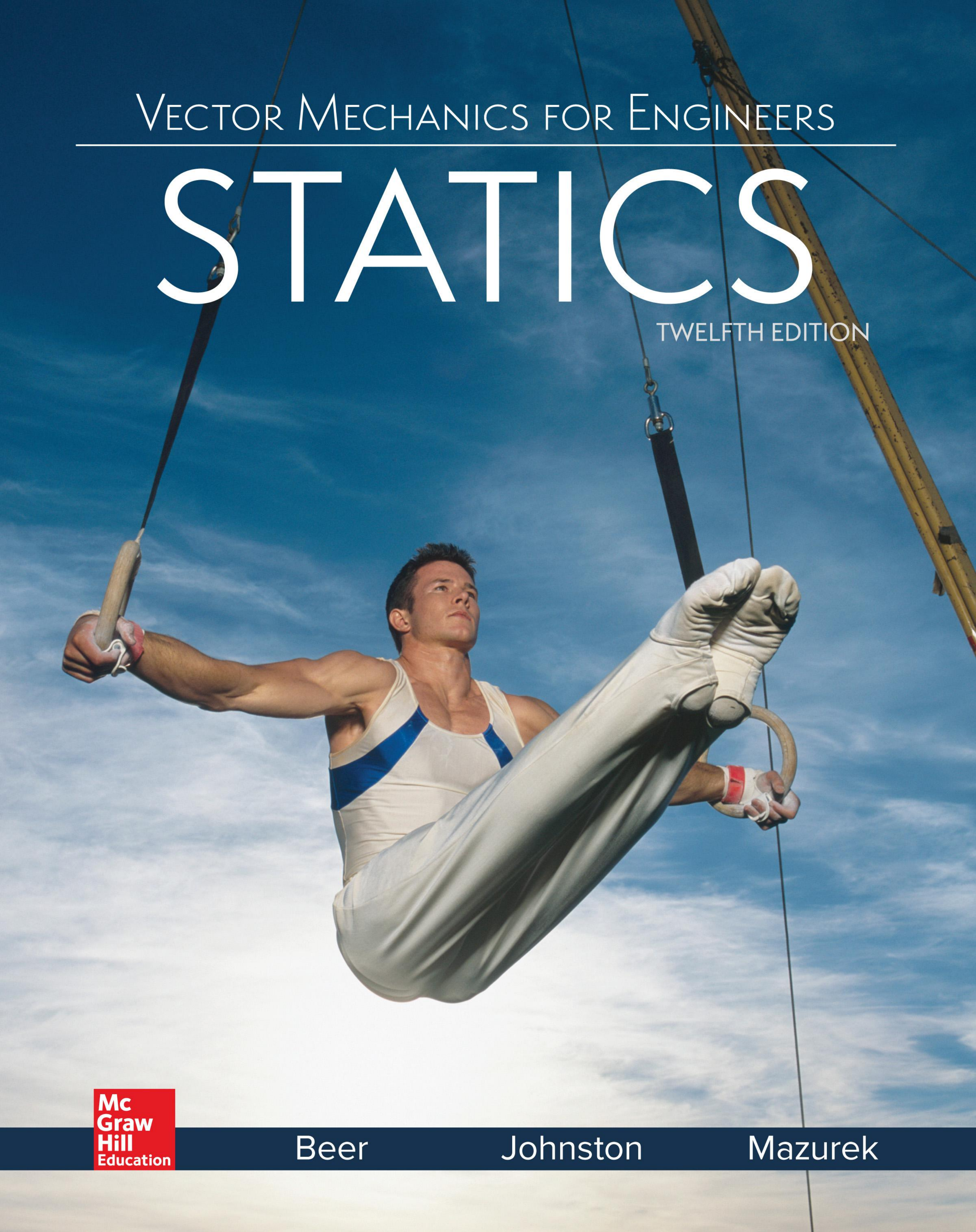

Vector Mechanics for Engineers

STATICS

TWELFTH EDITION

McGraw Hill Education

Beer Johnston Mazurek

Twelfth Edition

Vector Mechanics For Engineers

Statics

Ferdinand P. Beer
Late of Lehigh University

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Mc Graw Hill Education

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Preface

Objectives

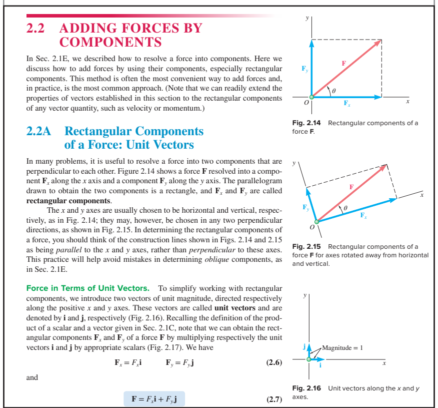
A primary objective in a first course in mechanics is to help develop a student's ability first to analyze problems in a simple and logical manner, and then to apply basic principles to its solution. A strong conceptual understanding of these basic mechanics principles is essential for successfully solving mechanics problems. We hope that this text, as well as the proceeding volume, *Vector Mechanics for Engineers: Dynamics,* will help instructors achieve these goals.†

General Approach

Vector analysis is introduced early in the text and is used in the presentation and discussion of the fundamental principles of mechanics. Vector methods are also used to solve many problems, particularly three-dimensional problems where these techniques result in a simpler and more concise solution. The emphasis in this text, however, remains on the correct understanding of the principles of mechanics and on their application to the solution of engineering problems, and vector analysis is presented chiefly as a convenient tool.‡

Practical Applications Are Introduced Early. One of the characteristics of the approach used in this book is that mechanics of *particles* is clearly separated from the mechanics of *rigid bodies.* This approach makes it possible to consider simple practical applications at an early stage and to postpone the introduction of the more difficult concepts. For example:

- In *Statics,* statics of particles is treated first (Chap. 2); after the rules of addition and subtraction of vectors are introduced, the principle of equilibrium of a particle is immediately applied to practical situations involving only concurrent forces. The statics of rigid bodies is considered in Chaps. 3 and 4. In Chap. 3, the vector and scalar products of two vectors are introduced and used to define the moment of a force about a point and about an axis. The presentation of these new concepts is followed by a thorough and rigorous discussion of equivalent systems of forces leading, in Chap. 4, to many practical applications involving the equilibrium of rigid bodies under general force systems.
- In *Dynamics,* the same division is observed. The basic concepts of force, mass, and acceleration, of work and energy, and of impulse and momentum are introduced and first applied to problems involving only particles. Thus, students can familiarize themselves with the three basic methods used in dynamics and learn their respective advantages before facing the difficulties associated with the motion of rigid bodies.



2.2 ADDING FORCES BY COMPONENTS

In Sec. 2.1E, we described how to resolve a force into components. Here we discuss how to add forces by using their components, especially rectangular components. This method is often the most convenient way to add forces and, in practice, is the most common approach. (Note that we can readily extend the properties of vectors established in this section to the rectangular components of any vector quantity, such as velocity or momentum.)

2.2A Rectangular Components of a Force: Unit Vectors

In many problems, it is useful to resolve a force into two components that are perpendicular to each other. Figure 2.14 shows a force **F** resolved into a component $\mathbf{F}_x$ along the x axis and a component $\mathbf{F}_y$ along the y axis. The parallelogram drawn to obtain the two components is a rectangle, and $\mathbf{F}_x$ and $\mathbf{F}_y$ are called **rectangular components**.

The x and y axes are usually chosen to be horizontal and vertical, respectively, as in Fig. 2.14; they may, however, be chosen in any two perpendicular directions, as shown in Fig. 2.15. In determining the rectangular components of a force, you should think of the construction lines shown in Figs. 2.14 and 2.15 as being *parallel* to the x and y axes, rather than *perpendicular* to these axes. This practice will help avoid mistakes in determining *oblique* components, as in Sec. 2.1E.

Force in Terms of Unit Vectors. To simplify working with rectangular components, we introduce two vectors of unit magnitude, directed respectively along the positive x and y axes. These vectors are called **unit vectors** and are denoted by **i** and **j**, respectively (Fig. 2.16). Recalling the definition of the product of a scalar and a vector given in Sec. 2.1C, note that we can obtain the rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$ of a force **F** by multiplying respectively the unit vectors **i** and **j** by appropriate scalars (Fig. 2.17). We have

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j} \tag{2.6}$$

and

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \tag{2.7}$$

Fig. 2.14 Rectangular components of a force **F**.

Fig. 2.15 Rectangular components of a force **F** for axes rotated away from horizontal and vertical.

Fig. 2.16 Unit vectors along the x and y axes.

†Both texts also are available in a single volume, *Vector Mechanics for Engineers: Statics and Dynamics,* eleventh edition.
‡In a parallel text, *Mechanics for Engineers: Statics,* fifth edition, the use of vector algebra is limited to the addition and subtraction of vectors.

New Concepts Are Introduced in Simple Terms. Since this text is designed for the first course in statics, new concepts are presented in simple terms and every step is explained in detail. On the other hand, by discussing the broader aspects of the problems considered, and by stressing methods of general applicability, a definite maturity of approach is achieved. For example, the concepts of partial constraints and statical indeterminacy are introduced early and are used throughout.

Fundamental Principles Are Placed in the Context of Simple Applications. The fact that mechanics is essentially a *deductive* science based on a few fundamental principles is stressed. Derivations have been presented in their logical sequence and with all the rigor warranted at this level. However, the learning process being largely *inductive,* simple applications are considered first. For example:

- The statics of particles precedes the statics of rigid bodies, and problems involving internal forces are postponed until Chap. 6.
- In Chap. 4, equilibrium problems involving only coplanar forces are considered first and solved by ordinary algebra, while problems involving three-dimensional forces and requiring the full use of vector algebra are discussed in the second part of the chapter.

Systematic Problem-Solving Approach. All the sample problems are solved using the steps of ***S***trategy, ***M***odeling, ***A***nalysis, and ***R***eflect & ***T***hink, or the "SMART" approach. This methodology is intended to give students confidence when approaching new problems, and students are encouraged to apply this approach in the solution of all assigned problems.

Free-Body Diagrams Are Used Both to Solve Equilibrium Problems and to Express the Equivalence of Force Systems. Free-body diagrams are introduced early, and their importance is emphasized throughout the text. They are used not only to solve equilibrium problems but also to express the equivalence of two systems of forces or, more generally, of two systems of vectors. The advantage of this approach becomes apparent in the study of the dynamics of rigid bodies, where it is used to solve three-dimensional as well as two-dimensional problems. By placing the emphasis on "free-body-diagram equations" rather than on the standard algebraic equations of motion, a more intuitive and more complete understanding of the fundamental principles of dynamics can be achieved. This approach, which was first introduced in 1962 in the first edition of *Vector Mechanics for Engineers,* has now gained wide acceptance among mechanics teachers in this country. It is, therefore, used in preference to the method of dynamic equilibrium and to the equations of motion in the solution of all sample problems in this book.

A Four-Color Presentation Uses Color to Distinguish Vectors. Color has been used, not only to enhance the quality of the illustrations, but also to help students distinguish among the various types of vectors they will encounter. While there was no intention to "color code" this text, the same color is used in any given chapter to represent vectors of the same type. Throughout *Statics,* for example, red is used exclusively to represent forces and couples, while position vectors are shown in blue and dimensions in black. This makes it easier for the students to identify the forces acting on a given particle or rigid body and to follow the discussion of sample problems and other examples given in the text.

17.1 ENERGY METHODS FOR A RIGID BODY

We now use the principle of work and energy to analyze the plane motion of rigid bodies. As we pointed out in Chap. 13, the method of work and energy is particularly well-adapted to solving problems involving velocities and displacements. Its main advantage is that the work of forces and the kinetic energy of objects are scalar quantities.

17.1A Principle of Work and Energy

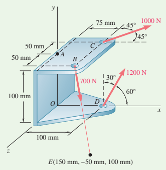
To apply the principle of work and energy to the motion of a rigid body, we again assume that the rigid body is made up of a large number n of particles of mass Δm_i. From Eq. (14.30) of Sec. 14.2B, we have

Principle of work and energy, rigid body

$$T_1 + U_{1\to 2} = T_2 \qquad (17.1)$$

where T_1, T_2 = the initial and final values of total kinetic energy of particles forming the rigid body

$U_{1\to 2}$ = work of all forces acting on various particles of the body



Sample Problem 3.10

Three cables are attached to a bracket as shown. Replace the forces exerted by the cables with an equivalent force-couple system at A.

STRATEGY: First determine the relative position vectors drawn from point A to the points of application of the various forces and resolve the forces into rectangular components. Then, sum the forces and moments.

MODELING and ANALYSIS: Note that $\mathbf{F}_B = (700\text{ N})\boldsymbol{\lambda}_{BE}$, where

$$\boldsymbol{\lambda}_{BE} = \frac{\overrightarrow{BE}}{BE} = \frac{75\mathbf{i} - 150\mathbf{j} + 50\mathbf{k}}{175}$$

Using meters and newtons, the position and force vectors are

$$\mathbf{r}_{B/A} = \overrightarrow{AB} = 0.075\mathbf{i} + 0.050\mathbf{k} \qquad \mathbf{F}_B = 300\mathbf{i} - 600\mathbf{j} + 200\mathbf{k}$$
$$\mathbf{r}_{C/A} = \overrightarrow{AC} = 0.075\mathbf{i} - 0.050\mathbf{k} \qquad \mathbf{F}_C = 707\mathbf{i} - 707\mathbf{k}$$
$$\mathbf{r}_{D/A} = \overrightarrow{AD} = 0.100\mathbf{i} - 0.100\mathbf{j} \qquad \mathbf{F}_D = 600\mathbf{i} + 1039\mathbf{j}$$

The force-couple system at A equivalent to the given forces consists of a force $\mathbf{R} = \Sigma\mathbf{F}$ and a couple $\mathbf{M}_A^R = \Sigma(\mathbf{r} \times \mathbf{F})$. Obtain the force $\mathbf{R}$ by adding respectively the x, y, and z components of the forces:

$$\mathbf{R} = \Sigma\mathbf{F} = (1607\text{ N})\mathbf{i} + (439\text{ N})\mathbf{j} - (507\text{ N})\mathbf{k}$$

(continued)

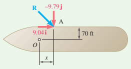


Fig. 3 The point of application of a single tugboat to create the same effect as the given force system.

Remark: Because all the forces are contained in the plane of the figure, you would expect the sum of their moments to be perpendicular to that plane. Note that you could obtain the moment of each force component directly from the diagram by first forming the product of its magnitude and perpendicular distance to O and then assigning to this product a positive or a negative sign, depending upon the sense of the moment.

b. Single Tugboat. The force exerted by a single tugboat must be equal to $\mathbf{R}$, and its point of application A must be such that the moment of $\mathbf{R}$ about O is equal to $\mathbf{M}_O^R$ (Fig. 3). Observing that the position vector of A is

$$\mathbf{r} = x\mathbf{i} + 70\mathbf{j}$$

you have

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O^R$$
$$(x\mathbf{i} + 70\mathbf{j}) \times (9.04\mathbf{i} - 9.79\mathbf{j}) = -1035\mathbf{k}$$
$$-x(9.79)\mathbf{k} - 633\mathbf{k} = -1035\mathbf{k} \qquad x = 41.1\text{ ft}$$

REFLECT and THINK: Reducing the given situation to that of a single force makes it easier to visualize the overall effect of the tugboats in maneuvering the ocean liner. But in practical terms, having four boats applying force allows for greater control in slowing and turning a large ship in a crowded harbor.

A Careful Balance Between SI and U.S. Customary Units Is Consistently Maintained. Because of the current trend in the American government and industry to adopt the international system of units (SI metric units), the SI units most frequently used in mechanics are introduced in Chap. 1 and are used throughout the text. Approximately half of the sample problems and 60 percent of the homework problems are stated in these units, while the remainder are in U.S. customary units. The authors believe that this approach will best serve the need of students, who, as engineers, will have to be conversant with both systems of units.

It also should be recognized that using both SI and U.S. customary units entails more than the use of conversion factors. Since the SI system of units is an absolute system based on the units of time, length, and mass, whereas the U.S. customary system is a gravitational system based on the units of time, length, and force, different approaches are required for the solution of many problems. For example, when SI units are used, a body is generally specified by its mass expressed in kilograms; in most problems of statics it will be necessary to determine the weight of the body in newtons, and an additional calculation will be required for this purpose. On the other hand, when U.S. customary units are used, a body is specified by its weight in pounds and, in dynamics problems, an additional calculation will be required to determine its mass in slugs (or lb·s^2/ft). The authors, therefore, believe that problem assignments should include both systems of units.

The *Instructor's and Solutions Manual* provides six different lists of assignments so that an equal number of problems stated in SI units and in U.S. customary units can be selected. If so desired, two complete lists of assignments can also be selected with up to 75 percent of the problems stated in SI units.

Optional Sections Offer Advanced or Specialty Topics. A large number of optional sections have been included. These sections are indicated by asterisks and thus are easily distinguished from those which form the core of the basic statics course. They can be omitted without prejudice to the understanding of the rest of the text.

Among the topics covered in these additional sections are the reduction of a system of forces to a wrench, applications to hydrostatics, equilibrium of cables, products of inertia and Mohr's circle, the determination of the principal axes and the mass moments of inertia of a body of arbitrary shape, and the method of virtual work. The sections on the inertia properties of three-dimensional bodies are primarily intended for students who will later study in dynamics the three-dimensional motion of rigid bodies.

The material presented in the text and most of the problems require no previous mathematical knowledge beyond algebra, trigonometry, and elementary calculus; all the elements of vector algebra necessary to the understanding of the text are carefully presented in Chaps. 2 and 3. In general, a greater emphasis is placed on the correct understanding of the basic mathematical concepts involved than on the nimble manipulation of mathematical formulas. In this connection, it should be mentioned that the determination of the centroids of composite areas precedes the calculation of centroids by integration, thus making it possible to establish the concept of the moment of an area firmly before introducing the use of integration.

