Beginning & Intermediate Algebra

SIXTH EDITION

Julie Miller Molly O'Neill Nancy Hyde

McGraw Hill

Beginning & Intermediate Algebra

SIXTH EDITION

Julie Miller
Professor Emerita,
Daytona State College

Molly O'Neill
Professor Emerita,
Daytona State College

Nancy Hyde
Professor Emerita,
Broward College

Mc Graw Hill

BEGINNING & INTERMEDIATE ALGEBRA, SIXTH EDITION

Published by McGraw Hill LLC, 1325 Avenue of the Americas, New York, NY 10019.

This book is printed on acid-free paper.

1 2 3 4 5 6 7 8 9 LWI 26 25 24 23 22 21

ISBN 978-1-260-67353-1 (bound edition)
MHID 1-260-67353-7 (bound edition)
ISBN 978-1-264-12120-5 (loose-leaf edition)
MHID 1-264-12120-2 (loose-leaf edition)
ISBN 978-1-264-12096-3 (instructor's edition)
MHID 1-264-12096-6 (instructor's edition)

Portfolio Managers: *Brittney Merriman and Jennifer Morales*
Product Developer: *Megan Platt*
Marketing Manager: *Noah Evans*
Content Project Manager: *Jane Mohr*
Buyer: *Sandy Ludovissy*
Designer: *David W. Hash*
Content Licensing Specialist: *Lorraine Buczek*
Cover Image: ©*GaudiLab/Shutterstock*
Compositor: *Aptara®, Inc.*

Library of Congress Cataloging-in-Publication Data

Names: Miller, Julie, 1962- author. | O'Neill, Molly, 1953- author. | Hyde, Nancy, author.
Title: Beginning & intermediate algebra / Julie Miller, Professor Emerita, Daytona State College, Molly O'Neill, Professor Emerita, Daytona State College, Nancy Hyde, Professor Emerita, Broward College.
Other titles: Beginning and intermediate algebra
Description: Sixth edition. | New York, NY : McGraw Hill Education, [2022] | Includes index.
Identifiers: LCCN 2021003132 (print) | LCCN 2021003133 (ebook) | ISBN 9781260673531 (hardcover) | ISBN 9781264120963 (hardcover) | ISBN 9781264121205 (spiral bound) | ISBN 9781264121243 (pdf) | ISBN 9781264120994 (ebook other)
Subjects: LCSH: Algebra—Textbooks.
Classification: LCC QA152.3 .M57 2022 (print) | LCC QA152.3 (ebook) | DDC 512.9—dc23
LC record available at https://lccn.loc.gov/2021003132
LC ebook record available at https://lccn.loc.gov/2021003133

Letter from the Authors

Dear Colleagues,

Across the country, Developmental Math courses are in a state of flux, and we as instructors are at the center of it all. As many of our institutions are grappling with the challenges of placement, retention, and graduation rates, we are on the front lines with our students—supporting all of them in their educational journey.

Flexibility—No Matter Your Course Format!

The three of us each teach differently, as do many of our current users. The Miller/O'Neill/Hyde series is designed for successful use in a variety of course formats, both traditional and modern—classroom lecture settings, flipped classrooms, hybrid classes, and online-only classes.

Ease of Instructor Preparation

We've all had to fill in for a colleague, pick up a last-minute section, or find ourselves running across campus to yet a different course. The Miller/O'Neill/Hyde series is carefully designed to support instructors teaching in a variety of different settings and circumstances. Experienced, senior faculty members can draw from a massive library of static and algorithmic content found in ALEKS to meticulously build assignments and assessments sharply tailored to individual student needs. Newer instructors and part-time adjunct instructors, on the other hand, will find support through a wide range of digital resources and prebuilt assignments ready to go on Day One. With these tools, instructors with limited time to prepare for class can still facilitate successful student outcomes.

Many instructors want to incorporate discovery-based learning and groupwork into their courses but don't have time to write or find quality materials. Each section of the text has numerous discovery-based activities that we have tested in our own classrooms. These are found in the text and Student Resource Manual along with other targeted worksheets for additional practice and materials for a student portfolio.

Student Success—Now and in the Future

Too often our math placement tests fail our students, which can lead to frustration, anxiety, and often withdrawal from their education journey. We encourage you to learn more about ALEKS Placement, Preparation, and Learning (ALEKS PPL), which uses adaptive learning technology to place students appropriately. No matter the skills they come in with, the Miller/O'Neill/Hyde series provides resources and support that uniquely position them for success in that course and for their next course. Whether they need a brush-up on their basic skills, ADA supportive materials, or advanced topics to help them cross the bridge to the next level, we've created a support system for them.

We hope you are as excited as we are about the series and the supporting resources and services that accompany it. Please reach out to any of us with any questions or comments you have about our texts.

Julie Miller Molly O'Neill Nancy Hyde

About the Authors

Photo courtesy of Molly O'Neill

Julie Miller is from Daytona State College, where she taught developmental and upper-level mathematics courses for 20 years. Prior to her work at Daytona State College, she worked as a software engineer for General Electric in the area of flight and radar simulation. Julie earned a Bachelor of Science in Applied Mathematics from Union College in Schenectady, New York, and a Master of Science in Mathematics from the University of Florida. In addition to this textbook, she has authored textbooks for college algebra, trigonometry, and precalculus, as well as several short works of fiction and nonfiction for young readers.

"My father is a medical researcher, and I got hooked on math and science when I was young and would visit his laboratory. I can remember using graph paper to plot data points for his experiments and doing simple calculations. He would then tell me what the peaks and features in the graph meant in the context of his experiment. I think that applications and hands-on experience made math come alive for me, and I'd like to see math come alive for my students."

—*Julie Miller*

Molly O'Neill is also from Daytona State College, where she taught for 22 years in the School of Mathematics. She has taught a variety of courses from developmental mathematics to calculus. Before she came to Florida, Molly taught as an adjunct instructor at the University of Michigan–Dearborn, Eastern Michigan University, Wayne State University, and Oakland Community College. Molly earned a Bachelor of Science in Mathematics and a Master of Arts and Teaching from Western Michigan University in Kalamazoo, Michigan. Besides this textbook, she has authored several course supplements for college algebra, trigonometry, and precalculus and has reviewed texts for developmental mathematics.

"I differ from many of my colleagues in that math was not always easy for me. But in seventh grade I had a teacher who taught me that if I follow the rules of mathematics, even I could solve math problems. Once I understood this, I enjoyed math to the point of choosing it for my career. I now have the greatest job because I get to do math every day and I have the opportunity to influence my students just as I was influenced. Authoring these texts has given me another avenue to reach even more students."

—*Molly O'Neill*

Nancy Hyde served as a full-time faculty member of the Mathematics Department at Broward College for 24 years. During this time she taught the full spectrum of courses from developmental math through differential equations. She received a Bachelor of Science in Math Education from Florida State University and a Master's degree in Math Education from Florida Atlantic University. She has conducted workshops and seminars for both students and teachers on the use of technology in the classroom. In addition to this textbook, she has authored a graphing calculator supplement for *College Algebra*.

"I grew up in Brevard County, Florida, where my father worked at Cape Canaveral. I was always excited by mathematics and physics in relation to the space program. As I studied higher levels of mathematics I became more intrigued by its abstract nature and infinite possibilities. It is enjoyable and rewarding to convey this perspective to students while helping them to understand mathematics."

—*Nancy Hyde*

Dedication

To Our Students

Julie Miller · Molly O'Neill · Nancy Hyde

The Miller/O'Neill/Hyde Developmental Math Series

Julie Miller, Molly O'Neill, and Nancy Hyde originally wrote their developmental math series because students were entering their College Algebra course underprepared. The students were not mathematically mature enough to understand the concepts of math, nor were they fully engaged with the material. The authors began their developmental mathematics offerings with Intermediate Algebra to help bridge that gap. This in turn evolved into several series of textbooks from Prealgebra through Precalculus to help students at all levels before Calculus.

What sets all of the Miller/O'Neill/Hyde series apart is that they address course content through an author-created digital package that maintains a consistent voice and notation throughout the program. This consistency—in videos, PowerPoints, Lecture Notes, and Integrated Video and Study Guides—coupled with the power of ALEKS, ensures that students master the skills necessary to be successful in Developmental Math through Precalculus and prepares them for the Calculus sequence.

Developmental Math Series

The Developmental Math series is traditional in approach, delivering a purposeful balance of skills and conceptual development. It places a strong emphasis on conceptual learning to prepare students for success in subsequent courses.

Basic College Mathematics, Third Edition
Prealgebra, Third Edition
Prealgebra & Introductory Algebra, Second Edition
Beginning Algebra, Sixth Edition
Beginning & Intermediate Algebra, Sixth Edition
Intermediate Algebra, Sixth Edition
Developmental Mathematics: Prealgebra, Beginning Algebra, & Intermediate Algebra, Second Edition

The Miller/Gerken College Algebra/Precalculus Series

The Precalculus series serves as the bridge from Developmental Math coursework to future courses by emphasizing the skills and concepts needed for Calculus.

College Algebra with Corequisite Support, First Edition
College Algebra, Second Edition
College Algebra and Trigonometry, First Edition
Precalculus, First Edition

Acknowledgments

The author team most humbly would like to thank all the people who have contributed to this project and the Miller/O'Neill/Hyde Developmental Math series as a whole.

First and foremost, our utmost gratitude to Sarah Alamilla for her close partnership, creativity, and collaboration throughout this revision. Special thanks to our team of digital contributors for their thousands of hours of work: to Kelly Jackson, Jody Harris, Lizette Hernandez Foley, Lisa Rombes, Kelly Kohlmetz, and Leah Rineck for their devoted work. To Donna Gerken: thank you for the countless grueling hours working through spreadsheets to ensure thorough coverage of our content in ALEKS. To our digital authors, Linda Schott, Michael Larkin, and Alina Coronel: thank you for digitizing our content so it could be brought into ALEKS. We also offer our sincerest appreciation to the outstanding video talent: Alina Coronel, Didi Quesada, Tony Alfonso, and Brianna Ashley. So many students have learned from you! To Jennifer Blue, Carey Lange, John Murdzek, and Kevin Campbell: thank you so much for ensuring accuracy in our manuscripts.

We also greatly appreciate the many people behind the scenes at McGraw Hill without whom we would still be on page 1. To Megan Platt, our product developer: thank you for being our help desk and handling all things math, English, and editorial. To Brittney Merriman, our portfolio manager and team leader: thank you so much for leading us down this path. Your insight, creativity, and commitment to our project have made our job easier.

To the marketing team, Michele McTighe, Noah Evans, and Mary Ellen Rahn: thank you for your creative ideas in making our books come to life in the market. Thank you as well to Debbie McFarland, Justin Washington, and Sherry Bartel for continuing to drive our long-term content vision through their market development efforts. And many thanks to the team at ALEKS for creating its spectacular adaptive technology and for overseeing the quality control.

To the production team: Jane Mohr, David Hash, Lorraine Buczek, and Sandy Ludovissy—thank you for making the manuscript beautiful and for keeping the unruly authors on track. To our copyeditor Kevin Campbell and proofreader John Murdzek, who have kept a watchful eye over our manuscripts—the two of you are brilliant. To our compositor Manvir Singh and his team at Aptara, you've been a dream to work with. And finally, to Kathleen McMahon and Caroline Celano, thank you for supporting our projects for many years and for the confidence you've always shown in us.

Most importantly, we give special thanks to the students and instructors who use our series in their classes.

Julie Miller
Molly O'Neill
Nancy Hyde

Contents

Chapter 15 (Online)

Additional Topics 15-1

Online Appendix A Review of Beginning Algebra A-1

To the Student

Take a deep breath and know that you aren't alone. Your instructor, fellow students, and we, your authors, are here to help you learn and master the material for this course and prepare you for future courses. You may feel like math just isn't your thing, or maybe it's been a long time since you've had a math class—that's okay!

We wrote the text and all the supporting materials with you in mind. Most of our students aren't really sure how to be successful in math, but we can help with that.

As you begin your class, we'd like to offer some specific suggestions:

1. **Attend class.** Arrive on time and be prepared. If your instructor has asked you to read prior to attending class—do it. How often have you sat in class and thought you understood the material, only to get home and realize you don't know how to get started? By reading and trying a couple of Skill Practice exercises, which follow each example, you will be able to ask questions and gain clarification from your instructor when needed.
2. **Be an *active* learner.** Whether you are at lecture, watching an author lecture or exercise video, or are reading the text, pick up a pencil and work out the examples given. Math is learned only by doing; we like to say, "Math is not a spectator sport." If you like a bit more guidance, we encourage you to use the Integrated Video and Study Guide. It was designed to provide structure and note-taking for lectures and while watching the accompanying videos.
3. **Schedule time to do some math every day.** Exercise, foreign language study, and math are three things that you must do every day to get the results you want. If you are used to cramming and doing all of your work in a few hours on a weekend, you should know that even mathematicians start making silly errors after an hour or so! Check your answers. Skill Practice exercises all have the answers at the bottom of that page. Odd-numbered exercises throughout the text have answers in the back of the text. If you didn't get it right, don't throw in the towel. Try again, revisit an example, or bring your questions to class for extra help.
4. **Prepare for quizzes and exams.** Each chapter has a set of Chapter Review Exercises at the end to help you integrate all of the important concepts. In addition, there is a detailed Chapter Summary and a Chapter Test. If you use ALEKS, use all of the tools available within the program to test your understanding.
5. **Use your resources.** This text comes with numerous supporting resources designed to help you succeed in this class and in your future classes. Additionally, your instructor can direct you to resources within your institution or community. Form a student study group. Teaching others is a great way to strengthen your own understanding, and they might be able to return the favor if you get stuck.

We wish you all the best in this class and in your educational journey!

Julie Miller　　Molly O'Neill　　Nancy Hyde

Student Guide to the Text

Clear, Precise Writing

Learning from our own students, we have written this text in simple and accessible language. Our goal is to keep you engaged and supported throughout your coursework.

