Math in Our World: A Quantitative Reasoning Approach

second edition

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McGraw Hill

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About the Authors

Dave Sobecki

Dave was born and raised in Cleveland, and in spite of starting college with a major in creative writing, he somehow ended up with a Doctorate degree in math. Go figure. Dave spent two years at Franklin and Marshall College in Lancaster, Pennsylvania, followed by 22 years at the Hamilton campus of Miami University in southwest Ohio. He recently relocated to North Port, Florida. Dave has won a number of teaching awards in his career, and has written or co-authored either nine or nineteen books, depending on how you count them. He's also working on his first novel. When not teaching or writing, Dave's passions include Ohio State football, Cleveland Indians baseball, heavy metal music, travel, golf, collecting fine art, and most importantly spending time with his wife Cat and over-attached retrievers, Macleod and Tessa.

Courtesy of Dave Sobecki

Brian Mercer

Brian is a tenured professor at Parkland College in Champaign, Illinois, where he has taught developmental and transfer math courses for 22 years. He began writing in 1999 and has currently co-authored eight textbooks, with others in the planning stages. Outside of the classroom and away from the computer, Brian is kept educated, entertained, and ever-busy by his wonderful wife Nikki, their two children, Charlotte (13) and Jake (12), and dog Molly. He is an avid St. Louis Cardinals fan and enjoys playing softball and golf in the summertime with colleagues and friends.

Courtesy of Brian Mercer

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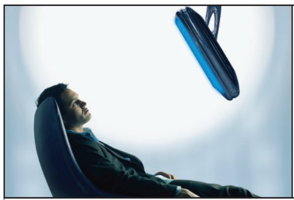

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Units 9, 10, and 11 are available online. Though not included in this desk copy, they can be added to custom versions of the text built through Create or accessed in the Instructor Resources area of ALEKS.

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A Letter to Students

This book has the term "Quantitative Reasoning" in the title, so we probably ought to begin by describing what that means. But nothing is for free in this course, so before we do that, we're going to ask you to think about an answer to this question: Why are you taking this class? For that matter, why are you in college at all? It would be great if "To get an education" was your answer, or at the very least crossed your mind. It might be even better if you thought "To get smarter." That should be the goal of everyone that's going to school, really.

So can you actually get smarter? OF COURSE YOU CAN. You almost certainly know that you can get stronger and more fit than you currently are by exercising your body. Then why in the world wouldn't you think you can get smarter by exercising your brain? You can, but not if you occupy a chair and expect your professor to just magically fill your head with math. That's not mental exercise. True mental exercise requires challenging yourself to think, to solve problems, to make connections, and to explain your understanding of concepts. Which brings us back to our part of the bargain, a description of what quantitative reasoning means.

At its core, this course is about exercising your brain by challenging you to really LEARN, not just to memorize formulas or mimic step-by-step procedures. When you hear the word "reasoning," it probably brings to mind something like "figuring stuff out." By including the term "Quantitative Reasoning" in the title of this book, we're simply pointing out the fact that we're not going to just ask you to do a bunch of calculations, simplify a bunch of expressions, or solve a bunch of equations. Instead, the focus is going to be practicing your reasoning skills by studying situations where mathematical thinking can help you be smarter and more successful, both as a student and as a citizen of 21st-century planet earth.

Certainly being comfortable with numbers is important in a monetary society. But it goes beyond that: Someone once told us that nobody is going to pay people to solve problems that have already been solved, and that kind of became a framework for this course. We gathered as much information as we could about what employers want college graduates to be able to do these days, and three things consistently jumped out at us: being able to work effectively in groups, being able to solve problems and figure stuff out without always being told what to do, and being able to use technology well. It's those three things that became the key principles of this book.

The Approach: What Truly Makes This Course Different

The thing that makes this a course in quantitative reasoning, rather than a survey of mathematical topics, is the way the material is presented, and more importantly, how we expect you to interact with the material. If you're dead-set on sitting in the back of the room and being talked at for an hour, you'll find out pretty quickly that you're in the wrong place: Active participation is required. You won't find large blocks of text and 8 to 10 solved examples per section in this book, like you're probably used to seeing in other math textbooks. What you will find is a series of activities designed to encourage (and by "encourage" we mean "force") you to take responsibility for your own learning. Discovery learning and productive struggle are the key phrases that describe our philosophy.

Discovery learning is the process of learning new ideas on your own, rather than having them presented to you. That doesn't mean that you're expected to "teach this course to yourself." Your professor is by far the most valuable resource you'll have on this journey. But his or her job is to help you to learn, not to show you exactly what to do. In a very real sense, the goal of this course is to help you become mathematically literate. We can define mathematical literacy as the quantitative and reasoning skills anyone should have to be successful in society. Think about literacy in the reading sense. You can't teach someone to read by just reading to them: At some point you have to guide them through reading on their own, and that's the role of your professor in this course.

Productive struggle is described really well by the exercise analogy from earlier: The best way to really learn something, and to make your brain stronger in the process, is to have to work at it. If you get every question in this book right on the first try, then we've failed as writers, to be honest. Life can be a hard place, especially when you're responsible for your own care and feeding, and we can promise you that you will get NOWHERE in this world if you give up every time you encounter some resistance. In our view, the single most valuable skill you'll practice in this course is GRIT: the ability to overcome obstacles, and keep trying until you reach a goal.

Because working effectively in groups is so important, the book has been designed as a series of in-class activities for you to work through in a group setting. There are two things you need to do in order to maximize your chance for success in this course: Be willing to put in a lot of work, and be willing to learn with and learn from your group mates, as well as to help teach them from time to time. If we were pure evil and decided to kidnap you and dump you out in the wilderness somewhere, you surely would appreciate not being alone, right? As you wander the path toward an adult education, having peers there with you should be comforting, and should be a resource that you take advantage of whenever possible.

Once class is over, there will be a lot of work you're expected to do on your own, in a variety of different settings. If you turn ahead to one of the pages in the book marked "Portfolio," you can see the different categories of homework, and no doubt your professor will talk about them on the first day of class. If you keep thinking back to the exercise analogy, you'll have a much easier time getting motivated to keep up with the homework. Why are you in this class? To get smarter. That will happen if you work hard toward the goal, and are never, ever afraid to think on your own. You were smart enough to make the decision to further your education in college, so you're smart enough to think, learn, and succeed in quantitative reasoning. Set a goal, work toward it, and believe in yourself. And don't be afraid to drop us a line if you have questions about the book, suggestions for improvement, or just want to share your thoughts on how the course is going.

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The Content

The backbone of any well-designed math course is content, so let's talk about the content that we've chosen as the vehicle for becoming quantitatively literate.

Unit 1: Mathematical Reasoning and Problem Solving

In our view, the ideal way to begin this course is to explicitly talk about the different kinds of reasoning that humans do, and to define and practice the types of problem-solving skills that will form the backbone of the course. We also encourage students to practice graph interpretation and estimation skills that should be (but are not) possessed by everyone in a technological and monetary-based society.

Unit 2: Consumer Math

Did we mention that we live in a monetary society? Consumer math is kind of the low-hanging fruit of quantitative reasoning content: Everyone who's ever spent their own money on anything has a natural understanding of how financial literacy is important. The focus of this unit isn't "here's how to do financial calculations"; it's "let's understand the concepts and situations that lead to important financial considerations." While we absolutely will study formulas that calculate interest, loan payments, and the like, we certainly will not just provide the formulas, ask students to plug a bunch of numbers in, and call that education. Instead, students are asked to think like consumers, and develop a true understanding of the math that serves us well in navigating the rocky financial waters of modern society.

Unit 3: Probability

Almost everything that we do is governed at least in part by chance and probability, so a book devoted to quantitative reasoning should go beyond simply calculating probabilities and odds: The focus should be on being able to interpret the story told by those calculations. We certainly feel that performing probability calculations is important, as long as the focus is on understanding rather than formulas. But we also believe that it's important to move beyond simple probability calculations, incorporating technology to allow us to study complex outcomes in a complex world.

Unit 4: Statistics

The logical follow-up to a study of probability is continuing the mindset by developing key ideas in the study of basic statistics. The focus is different methods of gathering, organizing, and interpreting data. Anyone can find the average (whatever that means) of a list of numbers, but that's a hollow exercise without the ability to interpret the story told by that statistic. While we don't feel that quantitative reasoning necessarily requires high-level statistical analysis, we do feel that our society will be much the better if everyone has experience with sifting through the mountain of information they're exposed to every day, and separating the gold nuggets from the cow pies, if you will. So a major focus of, and the culmination of the unit, is recognizing ways that statistics can be manipulated to tell a story that has at best a passing resemblance to the truth. While politicians and advertisers may not appreciate these efforts, we think that students interested in getting an education will.

