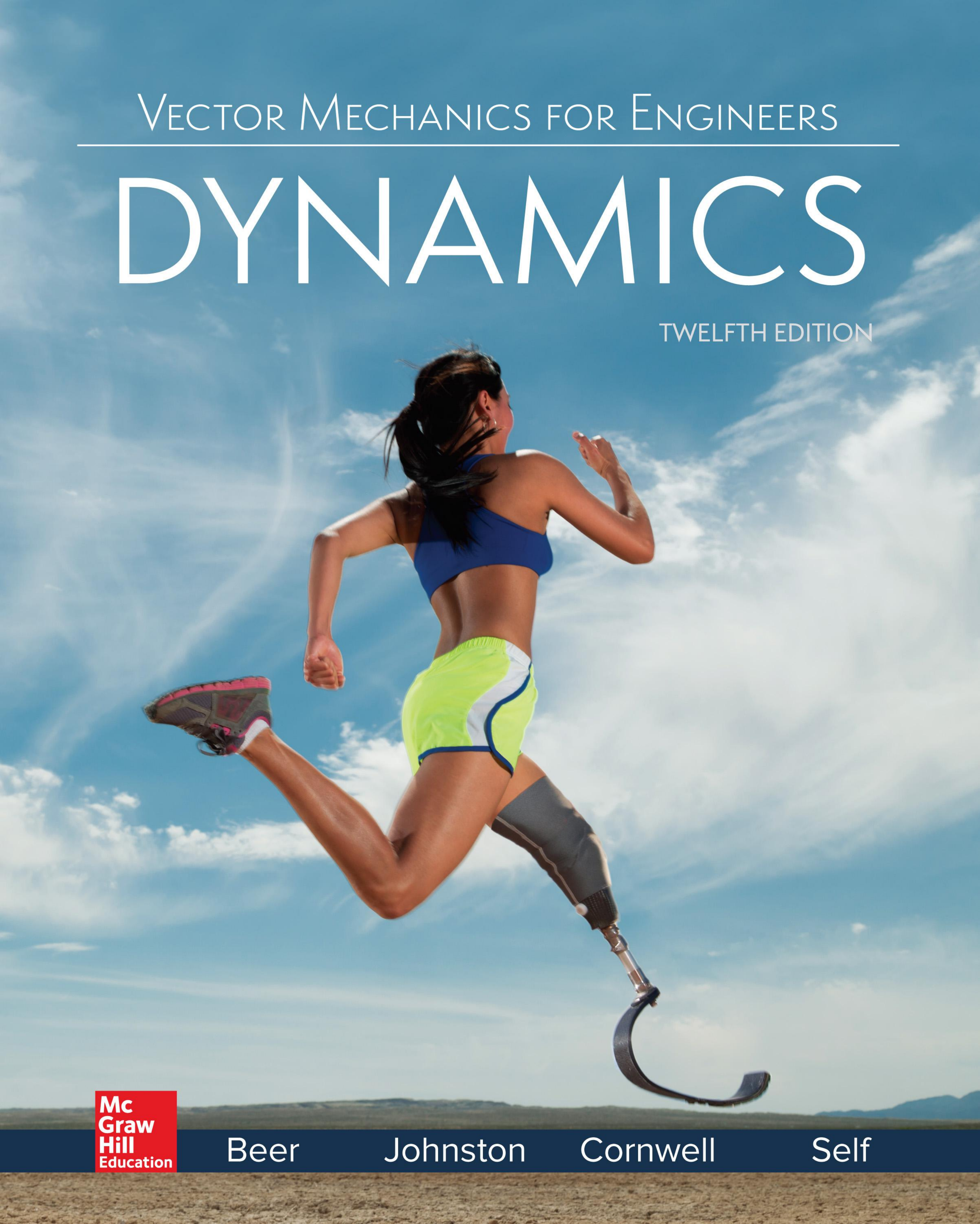
Vector Mechanics for Engineers

DYNAMICS

TWELFTH EDITION



Mc Graw Hill Education

Beer Johnston Cornwell Self

Vector Mechanics For Engineers

Dynamics

Twelfth Edition

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Ferdinand P. Beer
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VECTOR MECHANICS FOR ENGINEERS: DYNAMICS, TWELFTH EDITION

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Preface

Objectives

A primary objective in a first course in mechanics is to help develop a student's ability first to analyze problems in a simple and logical manner, and then to apply basic principles to their solutions. A strong conceptual understanding of these basic mechanics principles is essential for successfully solving mechanics problems. We hope that this text, as well as the preceding volume, *Vector Mechanics for Engineers: Statics,* will help instructors achieve these goals.†

General Approach

Vector algebra was introduced at the beginning of the first volume and is used in the presentation of the basic principles of statics, as well as in the solution of many problems, particularly three-dimensional problems. Similarly, the concept of vector differentiation will be introduced early in this volume, and vector analysis will be used throughout the presentation of dynamics. This approach leads to more concise derivations of the fundamental principles of

2.2 ADDING FORCES BY COMPONENTS

In Sec. 2.1E, we described how to resolve a force into components. Here we discuss how to add forces by using their components, especially rectangular components. This method is often the most convenient way to add forces and, in practice, is the most common approach. (Note that we can readily extend the properties of vectors established in this section to the rectangular components of any vector quantity, such as velocity or momentum.)

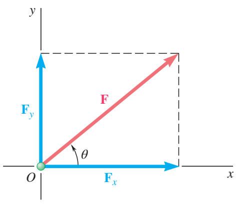


Fig. 2.14 Rectangular components of a force **F**.

2.2A Rectangular Components of a Force: Unit Vectors

In many problems, it is useful to resolve a force into two components that are perpendicular to each other. Figure 2.14 shows a force **F** resolved into a component $\mathbf{F}_x$ along the x axis and a component $\mathbf{F}_y$ along the y axis. The parallelogram drawn to obtain the two components is a rectangle, and $\mathbf{F}_x$ and $\mathbf{F}_y$ are called **rectangular components**.

The x and y axes are usually chosen to be horizontal and vertical, respectively, as in Fig. 2.14; they may, however, be chosen in any two perpendicular directions, as shown in Fig. 2.15. In determining the rectangular components of a force, you should think of the construction lines shown in Figs. 2.14 and 2.15 as being *parallel* to the x and y axes, rather than *perpendicular* to these axes. This practice will help avoid mistakes in determining *oblique* components, as in Sec. 2.1E.

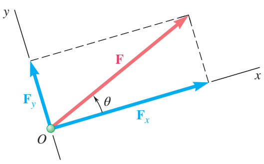
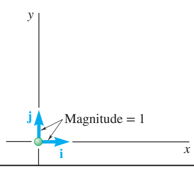


Fig. 2.15 Rectangular components of a force **F** for axes rotated away from horizontal and vertical.

Force in Terms of Unit Vectors. To simplify working with rectangular components, we introduce two vectors of unit magnitude, directed respectively along the positive x and y axes. These vectors are called **unit vectors** and are denoted by **i** and **j**, respectively (Fig. 2.16). Recalling the definition of the product of a scalar and a vector given in Sec. 2.1C, note that we can obtain the rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$ of a force **F** by multiplying respectively the unit vectors **i** and **j** by appropriate scalars (Fig. 2.17). We have

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j} \tag{2.6}$$



†Both texts also are available in a single volume, *Vector Mechanics for Engineers: Statics and Dynamics,* eleventh edition.

‡In a parallel text, *Mechanics for Engineers: Dynamics,* fifth edition, the use of vector algebra is limited to the addition and subtraction of vectors, and vector differentiation is omitted.

mechanics. It also makes it possible to analyze many problems in kinematics and kinetics which could not be solved by scalar methods. The emphasis in this text, however, remains on the correct understanding of the principles of mechanics and on their application to the solution of engineering problems, and vector analysis is presented chiefly as a convenient tool.‡

Practical Applications Are Introduced Early. One of the characteristics of the approach used in this book is that mechanics of *particles* is clearly separated from the mechanics of *rigid bodies.* This approach makes it possible to consider simple practical applications at an early stage and to postpone the introduction of the more difficult concepts. For example:

- In *Statics,* the statics of particles is treated first, and the principle of equilibrium of a particle was immediately applied to practical situations involving only concurrent forces. The statics of rigid bodies is considered later, at which time the vector and scalar products of two vectors were introduced and used to define the moment of a force about a point and about an axis.
- In *Dynamics,* the same division is observed. The basic concepts of force, mass, and acceleration, of work and energy, and of impulse and momentum are introduced and first applied to problems involving only particles. Thus, students can familiarize themselves with the three basic methods used in dynamics and learn their respective advantages before facing the difficulties associated with the motion of rigid bodies.

New Concepts Are Introduced in Simple Terms. Since this text is designed for the first course in dynamics, new concepts are presented in simple terms and every step is explained in detail. On the other hand, by discussing the broader aspects of the problems considered, and by stressing methods of general applicability, a definite maturity of approach has been achieved. For example, the concept of potential energy is discussed in the general case of

17.1 ENERGY METHODS FOR A RIGID BODY

We now use the principle of work and energy to analyze the plane motion of rigid bodies. As we pointed out in Chap. 13, the method of work and energy is particularly well-adapted to solving problems involving velocities and displacements. Its main advantage is that the work of forces and the kinetic energy of objects are scalar quantities.

17.1A Principle of Work and Energy

To apply the principle of work and energy to the motion of a rigid body, we again assume that the rigid body is made up of a large number n of particles of mass Δm_i. From Eq. (14.30) of Sec. 14.2B, we have

Principle of work and energy, rigid body

$$T_1 + U_{1\to2} = T_2 \tag{17.1}$$

where T_1, T_2 = the initial and final values of total kinetic energy of particles forming the rigid body
$U_{1\to2}$ = work of all forces acting on various particles of the body

Photo 17.1 The work done by friction reduces the kinetic energy of the wheel.

Just as we did in Chap. 13, we can express the work done by nonconservative forces as $U^{NC}_{1\to2}$, and we can define potential energy terms for conservative forces. Then we can express Eq. (17.1) as

$$T_1 + V_{g_1} + V_{e_1} + U^{NC}_{1\to2} = T_2 + V_{g_2} + V_{e_2} \tag{17.1'}$$

where V_{g_1} and V_{g_2} are the initial and final gravitational potential energy of the center of mass of the rigid body with respect to a reference point or datum, and V_{e_1} and V_{e_2} are the initial and final values of the elastic energy associated with springs in the system.

We obtain the total kinetic energy

a conservative force. Also, the study of the plane motion of rigid bodies is designed to lead naturally to the study of their general motion in space. This is true in kinematics as well as in kinetics, where the principle of equivalence of external and effective forces is applied directly to the analysis of plane motion, thus facilitating the transition to the study of three-dimensional motion.

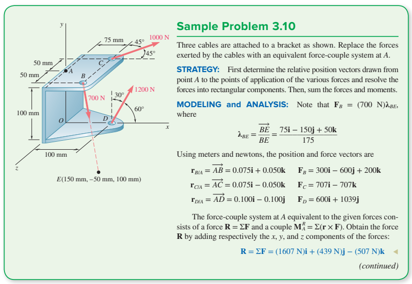
Fundamental Principles Are Placed in the Context of Simple Applications. The fact that mechanics is essentially a *deductive* science based on a few fundamental principles is stressed. Derivations have been presented in their logical sequence and with all the rigor warranted at this level. However, the learning process being largely *inductive*, simple applications are considered first. For example:

- The kinematics of particles (Chap. 11) precedes the kinematics of rigid bodies (Chap. 15).
- The fundamental principles of the kinetics of rigid bodies are first applied to the solution of two-dimensional problems (Chaps. 16 and 17), which can be more easily visualized by the student, while three-dimensional problems are postponed until Chap. 18.

The Presentation of the Principles of Kinetics Is Unified. The twelfth edition of *Vector Mechanics for Engineers* retains the unified presentation of the principles of kinetics which characterized the previous eleven editions. The concepts of linear and angular momentum are introduced in Chap. 12 so that Newton's second law of motion can be presented not only in its conventional form $\mathbf{F} = m\mathbf{a}$, but also as a law relating, respectively, the sum of the forces acting on a particle and the sum of their moments to the rates of change of the linear and angular momentum of the particle. This makes possible an earlier introduction of the principle of conservation of angular momentum and a more meaningful discussion of the motion of a particle under a central force (Sec. 12.3A). More importantly, this approach can be readily extended to the study of the motion of a system of particles (Chap. 14) and leads to a more concise and unified treatment of the kinetics of rigid bodies in two and three dimensions (Chaps. 16 through 18).

Systematic Problem-Solving Approach. All the sample problems are solved using the steps of ***S***trategy, ***M***odeling, ***A***nalysis, and ***R***eflect & ***T***hink, or the "SMART" approach. This methodology is intended to give students confidence when approaching new problems, and students are encouraged to apply this approach in the solution of all assigned problems.

