, when you need it.

The ALEKS Instructor Module offers a modern, intuitive interface to manage courses and track student progress.

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Supplements

McGraw-Hill Education Connect plus+ MATH

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- McGraw-Hill's Connect is a web-based assignment and assessment platform that helps students connect to their coursework and prepares them to succeed in and beyond the course.
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- Connect also links students to an interactive eBook with access to a variety of media assets and a place to study, highlight, and keep track of class notes.

To learn more, contact your sales rep or visit www.connectmath.com.

ALEKS®

New in ALEKS® are the **Liberal Arts Math** and **Quantitative Reasoning** course areas. These course products cover topic areas such as personal finance, probability, statistics, algebra, and symbolic logic. They emphasize "real-world" applications of concepts, while still providing a solid mathematical foundation.

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Instructor's Testing and Resource Online

This computerized test bank, available online to adopting instructors, utilizes TestGen® cross-platform test generation software to quickly and easily create customized exams. Using hundreds of test items taken directly from the text, TestGen allows rapid test creation and flexibility for instructors to create their own questions from scratch with ability to randomize number values. Powerful search and sort functions help quickly locate questions and arrange them in any order, and built-in mathematical templates let instructors insert stylized text, symbols, graphics, and equations directly into questions without need for a separate equation editor.

Student and Instructor Solution Manuals

provides comprehensive, worked-out solutions to all exercises in the text.

Lecture and Exercise Videos

introduce concepts, definitions, theorems, formulas, and problem-solving procedures to helps students comprehend topics.

Corequisite Workbook

provides corequisite remediation of the necessary skills for a liberal arts math course. The included topics are largely independent of one another and may be used in any order that works best for the instructor.

Updated Content

Global Changes

- As the quantitative reasoning movement has gained steam nationwide, the main goal of this revision was to emphasize the critical thinking and discovery learning typified by QR while maintaining the traditional format of a liberal arts math book. This was addressed in several ways. Many solved examples were replaced or rewritten to encourage more thinking and less computing, as were many Try This One questions. There's more emphasis on appropriate use of technology, both spreadsheets and calculators, freeing instructors and students to focus on interpretation and application. The critical thinking exercises have also been expanded. Finally, a new category of exercise was added in several sections: Using Technology. These are questions specifically designed for students to create spreadsheets to aid in solving interesting problems.
- I've always tried to base as many applications as possible on real, current data. That's a good thing. The downside is having to update or replace all of the problems with each revision. It's a lot of work, but well worth it, especially for information age students, many of whom feel like anything that happened before 2012 is ancient history.
- As always, I reevaluated every single passage in the book, looking for opportunities to improve readability and tone. I also tried to add a little more humor this time around—learning is supposed to be fun, right?
- Next is a list of some chapter-specific changes. Note that when listing the number of "improved" example questions, I'm not talking about simple rewording, or updating data. These are problems that were significantly improved, in most cases by adding a further critical thinking element.

Chapter 1 Problem Solving

- Chapter opener was updated with new questions.
- The section on estimation was revamped to put emphasis on when an estimate is an overestimate or underestimate, and when each would be desirable.
- Four totally new examples, and seven improved examples

Chapter 2 Sets Theory

- Sometimes example problems using abstract sets are the simplest way to introduce procedures and concepts, but I made an effort to minimize the number of abstract examples.
- Cartesian product was moved from a topic in lesson 2-2 to a series of discovery questions in the Critical Thinking exercises.
- Three totally new examples, and 10 improved examples

Chapter 3 Logic

- In general this chapter needs far less updating when shooting for more reasoning, as of course the entire chapter is all about logical reasoning. The most significant change is a new feature that illustrates a technique for simplifying complicated truth tables: students are encouraged to cover up all but the relevant columns at each stage with strips of paper, and this is graphically reflected in solved examples.
- Two totally new examples, and three improved examples

Chapter 4 Numeration Systems

- New chapter opener on computer graphics and base number systems
- When studying the history of numeration systems, more emphasis is put on comparing the efficiency of different types of systems.
- One totally new example, and three improved examples

Chapter 5 The Real Number System

- I feel like this chapter already had a pretty good emphasis on a deeper understanding of the real number system, so less updates were made than typical.
- Four totally new examples, and four improved examples

Chapter 6 Topics in Algebra

- This represents a major change in content and organization. The algebra review portion of *Math in Our World* used to span both Chapters 6 and 7. This has been streamlined into a single chapter. The algebraic topics that are most likely to be needed for other chapters in the book were left in the print version, with the remaining content shifted to an online algebra review supplement.
- One totally new example, and two improved examples

Chapter 7 Consumer Math

- The most obvious change is a completely new section on personal budgeting.
- The coverage of converting percentages between formats was streamlined.
- As financial calculations get more complicated, optional spreadsheet calculators have been created to allow students to save time on number crunching. These are available in instructor resources and can be shared with students if so desired.
- In the coverage of student loans, the difference between capitalized and non-capitalized interest has been clarified.
- Two new Sidelights have been added, one on retirement savings and another that studies how much of a mortgage payment goes directly to interest based on the age of the loan.
- The coverage of stock tables has been completely revamped to reflect the information that is most typically available in modern online stock listings.
- Coverage of stock splits has been added.
- Seven completely new examples, and seven improved examples

Chapter 8 Measurement

- Improved coverage of dimensional analysis and conversion factors
- Three completely new examples

Chapter 9 Geometry

- Chapter opener was updated with a recent use of geometry in my world.
- Substantial changes were made to coverage of perimeter, area, and volume, with more formulas developed in example problems, and less supplied. This chapter has some of the best examples of the new emphasis on reasoning and critical thinking (if I do say so myself).
- Seven totally new examples, and four improved examples

Chapter 10 Probability and Counting Techniques

- In previous editions, counting techniques were covered first, making it pretty hard to motivate the topic. The basic concepts of probability now come first, and counting techniques are introduced as tools for helping to study probability.
- Eight totally new examples, and 10 improved examples; this chapter has MANY examples of the new emphasis on reasoning.

Chapter 11 Statistics

- Chapter opener was expanded to include a third data set, allowing for further exploration of correlation.
- Coverage of bar graphs and pie charts refocused on when it's appropriate to use each type rather than just creating or reading graphs.
- Expanded and improved coverage of stem and leaf plots
- The section on measures of average has perhaps the best examples of the focus on reasoning. Far more attention is given to when particular measures are most appropriate, and when measures of average can be deceiving.
- More attention is given to contrasting descriptive and inferential statistics.
- A fun new sidelight on spurious correlations was added.
- Twelve completely new examples, and 11 improved examples

Chapter 12 Voting Methods

- More focus on comparing results using different voting methods
- More focus on understanding what the standard divisor and standard quota really are, allowing for an easier understanding of apportionment methods
- New sidelight on Alexander Hamilton added.
- Two totally new examples, and seven improved examples

Chapter 13 Graph Theory

- One totally new example, and eight improved examples

Chapter 14 Other Mathematical Systems

- The study of other systems is so based on mental flexibility and thinking outside the base 10 box that I didn't feel many changes were useful.
- One totally new example

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At McGraw-Hill Higher Education, thanks to my team: Brand managers Adam Rooke (miss you buddy) and Chrissy Skogen; Product Developer Nora Devlin; marketing guru Noah Evans; Project Manager Peggy Selle; Designer Tara McDermott; Director of Digital Content Cynthia Northrup; and Annie Clarke, who helps me stay on top of a hectic travel schedule. Finally, thanks to the wonderful national sales and marketing force. It's a pleasure to work with you, my friends.

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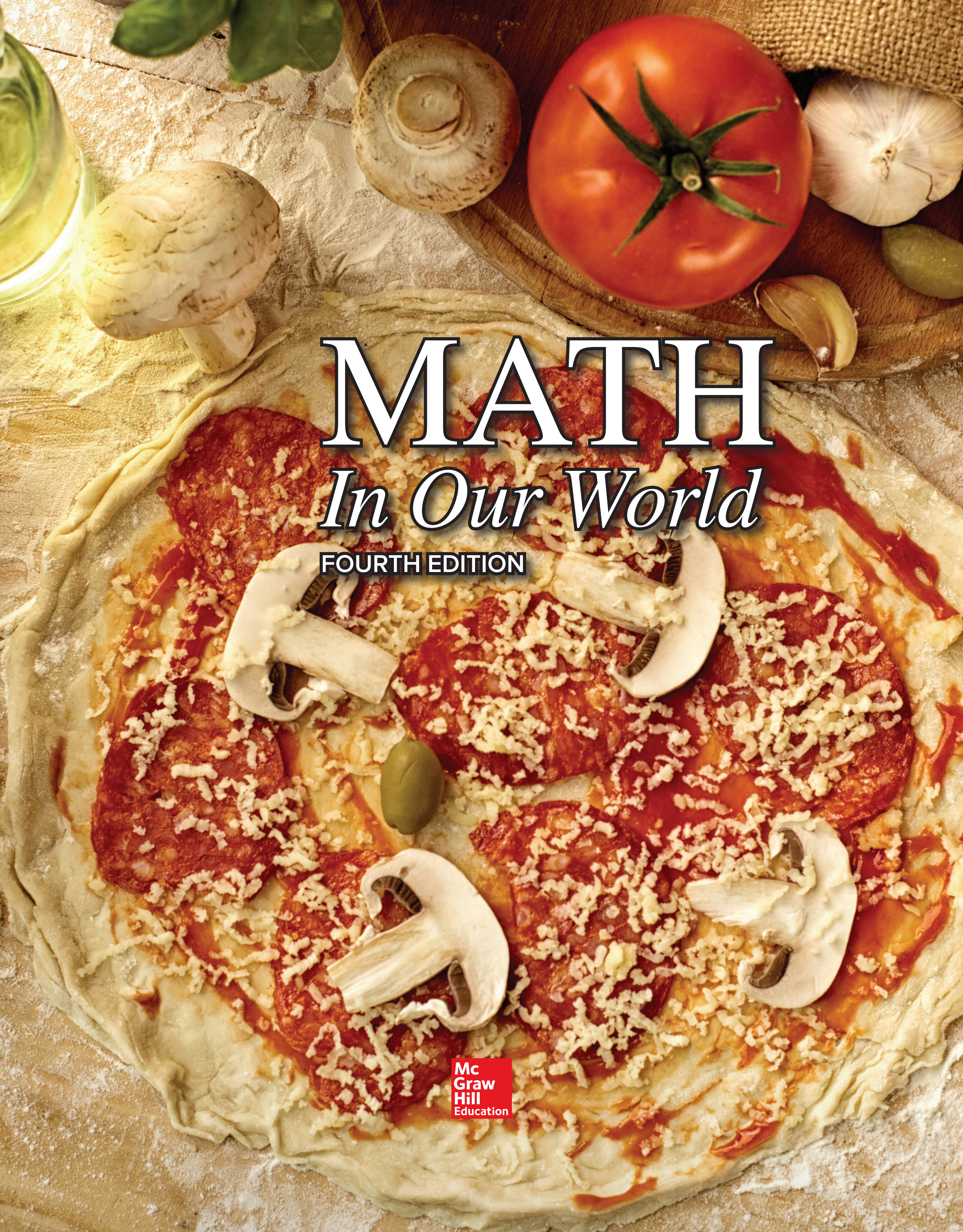

MATH
In Our World

FOURTH EDITION

McGraw Hill Education

CHAPTER

1

Problem Solving

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Outline

Math in Criminal Justice

In traditional cops-and-robbers movies, crime fighters use guns and fists to catch criminals, but in real life, often it's brain power that brings the bad guys to justice. *Law & Order: SVU* is one of the longest-running investigative shows ever. While the folks who make that show are not above the occasional foot chase, more often crimes are solved and prosecuted by gathering and organizing as much relevant information as possible, then using logic and intuition to formulate a plan. If all goes well, this leads investigators to a suspect. At that point, it's up to the prosecution to lead jurors to use reasoning skills to get to the truth.