Unit 5: Math Modeling

It's the sad truth that most college math students honestly believe that algebra was developed as a completely abstract field, with applications following along at some later point. We know, of course, that this is completely backward, but traditional curriculum design tends to foster this all-too-common misconception. In Unit 5, we cover important algebra topics from an applied approach, hopefully allowing students to finally see how the algebra that they've learned in the past is useful and relevant.

Unit 6: Set Theory

Humans have a natural tendency to categorize things in groups, from numbers to people. In a very real sense, the fundamentals of set theory are central to pretty much everything we do in math. So we study set theory both as an accessible way to apply mathematical thinking to our world and as a vehicle for thinking about the type of organized thought that's useful in any math-related field.

Unit 7: Logic

Digging further into the fundamentals of mathematical and scientific thought, we cover formal logic in a way that helps students to always remember that the *p*s and *q*s they're working with aren't letters, but statements about the world in which they live. We feel like the way to keep students engaged in the study of truth values for statements and validity of arguments is to ask them to build the rules for truth values through examples, rather than providing and illustrating the rules. All of us hope that through math, students learn to think logically and analytically: An explicit study of logical thought helps to foster that type of thinking.

Unit 8: Measurement

Measuring sizes is not only one of the oldest applications of math; it's also one of the most common even today. The study of mathematics should, of course, be part of any college program for a variety of reasons; in terms of direct application to many fields of study, working with measurement and units ranks highly. Ask any instructor of general sciences how much more they could accomplish if they didn't have to spend weeks covering key ideas in measurement and unit conversion, and you'll gain an appreciation for how useful this topic can be for students.

Unit 9: Voting Methods

Many students think that voting is simply casting a ballot and adding up the votes to see who got the most. The study of different voting methods provides an accessible example of mathematical thinking in society. It also opens students' eyes to the wide variety of places where voting is used, and it helps them to understand the complexities inherent in the topic.

Unit 10: Graph Theory

Think you know what a graph is? If you're thinking in the algebraic sense, you might be surprised to find out that there are completely different types of graphs that have nothing to do with x and y. You might be even more surprised to find out the wealth of interesting problems that can be studied using the theory of graphs. And break out those colored markers! Graph coloring is a thing.

Unit 11: Numeration Systems

Think you know what a number is? Is it different from a numeral? Are there different ways to represent numbers? This unit goes to the very foundation of everything that math was built upon: numbers. Your mind just might get blown when you study systems that use completely different symbols to represent numbers, and systems that use our familiar symbols in completely different ways.

One of the best things about a course of this nature is that the material is not completely linear. It's perfectly possible for instructors to choose the content that's most relevant for his or her students, and reordering of some of the topics is reasonable as well. We've devised an ordering that works well for us, but colleges or instructors can certainly choose a topic list and order that best meets the needs of their student populations.

What's New in This Edition?

Writing the first edition of a math text, especially in an evolving area like quantitative reasoning (QR), is part art and part science. You use your training and experience as an instructor to decide on the approach and the most appropriate topics. You travel a lot and talk to anyone who doesn't run away when they see you coming to gather more professional opinions. You count on your crack publisher's team to conduct surveys and focus groups. Then you put it all together and make some educated guesses, hoping that the result hits the mark.

We feel like we did a pretty good job of that in the first edition of this book. Then we set out to do it all again, gathering feedback from a multitude of sources. The result is the second edition, which we believe addresses the most important concerns expressed to us by colleagues from around the country.

First, let's talk about content. When developing the first edition, we chose the eight main topics that our research indicated were most commonly taught in QR courses. Of course, that left out several topics that an awful lot of folks think are important. So we started to develop ancillary materials for some of our partners almost immediately, and those grew into the new topics introduced for the second edition. Those new topics include:

- An expansion of the math modeling unit to include more algebraic coverage
- New units on Voting Methods, Graph Theory, and Numeration Systems

Next, we were struck by the number of instructors who wanted a bit more structure, and additional text that would help students navigate some of the more challenging topics. We feel strongly about keeping the main focus on discovery learning, but decided that strategically placing some solved examples throughout the lessons would be beneficial to both students and instructors. There are several reasons we felt that adding solved examples was a good idea:

- Help speed up class time.
- Help with concepts that students might have a hard time discovering on their own.
- Provide models of problem-solving techniques that students can use as a reference.
- Provide samples of procedures simple enough that discovery time could be better spent elsewhere.

Each of the solved examples is also accompanied by a video lecture that walks through the problem and solution, which can save class time as well as provide an extra out-of-class resource for students.

We've also greatly expanded the Prep Skills that begin each lesson. As more schools are introducing corequisite courses, it becomes more important than ever to clearly identify the skills needed for specific lessons. After writing each lesson, we identified those necessary skills. There is a full corequisite guide that accompanies this book, but we decided it would benefit students to get a bit of instruction on key skills right on the page of their main text. These are followed once again by Prep Skills questions, which are of course available online. These questions also have solutions videos, providing another valuable student resource.

Finally, the Portfolio section of each lesson has been expanded to include answers to questions found in the Prep Skills portion. It also covers two pages now so that when students turn in the first page of Applications, they retain the Portfolio pages for reference.

We hope that you find this revision to be an upgrade, and we'd love to hear your thoughts on how we did.

Acknowledgments

So many people were involved in bringing this project to life and improving the second edition that it's tempting to just thank the whole world and move on, but some folks deserve special recognition. We sincerely hope we didn't omit anyone in this category. If we did, blame it on Dave, and he owes you some beverages of your choice.

We've spent something like 5 or 6 years traveling all over the country visiting conferences and campuses and conducting focus groups and symposia. We thank everyone who attended and shared their ideas on improving college math for non-STEM students. You are the main reason this project exists.

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Vendor: Please add ALEKS title where indicated using the same font that was used for CREATE.

Shouldn't there be a title here that says "ALEKS" like all the other items?

Supplements

This text is available with the adaptive online learning platform ALEKS. Rooted in 20 years of research and analytics, ALEKS identifies what students know, what they don't know, and what they are ready to learn, then personalizes a learning path tailored to each student. This personalization keeps students focused on what they need to learn and prepares them to make real-world connections. ALEKS also delivers textbook-aligned exercises, videos, and the many author-created digital supplements that accompany this text.

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This computerized test bank, available online to adopting instructors, utilizes TestGen® cross-platform test generation software to quickly and easily create customized exams. Using hundreds of test items taken directly from the text, TestGen allows rapid test creation and flexibility for instructors to create their own questions from scratch with the ability to randomize number values. Powerful search and sort functions help quickly locate questions and arrange them in any order, and built-in mathematical templates let instructors insert stylized text, symbols, graphics, and equations directly into questions without need for a separate equation editor.

Videos

To ensure instructors have the support needed to help students learn concepts, a Solution Video is available for every exercise found in the text, including Prep Skills exercises. The intent in providing a full suite of videos is to support students as they discover concepts on their own.

In addition to these Solution Videos, DYGI (Did you Get it?) videos help students check their conceptual understanding, while Tech Hints videos provide extra help with technology, such as Excel. This video suite was expanded for the second edition to provide instructors more resources to deliver a discovery-based learning course in an online or hybrid environment.

Solved Examples

New in this edition, many of the more challenging topics now include a Solved Example so that students have additional structure and support as they are learning concepts on their own. Intended to help students with concepts that they might have a hard time discovering independently, these examples also serve as a helpful resource for students in an online or hybrid environment. Each of the solved examples is also accompanied by a video lecture that walks through the problem and solution, which can save class time as well as provide an extra out-of-class resource for students.

Corequisite Workbook

A corequisite workbook accompanying this text provides complete coverage of the prerequisite skills necessary for each unit and lesson. Individual sections of the workbook are available in the online resources area and can serve as worksheets for students needing review in specific skill areas.

Additional Resources

PowerPoint decks for students and instructors are available for each unit of the text. Tech templates to help students solve exercises in Excel are available in certain lessons. These and other resources can also be found in the online resources area.

McGraw Hill ALEKS®

Personalized for your students, flexible for your approach.

Every student has different needs and enters your course with varied levels of preparation. ALEKS® pinpoints what students already know, what they don't and, most importantly, what they're ready to learn next. ALEKS® provides flexibility and control when shaping your course so you can reach every student.

ALEKS® Keeps Students Focused with a Personalized Learning Path

ALEKS® creates an optimized path with an ongoing cycle of learning and assessment, celebrating students' small wins along the way with positive real-time feedback. Rooted in research and analytics, ALEKS® improves student outcomes by fostering better preparation, increased motivation and knowledge retention.

