Fundamentals of Structural Analysis

Sixth Edition

Kenneth M. Leet
Professor Emeritus, Northeastern University

Chia-Ming Uang
Professor, University of California, San Diego

Joel T. Lanning
Assistant Professor, University of California, Irvine

Mc Graw Hill

FUNDAMENTALS OF STRUCTURAL ANALYSIS, SIXTH EDITION

This book is printed on acid-free paper.

1 2 3 4 5 6 7 8 9 LCR 24 23 22 21 20

ISBN 978-1-260-47724-5 (bound edition)
MHID 1-260-47724-X (bound edition)
ISBN 978-1-260-58866-8 (loose-leaf edition)
MHID 1-260-58866-1 (loose-leaf edition)

Executive Portfolio Manager: *Michelle Hentz*
Portfolio Manager: *Beth Bettcher*
Product Development Coordinator: *Theresa Collins*
Marketing Manager: *Shannon O'Donnell*
Content Project Managers: *Jane Mohr and Sandra Schnee*
Buyer: *Laura Fuller*
Design: *Beth Blech*
Content Licensing Specialist: *Beth Cray*
Cover Image: *Sinotech Engineering Consultants, Ltd.*
Cover Information: *Danjiang Bridge. Architect: Zaha Hadid Architects; Designer: Sinotech Engineering Consultants/ Leonhardt Andra und Partner Beratende Ingenieure; Owner: Directorate General of Highways, MOTC, Taiwan*
Compositor: *Aptara®, Inc*.

Library of Congress Cataloging-in-Publication Data

Names: Leet, Kenneth, author. | Uang, Chia-Ming, author. | Lanning, Joel, author.
Title: Fundamentals of structural analysis / Kenneth M. Leet, Professor Emeritus, Northeastern University, Chia-Ming Uang, Professor, University of California, San Diego, Joel T. Lanning, Assistant Professor, University of California, Irvine, Irvine.
Description: Sixth edition. | Dubuque, IA : McGraw-Hill Education, 2021. | Includes index.
Identifiers: LCCN 2019029117 (print) | LCCN 2019029118 (ebook) | ISBN 9781260477245 (hardcover) | ISBN 9781260588668 (spiral bound) | ISBN 9781260570441 (hardcover) | ISBN 9781260588675 (ebook other)
Subjects: LCSH: Structural analysis (Engineering)
Classification: LCC TA645 .L34 2020 (print) | LCC TA645 (ebook) | DDC 624.1/7—dc23
LC record available at https://lccn.loc.gov/2019029117
LC ebook record available at https://lccn.loc.gov/2019029118

The Internet addresses listed in the text were accurate at the time of publication. The inclusion of a website does not indicate an endorsement by the authors or McGraw-Hill Education, and McGraw-Hill Education does not guarantee the accuracy of the information presented at these sites.

mheducation.com/highered

For Kenneth M. Leet

ABOUT THE AUTHORS

Kenneth M. Leet is a late Professor of structural engineering at Northeastern University. He received his Ph.D. in structural engineering from the Massachusetts Institute of Technology. As a professor of civil engineering at Northeastern University, he taught graduate and undergraduate courses in reinforced concrete design, structural analysis, foundations, plates and shells, and capstone courses on comprehensive engineering projects for over 30 years. Professor Leet was given an Excellence in Teaching award at Northeastern University in 1992. He was also a faculty member for ten years at Drexel University in Philadelphia.

In addition to being the author of the first edition of this book on structural analysis, originally published by Macmillan in 1988, he is the author of *Fundamentals of Reinforced Concrete,* published by McGraw-Hill.

Chia-Ming Uang is a Professor of structural engineering at the University of California, San Diego (UCSD). He received a B.S. degree in civil engineering from National Taiwan University and M.S. and Ph.D. degrees in civil engineering from the University of California, Berkeley.

Uang also coauthors the text *Ductile Design of Steel Structures* for McGraw-Hill. He received the UCSD Academic Senate Distinguished Teaching Award in 2004. He is also the recipient of the ASCE Raymond C. Reese Research Prize in 2001, the ASCE Moisseiff Award in 2004 and 2014, the AISC Special Achievement Award in 2007, and the T.R. Higgins Lectureship Award in 2015.

Joel T. Lanning is an Assistant Professor of Teaching in the area of structural engineering at the University of California, Irvine (UCI) and is a registered Civil Engineer in California. He received a B.S. degree in civil engineering from the Ohio State University and M.S. and Ph.D. degrees in structural engineering from the University of California, San Diego. Lanning is an award-winning educator passionate about teaching and developing effective instructional strategies in structural engineering.

TABLE OF CONTENTS

PREFACE

This text introduces engineering and architectural students to the basic techniques required for analyzing the majority of structures and the elements of which most structures are composed, including beams, frames, trusses, arches, and cables. Although the authors assume that readers have completed basic courses in statics and strength of materials, we briefly review the basic techniques from these courses the first time we mention them. To clarify the discussion, we use many carefully chosen examples to illustrate the various analytic techniques introduced, and whenever possible, we select examples confronting engineers in real-life professional practice.

Features of This Text

1. **Review topics.** Chapters 3, 4, and 5 provide a useful review of fundamental skills of statics and basic structures like trusses and beams. This edition features many revisions in these chapters aimed at helping students more quickly and effectively refresh their skills in these topics.
2. **Consistency.** In this edition, over 300 figures have been revised, sometimes extensively, to provide even more consistency and accuracy in the presentation of important aspects of the analysis methods.
3. **Expanded treatment of design loads.** Chapter 2 is devoted to a discussion of loads based on the most recent ANSI/ASCE 7 Standard. This includes dead and live loads, snow, wind, earthquake, and tsunami loadings. Further, a discussion on natural hazards and the ASCE Standard's probabilistic approach to natural hazard design loads is added. The presentation aims to provide students with a basic understanding of how design loads are determined for practical design of multistory buildings, bridges, and other structures.
4. **New homework problems and a new focus on loadings.** A substantial number of the problems are new or revised for this edition (in both metric and U.S. Customary System units), and many are typical of analysis problems encountered in practice. The many choices enable the instructor to select problems suited for a particular class or for a particular emphasis. Further, many problems have been added which require the student to calculate the applied loading on a structure (using Chapter 2 as a reference) in addition to performing the analysis. These unique problems provide students a more realistic experience as structural engineers are responsible for both critical tasks.

5. **Computer problems and applications.** Computer problems, some new to this edition, provide readers with a deeper understanding of the structural behavior of trusses, frames, arches, and other structural systems. These carefully tailored problems illustrate significant aspects of structural behavior that, in the past, experienced designers needed many years of practice to understand and to analyze correctly. The computer problems are identified with a computer screen icon and begin in Chapter 4 of the text. The computer problems can be solved using the Educational Version of the commercial software RISA-2D that is available to users at the textbook website. However, any software that produces shear, moment, and axial load diagrams, and deflected shapes can be used to solve the problems. An overview on the use of the RISA-2D software and an author-written tutorial are also available at the textbook website.
6. **Problem solutions have been carefully checked for accuracy.** The authors have carried out multiple checks on the problem solutions.
7. **Textbook website.** A text-specific website is available to users (http://mhhe.com/leet6e). The site offers an array of tools, including lecture slides, an image bank of the text's art, helpful web links, and the RISA-2D educational software.

Contents and Sequence of Chapters

We present the topics in this book in a carefully planned sequence to facilitate the student's study of analysis. In addition, we tailor the explanations to the level of students at an early stage in their engineering education. These explanations are based on the authors' many years of experience teaching analysis.

Chapter 1 provides a historical overview of structural engineering and a brief explanation of the interrelationship between analysis and design. We also describe the essential characteristics of basic structures, detailing both their advantages and their disadvantages.

Chapter 2 on code-specified design loads is described above in *Features of This Text*.

Chapters 3, 4, and 5 cover the basic techniques in statics required to determine bar forces in determinate trusses, and shear and moment in determinate beams and frames. Methods to identify if the structure is determinate are also presented.

Chapter 6 interrelates the behavior of arches and cables, and covers their special characteristics (of acting largely in direct stress and using materials efficiently).

Chapters 7 and 8 provide methods used to compute the deflections of structures. One direct application is to use it to analyze indeterminate structures by the method of consistent deformations in Chapter 9.

Chapters 9, 10, and 11 introduce three classical methods for analyzing indeterminate structures. The method of consistent deformations in Chapter 9 is classified as a flexibility method, while the slope-deflection and moment distribution methods in the other two chapters are classified as the stiffness method.

Chapter 12 introduces the concept of influence lines and covers methods for positioning live load that can vary in space on determinate and indeterminate structures to maximize the internal force at a specific section of a beam, frame, or bars of a truss. Engineers use this important concept to design bridges or other structures subject to moving loads or to live loads whose position on the structure can change.

Chapter 13 gives approximate methods of analysis, used to estimate the value of forces at selected points in highly indeterminate structures. With approximate methods, designers can perform preliminary member sizing, verify the accuracy of computer analysis results, or check the results of more traditional, lengthy hand analyses described in earlier chapters.

Chapters 14, 15, and 16 introduce matrix methods of analysis. Chapter 14 extends the general direct stiffness method to a variety of simple structures. The matrix formulation of the stiffness method, which is the basis of modern structural analysis software, is applied to the analysis of trusses (Chapter 15) and to the analysis of beams and frames (Chapter 16).

ACKNOWLEDGMENTS

This text was originally authored by Kenneth M. Leet and was published by Macmillan in 1988. Dionisio P. Bernal at Northeastern University contributed Chapters 15 and 16. Anne Gilbert served as a coauthor in the third, fourth, and fifth editions.

For their assistance with the first McGraw-Hill edition, we thank Amy Hill, Gloria Schiesl, Eric Munson, and Patti Scott of McGraw-Hill and Jeff Lachina of Lachina Publishing Services.

For their assistance with the second and third editions, we thank Amanda Green, Suzanne Jeans, Jane Mohr, and Gloria Schiesl of McGraw-Hill; Rose Kernan of RPK Editorial Services Inc.; and Patti Scott, who edited the second edition.

For their assistance with the fourth edition, we thank Debra Hash, Peter Massar, Lorraine Buczek, Joyce Watters, and Robin Reed of McGraw-Hill, and Rose Kernan of RPK Editorial Services Inc.

For their assistance with the fifth edition, we thank Thomas Scaife, Jolynn Kilburg, Chelsea Haupt, and Jane Mohr of McGraw-Hill Education.

For their help with this sixth edition, we thank Jane Mohr, Beth Blech, and Beth Cray of McGraw-Hill Education, and Beth Baugh of Baugh Editorial Services.

We also wish to thank RISA for providing an educational version of the RISA-2D computer program with its many options for presenting results. Mr. Muslim Abdulkarim assisted in preparing the answers for the sixth edition.

We would like to thank the following reviewers for their much appreciated comments and advice:

Jeffrey A. Laman, *Pennsylvania State University*
Thomas H. Miller, *Oregon State University*
W. M. Kim Roddis, *George Washington University*
James M. Thompson, *Carnegie Mellon University*

Kenneth M. Leet
Emeritus Professor
Northeastern University

Chia-Ming Uang
Professor
University of California,
San Diego

Joel T. Lanning
Assistant Professor,
University of California,
Irvine

Fundamentals of Structural Analysis

Jet Lowe/Library of Congress

The Brooklyn Bridge

Opened in 1883 at a cost of $9 million, this bridge was heralded as the "Eighth Wonder of the World." The center span, which rises 135 ft above the surface of the East River, spans nearly 1600 ft between towers. Designed in part by engineering judgment and in part by calculations, the bridge is able to support more than three times the original design load. The large masonry towers are supported on pneumatic caissons 102 by 168 ft in plan. In 1872 Colonel Washington A. Roebling, the director of the project, was paralyzed by caissons disease while supervising the construction of one of the submerged piers. Disabled for life, he directed the balance of the project from bed with the assistance of his wife and engineering staff.

CHAPTER 1

Introduction

1.1 Overview of the Text

As an engineer or architect involved with the design of buildings, bridges, and other structures, you will be required to make many technical decisions about structural systems. These decisions include (1) selecting an efficient, economical, and attractive structural form; (2) evaluating its safety, that is, its strength and stiffness; and (3) planning its erection under temporary construction loads.

To design a structure, you will learn to carry out a *structural analysis* that establishes the internal forces and deflections at all points produced by the design loads. Designers determine the internal forces in key members in order to size both members and the connections between members. And designers evaluate deflections to ensure a serviceable structure—one that does not deflect or vibrate excessively under load so that its function is impaired.

Analyzing Basic Structural Elements

During previous courses in statics and strength of materials, you developed some background in structural analysis when you computed the bar forces in trusses and constructed shear and moment curves for beams. You will now broaden your background in structural analysis by applying, in a systematic way, a variety of techniques for determining the forces in and the deflections of a number of basic structural elements: beams, trusses, frames, arches, and cables. These elements represent the basic components used to form more complex structural systems.

Moreover, as you work analysis problems and examine the distribution of forces in various types of structures, you will understand more about how structures are stressed and deformed by load. And you will gradually develop a clear sense of which structural configuration is optimal for a particular design situation.

Further, as you develop an almost intuitive sense of how a structure behaves, you will learn to estimate with a few simple computations the approximate values of forces at the most critical sections of the structure.

This ability will serve you well, enabling you (1) to verify the accuracy of the results of a computer analysis of large, complex structures and (2) to estimate the preliminary design forces needed to size individual components of multimember structures during the early design phase when the tentative configuration and proportions of the structure are being established.

Analyzing Two-Dimensional Structures

As you may have observed while watching the erection of a multistory building frame, when the structure is fully exposed to view, its structure is a complex three-dimensional system composed of beams, columns, slabs, walls, and diagonal bracing. Although load applied at a particular point in a three-dimensional structure will stress all adjacent members, most of the load is typically transmitted through certain key members directly to other supporting members or into the foundation.

Once the behavior and function of the various components of most three-dimensional structures are understood, the designer can typically simplify the analysis of the actual structure by subdividing it into smaller two-dimensional subsystems that act as beams, trusses, or frames. This procedure also significantly reduces the complexity of the analysis because two-dimensional structures are much easier and faster to analyze than three-dimensional structures. Since with few exceptions (e.g., geodesic domes constructed of light tubular bars) designers typically analyze a series of simple two-dimensional structures—even when they are designing the most complex three-dimensional structures—we will devote a large portion of this book to the analysis of two-dimensional or *planar* structures, those that carry forces lying in the plane of the structure.

Once you clearly understand the basic topics covered in this text, you will have acquired the fundamental techniques required to analyze most buildings, bridges, and structural systems typically encountered in professional practice. Of course, before you can design and analyze with confidence, you will require some months of actual design experience in an engineering office to gain further understanding of the total design process from a practitioner's perspective.

For those of you who plan to specialize in structures, mastery of the topics in this book will provide you with the basic structural principles required in more advanced analysis courses—those covering, for example, matrix methods or plates and shells. Further, because design and analysis are closely interrelated, you will use again many of the analytical procedures in this text for more specialized courses in steel, reinforced concrete, and bridge design.