Call-Outs

Just as your instructor will share tips and math advice in class, we provide call-outs throughout the text to offer tips and warn against common mistakes.

- Tip boxes offer additional insight into a concept or procedure.
- Avoiding Mistakes help fend off common student errors.
- For Review boxes positioned strategically throughout the text remind students of key skills relating to the current topic.

Examples

- Each example is step-by-step, with thorough annotation to the right explaining each step.
- Following each example is a similar **Skill Practice** exercise to give you a chance to test your understanding. You will find the answer at the bottom of the page—providing a quick check.

Exercise Sets

Each type of exercise is built so you can successfully learn the materials and show your mastery on exams.

- **Activities for discovery-based learning** appear before the exercise sets to walk students through the concepts presented in each section of the text.
- **Study Skills Exercises** integrate your studies of math concepts with strategies for helping you grow as a student overall.
- **Vocabulary and Key Concept Exercises** check your understanding of the language and ideas presented within the section.
- **Prerequisite Review** exercises keep fresh your knowledge of math content already learned by providing practice with concepts explored in previous sections.
- **Concept Exercises** assess your comprehension of the specific math concepts presented within the section.
- **Mixed Exercises** evaluate your ability to successfully complete exercises that combine multiple concepts presented within the section.
- **Expanding Your Skills** challenge you with advanced skills practice exercises around the concepts presented within the section.
- **Problem Recognition Exercises** appear in strategic locations in each chapter of the text. These will require you to distinguish between similar problem types and to determine what type of problem-solving technique to apply.
- **Technology Exercises** appear where appropriate.

End-of-Chapter Materials

The features at the end of each chapter are perfect for reviewing before test time.

- **Section-by-section summaries** provide references to key concepts, examples, and vocabulary.
- **Chapter Review Exercises** provide additional opportunities to practice material from the entire chapter.
- **Chapter tests** are an excellent way to test your complete understanding of the chapter concepts.

How Will Miller/O'Neill/Hyde Help Your Students *Get Better Results?*

Clarity, Quality, and Accuracy

Julie Miller, Molly O'Neill, and Nancy Hyde know what students need to be successful in mathematics. Better results come from clarity in their exposition, quality of step-by-step worked examples, and accuracy of their exercise sets; but it takes more than just great authors to build a textbook series to help students achieve success in mathematics. Our authors worked with a strong team of mathematics instructors from around the country to ensure that the clarity, quality, and accuracy you expect from the Miller/O'Neill/Hyde series was included in this edition.

Exercise Sets

Comprehensive sets of exercises are available for every student level. Julie Miller, Molly O'Neill, and Nancy Hyde worked with a board of advisors from across the country to offer the appropriate depth and breadth of exercises for your students. **Problem Recognition Exercises** were created to improve student performance while testing.

Practice exercise sets help students progress from skill development to conceptual understanding. Student tested and instructor approved, the Miller/O'Neill/Hyde exercise sets will help your students *get better results*.

- **Activities for Discovery-Based Learning**
- **Prerequisite Review Exercises**
- **Problem Recognition Exercises**
- **Skill Practice Exercises**
- **Study Skills Exercises**
- **Mixed Exercises**
- **Expanding Your Skills Exercises**
- **Vocabulary and Key Concepts Exercises**
- **Technology Exercises**

Step-By-Step Pedagogy

This text provides enhanced step-by-step learning tools to help students *get better results*.

- **For Review** tips placed in the margin guide students back to related prerequisite skills needed for full understanding of course-level topics.
- **Worked Examples** provide an "easy-to-understand" approach, clearly guiding each student through a step-by-step approach to master each practice exercise for better comprehension.
- **TIPs** offer students extra cautious direction to help improve understanding through hints and further insight.
- **Avoiding Mistakes** boxes alert students to common errors and provide practical ways to avoid them. Both of these learning aids will help students get better results by showing how to work through a problem using a clearly defined step-by-step methodology that has been class tested and student approved.

Get Better Results

Formula for Student Success

Step-by-Step Worked Examples

- Do you get the feeling that there is a disconnect between your students' class work and homework?
- Do your students have trouble finding worked examples that match the practice exercises?
- Do you prefer that your students see examples in the textbook that match the ones you use in class?

Miller/O'Neill/Hyde's *Worked Examples* offer a clear, concise methodology that replicates the mathematical processes used in the authors' classroom lectures.

Step 2 Write each term as the product of the GCF and another factor.
Step 3 Use the distributive property to remove the GCF.
Note: To check the factorization, multiply the polynomials to remove parentheses.

Classroom Examples: Exercises 18 and 22

Example 5 Factoring Out the Greatest Common Factor

Factor out the GCF.

a. $4x - 20$ **b.** $6w^2 + 3w$

Instructor Note: Tell students that an expression is factored only if the GCF has been removed. The expression $2(2x - 10)$ is not factored completely.

Solution:

a. $4x - 20$	The GCF is 4.
$= 4(x) - 4(5)$	Write each term as the product of the GCF and another factor.
$= 4(x - 5)$	Use the distributive property to factor out the GCF.

TIP: Any factoring problem can be checked by multiplying the factors:

Check: $4(x - 5) = 4x - 20$ ✓

Answer
6. $(x + 2)$

Classroom Examples

To ensure that the classroom experience also matches the examples in the text and the practice exercises, we have included references to even-numbered exercises to be used as Classroom Examples. These exercises are highlighted in the Practice Exercises at the end of each section.

Classroom Example: Exercise 88

Ryan McVay/Photodisc/Getty Images

Example 6 Adding Real Numbers in Applications

a. It is common for newborn infants to fluctuate in weight. Elise and Benjamin's baby lost 7 oz the first week after birth and gained 10 oz the second week. Write a mathematical expression to describe this situation, and then simplify the result.

b. A student has \$120 in her checking account. After depositing her paycheck of \$215, she writes a check for \$255 to cover her portion of the rent and another check for \$294 to cover her car payment. Write a mathematical expression to describe this situation, and then simplify the result.

Solution:

a. $-7 + 10$	A loss of 7 oz can be interpreted as -7 oz.
$= 3$	The infant had a net gain of 3 oz.
b. $\underbrace{120 + 215} + (-255) + (-294)$	Writing a check is equivalent to adding a negative amount to the bank account.

Get Better Results

Quality Learning Tools

For Review Boxes

Throughout the text, just-in-time tips and reminders of prerequisite skills appear in the margin alongside the concepts for which they are needed. References to prior sections are given for cases where more comprehensive review is available earlier in the text.

FOR REVIEW

Recall that to subtract fractions, write each fraction with the LCD.

$$\frac{5}{6}-\frac{3}{4}=\frac{5\cdot 2}{6\cdot 2}-\frac{3\cdot 3}{4\cdot 3}$$
$$=\frac{10}{12}-\frac{9}{12}$$
$$=\frac{1}{12}$$

TIP and Avoiding Mistakes Boxes

TIP and **Avoiding Mistakes** boxes have been created based on the authors' classroom experiences—they have also been integrated into the **Worked Examples.** These pedagogical tools will help students get better results by learning how to work through a problem using a clearly defined step-by-step methodology.

Example 12 Clearing Parentheses and Combining *Like* Terms

Classroom Example: Exercise 112

Simplify by clearing parentheses and combining *like* terms.

$$-7a-4[3a-2(a+6)]-4$$

Solution:

$-7a-4[3a-2(a+6)]-4$	
$=-7a-4[3a-2a-12]-4$	Apply the distributive property to clear the innermost parentheses.
$=-7a-4[a-12]-4$	Simplify within brackets by combining *like* terms.
$=-7a-4a+48-4$	Apply the distributive property to clear the brackets.
$=-11a+44$	Combine *like* terms.

Avoiding Mistakes

First clear the innermost parentheses and combine *like* terms within the brackets. Then use the distributive property to clear the brackets.

Skill Practice Clear the parentheses and combine *like* terms.

23. $6-5[-2y-4(2y-5)]$

Answers

21. $5x-1$ **22.** $-11x-7y$
23. $50y-94$

Avoiding Mistakes Boxes:

Avoiding Mistakes boxes are integrated throughout the textbook to alert students to common errors and how to avoid them.

TIP: To simplify square roots, it is advisable to become familiar with the following perfect squares and square roots.

$0^2=0 \longrightarrow \sqrt{0}=0$	$7^2=49 \longrightarrow \sqrt{49}=7$
$1^2=1 \longrightarrow \sqrt{1}=1$	$8^2=64 \longrightarrow \sqrt{64}=8$
$2^2=4 \longrightarrow \sqrt{4}=2$	$9^2=81 \longrightarrow \sqrt{81}=9$
$3^2=9 \longrightarrow \sqrt{9}=3$	$10^2=100 \longrightarrow \sqrt{100}=10$
$4^2=16 \longrightarrow \sqrt{16}=4$	$11^2=121 \longrightarrow \sqrt{121}=11$
$5^2=25 \longrightarrow \sqrt{25}=5$	$12^2=144 \longrightarrow \sqrt{144}=12$
$6^2=36 \longrightarrow \sqrt{36}=6$	$13^2=169 \longrightarrow \sqrt{169}=13$

TIP Boxes

Teaching tips are usually revealed only in the classroom. Not anymore! TIP boxes offer students helpful hints and extra direction to help improve understanding and provide further insight.

Get Better Results

Better Exercise Sets and Better Practice Yields Better Results

- Do your students have trouble with problem solving?
- Do you want to help students overcome math anxiety?
- Do you want to help your students improve performance on math assessments?

Problem Recognition Exercises

Problem Recognition Exercises present a collection of problems that look similar to a student upon first glance, but are actually quite different in the manner of their individual solutions. Students sharpen critical thinking skills and better develop their "solution recall" to help them distinguish the method needed to solve an exercise—an essential skill in mathematics.

Problem Recognition Exercises were tested in the authors' developmental mathematics classes and were created to improve student performance on tests.

Problem Recognition Exercises

Addition and Subtraction of Real Numbers

1. State the rule for adding two negative numbers. Add their absolute values and apply a negative sign.

2. State the rule for adding a negative number to a positive number. Subtract the smaller absolute value from the larger absolute value. Apply the sign of the number with the larger absolute value.

For Exercises 3–10, perform the indicated operations.

3. **a.** $14 + (-8)$ 6 **b.** $-14 + 8$ −6 **c.** $-14 + (-8)$ −22 **d.** $14 - (-8)$ 22 **e.** $-14 - 8$ −22

4. **a.** $-5 - (-3)$ −2 **b.** $-5 + (-3)$ −8 **c.** $-5 - 3$ −8 **d.** $-5 + 3$ −2 **e.** $5 - (-3)$ 8

5. **a.** $-25 + 25$ 0 **b.** $25 - 25$ 0 **c.** $25 - (-25)$ 50 **d.** $-25 - (-25)$ 0 **e.** $-25 + (-25)$ −50

6. **a.** $\frac{1}{2} + \left(-\frac{2}{3}\right)$ $-\frac{1}{6}$ **b.** $-\frac{1}{2} + \left(\frac{2}{3}\right)$ $\frac{1}{6}$ **c.** $-\frac{1}{2} + \left(-\frac{2}{3}\right)$ $-\frac{7}{6}$ **d.** $\frac{1}{2} - \left(-\frac{2}{3}\right)$ $\frac{7}{6}$ **e.** $-\frac{1}{2} - \frac{2}{3}$ $-\frac{7}{6}$

7. **a.** $3.5 - 7.1$ −3.6 **b.** $3.5 - (-7.1)$ 10.6 **c.** $-3.5 + 7.1$ 3.6 **d.** $-3.5 - (-7.1)$ 3.6 **e.** $-3.5 + (-7.1)$ −10.6

8. **a.** $6 - 1 + 4 - 5$ 4 **b.** $6 - (1 + 4) - 5$ −4 **c.** $6 - (1 + 4 - 5)$ 6 **d.** $(6 - 1) - (4 - 5)$ 6

9. **a.** $-100 - 90 - 80$ −270 **b.** $-100 - (90 - 80)$ −110 **c.** $-100 + (90 - 80)$ −90 **d.** $-100 - (90 + 80)$ −270

10. **a.** $-8 - (-10) + 20^2$ 402 **b.** $-8 - (-10 + 20)^2$ −108 **c.** $[-8 - (-10) + 20]^2$ 484 **d.** $[-8 - (-10)]^2 + 20$ 24

Get Better Results

Student Centered Applications

The Miller/O'Neill/Hyde Board of Advisors partnered with our authors to bring the *best applications* from every region in the country! These applications include real data and topics that are more relevant and interesting to today's student.

Concept 4: Applications Involving Addition of Real Numbers

Burke/Triolo/Brand X Pictures/Jupiterimages

85. The temperature in Minneapolis, Minnesota, began at −5°F (5° below zero) at 6:00 A.M. By noon, the temperature had risen 13°, and by the end of the day, the temperature had dropped 11° from its noontime high. Write an expression using addition that describes the change in temperatures during the day. Then evaluate the expression to give the temperature at the end of the day.
$-5 + 13 + (-11)$; -3°F

86. The temperature in Toronto, Ontario, Canada, began at 4°F. A cold front went through at noon, and the temperature dropped 9°. By 4:00 P.M., the temperature had risen 2° from its noontime low. Write an expression using addition that describes the changes in temperature during the day. Then evaluate the expression to give the temperature at the end of the day. $4 + (-9) + 2$; -3°F

87. For 4 months, Amara monitored her weight loss or gain. Her records showed that she lost 8 lb, gained 1 lb, gained 2 lb, and lost 5 lb. Write an expression using addition that describes Amara's total loss or gain and evaluate the expression. Interpret the result. **(See Example 6.)**
$-8 + 1 + 2 + (-5) = -10$; Amara lost 10 lb.