This same strategy is the essence of problem solving in many walks of life other than criminal justice. Students in math classes often ask, "When am I going to use what I learn?" The best answer to that question is, "Every day!" Math classes are not only about facts and formulas: they're also about exercising your mind, training your brain to think logically, and learning effective strategies for solving problems. And not just math problems. Every day of our lives, we face a wide variety of problems: they pop up in our jobs, in school, and in our personal lives. Which computer should you buy? What should you do when your car starts making an awful noise? What would be a good topic for a research paper? How can you get all your work done in time to go to that party Friday night?

Chapter 1 of this book is dedicated to the most important topic we'll cover: an introduction to some of the classic techniques of problem solving. These techniques will prove to be useful tools that you can apply throughout the rest of this course, and in the rest of your education. But more importantly, they can be applied just as well to situations outside the classroom.

And this brings us back to our friends from *SVU*. The logic and reasoning that they use to identify suspects and prove their guilt are largely based on problem-solving skills we'll study in this chapter. By the time you've finished the chapter, you should be able to evaluate the situations below, all based on episodes from Season 17 of *SVU*. In each case, you should identify the type of reasoning, inductive or deductive, that was used. Then discuss whether you think this evidence would be likely to lead to a conviction.

1. A corrections officer is accused of sexually assaulting several inmates. Many of the victims have been blackmailed into keeping quiet about the attacks. Video footage shows the officer escorting several of the alleged victims into a closed room, and his partner appearing to stand guard outside. (Episode 22: "Intersecting Lives")
2. When DNA evidence found on a murder suspect leads to a familial match, detectives are forced to try and obtain DNA samples from several members of the family to identify the killer. (Episode 8: "Melancholy Pursuit")
3. A contestant on a reality dating show claims to have been raped by another contestant. Because so much of their activities were being filmed, detectives were able to find video showing that the accused was in a different place at the time of the attack. (Episode 21: "Assaulting Reality")
4. A series of attacks occur in an area near a shelter, and the attacks match the specifics of similar crimes committed by an ex-con living in the shelter, so one of the SVU detectives goes undercover at the shelter. (Episode 19: "Sheltered Outcasts")

For answers, see Math in Criminal Justice Revisited on page 38

Section 1-1 The Nature of Mathematical Reasoning

LEARNING OBJECTIVES

1. Identify two types of reasoning.
2. Use inductive reasoning to make conjectures.
3. Find a counterexample to disprove a conjecture.
4. Explain the difference between inductive and deductive reasoning.
5. Use deductive reasoning to prove a conjecture.

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A big part of being an adult is making decisions on your own—every day is full of them, from the simple, like what to eat for breakfast, to the critical, like a choice of major, career, job, or mate. How far do you think you'll get in life if you flip a coin to make every decision? I'm going to say, "Not very." Instead, it's important to be able to analyze a situation based on logical thinking. What are the possible outcomes of making that decision? How likely is it that each choice will have positive or negative consequences? We call the process of logical thinking **reasoning**. It doesn't take a lot of imagination to understand how important reasoning is in everyone's life.

You may not realize it, but every day in your life, you use two types of reasoning to make decisions and solve problems: *inductive reasoning* or *induction*, and *deductive reasoning* or *deduction*.

> **Inductive reasoning** is the process of reasoning that arrives at a general conclusion based on the observation of specific examples.

For example, suppose that your instructor gives a surprise quiz every Friday for the first four weeks of your math class. At this point, you might make a **conjecture**, or educated guess, that you'll have a surprise quiz the next Friday as well. As a result, you'd probably study before that class.

1. Identify two types of reasoning.

This is an example of inductive reasoning. By observing certain events for four *specific* Fridays, you arrive at a general conclusion. Inductive reasoning is useful in everyday life, and it is also useful as a problem-solving tool in math, as shown in Example 1.

Example 1 Using Inductive Reasoning to Find a Pattern

A game show contestant is given the following string of numbers:

$$1, 2, 4, 5, 7, 8, 10, \_\_\_, \_\_\_, \_\_\_$$

She'll win $1,500 if she can continue the pattern and fill in the three blanks. Use inductive reasoning to find a correct answer. How confident are you in your answer? 100%? Or less than that?

SOLUTION

To find patterns in strings of numbers, it's often helpful to think about operations that can turn a number into the next one. In this case, we can use addition to find a regular pattern:

$$1 \underset{+1}{\smile} 2 \underset{+2}{\smile} 4 \underset{+1}{\smile} 5 \underset{+2}{\smile} 7 \underset{+1}{\smile} 8 \underset{+2}{\smile} 10 \underset{+1}{\smile} \_\_ \underset{+2}{\smile} \_\_ \underset{+1}{\smile} \_\_$$

The pattern seems to be to add 1, then add 2, then add 1, then add 2, etc. So a reasonable conjecture for the next three numbers is 11, 13, and 14.

Now let's talk about confidence. I like this answer a lot, but I'm not 100% confident. I made an educated guess based on just seven numbers. There's no guarantee that this is the ONLY correct answer.

Try This One 1

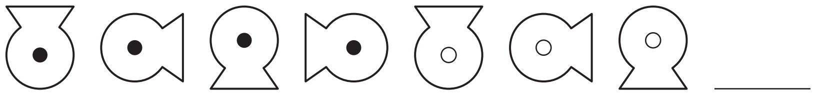
Use inductive reasoning to find a pattern and make a reasonable conjecture for the next three numbers by using that pattern.

1, 4, 2, 5, 3, 6, 4, 7, 5, ___, ___, ___

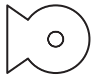
Example 2

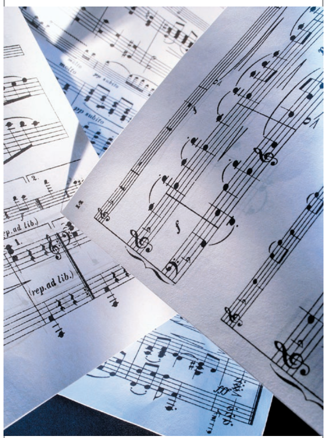
Using Inductive Reasoning to Find a Pattern

Make a reasonable conjecture for the next figure in the sequence.



SOLUTION

In the first four figures, the flat part goes from facing up to right, down, then left. There's also a solid circle • in each figure. The sequence then repeats with an open circle ∘ in each figure, so in the next one, the flat part should face left and have an open circle: 



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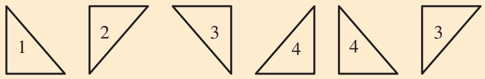
Recognizing, describing, and creating patterns are important in many fields. Many types of patterns are used in music like following an established pattern, altering an established pattern, and producing variations on a familiar pattern.

Try This One 2

Make a reasonable conjecture for the next figure in the sequence.



Example 3

Using Inductive Reasoning to Make a Conjecture

a. When two odd numbers are added, will the result always be an even number? Use inductive reasoning to determine your answer.
b. How many pairs of numbers would you need to try in order to be CERTAIN that your conjecture is true?

SOLUTION

a. First, let's try several specific examples of adding two odd numbers:

$$3 + 7 = 10 \qquad 25 + 5 = 30$$
$$5 + 9 = 14 \qquad 1 + 27 = 28$$
$$19 + 9 = 28 \qquad 21 + 33 = 54$$

Since all the answers are even, it seems reasonable to conclude that the sum of two odd numbers will be an even number.

b. Did we really slip in a trick question this early in the book? You bet. This is a very important point about inductive reasoning: you can try specific examples all day and always get an even sum, but that can never guarantee that it will ALWAYS happen. For that, we're going to need deductive reasoning.

✓ 2. Use inductive reasoning to make conjectures.

Try This One 3

If two odd numbers are multiplied, is the result always odd, always even, or sometimes odd and sometimes even? Use inductive reasoning to answer.

One number is *divisible* by another number if the remainder is zero after dividing. For example, 16 is divisible by 8 because $16 \div 8$ has remainder zero, but 17 is not divisible by 8 because $17 \div 8$ has remainder 1.

Example 4 Using Inductive Reasoning to Test a Conjecture

Use inductive reasoning to decide if the following conjecture is likely to be true: any four-digit number is divisible by 11 if the difference between the sum of the first and third digits and the sum of the second and fourth digits is divisible by 11.

Math Note

Once again, inductive reasoning can't tell us for certain that the conjecture is true. On the other hand, in this case it's at least *possible* to try every four-digit number. But on the fun scale, that would rank somewhere in between a root canal and paying taxes.

SOLUTION

Let's make up a few examples. For 1,738, the sum of the first and third digits is $1 + 3 = 4$, and the sum of the second and fourth digits is $7 + 8 = 15$. The difference is $15 - 4 = 11$, so if the conjecture is true, 1,738 should be divisible by 11. To check: $1{,}738 \div 11 = 158$ (with no remainder).

For 9,273, $9 + 7 = 16$, $2 + 3 = 5$, and $16 - 5 = 11$. So if the conjecture is true, 9,273 should be divisible by 11. To check: $9{,}273 \div 11 = 843$ (with no remainder).

Let's look at one more example. For 7,161, $7 + 6 = 13$, $1 + 1 = 2$, and $13 - 2 = 11$. Also $7{,}161 \div 11 = 651$ (with no remainder), so the conjecture is true for this example as well. While we can't be positive based on three examples, inductive reasoning indicates that the conjecture is likely to be true.

Try This One 4

Use inductive reasoning to decide if the following conjecture is likely to be true: if the sum of the digits of a number is divisible by 3, then the number itself is divisible by 3.

Inductive reasoning can definitely be a useful tool in decision making, and we use it very often in our lives. But we've talked about a pretty serious drawback: because you can very seldom verify conclusions for every possible case, you can't be positive that the conclusions you're drawing are correct. In the example of the class in which a quiz is given on four consecutive Fridays, even if that continues for 10 more weeks, there's still a chance that there won't be a quiz the following Friday. And if there's even one Friday on which a quiz is not given, then the conjecture that there will be a quiz every Friday proves to be false.

This is a useful observation: while it's not often easy to prove that a conjecture is true, it's much simpler to prove that one is false. All you need is to find one specific example that contradicts the conjecture. This is known as a **counterexample**. In the quiz example, just one Friday without a quiz serves as a counterexample: it proves that your conjecture that there would be a quiz every Friday is false. In Example 5, we'll use this idea to prove that a conjecture is false.

Example 5 Finding a Counterexample

Find a counterexample that proves the conjecture below is false.

Conjecture: A number is divisible by 3 if the last two digits are divisible by 3.

SOLUTION

We'll pick a few numbers at random whose last two digits are divisible by 3, then divide the original number by 3, and see if there's a remainder.

1,527: Last two digits, 27, divisible by 3; $1{,}527 \div 3 = 509$
11,745: Last two digits, 45, divisible by 3; $11{,}745 \div 3 = 3{,}915$

At this point, you might start to suspect that the conjecture is true, but you shouldn't! We've only checked two cases, and there are infinitely many possibilities.

1,136: Last two digits, 36, divisible by 3; $1{,}136 \div 3 = 378\frac{2}{3}$

This counterexample shows that the conjecture is false.