1.2 The Design Process: Relationship of Analysis to Design

The design of any structure—whether it is the framework for a space vehicle, a high-rise building, a suspension bridge, an offshore oil drilling platform, a tunnel, or whatever—is typically carried out in alternating steps of *design* and *analysis*. Each step supplies new information that permits the designer to proceed to the next phase. The process continues until the analysis indicates that no changes in member sizes are required. The specific steps of the procedure are described below.

Conceptual Design

A project begins with a specific need of a client. For example, a developer may authorize an engineering or architectural firm to prepare plans for a sports complex to house a regulation football field, as well as seating 60,000 people, parking for 4000 cars, and space for essential facilities. In another case, a city may retain an engineer to design a bridge to span a 2000-ft-wide river and to carry a certain hourly volume of traffic.

The designer begins by considering all possible layouts and structural systems that might satisfy the requirements of the project. Often, architects and engineers consult as a team at this stage to establish layouts that lend themselves to efficient structural systems in addition to meeting the architectural (functional and aesthetic) requirements of the project. The designer next prepares sketches of an architectural nature showing the main structural elements of each design, although details of the structural system at this point are often sketchy.

Preliminary Design

In the preliminary design phase, the engineer selects from the conceptual design several of the structural systems that appear most promising, and sizes their main components. This preliminary proportioning of structural members requires an understanding of structural behavior and a knowledge of the loading conditions (dead, live, wind, and so forth) that will most likely affect the design. At this point, the experienced designer may make a few rough computations to estimate the proportions of each structure at its critical sections.

Analysis of Preliminary Designs

At this next stage, the precise loads the structure will carry are not known because the exact size of members and the architectural details of the design are not finalized. Using estimated values of load, the engineer carries out an analysis of the several structural systems under consideration to determine the forces at critical sections and the deflections at any point that influence the serviceability of the structure.

The true weight of the members cannot be calculated until the structure is sized exactly, and certain architectural details will be influenced, in turn, by the structure. For example, the size and weight of mechanical equipment cannot be determined until the volume of the building is established, which in turn depends on the structural system. The designer, however, knows from past experience with similar structures how to estimate values for load that are fairly close approximations of final values.

Redesign of the Structures

Using the results of the analysis of preliminary designs, the designer recomputes the proportions of the main elements of all structures. Although each analysis is based on estimated values of load, the forces established at this stage are probably indicative of what a particular structure must carry, so that proportions are unlikely to change significantly even after the details of the final design are established.

Evaluation of Preliminary Designs

The various preliminary designs are next compared with regard to cost, availability of materials, appearance, maintenance, time for construction, and other pertinent considerations. The structure best satisfying the client's established criteria is selected for further refinement in the final design phase.

Final Design and Analysis Phases

In the final phase, the engineer makes any minor adjustments to the selected structure that will improve its economy or appearance. Now the designer carefully estimates dead loads and considers specific positions of the live load that will maximize stresses at specific sections. As part of the final analysis, the strength and stiffness of the structure are evaluated for all significant loads and combinations of load, dead and live, including wind, snow, earthquake, temperature change, and settlements. If the results of the final design confirm that the proportions of the structure are adequate to carry the design forces, the design is complete. On the other hand, if the final design reveals certain deficiencies (e.g., certain members are overstressed, the structure is unable to resist lateral wind loads efficiently, members are excessively flexible, or costs are over budget), the designer will either have to modify the configuration of the structure or consider an alternate structural system.

Steel, reinforced concrete, wood, and metals, such as aluminum, are all analyzed in the same manner. The different properties of materials are taken into account during the design process. When members are sized, designers refer to design codes, which take into account each material's special properties.

This text is concerned primarily with the *analysis* of structures as detailed above. Design is covered in separate courses in most engineering programs; however, since the two topics are so closely interrelated, we will necessarily touch upon some design issues.

1.3 Strength and Serviceability

The designer must proportion structures so that they will neither fail nor deform excessively under any possible loading conditions. Members are always designed with a capacity for load significantly greater than that required to support anticipated *service loads* (the real loads or the loads specified by design code). This additional capacity also provides a factor of safety against accidental overload.

Although structures must be designed with an adequate factor of safety to reduce the probability of failure to an acceptable level, the engineer must also ensure that the structure has sufficient stiffness to function usefully under all loading conditions. For example, floor beams under service loads must not sag excessively or vibrate under live load. Excessively large deflections of beams may produce cracking of masonry walls and plaster ceilings, or may damage equipment that becomes misaligned. High-rise buildings must not sway excessively under wind loads (or the building may cause motion sickness in the inhabitants of upper floors). Excessive movements of a building not only are disturbing to the occupants, who become concerned about the safety of the structure, but also may lead to cracking of exterior curtain walls and windows. Photo 1.1 shows an office building whose facade was constructed of large floor-to-ceiling glass panels. Shortly after the high-rise building was completed, larger than anticipated wind loads caused many glass panels to crack and fall out. The falling glass constituted an obvious danger to pedestrians in the street below. After a thorough investigation and further testing, all the original glass panels were removed. To correct the design deficiencies, the structure of the building was stiffened, and the facade was reconstructed with thicker, tempered glass panels. The dark areas in Photo 1.1 show the temporary plywood panels used to enclose the building during the period in which the original glass panels were removed and replaced by the more durable, tempered glass. Similarly, for seismic design of multistory buildings the designer also needs to ensure that the relative lateral deflection between two adjacent floors is not excessive.

Photo 1.1: Wind damage. Shortly after thermopane windows were installed in this high-rise office building, they began failing and falling out, scattering broken glass on passers-by beneath.

Before the building could be occupied, the structural frame had to be stiffened and all the original glass panels had to be replaced by thicker, tempered glass—costly procedures that delayed the opening of the building for several years.

Kenneth Leet

1.4 Historical Development of Structural Systems

To give you some historical perspective on structural engineering, we will briefly trace the evolution of structural systems from those trial-and-error designs used by the ancient Egyptians and Greeks to the highly sophisticated configurations used today. The evolution of structural forms is closely related to the materials available, the state of construction technology, the designer's knowledge of structural behavior (and much later, analysis), and the skill of the construction worker.

For their great engineering feats, the early Egyptian builders used stone quarried from sites along the Nile to construct temples and pyramids. Since the tensile strength of stone, a brittle material, is low and highly variable (because of a multitude of internal cracks and voids), beam spans in temples had to be short (Figure 1.1) to prevent bending failures. Since this *post-and-lintel* system—massive stone beams balanced on relatively thick stone columns—has only a limited capacity for horizontal or eccentric vertical loads, buildings had to be relatively low. For stability, columns had to be thick—a slender column will topple more easily than a stocky column.

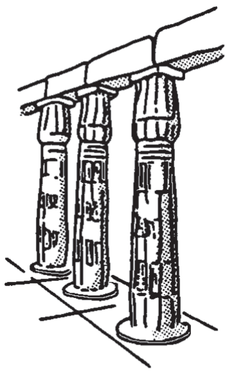


Figure 1.1: Early post-and-lintel construction as seen in an Egyptian temple.

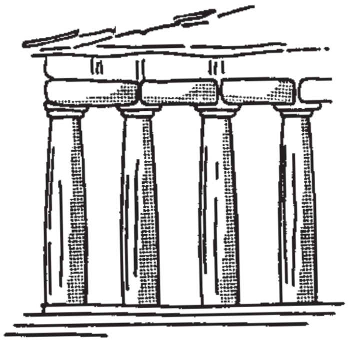


Figure 1.2: Front of Parthenon, where columns were tapered and fluted for decoration.

The Greeks, greatly interested in refining the aesthetic appearance of the stone column, used the same type of post-and-lintel construction in the Parthenon (about 400 B.C.), a temple considered one of the most elegant examples of stone construction of all time (Figure 1.2). Even up to the early twentieth century, long after post-and-lintel construction was superseded by steel and reinforced concrete frames, architects continued to impose the facade of the classic Greek temple on the entrance of public buildings. The classic tradition of the ancient Greeks was influential for centuries after their civilization declined.

Gifted as builders, Roman engineers made extensive use of the arch, often employing it in multiple tiers in coliseums, aqueducts, and bridges (Photo 1.2). The curved shape of the arch allows a departure from rectangular lines and permits much longer clear spans than are possible with masonry post-and-lintel construction. The stability of the masonry arch requires that (1) its entire cross section be stressed in compression under all loading conditions, and (2) abutments or end walls have sufficient strength to resist the large diagonal thrust at the base of the arch. The Romans, largely by trial and error, also developed a method of enclosing an interior space by a masonry dome, as seen in the Pantheon still standing in Rome.

During the Gothic period of great cathedral buildings (Chartres and Notre Dame in France, for example), the arch was refined by trimming away excess material, and its shape became far more elongated. The vaulted roof, a three-dimensional form of the arch, also appeared in the construction of cathedral roofs. Arch-like masonry elements, termed *flying buttresses*, were used together with piers (thick masonry columns) or walls to carry the thrust of vaulted roofs to the ground (Figure 1.3). Engineering in this period was

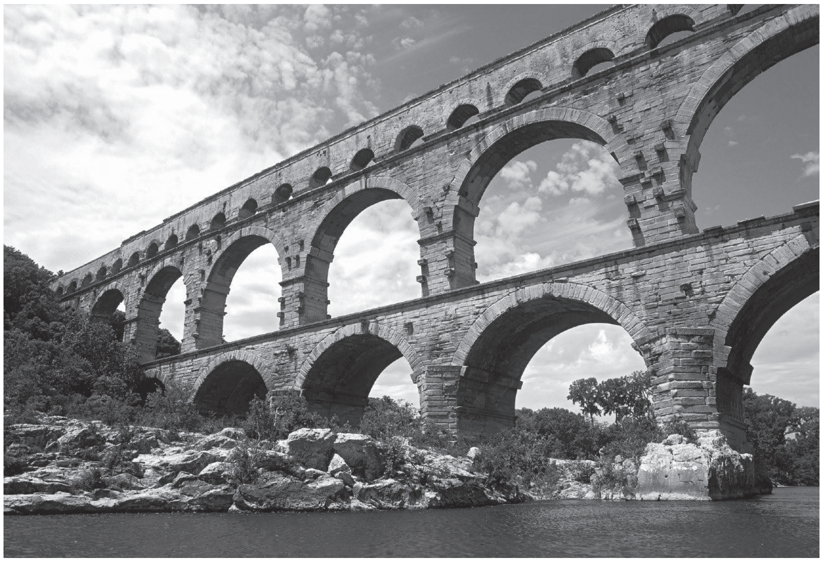


Photo 1.2: Romans pioneered in the use of arches for bridges, buildings, and aqueducts. Pont-du-Gard. Roman aqueduct built in 19 B.C. to carry water across the Gardon Valley to Nimes. Spans of the first- and second-level arches are 53 to 80 ft. (Near Remoulins, France.)
Apply Pictures/Alamy

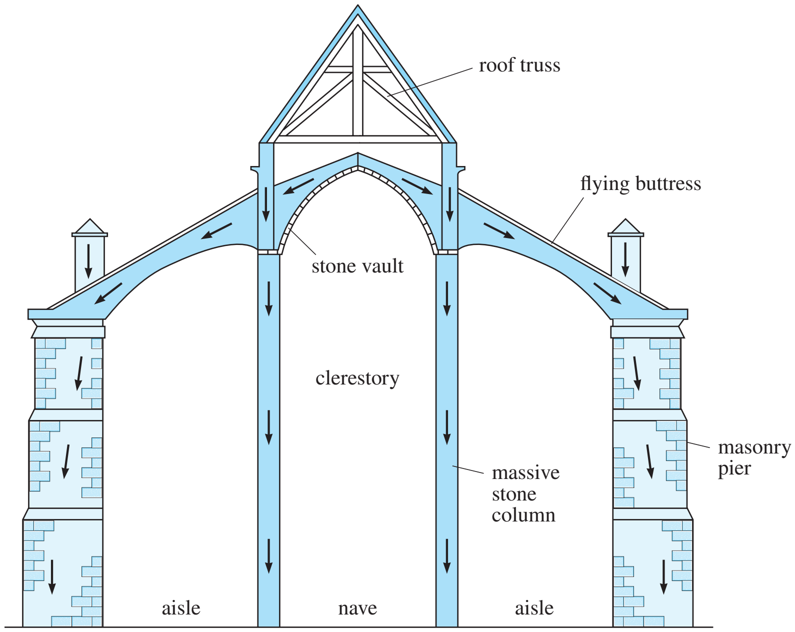


Figure 1.3: Simplified cross section showing the main structural elements of Gothic construction. Exterior masonry arches, called *flying buttresses*, were used to stabilize the arched stone vault over the nave. The outward thrust of the arched vault is transmitted through the flying buttresses to deep masonry piers on the exterior of the building. Typically the piers broaden toward the base of the building. For the structure to be stable, the masonry must be stressed in compression throughout. Arrows show the flow of forces.

highly empirical based on what master masons learned and passed on to their apprentices; these skills were passed down through the generations.

Although magnificent cathedrals and palaces were constructed for many centuries in Europe by master builders, no significant change occurred in construction technology until cast iron was produced in commercial quantities in the mid-eighteenth century. The introduction of cast iron made it possible for engineers to design buildings with shallow but strong beams, and columns with compact cross sections, permitting the design of lighter structures with longer open spans and larger window areas. The massive bearing walls required for masonry construction were no longer needed. Later, steels with high tensile and compressive strengths permitted the construction of taller structures and eventually led to the skyscraper of today.

In the late nineteenth century, the French engineer Eiffel constructed many long-span steel bridges in addition to his innovative Eiffel Tower, the internationally known landmark in Paris (Photo 1.3). With the development of high-strength steel cables, engineers were able to construct long-span suspension bridges. The Verrazano Bridge at the entrance of New York harbor—one of the longest bridges in the world—spans 4260 ft between towers.

The addition of steel reinforcement to concrete enabled engineers to convert unreinforced concrete (a brittle, stonelike material) into tough, ductile structural members. Reinforced concrete, which takes the shape of the temporary forms into which it is poured, allows a large variety of forms to be constructed. Since reinforced concrete structures are *monolithic*, meaning they act as one continuous unit, they are highly indeterminate.

Reinforced concrete is also used to *precast* individual structural components like beams, slabs, and wall panels. Both precast and monolithic

Photo 1.3: The Eiffel Tower, constructed of wrought iron in 1889, dominates the skyline of Paris in this early photograph. The tower, the forerunner of the modern steel frame building, rises to a height of 984 ft (300 m) from a 330-ft (100.6-m) square base. The broad base and the tapering shaft provide an efficient structural form to resist the large overturning forces of the wind. At the top of the tower where the wind forces are the greatest, the width of the building is smallest.