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In a single platform, ALEKS® provides a balance of personalized, adaptive practice and non-adaptive assignments for application and assessment. This supports students individually while allowing you to create assignments that align to your course goals.

By differentiating instruction for students, ALEKS® unlocks the power of instructional time, group work, and applications and prepares students to make real-world connections.

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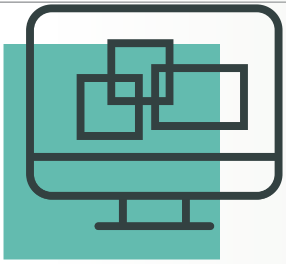


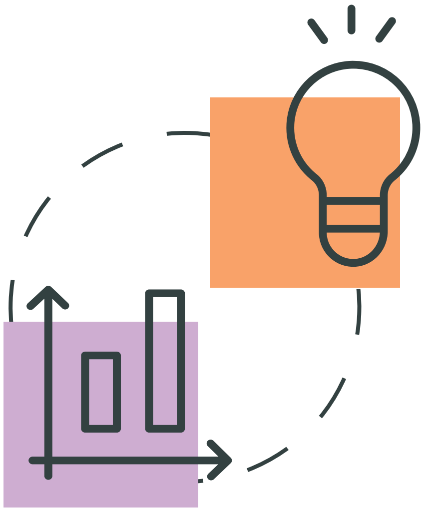
ALEKS® enables you to structure your course regardless of your instruction style and format. From a traditional classroom, to various corequisite models, to an online prep course before the start of the term, ALEKS® can supplement your instruction or play a lead role in delivering the content.

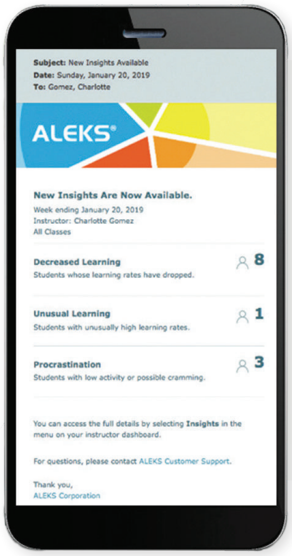
Teaching online? Respondus integration with ALEKS® supports secure testing with LockDown Browser and Respondus Monitor for Tests, Quizzes, and Scheduled Knowledge Checks.

Outcomes & Efficacy

Our commitment to improve student outcomes services a wide variety of implementation models and best practices, from lecture-based to labs and corequisites to summer prep courses. Our case studies illustrate our commitment to help you reach your course goals and our research demonstrates our drive to support all students, regardless of their math background and preparation level.

*visit **bit.ly/outcomesandefficacy** to review empirical data from ALEKS® users around the country





Turn Data Into Actionable Insights

ALEKS® Reports are designed to inform your instruction and create more meaningful interactions with your students when they need it the most. ALEKS® Insights alert you when students might be at risk of falling behind so that you can take immediate action. Insights summarize students exhibiting at least one of four negative behaviors that may require intervention including Failed Topics, Decreased Learning, Unusual Learning and Procrastination & Cramming.

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author
reviewed

Unit 1
Everyone Has Problems
(Problem Solving)

NBC/Photofest

Outline

Math in Criminal Investigation

In traditional cops-and-robbers movies, crime fighters use guns and fists to catch criminals, but in real life, often it's brain power that brings the bad guys to justice. *Law & Order: SVU* is one of the longest-running investigative shows ever. While the folks who make that show are not above the occasional foot chase, more often crimes are solved and prosecuted by gathering and organizing as much relevant information as possible, then using logic and intuition to formulate a plan. If all goes well, this leads investigators to a suspect. At that point, it's up to the prosecution to lead jurors to use reasoning skills to get to the truth.

This same strategy is the essence of problem solving in many walks of life other than criminal justice. Students in math classes often ask, "When am I going to use what I learn?" The best answer to that question is, "Every day!" Math classes are not only about facts and formulas: They're also about exercising your mind, training your brain to think logically, and learning effective strategies for solving problems. And not just math problems. Every day of our lives, we face a wide variety of problems: They pop up in our jobs, in school, and in our personal lives. Which computer should you buy? What should you do when your car starts making an awful noise? What would be a good topic for a research paper? How can you get all your work done in time to go to that party Friday night?

Unit 1 of this book is dedicated to the most important topic we'll cover: an introduction to some of the classic techniques of problem solving. These techniques will prove to be useful tools that you can apply throughout the rest of this course, and in the rest of your education. But more importantly, they can be applied just as well to situations outside the classroom.

And this brings us back to our friends from *SVU*. The logic and reasoning that they use to identify suspects and prove their guilt are largely based on problem-solving skills we'll study in this unit. By the time you've finished the unit, you should be able to evaluate the situations below, all based on episodes from Season 17 of *SVU*. In each case, you should identify the type of reasoning, inductive or deductive, that was used. Then discuss whether you think this evidence would be likely to lead to a conviction.

1. A corrections officer is accused of sexually assaulting several inmates. Many of the victims have been blackmailed into keeping quiet about the attacks. Video footage shows the officer escorting several of the alleged victims into a closed room and his partner appearing to stand guard outside. (Episode 22: "Intersecting Lives")
2. When DNA evidence found on a murder suspect leads to a familial match, detectives are forced to try and obtain DNA samples from several members of the family to identify the killer. (Episode 8: "Melancholy Pursuit")
3. A contestant on a reality dating show claims to have been raped by another contestant. Because so many of their activities were being filmed, detectives were able to find video showing that the accused was in a different place at the time of the attack. (Episode 21: "Assaulting Reality")
4. A series of attacks occur in an area near a shelter, and the attacks match the specifics of similar crimes committed by an ex-con living in the shelter, so one of the SVU detectives goes undercover at the shelter. (Episode 19: "Sheltered Outcasts")

For answers, see Math in Criminal Investigation Revisited on page 53.

Lesson 1-1 Prep Skills

SKILL 1: RECOGNIZE PATTERNS IN NUMBERS

Recognizing a pattern in a list of numbers can allow us to predict the next few numbers in the list. One common pattern that can be easy to recognize is when every number in the list is obtained by adding, subtracting, multiplying, or dividing a constant by the previous number.

- 2, 5, 8, 11, 14 — Adding 3 to each number will lead to the next number.
- 5, 1, −3, −7, −11 — Subtracting 4 from each number will lead to the next number.
- 2, 20, 200, 2,000 — Multiplying each number by 10 will lead to the next number.
- 20, 10, 5, 2.5, 1.25 — Dividing each number by 2 will lead to the next number.

SKILL 2: IDENTIFY DIVISIBILITY OF NUMBERS

If a number is divisible by another number, it means one number divides evenly into the other number. (That is, if you divide the bigger number by the smaller, there's no remainder.) 6 is divisible by 3 because 3 divides into 6 exactly 2 times with no remainder. This can be written as 6 is divisible by 3 because $3 \times 2 = 6$.

There are some interesting tests for divisibility for larger numbers, but divisibility tests for small numbers basically involve remembering our multiplication tables.

- Is 18 divisible by 2? Yes, because $2 \times 9 = 18$. — Note that 18 is an even number and that all even numbers are divisible by 2.
- Is 18 divisible by 3? Yes, because $3 \times 6 = 18$. — Note that 18 is an even multiple of 3. If you start counting by 3, you would land exactly on 18. (3, 6, 9, 12, 15, **18**)
- Is 18 divisible by 5? No. If you divide 18 by 5, the result isn't a whole number. — Note that 18 is not an even multiple of 5. If you start counting by 5, you would not land exactly on 18. (5, 10, 15, 20)

SKILL 3: SIMPLIFY ALGEBRAIC EXPRESSIONS

Two of the fundamental tools used in simplifying algebraic expressions are the distributive property and combining like terms. Assessing and keeping track of the number of terms is critical. Terms are constants, variables, or expressions that are added or subtracted.

- $6x + 5 - 2x = 4x + 5$ — Combine like terms.
- $-3(2c + 7) = -6c - 21$ — Use the distributive property.
- $7y - 5(3y - 8) = 7y - 15y + 40$ — Distribute the −5.
  $= -8y + 40$ — Combine like terms.