88. Alan just started an online business. His profit/loss records for the past 5 months show that he had a profit of \$200, a profit of \$750, a loss of \$340, a loss of \$290, and a profit of \$900. Write an expression using addition that describes Alan's total profit or loss and evaluate the expression. Interpret the result.
$200 + 750 + (-340) + (-290) + 900 = 1220$; Alan had a profit of \$1220.

89. Yoshima has \$52.23 in her checking account. She writes a check for groceries for \$52.95. **(See Example 6.)**

a. Write an addition statement that expresses Yoshima's transaction. $52.23 + (-52.95)$

90. Mohammad has \$40.02 in his checking account. He writes a check for a pair of shoes for \$40.96.

a. Write an addition statement that expresses Mohammad's transaction. $40.02 + (-40.96)$

Activities

Each section of the text ends with an activity that steps the student through the major concepts of the section. The purpose of the activities is to promote active, discovery-based learning for the student. The implementation of the activities is flexible for a variety of delivery methods. For face-to-face classes, the activities can be used to break up lecture by covering the exercises intermittently during the class. For the flipped classroom and hybrid classes, students can watch the videos and try the activities. Then, in the classroom, the instructor can go over the activities or have the students compare their answers in groups. For online classes, the activities provide great discussion questions.

Section 2.8 Activity

Inequality statements surround us in day-to-day life. In Exercises A.1–A.4, let x represent the unknown quantity, and write a mathematical inequality to represent the given statement.

A.1. In the United States, a person must be at least 18 years old to vote. $x \geq 18$ yr

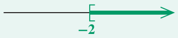
A.2. A child younger than 2 years old can fly free on most airlines. $x < 2$ yr

A.3. The normal range for hemoglobin for an adult female is between 12.0 grams per deciliter (g/dL) and 15.2 g/dL, inclusive. $12.0 \leq x \leq 15.2$ g/dL

A.4. To receive a discount at the zoo, a child must be at most 6 years old. $x \leq 6$ yr

A.5. a. Graph the inequality $x > -2$.

b. Graph the inequality $x \geq -2$.



c. Explain when to use a parenthesis, (or), versus a bracket, [or], when graphing an inequality.
Use a parenthesis when an "endpoint" is not included in the solution set. Use a bracket when an "endpoint" is included in the solution set.

d. Write the inequality $x > -2$ in interval notation. $(-2, \infty)$

e. Write the inequality $x \geq -2$ in interval notation. $[-2, \infty)$

f. Explain when to use a parenthesis, (or), versus a bracket, [or], when using interval notation.
Use a parenthesis when an "endpoint" is not included in the solution set and for ∞ and $-\infty$. Use a bracket when an "endpoint" is included in the solution set.

A.6. a. Graph the inequality $x < 3$.

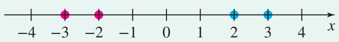
b. Graph the inequality $3 > x$.

c. What conclusion can you draw about the inequalities $x < 3$ and $3 > x$ based on their graphs?
The inequalities $x < 3$ and $3 > x$ are equivalent, which means that they have the same solution set.

d. Write the inequality $x < 3$ in interval notation. $(-\infty, 3)$

e. Write the inequality $3 > x$ in interval notation. $(-\infty, 3)$

A.7. Refer to the number line to fill in the blanks with < or >.



a. 2 □ 3 <

b. −2 □ −3 >

A.8. The following inequalities can each be solved in one step. Solve the inequality. Graph the solution set and write the set in interval notation.

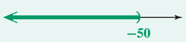
a. $5x > 10$

$(2, \infty)$

b. $x + 5 > 10$

$(5, \infty)$

c. $\dfrac{x}{-5} > 10$



$(-\infty, -50)$

d. $-5x > 10$

$(-\infty, -2)$

Get Better Results

Additional Supplements

Lecture Videos Created by the Authors

Julie Miller began creating these lecture videos for her own students to use when they were absent from class. The student response was overwhelmingly positive, prompting the author team to create the lecture videos for their entire developmental math book series. In these videos, the authors walk students through the learning objectives using the same language and procedures outlined in the book. Students learn and review right alongside the author! Students can also access the written notes that accompany the videos.

Integrated Video and Study Workbooks

The Integrated Video and Study Workbooks were built to be used in conjunction with the Miller/O'Neill/Hyde Developmental Math series online lecture videos. These new video guides allow students to consolidate their notes as they work through the material in the book, and they provide students with an opportunity to focus their studies on particular topics that they are struggling with rather than entire chapters at a time. Each video guide contains written examples to reinforce the content students are watching in the corresponding lecture video, along with additional written exercises for extra practice. There is also space provided for students to take their own notes alongside the guided notes already provided. By the end of the academic term, the video guides will not only be a robust study resource for exams, but will serve as a portfolio showcasing the hard work of students throughout the term.

Dynamic Math Animations

The authors have constructed a series of animations to illustrate difficult concepts where static images and text fall short. The animations leverage the use of on-screen movement and morphing shapes to give students an interactive approach to conceptual learning. Some provide a virtual laboratory for which an application is simulated and where students can collect data points for analysis and modeling. Others provide interactive question-and-answer sessions to test conceptual learning.

Exercise Videos

The authors, along with a team of faculty who have used the Miller/O'Neill/Hyde textbooks for many years, have created exercise videos for designated exercises in the textbook. These videos cover a representative sample of the main objectives in each section of the text. Each presenter works through selected problems, following the solution methodology employed in the text.

The video series is available online as part of ALEKS 360. The videos are closed-captioned for the hearing impaired and meet the Americans with Disabilities Act Standards for Accessible Design.

Student Resource Manual

The *Student Resource Manual (SRM),* created by the authors, is a printable, electronic supplement available to students through ALEKS. Instructors can also choose to customize this manual and package with their course materials. With increasing demands on faculty schedules, this resource offers a convenient means for both full-time and adjunct faculty to promote active learning and success strategies in the classroom.

This manual supports the series in a variety of different ways:

- Additional group activities developed by the authors to supplement what is already available in the text
- Discovery-based classroom activities written by the authors for each section
- Excel activities that not only provide students with numerical insights into algebraic concepts, but also teach simple computer skills to manipulate data in a spreadsheet

- Worksheets for extra practice written by the authors, including Problem Recognition Exercise Worksheets
- Lecture Notes designed to help students organize and take notes on key concepts
- Materials for a student portfolio

Annotated Instructor's Edition

In the *Annotated Instructor's Edition* (*AIE*), answers to all exercises appear adjacent to each exercise in a color used *only* for annotations. The *AIE* also contains Instructor Notes that appear in the margin. These notes offer instructors assistance with lecture preparation. In addition, there are Classroom Examples referenced in the text that are highlighted in the Practice Exercises. Also found in the *AIE* are icons within the Practice Exercises that serve to guide instructors in their preparation of homework assignments and lessons.

PowerPoints

The PowerPoints present key concepts and definitions with fully editable slides that follow the textbook. An instructor may project the slides in class or post to a website in an online course.

Test Bank

Among the supplements is a computerized test bank using the algorithm-based testing software TestGen® to create customized exams quickly. Hundreds of text-specific, open-ended, and multiple-choice questions are included in the question bank.

ALEKS PPL: Pave the Path to Graduation with Placement, Preparation, and Learning

- **Success in College Begins with Appropriate Course Placement:** A student's first math course is critical to his or her success. With a unique combination of adaptive assessment and personalized learning, ALEKS Placement, Preparation, and Learning (PPL) accurately measures the student's math foundation and creates a personalized learning module to review and refresh lost knowledge. This allows the student to be placed and successful in the right course, expediting the student's path to complete their degree.
- **The Right Placement Creates Greater Value:** Students invest thousands of dollars in their education. ALEKS PPL helps students optimize course enrollment by avoiding courses they don't need to take and helping them pass the courses they do need to take. With more accurate student placement, institutions will retain the students that they recruit initially, increasing their recruitment investment and decreasing their DFW rates. Understanding where your incoming students are placing helps to plan and develop course schedules and allocate resources efficiently.
- **See ALEKS PPL in Action:** http://bit.ly/ALEKSPPL

Mc Graw Hill create®

McGraw Hill Create allows you to select and arrange content to match your unique teaching style, add chapters from McGraw Hill textbooks, personalize content with your syllabus or lecture notes, create a cover design, and receive your PDF review copy in minutes! Order a print or eBook for use in your course, and update your material as often as you'd like. Additional third-party content can be selected from a number of special collections on Create. Visit **McGraw Hill Create** to browse Create Collections: **http://create.mheducation.com**.

Our Commitment to Market Development and Accuracy

McGraw Hill's Development Process is an ongoing, market-oriented approach to building accurate and innovative print and digital products. We begin developing a series by partnering with authors who have a vision for positively impacting student success. Next, we share these ideas and the manuscript with instructors to review and provide feedback to ensure that the authors' ideas represent the needs of that discipline. Throughout multiple drafts, we help our authors to incorporate ideas and suggestions from reviewers to ensure that the series follows the pulse of today's classroom. With all editions, we commit to accuracy in the print text, supplements, and online platforms. In addition to involving instructors as we develop our content, we also perform accuracy checks throughout the various stages of development and production. Through our commitment to this process, we are confident that our series features content that has been thoughtfully developed and vetted to meet the needs of both instructors and students.

Acknowledgments and Reviewers

Paramount to the development of this series was the invaluable feedback provided by the instructors from around the country who reviewed the manuscript or attended a market development event over the course of the several years the text was in development.

Maryann Faller, *Adirondack Community College*
Albert Miller, *Ball State University*
Debra Pearson, *Ball State University*
Patricia Parkison, *Ball State University*
Robin Rufatto, *Ball State University*
Melanie Walker, *Bergen Community College*
Robert Fusco, *Bergen Community College*
Latonya Ellis, *Bishop State Community College*
Ana Leon, *Bluegrass Community College & Technical College*
Kaye Black, *Bluegrass Community College & Technical College*
Barbara Elzey, *Bluegrass Community College & Technical College*
Cheryl Grant, *Bowling Green State University*
Beth Rountree, *Brevard College*
Juliet Carl, *Broward College*
Lizette Foley, *Broward College*
Angie Matthews, *Broward College*
Mitchel Levy, *Broward College*
Jody Harris, *Broward College*
Michelle Carmel, *Broward College*
Antonnette Gibbs, *Broward College*
Kelly Jackson, *Camden Community College*
Elizabeth Valentine, *Charleston Southern University*
Adedoyin Adeyiga, *Cheyney University of Pennsylvania*
Dot French, *Community College of Philadelphia*
Brad Berger, *Copper Mountain College*
Donna Troy, *Cuyamaca College*
Brianna Kurtz, *Daytona State College–Daytona Beach*
Jennifer Walsh, *Daytona State College–Daytona Beach*
Marc Campbell, *Daytona State College–Daytona Beach*
Richard Rupp, *Del Mar College*
Joseph Hernandez, *Delta College*
Randall Nichols, *Delta College*
Thomas Wells, *Delta College*
Paul Yun, *El Camino College*
Catherine Bliss, *Empire State College–Saratoga Springs*
Laurie Davis, *Erie Community College*
Linda Kuroski, *Erie Community College*
David Usinski, *Erie Community College*
Ron Bannon, *Essex County College*
David Platt, *Front Range Community College*
Alan Dinwiddie, *Front Range Community College*
Shanna Goff, *Grand Rapids Community College*
Betsy McKinney, *Grand Rapids Community College*
Cathy Gardner, *Grand Valley State University*
Jane Mays, *Grand Valley State University*
John Greene, *Henderson State University*
Fred Worth, *Henderson State University*
Ryan Baxter, *Illinois State University*
Angela Mccombs, *Illinois State University*
Elisha Van Meenen, *Illinois State University*
Teresa Hasenauer, *Indian River State College*

Tiffany Lewis, *Indian River State College*
Deanna Voehl, *Indian River State College*
Joe Jordan, *John Tyler Community College*
Sally Copeland, *Johnson County Community College*
Nancy Carpenter, *Johnson County Community College*
Susan Yellott, *Kilgore College*
Kim Miller, *Labette Community College*
Michelle Hempton, *Lansing Community College*
Michelle Whitmer, *Lansing Community College*
Kuen Lee, *Los Angeles Trade Tech*
Nic Lahue, *MCC-Longview Community College*
Jason Pallett, *MCC-Longview Community College*
Janet Wyatt, *MCC-Longview Community College*
Rene Barrientos, *Miami Dade College—Kendall*
Nelson De La Rosa, *Miami Dade College—Kendall*
Jody Balzer, *Milwaukee Area Technical College*
Shahla Razavi, *Mt. San Jacinto College*
Shawna Bynum, *Napa Valley College*
Tammy Ford, *North Carolina A & T University*
Ebrahim Ahmadizadeh, *Northampton Community College*
Christine Wetzel-Ulrich, *Northampton Community College*
Sharon Totten, *Northeast Alabama Community College*
Rodolfo Maglio, *Northeastern Illinios University*
Christine Copple, *Northwest State Community College*
Sumitana Chatterjee, *Nova Community College*
Charbel Fahed, *Nova Community College*
Ken Hirschel, *Orange County Community College*
Linda K. Schott, *Ozarks Technical Community College*
Matthew Harris, *Ozarks Technical Community College*
Daniel Kopsas, *Ozarks Technical Community College*
Andrew Aberle, *Ozarks Technical Community College*
Alan Papen, *Ozarks Technical Community College*
Angela Shreckhise, *Ozarks Technical Community College*
Jacob Lewellen, *Ozarks Technical Community College*
Marylynne Abbott, *Ozarks Technical Community College*
Jeffrey Gervasi, *Porterville College*
Stewart Hathaway, *Porterville College*
Lauran Johnson, *Richard Bland College*
Matthew Nickodemus, *Richard Bland College*
Cameron English, *Rio Hondo College*
Lydia Gonzalez, *Rio Hondo College*
Mark Littrell, *Rio Hondo College*
Matthew Pitassi, *Rio Hondo College*
Wayne Lee, *Saint Philips College*
Paula Looney, *Saint Philips College*
Fred Bakenhus, *Saint Philips College*
Lydia Casas, *Saint Philips College*
Gloria Guerra, *Saint Philips College*
Sounny Slitine, *Saint Philips College*
Jessica Lopez, *Saint Philips College*
Lorraine Lopez, *San Antonio College*
Peter Georgakis, *Santa Barbara City College*
Sandi Nieto-Navarro, *Santa Rosa Junior College*
Steve Drucker, *Santa Rosa Junior College*
Jean-Marie Magnier, *Springfield Tech Community College*
Dave Delrossi, *Tallahassee Community College*
Natalie Johnson, *Tarrant County College South*
Marilyn Peacock, *Tidewater Community College*
Yvonne Aucoin, *Tidewater Community College*
Cynthia Harris, *Triton College*
Jennifer Burkett, *Triton College*
Christyn Senese, *Triton College*
Jennifer Dale, *Triton College*
Patricia Hussey, *Triton College*
Glenn Jablonski, *Triton College*
Myrna La Rosa, *Triton College*
Michael Maltenfort, *Truman College*
Abdallah Shuaibi, *Truman College*
Marta Hidegkuti, *Truman College*
Sandra Wilder, *University of Akron*
Sandra Jovicic, *University of Akron*
Edward Migliore, *University of California–Santa Cruz*
Kelly Kohlmetz, *University of Wisconsin—Milwaukee*
Leah Rineck, *University of Wisconsin—Milwaukee*
Carolann Van Galder, *University of Wisconsin—Rock County*
Claudia Martinez, *Valencia College*
Stephen Toner, *Victor Valley Community College*
David Cooper, *Wake Tech Community College*
Karlata Elliott, *Wake Tech Community College*
Laura Kalbaugh, *Wake Tech Community College*
Kelly Vetter, *Wake Tech Community College*
Jacqui Fields, *Wake Tech Community College*
Jennifer Smeal, *Wake Tech Community College*
Shannon Vinson, *Wake Tech Community College*
Kim Walaski, *Wake Tech Community College*
Lisa Rombes, *Washtenaw Community College*
Maziar Ouliaeinia, *Western Iowa Tech Community College*
Keith McCoy, *Wilbur Wright College*

