PRINCIPLES AND APPLICATIONS OF ELECTRICAL ENGINEERING

Seventh Edition

Giorgio Rizzoni
The Ohio State University

James Kearns
York College of Pennsylvania

Mc Graw Hill

To our families

Mc Graw Hill

PRINCIPLES AND APPLICATIONS OF ELECTRICAL ENGINEERING, SEVENTH EDITION

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About the Authors

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Dr. Rizzoni's research interests are in the dynamics and control of future ground vehicle propulsion systems, including advanced engines, alternative fuels, electric and hybrid-electric drivetrains, energy storage systems, and fuel cell systems. He has contributed to the development of a graduate curriculum in these areas and has served as the director of three U.S. Department of Energy Graduate Automotive Technology Education Centers of Excellence: *Hybrid Drivetrains and Control Systems* (1998–2004), *Advanced Propulsion Systems* (2005–2011), and *Energy Efficient Vehicles for Sustainable Mobility* (2011–2016).

In 1999 Dr. Rizzoni established an automotive industry research consortium that today sees the participation of over 20 automotive OEMs and suppliers; in 2008 he created the SMART@CAR consortium, focusing on plug-in hybrid and electric vehicles and vehicle-grid interaction, with funding from electric utilities, automotive OEMS, and electronics suppliers. Through the Ohio Third Frontier Wright Project Program he created a *Center of Excellence for Commercial Hybrid Vehicles* in 2009, and a *Center of Excellence for Energy Storage Technology* in 2010.

Dr. Rizzoni is a Fellow of IEEE (2004), a Fellow of SAE (2005), a recipient of the 1991 National Science Foundation Presidential Young Investigator Award, and of several other technical and teaching awards.

The OSU Center for Automotive Research

The OSU Center for Automotive Research, CAR, is an interdisciplinary research center in the OSU College of Engineering founded in 1991 and located in a 50,000 ft^2 building complex on the west campus of OSU. CAR conducts interdisciplinary research in collaboration with the OSU colleges of Engineering, Medicine, Business, and Arts and Sciences, and with industry and government partners. CAR research aims to: develop efficient vehicle propulsion and energy storage systems; develop new sustainable mobility concepts; reduce the impact of vehicles on the environment; improve vehicle safety and reduce occupant and pedestrian injuries; increase vehicle autonomy and intelligence; and create quieter and more comfortable automobiles. A team of 50 administrative and research staff supports some 40 faculty, 120 graduate and 300 undergraduate students and maintains and makes use of advanced experimental facilities. Dr. Rizzoni has led CAR for over a decade, growing its research expenditures from $1M per year to over $10M today, and engaging CAR in a broad range of technology commercialization activities, start-up company incubation and spin-out, as well as providing a broad range of engineering services to the automotive industry.

CAR is also the home of the OSU Motorsports program, which supports the activities of five student vehicle competition programs of several student vehicle competition programs including: the Buckeye Bullet (holder of all current U.S. and FIA electric vehicle land speed records), the EcoCAR hybrid-electric vehicle team, the Formula Buckeyes and Baja Buckeyes SAE teams, and the Buckeye Current electric motorcycle racing team.

Jim Kearns is an Associate Professor of Electrical & Computer Engineering at York College of Pennsylvania. He received a B.S. in Mechanical Engineering (SEAS) and a B.S. in Economics (Wharton) from the University of Pennsylvania in 1982. Subsequently, he received his M.E. from Carnegie-Mellon University in 1984, and his Ph.D. from the Georgia Institute of Technology in 1990, both in Mechanical Engineering. While at Georgia Tech he was the recipient of a Presidential Fellowship. Subsequently, he worked as a Postdoctoral Fellow at the Applied Research Laboratory of the University of Texas—Austin.

In 1992, Dr. Kearns took his first teaching position at the Universidad del Turabo in Gurabo, Puerto Rico, where he worked with a small group of faculty and staff to build and develop a new school of engineering. In addition to other duties, he was tasked with developing a curriculum on electromechanics. During this time Dr. Kearns spent his summers at Sandia National Laboratories as a University Fellow.

In 1996, Dr. Kearns was the second full-time engineering faculty member hired by York College of Pennsylvania to (once again) develop a new engineering program with an emphasis on Mechatronics. As a result of that work, Jim was asked in 2003 to develop new electrical and computer engineering programs at YCP. Jim served as program coordinator until July 2010.

Throughout Dr. Kearns professional career he has been involved in teaching and research related to physical acoustics and electromechanical systems. His interest in electrical engineering began during his Ph.D. studies, when he built spark generators, DC power supplies, and signal amplifiers for his experiments. His steady pursuit of electromechanical engineering education has been the hallmark of his professional career. Dr. Kearns has been involved in a variety of pedagogical activities, including the development and refinement of techniques in electrical engineering education.

Dr. Kearns is a member of IEEE and ASEE. He is active in faculty governance at York College, where he is a past chair of its Tenure and Promotion committee and its Student Welfare committee. Dr. Kearns recently completed a four-year term as Vice-President and then President of the York College Academic Senate.

About the Cover

On the cover, an image of the Venturi Buckeye Bullet 3 at the Bonneville Salt Flats, UT, in 2016. On Monday, September 19, 2016, The Ohio State University's Venturi Buckeye Bullet 3 student team and driver Roger Schroer took this electric streamliner vehicle to a world record two-way average top speed of 341.4 miles per hour (549.4 kilometers per hour). The vehicle was designed and built by a team of engineering students at The Ohio State University, advised by Dr. Giorgio Rizzoni.

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Preface

The pervasive presence of electronic devices and instrumentation in all aspects of engineering design and analysis is one of the manifestations of the electronic revolution that has characterized the last 60 years. Every aspect of engineering practice, and of everyday life, has been affected in some way or another by electrical and electronic devices and instruments. Laptop and tablet computers along with so-called "smart" phones and touchscreen interfaces are perhaps the most obvious manifestations. These devices, and their underlying technology, have brought about a revolution in computing, communication, and entertainment. They allow us to store, process, and share professional and personal data and to access audio (most notably, music) and video of every variety. These advances in electrical engineering technology have had enormous impacts on all other fields of engineering, including mechanical, industrial, computer, civil, aeronautical, aerospace, chemical, nuclear, materials, and biological engineering. This rapidly expanding electrical and electronic technology has been adopted, leveraged, and incorporated in engineering designs across all fields. As a result, engineers work on projects requiring effective communication across multiple disciplines, one of which is nearly always electrical engineering.

0.1 OBJECTIVES

Engineering education and professional practice continue to undergo profound changes in an attempt to best utilize relevant advances in electronic technology. The need for textbooks and other learning resources that relate these advances to engineering disciplines beyond electrical and computer engineering continues to grow. This fact is evident in the ever-expanding application and integration of electronics and computer technologies in commercial products and processes. This textbook and its associated learning resources represent one effort to make the principles of electrical and computer engineering accessible to students in various engineering disciplines.

The principal objective of the book is to present the *principles* of electrical, electronic, and electromechanical engineering to an audience of engineering majors enrolled in introductory and more advanced or specialized electrical engineering courses.

A second objective is to present these principles with a focus on important results and common yet effective *analytical and computational* tools to solve practical problems.

Finally, a third objective of the book is to illustrate, by way of concrete, fully worked examples, a number of relevant *applications* of electrical engineering. These examples are drawn from the authors' industrial research experience and from ideas contributed by practicing engineers and industrial partners.

These three objectives are met through the use of various pedagogical features and methods.

0.2 ORGANIZATION

The basic organizational structure of a generic chapter remains essentially unchanged from the previous edition. Example problems and associated methods and procedures of problem solving remain organized so that students are able to easily and efficiently locate them when doing homework and preparing for exams.

Additional unguided exercises are provided to test student understanding. Relevant and stimulating applications to practical measurement challenges are included in nearly every chapter.

A continued and enhanced emphasis on problem solving can be found in this edition. All the highlighted *Focus on Problem Solving* boxes have been reviewed and revised to clarify and add additional detail to the steps needed by students to successfully complete end-of-chapter homework problems.

An effort was also made to reduce the aesthetic complexity of the book, without sacrificing technical content or overall aesthetic appeal. Effective reading is promoted by less clutter and visual "noise." A thorough, exhaustive, page-by-page search was made to locate errors in the text, equations, figures, references to equations and figures, examples, and homework problems.

The book is now divided into five major parts:

I. **Circuit Analysis**
II. **Systems and Instrumentation**
III. **Analog Electronics**
IV. **Digital Electronics**
V. **Electric Power and Machines**

The pedagogical enhancements made within each part are discussed below.

0.3 PEDAGOGY AND CONTENT

Part I: Circuit Analysis

Once again, the first part of the book has undergone a significant revision from the previous edition.

Chapter 1 begins with an emphasis on developing a student's ability to recognize structure within a circuit diagram. It is the authors' experience that this ability is key to student success. Yet, many books contain little content on developing this ability. The result is that many students wander into more difficult topics still viewing a circuit as simply an unruly collection of wires and elements.

The approach taken in this book is to encourage students to initially *focus on nodes*, rather than elements, in a circuit. For example, some of the earliest exercises in this book ask students to count the number of nodes in a circuit diagram. One immediate advantage of this patient approach is that students learn to disregard the particular aesthetic structure of a circuit diagram and instead focus on the technical structure and content. Chapter 1 also immediately engages students in the terminology, laws, and methods needed to solve basic DC problems and introduces the first of many electromechanical analogies.

Chapter 2 introduces students to more sophisticated analytic methods with a focus on appreciating the implications and utility of equivalent networks. The students' skill at recognizing circuit structure is further developed by the introduction of elements in series and parallel, applied to the more general concept of equivalent resistance between two nodes. The principle of superposition and the *source-load perspective* followed by Thévenin and Norton equivalent networks complete Chapter 2. The section on the source-load perspective revisits the concepts of voltage and current division to develop their graphical solution as the intersection of a source's load line with the load's *v-i* relation. This section is not essential but it can be very helpful to students when introduced prior to the usually difficult topic of Thévenin equivalent networks.

Methods of Problem Solving were enhanced and clarified. Throughout these chapters students are encouraged to think of problem solving in two steps: first **simplify**; then **solve**. In addition to being an effective problem-solving method, this method provides context for the power and importance of equivalent networks in general, and Thévenin's theorem, in particular. Chapter 3 continues the emphasis on equivalent networks applied to AC circuit analysis. In the following chapters on transient analysis and frequency response, foundational first- and second-order circuit *archetypes* are identified. Students are encouraged to continue to use Thévenin and Norton equivalent networks to simplify, when possible, transient circuit problems to these archetypes, which, in effect, become targets for students.

Finally, emphasis continues to be placed on visualizing phasors in the complex plane and understanding the key role of the unit phasor and Euler's theorem. Throughout the chapter on AC circuits students are encouraged to focus on the concepts of impedance and power triangles, and their similarity. Single-phase AC power concepts are now addressed in the chapter on AC circuits, whereas material related to transformers and three-phase power were moved to Part V.

Part II: Systems and Instrumentation

This part of the textbook brings together all of the material related to measurement and instrumentation found in the sixth edition and represents a significant change. The chapter on operational amplifiers continues to emphasize three amplifier archetypes (the unity-gain buffer, the inverting amplifier, and the noninverting amplifier) before introducing variations and applications, which are now more readily related to issues and challenges commonly encountered when conducting measurements using electronic instrumentation. The discussion of instrumentation amplifiers, in particular, was expanded and clarified. It is hoped that the reorganization of this material will bring greater relevance and practicality to students at an early stage of their study and allow instructors to complete this material and that in Part I in a one-semester course.

Part III: Analog Electronics

While much of the content on electronics is unchanged from the sixth edition, the problem-solving strategies and techniques for transistor circuits were further enhanced and clarified. The focus on simple but useful circuit examples was not changed.

The emphasis on large-signal models of BJTs and FETs and their applications was retained; however, an appropriate, but limited, presentation of small-signal models was included to support the discussion of AC amplifiers. These chapters present an uncomplicated and practical treatment of the analysis and design of simple amplifiers and switching circuits.

The chapter on power electronics is no longer included in the textbook but can be found in the online resources that support the book.

Part IV: Digital Electronics

The chapters on digital electronics remain largely unchanged except for a needed update of the material on encoders, gate arrays, and programmable logic devices. A greater number of end-of-chapter problems are now included in the chapter on digital systems.

It should be noted that the chapters on communication systems have been removed from the textbook but can be found in the online resources that support it.

Part V: Electric Power and Machines

Part V reflects a change in the organization of the book that brings together those aspects of electrical engineering that are related to electric power systems. Every instructor understands that there is no unique way of presenting introductory electrical engineering material, and the positioning of Part V in the book is somewhat arbitrary, as the section could really be placed anywhere after Section I. Chapter 13 covers the fundamentals of electric power systems, largely unchanged from previous editions, introducing AC power, complex power, and elements of three-phase power systems. Chapters 14 and 15 offer an introductory treatment of electrical machines, with focus on DC, and AC synchronous and induction machines. Two ancillary chapters are available online for instructors who wish to have a more in-depth treatment of electromechanical systems: one on power electronics, which introduces devices and systems for electric power conversion; the other on special-purpose electric machines, which presents a survey of electric machines commonly used in industrial systems and consumer products, such as step motors, brushless DC machines, switched reluctance machines, and single-phase AC machines. The content of Chapters 14 and 15 and of the ancillary chapters was developed by the first author for use in a required junior-year system dynamics course for mechanical engineers, and in a technical elective on mechatronics systems.

0.4 NOTATION

The notation used in this book for various symbols (variables, parameters, and units) has been updated but still follows generally accepted conventions. Distinctions in notation can be subtle. Luckily, very often the context in which a symbol appears makes its meaning clear. When the meaning of a symbol is not clear from its context a correct reading of the notation is important. A reasonably complete listing of the symbols used in this book and their notation is presented below.

For example, an uppercase roman font is used for units such as volts (V) and amperes (A). An uppercase italics math font is used for real parameters and variables such as resistance (R) and DC voltage (V). Notice the difference between the variable V and the unit V. Further, an uppercase bold math font is used for complex quantities such as voltage and current phasors ($\mathbf{V}$ and $\mathbf{I}$) as well as impedance ($\mathbf{Z}$), conductance ($\mathbf{Y}$), and frequency response functions ($\mathbf{H}$ and $\mathbf{G}$). Lowercase italic symbols are, in general, time dependent variables, such as voltage (v or $v(t)$) and current (i or $i(t)$), where (t) is an explicit indication of time dependence. Lowercase italic variables may represent constants in specific cases. Uppercase italic variables are reserved for constant (time-invariant) values exclusively.

Various subscripts are also used to denote particular instances or multiple occurrences of parameters and variables. Exponents are italicized superscripts.

Finally, in electrical engineering the imaginary unit $\sqrt{-1}$ is always represented by *j* rather than *i*, which is used by mathematicians. The reason for the use of *j* instead of *i* should be obvious!

Quantity	Symbol	Description
Voltage	v or $v(t)$	Time Dependent and Real
	V	Time Invariant and Real
	$\mathbf{V}$	Complex Phasor
Effective (rms) voltage	$\tilde{V}$	Time Invariant and Real
Current	i or $i(t)$	Time Dependent and Real
	I	Time Invariant and Real
	$\mathbf{I}$	Complex Phasor
Effective (rms) current	$\tilde{I}$	Time Invariant and Real
Volts	V	Unit of voltage
Amperes	A	Unit of current
Resistance	R	Real
Inductance	L	Real
Capacitance	C	Real
Reactance	X	Frequency Dependent and Real
Impedance	$\mathbf{Z}$	Frequency Dependent and Complex
Conductance	$\mathbf{Y}$	Frequency Dependent and Complex
Transfer Function	$\mathbf{G}$ or $\mathbf{H}$	Frequency Dependent and Complex
Cyclical Frequency	f	Time Invariant and Real
Angular Frequency	ω	Time Invariant and Real
Angle	θ	Time Invariant and Real
Amplitude	A	Time Invariant and Real

0.5 SYSTEM OF UNITS

This book employs the International System of Units (also called SI, from the French Système International des Unitès). SI units are adhered to by virtually all professional engineering societies and are based upon the seven fundamental quantities listed in Table 0.1. All other units are derived from these base units. An example of a derived unit is the radian, which is a measure of plane angles. In this book, angles are in units of radians unless explicitly given otherwise as degrees.

