FLUID MECHANICS FOR CHEMICAL ENGINEERS

FOURTH EDITION

Noel de Nevers

Department of Chemical Engineering
University of Utah

Geoffrey D. Silcox

Department of Chemical Engineering
University of Utah

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ABOUT THE AUTHORS

Noel de Nevers received a B.S. from Stanford in 1954 and M.S. and Ph.D. degrees from the University of Michigan in 1956 and 1959, all in chemical engineering.

He worked for the research arms of the Chevron Oil Company from 1958 to 1963 in the areas of chemical process development, chemical and refinery process design, and secondary recovery of petroleum. He was on the faculty of the University of Utah from 1963 until his untimely death in 2019, in the Department of Chemical Engineering, becoming emeritus in 2002.

He worked for the National Reactor Testing Site, Idaho Falls, Idaho, on nuclear problems, for the U.S. Army Harry Diamond Laboratory, Washington, DC, on weapons, and for the Office of Air Programs of the U.S. EPA in Durham, NC, on air pollution.

He was a Fulbright student of Chemical Engineering at the Technical University of Karlsruhe, Germany, in 1954–1955, a Fulbright lecturer on Air Pollution at the Universidad del Valle, in Cali, Colombia, in the summer of 1974, and at the Universidad de la República, Montevideo Uruguay and the Universidad Naciónal Mar del Plata, Argentina in the autumn of 1996.

His areas of research and publication are in fluid mechanics, thermodynamics, air pollution, technology and society, energy and energy policy, and explosions and fires. He regularly consulted on air pollution problems, explosions, fires and toxic exposures.

In 1993 he received the Corcoran Award from the Chemical Engineering Division of the American Society for Engineering Education for the best paper ("'Product in the Way' Processes") that year in *Chemical Engineering Education*.

In 2017 his textbook, *Air Pollution Control Engineering,* Third Edition, was issued by Waveland Press.

In 2012 his textbook, *Physical and Chemical Equilibrium for Chemical Engineers,* Second Edition, was issued by John Wiley.

In addition to his serious work he has three "de Nevers's Laws" in the latest "Murphy's Laws" compilation, and won the title "Poet Laureate of Jell-O Salad" at the Last Annual Jell-O Salad Festival in Salt Lake City in 1983. He was the official discoverer of Private Arch in Arches National Park.

Geoffrey D. Silcox received a B.S. degree in chemistry in 1976 and a Ph.D. degree in chemical engineering in 1985, both from the University of Utah. He is currently Professor (Lecturer) in the Department of Chemical Engineering at the University of Utah. He has served as Associate Chair since 2000, Chair of the Undergraduate Committee since 2003, and Chair of the Chemical Engineering ABET Committee since 2006. Before joining the university in 1987, he worked as a research engineer and group leader for Energy and Environmental Research Corporation, Irvine, California. His recent areas of research and publication include air pollution and the chemistry of mercury in combustion flue gases.

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Appendixes

NOTATION

a	acceleration	ft / s^2	m / s^2
a	some arbitrary direction or length (Sec. 2.1)	ft	m
a	resistance of filter medium (Sec. 11.4)	1 / ft	1 / m
a_x, a_y, a_z	x, y, and z components of acceleration	ft / s^2	m / s^2
a_c	centrifugal acceleration	ft / s^2	m / s^2
a, b, c, d	exponents in algebraic procedure (Sec. 9.3)	—	—
A	area or cross-sectional area perpendicular to flow	ft^2	m^2
A	independent variable (Sec. 9.3)	various	various
A, B, C, D	arbitrary constants	various	various
b	background concentration (Sec. 3.6)	lbm / ft^3	kg / m^3
B	dependent variable (Sec. 9.3)	various	various
c	volume fraction in hindered settling	—	—
c	speed of light (Chap. 4 only)	ft / s	m / s
c	speed of sound	ft / s	m / s
c	concentration	lbm / ft^3	kg / m^3
C	heat capacity	Btu / lbm · °F or Btu / lbmol · °F	J / kg · K or J / mol · K
C_d	drag coefficient (Sec. 6.13)	—	—
C_i	constants of integration	various	various
C_l	lift coefficient (Sec. 6.13)	—	—
C_f	integrated drag coefficient (Sec. 17.2)	—	—
C'_f	local drag coefficient (Sec. 17.2)	—	—
C_P	heat capacity at constant pressure	Btu / lbm · °F or Btu / lbmol · °F	J / kg · K or J / mol · K
C_v	orifice or venturi coefficient (Sec. 5.8)	—	—
C_V	heat capacity at constant volume	Btu / lbm · °F or Btu / lbmol · °F	J / kg · K or J / mol · K
CC	capital cost factor (Sec. 6.12)	1 / yr	1 / yr
D	diameter	ft	m
D_p	particle diameter	ft	m
D / Dt	Stokes or substantive or convective derivative	1 / s	1 / s
$\mathscr{D}$	diffusivity, molecular or turbulent	ft^2 / s	m^2 / s
erf	gauss error function (see Fig. 19.5)	—	—
E	energy	Btu or equivalent	J
E	voltage	volt	volt
E_s	surface energy	ft · lbf / ft^2	J / m^2
f	Fanning friction factor (Sec. 6.4)	—	—
$f_{\text{P.M.}}$	friction factor for porous medium (Sec. 11.1)	—	—

$f(n)$	spectrum function (Sec. 18.4)	1 / hertz	1 / hertz
°F	temperature or temperature interval, degrees Fahrenheit	°F	
F	force	lbf	N
$\mathscr{F}$	friction heating per lbm	ft · lbf / lbm or equivalent	J / kg
F_x, F_y, F_z	x, y, and z components of force	lbf	N
F_θ	tangential component of force	lbf	N
$F_I, F_V, F_G, F_S, F_E, F_P$	inertia, viscous, gravity, surface, elastic, and pressure forces (Sec. 9.3)	lbf	N
$F(n)$	spectrum function (Sec. 18.4)	—	—
$\mathscr{F}r$	Froude number	—	—
g	acceleration of gravity	$\mathrm{ft / s^2}$	$\mathrm{m / s^2}$
g_c	conversion factor $= 1 = 32.2\ \mathrm{lbm \cdot ft / lbf \cdot s^2}$	—	—
h	height or depth	ft	m
h	enthalpy per unit mass or mole $(u + Pv)$	Btu / lbm or Btu / lbmol	J / kg or J / mol
h_c	centroid depth measured from free surface (Prob. 2.26)	ft	m
H	enthalpy $(U + PV)$	Btu	J
H	height	ft	m
H	effective stack height (Chap. 19)	ft	m
H	mixing height (Chap. 3)	ft	m
hp	horsepower	ft · lbf / s	
$\mathscr{H}e$	Hedstrom number (Chap. 13)	—	—
HR	hydraulic radius	ft	m
i, j, k	unit vectors in the x, y, and z directions	—	—
I	electric current (dQ / dt)	amp	amp
I	angular moment of inertia (Chap. 7)	$\mathrm{lbm \cdot ft^2}$	$\mathrm{kg \cdot m^2}$
I_{sp}	specific impulse	lbf · s / lbm	N · s / kg
J_x, J_y, J_z	x, y, and z components of the electric current density (Sec. 16.3)	$\mathrm{amp / m^2}$	$\mathrm{amp / m^2}$
k	number of independent dimensions (Sec. 9.3)	—	—
k	ratio of specific heats, C_P / C_V (Sec. 8.1)	—	—
k	thermal conductivity (Sec. 16.3)	Btu / hr · °F · ft	W / m · K
k	permeability (Sec. 16.3 and Chap. 11)	$\mathrm{ft^2}$	$\mathrm{m^2}$
k	ratio of radii in a Couette viscometer	—	—
k	turbulent ke per unit mass	Btu / lbm	J / kg
ke	kinetic energy per unit mass	Btu / lbm	J / kg
K	arbitrary constant in "power law" (Chap. 13)	$\mathrm{lbf \cdot s^n / ft^2}$	$\mathrm{N \cdot s^n / m^2}$
K	bulk modulus (Sec. 8.1)	$\mathrm{lbf / in^2}$	Pa
K	resistance coefficient (Sec. 6.9)	—	—
K	arbitrary constant in jet equation (Chap. 19)	—	—
KE	kinetic energy	Btu	J
l	length	ft	m
L	length or lever arm	ft	m
L	angular momentum (Chap. 7)	$\mathrm{lbm \cdot ft^2 / s}$	$\mathrm{kg \cdot m^2 / s}$
m	mass	lbm	kg
$\dot{m}$	mass flow rate	lbm / s	kg / s
M	molecular weight	lbm / lbmol	g / mol
$\mathscr{M}$	Mach number	—	—
n	number of independent variables	—	—
n	number of moles	lbmol	mol
n	arbitrary power in "power law" (Chap. 13)	—	—
n	frequency	cyc / s	hertz

n	constant in Chézy Eq. (Chap. 6)	—	—
N	$4f\,\Delta x\,/\,D$ (Sec. 8.4)	—	—
N	rotation rate (rpm or rps)	1 / min; 1 / s	1 / min; 1 / s
pe	potential energy per unit mass	Btu / lbm	J / kg
P	pressure	lbf / in^2	Pa
Po	power	ft · lbf / s	W
PE	potential energy	Btu	J
PC	pumping cost (Sec. 6.12)	\$ / yr · hp	
PP	purchased price factor for a pipe (Sec. 6.12)	\$ / in · ft	
q	emission rate per unit area (Sec. 3.6)	lbm / s · ft^2	kg / s · m^2
q_x, q_y, q_z	*x, y,* and *z* components of heat flux (Sec. 16.3)	Btu / h · ft^2	W / m^2
Q	volumetric flow rate	ft^3 / s	m^3 / s
Q	heat	Btu	J
Q	charge	coul	coul
r	radius	ft	m
R	universal gas constant	See inside back cover	See inside back cover
R	correlation coefficient (Sec. 18.5)	—	—
R	radius of curvature (Chap. 14)	ft	m
$\mathscr{R}$	Reynolds number	—	—
$\mathscr{R}_p$	particle Reynolds number	—	—
$\mathscr{R}_{\text{P.M.}}$	Reynolds number for porous media	—	—
$\mathscr{R}_x$	Reynolds number based on distance from leading edge	—	—
$\mathscr{R}_{\text{power law}}$	Reynolds number for power law fluids	—	—
$\mathscr{R}_{\text{Bingham}}$	Reynolds number for Bingham plastics	—	—
$\mathscr{R}_{\text{impeller}}$	Reynolds number for a mixer impeller	—	—
s	entropy per unit mass or per mole	Btu / lbm · °R or Btu / lbmol · °R	J / kg · K or J / mol · K
s	cake compressibility coefficient (Sec. 11.4)	—	—
SG	specific gravity	—	—
t	time	s	s
t	wall thickness (Sec. 2.4)	ft	m
T	absolute temperature	°R or K	K
T	relative intensity of turbulence (Sec. 18.4)	—	—
u	internal energy per unit mass or per mole	Btu / lbm or Btu / lbmol	J / kg or J / mol
u^*	friction velocity (Sec. 17.4)	ft / s	m / s
u^+	$V_x\,/\,u^*$ (Sec. 17.4)	—	—
U	internal energy	Btu	J
υ	volume per unit mass	ft^3 / lbm	m^3 / kg
υ	fluctuating component of velocity (Chaps. 17 and 18)	ft / s	m / s
V	velocity	ft / s	m / s
V	volume	ft^3	m^3
V_x, V_y, V_z	*x, y,* and *z* components of velocity	ft / s	m / s
V_θ	tangential component of velocity	ft / s	m / s
V_r	radial velocity	ft / s	m / s
V_{avg}	average velocity	ft / s	m / s
$V_{\text{centerline}}$	centerline velocity in a pipe	ft / s	m / s
V_∞	free-stream velocity	ft / s	m / s
V_s	superficial velocity (Sec. 11.1)	ft / s	m / s
V_I	interstitial velocity (Sec. 11.1)	ft / s	m / s
V_{mf}	minimum fluidizing velocity (Sec. 11.5)	ft / s	m / s
W	work	ft · lbf	J
W	weight	lbf	N

W	width	ft	m
$W_{\text{n.f.}}$	non-flow work (excluding injection work)	ft · lbf	J
W	volumetric solids content of slurry (Sec. 11.4)	—	—
x, y, z	directions of coordinate axes, or lengths	ft	m
x	distance	ft	m
y	distance perpendicular to the flow direction	ft	m
y^+	$(r_{\text{wall}} - r)u^* / \nu$ (Sec. 17.4)	—	—
z	elevation	ft	m
α	coefficient of thermal expansion	1 / °F	1 / K
α	specific resistance of filter cake (Sec. 11.4)	1 / lbf	1 / N
α	small angle, jet angle (Chap. 19)	rad	rad
α	thermal diffusivity	ft^2 / s	m^2 / s
α	constant in Chézy Eq. (Chap. 6)	—	—
β	isothermal compressibility $= 1 / (\text{bulk modulus})$	1 / (lbf / in^2)	1 / Pa
γ	specific weight $= \rho g$	lbf / ft^3	N / m^3
Γ	torque	ft · lbf	N · m
δ	boundary-layer thickness (Chap. 17)	ft	m
δ^*	displacement thickness (Sec. 17.2)	ft	m
ε	absolute roughness	ft	m
ε	porosity or void fraction or volume fraction of gas	—	—
ε	eddy (kinematic) viscosity	ft^2 / s	m^2 / s
ε	turbulent dissipation rate	ft^2 / s^3	m^2 / s^3
ζ	vorticity $= 2\omega$	1 / s	1 / s
η	efficiency	—	—
η	$y(V_x / \nu_x)^{1/2}$ (Sec. 17.2)	—	—
η	viscosity (non-Newtonian fluids)	lbm / ft · s or cP	Pa · s
θ	angle	rad	rad
θ	momentum thickness (Sec. 17.2)	ft	m
θ	contact angle (Sec. 17.3)	rad	rad
μ	viscosity	lbm / ft · s or cP	Pa · s
ν	kinematic viscosity (μ / ρ)	ft^2 / s or cSt	m^2 / s
π	number of dimensionless groups (Chap. 9)	—	—
ρ	density	lbm / ft^3	kg / m^3
ρ	resistivity (Sec. 16.3)	—	ohm · m
σ	surface tension	lbf / ft	N / m
σ	stress	lbf / in^2	Pa
σ	shear rate	1 / s	1 / s
$\sigma_x, \sigma_y, \sigma_z$	turbulent dispersion coefficients (Chap. 19)	ft	m
σ_{xx}	normal stress in the *x* direction	lbf / in^2	Pa
τ	shear stress	lbf / in^2	Pa
τ_{xy}	shear stress in the *x* direction on a face perpendicular to the *y* axis	lbf / in^2	Pa
τ_{wall}	shear stress at a solid wall	lbf / in^2	Pa
τ_0	shear stress at a solid surface	lbf / in^2	Pa
τ_{yield}	yield stress for a Bingham fluid	lbf / in^2	Pa
ϕ	potential	ft^2 / s for fluid flow	m^2 / s for fluid flow
$\phi(t)$	arbitrary function of time (Sec. 16.2)	—	—
ψ	stream function	ft^2 / s	m^2 / s
ω	angular velocity	rad / s	rad / s

Superscripts

$*$	sonic condition (Chap. 8)		
$\overline{X}$	time average of *X*	various	various

Subscripts			
R	reservoir state in Chap. 8		
S	isentropic condition (speed of sound)		
1, 2	arbitrary states		
x, y	conditions before and after a normal shock in Chap. 8		
Vector			
boldface	indicates a vector	various	various
∇	$\nabla = \mathbf{i}\dfrac{\partial}{\partial x} + \mathbf{j}\dfrac{\partial}{\partial y} + \mathbf{k}\dfrac{\partial}{\partial z}$	1 / ft	1 / m

PREFACE

This book presents an introduction to fluid mechanics for undergraduate chemical engineering students.