Free-Body Diagrams Are Used Both to Solve Equilibrium Problems and to Express the Equivalence of Force Systems. Free-body diagrams are introduced early in Statics, and their importance is emphasized throughout. They are used not only to solve equilibrium problems but also to express the equivalence of two systems of forces or, more generally, of two systems of vectors. In dynamics we will introduce a kinetic diagram, which is a pictorial representation of inertia terms. The advantage of this approach becomes apparent in the study of the dynamics of rigid bodies, where it is used to solve three-dimensional as well as two-dimensional problems. By placing the emphasis on the free-body diagram and kinetic diagram, rather than on the standard algebraic equations of motion, a more intuitive and more complete understanding of the fundamental principles of dynamics can be achieved. This approach, which was first introduced in 1962 in the first edition of *Vector Mechanics for Engineers,* has now gained wide acceptance among mechanics


Sample Problem 3.10

Three cables are attached to a bracket as shown. Replace the forces exerted by the cables with an equivalent force-couple system at A.

STRATEGY: First determine the relative position vectors drawn from point A to the points of application of the various forces and resolve the forces into rectangular components. Then, sum the forces and moments.

MODELING and ANALYSIS: Note that $\mathbf{F}_B = (700\ \text{N})\boldsymbol{\lambda}_{BE}$, where

$$\boldsymbol{\lambda}_{BE} = \frac{\overrightarrow{BE}}{BE} = \frac{75\mathbf{i} - 150\mathbf{j} + 50\mathbf{k}}{175}$$

Using meters and newtons, the position and force vectors are

$$\mathbf{r}_{B/A} = \overrightarrow{AB} = 0.075\mathbf{i} + 0.050\mathbf{k} \qquad \mathbf{F}_B = 300\mathbf{i} - 600\mathbf{j} + 200\mathbf{k}$$
$$\mathbf{r}_{C/A} = \overrightarrow{AC} = 0.075\mathbf{i} - 0.050\mathbf{k} \qquad \mathbf{F}_C = 707\mathbf{i} - 707\mathbf{k}$$
$$\mathbf{r}_{D/A} = \overrightarrow{AD} = 0.100\mathbf{i} - 0.100\mathbf{j} \qquad \mathbf{F}_D = 600\mathbf{i} + 1039\mathbf{j}$$

The force-couple system at A equivalent to the given forces consists of a force $\mathbf{R} = \Sigma\mathbf{F}$ and a couple $\mathbf{M}_A^R = \Sigma(\mathbf{r} \times \mathbf{F})$. Obtain the force $\mathbf{R}$ by adding respectively the x, y, and z components of the forces:

$$\mathbf{R} = \Sigma\mathbf{F} = (1607\ \text{N})\mathbf{i} + (439\ \text{N})\mathbf{j} - (507\ \text{N})\mathbf{k}$$

(continued)

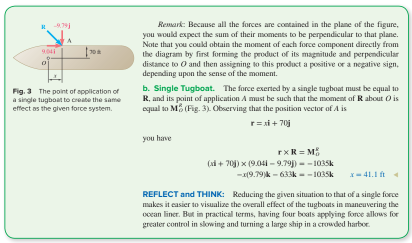


Fig. 3 The point of application of a single tugboat to create the same effect as the given force system.

Remark: Because all the forces are contained in the plane of the figure, you would expect the sum of their moments to be perpendicular to that plane. Note that you could obtain the moment of each force component directly from the diagram by first forming the product of its magnitude and perpendicular distance to O and then assigning to this product a positive or a negative sign, depending upon the sense of the moment.

b. Single Tugboat. The force exerted by a single tugboat must be equal to $\mathbf{R}$, and its point of application A must be such that the moment of $\mathbf{R}$ about O is equal to $\mathbf{M}_O^R$ (Fig. 3). Observing that the position vector of A is

$$\mathbf{r} = x\mathbf{i} + 70\mathbf{j}$$

you have

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O^R$$
$$(x\mathbf{i} + 70\mathbf{j}) \times (9.04\mathbf{i} - 9.79\mathbf{j}) = -1035\mathbf{k}$$
$$-x(9.79)\mathbf{k} - 633\mathbf{k} = -1035\mathbf{k} \qquad x = 41.1\ \text{ft}$$

REFLECT and THINK: Reducing the given situation to that of a single force makes it easier to visualize the overall effect of the tugboats in maneuvering the ocean liner. But in practical terms, having four boats applying force allows for greater control in slowing and turning a large ship in a crowded harbor.

teachers in this country. It is, therefore, used in preference to the method of dynamic equilibrium and to the equations of motion in the solution of all sample problems in this book.

A Careful Balance between SI and U.S. Customary Units Is Consistently Maintained. Because of the current trend in the American government and industry to adopt the international system of units (SI metric units), the SI units most frequently used in mechanics are introduced in Chap. 1 and are used throughout the text. Approximately half of the sample problems and 60 percent of the homework problems are stated in these units, while the remainder are in U.S. customary units. The authors believe that this approach will best serve the need of the students, who, as engineers, will have to be conversant with both systems of units.

It also should be recognized that using both SI and U.S. customary units entails more than the use of conversion factors. Since the SI system of units is an absolute system based on the units of time, length, and mass, whereas the U.S. customary system is a gravitational system based on the units of time, length, and force, different approaches are required for the solution of many problems. For example, when SI units are used, a body is generally specified by its mass expressed in kilograms; in most problems of statics it will be necessary to determine the weight of the body in newtons, and an additional calculation will be required for this purpose. On the other hand, when U.S. customary units are used, a body is specified by its weight in pounds and, in dynamics problems, an additional calculation will be required to determine its mass in slugs (or $\text{lb}\cdot\text{s}^2/\text{ft}$). The authors, therefore, believe that problem assignments should include both systems of units.

The *Instructor's and Solutions Manual* provides six different lists of assignments so that an equal number of problems stated in SI units and in U.S. customary units can be selected. If so desired, two complete lists of assignments can also be selected with up to 75 percent of the problems stated in SI units.

Optional Sections Offer Advanced or Specialty Topics. A large number of optional sections have been included. These sections are indicated by asterisks and thus are easily distinguished from those which form the core of the basic dynamics course. They can be omitted without prejudice to the understanding of the rest of the text.

The topics covered in the optional sections include graphical methods for the solution of rectilinear-motion problems, the trajectory of a particle under a central force, the deflection of fluid streams, problems involving jet and rocket propulsion, the kinematics and kinetics of rigid bodies in three dimensions, damped mechanical vibrations, and electrical analogues. These topics will be found of particular interest when dynamics is taught in the junior year.

The material presented in the text and most of the problems require no previous mathematical knowledge beyond algebra, trigonometry, elementary calculus, and the elements of vector algebra presented in Chaps. 2 and 3 of the volume on statics.[†] However, special problems are included, which make

[†]Some useful definitions and properties of vector algebra have been summarized in Appendix A at the end of this volume for the convenience of the reader. Also, Secs. 9.5 and 9.6 of the volume on statics, which deal with the moments of inertia of masses, have been reproduced in Appendix B.

use of a more advanced knowledge of calculus, and certain sections, such as Secs. 19.5A and 19.5B on damped vibrations, should be assigned only if students possess the proper mathematical background. In portions of the text using elementary calculus, a greater emphasis is placed on the correct understanding and application of the concepts of differentiation and integration, than on the nimble manipulation of mathematical formulas. In this connection, it should be mentioned that the determination of the centroids of composite areas precedes the calculation of centroids by integration, thus making it possible to establish the concept of moment of area firmly before introducing the use of integration.

Guided Tour

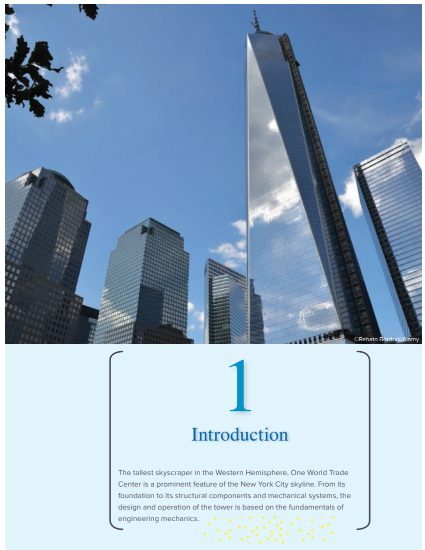
Chapter Introduction. Each chapter begins with a list of learning objectives and an outline that previews chapter topics. An introductory section describes the material to be covered in simple terms, and how it will be applied to the solution of engineering problems.

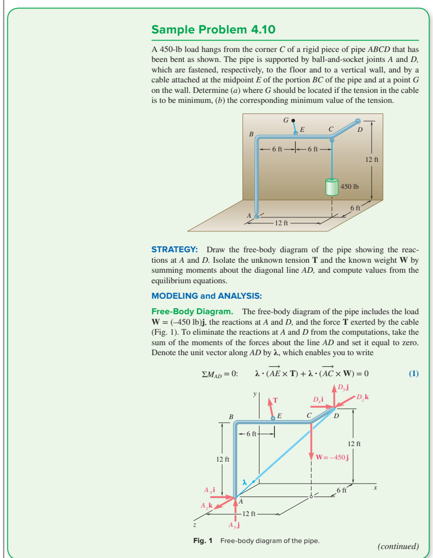
Chapter Lessons. The body of the text is divided into sections, each consisting of one or more sub-sections, several sample problems, and a large number of end-of-section problems for students to solve. Each section corresponds to a well-defined topic and generally can be covered in one lesson. In a number of cases, however, the instructor will find it desirable to devote more than one lesson to a given topic. *The Instructor's and Solutions Manual* contains suggestions on the coverage of each lesson.

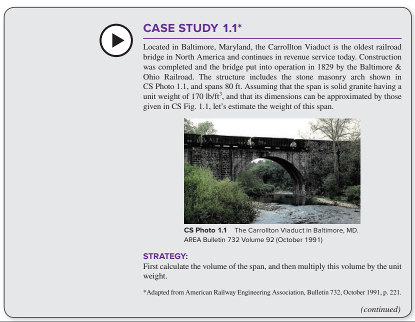
Sample Problems. The Sample Problems are set up in much the same form that students will use when solving assigned problems, and they employ the SMART problem-solving methodology that students are encouraged to use in the solution of their assigned problems. They thus serve the double purpose of reinforcing the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions. In addition, in-problem references and captions have been added to the sample problem figures for contextual linkage to the step-by-step solution. In the digital version, many Sample Problems now have simulations to help students visualize the problem. Enhanced digital content is indicated by a ▶ within the text.

Solving Problems on Your Own. A section entitled *Solving Problems on Your Own* is included for each lesson, between the sample problems and the problems to be assigned. The purpose of these sections is to help students organize in their own minds the preceding theory of the text and the solution methods of the sample problems so that they can more successfully solve the homework problems. Also included in these sections are specific suggestions and strategies that will enable the students to more efficiently attack any assigned problems.

▶ **Case Studies.** Statics and dynamics principles are used extensively in engineering applications, particularly for the designing of solutions to problems and for failure analysis when those solutions do not work as planned. Much can be learned from the historical successes and failures of past designs, and unique insight can be gained by studying how engineers developed different products and structures. To this end, real-world Case Studies have been introduced in this revision to provide relevance and application to the principles of engineering mechanics being discussed. The Case Studies are developed using the SMART problem-solving methodology to present the story. In this way, they serve as both a practical illustration of the concepts linked to some real-world situation and reinforce the consistent five-step approach to solving engineering problems.

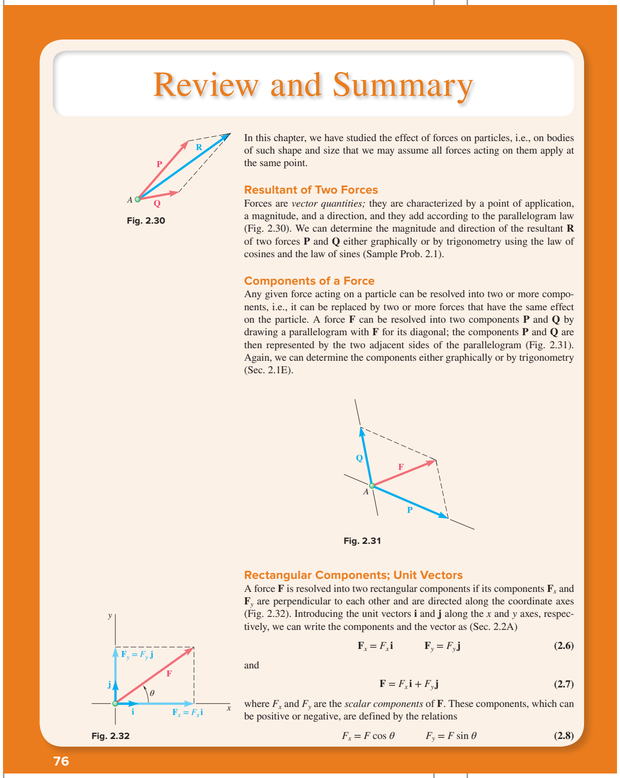






NEW!

Approximately 300 of the homework problems in the text are new or revised.



Review and Summary

Fig. 2.30

In this chapter, we have studied the effect of forces on particles, i.e., on bodies of such shape and size that we may assume all forces acting on them apply at the same point.