Since quantities often need to be described in large multiples or small fractions of a unit, the standard prefixes listed in Table 0.2 are used to denote SI units in powers of 10. In general, engineering units are expressed in powers of 10 that are multiples of 3. For example, 10^{-4} s would be expressed as 100×10^{-6} s, or 100 μs.

Tables 0.1 and 0.2 are useful references when reading this book.

Table 0.1 SI units

Quantity	Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Substance	Mole	mol
Luminous intensity	Candela	cd

Table 0.2 Standard prefixes

Prefix	Symbol	Power
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

0.6 ADDITIONAL FEATURES OF THE SEVENTH EDITION

Pedagogy

The seventh edition continues to offer all the time-tested pedagogical features available in the earlier editions.

- **Learning Objectives** offer an overview of key chapter ideas. Each chapter opens with a list of major objectives, and throughout the chapter the learning objective icon indicates targeted references to each objective.
- **Focus on Problem Solving** sections summarize important methods and procedures for the solution of common problems and assist the student in developing a methodical approach to problem solving.
- **Clearly Illustrated Examples** illustrate relevant applications of electrical engineering principles. The examples are fully integrated with the Focus on Problem Solving material, and each one is organized according to a prescribed set of logical steps.
- **Check Your Understanding** exercises follow each set of examples and allow students to confirm their mastery of concepts.
- **Make the Connection** sidebars present analogies that illuminate electrical engineering concepts using other concepts from engineering disciplines.
- **Focus on Measurements** boxes emphasize the great relevance of electrical engineering to the science and practice of measurement.

Instructor Resources on Connect:

Instructors have access to these files, which are housed in Connect.

- **PowerPoint presentation slides** of important figures from the text
- **Instructor's Solutions Manual** with complete solutions

Remote Proctoring & Browser-Locking Capabilities

New remote proctoring and browser-locking capabilities, hosted by Proctorio within Connect, provide control of the assessment environment by enabling security options and verifying the identity of the student.

Seamlessly integrated within Connect, these services allow instructors to control students' assessment experience by restricting browser activity, recording students' activity, and verifying students are doing their own work.

Instant and detailed reporting gives instructors an at-a-glance view of potential academic integrity concerns, thereby avoiding personal bias and supporting evidence-based claims.

Writing Assignment

Available within McGraw Hill Connect®, the Writing Assignment tool delivers a learning experience to help students improve their written communication skills and conceptual understanding. As an instructor you can assign, monitor, grade, and provide feedback on writing more efficiently and effectively.

0.7 ACKNOWLEDGMENTS

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Throughout the preparation of this edition, Kathryn Rizzoni has provided editorial support and has served as an interface to the editorial staff at MHHE. We are grateful for her patience, her time invested in the project, her unwavering encouragement, her kind words, and her willingness to discuss gardening and honeybees.

The book has been critically reviewed by:

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- N. Jill Schoof—Maine Maritime Academy

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- Tom Sullivan—Carnegie Mellon University
- Dr. James Moscola—York College of Pennsylvania
- Dr. Jason Forsyth—James Madison University

In addition, we would like to thank the many colleagues who have pointed out errors and inconsistencies and who have made other valuable suggestions.

Comments by Giorgio Rizzoni

As always, a new edition represents a new era. I am truly grateful to my friend and co-author, Jim Kearns, for taking on a new challenge and for bringing his perspective and experience to the book. Jim and I share a passion for teaching, and throughout this project we have invariably agreed on which course to take. It is not easy to find a suitable co-author in the life of a project of this magnitude, and I have been fortunate to find a friend willing to undertake a new journey with me.

When the first edition of the book was nearing, so was the birth of our first child, Alessandro (Alex). The second and third editions were marked by the births of Maria Caterina (Cat), and Michael. Time passes, Kathryn continues to be my best friend and partner, and now Cat is a successful industrial designer, Alex is studying electrical engineering (imagine that!), and Michael computer science and engineering. The years go by, but my family continues to be an endless source of joy, pleasant surprises and, always, smiles. Many thanks to Kathryn, Alex, Cat, and Michael for always being there to support and encourage me.

Comments by James Kearns

My association with this remarkable book continues to be a great privilege and honor. Its contents continue to reflect the enormous effort and expertise of the principal author, and my dear friend, Dr. Giorgio Rizzoni. His leadership and vision were essential to the creation of this new edition. I remain awestruck by his seemingly unbounded energy and enthusiasm and humbled by his kind, considerate, and generous ways.

As with all things, the love and support of my family and friends sustained me throughout this work. My children, Kevin, Claire, and Caroline, continue to bless, inspire, and inform my daily life.

Finally, I wish to once again thank my parents for their many years of unconditional love and support. Despite having lost both of them in recent years they remain very much alive within me and present in my work. Can anyone ever begin to measure or repay the gift of loving parents?

Guided Tour

transformations) discussed in Chapters 1 and 2. The only difference is that these relationships now involve phasors, that is, complex quantities.

The average and effective (root-mean-square) amplitude of a waveform are introduced in this chapter. An effective value represents the equivalent DC value required to supply or dissipate the same power as the AC waveform and thus provides a means of comparing different waveforms. This rather extensive chapter concludes with an introduction to single-phase AC power and the concepts of power factor, apparent, real and reactive power, power triangles and power factor correction.

In this chapter and throughout the book, angles are given in units of radians, unless indicated otherwise.

Learning Objectives offer an overview of key chapter ideas. Each chapter opens with a list of major objectives, and throughout the chapter the learning objective icon indicates targeted references to each objective.

LO Learning Objectives

Students will learn to...

1. Compute current, voltage, and energy of capacitors and inductors. *Section 3.2.*
2. Calculate the average and effective (root-mean-square) value of an arbitrary periodic waveform. *Section 3.3.*
3. Convert time-domain sinusoidal voltages and currents to phasor notation, and vice versa; and represent circuits using impedances. *Sections 3.4 and 3.5.*
4. Apply DC circuit analysis methods to AC circuits in phasor form. *Section 3.6.*
5. Compute average AC power and the power factor of a complex load. *Section 3.7.*
6. Compute apparent, real and reactive power for complex loads and draw a power triangle. *Section 3.8.*
7. Compute the capacitance required to correct the power factor of a complex load. *Section 3.9.*

3.1 CIRCUITS CONTAINING ENERGY STORAGE ELEMENTS

The resistive circuits studied in Chapters 1 and 2 had no dependence on time. The sources had constant (DC) values and the *i-v* relationship for resistors (Ohm's law) had no time dependence. As a result, all the equations obtained in those chapters

De Morgan's laws state that every SOP expression has an equivalent POS form. A simple example of a POS expression is $(W + Y) \cdot (Y + Z)$. For any particular logical expression one of the two forms may lead to a realization involving a smaller number of gates.

FOCUS ON PROBLEM SOLVING

LO

PRODUCT-OF-SUMS REALIZATIONS

1. Group 0s in subgroups exactly as is done for 1s when seeking an SOP expression.
2. Produce a complemented Karnaugh map by swapping X with $\overline{X}$, Y with $\overline{Y}$, and Z with $\overline{Z}$.
3. Each subgroup of 0s represents a *sum* of the complemented Karnaugh map's elements.
4. Form the product of those sums.

Focus on Problem Solving sections summarize important methods and procedures for the solution of common problems and assist the student in developing a methodical approach to problem solving.

EXAMPLE 3.5 **Calculating Inductor Current From Voltage**

Problem

Use a time plot of the voltage across an inductor and its initial current to calculate the current through it as a function of time.

Solution

Known Quantities: Inductor voltage; initial condition (current at $t = 0$); inductance value.

Find: Inductor current.

Schematics, Diagrams, Circuits, and Given Data:

$$v(t) = \begin{cases} 0\text{ V} & t < 0\text{ s} \\ -10\text{ mV} & 0 < t < 1\text{ s} \\ 0\text{ V} & t > 1\text{ s} \end{cases}$$

$$L = 10\text{ mH}; \qquad i_L(t = 0) = I_0 = 0\text{ A}$$

The voltage across the inductor is plotted in Figure 3.16(a).

Analysis: Use the integral i-v relationship for an inductor to obtain the current through it:

$$i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau \qquad t \geq t_0$$

$$= \begin{cases} I_0 + \frac{1}{L}\int_0^t (-10 \times 10^{-3})\,d\tau = 0 + \frac{-10^{-2}}{10^{-2}}t = -t\text{ A} & 0 \leq t \leq 1\text{ s} \\ -1\text{ A} & t \geq 1\text{ s} \end{cases}$$

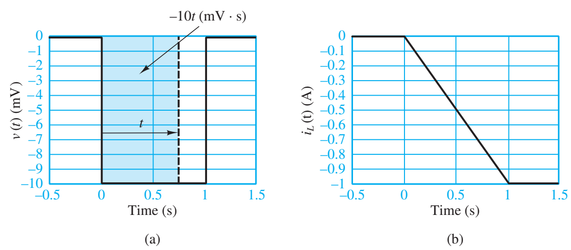


Figure 3.16

The inductor current is plotted in Figure 3.16b.

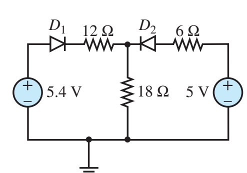
Comments: The inductor voltage can change instantaneously and thus be discontinuous!

Clearly illustrated examples present relevant applications of electrical engineering principles. The examples are fully integrated with the Focus on "Problem" Solving material, and each one is organized according to a prescribed set of logical steps.

Check Your Understanding exercises follow each set of examples and allow students to confirm their mastery of concepts.

CHECK YOUR UNDERSTANDING

Determine which of the diodes conduct in the circuit shown below. Each diode has an offset voltage of 0.6 V.

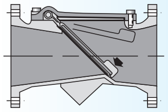


Answer: Both diodes conduct.

Make the Connection sidebars present analogies that illuminate electrical engineering concepts using concepts from other engineering disciplines.

(Concluded)

The second figure below depicts a flapper check valve. The principle is similar to that described above for the swing check valve in that fluid flow is permitted from left to right, but not in the reverse direction. The response of the flapper checkvalve is faster than the swing check valve due to the shorter travel distance of the flapper.



Flapper check valve

Diode circuits are much easier to understand when the behavior of the diode is visualized to be similar to that of a check valve, with the pressure difference across the valve orifice being analogous to the voltage across the diode and the fluid flow rate being analogous to the current through the diode. Charge flows only when the voltage across the diode is positive or *forward-biased,* and no charge flows when the diode voltage is negative or *reverse-biased.*

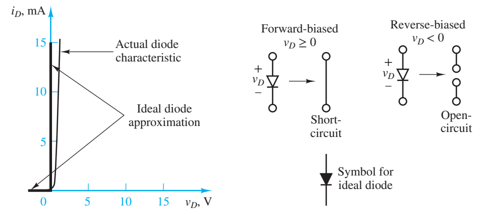


Figure 8.11 Large-signal on/off ideal diode model

($v_D \geq 0$). Due to its simplicity, the ideal diode model can be very useful in circuit analysis.

> Ideal diodes are represented by the solid black triangle symbol shown in Figure 8.11.

A general method for analyzing diode circuits is illustrated using the circuit shown in Figure 8.12, which contains a 1.5-V battery, an ideal diode, and a 1-kΩ resistor. The method is simply to assume that the ideal diode is forward-biased ($v_D \geq 0$) and thus equivalent to a short-circuit, as indicated in Figure 8.13. Under this assumption, $v_D = 0$ such that the loop current is $i_D = 1.5\text{ V}/1\text{ k}\Omega = 1.5$ mA. Since the resulting direction of the current and the diode voltage are consistent with the assumption of a conducting diode ($v_D \geq 0$, $i_D > 0$), the assumption is correct. If the assumption had resulted in diode current and voltage that contradict the assumption, then the assumption would have been deemed incorrect, and the opposite assumption of a nonconducting diode could be tested, and presumably found to be true.

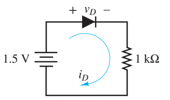


Figure 8.12 Circuit containing ideal diode

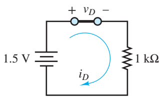


Figure 8.13 Circuit of Figure 8.12, assuming that the ideal diode conducts

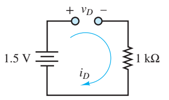


Figure 8.14 Circuit of Figure 8.12, assuming that the ideal diode does not conduct

To test the opposite assumption, assume the ideal diode is reverse-biased ($v_D < 0$) and thus equivalent to an open-circuit, as shown in Figure 8.14. Since the loop does not form a closed path, the current i_D must be zero and thus Ohm's law requires the voltage across the resistor to also be zero. Then, KVL requires that $v_D = 1.5$ V. However, this result contradicts the assumption that the ideal diode is reverse-biased. Thus, the assumption is deemed incorrect.

The method can be applied to more complicated circuits involving multiple diodes by simply testing all the possible combinations of forward- and reverse-biased assumptions for the diodes. In such cases, it is helpful to consider which

Diode Peak Detector Circuit for Capacitive Displacement Transducer

Another common application of semiconductor diodes, the *peak detector*, is very similar in appearance to the half-wave rectifier with capacitive filtering as shown in Figure 8.56. One of its more classic applications is in the demodulation of amplitude-modulated (AM) signals.

In Chapter 3, a capacitive displacement transducer was introduced in the two Focus on Measurements boxes, "Capacitive Displacement Transducer and Microphone." It took the form of a parallel-plate capacitor composed of a fixed plate and a movable plate. The capacitance of this variable capacitor was shown to be a function of displacement; that is, it was shown that a movable-plate capacitor can serve as a linear transducer. Recall the expression derived in Chapter 3

$$C = \frac{8.854 \times 10^{-3} A}{x} \quad \text{pF}$$

where C is the capacitance in picofarads, A is the area of the plates in square millimeters, and x is the variable separation distance in millimeters. The nominal plate separation is d. If the capacitor is placed in an AC circuit, its impedance will be determined by the expression

$$Z_C = \frac{1}{j\omega C}$$

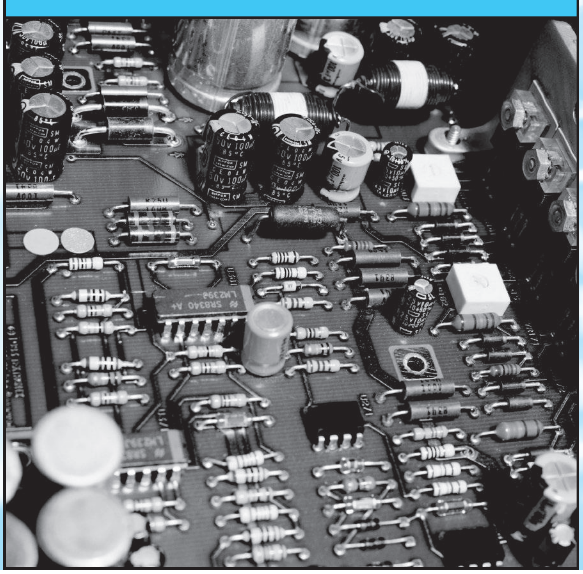
FOCUS ON MEASUREMENTS

Focus on Measurements boxes emphasize the great relevance of electrical engineering to the science and practice of measurement.