Throughout the text, emphasis is placed on the connection between physical reality and the mathematical models of reality, which we manipulate. The ultimate test of a mathematical solution is its ability to predict the results of future experiments. Because a mathematically correct consequence of inapplicable assumptions is often simply wrong, the text occasionally offers intentionally wrong solutions to caution the student.

The simplest mathematical approaches are used, consistent with technical rigor.

Considerable attention is paid to the units of quantities in the equations because students usually have trouble with them, and because this reminds them that each symbol in our equations stands for a real physical quantity.

The book is divided into four sections. Section I, preliminaries, provides background for the study of flowing fluids. It includes a separate chapter on the balance equation. One might think that this is such a simple topic that it deserves only a few lines. However, it is a continual source of trouble to students. Furthermore, it is the most all-pervasive concept of chemical engineering, forming the basic mathematical framework for the application of the laws of thermodynamics, Newtonian mechanics, stoichiometry, and for the study of chemically reacting systems. There is also a chapter on the first law of thermodynamics. In the undergraduate program at the University of Utah, the students study basic engineering thermodynamics before they are introduced to fluid mechanics; thus, Chapter 4 is merely a review for them.

Section II discusses flows that are practically one-dimensional or can be treated as such. This organization of the book is radically different from the organization of fluids books written by mechanical and civil engineers, who begin with three-dimensional fluid mechanics and work their way down to one-dimensional fluid mechanics. The reasons for this organization, which fits better with the background of chemical engineers, are spelled out in Section 1.11. Sections I and II are the core of the book, covering all the basic ideas in fluid mechanics, and many of the problems of greatest interest to chemical engineers.

Section III discusses some other topics that can be viewed by the methods of one-dimensional fluid mechanics. These six chapters introduce other areas of fluid mechanics that are of great practical interest to some chemical engineers but that are not covered in an introductory course for want of time. They can be assigned, in any order, as supplementary reading, or covered briefly in class, introducing students to the terminology and basic ideas of these fields and helping them to read related matters in the current literature.

Section IV introduces the student to two- and three-dimensional fluid mechanics. It shows the relations between the methods used for these flows and the simpler approaches used in Sections II and III. It shows what two- and three-dimensional problems can be solved by hand (a small number) and shows the basis on which most such problems are currently solved by Computational Fluid Mechanics programs. A separate chapter introduces the student to mixing, which is basic chemical engineering, but not routinely covered in fluid mechanics texts.

The fourth edition includes numerous new and revised examples. Chapters 2 (Fluid Statics) and 6 (Fluid Friction in Steady, 1-D Flow) introduce hydraulic fracturing though calculations of the minimum surface pressure required to fracture rock and the surface pressure required to steadily force fluid into the rock. Chapter 10 (Pumps, Compressors, and Turbines) includes new material on wind turbines. It also describes three new positive displacement pumps including peristaltic pumps and a one-use liquid dispenser. The latter is an example of the book's continued insightful use of household items to illustrate key concepts, phenomena, and equipment in fluid mechanics. There are about 30 new problems that draw on the new material. Also new to the fourth edition is a chapter on microfluidics (Chap. 21), which has important applications in biology, medicine, and chemical analysis.

Where necessary, older examples have been updated. For example, a coffee percolator in Chap. 2 has been replaced with a modern coffer maker, and in Chap. 7 the jet engine shown in Fig. 7.15 has been replaced with a modern, more efficient design.

Computers do not make hand calculations unnecessary. No new or unfamiliar computer solution should be believed until manual plausibility checks have shown that the computer is indeed solving the problem we think it is solving and that its solution is physically reasonable. Simply plugging values into available computer packages does not build physical insight, which is one of the most important tools of the successful engineer. Good pedagogy begins with hand solutions of simplified versions of the real problem, which build physical insight and some understanding of physical magnitudes, followed by computer solutions, which can relax the simplifications and cover a wider variety of conditions, followed by manual plausibility checks of the computer solutions.

After an initial rush of enthusiasm for SI, engineering educators seem to be deciding that the English system of units is not likely to vanish overnight. For this reason our students must become like educated Europeans, who speak more than one language fluently and can read and understand one or two additional languages. Our students must be fluent in SI and in the English system of units and must understand traditional metric and cgs, and be able to read and understand texts using the slug and the poundal. This book has a long discussion of these various systems of units. Examples are presented in both SI and English units. This is unlikely to please purists of any persuasion, but it probably serves our students as well as any other approach and better than some.

Our goal is to present a text that average chemical engineering undergraduates can read and understand and from which they can attack a variety of meaningful problems. We have tried to help the student develop a physical insight into the processes of fluid mechanics and develop the understanding that the equations on these pages truly describe what nature does. We have tried to choose examples from the student's own experiences, or that relate to things they can observe in their everyday lives. The home is a wonderful place to observe the principles of chemical engineering; good teachers help students interpret what they see in the home in terms of chemical engineering principles.

The true test of the quality of a textbook is whether it becomes the most worn and tattered book on a practicing engineer's bookshelf. Former students tell us that the first three editions of this book pass that test. We hope copies of this edition will become even more worn and tattered.

For instructor resources, visit the text's website at http://www.mhhe.com/denevers4e.

Acknowledgments
We thank the many secretaries who worked on various editions of this book, the faculty who have reviewed earlier editions, and the many students who have used it and given us their criticisms and comments.

Noel de Nevers
Geoffrey D. Silcox

CHAPTER 1

INTRODUCTION

1.1 WHAT IS FLUID MECHANICS?

Mechanics is the study of forces and motions. Therefore, fluid mechanics is the study of forces and motions in fluids. But what is a fluid? We all can think of some things that obviously are fluids: air, water, gasoline, lubricating oil, and milk. We also can think of some things that obviously are not fluids: steel, diamonds, rubber bands, and paper. These we call solids. But there are some very interesting intermediate types of matter: gelatin, peanut butter, cold cream, mayonnaise, toothpaste, roofing tar, library paste, bread dough, and auto grease.

To decide what we mean by the word “fluid,” we first have to consider the idea of shear stress. It is easiest to discuss shear stress in comparison with tensile stress and compressive stress; see Fig. 1.1.

In Fig. 1.1(*a*) a rope is holding up a weight. The weight exerts a force that tends to pull the rope apart. A stress is the ratio of the applied force to the area over which it is exerted (force/area). Thus, the stress in the rope is the force exerted by the weight divided by the cross-sectional area of the rope. The force that tries to pull things apart is called a *tensile force,* and the stress it causes is called a *tensile stress.*

In Fig. 1.1(*b*) a steel column is holding up a weight. The weight exerts a force that tends to crush the column. This kind of force is called a *compressive force,* and the stress in the column, the force divided by the cross-sectional area of the column, is called a *compressive stress.*

In Fig. 1.1(*c*) some glue is holding up a weight. The weight exerts a force that tends to pull the weight down the walls and thus to *shear* the glue. This force, which

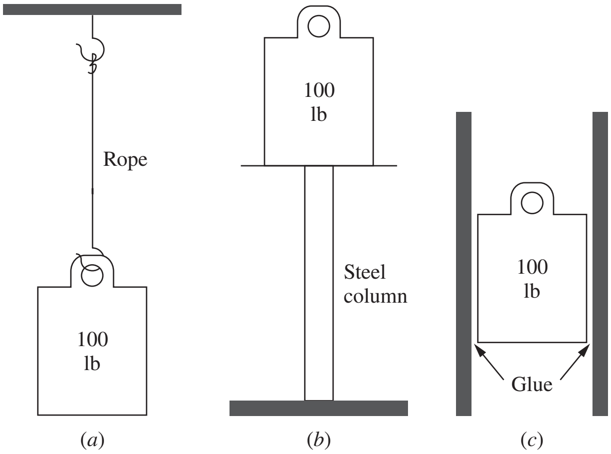

FIGURE 1.1
Comparison of tensile, compressive, and shear stresses. (*a*) The rope is in tensile stess; (*b*) the column is in compressive stress; and (*c*) the glue is in shear stress.

tends to make one surface *slide* parallel to an adjacent surface, is called a *shear force,* and the stress in the glue, the force divided by the area of the glue joint, is called a *shear stress.*

A more detailed examination of these examples would show that all three kinds of stress are present in each case, but those we have identified are the main ones. (For more information on this topic, see any text on strength of materials.)

Solids are substances that can permanently resist very large shear forces. When subject to a shear force, they move a short distance (elastic deformation), thereby setting up internal shear stresses that resist the external force, *and then they stop moving*. Materials that obviously are fluids cannot permanently resist a shear force, no matter how small. When subject to a shear force, they start to move and *keep on moving as long as the force is applied.*

Substances intermediate between solids and fluids can permanently resist a small shear force but cannot permanently resist a large one. For example, if we put a "blob" of any obvious liquid on a vertical wall, gravity will make it run down the wall. If we attach a piece of steel or diamond securely to a wall, it will remain there, no matter how long we wait. If we attach some peanut butter to a wall, it will probably stay, but if we increase the shear stress on the peanut butter by spreading it with a knife, it will flow like a fluid. We cannot spread steel with a knife as we spread peanut butter.

If, as shown above, the relevant difference between peanut butter and steel is the magnitude of the shear stress that the material can resist, then the difference is one of degree, not of kind. At extreme shear stresses steel can be made to "flow like a fluid." In the remainder of this book we will be talking mostly about materials such as air and water, which cannot permanently resist any shear force. However, it is well to keep our minds open to other possibilities of "fluid" behavior [1]. (Numbers in brackets refer to items listed in the References at the end of the chapter.)

1.2 WHAT GOOD IS FLUID MECHANICS?

The problems in fluid mechanics are basically no different from those in "ordinary" mechanics (the mechanics of solids) or in thermodynamics. Therefore, in principle one can solve problems in fluid mechanics with the same methods used to solve problems in mechanics or thermodynamics. However, for many of the problems involving the flow of fluids (or the movement of bodies through fluids), we use a combination of the problem-solving methods of mechanics and thermodynamics. Furthermore, the methods that work for hydraulics problems (dams, canals, locks, river flow, etc.) are applicable, with slight modifications, to aerodynamics problems (airplanes, rockets, wind forces on bridges, etc.) and to problems of special interest to chemical engineers such as the flow in chemical reactors, in distillation columns, or in polymer extrusion dies. Therefore, it makes sense to combine the study of this class of similar problems into one discipline, which we call fluid mechanics.

Consider the important fluids in our lives: the air we breathe, the water we drink, many of the foods we consume, most of the fuels for heating our houses or propelling our vehicles, and the various fluids in our bodies that make up our internal environment. Without some idea of the behavior of fluids, we can have only a very limited understanding of how the world works.

Some of the subdivisions and applications of fluid mechanics are:

1. Hydraulics: the flow of water in rivers, pipes, canals, pumps, and turbines.
2. Aerodynamics: the flow of air around airplanes, rockets, projectiles, and structures.
3. Meteorology: the flow of the atmosphere.
4. Particle dynamics: the flow of fluids around particles, the interaction of particles and fluids (i.e., dust settling, slurries, pneumatic transport, fluidized beds, air pollutant particles, and corpuscles in our blood).
5. Hydrology: the flow of water and water-borne pollutants in the ground.
6. Reservoir mechanics: the flow of oil, gas, and water in petroleum reservoirs.
7. Multiphase flow: coffee makers, oil wells, fuel injectors, combustion chambers, and sprays.
8. Combinations of fluid flow: with chemical reactions in combustion, with electromagnetic phenomena in magnetohydrodynamics, and with mass transport in distillation or drying.
9. Viscosity-dominated flows: lubrication, injection molding, wire coating, lava, and continental drift.

1.3 BASIC IDEAS IN FLUID MECHANICS

Fluid mechanics is based largely on working out the detailed consequences of four basic ideas:

1. The principle of the conservation of mass.
2. The first law of thermodynamics (the principle of the conservation of energy).
3. The second law of thermodynamics.
4. Newton's second law of motion, which may be summarized in the form $F = ma$.

Each of these four ideas is a generalization of experimental data. None of them can be deduced from the others or from any other prior principle. None of them can be "proven" mathematically. Rather, they stand on their ability to predict correctly the results of any experiment ever run to test them.

Sometimes, in fluid mechanics, we may start with these four ideas and the measured physical properties of the fluid(s) and proceed directly to solve mathematically for the desired forces, velocities, etc. This is generally possible only in the case of very simple flows. The observed behavior of a great many fluid flows is too complex to be solved directly from these four principles, so we must resort to experimental tests. Through the use of techniques called dimensional analysis (Chap. 9), we often can use the results of one experiment to predict the results of a much different experiment. Thus, careful experimental work is very important in fluid mechanics. With modern computers we can find useful numerical solutions to problems that would previously have required experimental tests. The methods for doing that are outlined in Part IV of this book. As computers become faster and cheaper, we will see additional complex fluid mechanics problems solved on computers. Ultimately, though, the computer solutions must be tested experimentally.