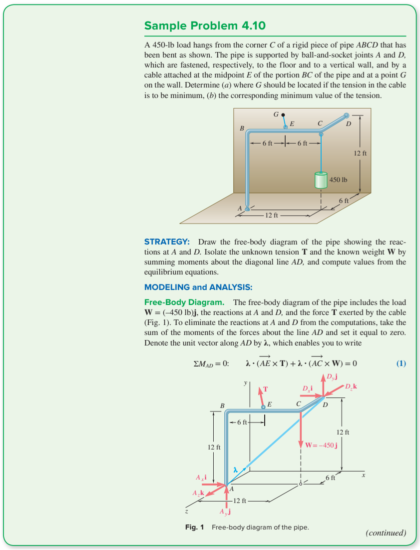
Guided Tour

Chapter Introduction. Each chapter begins with a list of learning objectives and an outline that previews chapter topics. An introductory section describes the material to be covered in simple terms, and how it will be applied to the solution of engineering problems.

Chapter Lessons. The body of the text is divided into sections, each consisting of one or more sub-sections, several sample problems, and a large number of end-of-section problems for students to solve. Each section corresponds to a well-defined topic and generally can be covered in one lesson. In a number of cases, however, the instructor will find it desirable to devote more than one lesson to a given topic. *The Instructor's and Solutions Manual* contains suggestions on the coverage of each lesson.

Concept Applications. Concept Applications are used within selected theory sections to amplify certain topics, and they are designed to reinforce the specific material being presented and facilitate its understanding.

Sample Problems. The Sample Problems are set up in much the same form that students will use when solving assigned problems, and they employ the SMART problem-solving methodology that students are encouraged to use in the solution of their assigned problems. They thus serve the double purpose of amplifying the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions. In addition, in-problem references and captions have been added to the sample problem figures for contextual linkage to the step-by-step solution. In the digital version, many Sample Problems now have simulations to help students visualize the problem. Enhanced digital content is indicated by a ▶ within the text.


Sample Problem 4.10

A 450-lb load hangs from the corner *C* of a rigid piece of pipe *ABCD* that has been bent as shown. The pipe is supported by ball-and-socket joints *A* and *D*, which are fastened, respectively, to the floor and to a vertical wall, and by a cable attached at the midpoint *E* of the portion *BC* of the pipe and at a point *G* on the wall. Determine (*a*) where *G* should be located if the tension in the cable is to be minimum, (*b*) the corresponding minimum value of the tension.

STRATEGY: Draw the free-body diagram of the pipe showing the reactions at *A* and *D*. Isolate the unknown tension **T** and the known weight **W** by summing moments about the diagonal line *AD*, and compute values from the equilibrium equations.

MODELING and ANALYSIS:

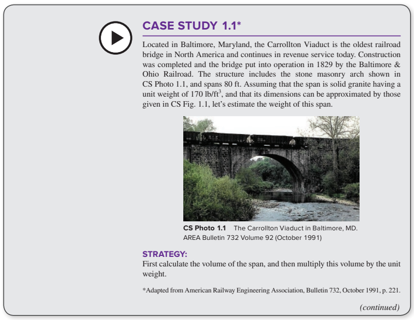
Free-Body Diagram. The free-body diagram of the pipe includes the load **W** = (–450 lb)**j**, the reactions at *A* and *D*, and the force **T** exerted by the cable (Fig. 1). To eliminate the reactions at *A* and *D* from the computations, take the sum of the moments of the forces about the line *AD* and set it equal to zero. Denote the unit vector along *AD* by **λ**, which enables you to write

$$\Sigma M_{AD} = 0: \quad \boldsymbol{\lambda} \cdot (\overrightarrow{AE} \times \mathbf{T}) + \boldsymbol{\lambda} \cdot (\overrightarrow{AC} \times \mathbf{W}) = 0 \qquad (1)$$

Fig. 1 Free-body diagram of the pipe.

(continued)

▶ **Case Studies.** Statics principles are used extensively in engineering applications, particularly for design and for analysis of engineering failures. Much can be learned from the historical successes and failures of past designs, and unique insight can be gained by studying how engineers developed different products and structures. To this end, real-world Case Studies have been introduced in this text to provide relevancy and application to the principles of engineering mechanics being discussed. These are developed using the SMART problem-solving methodology to both present the story behind each Case Study, as well as to analyze some aspect of the situation. In some instances, these Case Studies are examined further in the accompanying digital content through Connect®. The digital content also provides additional cases that are developed in their entirety.


CASE STUDY 1.1*

Located in Baltimore, Maryland, the Carrollton Viaduct is the oldest railroad bridge in North America and continues in revenue service today. Construction was completed and the bridge put into operation in 1829 by the Baltimore & Ohio Railroad. The structure includes the stone masonry arch shown in CS Photo 1.1, and spans 80 ft. Assuming that the span is solid granite having a unit weight of 170 lb/ft^3, and that its dimensions can be approximated by those given in CS Fig. 1.1, let's estimate the weight of this span.

CS Photo 1.1 The Carrollton Viaduct in Baltimore, MD. AREA Bulletin 732 Volume 92 (October 1991)

STRATEGY:
First calculate the volume of the span, and then multiply this volume by the unit weight.

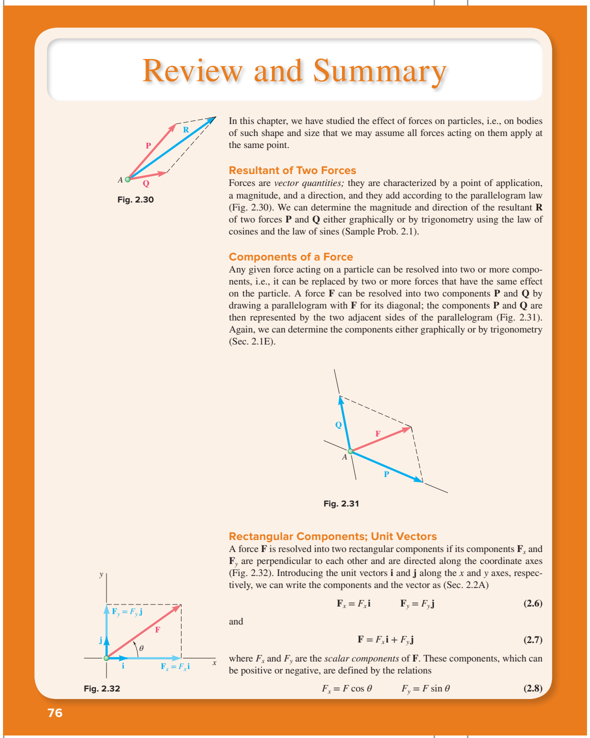
*Adapted from American Railway Engineering Association, Bulletin 732, October 1991, p. 221.

(continued)

Solving Problems on Your Own. A section entitled *Solving Problems on Your Own* is included for each lesson, between the sample problems and the problems to be assigned. The purpose of these sections is to help students organize in their own minds the preceding theory of the text and the solution methods of the sample problems so that they can more successfully solve the

NEW!

Over 350 of the homework problems in the text are new or revised.



Review and Summary

In this chapter, we have studied the effect of forces on particles, i.e., on bodies of such shape and size that we may assume all forces acting on them apply at the same point.

Fig. 2.30

Resultant of Two Forces

Forces are *vector quantities;* they are characterized by a point of application, a magnitude, and a direction, and they add according to the parallelogram law (Fig. 2.30). We can determine the magnitude and direction of the resultant **R** of two forces **P** and **Q** either graphically or by trigonometry using the law of cosines and the law of sines (Sample Prob. 2.1).

Components of a Force

Any given force acting on a particle can be resolved into two or more components, i.e., it can be replaced by two or more forces that have the same effect on the particle. A force **F** can be resolved into two components **P** and **Q** by drawing a parallelogram with **F** for its diagonal; the components **P** and **Q** are then represented by the two adjacent sides of the parallelogram (Fig. 2.31). Again, we can determine the components either graphically or by trigonometry (Sec. 2.1E).

Fig. 2.31

Rectangular Components; Unit Vectors

A force **F** is resolved into two rectangular components if its components $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular to each other and are directed along the coordinate axes (Fig. 2.32). Introducing the unit vectors **i** and **j** along the x and y axes, respectively, we can write the components and the vector as (Sec. 2.2A)

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j} \tag{2.6}$$

and

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \tag{2.7}$$

where F_x and F_y are the *scalar components* of **F**. These components, which can be positive or negative, are defined by the relations

$$F_x = F\cos\theta \qquad F_y = F\sin\theta \tag{2.8}$$

Fig. 2.32

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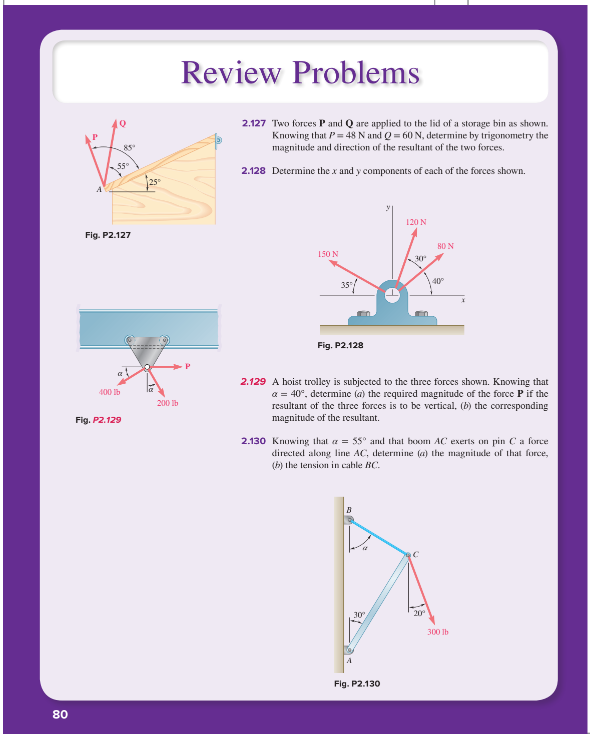

Review Problems

2.127 Two forces **P** and **Q** are applied to the lid of a storage bin as shown. Knowing that $P = 48$ N and $Q = 60$ N, determine by trigonometry the magnitude and direction of the resultant of the two forces.

Fig. P2.127

2.128 Determine the x and y components of each of the forces shown.

Fig. P2.128

2.129 A hoist trolley is subjected to the three forces shown. Knowing that $\alpha = 40°$, determine (*a*) the required magnitude of the force **P** if the resultant of the three forces is to be vertical, (*b*) the corresponding magnitude of the resultant.

Fig. *P2.129*

2.130 Knowing that $\alpha = 55°$ and that boom AC exerts on pin C a force directed along line AC, determine (*a*) the magnitude of that force, (*b*) the tension in cable BC.

Fig. P2.130

80

homework problems. Also included in these sections are specific suggestions and strategies that will enable the students to more efficiently attack any assigned problems.

Homework Problem Sets. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and to help students understand the principles of mechanics. The problems are grouped according to the portions of material they illustrate and, in general, are arranged in order of increasing difficulty. Problems requiring special attention are indicated by asterisks. Answers to 70 percent of the problems are given at the end of the book. Problems for which the answers are given are set in straight type in the text, while problems for which no answer is given are set in italic and red font color.

Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in that chapter. Marginal notes are used to help students organize their review work, and cross-references have been included to help them find the portions of material requiring their special attention.

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.

Computer Problems. Accessible through Connect are problem sets for each chapter that are designed to be solved with computational software. Many of these problems are relevant to the design process; they may involve the analysis of a structure for various configurations and loadings of the structure, or the determination of the equilibrium positions of a given mechanism that may require an iterative method of solution. Developing the algorithm required to solve a given mechanics problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply their computer skills to the solution of a meaningful engineering problem.

Digital Resources

Connect® is a highly reliable, easy-to-use homework and learning management solution that embeds learning science and award-winning adaptive tools to improve student results.

Analytics Connect Insight is Connect's one-of-a-kind visual analytics dashboard. Now available for both instructors and students, it provides at-a-glance information regarding student performance, which is immediately actionable. By presenting assignment, assessment, and topical performance results together with a time metric that is easily visible for aggregate or individual results, Connect InSight gives the user the ability to take a just-in-time approach to teaching and learning, which was never before available. Connect Insight presents data that empower students and help instructors improve class performance in a way that is efficient and effective.

Autograded Free-Body Diagram Problems

NEW!

- Within Connect, algorithmic end-of-chapter problems include our new Free-Body Diagram Drawing tool. The Free-Body Diagram Tool allows students to draw free-body diagrams that are auto graded by the system. Student's receive immediate feedback on their diagrams to help student's solidify their understanding of the physical situation presented in the problem.

Case Study Interactives

New digital content has been added throughout the text to enhance student learning. This includes a more in-depth discussion of the new Case Studies, as well as interactive questions embedded in these video explorations to make students *think* about the problem rather than just viewing the video. Within the text, simulations and short videos have been added to help students visualize topics, such as zero-force members and the motion of different linkages.

Find the following instructor resources available through Connect:

- **Instructor's and Solutions Manual.** *The Instructor's and Solutions Manual* that accompanies the eleventh edition features solutions to all end of chapter problems. This manual also features a number of tables designed to assist instructors in creating a schedule of assignments for their course. The various topics covered in the text have been listed in Table I and a suggested number of periods to be spent on each topic has been indicated. Table II prepares a brief description of all groups of problems and a classification of the problems in each group according to the units used. Sample lesson schedules are shown in Tables III, IV, and V, together with various alternative lists of assigned homework problems.
- **Lecture PowerPoint Slides** for each chapter that can be modified. These generally have an introductory application slide, animated worked-out problems that you can do in class with your students, concept questions, and "what-if?" questions at the end of the units.

- **Textbook images**
- **Computer Problem sets** for each chapter that are designed to be solved with computational software.
- **C.O.S.M.O.S.,** the Complete Online Solutions Manual Organization System that allows instructors to create custom homework, quizzes, and tests using end-of-chapter problems from the text.

SMARTBOOK® SmartBook helps students study more efficiently by highlighting where in the chapter to focus, asking review questions and pointing them to resources until they understand.

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List of Symbols

a	Constant; radius; distance
A, B, C, . . .	Reactions at supports and connections
$A, B, C, \ldots$	Points
A	Area
b	Width; distance
c	Constant
C	Centroid
d	Distance
e	Base of natural logarithms
F	Force; friction force
g	Acceleration of gravity
G	Center of gravity; constant of gravitation
h	Height; sag of cable
i, j, k	Unit vectors along coordinate axes
$I, I_x, \ldots$	Moments of inertia
$\bar{I}$	Centroidal moment of inertia
$I_{xy}, \ldots$	Products of inertia
J	Polar moment of inertia
k	Spring constant
k_x, k_y, k_O	Radii of gyration
$\bar{k}$	Centroidal radius of gyration
l	Length
L	Length; span
m	Mass
M	Couple; moment
$\mathbf{M}_O$	Moment about point O
$\mathbf{M}_O^R$	Moment resultant about point O
M	Magnitude of couple or moment; mass of earth
M_{OL}	Moment about axis OL
N	Normal component of reaction
O	Origin of coordinates
p	Pressure
P	Force; vector
Q	Force; vector
r	Position vector
r	Radius; distance; polar coordinate
R	Resultant force; resultant vector; reaction
R	Radius of earth
s	Position vector
s	Length of arc; length of cable
S	Force; vector
t	Thickness
T	Force
T	Tension
U	Work
V	Vector product; shearing force
V	Volume; potential energy; shear
w	Load per unit length
W, W	Weight; load
x, y, z	Rectangular coordinates; distances
$\bar{x}, \bar{y}, \bar{z}$	Rectangular coordinates of centroid or center of gravity
α, β, γ	Angles
γ	Specific weight
δ	Elongation
$\delta\mathbf{r}$	Virtual displacement
δU	Virtual work
$\boldsymbol{\lambda}$	Unit vector along a line
η	Efficiency
θ	Angular coordinate; angle; polar coordinate
μ	Coefficient of friction
ρ	Density
ϕ	Angle of friction; angle


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1

Introduction

The tallest skyscraper in the Western Hemisphere, One World Trade Center is a prominent feature of the New York City skyline. From its foundation to its structural components and mechanical systems, the design and operation of the tower is based on the fundamentals of engineering mechanics.

Introduction

1.1 WHAT IS MECHANICS?
1.2 FUNDAMENTAL CONCEPTS AND PRINCIPLES
1.3 SYSTEMS OF UNITS
1.4 CONVERTING BETWEEN TWO SYSTEMS OF UNITS
1.5 METHOD OF SOLVING PROBLEMS
1.6 NUMERICAL ACCURACY

Objectives

- **Define** the science of mechanics and examine its fundamental principles.
- **Discuss** and compare the International System of Units and U.S. customary units.
- **Discuss** how to approach the solution of mechanics problems, and introduce the SMART problem-solving methodology.
- **Examine** factors that govern numerical accuracy in the solution of a mechanics problem.

1.1 WHAT IS MECHANICS?

Mechanics is defined as the science that describes and predicts the conditions of rest or motion of bodies under the action of forces. It consists of the mechanics of *rigid bodies,* mechanics of *deformable bodies,* and mechanics of *fluids.*

The mechanics of rigid bodies is subdivided into **statics** and **dynamics**. Statics deals with bodies at rest; dynamics deals with bodies in motion. In this text, we assume bodies are perfectly rigid. In fact, actual structures and machines are never absolutely rigid; they deform under the loads to which they are subjected. However, because these deformations are usually small, they do not appreciably affect the conditions of equilibrium or the motion of the structure under consideration. They are important, though, as far as the resistance of the structure to failure is concerned. Deformations are studied in a course in mechanics of materials, which is part of the mechanics of deformable bodies. The third division of mechanics, the mechanics of fluids, is subdivided into the study of *incompressible fluids* and of *compressible fluids.* An important subdivision of the study of incompressible fluids is *hydraulics,* which deals with applications involving water.

Mechanics is a physical science, because it deals with the study of physical phenomena. However, some teachers associate mechanics with mathematics, whereas many others consider it as an engineering subject. Both of these views are justified in part. Mechanics is the foundation of most engineering sciences and is an indispensable prerequisite to their study. However, it does not have the *empiricism* found in some engineering sciences, i.e., it does not rely on experience or observation alone. The rigor of mechanics and the emphasis it places on deductive reasoning makes it resemble mathematics. However, mechanics is not an *abstract* or even a *pure* science; it is an *applied* science.