PREP SKILLS QUESTIONS

1. Fill in the blanks with the most likely next two numbers.

   7, 11, 15, 19, 23, ______, ______

2. Fill in the blanks with the most likely next two numbers.

   1, 2, 4, 8, 16, ______, ______

3. Which of the following numbers are divisible by 3?

   12, 14, 19, 21, 36, 40

4. Which of the following numbers are divisible by 5?

   25, 36, 55, 70, 100, 111

5. Simplify the expression: $2(3a + 7) - 4a + 2$

6. Simplify the expression: $5y + 6 - 7(2y - 4)$

Lesson 1-1 Be Reasonable (Inductive and Deductive Reasoning)

LEARNING OBJECTIVES

- ☐ 1. Explain the difference between inductive and deductive reasoning.
- ☐ 2. Use inductive reasoning to make conjectures.
- ☐ 3. Use deductive reasoning to prove or disprove a conjecture.

foodfolio/Alamy Stock Photo

We live in an epoch where rational reasoning associated with evidence isn't universally accepted and is, in fact, in jeopardy. That worries me a lot.
—Rainer Weiss

A big part of being an adult is making decisions on your own—every day is full of them, from the simple, like what to eat for breakfast, to the critical, like a choice of major or career. If you make every decision based on a coin flip, you won't get very far in life. Instead, it's important to be able to analyze a situation based on logical thinking. What are the possible outcomes of making that decision? How likely is it that each choice will have positive or negative consequences? We call the process of logical thinking **reasoning.** It doesn't take a lot of imagination to understand how important reasoning is in everyone's life. So in this lesson, we'll study the process of reasoning, which I think we can all agree is a perfectly reasonable thing to do.

0. Write about an important decision you've made where you had to use reasoning.

1-1 Class

Consider this scenario: A friend invites you to check out their new apartment a few blocks away from where you live. It's a nice day, so you decide to walk over. On the way, you notice a dog loose in someone's yard, but it seems like a nice doggie so you don't think much of it—until the dog runs over and bites you on the leg. Owwww!

1. Discuss whether or not you'd be likely to take the same path the next time you visit your friend. Include both an argument for why you would take the same route and an argument for why you would not.

David Sobecki

Answers vary. Sample argument in favor: isolated incident, unlikely to happen again. Sample argument against: You could get bitten again if you walk by the same house.

2. Suppose that you decide to take the same path the next day. When passing by the same dog at the same house, he again runs over and bites you on the leg. Discuss how this might affect your thoughts on this particular route to your friend's apartment.

Answers vary, but you would certainly think that most folks would decide not to take that route anymore.

Assuming that you would find a different way to get to your friend's place next time, you have used what we call *inductive reasoning*.

Inductive reasoning is the process of reasoning that arrives at a general conclusion based on the observation of specific examples.

In this case, you are using two specific events (the dog bites) to draw a general conclusion (that walking along a different route would be a good idea). That's what makes this way of thinking inductive reasoning.

Now let's think about another scenario. You're still walking on the shortest path to your friend's new apartment, but on one street, you notice that there's a big hole in the sidewalk where construction is being done, with no way to get around it.

3. Discuss whether or not you'd be likely to take the same path the next day. Would you have to fall into the hole to decide?

You would certainly hope not, on both counts.

A_Lesik/Shutterstock

There's a clear difference between these two scenarios: In the second, you wouldn't need to fall into the hole twice to decide that a different route would be a good idea. At least I hope you wouldn't. Instead, you would deduce from a known principle (I like to call it "gravity") that walking along this sidewalk wouldn't work. The use of the word "deduce" was important there: We call this type of reasoning *deductive reasoning*.

Deductive reasoning is the process of reasoning that arrives at a conclusion based on previously accepted general statements. It doesn't rely on specific examples.

Before we start using these two methods of reasoning to draw conclusions, let's make sure you understand the difference between the two. In Questions 4–8, decide whether inductive or deductive reasoning was used to draw the conclusion, and explain your choice in your own words.

4. The last six times we played our archrival in football, we won, so I know we're going to win on Saturday.

Inductive reasoning. The conclusion is based on six specific instances—the six previous games.

Even though this is in the Class portion of the lesson, it's a good idea to let students discuss these in pairs or in groups before going over them in class.

5. There is no mail delivery on holidays. Tomorrow is Labor Day, so I know my student loan check won't show up.

Deductive reasoning. The fact that mail doesn't get delivered on legal holidays is a known rule.

6. The syllabus states that any final average between 80 and 90% will result in a B. I got 80 and 82 on my first two tests, and if I get 78% on my final, my overall average will be 80.1%. That means I'll get a B.

Deductive. Don't be misled by the specific scores being given. The conclusion is based on the stated premise that a final average between 80 and 90 will result in a B.

7. Everyone I know in my sorority got at least a 2.5 GPA last semester, so I'm sure I'll get at least a 2.5 this semester.

Inductive. The conclusion is based on the specific experiences of some number of people.

8. All birds can fly. An ostrich is a bird. Therefore, ostriches can fly.

Deductive. It doesn't matter that the conclusion is false: It's based on the premise that all birds can fly, not specific examples.

You can lay some groundwork for our study of logic here if you like. The conclusion here isn't true, but it was still drawn using deductive reasoning. It just so happens that the "known rule" here is incorrect.

Did You Get It

Try this problem to see if you understand the concepts we just studied. The answer can be found at the end of the portfolio section.

1. Decide whether inductive or deductive reasoning was used to draw each conclusion.
   a. I've never met a golden retriever with a nasty disposition. I bet there aren't any.
   b. An apple a day keeps the doctor away. I eat an apple every day, so Dr. Phil has never come to my house.
   c. Today is Friday, so it will be Monday in three days.

1-1 Group

1. Numerous studies have shown that one of the best ways to do better in college classes is to study in pairs or groups. To help you get started, if you feel comfortable sharing contact information, exchange the information in the table below. The group you're in now will be your small group for the first unit of this course. When you get used to meeting in class, you'll likely find that meeting outside of class to study and work on homework is a good idea as well, so include some study times that would be convenient for you to meet.

Name	Phone Number	Email	Available times

This marks the first time students will formally introduce themselves to their groups and work on the Group portion of a lesson. Set the tone for the class by walking around, asking the groups questions about their work, and encouraging groups that are not working together. Make sure they're not rushing through the problems without really thinking about them. The most common sentence I say in my classes is "Talk to each other, people!" Students tend to think that group work in math means comparing answers. Convince them to reason out loud!

In inductive reasoning, we're drawing conclusions based on observing specific examples or occurrences. An important aspect of being able to do this effectively is recognizing patterns. So we'll start there in our study of using inductive reasoning. Consider this list of numbers:

3, 6, 9, 12, 15

If you were asked to make an educated guess as to what number might come next on the list, a good place to start would be to recognize that after the first number, all others come from adding 3 to the previous number on the list. In that case, the most logical guess for the next number comes from adding 3 to 15, of course getting 18.

Math Note

The word **conjecture** is basically a synonym for "educated guess." So we would say that we *conjectured* that the next number on the list 3, 6, 9, 12, 15 would be 18.

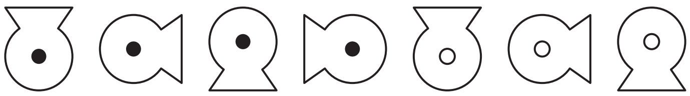
2. Use inductive reasoning to make a conjecture about the next number on this list:

1, 2, 4, 5, 7, 8, 10, 11, 13

Explain how you decided on your answer.

14. The pattern is add one, then add two, then go back to adding one and repeat. The last number added was 2, so this time it'll be 1.

3. Make a reasonable conjecture for the next figure in this sequence, and briefly describe your reasoning.

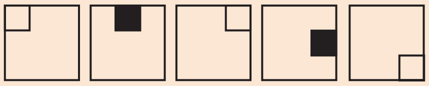


The next figure will be the same shape, but have the flat side facing to the left. Also, the circle inside will be white. With each successive picture, the figure rotates a quarter-turn clockwise. The first four had a black circle inside, and the next four should have a white circle.

4. Think of a way that you have used inductive reasoning recently to draw a conclusion of some sort, and describe that thought process. Each member of the group should come up with their own example.

Answers vary.

Did You Get It

2. Make a reasonable conjecture for the next two items on each list.
   a. 0 −2 4 −6 8 −10
   b. 26 y 24 w 22 u 20 s
   c. 

You could make a very real case that inductive reasoning is the essence of where scientific advances come from. You make some observations, then use inductive reasoning to draw a conclusion, and test to see if it seems to hold true. Let's see how this works.

5. If you add two odd numbers together, what will the result look like? Use inductive reasoning to make a conjecture.

The best approach is to pick several pairs of odd numbers and add them together. Some samples:

$3 + 7 = 10$, $11 + 15 = 26$, $1 + 3 = 4$, $19 + 111 = 140$, $51 + 51 = 102$

The only connection between the sums is that they're all even.

6. How sure are you that your conjecture is true? Discuss.

Not particularly. I tried five pairs of odd numbers, and it worked each time. But there are infinitely many pairs to choose from, and I tested FIVE. That doesn't give me a ton of confidence.