Aaron Roeth Photography

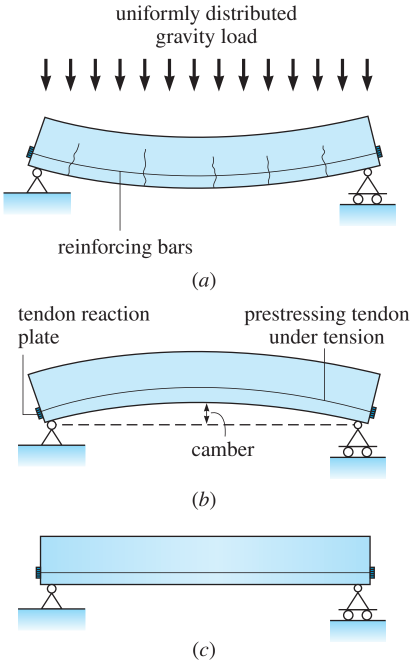


Figure 1.4: (*a*) Reinforced concrete beams utilize steel reinforcing bars to carry tensile bending stress, but small cracks will still occur; (*b*) a prestress concrete beam is provided with an axial compressive load using a tensioned steel cable, or tendon, before gravity loads are applied. Depending on the location of the tendon along the cross section, a camber, or initial upward beam deflection, may be introduced; (*c*) upon loading, the prestressed beam still undergoes tensile bending stress, but the prestressing compressive load counteracts it. Meanwhile, the beam experiences a reduced net downward deflection due to the cambering.

reinforced concrete systems are, nowadays, commonly *prestressed*. This construction method is used to overcome concrete's lack of tensile strength by including high-strength steel cables, or tendons, inside the structural members (Figure 1.4). After the concrete is cured, the tendons are tensioned and each end is fixed to the outside, creating a compressive stress on the concrete. This initial compressive stress is strategically located along the cross section and along the beam by the placement of the tendons. In Figure 1.4*b* the tendon is located on the bottom of the cross section to counteract the tensile bending stress caused by the uniform gravity loading (shown in Figure 1.4*c*). Afterward, the beam deflection is greatly reduced (Figure 1.4*c*). Prestressing allows engineers to design very thin slabs and long-span beams for building and bridge applications.

Until improved methods of indeterminate analysis enabled designers to predict the internal forces in reinforced concrete construction, design remained semi-empirical; that is, simplified computations were based on observed behavior and testing as well as on the principles of mechanics. With the introduction in the early 1930s of *moment distribution* by Hardy Cross, engineers acquired a relatively simple technique to analyze continuous structures. As designers became familiar with moment distribution, they were able to analyze indeterminate frames, and the use of reinforced concrete as a building material increased rapidly.

The introduction of welding in the late nineteenth century facilitated the joining of steel members—welding eliminated heavy plates and angles required by earlier riveting methods—and simplified the construction of rigid-jointed steel frames.

In recent years, the computer and research in materials science have produced major changes in the engineer's ability to construct special-purpose structures, such as space vehicles. The introduction of the computer and the subsequent development of the direct stiffness method for beams, plates, and shell elements permitted designers to analyze many complex structures rapidly and accurately. Structures that even in the 1950s took teams of engineers months to analyze can now be analyzed more accurately in minutes by one designer using a computer.

1.5 Basic Structural Elements

All structural systems are composed of a number of basic structural elements—beams, columns, hangers, trusses, and so forth. In this section we describe the main characteristics of these basic elements so that you will understand how to use them most effectively.

Hangers, Suspension Cables—Axially Loaded Members in Tension

Since all cross sections of axially loaded members are uniformly stressed, the material is used at optimum efficiency. The capacity of tension members is a direct function of the tensile strength of the material. When members are constructed of high-strength materials, such as alloyed steels, even members with small cross sections have the capacity to support large loads (Figure 1.5).

As a negative feature, members with small cross sections are very flexible and tend to vibrate easily under moving loads. To reduce this tendency to vibrate, most building codes specify that certain types of tension members have a minimum amount of flexural stiffness by placing an upper limit on their *slenderness ratio* l/r, where l is the length of member and r is the radius of gyration. By definition $r = \sqrt{I/A}$, where I equals the moment of inertia and A equals the area of the cross section.

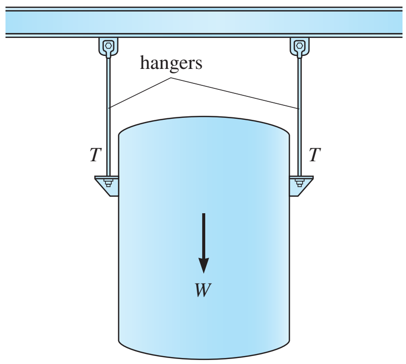


Figure 1.5: Chemical storage tank supported by tension hangers carrying force T.

Columns—Axially Loaded Members in Compression

Columns also carry load in direct stress very efficiently. The capacity of a compression member is a function of its slenderness ratio l/r. If l/r is large, the member is slender and will fail by buckling at a low stress level—often with little warning. If l/r is small, the member is stocky and its capacity for axial load is high. The capacity of an axially loaded column also depends on the restraint at its ends. For example, a slender cantilever column—fixed at one end and free at the other—will support a load that is one-fourth as large as that of an identical column with two pinned ends (Figure 1.6*b*, *c*).

In fact, columns supporting pure axial load occur only in idealized situations. In actual practice, the initial slight crookedness of columns or an eccentricity of the applied load creates bending moments that must be taken into account by the designer. Also in reinforced concrete or welded building frames where beams and columns are connected by rigid joints, columns carry both axial load and bending moment. These members are called *beam-columns* (Figure 1.6*d*).

Beams—Members Carrying Bending Moment and Shear

Beams are flexural members that are loaded perpendicular to their longitudinal axis (Figure 1.7*a*). As the *transverse* load is applied, a beam bends and deflects into a shallow curve. At a typical section of a beam, internal forces of shear V and moment M develop (Figure 1.7*b*). Except in short, heavily loaded beams, the shear stresses τ produced by V are relatively small, but the longitudinal bending stresses produced by M are large. If the beam behaves elastically, the bending stresses on a cross section (compression on the top and tension on the bottom) vary linearly from a horizontal axis passing through the centroid of the cross section. The bending stresses are directly proportional to the moment, and vary in magnitude along the axis of the beam.

Shallow beams are relatively inefficient in carrying load because the arm between the forces C and T that make up the internal couple is small. To increase the length of the arm, material is often removed from the center of the cross section and concentrated at the top and bottom surfaces, producing an I-shaped section (Figure 1.7*c* and *d*).

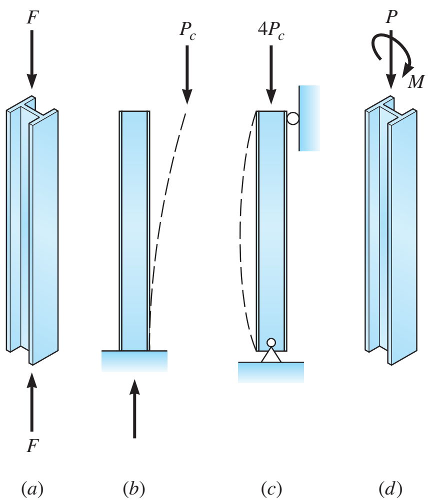
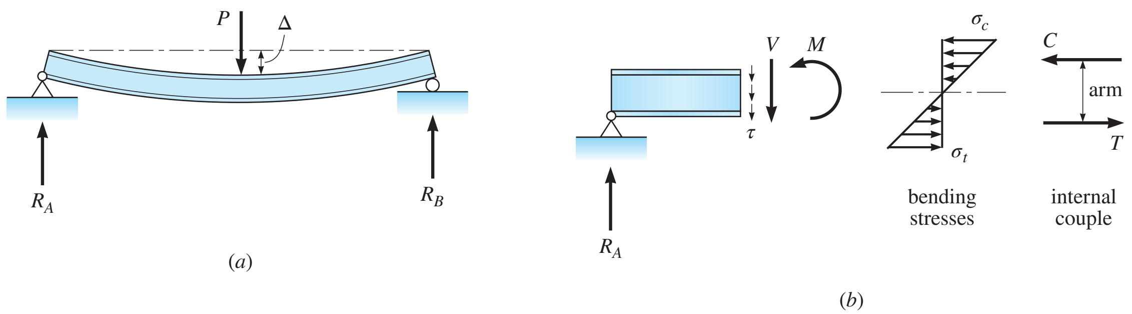


Figure 1.6: (*a*) Axially loaded column; (*b*) cantilever column with buckling load P_c; (*c*) pin-supported column with buckling load $4P_c$; (*d*) beam-column.

Planar Trusses—All Members Axially Loaded

A truss is a structural system composed of slender bars whose ends are assumed to be connected by frictionless pin joints. If pin-jointed trusses are



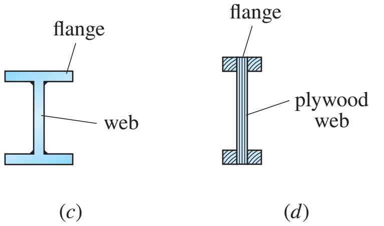


Figure 1.7: (*a*) Beam deflects into a shallow curve; (*b*) internal forces (shear *V* and moment *M*); (*c*) I-shaped steel section; (*d*) glue-laminated wood I-beam.

loaded at the joints only, direct or axial stress develops in all bars. Thus the material is used at optimum efficiency. Typically, truss bars are assembled in a triangular pattern—the simplest stable geometric configuration (Figure 1.8*a*). In the nineteenth century, trusses were often named after the designers who established a particular configuration of bars (Figure 1.8*b*).

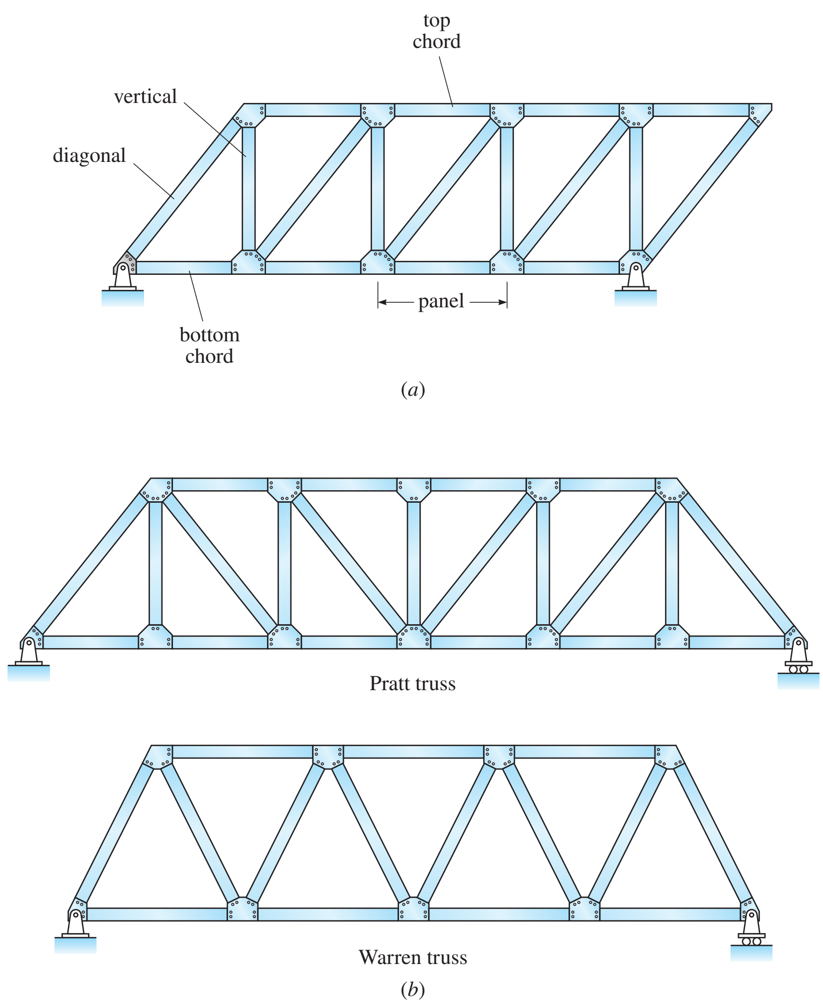


Figure 1.8: (*a*) Assembly of triangular elements to form a truss; (*b*) two common types of trusses named after the original designer.

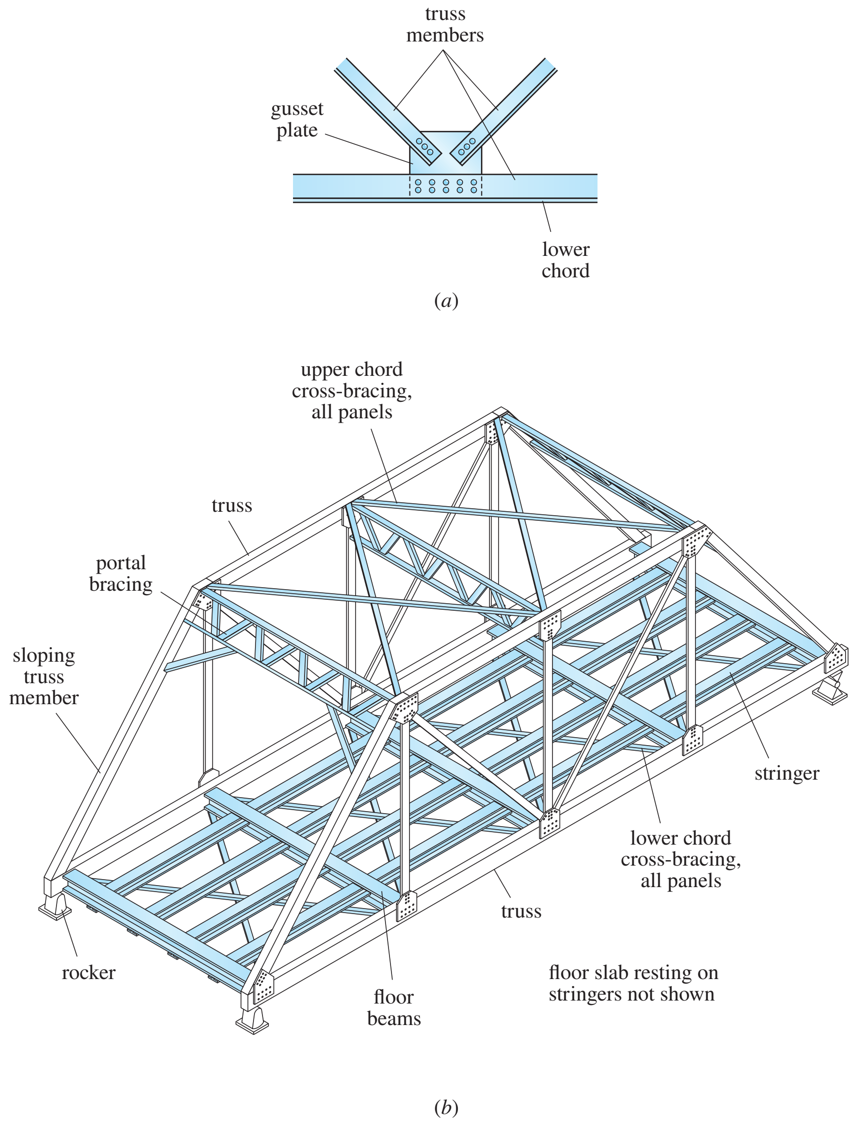


Figure 1.9: (*a*) Bolted joint detail; (*b*) truss bridge showing cross-bracing needed to stabilize the two main trusses.

The behavior of a truss is similar to that of a beam in which the solid beam web (which transmits the shear) is replaced by a series of vertical and diagonal bars. By eliminating the solid web, the designer can reduce the deadweight of the structure significantly. Since trusses are much lighter than beams of the same capacity, trusses are easier to erect. Although most truss joints are formed by welding or bolting the ends of the bars to a connection (or gusset) plate (Figure 1.9*a*), an analysis of the truss based on the assumption of pinned joints produces an acceptable result.