These four ideas are applied to fluid mechanical problems as follows. This introductory chapter launches our study and defines some important terms. Then Part I of the book, Chaps. 2–4, deals with preliminaries. We will need these in our study of moving fluids, and they provide direct solutions and/or insight into many practical problems. Parts II and III, Chaps. 5–14, deal with the flow of fluids that are one-dimensional or can be treated as if they were. Part IV, Chaps. 15–20, deals with two- and three-dimensional fluid mechanics. Each of these sections will be described as we begin them.

Students using this book should have previously completed a course in elementary thermodynamics. Chapters 3 and 4 should serve as a review of matter previously covered; they are included because the principles involved are central to fluid mechanics. It is assumed that the student is familiar with the second law of thermodynamics, which is used occasionally. Remember that this entire book is devoted to the application of the four basic ideas and the results of experimental tests to fluid-flow problems. Although the details can become quite involved, the basic ideas are few.

1.4 LIQUIDS AND GASES

Fluids are of two types, liquids and gases. On the molecular level these are quite different. In liquids the molecules are close together and are held together by significant forces of attraction; in gases the molecules are relatively far apart and have very weak forces of attraction. As a rule, the specific volumes of gases are ≈1000 times those of liquids, which means that the average intermolecular distance (center to center of the molecules) is roughly 10 times as far in a typical gas as in a typical liquid. As temperature and pressure increase, these differences become less and less, until the liquid and gas become identical at the critical temperature and pressure. The difference between the behavior of liquids and gases is most marked when these fluids are expanded. Suppose that some fluid completely fills the space below the piston in Fig. 1.2. When we raise the piston, the volume occupied by the fluid is increased. If the fluid is a gas, it will expand readily, filling all the space vacated by the piston; gases can expand without limit to occupy space made available to them. But, if the fluid is a liquid, then as the piston is raised, the liquid can expand only a small amount, and then it can expand no more. What fills the space between the piston and the liquid? Part of the liquid must turn into a gas by boiling, and this gas expands to fill the vacant space. This can be explained on the molecular level by saying that there is a maximum distance between molecules over which the attractive forces hold them together to form a liquid and that, when the molecules separate more than this distance, they cease behaving as a liquid and behave as a gas.

Because of their closer molecular spacing, liquids normally have higher densities, viscosities, refractive indices, etc., than gases (see Prob. 1.2). In engineering this frequently leads to quite different behaviors of liquids and gases, as we will see.

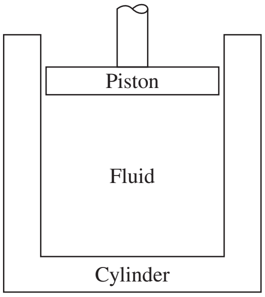

FIGURE 1.2
Piston and cylinder. If the fluid is a gas, we can move the piston up and down as much as we like, and the gas will expand or contract to fill the volume available. If the fluid is a liquid, we can move the piston down very little without producing extreme pressures; if we move it up, the liquid must partly evaporate to produce a gas to fill the space.

1.5 PROPERTIES OF FLUIDS

The physical properties of fluids that will enter our calculations most often are density, viscosity, and surface tension.

1.5.1 Density

The *density* ρ is defined the mass per unit volume:

$$\rho = \frac{m}{V} \tag{1.1}$$

We are all aware of the differences in density between various materials, such as that between lead and wood. How can we measure the density of a material? If we want to know the density of a liquid, we can weigh a bottle of known volume (determine its mass), fill it with the liquid, weigh it again, and compute the density with the aid of Eq. 1.1. (This is one of the standard laboratory methods of determining liquid density; the special weighing bottles designed for this purpose are called *pyncnometers,* Prob. 1.5) If we want to know the density of a cubical solid block, we can measure the length of its sides, compute its volume, weigh it, and apply these results to Eq. 1.1.

Now suppose that we are asked to determine the density of a piece of Swiss cheese. If we have a large block of the cheese, we can cut off a cube, measure its sides, compute its volume, weigh it, and then calculate its density. This is an average density, one that includes the density of the air in the holes in the cheese. As long as we are dealing with large pieces of cheese, it is a satisfactory density. Suppose, however, we are asked to find the density at some point inside a large block of the cheese. If we can cut the cheese open, and if we find that the point in question is in the solid cheese and not in one of its holes, we can find the density easily enough or, if the point in question is in a hole, we find the density of the air in the hole. But if the point is on the surface of a hole, the problem is more difficult. Then the density is discontinuous; see Fig. 1.3. There is no meaningful single value of the density at *x.*

Why this long discussion about the density of Swiss cheese? Because the world is full of holes! Atomic physics tells us that even in a solid bar of steel, the space

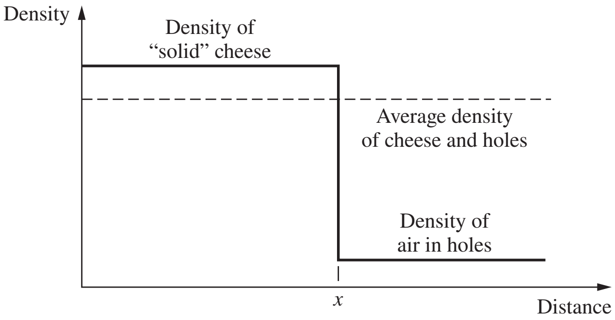

FIGURE 1.3
The density of Swiss cheese is not uniform from point to point, but has local point densities and an average density.

occupied by the electrons, protons, and neutrons is a very small fraction of the total space; the rest presumably is empty. Furthermore, even at the molecular level, there are holes; in a typical gas the space actually occupied by the individual gas molecules at any instant is a small fraction of the total space. Thus, in any attempt to speak of density at a given point we are in the same trouble as with the Swiss cheese. Therefore, we must restrict the definition of density to samples large enough to average out the holes. This causes no problem in fluid mechanics, because of the size of the samples normally used, but it indicates that the concept of density does not readily apply to samples of molecular and subatomic sizes.

In addition, we must be careful in defining the densities of composite materials. For example, a piece of reinforced concrete consists of several parts with different densities. In discussing such materials, we must distinguish between the *particle densities* of the individual pebbles or steel reinforcing bars and the *bulk density* of the mixed mass. When we refer to bulk density, our sample must be large compared with the dimensions of one particle. Some examples of composite solid materials are cast iron, fiberglass-reinforced plastics, and wood. Some examples of composite liquids are slurries, such as muds, milkshakes, and toothpaste, and emulsions, such as homogenized milk, mayonnaise, and cold cream. Smokes and clouds behave as composite gases.

Example 1.1. A typical mud is 70 wt. % sand and 30 wt. % water. What is its density? The sand is practically pure quartz (SiO_2), for which $\rho_{\text{sand}} = 165\ \text{lbm}/\text{ft}^3\ (2.65\ \text{g}/\text{cm}^3)$. See App. E for the properties of water used in all examples and problems.

Here we assume that there is no volume change on mixing sand and water. There are volume changes on mixing for some substances like ethanol and water, but they are small enough to ignore for most problems, including this one. Then

$$\rho = \frac{m}{V} = \frac{m_{\text{sand}} + m_{\text{water}}}{V_{\text{sand}} + V_{\text{water}}} = \frac{m_{\text{sand}} + m_{\text{water}}}{(m/\rho)_{\text{sand}} + (m/\rho)_{\text{water}}} \tag{1.A}$$

[Every equation in this book has a number. Those, like this one, that are parts of examples or in other ways specific to some situation are identified with number-letter combinations, such as (l.A). General equations have number-number combinations, such as (1.1).]

We could simplify Eq. 1.A algebraically, but a more intuitive approach is to choose as our *basis* 100 lbm of mud, and substitute into Eq. 1.A, finding

$$\rho = \frac{m_{\text{sand}} + m_{\text{water}}}{\left(\frac{m}{\rho}\right)_{\text{sand}} + \left(\frac{m}{\rho}\right)_{\text{water}}} = \frac{70 \text{ lbm} + 30 \text{ lbm}}{\left(\frac{70 \text{ lbm}}{165 \text{ lbm}/\text{ft}^3}\right)_{\text{sand}} + \left(\frac{30 \text{ lbm}}{62.3 \text{ lbm}/\text{ft}^3}\right)_{\text{water}}}$$

$$= 110.4 \frac{\text{lbm}}{\text{ft}^3} = 1769 \frac{\text{kg}}{\text{m}^3} \tag{1.B}$$

■

The ■ indicates the end of an example.

1.5.2 Specific Gravity

Specific gravity of liquids and solids (SG) is defined as

$$\text{SG} = \frac{\text{density}}{\text{density of water at some specified temperature and pressure}} \tag{1.2}$$

This definition has the merit of being a ratio and, hence, a pure number, which is independent of the system of units chosen. Occasionally it leads to confusion, because some specific gravities are referred to water at 60°F, some to water at 70°F, and some to water at 39°F = 4°C (all at a pressure of 1 atm). The differences are small but great enough to cause trouble.

If the temperature of the water is specified as 39°F = 4°C, then the density of water is 1.000 g/cm^3. (The gram was defined to make this number come out 1.000). Thus, if this basis of measurement is chosen, then specific gravities become numerically identical with densities expressed in g/cm^3 or kg / L or metric tons/m^3. The mud in Example 1.1 has SG = 1.769.

Many process industries use special scales of fluid density, which are usually referred to as *gravities.* Some of them are the API gravity (American Petroleum Institute) for oil and petroleum products (Prob. 1.6), Brix gravity for the sugar industry, and Baumé gravity for sulfuric acid. Each scale is directly convertible to density; conversion tables and formulae are available in handbooks.

Specific gravities of gases are normally defined as

$$\begin{pmatrix}\text{SG of}\\ \text{a gas}\end{pmatrix} = \left(\frac{\text{density of the gas}}{\text{density of air}}\right)_{\text{Both at the same temperature and pressure}} \tag{1.3}$$

For ideal gases the specific gravity of any gas = $(M_{\text{gas}} / M_{\text{air}})$.

Throughout this text we use liquid and solid specific gravities referred to water at 4°C. Thus a liquid with a specific gravity of 0.8 is a liquid with a density of 0.8 g/cm^3.

1.5.3 Viscosity

Viscosity is a measure of internal, frictional resistance to flow. If we tip over a glass of water on the dinner table, the water will spill out before we can stop it. If we tip over an open jar of honey, we probably can set it upright again before much honey

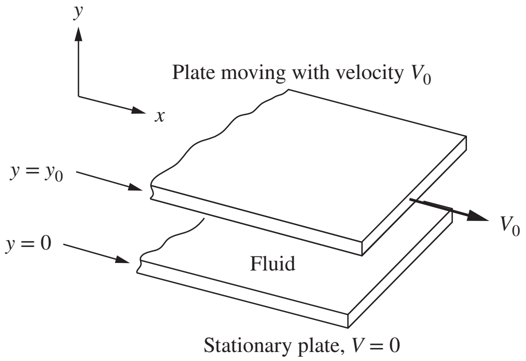

FIGURE 1.4
The sliding-plate experiment.

flows out; this is possible because the honey has much more resistance to flow, more viscosity, than water. A more precise definition of viscosity is possible in terms of the following conceptual experiment.

Consider two long, solid plates separated by a thin film of fluid (see Fig. 1.4). This apparatus is easy to grasp conceptually and mathematically but difficult to use, because the fluid leaks out at the edges and gravity pulls the two plates together. Other devices that are more complex mathematically but easier to use are actually used to measure viscosities (see Example 1.2 and Chaps. 6 and 13). If we slide the upper plate steadily in the x direction with velocity V_0, a force will be required to overcome the internal friction in the fluid between the plates. This force will be different for different velocities, different plate sizes, different fluids, and different distances between the plates. We can eliminate the effect of different plate sizes, however, by measuring the force per unit area of the plate, which we define as the *shear stress* τ.

It has been demonstrated experimentally that at low values of V_0, the velocity profile in the fluid between the plates is linear, i.e.,

$$V = \frac{V_0 \, y}{y_0} \tag{1.C}$$

so that

$$\sigma = \begin{pmatrix}\text{shear rate, rectangular} \\ \text{coordinates}\end{pmatrix} = \frac{dV}{dy} = \frac{V_0}{y_0} \tag{1.D}$$

It also has been demonstrated experimentally that for most fluids, the results of this experiment can be shown most conveniently on a plot of τ versus dV / dy (see Fig. 1.6). As shown here, dV / dy is simply a velocity divided by a distance. In more complex geometries, it is the limiting value of such a ratio at a point. It is commonly called *shear rate, the rate of strain,* and *rate of shear deformation,* all of which mean exactly the same thing.

Example 1.2. Figure 1.5 shows a cutaway photograph of a concentric-cylinder ("cup and bob") viscometer also called a *Couette viscometer.* An inner cylinder (the bob) rotates inside a stationary outer cylinder (the cup). The shaft that drives the bob is instrumented to record both the angular velocity and the applied torque. The solid bob has $D_1 = 25.15$ mm and $L = 92.27$ mm. The surrounding cup has $D_2 = 27.62$ mm and is longer than the bob. When the bob is driven at 10 rpm, the observed torque (tangential force times radius) is $\Gamma = 0.005$ N·mm. What are τ and dV / dy?

This viscometer is simply the device in Fig. 1.4, wrapped around a cylinder. In this form, the leakage-at-the-edges problem and the difficulty of keeping

FIGURE 1.5
Cutaway photograph of a concentric-cylinder viscometer. This is simply the sliding-plate arrangement in Fig. 1.4, wrapped around a cylinder, thus eliminating the leaky edges in Fig. 1.4. The drive mechanism at the top holds the outer cylinder fixed and rotates the inner closed cylindrical bob. It provides a measured, controllable rotation rate and simultaneously measures the torque required to produce that rotation. The two flexible hoses circulate constant-temperature water or other fluid, to hold the whole apparatus at a constant temperature. Example 1.2 shows the dimensions of this device. (Courtesy of Brookfield Engineering Company.)

the distance between the two surfaces constant are solved. (Fluid forces hold the rotating inner cylinder properly centered inside the outer cylinder.) Here we must replace the *y*s in Eq. 1.5 with *r*s, because the velocity is changing in the radial direction. $\Delta y = y_0$ is replaced by

$$\Delta r = 0.5(D_2 - D_1) = 0.5(27.62 - 25.15) = 1.235 \text{ mm} \tag{1.E}$$

and

$$V_0 = \pi D_1 \cdot \text{rpm} = \pi \cdot 25.15 \text{ mm} \cdot \frac{10}{\text{min}} = 790.1 \frac{\text{mm}}{\text{min}} = 13.17 \frac{\text{mm}}{\text{s}} \tag{1.F}$$

Thus,

$$\frac{dV}{dr} = \frac{V_0}{\Delta r} = \frac{13.17 \text{ mm/s}}{1.235 \text{ mm}} = 10.66 \frac{1}{\text{s}} \tag{1.G}$$

This is a linearized approximation of a cylindrical problem that understates the correct value, which is 12.26 (1 / s) (see Prob. 1.10), a difference of 15%. We will use the correct (cylindrical) value in the rest of this chapter.

