Intermediate Algebra

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SIXTH EDITION

Julie Miller
Professor Emerita,
Daytona State College

Molly O'Neill
Professor Emerita,
Daytona State College

Nancy Hyde
Professor Emerita,
Broward College

Mc Graw Hill

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Letter from the Authors

Dear Colleagues,

Across the country, Developmental Math courses are in a state of flux, and we as instructors are at the center of it all. As many of our institutions are grappling with the challenges of placement, retention, and graduation rates, we are on the front lines with our students—supporting all of them in their educational journey.

Flexibility—No Matter Your Course Format!

The three of us each teach differently, as do many of our current users. The Miller/O'Neill/Hyde series is designed for successful use in a variety of course formats, both traditional and modern—classroom lecture settings, flipped classrooms, hybrid classes, and online-only classes.

Ease of Instructor Preparation

We've all had to fill in for a colleague, pick up a last-minute section, or find ourselves running across campus to yet a different course. The Miller/O'Neill/Hyde series is carefully designed to support instructors teaching in a variety of different settings and circumstances. Experienced, senior faculty members can draw from a massive library of static and algorithmic content found in ALEKS to meticulously build assignments and assessments sharply tailored to individual student needs. Newer instructors and part-time adjunct instructors, on the other hand, will find support through a wide range of digital resources and prebuilt assignments ready to go on Day One. With these tools, instructors with limited time to prepare for class can still facilitate successful student outcomes.

Many instructors want to incorporate discovery-based learning and groupwork into their courses but don't have time to write or find quality materials. Each section of the text has numerous discovery-based activities that we have tested in our own classrooms. These are found in the text and Student Resource Manual along with other targeted worksheets for additional practice and materials for a student portfolio.

Student Success—Now and in the Future

Too often our math placement tests fail our students, which can lead to frustration, anxiety, and often withdrawal from their education journey. We encourage you to learn more about ALEKS Placement, Preparation, and Learning (ALEKS PPL), which uses adaptive learning technology to place students appropriately. No matter the skills they come in with, the Miller/O'Neill/Hyde series provides resources and support that uniquely position them for success in that course and for their next course. Whether they need a brush-up on their basic skills, ADA supportive materials, or advanced topics to help them cross the bridge to the next level, we've created a support system for them.

We hope you are as excited as we are about the series and the supporting resources and services that accompany it. Please reach out to any of us with any questions or comments you have about our texts.

Julie Miller   Molly O'Neill   Nancy Hyde

About the Authors

Photo courtesy of Molly O'Neill

Julie Miller is from Daytona State College, where she taught developmental and upper-level mathematics courses for 20 years. Prior to her work at Daytona State College, she worked as a software engineer for General Electric in the area of flight and radar simulation. Julie earned a Bachelor of Science in Applied Mathematics from Union College in Schenectady, New York, and a Master of Science in Mathematics from the University of Florida. In addition to this textbook, she has authored textbooks for college algebra, trigonometry, and precalculus, as well as several short works of fiction and nonfiction for young readers.

"My father is a medical researcher, and I got hooked on math and science when I was young and would visit his laboratory. I can remember using graph paper to plot data points for his experiments and doing simple calculations. He would then tell me what the peaks and features in the graph meant in the context of his experiment. I think that applications and hands-on experience made math come alive for me, and I'd like to see math come alive for my students."

—*Julie Miller*

Molly O'Neill is also from Daytona State College, where she taught for 22 years in the School of Mathematics. She has taught a variety of courses from developmental mathematics to calculus. Before she came to Florida, Molly taught as an adjunct instructor at the University of Michigan–Dearborn, Eastern Michigan University, Wayne State University, and Oakland Community College. Molly earned a Bachelor of Science in Mathematics and a Master of Arts and Teaching from Western Michigan University in Kalamazoo, Michigan. Besides this textbook, she has authored several course supplements for college algebra, trigonometry, and precalculus and has reviewed texts for developmental mathematics.

"I differ from many of my colleagues in that math was not always easy for me. But in seventh grade I had a teacher who taught me that if I follow the rules of mathematics, even I could solve math problems. Once I understood this, I enjoyed math to the point of choosing it for my career. I now have the greatest job because I get to do math every day and I have the opportunity to influence my students just as I was influenced. Authoring these texts has given me another avenue to reach even more students."