The purpose of mechanics is to explain and predict physical phenomena and thus to lay the foundations for engineering applications. You need to know statics to determine how much force will be exerted on a point in a bridge design and whether the structure can withstand that force. Determining the force a dam needs to withstand from the water in a river requires statics. You need statics to calculate how much weight a crane can lift, how much force a locomotive needs to pull a freight train, or how much force a circuit board in a computer can withstand. The concepts of dynamics enable you to analyze the flight characteristics of a jet, design a building to resist earthquakes, and mitigate shock and vibration to passengers inside a vehicle. The concepts of dynamics enable you to calculate how much force you need to send a satellite into orbit, accelerate a 200,000-ton cruise ship, or design a toy truck that doesn't break. You will not learn how to do these things in this course, but the ideas and methods you learn here will be the underlying basis for the engineering applications you will learn in your work.

1.2 FUNDAMENTAL CONCEPTS AND PRINCIPLES

Although the study of mechanics goes back to the time of Aristotle (384–322 B.C.) and Archimedes (287–212 B.C.), not until Newton (1642–1727) did anyone develop a satisfactory formulation of its fundamental principles. These principles were later modified by d'Alembert, Lagrange, and Hamilton. Their validity remained unchallenged until Einstein formulated his **theory of relativity** (1905). Although its limitations have now been recognized, **newtonian mechanics** still remains the basis of today's engineering sciences.

The basic concepts used in mechanics are *space, time, mass,* and *force.* These concepts cannot be truly defined; they should be accepted on the basis of our intuition and experience and used as a mental frame of reference for our study of mechanics.

The concept of **space** is associated with the position of a point *P*. We can define the position of *P* by providing three lengths measured from a certain reference point, or *origin,* in three given directions. These lengths are known as the *coordinates* of *P*.

To define an event, it is insufficient to indicate its position in space. We also need to specify the **time** of the event.

We use the concept of **mass** to characterize and compare bodies on the basis of certain fundamental mechanical experiments. Two bodies of the same mass, for example, are attracted by the earth in the same manner; they also offer the same resistance to a change in translational motion.

A **force** represents the action of one body on another. A force can be exerted by actual contact, like a push or a pull, or at a distance, as in the case of gravitational or magnetic forces. A force is characterized by its *point of application,* its *magnitude,* and its *direction;* a force is represented by a *vector* (Sec. 2.1B).

In newtonian mechanics, space, time, and mass are absolute concepts that are independent of each other. (This is not true in **relativistic mechanics**, where the duration of an event depends upon its position and the mass of a body varies with its velocity.) On the other hand, the concept of force is not independent of the other three. Indeed, one of the fundamental principles of newtonian mechanics listed below is that the resultant force acting on a body is related to the mass of the body and to the manner in which its velocity varies with time.

In this text, you will study the conditions of rest or motion of particles and rigid bodies in terms of the four basic concepts we have introduced. By **particle**, we mean a very small amount of matter, which we assume occupies a single point in space. A **rigid body** consists of a large number of particles occupying fixed positions with respect to one another. The study of the mechanics of particles is therefore a prerequisite to that of rigid bodies. Besides, we can use the results obtained for a particle directly in a large number of problems dealing with the conditions of rest or motion of actual bodies.

The study of elementary mechanics rests on six fundamental principles, based on experimental evidence.

- **The Parallelogram Law for the Addition of Forces.** Two forces acting on a particle may be replaced by a single force, called their *resultant,* obtained by drawing the diagonal of the parallelogram with sides equal to the given forces (Sec. 2.1A).
- **The Principle of Transmissibility.** The conditions of equilibrium or of motion of a rigid body remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action (Sec. 3.1B).
- **Newton's Three Laws of Motion.** Formulated by Sir Isaac Newton in the late seventeenth century, these laws can be stated as follows:

  FIRST LAW. If the resultant force acting on a particle is zero, the particle remains at rest (if originally at rest) or moves with constant speed in a straight line (if originally in motion) (Sec. 2.3B).
  SECOND LAW. If the resultant force acting on a particle is not zero, the particle has an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.
  As you will see in Sec. 12.1, this law can be stated as

$$\mathbf{F} = m\mathbf{a} \qquad \textbf{(1.1)}$$

  where $\mathbf{F}$, m, and $\mathbf{a}$ represent, respectively, the resultant force acting on the particle, the mass of the particle, and the acceleration of the particle expressed in a consistent system of units.
  THIRD LAW. The forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense (Chap. 6, Introduction).

- **Newton's Law of Gravitation.** Two particles of mass M and m are mutually attracted with equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ of magnitude F (Fig. 1.1), given by the formula

$$F = G\frac{Mm}{r^2} \qquad \textbf{(1.2)}$$

  where r = the distance between the two particles and G = a universal constant called the *constant of gravitation.* Newton's law of gravitation introduces the idea of an action exerted at a distance and extends the range of application of Newton's third law: the action $\mathbf{F}$ and the reaction $-\mathbf{F}$ in Fig. 1.1 are equal and opposite, and they have the same line of action.

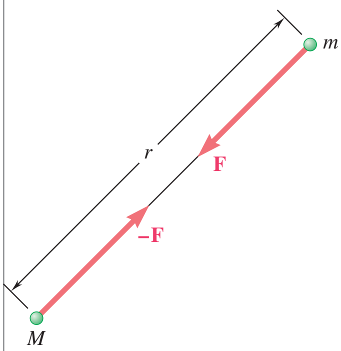


Fig. 1.1 From Newton's law of gravitation, two particles of masses M and m exert forces upon each other of equal magnitude, opposite direction, and the same line of action. This also illustrates Newton's third law of motion.

A particular case of great importance is that of the attraction of the earth on a particle located on its surface. The force $\mathbf{F}$ exerted by the earth on the particle is defined as the **weight W** of the particle. Suppose we set M equal to the mass of the earth, m equal to the mass of the particle, and r equal to the earth's radius R. Then, introducing the constant

$$g = \frac{GM}{R^2} \qquad \textbf{(1.3)}$$

we can express the magnitude W of the weight of a particle of mass m as[†]

$$W = mg \qquad \textbf{(1.4)}$$

The value of R in formula (1.3) depends upon the elevation of the point considered; it also depends upon its latitude, because the earth is not truly spherical. The value of g therefore varies with the position of the point considered.

[†] A more accurate definition of the weight **W** should take into account the earth's rotation.

However, as long as the point actually remains on the earth's surface, it is sufficiently accurate in most engineering computations to assume that g equals 9.81 m/s^2 or 32.2 ft/s^2.

The principles we have just listed will be introduced in the course of our study of mechanics as they are needed. The statics of particles carried out in Chap. 2 will be based on the parallelogram law of addition and on Newton's first law alone. We introduce the principle of transmissibility in Chap. 3 as we begin the study of the statics of rigid bodies, and we bring in Newton's third law in Chap. 6 as we analyze the forces exerted on each other by the various members forming a structure. We introduce Newton's second law and Newton's law of gravitation in dynamics. We will then show that Newton's first law is a particular case of Newton's second law (Sec. 12.1) and that the principle of transmissibility could be derived from the other principles and thus eliminated (Sec. 16.1D). In the meantime, however, Newton's first and third laws, the parallelogram law of addition, and the principle of transmissibility will provide us with the necessary and sufficient foundation for the entire study of the statics of particles, rigid bodies, and systems of rigid bodies.

As noted earlier, the six fundamental principles listed previously are based on experimental evidence. Except for Newton's first law and the principle of transmissibility, they are independent principles that cannot be derived mathematically from each other or from any other elementary physical principle. On these principles rests most of the intricate structure of newtonian mechanics. For more than two centuries, engineers have solved a tremendous number of problems dealing with the conditions of rest and motion of rigid bodies, deformable bodies, and fluids by applying these fundamental principles. Many of the solutions obtained could be checked experimentally, thus providing a further verification of the principles from which they were derived. Only in the twentieth century has Newton's mechanics been found to be at fault, in the study of the motion of atoms and the motion of the planets, where it must be supplemented by the theory of relativity. On the human or engineering scale, however, where velocities are small compared with the speed of light, Newton's mechanics have yet to be disproved.

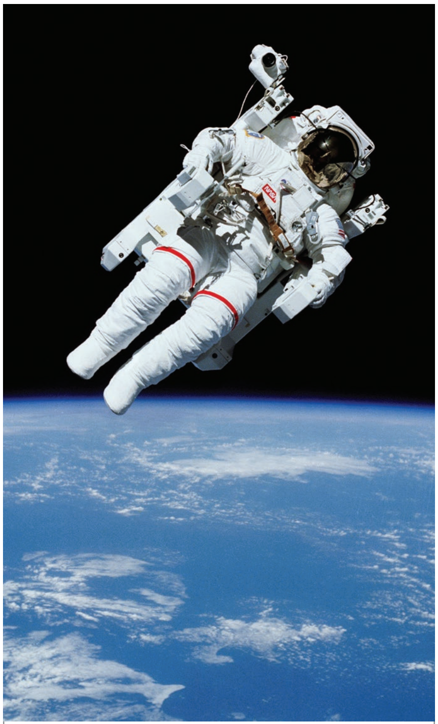


Photo 1.1 When in orbit of the earth, people and objects are said to be *weightless,* even though the gravitational force acting is approximately 90% of that experienced on the surface of the earth. This apparent contradiction will be resolved in Chapter 12 when we apply Newton's second law to the motion of particles.
Source: NASA

1.3 SYSTEMS OF UNITS

Associated with the four fundamental concepts just discussed are the so-called *kinetic units,* i.e., the units of *length, time, mass,* and *force.* These units cannot be chosen independently if Eq. (1.1) is to be satisfied. Three of the units may be defined arbitrarily; we refer to them as **basic units**. The fourth unit, however, must be chosen in accordance with Eq. (1.1) and is referred to as a **derived unit**. Kinetic units selected in this way are said to form a **consistent system of units**.

International System of Units (SI Units).[†] In this system, which will be in universal use after the United States has completed its conversion to SI units, the base units are the units of length, mass, and time, and they are called, respectively, the **meter** (m), the **kilogram** (kg), and the **second** (s). All three are arbitrarily defined. The second was originally chosen to represent 1/86 400 of the mean solar day, but it is now defined as the duration of 9 192 631 770 cycles of the radiation corresponding to the transition between two levels of the fundamental state of the cesium-133 atom. The meter, originally defined as one

[†]SI stands for *Système International d'Unités* (French).

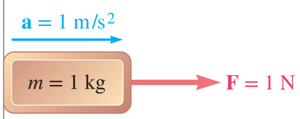


Fig. 1.2 A force of 1 newton applied to a body of mass 1 kg provides an acceleration of 1 m/s^2.

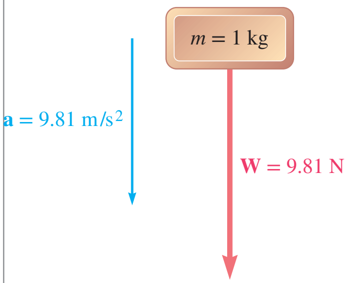


Fig. 1.3 A body of mass 1 kg experiencing an acceleration due to gravity of 9.81 m/s^2 has a weight of 9.81 N.

ten-millionth of the distance from the equator to either pole, is now defined as 1 650 763.73 wavelengths of the orange-red light corresponding to a certain transition in an atom of krypton-86. (The newer definitions are much more precise, and with today's modern instrumentation, are easier to verify as a standard.) The kilogram, which is approximately equal to the mass of 0.001 m^3 of water, is defined as the mass of a platinum-iridium standard kept at the International Bureau of Weights and Measures at Sèvres, near Paris, France. The unit of force is a derived unit. It is called the **newton** (N) and is defined as the force that gives an acceleration of 1 m/s^2 to a body of mass 1 kg (Fig. 1.2). From Eq. (1.1), we have

$$1\ \mathrm{N} = (1\ \mathrm{kg})(1\ \mathrm{m/s^2}) = 1\ \mathrm{kg \cdot m/s^2} \qquad \textbf{(1.5)}$$

The SI units are said to form an *absolute* system of units. This means that the three base units chosen are independent of the location where measurements are made. The meter, the kilogram, and the second may be used anywhere on the earth; they may even be used on another planet and still have the same significance.

The *weight* of a body, or the *force of gravity* exerted on that body, like any other force, should be expressed in newtons. From Eq. (1.4), it follows that the weight of a body of mass 1 kg (Fig. 1.3) is

$$\begin{aligned} W &= mg \\ &= (1\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) \\ &= 9.81\ \mathrm{N} \end{aligned}$$

Multiples and submultiples of the fundamental SI units are denoted through the use of the prefixes defined in Table 1.1. The multiples and submultiples of the units of length, mass, and force most frequently used in engineering are, respectively, the *kilometer* (km) and the *millimeter* (mm); the *megagram*‡ (Mg) and the *gram* (g); and the *kilonewton* (kN). According to Table 1.1, we have

$$\begin{aligned} 1\ \mathrm{km} &= 1000\ \mathrm{m} \qquad & 1\ \mathrm{mm} &= 0.001\ \mathrm{m} \\ 1\ \mathrm{Mg} &= 1000\ \mathrm{kg} \qquad & 1\ \mathrm{g} &= 0.001\ \mathrm{kg} \\ 1\ \mathrm{kN} &= 1000\ \mathrm{N} \end{aligned}$$

The conversion of these units into meters, kilograms, and newtons, respectively, can be effected by simply moving the decimal point three places to the right or to the left. For example, to convert 3.82 km into meters, move the decimal point three places to the right:

$$3.82\ \mathrm{km} = 3820\ \mathrm{m}$$

Similarly, to convert 47.2 mm into meters, move the decimal point three places to the left:

$$47.2\ \mathrm{mm} = 0.0472\ \mathrm{m}$$

Using engineering notation, you can also write

$$\begin{aligned} 3.82\ \mathrm{km} &= 3.82 \times 10^{3}\ \mathrm{m} \\ 47.2\ \mathrm{mm} &= 47.2 \times 10^{w-3}\ \mathrm{m} \end{aligned}$$

The multiples of the unit of time are the *minute* (min) and the *hour* (h). Because 1 min = 60 s and 1 h = 60 min = 3600 s, these multiples cannot be converted as readily as the others.

‡Also known as a *metric ton*.

Table 1.1 SI Prefixes

Multiplication Factor	Prefix[†]	Symbol
$1\ 000\ 000\ 000\ 000 = 10^{12}$	Tera	T
$1\ 000\ 000\ 000 = 10^{9}$	Giga	G
$1\ 000\ 000 = 10^{6}$	Mega	M
$1\ 000 = 10^{3}$	Kilo	k
$100 = 10^{2}$	Hecto[‡]	h
$10 = 10^{1}$	Deka[‡]	da
$0.1 = 10^{-1}$	Deci[‡]	d
$0.01 = 10^{-2}$	Centi[‡]	c
$0.001 = 10^{-3}$	Milli	m
$0.000\ 001 = 10^{-6}$	Micro	μ
$0.000\ 000\ 001 = 10^{-9}$	Nano	n
$0.000\ 000\ 000\ 001 = 10^{-12}$	Pico	p
$0.000\ 000\ 000\ 000\ 001 = 10^{-15}$	Femto	f
$0.000\ 000\ 000\ 000\ 000\ 001 = 10^{-18}$	Atto	a

[†]The first syllable of every prefix is accented, so that the prefix retains its identity. Thus, the preferred pronunciation of kilometer places the accent on the first syllable, not the second.

[‡]The use of these prefixes should be avoided, except for the measurement of areas and volumes and for the nontechnical use of centimeter, as for body and clothing measurements.

By using the appropriate multiple or submultiple of a given unit, you can avoid writing very large or very small numbers. For example, it is usually simpler to write 427.2 km rather than 427 200 m and 2.16 mm rather than 0.002 16 m.[†]

Units of Area and Volume. The unit of area is the *square meter* (m^2), which represents the area of a square of side 1 m; the unit of volume is the *cubic meter* (m^3), which is equal to the volume of a cube of side 1 m. In order to avoid exceedingly small or large numerical values when computing areas and volumes, we use systems of subunits obtained by respectively squaring and cubing not only the millimeter, but also two intermediate submultiples of the meter: the *decimeter* (dm) and the *centimeter* (cm). By definition,

$$1\text{ dm} = 0.1\text{ m} = 10^{-1}\text{ m}$$
$$1\text{ cm} = 0.01\text{ m} = 10^{-2}\text{ m}$$
$$1\text{ mm} = 0.001\text{ m} = 10^{-3}\text{ m}$$

Therefore, the submultiples of the unit of area are

$$1\text{ dm}^2 = (1\text{ dm})^2 = (10^{-1}\text{ m})^2 = 10^{-2}\text{ m}^2$$
$$1\text{ cm}^2 = (1\text{ cm})^2 = (10^{-2}\text{ m})^2 = 10^{-4}\text{ m}^2$$
$$1\text{ mm}^2 = (1\text{ mm})^2 = (10^{-3}\text{ m})^2 = 10^{-6}\text{ m}^2$$

Similarly, the submultiples of the unit of volume are

$$1\text{ dm}^3 = (1\text{ dm})^3 = (10^{-1}\text{ m})^3 = 10^{-3}\text{ m}^3$$
$$1\text{ cm}^3 = (1\text{ cm})^3 = (10^{-2}\text{ m})^3 = 10^{-6}\text{ m}^3$$
$$1\text{ mm}^3 = (1\text{ mm})^3 = (10^{-3}\text{ m})^3 = 10^{-9}\text{ m}^3$$

Note that when measuring the volume of a liquid, the cubic decimeter (dm^3) is usually referred to as a *liter* (L).

[†]Note that when more than four digits appear on either side of the decimal point to express a quantity in SI units—as in 427 000 m or 0.002 16 m—use spaces, never commas, to separate the digits into groups of three. This practice avoids confusion with the comma used in place of a decimal point, which is the convention in many countries.

Table 1.2 shows other derived SI units used to measure the moment of a force, the work of a force, etc. Although we will introduce these units in later chapters as they are needed, we should note an important rule at this time: When a derived unit is obtained by dividing a base unit by another base unit, you may use a prefix in the numerator of the derived unit, but not in its denominator. For example, the constant k of a spring that stretches 20 mm under a load of 100 N is expressed as

$$k = \frac{100\text{ N}}{20\text{ mm}} = \frac{100\text{ N}}{0.020\text{ m}} = 5000\text{ N/m or } k = 5\text{ kN/m}$$

but never as $k = 5$ N/mm.