The shear stress at the surface of the inner cylinder is

$$\tau = \frac{F}{A} = \frac{\Gamma / r_1}{\pi D_1 L} = \frac{0.005 \text{ N·mm} / (0.5 \cdot 25.15 \text{ mm})}{\pi \cdot 25.15 \text{ mm} \cdot 92.37 \text{ mm}} = 5.45 \cdot 10^{-8} \frac{\text{N}}{\text{mm}^2} = 0.0545 \frac{\text{N}}{\text{m}^2} \tag{1.H}$$

■

This example ignores the stress on the bottom surface of the bob, a small effect, for which a correction is made in real viscosity measurements. The whole device is shown immersed in a constant-temperature bath, because the results are very temperature dependent.

The experiment in Example 1.2 can be repeated at different rotational speeds and the results plotted as shown in Fig. 1.6. Four different kinds of curve are shown as experimental results in the figure. All four of these results are observed in nature. The most common behavior is that represented by the straight line through the origin in the figure. This line is called Newtonian because it is described by *Newton's law of viscosity*

$$\tau = \mu \frac{dV}{dy} \qquad \text{[Newtonian fluids]} \tag{1.4}$$

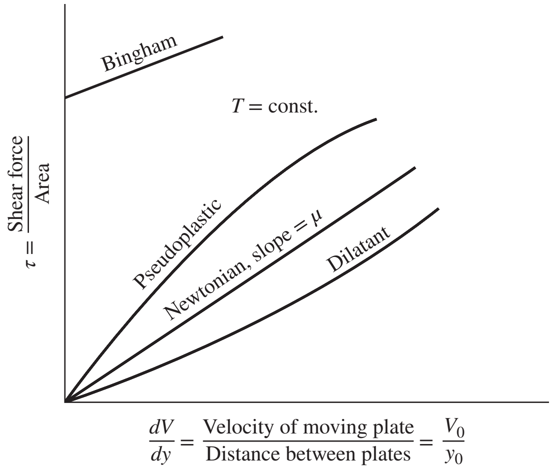

FIGURE 1.6
Possible outcomes of the sliding-plate experiment at constant temperature and pressure.

This equation says that the shear stress τ is linearly proportional to the velocity gradient dV/dy. It is also the definition of viscosity, because we can rearrange it to

$$\mu = \frac{\tau}{dV/dy} \tag{1.5}$$

Here μ is called the *viscosity* or the *coefficient of viscosity*. [We occasionally see this equation written with a minus sign in front of the τ. This is done so that the equation will have the same form as the heat-conduction and mass-diffusion equations ([2], pp. 266, 515). Since the shear stress acts in one direction on the rotating cylinder and in the opposite direction on the fluid adjacent to it, we can introduce this minus sign and reverse our idea of the direction of τ so that the result is always the same as in Eq. (1.5).] For Example 1.2, we would calculate

$$\mu = \frac{\tau}{dV/dy} = \frac{0.0545\ \text{N}/\text{m}^2}{12.26/\text{s}} = 0.0044\ \frac{\text{N}\cdot\text{s}}{\text{m}^2} \tag{1.I}$$

For fluids such as air the value of μ is very low; therefore, their observed behavior is represented in Fig. 1.6 by a straight line through the origin, very close to the dV/dy axis. For fluids such as corn syrup the value of μ is very large, and the straight line through the origin is close to the τ axis.

Fluids that exhibit this behavior in the sliding-plate experiment or its cylindrical equivalent (i.e., fluids that obey Newton's law of viscosity) are called *Newtonian fluids*. All the others are called *non-Newtonian fluids*. Which fluids are Newtonian? All gases are Newtonian. All pure liquids for which we can write a simple chemical formula are Newtonian, such as water, benzene, ethyl alcohol, carbon tetrachloride, and hexane. Most dilute solutions of simple molecules in water or organic solvents are Newtonian, such as solutions of inorganic salts or sugar in water or benzene. Which fluids are non-Newtonian? Generally, non-Newtonian fluids are complex mixtures: slurries, pastes, gels, polymer solutions, etc. (some authors refer to them as *complex fluids*). Most non-Newtonian fluids are mixtures with constituents of very different sizes. For example, toothpaste consists of solid particles suspended in an aqueous solution of various polymers. The solid particles are much, much bigger than water molecules, and the polymer molecules are much bigger than water molecules.

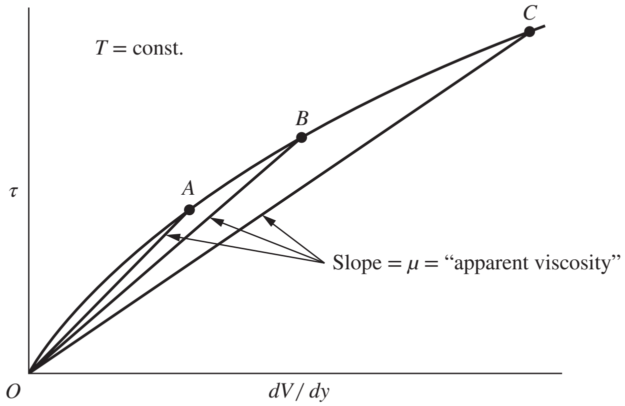

FIGURE 1.7
The "apparent viscosity" of a pseudoplastic fluid decreases as the shear rate increases.

In discussing non-Newtonian fluids, we must agree on what we mean by viscosity. If we retain the definition given by Eq. (1.5), then the viscosity can no longer be considered a constant independent of dV/dy for a given temperature, but must be considered a function of dV/dy. This is shown in Fig. 1.7. Here each of the lines OA, OB, and OC has slope μ, so the viscosity is decreasing with increasing dV/dy. (Viscosities defined as the slopes in Fig. 1.7 are often called *apparent viscosities.*) Using this definition, we can observe that there are three common types of non-Newtonian fluid (Fig. 1.6):

1. *Pseudoplastic fluids* show an apparent viscosity that decreases with increasing velocity gradient. Examples are most slurries, muds, polymer solutions, solutions of natural gums, and blood. These fluids are referred to as *shear thinning* fluids. This is the most common type of non-Newtonian behavior.
2. *Bingham fluids,* sometimes called *Bingham plastics,* resist a small shear stress indefinitely but flow easily under larger shear stresses. One may say that at low stresses the viscosity is infinite and at higher stresses the viscosity decreases with increasing velocity gradient. Examples are bread dough, toothpaste, applesauce, some paints, jellies, and some slurries.
3. *Dilatant fluids* show a viscosity that increases with the increasing velocity gradient. This behavior is called *shear thickening;* it is uncommon, but starch suspensions and some muds behave this way. For these materials the liquid lubricates the passage of one solid particle over another; at high shear rate the lubrication breaks down, and the particles have more resistance to slipping past each other.

So far, we have assumed that the curve of τ versus dV/dy is not a function of time; i.e., if we move the sliding plate at a constant speed, we will always require the same force. This is true of most fluids, but not of all. A more complete picture

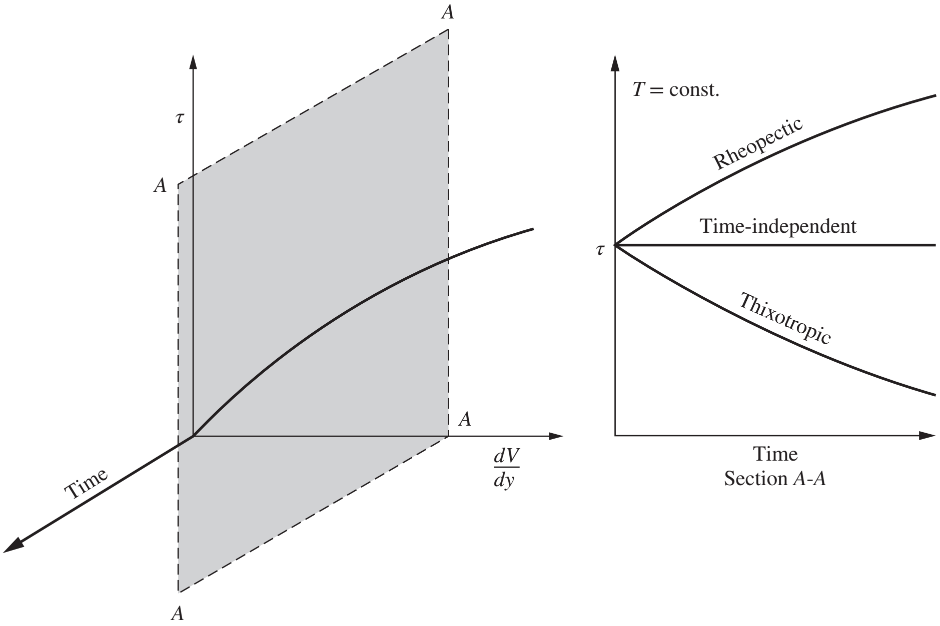

FIGURE 1.8
The viscosity of fluids can be independent of time of shearing or can increase or decrease with time as the fluid is sheared.

is given in Fig. 1.8. In Fig. 1.8 we see a constant dV/dy slice out of the solid constructed of τ versus dV/dy versus time. We see three possibilities:

1. The viscosity can remain constant with time, in which case the fluid is called *time independent.*
2. The viscosity can decrease with time, in which case the fluid is called *thixotropic.*
3. The viscosity can increase with time, in which case the fluid is called *rheopectic.*

All Newtonian fluids are time independent, as are most non-Newtonian fluids. Many thixotropic fluids are known, almost all of which are slurries or solutions of polymers, and a few examples of rheopectic fluids are known.

In addition, some fluids, called *viscoelastic fluids,* can show not only the kinds of behavior represented in Figs. 1.6 and 1.8 but also elastic properties, which allow them to "spring back" when a shear force is released. The most common examples of viscoelastic fluids are egg whites, cookie dough, and the rubber cement sold at stationery stores. Rubber cement's viscoelastic properties can be demonstrated most easily by starting to pour a little out of the bottle and then snapping it back into the bottle with a quick jerk of the hand. The same can be done with egg white. This is quite impossible with any ordinary fluid such as water; try it!

These strange types of fluid behavior are of considerable practical use. A good toothpaste must be a Bingham fluid, so that it can easily be squeezed out of the tube but will not drip off the toothbrush the way water or honey would. A good paint should be a thixotropic Bingham fluid, so that in the can it will be very viscous and the pigment will not settle to the bottom, but when it is stirred, it will become less viscous and can easily be brushed onto a surface. In addition, the brushing should temporarily reduce the viscosity so that the paint will flow sideways (under the influence of surface tension; see below) and fill in the brush marks (called *leveling* in the paint industry); then, as it stands, its viscosity should increase, so that it will not form drops and run down the wall.

Most engineering applications of fluid flow involve water, air, gases, and simple fluids. Therefore, most fluid-flow problems have to do with Newtonian fluids, as do most of the problems in this book. Non-Newtonian fluids (Chap. 13) are important, however, precisely because of their non-Newtonian behavior.

The viscosity of simple gases, such as helium, can be calculated for all temperatures and pressures from the kinetic theory of gases using only one experimental measurement for each gas [2]. For the viscosities of most gases and all liquids, several experimental data points are required, although ways of predicting viscosity change with changing temperature and pressure are available [3]. As a general rule, the viscosity of gases *increases slowly* with increasing temperature, and the viscosity of liquids *decreases rapidly* with increasing temperature. The viscosity of both gases and liquids is practically independent of pressure at low and moderate pressures.

The basic unit of viscosity is the *poise,* where $\text{P} = 1\ \text{g}/(\text{cm}\cdot\text{s}) = 0.1\ \text{Pa}\cdot\text{s} = 6.72\cdot 10^{-2}\ \text{lbm}/(\text{ft}\cdot\text{s})$ [See App. D for conversion factors.] The poise is widely used for materials like high-polymer solutions and molten polymers. However, it is too large a unit for most common fluids. By sheer coincidence the viscosity of pure water at about 68°F = 20°C is 0.01 poise; for that reason the common unit of viscosity in the United States is the centipoise, $\text{cP} = 0.01\ \text{P} = 0.01\ \text{g}/(\text{cm}\cdot\text{s}) = 0.001\ \text{N}\cdot\text{s}/\text{m}^2 = 0.001\ \text{Pa}\cdot\text{s} = 6.72\cdot 10^{-4}\ \text{lbm}/(\text{ft}\cdot\text{s})$. Hence, the viscosity of a fluid expressed in centipoise is the same as the ratio of its viscosity to that of water at room temperature. The viscosities of some common liquids and gases are shown in App. A.1. The computed viscosity of the fluid in Example 1.2 is 4.4 cP.

1.5.4 Kinematic Viscosity

In many engineering problems, viscosity appears only in the relation (viscosity/density). Therefore, to save writing, we define

$$\text{Kinematic viscosity} = \nu = \mu / \rho \tag{1.6}$$

The most common unit of kinematic viscosity is the centistoke (cSt):

$$1\ \text{cSt} = \frac{1\ \text{cP}}{1\ \text{g}/\text{cm}^3} = 10^{-6}\,\frac{\text{m}^2}{\text{s}} = 1.08\cdot 10^{-5}\,\frac{\text{ft}^2}{\text{s}} \tag{1.J}$$

at 68°F = 20°C, water has a kinematic viscosity of $1.004 \approx 1$ cSt. To avoid confusion over which viscosity is being used, some writers refer to the viscosity μ as the *absolute viscosity.* The kinematic viscosity has the same dimension ($\text{length}^2/\text{time}$) as the thermal diffusivity and the molecular diffusivity; in many problems, it acts the same way as them. In Chap. 6, we will see some examples of the practical convenience of the kinematic viscosity.