—*Molly O'Neill*

Nancy Hyde served as a full-time faculty member of the Mathematics Department at Broward College for 24 years. During this time she taught the full spectrum of courses from developmental math through differential equations. She received a Bachelor of Science in Math Education from Florida State University and a Master's degree in Math Education from Florida Atlantic University. She has conducted workshops and seminars for both students and teachers on the use of technology in the classroom. In addition to this textbook, she has authored a graphing calculator supplement for *College Algebra*.

"I grew up in Brevard County, Florida, where my father worked at Cape Canaveral. I was always excited by mathematics and physics in relation to the space program. As I studied higher levels of mathematics I became more intrigued by its abstract nature and infinite possibilities. It is enjoyable and rewarding to convey this perspective to students while helping them to understand mathematics."

—*Nancy Hyde*

Dedication

To Our Students

Julie Miller ⚘ Molly O'Neill ⚘ Nancy Hyde

The Miller/O'Neill/Hyde Developmental Math Series

Julie Miller, Molly O'Neill, and Nancy Hyde originally wrote their developmental math series because students were entering their College Algebra course underprepared. The students were not mathematically mature enough to understand the concepts of math, nor were they fully engaged with the material. The authors began their developmental mathematics offerings with Intermediate Algebra to help bridge that gap. This in turn evolved into several series of textbooks from Prealgebra through Precalculus to help students at all levels before Calculus.

What sets all of the Miller/O'Neill/Hyde series apart is that they address course content through an author-created digital package that maintains a consistent voice and notation throughout the program. This consistency—in videos, PowerPoints, Lecture Notes, and Integrated Video and Study Guides—coupled with the power of ALEKS, ensures that students master the skills necessary to be successful in Developmental Math through Precalculus and prepares them for the Calculus sequence.

Developmental Math Series

The Developmental Math series is traditional in approach, delivering a purposeful balance of skills and conceptual development. It places a strong emphasis on conceptual learning to prepare students for success in subsequent courses.

Basic College Mathematics, Third Edition
Prealgebra, Third Edition
Prealgebra & Introductory Algebra, Second Edition
Beginning Algebra, Sixth Edition
Beginning & Intermediate Algebra, Sixth Edition
Intermediate Algebra, Sixth Edition
Developmental Mathematics: Prealgebra, Beginning Algebra, & Intermediate Algebra, Second Edition

The Miller/Gerken College Algebra/Precalculus Series

The Precalculus series serves as the bridge from Developmental Math coursework to future courses by emphasizing the skills and concepts needed for Calculus.

College Algebra with Corequisite Support, First Edition
College Algebra, Second Edition
College Algebra and Trigonometry, First Edition
Precalculus, First Edition

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To the marketing team, Michele McTighe, Noah Evans, and Mary Ellen Rahn: thank you for your creative ideas in making our books come to life in the market. Thank you as well to Debbie McFarland, Justin Washington, and Sherry Bartel for continuing to drive our long-term content vision through their market development efforts. And many thanks to the team at ALEKS for creating its spectacular adaptive technology and for overseeing the quality control.

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Most importantly, we give special thanks to the students and instructors who use our series in their classes.

Julie Miller
Molly O'Neill
Nancy Hyde

Contents

To the Student

Take a deep breath and know that you aren't alone. Your instructor, fellow students, and we, your authors, are here to help you learn and master the material for this course and prepare you for future courses. You may feel like math just isn't your thing, or maybe it's been a long time since you've had a math class—that's okay!

We wrote the text and all the supporting materials with you in mind. Most of our students aren't really sure how to be successful in math, but we can help with that.