PRINCIPLES AND APPLICATIONS OF ELECTRICAL ENGINEERING

PART I

CIRCUITS



Arthur S. Aubry/Stockbyte/Getty Images

Chapter 1 Fundamentals of Electric Circuits

Chapter 2 Equivalent Networks

Chapter 3 AC Network Analysis

Chapter 4 Transient Analysis

Chapter 5 Frequency Response and System Concepts

CHAPTER 1

FUNDAMENTALS OF ELECTRIC CIRCUITS

Chapter 1 is the foundation for the entire book and presents the fundamental laws that govern the behavior of electric circuits. Basic features and terminology of electric circuits, such as nodes, branches, meshes, and loops, are defined, and the three fundamental laws of circuit analysis, Kirchhoff's current and voltage laws and Ohm's law, are introduced. The concept of electric power and the passive sign convention are introduced along with basic circuit elements—sources and resistors. Basic analytic techniques of node voltage and mesh current analyses are introduced along with some engineering applications.

Learning Objectives

Students will learn to...

1. Identify the principal *features of electric circuits or networks*: nodes, loops, meshes, and branches. *Section 1.1.*
2. Apply definitions of charge, current and voltage. *Section 1.2.*
3. Identify *sources* and their *i-υ characteristics*. *Section 1.3.*
4. Apply the *passive sign convention* to compute the power consumed or supplied by circuit elements. *Section 1.4.*
5. Apply *Kirchhoff's laws* to simple electric circuits. *Section 1.5.*
6. Apply *Ohm's law* to calculate unknown voltages and currents in simple circuits. *Section 1.6.*
7. Apply the *Node Voltage method* to solve for unknown voltages and currents in resistive networks. *Section 1.7.*
8. Apply the *Mesh Current method* to solve for unknown voltages and currents in resistive networks. *Section 1.8.*
9. Apply the Node Voltage and Mesh Current methods to solve for unknown voltages and currents in resistive networks with *dependent sources*. *Section 1.9.*

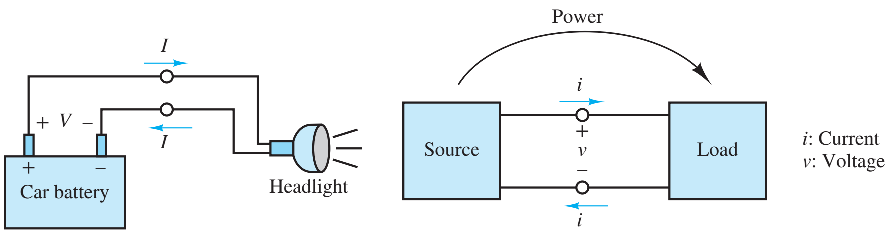
1.1 FEATURES OF NETWORKS AND CIRCUITS

"A *network* can be defined as a collection of interconnected objects. In an electric network, *elements*, such as resistors, are connected by wires. An electric *circuit* can be defined as an electric network within which at least one closed path exists and around which electric charge may flow. All electric circuits are networks but not all electric networks contain a circuit. In this book, a circuit is any network that contains at least one complete and closed path.

There are two principal quantities within a circuit: current and voltage. *The primary objective of circuit analysis is to determine one or more unknown currents and voltages.* Once these currents and voltages are determined, any other aspect of the circuit, such as its power requirements, efficiency, and speed of response, can be computed.

Two useful concepts for circuit analysis are those of a source and of a load. In general, the load is the circuit element or segment of interest to the designer or user of the circuit. By default, the source is everything else not included in the load. Typically, the source provides energy and the load consumes it for some purpose, such as the lifting of a weight. For example, consider the simple physical circuit of a headlight attached to a car battery as shown in Figure 1.1(a). For the driver of the car, the headlight may be the circuit element of interest since it enables the driver to see the road at night. From this perspective, the headlight is the load and the battery is the source as shown in Figure 1.1(b), which is intuitively appealing because power flows from the source (the battery) to the load (the headlight). However, in general, it is not required nor necessarily true that power flows in this manner. Electric power is discussed later in this chapter.

The use of the term *source* can be confusing at times because, as will be discussed later in this chapter, there are circuit elements known as *ideal voltage and current sources*, which have well-defined attributes and circuit symbols. These



(a) Physical representation (b) Conceptual representation

Figure 1.1 (a) Physical model and (b) generalized conceptual representation of an electrical system. See the notation rules for V, I, v, and i listed in the preface.

ideal sources, along with other circuit elements, are often the constituents of the source portion of a circuit, as well as the load portion. In this book, ideal sources are referred to as either voltage or current sources, explicitly, to avoid confusion.

Other key conceptual features of electric circuits are the *ideal wire*, *node*, *branch*, *loop*, and *mesh.* The concept of a node (see below) is particularly useful for correctly interpreting circuit diagrams and constructing circuit prototypes on breadboards.

Many students struggle with circuit analysis simply because they lack an organizing perspective with which to interpret circuit diagrams. One particularly helpful perspective is to see electric circuits as comprised of elements situated between nodes. This perspective enables students to see beyond the particular aesthetic presentation of a circuit diagram, to see its substance and not be fooled by its appearance. Once the concept of a node is well understood circuits that previously appeared complicated often become meaningful and clear.

Ideal Wire

Electric circuit and network *diagrams* are used to model actual electric circuits and networks. These diagrams contain *elements* connected by *ideal wires*. An ideal wire is able to conduct electric charge without any loss of electric potential. In other words, no work is required to move an electric charge along an ideal wire. Luckily, in many applications, actual wires are well approximated by ideal wires. However, there are applications where wiring accounts for significant losses of potential (e.g., long-distance transmission lines and microscopic integrated circuits). In these applications, the ideal wire approximation must be used with care. In this book, all wires in circuit and network diagrams are ideal, unless indicated otherwise.

Node

A **node** consists of one or more ideal wires connected together such that an electric charge can travel between any two points on the node without traversing a circuit element, such as a resistor. Thus, every point on a node has the same electric potential, which is known as the *node voltage* and its value is relative to a reference potential.

A *junction* is a point where two or more wires are joined together. A node may contain one or more junctions or none at all, such as when a single wire directly connects two elements. A junction is part of a node but is not a node itself.

It is crucial to correctly identify and count nodes in the analysis of electric circuits. Figure 1.2 illustrates a helpful way to mark nodes. There are three nodes in Figure 1.2(a) and two nodes in Figure 1.2(b). It is sometimes convenient to use the concept of a **supernode**, which is simply a closed boundary enclosing two or more nodes, as shown in Figure 1.2(c).

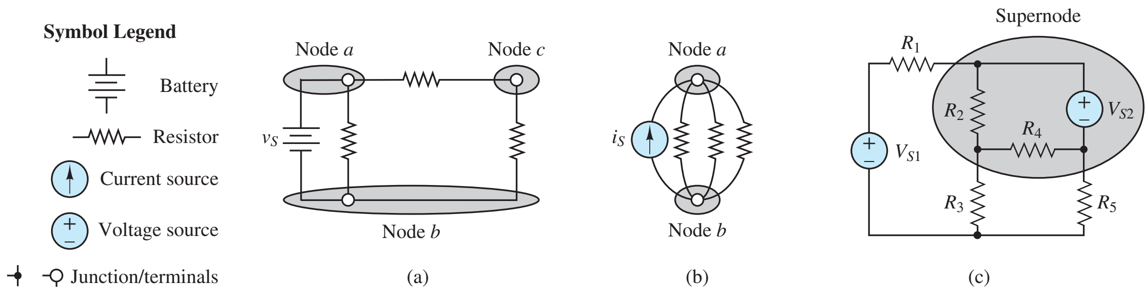


Figure 1.2 Illustrating nodes and supernodes in circuit diagrams

It is also important to realize that since no work is required to move an electric charge along an ideal wire, the length and shape of an ideal wire has no impact on the behavior of a circuit. Likewise, since nodes are comprised of ideal wires, the extent and shape of a node has no impact on the behavior of a circuit. As a result, a node may be redrawn in any manner as long as the newly drawn node is attached to the same elements as the original node. Circuit diagrams are typically drawn, by convention, in a rectangular manner, with all wires drawn either side to side or up and down. However, many students find it helpful to redraw circuits so as to clarify the number and location of nodes in a circuit. Figure 1.3 shows two identical circuits drawn in two different ways. Can you tell that these circuits have the same number of nodes?

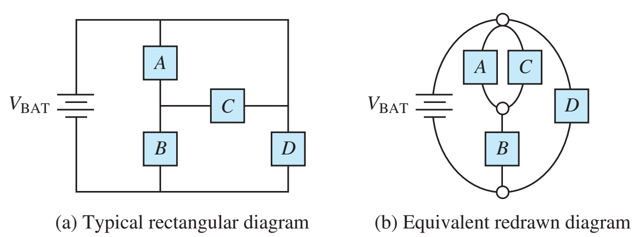
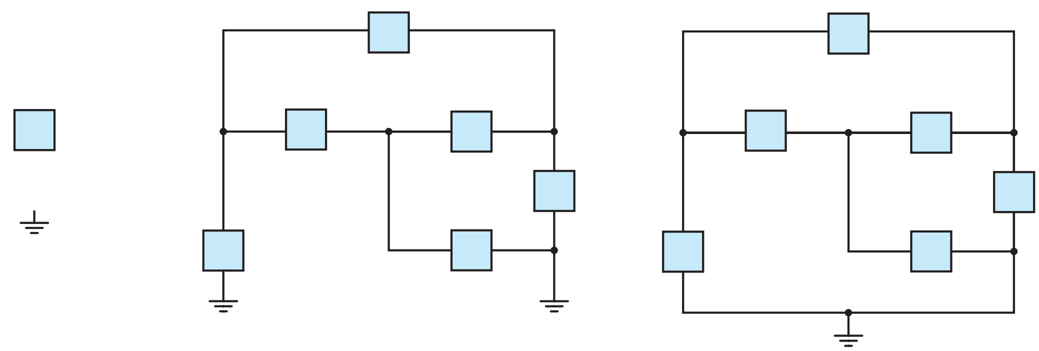


Figure 1.3 (a) A typical rectangular circuit diagram and (b) an equivalent redrawn diagram. A circuit can be redrawn to have almost any appearance; however, the behavior of the circuit is unchanged as long as the number of nodes and the elements between those nodes remain unchanged.

Keep in mind that all forms of potential, including voltage, are relative quantities. For this reason, it is important to refer to the voltage *across* an element. In circuit diagrams, the voltage across an element is indicated by the paired symbols + and −. Taken together as a single symbol they indicate the *assumed polarity* of the voltage.

Sometimes it is convenient to establish a *reference node.* Any one node in a network can be designated as the reference node. Then, all other node voltages are determined relative to that reference node. The value of the reference node can be chosen freely, although a value of zero is usually chosen, for simplicity. It is often true that a smart choice of reference node will simplify the analysis that follows. A good rule of thumb is to select a node that is connected to a large number of elements.

A reference node is designated by the symbol shown in Figure 1.4(a). This symbol is also used to designate *earth ground* in applications. To reduce the apparent complexity of some circuits, multiple reference symbols are used to minimize the amount of displayed reference node wiring. It is simply understood that all nodes to which these symbols are attached are, in fact, connected by ideal wires and therefore part of one large reference node. Figure 1.4(b) and (c) illustrate this practice.



(a) Generic element and reference node symbol

(b) Circuit with two reference node symbol

(c) Equivalent circuit

Figure 1.4 There can be one and only one reference node in a network although the reference node symbol may appear more than once in order to reduce the amount of displayed reference node wiring. The reference node symbol is also used to designate a connection to earth ground in practical circuits.

> Elements that sit between the same two nodes are said to be in *parallel.*

Branch

A **branch** is defined in this book as a single electrical pathway, consisting of wires and elements. A branch may contain one or more circuit elements as shown in Figure 1.5. By definition, the current *through* any one element in a branch is the

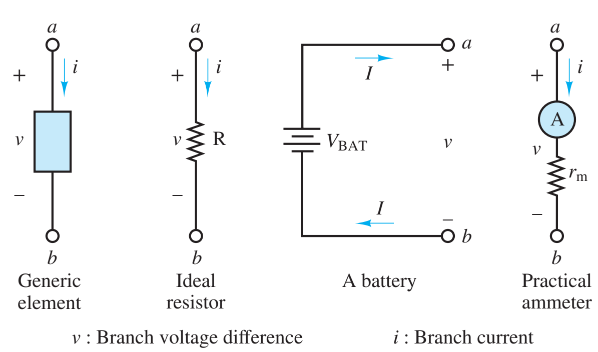


Figure 1.5 Examples of circuit branches

same as the current through every other element in that branch; that is, there is one current in a branch, the *branch current.*

> Elements that sit along the same branch are said to be in *series.*

Loop

A **loop** is any closed pathway. Figure 1.6(a) shows that different loops in the same circuit may share common elements and branches. It is interesting, and perhaps initially confusing, to note that a loop does not necessarily have to correspond to a closed electrical pathway, consisting of wires and elements. Figure 1.6(b) shows one example in which a loop passes directly from node a to node c.

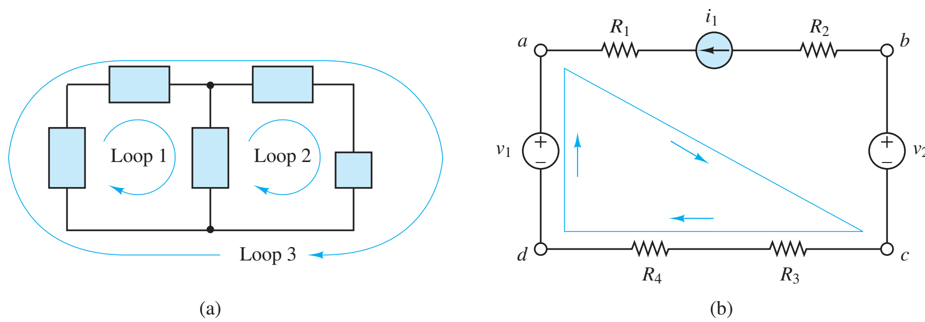


Figure 1.6 Examples of loops. How many nodes are in each of these circuits? [Answers: (a) 4; (b) 7]

Mesh

A **mesh** is a closed electrical pathway that does not contain other closed electrical pathways. In Figure 1.6(a), loops 1 and 2 are meshes, but loop 3 is not a mesh because it contains loops 1 and 2. The circuit in Figure 1.6(b) has one mesh. Figure 1.7 illustrates how simple it is to visualize meshes.

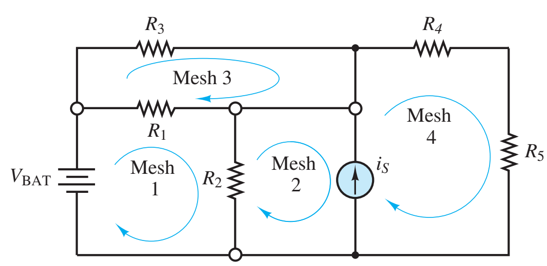


Figure 1.7 Circuit with four meshes. How many different closed electrical pathways are in this circuit? [Answer: 14]

EXAMPLE 1.1

Problem

Identify the branch and node voltages and the loop and mesh currents in the circuit of Figure 1.8.

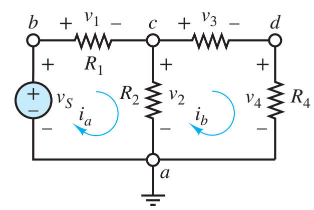


Figure 1.8

Solution

The following node and branch voltages may be identified:

Node voltages	Branch voltages	Relationship
$v_a = 0$ (reference)		
v_b	v_S	$v_S = v_b - v_a$
	v_1	$v_1 = v_b - v_c$
v_c	v_2	$v_2 = v_c - v_a$
	v_3	$v_3 = v_c - v_d$
v_d	v_4	$v_4 = v_d - v_a$

Comments: Currents i_a and i_b are mesh currents.