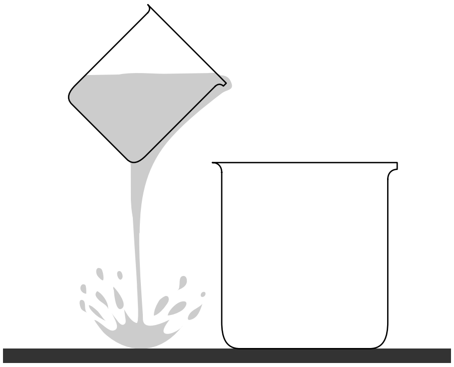

FIGURE 1.9
Disheartening effect of surface tension. The water dribbles down the surface of the container.

1.5.5 Surface Tension

Liquids behave *as if* they were surrounded by a skin that tends to shrink, or contract, like a sheet of stretched rubber, a phenomenon known as *surface tension.* It is seen in many everyday events, the most disheartening of which is the tendency of water, when poured slowly from a glass, to dribble down the edge of the glass (see Fig. 1.9).

Surface tension is caused by the attractive forces in liquids. All of the molecules attract each other; those in the center are attracted equally in all directions, but those at the surface are drawn toward the center because there are no liquid molecules in the other direction to pull them outward (see Fig. 1.10). The "effort" of each molecule to reach the center causes the fluid to try to take a shape that will have the greatest number of molecules nearest the center, a sphere (Prob. 1.11). Any other shape has more surface per unit volume; therefore, regardless of the shape of a liquid, the attractive forces tend to pull the liquid into a sphere. Other forces such as gravity often oppose surface tension forces, so the spherical shape is only seen for small systems, such as small water drops on a water-repellent surface. The fluid thus tries to decrease its surface area to a minimum. (An analogous situation in two dimensions is observable in the behavior of some army ants. They travel in large groups, and, viewed from above, the swarm often looks like a circle. The reason appears to be that the ants are attracted by the scent of other ants and, hence all try to get to the place where the scent is strongest, the center. The ants all stay in one plane, so the result is the plane figure with the smallest possible ratio of perimeter to area—a circle [4].)

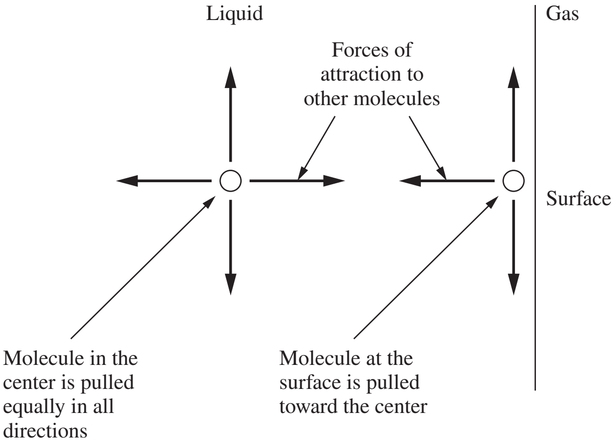

FIGURE 1.10
Surface tension is caused by the attractive forces between molecules.

The tendency of a surface to contract can be measured with the device

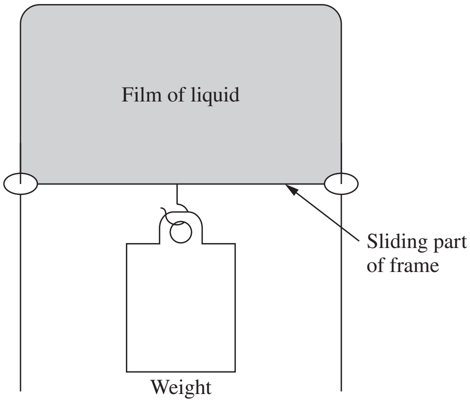

FIGURE 1.11
A very simple way to measure surface tension; see Example 1.3.

shown in Fig. 1.11. A wire frame with one movable side is dipped into a liquid and carefully removed with a film of liquid in the space formed by the frame. The film tries to assume a spherical shape, but since it adheres to the wire, it draws the movable part of the frame inward. The force necessary to resist this motion is measured by a weight. It is found experimentally that the ratio of the force to the length of the sliding part of the wire is always the same for a given liquid at a given temperature, regardless of the size of the apparatus. The liquid film in the frame has two surfaces (front and back), so the force-to-length ratio of *one* of the surfaces is exactly one-half of the total measurement. The surface tension of the liquid is then defined as

$$\text{Surface tension} = \frac{\text{force of one film}}{\text{length}} \qquad \text{or} \qquad \sigma = \frac{F}{l} \tag{1.7}$$

Example 1.3. The device in Fig. 1.10 has a sliding part 10 cm long. The mass needed to resist the inward pull of the fluid is 0.6 g, which exerts a force of 0.005 89 N. What is the surface tension of the fluid?

From Eq. 1.7,

$$\sigma = \frac{F(\text{one film})}{l} = \frac{0.005\,89\ \text{N}/2}{0.1\ \text{m}} = 0.0294\ \frac{\text{N}}{\text{m}} = 0.000\,168\ \frac{\text{lbf}}{\text{in}} \tag{1.K}$$

■

The device shown in Fig. 1.11 is easy to understand but not very practical as a measuring device; more practical ones are discussed in Chap. 14.

Surface tension is very slightly influenced by what the surrounding gas is—air or water vapor or some other gas. Typical values of the surface tension of liquids exposed to air are shown in Table 1.1. The traditional unit of surface tension is the dyne / cm = 0.001 N / m. At 68°F = 20°C, most organic liquids have about the same surface tension (≈25 dyne / cm) whereas that of water is about 3 times higher, and that of mercury is 20 times higher.

We indicated that the liquid adheres to the solid in the apparatus shown in Fig. 1.11. Liquids adhere strongly to some solids and not to others. For example, water adheres strongly to glass but very weakly to polyethylene. This greatly complicates the whole subject of surface tension; the phenomenon shown in Fig. 1.9 occurs much more often with glass, ceramic, or metal cups than with polyethylene or Teflon cups.

TABLE 1.1
Surface tensions of pure fluids exposed to air at 68°F = 20°C

Fluid	Surface tension, dyne / cm = 0.001 N / m	Surface tension, lbf / in
Acetic acid	27.8	0.000 159
Acetone	23.7	0.000 135
Benzene	28.25	0.000 161
Carbon tetrachloride	26.95	0.000 154
Ethyl alcohol	22.75	0.000 130
n-Octane	21.8	0.000 124
Toluene	28.5	0.000 163
Water	72.74	0.000 415
Mercury	484	0.002 763

Lide, David R. *Handbook of Chemistry and Physics*. Boca Raton, FL: CRC Press, 2003; and various other handbooks.

Two important effects attributable to surface tension are the capillary rise of liquids in small tubes and porous wicks (without which candles, kerosene lanterns, or copper sweat-solder fittings would not work at all) and the tendency of jets of liquid to break up into drops (as from a garden hose or gasoline or diesel fuel injector or in an ink-jet printer). Surface tension effects are very important in systems involving large surface areas, such as emulsions (mayonnaise, cold cream, water-based paints) and multiphase flow through porous media (oil fields); see Chap. 14 and references [5,6].

1.6 PRESSURE

Pressure is defined as a compressive stress or compressive force per unit area. In a stationary fluid (liquid or gas) the compressive force per unit area is the same in all directions. In a solid or in a moving fluid, the compressive force per unit area at some point is not necessarily the same in all directions. We can visualize why by squeezing a rubber eraser between our fingers; see Fig. 1.12. As we squeeze the eraser, it becomes thinner and longer, as shown. If we analyze the stresses in the eraser, we find that in the y direction the eraser is in compression, whereas in the x direction it is in tension. (This seems strange, but the eraser has been stretched in the x direction, and its elastic forces will pull it back when we let go; hence the tension.) The contraction in one direction and expansion in another in an elastic solid is described in terms of Poisson's ratio, discussed in any text on strength of materials. Because the tensile and compressive forces are at right angles to each other, there is also a strong shear stress at 45° to the x axis.

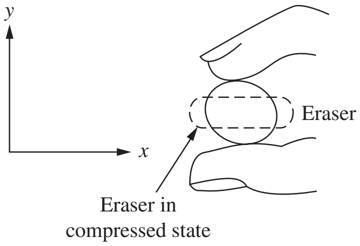

FIGURE 1.12
The response of an elastic solid to compression in one direction.

What would happen if we held our fingers in a cup of water and tried to squeeze the water between our fingers? Obviously, the water would run out from between our fingers, and our fingers would come together. Why? When we start to squeeze the water, it behaves like the eraser, setting up internal shear and tensile forces in the same directions as the eraser. However, ordinary fluids cannot permanently resist shear forces, so the water begins to flow and finally flows away. The eraser also flowed, until it had taken up a new shape, in which its internal tensile and shear resistance were enough to hold our fingers apart. Water cannot set up such resistance and so it simply flows away.

If we really wanted to squeeze the water, we would put it in some container that would prevent its flowing out to the side. If we did this with the eraser, then as we compressed it from the top, it would press out on the sides of the container. So also does water.

The foregoing is a description of why the pressure at a point in a fluid at rest is the same in all directions. It is not a proof of that fact; for a proof, see App. B.1.

What we mean by pressure is not so clear for a solid as it is for a liquid or a gas. The compressive stresses at a given point in a solid are not the same in all directions. The usual definition of pressure in a solid is as follows: Pressure at a point is the average of the compressive stresses measured in three perpendicular directions. Since, as we have seen, these three stresses are all the same in a fluid at rest, the two definitions are the same. For a fluid in motion, the three perpendicular compressive stresses may not be the same. However, for this difference to be significant, the shear stresses must be very large, well outside the range of normal problems in fluid mechanics. Therefore, we normally extend the notion that pressure in a fluid at rest is the same in all directions to fluids in motion, with the reservation that at very high shear stresses (such as in the flow of metals or polymer melts through forming dies) this is not necessarily true. For polymer solutions and polymer melts the differences between the compressive stresses in directions at right angles to one another can be very significant and can lead to behavior quite different from the behavior of simple fluids; see [7].

In the solution of many problems, particularly those involving gases, it is most convenient to deal with pressures in an absolute sense, i.e., pressures relative to a compressive stress of zero; these are called *absolute pressures.* In the solution of many other problems, particularly those involving liquids with free surfaces, such as are encountered in rivers, lakes, and open or vented tanks, it is more convenient to deal with pressures above an arbitrary datum, the local atmospheric pressure. Pressures relative to the local atmospheric pressure are called *gauge pressures.*

Because both systems of measurement are in common use, it is necessary to make clear which kind of pressure we mean when we write "a pressure of $15\ \text{lb}/\text{in}^2$" [This unit is also called psi (pounds per square inch)]. It is usual to say "15 psi absolute" or "15 psia" for absolute pressure and "15 psi gauge" or "15 psig" for gauge pressure. The SI unit of pressure is the pascal, $\text{Pa} = \text{N}/\text{m}^2$. There does not seem to be a common set of abbreviations for Pascal absolute and Pascal gauge, so these must be written out.

Another two-datum situation familiar to the reader is found in the measurement of elevation. Mountain tops, road routes, and rivers are normally surveyed relative to

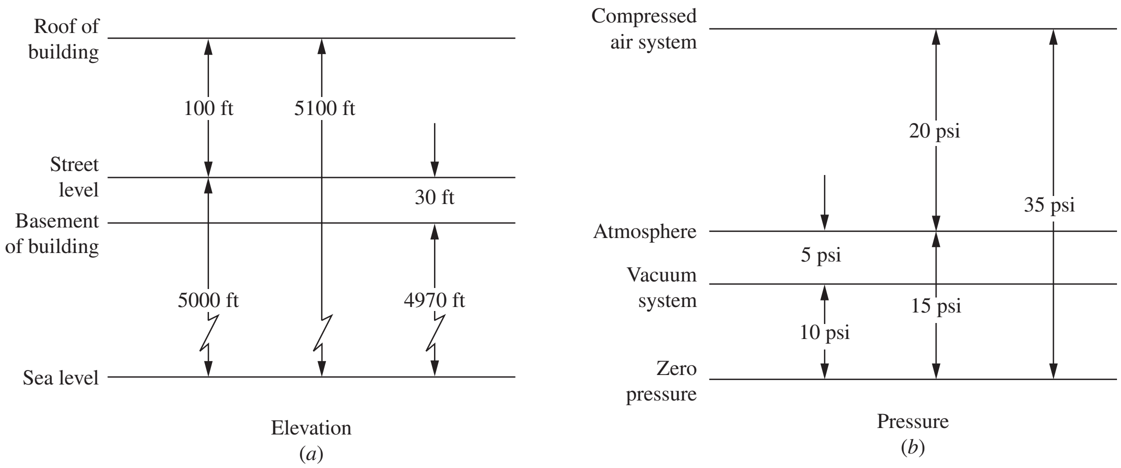

FIGURE 1.13
The relation between gauge and absolute pressure, and a comparison with elevation measurements.

mean sea level, which serves as an "absolute" datum, but most buildings are designed and constructed relative to some local elevation (usually a marker in the street); see Fig. 1.13. In both cases the most common measuring method gives answers in terms of the local datum. Most pressure gauges read the difference between the measured pressure and the local atmospheric pressure. For instance, the pressure gauge on the compressed air system in the figure would read 20 psig = 137.8 kPa gauge; the building height (by tape measure or transits) might be given as 100 ft = 30.5 m elevation. Both such measurements usually involve negative values, based on the local datum; the basement has a negative elevation relative to the street, $-30\text{ ft} = -9.15\text{ m}$, and the vacuum system has a negative pressure relative to the atmosphere, $-5\text{ psig} = -34.5\text{ kPa}$ gauge.

Negative elevations relative to sea level can exist; the Dead Sea, for instance, is about 1200 feet (366 m) below sea level. Can negative absolute pressures exist? Certainly; a negative absolute pressure is a negative compressive stress, i.e., a tensile stress. These occur often in solids, very rarely in liquids, never in gases. They are rare in liquids because all liquids possess a finite vapor pressure. If the pressure of a liquid is reduced below its vapor pressure, the liquid boils and thus replaces the low pressure with the equilibrium vapor pressure of the liquid. However, this boiling never takes place spontaneously in an absolutely pure liquid [8], but rather occurs around small particles of impurities or at the wall of the container. (Most people have observed this phenomenon when they pour a cold carbonated drink into a glass; the bubbles form mostly at the edge of the glass, not in the bulk of the liquid. It can be shown dramatically by dropping some sugar into a cold, fresh glass of soft drink; do this over a sink!) Thus, if a liquid is very pure and the surfaces of its container are very smooth, the liquid can exist in tension at a negative absolute pressure. This situation is unstable, and a slight disturbance can cause the liquid to boil [9].