> You might consider tying this into our forthcoming study of probability. Ask students to give a percentage of how confident they are that their conjecture is right.

Here's the problem with your conjecture, and it's HUGELY important in mathematical reasoning: You're guessing that the sum of EVERY pair of odd numbers has a certain property. ***But there's no way that you can actually test every pair of numbers because (of course) there are infinitely many to test.*** And that's the big issue with inductive reasoning: It's an incredibly useful tool in decision making, but in most cases you can't verify a conclusion for every possible case. So you can't be 100% certain that your conclusion is valid. We'll come back to your conjecture later in the lesson. For now, we'll look at a different problem.

Suppose that your history professor gives a surprise quiz every Friday for the first 4 weeks of class. At that point, you might use inductive reasoning to conjecture that you'll have a quiz every Friday. But you can't look into the future and know for sure, so it's entirely possible that your conjecture is not true. In fact, the first time you DON'T have a quiz on Friday, your conjecture has proven to be false.

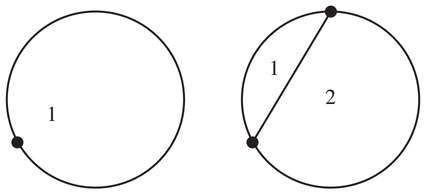
This is a really useful observation: While it can be difficult to prove that a conjecture is true, it's much simpler to prove that one is false. All you need to do is find one specific example that contradicts the conjecture (like the first Friday without a quiz). This is known as a *counterexample.*

A **counterexample** is a single specific example that violates a conjecture. Even one counterexample is enough to prove that a conjecture is false.

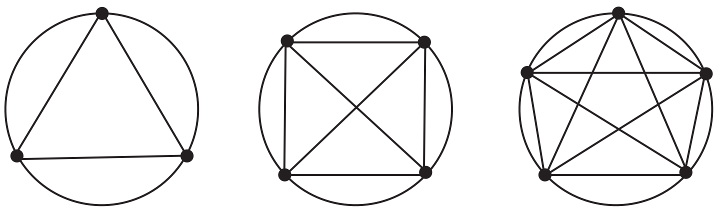
7. This conjecture is false (Trust me, I'm a doctor.): A whole number greater than 10 can be divided by 3 with no remainder if the last two digits can be divided by 3 with no remainder. Find a counterexample that proves it's not true. (Hint: Pick some random numbers whose last two digits can be divided evenly by 3.)

103 is the smallest number that works as a counterexample.

A chord is a line connecting two points on a circle. If you draw a chord connecting two points, it divides the circle into two sections. With only one point, there's no chord to draw, and the circle has just one section:



8. Using the diagrams below, count the number of sections a circle is divided into by chords connecting 3, 4, and 5 points. Numbering the sections as you count them is a big help.

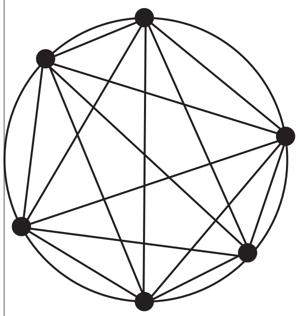


Number of Points	1	2	3	4	5
Number of Sections	1	2	4	8	16

9. Use your results from Question 8 to make a conjecture as to the number of sections with 6 points. Explain your reasoning.

The most logical guess is 32, as the number of sections appears to keep doubling.

10. Use the diagram to check your conjecture. What does that tell you about conjectures made using inductive reasoning?



There are 31 sections. Oops. It's entirely possible you can think logically and make what seems to be a correct conjecture, but it can still turn out to be false.

This is easily one of the most important questions in the lesson. Every human being should understand that you can't prove something is true by looking at a handful of specific examples.

Question 10 illustrates the essence of math and science we mentioned earlier: We have an idea that something might be true, and we use inductive reasoning to test it. But the result shows why inductive reasoning can't be used to prove results: What appears to be true after looking at several examples can still turn out to be false. We'll explore this idea more in the Technology part of tonight's homework. Doing so will require using formulas to perform calculations in Excel; if you're not familiar with how to do this, these directions will help, and the video in class resources will help even more.

Using Technology: Entering a Formula in Excel

To enter a formula in Excel:

1. Set up a separate column for each quantity involved in the calculation.
2. Identify the column where you want to store the result of the calculation.
3. In the first cell of that column, type = to indicate that you're entering a formula.
4. Type the calculation that you want completed after the = sign. When you want to include the number in a particular cell as part of the calculation, you can either type the column and row of that cell (A3, for example), or you can use the cursor to click on the cell and it will be entered into your formula automatically.
5. When you've completed the formula, press ENTER.

See the Lesson 1-1 Using Tech video in the class resources for further information.

It's a good idea to take some class time to demonstrate Excel procedures, especially early in the class. Some students will have no familiarity with spreadsheets.

Did You Get It

3. Find a counterexample to prove that each conjecture is false.
   a. No month has more than eight letters in it.
   b. The sum of any two positive numbers with two digits also has two digits.

11. When you prove that a conjecture is false using a counterexample, are you using inductive or deductive reasoning? Discuss.

This is an interesting question to discuss. The answer is deductive: It's a known rule that something is not ALWAYS true if there's at least one case where it's not.

So inductive reasoning can be used to give you a really good idea that something may be true, but most often it cannot be used to *prove* that it's true. Our next step will be to look at how inductive and deductive reasoning can work together to both make a conjecture, and then prove it.

12. Pick a random number, and add 50 to it. Then multiply by 2 and subtract the original number. What is the result?

Answers vary depending on the number, of course. Sample calculation: $33 + 50 = 83$; $83 \cdot 2 = 166$; $166 - 33 = 133$. The result is 100 more than the original number.

13. Repeat Question 12 for several other beginning numbers until you're able to use inductive reasoning to make a conjecture as to what the result will always look like.

It appears that the result always works out to be 100 more than the original number.

Now let's use deductive reasoning to prove your conjecture. So far, you've used *specific* numbers in developing your conjecture: That's what makes what you've done so far inductive reasoning. In order to turn the same idea into deductive reasoning, you need to do these calculations on a number that's nonspecific. Sounds impossible, right? Not so fast, my friend. Rather than use another specific number, we'll use a symbol (a letter in this case) to stand in for ALL possible numbers. If that sounds like a variable to you, you're absolutely right! Good job. The letter we choose will represent a quantity (the number we pick) that can VARY. We want our symbol to stand for any old number, so let's use the letter *a*.

14. Add 50 to our variable number. The result will be an **algebraic expression,** which is a combination of numbers and variables using operations like addition, subtraction, multiplication, division, powers, and roots.

$a + 50$

15. Next, multiply your answer to Question 14 by 2. Please don't forget that the distributive property exists.

$2(a + 50) = 2a + 100$

16. Finally, subtract the original number from your answer to Question 15. What is the result?

$2a + 100 - a = a + 100$ This is 100 more than the original number.

1-1 Class (Again)

1. Explain why Questions 14–16 in the Group portion prove the conjecture, while Questions 12 and 13 just indicated that it was likely to be true.

In Questions 12 and 13, we were using specific numbers. There's no way we could test the conjecture using EVERY number, so we can't prove the conjecture that way. In Questions 14–16, however, we used a symbol that represents EVERY number because it's not specific. So deductive reasoning then tells us that the conjecture is always true.

This is where we really bring home the key idea between having a pretty darn good idea that something is true and PROVING that it's true, which is why we labeled the remainder of the lesson as class.

Did You Get It

4. Think of any number. Multiply that number by 3, then add 30, and divide the result by 3. Next subtract the original number. What is the result?
   a. Use inductive reasoning to make a conjecture for the answer.
   b. Use deductive reasoning to prove your conjecture.

Now let's get back to your earlier conjecture about adding odd numbers. You DID conjecture that the result is always even, right? Good job. Go buy yourself a muffin or something. Next, we're going to think about how we would go about proving this conjecture.

Solved Example

Prove that the sum of any two odd numbers is always even.

SOLUTION

The most important thing to understand here is what you CAN'T do to prove a conjecture like this: Just pick a bunch of *specific pairs* of odd numbers and add them, then check to see if the result is even. No matter how many pairs you choose, even if the results keeps coming up even, this can NEVER prove that it will ALWAYS happen. For that, we'll need to add together two arbitrary odd numbers.