EXAMPLE 1.2 Counting Nodes in a Network

Problem

Count the total number of nodes in each of the four networks.

Solution

Known Quantities: Wires and elements.

Find: The number of nodes in each network diagram.

Schematics, Diagrams, Circuits, and Given Data: Figure 1.9 contains four elements: two resistors and two ideal voltage sources, one independent and one dependent. Figure 1.10 contains five elements: four resistors and one independent ideal voltage source. Figure 1.11 contains five elements: four resistors and one operational amplifier. Figure 1.12 contains three elements: two headlamps and one 12-V battery.

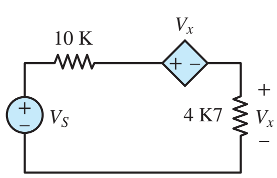


Figure 1.9

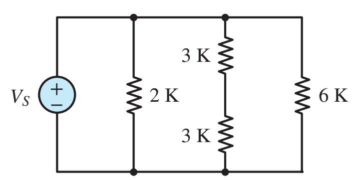


Figure 1.10

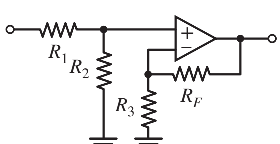


Figure 1.11

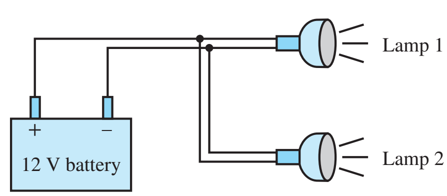


Figure 1.12

Assumptions: All wires are ideal.

Analysis:
In Figure 1.9, all four elements are in a single electrical loop. There is one node between each pair of elements. Thus, there are *four nodes* in this network.

In Figure 1.10, the voltage source and the 2K and 6K resistors sit between two large nodes, one along the top of the network and the other along the bottom of the network. Each of these nodes contains two wire junctions. Another node is between the two 3K resistors. Thus, there are *three nodes* in this network.

In Figure 1.11, there is one node to the left of R_1 and one node to the right of the operational amplifier and R_F. A third node is located between R_1, R_2, and the + (noninverting) terminal of the operational amplifier. A fourth node is located between R_3, R_F, and the − (inverting) terminal of the operational amplifier. A fifth, and final, node is the reference node, which is between R_2 and R_3. Thus, there are *five nodes* in this network.

In Figure 1.12, there is one node between the positive + battery terminal and one terminal on each headlamp. There is another node between the negative − battery terminal and the second terminal on each headlamp. Thus, there are *two nodes* in this network.

Comments: The notation K is short for kΩ. The placement of K indicates the location of a decimal point. For example, 4K7 is 4.7 kΩ.

1.2 CHARGE, CURRENT AND VOLTAGE

Charles Coulomb (1736–1806). (*INTERFOTO/Personalities/ Alamy Stock Photo*)

The earliest accounts of electricity date from about 2,500 years ago, when it was discovered that a piece of amber was capable of attracting very light objects, such as feathers. The word *electricity* originated about 600 B.C.; it comes from *elektron*, which was the ancient Greek word for amber. Following the work of Alessandro Volta and his invention of the copper-zinc battery, it was determined that static electric effects and the current in metal wires connected to a battery were both due to the same fundamental nature of matter, namely, the atomic structure of matter, consisting of a nucleus—neutrons and protons—surrounded by electrons.

The unit of charge, the **coulomb (C)**, is named after Charles Coulomb. The unit of current, the ampere (A), is named after the French scientist André-Marie Ampère.

The fundamental electric quantity is **charge**. The unit of charge is the **coulomb (C)**. The electron and proton each carry one unit of charge but of opposite sign. By convention, the electron is deemed to be negatively charged.

$$q_e = -1.602 \times 10^{-19}\,\text{C} \qquad q_p = +1.602 \times 10^{-19}\,\text{C} \tag{1.1}$$

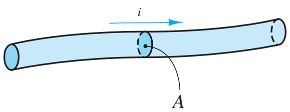


Figure 1.13 Current in an electric conductor is defined as the net flow rate of charge through the cross-sectional area *A*.

Electrons and protons are often referred to as **elementary charges**. The amount of charge associated with an electron may seem rather small. However, typical currents involve the flow of large numbers of charged particles.

Current

Electric current is defined as the rate at which charge passes through an area, such as the cross-sectional area of a wire. Figure 1.13 depicts a macroscopic view of current i in a wire, where Δq units of charge flow through the cross-sectional area A in a period Δt. The resulting current i is

$$i \equiv \frac{\Delta q}{\Delta t} \quad \frac{\text{C}}{\text{s}} \tag{1.2}$$

The arrow symbol associated with the current i is its *assumed* direction through the wire segment. A negative value for i would indicate a direction opposite to the assumed direction. When large numbers of discrete charges cross A in a very small period, the current i can be written in differential form.

$$i \equiv \frac{dq}{dt} \quad \frac{\text{C}}{\text{s}} \tag{1.3}$$

The unit of current is the **ampere**, where 1 ampere (A) = 1 coulomb/second (C/s). By convention, in electrical engineering positive current is the direction of positive charge flow. This convention can be confusing since the mobile charge carriers in metal wires and many other conductors are electrons from the *conduction band* of the material. However, when an electron travels in one direction the effect on the distribution of *net charge* is the same as if a proton had travelled in the opposite direction. In other words, positive current represents the *relative* flow of positive charges.

Voltage

Typically, work is required to move charge between two nodes in a circuit. The total *work per unit charge* is called **voltage**, and the unit of voltage is the **volt** in honor of Alessandro Volta.

$$1 \text{ volt (V)} = 1 \frac{\text{joule (J)}}{\text{coulomb (C)}}$$

The voltage, or **potential difference**, across two nodes in a circuit is the energy (in joules) per unit charge (1 coulomb) needed to move charge from one node to the other. The direction, or *polarity*, of the voltage is related to whether energy is being gained or lost by the charge in the process.

Note that the word *potential* is quite appropriate as a synonym of voltage, in that voltage is the potential energy per unit charge across two nodes in a circuit. If the lightbulb is disconnected from the circuit, a voltage v_{ab} still exists across the battery terminals, as illustrated in Figure 1.14. This voltage represents work done on the battery to separate positive ions from negative ions. The potential energy associated with the separated ions is available to do work on an external element attached to the battery terminals. That work is expressed as positive charge flowing (current) through the element from high to low potential. Thus, the battery is able to *supply* energy to the attached element and, likewise, the attached element is able to *consume* or *dissipate* energy from the battery.

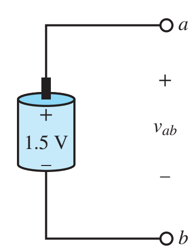


Figure 1.14 The voltage v_{ab} across the open terminals of the battery represents the potential energy available to move charge from a to b once a closed circuit is established.

The Reference Node and Ground

Earth ground represents a specific, and usually clearly marked, node in many circuits. Residential electric circuits are connected to earth ground through a large conductor, such as a copper spike or water pipe, that is buried in the earth. When present in a circuit diagram, earth ground is always chosen as the reference node

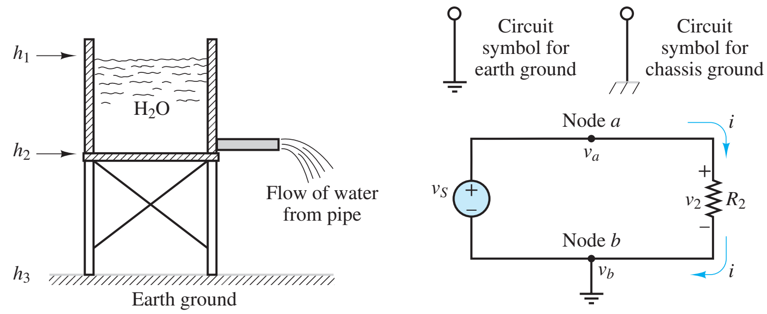


Figure 1.15 An analogy between water flow and electric current illustrates the relation between potential differences and a ground reference potential.

because the earth's potential is relatively stable and uniform due to its ability to store and distribute large quantities of charge. In circuits where earth ground is not present, some other relatively large conductor can serve as a stable ground node, such as a metal enclosure or chassis of an instrument.

In practice, the voltage value assigned to a reference node, such as earth ground, while typically zero, is not consequential. A simple analogy with fluid flow illustrates this rule. Consider a tank of water, as shown in Figure 1.15, located at a certain height above the ground. The potential energy difference per unit mass due to gravity $u_{12} = g(h_1 - h_2)$ is completely analogous to the potential energy difference per unit charge $v_a - v_b$. Now assume that the height h_3 at ground level is chosen to be the zero potential energy reference. Is the flow of water in the pipe changed due to this choice? Of course not. Is the flow of water in the pipe dependent upon the height $h_2 - h_3$ of the support structure? Again, the answer is no. The truth of these statements is demonstrated by rewriting the *head* of the water tank $h_1 - h_2$ as $(h_1 - h_3) - (h_2 - h_3)$ such that

$$u_{12} = g(h_1 - h_2) = [g(h_1 - h_3)] - [g(h_2 - h_3)] = u_{13} - u_{23}$$

Even though the values of u_{13} and u_{23} depend upon h_3, the difference between them u_{12} does *not* depend upon h_3. It is the change in potential energy that matters in the water tank problem. So it is with electric circuits. The voltage *across* an element does not depend upon the selection of a reference node nor upon the arbitrary voltage value assigned to the reference node.

Another familiar scenario is that of a skydiver leaping from an airplane and parachuting to the surface below (see Figure 1.16). To quantify the potential energy U of the skydiver it is first necessary to choose a reference height h_0 such that $U = mg\Delta h = mg(h - h_0)$, where h represents the height of the skydiver. One possible choice for a reference height is the height of the airplane such that the potential energy of the skydiver is negative ($U < 0$)? However, such a choice would be strange and perhaps misleading. The surface of the earth is a more meaningful reference to the skydiver, who knows that a soft landing depends upon dissipating most of the initial potential energy through collisions with air molecules rather than through a collision with the surface. The skydiver knows that her fate is unchanged by her choice of reference; however, some choices are more meaningful than others. So it often is with electric circuits.

Figure 1.16 A skydiver understands all too well that her fate is unchanged by the choice of reference potential.

EXAMPLE 1.3 Charge and Current in a Conductor

Problem

Find the total charge in a cylindrical conductor (solid wire) and compute the current through the wire.

Solution

Known Quantities: Conductor geometry, charge density, charge carrier velocity.

Find: Total charge of carriers Q; current in the wire I.

Schematics, Diagrams, Circuits, and Given Data:

Conductor length: $L = 1$ m.

Conductor diameter: $2r = 2 \times 10^{-3}$ m.

Charge density: $n = 10^{29}$ carriers/m^3.

Charge of one electron: $q_e = -1.602 \times 10^{-19}$.

Charge carrier average net speed: $u = 19.9 \times 10^{-6}$ m/s.

Assumptions: None.

Analysis: To compute the total charge in the conductor, first determine the volume of the conductor:

Volume = length × cross-sectional area

$$\text{Vol} = L \times \pi r^2 = (1 \text{ m}) \left[\pi \left(\frac{2 \times 10^{-3}}{2}\right)^2 \text{m}^2\right] = \pi \times 10^{-6} \text{ m}^3$$

Next, compute the number of carriers (electrons) in the conductor and the total charge:

Number of carriers = volume × carrier density

$$N = \text{Vol} \times n = (\pi \times 10^{-6} \text{ m}^3)\left(10^{29} \frac{\text{carriers}}{\text{m}^3}\right) = \pi \times 10^{23} \text{ carriers}$$

Charge = number of carriers × charge/carrier

$$Q = N \times q_e = (\pi \times 10^{23} \text{ carriers}) \times \left(-1.602 \times 10^{-19} \frac{\text{C}}{\text{carrier}}\right) = -50.33 \times 10^3 \text{ C}$$

To compute the current, consider the average net speed of the charge carriers and the charge density per unit length of the conductor:

Current = carrier charge density per unit length × carrier average net speed

$$I = \left(\frac{Q}{L} \ \frac{\text{C}}{\text{m}}\right) \times \left(u \ \frac{\text{m}}{\text{s}}\right) = \left(-50.33 \times 10^3 \ \frac{\text{C}}{\text{m}}\right)\left(19.9 \times 10^{-6} \ \frac{\text{m}}{\text{s}}\right) = -1 \text{ A}$$

Comments: Charge carrier density is a function of material properties. Carrier average net speed is a function of the applied electric field.

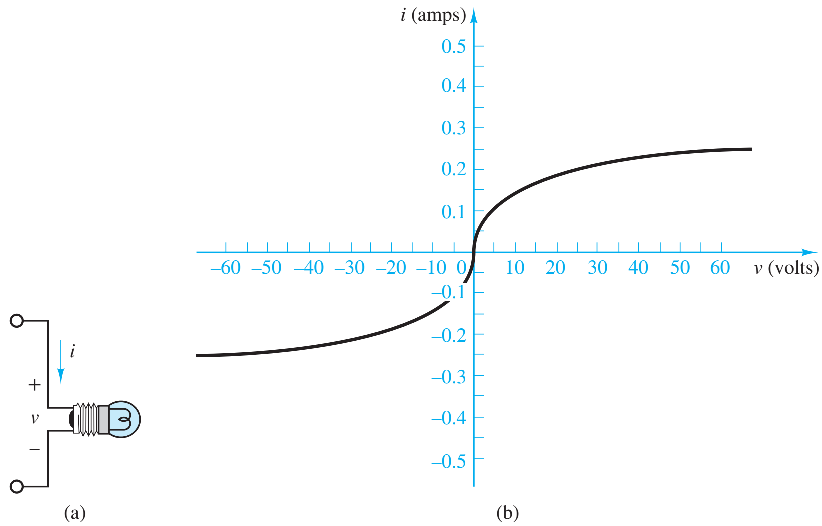


Figure 1.17 (a) Depiction of how to measure the *i*-*υ* characteristic of an incandescent (tungsten filament) lightbulb; (b) typical *i*-*υ* plot of such a lightbulb

1.3 *i*-*υ* CHARACTERISTICS AND SOURCES

It is possible to create an *i*-*υ* plot for any circuit element. The functional relationship between i and υ may be quite complex and not easily expressed in a closed mathematical form, such as $i = f(\upsilon)$ However, the plot of the ***i*-*υ* characteristic** (or **volt-ampere characteristic**) for most circuit elements is either known or can be determined experimentally.

For example, consider the incandescent (tungsten filament) lightbulb shown in Figure 1.17(a). The *i*-*υ* characteristic of the lightbulb can be determined by varying the voltage over some predetermined range and recording the resulting current for each particular voltage in that range. The plot of the *i*-*υ* data will be similar to that shown in Figure 1.17(b). A positive voltage across the bulb results in a positive current through it, and conversely, a negative voltage across the bulb results in a negative current through it. In both cases charge flows from high to low potential, releasing energy that is dissipated by the bulb as light and heat.

The *i*-*υ* characteristics of ideal voltage and current sources are simple yet helpful visual aids. An ideal source is one that can provide any amount of energy without affecting the behavior of the source itself. **Ideal sources** are divided into two types: voltage sources and current sources.