Although trusses are very stiff in their own plane, they are very flexible when loaded perpendicular to their plane. For this reason, the compression chords of trusses must be stabilized by cross-bracing (Figure 1.9*b*).

For example, in buildings, the roof or floor systems attached to the joints of the upper chord serve as lateral supports to prevent lateral buckling of this member.

Arches—Curved Members Stressed Mainly in Direct Compression

Arches typically are stressed in compression under their dead load. Because of their efficient use of material, arches have been constructed with spans of more than 2000 ft. To be in pure compression, an efficient state of stress, the arch must be shaped so that the resultant of the internal forces on each section passes through the centroid. For a given span and rise, only one shape of arch exists in which direct stress will occur for a particular load pattern. For other loading conditions, bending moments develop that can produce large deflections in slender arches. The selection of the appropriate arch shape by the early builders in the Roman and Gothic periods represented a rather sophisticated understanding of structural behavior. (Since historical records report many failures of masonry arches, obviously not all builders understood arch action.)

Because the base of the arch intersects the end supports (called *abutments*) at an acute angle, the internal force at that point exerts a horizontal as well as a vertical thrust on the abutments. When spans are large, loads are heavy, and the slope of the arch is shallow, the horizontal component of the thrust is large. Unless natural rocks exist to resist the horizontal thrust (Figure 1.10*a*), either massive abutments must be constructed (Figure 1.10*b*), or the ends of the arch must be tied together by a tension member (Figure 1.10*c*), or the abutment must be supported on piles (Figure 1.10*d*).

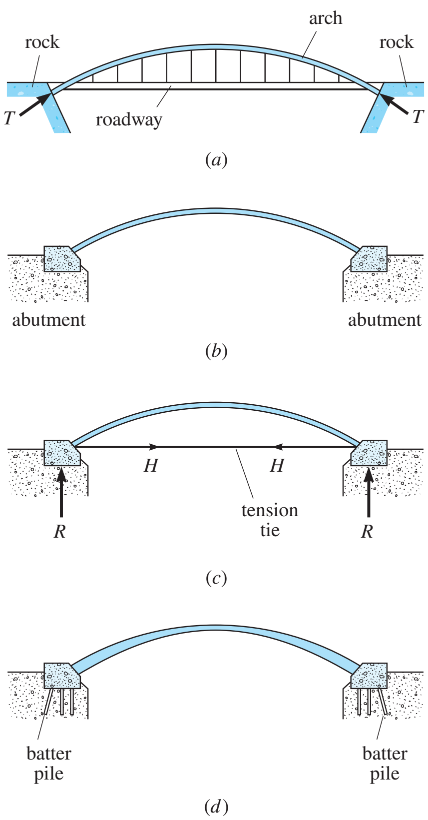


Figure 1.10: (*a*) Fixed-end arch carries roadway over a canyon where rock walls provide a natural support for arch thrust *T;* (*b*) large abutments provided to resist arch thrust; (*c*) tension tie added at base to carry horizontal thrust, foundations designed only for vertical reaction *R;* (*d*) foundation placed on piles, batter piles used to transfer horizontal component of thrust into ground.

Cables—Flexible Members Stressed in Tension

Cables are very flexible members composed of a group of high-strength steel wires twisted together mechanically. By drawing alloyed steel bars through dies—a process that aligns the molecules of the metal—manufacturers are able to produce wire with a tensile strength reaching as high as 270,000 psi. Since cables have no bending stiffness, they can only carry direct tensile stress (they would obviously buckle under the smallest compressive force). Because of their high tensile strength and efficient manner of transmitting load (by direct stress), cable structures have the strength to support the large loads of long-span structures more economically than most other structural elements. For example, when distances to be spanned exceed 2000 ft, designers usually select suspension or cable-stayed bridges (Photo 1.4). Cables can be used in the construction of roofs as well as guyed towers.

Under its own deadweight (a uniform load acting along the arc of the cable), the cable takes the shape of a catenary (Figure 1.11*a*). If the cable carries a load distributed uniformly over the horizontal projection of its span, it will assume the shape of a *parabola* (Figure 1.11*b*). When the *sag* (the vertical distance between the cable chord and the cable at midspan) is small (Figure 1.11*a*), the cable shape produced by its dead load may be closely approximated by a parabola.

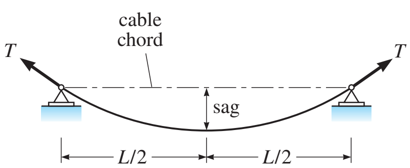
Because of a lack of bending stiffness, cables undergo large changes in shape when concentrated loads are applied. The lack of bending stiffness also

(*a*)

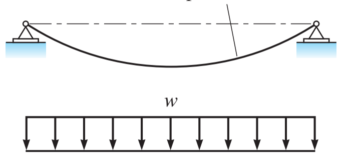
(*b*)

Photo 1.4: (*a*) Golden Gate Bridge (San Francisco Bay Area). Opened in 1937, the main span of 4200 ft was the longest single span at that time and retained this distinction for 29 years. Principal designer was Joseph Strauss who had previously collaborated with Ammann on the George Washington Bridge in New York City; (*b*) Rhine River Bridge at Flehe, near Dusseldorf, Germany. Single-tower design. The single line of cables is connected to the center of the deck, and there are three traffic lanes on each side. This arrangement depends on the torsional stiffness of the deck structure for overall stability.

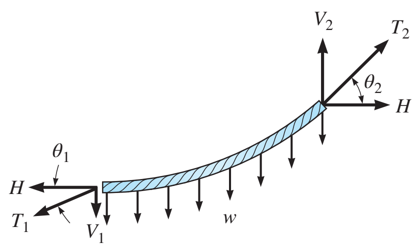
(*a*) Thinkstock/Getty Images; (*b*) Courtesy of the Godden Collection, NISEE, University of California, Berkeley



(*a*)



(*b*)



(*c*)

Figure 1.11: (*a*) Cable in the shape of a catenary under dead load; (*b*) parabolic cable produced by a uniform load; (*c*) free-body diagram of a section of cable carrying a uniform vertical load; equilibrium in horizontal direction shows that the horizontal component of cable tension H is constant.

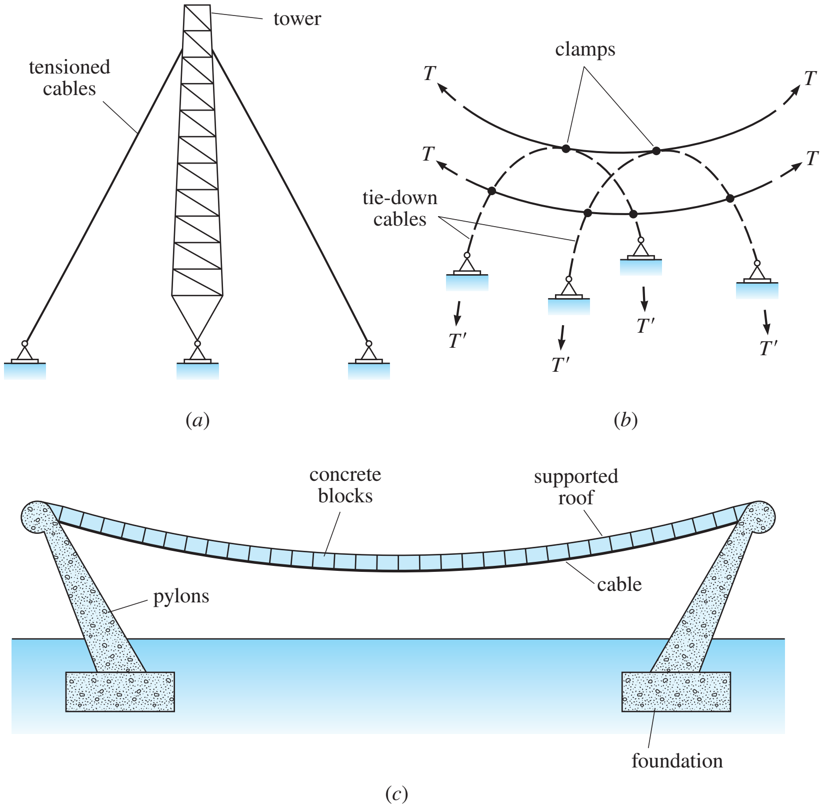


Figure 1.12: Techniques to stiffen cables: (*a*) guyed tower with pretensioned cables stressed to approximately 50 percent of their ultimate tensile strength; (*b*) three-dimensional net of cables; tie-down cables stabilize the upward-sloping cables; (*c*) cable roof paved with concrete blocks to hold down cable to eliminate vibrations. Cables are supported by massive pylons (columns) at each end.

makes it very easy for small disturbing forces (e.g., wind) to induce oscillations (flutter) into cable-supported roofs and bridges. To utilize cables effectively as structural members, engineers have devised a variety of techniques to minimize deformations and vibrations produced by live loads. Techniques to stiffen cables include (1) pretensioning, (2) using tie-down cables, and (3) adding extra dead load (Figure 1.12).

As part of the cable system, supports must be designed to resist the cable end reactions. Where solid rock is available, cables can be anchored economically by grouting the anchorage into rock (Figure 1.13). If rock is not available, heavy foundations must be constructed to anchor the cables. In the case of suspension bridges, large towers are required to support the cable, much as a clothes pole props up a clothesline.

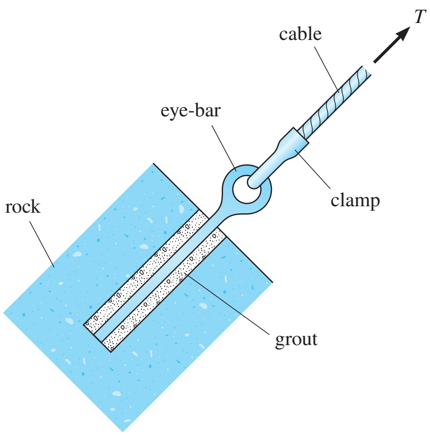


Figure 1.13: Detail of a cable anchorage into rock.

Rigid Frames—Members Stressed by Moment and Axial Load

Rigid frames are also commonly called moment frames in structural design. Examples of rigid frames (structures with rigid joints) are shown in Figure 1.14*a* and *b*. Members of a rigid frame, which typically carry moment and axial load, are called *beam-columns*. For a joint to be rigid, the angle between the members framing into a joint needs to remain essentially unchanged when the members are loaded. Rigid joints in reinforced concrete structures are simple to

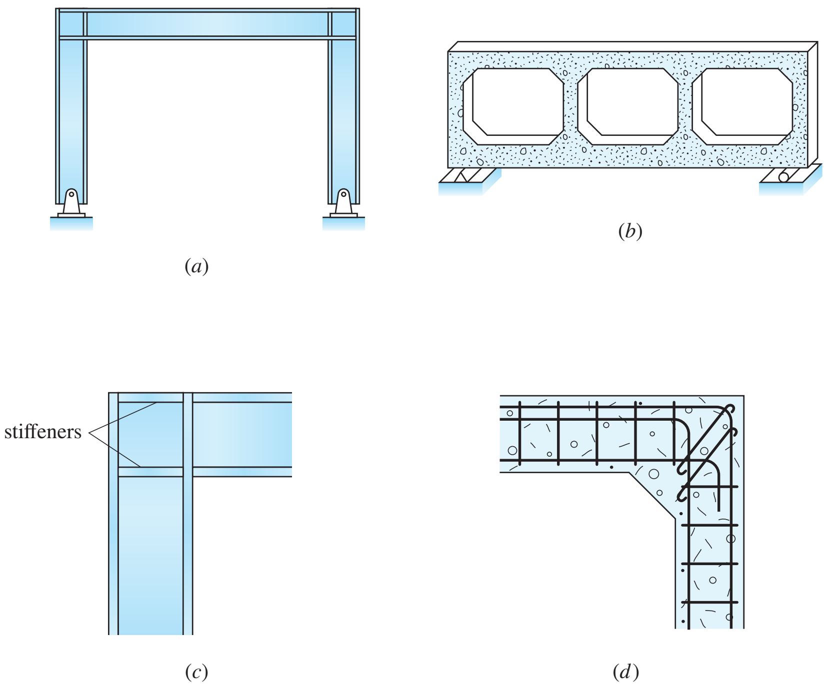


Figure 1.14: Rigid-jointed structures: (*a*) one-story rigid frame; (*b*) Vierendeel truss, loads transmitted both by direct stress and bending; (*c*) details of a welded joint at the corner of a steel rigid frame; (*d*) reinforcing detail for corner of concrete frame in (*b*).

construct because of the monolithic nature of poured concrete. However, rigid joints fabricated from steel beams with wide flanges (Figure 1.7*c*) often require stiffening plates to transfer the large forces in the flanges between members framing into the joint (Figure 1.14*c*). Although joints can be formed by bolting, welding greatly simplifies the fabrication of rigid joints in steel frames.

Plates or Slabs—Load Carried by Bending

Plates are planar elements whose depth (or thickness) is small compared to their length and width. They are typically used as floors in buildings and bridges or as walls for storage tanks. The behavior of a plate depends on the position of supports along the boundaries. If rectangular plates are supported on two opposite edges, they bend in single curvature (Figure 1.15*a*). If supports are continuous around the boundaries, double curvature bending occurs and the deflection is less.

Since slabs are flexible owing to their small depth, the distance they can span without sagging excessively is relatively small. (For example, typical reinforced concrete slabs can span approximately 12 to 16 ft.) If spans are large, slabs are typically supported on beams or stiffened by adding ribs (Figure 1.15*b*). Alternatively, concrete slabs can be prestressed.

If the connection between a slab and the supporting beam is properly designed, the two elements act together (a condition called *composite action*) to form a T-beam (Figure 1.15*c*). When the slab acts as the flange of a rectangular beam, the stiffness of the beam will increase by a factor of approximately 2.