In order to do that effectively, we need to think about what exactly makes a number even or odd. A number is even exactly when it can be written as 2 times some other integer. So we can write an arbitrary even number by writing it as $2n$, where n is some integer. Next, think about the fact that every odd number is exactly one less than an even number. One is one less than 2, 13 is one less than 14, 157 is one less than 158, and so on. Since an arbitrary even number looks like $2n$, we can write an arbitrary odd number as one less than that, which of course is $2n - 1$. Now we're ready to prove our conjecture.

Two arbitrary odd numbers can be written as $2n - 1$ and $2m - 1$, where n and m are integers. Let's add them:

$$(2n - 1) + (2m - 1)$$

Now we need to do some algebra.

$(2n - 1) + (2m - 1) = 2n + 2m - 2$	Combine the like terms −1 and −1
$= 2(n + m - 1)$	Factor 2 out of each term
$= 2(\text{Some integer})$	Since n and m are integers, so is $n + m - 1$

Did You Get It

5. Prove that the sum of two even numbers is always even.

2. Explain why what we did in the solved example proves that the sum of two odd numbers is ALWAYS even.

The result is the sum of ANY two odd numbers, and we were able to write it as 2 times a whole number. So the result has to be even.

3. The owner of a growing business has noticed that as she hires more employees, the company's weekly profit grows. She hires a consultant to run some numbers, and he informs her that the formula below is a good model of the profit in dollars she can expect to make each week. The variable x represents the number of employees. If she started with six employees, use inductive reasoning to investigate whether or not the weekly profit will keep going up if she keeps hiring more employees. Write your conclusion in the form of a sentence.

$-10x^2 + 480x + 308$

Answers should vary depending on what number of employees they choose to test for. The function is designed so that the profit increases up until 24 employees, where it reaches a max and then begins to decrease.

I actually prefer when students leave with the impression that this conjecture is true, leaving them to find out otherwise in the Technology assignment. So if they say true, let it go and tell them they'll have a chance to confirm their conjecture in the homework.

1-1 Portfolio

Name ____

Check each box when you've completed the task. Remember that your instructor will want you to turn in the portfolio pages you create.

Technology

1. ☐ Spreadsheets can be a hugely useful tool for using inductive reasoning since you can set up a spreadsheet to test the results of calculations for many different values quickly. We'll be using a spreadsheet to test some of the conjectures from this lesson. The goal will be to use formulas to test conjectures for many different values, similar to the sample below. A template to help you get started can be found in the online resources for this lesson, along with detailed instructions.

	A	B	C	D
1	Original number	Add 50	Multiply by 2	Subtract original number
2	56	106	212	156
3				

Source: Microsoft

Online Practice

1. ☐ Include any written work from the online assignment along with any notes or questions about this lesson's content.

Applications

1. ☐ Complete the applications problems.

Reflections

Type a short answer to each question.

1. ☐ Describe the difference between inductive and deductive reasoning in your own words.

2. ☐ Which type of reasoning (inductive/deductive) do you prefer, and why?

3. ☐ Why can inductive reasoning never be used to PROVE a conjecture?

4. ☐ Name one thing you learned or discovered in this lesson that you found particularly interesting.

5. ☐ What questions do you have about this lesson?

Looking Ahead

1. ☐ Complete the prep skills for Lesson 1-2.

2. ☐ Read the opening paragraph in Lesson 1-2 carefully and answer Question 0 in preparation for that lesson.

Answers to "Did You Get It?"

1. a. Inductive **b.** Deductive **c.** Deductive **2. a.** 12 −14 **b.** 18 q **c.** 

3. a. September has nine letters. **b.** If you pick two numbers with a sum more than 100, the sum has more than two digits. **4. a.** The result is always 10.

b. $3a \Rightarrow 3a + 30 \Rightarrow \frac{3a + 30}{3} \Rightarrow \frac{3a + 30}{3} - a = a + 10 - a = 10$

5. See video in the online class resources for the proof, which looks a lot like the solution to Solved Example ~~1~~ 9.

Answers to Prep Skills Questions

1. 27, 31
2. 32, 64
3. 12, 21, 36
4. 25, 55, 70, 100
5. $2a + 16$
6. $-9y + 34$

1-1 Applications

Name ______________________________

In Questions 1–4, decide whether inductive or deductive reasoning was used to draw a conclusion. Briefly explain your choice. An actual sentence or two would be nice.

1. The last four congressional representatives from this district were all Republicans. I don't know why the Democratic candidate is even bothering to run this year.

Inductive reasoning. Based on four specific elections, this draws a general conclusion.

2. Working as a nurse in a hospital requires at least a 2-year degree in this state, so when I was in the emergency room last week I asked the nurse where he went to college.

Deductive. If it's a rule or law that a degree is required, then the nurse must have gone to college.

3. Experts say that opening email attachments that come from unknown senders is the easiest way to get a virus on your computer. Shauna constantly opens attachments from people she doesn't know, so she'll probably end up with a virus on her system.

Deductive. Whether or not the premise set forth by "experts" is true, the conclusion is drawn from a rule that is being accepted, not specific instances.

4. Every time Beth sold back her textbooks, she got about 10% of what she paid for them; so this semester she realized it wouldn't be worth the effort to sell back her books at all.

Inductive. Beth is basing her conclusion on her own past experiences.

1-1 Applications

Name ______________________________

For each conjecture in Questions 5–7, if it's false, provide a counterexample. If it's true, use deductive reasoning to prove it.

5. The square of every real number is greater than the number itself.

This is false. Any number between zero and one will provide a counterexample.

6. Pick two even numbers and subtract the larger number minus the smaller number. The result is even as well.

True. If the first is $2n$ and the second is $2m$, then the difference is $2n - 2m$. We can factor out 2 and get $2(n - m)$, which shows that the result is even because $n - m$ is a whole number.

7. When an even number is added to the product of two odd numbers, the result will be even.

This is false. $3(11) + 4 = 37$, which of course is not even.

1-1 Applications

Name ______________________________

8. Suppose that you drive a certain distance at 20 miles per hour, then turn around and drive back the same distance at 60 miles per hour. Choose at least four different distances and find the average speed for the whole trip. Then use inductive reasoning to make a conjecture as to what the average speed is in general. (Hint: You'll need the formula distance = speed times time repeatedly!)

Distance: 60 miles. It will take 3 hours out and 1 hour back in, for a total of 4 hours. Average speed is 120 mi/4 hr = 30 mi/hr.

Distance: 120 miles. It will take 6 hours out and 2 hours back in, for a total of 8 hours. Average speed is 240 mi/8 hr = 30 mi/hr.

Etc. It always works out to be 30 mph.

9. Use algebra to prove your conjecture from Question 8. (Hint: Use the exact same steps you did for each calculation in Question 8, but use a variable for the distance rather than a distance you chose.)

Distance: d miles. It will take d/20 hours out and d/60 hours back in. Total time is

$$\frac{d}{20} + \frac{d}{60} = \frac{4d}{60} = \frac{d}{15}\text{ hr}$$

Average speed is $\dfrac{2d\text{ mi}}{\frac{d}{15}\text{ hr}} = 2d\text{ mi} \cdot \dfrac{15}{d\text{ hr}} = 30\,\dfrac{\text{mi}}{\text{hr}}$

Lesson 1-2 Prep Skills

SKILL 1: RECOGNIZE PLACE VALUES

The place value of a digit in a number depends on where that digit is located in the number. The chart below should help.

Place Value																		
Millions			,	Thousands			,	Ones			.	Decimals					
hundred millions	ten millions	millions		hundred thousands	ten thousands	thousands		hundreds	tens	ones		tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths

The number 342.7 has four digits.

The 3 is in the hundreds place. Notice that the 3 is three places to the left of the decimal.

The 4 is in the tens place. Notice that the 4 is two places to the left of the decimal.

The 2 is in the ones place. Notice that the 2 is one place to the left of the decimal.

The 7 is in the tenths place. Notice that the 7 is one place to the right of the decimal.

SKILL 2: ROUND NUMBERS

Rounding numbers

Rounding numbers involves a fairly simple procedure.

Step 1. Locate the digit in the desired place.

Step 2. Look one place to the right.

Step 3. If the digit to the right of the desired place is 4 or lower, the digit in the desired place remains unchanged, rounding down. If the digit to the right is 5 or higher (5, 6, 7, 8, or 9), the digit in the desired place increases by one, rounding up.

To round 34.726 to the nearest hundredth:

Step 1. The digit in the hundredths place is 2. Notice that the 2 is two places to the right of the decimal.

Step 2. The digit to the right of the 2 is 6.

Step 3. Since 6 is greater than 5, the digit in the desired place should be increased by one (rounded up). We round up because our procedure helps us see that 34.726 is closer to 34.73 than it is to 34.72.

So 34.726 rounded to the nearest hundredth is 34.73.