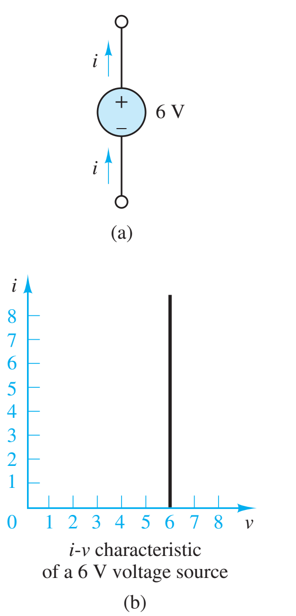


Figure 1.18 (a) An ideal voltage source; and (b) a typical *i*-*υ* characteristic, which indicates energy is supplied by the source to the flowing charge

Ideal Voltage Sources

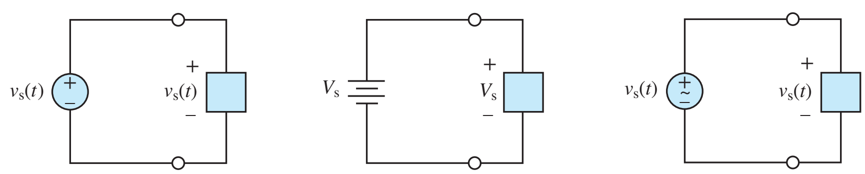
An **ideal voltage source** generates a prescribed voltage across its terminals independent of the current through its terminals. The circuit symbol for an ideal voltage source is shown in Figure 1.18(a). Notice that the current is defined as being directed from low to high potential. In other words, the voltage source is *supplying* energy to the flowing charge.

A typical *i-v* characteristic is shown in Figure 1.18(b). The current supplied by the source is determined by the circuit connected to it. It is important to recognize that an ideal voltage source guarantees a particular *change* in voltage from the node attached to its − terminal to the node attached to its + terminal. (Note: The + and − polarity markers do *not* indicate positive and negative voltage values relative to some zero reference. Do not make this mistake when solving problems!)

> An ideal voltage source provides a prescribed voltage across its terminals independent of the current through those terminals. The amount of current through the source is determined by the circuit connected to it.

Various types of batteries, electronic power supplies, and function generators approximate ideal voltage sources when used in proper circumstances. However, all such real devices have limits on the amount of current that can be supplied without impacting the voltage across the source. This behavior can be seen in a typical 12-V car battery. A digital voltmeter can be used to observe the voltage across a car battery as various electrical devices in the car are turned on and off. Very little change in the battery voltage will be observed, even when power windows are engaged. However, when the car is started, the battery voltage will drop significantly during the short period needed for the engine to start.

Figure 1.19 depicts various symbols for voltage sources. The output voltage of an ideal source can be a function of time. In this book, unless otherwise noted, a generic voltage source is denoted by a lowercase v. If it is necessary to emphasize that the source produces a time-varying voltage, then the notation $v(t)$ is employed. Finally, a constant, or dc voltage source is denoted by the uppercase character V.



General symbol for ideal voltage source

A special case: constant (DC) voltage source (ideal battery)

A special case: sinusoidal voltage source, $v_s(t) = V \cos \omega t$

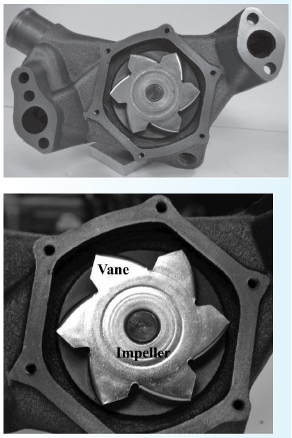
Figure 1.19 Three common ideal voltage sources

Ideal Current Sources

An **ideal current source** generates a prescribed current through its terminals independent of the voltage across its terminals. The circuit symbol for an ideal current source is shown in Figure 1.20(a). Notice that the current is defined as being

Hydraulic Analog of a Voltage Source

The role played by a voltage source in an electric circuit is very similar to that played by a velocity pump in a hydraulic circuit. In a velocity or rotodynamic pump, such as a centrifugal pump, impeller vanes add kinetic energy (velocity) to the fluid flow. This increase in kinetic energy is translated to an increase in pressure *across* the pump. The pressure difference *across* the pump is analogous to the voltage, or potential difference, *across* the voltage source.



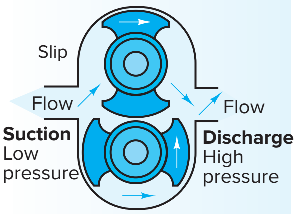
A centrifugal pump (*Giorgio Rizzoni*).

MAKE THE CONNECTION

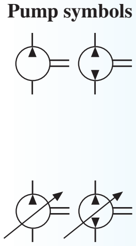
Hydraulic Analog of a Current Source

The role played by a current source in an electric circuit is very similar to that of a positive displacement pump in a hydraulic circuit. In a positive displacement pump, such as a peristaltic or reciprocating pump, an internal mechanism, such as a roller, piston, or diaphragm, forces a particular volume of fluid to be pumped *through* a hydraulic line. The volume flow rate *through* the pump is analogous to the charge flow rate *through* the current source.

Positive displacement pump



Pump symbols



Left: Fixed capacity pump. **Right:** Fixed capacity pump with two directions of flow.

Left: Variable capacity pump. **Right:** Variable capacity pump with two directions of flow.

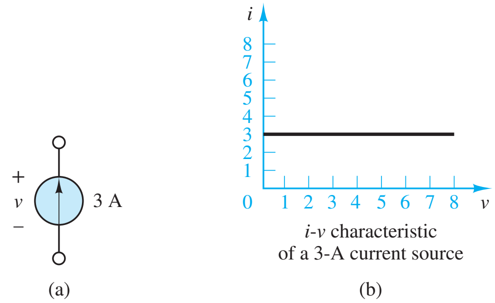


Figure 1.20 (a) An ideal current source; and (b) a typical *i-v* characteristic, which indicates energy is supplied by the source to the flowing charge

directed from low to high potential. In other words, the current source is *supplying* energy to the flowing charge.

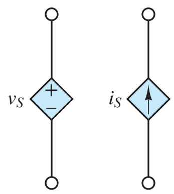
A typical *i-v* characteristic is shown in Figure 1.20(b). The voltage *across* the current source is determined by the circuit connected to it. It is important to recognize that an ideal current source guarantees a particular current *through* its terminals, such that the current entering the − terminal is the same as the current exiting the + terminal. (Again, the + and − polarity markers do not indicate positive and negative voltage values relative to some zero reference. Do not make this mistake when solving problems!)

An ideal current source provides a prescribed current through its terminals independent of the voltage across those terminals. The amount of voltage across the source is determined by the circuit connected to it.

Practical approximations to ideal current sources are not as common nor numerous as those for ideal voltage sources. However, in general, an ideal voltage source in series with an output resistance that is large in comparison to the input resistance of the circuit attached to its terminals provides a nearly constant current and thus approximates an ideal current source. A battery charger is a common and approximate example of an ideal current source.

Dependent (Controlled) Sources

The ideal *independent* sources described above are able to generate a prescribed voltage or current independent of the circuit attached to its terminals. Another category of sources, whose output (current or voltage) depends on some other voltage or current in a circuit, is known as **dependent** (or **controlled**) **sources**. As shown in Figure 1.21, the circuit symbols for these sources are diamonds to distinguish them from independent sources. The table illustrates the relationship between the source voltage v_S or source current i_S and the circuit voltage v_x or circuit current i_x, which they depend upon and which can be any voltage or current elsewhere in the overall circuit.



Source type	Relationship
Voltage controlled voltage source (VCVS)	$v_S = \mu v_x$
Current controlled voltage source (CCVS)	$v_S = r i_x$
Voltage controlled current source (VCCS)	$i_S = g v_x$
Current controlled current source (CCCS)	$i_S = \beta i_x$

Figure 1.21 Symbols for dependent sources

Dependent sources are very useful in describing the behavior of transistors and other electronic devices.

1.4 POWER AND THE PASSIVE SIGN CONVENTION

The power supplied or dissipated by a circuit element can be represented by the following relationship:

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{\text{work}}{\text{charge}}\,\frac{\text{charge}}{\text{time}} = \text{voltage} \times \text{current} \tag{1.4}$$

Thus,

LO

Electric power, P, is the product of voltage *across* an element and current *through* it.

$$\boxed{P = vi} \tag{1.5}$$

The unit of voltage (joules per coulomb) multiplied by that of current (coulombs per second) equals the unit of power (joules per second, or watts).

The power associated with a circuit element can be positive or negative. Positive power is, by convention, the rate at which energy is transferred from the flowing charge to an element. Negative power implies energy is transferred by an element to the flowing charge. Consider Figure 1.22(a), in which electric charge flows from low to high potential. Clearly, work has been done *by* element A *on* the flowing charge as its potential is raised. The rate at which this work is done *by* element A is its power. In this case, power is considered negative when energy is *supplied* or *released* by the element. In Figure 1.22(b), charge flows from high to low potential. Here, work has been done *on* element B *by* the flowing charge as its potential is lowered. The rate at which this work is done *on* element B is its power. In this case, power is considered positive when energy is *dissipated* or *stored* by the element.

LO

In the *passive sign convention*, current is directed from high to low potential. In this convention, energy is released by the flowing charge and consumed or stored by the element. The rate at which energy is transferred from the flowing charge to the element is considered positive power.

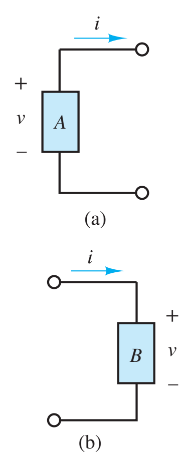


Figure 1.22 Assuming positive values for i and v, the active sign convention shown in (a) implies energy is supplied or released by element A while the passive sign convention shown in (b) implies energy is consumed or stored by element B.

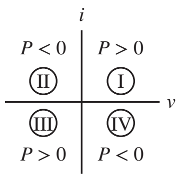


Figure 1.23 The four quadrants of a generic *i-v* plot, where *i* and *v* are assumed to observe the passive sign convention

Figure 1.23 shows the four quadrants of a generic *i-v* plot, where *i* and *v* are assumed to observe the passive sign convention. Power is positive in the first and third quadrants; negative in the second and fourth quadrants. The *i-v* plot of a typical incandescent lightbulb shown in Figure 1.17(b) reveals that its power is always positive. In other words, the lightbulb always dissipates energy.

Passive elements are defined as those that do not require an external source of energy to *enable* them. Common passive elements are resistors, capacitors, inductors, diodes, and electric motors. Passive elements can dissipate energy (e.g., resistors) and/or store and release energy (e.g., capacitors and inductors).

Active elements, on the other hand, are defined as those that do require an external source of energy to be enabled. Common active elements are transistors, amplifiers, and voltage and current sources. There are electronic devices that can operate either as passive or active elements. For example, a photodiode can act either as a light sensor (passive element) or as a solar cell (active element).

The electrical engineering community has uniformly adopted the passive sign convention. All the constitutive laws (e.g., Ohm's law) introduced in this book assume that convention. It is often necessary to assume directions for unknown currents and/or assume polarities for unknown voltages when solving circuit problems. It is important that these assumptions be made in accord with the passive sign convention. As long as the passive sign convention is observed it is not necessary to foresee actual current directions nor actual voltage polarities. Instead, when a current direction or voltage polarity is assumed incorrectly, the solution will yield a negative result, indicating that the assumed direction or polarity is opposite the actual.

LO

FOCUS ON PROBLEM SOLVING

THE PASSIVE SIGN CONVENTION

1. Assign a current through each passive element. The direction of each current can be assumed arbitrarily.
2. For each *passive* element, assign a voltage across the element such that the assigned current through the element is directed from high to low potential. Other valid descriptions are that current enters the + terminal or exits the − terminal of the element.
3. The power associated with each passive element is equal to *vi*. Positive power indicates that the element is either dissipating or storing energy.

EXAMPLE 1.4 Use of the Passive Sign Convention

Problem

Apply the passive sign convention to solve for the voltages and *mesh current* in the circuit of Figure 1.24.

Solution

Known Quantities: Voltage of the battery and the power dissipated by elements 1 and 2.

Find: Mesh current and the voltage across each load.

Schematics, Diagrams, Circuits, and Given Data: Figure 1.25(a) and (b). The voltage of the battery is $V_B = 12\,\text{V}$. The power dissipated by element 1 is $P_1 = 0.8$ W and by element 2 is $P_2 = 0.4$ W.

Assumptions: None.

Analysis: This problem can be solved using the passive sign convention in two different approaches. The first approach assumes a clockwise mesh current, while the second approach assumes a counterclockwise current. For either approach, the passive sign convention is used to label the change in voltage across each load. Figure 1.25(a) and (b) show the result of these two approaches. Notice that the change in voltage across each element was chosen so that the assumed current is directed from high to low potential.

The polarity of the battery is indicated by the alternating sequence of long and short bars. The positive and negative terminals of the battery are connected to a long and short bar, respectively.

A four-step solution using the first approach, as depicted in Figure 1.25(a), is given below.

1. Assume a clockwise direction for the current.
2. Label the change in voltage across each (passive) element so that the current is directed from high to low potential.
3. Express the power dissipated by each element using the relation $P = \upsilon i$, which is valid when the passive sign convention is observed.

$$P_1 = \upsilon_1 i = 0.8\,\text{W}$$
$$P_2 = \upsilon_2 i = 0.4\,\text{W}$$

   The power associated with the battery is expressed as $P_B = -V_B i$, which requires a negative sign $-\upsilon i$ because the current through the battery is directed from low to high potential, opposite of the passive sign convention.

4. Conservation of energy requires that the total power associated with the circuit be zero. Thus,

$$P_1 + P_2 + P_B = 0$$
$$P_B = -P_1 - P_2 = -0.8\,\text{W} - 0.4\,\text{W} = -1.2\,\text{W} = -V_B i$$

   It is now possible to use the three υi equations to solve for the three unknown variables i, υ_1, and υ_2. Since $V_B = 12\,\text{V}$, the current i is:

$$i = \frac{-1.2\,\text{W}}{-12\,\text{V}} = 0.1\,\text{A}$$

   As a result, the change in voltage across each element is:

$$\upsilon_1 = \frac{0.8\ \text{W}}{0.1\ \text{A}} = 8\,\text{V}$$
$$\upsilon_2 = \frac{0.4\ \text{W}}{0.1\ \text{A}} = 4\,\text{V}$$

A four-step solution using the second approach, as depicted in Figure 1.25(b), is given below.

1. Assume a counterclockwise direction for the current.
2. Label the change in voltage across each (passive) element so that the current is directed from high to low potential.

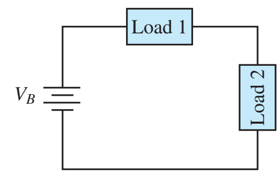


Figure 1.24

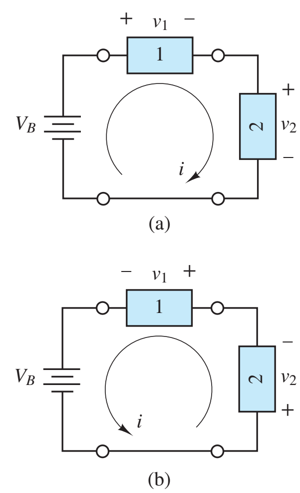


Figure 1.25

3. Express the power dissipated by each element using the relation $P = \upsilon i$, which is valid when the passive sign convention is observed.

$$P_1 = \upsilon_1 i = 0.8\text{ W}$$
$$P_2 = \upsilon_2 i = 0.4\text{ W}$$

The power associated with the battery is expressed here as $P_B = +V_B i$, which now requires a positive sign $+\upsilon i$ because the current through the battery is directed from high to low potential.