3. Find a counterexample to disprove a conjecture.

Try This One 5

Find a counterexample to disprove the conjecture that the name of every month in English contains either the letter y or the letter r.

CAUTION

Remember: One counterexample is enough to show that a conjecture is false. But one positive example is *never* enough to show that a conjecture is true.

Example 6 Making and Testing a Conjecture

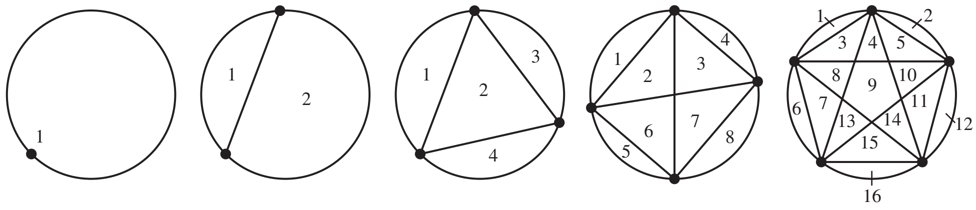
Math Note

Example 6 illustrates what is, in some sense, the very essence of math and science: we have an idea that something might be true, and we use inductive reasoning to test it. But the result of the example shows why inductive reasoning can't be used to *prove* results: what appears to be true after looking at several examples can still turn out to be false.

Use inductive reasoning to make a conjecture about the number of sections a circle is divided into when a given number of points on the circle are connected by chords. (A chord is a line connecting two points on a circle.) Then test the conjecture with one further example.

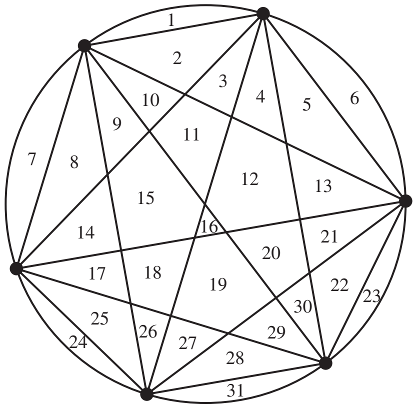
SOLUTION

We'll draw several circles, connect the points with chords, and then count the sections.



Point(s)	1	2	3	4	5	6
Sections	1	2	4	8	16	?

Looking at the pattern in the number of sections, we see that a logical guess for the next number is 32. It looks like the number of sections keeps doubling. In fact, the number of sections appears to be 2 raised to the power of 1 less than the number of points. This will be our conjecture. Let's see how we did by checking with six points:



Uh oh . . . there are 31 sections! It looks like our conjecture is not true.

Try This One 6

Given that there are 31 sections with 6 points, guess how many there will be with 7, and then check your answer.

The other method of reasoning that we will study is called *deductive reasoning*, or *deduction.*

Deductive reasoning is the process of reasoning that arrives at a conclusion based on previously accepted general statements. It's based on overall rules, NOT specific examples.

Here's an example of deductive reasoning. At many colleges, a student has to be registered for at least 12 hours to be considered full-time. So we accept the statement

Sidelight **Of Fuzzy Dogs and Inductive Reasoning**

Here's a clever way to understand the difference between inductive and deductive reasoning. Suppose a friend invites you over to his new apartment, and while walking over there, you come across a house with a loose dog in the yard. As you pass, the dog runs over and bites you on the ankle. Ouch! If you're walking back to your friend's place again a week later, you might decide to take the same path—hey, it's the quickest way to get there, and the dog bite was an isolated incident. But as you're passing by the same house, the same dog runs over and bites you again. At that point, you'd probably decide, based on two specific incidents, to walk a different way next time. That's inductive reasoning.

On the other hand, suppose that instead of a loose dog, in front of that house there's a big hole where the sidewalk used to be, with no way to get around it. Would you have to fall into the hole twice to decide that taking a different route would be a good idea? Probably not . . . you're not a goof, so you know that a known principle (I call it "gravity") dictates that you can't walk over a big hole without falling in. That's deductive reasoning.

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4. Explain the difference between inductive and deductive reasoning.

"A student is full-time if he or she is registered for at least 12 hours" as true. If we're then told that a particular student has full-time status, we can conclude that she or he is registered for at least 12 hours. The key is that we can be *positive* that this is true. On the other hand, you might have 10 friends who are full-time students, and all of them might be taking 15 or more hours of classes. If you concluded that "full-time" means more than 14 hours, you'd be using inductive reasoning, not deductive. And in this case, you'd probably be wrong.

So that's what separates deductive reasoning from inductive reasoning. In Example 7, we'll look at an incredibly important aspect of these types of reasoning. We'll use inductive reasoning to MAKE a conjecture, and deductive reasoning to PROVE it.

Example 7 Using Deductive Reasoning to Prove a Conjecture

Math Note

Here's another way to think about the difference between inductive and deductive reasoning: inductive reasoning begins with *specific* examples, and leads to a more *general* conclusion. Deductive reasoning starts with a *general* principle, and uses it to draw conclusions about *specific* examples. In short:

Inductive: Specific → General
Deductive: General → Specific

Consider the following problem: think of any number. Multiply that number by 2, then add 6, and divide the result by 2. Next subtract the original number. What is the result?

(a) Use inductive reasoning to make a conjecture for the answer.
(b) Use deductive reasoning to prove your conjecture.

SOLUTION

(a) Let's begin by picking a few specific numbers randomly, and performing the described operations to see what the result looks like.

Number:	12	5	43
Multiply by 2:	$2 \times 12 = 24$	$2 \times 5 = 10$	$2 \times 43 = 86$
Add 6:	$24 + 6 = 30$	$10 + 6 = 16$	$86 + 6 = 92$
Divide by 2:	$30 \div 2 = 15$	$16 \div 2 = 8$	$92 \div 2 = 46$
Subtract the original number:	$15 - 12 = 3$	$8 - 5 = 3$	$46 - 43 = 3$
Result:	3	3	3

At this point, you may be tempted to conclude that the result is always 3. But this is just a conjecture: we've tried only three of infinitely many possible numbers! As usual when using inductive reasoning, we can't be completely sure that our conjecture is always true. But at this point, it seems like it would at least be worth the effort to see if we can prove that our conjecture is true.

(b) The problem with the inductive approach is that it requires using specific numbers, and we know that we can't check every possible number. Instead, we'll choose an *arbitrary* number and call it a. Think of that as standing for "any old number." If we can show that the result is 3 in this case, that will tell us that this is the result for *every* number. Remember, we'll be doing the exact same operations, just on an arbitrary number a.

Number:	a
Multiply by 2:	$2a$
Add 6:	$2a + 6$
Divide by 2:	$\dfrac{2a+6}{2} = \dfrac{2a}{2} + \dfrac{6}{2} = a + 3$
Subtract the original number:	$a + 3 - a = 3$

Now we know for sure that the result will always be 3, and our conjecture is proved.

Try This One 7

5. Use deductive reasoning to prove a conjecture.

Consider the following problem: think of any number. Multiply that number by 3, then add 30, and divide the result by 3. Next subtract the original number. What is the result?

(a) Use inductive reasoning to make a conjecture for the answer.
(b) Use deductive reasoning to prove your conjecture.

Sidelight **Making an Arbitrary Point**

In common usage, the word *arbitrary* is often misinterpreted as a synonym for *random.* When reaching into a bag of potato chips, you make a random selection, and some people would also call this an arbitrary selection. But in math, the word *arbitrary* means something very different. When randomly selecting that chip, you have still chosen a specific chip—it is probably not representative of every chip in the bag. Some chips will be bigger, and others smaller. Maybe you got that one gross little burned chip. Yuck.

When we use *arbitrary* in math, we're referring to a nonspecific item that is able to represent *all* such items. In the series of calculations we looked at above, we could never be sure that the result will always be 3 by choosing specific numbers. Why? Because we'd have to try it for *every* number, which is, of course, impossible. You have better things to do than spend the rest of your life testing number after number. The value of performing the calculation on an *arbitrary* number *a* is that this one calculation proves what will happen for *every* number you choose. It is absolutely crucial in the study of mathematics to understand that choosing specific numbers can almost never *prove* a result, because you can't try every number. Instead, we'll rely on using arbitrary numbers and deductive reasoning. Now ponder this deep question: is there such a thing as an arbitrary potato chip?

Let's try another example. Try to focus on the difference between inductive and deductive reasoning, and the fact that inductive reasoning is great for giving you an idea about what the truth might be for a given situation, but deductive reasoning is needed for proof.

Example 8 Using Deductive Reasoning to Prove a Conjecture

Use inductive reasoning to arrive at a general conclusion, and then prove your conclusion is true by using deductive reasoning.

Pick a number:
Add 50:
Multiply by 2:
Subtract the original number:
Result:

SOLUTION

Approach: Induction

Try a couple different numbers and make a conjecture.

Original number:	12	50
Add 50:	$12 + 50 = 62$	$50 + 50 = 100$
Multiply by 2:	$62 \times 2 = 124$	$100 \times 2 = 200$
Subtract the original number:	$124 - 12 = 112$	$200 - 50 = 150$
Result:	112	150

A reasonable conjecture is that the final answer is 100 more than the original number.

Approach:	Deduction
Pick an arbitrary number:	a
Add 50:	$a + 50$
Multiply by 2:	$2(a + 50) = 2a + 100$
Subtract the original number:	$2a + 100 - a$
Result:	$a + 100$

Our conjecture was right: the final answer is always 100 more than the original number.

Calculator Guide

If you use a calculator to try the repeated operations in Example 8, you'll need to press [=] (scientific calculator) or [ENTER] (graphing calculator) after every operation, or you'll get the wrong result. Suppose you simply enter the whole string:

Standard Scientific Calculator

12 [+] 50 [×] 2 [−] 12 [=]

Standard Graphing Calculator

12 [+] 50 [×] 2 [−] 12 [ENTER]

The result is 100, which is incorrect. In Chapter 5, we'll find out why when we study the order of operations.

Math Note

You probably recognized the deductive approach to proving our conjecture in Example 8 as algebra, where we use a symbol (in this case the letter *a*) to represent an arbitrary number. If you need some help brushing up on algebra, we have you covered: Chapter 6 reviews some key elements of algebra, and there are online resources as well.

Try This One 8

Arrive at a conclusion by using inductive reasoning, and then try to prove your conclusion by using deductive reasoning.

Pick a number:
Add 16:
Multiply by 3:
Add 2:
Subtract twice the original number:
Subtract 50:
Result:

Now that we've seen how inductive and deductive reasoning can be used, let's review by distinguishing between the two types of reasoning in some real situations.

Example 9 Comparing Inductive and Deductive Reasoning

The last six times we played our archrival in football, we won, so I know we're going to win on Saturday. Did I use inductive or deductive reasoning?

SOLUTION

This conclusion is based on six specific occurrences, not a general rule that we know to be true. (No team wins *every* game!). I used inductive reasoning.

Try This One 9

There is no mail delivery on holidays. Tomorrow is Labor Day so I know my student loan check won't show up. Did I use inductive or deductive reasoning?

Example 10 Comparing Inductive and Deductive Reasoning

The syllabus states that any final average between 80 and 90% will result in a B. If I get 78% on my final, my overall average will be 80.1%, so I'll get a B. Did I use inductive or deductive reasoning?

SOLUTION

Although we're talking about a specific person's grade, the conclusion that I'll get a B is based on a general rule: all scores in the 80s earn a B. So this is deductive reasoning.