Create More Lightbulb Moments.

Every student has different needs and enters your course with varied levels of preparation. ALEKS® pinpoints what students already know, what they don't and, most importantly, what they're ready to learn next. Optimize your class engagement by aligning your course objectives to ALEKS® topics and layer on our textbook as an additional resource for students.

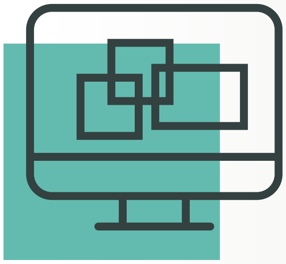
ALEKS® Creates a Personalized and Dynamic Learning Path

ALEKS® creates an optimized path with an ongoing cycle of learning and assessment, celebrating students' small wins along the way with positive real-time feedback. Rooted in research and analytics, ALEKS® improves student outcomes by fostering better preparation, increased motivation and knowledge retention.

*visit **bit.ly/whatmakesALEKSunique** to learn more about the science behind the most powerful adaptive learning tool in education!

Preparation & Retention

The more prepared your students are, the more effective your instruction is. Because ALEKS® understands the prerequisite skills necessary for mastery, students are better prepared when a topic is presented to them. ALEKS® provides personalized practice and guides students to what they need to learn next to achieve mastery. ALEKS® improves knowledge and student retention through periodic knowledge checks and personalized learning paths. This cycle of learning and assessment ensures that students remember topics they have learned, are better prepared for exams, and are ready to learn new content as they continue into their next course.



Flexible Implementation: Your Class Your Way!

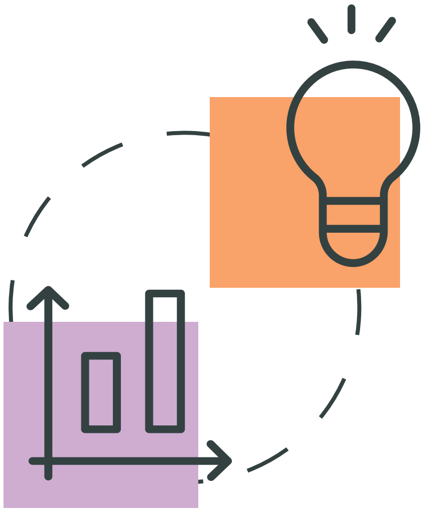
ALEKS® enables you to structure your course regardless of your instruction style and format. From a traditional classroom, to various co-requisite models, to an online prep course before the start of the term, ALEKS® can supplement your instruction or play a lead role in delivering the content.

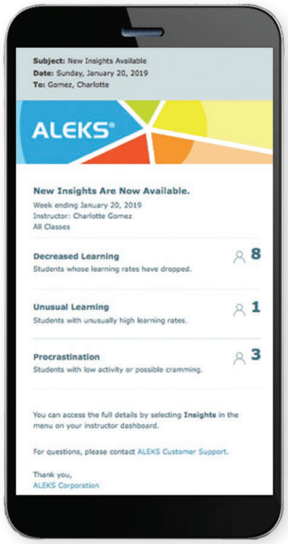
*visit **bit.ly/ALEKScasestudies** to see how your peers are delivering better outcomes across various course models!

Outcomes & Efficacy

Our commitment to improve student outcomes services a wide variety of implementation models and best practices, from lecture-based to labs and co-reqs to summer prep courses. Our case studies illustrate our commitment to help you reach your course goals and our research demonstrates our drive to support all students, regardless of their math background and preparation level.

*visit **bit.ly/outcomesandefficacy** to review empirical data from ALEKS® users around the country





Turn Data Into Actionable Insights

ALEKS® Reports are designed to inform your instruction and create more meaningful interactions with your students when they need it the most. ALEKS® Insights alert you when students might be at risk of falling behind so that you can take immediate action. Insights summarize students exhibiting at least one of four negative behaviors that may require intervention including Failed Topics, Decreased Learning, Unusual Learning and Procrastination & Cramming.

DIGITAL EDGE50 AWARDS 2019

Winner of 2019 Digital Edge 50 Award for Data Analytics!

bit.ly/ALEKS_MHE

The Set of Real Numbers

1

CHAPTER OUTLINE

Numbers in Our World

Zephyr P/Shutterstock

Imagine a world where the only numbers known are the counting or natural numbers (1, 2, 3, 4, . . .). Now imagine that you want to sell only a fraction of your land to another person, or that you owe twenty dollars and fifteen cents to your bank. How could these values be written without formal numerical symbols? Living in such a world would deter the growth of a complex society like ours.

It is difficult to fathom that the use of zero, fractions, and negative numbers was formally accepted only about a thousand years ago! Before that time, communicating about parts of items, the absence of value, and owing money to a lender was likely done in creative but arduous ways. When we talk about a *third* of a parcel of land, temperatures below *zero* such as *negative* 20 degrees, and the number π, we make use of the many subsets of the **set of real numbers**.

Real numbers enable us to talk about parts of things, to expand our thinking to explain phenomena in precise ways, and to operate with numbers in a consistent and predictable manner. In this chapter we will explore how real numbers are used, and the way they open the door to algebra.

Section 1.1 Fractions

Concepts

1. Basic Definitions
2. Prime Factorization
3. Simplifying Fractions to Lowest Terms
4. Multiplying Fractions
5. Dividing Fractions
6. Adding and Subtracting Fractions
7. Operations on Mixed Numbers

1. Basic Definitions

The study of algebra involves many of the operations and procedures used in arithmetic. Therefore, we begin this text by reviewing the basic operations of addition, subtraction, multiplication, and division on fractions and mixed numbers.

We begin with the numbers used for counting:

the **natural numbers:** 1, 2, 3, 4, . . .

and

the **whole numbers:** 0, 1, 2, 3, . . .

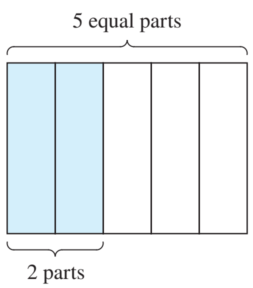
Whole numbers are used to count the number of whole units in a quantity. A fraction is used to express part of a whole unit. If a child gains $2\frac{1}{2}$ lb, the child has gained two whole pounds plus a portion of a pound. To express the additional half pound mathematically, we may use the fraction $\frac{1}{2}$.

A Fraction and Its Parts

Fractions are numbers of the form $\frac{a}{b}$, where $\frac{a}{b} = a \div b$ and b does not equal zero.

In the fraction $\frac{a}{b}$, the **numerator** is a, and the **denominator** is b.

The denominator of a fraction indicates how many equal parts divide the whole. The numerator indicates how many parts are being represented. For instance, suppose Jack wants to plant carrots in $\frac{2}{5}$ of a rectangular garden. He can divide the garden into five equal parts and use two of the parts for carrots (Figure 1-1).

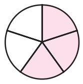
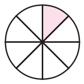


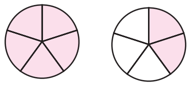
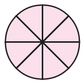
The shaded region represents $\frac{2}{5}$ of the garden.

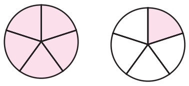
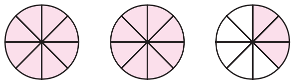
Figure 1-1

Proper Fractions, Improper Fractions, and Mixed Numbers

1. If the numerator of a fraction is less than the denominator, the fraction is a **proper fraction**. A proper fraction represents a quantity that is less than a whole unit.
2. If the numerator of a fraction is greater than or equal to the denominator, then the fraction is an **improper fraction**. An improper fraction represents a quantity greater than or equal to a whole unit.
3. A **mixed number** is a whole number added to a proper fraction.

Proper Fractions: $\frac{3}{5}$  $\frac{1}{8}$ 

Improper Fractions: $\frac{7}{5}$  $\frac{8}{8}$ 

Mixed Numbers: $1\frac{1}{5}$  $2\frac{3}{8}$ 

2. Prime Factorization

To perform operations on fractions it is important to understand the concept of a factor. For example, when the numbers 2 and 6 are multiplied, the result (called the **product**) is 12.

$$\underbrace{2 \times 6}_{\text{factors}} = \underbrace{12}_{\text{product}}$$

The numbers 2 and 6 are said to be **factors** of 12. (In this context, we refer only to natural number factors.) The number 12 is said to be factored when it is written as the product of two or more natural numbers. For example, 12 can be factored in several ways:

$$12 = 1 \times 12 \qquad 12 = 2 \times 6 \qquad 12 = 3 \times 4 \qquad 12 = 2 \times 2 \times 3$$

A natural number greater than 1 that has only two factors, 1 and itself, is called a **prime number**. The first several prime numbers are 2, 3, 5, 7, 11, and 13. A natural number greater than 1 that is not prime is called a **composite number**. That is, a composite number has factors other than itself and 1. The first several composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, and 16.

Avoiding Mistakes

The number 1 is neither prime nor composite.

Example 1 Writing a Natural Number as a Product of Prime Factors

Write each number as a product of prime factors.

a. 12 **b.** 30

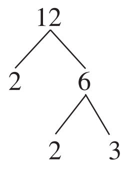
Solution:

a. $12 = 2 \times 2 \times 3$

Divide 12 by prime numbers until the result is also a prime number.

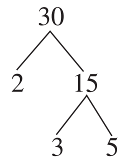
$$\begin{array}{r} 2\overline{)12} \\ 2\overline{)6} \\ 3 \end{array}$$

Or use a factor tree



b. $30 = 2 \times 3 \times 5$

$$\begin{array}{r} 2\overline{)30} \\ 3\overline{)15} \\ 5 \end{array}$$



Skill Practice Write the number as a product of prime factors.

1. 40 **2.** 72

Answers

1. $2 \times 2 \times 2 \times 5$
2. $2 \times 2 \times 2 \times 3 \times 3$

3. Simplifying Fractions to Lowest Terms

The process of factoring numbers can be used to reduce or simplify fractions to lowest terms. A fractional portion of a whole can be represented by infinitely many fractions. For example, Figure 1-2 shows that $\frac{1}{2}$ is equivalent to $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, and so on.

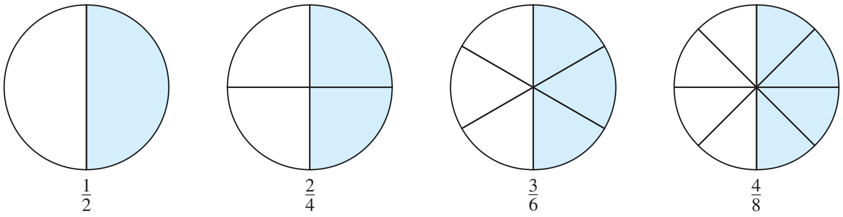


Figure 1-2

The fraction $\frac{1}{2}$ is said to be in **lowest terms** because the numerator and denominator share no common factor other than 1.

To simplify a fraction to lowest terms, we use the following important principle.

Fundamental Principle of Fractions

Suppose that a number, c, is a common factor in the numerator and denominator of a fraction. Then

$$\frac{a \times c}{b \times c} = \frac{a}{b} \times \frac{c}{c} = \frac{a}{b} \times 1 = \frac{a}{b}$$

To simplify a fraction, we begin by factoring the numerator and denominator into prime factors. This will help identify the common factors.

Example 2 Simplifying a Fraction to Lowest Terms

Simplify $\frac{45}{30}$ to lowest terms.

Solution:

$\frac{45}{30} = \frac{3 \times 3 \times 5}{2 \times 3 \times 5}$	Factor the numerator and denominator.
$= \frac{3}{2} \times \frac{3}{3} \times \frac{5}{5}$	Apply the fundamental principle of fractions.
$= \frac{3}{2} \times 1 \times 1$	Any nonzero number divided by itself is 1.
$= \frac{3}{2}$	Any number multiplied by 1 is itself.