As you begin your class, we'd like to offer some specific suggestions:

1. **Attend class.** Arrive on time and be prepared. If your instructor has asked you to read prior to attending class—do it. How often have you sat in class and thought you understood the material, only to get home and realize you don't know how to get started? By reading and trying a couple of Skill Practice exercises, which follow each example, you will be able to ask questions and gain clarification from your instructor when needed.
2. **Be an *active* learner.** Whether you are at lecture, watching an author lecture or exercise video, or are reading the text, pick up a pencil and work out the examples given. Math is learned only by doing; we like to say, "Math is not a spectator sport." If you like a bit more guidance, we encourage you to use the Integrated Video and Study Guide. It was designed to provide structure and note-taking for lectures and while watching the accompanying videos.
3. **Schedule time to do some math every day.** Exercise, foreign language study, and math are three things that you must do every day to get the results you want. If you are used to cramming and doing all of your work in a few hours on a weekend, you should know that even mathematicians start making silly errors after an hour or so! Check your answers. Skill Practice exercises all have the answers at the bottom of that page. Odd-numbered exercises throughout the text have answers in the back of the text. If you didn't get it right, don't throw in the towel. Try again, revisit an example, or bring your questions to class for extra help.
4. **Prepare for quizzes and exams.** Each chapter has a set of Chapter Review Exercises at the end to help you integrate all of the important concepts. In addition, there is a detailed Chapter Summary and a Chapter Test. If you use ALEKS, use all of the tools available within the program to test your understanding.
5. **Use your resources.** This text comes with numerous supporting resources designed to help you succeed in this class and in your future classes. Additionally, your instructor can direct you to resources within your institution or community. Form a student study group. Teaching others is a great way to strengthen your own understanding, and they might be able to return the favor if you get stuck.

We wish you all the best in this class and in your educational journey!

Julie Miller   Molly O'Neill   Nancy Hyde

Student Guide to the Text

Clear, Precise Writing
Learning from our own students, we have written this text in simple and accessible language. Our goal is to keep you engaged and supported throughout your coursework.

Call-Outs
Just as your instructor will share tips and math advice in class, we provide call-outs throughout the text to offer tips and warn against common mistakes.

- Tip boxes offer additional insight into a concept or procedure.
- Avoiding Mistakes help fend off common student errors.
- For Review boxes positioned strategically throughout the text remind students of key skills relating to the current topic.

Examples
- Each example is step-by-step, with thorough annotation to the right explaining each step.
- Following each example is a similar **Skill Practice** exercise to give you a chance to test your understanding. You will find the answer at the bottom of the page—providing a quick check.

Exercise Sets
Each type of exercise is built so you can successfully learn the materials and show your mastery on exams.
- **Activities for discovery-based learning** appear before the exercise sets to walk students through the concepts presented in each section of the text.
- **Study Skills Exercises** integrate your studies of math concepts with strategies for helping you grow as a student overall.
- **Vocabulary and Key Concept Exercises** check your understanding of the language and ideas presented within the section.
- **Prerequisite Review** exercises keep fresh your knowledge of math content already learned by providing practice with concepts explored in previous sections.
- **Concept Exercises** assess your comprehension of the specific math concepts presented within the section.
- **Mixed Exercises** evaluate your ability to successfully complete exercises that combine multiple concepts presented within the section.
- **Expanding Your Skills** challenge you with advanced skills practice exercises around the concepts presented within the section.
- **Problem Recognition Exercises** appear in strategic locations in each chapter of the text. These will require you to distinguish between similar problem types and to determine what type of problem-solving technique to apply.
- **Technology Exercises** appear where appropriate.

End-of-Chapter Materials
The features at the end of each chapter are perfect for reviewing before test time.
- **Section-by-section summaries** provide references to key concepts, examples, and vocabulary.
- **Chapter Review Exercises** provide additional opportunities to practice material from the entire chapter.
- **Chapter tests** are an excellent way to test your complete understanding of the chapter concepts.

Get Better Results

How Will Miller/O'Neill/Hyde Help Your Students *Get Better Results?*

Clarity, Quality, and Accuracy

Julie Miller, Molly O'Neill, and Nancy Hyde know what students need to be successful in mathematics. Better results come from clarity in their exposition, quality of step-by-step worked examples, and accuracy of their exercise sets; but it takes more than just great authors to build a textbook series to help students achieve success in mathematics. Our authors worked with a strong team of mathematics instructors from around the country to ensure that the clarity, quality, and accuracy you expect from the Miller/O'Neill/Hyde series was included in this edition.

Exercise Sets

Comprehensive sets of exercises are available for every student level. Julie Miller, Molly O'Neill, and Nancy Hyde worked with a board of advisors from across the country to offer the appropriate depth and breadth of exercises for your students. **Problem Recognition Exercises** were created to improve student performance while testing.