U.S. Customary Units. Most practicing American engineers still commonly use a system in which the base units are those of length, force, and time. These units are, respectively, the *foot* (ft), the *pound* (lb), and the *second* (s). The second is the same as the corresponding SI unit. The foot is defined as 0.3048 m. The pound is defined as the *weight* of a platinum standard, called the *standard pound,* which is kept at the National Institute of Standards and Technology outside Washington, DC, the mass of which is 0.453 592 43 kg. Because the weight of a body depends upon the earth's gravitational attraction, which varies with location, the standard pound should be placed at sea level and at a latitude of 45° to properly define a force of 1 lb. Thus, the U.S. customary units do not form an absolute system of units. Because they depend upon the gravitational attraction of the earth, they form a *gravitational* system of units.

Although the standard pound also serves as the unit of mass in commercial transactions in the United States, it cannot be used that way in engineering

Table 1.2 Principal SI Units Used in Mechanics

Quantity	Unit	Symbol	Formula
Acceleration	Meter per second squared	. . .	m/s^2
Angle	Radian	rad	†
Angular acceleration	Radian per second squared	. . .	rad/s^2
Angular velocity	Radian per second	. . .	rad/s
Area	Square meter	. . .	m^2
Density	Kilogram per cubic meter	. . .	kg/m^3
Energy	Joule	J	N·m
Force	Newton	N	$kg{\cdot}m/s^2$
Frequency	Hertz	Hz	s^{-1}
Impulse	Newton-second	. . .	kg·m/s
Length	Meter	m	‡
Mass	Kilogram	kg	‡
Moment of a force	Newton-meter	. . .	N·m
Power	Watt	W	J/s
Pressure	Pascal	Pa	N/m^2
Stress	Pascal	Pa	N/m^2
Time	Second	s	‡
Velocity	Meter per second	. . .	m/s
Volume			
Solids	Cubic meter	. . .	m^3
Liquids	Liter	L	10^{-3} m^3
Work	Joule	J	N·m

†Supplementary unit (1 revolution = 2π rad = 360°).
‡Base unit.

computations, because such a unit would not be consistent with the base units defined in the preceding paragraph. Indeed, when acted upon by a force of 1 lb—that is, when subjected to the force of gravity—the standard pound has the acceleration due to gravity, $g = 32.2\ \text{ft/s}^2$ (Fig. 1.4), not the unit acceleration required by Eq. (1.1). The unit of mass consistent with the foot, the pound, and the second is the mass that receives an acceleration of $1\ \text{ft/s}^2$ when a force of 1 lb is applied to it (Fig. 1.5). This unit, sometimes called a *slug*, can be derived from the equation $F = ma$ after substituting 1 lb for F and $1\ \text{ft/s}^2$ for a. We have

$$F = ma \qquad 1\ \text{lb} = (1\ \text{slug})(1\ \text{ft/s}^2)$$

This gives us

$$1\ \text{slug} = \frac{1\ \text{lb}}{1\ \text{ft/s}^2} = 1\ \text{lb}\cdot\text{s}^2/\text{ft} \tag{1.6}$$

Comparing Figs. 1.4 and 1.5, we conclude that the slug is a mass 32.2 times larger than the mass of the standard pound.

The fact that, in the U.S. customary system of units, bodies are characterized by their weight in pounds rather than by their mass in slugs is convenient in the study of statics, where we constantly deal with weights and other forces and only seldom deal directly with masses. However, in the study of dynamics, where forces, masses, and accelerations are involved, the mass m of a body is expressed in slugs when its weight W is given in pounds. Recalling Eq. (1.4), we write

$$m = \frac{W}{g} \tag{1.7}$$

where g is the acceleration due to gravity ($g = 32.2\ \text{ft/s}^2$).

Other U.S. customary units frequently encountered in engineering problems are the *mile* (mi), equal to 5280 ft; the *inch* (in.), equal to (1/12) ft; and the *kilopound* (kip), equal to 1000 lb. The *ton* is often used to represent a mass of 2000 lb but, like the pound, must be converted into slugs in engineering computations.

The conversion into feet, pounds, and seconds of quantities expressed in other U.S. customary units is generally more involved and requires greater attention than the corresponding operation in SI units. For example, suppose we are given the magnitude of a velocity $v = 30$ mi/h and want to convert it to ft/s. First we write

$$v = 30\frac{\text{mi}}{\text{h}}$$

Because we want to get rid of the unit miles and introduce instead the unit feet, we should multiply the right-hand member of the equation by an expression containing miles in the denominator and feet in the numerator. However, because we do not want to change the value of the right-hand side of the equation, the expression used should have a value equal to unity. The quotient (5280 ft)/(1 mi) is such an expression. Operating in a similar way to transform the unit hour into seconds, we have

$$v = \left(30\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)$$

Carrying out the numerical computations and canceling out units that appear in both the numerator and the denominator, we obtain

$$v = 44\frac{\text{ft}}{\text{s}} = 44\ \text{ft/s}$$

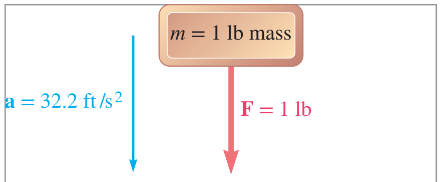


Fig. 1.4 A body of 1 pound mass acted upon by a force of 1 pound has an acceleration of $32.2\ \text{ft/s}^2$.

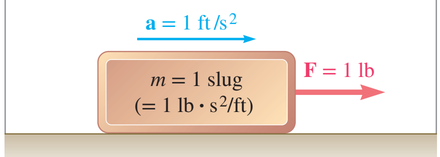


Fig. 1.5 A force of 1 pound applied to a body of mass 1 slug produces an acceleration of $1\ \text{ft/s}^2$.

1.4 CONVERTING BETWEEN TWO SYSTEMS OF UNITS

In many situations, an engineer might need to convert into SI units a numerical result obtained in U.S. customary units or vice versa. Because the unit of time is the same in both systems, only two kinetic base units need to be converted. Thus, because all other kinetic units can be derived from these base units, only two conversion factors need to be remembered.

Units of Length. By definition, the U.S. customary unit of length is

$$1\text{ ft} = 0.3048\text{ m} \tag{1.8}$$

It follows that

$$1\text{ mi} = 5280\text{ ft} = 5280(0.3048\text{ m}) = 1609\text{ m}$$

or

$$1\text{ mi} = 1.609\text{ km} \tag{1.9}$$

Also,

$$1\text{ in.} = \frac{1}{12}\text{ ft} = \frac{1}{12}(0.3048\text{ m}) = 0.0254\text{ m}$$

or

$$1\text{ in.} = 25.4\text{ mm} \tag{1.10}$$

Units of Force. Recall that the U.S. customary unit of force (pound) is defined as the weight of the standard pound (of mass 0.4536 kg) at sea level and at a latitude of 45° (where $g = 9.807\text{ m/s}^2$). Then, using Eq. (1.4), we write

$$\begin{aligned} W &= mg \\ 1\text{ lb} &= (0.4536\text{ kg})(9.807\text{ m/s}^2) = 4.448\text{ kg}\cdot\text{m/s}^2 \end{aligned}$$

From Eq. (1.5), this reduces to

$$1\text{ lb} = 4.448\text{ N} \tag{1.11}$$

Units of Mass. The U.S. customary unit of mass (slug) is a derived unit. Thus, using Eqs. (1.6), (1.8), and (1.11), we have

$$1\text{ slug} = 1\text{ lb}\cdot\text{s}^2/\text{ft} = \frac{1\text{ lb}}{1\text{ ft/s}^2} = \frac{4.448\text{ N}}{0.3048\text{ m/s}^2} = 14.59\text{ N}\cdot\text{s}^2/\text{m}$$

Again, from Eq. (1.5),

$$1\text{ slug} = 1\text{ lb}\cdot\text{s}^2/\text{ft} = 14.59\text{ kg} \tag{1.12}$$

Although it cannot be used as a consistent unit of mass, recall that the mass of the standard pound is, by definition,

$$1\text{ pound mass} = 0.4536\text{ kg} \tag{1.13}$$

We can use this constant to determine the *mass* in SI units (kilograms) of a body that has been characterized by its *weight* in U.S. customary units (pounds).

To convert a derived U.S. customary unit into SI units, simply multiply or divide by the appropriate conversion factors. For example, to convert the moment of a force that is measured as $M = 47$ lb·in. into SI units, use formulas (1.10) and (1.11) and write

$$M = 47 \text{ lb·in.} = 47(4.448 \text{ N})(25.4 \text{ mm})$$
$$= 5310 \text{ N·mm} = 5.31 \text{ N·m}$$

You can also use conversion factors to convert a numerical result obtained in SI units into U.S. customary units. For example, if the moment of a force is measured as $M = 40$ N·m, follow the procedure at the end of Sec. 1.3 to write

$$M = 40 \text{ N·m} = (40 \text{ N·m})\left(\frac{1 \text{ lb}}{4.448 \text{ N}}\right)\left(\frac{1 \text{ ft}}{0.3048 \text{ m}}\right)$$

Carrying out the numerical computations and canceling out units that appear in both the numerator and the denominator, you obtain

$$M = 29.5 \text{ lb·ft}$$

The U.S. customary units most frequently used in mechanics are listed in Table 1.3 with their SI equivalents.

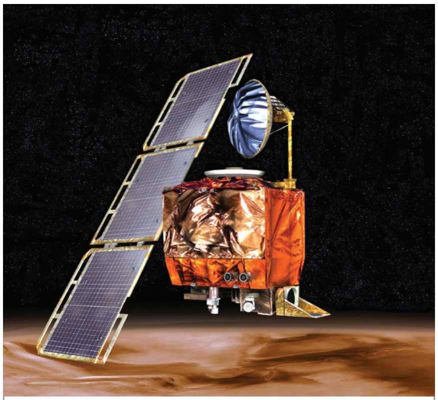


Photo 1.2 In 1999, The *Mars Climate Orbiter* entered orbit around Mars at too low an altitude and disintegrated. Investigation showed that the software on board the probe interpreted force instructions in newtons, but the software at mission control on the earth was generating those instructions in terms of pounds.
Source: NASA/JPL-Caltech

1.5 METHOD OF SOLVING PROBLEMS

You should approach a problem in mechanics as you would approach an actual engineering situation. By drawing on your own experience and intuition about physical behavior, you will find it easier to understand and formulate the problem. Once you have clearly stated and understood the problem, however, there is no place in its solution for arbitrary methodologies.

The solution must be based on the six fundamental principles stated in Sec. 1.2 or on theorems derived from them.

Every step you take in the solution must be justified on this basis. Strict rules must be followed, which lead to the solution in an almost automatic fashion, leaving no room for your intuition or "feeling." After you have obtained an answer, you should check it. Here again, you may call upon your common sense and personal experience. If you are not completely satisfied with the result, you should carefully check your formulation of the problem, the validity of the methods used for its solution, and the accuracy of your computations.

In general, you can usually solve problems in several different ways; there is no one approach that works best for everybody. However, we have found that students often find it helpful to have a general set of guidelines to use for framing problems and planning solutions. In the Sample Problems throughout this text, we use a four-step method for approaching problems, which we refer to as the SMART methodology: **S**trategy, **M**odeling, **A**nalysis, and **R**eflect and **T**hink.

1. **Strategy.** The statement of a problem should be clear and precise, and it should contain the given data and indicate what information is required. The first step in solving the problem is to decide what concepts you have learned that apply to the given situation and to connect the data to the required information. It is often useful to work backward from the information you are trying to find: Ask yourself what quantities you need to

Table 1.3 U.S. Customary Units and Their SI Equivalents

Quantity	U.S. Customary Unit	SI Equivalent
Acceleration	ft/s^2	$0.3048\ m/s^2$
	$in./s^2$	$0.0254\ m/s^2$
Area	ft^2	$0.0929\ m^2$
	in^2	$645.2\ mm^2$
Energy	ft·lb	1.356 J
Force	kip	4.448 kN
	lb	4.448 N
	oz	0.2780 N
Impulse	lb·s	4.448 N·s
Length	ft	0.3048 m
	in.	25.40 mm
	mi	1.609 km
Mass	oz mass	28.35 g
	lb mass	0.4536 kg
	slug	14.59 kg
	ton	907.2 kg
Moment of a force	lb·ft	1.356 N·m
	lb·in.	0.1130 N·m
Moment of inertia		
Of an area	in^4	$0.4162 \times 10^6\ mm^4$
Of a mass	$lb \cdot ft \cdot s^2$	$1.356\ kg \cdot m^2$
Momentum	lb·s	4.448 kg·m/s
Power	ft·lb/s	1.356 W
	hp	745.7 W
Pressure or stress	lb/ft^2	47.88 Pa
	lb/in^2 (psi)	6.895 kPa
Velocity	ft/s	0.3048 m/s
	in./s	0.0254 m/s
	mi/h (mph)	0.4470 m/s
	mi/h (mph)	1.609 km/h
Volume	ft^3	$0.02832\ m^3$
	in^3	$16.39\ cm^3$
Liquids	gal	3.785 L
	qt	0.9464 L
Work	ft·lb	1.356 J

know to obtain the answer, and if some of these quantities are unknown, how can you find them from the given data.

2. **Modeling.** The first step in modeling is to define the system; that is, clearly define what you are setting aside for analysis. After you have selected a system, draw a neat sketch showing all quantities involved, with a separate diagram for each body in the problem. For equilibrium problems, indicate clearly the forces acting on each body along with any relevant geometrical data, such as lengths and angles. (These diagrams are known as **free-body diagrams** and are described in detail in Sec. 2.3C and the beginning of Chap. 4.)
3. **Analysis.** After you have drawn the appropriate diagrams, use the fundamental principles of mechanics listed in Sec. 1.2 to write equations expressing the conditions of rest or motion of the bodies considered. Each equation should be clearly related to one of the free-body diagrams and should be numbered. If you do not have enough equations to solve for the unknowns, try selecting another system, or reexamine your strategy to see if you can apply other principles to the problem. Once you

have obtained enough equations, you can find a numerical solution by following the usual rules of algebra, neatly recording each step and the intermediate results. Alternatively, you can solve the resulting equations with your calculator or a computer. (For multipart problems, it is sometimes convenient to present the Modeling and Analysis steps together, but they are both essential parts of the overall process.)

4. **Reflect and Think.** After you have obtained the answer, check it carefully. Does it make sense in the context of the original problem? For instance, the problem may ask for the force at a given point of a structure. If your answer is negative, what does that mean for the force at the point?

You can often detect mistakes in *reasoning* by checking the units. For example, to determine the moment of a force of 50 N about a point 0.60 m from its line of action, we write (Sec. 3.3A)

$$M = Fd = (30\text{ N})(0.60\text{ m}) = 30\text{ N}\cdot\text{m}$$

The unit N·m obtained by multiplying newtons by meters is the correct unit for the moment of a force; if you had obtained another unit, you would know that some mistake had been made.

You can often detect errors in *computation* by substituting the numerical answer into an equation that was not used in the solution and verifying that the equation is satisfied. The importance of correct computations in engineering cannot be overemphasized.

CASE STUDY 1.1*

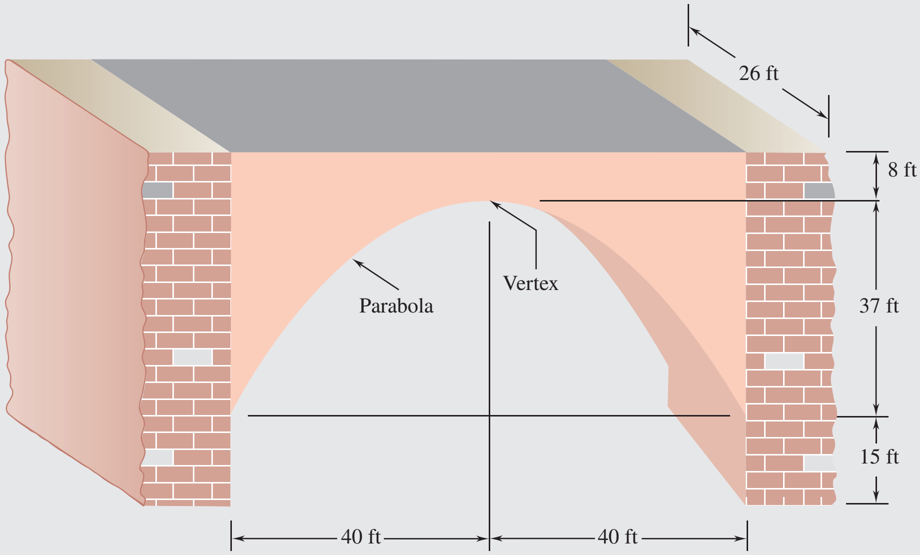
Located in Baltimore, Maryland, the Carrollton Viaduct is the oldest railroad bridge in North America and continues in revenue service today. Construction was completed and the bridge put into operation in 1829 by the Baltimore & Ohio Railroad. The structure includes the stone masonry arch shown in CS Photo 1.1, and spans 80 ft. Assuming that the span is solid granite having a unit weight of 170 lb/ft^3, and that its dimensions can be approximated by those given in CS Fig. 1.1, let's estimate the weight of this span.

CS Photo 1.1 The Carrollton Viaduct in Baltimore, MD.
AREA Bulletin 732 Volume 92 (October 1991)

STRATEGY:
First calculate the volume of the span, and then multiply this volume by the unit weight.

*Adapted from American Railway Engineering Association, Bulletin 732, October 1991, p. 221.

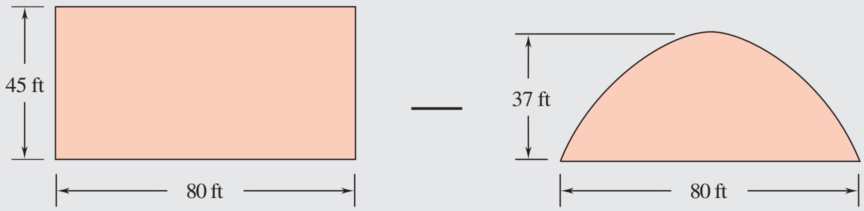
(continued)



CS Fig. 1.1 Assumed arch span geometry.

MODELING:

The span can be represented by a body where a parabolic portion has been removed from a rectangular portion as shown in CS Fig. 1.2 (with both parts having a depth of 26 ft).