Skill Practice Simplify to lowest terms.

3. $\frac{20}{50}$

Answer

3. $\frac{2}{5}$

In Example 2, we showed numerous steps to reduce fractions to lowest terms. However, the process is often simplified by dividing out the *greatest common factor* from the numerator and denominator. The **greatest common factor (GCF)** of the numerator and denominator of a fraction is the largest factor that is common to both values. For example, consider the fraction $\frac{45}{30}$. To find the greatest common factor of 45 and 30, factor each number into its prime factors. Then identify the common factors.

$45 = 3 \times 3 \times 5$ The factors that are common to both lists are circled.

$30 = 2 \times 3 \times 5$ The GCF is 3×5, which is 15.

Thus, to reduce the fraction $\frac{45}{30}$ to lowest terms, we have the following:

$\frac{45}{30} = \frac{3 \times 15}{2 \times 15}$ The greatest common factor of 45 and 30 is 15.

$= \frac{3 \times \cancel{15}^{1}}{2 \times \cancel{15}_{1}}$ The symbol / is often used to show that a common factor has been divided out.

$= \frac{3}{2}$ Notice that "dividing out" the common factor of 15 has the same effect as dividing the numerator and denominator by 15. This is often done mentally.

$\frac{\cancel{45}^{3}}{\cancel{30}_{2}} = \frac{3}{2}$ ← 45 divided by 15 equals 3. ← 30 divided by 15 equals 2.

Example 3 **Simplifying a Fraction to Lowest Terms**

Simplify $\frac{14}{42}$ to lowest terms.

Solution:

$\frac{14}{42} = \frac{1 \times 14}{3 \times 14}$ The greatest common factor of 14 and 42 is 14.

$= \frac{1 \times \cancel{14}^{1}}{3 \times \cancel{14}_{1}}$

$= \frac{1}{3}$ $\frac{\cancel{14}^{1}}{\cancel{42}_{3}} = \frac{1}{3}$ ← 14 divided by 14 equals 1. ← 42 divided by 14 equals 3.

Avoiding Mistakes

In Example 3, the common factor 14 in the numerator and denominator simplifies to 1. It is important to remember to write the factor 1 in the numerator. The simplified form of the fraction is $\frac{1}{3}$.

Skill Practice Simplify to lowest terms.

4. $\frac{32}{12}$

4. Multiplying Fractions

Multiplying Fractions

If b is not zero and d is not zero, then

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

To multiply fractions, multiply the numerators and multiply the denominators.

Answer

4. $\frac{8}{3}$

Example 4 Multiplying Fractions

Multiply the fractions: $\frac{1}{4} \times \frac{1}{2}$

Solution:

$$\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$$ Multiply the numerators. Multiply the denominators.

Notice that the product $\frac{1}{4} \times \frac{1}{2}$ represents a quantity that is $\frac{1}{4}$ of $\frac{1}{2}$. Taking $\frac{1}{4}$ of a quantity is equivalent to dividing the quantity by 4. One-half of a pie divided into four pieces leaves pieces that each represent $\frac{1}{8}$ of the pie (Figure 1-3).

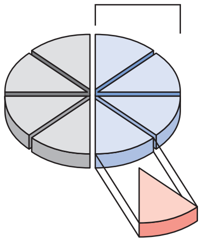


Figure 1-3

Skill Practice Multiply.

5. $\frac{2}{7} \times \frac{3}{5}$

Example 5 Multiplying Fractions

Multiply the fractions.

a. $\frac{7}{10} \times \frac{15}{14}$ **b.** $\frac{2}{13} \times \frac{13}{2}$ **c.** $5 \times \frac{1}{5}$

Solution:

a. $\frac{7}{10} \times \frac{15}{14} = \frac{7 \times 15}{10 \times 14}$ Multiply the numerators. Multiply the denominators.

$= \frac{\overset{1}{\cancel{7}} \times \overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}} \times \underset{2}{\cancel{14}}}$ Divide out the common factors.

$= \frac{3}{4}$ Multiply.

b. $\frac{2}{13} \times \frac{13}{2} = \frac{2 \times 13}{13 \times 2} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{13}}}{\underset{1}{\cancel{13}} \times \underset{1}{\cancel{2}}} = \frac{1}{1} = 1$

Multiply $1 \times 1 = 1$.

Multiply $1 \times 1 = 1$.

c. $5 \times \frac{1}{5} = \frac{5}{1} \times \frac{1}{5}$ The whole number 5 can be written as $\frac{5}{1}$.

$= \frac{\overset{1}{\cancel{5}} \times 1}{1 \times \underset{1}{\cancel{5}}} = \frac{1}{1} = 1$ Divide out the common factors and multiply.

TIP: The same result can be obtained by dividing out common factors *before* multiplying.

$$\frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{14}}} = \frac{3}{4}$$

Skill Practice Multiply.

6. $\frac{8}{9} \times \frac{3}{4}$ **7.** $\frac{4}{5} \times \frac{5}{4}$ **8.** $10 \times \frac{1}{10}$

Answers

5. $\frac{6}{35}$ **6.** $\frac{2}{3}$ **7.** 1 **8.** 1

5. Dividing Fractions

Before we divide fractions, we need to know how to find the reciprocal of a fraction. Notice from Example 5 that $\frac{2}{13} \times \frac{13}{2} = 1$ and $5 \times \frac{1}{5} = 1$. The numbers $\frac{2}{13}$ and $\frac{13}{2}$ are said to be reciprocals because their product is 1. Likewise the numbers 5 and $\frac{1}{5}$ are reciprocals.

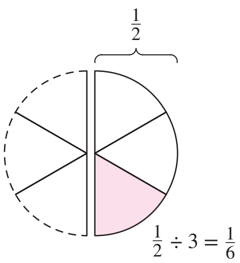
The Reciprocal of a Number

Two nonzero numbers are **reciprocals** of each other if their product is 1. Therefore, the reciprocal of the fraction

$$\frac{a}{b} \text{ is } \frac{b}{a} \qquad \text{because} \qquad \frac{a}{b} \times \frac{b}{a} = 1$$

Number	Reciprocal	Product
$\frac{2}{15}$	$\frac{15}{2}$	$\frac{2}{15} \times \frac{15}{2} = 1$
$\frac{1}{8}$	$\frac{8}{1}$ (or equivalently 8)	$\frac{1}{8} \times 8 = 1$
$6 \left(\text{or equivalently } \frac{6}{1}\right)$	$\frac{1}{6}$	$6 \times \frac{1}{6} = 1$

To understand the concept of dividing fractions, consider a pie that is half-eaten. Suppose the remaining half must be divided among three people, that is, $\frac{1}{2} \div 3$. However, dividing by 3 is equivalent to taking $\frac{1}{3}$ of the remaining $\frac{1}{2}$ of the pie (Figure 1-4).



$$\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Figure 1-4

This example illustrates that dividing two numbers is equivalent to multiplying the first number by the reciprocal of the second number.

Dividing Fractions

Let a, b, c, and d be numbers such that b, c, and d are not zero. Then,

multiply

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

reciprocal

To divide fractions, multiply the first fraction by the reciprocal of the second fraction.

Example 6 Dividing Fractions

Divide the fractions.

a. $\dfrac{8}{5} \div \dfrac{3}{10}$ **b.** $\dfrac{12}{13} \div 6$

Solution:

a. $\dfrac{8}{5} \div \dfrac{3}{10} = \dfrac{8}{5} \times \dfrac{10}{3}$ Multiply by the reciprocal of $\frac{3}{10}$, which is $\frac{10}{3}$.

$= \dfrac{8 \times \overset{2}{\cancel{10}}}{\underset{1}{\cancel{5}} \times 3} = \dfrac{16}{3}$ Divide out the common factors and multiply.

b. $\dfrac{12}{13} \div 6 = \dfrac{12}{13} \div \dfrac{6}{1}$ Write the whole number 6 as $\frac{6}{1}$.

$= \dfrac{12}{13} \times \dfrac{1}{6}$ Multiply by the reciprocal of $\frac{6}{1}$, which is $\frac{1}{6}$.

$= \dfrac{\overset{2}{\cancel{12}} \times 1}{13 \times \underset{1}{\cancel{6}}} = \dfrac{2}{13}$ Divide out the common factors and multiply.

Avoiding Mistakes

Always check that the final answer is in lowest terms.

Skill Practice Divide.

9. $\dfrac{12}{25} \div \dfrac{8}{15}$ **10.** $\dfrac{1}{4} \div 2$

6. Adding and Subtracting Fractions

Adding and Subtracting Fractions

Two fractions can be added or subtracted if they have a common denominator. Let *a*, *b*, and *c* be numbers such that *b* does not equal zero. Then,

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

To add or subtract fractions with the same denominator, add or subtract the numerators and write the result over the common denominator.

Example 7 Adding and Subtracting Fractions With the Same Denominator

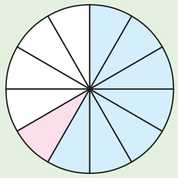
Add or subtract as indicated.

a. $\dfrac{1}{12} + \dfrac{7}{12}$ **b.** $\dfrac{13}{5} - \dfrac{3}{5}$

Answers

9. $\frac{9}{10}$ **10.** $\frac{1}{8}$

TIP: The sum $\frac{1}{12}+\frac{7}{12}$ can be visualized as the sum of the pink and blue sections of the figure.



Solution:

a. $\frac{1}{12}+\frac{7}{12}=\frac{1+7}{12}$ Add the numerators.

$=\frac{8}{12}$

$=\frac{2}{3}$ Simplify to lowest terms.

b. $\frac{13}{5}-\frac{3}{5}=\frac{13-3}{5}$ Subtract the numerators.

$=\frac{10}{5}$ Simplify.

$=2$ Simplify to lowest terms.

Skill Practice Add or subtract as indicated.

11. $\frac{2}{3}+\frac{5}{3}$ **12.** $\frac{5}{8}-\frac{1}{8}$

In Example 7, we added and subtracted fractions with the same denominators. To add or subtract fractions with different denominators, we must first become familiar with the idea of the least common multiple between two or more numbers. The **least common multiple (LCM)** of two numbers is the smallest whole number that is a multiple of each number. For example, the LCM of 6 and 9 is 18.

multiples of 6: 6, 12, 18, 24, 30, 36, . . .
multiples of 9: 9, 18, 27, 36, 45, 54, . . .

Listing the multiples of two or more given numbers can be a cumbersome way to find the LCM. Therefore, we offer the following method to find the LCM of two numbers.

Finding the LCM of Two Numbers

Step 1 Write each number as a product of prime factors.

Step 2 The LCM is the product of unique prime factors from *both* numbers. Use repeated factors the maximum number of times they appear in *either* factorization.

Example 8 Finding the LCM of Two Numbers

Find the LCM of 9 and 15.

Solution:

	3's	5's
9 =	3×3	
15 =	$3\times$	5

LCM $= 3\times3\times5 = 45$

For the factors of 3 and 5, we circle the greatest number of times each occurs. The LCM is the product.

Skill Practice Find the LCM.

13. 10 and 25

Answers

11. $\frac{7}{3}$ or $2\frac{1}{3}$ **12.** $\frac{1}{2}$ **13.** 50

To add or subtract fractions with *different* denominators, we must first write each fraction as an equivalent fraction with a common denominator. A common denominator may be *any* common multiple of the denominators. However, we will use the least common denominator. The **least common denominator (LCD)** of two or more fractions is the LCM of the denominators of the fractions. The following example uses the fundamental principle of fractions to rewrite fractions with the desired denominator. *Note:* Multiplying the numerator and denominator by the *same* nonzero quantity will not change the value of the fraction.

Example 9 Writing Equivalent Fractions and Subtracting Fractions

a. Write each of the fractions $\frac{1}{9}$ and $\frac{1}{15}$ as an equivalent fraction with the LCD as its denominator.

b. Subtract $\dfrac{1}{9} - \dfrac{1}{15}$.

Solution:

From Example 8, we know that the LCM for 9 and 15 is 45. Therefore, the LCD of $\frac{1}{9}$ and $\frac{1}{15}$ is 45.

a. $\dfrac{1}{19} = \dfrac{}{45}$ $\qquad \dfrac{1 \times 5}{9 \times 5} = \dfrac{5}{45}$ $\qquad$ So, $\dfrac{1}{9}$ is equivalent to $\dfrac{5}{45}$.

What number must we multiply 9 by to get 45?

Multiply numerator and denominator by 5.

$\dfrac{1}{15} = \dfrac{}{45}$ $\qquad \dfrac{1 \times 3}{15 \times 3} = \dfrac{3}{45}$ $\qquad$ So, $\dfrac{1}{15}$ is equivalent to $\dfrac{3}{45}$.

What number must we multiply 15 by to get 45?

Multiply numerator and denominator by 3.

b. $\dfrac{1}{9} - \dfrac{1}{15}$

$= \dfrac{5}{45} - \dfrac{3}{45}$ Write $\frac{1}{9}$ and $\frac{1}{15}$ as equivalent fractions with the same denominator.

$= \dfrac{2}{45}$ Subtract.

Skill Practice

14. Write each of the fractions $\frac{5}{8}$ and $\frac{5}{12}$ as an equivalent fraction with the LCD as its denominator.

15. Subtract. $\dfrac{5}{8} - \dfrac{5}{12}$

Answers

14. $\dfrac{5}{8} = \dfrac{15}{24}$ and $\dfrac{5}{12} = \dfrac{10}{24}$ **15.** $\dfrac{5}{24}$

Example 10 Adding and Subtracting Fractions

Simplify. $\frac{5}{12}+\frac{3}{4}-\frac{1}{2}$

Solution:

$\frac{5}{12}+\frac{3}{4}-\frac{1}{2}$

To find the LCD, we have:

	2's	3's
12 =	2×2	3
4 =	2×2	
2 =	2	

LCD $= 2\times 2\times 3 = 12$

$=\frac{5}{12}+\frac{3\times 3}{4\times 3}-\frac{1\times 6}{2\times 6}$ Write each fraction as an equivalent fraction with the LCD as its denominator.