Practice exercise sets help students progress from skill development to conceptual understanding. Student tested and instructor approved, the Miller/O'Neill/Hyde exercise sets will help your students *get better results*.

- **Activities for Discovery-Based Learning**
- **Prerequisite Review Exercises**
- **Problem Recognition Exercises**
- **Skill Practice Exercises**
- **Study Skills Exercises**
- **Mixed Exercises**
- **Expanding Your Skills Exercises**
- **Vocabulary and Key Concepts Exercises**
- **Technology Exercises**

Step-By-Step Pedagogy

This text provides enhanced step-by-step learning tools to help students *get better results*.

- **For Review** tips placed in the margin guide students back to related prerequisite skills needed for full understanding of course-level topics.
- **Worked Examples** provide an "easy-to-understand" approach, clearly guiding each student through a step-by-step approach to master each practice exercise for better comprehension.
- **TIPs** offer students extra cautious direction to help improve understanding through hints and further insight.
- **Avoiding Mistakes** boxes alert students to common errors and provide practical ways to avoid them. Both of these learning aids will help students get better results by showing how to work through a problem using a clearly defined step-by-step methodology that has been class tested and student approved.

Get Better Results

Formula for Student Success

Step-by-Step Worked Examples

- Do you get the feeling that there is a disconnect between your students' class work and homework?
- Do your students have trouble finding worked examples that match the practice exercises?
- Do you prefer that your students see examples in the textbook that match the ones you use in class?

Miller/O'Neill/Hyde's *Worked Examples* offer a clear, concise methodology that replicates the mathematical processes used in the authors' classroom lectures.

Example 1 Determining the Order of a Matrix

Determine the order of each matrix.

a. $\begin{bmatrix} 2 & -4 & 1 \\ 5 & \pi & \sqrt{7} \end{bmatrix}$ **b.** $\begin{bmatrix} 1.9 \\ 0 \\ 7.2 \\ -6.1 \end{bmatrix}$ **c.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **d.** $[a \quad b \quad c]$

Solution:

a. This matrix has two rows and three columns. Therefore, it is a 2×3 matrix.

b. This matrix has four rows and one column. Therefore, it is a 4×1 matrix. A matrix with one column is called a **column matrix**.

c. This matrix has three rows and three columns. Therefore, it is a 3×3 matrix. A matrix with the same number of rows and columns is called a **square matrix**.

d. This matrix has one row and three columns. Therefore, it is a 1×3 matrix. A matrix with one row is called a **row matrix**.

Skill Practice Determine the order of the matrix.

1. $\begin{bmatrix} -5 & 2 \\ 1 & 3 \end{bmatrix}$ **2.** $[4 \quad -8]$ **3.** $\begin{bmatrix} 5 \\ 10 \end{bmatrix}$ **4.** $\begin{bmatrix} 2 & -0.5 \\ -1 & 6 \end{bmatrix}$

Classroom Examples

To ensure that the classroom experience also matches the examples in the text and the practice exercises, we have included references to even-numbered exercises to be used as Classroom Examples. These exercises are highlighted in the Practice Exercises at the end of each section.

Example 5 Finding the x- and y-Intercepts of a Line

Given $2x + 4y = 8$, find the x- and y-intercepts. Then graph the equation.

Solution:

To find the x-intercept, substitute $y = 0$.

$$2x + 4y = 8$$
$$2x + 4(0) = 8$$
$$2x = 8$$
$$x = 4$$

The x-intercept is $(4, 0)$.

To find the y-intercept, substitute $x = 0$.

$$2x + 4y = 8$$
$$2(0) + 4y = 8$$
$$4y = 8$$
$$y = 2$$

The y-intercept is $(0, 2)$.

Get Better Results

Quality Learning Tools

For Review Boxes

Throughout the text, just-in-time tips and reminders of prerequisite skills appear in the margin alongside the concepts for which they are needed. References to prior sections are given for cases where more comprehensive review is available earlier in the text.