Resultant of Two Forces

Forces are *vector quantities;* they are characterized by a point of application, a magnitude, and a direction, and they add according to the parallelogram law (Fig. 2.30). We can determine the magnitude and direction of the resultant **R** of two forces **P** and **Q** either graphically or by trigonometry using the law of cosines and the law of sines (Sample Prob. 2.1).

Components of a Force

Any given force acting on a particle can be resolved into two or more components, i.e., it can be replaced by two or more forces that have the same effect on the particle. A force **F** can be resolved into two components **P** and **Q** by drawing a parallelogram with **F** for its diagonal; the components **P** and **Q** are then represented by the two adjacent sides of the parallelogram (Fig. 2.31). Again, we can determine the components either graphically or by trigonometry (Sec. 2.1E).

Fig. 2.31

Rectangular Components; Unit Vectors

A force **F** is resolved into two rectangular components if its components $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular to each other and are directed along the coordinate axes (Fig. 2.32). Introducing the unit vectors **i** and **j** along the x and y axes, respectively, we can write the components and the vector as (Sec. 2.2A)

$$\mathbf{F}_x = F_x\mathbf{i} \qquad \mathbf{F}_y = F_y\mathbf{j} \tag{2.6}$$

and

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \tag{2.7}$$

where F_x and F_y are the *scalar components* of **F**. These components, which can be positive or negative, are defined by the relations

Fig. 2.32

$$F_x = F\cos\theta \qquad F_y = F\sin\theta \tag{2.8}$$

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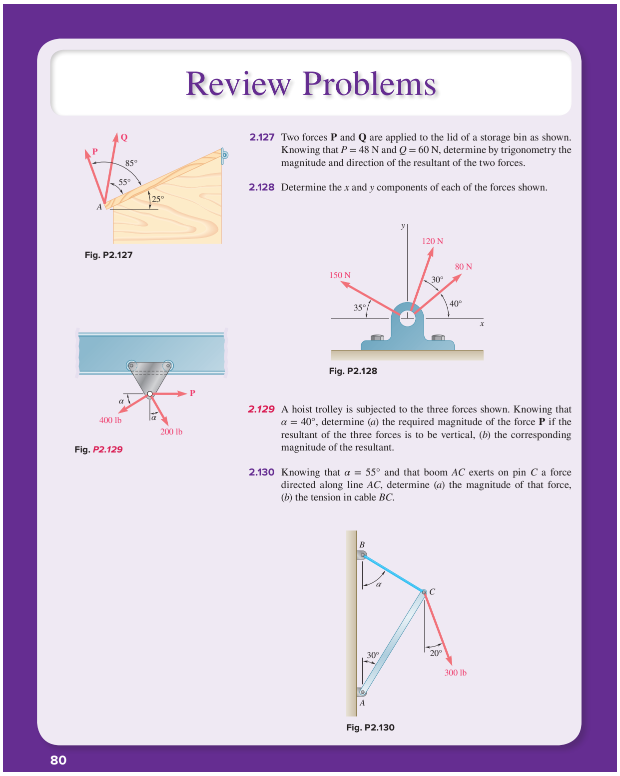
In some instances, these Case Studies are examined further in the accompanying digital content through Connect®. The digital content also provides additional cases that are developed in their entirety.

Homework Problem Sets. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and to help students understand the principles of mechanics. The problems are grouped according to the portions of material they illustrate and, in general, are arranged in order of increasing difficulty. Problems requiring special attention are indicated by asterisks. Answers to 70 percent of the problems are given at the end of the book. Problems for which the answers are given are set in straight type in the text, while problems for which no answer is given are set in italic and red font color.

Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in that chapter. Marginal notes are used to help students organize their review work, and cross-references have been included to help them find the portions of material requiring their special attention.



Review Problems

2.127 Two forces **P** and **Q** are applied to the lid of a storage bin as shown. Knowing that $P = 48$ N and $Q = 60$ N, determine by trigonometry the magnitude and direction of the resultant of the two forces.

Fig. P2.127

2.128 Determine the x and y components of each of the forces shown.

Fig. P2.128

Fig. *P2.129*

2.129 A hoist trolley is subjected to the three forces shown. Knowing that $\alpha = 40°$, determine (*a*) the required magnitude of the force **P** if the resultant of the three forces is to be vertical, (*b*) the corresponding magnitude of the resultant.

2.130 Knowing that $\alpha = 55°$ and that boom AC exerts on pin C a force directed along line AC, determine (*a*) the magnitude of that force, (*b*) the tension in cable BC.

Fig. P2.130

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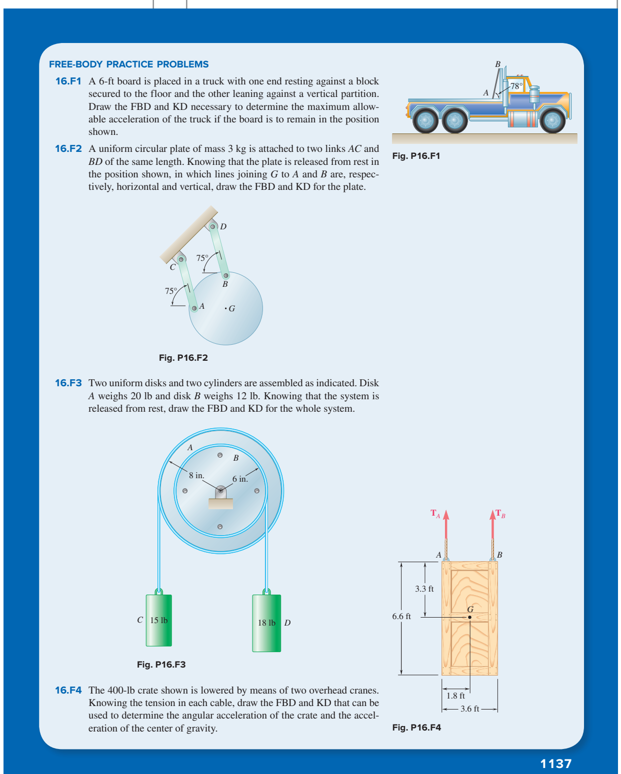
[†]Hestenes, D., Wells, M., and Swakhamer, G (1992). The force concept inventory. *The Physics Teacher,* 30: 141–158.
Streveler, R. A., Litzinger, T. A., Miller, R. L., and Steif, P. S. (2008). Learning conceptual knowledge in the engineering sciences: Overview and future research directions, *JEE,* 279–294.

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.

Concept Questions. Educational research has shown that students can often choose appropriate equations and solve algorithmic problems without having a strong conceptual understanding of mechanics principles.† To help assess and develop student conceptual understanding, we have included Concept Questions, which are multiple choice problems that require few, if any, calculations. Each possible incorrect answer typically represents a common misconception (e.g., students often think that a vehicle moving in a curved path at constant speed has zero acceleration). Students are encouraged to solve these problems using the principles and techniques discussed in the text and to use these principles to help them develop their intuition. Mastery and discussion of these Concept Questions will deepen students' conceptual understanding and help them to solve dynamics problems.

Free Body and Impulse-Momentum Diagram Practice Problems. Drawing diagrams correctly is a critical step in solving kinetics problems in dynamics. A new type of problem has been added to the text to emphasize the importance of drawing these diagrams. In Chaps. 12 and 16 the Free Body Practice Problems require students to draw a free-body diagram (FBD) showing the applied forces and an equivalent diagram called a "kinetic diagram" (KD) showing $m\mathbf{a}$ or its components and $\bar{I}\alpha$. These diagrams provide students with a pictorial representation of Newton's second law and are critical in helping students to correctly solve kinetic problems. In Chaps. 13 and 17 the Impulse-Momentum Diagram Practice Problems require students to draw diagrams showing the momenta of the bodies before impact, the impulses exerted on the body during impact, and the final momenta of the bodies. The answers to all of these questions can be accessed through Connect.



FREE-BODY PRACTICE PROBLEMS

16.F1 A 6-ft board is placed in a truck with one end resting against a block secured to the floor and the other leaning against a vertical partition. Draw the FBD and KD necessary to determine the maximum allowable acceleration of the truck if the board is to remain in the position shown.

16.F2 A uniform circular plate of mass 3 kg is attached to two links *AC* and *BD* of the same length. Knowing that the plate is released from rest in the position shown, in which lines joining *G* to *A* and *B* are, respectively, horizontal and vertical, draw the FBD and KD for the plate.

Fig. P16.F1

Fig. P16.F2

16.F3 Two uniform disks and two cylinders are assembled as indicated. Disk *A* weighs 20 lb and disk *B* weighs 12 lb. Knowing that the system is released from rest, draw the FBD and KD for the whole system.

Fig. P16.F3

16.F4 The 400-lb crate shown is lowered by means of two overhead cranes. Knowing the tension in each cable, draw the FBD and KD that can be used to determine the angular acceleration of the crate and the acceleration of the center of gravity.

Fig. P16.F4

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Computer Problems. Accessible through Connect are problem sets for each chapter that are designed to be solved with computational software. Many of these problems are relevant to the design process; they may involve the analysis of a structure for various configurations and loadings of the structure, or the determination of the equilibrium positions of a given mechanism that may require an iterative method of solution. Developing the algorithm required to solve a given mechanics problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply their computer skills to the solution of a meaningful engineering problem.

Digital Resources

Connect® is a highly reliable, easy-to-use homework and learning management solution that embeds learning science and award-winning adaptive tools to improve student results.

Analytics Connect Insight is Connect's one-of-a-kind visual analytics dashboard. Now available for both instructors and students, it provides at-a-glance information regarding student performance, which is immediately actionable. By presenting assignment, assessment, and topical performance results together with a time metric that is easily visible for aggregate or individual results, Connect InSight gives the user the ability to take a just-in-time approach to teaching and learning, which was never before available. Connect Insight presents data that empower students and help instructors improve class performance in a way that is efficient and effective.

Autograded Free-Body Diagram Problems

NEW!

- Within Connect, algorithmic end-of-chapter problems include our new Free-Body Diagram Drawing tool. The Free-Body Diagram Tool allows students to draw free-body diagrams that are auto graded by the system. Student's receive immediate feedback on their diagrams to help student's solidify their understanding of the physical situation presented in the problem.

Case Study Interactives

New digital content has been added throughout the text to enhance student learning. This includes a more in-depth discussion of the new Case Studies, as well as interactive questions embedded in these video explorations to make students *think* about the problem rather than just viewing the video. Within the text, simulations and short videos have been added to help students visualize topics, such as zero-force members and the motion of different linkages.

Find the following instructor resources available through Connect:

- **Instructor's and Solutions Manual.** *The Instructor's and Solutions Manual* that accompanies the eleventh edition features solutions to all end of chapter problems. This manual also features a number of tables designed to assist instructors in creating a schedule of assignments for their course. The various topics covered in the text have been listed in Table I and a suggested number of periods to be spent on each topic has been indicated. Table II prepares a brief description of all groups of problems and a classification of the problems in each group according to the units used. Sample lesson schedules are shown in Tables III, IV, and V, together with various alternative lists of assigned homework problems.
- **Lecture PowerPoint Slides** for each chapter that can be modified. These generally have an introductory application slide, animated worked-out problems that you can do in class with your students, concept questions, and "what-if?" questions at the end of the units.

- **Textbook images**
- **Computer Problem sets** for each chapter that are designed to be solved with computational software.
- **C.O.S.M.O.S.,** the Complete Online Solutions Manual Organization System that allows instructors to create custom homework, quizzes, and tests using end-of-chapter problems from the text.

McGraw Hill Education SMARTBOOK® SmartBook helps students study more efficiently by highlighting where in the chapter to focus, asking review questions and pointing them to resources until they understand.