Try This One 10

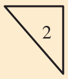
Everyone I know in my sorority got at least a 2.5 GPA last semester, so I'm sure I'll get at least a 2.5 this semester. Did I use inductive or deductive reasoning?

Remember that both inductive reasoning and deductive reasoning are useful tools for problem solving. But the biggest difference between them is that conclusions drawn from inductive reasoning, no matter how reasonable, are still at least somewhat uncertain. In Problems 67–72, we'll distinguish between *weak* and *strong* inductive arguments. But conclusions drawn by using deductive reasoning can be considered definitely true, as long as the general rules used to draw the conclusion are known to be true.

In addition, you should take a minute or two to think about the fact that to disprove a conjecture, you only need to find *one specific example* for which it's not true. But to prove a conjecture, you have to show that it's true in *every* possible case.

Answers to Try This One

1 Pattern: every entry is 1 more than the one that comes two spots before it. The next three numbers are 8, 6, 9.

2 

3 Always odd

4 True

5 June has neither a y nor an r.

6 There are 57.

7 10

8 The result is the original number.

9 Deductive

10 Inductive

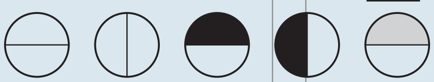
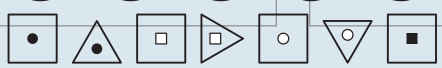
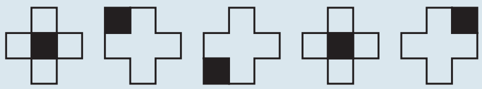
Exercise Set 1-1

Writing Exercises

1. Explain the difference between inductive and deductive reasoning.
2. What is meant by the term *conjecture*?
3. Give an example of a decision you made based on inductive reasoning that turned out well, and one that turned out poorly.
4. What is a counterexample? What are counterexamples used for?
5. Explain why you can never be sure that a conclusion you arrived at using inductive reasoning is true.
6. Explain the difference between an arbitrary number and a number selected at random.
7. Take another look at the opener for Chapter 1 on page 2. How do the terms inductive and deductive reasoning apply to evidence in court?
8. Describe the difference between being confident that a conjecture is true, and being CERTAIN that it's true.

Computational Exercises

For Exercises 9–18, use inductive reasoning to find a pattern, and then make a reasonable conjecture for the next number or item in the sequence.

9. 1 2 4 7 11 16 22 29 ____
10. 6 10 22 58 166 490 ____
11. 10 20 11 18 12 16 13 14 14 12 15 ____
12. 2 3 8 63 3,968 ____
13. 100 99 97 94 90 85 79 ____
14. 9 12 11 14 13 16 15 18 ____
15.  ____
16.  ____
17.  ____
18.  ____

For Exercises 19–22, find a counterexample to show that each statement is false.

19. The sum of any three odd numbers is even.
20. When an even number is added to the product of two odd numbers, the result will be even.
21. When an odd number is squared and divided by 2, the result will be a whole number.

22. When any number is multiplied by 6 and the digits of the answer are added, the sum will be divisible by 6.

For Exercises 23–26, use inductive reasoning to make a conjecture about a rule that relates the number you selected to the final answer. Try to prove your conjecture by using deductive reasoning.

23. Pick a number:
    Double it:
    Subtract 20 from the answer:
    Divide by 2:
    Subtract the original number:
    Result:
24. Pick a number:
    Multiply it by 9:
    Add 21:
    Divide by 3:
    Subtract three times the original number:
    Result:
25. Pick a number:
    Add 6:
    Multiply the answer by 9:
    Divide the answer by 3:
    Subtract 3 times the original number:
    Result:
26. Pick an even number:
    Multiply it by 4:
    Add 8 to the product:
    Divide the answer by 2:
    Subtract 2 times the original number:
    Result:

For Exercises 27–36, use inductive reasoning to find a pattern for the answers. Then use the pattern to guess the result of the final calculation, and perform the operation to see if your answer is correct.

27. $12{,}345{,}679 \times 9 = 111{,}111{,}111$
    $12{,}345{,}679 \times 18 = 222{,}222{,}222$
    $12{,}345{,}679 \times 27 = 333{,}333{,}333$
    $\vdots$
    $12{,}345{,}679 \times 72 = ?$
28. $0^2 + 1 = 1$
    $1^2 + 3 = 2^2$
    $2^2 + 5 = 3^2$
    $3^2 + 7 = 4^2$
    $4^2 + 9 = 5^2$
    $5^2 + 11 = ?$
29. $999{,}999 \times 1 = 0{,}999{,}999$
    $999{,}999 \times 2 = 1{,}999{,}998$
    $999{,}999 \times 3 = 2{,}999{,}997$
    $\vdots$
    $999{,}999 \times 9 = ?$
30. $1 = 1^2$
    $1 + 2 + 1 = 2^2$
    $1 + 2 + 3 + 2 + 1 = 3^2$
    $\vdots$
    $1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = ?$
31. $9 \times 9 = 81$
    $99 \times 99 = 9{,}801$
    $999 \times 999 = 998{,}001$
    $9{,}999 \times 9{,}999 = 99{,}980{,}001$
    $99{,}999 \times 99{,}999 = ?$
32. $1 \times 8 + 1 = 9$
    $12 \times 8 + 2 = 98$
    $123 \times 8 + 3 = 987$
    $1{,}234 \times 8 + 4 = 9{,}876$
    $12{,}345 \times 8 + 5 = ?$
33. $1 \cdot 1 = 1$
    $11 \cdot 11 = 121$
    $111 \cdot 111 = 12{,}321$
    $1{,}111 \cdot 1{,}111 = 1{,}234{,}321$
    $11{,}111 \cdot 11{,}111 = ?$
34. $9 \cdot 91 = 819$
    $8 \cdot 91 = 728$
    $7 \cdot 91 = 637$
    $6 \cdot 91 = 546$
    $5 \cdot 91 = ?$
35. Explain what happens when the number 142,857 is multiplied by the numbers 2 through 8.
36. A Greek mathematician named Pythagoras is said to have discovered the following number pattern. Find the next three sums by using inductive reasoning. Don't just add!
    $1 = 1$
    $1 + 3 = 4$
    $1 + 3 + 5 = 9$
    $1 + 3 + 5 + 7 = 16$
    $1 + 3 + 5 + 7 + 9 = ?$
    $1 + 3 + 5 + 7 + 9 + 11 = ?$
    $1 + 3 + 5 + 7 + 9 + 11 + 13 = ?$
37. Use inductive reasoning to make a conjecture about the next three sums, and then perform the calculations to verify that your conjecture is true.

$$1 + \frac{1}{2} = \frac{3}{2}$$
$$1 + \frac{1}{2} + \frac{1}{2 \cdot 3} = \frac{5}{3}$$
$$1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} = \frac{7}{4}$$
$$1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} = ?$$
$$1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} = ?$$
$$1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} = ?$$

38. Use inductive reasoning to determine the unknown sum, then perform the calculation to verify your answer.
$2 = 1(2)$
$2 + 4 = 2(3)$
$2 + 4 + 6 = 3(4)$
$2 + 4 + 6 + 8 = 4(5)$
$2 + 4 + 6 + 8 + 10 + 12 + 14 = ?$

In Exercises 39–42, use inductive reasoning to find a pattern, then make a reasonable conjecture for the next three items in the pattern. You may have to think outside the box on some of them. That's a good thing.

39. d b e c f d ___ ___ ___
40. a b c e d f i g h o ___ ___ ___
41. J F M A ___ ___ ___
42. D N O S A ___ ___ ___

Applications in Our World

In Exercises 43–58, determine whether the type of reasoning used is inductive or deductive reasoning.

43. The last four congressional representatives from this district were all Republicans. I don't know why the Democratic candidate is even bothering to run this year.
44. I know I'll have to work a double shift today because I have a migraine and every time I have a migraine I get stuck pulling a double.
45. If class is canceled, I go to the beach with my friends. I didn't go to the beach with my friends yesterday; so class wasn't canceled.
46. On Christmas Day, movie theaters and Chinese restaurants are always open, so this Christmas Day we can go to a movie and get some Chinese takeout.
47. For the first three games this year, the parking lot was packed with tailgaters, so we'll have to leave extra early to find a spot this week.
48. Every time Beth sold back her textbooks, she got about 10% of what she paid for them; so this semester she decided to not bother selling her books back.
49. Experts say that opening e-mail attachments that come from unknown senders is the easiest way to get a virus on your computer. Shauna constantly opens attachments from people she doesn't know, so she'll probably end up with a virus on her system.
50. Whenever Marcie let friends set her up on a blind date, the guy turned out to be a total loser. This time, when a friend offered to fix her up, she decided the guy would be a loser, so she declined.
51. Dr. Spalsbury's policy is that any student whose cell phone goes off during class will be asked to leave. So when Ericka forgot to turn hers off and it rang during a quiz, out the door she went.
52. While experimenting on learning in mice, a biology student was able to successfully train six different mice to finish a maze, so she was really surprised when the next one was unable to learn the maze.
53. Since Josie ate a diet of mostly foods high in saturated fat, she was not surprised when her doctor said her cholesterol levels were too high.
54. In the past, even when Chris followed a recipe, her meal was either burned or underdone. Now her party guests know to eat before they attend her dinners so they won't starve all evening.
55. Working as a nurse in a hospital requires at least a two-year degree in this state, so when I was in the emergency room last week I asked the nurse where he went to college.
56. Marathon runners should eat extra carbs before a big race, and since Mark did not eat enough carbs before the race, he felt sluggish the entire time.
57. Organizing chapter contents in your own words before the test will decrease the amount of study you have to do before a test. When Scott tried this method, he was pleasantly surprised at how fast he was able to study.
58. The last several network dramas I've followed have been canceled just when I started getting into them. So I'm not going to bother watching the new one they're advertising even though it looks good, because I don't want to be disappointed when it gets canceled.

Critical Thinking

59. Do a Google search for the string "studies texting while driving." Suppose that you've driven while texting 10 times in the past without any incident. How likely would you be to text while driving if you use (a) inductive reasoning, and (b) deductive reasoning based on your Google search? Describe your reasoning in each case.
60. Just about everyone had a conversation like this with their parents at some point in their childhood: "But all my friends are doing it!" "If your friends jumped, off a bridge, would you jump, too?" Describe how arguments like this apply to inductive and deductive reasoning. Specifically, what type of reasoning is each person using, and who in your opinion makes a stronger argument?
61. (a) Find a likely candidate for the next two numbers in the following sequence: 2, 4, 8, . . .
(b) Was your answer 16 and 32? How did you get that answer? Can you find a formula with variable n that provides the numbers in your sequence?
(c) My answer is 14 and 22. How did I get that answer? Can you find a formula with variable n that provides the numbers in my sequence? (*Note:* The last question is NOT easy!)

(d) Fill in the following table by substituting the given values of n into the formula. Can you answer the last question in part (c) now? Based on all parts of this problem, what can you conclude about finding a pattern when you have just a bit of information to use?

n	1	2	3	4	5
$n^2 - n + 2$					

62. (a) Find a likely candidate for the next two numbers in the following sequence: 3, 9, 27, . . .
(b) Was your answer 81 and 243? How did you get that answer? Can you find a formula with variable n that provides the numbers in your sequence?
(c) My answer is 57 and 99. How did I get that answer? (*Hint:* Find differences between the first two pairs of terms and look for a pattern.) Can you find a formula with variable n that provides the numbers in my sequence? (*Note:* This is NOT easy!)
(d) Fill in the following table by substituting the given values of n into the formula. Can you answer the last question in part (c) now? Based on all parts of this problem, what can you conclude about finding a pattern when you have just a bit of information to use?

n	1	2	3	4	5
$6n^2 - 12n + 9$					

63. (a) In several of the problems in this section, you looked at a string of numbers then decided what the next number would be. This time, write a string of five numbers with a pattern so that the next number in the string would be 10.
(b) Next, write a string of five numbers with a pattern so that the next two numbers in the string would be 10 and 13.
(c) Finally, write a string of five numbers with a pattern so that the next three numbers in the string would be 10, 13, and 17.