1.7 FORCE, MASS, AND WEIGHT

In fluid mechanics we are often concerned with forces, masses, and weights. The problem of units of force and mass is discussed in the next section. An unbalanced force makes things change speed or direction. Most forces in the world are balanced by opposite forces (a building exerts a force on the ground; the ground exerts an equal and opposite force on the building; neither moves). To make anything start moving or stop moving, we must exert an unbalanced force.

Mass is an indication of how much matter is present. The more matter, the more mass. (We may think of matter in any size, as bricks, molecules, atoms, nucleons, quarks, etc.) Mass is also an indicator of how hard it is to get some amount of matter moving or how hard it is to stop it once it is moving. We can all stop a baseball moving 50 ft / s (15.2 m / s) with little more damage than a possible sore hand. If we step in front of an automobile moving at the same speed, we will certainly be killed. The auto has much more mass; it is much harder to stop.

Weight is a force—the force that a body exerts due to the acceleration of gravity. When there is no gravity, there is no weight (e.g., in earth satellites there is no apparent gravity; this state is referred to as *weightlessness*).

1.8 UNITS AND CONVERSION FACTORS

Engineering is about real physical things, which can be measured and described in terms of those units of measure. Most engineering calculations involve these units of measure. It would be simple if there were only one set of such units that the whole world agreed upon and used, but that is not the case today. In the United States, most measurements use the English system of units, based on the foot, the pound, and the °F, but most of the world uses the metric (or SI) system of units based on the meter, the kg, and the °C. The metric system has been legally accepted in the United States since 1866, and it has been the declared policy of the U.S. government to convert to metric since 1975 [10]. Progress has been disappointingly slow.

The situation is similar with languages; it would be easier if we all spoke one language. But we do not; the world has many languages. Educated Europeans all speak at least two languages well and generally can read one or two more. Similarly, U.S. engineers must be fluent in English and in metric units and be able to understand older literature written in the centimeter-gram-second (cgs) system, and in variant English systems that use the poundal or the slug, and in specialized industrial units, like the 42-gal barrel for petroleum products or pressure differences expressed in inches of water. U.S. engineers must even deal with mixed systems, like automotive air pollutant emissions expressed in grams per mile. Furthermore, they must understand the differences between the common-use version of metric and SI, discussed below; they will be better able to deal with those differences if they understand why the differences arise.

In fluid mechanics we most often deal with dimensioned quantities, such as 12 ft / s (= 3.66 m / s), rather than with pure numbers such as 12 or 3.66. We often drop the units, for example, “I was driving 60,” which in the United States normally means 60 mi / h, but in the rest of the world means 60 km / h. This is poor practice, but common. In 1999 [11] a $125 million NASA Mars probe was destroyed because someone failed to check their units. In technical work we always make clear the units in which any value is expressed! To become competent at solving fluid mechanics problems, we must become virtually infallible in the handling of such units and their conversion factors. For most engineers the major sources of difficulties with units and conversion factors are carelessness and the simultaneous appearance of force and mass in the same equation.

A useful "system" for avoiding carelessness and consistently converting the dimensions of engineering quantities from one set of units to another has two rules:

1. *Always* (repeat, *always*) include the dimensions with any engineering quantity you write down.
2. Convert the dimensions you have written down to the dimensions you want in your answer by multiplying or dividing by 1.

Example 1.4. We are required to convert a speed of 327 mi / h to a speed in ft / s. The first step is to write the equation

$$\text{Speed} = 327\ \text{mi}/\text{h} \tag{1.L}$$

This is not the same as 327 km / h or 327. If we omit the dimensions, our equation is meaningless. We now write, as an equation, the definition of a mile:

$$1\ \text{mi} = 5280\ \text{ft} \tag{1.M}$$

Dividing both sides of this equation by 1 mi, we find

$$\frac{1\ \text{mi}}{1\ \text{mi}} = 1 = \frac{5280\ \text{ft}}{\text{mi}} \tag{1.N}$$

You may not be used to thinking of 5280 ft / mi as being the same thing as 1, but Eq. 1.N shows that they are the same. Similarly, we write the definition of an hour as an equation,

$$1\ \text{h} = 3600\ \text{s} \tag{1.O}$$

and divide both sides by 3600 s to find

$$\frac{3600\ \text{s}}{3600\ \text{s}} = 1 = \frac{\text{h}}{3600\ \text{s}} \tag{1.P}$$

Again, you may not be used to thinking of 1 h / 3600 s as the same thing as 1, but it is. Now let us return to Eq. 1.L and multiply both sides by 1 twice, choosing our equivalents of 1 from Eqs. 1.N and l.P:

$$\text{Speed} \cdot 1 \cdot 1 = \frac{327\ \text{mi}}{\text{h}} \cdot \frac{5280\ \text{ft}}{\text{mi}} \cdot \frac{\text{h}}{3600\ \text{s}} \tag{1.Q}$$

We can now cancel the two 1's on the left side, because they do not change the value of "Speed," and we can cancel the units that appear both above and below the line on the right side to find

$$\text{Speed} = \frac{327\ \cancel{\text{mi}}}{\cancel{\text{h}}} \cdot \frac{5280\ \text{ft}}{\cancel{\text{mi}}} \cdot \frac{\cancel{\text{h}}}{3600\ \text{s}} = \frac{327 \cdot 5280\ \text{ft}}{3600\ \text{s}} = 480\ \frac{\text{ft}}{\text{s}} = 146\ \frac{\text{m}}{\text{s}} \tag{1.R}$$

■

This was an easy example, one you could certainly solve without going into as much detail as shown here, but it illustrates the procedure to be used in more complicated problems.

Example 1.5. Suppose Time equals 2.6 h. How many seconds is this? Again we begin by writing Time with its dimension as an equation:

$$\text{Time} = 2.6\ \text{h} \tag{1.S}$$

We want to know its value in seconds, so we divide by 1,

$$\text{Time} = 2.6\ \text{h} \cdot \frac{3600\ \text{s}}{\text{h}} = 2.6 \cdot 3600\ \text{s} = 9360\ \text{s} \tag{1.T}$$

■

How did we know to multiply by 1 h / 3600 s in Example 1.4 and to divide by 1 h / 3600 s in Example 1.5? In each case, we chose the value of 1 that allowed us to cancel the unwanted dimension. Three ideas are involved here:

1. Dimensions are treated as algebraic quantities and multiplied or divided accordingly.
2. Multiplying or dividing any quantity by 1 does not change its value.
3. Any dimensioned equation can be converted to 1 = 1 by dividing through by either side.

Using the last procedure, we can write

$$1 = \frac{60\ \text{s}}{\text{min}} = \frac{12\ \text{in}}{\text{ft}} = \frac{7000\ \text{gr}}{\text{lbm}} = \frac{\text{mi}^2}{640\ \text{acres}} = \frac{\text{Btu}}{252\ \text{cal}} = \frac{\text{W}}{\text{VA}} = \text{etc.} \tag{1.8}$$

and as many other values of 1 as we like.

The previous examples did not involve the unit conversions that cause difficulties, the ones involving force and mass or thermal and mechanical energies. If everyone always used SI, we would never have those difficulties. In SI there is no difficulty with the units of force and mass; force is measured in newtons (N) and mass in kilograms (kg), and the only unit of energy is the mechanical-energy unit, the joule, where $J = N \cdot m$.

Unfortunately, in the English system (and in the traditional metric system as it is used by the public in Europe), there is difficulty with force-mass unit conversion. If we ask a typical European male what he weighs, he might well respond “80 kilos,” meaning 80 kg. If he were speaking in SI, he would not use kg as a unit of weight, because weight is a force and the SI unit of force is the newton. He should respond, “784.8 newtons” because that is the weight of an 80 kg mass in a standard gravitational field of $9.807\ \text{m/s}^2 = 32.17\ \text{ft/s}^2$. It is hard enough to teach novice engineers the difference between weight and mass; it is probably impossible to get the general public to take the view that a mass of 80 kg does not exert a force of 80 kg. To make this come out right, we need to decide that there are really *two* kilogram units, the kilogram-mass (kgm) and the kilogram-force (kgf). We can define these so that one kgm exerts a force of one kgf at standard gravity. That is what most of the people in the world actually do. Similarly, in the English system of units we need two kinds of pounds; pound-mass (lbm) and pound-force (lbf). Again we have defined these so that one lbm has a weight of (exerts a force of) one lbf at standard gravity.

Why does this cause problems? Because the kgm and kgf look like the same thing, so we are tempted to believe they are the same thing, and the lbm and the lbf look like the same thing, so we are tempted to believe they are the same thing. That is wrong. It is a trap for the unwary. They are not the same. This leads to serious errors in engineering calculations.

Newton's second law of motion is

$$F = ma \tag{1.9}$$

where F is force, m is mass, and a is acceleration. The pound-force (lbf) is defined as that force which, acting on a mass of 1 lbm, produces an acceleration of 32.2 ft / s^2. Substituting this definition into the last equation, we find

$$1 \text{ lbf} = \text{lbm} \cdot 32.2 \frac{\text{ft}}{\text{s}^2} \tag{1.U}$$

Dividing both sides of this by 1 lbf, we find

$$1 = \frac{\text{lbf}}{\text{lbf}} = 32.2 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \tag{1.V}$$

If we then make the mistake of canceling the lbm on the top and the lbf on the bottom right-hand side, we will conclude that $1 = 32.2$ ft / s^2. This is clearly wrong, and if we do it in a problem we will find that the dimensions do not check and the numerical value of the answer will be wrong by a factor of 32.2 (if we use English units) or 9.8 (if we use metric units). Similarly, in the traditional metric system, we have

$$1 \text{ kfg} = \text{kgm} \cdot 9.8 \text{ m} / \text{s}^2 \tag{1.W}$$

and if we divide both sides by kgf, we find

$$1 = \frac{\text{kgf}}{\text{kgf}} = 9.8 \frac{\text{kgm} \cdot \text{m}}{\text{kgf} \cdot \text{s}^2} \tag{1.X}$$

If we then cancel kgm and kgf on the right side, we will conclude that $1 = 9.8$ m / s^2, which is equally absurd.

How can we get out of this difficulty? One way is to always work exclusively in SI. In that case kg will always mean kgm, and kgf will never appear. Instead the unit of force will always be the N = (1 / 9.8) kgf. However, then we will be unable to deal with the public, who speak (unintentionally) in kgf and lbf, or to deal with those parts of the engineering literature that use kgf and lbf. The other way is to decide we must live with the kgf and lbf, and so we will regularly have to use the force-mass conversion factor whenever units of force and of mass occur in the same equation. This conversion factor has the following values:

$$1 = 9.8 \frac{\text{kgm} \cdot \text{m}}{\text{kgf} \cdot \text{s}^2} = 1.0 \frac{\text{kgm} \cdot \text{m}}{\text{N} \cdot \text{s}^2} = 32.2 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \tag{1.10}$$

Furthermore, we must know some history to understand the older literature. First, we must know that many older textbooks and articles used the symbol g_c to stand for this force-mass conversion factor. So whenever we see a g_c written into an equation, we must recognize it as a reminder that we must use the force-mass conversion factor. We must not confuse g_c, the force-mass conversion factor, with g, the acceleration of gravity; they are *not* the same.

Second, we should recognize that engineers using English units have tried to evade this difficulty by inventing two new units, the slug (1 slug = 32.2 lbm = 14.6 kg) and the poundal (pdl) (1 pdl = lbf / 32.2 = 0.138 N = 0.014 kgf). Using these, we have the following force-mass conversion factors:

$$1 = 9.8\,\frac{\text{kgm}\cdot\text{m}}{\text{kgf}\cdot\text{s}^2} = 1.0\,\frac{\text{kgm}\cdot\text{m}}{\text{N}\cdot\text{s}^2} = 32.2\,\frac{\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 1.0\,\frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 1.0\,\frac{\text{lbm}\cdot\text{ft}}{\text{pdl}\cdot\text{s}^2} \qquad (1.11)$$

Chemical engineers rarely use the slug or the poundal, but other branches of engineering do; we must recognize them when we see them.

The kgf and the lbf have been around a long time, in spite of the efforts of scientists and engineers to replace them with the newton or the poundal. They survive because they seem natural to nonscientific users. Probably they will continue to be widely used, in spite of the efforts of the scientific community to replace them. Prudent engineers will learn to live with this fact, to use them when it seems appropriate, and to understand why they came about.

The second difficulty with units concerns mechanical and thermal units of energy. In SI the only unit of energy is the joule, 1 J = 1 N · m. This is clearly a mechanical unit, the product of a force and a distance. If we are transferring thermal energy (e.g., heating our houses or our soup), it seems natural to base the measurements on the quantity of thermal energy required to raise the temperature of some reference substance by some finite temperature interval. In the English system this quantity is the British thermal unit (Btu), which is the quantity of thermal energy required to raise the temperature of 1 lbm of water by 1°F. In the metric system the unit is the calorie (cal), which is the quantity of thermal energy required to raise the temperature of 1 g of water 1°C, or the kcal (kcal = 1000 cal; this is the "calorie" used in describing the energy content of foods). If we want to use the calorie or the Btu, then we need to convert from joules to calories or ft-lbf to Btu:

$$1 = \frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}} = \frac{\text{Btu}}{1055\ \text{J}} = \frac{\text{cal}}{4.18\ \text{J}} = \frac{\text{kcal}}{4180\ \text{J}} = \frac{\text{kcal}}{4.18\ \text{kJ}} \qquad (1.12)$$

The Btu and the cal (or kcal) seem likely to continue in common usage; the Btu appears on almost all U.S. heating appliance and fuel bills (sometimes natural gas bills use the *therm* = 10^5 Btu or the *Dekatherm* = 10 therm = 10^6 Btu), and kcal appears on numerous food products.

In summary, if we can do all our work in SI, we need never be concerned about force-mass conversions (N = kg · m / s) or energy conversions (J = N · m = W · s). If we are confronted with problems (or literature, or current U.S. legal definitions) involving the kgf, lbf, cal, kcal, or Btu, we must follow the rules outlined above. Always write down the dimensions, treat the dimensions as algebraic quantities, and multiply by 1 as often as needed to get the quantities into the desired set of units, using the appropriate values of the force-mass conversion factor and the thermal-mechanical energy conversion factor. Even in SI, if we stray from the basic units (m, kg, s, A, K, mol, and cd), we will need conversion factors such as

$$1 = \frac{1000\ \text{g}}{\text{kg}} = \frac{100\ \text{cm}}{\text{m}} = \frac{1000\ \text{mV}}{\text{V}} \tag{1.13}$$

Example 1.6. A mass of 10 lbm (4.54 kgm) is acted on by a force of 3.5 lbf (15.56 N or 1.59 kgf). What is the acceleration in ft/min^2?