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List of Symbols

$\mathbf{a}, a$	Acceleration
a	Constant; radius; distance; semimajor axis of ellipse
$\bar{\mathbf{a}}, \bar{a}$	Acceleration of mass center
$\mathbf{a}_{B/A}$	Acceleration of B relative to frame in translation with A
$\mathbf{a}_{P/\mathcal{F}}$	Acceleration of P relative to rotating frame $\mathcal{F}$
$\mathbf{a}_c$	Coriolis acceleration
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$	Reactions at supports and connections
$A, B, C, \ldots$	Points
A	Area
b	Width; distance; semiminor axis of ellipse
c	Constant; coefficient of viscous damping
C	Centroid; instantaneous center of rotation; capacitance
d	Distance
$\mathbf{e}_n, \mathbf{e}_t$	Unit vectors along normal and tangent
$\mathbf{e}_r, \mathbf{e}_\theta$	Unit vectors in radial and transverse directions
e	Coefficient of restitution; base of natural logarithms
E	Total mechanical energy; voltage
f	Scalar function
f_f	Frequency of forced vibration
f_n	Natural frequency
$\mathbf{F}$	Force; friction force
g	Acceleration of gravity
G	Center of gravity; mass center; constant of gravitation
h	Angular momentum per unit mass
$\mathbf{H}_O$	Angular momentum about point O
$\dot{\mathbf{H}}_G$	Rate of change of angular momentum $\mathbf{H}_G$ with respect to frame of fixed orientation
$(\dot{\mathbf{H}}_G)_{Gxyz}$	Rate of change of angular momentum $\mathbf{H}_G$ with respect to rotating frame $Gxyz$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors along coordinate axes
i	Current
$I, I_x, \ldots$	Moments of inertia
$\bar{I}$	Centroidal moment of inertia
$I_{xy}, \ldots$	Products of inertia
J	Polar moment of inertia
k	Spring constant
k_x, k_y, k_O	Radii of gyration
$\bar{k}$	Centroidal radius of gyration
l	Length
$\mathbf{L}$	Linear momentum
L	Length; inductance
m	Mass
m'	Mass per unit length
$\mathbf{M}$	Couple; moment
$\mathbf{M}_O$	Moment about point O
M_O^R	Moment resultant about point O

M	Magnitude of couple or moment; mass of earth
M_{OL}	Moment about axis OL
n	Normal direction
$\mathbf{N}$	Normal component of reaction
O	Origin of coordinates
$\mathbf{P}$	Force; vector
$\dot{\mathrm{P}}$	Rate of change of vector **P** with respect to frame of fixed orientation
q	Mass rate of flow; electric charge
$\mathbf{Q}$	Force; vector
$\dot{\mathbf{Q}}$	Rate of change of vector **Q** with respect to frame of fixed orientation
$(\dot{\mathbf{Q}})_{Oxyz}$	Rate of change of vector **Q** with respect to frame $Oxyz$
$\mathbf{r}$	Position vector
$\mathbf{r}_{B/A}$	Position vector of B relative to A
r	Radius; distance; polar coordinate
$\mathbf{R}$	Resultant force; resultant vector; reaction
R	Radius of earth; resistance
$\mathbf{s}$	Position vector
s	Length of arc
t	Time; thickness; tangential direction
$\mathbf{T}$	Force
T	Tension; kinetic energy
$\mathbf{u}$	Velocity
u	Variable
U	Work
U_{1-2}^{NC}	work done by non-conservative forces
$\mathbf{v}, v$	Velocity
v	Speed
$\overline{\mathbf{v}}, \overline{v}$	Velocity of mass center
$\mathbf{v}_{B/A}$	Velocity of B relative to frame in translation with A
$\mathbf{v}_{P/\mathcal{F}}$	Velocity of P relative to rotating frame $\mathcal{F}$
$\mathbf{V}$	Vector product
V	Volume; potential energy
w	Load per unit length
$\mathbf{W}, W$	Weight; load
x, y, z	Rectangular coordinates; distances
$\dot{x}, \dot{y}, \dot{z}$	Time derivatives of coordinates x, y, z
$\overline{x}, \overline{y}, \overline{z}$	Rectangular coordinates of centroid, center of gravity, or mass center
$\boldsymbol{\alpha}, \alpha$	Angular acceleration
α, β, γ	Angles
γ	Specific weight
δ	Elongation
ε	Eccentricity of conic section or of orbit
$\boldsymbol{\lambda}$	Unit vector along a line
η	Efficiency
θ	Angular coordinate; Eulerian angle; angle; polar coordinate
μ	Coefficient of friction
ρ	Density; radius of curvature
τ	Periodic time

τ_n	Period of free vibration
ϕ	Angle of friction; Eulerian angle; phase angle; angle
φ	Phase difference
ψ	Eulerian angle
$\boldsymbol{\omega}, \omega$	Angular velocity
ω_f	Circular frequency of forced vibration
ω_n	Natural circular frequency
$\boldsymbol{\Omega}$	Angular velocity of frame of reference

Vector Mechanics For Engineers

Dynamics

11

Kinematics of Particles

The motion of the paraglider can be described in terms of its *position, velocity,* and *acceleration.* When landing, the pilot of the paraglider needs to consider the wind velocity and the *relative motion* of the glider with respect to the wind. The study of motion is known as *kinematics* and is the subject of this chapter.

Introduction
11.1 RECTILINEAR MOTION OF PARTICLES
11.1A Position, Velocity, and Acceleration
11.1B Determining the Motion of a Particle
11.2 SPECIAL CASES AND RELATIVE MOTION
11.2A Uniform Rectilinear Motion
11.2B Uniformly Accelerated Rectilinear Motion
11.2C Motion of Several Particles
*11.3 GRAPHICAL SOLUTIONS
11.4 CURVILINEAR MOTION OF PARTICLES
11.4A Position, Velocity, and Acceleration Vectors
11.4B Derivatives of Vector Functions
11.4C Rectangular Components of Velocity and Acceleration
11.4D Motion Relative to a Frame in Translation
11.5 NON-RECTANGULAR COMPONENTS
11.5A Tangential and Normal Components
11.5B Radial and Transverse Components

Objectives

- **Describe** the basic kinematic relationships between position, velocity, acceleration, and time.
- **Solve** problems using these basic kinematic relationships and calculus or graphical methods.
- **Define** position, velocity, and acceleration in terms of Cartesian, tangential and normal, and radial and transverse coordinates.
- **Analyze** the relative motion of multiple particles by using a translating coordinate system.
- **Determine** the motion of a particle that depends on the motion of another particle.
- **Determine** which coordinate system is most appropriate for solving a curvilinear kinematics problem.
- **Calculate** the position, velocity, and acceleration of a particle undergoing curvilinear motion using Cartesian, tangential and normal, and radial and transverse coordinates.

Introduction

Chaps. 1 to 10 were devoted to **statics**; that is, to the analysis of bodies at rest. We now begin the study of **dynamics**, which is the part of mechanics that deals with the analysis of bodies in motion.

Although the study of statics goes back to the time of the Greek philosophers, the first significant contribution to dynamics was made by Galileo (1564–1642). Galileo's experiments on uniformly accelerated bodies led Newton (1642–1727) to formulate his fundamental laws of motion.

Dynamics includes two broad areas of study:

1. **Kinematics**, which is the study of the geometry of motion. The principles of kinematics relate the displacement, velocity, acceleration, and time of a body's motion, without reference to the cause of the motion.
2. **Kinetics**, which is the study of the relation between the forces acting on a body, the mass of the body, and the motion of the body. We use kinetics to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Chaps. 11 through 14 describe the **dynamics of particles**; in Chap. 11, we consider the **kinematics of particles**. The use of the word *particles* does not mean that our study is restricted to small objects; rather, it indicates that in these first chapters we study the motion of bodies—possibly as large as cars, rockets, or airplanes—without regard to their size or shape. By saying that we analyze the bodies as particles, we mean that we consider only their motion as an entire unit; we neglect any rotation about their own centers of mass. In some cases, however, such a rotation is not negligible, and we cannot treat the bodies as particles. Such motions are analyzed in later chapters dealing with the **dynamics of rigid bodies**.

In the first part of Chap. 11, we describe the rectilinear motion of a particle; that is, we determine the position, velocity, and acceleration of a particle at every instant as it moves along a straight line. We first use general methods of analysis to study the motion of a particle; we then consider two important particular cases, namely, the uniform motion and the uniformly accelerated motion of a particle (Sec. 11.2). We then discuss the simultaneous motion of several particles and introduce the concept of the relative motion of one particle with respect to another. The first part of this chapter concludes with a study of graphical methods of analysis and their application to the solution of problems involving the rectilinear motion of particles.

In the second part of this chapter, we analyze the motion of a particle as it moves along a curved path. We define the position, velocity, and acceleration of a particle as vector quantities and introduce the derivative of a vector function to add to our mathematical tools. We consider applications in which we define the motion of a particle by the rectangular components of its velocity and acceleration; at this point, we analyze the motion of a projectile (Sec. 11.4C). Then, we examine the motion of a particle relative to a reference frame in translation. Finally, we analyze the curvilinear motion of a particle in terms of components other than rectangular. In Sec. 11.5, we introduce the tangential and normal components of an object's velocity and acceleration, and then examine the radial and transverse components of an object's motion.

11.1 RECTILINEAR MOTION OF PARTICLES

A particle moving along a straight line is said to be in **rectilinear motion**. The only variables we need to describe this motion are the time, t, and the distance along the line, x, as a function of time. With these variables, we can define the particle's position, velocity, and acceleration, which completely describe the particle's motion. When we study the motion of a particle moving in a plane (two dimensions) or in space (three dimensions), we will use a more general position vector rather than simply the distance along a line.

11.1A Position, Velocity, and Acceleration

At any given instant t, a particle in rectilinear motion occupies some position on the straight line. To define the particle's position P, we choose a fixed origin O on the straight line and a positive direction along the line. We measure the distance x from O to P and record it with a plus or minus sign, according to whether we reach P from O by moving along the line in the positive or negative direction. The distance x, with the appropriate sign, completely defines the position of the particle; it is called the **position coordinate** of the particle. For example, the position coordinate corresponding to P in Fig. 11.1a is $x = +5$ m; the coordinate corresponding to P' in Fig. 11.1b is $x' = -2$ m.

When we know the position coordinate x of a particle for every value of time t, we say that the motion of the particle is known. We can provide a "timetable" of the motion in the form of an equation in x and t, such as $x = 6t^2 - t^3$, or in the form of a graph of x versus t, as shown in Fig. 11.6. The units most often used to measure the position coordinate x are the meter (m) in the SI system of units[†] and the foot (ft) in the U.S. customary system of units. Time t is usually measured in seconds (s).

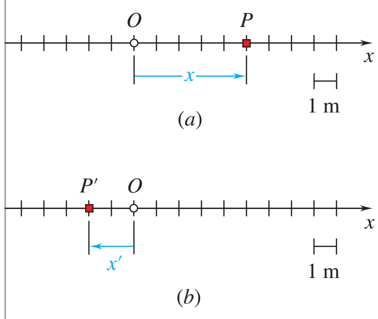


Fig. 11.1 Position is measured from a fixed origin. (a) A positive position coordinate; (b) a negative position coordinate.

[†]See Sec. 1.3.

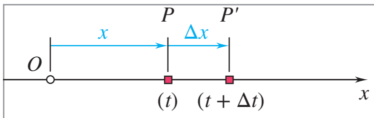


Fig. 11.2 A small displacement Δx from time t to time $t + \Delta t$.

Photo 11.1 The motion of this solar car can be described by its position, velocity, and acceleration.

Source: Stefano Paltera/NREL

Now consider the position P occupied by the particle at time t and the corresponding coordinate x (Fig. 11.2). Consider also the position P' occupied by the particle at a later time $t + \Delta t$. We can obtain the position coordinate of P' by adding the small displacement Δx to the coordinate x of P. This displacement is positive or negative according to whether P' is to the right or to the left of P. We define the **average velocity** of the particle over the time interval Δt as the quotient of the displacement Δx and the time interval Δt as

$$\text{Average velocity} = \frac{\Delta x}{\Delta t}$$

If we use SI units, Δx is expressed in meters and Δt in seconds; the average velocity is then expressed in meters per second (m/s). If we use U.S. customary units, Δx is expressed in feet and Δt in seconds; the average velocity is then expressed in feet per second (ft/s).

We can determine the **instantaneous velocity** v of a particle at the instant t by allowing the time interval Δt to become infinitesimally small. Thus,

$$\text{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

The instantaneous velocity is also expressed in m/s or ft/s. Observing that the limit of the quotient is equal, by definition, to the derivative of x with respect to t, we have

Velocity of a particle along a line

$$v = \frac{dx}{dt} \tag{11.1}$$

We represent the velocity v by an algebraic number that can be positive or negative.‡ A positive value of v indicates that x increases, i.e., that the particle moves in the positive direction (Fig. 11.3*a*). A negative value of v indicates that x decreases; that is, that the particle moves in the negative direction (Fig. 11.3*b*). The magnitude of v is known as the **speed** of the particle.

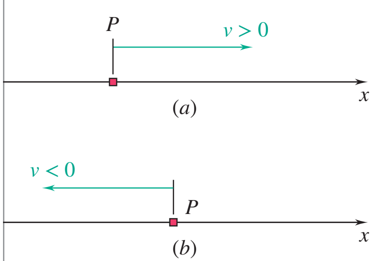


Fig. 11.3 In rectilinear motion, velocity can be only (*a*) positive or (*b*) negative along the line.

Consider the velocity v of the particle at time t and also its velocity $v + \Delta v$ at a later time $t + \Delta t$ (Fig. 11.4). We define the **average acceleration** of the particle over the time interval Δt as the quotient of Δv and Δt as

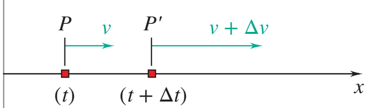


Fig. 11.4 A change in velocity from v to $v + \Delta v$ corresponding to a change in time from t to $t + \Delta t$.

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t}$$

If we use SI units, Δv is expressed in m/s and Δt in seconds; the average acceleration is then expressed in m/s^2. If we use U.S. customary units, Δv is expressed in ft/s and Δt in seconds; the average acceleration is then expressed in ft/s^2.

We obtain the **instantaneous acceleration** a of the particle at the instant t by again allowing the time interval Δt to approach zero. Thus,

$$\text{Instantaneous acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

‡As you will see in Sec. 11.4A, velocity is actually a vector quantity. However, because we are considering here the rectilinear motion of a particle where the velocity has a known and fixed direction, we need only specify its sense and magnitude. We can do this conveniently by using a scalar quantity with a plus or minus sign. This is also true of the acceleration of a particle in rectilinear motion.