CS Fig. 1.2 Modeling the arch span.

ANALYSIS:

Volume of the Span, *V*. Removing the parabolic region from the rectangle,

$$V = \left[(80\text{ ft})(45\text{ ft}) - \frac{2}{3}(80\text{ ft})(37\text{ ft})\right](26\text{ ft}) = 42{,}300\text{ ft}^3$$

Weight of the Span, *W*. Multiplying the volume by the unit weight,

$$W = (170\text{ lb/ft}^3)(42{,}300\text{ ft}^3) = 7.19 \times 10^6\text{ lb}$$

REFLECT AND THINK:

Though completed in 1829, regular locomotive usage didn't begin on this bridge until 1831 with the steam-powered *York,* which weighed approximately 7000 lb. (Up to that point, trains had been pulled by horses.) Then in 1832, there was initially concern regarding the ability of the stone arch to support a newer and heavier locomotive, the 13,000-lb *Atlantic*.* As our knowledge of engineering mechanics has progressed since then, we better understand that a massive arch

(continued)

like this can indeed sustain such loads quite easily. This is illustrated by the modern-day coal cars shown crossing this same span in CS Photo 1.1, where each car has a rated weight of 263,000 lb. Arches derive load-carrying capacity through compression and are well-suited for stone masonry construction, because it provides high compressive strength. And while trains traversing the bridge would tend to introduce other types of effects into the span, the massiveness of the span itself (which we estimated to be 7.19×10^6 lb) far exceeds the car loads and therefore keeps the barrel (or portal) of the arch in compression.

1.6 NUMERICAL ACCURACY

The accuracy of the solution to a problem depends upon two items: (1) the accuracy of the given data and (2) the accuracy of the computations performed. The solution cannot be more accurate than the less accurate of these two items.

For example, suppose the loading of a bridge is known to be 75,000 lb with a possible error of 100 lb either way. The relative error that measures the degree of accuracy of the data is

$$\frac{100\ \text{lb}}{75,000\ \text{lb}} = 0.0013 = 0.13\%$$

In computing the reaction at one of the bridge supports, it would be meaningless to record it as 14,322 lb. The accuracy of the solution cannot be greater than 0.13%, no matter how precise the computations are, and the possible error in the answer may be as large as (0.13/100)(14,322 lb) $\approx$ 20 lb. The answer should be properly recorded as 14,320 $\pm$ 20 lb.

In engineering problems, the data are seldom known with an accuracy greater than 0.2%. It is therefore seldom justified to write answers with an accuracy greater than 0.2%. A practical rule is to use four figures to record numbers beginning with a “1” and three figures in all other cases. Unless otherwise indicated, you should assume the data given in a problem are known with a comparable degree of accuracy. A force of 40 lb, for example, should be read as 40.0 lb, and a force of 15 lb should be read as 15.00 lb.

Electronic calculators are widely used by practicing engineers and engineering students. The speed and accuracy of these calculators facilitate the numerical computations in the solution of many problems. However, you should not record more significant figures than can be justified merely because you can obtain them easily. As noted previously, an accuracy greater than 0.2% is seldom necessary or meaningful in the solution of practical engineering problems.

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2

Statics of Particles

Many engineering problems can be solved by considering the equilibrium of a “particle.” In the case of this beam that is being hoisted into position, a relation between the tensions in the various cables involved can be obtained by considering the equilibrium of the hook to which the cables are attached.

Objectives

- **Describe** force as a vector quantity.
- **Examine** vector operations useful for the analysis of forces.
- **Determine** the resultant of multiple forces acting on a particle.
- **Resolve** forces into components.
- **Add** forces that have been resolved into rectangular components.
- **Introduce** the concept of the free-body diagram.
- **Use** free-body diagrams to assist in the analysis of planar and spatial particle equilibrium problems.

Introduction

2.1 ADDITION OF PLANAR FORCES
2.1A Force on a Particle: Resultant of Two Forces
2.1B Vectors
2.1C Addition of Vectors
2.1D Resultant of Several Concurrent Forces
2.1E Resolution of a Force into Components
2.2 ADDING FORCES BY COMPONENTS
2.2A Rectangular Components of a Force: Unit Vectors
2.2B Addition of Forces by Summing *x* and *y* Components
2.3 FORCES AND EQUILIBRIUM IN A PLANE
2.3A Equilibrium of a Particle
2.3B Newton's First Law of Motion
2.3C Free-Body Diagrams and Problem Solving
2.4 ADDING FORCES IN SPACE
2.4A Rectangular Components of a Force in Space
2.4B Force Defined by Its Magnitude and Two Points on Its Line of Action
2.4C Addition of Concurrent Forces in Space
2.5 FORCES AND EQUILIBRIUM IN SPACE

Introduction

In this chapter, you will study the effect of forces acting on particles. By the word "particle" we do not mean only tiny bits of matter, like an atom or an electron. Instead, we mean that the sizes and shapes of the bodies under consideration do not significantly affect the solutions of the problems. Another way of saying this is that we assume all forces acting on a given body act at the same point. This does not mean the object must be tiny—if you were modeling the mechanics of the Milky Way galaxy, for example, you could treat the sun and the entire Solar System as just a particle.

Our first step is to explain how to replace two or more forces acting on a given particle by a single force having the same effect as the original forces. This single equivalent force is called the *resultant* of the original forces. After this step, we will derive the relations among the various forces acting on a particle in a state of *equilibrium*. We will use these relations to determine some of the forces acting on the particle.

The first part of this chapter deals with forces contained in a single plane. Because two lines determine a plane, this situation arises any time we can reduce the problem to one of a particle subjected to two forces that support a third force, such as a crate suspended from two chains or a traffic light held in place by two cables. In the second part of this chapter, we examine the more general case of forces in three-dimensional space.

2.1 ADDITION OF PLANAR FORCES

Many important practical situations in engineering involve forces in the same plane. These include forces acting on a pulley, projectile motion, and an object in equilibrium on a flat surface. We will examine this situation first before looking at the added complications of forces acting in three-dimensional space.

2.1A Force on a Particle: Resultant of Two Forces

A force represents the action of one body on another. It is generally characterized by its **point of application**, its **magnitude**, and its **direction**. Forces acting on a given particle, however, have the same point of application. Thus, each force considered in this chapter is completely defined by its magnitude and direction.

The magnitude of a force is characterized by a certain number of units. As indicated in Chap. 1, the SI units used by engineers to measure the magnitude of a force are the newton (N) and its multiple the kilonewton (kN), which is equal to 1000 N. The U.S. customary units used for the same purpose are the pound (lb) and its multiple the kilopound (kip), which is equal to 1000 lb. We saw in Chap. 1 that a force of 445 N is equivalent to a force of 100 lb or that a force of 100 N equals a force of about 22.5 lb.

We define the direction of a force by its **line of action** and the **sense** of the force. The line of action is the infinite straight line along which the force acts; it is characterized by the angle it forms with some fixed axis (Fig. 2.1). The force itself is represented by a segment of that line; through the use of an appropriate scale, we can choose the length of this segment to represent the magnitude of the force. We indicate the sense of the force by an arrowhead. It is important in defining a force to indicate its sense. Two forces having the same magnitude and the same line of action but a different sense, such as the forces shown in Fig. 2.1*a* and *b*, have directly opposite effects on a particle.

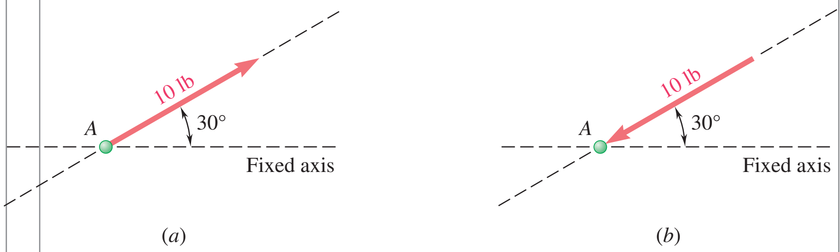


Fig. 2.1 The line of action of a force makes an angle with a given fixed axis. (*a*) The sense of the 10-lb force is away from particle *A*; (*b*) the sense of the 10-lb force is toward particle *A*.

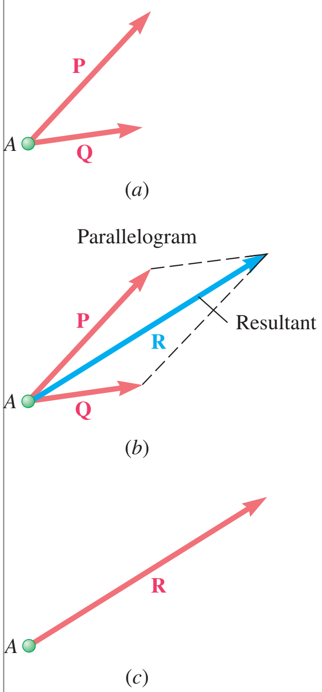


Fig. 2.2 (*a*) Two forces **P** and **Q** act on particle *A*. (*b*) Draw a parallelogram with **P** and **Q** as the adjacent sides and label the diagonal that passes through *A* as **R**. (*c*) **R** is the resultant of the two forces **P** and **Q** and is equivalent to their sum.

Photo 2.1 In its purest form, a tug-of-war pits two opposite and almost-equal forces against each other. Whichever team can generate the larger force, wins. As you can see, a competitive tug-of-war can be quite intense. ©DGB/Alamy

Experimental evidence shows that two forces **P** and **Q** acting on a particle *A* (Fig. 2.2*a*) can be replaced by a single force **R** that has the same effect on the particle (Fig. 2.2*c*). This force is called the **resultant** of the forces **P** and **Q**. We can obtain **R**, as shown in Fig. 2.2*b*, by constructing a parallelogram, using **P** and **Q** as two adjacent sides. **The diagonal that passes through *A* represents the resultant**. This method for finding the resultant is known as the **parallelogram law** for the addition of two forces. This law is based on experimental evidence; it cannot be proved or derived mathematically.

2.1B Vectors

We have just seen that forces do not obey the rules of addition defined in ordinary arithmetic or algebra. For example, two forces acting at a right angle to each other, one of 4 lb and the other of 3 lb, add up to a force of 5 lb acting at an angle between them, *not* to a force of 7 lb. Forces are not the only quantities that follow the parallelogram law of addition. As you will see later, *displacements, velocities, accelerations,* and *momenta* are other physical quantities possessing magnitude and direction that add according to the parallelogram law. All of these quantities can be represented mathematically by **vectors**. Those physical quantities that have magnitude but not direction, such as *volume, mass,* or *energy,* are represented by plain numbers often called **scalars** to distinguish them from vectors.

Vectors are defined as **mathematical expressions possessing magnitude and direction, which add according to the parallelogram law**. Vectors are represented by arrows in diagrams and are distinguished from scalar quantities in this text through the use of boldface type (**P**). In longhand writing, a vector may be denoted by drawing a short arrow above the letter used to represent it ($\vec{P}$). The magnitude of a vector defines the length of the arrow used to represent it. In this text, we use italic type to denote the magnitude of a vector. Thus, the magnitude of the vector **P** is denoted by *P*.

A vector used to represent a force acting on a given particle has a well-defined point of application—namely, the particle itself. Such a vector is said to be a *fixed,* or *bound,* vector and cannot be moved without modifying the conditions of the problem. Other physical quantities, however, such as couples (see Chap. 3), are represented by vectors that may be freely moved in space; these vectors are called *free* vectors. Still other physical quantities, such as forces acting on a rigid body (see Chap. 3), are represented by vectors that can be moved along their lines of action; they are known as *sliding* vectors.

Two vectors that have the same magnitude and the same direction are said to be *equal,* whether or not they also have the same point of application (Fig. 2.3); equal vectors may be denoted by the same letter.

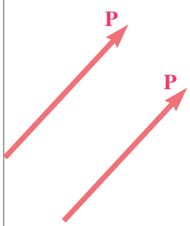


Fig. 2.3 Equal vectors have the same magnitude and the same direction, even if they have different points of application.

The *negative vector* of a given vector **P** is defined as a vector having the same magnitude as **P** and a direction opposite to that of **P** (Fig. 2.4); the negative of the vector **P** is denoted by **−P**. The vectors **P** and **−P** are commonly referred to as **equal and opposite** vectors. Thus, we have

$$\mathbf{P} + (-\mathbf{P}) = 0$$

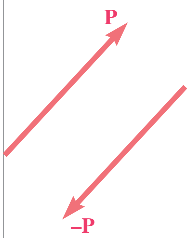


Fig. 2.4 The negative vector of a given vector has the same magnitude but the opposite direction of the given vector.

2.1C Addition of Vectors

By definition, vectors add according to the parallelogram law. Thus, we obtain the sum of two vectors **P** and **Q** by attaching the two vectors to the same point *A* and constructing a parallelogram, using **P** and **Q** as two adjacent sides (Fig. 2.5). The diagonal that passes through *A* represents the sum of the vectors **P** and **Q**, denoted by **P** + **Q**. The fact that the sign + is used for both vector and scalar addition should not cause any confusion if vector and scalar quantities are always carefully distinguished. Note that the magnitude of the vector **P** + **Q** is *not,* in general, equal to the sum *P* + *Q* of the magnitudes of the vectors **P** and **Q**.

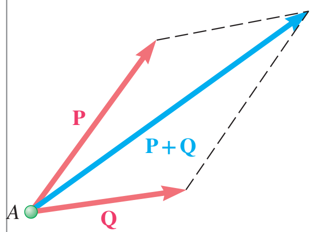


Fig. 2.5 Using the parallelogram law to add two vectors.

Because the parallelogram constructed on the vectors **P** and **Q** does not depend upon the order in which **P** and **Q** are selected, we conclude that the addition of two vectors is *commutative,* and we write

$$\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P} \tag{2.1}$$

From the parallelogram law, we can derive an alternative method for determining the sum of two vectors, known as the **triangle rule**. Consider Fig. 2.5, where the sum of the vectors **P** and **Q** has been determined by the parallelogram law. Because the side of the parallelogram opposite **Q** is equal to **Q** in magnitude and direction, we could draw only half of the parallelogram (Fig. 2.6*a*). The sum of the two vectors thus can be found by **arranging P and Q in tip-to-tail fashion and then connecting the tail of P with the tip of Q**. If we draw the other half of the parallelogram, as in Fig. 2.6*b*, we obtain the same result, confirming that vector addition is commutative.

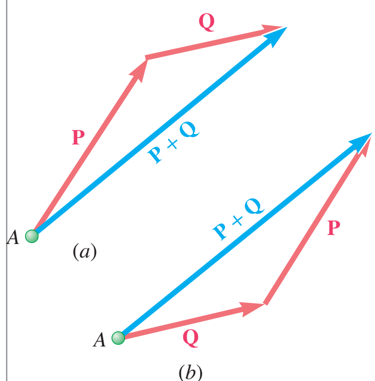


Fig. 2.6 The triangle rule of vector addition. (*a*) Adding vector **Q** to vector **P** equals (*b*) adding vector **P** to vector **Q**.

We define *subtraction* of a vector as the addition of the corresponding negative vector. Thus, we determine the vector **P** − **Q**, representing the

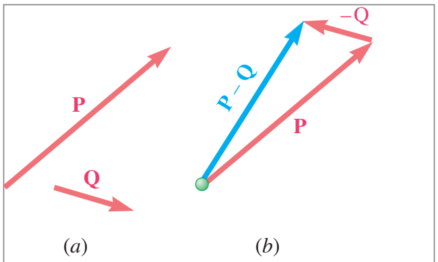


Fig. 2.7 Vector subtraction: Subtracting vector **Q** from vector **P** is the same as adding vector −**Q** to vector **P**.

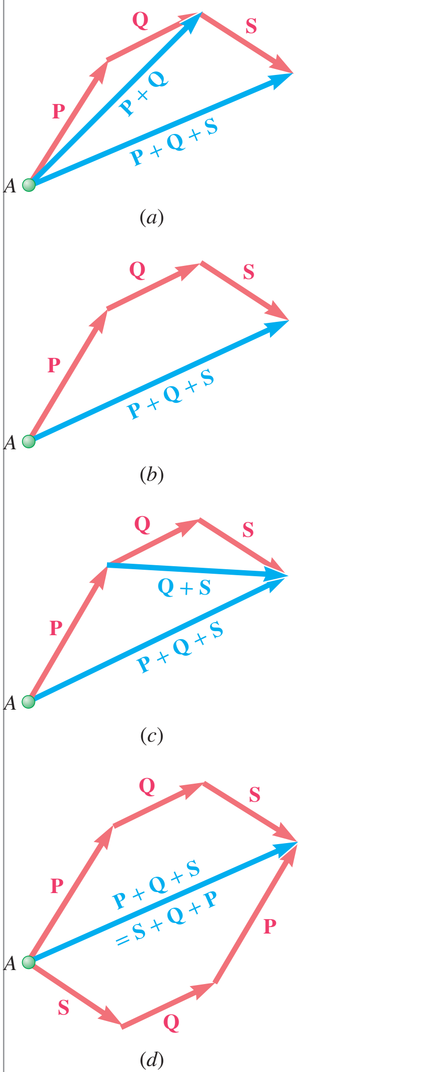


Fig. 2.8 Graphical addition of vectors. (*a*) Applying the triangle rule twice to add three vectors; (*b*) the vectors can be added in one step by the polygon rule; (*c*) vector addition is associative; (*d*) the order of addition is immaterial.

difference between the vectors **P** and **Q**, by adding to **P** the negative vector −**Q** (Fig. 2.7). We write

$$\mathbf{P} - \mathbf{Q} = \mathbf{P} + (-\mathbf{Q}) \qquad (2.2)$$

Here again we should observe that, although we use the same sign to denote both vector and scalar subtraction, we avoid confusion by taking care to distinguish between vector and scalar quantities.

We now consider the *sum of three or more vectors*. The sum of three vectors **P**, **Q**, and **S** is, *by definition*, obtained by first adding the vectors **P** and **Q** and then adding the vector **S** to the vector **P** + **Q**. We write

$$\mathbf{P} + \mathbf{Q} + \mathbf{S} = (\mathbf{P} + \mathbf{Q}) + \mathbf{S} \qquad (2.3)$$

Similarly, we obtain the sum of four vectors by adding the fourth vector to the sum of the first three. It follows that we can obtain the sum of any number of vectors by applying the parallelogram law repeatedly to successive pairs of vectors until all of the given vectors are replaced by a single vector.