Rearranging Eq. 1.9, we find

$$a = F / m \tag{1.14}$$

Substituting, we find

$$a = \frac{3.5}{10}\,\frac{\text{lbf}}{\text{lbm}} \tag{1.Y}$$

Here we want the acceleration in ft/s^2 so we multiply or divide by those equivalents of 1 that will convert the units:

$$a = \frac{3.5\ \text{lbf}}{10\ \text{lbm}} \cdot \frac{32.2\ \text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} = \frac{3.5 \cdot 32.2}{10}\,\frac{\text{ft}}{\text{s}^2} = 11.27\,\frac{\text{ft}}{\text{s}^2} = 3.44\,\frac{\text{m}}{\text{s}^2} \tag{1.Z}$$

or

$$a = \frac{15.56\ \text{N}}{4.54\ \text{kg}} \cdot \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2} = \frac{15.56}{4.54}\,\frac{\text{m}}{\text{s}^2} = 3.43\,\frac{\text{m}}{\text{s}^2} = 11.25\,\frac{\text{ft}}{\text{s}^2} \tag{1.AA}$$

or

$$a = \frac{1.59\ \text{kgf}}{4.54\ \text{kgm}} \cdot \frac{9.8\ \text{kgm} \cdot \text{m}}{\text{kgf} \cdot \text{s}^2} = \frac{1.59 \cdot 9.8}{4.54}\,\frac{\text{m}}{\text{s}^2} = 3.43\,\frac{\text{m}}{\text{s}^2}\ 11.26\,\frac{\text{ft}}{\text{s}^2} \tag{1.AB}$$

The difference between these three answers is due to round-off error in the conversion factors used. If more figures had been carried (e.g., kgf = 9.80650 N), the answers would have agreed exactly, but since we know the input data to only two significant figures, our best answer, in all three cases, should be $11.3\ \text{ft}/\text{s}^2$.

■

Example 1.6 will be the last example in this book to use the kgf. Clearly the method of dealing with kgm and kgf is just the same as the method of dealing with lbm and lbf. For the rest of this book, we will use either lbm and lbf, or SI.

Example 1.7. An aluminum cell (Hall-Héroult process) has a current of 50000 amp. If we assume it is 100% efficient, how much metallic aluminum does it produce per hour?

We first convert the current to gram equivalents per hour, using the necessary values of 1, one of which we take out of Prob. 1.16:

$$I = 50000\ \text{A} \cdot \frac{\text{C}}{\text{A} \cdot \text{s}} \cdot \frac{3600\ \text{s}}{\text{h}} \cdot \frac{\text{g equiv}}{95600\ \text{C}} = 1870\,\frac{\text{g equiv}}{\text{h}} \tag{1.AC}$$

For aluminum,

$$27 \text{ g} = 1 \text{ mol} \tag{1.AD}$$

and

$$1 \text{ mol} = 3 \text{ g equiv} \tag{1.AE}$$

therefore,

$$I = 1870 \frac{\text{g equiv}}{\text{h}} \cdot \frac{\text{mol}}{3 \text{ g equiv}} \cdot \frac{27 \text{ g}}{\text{mol}} \cdot \frac{\text{lbm}}{454 \text{ g}} = 37.1 \frac{\text{lbm}}{\text{h}} = 16.8 \frac{\text{kg}}{\text{h}} \tag{1.AF}$$

■

In solving Example 1.7, we multiplied by 1 six times. Nonetheless, the procedure is simple and straightforward. Each multiplication by 1 gets rid of an undesired dimension and brings us closer to an answer in the desired units. We saw that an apparently complex problem was really a simple conversion-of-units problem. In the course of our studies and our professional careers we will have to convert units as quickly and as easily as we now add and subtract. It will be easiest if we develop the habit of following the two rules given at the start of Sec. 1.8, namely:

1. Always include the dimensions with any engineering quantity you write.
2. Convert the dimensions you have written to the dimensions you want in your answer by multiplying or dividing by 1.

A short table of these conversion factors can be found in App. D. The American National Standard for Metric Practice [12] presents a much longer and more complete table, which reveals some additional complexity.

1.9 PRINCIPLES AND TECHNIQUES

As discussed in Sec. 1.3, there are very few underlying ideas in fluid mechanics. With these few ideas we can solve a great variety of problems. In so doing, we can focus our attention either on the application of principles or on the techniques of solving problems. The author recommends attention to the principles. In the 10 years following his graduation from college, the engineering business was revolutionized by the digital computer, the transistor, and the space industry, among other things. None of these amounted to much in 1954, and they were not part of undergraduate courses.

All these technologies rigidly obey Newton's laws and the laws of thermodynamics. Students who learned "cookbook" techniques for solving problems on 1954 were not well prepared for the technologies that appeared during the next 10 years, but those who learned the basic principles and how to apply them could adapt to any one of them. There seems to be little reason to believe that the pace of technological change will become slower in the future. If we concentrate on learning techniques, we may be faced in a few years with "technical obsolescence," but if we learn principles and their applications, we should have no such problem. The author believes that there will never be a surplus of people who *really* understand Newton's laws and the laws of thermodynamics.

1.10 ENGINEERING PROBLEMS

Although this book may fall into the hands of a practicing engineer, most of its readers will be college juniors; the following is addressed to them.

Engineering students start out in their freshman and sophomore years by doing "plug-in" problems. Given a problem statement, they select the appropriate formula either from the textbook or from their memory, and "plug in" the data in the problem to find the final answer. In their junior year they begin to find problems that can be readily reduced to plug-ins or to problems involving two or more equations that require some manipulations to be put in plug-in form. Furthermore, they may be exposed to problems that cannot be reduced to plug-ins and must be solved by trial and error. It is assumed that they can do simple plug-ins (such as gas-law calculations) without hesitation.

Instructors of third-year students would like to assign more complicated or difficult problems but generally cannot because:

1. The time required for them is too great—they cannot be done in the time that most students will devote to one homework problem.
2. The students would probably get intellectual indigestion on them. Therefore, at the third-year level most of the problems and examples in texts like this one are plug-ins or can be readily reduced to plug-ins.

When students start a senior laboratory or design course, they find their first real engineering problems. One of these may require 10 or 20 h of work and consist of 15 or 20 parts, each comparable to the problems and examples in this book. To deal with these problems, students break them into pieces small enough to handle as plug-ins. The interesting and exciting part of engineering is often the task of deciding how to divide a problem into reasonable pieces and how then to reassemble these pieces into a recognizable whole so that they fit together properly.

In the examples and problems in this book, there are numerous simple plug-in problems. They are included because their solutions give the reader some feel for the numerical values involved in fluid mechanics. There are also more complex problems, in which two or more basic principles are involved (such as the mass balance and the energy balance). In these, some manipulation is required to get the equations into plug-in form. The recommended procedure for solving such problems is this:

1. Make sure you understand precisely what the problem is; in particular, make sure you know precisely what is being asked for.
2. Decide which physical laws relate what you know to what you want to find.
3. Write the working form of these laws (as discussed later), and rearrange them to get the symbol for the quantity you seek standing alone to the left of the equal sign. In so doing, you will probably have to discard several terms in the physical-law equations. Discarding a term corresponds to making an assumption about the physical nature of the system (e.g., that a certain velocity is negligible). Thus, a list of such terms dropped is a list of assumptions made in solving the problem.

4. When step 3 is finished, the problem is reduced to a plug-in. Insert the given data, check the units, and find the numerical value of the answer.
5. Check the answer for plausibility: Does it indicate negative masses, velocities greater than the speed of light, or efficiencies greater than 100%? Does it pass the test of common sense, that is, do the results match your intuitive idea of what they should be? If not, is the difficulty with the calculations? or with your intuition? If neither is incorrect, perhaps you have made a new technical discovery! Also, re-examine the assumptions listed in step 3 to see whether they are consistent with the answer. If these checks are met, the answer probably is satisfactory.
6. If the problem is one that you may have to repeat with different data (such as the calculation of a fluid-flow rate from a measured pressure difference), then it might be worthwhile to see whether the answer can be put in a more convenient form, for example, some general plot or diagram.
7. Currently most engineers have access to a variety of computer programs. Many of these require the user to learn a new language. Some fluids books introduce these as tools for problems and examples. In this book I use only Microsoft Excel. Its advantages include extreme simplicity, high intuitive content, worldwide acceptance, and ease of file transfer. It maintains backward compatibility; Excel documents from 20 years ago are easily read by the current versions, which is not the case for Microsoft Word and many other programs. All of the examples in this book have been easily checked for numerical accuracy using Excel. When one of the examples in the 3rd edition was found to be in error, its Excel program was reviewed, and the error promptly found and easily corrected. The use of Excel to solve trial-and-error problems is explained in detail in Example 6.5, and then used without such detailed explanation in the remaining such solutions throughout the book.

   Such solutions are simpler and more intuitive than the solutions in other programming languages. I strongly urge all readers to become fluent in Excel. Once you do, you will find yourself preferring to do all routine numerical computations (e.g., balancing your checkbook or preparing your income tax) with it, in preference to all other methods.

In all engineering we must consider the degree of precision needed. Voltaire's famous dictum "The perfect is the enemy of the good!" describes the situation of the engineer. We could always spend more engineering effort, and do more testing, and thereby refine our design or our calculation a little more. But, in any real problem the engineer's time is one of the limiting resources. We would all like the conditions that the famous architect Kobori Enshu demanded and received from the Japanese dictator Hideyoshi for the Katsura Villa: no limit on expense, no limit on time, and no client visits until the job is done. Many believe the result to be the greatest achievement of Japanese architecture and garden planning [13]. (If you are ever in Kyoto, visit it and decide for yourself.) But most engineers (and other professionals) are always working with limited time and limited budgets as well as clients who want intermediate progress reports. For us the goal is always to do the best possible, within the time, budget, and other constraints imposed by the client (or codes and regulations). So engineers must allocate their time well, handling routine things swiftly, and concentrating on those that are not routine and that may be a source of trouble. Much of what you learn in this book is routine to practicing engineers. The goal of this book is that students not only learn to do those routine things but also learn the scientific basis of the solution of those routine problems. In so doing, you will learn how engineers and scientists have turned yesterday's difficult problems into today's routine ones. That will help you to develop the habits of mind that will turn today's difficult problems into tomorrow's routine problems.

You should consider your degree of confidence in the answer to a problem. If the calculation used physical property data that is accurate to no more than ±5%, then it makes no sense to report the answer to 3 or more significant figures. If the solution presented required really speculative calculating approaches, or questionable input data, the reader should be alerted to that fact.

In the problems at the end of each chapter, one or two need to be broken down into simpler ones before they can be solved. The practice gained in doing these is well worth the effort.

1.11 WHY THIS BOOK IS DIFFERENT FROM OTHER FLUID MECHANICS BOOKS

Most undergraduate fluid mechanics books are written by mechanical or civil engineers. Please look at one; your impression will be that those books and this one are about totally different subjects. The reasons they look so different are:

1. The fluid mechanics problems of greatest interest to mechanical and civil engineers (aerodynamics, flow around structures) are inherently two- or three-dimensional. They cannot be understood as or easily reduced to one-dimensional form. Most of the fluid mechanics problems of greatest interest to chemical engineers are inherently one-dimensional or can be understood and easily reduced to one-dimensional form. For this reason, civil and mechanical engineers start fluid mechanics as a three-dimensional study, and then derive the one-dimensional forms of greatest interest to chemical engineers from those three-dimensional forms.
2. Mechanical and civil engineers base most of their work on force and momentum. Those are the basic tools of the mechanical and civil engineer. Chemical engineers base most of their work on the conservation of mass and energy; the first course in chemical engineering is about mass and energy balances. Chemical engineers learn about force and momentum in physics but use them much less in their professional careers than they use mass and energy. The single most useful equation in fluid mechanics, Bernoulli's equation, can be found by starting with force and momentum, or with energy. Mechanical and civil engineers start with momentum. This book starts with energy. The energy approach makes much more sense to chemical engineers than does the momentum approach.
3. Momentum and force are vectors. For mechanical and civil engineers, fluid mechanics is inherently an exercise in vector calculus. Their books are full of vector equations. Many take the view that one of the main purposes of a fluid mechanics course is to immerse their students in the vector calculus, and make them exercise it. Mass and energy are scalars. Most of the quantities in chemical engineering are also scalars. Thus, chemical engineers have much less use of the vector calculus than do mechanical and civil engineers. Our graduate students are normally expected to become good at the vector calculus, but our undergraduates rarely use it.

For these reasons, this book uses scalars as much as possible and vectors only when necessary. It begins with the conservation of mass and energy and shows the vast range of practical fluid mechanical problems that can be solved with them, before it shows the momentum balance (which is inherently a vector balance) and shows the problems for which we need it. As a consequence, this book has far simpler mathematics than other fluids books. That does not mean that it sacrifices rigor; complexity is not rigor, or simplicity carelessness. In many cases the complete derivations are shown in appendices, with only the practical result shown in the main text.

In Parts II and III of this book, we cover the wide range of fluid mechanical problems of interest to chemical engineers that are best approached in a one-dimensional, energy-first approach. Then in Part IV, we introduce the two- or three-dimensional, momentum-first approach, and discuss some of the chemical engineering problems that are best approached that way.

Figure 1.14 shows a chemical processing plant, in which lower-price chemicals are converted to higher-price (more useful) chemicals for profit and social benefit (and jobs for chemical engineers!). Many readers of this book will participate in the design, construction, and/or operation of similar plants. In such a plant the fluid flows are almost entirely inside pipes, pumps, vessels, fractionators, reactors, etc. We keep them inside because they are too valuable to waste and/or because their release would be dangerous or polluting. Almost all the flows in such a plant are most easily studied, predicted, and managed by the one-dimensional, mass-and-energy balance approach that forms Parts II and III of this book.

FIGURE 1.14
The mercaptan manufacturing unit at the Borger, Texas, complex of the Chevron Phillips Chemical Company. This plant is full of fluid flows, almost all of which are inside pipes, pumps, distillation columns, and associated vessels. (Leonid Eremeychuk/Shutterstock.)