The instantaneous acceleration is also expressed in m/s^2 or ft/s^2. The limit of the quotient, which is by definition the derivative of v with respect to t, measures the rate of change of the velocity. We have

Acceleration of a particle along a line

$$a = \frac{dv}{dt} \tag{11.2}$$

or substituting for v from Eq. (11.1),

$$a = \frac{d^2x}{dt^2} \tag{11.3}$$

We represent the acceleration a by an algebraic number that can be positive or negative (see the footnote on the preceding page). A positive value of a indicates that the velocity (i.e., the algebraic number v) increases. This may mean that the particle is moving faster in the positive direction (Fig. 11.5*a*) or that it is moving more slowly in the negative direction (Fig. 11.5*b*); in both cases, Δv is positive. A negative value of a indicates that the velocity decreases; either the particle is moving more slowly in the positive direction (Fig. 11.5*c*), or it is moving faster in the negative direction (Fig. 11.5*d*).

Sometimes we use the term *deceleration* to refer to a when the speed of the particle (i.e., the magnitude of v) decreases; the particle is then moving more slowly. For example, the particle of Fig. 11.5 is decelerating in parts b and c; it is truly accelerating (i.e., moving faster) in parts a and d.

We can obtain another expression for the acceleration by eliminating the differential dt in Eqs. (11.1) and (11.2). Solving Eq. (11.1) for dt, we have $dt = dx/v$; substituting into Eq. (11.2) gives us

$$a = v\frac{dv}{dx} \tag{11.4}$$

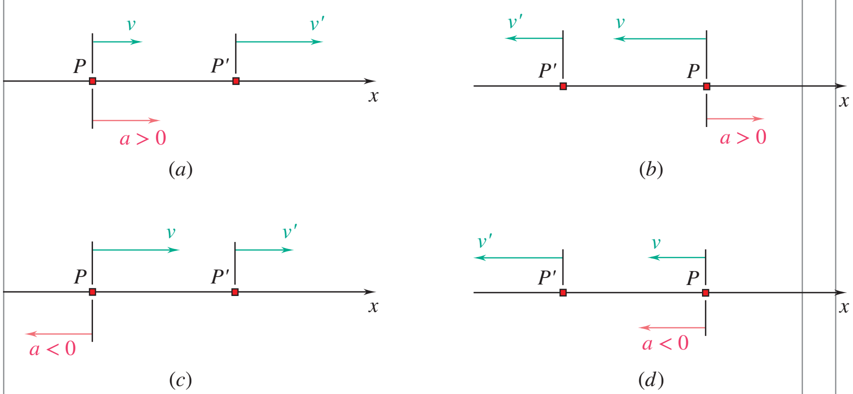


Fig. 11.5 Velocity and acceleration can be in the same or different directions. (*a, d*) When *a* and *v* are in the same direction, the particle speeds up; (*b, c*) when *a* and *v* are in opposite directions, the particle slows down.

Concept Application 11.1

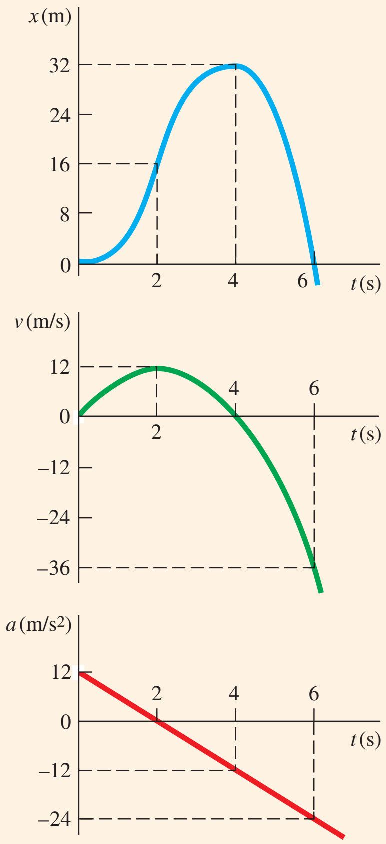


Fig. 11.6 Graphs of position, velocity, and acceleration as functions of time for Concept Application 11.1.

Consider a particle moving in a straight line, and assume that its position is defined by

$$x = 6t^2 - t^3$$

where t is in seconds and x in meters. We can obtain the velocity v at any time t by differentiating x with respect to t as

$$v = \frac{dx}{dt} = 12t - 3t^2$$

We can obtain the acceleration a by differentiating again with respect to t. Hence,

$$a = \frac{dv}{dt} = 12 - 6t$$

In Fig. 11.6, we have plotted the position coordinate, the velocity, and the acceleration. These curves are known as *motion curves*. Keep in mind, however, that the particle does not move along any of these curves; the particle moves in a straight line.

Because the derivative of a function measures the slope of the corresponding curve, the slope of the x–t curve at any given time is equal to the value of v at that time. Similarly, the slope of the v–t curve is equal to the value of a. Because $a = 0$ at $t = 2$ s, the slope of the v–t curve must be zero at $t = 2$ s; the velocity reaches a maximum at this instant. Also, because $v = 0$ at $t = 0$ and at $t = 4$ s, the tangent to the x–t curve must be horizontal for both of these values of t.

A study of the three motion curves of Fig. 11.6 shows that the motion of the particle from $t = 0$ to $t = \infty$ can be divided into four phases:

1. The particle starts from the origin, $x = 0$, with no velocity but with a positive acceleration. Under this acceleration, the particle gains a positive velocity and moves in the positive direction. From $t = 0$ to $t = 2$ s, x, v, and a are all positive.
2. At $t = 2$ s, the acceleration is zero; the velocity has reached its maximum value. From $t = 2$ s to $t = 4$ s, v is positive, but a is negative. The particle still moves in the positive direction but more slowly; the particle is decelerating.
3. At $t = 4$ s, the velocity is zero; the position coordinate x has reached its maximum value (32 m). From then on, both v and a are negative; the particle is accelerating and moves in the negative direction with increasing speed.
4. At $t = 6$ s, the particle passes through the origin; its coordinate x is then zero, while the total distance traveled because the beginning of the motion is 64 m (i.e., twice its maximum value). For values of t larger than 6 s, x, v, and a are all negative. The particle keeps moving in the negative direction—away from O—faster and faster.

11.1B Determining the Motion of a Particle

We have just seen that the motion of a particle is said to be known if we know its position for every value of the time t. In practice, however, a motion is seldom defined by a relation between x and t. More often, the conditions of the motion are specified by the type of acceleration that the particle possesses. For example, a freely falling body has a constant acceleration that is directed downward and equal to 9.81 m/s^2 or 32.2 ft/s^2, a mass attached to a stretched spring has an acceleration proportional to the instantaneous elongation of the spring measured from its equilibrium position, etc. In general, we can express the acceleration of the particle as a function of one or more of the variables x, v, and t. Thus, in order to determine the position coordinate x in terms of t, we need to perform two successive integrations.

Let us consider three common classes of motion.

1. $a = f(t)$. **The Acceleration Is a Given Function of *t*.** Solving Eq. (11.2) for dv and substituting $f(t)$ for a, we have

$$dv = a\,dt$$
$$dv = f(t)dt$$

Integrating both sides of the equation, we obtain

$$\int dv = \int f(t)\,dt$$

This equation defines v in terms of t. Note, however, that an arbitrary constant is introduced after the integration is performed. This is due to the fact that many motions correspond to the given acceleration $a = f(t)$. In order to define the motion of the particle uniquely, it is necessary to specify the **initial conditions** of the motion; that is, the value v_0 of the velocity and the value x_0 of the position coordinate at $t = 0$. Rather than use an arbitrary constant that is determined by the initial conditions, it is often more convenient to replace the indefinite integrals with **definite integrals**. Definite integrals have lower limits corresponding to the initial conditions $t = 0$ and $v = v_0$ and upper limits corresponding to $t = t$ and $v = v$. This gives us

$$\int_{v_0}^{v} dv = \int_0^t f(t)\,dt$$

$$v - v_0 = \int_0^t f(t)\,dt$$

which yields v in terms of t.

We can now solve Eq. (11.1) for dx as

$$dx = v\,dt$$

and substitute the expression obtained from the first integration for v. Then, we integrate both sides of this equation via the left-hand side with respect to x from $x = x_0$ to $x = x$ and the right-hand side with respect to t from $t = 0$ to $t = t$. In this way, we obtain the position coordinate x in terms of t; the motion is completely determined.

We will study two important cases in greater detail in Sec. 11.2: the case when $a = 0$, corresponding to a *uniform motion,* and the case when a = constant, corresponding to a *uniformly accelerated motion.*

2. $a = f(x)$. **The Acceleration Is a Given Function of** x. Rearranging Eq. (11.4) and substituting $f(x)$ for a, we have

$$v\,dv = a\,dx$$
$$v\,dv = f(x)dx$$

Because each side contains only one variable, we can integrate the equation. Denoting again the initial values of the velocity and of the position coordinate by v_0 and x_0, respectively, we obtain

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx$$
$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \int_{x_0}^{x} f(x)\,dx$$

which yields v in terms of x. We now solve Eq. (11.1) for dt, giving us

$$dt = \frac{dx}{v}$$

and substitute for v the expression just obtained. We can then integrate both sides to obtain the desired relation between x and t. However, in most cases, this last integration cannot be performed analytically, and we must resort to a numerical method of integration.

3. $a = f(v)$. **The Acceleration Is a Given Function of** v. We can now substitute $f(v)$ for a in either Eqs. (11.2) or (11.4) to obtain either

$$f(v) = \frac{dv}{dt} \qquad f(v) = v\frac{dv}{dx}$$
$$dt = \frac{dv}{f(v)} \qquad dx = \frac{v\,dv}{f(v)}$$

Integration of the first equation yields a relation between v and t; integration of the second equation yields a relation between v and x. Either of these relations can be used in conjunction with Eq. (11.1) to obtain the relation between x and t that characterizes the motion of the particle.

Sample Problem 11.1

The position of a particle moving along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in feet and t in seconds. Determine (a) the time at which the velocity is zero, (b) the position and distance traveled by the particle at that time, (c) the acceleration of the particle at that time, (d) the distance traveled by the particle from $t = 4$ s to $t = 6$ s.

STRATEGY: You need to use the basic kinematic relationships between position, velocity, and acceleration. Because the position is given as a function of time, you can differentiate it to find equations for the velocity and acceleration. Once you have these equations, you can solve the problem.

MODELING and ANALYSIS: Taking the derivative of position, you obtain

$$x = t^3 - 6t^2 - 15t + 40 \tag{1}$$
$$v = \frac{dx}{dt} = 3t^2 - 12t - 15 \tag{2}$$
$$a = \frac{dv}{dt} = 6t - 12 \tag{3}$$

These equations are graphed in Fig. 1.

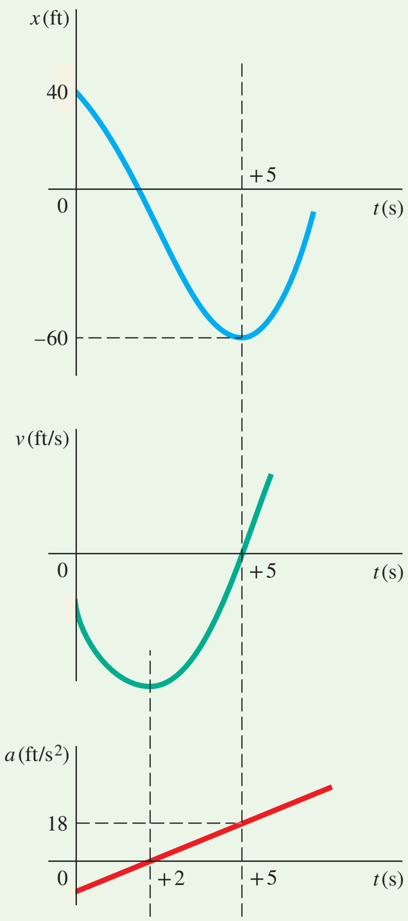


Fig. 1 Motion curves for the particle.

a. Time When $v = 0$. Set $v = 0$ in Eq. (2) for

$$3t^2 - 12t - 15 = 0 \qquad t = -1 \text{ s} \quad \text{and} \quad t = +5 \text{ s} \quad \blacktriangleleft$$

Only the root $t = +5$ s corresponds to a time after the motion has begun: for $t < 5$ s, $v < 0$ and the particle moves in the negative direction; for $t > 5$ s, $v > 0$ and the particle moves in the positive direction.

b. Position and Distance Traveled When $v = 0$. Substitute $t = +5$ s into Eq. (1), yielding

$$x_5 = (5)^3 - 6(5)^2 - 15(5) + 40 \qquad x_5 = -60 \text{ ft} \quad \blacktriangleleft$$

The initial position at $t = 0$ was $x_0 = +40$ ft. Because $v \neq 0$ during the interval $t = 0$ to $t = 5$ s, you have

$$\text{Distance traveled} = x_5 - x_0 = -60 \text{ ft} - 40 \text{ ft} = -100 \text{ ft}$$

Distance traveled = 100 ft in the negative direction ◀

c. Acceleration When $v = 0$. Substitute $t = +5$ s into Eq. (3) for

$$a_5 = 6(5) - 12 \qquad a_5 = +18 \text{ ft/s}^2 \quad \blacktriangleleft$$

d. Distance Traveled from $t = 4$ s to $t = 6$ s. The particle moves in the negative direction from $t = 4$ s to $t = 5$ s and in the positive direction from $t = 5$ s to $t = 6$ s; therefore, the distance traveled during each of these time intervals must be computed separately.