4. Conservation of energy requires that the total power associated with the circuit be zero. Thus,

$$P_1 + P_2 + P_B = 0$$
$$P_B = -P_1 - P_2 = -0.8\text{ W} - 0.4\text{ W} = -1.2\text{ W} = V_B i$$

It is now possible to use the three υi equations to solve for the three unknown variables i, υ_1, and υ_2. Since $V_B = 12$ V, the current i is:

$$i = \frac{-1.2\text{ W}}{12\text{ V}} = -0.1\text{ A}$$

As a result, the change in voltage across each element is:

$$\upsilon_1 = \frac{0.8\text{ W}}{-0.1\text{ A}} = -8\text{ V}$$
$$\upsilon_2 = \frac{0.4\text{ W}}{-0.1\text{ A}} = -4\text{ V}$$

Comments: Notice that the *actual* current present in the circuit and the *actual* change in voltage across each element is the same for each solution approach. For instance, using the first approach the current was found to be 0.1 A clockwise, while using the second approach the current was found to be −0.1 A counterclockwise. The negative sign found for the current in the second approach indicates that the actual current is directed clockwise, not counterclockwise. This example provides a good demonstration of the fact that it is not necessary to foresee the actual direction of unknown currents and voltages when solving a circuit problem. The important point is to observe the passive sign convention.

Also note that conservation of energy is required for electric circuits, just as it is for any other physical system. For electric circuits: *Power supplied always equals power consumed.*

EXAMPLE 1.5 Power Calculations

Problem

For the circuit shown in Figure 1.26, determine which components are consuming power and which are supplying power. Is conservation of power satisfied? Explain your answer.

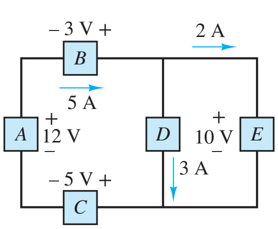


Figure 1.26

Solution

Known Quantities: All currents and voltages.

Find: Which components are consuming power, and which are supplying power? Verify conservation of power.

Analysis: The power associated with each element can be computed using $P = vi$ when the passive sign convention is observed or $P = -vi$ when it is not observed.

$$P_A = -(12\text{ V})(5\text{ A}) = -60\text{ W}$$
$$P_B = -(3\text{ V})(5\text{ A}) = -15\text{ W}$$
$$P_C = (5\text{ V})(5\text{ A}) = 25\text{ W}$$
$$P_D = (10\text{ V})(3\text{ A}) = 30\text{ W}$$
$$P_E = (10\text{ V})(2\text{ A}) = 20\text{ W}$$

Notice that the total power sums to zero. The same results can be expressed more literally as:

- *A* supplies 60 W
- *B* supplies 15 W
- *C* dissipates (consumes) 25 W
- *D* dissipates (consumes) 30 W
- *E* dissipates (consumes) 20 W
- Total power supplied equals 75 W
- Total power dissipated (consumed) equals 75 W
- Total power supplied = total power dissipated

Comments: Notice that whether power is calculated using $P = vi$ or $P = -vi$ depends entirely upon whether the passive sign convention is observed for any particular element.

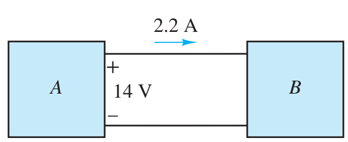
CHECK YOUR UNDERSTANDING

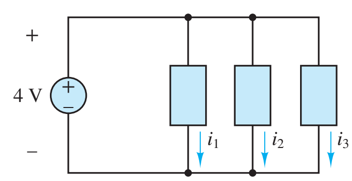
Compute the current through each of the headlamps shown in Figure 1.12 assuming each headlamp consumes 50 W. How much power is the battery providing?

Answers: $I_1 = I_2 = 4.17$ A; 100 W.

CHECK YOUR UNDERSTANDING

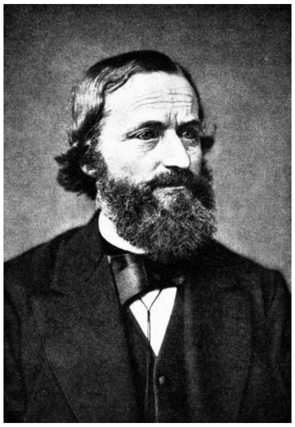
Determine which circuit element, *A* or *B*, in the figure on the left is supplying power and which is dissipating power. Also determine how much power is dissipated and supplied.





If the voltage source in the figure on the right supplies a total of 10 mW and $i_1 = 2$ mA and $i_2 = 1.5$ mA, what is the current i_3? If $i_1 = 1$ mA and $i_3 = 1.5$ mA, what is i_2?

Answers: *A* supplies 30.8 W; *B* dissipates 30.8 W. $i_3 = -1$ mA; $i_2 = 0$ mA.



Gustav Robert Kirchhoff (1824–1887) (*bilwissedition Ltd. & Co. KG/Alamy Stock Photo*)

1.5 KIRCHHOFF'S LAWS

Earlier in this chapter, a circuit was defined as an electric network within which at least one closed path exists and around which electric charge may flow. In fact, conservation of electric charge requires a closed path for any non-zero current.

> To have a non-zero current, there must be a closed electrical path (i.e., a circuit).

For example, Figure 1.27 depicts a simple circuit, composed of a battery (e.g., a 1.5-V lithium battery) and a lightbulb. Conservation of charge requires that the current i from the battery to the lightbulb is equal to the current from the lightbulb to the battery. No current (nor charge) is "lost" around the closed circuit. This principle was observed by the German scientist G. R. Kirchhoff[1] and is known as **Kirchhoff's current law (KCL)**. This law states that *the net sum of the currents crossing any closed boundary must equal zero*.

$$\sum_{n=1}^{N} i_n = 0 \qquad \text{Kirchhoff's current law (KCL)} \tag{1.6}$$

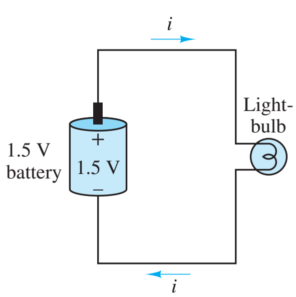


Figure 1.27 A simple electric circuit composed of a battery, a lightbulb, and two nodes

where the sign of currents entering the region surrounded by the closed boundary must be opposite to the sign of currents exiting the same region. In other words, the sum of currents "in" must equal the sum of currents "out."

$$\sum_{\text{in}} i = \sum_{\text{out}} i \qquad \text{Alternate KCL} \tag{1.7}$$

An application of Kirchhoff's current law is illustrated in Figure 1.28, where the simple circuit of Figure 1.27 has been augmented by the addition of two lightbulbs. The relationship between the currents is found by applying either version of KCL. To express the net sum of currents it is necessary to select a sign convention for currents entering and exiting a node. One possibility is to consider all currents entering a node as positive and all currents exiting a node as negative. (This particular sign convention is completely arbitrary.) The result of using this sign convention and applying the first version of KCL to node 1 is

$$i - i_1 - i_2 - i_3 = 0 \qquad \text{which is equivalent to} \qquad i = i_1 + i_2 + i_3$$

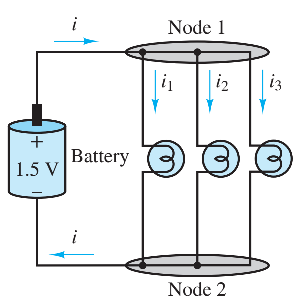


Figure 1.28 KCL applied at node 1 results in $i - i_1 - i_2 - i_3 = 0$, or equivalently $i = i_1 + i_2 + i_3$.

Note that the latter expression is exactly what would have been found if the alternate version of KCL had been applied. Also note that the result is the same if the opposite sign convention (i.e., currents entering and exiting the node are negative and positive, respectively) is used.

[1]Gustav Robert Kirchhoff (1824–1887), a German scientist, published the first systematic description of the laws of circuit analysis. His contribution—though not original in terms of its scientific content—forms the basis of all circuit analysis.

Consider again the simple circuit of a battery and a lightbulb shown in Figure 1.29. **Kirchhoff's voltage law** (**KVL**) states that *the net change in electric potential around a closed loop is zero.* In mathematical terms:

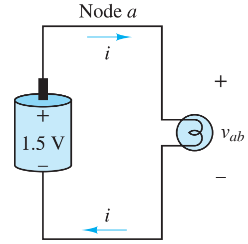


Figure 1.29 KVL applied clockwise from node b around the single loop circuit results in $1.5V - v_{ab} = 0$, or equivalently $v_{ab} = 1.5\ V$.

LO

$$\sum_{n=1}^{N} v_n = 0 \qquad \text{Kirchhoff's voltage law} \tag{1.8}$$

Here, v_n are the *changes* in voltage from *one node to another* around a closed loop.

When summing these changes in voltage, it is necessary to account for the polarity of the change. Changes in voltage from the minus sign − to the plus sign + are considered positive (i.e., a rise in voltage), while those from plus to minus are considered negative (i.e., a drop in voltage). These two symbols act together to indicate the *assumed* direction of the change in voltage from one node to another.

An alternate but equivalent expression for KVL is that the sum of all voltage rises around a loop must equal the sum of all voltage drops around the same loop.

LO

$$\sum_{\text{rises}} v = \sum_{\text{drops}} v \qquad \text{Alternate KVL} \tag{1.9}$$

In Figure 1.29, the *voltage across the lightbulb* is the change in electric potential from node a to node b. This change can also be expressed as the difference between two node voltages, v_a and v_b. The values of node voltages are relative to a reference node. Any single node may be chosen as the reference with its value set to zero, for simplicity. For the circuit in Figure 1.29 select node b as the reference and set its value as $v_b = 0$. Observe that the battery's positive terminal is 1.5 V *above the reference*, so that $v_a = 1.5$ V. In general, the battery guarantees that node a will always be 1.5 V above node b.

$$v_a = v_b + 1.5\ \text{V}$$
$$v_a = 1.5\ \text{V} \qquad \text{when} \qquad v_b = 0 \text{ acts as the reference.}$$

The notation used to express the *change* in voltage across the lightbulb, *from* node b *to* node a, is v_{ab}, where

$$v_{ab} \equiv v_a - v_b = 1.5\ \text{V}$$

EXAMPLE 1.6 Kirchhoff's Current Law Applied to an Automotive Electrical Harness

Problem

Figure 1.30 shows an automotive battery connected to a variety of elements in an automobile. The elements include headlights, taillights, starter motor, fan, power locks, and dashboard panel. The battery must supply enough current to satisfy each of the elements. Apply KCL to a model of the electrical system to find a relationship between the currents in the circuit.

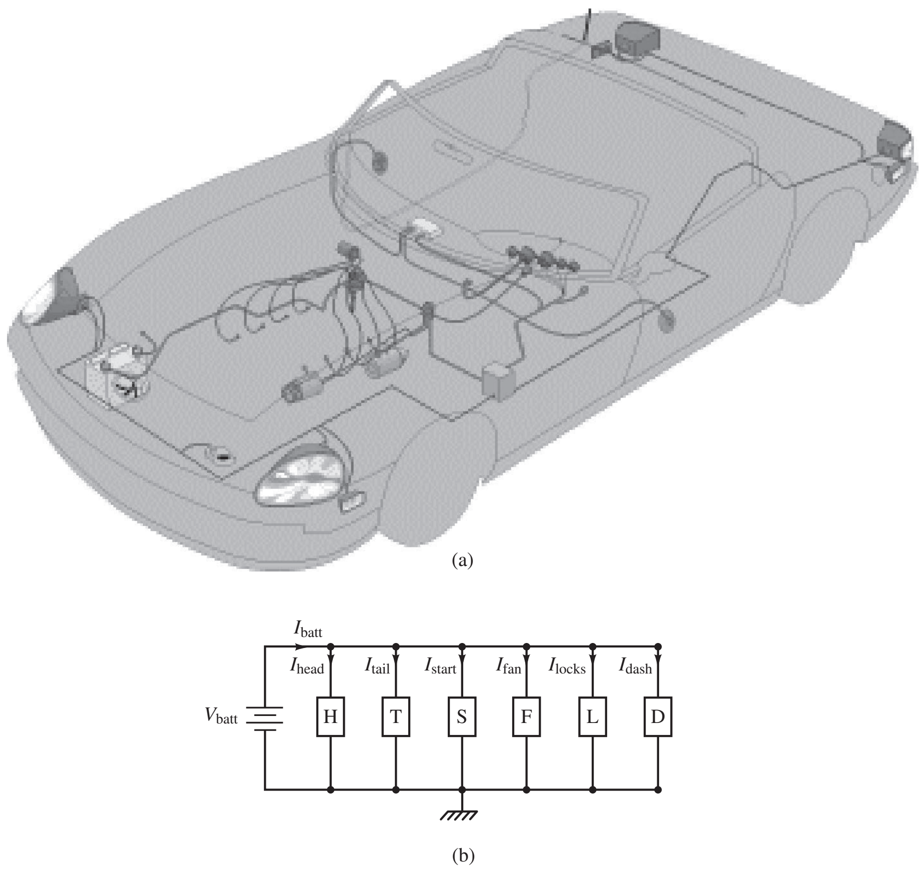


Figure 1.30 (a) Automotive electrical harness; (b) model electric circuit diagram

Solution

Known Quantities: Components of electrical harness: headlights, taillights, starter motor, fan, power locks, and dashboard panel.

Find: Expression relating battery current to harness currents.

Schematics, Diagrams, Circuits, and Given Data: Figure 1.30.

Assumptions: None.

Analysis: Figure 1.30(b) depicts the model electric circuit, illustrating that the current supplied by the battery is divided among the various elements. The application of KCL to the upper node yields

$$I_{batt} - I_{head} - I_{tail} - I_{start} - I_{fan} - I_{locks} - I_{dash} = 0$$

or

$$I_{batt} = I_{head} + I_{tail} + I_{start} + I_{fan} + I_{locks} + I_{dash}$$

EXAMPLE 1.7 Application of KCL

Problem

Determine the unknown currents in the circuit of Figure 1.31.

Solution

Known Quantities:

$$I_S = 5\text{ A} \qquad I_1 = 2\text{ A} \qquad I_2 = -3\text{ A} \qquad I_3 = 1.5\text{ A}$$

Find: I_0 and I_4.

Analysis: Two nodes are clearly shown in Figure 1.31 as node a and node b; the third node in the circuit is the reference node. Apply KCL at each of the three nodes.

At node a:

$$I_0 + I_1 + I_2 = 0 \qquad \text{from} \qquad \sum i_{\text{out}} = \sum i_{\text{in}}$$

$$I_0 + 2 - 3 = 0$$

$$\therefore \qquad I_0 = 1\text{ A}$$

Note that the assumed direction of all three currents is away from the node. However, I_2 has a negative value, which means that its actual direction is toward the node. The magnitude of I_2 is 3A.

At node b:

$$I_0 + I_1 + I_2 + I_S = I_3 + I_4 \qquad \text{from} \qquad \sum i_{\text{in}} = \sum i_{\text{out}}$$

$$1 + 2 - 3 + 5 = 1.5 + I_4$$

$$\therefore \qquad I_4 = 3.5\text{ A}$$

At the reference node: Assume that currents entering a node are positive and currents exiting a node are negative.

$$-I_S + I_3 + I_4 = 0$$

$$-5 + 1.5 + I_4 = 0$$

$$\therefore \qquad I_4 = 3.5\text{ A}$$

Comments: The result obtained at the reference node is exactly the same as that calculated at node b. Applying KCL to every node in a circuit will result in a redundant equation.

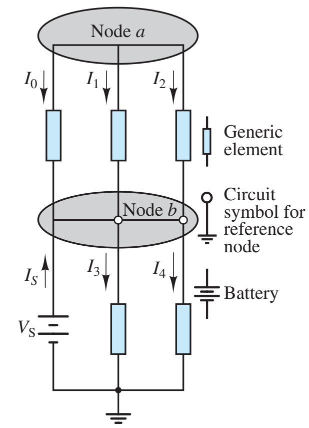


Figure 1.31 KCL yields $I_0 + I_1 + I_2 = 0$ at node a and $I_0 + I_1 + I_2 + I_S = I_3 + I_4$ at node b.

EXAMPLE 1.8 Application of KCL

Problem

Apply KCL to the circuit of Figure 1.32, using the concept of a supernode to determine the source current i_{s1}.