SKILL 3: PERFORM MENTAL ARITHMETIC

Performing mental arithmetic is just that. Mental. It's a useful skill that you have to PRACTICE. If you insist on always using a calculator, even for relatively simple operations, there's a pretty good chance that you'll never feel very good about your math skills. You may be surprised how practicing some mental arithmetic makes you feel much more comfortable with math in general. Use a calculator as a tool, not a crutch.

PREP SKILLS QUESTIONS

1. For the number 87.23, list the digit in each place value.

   a. Tens place b. Ones place c. Tenths place d. Hundredths place

2. Round each number to the designated place value.

   a. 8,431 to the nearest ten b. 11.46 to the nearest tenth c. $84.76 to the nearest dollar

3. Perform each calculation without using a calculator or computer.

   a. 12×8 b. 300×6 c. 25×8 d. $120 + 80$ e. $95 + 15$ f. $1,400 + 800$

Lesson 1-2 More or Less (Estimation and Interpreting Graphs)

LEARNING OBJECTIVES

- ☐ 1. Use rounding and mental arithmetic to estimate the answers to applied problems.
- ☐ 2. Obtain and interpret information from bar graphs, pie charts, and time series graphs.

The key to good decision making is evaluating the available information—the data—and combining it with your own estimates of pluses and minuses.

—Emily Oster

YinYang/Getty Images

Everyone likes buying items on sale, so we should all be familiar with the idea of finding a rough approximation for a sale price. If you're looking at a pair of shoes that normally sells for $70 and the store has a 40% off sale, you might figure that the shoes are a little more than half price, which would be $35, so they're probably around $40. We will call the process of finding an approximate answer to a math problem **estimation.** Estimation comes in handy in a wide variety of settings. When the auto repair shop technicians look over your car to see what's wrong, they can't know for sure what the exact cost will be until they've made the repairs, so they will give you an estimate. When you go to the grocery store and have only $20 to spend, you'll probably keep a rough estimate of the total as you add items to the cart. (Imagine buying a week's worth of groceries and keeping track of every price to the penny on your cell phone. Who has time for that?) If you plan on buying carpet for a room, you'd most likely measure the square footage and then estimate the total cost as you looked at different styles of carpet. You could find the exact cost if you really needed to, but often an estimate is good enough for you to make a sound buying decision.

0. Write about a time you've found estimation to be useful in your life.

1-2 Class

Rounding numbers is going to play a very significant role in our study of estimation, so we'll begin with a quick review of the rules for rounding. But of course, we'll do so in the context of an applied problem. (As you may have already noticed, that's kind of the point of this course.)

Solved Example 1

Let's say you're planning a group outing, with lunch included. The lunch is supposed to cost $3.95 per person, and 24 people sign up. Would $94.80 be a reasonable bill?

SOLUTION

The most precise approach would be to multiply $3.95 by 24 and see what the bill would be. But we're not asked if that amount is EXACTLY right—we're asked if it's reasonable. So a quick estimate would get the job done, and would be simpler than the actual multiplication, which pretty clearly would require a calculator.

Here's our strategy: First, we'll notice that $3.95 is pretty close to $4, and that 24 people is pretty close to 25 people. Why is that helpful? Because $4 times 25 is $100, and that doesn't require a calculator. Finally, we rounded the price up by a bit, and the number of customers up by a bit, so that $100 estimate has to be a bit on the high side. Bingo! A bill of $94.80 is just about what we'd expect.

Solved Example 1 does a nice job of illustrating a good, reliable procedure for doing estimation with numerical calculations:

1. Round the numbers being used to numbers that make the calculation simple.
2. Perform the operation or operations involved.

In order to practice rounding rules, we'll start with a very quick review of percents. This topic is covered in detail in Unit 2, so we'll just review the basics here. Percents are used to describe certain portions of a whole. As you know, 50% of some amount is half of it. We can use this idea and estimation to get a rough idea of certain percentages without needing a calculator.

1. There are about 212 million people in the U.S. over the age of 24. About half of them are female. Estimate the number of females over the age of 24.

Half of 212 is 106, so there are about 106 million females over age 24.

Emphasize that when estimating there is no single correct answer, and that if you use a calculator to get an estimate, you might as well just compute an exact value. The point is to do the arithmetic mentally.

2. Of those females, the U.S. Census Bureau reports that about 33% have a bachelor's degree or higher. Use this to estimate the number of females over age 24 who have a bachelor's degree or higher.

33% of 100 million would be 33 million. Since 106 million is a little bigger than 100 million, we'd estimate that maybe 35 million women in that age group have a bachelor's degree.

3. For males in that age group, about 12% have a graduate degree. About how many men is that?

12% of 100 million would be 12 million, so 12% of 106 million is probably something close to 13 million.

We often use the word "nearest" when talking about rounding numbers. If we're dealing with currency, we typically will want to round either to the ones place (which is dollars), or the hundredths place (which is cents). In that case, we'll use the terminology in Solved Example 2.

Solved Example 2

The purchasing manager for a resort hotel places an order for 1,155 new linen napkins, each of which costs $2.48.

a. Find the exact cost.

b. The hotel accountant needs the amount rounded to the nearest dollar. What is it?

c. The general manager, more of a "big picture" guy, wants to know the cost to the nearest hundred dollars. What should our friend the purchasing manager tell him?

SOLUTION

a. The exact cost is just 1,155 times $2.48, which is $2,864.40.

b. The number of cents is less than 50, so we'd round down to $2,864.

c. The digit in the tens place is 6, so we round the 8 in the hundreds place up to 9. The price to the nearest hundred dollars is $2,900.

4. A one-year investment in a certificate of deposit (CD) draws 2.1% interest, which mathematically means that to find the new value after one year, you multiply the original investment by 1.021. If the beginning balance of an investment is $8,412.35, find the value at the end of one year, rounded to the nearest cent.

1.021 · $8,412.35 = $8,589.00935, which rounds to $8,589.01.

One of the key concepts involved in rounding is *accuracy*. For our purposes, we'll define accuracy as how far off from the exact value of some quantity an estimate is. For the calculation in Question 4, you could round the initial investment to $10,000, and that would make the computation considerably easier. But because $10,000 is quite a bit different from the actual amount, your estimated value would be very different from the actual value. In that case, we'd say that your answer was inaccurate.

In short, when using rounding and estimation, it's always important to think about the level of accuracy as well as the difficulty level of getting an estimate.

5. The owner of an apartment complex needs to buy six new refrigerators. She's considering choosing a model that costs $579.95 each. By rounding the cost to the nearest hundred dollars, estimate the total cost.

Rounding to $600, we get 6 · $600 = $3,600.

6. Instead of rounding to the nearest hundred dollars, round to the nearest ten dollars and estimate the cost. Then discuss the pros and cons of rounding to the nearest hundred as opposed to the nearest ten.

Rounding to $580, we get 6 · $580 = $3,480. Since there was less rounding, the result is going to be more accurate. But the trade-off is that the computation was more difficult to do mentally. And if you need to use a calculator to get the estimate, there's no point in estimating!

Did You Get It

Try this problem to see if you understand the concepts we just studied. The answer can be found at the end of the portfolio section.

1. When shopping for his books this semester, Arlen has $400 in financial aid money allocated. He needs two paper books at $147 each, an e-book costing $79, and an online homework system access code for $82. Rounding each value to the nearest ten dollars, estimate if Arlen's aid will cover his expenses.

Questions 5 and 6 bring up an interesting point: How do you know what digit to round to when estimating? The correct answer is "five." Just kidding—there IS no correct answer. It depends on the individual numbers. Deciding on how much to round is really a trade-off: ease of calculation versus accuracy. In most cases you'll get a more accurate result if you round less, but the calculation will be a little harder. Since there's no exact rule, it's important to evaluate the situation and use good old-fashioned common sense. And remember, when you are estimating, there is no one correct answer.

7. Suppose that you're considering a new cell phone plan where you have to pay $179 upfront for the latest phone, but the monthly charge of $39.99 includes unlimited data, talk, and text. A different plan gives you the phone you want for free (aside from sales tax of $12.53, which you are responsible for), and the unlimited data, talk, and text plan is $61.20 per month. Use the type of estimation we've practiced to decide which is the better choice. Are there factors other than total cost you might consider? Explain your calculations and your decision.

Answers will vary on the most important part, but here's the computation for one year: $\$180 + 12 \cdot \$40 = \$180 + \$480 = \$660$. Compare to $\$10 + 12 \cdot \$60 = \$730$. Notice also that the first is an overestimate because of rounding up, while the second is an underestimate.

Notice that this question is intentionally very open-ended. No time frame is specified, so it's up to each group to decide how they want to estimate costs.