From $t = 4$ s to $t = 5$ s: $x_5 = -60$ ft

$$x_4 = (4)^3 - 6(4)^2 - 15(4) + 40 = -52 \text{ ft}$$

(continued)

$$\text{Distance traveled} = x_5 - x_4 = -60 \text{ ft} - (-52 \text{ ft}) = -8 \text{ ft}$$
$$= 8 \text{ ft in the negative direction}$$

From $t = 5$ s to $t = 6$ s: $x_5 = -60$ ft

$$x_6 = (6)^3 - 6(6)^2 - 15(6) + 40 = -50 \text{ ft}$$
$$\text{Distance traveled} = x_6 - x_5 = -50 \text{ ft} - (-60 \text{ ft}) = +10 \text{ ft}$$
$$= 10 \text{ ft in the positive direction}$$

Total distance traveled from $t = 4$ s to $t = 6$ s is 8 ft + 10 ft = 18 ft

REFLECT and THINK: The total distance traveled by the particle in the two-second interval is 18 ft, but because one distance is positive and one is negative, the net change in position is only 2 ft (in the positive direction). This illustrates the difference between total distance traveled and net change in position. Note that the maximum displacement occurs at $t = 5$ s, when the velocity is zero.

Sample Problem 11.2

You throw a ball vertically upward with a velocity of 10 m/s from a window located 20 m above the ground. Knowing that the acceleration of the ball is constant and equal to 9.81 m/s^2 downward, determine (*a*) the velocity v and elevation y of the ball above the ground at any time t, (*b*) the highest elevation reached by the ball and the corresponding value of t, (*c*) the time when the ball hits the ground and the corresponding velocity. Draw the v–t and y–t curves.

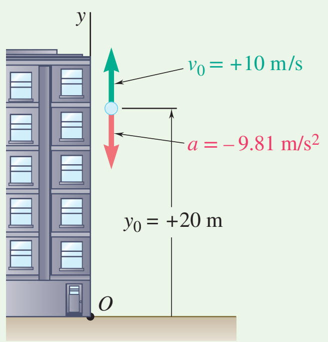


Fig. 1 Acceleration, initial velocity, and initial position of the ball.

STRATEGY: The acceleration is constant, so you can integrate the defining kinematic equation for acceleration once to find the velocity equation and a second time to find the position relationship. Once you have these equations, you can solve the problem.

MODELING and ANALYSIS: Model the ball as a particle with negligible drag.

a. Velocity and Elevation. Choose the y axis measuring the position coordinate (or elevation) with its origin O on the ground and its positive sense upward. The value of the acceleration and the initial values of v and y are as indicated in Fig. 1. Substituting for a in $a = dv/dt$ and noting that when $t = 0$, $v_0 = +10$ m/s, you have

$$\frac{dv}{dt} = a = -9.81 \text{ m/s}^2$$
$$\int_{v_0=10}^{v} dv = -\int_0^t 9.81\, dt$$
$$[v]_{10}^{v} = -[9.81t]_0^t$$
$$v - 10 = -9.81t$$
$$v = 10 - 9.81t \quad (1)$$

(continued)

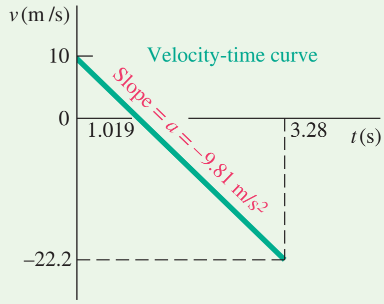


Fig. 2 Velocity of the ball as a function of time.

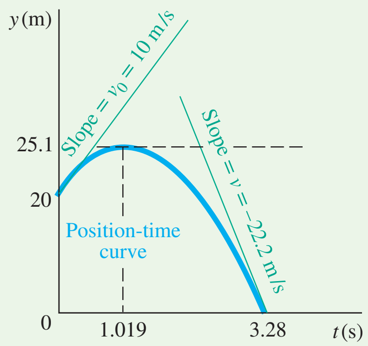


Fig. 3 Height of the ball as a function of time.

Substituting for v in $v = dy/dt$ and noting that when $t = 0$, $y_0 = 20$ m, you have

$$\frac{dy}{dt} = v = 10 - 9.81t$$

$$\int_{y_0=20}^{y} dy = \int_0^t (10 - 9.81t)\, dt$$

$$[y]_{20}^{y} = [10t - 4.905t^2]_0^t$$

$$y - 20 = 10t - 4.905\,t^2$$

$$y = 20 + 10t - 4.905\,t^2 \quad \textbf{(2)} \quad ◀$$

Graphs of these equations are shown in Figs. 2 and 3.

b. Highest Elevation. The ball reaches its highest elevation when $v = 0$. Substituting into Eq. (1), you obtain

$$10 - 9.81t = 0 \qquad t = 1.019 \text{ s} \quad ◀$$

Substituting $t = 1.019$ s into Eq. (2), you find

$$y = 20 + 10(1.019) - 4.905(1.019)^2 \qquad y = 25.1 \text{ m} \quad ◀$$

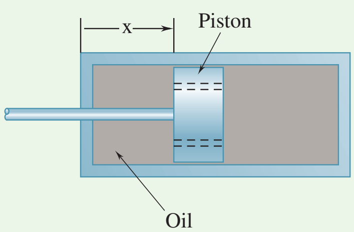
c. Ball Hits the Ground. The ball hits the ground when $y = 0$. Substituting into Eq. (2), you obtain

$$20 + 10t - 4.905t^2 = 0 \qquad t = -1.243 \text{ s} \quad \text{and} \quad t = +3.28 \text{ s} \quad ◀$$

Only the root $t = +3.28$ s corresponds to a time after the motion has begun. Carrying this value of t into Eq. (1), you find

$$v = 10 - 9.81(3.28) = -22.2 \text{ m/s} \qquad v = 22.2 \text{ m/s} \downarrow \quad ◀$$

REFLECT and THINK: When the acceleration is constant, the velocity changes linearly, and the position is a quadratic function of time. You will see in Sec. 11.2 that the motion in this problem is an example of free fall, where the acceleration in the vertical direction is constant and equal to $-g$.



Sample Problem 11.3

Many mountain bike shocks utilize a piston that travels in an oil-filled cylinder to provide shock absorption; this system is shown schematically. When the front tire goes over a bump, the cylinder is given an initial velocity v_0. The piston, which is attached to the fork, then moves with respect to the cylinder, and oil is forced through orifices in the piston. This causes the piston to decelerate at a rate proportional to the velocity at $a = -kv$. At time $t = 0$, the position of the piston is $x = 0$. Express (*a*) the velocity v in terms of t, (*b*) the position x in terms of t, (*c*) the velocity v in terms of x. Draw the corresponding motion curves.

(continued)

STRATEGY: Because the acceleration is given as a function of velocity, you need to use either $a = dv/dt$ or $a = v\,dv/dx$ and then separate variables and integrate. Which one you use depends on what you are asked to find. Because part *a* asks for v in terms of t, use $a = dv/dt$. You can integrate this again using $v = dx/dt$ for part *b*. Because part *c* asked for $v(x)$, you should use $a = v\,dv/dx$ and then separate the variables and integrate.

MODELING and ANALYSIS: Rotation of the piston is not relevant, so you can model it as a particle undergoing rectilinear motion.

a. *v* in Terms of *t*. Substitute $-kv$ for a in the fundamental formula defining acceleration, $a = dv/dt$. You obtain

$$-kv = \frac{dv}{dt} \qquad \frac{dv}{v} = -k\,dt \qquad \int_{v_0}^{v} \frac{dv}{v} = -k\int_0^t dt$$

$$\ln\frac{v}{v_0} = -kt \qquad\qquad v = v_0 e^{-kt}$$

b. *x* in Terms of *t*. Substitute the expression just obtained for v into $v = dx/dt$. You get

$$v_0 e^{-kt} = \frac{dx}{dt}$$

$$\int_0^x dx = v_0\int_0^t e^{-kt}dt$$

$$x = -\frac{v_0}{k}[e^{-kt}]_0^t = -\frac{v_0}{k}(e^{-kt} - 1)$$

$$x = \frac{v_0}{k}(1 - e^{-kt})$$

c. *v* in Terms of *x*. Substitute $-kv$ for a in $a = v\,dv/dx$. You have

$$-kv = v\frac{dv}{dx}$$

$$dv = -k\,dx$$

$$\int_{v_0}^{v} dv = -k\int_0^x dx$$

$$v - v_0 = -kx \qquad\qquad v = v_0 - kx$$

The motion curves are shown in Fig. 1.

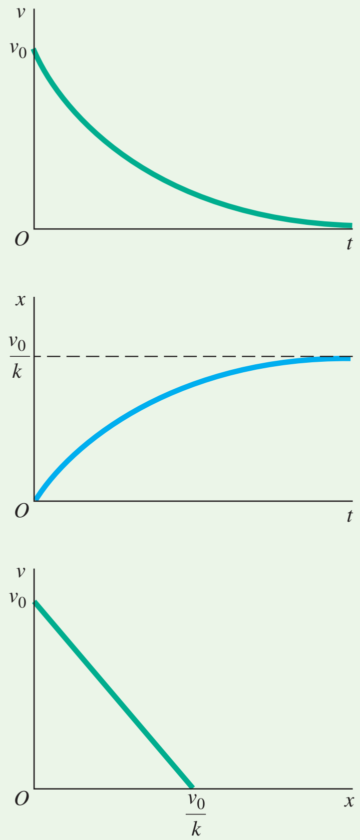


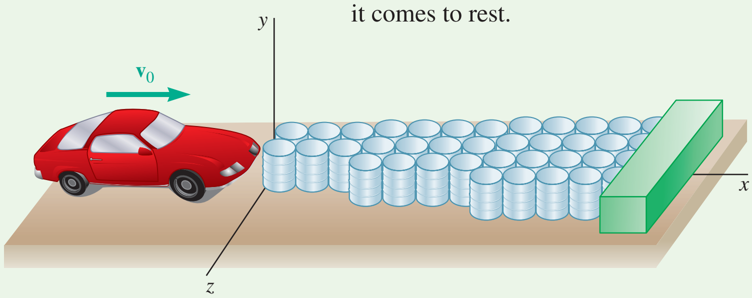
Fig. 1 Motion curves for the piston.

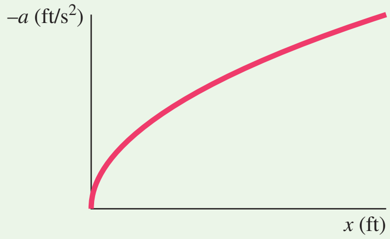
REFLECT and THINK: You could have solved part *c* by eliminating t from the answers obtained for parts *a* and *b*. You could use this alternative method as a check. From part *a*, you obtain $e^{-kt} = v/v_0$; substituting into the answer of part *b*, you have

$$x = \frac{v_0}{k}(1 - e^{-kt}) = \frac{v_0}{k}\left(1 - \frac{v}{v_0}\right) \qquad v = v_0 - kx \qquad \text{(checks)}$$

Sample Problem 11.4

An uncontrolled automobile traveling at 45 mph strikes a highway crash barrier square on. After initially hitting the barrier, the automobile decelerates at a rate proportional to the distance x the automobile has moved into the barrier; specifically, $a = -60\sqrt{x}$, where a and x are expressed in ft/s^2 and ft, respectively. Determine the distance the automobile will move into the barrier before it comes to rest.





STRATEGY: Because you are given the deceleration as a function of displacement, you should start with the basic kinematic relationship $a = v\,dv/dx$.