64. Refer to Problem 63.
(a) Write a string of three numbers so that the next number in the string would be $\frac{4}{81}$.
(b) Next, write a string of three numbers so that the next two numbers in the string would be $\frac{4}{81}$ and $-\frac{5}{243}$.
(c) Find a formula with variable n that provides the numbers in your sequence.

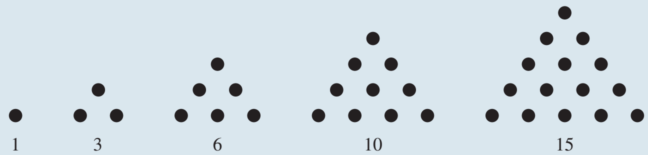
Problems 65 and 66 use the formula average speed = distance/time.

65. Suppose that you drive a certain distance at 20 miles per hour, then turn around and drive back the same distance at 60 miles per hour.
(a) Choose at least four different distances and find the average speed for the whole trip. Then use inductive reasoning to make a conjecture as to what the average speed is in general.
(b) Use algebra to prove your conjecture from part (a).

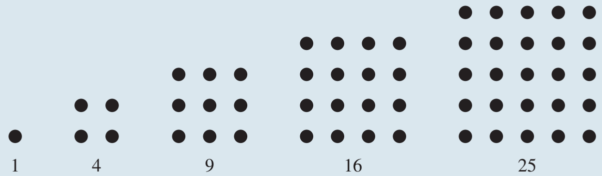
66. Suppose that you drive a certain distance at 40 miles per hour, then turn around and drive twice as far at 20 miles per hour.
(a) Choose at least four different distances and find the average speed for the whole trip. Then use inductive reasoning to make a conjecture as to what the average speed is in general.
(b) Use algebra to prove your conjecture from part (a).

All conclusions drawn from inductive reasoning aren't created equal. We can distinguish between a ***weak inductive argument****, where a conclusion is drawn from just a few specific instances, and a* ***strong inductive argument****, where a conclusion is drawn from a large amount of observations. In Problems 67–72, classify each argument as weak or strong induction, and discuss your reasoning.*

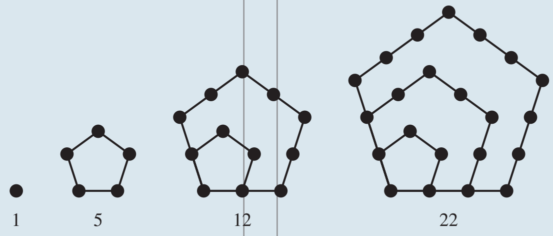
67. The Cubs lost the first six games of the season. They have NO chance tonight.
68. There are 52 people on my flight to the Bahamas, and 40 of them brought carry-on bags to avoid paying baggage fees. So I'm thinking that about 80% of air travelers bring carry-ons to avoid fees.
69. Public Policy Polling conducted a poll of 1,224 Americans in December of 2016 and reported that 75% of those surveyed disapprove of the job that Congress is doing. I conclude that a majority of Americans were unhappy with Congress at that point.
70. I played the first nine holes of a local golf course last night, and the grass was brown on all nine of them. I imagine all nine holes on the back nine are burned out, too.
71. There were 109,000 people at the Ohio State-Michigan State game last year. Almost all of them cheered when Ohio State scored, so the game must have been played in Ohio.
72. All three high school math teachers I had were kind of nerdy. I guess that all math teachers are nerds.
73. The numbers 1, 3, 6, 10, 15, . . . are called *triangular numbers* since they can be displayed as shown.



The numbers 1, 4, 9, 16, 25, . . . are called *square numbers* since they can be displayed as shown.



The numbers 1, 5, 12, 22, 35, . . . are called *pentagonal numbers* since they can be displayed as shown.



(a) Using inductive reasoning, find the next three triangular numbers.

(b) Using inductive reasoning, find the next three square numbers.

(c) Using inductive reasoning, find the next three pentagonal numbers.

(d) Using inductive reasoning, find the first four hexagonal numbers.

74. Refer to Exercise 73. The formula for finding triangular numbers is $\frac{n[1n-(-1)]}{2}$ or $\frac{n(n+1)}{2}$. The formula for finding square numbers is $\frac{n(2n-0)}{2}$ or n^2. The formula for pentagonal numbers is $\frac{n(3n-1)}{2}$. Find the formula for hexagonal numbers, using inductive reasoning.

Section 1-2 Estimation and Interpreting Graphs

LEARNING OBJECTIVES

1. Identify some uses for estimation.
2. Round numbers to a given level of accuracy.
3. Estimate the answers to problems in our world.
4. Use estimation to obtain information from graphs.

Everyone likes buying items on sale, so we should all be familiar with the idea of finding a rough approximation for a sale price. If you're looking at a pair of shoes that normally sells for $70 and the store has a 40% off sale, you might figure that the shoes are a little more than half price, which would be $35, so they're probably around $40. We will call the process of finding an approximate answer to a math problem **estimation**. Chances are that you use estimation a lot more than you realize, unless you carry a calculator with you everywhere you go. (Okay, so your cell phone has a calculator, but how often do you use it?)

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1. Identify some uses for estimation.

Estimation comes in handy in a wide variety of settings. When the auto repair shop technicians look over your car to see what's wrong, they can't know for sure what the exact cost will be until they've made the repairs, so they will give you an estimate. When you go to the grocery store and have only $20 to spend, you'll probably keep a rough estimate of the total as you add items to the cart. (Imagine buying a week's worth of groceries and keeping track of every price to the penny on your cell phone. Who has time for that?) If you plan on buying carpet for a room, you'd most likely measure the square footage and then estimate the total cost as you looked at different styles of carpet. You could find the exact cost if you really needed to, but often an estimate is good enough for you to make a sound buying decision.

Estimation is also a really useful tool in checking answers to math problems, particularly word problems. Let's say you're planning an outing for a student group you belong to, and lunch is included at $3.95 per person. If 24 people signed up and you were billed $94.80, you could use estimation to quickly figure that this is a reasonable bill. Since 24 is close to 25, and $3.95 is close to $4.00, the bill should be close to 25 × $4.00 = $100.

You also use estimation when rounding numbers for simplicity. If someone asks your height and age, you might say 5′11″ and 20, even if you're actually $5'10\frac{1}{2}''$ (everyone fudges a little) and 20 years, 4 months, and 6 days old.

Since the process of estimating uses rounding, we'll start with a brief review of rounding numbers. This is based on the concept of place value. The **place value** of a digit in a number tells the value of the digit in terms of ones, tens, hundreds, etc. For example, in the number 325, the 3 means 3 hundreds or 300 since its place value is hundreds. The 2 means 2 tens, or 20, and the 5 means 5 ones. A place value chart is shown here, along with instructions for rounding numbers.

Math Note

We often use the word *nearest* to describe the place value to round to. Instead of saying, "Round to the hundreds place," we might say, "Round to the nearest hundred."

Steps for Rounding Numbers

1. Locate the place-value digit of the number that is being rounded. Here is the place-value chart for whole numbers and decimals:

8,	9	8	5,	7	3	0,	2	6	1	.	2	3	5	6	7	8
billions	hundred-millions	ten-millions	millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	ones		tenths	hundreds	thousandths	ten-thousandths	hundred-thousandths	millionths

2a. If the digit to the right of the place-value digit is 0 through 4, don't change the place-value digit.

2b. If the digit to the right of the place-value digit is 5 through 9, add 1 to the place-value digit.

Note: When you round whole numbers, replace all digits to the right of the digit being rounded with zeros. When you round decimal numbers, drop all digits to the right of the digit that is being rounded.

Example 1 Rounding Numbers

Round each value as requested.

(a) $147.38 to the nearest 10 dollars
(b) According to Wikipedia, an average person's nails grow at the rate of 0.1181 inches per month. (That makes me well above average I guess. Yay me.) Round that length to the nearest hundredth of an inch.
(c) As of December 11, the losing candidate in the 2016 presidential election was credited with 65,432,202 votes. Round this to the nearest million.
(d) Is $1 million a lot of money? It depends who you ask. That amount is 0.23749% of the amount the federal government spent in 2015— in ONE HOUR. Seriously. Round that percentage to the nearest thousandth of a percent.

SOLUTION

(a) Rounding to the nearest ten dollars is the tens place, and the digit there is 4. The digit to the right of that is 7, which is more than 4. So we round the 4 up to 5, and replace the 7 with a zero, to get $150. You could include zeros in the two places after the decimal if it means a lot to you, but you can just leave them off as well.
(b) The digit in the hundredths place is 1, and it's followed by an 8. Again, this is more than 4, so we round up the 1 to a 2 to get 0.12 inch.
(c) The millions place is the second digit here, which is 5. The following digit (4) is less than 5, so we leave the 5 as is and replace the last six digits with zeros. The rounded result is 65,000,000 votes.
(d) Thousandths is the third digit after the decimal point, which in this case is 7. The following digit (4) is less than 5, so we keep the 7 and round to 0.237%.

Try This One 1

2. Round numbers to a given level of accuracy.

Round each value as requested.

(a) The average distance from Earth to the moon is 238,856 miles. Round to the nearest hundred miles.
(b) The moon is moving away from Earth at the rate of 1.488189 inches per year. Round to the nearest ten-thousandth of an inch.
(c) $8.93 to the nearest 10 cents.

Estimation

When we use estimation to simplify numerical calculations, we use two steps:

1. Round the numbers being used to numbers that make the calculation simple.
2. Perform the operation or operations involved.

Example 2 Estimating Total Cost When Shopping

The owner of an apartment complex needs to buy six refrigerators for a new building. She chooses a model that costs $579.99 per refrigerator. Estimate the total cost of all six and decide if your answer is an **overestimate** (more than the actual value) or an **underestimate** (less than the actual value).

SOLUTION

Step 1 Round the cost of the refrigerators. In this case, rounding up to $600 will make the calculation easy.

Step 2 Perform the calculation: $600 × 6 = $3,600. Our estimated cost is $3,600.

The actual cost will be a little less than $3,600—since we rounded the price up, our estimate will be high.

Try This One 2

At one ballpark, large frosty beverages cost $7.25 each. Estimate the cost of buying one for each member of a group consisting of four couples. Is the actual cost more or less than your answer?

Students often wonder, "How do I know what digit to round to?" It would be nice if there were an exact answer to that question, but there isn't—it depends on the individual numbers. In Example 2, the cost of the refrigerators could have been rounded from $579.99 to $580. Then the cost estimate would be $580 × 6 = $3,480. This is a much closer estimate because we rounded the cost to the nearest dollar, rather than the nearest $100. But the calculation is harder.

Deciding on how much to round is really a tradeoff: ease of calculation versus accuracy. In most cases you'll get a more accurate result if you round less, but the calculation will be a little harder. Since there's no exact rule, it's important to evaluate the situation and use good old-fashioned common sense. And remember, when you're estimating, there is no one correct answer.