If the given vectors are *coplanar,* i.e., if they are contained in the same plane, we can obtain their sum graphically. For this case, repeated application of the triangle rule is simpler than applying the parallelogram law. In Fig. 2.8*a*, we find the sum of three vectors **P**, **Q**, and **S** in this manner. The triangle rule is first applied to obtain the sum **P** + **Q** of the vectors **P** and **Q**; we apply it again to obtain the sum of the vectors **P** + **Q** and **S**. However, we could have omitted determining the vector **P** + **Q** and obtain the sum of the three vectors directly, as shown in Fig. 2.8*b*, by **arranging the given vectors in tip-to-tail fashion and connecting the tail of the first vector with the tip of the last one**. This is known as the **polygon rule** for the addition of vectors.

The result would be unchanged if, as shown in Fig. 2.8*c*, we had replaced the vectors **Q** and **S** by their sum **Q** + **S**. We may thus write

$$\mathbf{P} + \mathbf{Q} + \mathbf{S} = (\mathbf{P} + \mathbf{Q}) + \mathbf{S} = \mathbf{P} + (\mathbf{Q} + \mathbf{S}) \qquad (2.4)$$

which expresses the fact that vector addition is *associative*. Recalling that vector addition also has been shown to be commutative in the case of two vectors, we can write

$$\begin{aligned} \mathbf{P} + \mathbf{Q} + \mathbf{S} &= (\mathbf{P} + \mathbf{Q}) + \mathbf{S} = \mathbf{S} + (\mathbf{P} + \mathbf{Q}) \\ &= \mathbf{S} + (\mathbf{Q} + \mathbf{P}) = \mathbf{S} + \mathbf{Q} + \mathbf{P} \end{aligned} \qquad (2.5)$$

This expression, as well as others we can obtain in the same way, shows that the order in which several vectors are added together is immaterial (Fig. 2.8*d*).

Product of a Scalar and a Vector. It is convenient to denote the sum **P** + **P** by 2**P**, the sum **P** + **P** + **P** by 3**P**, and, in general, the sum of n equal vectors **P** by the product n**P**. Therefore, we define the product n**P** of a positive integer n and a vector **P** as a vector having the same direction as **P** and the magnitude nP. Extending this definition to include all scalars and recalling the definition of a negative vector given earlier, we define the product k**P** of a scalar k and a vector **P** as a vector having the same direction as **P** (if k is positive) or a direction opposite to that of **P** (if k is negative) and a magnitude equal to the product of P and the absolute value of k (Fig. 2.9).

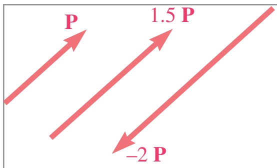


Fig. 2.9 Multiplying a vector by a scalar changes the vector's magnitude, but not its direction (unless the scalar is negative, in which case the direction is reversed).

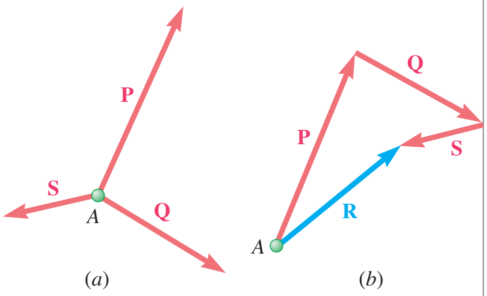


Fig. 2.10 Concurrent forces can be added by the polygon rule.

2.1D Resultant of Several Concurrent Forces

Consider a particle *A* acted upon by several coplanar forces, i.e., by several forces contained in the same plane (Fig. 2.10*a*). Because the forces all pass through *A*, they are also said to be *concurrent*. We can add the vectors representing the forces acting on *A* by the polygon rule (Fig. 2.10*b*). Because the use of the polygon rule is equivalent to the repeated application of the parallelogram law, the vector **R** obtained in this way represents the resultant of the given concurrent forces. That is, the single force **R** has the same effect on the particle *A* as the given forces. As before, the order in which we add the vectors **P**, **Q**, and **S** representing the given forces is immaterial.

2.1E Resolution of a Force into Components

We have seen that two or more forces acting on a particle may be replaced by a single force that has the same effect on the particle. Conversely, a single force **F** acting on a particle may be replaced by two or more forces that, together, have the same effect on the particle. These forces are called **components** of the original force **F**, and the process of substituting them for **F** is called **resolving the force F into components**.

Each force **F** can be resolved into an infinite number of possible sets of components. Sets of *two components* **P** *and* **Q** are the most important as far as practical applications are concerned. However, even then, the number of ways in which a given force **F** may be resolved into two components is unlimited (Fig. 2.11).

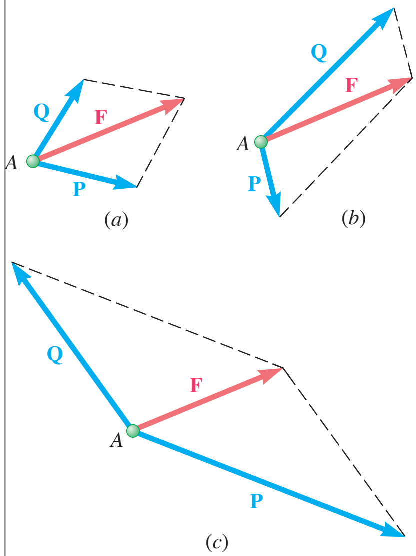


Fig. 2.11 Three possible sets of components for a given force vector **F**.

In many practical problems, we start with a given vector **F** and want to determine a useful set of components. Two cases are of particular interest:

1. **One of the Two Components, P, Is Known.** We obtain the second component, **Q**, by applying the triangle rule and joining the tip of **P** to the tip of **F** (Fig. 2.12). We can determine the magnitude and direction of **Q** graphically or by trigonometry. Once we have determined **Q**, both components **P** and **Q** should be applied at *A*.
2. **The Line of Action of Each Component Is Known.** We obtain the magnitude and sense of the components by applying the parallelogram law and drawing lines through the tip of **F** that are parallel to the given lines of action (Fig. 2.13). This process leads to two well-defined components, **P** and **Q**, which can be determined graphically or computed trigonometrically by applying the law of sines.

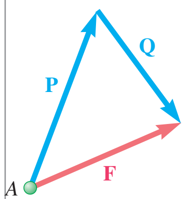


Fig. 2.12 When component **P** is known, use the triangle rule to find component **Q**.

You will encounter many similar cases; for example, you might know the direction of one component while the magnitude of the other component is to be as small as possible (see Sample Prob. 2.2). In all cases, you need to draw the appropriate triangle or parallelogram that satisfies the given conditions.

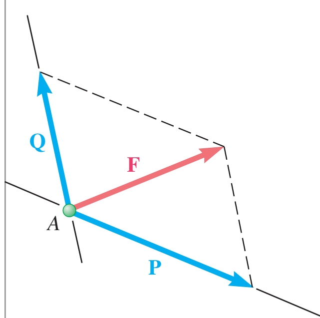
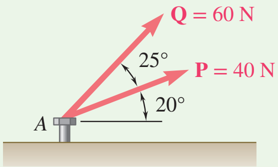


Fig. 2.13 When the lines of action are known, use the parallelogram rule to determine components **P** and **Q**.



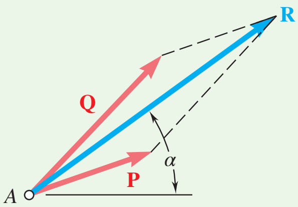


Fig. 1 Parallelogram law applied to add forces **P** and **Q**.

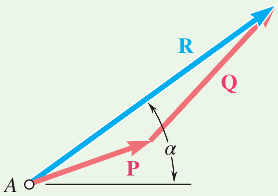


Fig. 2 Triangle rule applied to add forces **P** and **Q**.

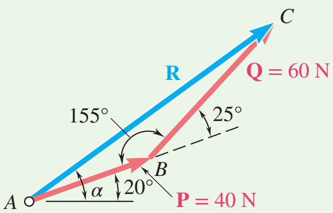


Fig. 3 Geometry of triangle rule applied to add forces **P** and **Q**.

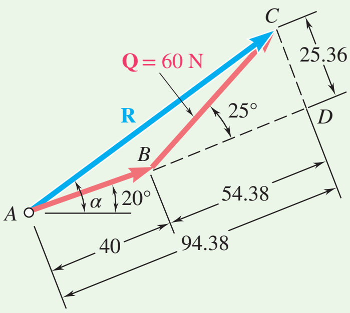


Fig. 4 Alternative geometry of triangle rule applied to add forces **P** and **Q**.

Sample Problem 2.1

Two forces **P** and **Q** act on a bolt *A*. Determine their resultant.

STRATEGY: Two lines determine a plane, so this is a problem of two coplanar forces. You can solve the problem graphically or by trigonometry.

MODELING: For a graphical solution, you can use the parallelogram rule or the triangle rule for addition of vectors. For a trigonometric solution, you can use the law of cosines and law of sines or use a right-triangle approach.

ANALYSIS:

Graphical Solution. Draw to scale a parallelogram with sides equal to **P** and **Q** (Fig. 1). Measure the magnitude and direction of the resultant. They are

$$R = 98 \text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98 \text{ N} \measuredangle 35° \quad ◀$$

You can also use the triangle rule. Draw forces **P** and **Q** in tip-to-tail fashion (Fig. 2). Again measure the magnitude and direction of the resultant. The answers should be the same.

$$R = 98 \text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98 \text{ N} \measuredangle 35° \quad ◀$$

Trigonometric Solution. Using the triangle rule again, you know two sides and the included angle (Fig. 3). Apply the law of cosines.

$$R^2 = P^2 + Q^2 - 2PQ \cos B$$
$$R^2 = (40 \text{ N})^2 + (60 \text{ N})^2 - 2(40 \text{ N})(60 \text{ N}) \cos 155°$$
$$R = 97.73 \text{ N}$$

Now apply the law of sines:

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60 \text{ N}} = \frac{\sin 155°}{97.73 \text{ N}} \tag{1}$$

Solving Eq. (1) for sin *A*, you obtain

$$\sin A = \frac{(60 \text{ N}) \sin 155°}{97.73 \text{ N}}$$

Using a calculator, compute this quotient, and then obtain its arc sine:

$$A = 15.04° \qquad \alpha = 20° + A = 35.04°$$

Use three significant figures to record the answer (c.f. Sec. 1.6):

$$\mathbf{R} = 97.7 \text{ N} \measuredangle 35.0° \quad ◀$$

Alternative Trigonometric Solution. Construct the right triangle *BCD* (Fig. 4) and compute

$$CD = (60 \text{ N}) \sin 25° = 25.36 \text{ N}$$
$$BD = (60 \text{ N}) \cos 25° = 54.38 \text{ N}$$

(continued)

Then, using triangle *ACD*, you have

$$\tan A = \frac{25.36\text{ N}}{94.38\text{ N}} \qquad A = 15.04°$$

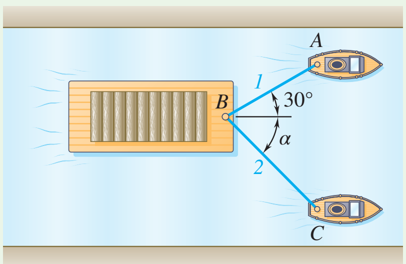
$$R = \frac{25.36}{\sin A} \qquad R = 97.73\text{ N}$$

Again,

$$\alpha = 20° + A = 35.04° \qquad \mathbf{R} = 97.7\text{ N} \measuredangle 35.0°$$ ◀

REFLECT and THINK: An analytical solution using trigonometry provides for greater accuracy. However, it is helpful to use a graphical solution as a check.

Sample Problem 2.2



Two tugboats are pulling a barge. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (*a*) the tension in each of the ropes, given that $\alpha = 45°$, (*b*) the value of α for which the tension in rope *2* is a minimum.

STRATEGY: This is a problem of two coplanar forces. You can solve the first part either graphically or analytically. In the second part, a graphical approach readily shows the necessary direction for rope *2*, and you can use an analytical approach to complete the solution.

MODELING: You can use the parallelogram law or the triangle rule to solve part (*a*). For part (*b*), use a variation of the triangle rule.

ANALYSIS: **a. Tension for $\alpha = 45°$.**

Graphical Solution. Use the parallelogram law. The resultant (the diagonal of the parallelogram) is equal to 5000 lb and is directed to the right. Draw the sides parallel to the ropes (Fig. 1). If the drawing is done to scale, you should measure

$$T_1 = 3700\text{ lb} \qquad T_2 = 2600\text{ lb}$$ ◀

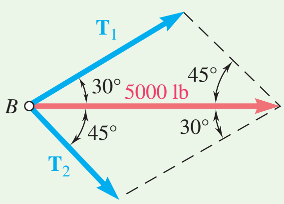


Fig. 1 Parallelogram law applied to add forces $\mathbf{T}_1$ and $\mathbf{T}_2$.

(continued)

Trigonometric Solution. Use the triangle rule. Note that the triangle in Fig. 2 represents half of the parallelogram shown in Fig. 1. Using the law of sines,

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000 \text{ lb}}{\sin 105°}$$

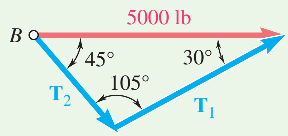


Fig. 2 Triangle rule applied to add forces $\mathbf{T}_1$ and $\mathbf{T}_2$.

With a calculator, compute and store the value of the last quotient. Multiply this value successively by sin 45° and sin 30°, obtaining

$$T_1 = 3660 \text{ lb} \qquad T_2 = 2590 \text{ lb}$$ ◀

b. Value of α for Minimum T_2. To determine the value of α for which the tension in rope *2* is a minimum, use the triangle rule again. In Fig. 3, line *1-1′* is the known direction of $\mathbf{T}_1$. Several possible directions of $\mathbf{T}_2$ are shown by the lines *2-2′*. The minimum value of T_2 occurs when $\mathbf{T}_1$ and $\mathbf{T}_2$ are perpendicular (Fig. 4). Thus, the minimum value of T_2 is

$$T_2 = (5000 \text{ lb}) \sin 30° = 2500 \text{ lb}$$

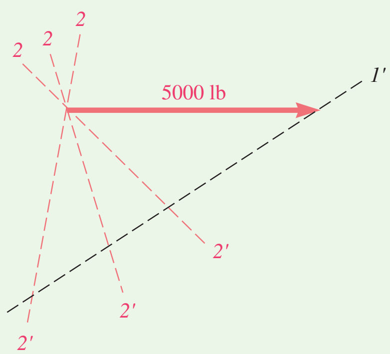


Fig. 3 Determination of direction of minimum $\mathbf{T}_2$.

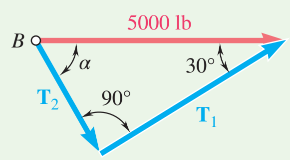


Fig. 4 Triangle rule applied for minimum $\mathbf{T}_2$.

Corresponding values of T_1 and α are

$$T_1 = (5000 \text{ lb}) \cos 30° = 4330 \text{ lb}$$

$$\alpha = 90° - 30° \qquad \alpha = 60°$$ ◀

REFLECT and THINK: Part (*a*) is a straightforward application of resolving a vector into components. The key to part (*b*) is recognizing that the minimum value of T_2 occurs when $\mathbf{T}_1$ and $\mathbf{T}_2$ are perpendicular.

SOLVING PROBLEMS ON YOUR OWN

The preceding sections were devoted to adding vectors by using the parallelogram law, triangle rule, and polygon rule with application to forces.

We presented two Sample Problems. In Sample Prob. 2.1, we used the parallelogram law to determine the resultant of two forces of known magnitude and direction. In Sample Prob. 2.2, we used it to resolve a given force into two components of known direction.

You will now be asked to solve problems on your own. Some may resemble one of the Sample Problems; others may not. What all Problems and Sample Problems in this section have in common is that they can be solved by direct application of the parallelogram law.

Your solution of a given problem should consist of the following steps:

1. Identify which forces are the applied forces and which is the resultant. It is often helpful to write the vector equation that shows how the forces are related. For example, in Sample Prob. 2.1 you could write

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

You may want to keep this relation in mind as you formulate the next part of the solution.

2. Draw a parallelogram with the applied forces as two adjacent sides and the resultant as the included diagonal (Fig. 2.2). Alternatively, you can use the **triangle rule** with the applied forces drawn in tip-to-tail fashion and the resultant extending from the tail of the first vector to the tip of the second (Fig. 2.6).

3. Indicate all dimensions. Using one of the triangles of the parallelogram or the triangle constructed according to the triangle rule, indicate all dimensions—whether sides or angles—and determine the unknown dimensions either graphically or by trigonometry.

4. Recall the laws of trigonometry. If you use trigonometry, remember that the law of cosines should be applied first if two sides and the included angle are known (Sample Prob. 2.1), and the law of sines should be applied first if one side and all angles are known (Sample Prob. 2.2).

If you have had prior exposure to mechanics, you might be tempted to ignore the solution techniques of this lesson in favor of resolving the forces into rectangular components. The component method is important and is considered in the next section, but use of the parallelogram law simplifies the solution of many problems and should be mastered first.