FOR REVIEW

Recall that the sum of an expression and its opposite is zero. For example:

$4y + (-4y) = 0$

TIP and Avoiding Mistakes Boxes

TIP and **Avoiding Mistakes** boxes have been created based on the authors' classroom experiences—they have also been integrated into the **Worked Examples.** These pedagogical tools will help students get better results by learning how to work through a problem using a clearly defined step-by-step methodology.

Example 7 Simplifying a Radical Expression

Simplify. $\dfrac{7\sqrt{50}}{10}$

Solution:

$\dfrac{7\sqrt{50}}{10} = \dfrac{7\sqrt{25 \cdot 2}}{10}$ 25 is the greatest perfect square in the radicand.

$= \dfrac{7 \cdot 5\sqrt{2}}{10}$ Simplify the radical.

$= \dfrac{7 \cdot \overset{1}{\cancel{5}}\sqrt{2}}{\underset{2}{\cancel{10}}}$ Simplify the fraction to lowest terms.

$= \dfrac{7\sqrt{2}}{2}$

Avoiding Mistakes

The expression $\frac{7\sqrt{2}}{2}$ cannot be simplified further because one factor of 2 is in the radicand and the other is outside the radical.

Skill Practice Simplify.

9. $\dfrac{2\sqrt{300}}{30}$

Avoiding Mistakes Boxes:

Avoiding Mistakes boxes are integrated throughout the textbook to alert students to common errors and how to avoid them.

TIP: When solving a literal equation for a specified variable, there is sometimes more than one way to express your final answer. This flexibility often presents difficulty for students. Students may leave their answer in one form, but the answer given in the text may look different. Yet both forms may be correct. To know if your answer is equivalent to the form given in the text, you must try to manipulate it to look like the answer in the book, a process called *form fitting.*

The literal equation from Example 4 can be written in several different forms. The quantity $(2A - b_2h)/h$ can be split into two fractions.

$$b_1 = \frac{2A - b_2h}{h} = \frac{2A}{h} - \frac{b_2h}{h} = \frac{2A}{h} - b_2$$

TIP Boxes

Teaching tips are usually revealed only in the classroom. Not anymore! TIP boxes offer students helpful hints and extra direction to help improve understanding and provide further insight.

Get Better Results

Better Exercise Sets and Better Practice Yield Better Results

- Do your students have trouble with problem solving?
- Do you want to help students overcome math anxiety?
- Do you want to help your students improve performance on math assessments?

Problem Recognition Exercises

Problem Recognition Exercises present a collection of problems that look similar to a student upon first glance, but are actually quite different in the manner of their individual solutions. Students sharpen critical thinking skills and better develop their "solution recall" to help them distinguish the method needed to solve an exercise—an essential skill in mathematics.

Problem Recognition Exercises were tested in the authors' developmental mathematics classes and were created to improve student performance on tests.

Problem Recognition Exercises

Rational Equations vs. Expressions

1. a. Simplify. $\frac{3}{w-5}+\frac{10}{w^2-25}-\frac{1}{w+5}$

b. Solve. $\frac{3}{w-5}+\frac{10}{w^2-25}-\frac{1}{w+5}=0$

c. Identify each problem in parts (a) and (b) as either an equation or an expression.

2. a. Simplify. $\frac{x}{2x+4}+\frac{2}{3x+6}-1$

b. Solve. $\frac{x}{2x+4}+\frac{2}{3x+6}=1$

c. Identify each problem in parts (a) and (b) as either an equation or an expression.

For Exercises 3–20, first ask yourself whether the problem is an expression to simplify or an equation to solve. Then simplify or solve as indicated.

3. $\frac{2}{a^2+4a+3}+\frac{1}{a+3}$

4. $\frac{1}{c+6}+\frac{4}{c^2+8c+12}$

5. $\frac{7}{y^2-y-2}+\frac{1}{y+1}-\frac{3}{y-2}=0$

6. $\frac{3}{b+2}-\frac{1}{b-1}-\frac{5}{b^2+b-2}=0$

7. $\frac{x}{x-1}-\frac{12}{x^2-x}$

8. $\frac{3}{5t-20}+\frac{4}{t-4}$

Get Better Results

Student-Centered Applications

The Miller/O'Neill/Hyde Board of Advisors partnered with our authors to bring the *best applications* from every region in the country! These applications include real data and topics that are more relevant and interesting to today's student.