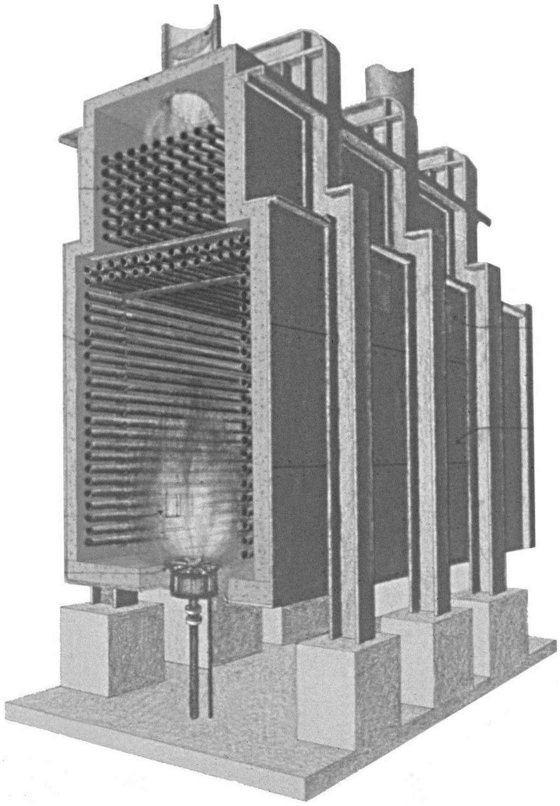

FIGURE 1.15
Cutaway drawing of a modern industrial furnace. The external steel frame supports the high-temperature refractory ceramic walls. There are multiple burners at the bottom, of which only one is shown. The flame heats the walls and the pipes through which the fluid being heated flows. Above the combustion chamber the hot gases pass over another bank of tubes, in which cooler fluid is warmed by the hot gases before they pass up the exhaust stacks seen at the top. (Courtesy of John Zink Co. LLC.)

Figure 1.15 shows a schematic of a "cabin-type" industrial furnace. These are widely used for pyrolysis and reforming reactions in chemical engineering. Fifty years ago these were designed by hand using the one-dimensional methods presented in Parts II and III. With the recent spectacular advances in computer power, such furnaces are now designed using the two- and three-dimensional fluid mechanics methods presented in Part IV. Those methods and their computer implementation were largely developed by aeronautical engineers, to deal with the inherently three-dimensional flow around airplanes. Furnace designers and other chemical engineers now use large computer codes to model the simultaneous three-dimensional fluid flow, heat transfer, and chemical reactions in such furnaces. The improvement in computational accuracy more than repays the additional cost and complexity. Part IV only introduces the basic ideas underlying such computations, and gives a bit of their history.

1.12 SUMMARY

1. Fluid mechanics is the study of forces and motions in fluids.
2. Fluids are substances that move continually when subjected to a shear force as long as the force is applied. Solids are substances that deform slightly when subjected to a shear force and then stop moving and permanently resist the force. There are, however, intermediate types of substance; the distinction between solid and liquid is one of degree rather than of kind.
3. Fluid mechanics is based on the principle of the conservation of matter, the first two laws of thermodynamics, Newton's laws of motion, and careful experiments.

4. Gases have weak intermolecular attractions and expand without limit. Liquids have much stronger intermolecular attractions and can expand very little. With increasing temperature and pressure, the differences between liquids and gases gradually disappear.
5. Density is mass per unit volume. Specific gravity of liquids is density / (density of water at 4°C). Specific gravity of gases is density / (density of air at the same T and P).
6. Viscosity is a measure of a fluid's resistance to flow. Most simple fluids are represented well by Newton's law of viscosity. The exceptions (non-Newtonian fluids) are generally complex mixtures, some of which are of great practical significance. Kinematic viscosity is viscosity divided by density.
7. Surface tension is a measure of a liquid's tendency to take a spherical shape, caused by the mutual attraction of the liquid's molecules.
8. Pressure is compressive force divided by area. It is the same in all directions for a fluid at rest and practically the same in all directions for most moving fluids.
9. In handling the units (dimensions) in this text, one should always write down the units of any dimensioned quantity and then multiply or divide by 1 to find the desired units in the answer.
10. Much of fluid mechanics can be based either on force and momentum, or on mass and energy. This book, for chemical engineers, bases most of fluid mechanics or on mass and energy, thus dealing mostly with scalars instead of vectors. Momentum and vectors are used where they are needed.

PROBLEMS

See the Common Units and Values for Problems and Examples in App. E. An asterisk (*) on the problem number indicates that the answer is in App. C.

1.1. In Sec. 1.3 the basic laws on which fluid mechanics rests are listed. How many of the basic laws of nature are not included in the list? To answer this question, make a list of what you consider to be the basic laws of nature. By basic laws, we mean laws that cannot be derived from other more basic ones; for example, Galileo's "laws of falling bodies" can be derived form Newton's laws and are not basic.

1.2. At low pressures there is a significant difference between the densities of liquids and of gases. For example, at 1 atm the densest gas known to the author is uranium hexafluoride, which has $M = 352$ g / mol; its normal boiling point is 56.2°C. Calculate its density in the gas phase at 1 atm and 56.2°C, assuming that it obeys the ideal gas law. The least dense liquid known to the author is liquid hydrogen, which at its normal boiling point, 20 K, has a density of 0.071 g / cm^3. Liquid helium also has a very low density, about 0.125 g / cm^3 (at 4 K). Excluding these remarkable materials, make a list of liquids that at 1 atm can exist at densities of less than 0.5 g / cm^3. A good source of data is *The Handbook of Chemistry and Physics,* CRC Press, Boca Raton, Florida, annual editions.

1.3.*For some oil and gas drilling operations, we need a high-density drilling fluid (called "drilling mud"). Repeat Example 1.1 for a mud that is 50 wt. % water, 50 wt. % $BaSO_4$ (barite), $SG_{\text{barite}} = 4.49$.

1.4. Why are specific gravities most often referred to the density of water at 4°C instead of 0°C?

1.5.*A special-purpose piece of laboratory glassware, called a *pyncnometer,* is used to measure liquid densities. It has a volume of 25 cc, and a mass of 17.24 g when it is full of air.

When filled with a liquid of unknown density, its mass = 45.00 g. What is the density of this liquid? How large an error do we make if we ignore the mass of the air that was in it when we weighed it and found $m = 17.24$ g?

1.6. The American Petroleum Institute (API) gravity (used extensively in the petroleum industry) is defined, in "degrees," by

$$\text{Deg API} = \frac{141.5}{\text{specific gravity}} - 131.5 \tag{1.AG}$$

Here the specific gravity is the ratio of the density of the liquid to that of water, both at 60°F. Sketch the relation between density in g/cm^3 and degrees API. What advantages of this scale might have led the petroleum industry to invent and adopt it?

1.7. Estimate the specific gravities (gas) for methane and propane. Their molecular weights are shown in App. E. (Commercial natural gas and commercial propane are mostly methane and propane, with small amounts of other substances, which may be ignored for this problem.) Which is more dangerous, a natural gas leak or a propane leak? Why?

1.8. What are the dimensions of dV/dy? What are the dimensions of shear stress? Shear stress in liquids is often called "momentum flux" [2]. Show that shear stress has the same dimensions as momentum / (area · time). What are the dimensions of viscosity?

1.9. List as many applications as you can of industrial, domestic, or other materials in which non-Newtonian viscosity behavior is desirable. In each case, specify why this behavior is desirable.

1.10. In Example 1.2 we replaced a cylindrical problem with a linear approximation. The velocity distribution for this flow, taking the cylindrical character into account (see Prob. 15.22 and also [2], p. 91), is

$$V_\theta = \omega\left(\frac{k^2}{1-k^2}\right)\cdot\left(\frac{R^2}{r} - r\right) \tag{l.AH}$$

where R is the radius of the outer cylinder, r is the local radius, $k = r_{\text{inner cylinder}}/R$, and ω is the angular velocity of the inner cylinder.

(*a*) Verify that this distribution shows a zero velocity at the radius of the outer, non-moving cylinder and shows $V_\theta = \omega kR$ at the surface of the inner, rotating cylinder.

(*b*) The shear rate in cylindrical coordinates, for a fluid whose velocity depends only on r (equivalent to dV/dy in rectangular coordinates), is given by

$$\sigma = \begin{pmatrix}\text{shear rate cylindrical}\\ \text{coordinates}\end{pmatrix} = r\frac{d}{dr}\left(\frac{V_\theta}{r}\right) \tag{1.AI}$$

Show that for the above velocity distribution, the shear rate at the surface of the inner cylinder is given by

$$\sigma = \omega\left(\frac{2}{1-k^2}\right) \tag{1.AJ}$$

(*c*) Show that the shear rate computed by Eq. 1.AJ using the values in Example 1.2 is 12.26 / s, which is 1.15 times the value for the flat approximation in Example 1.2. The manual for the viscometer shown in Fig. 1.5 provides formulae equivalent to those in this problem.

1.11.* Calculate the surface / volume of a sphere, a cube, and a right cylinder of height equal to diameter. Which has the least surface / volume?

1.12. A liquid under tensile stress is unstable [9]; a small disturbance can cause it to boil and thereby change to a stable state. Make a list of other unstable situations demonstrable in a chemistry or physics laboratory. The working criterion of instability is that a very small disturbance can cause a large effect.

1.13. Earth may be considered a sphere with a diameter of ≈ 8000 mi and an average SG of ≈ 5.5. What is its mass? What is its weight? Explain your answer.

1.14. A cubic foot of water at 68°F = 20°C weighs 62.3 lbf on earth.
(*a*) What is its density?
(*b*) What does it weigh on the moon ($g \approx 6$ ft / s^2)?
(*c*) What is its density on the moon?

1.15.* How many U.S. gallons are there in a cubic mile? The total proven oil reserves of the United States are roughly $30 \cdot 10^9$ bbl. How many cubic miles is this?

1.16. In electrochemical equations, it is common to write in the symbol $\mathscr{F}$ (called *Faraday's constant*) to remind the user to convert from moles of electrons to coulombs. This is just like the force-mass and thermal energy-mechanical energy conversion factor, namely,

$$\mathscr{F} = 1 = \frac{95\,600 \text{ C}}{\text{g equiv of electrons}} \tag{1.AK}$$

1 g equiv of electrons = $6.02 \cdot 10^{23}$ electrons. How many electrons are there in 1 C?

1.17. Older thermodynamics and fluids textbooks not only put the symbol g_c into equations to remind us to make the force-mass conversion but also put a J in equations to remind us to make the conversion from mechanical units of energy (e.g., ft · lbf) to thermal units of energy (e.g., Btu). Equation 1.11 shows the values of $g_c = 1$ for a variety of systems of units. Show the corresponding equation for J. (The use of the symbol g_c caused confusion because it is similar to g. Is there a symbol with which the J discussed in this problem can be confused?)

1.18.* As discussed in the text, the slug and the poundal were invented to make the conversion factor (mass length) / (force time^2) have a coefficient of 1. A new unit of length or a new unit of time could just as logically have been invented for this. Let us name those units the *toof* and the *dnoces.* What are the values of the toof and the dnoces in terms of the foot and the second?

1.19. In U.S. irrigation practice, water is measured in acre-feet, which is the volume of water that covers an acre of land, one foot deep. What is the mass of an acre-foot of water (1 mi^2 = 640 acres)? What is the mass of a hectare-meter (ha · m) of water (km^2 = 100 ha)? Why would the acre-foot be a practical measure of irrigation water?

1.20. Einstein's equation $E = mc^2$ indicates that the speed of light squared must be expressible in units of energy per unit mass. What is the value of the square of the speed of light in Btu / lbm? In J / kg? The speed of light $c \approx 186\,000$ mi / s $= 2.998 \cdot 10^8$ m / s.

1.21.* A common basis for comparing rocket fuel systems is the *specific impulse,* defined as lbf of thrust produced divided by lbm / s of fuel and oxidizer consumed (see Chap. 7). The common values are 250 to 400 lbf · s / lbm. We frequently see the specific impulse referred to simply as "300 s." Is 300 s the same thing as 300 lbf s / lbm? European engineers regularly express the same quantity in terms of the equivalent exhaust velocity of the rocket. If a rocket has a specific impulse of 300 lbf s / lbm, what is its equivalent exhaust velocity?

1.22. Most U.S. engineers work with heat fluxes with the unit Btu / (h · ft^2). In the rocket business the common unit is cal / (s · cm^2). How many Btu / (h · ft^2) is 1 cal / (s · cm^2)? The proper SI unit is J / (m^2 · s). How many Btu / (h · ft^2) = 1 J / (m^2 · s)?

1.23.* The Reynolds number, discussed in Chap. 6, is defined for a pipe as (velocity · diameter · density) / viscosity. What is the Reynolds number for water flowing at 10 ft / s in a pipe with a diameter of 6 in? What are its dimensions?

1.24. The flow of fluids through porous media (such as oil sands) is often described by *Darcy's equation* (see Chap. 11):

$$\frac{\text{Flow}}{\text{Area}} = \frac{\text{permeability}}{\text{viscosity}} \cdot \text{pressure gradient} \tag{1.AL}$$

The unit of permeability is the *darcy,* which is defined as that permeability for which a pressure gradient of 1 atm / cm for a fluid of 1 cP viscosity produces a flow of 1 cm^3 / s through an area of 1 cm^2. What are the dimensions of the darcy? What is its numerical value in the dimension? Give the answer both in English units and in SI units. See Chap. 11.

1.25.* What mass (weight?) would be needed in Example 1.3 if the liquid had been water?

1.26. Determine the value of X in the equation,

$$1.0 \frac{\text{Btu}}{\text{lbm} \cdot {}^\circ\text{F}} = X \frac{\text{cal}}{\text{g} \cdot {}^\circ\text{C}} \tag{1.AM}$$

1.27. In strict SI, the only unit of pressure is the Pascal (Pa). The most widely used derived unit is the bar $(\text{bar} = 10^5\ \text{Pa} = 0.1\ \text{MPa})$. What is the relation between the bar and the pressure of the atmosphere at sea level? Why is the bar a popular choice for a working SI derived unit?

1.28.* Air pollutant emissions from autos and trucks in the United States are reported in a mixed metric-English unit, g / mi. Suggest reasons why this might be a practical unit.

1.29. Many European pressure gauges give the pressure in kg / cm^2. Is this kgm or kgf? Why would this be a convenient unit of pressure?

1.30. In the third part of Example 1.6, what would have happened if we had taken the force-mass conversion factor as 32.2 lbm · ft / (lbf · s^2), instead of 9.8 kgm · m / (kgf · s^2)?

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