Problems

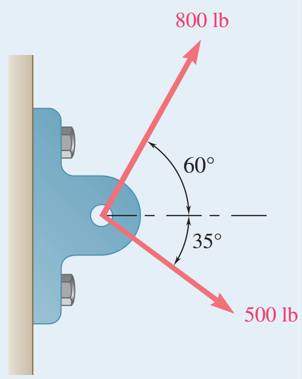


Fig. P2.2

2.1 Two forces are applied as shown to a hook. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

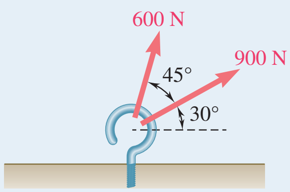


Fig. P2.1

2.2 Two forces are applied as shown to a bracket support. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

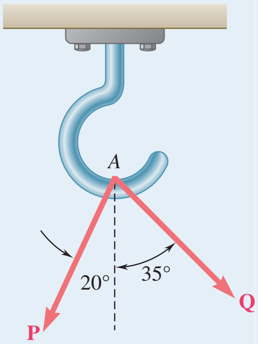


Fig. *P2.3* and P2.4

2.3 Two forces **P** and **Q** are applied as shown at point *A* of a hook support. Knowing that $P = 75$ N and $Q = 125$ N, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

2.4 Two forces **P** and **Q** are applied as shown at point *A* of a hook support. Knowing that $P = 60$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

2.5 A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.

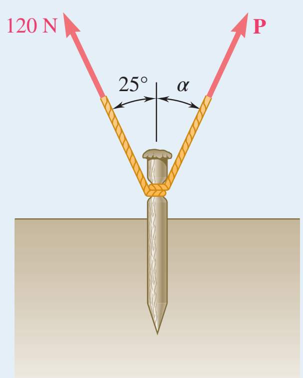


Fig. P2.5

2.6 A telephone cable is clamped at *A* to the pole *AB*. Knowing that the tension in the left-hand portion of the cable is $T_1 = 800$ lb, determine by trigonometry (*a*) the required tension T_2 in the right-hand portion if the resultant **R** of the forces exerted by the cable at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

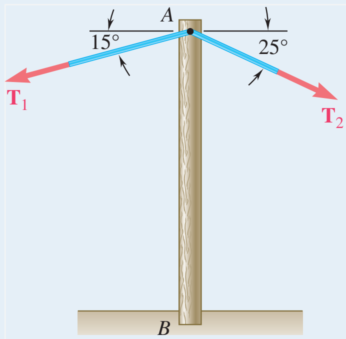


Fig. P2.6 and *P2.7*

2.7 A telephone cable is clamped at *A* to the pole *AB*. Knowing that the tension in the right-hand portion of the cable is $T_2 = 1000$ lb, determine by trigonometry (*a*) the required tension T_1 in the left-hand portion if the resultant **R** of the forces exerted by the cable at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

2.8 A disabled automobile is pulled by means of two ropes as shown. The tension in rope *AB* is 2.2 kN, and the angle α is 25°. Knowing that the resultant of the two forces applied at *A* is directed along the axis of the automobile, determine by trigonometry (*a*) the tension in rope *AC*, (*b*) the magnitude of the resultant of the two forces applied at *A*.

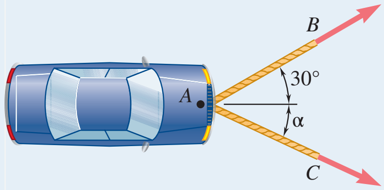


Fig. P2.8 and P2.9

2.9 A disabled automobile is pulled by means of two ropes as shown. Knowing that the tension in rope *AB* is 3 kN, determine by trigonometry the tension in rope *AC* and the value of α so that the resultant force exerted at *A* is a 4.8-kN force directed along the axis of the automobile.

2.10 Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

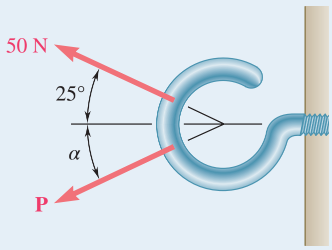


Fig. P2.10

2.11 A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20°$, determine by trigonometry (*a*) the required magnitude of the force **P** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

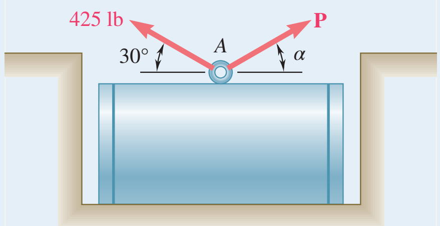


Fig. P2.11, *P2.12*, and P2.13

2.12 A steel tank is to be positioned in an excavation. Knowing that the magnitude of **P** is 500 lb, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

2.13 A steel tank is to be positioned in an excavation. Determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied at *A* is vertical, (*b*) the corresponding magnitude of **R**.

2.14 For the hook support of Prob. 2.10, determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied to the support is horizontal, (*b*) the corresponding magnitude of **R**.

2.15 The barge *B* is pulled by two tugboats *A* and *C*. At a given instant, the tension in cable *AB* is 4500 lb and the tension in cable *BC* is 2000 lb. Determine by trigonometry the magnitude and direction of the resultant of the two forces applied at *B* at that instant.

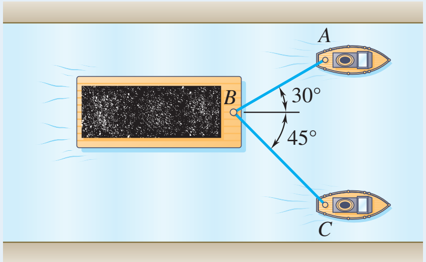


Fig. P2.15

2.16 Solve Prob. 2.1 by trigonometry.

2.17 Solve Prob. 2.4 by trigonometry.

2.18 For the stake of Prob. 2.5, knowing that the tension in one rope is 120 N, determine by trigonometry the magnitude and direction of the force **P** so that the resultant is a vertical force of 160 N.

2.19 Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 10 kN in member *A* and 15 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

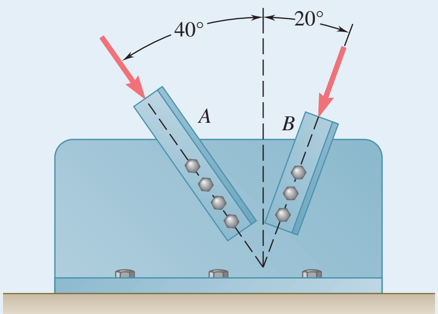


Fig. P2.19 and *P2.20*

2.20 Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member *A* and 10 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

2.2 ADDING FORCES BY COMPONENTS

In Sec. 2.1E, we described how to resolve a force into components. Here we discuss how to add forces by using their components, especially rectangular components. This method is often the most convenient way to add forces and, in practice, is the most common approach. (Note that we can readily extend the properties of vectors established in this section to the rectangular components of any vector quantity, such as velocity or momentum.)

2.2A Rectangular Components of a Force: Unit Vectors

In many problems, it is useful to resolve a force into two components that are perpendicular to each other. Figure 2.14 shows a force **F** resolved into a component $\mathbf{F}_x$ along the x axis and a component $\mathbf{F}_y$ along the y axis. The parallelogram drawn to obtain the two components is a rectangle, and $\mathbf{F}_x$ and $\mathbf{F}_y$ are called **rectangular components**.

The x and y axes are usually chosen to be horizontal and vertical, respectively, as in Fig. 2.14; they may, however, be chosen in any two perpendicular directions, as shown in Fig. 2.15. In determining the rectangular components of a force, you should think of the construction lines shown in Figs. 2.14 and 2.15 as being *parallel* to the x and y axes, rather than *perpendicular* to these axes. This practice will help avoid mistakes in determining *oblique* components, as in Sec. 2.1E.

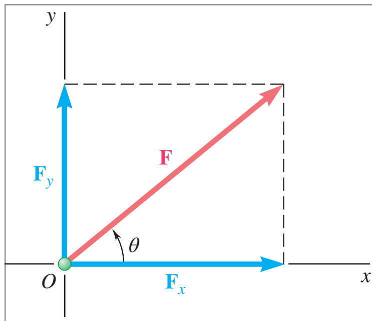


Fig. 2.14 Rectangular components of a force **F**.

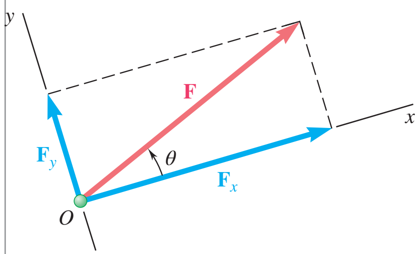


Fig. 2.15 Rectangular components of a force **F** for axes rotated away from horizontal and vertical.

Force in Terms of Unit Vectors. To simplify working with rectangular components, we introduce two vectors of unit magnitude, directed respectively along the positive x and y axes. These vectors are called **unit vectors** and are denoted by **i** and **j**, respectively (Fig. 2.16). Recalling the definition of the product of a scalar and a vector given in Sec. 2.1C, note that we can obtain the rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$ of a force **F** by multiplying respectively the unit vectors **i** and **j** by appropriate scalars (Fig. 2.17). We have

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j} \tag{2.6}$$

and

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \tag{2.7}$$

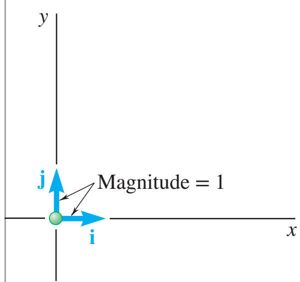


Fig. 2.16 Unit vectors along the x and y axes.

The scalars F_x and F_y may be positive or negative, depending upon the sense of $\mathbf{F}_x$ and of $\mathbf{F}_y$, but their absolute values are equal to the magnitudes of the component forces $\mathbf{F}_x$ and $\mathbf{F}_y$, respectively. The scalars F_x and F_y are called the **scalar components** of the force **F**, whereas the actual component forces $\mathbf{F}_x$ and $\mathbf{F}_y$ should be referred to as the **vector components** of **F**. However, when there exists no possibility of confusion, we may refer to the vector as well as the scalar components of **F** as simply the **components** of **F**. Note that the scalar component F_x is positive when the vector component $\mathbf{F}_x$ has the same sense as the unit vector **i** (i.e., the same sense as the positive x axis) and is negative when $\mathbf{F}_x$ has the opposite sense. A similar conclusion holds for the sign of the scalar component F_y.

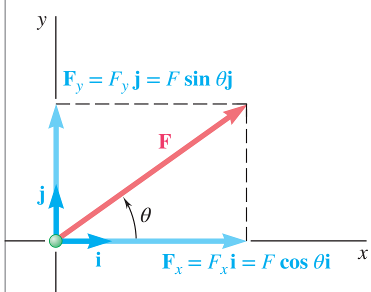


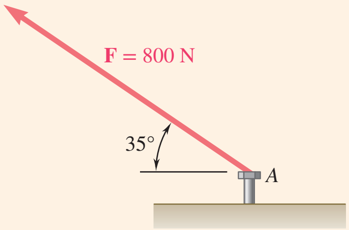
Fig. 2.17 Expressing the components of **F** in terms of unit vectors with scalar multipliers.

Scalar Components. Denoting by F the magnitude of the force **F** and by θ the angle between **F** and the x axis, which is measured counterclockwise from the positive x axis (Fig. 2.17), we may express the scalar components of **F** as

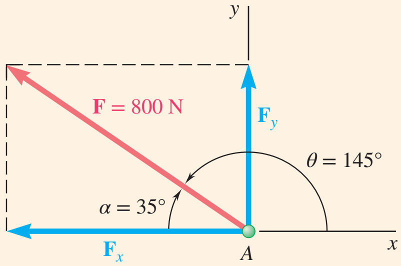
$$F_x = F\cos\theta \qquad F_y = F\sin\theta \tag{2.8}$$

These relations hold for any value of the angle θ from 0° to 360°, and they define the signs and absolute values of the scalar components F_x and F_y.

Concept Application 2.1



(*a*)



(*b*)

Fig. 2.18 (*a*) Force **F** exerted on a bolt; (*b*) rectangular components of **F**.

A force of 800 N is exerted on a bolt A, as shown in Fig. 2.18*a*. Determine the horizontal and vertical components of the force.

Solution In order to obtain the correct sign for the scalar components F_x and F_y, we could substitute the value $180° - 35° = 145°$ for θ in Eqs. (2.8). However, it is often more practical to determine by inspection the signs of F_x and F_y (Fig. 2.18*b*) and then use the trigonometric functions of the angle $\alpha = 35°$. Therefore,

$$F_x = -F\cos\alpha = -(800\text{ N})\cos 35° = -655\text{ N}$$
$$F_y = +F\sin\alpha = +(800\text{ N})\sin 35° = +459\text{ N}$$

The vector components of **F** are thus

$$\mathbf{F}_x = -(655\text{ N})\mathbf{i} \qquad \mathbf{F}_y = +(459\text{ N})\mathbf{j}$$

and we may write **F** in the form

$$\mathbf{F} = -(655\text{ N})\mathbf{i} + (459\text{ N})\mathbf{j}$$ ◀

Concept Application 2.2

A man pulls with a force of 300 N on a rope attached to the top of a building, as shown in Fig. 2.19*a*. What are the horizontal and vertical components of the force exerted by the rope at point A?

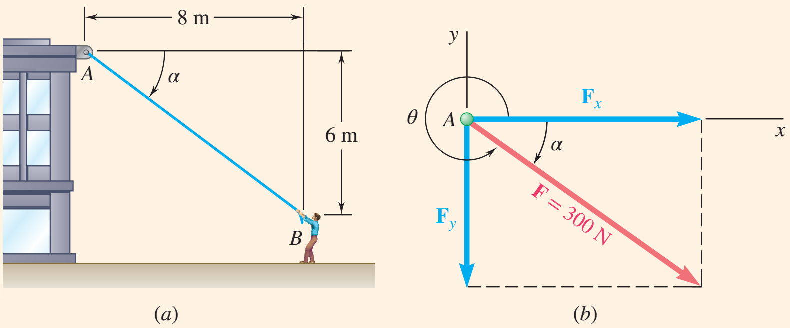


Fig. 2.19 (*a*) A man pulls on a rope attached to a building; (*b*) components of the rope's force **F**.

(continued)

Solution You can see from Fig. 2.19*b* that

$$F_x = +(300\text{ N})\cos\alpha \qquad F_y = -(300\text{ N})\sin\alpha$$

Observing that $AB = 10$ m, we find from Fig. 2.19*a* that

$$\cos\alpha = \frac{8\text{ m}}{AB} = \frac{8\text{ m}}{10\text{ m}} = \frac{4}{5} \qquad \sin\alpha = \frac{6\text{ m}}{AB} = \frac{6\text{ m}}{10\text{ m}} = \frac{3}{5}$$

We thus obtain

$$F_x = +(300\text{ N})\frac{4}{5} = +240\text{ N} \qquad F_y = -(300\text{ N})\frac{3}{5} = -180\text{ N}$$

This gives us a total force of

$$\mathbf{F} = (240\text{ N})\mathbf{i} - (180\text{ N})\mathbf{j}$$

Direction of a Force. When a force **F** is defined by its rectangular components F_x and F_y (see Fig. 2.17), we can find the angle θ defining its direction from

$$\tan\theta = \frac{F_y}{F_x} \tag{2.9}$$

We can obtain the magnitude F of the force by applying the Pythagorean theorem,

$$F = \sqrt{F_x^2 + F_y^2} \tag{2.10}$$

or by solving for F from one of the Eqs. (2.8).

Concept Application 2.3

A force $\mathbf{F} = (700\text{ lb})\mathbf{i} + (1500\text{ lb})\mathbf{j}$ is applied to a bolt A. Determine the magnitude of the force and the angle θ it forms with the horizontal.

Solution First draw a diagram showing the two rectangular components of the force and the angle θ (Fig. 2.20). From Eq. (2.9), you obtain

$$\tan\theta = \frac{F_y}{F_x} = \frac{1500\text{ lb}}{700\text{ lb}}$$

Using a calculator, enter 1500 lb and divide by 700 lb; computing the arc tangent of the quotient gives you $\theta = 65.0°$. Solve the second of Eqs. (2.8) for F to get

$$F = \frac{F_y}{\sin\theta} = \frac{1500\text{ lb}}{\sin 65.0°} = 1655\text{ lb}$$

The last calculation is easier if you store the value of F_y when originally entered; you may then recall it and divide it by sin θ.

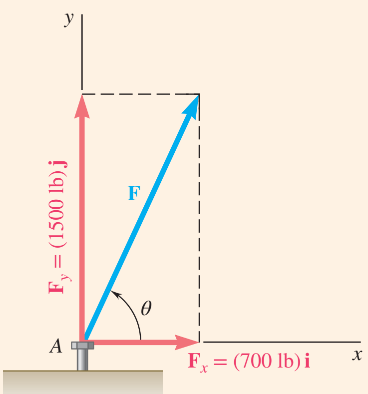
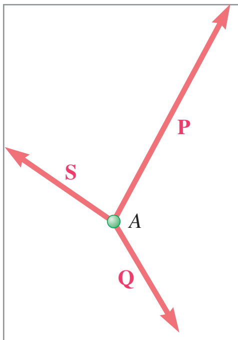
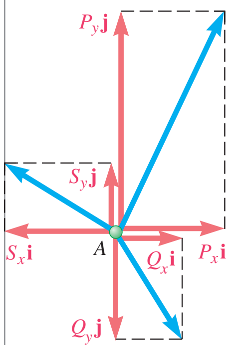


Fig. 2.20 Components of a force **F** exerted on a bolt.



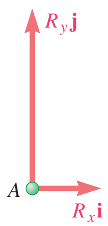
(a)

Fig. 2.21 (a) Three forces acting on a particle.



(b)

Fig. 2.21 (b) Rectangular components of each force.



(c)

Fig. 2.21 (c) Summation of the components.

2.2B Addition of Forces by Summing x and y Components

We described in Sec. 2.1A how to add forces according to the parallelogram law. From this law, we derived two other methods that are more readily applicable to the graphical solution of problems: the triangle rule for the addition of two forces and the polygon rule for the addition of three or more forces. We also explained that the force triangle used to define the resultant of two forces could be used to obtain a trigonometric solution.

However, when we need to add three or more forces, we cannot obtain any practical trigonometric solution from the force polygon that defines the resultant of the forces. In this case, the best approach is to obtain an analytic solution of the problem by resolving each force into two rectangular components.

Consider, for instance, three forces **P**, **Q**, and **S** acting on a particle A (Fig. 2.21a). Their resultant **R** is defined by the relation

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{S} \tag{2.11}$$

Resolving each force into its rectangular components, we have

$$\begin{aligned} R_x\mathbf{i} + R_y\mathbf{j} &= P_x\mathbf{i} + P_y\mathbf{j} + Q_x\mathbf{i} + Q_y\mathbf{j} + S_x\mathbf{i} + S_y\mathbf{j} \\ &= (P_x + Q_x + S_x)\mathbf{i} + (P_y + Q_y + S_y)\mathbf{j} \end{aligned}$$

From this equation, we can see that

$$R_x = P_x + Q_x + S_x \qquad R_y = P_y + Q_y + S_y \tag{2.12}$$

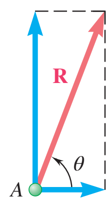
or for short,

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \tag{2.13}$$

We thus conclude that **when several forces are acting on a particle, we obtain the scalar components R_x and R_y of the resultant R by adding algebraically the corresponding scalar components of the given forces**. *(This result also applies to the addition of other vector quantities, such as velocities, accelerations, or momenta.)*

In practice, determining the resultant **R** is carried out in three steps, as illustrated in Fig. 2.21.