By corrugating plates, the designer can create a series of deep beams (called *folded plates*) that can span long distances. At Logan Airport in

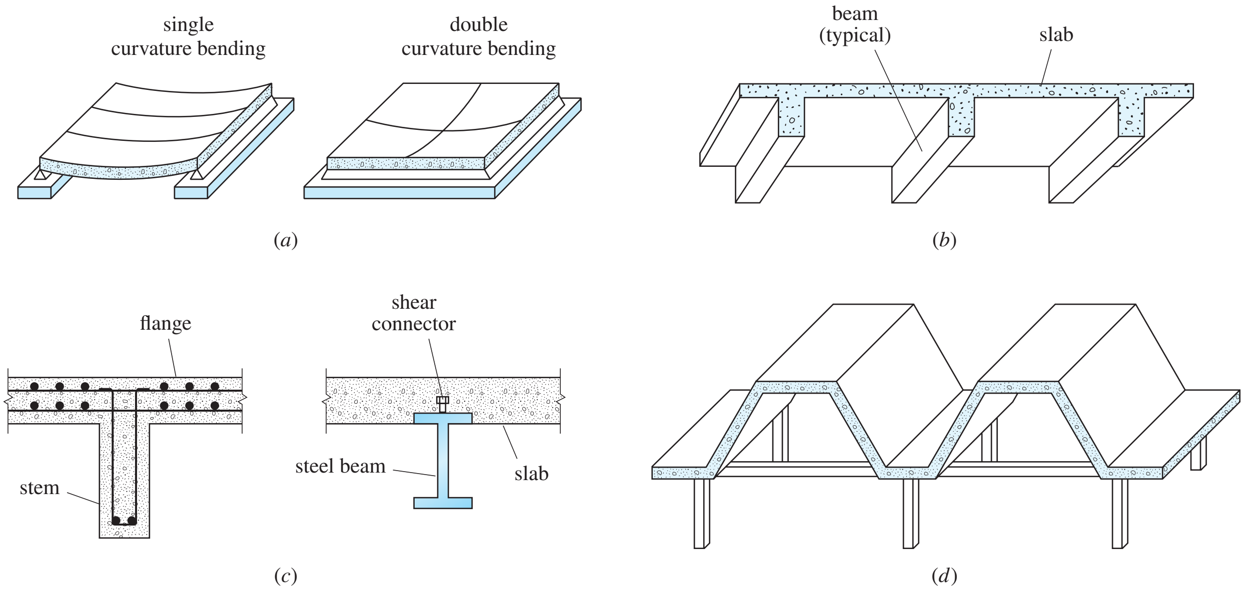


Figure 1.15: (*a*) Influence of boundaries on curvature; (*b*) beam and slab system; (*c*) slab and beams act as a unit: on left, concrete slab cast with stem to form a T-beam; right, shear connector joins concrete slab to steel beam, producing a composite beam; (*d*) a folded plate roof.

Boston, a prestressed concrete folded plate of the type shown in Figure 1.15*d* spans 270 ft to act as the roof of a hangar.

Thin Shells (Curved Surface Elements)—Stresses Acting Primarily in Plane of Element

Thin shells are three-dimensional curved surfaces. Although their thickness is often small (several inches is common in the case of a reinforced concrete shell), they can span large distances because of the inherent strength and stiffness of the curved shape. Spherical domes, which are commonly used to cover sports arenas and storage tanks, are one of the most common types of shells built.

Under uniformly distributed loads, shells develop in-plane stresses (called *membrane stresses*) that efficiently support the external load (Figure 1.16). In addition to the membrane stresses, which are typically small in magnitude, shear stresses perpendicular to the plane of the shell, bending moments, and torsional moments also develop. If the shell has boundaries that can equilibrate the membrane stresses at all points (Figure 1.17*a* and *b*), the majority of the load will be carried by the membrane stresses. But if the shell boundaries cannot provide reactions for the membrane stresses (Figure 1.17*c* and *d*), the region of the shell near the boundaries will deform. Since these deformations create shear normal to the surface of the shell as well as moments, the shell must be thickened or an edge member supplied. Rings can also be used to provide reactions for the membrane stresses (Figure 1.17*e*). Figure 1.17*f* shows a cylindrical shell with edge beams to carry the member stresses. In most shells, boundary shear and moments drop rapidly with distance from the edge.

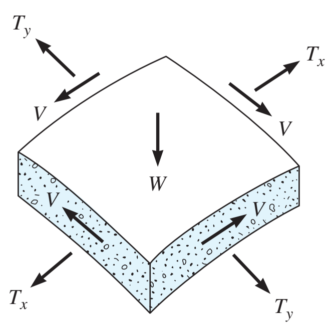


Figure 1.16: Membrane stresses acting on a small shell element.

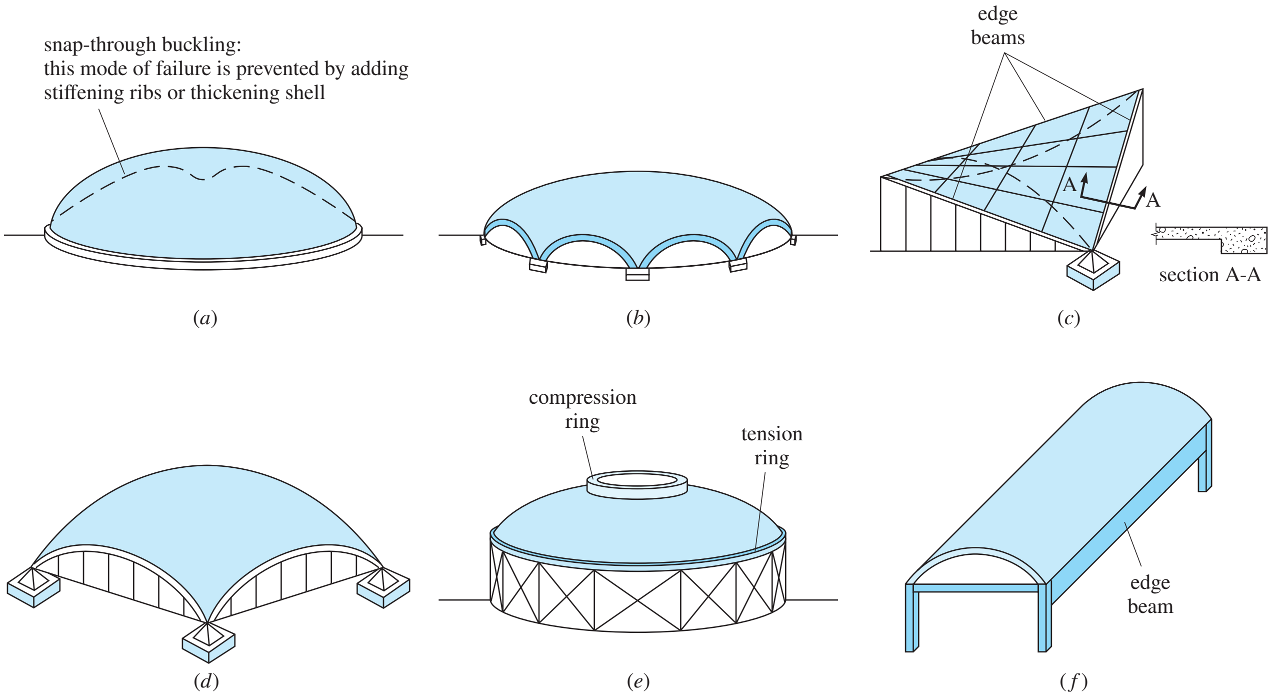


Figure 1.17: Commonly constructed types of shells: (*a*) spherical dome supported continuously. Boundary condition for membrane action is provided; (*b*) modified dome with closely spaced supports. Due to openings, the membrane condition is disturbed somewhat at the boundaries. Shell must be thickened or edge beams supplied at openings; (*c*) hyperbolic paraboloid. Straight-line generators form this shell. Edge members are needed to supply the reaction for the membrane stresses; (*d*) dome with widely spaced supports. Membrane forces cannot develop at the boundaries. Edge beams and thickening of shell are required around the perimeter; (*e*) dome with a compression ring at the top and a tension ring at the bottom. These rings provide reactions for membrane stresses. Columns must carry only vertical load; (*f*) cylindrical shell.

The ability of thin shells to span large unobstructed areas has always excited great interest among engineers and architects. However, the great expense of forming the shell, the acoustical problems, the difficulty of producing a watertight roof, and problems of buckling at low stresses have restricted their use. In addition, thin shells are not able to carry heavy concentrated loads without the addition of ribs or other types of stiffeners.

1.6 Assembling Basic Elements to Form a Stable Structural System

One-Story Building

To illustrate how the designer combines the basic structural elements (described in Section 1.5) into a stable structural system, we will discuss in

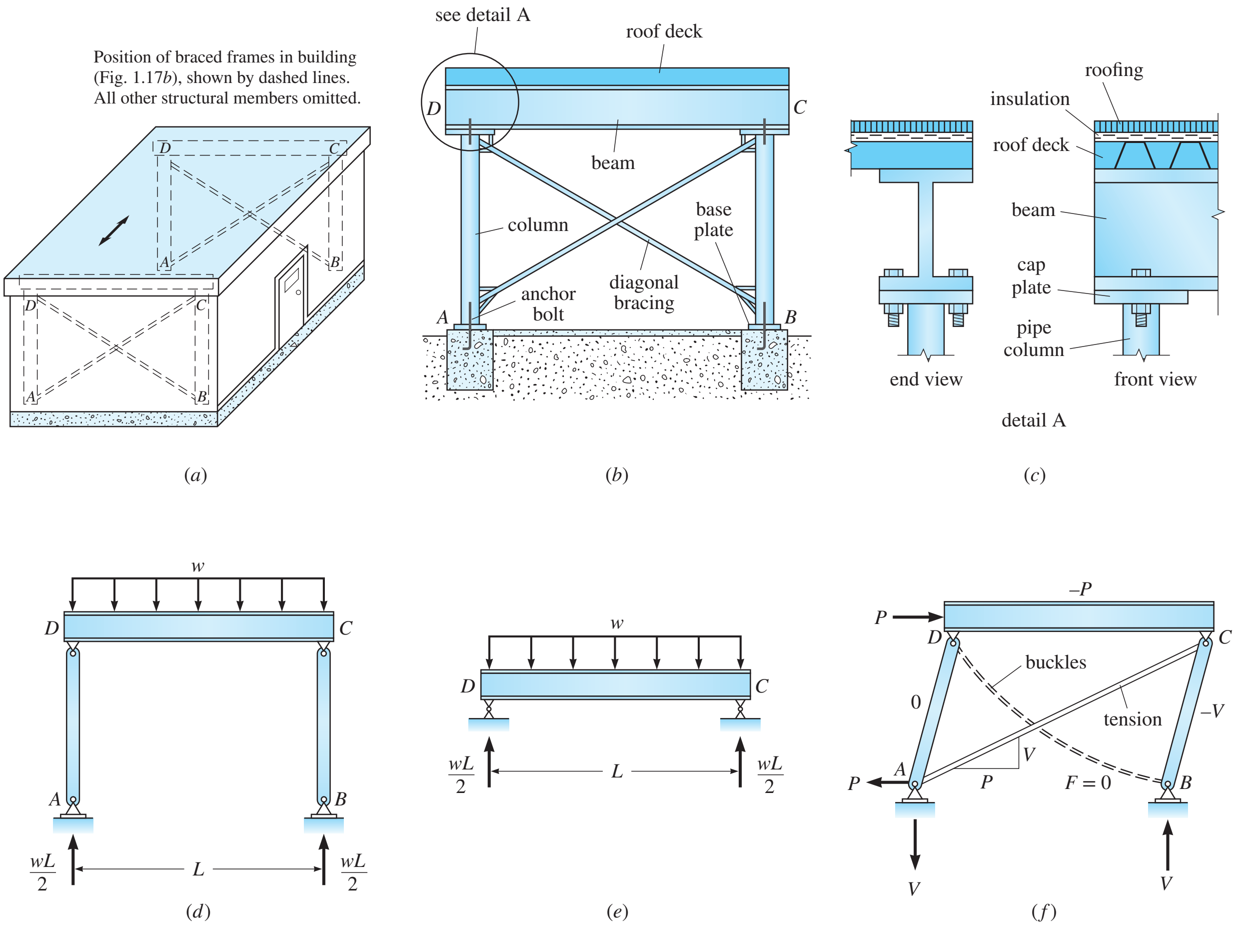


Figure 1.18: (*a*) Three-dimensional view of building (arrow indicates direction in which roof deck spans); (*b*) details of cross-braced frame with bolted joints; (*c*) details of beam-to-column connections; (*d*) idealized model of structural system transmitting gravity loads from roof; (*e*) model of beam *CD*; (*f*) idealized model of truss system for transmitting lateral load acting to the right. Diagonal member *DB* buckles and is ineffective.

detail the behavior of a simple structure, considering the one-story, boxlike structure in Figure 1.18*a*. This building, representing a small storage facility, consists of structural steel frames covered with light-gage corregated metal panels. Although the exterior masonry or windows and wall panels of the building, connected to the structural frame, increase the stiffness of the structure, this interaction is typically neglected.

In Figure 1.18*b*, we show one of the steel frames located just inside the end wall (labeled *ABCD* in Figure 1.18*a*) of the building. Here the metal roof deck is supported on beam *CD* that spans between two round hollow structural section or pipe columns located at the corners of the building. As shown in Figure 1.18*c*, the ends of the beam are connected to the tops of the columns by bolts that pass through the bottom flange of the beam and a cap plate welded to the top of the column. Since this type of connection cannot transmit moment effectively between the end of the beam and the top of the column, the designer assumes that this type of a connection acts as a hinge.

Because these bolted joints are not rigid, additional light members (often round bars or steel angle members) are run diagonally between adjacent columns in the plane of the frame, serving to stabilize the structure further. Without this diagonal bracing (Figure 1.18*b*), the resistance of the frame to lateral loads would be small, and the structure would lack lateral stiffness. Designers insert similar cross-bracing in the other three walls—and sometimes in the plane of the roof.

The frame is connected to the foundation by bolts that pass through a light steel baseplate, welded to the bottom of the column. The bottom ends of the bolts, called *anchor bolts*, are embedded in concrete piers located directly under the column. Typically, designers assume that a simple bolted connection of this type acts as a *pin support*; that is, the connection prevents the base of the column from displacing vertically and horizontally, but it does not have sufficient stiffness to prevent rotation. (Students often wrongly assume that a flat baseplate bolted to a concrete pier produces a fixed-end condition, but they are not taking into account the large loss of rotational restraint induced by even small flexural deformations of the plate.)

Although the bolted connection does have the capacity to provide a small but uncertain amount of rotational restraint to the base of the column, the designer usually treats it conservatively as a *frictionless pin*. However, it is usually unnecessary to achieve a fixed support at the column base to increase stiffness; doing so is expensive to construct as it requires the use of a heavy and stiffened baseplate and the foundation must be massive. The required lateral stiffness of the building can be provided by properly sizing the diagonal members.

Design of Frame for Gravity Load. To analyze this small frame for gravity load, the designer assumes the weight of the roof and any vertical live load (e.g., snow or ice) are carried by the roof deck (acting as a series of small parallel beams) to the frame shown in Figure 1.18*d*. This frame is idealized by the designer as a beam connected by a pinned joint to the columns. *The designer neglects the diagonal bracing—assumed to be inactive when vertical load acts*. Since no moments are assumed to develop at the ends of the beam, the designer analyzes the beam as a simply supported member with a uniform load (Figure 1.18*e*). Because the reactions of the beam are applied directly over the centerlines of the columns, the designer assumes that the column carries only direct stress and behaves as an axially loaded compression member.

Design for Lateral Load. The designer next checks for lateral loads. If a lateral load *P* (produced by wind or earthquake, for example) is applied to the top of the roof (Figure 1.18*f*), the designer can assume that one of the diagonals acting together with the roof beam and columns forms a truss. If the diagonals are light flexible members, only the diagonal running from *A* to *C*, which stretches and develops tensile stresses as the frame displaces to the right, is assumed to be effective. The opposite diagonal *BD* is assumed to buckle because it is slender and placed in compression by the lateral movement of the frame. If the wind reverses direction, the other diagonal *BD* would become effective, and diagonal *AC* would buckle. Such frame is called tension-only braced frame. If more stocky diagonal members are used, then one diagonal member will be designed as a tension member and the other member will be designed as a compression member that considers buckling.