$=\frac{5}{12}+\frac{9}{12}-\frac{6}{12}$

$=\frac{5+9-6}{12}$ Add and subtract the numerators.

$=\frac{8}{12}$ Simplify to lowest terms.

$=\frac{2}{3}$

Skill Practice Add.

16. $\frac{2}{3}+\frac{1}{2}+\frac{5}{6}$

7. Operations on Mixed Numbers

Recall that a mixed number is a whole number added to a fraction. The number $3\frac{1}{2}$ represents the sum of three wholes plus a half, that is, $3\frac{1}{2} = 3+\frac{1}{2}$. For this reason, any mixed number can be converted to an improper fraction by using addition.

$$3\tfrac{1}{2} = 3+\frac{1}{2} = \frac{6}{2}+\frac{1}{2} = \frac{7}{2}$$

TIP: A shortcut to writing a mixed number as an improper fraction is to multiply the whole number by the denominator of the fraction. Then add this value to the numerator of the fraction, and write the result over the denominator.

$3\frac{1}{2}$ ⟶ Multiply the whole number by the denominator: $3\times 2 = 6$
Add the numerator: $6+1 = 7$
Write the result over the denominator: $\frac{7}{2}$

To add, subtract, multiply, or divide mixed numbers, we will first write the mixed number as an improper fraction.

Answer

16. 2

Example 11 **Operations on Mixed Numbers**

Subtract. $5\frac{1}{3} - 2\frac{1}{4}$

Solution:

$$5\frac{1}{3} - 2\frac{1}{4}$$

$$= \frac{16}{3} - \frac{9}{4}$$ Write the mixed numbers as improper fractions.

$$= \frac{16 \times 4}{3 \times 4} - \frac{9 \times 3}{4 \times 3}$$ The LCD is 12. Multiply numerators and denominators by the missing factors from the denominators.

$$= \frac{64}{12} - \frac{27}{12}$$

$$= \frac{37}{12} \text{ or } 3\frac{1}{12}$$ Subtract the fractions.

Skill Practice Subtract.

17. $2\frac{3}{4} - 1\frac{1}{3}$

TIP: An improper fraction can also be written as a mixed number. Both answers are acceptable. Note that

$$\frac{37}{12} = \frac{36}{12} + \frac{1}{12} = 3 + \frac{1}{12}, \text{ or } 3\frac{1}{12}$$

This can easily be found by dividing.

$$\frac{37}{12} \longrightarrow \begin{array}{r} 3 \\ 12\overline{)37} \\ \underline{-36} \\ 1 \end{array} \qquad \overbrace{3}^{\text{quotient}}\frac{\overset{\text{remainder}}{1}}{\underset{\text{divisor}}{12}}$$

Example 12 **Operations on Mixed Numbers**

Divide. $7\frac{1}{2} \div 3$

Solution:

$$7\frac{1}{2} \div 3$$

$$= \frac{15}{2} \div \frac{3}{1}$$ Write the mixed number and whole number as fractions.

$$= \frac{\overset{5}{\cancel{15}}}{2} \times \frac{1}{\underset{1}{\cancel{3}}}$$ Multiply by the reciprocal of $\frac{3}{1}$, which is $\frac{1}{3}$.

$$= \frac{5}{2} \text{ or } 2\frac{1}{2}$$ The answer may be written as an improper fraction or as a mixed number.

Avoiding Mistakes

Remember that when dividing (or multiplying) fractions, a common denominator is not necessary.

Answer

17. $\frac{17}{12}$ or $1\frac{5}{12}$

Skill Practice Divide.

18. $5\frac{5}{6} \div 3\frac{2}{3}$

Answer

18. $\frac{35}{22}$ or $1\frac{13}{22}$

Section 1.1 Activity

A.1. Consider the numbers $\frac{2}{7}$, $3\frac{3}{8}$, and $\frac{42}{5}$.

a. Identify the proper fraction.

b. Identify the numerator and denominator of the proper fraction.

c. Identify the improper fraction.

d. Write the improper fraction as a mixed number.

e. Write the mixed number as an improper fraction.

A.2. a. List the prime numbers between 1 and 30.

b. Write the prime factorization of 132.

c. Write the prime factorization of 231.

d. Use the results of parts (b) and (c) to reduce the fraction $\frac{132}{231}$ to lowest terms.

A.3. a. Explain the similarities and differences between the procedures to multiply fractions and divide fractions.

b. A piece of piping is $\frac{5}{6}$ ft long. How many pieces are needed to cover a distance of 15 ft?

c. A recipe calls for $2\frac{3}{4}$ cups of flour. If a chef wants to make $\frac{3}{4}$ of the recipe, how much flour does she need?

A.4. a. Identify the least common denominator of the fractions $\frac{4}{45}$ and $\frac{3}{35}$.

b. Write each fraction as an equivalent fraction with the least common denominator from part (a).

c. Add the fractions and express the sum in lowest terms. $\frac{4}{45} + \frac{3}{35}$

d. A welder has a piece of steel that is $\frac{15}{16}$ yd long. She uses $\frac{3}{10}$ yd to make a repair of a gate mechanism. Does she have enough left over to repair part of the fence that requires $\frac{5}{8}$ yd of steel?

A.5. a. Convert the mixed number to an improper fraction. $4\frac{1}{3}$

b. Convert the improper fraction to a mixed number. $\frac{5}{2}$

c. Peggy buys $4\frac{1}{3}$ lb of smoked turkey, $2\frac{1}{2}$ lb of roast beef, and 3 lb of ham for a party. How much meat did she buy?

Section 1.1 Practice Exercises

Study Skills Exercise

Mindset plays an important role in your approach to learning mathematics. Mindset consists of your thoughts, beliefs, and attitudes about your abilities based on previous experiences throughout your lifetime. There are two types of mindsets: fixed mindsets and growth mindsets. People with a fixed mindset believe that they are born with a certain amount of intelligence that cannot be changed despite their actions. On the other hand, a person with a growth mindset believes that intelligence is dynamic and can be increased with effort and learning. What type of mindset do you have? Think about these questions:

- Have you said to yourself, "I'm just not good at math"?
- Do you believe you lack the skills required to understand math?
- Can you recall an experience that positively impacted your self-confidence in mathematics?

Prerequisite Review

R.1. **a.** Write 8 as a product of prime factors.
b. Write 12 as a product of prime factors.
c. Identify the greatest common factor (GCF) of 8 and 12.

R.2. **a.** Write 15 as a product of prime factors.
b. Write 9 as a product of prime factors.
c. Identify the GCF of 15 and 9.

R.3. **a.** Write the first six multiples of 8 (i.e. 8, 16, …).
b. Write the first six multiples of 12.
c. Identify the least common multiple (LCM) of 8 and 12.

R.4. **a.** Write the first six multiples of 15.
b. Write the first six multiples of 9.
c. Identify the LCM of 15 and 9.

R.5. Find the GCF and LCM of 18 and 24.

R.6. Find the GCF and LCM of 24 and 40.

Vocabulary and Key Concepts

1. **a.** A ________ is the result of multiplying two or more numbers.

b. The numbers being multiplied in a product are called ________.

c. Given a fraction $\frac{a}{b}$ with $b \neq 0$, the value a is the ________ and ________ is the denominator.

d. A fraction is said to be in ________ terms if the numerator and denominator share no common factor other than 1.

e. The fraction $\frac{4}{4}$ can also be written as the whole number ________, and the fraction $\frac{4}{1}$ can be written as the whole number ________.

f. Two nonzero numbers $\frac{a}{b}$ and $\frac{b}{a}$ are ________ because their product is 1.

g. The least common multiple (LCM) of two numbers is the smallest whole number that is a ________ of both numbers.

h. The ________ common denominator of two or more fractions is the LCM of their denominators.

Concept 1: Basic Definitions

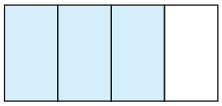
For Exercises 2–10, identify the numerator and denominator of each fraction. Then determine if the fraction is a proper fraction or an improper fraction.

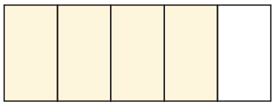
2. $\frac{7}{8}$ **3.** $\frac{2}{3}$ **4.** $\frac{9}{5}$

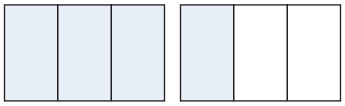
5. $\frac{5}{2}$ **6.** $\frac{6}{6}$ **7.** $\frac{4}{4}$

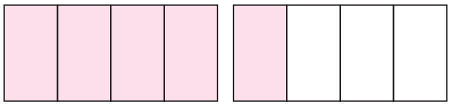
8. $\frac{12}{1}$ **9.** $\frac{5}{1}$ **10.** $\frac{6}{7}$

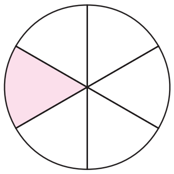
For Exercises 11–18, write a proper or improper fraction associated with the shaded region of each figure.

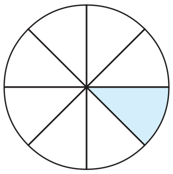
11. 

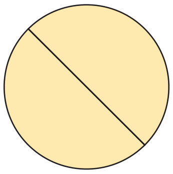
12. 

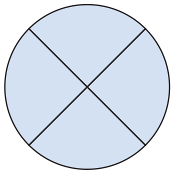
13. 

14. 

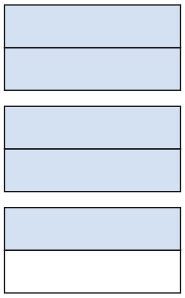
15. 

16. 

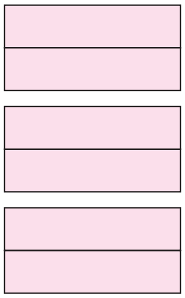
17. 

18. 

For Exercises 19–22, write both an improper fraction and a mixed number associated with the shaded region of each figure.

19. 

20. 

21. 

22. 

23. Explain the difference between the set of whole numbers and the set of natural numbers.

24. Explain the difference between a proper fraction and an improper fraction.

25. Write a fraction that simplifies to $\frac{1}{2}$. (Answers may vary.)

26. Write a fraction that simplifies to $\frac{1}{3}$. (Answers may vary.)

Concept 2: Prime Factorization

For Exercises 27–34, identify each number as either a prime number or a composite number.

27. 5 **28.** 9 **29.** 4 **30.** 2

31. 39 **32.** 23 **33.** 53 **34.** 51

For Exercises 35–42, write each number as a product of prime factors. **(See Example 1.)**

35. 36 **36.** 70 **37.** 42 **38.** 35

39. 110 **40.** 136 **41.** 135 **42.** 105

Concept 3: Simplifying Fractions to Lowest Terms

For Exercises 43–54, simplify each fraction to lowest terms. **(See Examples 2–3.)**

43. $\frac{3}{15}$ **44.** $\frac{8}{12}$ **45.** $\frac{16}{6}$ **46.** $\frac{20}{12}$

47. $\frac{42}{48}$ **48.** $\frac{35}{80}$ **49.** $\frac{48}{64}$ **50.** $\frac{32}{48}$

51. $\frac{110}{176}$ **52.** $\frac{70}{120}$ **53.** $\frac{200}{150}$ **54.** $\frac{210}{119}$

Concepts 4–5: Multiplying and Dividing Fractions

For Exercises 55–56, determine if the statement is true or false. If it is false, rewrite as a true statement.

55. When multiplying or dividing fractions, it is necessary to have a common denominator.

56. When dividing two fractions, it is necessary to multiply the first fraction by the reciprocal of the second fraction.

For Exercises 57–68, multiply or divide as indicated. **(See Examples 4–6.)**

57. $\frac{10}{13} \times \frac{26}{15}$ **58.** $\frac{15}{28} \times \frac{7}{9}$ **59.** $\frac{3}{7} \div \frac{9}{14}$ **60.** $\frac{7}{25} \div \frac{1}{5}$

61. $\frac{9}{10} \times 5$ **62.** $\frac{3}{7} \times 14$ **63.** $\frac{12}{5} \div 4$ **64.** $\frac{20}{6} \div 5$

65. $\frac{5}{2} \times \frac{10}{21} \times \frac{7}{5}$ **66.** $\frac{55}{9} \times \frac{18}{32} \times \frac{24}{11}$ **67.** $\frac{9}{100} \div \frac{13}{1000}$ **68.** $\frac{1000}{17} \div \frac{10}{3}$

69. Gus decides to save $\frac{1}{3}$ of his pay each month. If his monthly pay is \$2112, how much will he save each month?

70. Stephen's take-home pay is \$4200 a month. If he budgeted $\frac{1}{4}$ of his pay for rent, how much is his rent?

71. In Professor Foley's Beginning Algebra class, $\frac{5}{6}$ of the students passed the first test. If there are 42 students in the class, how many passed the test?

72. Shontell had only enough paper to print out $\frac{3}{5}$ of her book report before school. If the report is 10 pages long, how many pages did she print out?

73. Marty will reinforce a concrete walkway by cutting a steel rod (called rebar) that is 4 yd long. How many pieces can he cut if each piece must be $\frac{1}{2}$ yd in length?

74. There are 4 cups of oatmeal in a box. If each serving is $\frac{1}{3}$ of a cup, how many servings are contained in the box?

75. Anita buys 6 lb of mixed nuts to be divided into decorative jars that will each hold $\frac{3}{4}$ lb of nuts. How many jars will she be able to fill?

76. Beth has a $\frac{7}{8}$-in. nail that she must hammer into a board. Each strike of the hammer moves the nail $\frac{1}{16}$ in. into the board. How many strikes of the hammer must she make to drive the nail completely into the board?