Sidelight **Just How Big Are Big Numbers?**

Back when we were still living in caves, really large numbers probably weren't of much use. Early humans could use their fingers and toes to count their families and possessions, and I imagine that was good enough. How the world has changed! In the 21st century, we're bombarded with large numbers from every direction, and being able to have some perspective on the size of those numbers is a useful skill. The truth is that most people have absolutely no idea how big a number like a million actually is.

One million is a one followed by six zeros (1,000,000). If you wanted to count to a million and you counted one number each second with no time off to eat or sleep, it would take you just about $11\frac{1}{2}$ days. Wow. A stack of one million pennies would be almost a mile high; a pile of one million dollar bills would weigh almost a ton. As of this writing, the federal minimum wage is $7.25 per hour. If you worked 40 hours a week at a minimum wage job, it would take you over 66 years to make one million dollars—before taxes!

Add three more zeros to the end of a million and you get one billion (1,000,000,000); that makes a billion equal to 1,000 million. In 2010, the federal government spent about $7\frac{1}{2}$ billion dollars—per DAY. So how big is a billion?

Counting to one billion by ones would take you about 32 years (no rest or sleep, of course). A billion pennies would make a stack almost 1,000 miles high; a pile of one billion dollar bills would be about the size of a medium-sized office building, and weigh over a thousand tons. And guess what? From some perspectives, a billion isn't even that much.

In 2016, the federal government spent 433 billion dollars just paying *interest* on the national debt. (And guess where that money is coming from?) Here's the actual amount of money spent by our government in 2016: $3,540,000,000,000. I would be willing to bet that 80% of the population can't even read that number out loud, let alone have any perspective on just how big it is. For the record, that's 3 trillion, 540 billion dollars. It would take over 100,000 YEARS to count that high. Everyone knows that a person making a million dollars a year is wealthy, but he or she would have to work for 3.54 million years to make the amount of money the feds spend in one year.

Once you get past a trillion, things get just plain silly. A quadrillion is one followed by 15 zeros. Eighteen zeros gives you a quintillion, and 21 zeros a sextillion. The entire Earth weighs about 6 sextillion, 570 quintillion tons; the weight of all the people on Earth is a mere 525 million tons. The largest number with a name ending in -illion that I know of is the vigintillion, which has 63 zeros.

So is that the biggest number of all? Not even close. A nine-year-old girl came up with the name "googol" in 1938 to describe one followed by 100 zeros. Scientists think this is more than the total number of protons in our universe. But if you want to make a googol look small, go up to a googolplex, which is one followed by a googol of zeros. It's almost impossible to imagine how large a number this is, but writing it out would require a piece of paper far longer than the known universe.

Finally, here's an easy way to show that there IS no biggest number: give me any number, and I can give you a bigger one by adding one to it. Maybe a million isn't so big after all.

Example 3 Estimating the Cost of a Cell Phone

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You're considering a new cell phone plan where you have to pay $179 up front for the latest phone, but the monthly charge of $39.99 includes unlimited minutes, data, and messaging. Estimate the cost of the phone for 1 year if there are no additional charges. If you were trying to decide if you could afford this phone in your budget, would you want an overestimate or an underestimate?

SOLUTION

Step 1 We can round the cost of the phone to $180 and the monthly charge to $40.

Step 2 The monthly cost estimate for 1 year will be $\$40 \times 12 = \$480$; add the estimated cost of the phone to get an estimated total cost of $\$480 + \$180 = \$660$.

When building a budget, you'd want an overestimate to make sure that you don't end up spending more than you were planning on.

Try This One 3

A rental car company charges a rate of $78 per week to rent an economy car. For an upfront fee of $52, you can upgrade to a midsize car. Estimate the cost of renting a midsize car for 3 weeks. When planning a vacation, would you want an overestimate or an underestimate of the cost?

Example 4 Estimating Remodeling Costs

Math Note

While rounding up or down to intentionally get either an over- or underestimate can be useful, for calculations involving addition and multiplication, rounding one quantity up and the other down can reduce the total error.

The Osbueño family plans on remodeling the living room. They will be replacing 21 square yards of carpet at a cost of \$23 per square yard (installed), and they also need to have 26 linear feet of crown molding installed. They'd like to keep the total cost around \$1,000. Estimate the cost per foot of crown molding that they can afford.

SOLUTION

First, we'll estimate how much is going to be spent on carpet: we can round the 21 square yards down to 20, and the \$23 per square yard up to \$25, giving us $20 \times \$25 = \$500$. So the Osbueños will have about \$500 left to spend on crown molding.

We can round the 26 linear feet down to 25, and use division to estimate the price per foot: $\$500 \div 25 = \$20$, so they should be looking for crown molding that costs no more than \$20 per linear foot installed.

Try This One 4

3. Estimate the answers to problems in our world.

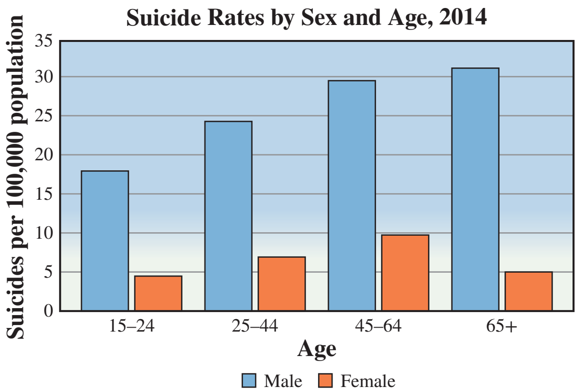
Next up for the Osbueños: a bedroom remodel. They will have 28 linear feet of wall painted at \$12 per foot, and need 19 square yards of carpet. If the budget for the bedroom is \$900, estimate the price per square yard of carpet they can afford.

In our world, useful or interesting information is often displayed in graphical form. In Examples 5–7, we'll illustrate how estimation applies to interpreting graphical information.

A **bar graph** is used to compare amounts or percentages using either vertical or horizontal bars of various lengths; the lengths correspond to the amounts or percentages, with longer bars representing larger amounts.

Example 5 Getting Information from a Bar Graph

The graph shows the number of suicides for every 100,000 people in the United States in 2015, broken down by both age and sex. Use the graph to find about how many suicides there were per 100,000 males in the 15–24 age range, and also to estimate the largest discrepancy between males and females in any age range. How confident are you in these estimates?



Source: Centers for Disease Control

SOLUTION

The blue bars illustrate the rate for males, so we need to estimate the height of the first blue bar (the first pair of bars corresponds to the 15–24 age group). That bar appears to be a little more than halfway between 15 and 20, so I'd estimate that height to be 18, and say that there were about 18 suicides for every 100,000 males in that age range.

The biggest discrepancy between males and females is in the 65 and over age range. The blue bar looks to be about height 31, while the female is right on 5, so there were about 26 more suicides for every 100,000 males than females in that age range.

Confidence is in the eye of the beholder, but because the grid only has lines marked every 5 units, it's hard to be much more precise than to the nearest whole number. So if we're interested in accuracy greater than that, I wouldn't be terribly confident.

Try This One 5

Which difference in suicide rate is greater for females: between the 15–24 and 25–44 groups, or between the 45–64 and 65+ age groups? By how much?

A **pie chart**, also called a **circle graph**, is constructed by drawing a circle and dividing it into parts called sectors, according to the size of the percentage of each portion in relation to the whole.

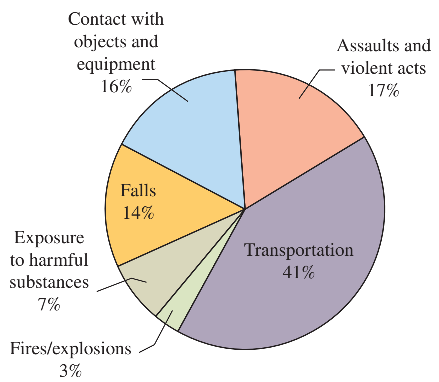
Example 6 Getting Information from a Pie Chart

Math Note

To change a percent to decimal form, move the decimal point two places to the left and drop the percent sign: 18 percent means "18 per hundred," which is 18/100, or 0.18. We'll study percents in detail in Section 7-1.

The pie chart shown represents the number of fatal occupational injuries in the United States for 2013. If the total number of fatal injuries was 4,585 for the year, estimate how many resulted from assaults and violent acts.

Fatal Occupational Injuries



Source: U.S. Bureau of Labor Statistics

SOLUTION

The sector labeled "Assaults and violent acts" indicates that 17% of the total fatal occupational injuries resulted from assaults and violent acts. So we need to find 17% of 4,585. To find a percentage, we multiply the decimal equivalent of the percentage by the total amount. In this case, we get $0.17 \times 4{,}585 = 779.45$. Since 0.45 fatal injury makes no sense, we round to 779.

Try This One 6

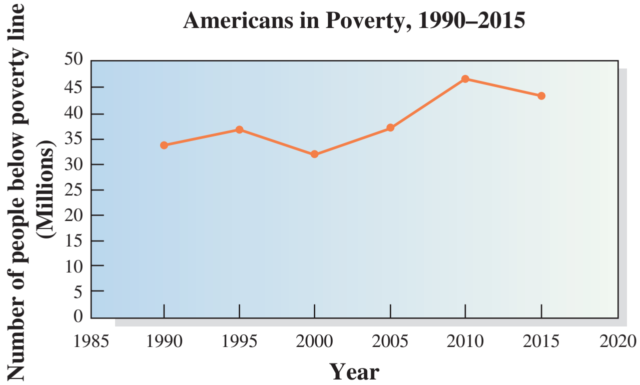
Using the pie chart shown in Example 6, find the approximate number of fatal occupational injuries that resulted from transportation incidents.

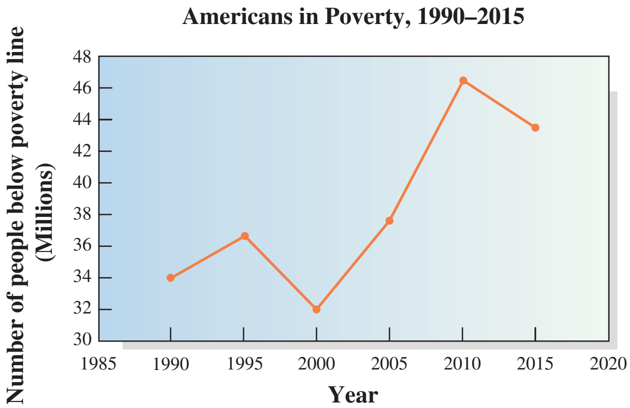
A **time series graph** or **line graph** shows how the value of some variable quantity changes over a specific time period.

Example 7 Estimating Information from a Line Graph

The graphs below each illustrate the number of Americans that were living in poverty according to federal guidelines that determine the poverty level.

(a) Find the approximate number of Americans living in poverty in 2015.
(b) Find the average rate at which the number of folks living in poverty changed between 1995 and 2000.
(c) Estimate the year in which the number of Americans living in poverty first topped 40 million.
(d) Which of the two graphs do you think gives a more accurate view of the poverty rate?

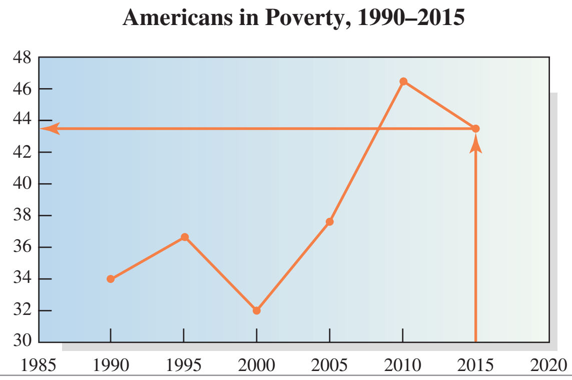




Source: Statista.com

SOLUTION

(a) First, find the year 2015 along the horizontal axis, and move straight upward until hitting the graph. Then move horizontally to the left to find the height on the vertical axis as shown.