Load Path. As we have illustrated in this simple problem, under certain types of loads, certain members come into play to transmit the loads into the supports. As long as the designer understands how to select a logical path for these loads, the analysis can be greatly simplified by eliminating members that are not effective. In selecting and laying out a structural system, therefore, it is essential that the designer clearly identifies the *load paths* to make sure that not only gravity loads but also lateral loads can be properly transmitted from the superstructure to the foundation.

1.7 Analyzing by Computer

Until the late 1950s, the analysis of certain types of indeterminate structures was a long, tedious procedure. The analysis of a structure with many joints and members (a space truss, for example) might require many months of computations by a team of experienced structural engineers. Moreover, since a number of simplifying assumptions about structural behavior were often required, the accuracy of the final results was uncertain. Today computer programs are available that can analyze most structures rapidly and accurately. Some exceptions still exist. If the structure is an unusual shape and complex—a thick-walled nuclear containment vessel or the hull of a submarine—the computer analysis can still be complicated and time-consuming.

Most computer programs for analyzing structures are written to produce a *first-order analysis*; that is, they assume (1) linear-elastic behavior, (2) that member forces are unaffected by the deformations (change in geometry) of the structure, and (3) that no reduction in flexural stiffness is produced in columns by compression forces.

The classical methods of analysis covered in this book produce a first-order analysis, suitable for the majority of structures, such as trusses, continuous beams, and frames, encountered in engineering practice. When a first-order analysis is used, structural design codes provide simplified procedures needed to adjust required member forces that may be underestimated.

While more complicated to use, second-order programs that do account for inelastic behavior, changes in geometry, and other effects influencing the magnitude of forces in members are more precise and produce a more accurate analysis. For example, long slender arches under moving loads can undergo changes in geometry that increase bending moments significantly. For structures of this type, a second-order analysis is essential.

Although computer structural analysis is routinely used in design office nowadays, the computer output and the resulting building design are as good as the assumptions made in the computer model and the accuracy of the input data. A structure with an ill-defined load path is not likely to perform well, especially under overloads. This is also true when the structure lacks redundancy. In 1977, the failure of the large three-dimensional space truss (Chapter 15 opening photo) supporting the 300-ft by 360-ft roof of the Hartford Civic Center Arena is an example of a structural design in which the designers relied on an incomplete computer analysis and failed to produce a safe structure. Among the factors contributing to this disaster were inaccurate data (the designer underestimated the deadweight of the roof by 1.5 million lb) and the inability of the computer program to predict the buckling load of the compression members in the truss. In other words, the presumption existed in the program that the structure was stable—an assumption in the majority of early computer programs used for analyzing structures. Shortly after a winter storm deposited a heavy load of rain-soaked snow and ice on the roof, the buckling of certain slender compression members in the roof truss precipitated a sudden collapse of the entire roof. Fortunately, the failure occurred several hours after a crowd of 5000 sports fans attending a basketball game had left the building. Had the failure taken place several hours sooner (when the building was occupied), hundreds of people would have been killed. Although no loss of life occurred, the facility was unusable for a considerable period, and large sums of money were required to clear the wreckage, to redesign the building, and to reconstruct the arena.

As the powers of computers and structural analysis software have increased tremendously over the past few decades, designers nowadays have the luxury to design very complicated structures that were not possible before. With the help of these "black box" tools, however, designers are also faced with an ever greater challenge and responsibility to properly prepare their models, interpret, and, more importantly, judge the accuracy of the computer analysis results and use them to anticipate all potential failure modes. An essential part of establishing such knowledge and "engineering intuition" is to study classical methods of structural analysis, which are the main focus of this textbook.

1.8 Preparation of Computations

Preparation of a set of clear, complete computations for each analysis is an important responsibility of the engineer. A well-organized set of computations not only will reduce the possibility of computational error, but also will provide essential information if the strength of an existing structure must be investigated at some future time. For example, the owner of a building may wish to determine if one or more additional floors can be added to an existing structure without overstressing the structural frame and foundations. If the original computations are complete and the engineer can determine the design loads, the design strengths, and the assumptions upon which the original analysis and design were based, evaluation of the modified structure's strength is facilitated.

Occasionally, a structure fails (in the worst case, lives are lost) or proves unsatisfactory in service (e.g., floors sag or vibrate, walls crack). In these situations, the original computations will be examined closely by all parties to establish the liability of the designer. A sloppy or incomplete set of computations can damage an engineer's reputation.

Therefore, in solving the homework problems in this book, students should consider each assignment as an opportunity to improve the skills required to produce computations of professional quality. With this objective in mind, the following suggestions are offered:

1. State the objective of the analysis in a short sentence.
2. Prepare a clear sketch of the structure, showing all loads and dimensions. Use a sharp pencil and a straightedge to draw lines. Figures and numbers that are neat and clear have a more professional appearance.
3. *Include all steps of your computations*. Computations cannot easily be checked by another engineer unless all steps are shown. Provide a word or two stating what is being done, as needed for clarification.
4. *Check the results* of your computations by making a static check (i.e., writing additional equilibrium equations).
5. If the structure is complex, check the computations by making an approximate analysis (Chapter 13).
6. Verify that the direction of the deflections is consistent with the direction of the applied forces. If a structure is analyzed by a computer, the deformed shape of the structure can be easily obtained.

Summary

- To begin our study of structural analysis, we reviewed the relationship between planning, design, and analysis. In this interrelated process, the structural engineer first establishes one or more initial configurations of possible structural forms while considering the appropriate load paths, estimates deadweights, selects critical design loads, and analyzes the structure. Once the structure is analyzed, major members are resized. If the results of the design confirm that the initial assumptions were correct, the design is complete. If there are large differences between the initial and final proportions, the design is modified, and the analysis and sizing repeated. This process continues until final results confirm that the proportions of the structure require no modifications.
- The characteristics of common structural elements that comprise typical buildings and bridges are reviewed. These include beams, trusses, arches, frames with rigid joints, cables, and shells.
- Although most structures are three-dimensional, the designer who develops an understanding of structural behavior can often divide the structure into a series of simpler planar structures for analysis. The designer is able to select a simplified and idealized model that accurately represents the essentials of the real structure.
- Since most structures are analyzed by computer, structural engineers must develop an understanding of structural behavior so they can, with a few simple computations, verify that the results of the computer analysis are reasonable. Structural failures not only involve high costs, but also may result in injury to the public or loss of life.

Taichiro Okazaki

Whole Building as Debris in the Tsunami Following the 2011 Tohoku Earthquake in Japan

The 2011 Tohoku Earthquake (magnitude 9.0) and subsequent tsunami caused widespread damage and casualties throughout the north-eastern coast of Japan. Lateral and buoyant forces of waves and flood waters, acting together, uplifted this entire building including its foundation, and carried it away. Note the footers and the ground on the right are intact, indicating the structure did not simply overturn. This and other recent tsunamis have highlighted the need for designers to consider combined effects of lateral and vertical loads imparted to structures due to these powerful waves.

CHAPTER 2

Design Loads and Structural Framing

Chapter Objectives

- Learn the importance of codes for the determination of the governing design loads as they relate to life safety and serviceability and apply to a building's structural framing system.
- Understand that code prescribed loads generate minimum design forces, which are either applied statically or dynamically in the analysis of the building's structural systems.
- Become familiar with dead and live loads, calculate floor material's self weight, select live loads based on a building's occupancy use, and learn tributary area method for calculating forces on beams, girders, or columns.
- Understand the effects of natural hazards including wind, earthquakes, and tsunamis on building structures and determine the design loads for these hazards.

2.1 Building and Design Code

A code is a set of technical specifications and standards that control major details of analysis, design, and construction of buildings, equipment, and bridges. The purpose of codes is to produce safe, economical structures so that the public will be protected from poor or inadequate design and construction.

Two types of codes exist. One type, called a *structural code*, is written by engineers and other specialists who are concerned with the design of a particular class of structure (e.g., buildings, highway bridges, or nuclear power plants) or who are interested in the proper use of a specific material (steel, reinforced concrete, aluminum, or wood). Typically, structural codes specify design loads, allowable stresses for various types of members, design assumptions, and requirements for materials. Examples of structural codes frequently used by structural engineers include the following:

1. *LRFD Bridge Design Specifications* by the American Association of State Highway and Transportation Officials (AASHTO) cover the design and analysis of highway bridges.

Building codes exist for the purpose of protecting public health, safety, and welfare in the construction and occupancy of buildings and structures, and do so by sets of rules specifying the minimum loads and requirements. Their improvement unfortunately sometimes follows lessons from disasters. For instance, the 1666 London Fire destroyed over 80 percent of the city's homes and gave way to the first thorough building code. In the United States, it was not until after the destructive 1906 San Francisco and 1933 Long Beach Earthquakes that California set in motion The Field and Riley Acts which enforced the country's first earthquake resistant construction practices. Even today, recent disastrous tsunamis in Indonesia and Japan have highlighted the need for consideration of tsunami wave and flooding loads. Engineers and scientists, however, are continuously working to develop building codes ahead of potential future problems.

2. *Manual for Railway Engineering* by the American Railway Engineering and Maintenance of Way Association (AREMA) covers the design and analysis of railroad bridges.
3. *Building Code Requirements for Structural Concrete* (ACI 318) by the American Concrete Institute (ACI) cover the analysis and design of concrete structures.
4. *Specification for Structural Steel Buildings* by the American Institute of Steel Construction (AISC) covers the analysis and design of steel structures.
5. *National Design Specification for Wood Construction* by the American Forest & Paper Association (AFPA) covers the analysis and design of wood structures.

The second type of code, called a *building code*, is established to cover construction in a given region (often a city or a state). A building code contains provisions pertaining to architectural, structural, mechanical, and electrical requirements. The objective of a building code is also to protect the public by accounting for the influence of local conditions on construction. Those provisions of particular concern to the structural designer cover such topics as soil conditions (bearing pressures), live loads, wind pressures, snow and ice loads, and earthquake forces. Today many building codes adopt the provisions of the ASCE/SEI 7-16 Standard, *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*, published by the American Society of Civil Engineers (ASCE) or the more recent *International Building Code* by the International Code Council.

As new systems evolve, as new materials become available, or as repeated failures of accepted systems occur, the contents of codes are reviewed and updated. In recent years the large volume of research on structural behavior and materials has resulted in frequent changes to both types of codes.

Most codes make provision for the designer to depart from prescribed standards if the designer can show by tests or analytical studies that such changes produce a safe design.

2.2 Loads

Structures must be proportioned so that they will not fail or deform excessively under load. Therefore, an engineer must take great care to anticipate the probable loads a structure must carry. Although the design loads specified by the codes are generally satisfactory for most buildings, the designer must also decide if these loads apply to the specific structure under consideration. For example, if the shape of a building is unusual (and induces increased wind speeds), wind forces may deviate significantly from the minimum prescribed by a building code. In such cases, the designer should conduct wind tunnel tests on models to establish the appropriate design forces. The designer should also try to foresee if the function of a structure (and consequently the loads it must carry) will change in the future. For example, if the possibility exists that heavier equipment may be introduced into an area that is originally designed for a smaller load, the designer may decide to increase the design loads specified by the code. Designers typically differentiate between two types of gravity load: live load and dead load.

2.3 Dead Loads and Gravity Framing

The load associated with the weight of the structure and its permanent components (floors, ceilings, ducts, and so forth) is called the *dead load*. When designing a structure, the member dead loads must first be estimated since member sizes are initially unknown, yet must still be accounted for in the total load carried by the structure. After members are sized and architectural details finalized, the dead load can be computed more accurately. If the computed value of dead load is approximately equal to (or slightly less than) the initial estimate of its value, the analysis is finished. But if a large difference exists between the estimated and computed values of dead load, the designer should revise the computations, using the improved value of deadweight.

In most buildings the space directly under each floor is occupied by a variety of utility lines and supports for fixtures including air ducts, water and sewage pipes, electrical conduit, and lighting fixtures. Rather than attempt to account for the actual weight and position of each item, designers typically add an additional 10 to 15 lb/ft^2 (0.479 to 0.718 kN/m^2) to the weight of the floor system to ensure that the strength of the floor, columns, and other structural members will be adequate.

Distribution of Dead Load to Framed Floor Systems

Many floor systems consist of a reinforced concrete slab supported on a rectangular grid of beams. The supporting beams reduce the span of the slab and permit the designer to reduce the depth and weight of the floor system. The distribution of dead loads to a floor beam depends on the geometric configuration of the beams forming the grid. To develop an insight into how load from a particular region of a slab is transferred to supporting beams, we will examine the three cases shown in Figure 2.1. In the first case, the edge beams support a *square slab* loaded with a uniform area load, q, which has units of force per area (Figure 2.1*a*). From symmetry we can infer that each of the four beams along the outside edges of the slab carries the same triangular line load, with units of force per length. In fact, if a slab with the same area of uniformly distributed reinforcement in the x and y directions were loaded to failure by a uniform load, large cracks would open along the main diagonals, confirming that each beam supports the load on a triangular area. The area of slab that is supported by a particular beam is termed as the beam's *tributary area*. Later in this chapter we will extend the application of the tributary area of the beams (and columns) to other gravity loads.