Concept 6: Adding and Subtracting Fractions

For Exercises 77–80, add or subtract as indicated. **(See Example 7.)**

77. $\frac{5}{14} + \frac{1}{14}$ **78.** $\frac{9}{5} + \frac{1}{5}$ **79.** $\frac{17}{24} - \frac{5}{24}$ **80.** $\frac{11}{18} - \frac{5}{18}$

For Exercises 81–84, find the least common multiple for each list of numbers. **(See Example 8.)**

81. 6, 15

82. 12, 30

83. 20, 8, 4

84. 24, 40, 30

For Exercises 85–100, add or subtract as indicated. **(See Examples 9–10.)**

85. $\frac{1}{8}+\frac{3}{4}$

86. $\frac{3}{16}+\frac{1}{2}$

87. $\frac{11}{8}-\frac{3}{10}$

88. $\frac{12}{35}-\frac{1}{10}$

89. $\frac{7}{26}-\frac{2}{13}$

90. $\frac{25}{24}-\frac{5}{16}$

91. $\frac{7}{18}+\frac{5}{12}$

92. $\frac{3}{16}+\frac{9}{20}$

93. $\frac{5}{4}-\frac{1}{20}$

94. $\frac{7}{6}-\frac{1}{24}$

95. $\frac{5}{12}+\frac{5}{16}$

96. $\frac{3}{25}+\frac{8}{35}$

97. $\frac{1}{6}+\frac{3}{4}-\frac{5}{8}$

98. $\frac{1}{2}+\frac{2}{3}-\frac{5}{12}$

99. $\frac{4}{7}+\frac{1}{2}+\frac{3}{4}$

100. $\frac{9}{10}+\frac{4}{5}+\frac{3}{4}$

101. For his famous brownie recipe, Chef Alfonso combines $\frac{2}{3}$ cup granulated sugar with $\frac{1}{4}$ cup brown sugar. What is the total amount of sugar in his recipe?

102. Chef Alfonso eats too many of his brownies and his waistline increased by $\frac{3}{4}$ in. during one month and $\frac{3}{8}$ in. the next month. What was his total increase for the 2-month period?

103. Currently the most popular smartphone has a thickness of $\frac{9}{25}$ in. The second most popular is $\frac{1}{2}$ in. thick. How much thicker is the second most popular smartphone?

104. The diameter of a penny is $\frac{3}{4}$ in. while the dime is $\frac{7}{10}$ in. How much larger is the penny than the dime?

Concept 7: Operations on Mixed Numbers

For Exercises 105–118, perform the indicated operations. **(See Examples 11–12.)**

105. $3\frac{1}{5}\times 2\frac{7}{8}$

106. $2\frac{1}{2}\times 1\frac{4}{5}$

107. $1\frac{2}{9}\div 7\frac{1}{3}$

108. $2\frac{2}{5}\div 1\frac{2}{7}$

109. $1\frac{2}{9}\div 6$

110. $2\frac{2}{5}\div 2$

111. $2\frac{1}{8}+1\frac{3}{8}$

112. $1\frac{3}{14}+1\frac{1}{14}$

113. $3\frac{1}{2}-1\frac{7}{8}$

114. $5\frac{1}{3}-2\frac{3}{4}$

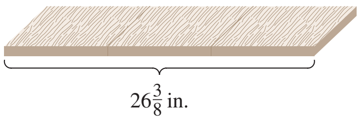
115. $1\frac{1}{6}+3\frac{3}{4}$

116. $4\frac{1}{2}+2\frac{2}{3}$

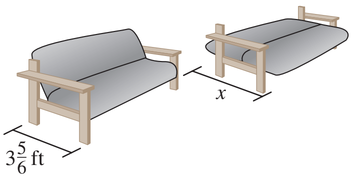
117. $1-\frac{7}{8}$

118. $2-\frac{3}{7}$

119. A board $26\frac{3}{8}$ in. long must be cut into three pieces of equal length. Find the length of each piece.



120. A futon, when set up as a sofa, measures $3\frac{5}{6}$ ft wide. When it is opened to be used as a bed, the width is increased by $1\frac{3}{4}$ ft. What is the total width of this bed?



121. A plane trip from Orlando to Detroit takes $2\frac{3}{4}$ hr. If the plane traveled for $1\frac{1}{6}$ hr, how much time remains for the flight?

122. Silvia manages a sub shop and needs to prepare smoked turkey sandwiches. She has $3\frac{3}{4}$ lb of turkey in the cooler, and each sandwich requires $\frac{3}{8}$ lb of turkey. How many sandwiches can she make?

123. José's catering company plans to prepare two different shrimp dishes for an upcoming event. One dish requires $1\frac{1}{2}$ lb of shrimp and the other requires $\frac{3}{4}$ lb of shrimp. How much shrimp should José order for the two dishes?

124. Ayako took a trip to the store $5\frac{1}{2}$ mi away. If she rode the bus for $4\frac{5}{6}$ mi and walked the rest of the way, how far did she have to walk?

125. If Tampa, Florida, averages $6\frac{1}{4}$ in. of rain during each summer month, how much total rain would be expected in June, July, August, and September?

126. Pete started working out and found that he lost approximately $\frac{3}{4}$ in. off his waistline every month. How much would he lose around his waist in 6 months?

Section 1.2 Introduction to Algebra and the Set of Real Numbers

Concepts

1. Variables and Expressions
2. The Set of Real Numbers
3. Inequalities
4. Opposite of a Real Number
5. Absolute Value of a Real Number

1. Variables and Expressions

Doctors promote daily exercise as part of a healthy lifestyle. Aerobic exercise is exercise for the heart. During aerobic exercise, the goal is to maintain a heart rate level between 65% and 85% of an individual's maximum recommended heart rate. The maximum recommended heart rate, in beats per minute, for an adult of age a is given by:

$$\text{Maximum recommended heart rate} = 220 - a$$

In this example, value a is called a **variable**. This is a symbol or letter, such as x, y, z, a, and the like, that is used to represent an unknown number that is subject to change. The number 220 is called a **constant**, because it does not vary. The quantity $220 - a$ is called an algebraic expression. An algebraic **expression** is a collection of variables and constants under algebraic operations. For example, $\frac{3}{x}$, $y + 7$, and $t - 1.4$ are algebraic expressions.

The symbols used in algebraic expressions to show the four basic operations are shown here:

Addition $a + b$

Subtraction $a - b$

Multiplication $a \times b$, $a \cdot b$, $(a)b$, $a(b)$, $(a)(b)$, ab

(*Note:* We rarely use the notation $a \times b$ because the symbol $\times$ may be confused with the variable x.)

Division $a \div b$, $\dfrac{a}{b}$, a/b, $b\overline{)a}$

The value of an algebraic expression depends on the values of the variables within the expression.

Example 1 Evaluating an Algebraic Expression

The expression $220 - a$ represents the maximum recommended heart rate for an adult of age a. Determine the maximum heart rate for:

a. A 20-year-old **b.** A 45-year-old

Solution:

a. In the expression $220 - a$, the variable, a, represents the age of the individual. To calculate the maximum recommended heart rate for a 20-year-old, we substitute 20 for a in the expression.

$220 - a$

$220 - (\quad)$	When substituting a number for a variable, use parentheses.
$= 220 - (20)$	Substitute $a = 20$.
$= 200$	Subtract.

The maximum recommended heart rate for a 20-year-old is 200 beats per minute.

b. $220 - a$

$220 - (\quad)$	When substituting a number for a variable, use parentheses.
$= 220 - (45)$	Substitute $a = 45$.
$= 175$	Subtract.

The maximum recommended heart rate for a 45-year-old is 175 beats per minute.

Skill Practice

1. After dining out at a restaurant, the recommended minimum amount for tipping the server is 15% of the cost of the meal. This can be represented by the expression $0.15c$, where c is the cost of the meal. Compute the tip for a meal that costs:

a. \$18 **b.** \$46

Example 2 Evaluating Algebraic Expressions

Evaluate the algebraic expression when $p = 4$ and $q = \frac{3}{4}$.

a. $100 - p$ **b.** pq

Solution:

a. $100 - p$

$100 - (\quad)$	When substituting a number for a variable, use parentheses.
$= 100 - (4)$	Substitute $p = 4$ in the parentheses.
$= 96$	Subtract.

Answers

1. a. \$2.70 **b.** \$6.90

b. pq

$= (\quad)(\quad)$	When substituting a number for a variable, use parentheses.
$= (4)\left(\frac{3}{4}\right)$	Substitute $p = 4$ and $q = \frac{3}{4}$.
$= \frac{\overset{1}{\cancel{4}}}{1} \cdot \frac{3}{\underset{1}{\cancel{4}}}$	Write the whole number as a fraction.
$= \frac{3}{1}$	Multiply fractions.
$= 3$	Simplify.

Skill Practice Evaluate the algebraic expressions when $x = 5$ and $y = 2$.

2. $20 - y$ **3.** xy

2. The Set of Real Numbers

Typically, the numbers represented by variables in an algebraic expression are all part of the set of **real numbers**. These are the numbers that we work with on a day-to-day basis. The real numbers encompass zero, all positive, and all negative numbers, including those represented by fractions and decimal numbers. The set of real numbers can be represented graphically on a horizontal number line with a point labeled as 0. Positive real numbers are graphed to the right of 0, and negative real numbers are graphed to the left of 0. Zero is neither positive nor negative. Each point on the number line corresponds to exactly one real number. For this reason, this number line is called the *real number line* (Figure 1-5).

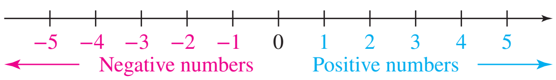


Figure 1-5

Example 3 Plotting Points on the Real Number Line

Plot the numbers on the real number line.

a. -3 **b.** $\frac{3}{2}$ **c.** -4.7 **d.** $\frac{16}{5}$

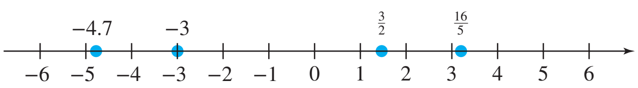
Solution:

a. Because -3 is negative, it lies three units to the left of 0.

b. The fraction $\frac{3}{2}$ can be expressed as the mixed number $1\frac{1}{2}$, which lies halfway between 1 and 2 on the number line.

c. The negative number -4.7 lies $\frac{7}{10}$ unit to the left of -4 on the number line.

d. The fraction $\frac{16}{5}$ can be expressed as the mixed number $3\frac{1}{5}$, which lies $\frac{1}{5}$ unit to the right of 3 on the number line.

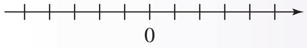


Answers

2. 18 **3.** 10

Skill Practice Plot the numbers on the real number line.

4. $\{-1, \frac{3}{4}, -2.5, \frac{10}{3}\}$



In mathematics, a well-defined collection of elements is called a **set**. "Well-defined" means the set is described in such a way that it is clear whether an element is in the set. The symbols { } are used to enclose the elements of the set. For example, the set {A, B, C, D, E} represents the set of the first five letters of the alphabet.

Several sets of numbers are used extensively in algebra and are *subsets* (or part) of the set of real numbers.

TIP: The natural numbers are used for counting. For this reason, they are sometimes called the "counting numbers."

Natural Numbers, Whole Numbers, and Integers

The set of **natural numbers** is $\{1, 2, 3, \ldots\}$

The set of **whole numbers** is $\{0, 1, 2, 3, \ldots\}$

The set of **integers** is $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Notice that the set of whole numbers includes the natural numbers. Therefore, every natural number is also a whole number. The set of integers includes the set of whole numbers. Therefore, every whole number is also an integer.

Fractions are also among the numbers we use frequently. A number that can be written as a fraction whose numerator is an integer and whose denominator is a nonzero integer is called a *rational number*.

Rational Numbers

The set of **rational numbers** is the set of numbers that can be expressed in the form $\frac{p}{q}$, where both p and q are integers and q does not equal 0.

We also say that a rational number $\frac{p}{q}$ is a *ratio* of two integers, p and q, where q is not equal to zero.

Example 4 Identifying Rational Numbers

Show that the following numbers are rational numbers by finding an equivalent ratio of two integers.

a. $\dfrac{-2}{3}$ **b.** -12 **c.** 0.5 **d.** $0.\overline{6}$

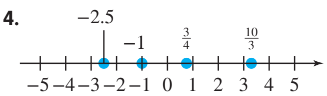
Solution:

a. The fraction $\frac{-2}{3}$ is a rational number because it can be expressed as the ratio of -2 and 3.

b. The number -12 is a rational number because it can be expressed as the ratio of -12 and 1, that is, $-12 = \frac{-12}{1}$. In this example, we see that an integer is also a rational number.

Answer

4.



c. The terminating decimal 0.5 is a rational number because it can be expressed as the ratio of 5 and 10, that is, $0.5 = \frac{5}{10}$. In this example, we see that a terminating decimal is also a rational number.

d. The numeral $0.\overline{6}$ represents the nonterminating, repeating decimal 0.6666666 . . . . The number $0.\overline{6}$ is a rational number because it can be expressed as the ratio of 2 and 3, that is, $0.\overline{6} = \frac{2}{3}$. In this example, we see that a repeating decimal is also a rational number.

Skill Practice Show that each number is rational by finding an equivalent ratio of two integers.

5. $\frac{3}{7}$ **6.** -5 **7.** 0.3 **8.** $0.\overline{3}$

TIP: A rational number can be represented by a terminating decimal or by a repeating decimal.

Some real numbers, such as the number π, cannot be represented by the ratio of two integers. These numbers are called irrational numbers and in decimal form are nonterminating, nonrepeating decimals. The value of π, for example, can be approximated as $\pi \approx 3.1415926535897932$. However, the decimal digits continue forever with no repeated pattern. Another example of an irrational number is $\sqrt{3}$ (read as "the positive square root of 3"). The expression $\sqrt{3}$ is a number that when multiplied by itself is 3. There is no rational number that satisfies this condition. Thus, $\sqrt{3}$ is an irrational number.