The height is a bit below 44, so we estimate that there were about 43.5 million people living in poverty in 2015.

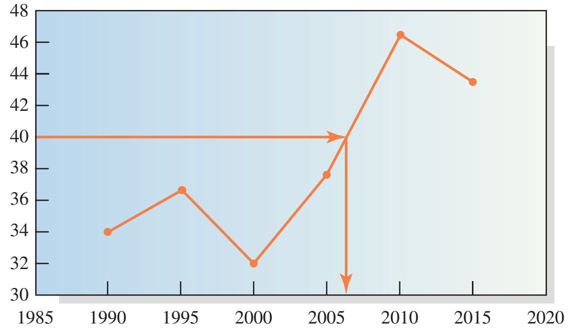
(b) The point corresponding to 1995 is almost halfway between heights 36 and 38, so we can estimate about 37 million people in poverty in 1995. The height of the point for 2000 is almost right on height 32, so we can estimate 32 million people in 2000. Subtracting, we find that the approximate change in number of people in poverty is 32 million − 37 million, or −5 million (the negative represents a decrease in amount). This occurred over a span of 5 years, so the rate of change is

$$\frac{-5 \text{ million people}}{5 \text{ years}} = -1 \text{ million people per year}$$

In other words, between 1995 and 2000, the number of people living in poverty decreased by an average of about a million people per year.

(c) This question is basically the opposite of part (a): we're given a number of people (a height on the graph), and are asked to find the corresponding year. So we locate height 40 on the vertical axis (which corresponds to 40 million people), and move across to the graph, then move down to find the year on the horizontal axis.

Americans in Poverty, 1990–2015



It certainly happened after 2005. Either 2006 or 2007 would be good guesses, but I'd probably go with 2006.

(d) This question brings up a really interesting point about the scale for graphs. Notice that the vertical axis on the first graph starts at zero, resulting in a lot of "dead space" near the bottom of the graph. The second starts at 30. That makes it a little easier to read the graph precisely, which is good. But the first graph provides a more accurate picture of the poverty situation: it shows that the number in poverty fluctuated over 25 years, but not THAT much. The second graph makes the change look much more drastic than it actually was.

Try This One 7

4. Use estimation to obtain information from graphs.

(a) About how many people lived in poverty in 2010?

(b) Find the average rate of change in the number of people living in poverty over the entire span from 1990 to 2015.

(c) Do you think your answer to part (b) provides a complete picture of the change in poverty over that time span? Why or why not?

(d) Estimate the year or years in which there were about 44 million people living in poverty.

Answers to Try This One

1 (a) 238,900 miles
(b) 1.4882 inches
(c) $8.90
2 About $56; actual cost is more
3 About $290; you'd want an overestimate
4 About $30 per square yard
5 The difference between 45–64 and 65+ is about 2 or 2.5 suicides per 100,000 people greater than the difference between 15–24 and 25–44.
6 About 1,880
7 (a) About 46.5 million
(b) About 380,000 more people per year
(c) It provides an incomplete picture, because it totally ignores all of the ups and downs in between those two years.
(d) Around 2008 and 2014

Exercise Set 1-2

Writing Exercises

1. Think of three situations in our world where you could use estimation.
2. Explain why an exact answer to a math problem isn't always necessary.
3. How can estimation be used as a quick check to see if the answer to a math problem is reasonable?
4. Describe the rules for rounding numbers to a given place.
5. Explain why there is never a single, correct answer to a question that asks you to estimate some quantity.
6. Explain how to estimate the size of a quantity from a bar graph.
7. How is information described in a pie chart? What sort of information works well with pie charts?
8. How can you tell when a quantity is getting larger over time from looking at a time-series graph?
9. Think of a situation where you'd most likely want an overestimate, and one where you'd most likely want an underestimate.
10. When information is presented in the form of a bar graph or time-series graph, you could get more exact values if all of the data were just listed out in table form. Then why not always do that? Why bother with graphs?

Computational Exercises

For Exercises 11–30, round the number to the place value given.

11. 2,861 (hundreds)
12. 732.6498 (thousandths)
13. 3,261,437 (ten-thousands)
14. 9,347 (tens)
15. 62.67 (ones)
16. 45,371,999 (millions)
17. 218,763 (hundred-thousands)
18. 923 (hundreds)
19. 3.671 (hundredths)
20. 56.3 (ones)
21. 327.146 (tenths)
22. 83,261,000 (millions)
23. 5,462,371 (ten-thousands)
24. 7.8662 (thousandths)
25. 272,341 (hundred-thousands)
26. 63.715 (tenths)
27. 264.97348 (ten-thousandths)
28. 1,655,432 (thousands)
29. 482.6002 (hundredths)
30. 426.861356 (hundred-thousandths)

For Exercises 31–34, estimate the result of the computation by rounding the numbers involved, then use a calculator to find the exact value and find the percent error. (Note: Percent error is the amount of error divided by the exact value, written in percentage form.)

31. $-4.21(7.38 + 3.51)$
32. $10.24(-8.93 + 2.77)$
33. $\dfrac{\sqrt{9.36}}{7.423-9.1}$
34. $\dfrac{47.256-9.90}{\sqrt{24.501}}$

Applications in Our World

For Exercises 35–50, supply the requested estimate, then describe your answer as an overestimate or underestimate.

35. Estimate the total cost of eight high-intensity LED lightbulbs on sale for $16.99 each.
36. Estimate the cost of five months of HD cable at $39.95 per month.
37. Estimate the time it would take you to drive 237 miles at 37 miles per hour.
38. Estimate the distance you can travel in 3 hours 25 minutes if you drive on average 42 miles per hour.
39. Estimate the sale price of a futon you saw on eBay that costs $178.99 and is now on sale for 60% off.
40. Estimate the sale price of a sweater that costs $42.99, on sale for 15% off.
41. Estimate the total cost of the following meal at McDonald's:

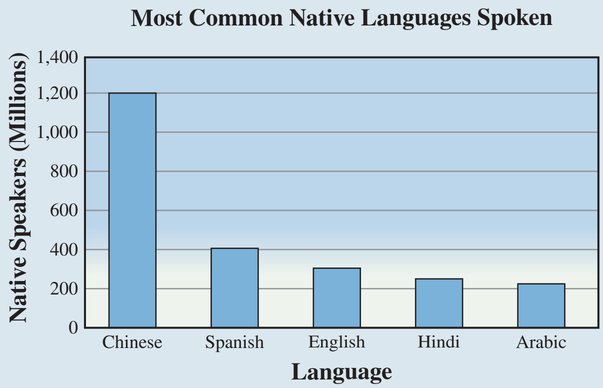
Quarter pounder with cheese	$3.89
Large fries	$1.89
Small shamrock shake	$1.29

42. Estimate the total cost of the following items for a dorm room:

Loft bed	$159.95
Beanbag chair	$49.95
Storage cubes	$29.95
Lava lamp	$19.95

43. A group of five architecture students enters a design for an eco-friendly building in a contest, and wins third place, with a prize of $950. Estimate how much each student will get.
44. A biology lab houses 47 rats for experiments, and they go through about 105 pounds of food each week. Estimate how much food the average rat eats per week.
45. If Erin earns $48,300.00 per year, estimate how much she earns per hour. Assume that she works 40 hours per week and 50 weeks per year.
46. If Jamaal earns $8.75 per hour, estimate how much he would earn per year. Assume that he works 40 hours per week and 50 weeks per year.
47. Estimate the cost of putting up a decorative border in your family room if the room is 24 feet long and 18 feet wide and the border costs $5.95 every 10 feet.
48. Estimate the cost of painting a concrete patio if it's a 12 foot by 16 foot rectangle, and a quart of paint that covers 53 square feet costs $11.99.
49. The Green Party at a large university plans to line both sides of a 30-foot-long hallway with posters endorsing a candidate for state senate. Each of the posters costs them $4.95, they're 2 feet wide, and there will be 5 feet between posters. Estimate how much this will cost.
50. Estimate your cost to live in an apartment for 1 year if the rent is $365.00 per month and utilities are $62.00 per month.

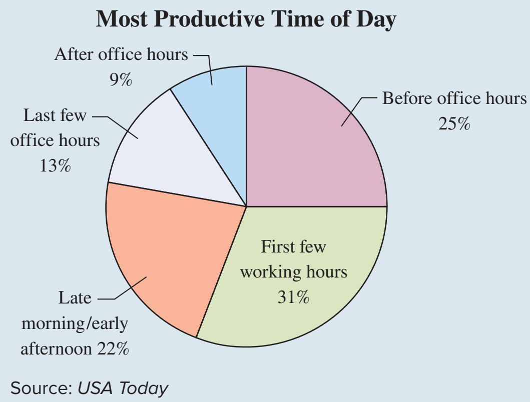
Use the information shown in the bar graph for Exercises 51–54. The graph shows the number of people (in millions) that are native speakers of the top five most common languages in the world.



Source: Ethnologue

51. Estimate the number of native English speakers in the world.
52. Estimate the number of native Chinese speakers in the world.
53. Estimate the difference in the number of native speakers between the first and fifth most common languages.
54. Which has more native speakers: Chinese, or the next four languages combined?

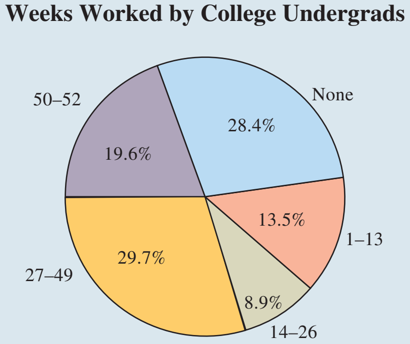
Use the information shown in the graph for Exercises 55–58. The graph represents a survey of 1,385 office workers and shows the percent of people who indicated what time of day is most productive for them.



Source: *USA Today*

55. Estimate the number of people who feel they are most productive outside normal office hours.
56. Estimate the number of people who feel they are most productive before late morning.
57. How many more people feel they're most productive in the first few working hours compared to those that feel they're most productive in the last few office hours?
58. How many times more people are most productive before office hours compared to after?

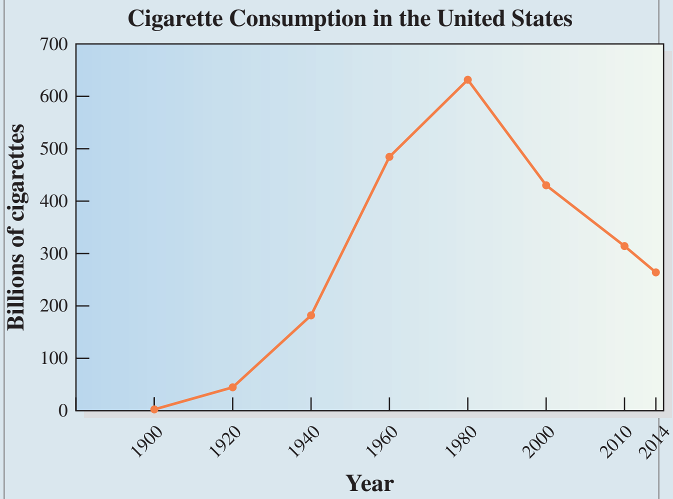
The next graph covers Exercises 59–62. It shows the percentage of college undergraduate students who worked at least part-time for a certain number of weeks out of the year in 2013.