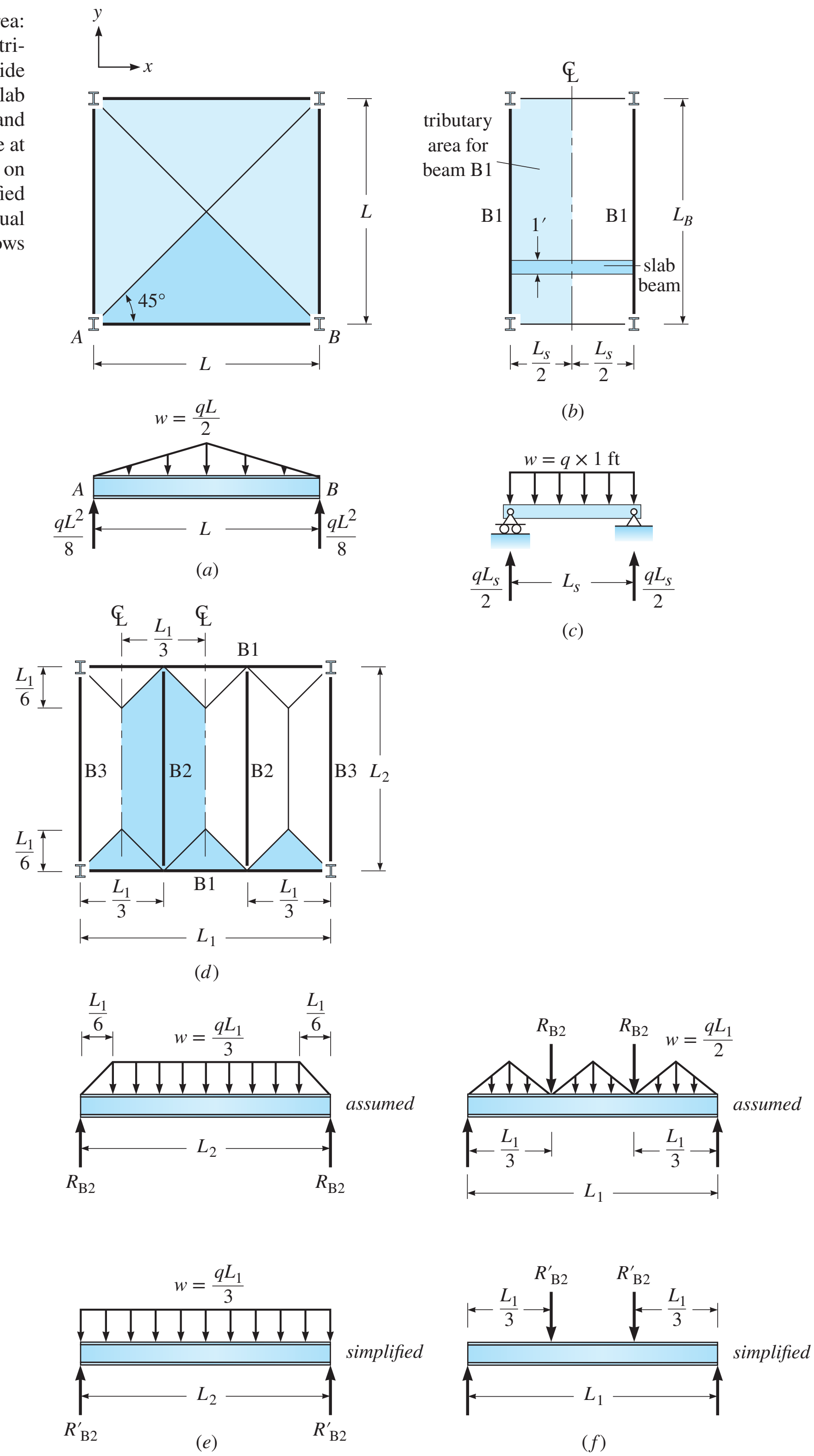


Figure 2.1: Concept of tributary area: (*a*) square slab, all edge beams support a triangular area; (*b*) two edge beams divide load equally; (*c*) load on a 1-ft width of slab in (*b*); (*d*) tributary areas for beams B1 and B2 shown shaded, all diagonal lines slope at 45°; (*e*) top figure shows actual load on beam B2 and bottom figure shows simplified load distribution; (*f*) top figure shows actual load on beam B1 and bottom figure shows simplified load distribution.

In the second case, we consider a rectangular slab, again loaded with a uniform area load q, supported on opposite sides by two parallel beams (Figure 2.1*b*). In this case, if we imagine a 1-ft-wide strip of slab that acts as a beam spanning a distance L_s between two edge beams (Figure 2.1*b*), we can see that the load on the slab divides equally between the supporting edge beams; that is, each foot of each edge beam, B1, carries a uniformly distributed line load, $w = qL_s/2$ (Figure 2.1*c*), and the tributary area for each beam is a rectangular area that extends out from the beam a distance $L_s/2$ to the centerline of the slab.

For the third case, shown in Figure 2.1*d*, a slab, carrying a uniformly distributed area load q, is supported on a rectangular grid of beams. The tributary area for both an interior and an exterior beam is shown shaded in Figure 2.1*d*. Each interior beam B2 (Figure 2.1*d*) carries a trapezoidal load. The edge beam B1, which is loaded at the third points by the reactions from the two interior beams, also carries smaller amounts of load from three triangular areas of slab (Figure 2.1*f*). If the ratio of the long to short side of a panel is approximately 2 or more, the actual load distributions on beam B2 can be simplified by assuming conservatively that the total load per foot, $w = qL_1/3$, is uniformly distributed over the entire length (Figure 2.1*e*), producing the reaction R'_{B2}. In the case of beam B1, we can simplify the analysis by assuming the reaction R'_{B2} from the uniformly loaded B2 beams is applied as a concentrated load at the third points (Figure 2.1*f*).

Table 2.1*a* lists the unit weights of a number of commonly used construction materials, and Table 2.1*b* contains the weights of building components that are frequently specified in building construction. We will make use of these tables in examples and problems.

Examples 2.1 and 2.2 introduce computations for dead load.

EXAMPLE 2.1

A three-ply asphalt felt and gravel roof over 2-in.-thick insulation board is supported by 18-in.-deep precast reinforced concrete beams with 3-ft-wide flanges (Figure 2.2). If the insulation weighs 3 lb/ft^2 and the asphalt roofing weighs $5\frac{1}{2}$ lb/ft^2, determine the total dead load, per foot of length, each beam must support.

Solution

Weight of beam is as follows:

Flange $\quad \frac{4}{12}\text{ ft} \times \frac{36}{12}\text{ ft} \times 1\text{ ft} \times 150\text{ lb/ft}^3 = 150\text{ lb/ft}$

Stem $\quad \frac{10}{12}\text{ ft} \times \frac{14}{12}\text{ ft} \times 1\text{ ft} \times 150\text{ lb/ft}^3 = 145\text{ lb/ft}$

Insulation $\quad 3\text{ lb/ft}^2 \times 3\text{ ft} \times 1\text{ ft} = 9\text{ lb/ft}$

Roofing $\quad 5\frac{1}{2}\text{ lb/ft}^2 \times 3\text{ ft} \times 1\text{ ft} = 16.5\text{ lb/ft}$

Total = 320.5 lb/ft,
round to 0.321 kip/ft

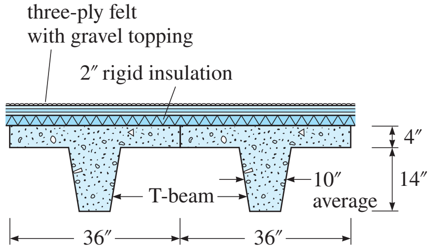


Figure 2.2: Cross section of reinforced concrete beams.

EXAMPLE 2.2

The steel framing plan of a small building is shown in Figure 2.3*a*. The floor consists of a 5-in.-thick reinforced concrete slab supported on steel beams (see section 1-1 in Figure 2.3*b*). Beams are connected to each other and to the corner columns by clip angles; see Figure 2.3*c*. The clip angles are assumed to provide the equivalent of a pin support for the beams; that is, they can transmit vertical load but no moment. An acoustical board ceiling, which weighs 1.5 lb/ft^2, is suspended from the concrete slab by closely spaced supports, and it can be treated as an additional uniform load on the slab. To account for the weight of ducts, piping, conduit, and so forth, located between the slab and ceiling (and supported by hangers from the slab), an additional dead load allowance of 20 lb/ft^2 is assumed. The designer initially estimates the weight of beams B1 at 30 lb/ft and the 24-ft girders B2 on column lines 1 and 2 at 50 lb/ft. Establish the magnitude of the dead load distribution on beam B1 and girder B2.

Solution

We will assume that all load between panel centerlines on either side of beam B1 (the tributary area) is supported by beam B1 (see the shaded area in Figure 2.3*a*). In other words, as previously discussed, to compute the dead load applied by the slab to the beam, we treat the slab as a series of

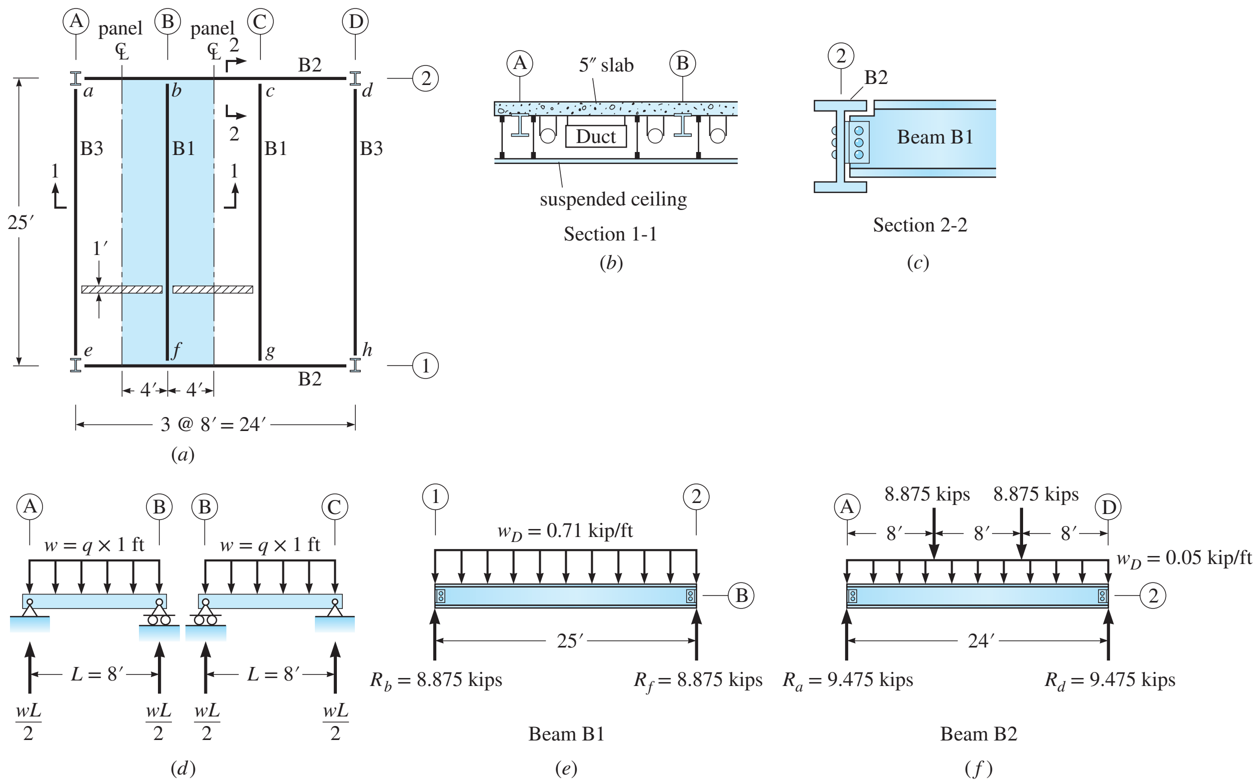


Figure 2.3: Determination of dead load for beam and girder.

closely spaced, 1-ft-wide, simply supported beams, spanning between the steel beams on column lines A and B, and between B and C (see the cross-hatched area in Figure 2.3*a*). One-half of the load, $qL/2$, will go to each supporting beam (Figure 2.3*d*), and the total slab reaction applied per foot of steel beam equals $qL = 8q$ (Figure 2.3*e*).

Total dead load applied per foot to beam B1:

Weight of slab	$1 \text{ ft} \times 1 \text{ ft} \times \frac{5}{12} \text{ ft} \times 8 \text{ ft} \times 150 \text{ lb/ft}^3 = 500 \text{ lb/ft}$
Weight of ceiling	$1.5 \text{ lb/ft}^2 \times 8 \text{ ft} = 12 \text{ lb/ft}$
Weight of ducts, etc.	$20 \text{ lb/ft}^2 \times 8 \text{ ft} = 160 \text{ lb/ft}$
Estimated weight of beam	$= 30 \text{ lb/ft}$
	Total = 702 lb/ft, round to 0.71 kip/ft

Sketches of each beam with its applied loads are shown in Figure 2.3*e* and *f*. The reactions (8.875 kips) from the B1 beams are applied as concentrated loads to the third points of girder B2 on column line 2 (Figure 2.3*f*). The uniform load of 0.05 kip/ft is the estimated weight of girder B2.

Tributary Areas of Columns

To determine the gravity loads transmitted into a column from a floor slab, the designer can either (1) determine the reactions of the beams framing into the column or (2) multiply the tributary area of the floor surrounding the column by the magnitude of the load per unit area acting on the floor. The *tributary area* of a column is defined as *the area surrounding the column that is bounded by the panel centerlines*. Use of tributary areas is the more common procedure of the two methods for computing column loads. In Figure 2.4 the tributary areas are shaded for corner column A1, interior column B2, and exterior column C1. Exterior columns located on the perimeter of a building also support the exterior walls as well as floor loads.

As you can see by comparing tributary areas for the floor system in Figure 2.4, when column spacing is approximately the same length in both directions, interior columns support approximately four times more floor dead load than corner columns. When we use the tributary areas to establish column loads, we do not consider the position of floor beams, but we do include an allowance for their weight.

Use of tributary areas is the more common procedure of the two methods for computing columns loads because designers also need the tributary areas to compute live loads, given that design codes specify that the percentage of *live load* transmitted to a column is an inverse function of the tributary areas; that is, as the tributary areas increase, the live load reduction increases. For columns supporting large areas, this reduction can reach a maximum of 40 to 50 percent. We will cover the ASCE7 standard for live load reduction in Section 2.4.

TABLE 2.1 Typical Design Dead Loads

(a) Material Weights

Substance	Weight, lb/ft^3 (kN/m^3)
Steel	490 (77.0)
Aluminum	165 (25.9)
Reinforced concrete:	
Normal weight	150 (23.6)
Light weight	90–120 (14.1–18.9)
Brick	120 (18.9)
Wood	
Southern pine	37 (5.8)
Douglas fir	34 (5.3)
Plywood	36 (5.7)

(b) Building Component Weights

Component	Weight, lb/ft^2 (kN/m^2)
Ceilings	
Gypsum plaster on suspended metal lath	10 (0.48)
Acoustical fiber tile on metal lath and channel ceiling	5 (0.24)
Floors	
Reinforced concrete slab per inch of thickness	
Normal weight	$12\frac{1}{2}$ (0.60)
Lightweight	7.5–10 (0.36–0.48)
Roofs	
Three-ply felt tar and gravel	$5\frac{1}{2}$ (0.26)
2-in. insulation	3 (0.14)
Walls and partitions	
Gypsum board (1-in. thick)	4(0.19)
Brick (per inch of thickness)	10 (0.48)
Hollow concrete masonry unit (12-in. thick)	
Heavy aggregate	85 (4.06)
Light aggregate	55 (2.63)
Hollow clay tile (6-in. thick)	30 (1.44)
2×4 studs at 16 in. on center, $\frac{1}{2}$-in. gypsum wall on both sides	8 (0.38)

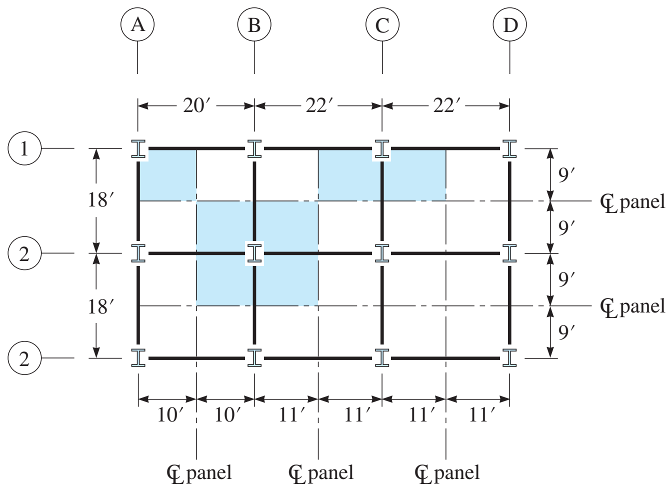


Figure 2.4: Tributary area of columns A1, B2, and C1 shown shaded.