1. Resolve the given forces (Fig. 2.21a) into their x and y components (Fig. 2.21b).
2. Add these components to obtain the x and y components of **R** (Fig. 2.21c).
3. Apply the parallelogram law to determine the resultant $\mathbf{R} = R_x\,\mathbf{i} + R_y\,\mathbf{j}$ (Fig. 2.21d).



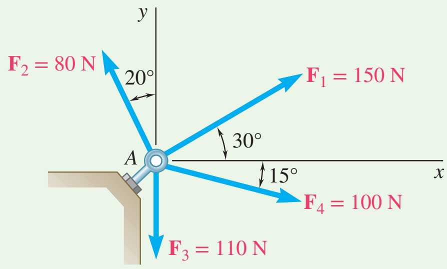
(d)

Fig. 2.21 (d) Determining the resultant from its components.

The procedure just described is most efficiently carried out if you arrange the computations in a table (see Sample Prob. 2.3). Although this is the only practical analytic method for adding three or more forces, it is also often preferred to the trigonometric solution in the case of adding two forces.

Sample Problem 2.3

Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.



STRATEGY: The simplest way to approach a problem of adding four forces is to resolve the forces into components.

MODELING: As we mentioned, solving this kind of problem is usually easier if you arrange the components of each force in a table. In the table below, we entered the x and y components of each force as determined by trigonometry (Fig. 1). According to the convention adopted in this section, the scalar number representing a force component is positive if the force component has the same sense as the corresponding coordinate axis. Thus, x components acting to the right and y components acting upward are represented by positive numbers.

ANALYSIS:

Force	Magnitude, N	x Component, N	y Component, N
$\mathbf{F}_1$	150	+129.9	+75.0
$\mathbf{F}_2$	80	−27.4	+75.2
$\mathbf{F}_3$	110	0	−110.0
$\mathbf{F}_4$	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$

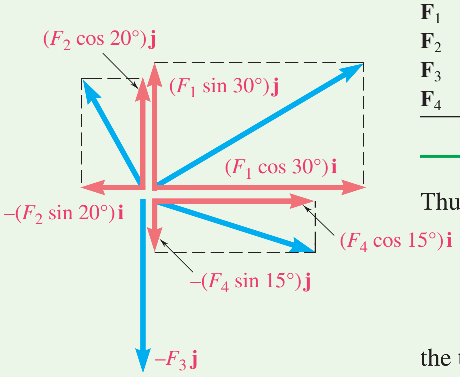


Fig. 1 Rectangular components of each force.

Thus, the resultant **R** of the four forces is

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} \qquad \mathbf{R} = (199.1\text{ N})\mathbf{i} + (14.3\text{ N})\mathbf{j} \blacktriangleleft$$

You can now determine the magnitude and direction of the resultant. From the triangle shown in Fig. 2, you have

$$\tan \alpha = \frac{R_y}{R_x} = \frac{14.3\text{ N}}{199.1\text{ N}} \qquad \alpha = 4.1^\circ$$

$$R = \frac{14.3\text{ N}}{\sin \alpha} = 199.6\text{ N} \qquad \mathbf{R} = 199.6\text{ N} \measuredangle 4.1^\circ \blacktriangleleft$$

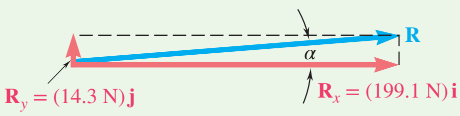


Fig. 2 Resultant of the given force system.

REFLECT and THINK: Arranging data in a table not only helps you keep track of the calculations, but also makes things simpler for using a calculator on similar computations.

SOLVING PROBLEMS ON YOUR OWN

You saw in the preceding lesson that we can determine the resultant of two forces either graphically or from the trigonometry of an oblique triangle.

A. When three or more forces are involved, the best way to determine their resultant R is by first resolving each force into **rectangular components.** You may encounter either of two cases, depending upon the way in which each of the given forces is defined.

Case 1. The force F is defined by its magnitude *F* and the angle α it forms with the *x* axis. Obtain the *x* and *y* components of the force by multiplying *F* by $\cos \alpha$ and $\sin \alpha$, respectively (Concept Application 2.1).

Case 2. The force F is defined by its magnitude *F* and the coordinates of two points *A* and *B* on its line of action (Fig. 2.19). Find the angle α that **F** forms with the *x* axis by trigonometry, and then use the process of Case 1. However, you can also find the components of **F** directly from proportions among the various dimensions involved without actually determining α (Concept Application 2.2).

B. Rectangular components of the resultant. Obtain the components R_x and R_y of the resultant by adding the corresponding components of the given forces algebraically (Sample Prob. 2.3).

You can express the resultant in vectorial form using the unit vectors **i** and **j**, which are directed along the *x* and *y* axes, respectively:

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

Alternatively, you can determine the *magnitude and direction* of the resultant by solving the right triangle of sides R_x and R_y for R and for the angle that **R** forms with the *x* axis.

Problems

2.21 and 2.22 Determine the x and y components of each of the forces shown.

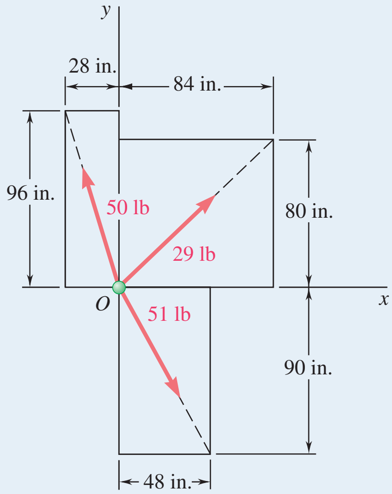


Fig. P2.21

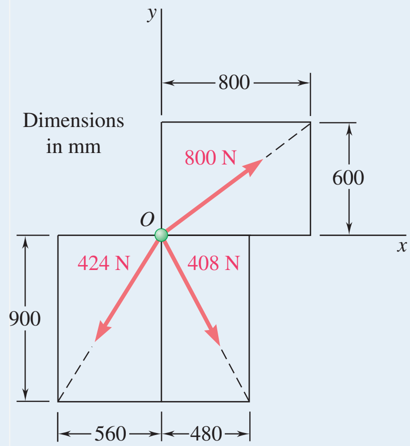


Fig. *P2.22*

2.23 and 2.24 Determine the x and y components of each of the forces shown.

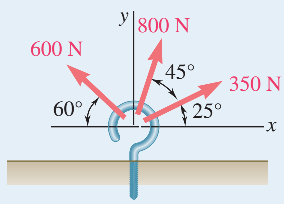


Fig. P2.23

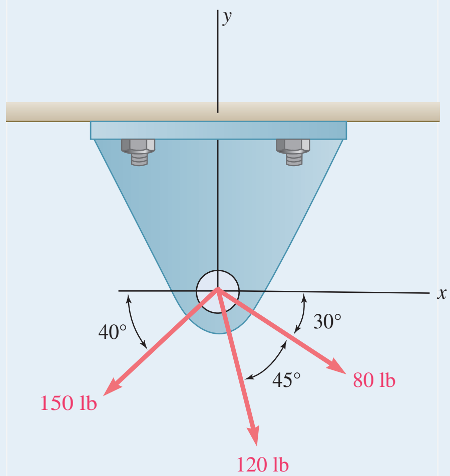


Fig. P2.24

2.25 Member BC exerts on member AC a force **P** directed along line BC. Knowing that **P** must have a 325-N horizontal component, determine (a) the magnitude of the force **P**, (b) its vertical component.

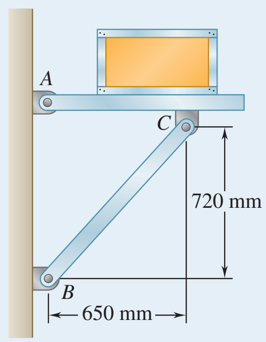


Fig. *P2.25*

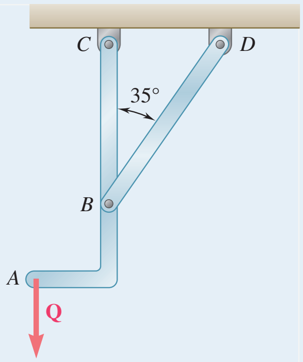


Fig. P2.26

2.26 Member BD exerts on member ABC a force **P** directed along line BD. Knowing that **P** must have a 300-lb horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

2.27 The hydraulic cylinder BC exerts on member AB a force **P** directed along line BC. Knowing that **P** must have a 600-N component perpendicular to member AB, determine (*a*) the magnitude of the force **P**, (*b*) its component along line AB.

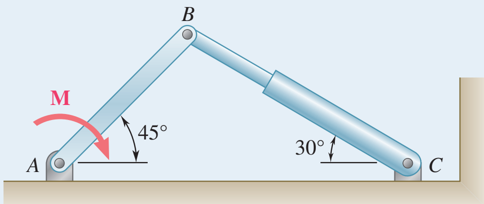


Fig. P2.27

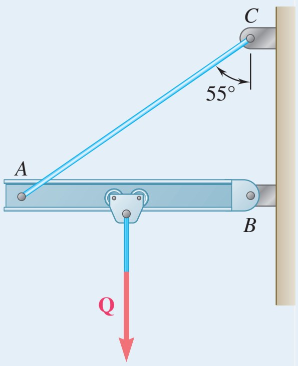


Fig. P2.28

2.28 Cable AC exerts on beam AB a force **P** directed along line AC. Knowing that **P** must have a 350-lb vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

2.29 The hydraulic cylinder BD exerts on member ABC a force **P** directed along line BD. Knowing that **P** must have a 750-N component perpendicular to member ABC, determine (*a*) the magnitude of the force **P**, (*b*) its component parallel to ABC.

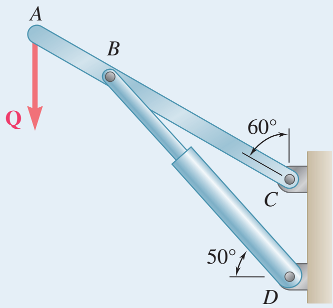


Fig. P2.29

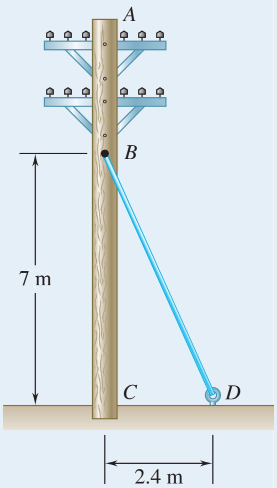


Fig. *P2.30*

2.30 The guy wire BD exerts on the telephone pole AC a force **P** directed along BD. Knowing that **P** must have a 720-N component perpendicular to the pole AC, determine (*a*) the magnitude of the force **P**, (*b*) its component along line AC.

2.31 Determine the resultant of the three forces of Prob. 2.21.

2.32 Determine the resultant of the three forces of Prob. 2.23.

2.33 Determine the resultant of the three forces of Prob. 2.24.

2.34 Determine the resultant of the three forces of Prob. 2.22.

2.35 Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.

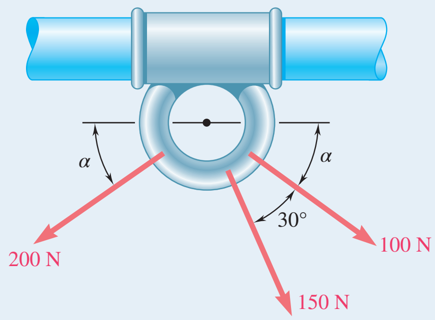


Fig. P2.35

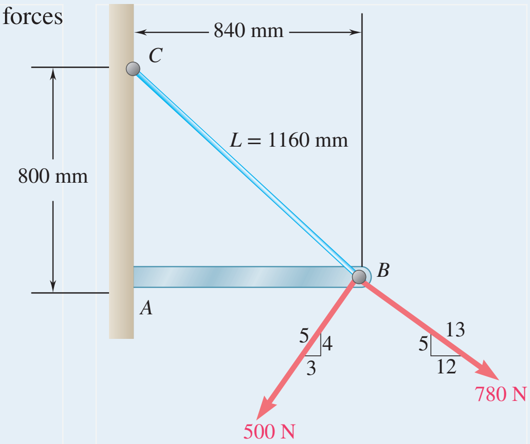


Fig. P2.36

2.36 Knowing that the tension in cable *BC* is 725 N, determine the resultant of the three forces exerted at point *B* of beam *AB*.

2.37 Knowing that $\alpha = 40°$, determine the resultant of the three forces shown.

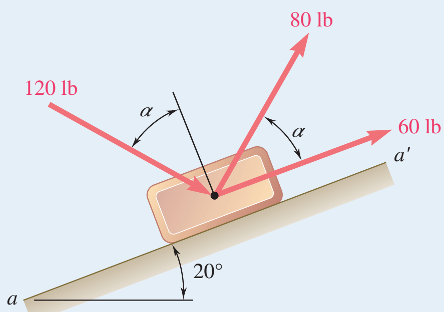


Fig. P2.37 and *P2.38*

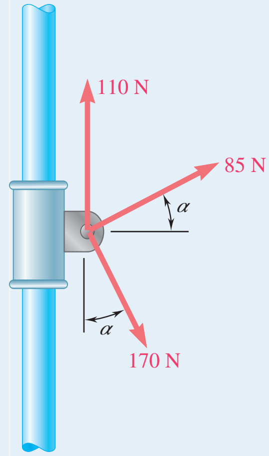


Fig. P2.39

2.38 Knowing that $\alpha = 75°$, determine the resultant of the three forces shown.

2.39 A collar that can slide on a vertical rod is subjected to the three forces shown. Determine (*a*) the required value of α if the resultant of the three forces is to be horizontal, (*b*) the corresponding magnitude of the resultant.

2.40 For the beam of Prob. 2.36, determine (*a*) the required tension in cable *BC* if the resultant of the three forces exerted at point *B* is to be vertical, (*b*) the corresponding magnitude of the resultant.

2.41 Determine (*a*) the required tension in cable *AC*, knowing that the resultant of the three forces exerted at point *C* of boom *BC* must be directed along *BC*, (*b*) the corresponding magnitude of the resultant.

2.42 For the block of Probs. 2.37 and 2.38, determine (*a*) the required value of α if the resultant of the three forces shown is to be parallel to the incline, (*b*) the corresponding magnitude of the resultant.

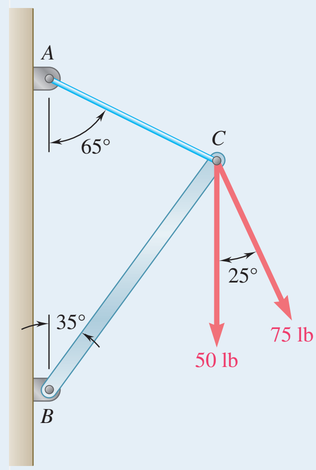


Fig. *P2.41*

Photo 2.2 Forces acting on the carabiner include the weight of the girl and her harness, and the force exerted by the pulley attachment. Treating the carabiner as a particle, it is in equilibrium because the resultant of all forces acting on it is zero.

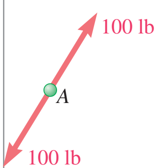


Fig. 2.22 When a particle is in equilibrium, the resultant of all forces acting on the particle is zero.

2.3 FORCES AND EQUILIBRIUM IN A PLANE

Now that we have seen how to add forces, we can proceed to one of the key concepts in this course: the equilibrium of a particle. The connection between equilibrium and the sum of forces is very direct: a particle can be in equilibrium only when the sum of the forces acting on it is zero.

2.3A Equilibrium of a Particle

In the preceding sections, we discussed methods for determining the resultant of several forces acting on a particle. Although it has not occurred in any of the problems considered so far, it is quite possible for the resultant to be zero. In such a case, the net effect of the given forces is zero, and the particle is said to be in **equilibrium**. We thus have the definition:

When the resultant of all the forces acting on a particle is zero, the particle is in equilibrium.

A particle acted upon by two forces is in equilibrium if the two forces have the same magnitude and the same line of action but opposite sense. The resultant of the two forces is then zero, as shown in Fig. 2.22.

Another case of equilibrium of a particle is represented in Fig. 2.23*a*, where four forces are shown acting on particle *A*. In Fig. 2.23*b*, we use the polygon rule to determine the resultant of the given forces. Starting from point *O* with $\mathbf{F}_1$ and arranging the forces in tip-to-tail fashion, we find that the tip of $\mathbf{F}_4$ coincides with the starting point *O*. Thus, the resultant **R** of the given system of forces is zero, and the particle is in equilibrium.

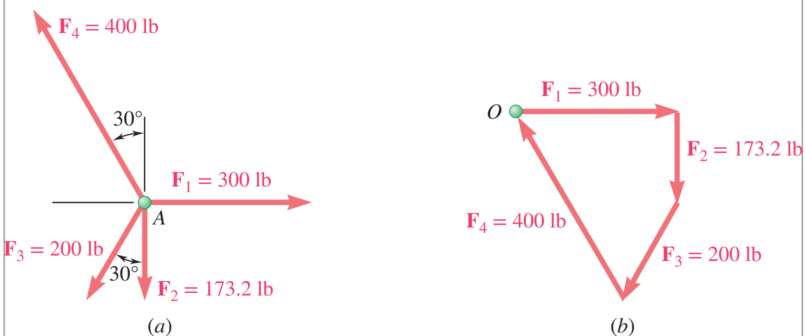


Fig. 2.23 (*a*) Four forces acting on particle *A*; (*b*) using the polygon law to find the resultant of the forces in (*a*), which is zero because the particle is in equilibrium.

The closed polygon drawn in Fig. 2.23*b* provides a *graphical* expression of the equilibrium of *A*. To express *algebraically* the conditions for the equilibrium of a particle, we write

Equilibrium of a particle $$\mathbf{R} = \Sigma\mathbf{F} = 0 \tag{2.14}$$

Resolving each force **F** into rectangular components, we have

$$\Sigma(F_x\mathbf{i} + F_y\mathbf{j}) = 0 \qquad \text{or} \qquad (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} = 0$$