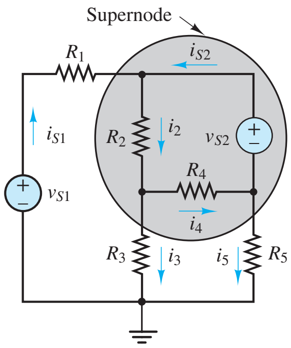


Figure 1.32 KCL applied at the boundary of the supernode yields $i_{S1} = i_3 + i_5$.

Solution

Known Quantities:

$$i_3 = 2\text{ A} \qquad i_5 = 0\text{ A}$$

Find: i_{S1}.

Analysis: Apply KCL at the closed boundary of the so-called supernode to obtain

$$i_{S1} = i_3 + i_5 \qquad \text{from} \qquad \sum i_{\text{in}} = \sum i_{\text{out}}$$

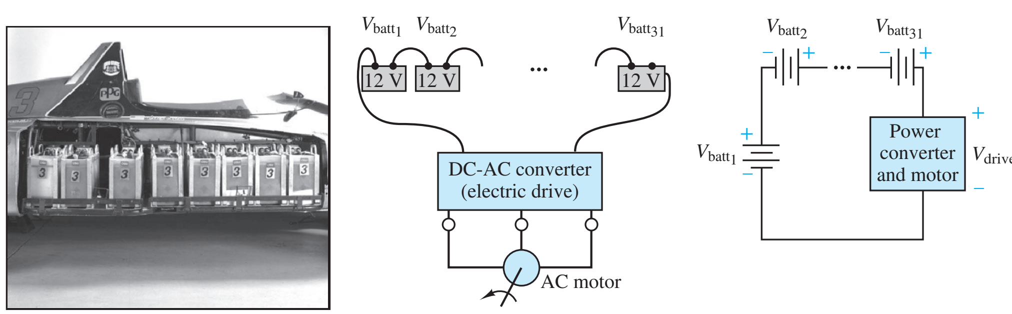
$$i_{S1} = 2 + 0 = 2\text{ A}$$

Comments: Notice that the same result for i_{S1} is obtained by applying KCL at the bottom node. This result is another example of a redundant equation that is sometimes obtained by applying KCL at two different closed boundaries or nodes.

EXAMPLE 1.9 Kirchhoff's Voltage Law—Electric Vehicle Battery Pack

Problem

Figure 1.33(a) depicts the battery pack in the Smokin' Buckeye electric race car, which consists of thirty-one 12-V batteries in series.



(a) (b) (c)

Figure 1.33 Electric vehicle battery pack illustrates KVL. (*Courtesy: David H. Koether Photography*)

Solution

Known Quantities: Nominal characteristics of **Optima™ lead-acid batteries**.

Find: Expression relating battery and electric motor drive voltages.

Schematics, Diagrams, Circuits, and Given Data: $V_{\text{batt}} = 12$ V; Figure 1.33(a), (b), and (c).

Assumptions: None.

Analysis: Figure 1.33(b) models the electric circuit, illustrating the batteries in series with the electric drive that powers the vehicle's 150-kW three-phase induction motor. Apply KVL around the circuit of Figure 1.33(c):

$$\sum_{n=1}^{31} V_{\text{batt}_n} - V_{\text{drive}} = 0$$

Thus, the electric drive is nominally supplied by a $31 \times 12 = 372$-V battery pack. In practice, the voltage across a lead-acid battery depends upon the state of charge of the battery. When fully charged, the battery pack of Figure 1.33(a) supplies closer to 400 V (i.e., roughly 13 V per battery).

EXAMPLE 1.10 Application of KVL

Problem

Determine the unknown voltage υ_2 by applying KVL to the circuit of Figure 1.34.

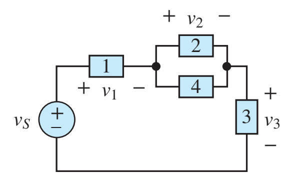


Figure 1.34 A circuit with four generic elements and one ideal voltage source

Solution

Known Quantities:

$$\upsilon_S = 12\text{ V} \qquad \upsilon_1 = 6\text{ V} \qquad \upsilon_3 = 1\text{ V}$$

Find: υ_2.

Analysis: Apply KVL starting at the reference node and proceeding clockwise around the large outer loop (the outer perimeter) of the circuit:

$$\begin{aligned} \upsilon_S - \upsilon_1 - \upsilon_2 - \upsilon_3 &= 0 \\ \upsilon_S - \upsilon_1 - \upsilon_3 &= \upsilon_2 \\ 12 - 6 - 1 = \upsilon_2 &= 5\text{ V} \end{aligned}$$

Comments: Note that υ_2 is the voltage across elements 2 and 4. These two elements are in *parallel* because they are located between the same two nodes. One can also say that the two branches that contain these elements are in parallel.

EXAMPLE 1.11 Application of KVL

Problem

Use KVL to determine the unknown voltages υ_1 and υ_4 in the circuit of Figure 1.35.

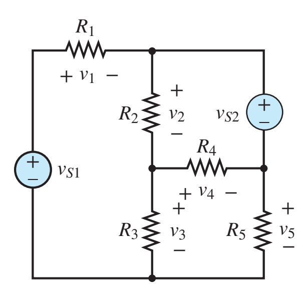


Figure 1.35 Circuit for Example 1.11

Solution

Known Quantities:

$$\upsilon_{S1} = 12\text{ V} \qquad \upsilon_{S2} = -4\text{ V} \qquad \upsilon_2 = 2\text{ V} \qquad \upsilon_3 = 6\text{ V} \qquad \upsilon_5 = 12\text{ V}$$

Find: υ_1, υ_4.

Analysis: To determine the unknown voltages, apply KVL clockwise around the left and upper-right meshes:

$$\begin{aligned} \upsilon_{S1} - \upsilon_1 - \upsilon_2 - \upsilon_3 &= 0 \\ \upsilon_2 - \upsilon_{S2} + \upsilon_4 &= 0 \end{aligned}$$

After substituting numerical values, the equations become:

$$12 - v_1 - 2 - 6 = 0$$
$$v_1 = 4 \text{ V}$$
$$2 - (-4) + v_4 = 0$$
$$v_4 = -6 \text{ V}$$

It is possible to solve for v_1 and v_4 using other loops in the circuit. For instance, apply KVL clockwise around the lower-right mesh to find v_4:

$$v_3 - v_4 - v_5 = 0$$
$$6 - v_4 - 12 = 0$$
$$v_4 = -6 \text{ V}$$

Or apply KVL clockwise around the outer most loop to find v_1:

$$v_{S1} - v_1 - v_{S2} - v_5 = 0$$
$$12 - v_1 - (-4) - 12 = 0$$
$$v_1 = 4 \text{ V}$$

Comments: Notice that there are seven closed wire loops in the circuit. KVL could be applied around any of these loops to find an equation. The key is to find two linearly independent equations that involve the two unknowns.

CHECK YOUR UNDERSTANDING

Apply KVL to each of the other three closed wire loops in Figure 1.35 that were not explored in Example 1.11. Compare the results to those found in the example. Are the results consistent?

CHECK YOUR UNDERSTANDING

Repeat the exercise of Example 1.7 when $I_0 = 0.5$ A, $I_2 = 2$ A, $I_3 = 7$ A, and $I_4 = -1$ A. Find I_1 and I_S.

Answers: $I_1 = -2.5$ A and $I_S = 6$ A

CHECK YOUR UNDERSTANDING

Use the result of Example 1.8 and the following data to compute the current i_{S2} in the circuit of Figure 1.32.

$$i_2 = 3 \text{ A} \qquad i_4 = 1 \text{ A}$$

Answer: $i_{S2} = 1$ A

1.6 RESISTANCE AND OHM'S LAW

When charge flows through a wire or circuit element, it encounters **resistance**, the magnitude of which depends on the *resistivity* of the material and the geometry of the wire or element. In practice, all circuit elements exhibit some resistance, which leads to energy dissipation in the form of heat. Whether this loss of electrical energy as heat is detrimental depends upon the purpose of the circuit element. For example, a typical electric toaster relies on the conversion of electrical energy to heat within its resistive coils to accomplish its purpose, the making of toast. All electric heaters rely upon this process, in one form or another. On the other hand, heat loss due to resistance in residential wiring is costly, and potentially dangerous. Resistance in microcircuitry generates heat that effectively limits the speed of microprocessors and the number and scale of transistors that can be packed into a given volume.

The resistance of a cylindrical wire segment, as shown in Figure 1.36(a), is given by

$$R = \rho \frac{l}{A} = \frac{l}{\sigma A} \tag{1.10}$$

where ρ and σ are the material properties *resistivity* and *conductivity*, respectively, and l and A are the segment length and cross-sectional area, respectively. As evident in the above equation, conductivity is simply the inverse of resistivity. The unit of resistance R is **ohms (Ω)**, where

$$1\ \Omega = 1\ \text{V/A} \tag{1.11}$$

The resistance of an actual wire or circuit element is usually accounted for in a circuit diagram by an **ideal resistor**, which lumps the entire distributed resistance R of the wire or element into one single element. Ideal resistors exhibit a linear i-v relationship known as **Ohm's law**, which is

$$\boxed{v = iR \qquad \text{Ohm's law}} \tag{1.12}$$

LO

In other words, the voltage across an ideal resistor is directly proportional to the current through it. The constant of proportionality is the resistance R. The circuit symbol and i-v characteristic for an ideal resistor are shown in Figure 1.36(b) and (c), respectively. Notice the passive sign convention used in the circuit symbol diagram, as appropriate, since a resistor is a passive element.

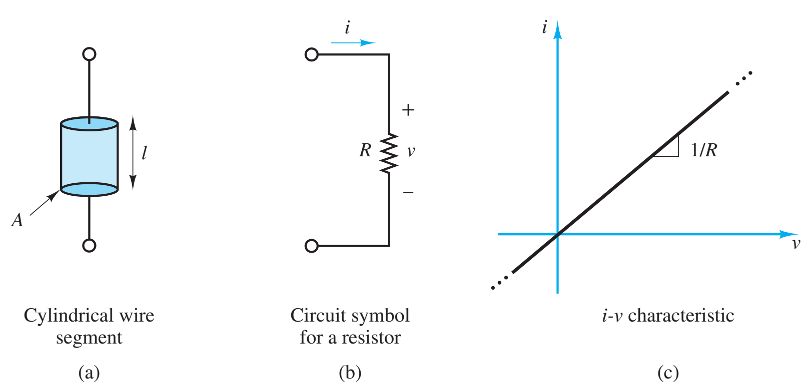


Figure 1.36 (a) Resistive wire segment; (b) ideal resistor circuit symbol; (c) the i-v relationship (Ohm's law) for an ideal resistor

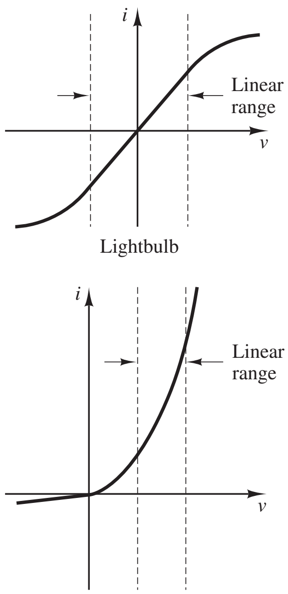


Figure 1.37 Piecewise linear segments within non-linear i-v characteristics

It is often convenient to define the *conductance*, G (unit is siemens, S), of a circuit element as the inverse of its resistance.

$$G = \frac{1}{R} \qquad \text{siemens (S)} \qquad \text{where} \qquad 1\ \text{S} = \frac{1A}{V} \tag{1.13}$$

In terms of conductance, Ohm's law is

$$i = Gv \tag{1.14}$$

Ohm's law is an *empirical* relationship that finds widespread application in electrical engineering. It is a simple yet powerful approximation of the physics of electrical conductors. However, the linear i-v relationship usually does not apply over very large ranges of voltage or current. For some conductors, Ohm's law does not approximate the i-v relationship even over modest ranges of voltage or current. Nonetheless, most conductors exhibit piecewise linear i-v characteristics for one or more ranges of voltage and current, as shown in Figure 1.37 for an incandescent lightbulb and a semiconductor diode.

Short- and Open-Circuits

Two convenient idealizations, the **short-circuit** and the **open-circuit**, are limiting cases of Ohm's law as the resistance approaches zero or infinity, respectively. Formally, a short-circuit is an element *across* which the voltage is zero, regardless of the current *through* it. Figure 1.38 depicts the circuit symbol for an ideal short-circuit.

In practice, any conductor will exhibit some resistance. For practical purposes, however, many elements approximate a short-circuit under certain conditions. For example, a large-diameter copper pipe is effectively a short-circuit in the context of a residential electric power supply, while in a low-power microelectronic circuit (e.g., an iPhone®) a typical ground plane is 35×10^{-6} m thick, which is adequate for a short-circuit in that context. A typical solderless breadboard is designed to accept 22-gauge solid jumper wires, which act effectively as short-circuits between elements on the breadboard. Table 1.1 lists the resistance per 1,000 ft of some commonly used wire, as specified by the *American Wire Gauge Standards.*

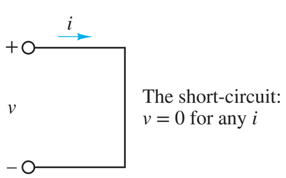


Figure 1.38 The short-circuit

Table 1.1 Resistance of copper wire

AWG size	Number of strands	Diameter per strand (in)	Resistance per 1,000 ft (Ω)
24	Solid	0.0201	28.4
24	7	0.0080	28.4
22	Solid	0.0254	18.0
22	7	0.0100	19.0
20	Solid	0.0320	11.3
20	7	0.0126	11.9
18	Solid	0.0403	7.2
18	7	0.0159	7.5
16	Solid	0.0508	4.5
16	19	0.0113	4.7
14	Solid	0.0641	2.52
12	Solid	0.0808	1.62
10	Solid	0.1019	1.02
8	Solid	0.1285	0.64
6	Solid	0.1620	0.4
4	Solid	0.2043	0.25
2	Solid	0.2576	0.16

The limiting case for Ohm's law when $R \to \infty$ is called an **open-circuit**. Formally, an open-circuit is an element *through* which the current is zero, regardless of the voltage *across* it. Figure 1.39 depicts the circuit symbol for an ideal open-circuit.

In practice, it is easy to approximate an open-circuit. For moderate voltage levels, any gap or break in a conducting path amounts to an open-circuit. However, at sufficiently high voltages such a gap will become ionized and its resistance will decrease suddenly and dramatically, effectively producing a short-circuit across the gap. If sufficient charge is available, the result will be arcing in which a pulse of charge jumps the gap. Subsequently, the pulse discharge results in a decrease in the voltage *across* the gap and the ionized path collapses. The result is that the gap has returned to its open-circuit approximation. This phenomenon is employed in spark plugs to ignite the air-fuel mixture in a spark-ignition internal combustion engine. Any insulating material will break down when a sufficiently high voltage is applied across it.

The *dielectric strength* is a measure of the maximum electric field (voltage per unit distance) that an insulating material can sustain without breaking down and allowing charge to flow. This measure is somewhat dependent upon temperature, pressure, and the material thickness; however, typical values are 3 kV/mm for air at sea level and room temperature, 10 kV/mm for window glass, 20 kV/mm for neoprene rubber, 30 kV/mm for pure water, and 60 kV/mm for PTFE, commonly known as Teflon.