MODELING and ANALYSIS: Model the car as a particle. First find the initial speed in ft/s,

$$v_0 = \left(45\frac{\text{mi}}{\text{hr}}\right)\left(\frac{1\text{ hr}}{3600\text{ s}}\right)\left(\frac{5280\text{ ft}}{\text{mi}}\right) = 66\frac{\text{ft}}{\text{s}}$$

Substituting $a = -60\sqrt{x}$ into $a = v\,dv/dx$ gives

$$a = -60\sqrt{x} = \frac{v\,dv}{dx}$$

Separating variables and integrating gives

$$v\,dv = -60\sqrt{x}\,dx \rightarrow \int_{v_0}^{0} v\,dv = -\int_0^x 60\sqrt{x}\,dx$$

$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = -40x^{3/2} \rightarrow x = \left(\frac{1}{80}(v_0^2 - v^2)\right)^{2/3} \quad \textbf{(1)}$$

Substituting $v = 0$, $v_0 = 66$ ft/s gives

$$d = 14.37 \text{ ft} \quad ◀$$

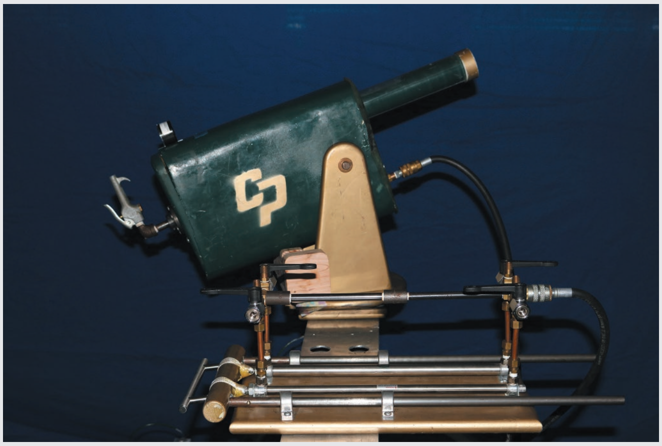
REFLECT and THINK: A distance of 14 ft seems reasonable for a barrier of this type. If you substitute d into the equation for a, you find a maximum deceleration of about 7 g's. Note that this problem would have been much harder to solve if you had been asked to find the time for the automobile to stop. In this case, you would need to determine $v(t)$ from Eq. (1). This gives $v = \sqrt{v_0^2 - 80x^{3/2}}$. Using the basic kinematic relationship $v = dx/dt$, you can easily show that

$$\int_0^t dt = \int_0^x \frac{dx}{\sqrt{v_0^2 - 80x^{3/2}}}$$

Unfortunately, there is no closed-form solution to this integral, so you would need to solve it numerically.

Case Study 11.1

People with mobility impairments often have difficulty participating in athletic and recreational activities, missing out on the social and physiological benefits such recreation has to offer. Some senior engineering students decided to design and build an adapted dart launcher to allow athletes with disabilities to play this fun game with their friends.



CS Photo 11.1 Adapted dart launcher.

The device they designed, shown in CS Photo 11.1, is similar to an air cannon. The athlete fills a pressure tank by moving a pneumatic cylinder back and forth; the displacement necessary is less than 150 mm and the required force is less than 20 N. Each pump supplies approximately 5 kN/m^2 pressure to the reservoir.

Assuming an adiabatic system, the acceleration a of the dart while in the launch tube can be expressed as

$$a = \frac{1}{m}\left[\frac{AP_0V_0}{V_0 + Ax} - f\right] \qquad (1)$$

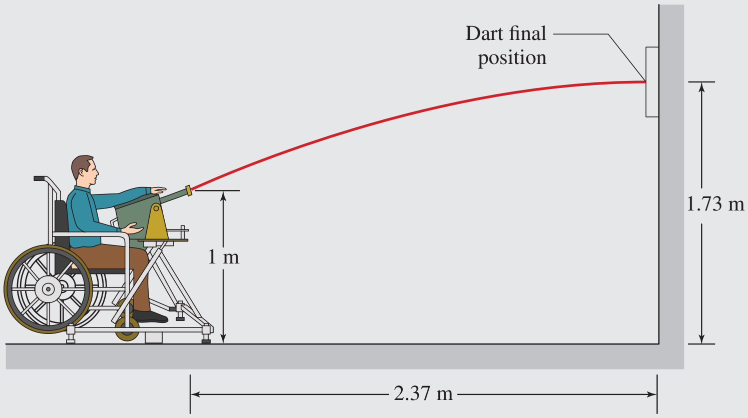
where A is the cross-sectional area of the launch tube, P_0 is the stored pressure in the reservoir, V_0 is the volume of the reservoir, x is the distance along the tube, and f is the friction force between the dart launch carriage and the tube wall.

During testing, the darts stick best when they hit perpendicular to the board. You can experiment with different launch angles (20°, 30°, 40°, and 45°) and the number of pumps to determine the best combination when trying to hit the bull's eye. CS Fig. 11.1 shows the location of the dart board with respect to the launcher.

The cannon has the following parameters: cannon inner radius $r = 20$ mm, cannon tube length $L = 330$ mm, volume of reservoir $V_0 = 0.0005$ m^3, mass of dart and piston $m = 0.2$ kg, friction force along walls $f = 0.1$ N, and launch height $y_0 = 1.0$ m.

STRATEGY: You are given acceleration as a function of position, so you should start with the basic relationship $a = v\,dv/dx$. You will then need to use projectile motion to analyze the flight of the dart toward the board.

(continued)



CS Fig. 11.1 Dart launch schematic—distance and height to bull's eye.

MODELING: Treat the air cannon as an adiabatic system (you will learn more about this in your thermodynamics class) and the dart as a particle, with drag considered negligible during flight.

ANALYSIS: Substituting the expression for a in Eq. (1) into $a = v\,dv/dx$, and then separating variables and integrating gives you:

$$\int_0^L \frac{1}{m}\left[\frac{AP_0V_0}{V_0 + Ax} - f\right] dx = \int_0^{v_0} v dv$$

Performing these integrations and solving for the exit velocity v_0 as a function of the system parameters gives

$$v_0 = \left[\frac{2}{m}\left(P_0V_0 \ln\left(\frac{V_0 + AL}{V_0}\right) - fL\right)\right]^{1/2} \quad (2)$$

The pressure P_0 is equal to the number of pumps N times the pressure per pump, 5000 N/m^2. You can calculate v_0 of the dart for a given number of pumps, and then use this to solve for the projectile motion of the dart after it leaves the cannon. For the x direction, you get

$$x = x_0 + (v_0)_x t, \text{ where } x_0 = 0 \text{ and } (v_0)_x = v_0 \cos(\theta)$$

Solving for the time gives

$$t = \frac{d}{v_0 \cos \theta}$$

Using this time, you can determine the vertical component of velocity $(v_{\text{hit}})_y$ using

$$(v_{\text{hit}})_y = v_0 \sin \theta - gt$$

and the height y where the dart hits the board

$$y = y_0 + v_0 \sin \theta t - \frac{1}{2}gt^2$$

The exit velocity v_0 for different pumps is shown in CS Table 11.1, and the y component of velocity (which we want close to zero to ensure it sticks) and the height of the dart when it hits the board are shown in CS Table 11.2 for

(continued)

CS Table 11.1: Exit Velocity of Dart

Number of Pumps	Velocity (m/s)
2	5.47
3	6.71
4	7.75
5	8.67
6	9.50
7	10.27
8	10.98

CS Table 11.2: Vertical Velocity and Height When the Dart Hits the Board for Different Pumps and at Different Launch Angles

Angle	Velocity and Position of Hit	Number of Pumps						
		2	3	4	5	6	7	8
20°	$(v_{hit})_y$ (m/s)	–2.66	–1.396	–0.542	0.1115	0.645	1.100	1.500
	y (m)	0.818	1.169	1.343	1.448	1.517	1.567	1.604
30°	$(v_{hit})_y$ (m/s)	–2.18	–0.651	0.411	1.238	1.925	2.52	3.04
	y (m)	1.138	1.550	1.750	1.880	1.961	2.02	2.06
40°	$(v_{hit})_y$ (m/s)	–2.04	–0.216	1.066	2.0722	2.91	3.64	4.29
	y (m)	1.417	1.945	2.21	2.36	2.47	2.54	2.60

launch angles of 20, 30, and 40 degrees. From this table, you can see that a launch angle at 30° with four pumps, which results in a vertical velocity component of 0.411 m/s and a height of 1.75 m, is the best combination.

REFLECT and THINK: Wheelchair dart competitions place the bull's-eye at a height of 1.37 m. How would this change the calculations performed above? A number of different variables have to be considered when designing the system, including cannon length, reservoir size, displacement and force required to pump the air cylinders, aesthetics, and athlete engagement. Athletes with mobility impairments may have limited range and force production to actuate the pump cylinder, and the design should provide them with the opportunity to get some exercise and to control how the dart is "thrown." Additionally, there are a number of safety considerations that have to be taken into account.

SOLVING PROBLEMS ON YOUR OWN

In the problems for this section, you will be asked to determine the **position, velocity,** and/or **acceleration** of a particle in **rectilinear motion**. As you read each problem, it is important to identify both the independent variable (typically t or x) and what is required (e.g., the need to express v as a function of x). You may find it helpful to start each problem by writing down both the given information and a simple statement of what is to be determined.

1. Determining $v(t)$ and $a(t)$ for a given $x(t)$. As explained in Sec. 11.1A, the first and second derivatives of x with respect to t are equal to the velocity and the acceleration, respectively, of the particle [Eqs. (11.1) and (11.2)]. If the velocity and acceleration have opposite signs, the particle can come to rest and then move in the opposite direction (Sample Prob. 11.1). Thus, when computing the total distance traveled by a particle, you should first determine if the particle comes to rest during the specified interval of time. Constructing a diagram similar to that of Sample Prob. 11.1, which shows the position and the velocity of the particle at each critical instant ($v = v_{max}$, $v = 0$, etc.), will help you to visualize the motion.

2. Determining $v(t)$ and $x(t)$ for a given $a(t)$. We discussed the solution of problems of this type in the first part of Sec. 11.1B. We used the initial conditions, $t = 0$ and $v = v_0$, for the lower limits of the integrals in t and v, but any other known state (e.g., $t = t_1$ and $v = v_1$) could be used instead. Also, if the given function $a(t)$ contains an unknown constant (e.g., the constant k if $a = kt$), you will first have to determine that constant by substituting a set of known values of t and a in the equation defining $a(t)$.

3. Determining $v(x)$ and $x(t)$ for a given $a(x)$. This is the second case considered in Sec. 11.1B and is illustrated in Sample Prob. 11.4. We again note that the lower limits of integration can be any known state (e.g., $x = x_1$ and $v = v_1$). In addition, because $v = v_{max}$ when $a = 0$, you can determine the positions where the maximum or minimum values of the velocity occur by setting $a(x) = 0$ and solving for x.

4. Determining $v(x)$, $v(t)$, and $x(t)$ for a given $a(v)$. This is the last case treated in Sec. 11.1B; the appropriate solution techniques for problems of this type are illustrated in Sample Prob. 11.3. All of the general comments for the preceding cases once again apply. Note that Sample Prob. 11.3 provides a summary of how and when to use the equations $v = dx/dt$, $a = dv/dt$, and $a = v\ dv/dx$.

(continued)

We can summarize these relationships in Table 11.1.

Table 11.1

If. . . .	Kinematic Relationship	Integrate
$a = a(t)$	$\frac{dv}{dt} = a(t)$	$\int_{v_0}^{v} dv = \int_0^t a(t)dt$
$a = a(x)$	$v\frac{dv}{dx} = a(x)$	$\int_{v_0}^{v} v\ dv = \int_{x_0}^{x} a(x)dx$
$a = a(v)$	$\frac{dv}{dt} = a(v)$	$\int_{v_0}^{v} \frac{dv}{a(v)} = \int_0^t dt$
	$v\frac{dv}{dx} = a(v)$	$\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{vdv}{a(v)}$

Problems[†]

CONCEPT QUESTIONS

11.CQ1 A bus travels the 100 miles between A and B at 50 mi/h and then another 100 miles between B and C at 70 mi/h. The average speed of the bus for the entire 200-mile trip is:
a. More than 60 mi/h.
b. Equal to 60 mi/h.
c. Less than 60 mi/h.

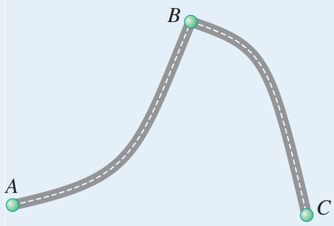


Fig. P11.CQ1

11.CQ2 Two cars A and B race each other down a straight road. The position of each car as a function of time is shown. Which of the following statements are true? (More than one answer can be correct.)
a. At time t_2, both cars have traveled the same distance.
b. At time t_1, both cars have the same speed.
c. Both cars have the same speed at some time $t < t_1$.
d. Both cars have the same acceleration at some time $t < t_1$.
e. Both cars have the same acceleration at some time $t_1 < t < t_2$.

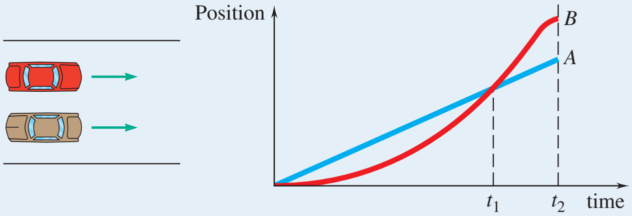


Fig. P11.CQ2

END-OF-SECTION PROBLEMS

11.1 A snowboarder starts from rest at the top of a double black diamond hill. As she rides down the slope, GPS coordinates are used to determine her displacement as a function of time: $x = 0.5t^3 + t^2 + 2t$, where x and t are expressed in feet and seconds, respectively. Determine the position, velocity, and acceleration of the boarder when $t = 5$ seconds.

11.2 The motion of a particle is defined by the relation $x = t^3 - 12t^2 + 36t + 30$, where x and t are expressed in feet and seconds, respectively. Determine the time, the position, and the acceleration of the particle when $v = 0$.