EXAMPLE 2.3

Using the tributary area method, compute the floor dead loads supported by columns A1 and B2 in Figure 2.4. The floor system consists of a 6-in.-thick reinforced concrete slab weighing 75 lb/ft^2. Allow 15 lb/ft^2 for the weight of floor beams, utilities, and a ceiling suspended from the floor. The precast exterior wall supported by the perimeter beams weighs 600 lb/ft.

Solution

Total floor dead load is

$$D = 75 + 15 = 90 \text{ lb/ft}^2 = 0.09 \text{ kip/ft}^2$$

Dead load to column A1 is as follows:

Tributary area $\quad A_t = 9 \times 10 = 90 \text{ ft}^2$

Floor dead load $\quad A_t D = 90 \times 0.09 \text{ kip/ft}^2 = 8.1 \text{ kips}$

Weight of exterior wall =
weight/ft (length) = (0.6 kip/ft)(10 + 9) = 11.4 kips

Total = 19.5 kips

Dead load to column B2 is as follows:

Tributary area = $18 \times 21 = 378 \text{ ft}^2$

Total dead load = $378 \text{ ft}^2 \times 0.09 \text{ kip/ft}^2 = 34.02 \text{ kips}$

2.4 Live Loads

Buildings

Loads that can be moved on or off a structure are classified as *live loads*. Live loads include the weight of people, furniture, machinery, and other equipment. Live loads can vary over time especially if the function of the building changes. The live loads specified by codes for various types of buildings represent a conservative estimate of the maximum load likely to be produced by the intended use and occupancy of the building. In each region of the country, building codes typically specify the design live load. Currently, many state and city building codes base the magnitude of live loads and design procedures on the ASCE standard, which has evolved over time by relating the magnitude of the design load to the successful performance of actual buildings. When sizing members, designers must also consider short-term construction live loads, particularly if these loads are large. In the past a number of building failures have occurred during construction when large piles of heavy construction material were concentrated in a small area of a floor or roof of a partially erected building, when the capacity of members, not fully bolted or braced, is below their potential load capacity.

The ASCE standard typically specifies a minimum value of uniformly distributed live load for various types of buildings (a portion of the ASCE minimum live load table is shown in Table 2.2). If certain structures, such as

TABLE 2.2 Typical Design Floor Live Loads, L_o

Occupancy Use	Live Load, lb/ft^2 (kN/m^2)
Assembly areas and theaters	
Fixed seats (fastened to floor)	60 (2.87)
Lobbies	100 (4.79)
Stage floors	150 (7.18)
Libraries	
Reading rooms	60 (2.87)
Stack rooms	150 (7.18)
Office buildings	
Lobbies	100 (4.79)
Offices	50 (2.40)
Residential (one- and two-family)	
Habitable attics and sleeping areas	30 (1.44)
Uninhabitable attics with storage	20 (0.96)
All other areas (except balconies)	40 (1.92)
Schools	
Classrooms	40 (1.92)
Corridors above the first floor	80 (3.83)
First-floor corridors	100 (4.79)

Source: A portion of the ASCE minimum live load.

parking garages, are also subjected to large concentrated loads, the standard may require that forces in members be investigated for both uniform and concentrated loads, and that the design be based on the loading condition that creates the greatest stresses. For example, the ASCE standard specifies that, in the case of parking garages for passengers vehicles, members be designed to carry either the forces produced by a uniformly distributed live load of 40 lb/ft^2 or a concentrated load of 3000 lb acting over an area of 4.5 in. by 4.5 in.—whichever is larger.

The ASCE standard specifies wall partitions to be live loads. Normally designers try to position beams directly under heavy masonry walls to carry their weight directly into supports. If an owner requires flexibility to move walls or partitions periodically in order to reconfigure office or laboratory space, the designer can add an appropriate allowance to the floor live load. If partitions are light (such as stud walls with $\frac{1}{2}$-in. gypsum board each side), a minimum additional uniform floor live load of 15 lb/ft^2 (0.479 kN/m^2) is required. Similarly, in a factory or a laboratory that houses heavy test equipment, the allowance may be three or four times larger.

The ASCE standard specifies the minimum design live load on roofs as a uniformly distributed 20 psf on ordinary flat, pitched, and curved roofs. However, roof design live loads must also include mechanical equipment, architectural features, as well as potential live loads that can occur during construction, maintenance, and the life of the structure.

Live Load Reduction

Recognizing that a member supporting a large tributary area is less likely to be loaded at all points by the maximum value of live load than a member supporting a smaller floor area, building codes permit live load reductions for members that have a large tributary area. For this situation, the ASCE standard permits a reduction of the design floor live loads L_o, as listed in Table 2.2, by the following equation when the *influence area* $K_{LL}A_T$ is larger than 400 ft^2 (37.2 m^2). However, the reduced live load must not be less than 50 percent of L_o for members supporting one floor or a section of a single floor, nor less than 40 percent of L_o for members supporting two or more floors:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \quad \text{U.S. customary units} \qquad (2.1a)$$

$$L = L_o\left(0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}}\right) \quad \text{SI units} \qquad (2.1b)$$

where L_o = design live load listed in Table 2.2
L = reduced value of live load
A_T = tributary area, ft^2 (m^2)
K_{LL} = live load element factor, equal to 4 for interior columns and exterior columns without cantilever slabs and 2 for interior beams and edge beams without cantilever slabs

The minimum uniformly distributed roof live loads are permitted to be reduced by ASCE standard as follows:

$$L_r = L_o R_1 R_2 \tag{2.2}$$

where L_o = design roof live load

L_r = reduced roof live load, with minimum of 12 psf $\leq L_r \leq$ 20 psf (0.58 m$^2 \leq L_r \leq$ 0.96 m^2 in SI units) for ordinary flat, pitched, and curved roofs

$R_1 = 1$ for $A_T \leq 200$ ft^2 (18.58 m^2); and $R_1 = 0.6$ for $A_T \geq 600$ ft^2 (55.74 m^2); $R_1 = 1.2 - 0.001A_T$ ($R_1 = 1.2 - 0.011A_T$ in SI units) for 200 ft$^2 < A_T <$ 600 ft^2 (18.58 m$^2 < A_T <$ 55.74m^2)

$R_2 = 1.0$ for flat roofs $F \leq 4$; $R_2 = 1.2 - 0.05F$ for $4 < F < 12$; and $R_2 = 0.6$ for $F \geq 12$; where F = number of inches of rise per foot of roof slope for pitched roofs in SI: $F = 0.12 \times$ slope, with slope expressed in percentage)

For a column or beam supporting more than one floor, the term A_T represents the sum of the tributary areas from all floors.

Note that the ASCE standard limits the amount of live load reduction for special occupancies. Reduction in live load is not permitted for public assembly areas or when the live load is high (>100 psf).

EXAMPLE 2.4

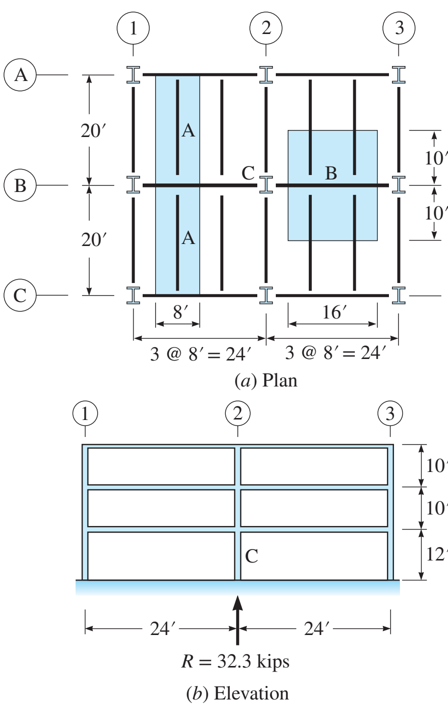


Figure 2.5: Live load reduction (*continues*).

For the three-story building shown in Figure 2.5*a* and *b*, calculate the design live load supported by (1) floor beam A, (2) girder B, and (3) the interior column C located at grid 2-B in the first story. Assume a 50 lb/ft^2 design live load, L_o, on all floors including the roof.

Solution

(1) Floor beam A

Span = 20 ft tributary area $A_T = 8(20) = 160$ ft^2 $K_{LL} = 2$

Determine if live loads can be reduced:

$$K_{LL}A_T = 2A_T = 2(160) = 320 \text{ ft}^2 < 400 \text{ ft}^2$$

therefore, no live load reduction is permitted.

Compute the uniform live load per foot to beam:

$$w = 50(8) = 400 \text{ lb/ft} = 0.4 \text{ kip/ft}$$

See Figure 2.5*d* for loads and reactions.

(2) Girder B

Girder B is loaded at each third point by the reactions of two floor beams. Its tributary area extends outward 10 ft from its longitudinal axis to the midpoint of the panels on each side of the girder (see shaded area in Figure 2.5*a*); therefore $A_T = 20(16) = 320$ ft^2.

$$K_{LL}A_T = 2(320) = 640 \text{ ft}^2$$

Since $K_{LL}A_T = 640\ \text{ft}^2 > 400\ \text{ft}^2$, a live load reduction is permitted. Use Equation 2.1*a*.

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) = 50\left(0.25 + \frac{15}{\sqrt{640}}\right) = 50(0.843) = 42.1\ \text{lb/ft}^2$$

Since $42.1\ \text{lb/ft}^2 > 0.5(50) = 25\ \text{lb/ft}^2$ (the lower limit), still use $w = 42.1\ \text{lb/ft}^2$.

$$\text{Load at third point} = 2\left[\frac{42.1}{1000}(8)(10)\right] = 6.736\ \text{kips}$$

The resulting design loads are shown in Figure 2.5*e*.

(3) Column C in the first story

The shaded area in Figure. 2.5*c* shows the tributary area of the interior column *for each floor*. Compute the tributary area for roof:

$$A_T = 20(24) = 480\ \text{ft}^2$$

The reduction for roof live load using Equation 2.2 is

$$R_1 = 1.2 - 0.001A_T = 0.72$$

$$R_2 = 1.0$$

and the reduced roof live load is

$$L_{\text{roof}} = L_oR_1R_2 = 50(0.72)(1.0) = 36.0\ \text{psf}$$

Compute the tributary area for the remaining two floors:

$$2A_T = 2(480) = 960\ \text{ft}^2$$

and

$$K_{LL}A_T = 4(960) = 3840\ \text{ft}^2 > 400\ \text{ft}^2$$

Therefore, reduce live load for two floors using Equation 2.1*a* (but not less than $0.4L_o$) is

$$L_{\text{floor}} = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) = 50\ \text{lb/ft}^2\left(0.25 + \frac{15}{\sqrt{3840}}\right)$$

$$= 24.6\ \text{lb/ft}^2$$

Since $24.6\ \text{lb/ft}^2 > 0.4 \times 50\ \text{lb/ft}^2 = 20\ \text{lb/ft}^2$ (the lower limit), use $L = 24.6\ \text{lb/ft}^2$.

Load to column $= A_T(L_{\text{roof}}) + 2A_T(L_{\text{floor}}) = 480(36.0) + 960(24.6) = 40{,}896\ \text{lb} = 40.9\ \text{kips}$.

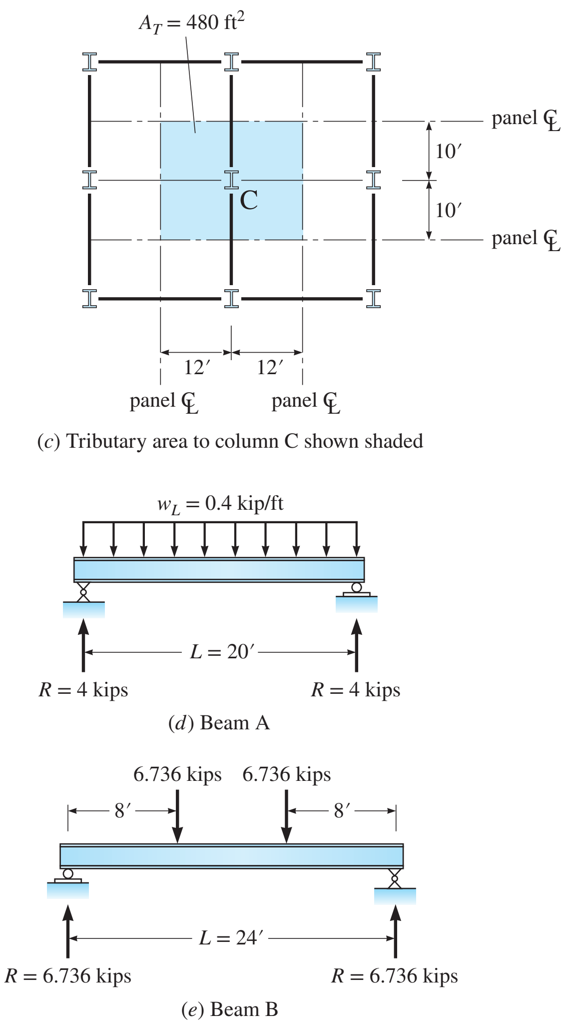


Figure 2.5: *Continued*

TABLE 2.3 Live Load Impact Factor

Loading Case	Impact Factor, *I* (percent)
Supports of elevators and elevator machinery	100
Supports of light machinery, shaft or motor-driven	20
Supports of reciprocating machinery or power-driven units	50
Hangers supporting floors and balconies	33
Cab-operated traveling crane support girders and their connections	25

Impact

Normally the values of live loads specified by building codes are treated as static loads because the majority of loads (desks, bookcases, filing cabinets, and so forth) are stationary. If loads are applied rapidly, they create additional impact forces. When a moving body (e.g., an elevator coming to a sudden stop) loads a structure, the structure deforms and absorbs the kinetic energy of the moving object. As an alternative to a dynamic analysis, moving loads are often treated as static forces and increased empirically by an impact factor. The magnitude of the impact factor I for a number of common structural supports is listed in Table 2.3.

EXAMPLE 2.5

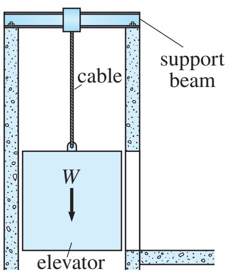


Figure 2.6: Beam supporting an elevator.

Determine the magnitude of the concentrated force for which the beam in Figure 2.6 supporting an elevator must be designed. The elevator, which weighs 3000 lb, can carry a maximum of six people with an average weight of 160 lb.

Solution

Read in Table 2.3 that an impact factor, I of 100 (percent) applies to all elevator loads. Therefore, the weight of the elevator and its passengers must be doubled.

$$\text{Total load} = D + L = 3000 + 6 \times 160 = 3960 \text{ lb}$$
$$\text{Design load} = (D + L)2 = 3960 \times 2 = 7920 \text{ lb}$$

Bridges

Highway bridge designs are governed by the AASHTO LRFD Bridge Design Specifications, which require that the engineer consider vehicular live load. The highway loading adopted in 1993, called HL-93, consists of a combination of the three-axle Design Truck (Figure 2.7*a*), or two-axel Design Tandem (Figure 2.7*b*), and the Design Lane Load (Figure 2.7*c*). The designer must consider the combination of these loads in various locations along the bridge span (see Section 12.8 and Figure 12.25).