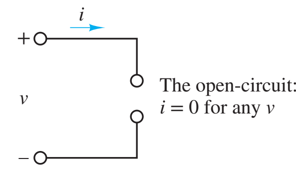
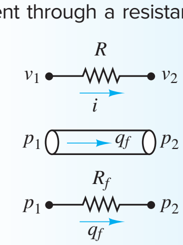


Figure 1.39 The open-circuit

Hydraulic Analog of Electrical Resistance

A useful analogy can be made between the electric current through electric components and the flow of incompressible fluids (e.g., water, oil) through hydraulic components. The fluid flow rate *through* a pipe is analogous to current *through* a conductor. Similarly, pressure drop *across* a pipe is analogous to voltage *across* a resistor. The resistance of the pipe to fluid flow is analogous to electrical resistance: The pressure difference across the pipe causes fluid flow, much as a potential difference across a resistor causes charge to flow. The figure below depicts how pipe flow is often modeled as current through a resistance.



Analogy between electrical and fluid resistance

Discrete Resistors

Various types of *discrete resistors* are used in laboratory experiments, tinkering projects, and commercial hardware, and are available in a wide range of nominal values, tolerances, and power ratings. Each type has a particular temperature range within which it is designed to operate. In fact, some discrete resistors (known as thermistors) are designed to be highly sensitive to temperature and to be used as temperature transducers.

The majority of discrete resistors have a cylindrical shape and are color coded for their nominal value and tolerance. Several common types of resistors are: *carbon composites*, in which the resistance is set by a mixture of carbon and ceramic powder (Figure 1.40); *carbon film*, in which the resistance is set by the length and width of a thin strip of carbon wrapped around an insulating core; and thin metal film, in which the resistance is set by the characteristics of a thin metal film also wrapped around an insulating core (Figure 1.41).

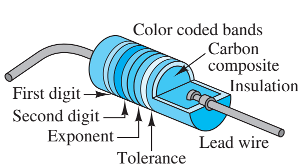


Figure 1.40 Carbon composite resistor

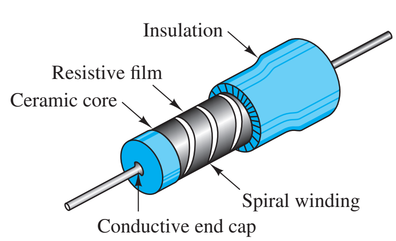


Figure 1.41 Thin-film resistor

Figure 1.42 Typical $\frac{1}{4}$-W resistors (*Jim Kearns*)

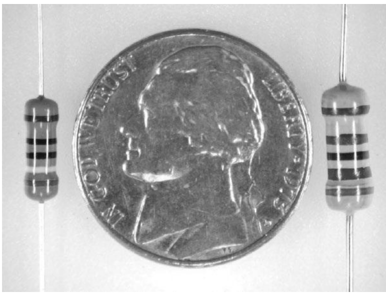
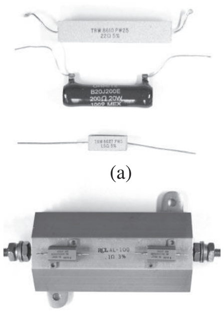


Figure 1.43 Typical $\frac{1}{2}$-W resistors (*Jim Kearns*)



(a)

(b)

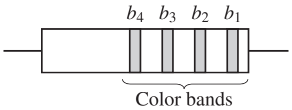
Figure 1.44 (a) 25-W, 20-W, and 5-W, and (b) two 5-W resistors sitting atop one 100-W resistor (*Jim Kearns*)

Discrete resistors are available with various power ratings, where the power rating scales with the size of the resistor itself. Figures 1.42 and 1.43 show (to scale) typical $\frac{1}{4}$-W and $\frac{1}{2}$-W resistors, respectively. Notice the bands along the length of each resistor. Discrete resistors are also available with typical power ratings of 1, 2, 5, 10 W, and larger. Many industrial power resistors are manufactured by winding wire, such as Nichrome, around a non-conducting core, such as ceramic, plastic, or fiberglass. Others are made of cylindrical sections of carbon. Power resistors are available in a variety of packages, such as cement or molded plastic, aluminum encasements with fins for wicking away heat, and enamel coatings. Typical power resistors are shown in Figure 1.44.

The value of a discrete resistor is determined by the resistivity, shape, and size of the conducting element. Table 1.2 lists the resistivity of many common materials.

Table 1.2 Resistivity of common materials at room temperature

Material	Resistivity (Ω-m)
Aluminum	2.733×10^{-8}
Copper	1.725×10^{-8}
Gold	2.271×10^{-8}
Iron	9.98×10^{-8}
Nickel	7.20×10^{-8}
Platinum	10.8×10^{-8}
Silver	1.629×10^{-8}
Carbon	3.5×10^{-5}



black	0	blue	6
brown	1	violet	7
red	2	gray	8
orange	3	white	9
yellow	4	silver	10%
green	5	gold	5%

Resistor value $= (b_1\, b_2) \times 10^{b_3}$; $b_4 =$ % tolerance in actual value

Figure 1.45 Resistor color code

The nominal value and tolerance are often color-coded on a discrete resistor. Typically, discrete resistors have four color bands, where the first two designate a two-digit integer, the third designates a multiplier of 10, and the fourth designates the tolerance. Occasionally, discrete resistors have five bands, where the first three designate a three-digit integer, and the remaining two designate the multiplier and the tolerance. The value of each color band is decoded using the system displayed in Figure 1.45 and Table 1.3.

$$(\text{Two- or three-digit integer}) \times 10^{\text{multiplier}}, \quad \text{in ohms } (\Omega)$$

Table 1.3 b_1b_2 **indicates the two-digit significand;** b_3 **indicates the multiplier**

b_1b_2	Code	b_3	Code	Ω	b_3	Code	kΩ	b_3	Code	kΩ	b_3	Code	kΩ
10	Brn-blk	1	Brown	100	2	Red	1.0	3	Orange	10	4	Yellow	100
12	Brn-red	1	Brown	120	2	Red	1.2	3	Orange	12	4	Yellow	120
15	Brn-grn	1	Brown	150	2	Red	1.5	3	Orange	15	4	Yellow	150
18	Brn-gry	1	Brown	180	2	Red	1.8	3	Orange	18	4	Yellow	180
22	Red-red	1	Brown	220	2	Red	2.2	3	Orange	22	4	Yellow	220
27	Red-vlt	1	Brown	270	2	Red	2.7	3	Orange	27	4	Yellow	270
33	Org-org	1	Brown	330	2	Red	3.3	3	Orange	33	4	Yellow	330
39	Org-wht	1	Brown	390	2	Red	3.9	3	Orange	39	4	Yellow	390
47	Ylw-vlt	1	Brown	470	2	Red	4.7	3	Orange	47	4	Yellow	470
56	Grn-blu	1	Brown	560	2	Red	5.6	3	Orange	56	4	Yellow	560
68	Blu-gry	1	Brown	680	2	Red	6.8	3	Orange	68	4	Yellow	680
82	Gry-red	1	Brown	820	2	Red	8.2	3	Orange	82	4	Yellow	820

For example, a resistor with four bands (yellow, violet, red, gold) has a nominal value of:

$$(\text{yellow})(\text{violet}) \times 10^{\text{red}} = 47 \times 10^2 = 4700\ \Omega = 4.7\ \text{k}\Omega$$

and a "gold" tolerance of ±5 percent. 4.7 kΩ is often shortened in practice to 4K7, where the letter K indicates the placement of the decimal point as well as the unit of kΩ. Likewise, 3.3 MΩ is often shortened to 3M3. Table 1.3 lists the standard nominal values established by the Electronic Industries Association (EIA) for a tolerance of 10 percent, commonly referred to as the E12 series. The number 12 indicates the number of logarithmic steps per decade of resistor values. Notice that the values in adjacent decades (columns) are different by a factor of 10.

Due to imperfect manufacturing the actual value of a discrete resistor is only approximately equal to its nominal value. The tolerance is a measure of the likely variation between the actual value and the nominal value. Other EIA series are E6, E24, E48, E96, and E192 for tolerances of 20%, 5%, 2%, 1%, and even finer tolerances, respectively.

Variable Resistors

The resistance of a variable resistor is not fixed but can vary with some other quantity. Examples of variable resistors are a photoresistor and a thermistor, in which the resistance varies with light intensity and temperature, respectively. Many useful sensors are based upon variable resistors.

Figure 1.46 shows a simple loop with a voltage source, a variable resistor R, and a fixed resistor R_0. Apply KVL around the loop:

$$\begin{aligned} v_S &= iR + iR_0 = i(R + R_0) \\ &= iR + v_0 \end{aligned}$$

Solve for i and substitute for it in the above equation:

$$i = \frac{v_S}{R + R_0} \qquad \text{and} \qquad v_0 = iR_0 = v_S\frac{R_0}{R + R_0}$$

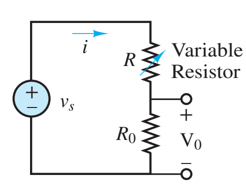


Figure 1.46 A variable resistor R in a series loop

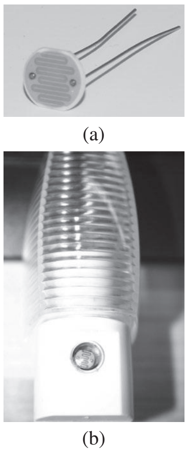


Figure 1.47 (a) A typical cadmium sulfide (CdS) cell. (b) A nightlight relies on a CdS cell to detect dark conditions. (*Jim Kearns*)

Figure 1.48 A typical negative temperature coefficient (NTC) thermistor (*Jim Kearns*)

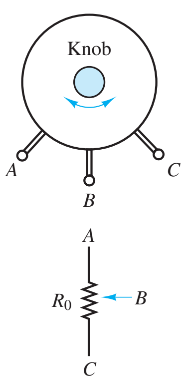


Figure 1.49 A potentiometer is a three-terminal resistive device with a fixed resistance R_0 between terminals A and C. The resistances between terminal B (the "wiper") and the other two terminals is set by the knob.

Now assume that the variable resistor has a range from $0\,\Omega$ to some value R_{max} that is much larger than R_0. When $R = 0$:

$$v_0 = v_S \frac{R_0}{R + R_0} = v_S \frac{R_0}{R_0} = v_S \qquad (R = 0)$$

When $R = R_{max}$:

$$v_0 = v_S \frac{R_0}{R + R_0} = v_S \frac{R_0}{R_{max} + R_0} \approx v_S \frac{R_0}{R_{max}} \approx 0 \qquad (R = R_{max})$$

Thus, as R varies from 0 to R_{max}, v_0 varies from v_S to 0. The changes in R can be observed as changes in v_0. Imagine that the variable resistor in Figure 1.46 is a photoresistor, such as a cadmium sulfide (CdS) cell shown in Figure 1.47(a), that has a very small resistance when the incident light intensity is bright and has a very large resistance when the incident light intensity is dim or dark. The result is that under bright conditions, $v_0 \approx v_S$, while under dark conditions, $v_0 \approx 0$. A nightlight, such as that shown in Figure 1.47(b), is a device that turns on when $v_0 \ll v_{ref}$ and turns off when $v_0 \gg v_{ref}$, where v_{ref} is some appropriate reference voltage, such as $v_S/2$.

Figure 1.48 shows a typical thermistor, which can be used in exactly the same manner as a CdS cell but which responds to changes in temperature.

Potentiometers

A potentiometer is a three-terminal device. Figure 1.49 depicts a potentiometer and its circuit symbol. A potentiometer has a fixed resistance R_0, typically formed by a tightly wound coil of wire, between terminals A and C. Terminal B is connected to a *wiper* that slides along the coil as the knob is turned. The arrow in the circuit symbol represents the position of the slider along the length of the coil R_0. The resistance from terminal B to the other two terminals is determined by the wiper position. As R_{BA} increases, R_{BC} decreases, and vice versa, such that the sum $R_{BA} + R_{BC}$ always equals R_0.

Figure 1.50(a) illustrates the use of a potentiometer symbol in a simple circuit. Figure 1.50(b) is an equivalent representation of the circuit, where the resistance between terminals A and B and that between terminals B and C are depicted as discrete resistors.

The ideal voltmeter reading v_{bc} can be calculated in a manner similar to that used in the preceding section on variable resistors. Apply KVL around the loop

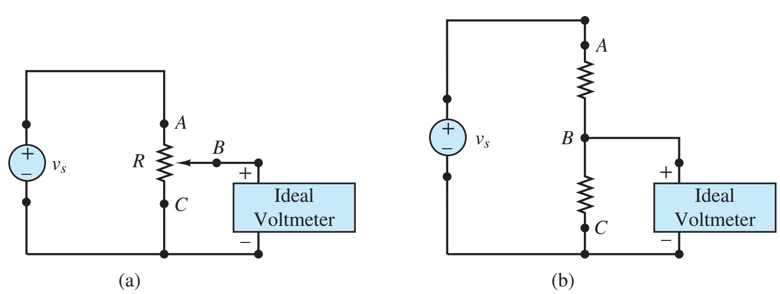


Figure 1.50 (a) A potentiometer in a simple circuit; (b) an equivalent circuit of (a), where $R = R_{AB} + R_{BC} = R_{AC}$

containing the voltage source and the two discrete resistors, using Ohm's law to express the change in voltage across each resistor. The result is

$$v_{BC} = v_S \frac{R_{BC}}{R_{BC} + R_{AB}}$$

This important result for two resistors in series is an example of *voltage division.* When the wiper is turned all the way to terminal C, $R_{BC} = 0$ and so $v_{BC} = 0$. When the wiper is turned all the way to terminal A, $R_{AB} = 0$ and so $v_{BC} = v_S$. In general, as the wiper is turned from terminal A to terminal C, the voltage across terminals B and C falls continuously from v_S to 0.

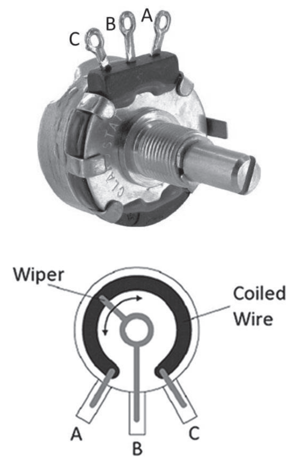


Figure 1.51 A typical $\frac{1}{2}$-W potentiometer and its internal construction (*Jim Kearns*)

Power Dissipation in Resistors

All discrete resistors have a power rating, which is not designated by a color band, but which tends to scale with the size of the resistor itself. Larger resistors typically have a larger power rating. The power consumed or dissipated by a resistor R is

$$\begin{aligned} P &= vi = (iR)i = i^2R > 0 \\ &= v\left(\frac{v}{R}\right) = \frac{v^2}{R} > 0 \end{aligned} \tag{1.15}$$

Remember that the voltage v and the current i are defined and linked by the *passive sign convention* and that power consumed by an element is positive. In the case of resistors, power is always positive and energy is dissipated as heat. The implication is that if the current through (or the voltage across) a resistor is too large, the power will exceed the resistor's rating and result in a smoking and/or burning resistor! The smell of an overheating resistor is well known to technicians and hobbyists alike.

> Positive power is power dissipated (i.e., consumed) by an element.

EXAMPLE 1.12 Using Resistor Power Ratings

Problem

For a given voltage across a resistor, determine the minimum allowed resistance for a $\frac{1}{4}$-W power rating.

Solution

Known Quantities: Resistor power rating 0.25 W. Voltages due to a battery across the resistor: 1.5 V and 3 V.

Find: The minimum allowed resistance for a $\frac{1}{4}$-W resistor.

Schematics, Diagrams, Circuits, and Given Data: Figures 1.52 and 1.53.

Analysis: The power dissipated by a resistor is

$$P_R = vi = v \cdot \frac{v}{R} = \frac{v^2}{R}$$

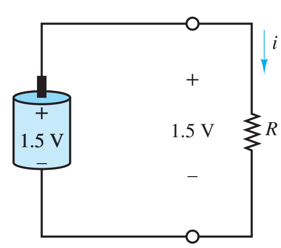


Figure 1.52