11.3 The vertical motion of mass A is defined by the relation $x = \cos(10t) - 0.1\sin(10t)$, where x and t are expressed in mm and seconds, respectively. Determine (a) the position, velocity, and acceleration of A when $t = 0.4$ s, (b) the maximum velocity and acceleration of A.

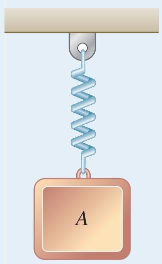


Fig. P11.3

[†]Answers to all problems set in straight type (such as **11.1**) are given at the end of the book. Answers to problems with a number set in italic type (such as ***11.6***) are not given.

11.4 A loaded railroad car is rolling at a constant velocity when it couples with a spring and dashpot bumper system. After the coupling, the motion of the car is defined by the relation $x = 60e^{-4.8t} \sin 16t$, where x and t are expressed in millimeters and seconds, respectively. Determine the position, the velocity, and the acceleration of the railroad car when (*a*) $t = 0$, (*b*) $t = 0.3$ s.

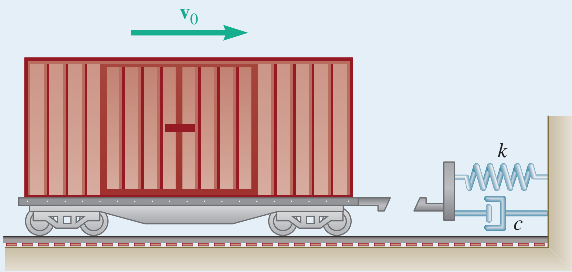


Fig. P11.4

11.5 A group of hikers uses a GPS while doing a 40-mile trek in Colorado. A curve fit to the data shows that their altitude can be approximated by the function $y(t) = 0.12t^5 - 6.75t^4 + 135t^3 - 1120t^2 + 3200t + 9070$, where y and t are expressed in feet and hours, respectively. During the 18-hour hike, determine (*a*) the maximum altitude that the hikers reach, (*b*) the total feet they ascend, (*c*) the total feet they descend. *Hint:* You will need to use a calculator or computer to solve for the roots of a fourth-order polynomial.

11.6 The motion of a particle is defined by the relation $x = t^3 - 6t^2 + 9t + 5$, where x is expressed in feet and t in seconds. Determine (*a*) when the velocity is zero, (*b*) the position, acceleration, and total distance traveled when $t = 5$ s.

11.7 A girl operates a radio-controlled model car in a vacant parking lot. The girl's position is at the origin of the xy coordinate axes, and the surface of the parking lot lies in the x–y plane. She drives the car in a straight line so that the x coordinate is defined by the relation $x(t) = 0.5t^3 - 3t^2 + 3t + 2$, where x and t are expressed in meters and seconds, respectively. Determine (*a*) when the velocity is zero, (*b*) the position and total distance traveled when the acceleration is zero.

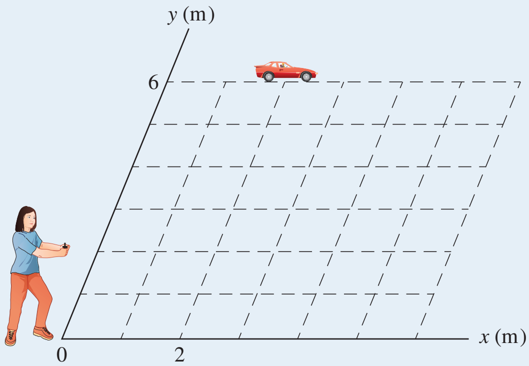


Fig. P11.7

11.8 The motion of a particle is defined by the relation $x = t^2 - (t - 2)^3$, where x and t are expressed in feet and seconds, respectively. Determine (*a*) the two positions at which the velocity is zero, (*b*) the total distance traveled by the particle from $t = 0$ to $t = 4$ s.

11.9 The brakes of a car are applied, causing it to slow down at a rate of 10 ft/s^2. Knowing that the car stops in 300 ft, determine (*a*) how fast the car was traveling immediately before the brakes were applied, (*b*) the time required for the car to stop.

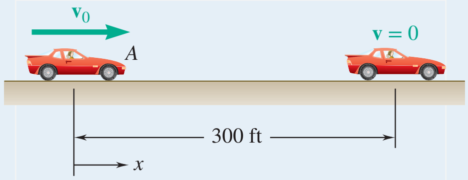


Fig. P11.9

11.10 The acceleration of a particle is defined by the relation $a = 3e^{-0.2t}$, where a and t are expressed in ft/s^2 and seconds, respectively. Knowing that $x = 0$ and $v = 0$ at $t = 0$, determine the velocity and position of the particle when $t = 0.5$ s.

11.11 The acceleration of a particle is defined by the relation $a = 9 - 3t^2$, where a and t are expressed in ft/s^2 and seconds, respectively. The particle starts at $t = 0$ with $v = 0$ and $x = 5$ ft. Determine (*a*) the time when the velocity is again zero, (*b*) the position and velocity when $t = 4$ s, (*c*) the total distance traveled by the particle from $t = 0$ to $t = 4$ s.

11.12 Many car companies are performing research on collision avoidance systems. A small prototype applies engine braking that decelerates the vehicle according to the relationship $a = -k\sqrt{t}$, where a and t are expressed in m/s^2 and seconds, respectively. The vehicle is traveling at 20 m/s when its radar sensors detect a stationary obstacle. Knowing that it takes the prototype vehicle 4 seconds to stop, determine (*a*) expressions for its velocity and position as a function of time, (*b*) how far the vehicle traveled before it stopped.

11.13 A Scotch yoke is a mechanism that transforms the circular motion of a crank into the reciprocating motion of a shaft (or vice versa). It has been used in a number of different internal combustion engines and in control valves. In the Scotch yoke shown, the acceleration of point A is defined by the relation $a = -1.8 \sin kt$, where a and t are expressed in m/s^2 and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0$ and $v = 0.6$ m/s when $t = 0$, determine the velocity and position of point A when $t = 0.5$ s.

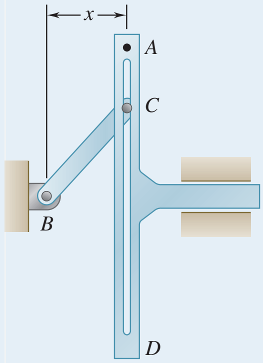


Fig. *P11.13* and *P11.14*

11.14 For the Scotch yoke mechanism shown, the acceleration of point A is defined by the relation $a = -1.08 \sin kt - 1.44 \cos kt$, where a and t are expressed in m/s^2 and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0.16$ m and $v = 0.36$ m/s when $t = 0$, determine the velocity and position of point A when $t = 0.5$ s.

11.15 A piece of electronic equipment that is surrounded by packing material is dropped so that it hits the ground with a speed of 4 m/s. After contact, the equipment experiences an acceleration of $a = -kx$, where k is a constant and x is the compression of the packing material. If the packing material experiences a maximum compression of 15 mm, determine the maximum acceleration of the equipment.

Fig. P11.15

11.16 A projectile enters a resisting medium at $x = 0$ with an initial velocity $v_0 = 1000$ ft/s and travels 3 in. before coming to rest. Knowing that the velocity of the projectile is defined by the relation $v = v_0 - kx$, where v is expressed in ft/s and x is in feet, determine (*a*) the initial acceleration of the projectile, (*b*) the time required for the projectile to penetrate 2.5 in. into the resisting medium.

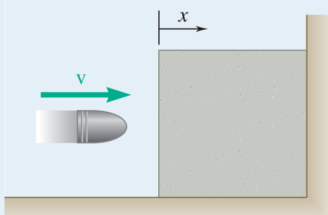


Fig. P11.16

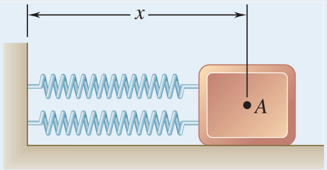


Fig. P11.17

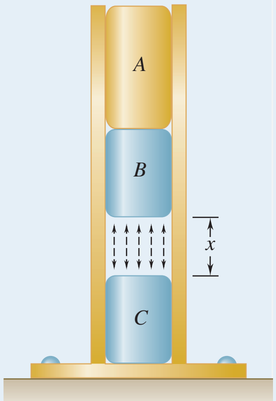


Fig. P11.18

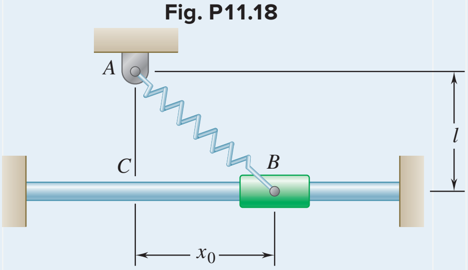


Fig. *P11.20*

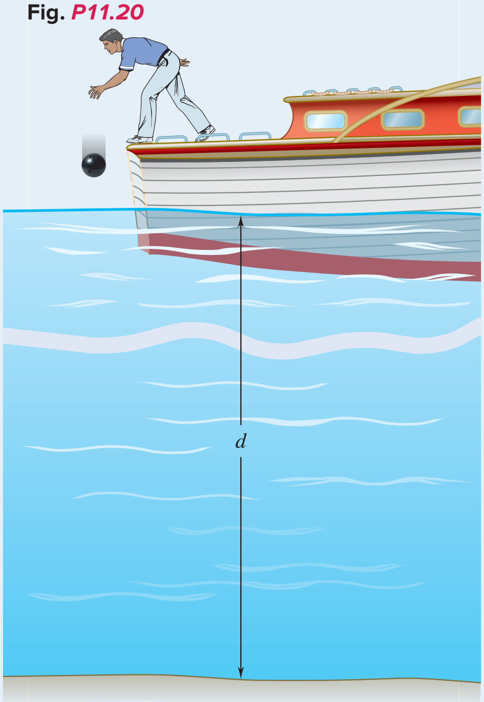


Fig. P11.23

11.17 Point A oscillates with an acceleration $a = 100(0.25 - x)$, where a and x are expressed in m/s^2 and meters, respectively. Knowing that the system starts at time $t = 0$ with $v = 0$ and $x = 0.2$ m, determine the position and the velocity of A when $t = 0.2$ s.

11.18 A brass (nonmagnetic) block A and a steel magnet B are in equilibrium in a brass tube under the magnetic repelling force of another steel magnet C located at a distance $x = 0.004$ m from B. The force is inversely proportional to the square of the distance between B and C. If block A is suddenly removed, the acceleration of block B is $a = -9.81 + k/x^2$, where a and x are expressed in m/s^2 and meters, respectively, and $k = 4 \times 10^{-4}\ m^3/s^2$. Determine the maximum velocity and acceleration of B.

11.19 Based on experimental observations, the acceleration of a particle is defined by the relation $a = -(0.1 + \sin x/b)$, where a and x are expressed in m/s^2 and meters, respectively. Knowing that $b = 0.8$ m and that $v = 1$ m/s when $x = 0$, determine (a) the velocity of the particle when $x = -1$ m, (b) the position where the velocity is maximum, (c) the maximum velocity.

11.20 A spring AB is attached to a support at A and to a collar. The unstretched length of the spring is l. Knowing that the collar is released from rest at $x = x_0$ and has an acceleration defined by the relation $a = -100(x - lx/\sqrt{l^2 + x^2})$, determine the velocity of the collar as it passes through point C.

11.21 The acceleration of a particle is defined by the relation $a = k(1 - e^{-x})$, where k is a constant. Knowing that the velocity of the particle is $v = +9$ m/s when $x = -3$ m and that the particle comes to rest at the origin, determine (a) the value of k, (b) the velocity of the particle when $x = -2$ m.

11.22 Starting from $x = 0$ with no initial velocity, a particle is given an acceleration $a = 0.8\sqrt{v^2 + 49}$, where a and v are expressed in ft/s^2 and ft/s, respectively. Determine (a) the position of the particle when $v = 24$ ft/s, (b) the speed and acceleration of the particle when $x = 40$ ft.

11.23 A ball is dropped from a boat so that it strikes the surface of a lake with a speed of 16.5 ft/s. While in the water the ball experiences an acceleration of $a = 10 - 0.8v$, where a and v are expressed in ft/s^2 and ft/s, respectively. Knowing that the ball takes 3 s to reach the bottom of the lake, determine (a) the depth of the lake, (b) the speed of the ball when it hits the bottom of the lake.

11.24 The acceleration of a particle is defined by the relation $a = -k\sqrt{v}$, where k is a constant. Knowing that $x = 0$ and $v = 81$ m/s at $t = 0$, and that $v = 36$ m/s when $x = 18$ m, determine (a) the velocity of the particle when $x = 20$ m, (b) the time required for the particle to come to rest.

11.25 The acceleration of a particle is defined by the relation $a = -kv^{2.5}$, where k is a constant. The particle starts at $x = 0$ with a velocity of 16 mm/s, and when $x = 6$ mm, the velocity is observed to be 4 mm/s. Determine (a) the velocity of the particle when $x = 5$ mm, (b) the time at which the velocity of the particle is 9 mm/s.