Source: U.S. Census Bureau

59. About what percent of students worked at least some during the year? Round to the nearest full percent.
60. About what percent of students worked less than half of the year? Round to the nearest full percent.
61. On one campus, 620 undergraduate students work 50 or more weeks. If this student body is average in terms of work habits, about how many undergrads would you expect there to be on the entire campus?
62. In a survey at one community college, 310 undergrads surveyed said they don't work at all. If this student body is average in terms of work habits, about how many undergrads would you expect were surveyed total?

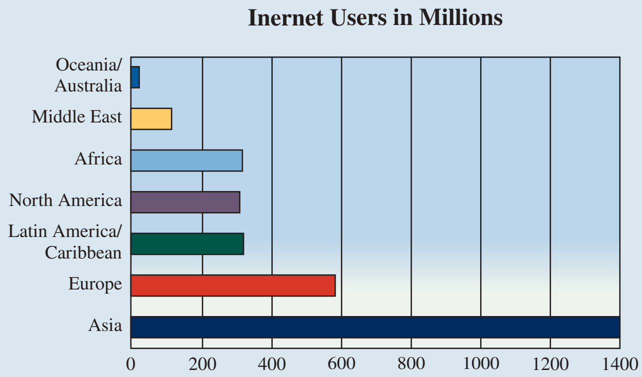
Use the line graph shown for Exercises 63–68. The graph shows annual cigarette consumption (in billions) for the United States for the years 1900 to 2014.

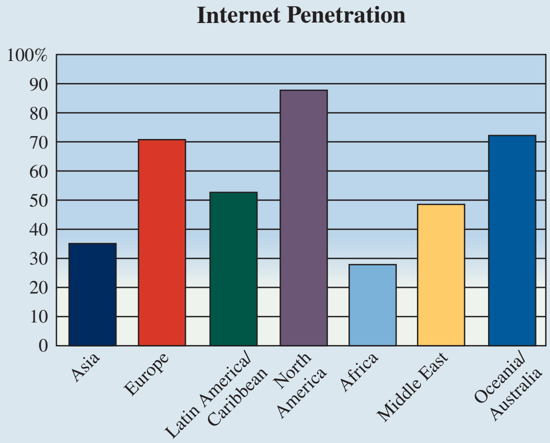


Source: www.infoplease.com

63. Estimate the number of cigarettes smoked in 1950.
64. Estimate the number of cigarettes smoked in 1985.
65. Estimate the year or years in which 200 billion cigarettes were smoked.
66. Estimate the year or years in which 400 billion cigarettes were smoked.
67. Find the average rate of change in cigarette consumption for the years shown when consumption was increasing.
68. Find the average rate of change in cigarette consumption for the years shown when consumption was decreasing.

The next two graphs describe Internet access by continent as of 2016. The first shows the NUMBER of people that have Internet access in the place where they live, in millions. The second shows Internet penetration, which is the PERCENTAGE of people on each continent that have Internet access where they live. Use the graphs to answer Exercises 69–74.

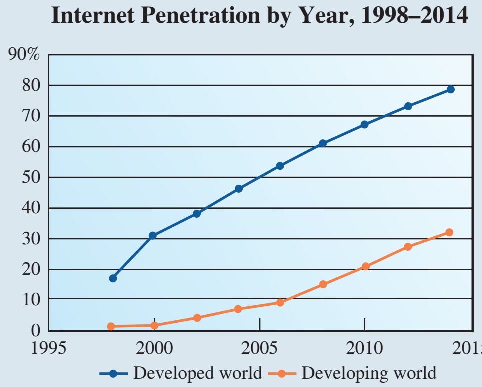




Source: www.internetworldstats.com

69. Which continent has the most people with Internet access at home? The least?
70. If you picked a person at random from each continent, on which continent would it be most likely that the person has Internet access at home? Least likely?
71. Estimate the number of Internet users in Europe, and the Internet penetration in Asia.
72. Estimate the Internet penetration in Africa and the number of Internet users in the Middle East.
73. What do you think accounts for the fact that North America has the longest bar on the second graph, but is middle of the pack on the first one?
74. Does Asia have more Internet users than the rest of the world combined? Justify your answer.

The next graph displays growth in Internet penetration from 1998 to 2014 in the developed world compared to growth in the developing world. Use this graph to answer Exercises 75–78

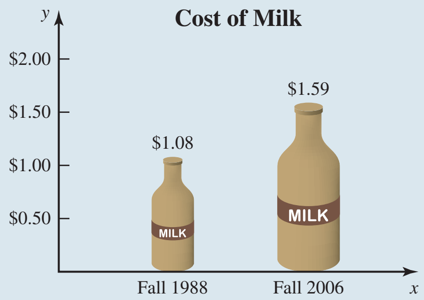


Source: International Telecommunications Union

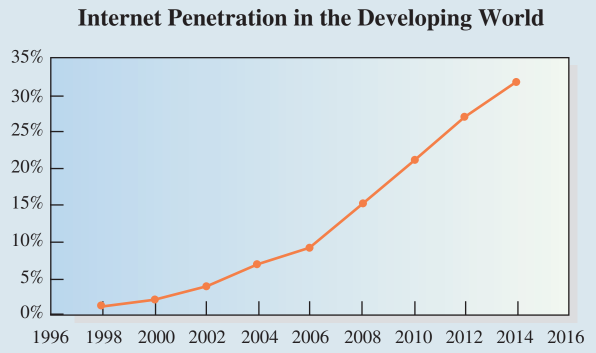
75. By how much did the percentage change from 1998 to 2014 in the developed world? What about the developing world?
76. Is the gap between the developed and developing worlds getting bigger or smaller? Discuss.
77. Which had a more significant increase between 1998 and 2014, the developed or developing world? Explain how you decided.
78. Which of the trends shown on the graph is likely to continue for the next 5–10 years? Explain how you decided.
79. In Jared's math class, he scored 84, 92, 79, and 86 on the first four tests. He needs at least an 80% overall test average to earn the B he's shooting for. Without doing any computation at all, make a guess as to the score you think he would need on the final (which is worth two test grades) to have an average of at least 80%. Then find what his average would be if he got the score you guessed. How did you do?
80. Entering a postseason basketball tournament, Marta's goal is to average at least 15 points per game for the tournament. (Marta is not the greatest team player in the world.) She scores 16, 12, 9, and 18 in the first four games. How many points do you think she will need to score in the remaining two games to reach her goal? First, make a guess without doing any computation. Then find what her average would be if she scored the number of points you guessed. How did you do?
81. One of the most valuable uses of estimation is to roughly keep track of the total cost of items when shopping. Use rounding to estimate the total cost of the following items at a grocery store: 4 cans of green beans at 79 cents each; 8 cups of yogurt at 49 cents each; a 29-ounce steak at \$5.80 per pound; 4 energy drinks at \$1.29 each; and 100 ounces of mineral water at \$3.08 per gallon. (*Hint:* You may need to look up conversions for units of weight and volume.)
82. Refer to Problem 81. In getting my swimming pool ready for summer, I usually buy the following supplies. Use rounding to estimate the total cost: 8 boxes of baking soda at 89 cents per box; 20 gallons of bleach at \$1.29 for 96 ounces; 8 pounds of chlorine stabilizer at \$11.99 for a 4-pound bottle; and four 24-can cases of refreshments at 60 cents per can.

Critical Thinking

83. Sometimes graphs are drawn in such a way as to support a conclusion that may or may not be true. Look at the graph and see if you can find anything misleading.



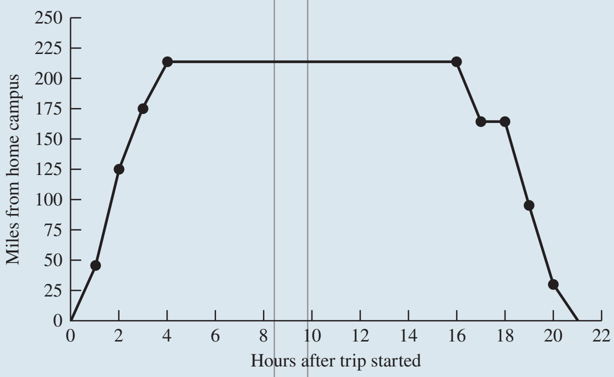
84. The choice of labeling on a graph can have a profound effect on how the information is perceived. Compare the graph to the one from Exercises 75–78. It contains the same information for the developing world as the earlier graph. Why does it appear to show a much sharper increase in Internet penetration?



Recall that the average speed of an object over some time period is calculated using the formula: speed = distance/time. The following graph describes an overnight road trip to go to an off-campus party at another college: the horizontal axis is hours driven, and the vertical axis is miles from the home campus. Use the graph to answer Exercises 85–90.

85. Estimate the average speed over the first hour of the trip, then the first 2 hours, the first 3 hours, and the first

4 hours. Based on your results, write a description of the trip.



86. How long did the road-trippers stay at the other school? How far away was it from their own campus?
87. Estimate the average speed from time 16 hours to 17 hours, then from 17 hours to 18 hours. What is the significance of the sign in the first answer? What happened between 17 and 18 hours?
88. Without doing any calculations, how can you tell if the average speed on the trip back was more or less than the average speed on the trip there?
89. Using your results from Exercises 85–88, what feature of the graph do you think corresponds to the average speed over a portion of the trip?
90. Find the average speed for the whole trip. Why is your answer nonsense? What can you conclude about using the average speed formula?

Section 1-3 Problem-Solving Strategies

LEARNING OBJECTIVES

1. State the four steps in the basic problem-solving procedure.
2. Solve problems using a diagram.
3. Solve problems using trial and error.
4. Solve problems involving money.
5. Solve problems using calculation.

Here's an idea that can help you understand your math classes better: once in a while, think about what some of the math words you take for granted actually mean in English. For example, have you ever thought about why we call math problems "problems?" In real life, when a problem confronts you, you probably think about different strategies you might use to overcome an obstacle and then decide on the best course of action. So why not use that same strategy in solving math problems? This is probably the single biggest reason why taking math classes is useful to *anyone*: math is all about learning and practicing problem-solving strategies.

A Hungarian mathematician named George Polya did a lot of research on the nature of problem solving in the first half of the 20th century. His biggest contribution to the field was an attempt to identify a series of steps that were fundamental to problem-solving strategies used by great thinkers throughout human history. One of his books, published in 1945 (and still a big seller on Amazon!), set forth these basic steps. *How to Solve It* is so widely read that it has been translated into at least 17 languages.

Polya's strategy isn't necessarily earth-shattering: its brilliance lies in its simplicity. It provides four basic steps that can be used as a framework for problem solving in any area, from math to home improvements.

1. State the four steps in the basic problem-solving procedure.

Polya's Four-Step Problem-Solving Procedure

Step 1 *Understand the problem.* The best way to start any problem is to write down information that's provided as you come to it. Especially with longer word problems, if you read the whole thing all at once and don't DO anything, it's easy to get overwhelmed. If you read the problem slowly and carefully, writing down information as it's provided, you'll always at least have a start on the problem. Another great idea: carefully identify and *write down* what it is they're asking you to find; this almost always helps you to devise a strategy.

Step 2 *Devise a plan to solve the problem.* This is where problem solving is at least as much art as science—there are many, many ways to solve problems. Some common strategies: making a list of possible outcomes; drawing a diagram; trial and error; finding a similar problem that you already know how to solve; and using arithmetic, algebra, or geometry.