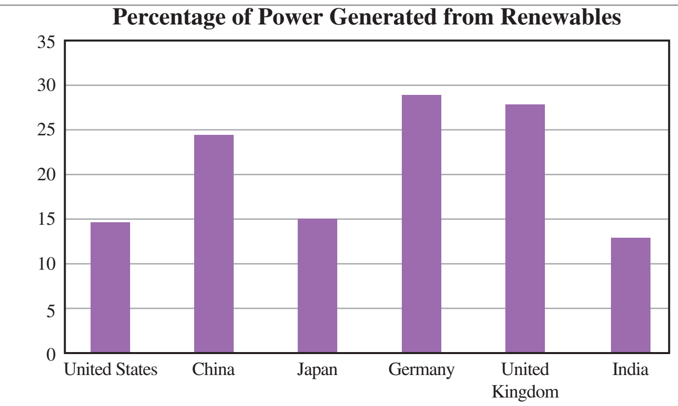
moodboard/SuperStock

1-2 Group

In this age of information, it's very common for interesting or useful information to be displayed in graphical form, rather than just listing out numbers. Why? Because this almost always makes it easier to get a quick overall view of what that data are telling us. Here's an example: The table below shows the percentage of total power generated that came from renewable sources for the countries with the six largest economies in the world as of 2019.

Country	% of Power from Renewable Sources
United States	14.7%
China	24.5%
Japan	15.0%
Germany	29.0%
United Kingdom	27.9%
India	12.8%

There is absolutely nothing wrong with this table. It's positively delightful. And if you want to read it line by line and concentrate on the percentages, you can get a very accurate comparison of how well these nations are doing in terms of transitioning to a green economy. But let's compare that to a **bar graph** representing the same data.



Source: Renewable Energy Policy Network for the 21st Century

Bar graphs are used to compare amounts or percentages using either vertical or horizontal bars of various lengths; the lengths correspond to the amounts or percentages, with longer bars representing larger amounts.

The graph makes it pretty easy to get an overall picture of what the data in the table indicate. First, we can easily see that Germany gets the highest percentage of its energy from renewables, and India gets the lowest percentage. We can also see that the percentages for China, Germany, and the U.K. are reasonably similar, as are the percentages for the U.S., Japan, and India.

So what does that have to do with estimation? In looking at the graph, we're visually estimating the percentages based on the heights of the bars, rather than getting an exact percentage like we did from the table.

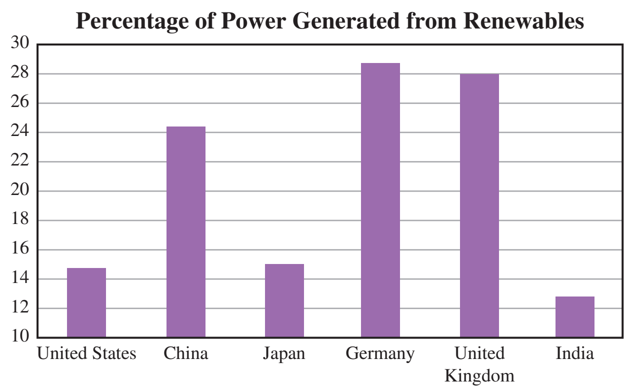
1. Without looking at the table, use the bar graph above to estimate the percentage of electricity generated by renewable sources for each country shown. Then compare your estimates to the exact percentages shown in the table. You should give some thought to how accurately you think you can reasonably round the percentages based on the graph.

Sample answers: United States 15%, China 25%, Japan 15%, Germany 29%, United Kingdom 28%, India 13%.

Math Note

Sometimes a graph will contain labels that provide the exact values of the data illustrated. In this case, we could have put the exact percentages at the top of each bar.

2. The same information is shown on the next bar graph. But we've altered the scale on the vertical axis. Describe several ways that this can change your perception of the differences in percentages for these countries.



Source: Renewable Energy Policy Network for the 21st Century

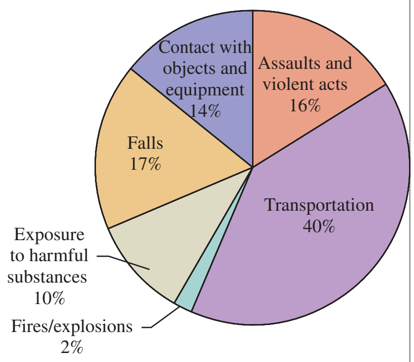
Answers can vary. The most obvious effect is that this makes the difference between the higher percentages and lower percentages appear much more dramatic.

Did You Get It

2. Use the second bar graph describing renewable energy production to estimate the percentages for the three top producers. Then describe your confidence in how accurate your estimates are compared to the ones you made from the first bar graph.

A **pie chart,** also called a **circle graph,** is constructed by drawing a circle and dividing it into parts called sectors, according to the size of the percentage of each portion in relation to the whole. The next series of questions will be based on the pie chart below, which illustrates the breakdown of different types of fatal workplace injuries in 2017.

Fatal Occupational Injuries



Source: U.S. Bureau of Labor Statistics

3. What percentage of workplace fatalities were caused by transportation incidents?

40%

4. What percentage of workplace fatalities were not due to exposure to substances or contact with objects and equipment?

76%

5. There were 5,147 workplace fatalities in that year. Use rounding and estimation to estimate how many were due to either falls or contact with objects and equipment. Explain how you got your estimate.

One possible approach: Those two categories add up to 31%, which we can round down to 30%. We can round the fatalities down to 5,000, and 30% of that is 1,500. So something a bit more than 1,500 is a good estimate.

6. Use a calculator to find the exact number of deaths caused by either falls or contact with objects and equipment. Discuss the accuracy of your estimate, and why it was either too low or too high.

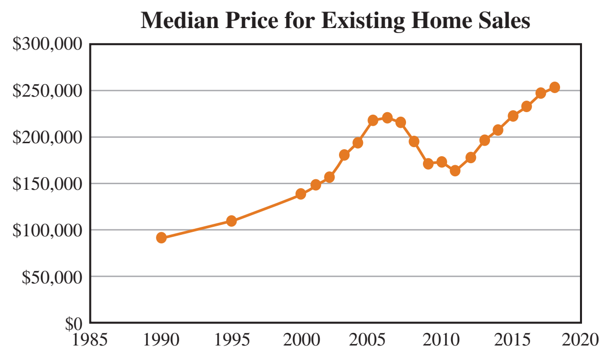
31% of 5,147 is 1,595.57. This rounds up to 1,596, but you could discuss with the students that in a case where only whole numbers make sense, like number of deaths, it may be more appropriate to round to the next lowest whole number. We can see that our estimate was too low, which it should have been since we rounded the percentage down to 30% and the total number of fatalities down to 5,000.

Did You Get It

3. Were there more fatal accidents resulting from transportation, or from the next three highest causes? What was the difference in percentages for these causes?

A **time series graph** or **line graph** shows how the value of some variable quantity changes over a specific time period. These are the types of graphs you're most likely to remember working with in algebra classes. And maybe you always wondered, "Why does anyone care about drawing graphs in algebra?" The answer to that question is exactly the point of studying different types of graphs. Graphs aren't just bad art; they're used to conveniently display useful numerical information.

The last handful of questions in this portion of the lesson are based on the time series graph below, which illustrates the median selling price of all existing homes in the United States from 1990 to 2018.



7. Estimate the median selling price in 1990 from the graph. Don't forget to consider how "rounded" you think your answer should be.

About $90,000

Math Note

Median is a measure of average that we'll study in Unit 4. It's kind of like the midpoint if you list out an entire string of numbers.

8. In which year between 2000 and 2010 did the median price reach its highest point? Estimate the highest median price, and write a brief explanation of how you decided on your answer.

Around 2006 the median price reached very close to $225,000. The height of each point on the graph represents the median price, so we find the highest price by finding the highest point on the graph.

9. Housing prices rose dramatically as the economy flourished in the early 2000s, but there were many issues that led to a major economic downturn. The period of increase later became known as the housing bubble. How long did it take for the median price to drop by $25,000 when the housing bubble burst? How about $50,000? Explain your answers.

It took about 2 years to drop by $25,000, and then only 1 more year after that to reach $50,000. The goal is to find the year corresponding to the first point on the graph after 2006 with height less than $200,000, then the first point after that with height less than $175,000.

10. What does the graph tell us about what the median price will be for 2020? Talk about this with your group, and give it some serious thought.

It doesn't TELL us anything. We can make a guess based on the pattern of the graph, in which case something around $260,000 would be reasonable. But as what happened after 2006 shows, the pattern could have completely changed after 2018, in which case it wouldn't be useful in predicting the height of the point for 2020.