Irrational Numbers

The set of **irrational numbers** is a subset of the real numbers whose elements cannot be written as a ratio of two integers.

Note: An irrational number cannot be written as a terminating decimal or as a repeating decimal.

The set of real numbers consists of both the rational and the irrational numbers. The relationship among these important sets of numbers is illustrated in Figure 1-6 along with numerical examples.

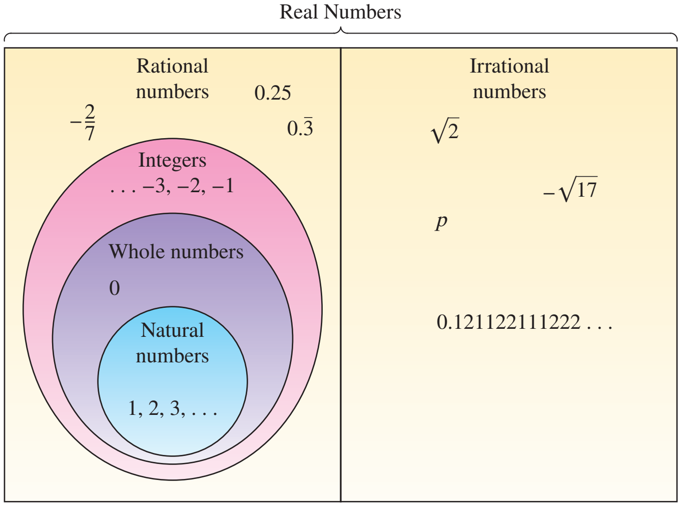


Figure 1-6

Answers

5. Ratio of 3 and 7
6. Ratio of −5 and 1
7. Ratio of 3 and 10
8. Ratio of 1 and 3

Example 5 Classifying Numbers by Set

Check the set(s) to which each number belongs. The numbers may belong to more than one set.

	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
5						
$\dfrac{-47}{3}$						
1.48						
$\sqrt{7}$						
0						

Solution:

	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
5	✓	✓	✓	✓ (ratio of 5 and 1)		✓
$\dfrac{-47}{3}$				✓ (ratio of −47 and 3)		✓
1.48				✓ (ratio of 148 and 100)		✓
$\sqrt{7}$					✓	✓
0		✓	✓	✓ (ratio of 0 and 1)		✓

Skill Practice Identify the sets to which each number belongs. Choose from: natural numbers, whole numbers, integers, rational numbers, irrational numbers, real numbers.

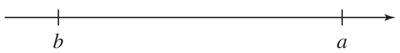
9. -4 **10.** $0.\overline{7}$ **11.** $\sqrt{13}$ **12.** 12 **13.** 1

3. Inequalities

The relative size of two real numbers can be compared using the real number line. Suppose a and b represent two real numbers. We say that a is less than b, denoted $a < b$, if a lies to the left of b on the number line.

$a < b$

We say that a is greater than b, denoted $a > b$, if a lies to the right of b on the number line.



$a > b$

Answers

9. Integers, rational numbers, real numbers
10. Rational numbers, real numbers
11. Irrational numbers, real numbers
12. Natural numbers, whole numbers, integers, rational numbers, real numbers
13. Natural numbers, whole numbers, integers, rational numbers, real numbers

Table 1-1 summarizes the relational operators that compare two real numbers a and b.

Table 1-1

Mathematical Expression	Translation	Example
$a < b$	a is less than b.	$2 < 3$
$a > b$	a is greater than b.	$5 > 1$
$a \leq b$	a is less than or equal to b.	$4 \leq 4$
$a \geq b$	a is greater than or equal to b.	$10 \geq 9$
$a = b$	a is equal to b.	$6 = 6$
$a \neq b$	a is not equal to b.	$7 \neq 0$
$a \approx b$	a is approximately equal to b.	$2.3 \approx 2$

The symbols $<$, $>$, $\leq$, $\geq$, and $\neq$ are called *inequality signs*, and the statements $a < b$, $a > b$, $a \leq b$, $a \geq b$, and $a \neq b$ are called **inequalities**.

Example 6 Ordering Real Numbers

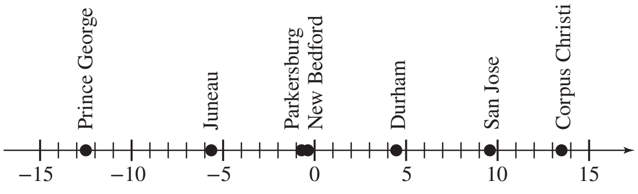
The average temperatures (in degrees Celsius) for selected cities in the United States and Canada in January are shown in Table 1-2.

Table 1-2

City	Temp (°C)
Prince George, British Columbia	−12.5
Corpus Christi, Texas	13.4
Parkersburg, West Virginia	−0.9
San Jose, California	9.7
Juneau, Alaska	−5.7
New Bedford, Massachusetts	−0.2
Durham, North Carolina	4.2

Plot a point on the real number line representing the temperature of each city. Compare the temperatures between the following cities, and fill in the blank with the appropriate inequality sign: < or >.

Solution:



a. Temperature of San Jose $\boxed{<}$ temperature of Corpus Christi

b. Temperature of Juneau $\boxed{>}$ temperature of Prince George

c. Temperature of Parkersburg $\boxed{<}$ temperature of New Bedford

d. Temperature of Parkersburg $\boxed{>}$ temperature of Prince George

Skill Practice Fill in the blanks with the appropriate inequality sign: $<$ or $>$.

14. -11 ________ 20 **15.** -3 ________ -6

16. 0 ________ -9 **17.** -6.2 ________ -1.8

4. Opposite of a Real Number

To gain mastery of any algebraic skill, it is necessary to know the meaning of key definitions and key symbols. Two important definitions are the *opposite* of a real number and the *absolute value* of a real number.

The Opposite of a Real Number

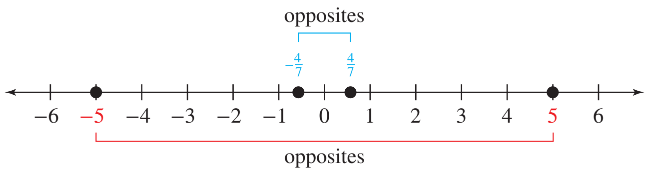
Two numbers that are the same distance from 0 but on opposite sides of 0 on the number line are called **opposites** of each other. Symbolically, we denote the opposite of a real number a as $-a$.

Example 7 Finding the Opposite of a Real Number

a. Find the opposite of 5. **b.** Find the opposite of $-\frac{4}{7}$.

Solution:

a. The opposite of 5 is -5. **b.** The opposite of $-\frac{4}{7}$ is $\frac{4}{7}$.



Skill Practice Find the opposite.

18. 224 **19.** -3.4

Example 8 Finding the Opposite of a Real Number

Evaluate each expression.

a. $-(0.46)$ **b.** $-\left(-\frac{11}{3}\right)$

Solution:

a. $-(0.46) = -0.46$ The expression $-(0.46)$ represents the opposite of 0.46.

b. $-\left(-\frac{11}{3}\right) = \frac{11}{3}$ The expression $-\left(-\frac{11}{3}\right)$ represents the opposite of $-\frac{11}{3}$.

Skill Practice Evaluate.

20. $-(2.8)$ **21.** $-\left(-\frac{1}{5}\right)$

Answers

14. $<$ **15.** $>$ **16.** $>$
17. $<$ **18.** -224 **19.** 3.4
20. -2.8 **21.** $\frac{1}{5}$

5. Absolute Value of a Real Number

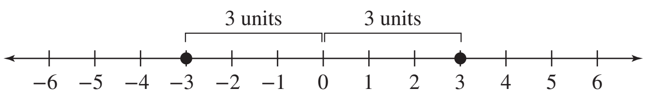
To define the addition of real numbers, we use the concept of absolute value.

Informal Definition of the Absolute Value of a Real Number

The **absolute value** of a real number a, denoted $|a|$, is the distance between a and 0 on the number line.

Note: The absolute value of any real number is positive or zero.

For example, $|3| = 3$ and $|-3| = 3$.



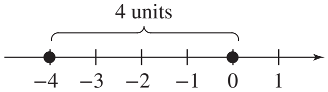
Example 9 Finding the Absolute Value of a Real Number

Evaluate the absolute value expressions.

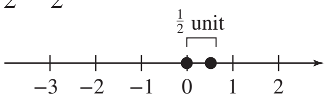
a. $|-4|$ **b.** $\left|\frac{1}{2}\right|$ **c.** $|-6.2|$ **d.** $|0|$

Solution:

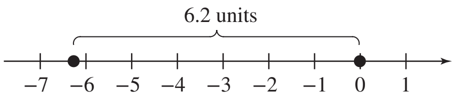
a. $|-4| = 4$ -4 is 4 units from 0 on the number line.



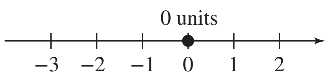
b. $\left|\frac{1}{2}\right| = \frac{1}{2}$ $\frac{1}{2}$ is $\frac{1}{2}$ unit from 0 on the number line.



c. $|-6.2| = 6.2$ -6.2 is 6.2 units from 0 on the number line.



d. $|0| = 0$ 0 is 0 units from 0 on the number line.



Skill Practice Evaluate.

22. $|-99|$ **23.** $\left|\frac{7}{8}\right|$ **24.** $|-1.4|$ **25.** $|1|$

The absolute value of a number a is its distance from 0 on the number line. The definition of $|a|$ may also be given symbolically depending on whether a is negative or nonnegative.

Answers

22. 99 **23.** $\frac{7}{8}$

24. 1.4 **25.** 1

Absolute Value of a Real Number

Let a be a real number. Then

1. If a is nonnegative (that is, $a \geq 0$), then $|a| = a$.
2. If a is negative (that is, $a < 0$), then $|a| = -a$.

This definition states that if a is a nonnegative number, then $|a|$ equals a itself. If a is a negative number, then $|a|$ equals the opposite of a. For example:

$|9| = 9$ Because 9 is positive, then $|9|$ equals the number 9 itself.

$|-7| = 7$ Because -7 is negative, then $|-7|$ equals the opposite of -7, which is 7.

Example 10 Comparing Absolute Value Expressions

Determine if the statements are true or false.

a. $|3| \leq 3$ **b.** $-|5| = |-5|$

Solution:

a. $|3| \leq 3$ $|3| \overset{?}{\leq} 3$ Simplify the absolute value.

$3 \overset{?}{\leq} 3$ True

b. $-|5| = |-5|$ $-|5| \overset{?}{=} |-5|$ Simplify the absolute values.

$-5 \overset{?}{=} 5$ False

Skill Practice Answer true or false.

26. $-|4| > |-4|$ **27.** $|-17| = 17$

Answers

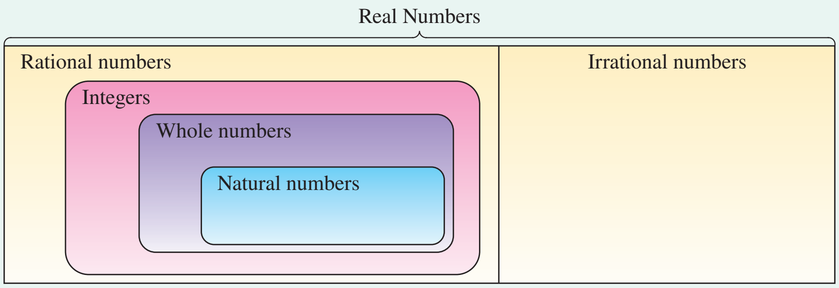
26. False **27.** True

Section 1.2 Activity

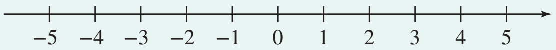
For Exercises A.1–A.4, consider the elements of set A.

$$A = \left\{0, -\frac{2}{3}, \pi, -4, 5, -2.\overline{6}, 3.1, 4\frac{4}{5}, -2.6, -3\right\}$$

A.1. Place each element of A into the correct subset in the diagram.



A.2. Plot the elements of A on the number line.



A.3. Fill in the blank with $<$, $>$, or $=$.

a. π ____ 3.1 **b.** $-2.\overline{6}$ ____ -2.6

A.4. Identify the opposite and absolute value of the given values.

a. -4 **b.** $-\frac{2}{3}$ **c.** 3.1 **d.** π

Section 1.2 Practice Exercises

Study Skills Exercise

Class notes serve as a summary of key concepts that can be used later to prepare for tests.

- Look over the notes that you took today.
- Do you understand what you wrote?
- Identify any material or challenging problems that you need to revisit.
- Add any comments to make your notes clearer to you or rewrite the notes in your own words.
- If there are any rules, definitions, or formulas in your notes, highlight them so that they can be easily found when studying for the test.

Remember that note-taking skills are learned by practice.

Prerequisite Review

R.1. **a.** Write the first six digits to the right of the decimal point for the repeating decimal $0.\overline{45}$.
b. Round $0.\overline{45}$ to the hundredths place.
c. Round $0.\overline{45}$ to the thousandths place.

R.2. **a.** Write the first six digits to the right of the decimal point for the repeating decimal $1.51\overline{9}$.
b. Round $1.51\overline{9}$ to the tenths place.
c. Round $1.51\overline{9}$ to the ten-thousandths place.

R.3. Order the numbers from least to greatest.
$6.13, 6.12999, 6.1\overline{3}, 6.\overline{1}$

R.4. Order the values from least to greatest.
$0.01\overline{35}, 0.0135, 0.01\overline{4}, 0.01349999$

For Exercises R.5–R.10, perform the indicated operations.

R.5. **a.** $\frac{3}{5} - \frac{2}{7}$ **b.** $\frac{3}{5} + \frac{2}{7}$

R.6. **a.** $\frac{5}{6} + \frac{3}{8}$ **b.** $\frac{5}{6} - \frac{3}{8}$