92. ^{99m}Tc is a radionuclide of technetium that is widely used in nuclear medicine. Although its half-life is only 6 hr, the isotope is continuously produced via the decay of its longer-lived parent ^{99}Mo (molybdenum-99), whose half-life is approximately 3 days. The ^{99}Mo generators (or "cows") are sold to hospitals in which the ^{99m}Tc can be "milked" as needed over a period of a few weeks. Once separated from its parent, the ^{99m}Tc may be chemically incorporated into a variety of imaging agents, each of which is designed to be taken up by a specific target organ within the body. Special cameras, sensitive to the gamma rays emitted by the technetium, are then used to record a "picture" (similar in appearance to an X-ray film) of the selected organ.

Suppose a technician prepares a sample of ^{99m}Tc-pyrophosphate to image the heart of a patient suspected of having had a mild heart attack. If the injection contains 10 millicuries (mCi) of ^{99m}Tc at 1:00 P.M., then the amount of technetium still present is given by

$$T(t) = 10e^{-0.1155t}$$

where $t > 0$ represents the time in hours after 1:00 P.M. and $T(t)$ represents the amount of ^{99m}Tc (in millicuries) still present.

a. How many millicuries of ^{99m}Tc will remain at 4:20 P.M. when the image is recorded? Round to the nearest tenth of a millicurie.

b. How long will it take for the radioactive level of the ^{99m}Tc to reach 2 mCi? Round to the nearest tenth of an hour.

Activities

Each section of the text ends with an activity that steps the student through the major concepts of the section. The purpose of the activities is to promote active, discovery-based learning for the student. The implementation of the activities is flexible for a variety of delivery methods. For face-to-face classes, the activities can be used to break up lecture by covering the exercises intermittently during the class. For the flipped classroom and hybrid classes, students can watch the videos and try the activities. Then, in the classroom, the instructor can go over the activities or have the students compare their answers in groups. For online classes, the activities provide great discussion questions.

Section 2.6 Activity

A.1. Given a set of ordered pairs, how can you determine whether the relation defines y as a function of x?

For Exercises A.2–A.3, consider the given relation.

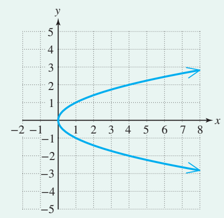
a. Do any two ordered pairs have the same x value but different y values?

b. Is the relation a function?

A.2. $\{(-5, 1), (3, 4), (-2, 6), (-5, 2), (0, -3)\}$

A.3. $\{(-1, 6), (2, 11), (8, 6), (-3, 1), (0.4, -0.5)\}$

A.4. **a.** For the graph given, draw a vertical line through the point (4, 2). Does the vertical line intersect the graph at any other point?



b. Does this graph define y as a function of x?

c. Using this example, explain how the vertical line test is used to determine if a graph defines y as a function of x.

A.5. Consider the equation $y = 2x + 1$.

a. If $x = 3$, what is the corresponding y value?

b. Write the result of part (a) as an ordered pair (x, y).

A.6. Consider the function defined by $f(x) = 2x + 1$.

a. Find $f(3)$. That is, evaluate the function for $x = 3$ by substituting 3 for x.

b. Write the result of part (a) as an ordered pair (x, y).

c. Refer to Exercise A.5 and compare the results.

A.7. Given $g(x) = \dfrac{4}{x-1}$, find the function values if possible.

a. $g(2)$ **b.** $g(-3)$

c. $g(0)$ **d.** $g(1)$

A.8. Refer to $g(x) = \dfrac{4}{x-1}$ from Exercise A.7.

a. What value(s) of x must be excluded from the domain of g? Why?

b. Write the domain of g in interval notation.

A.9. Given $h(x) = \sqrt{x+2}$, find the function values if possible.

a. $h(-1)$ **b.** $h(2)$

c. $h(7)$ **d.** $h(-6)$

Get Better Results

Additional Supplements

Lecture Videos Created by the Authors

Julie Miller began creating these lecture videos for her own students to use when they were absent from class. The student response was overwhelmingly positive, prompting the author team to create the lecture videos for their entire developmental math book series. In these videos, the authors walk students through the learning objectives using the same language and procedures outlined in the book. Students learn and review right alongside the author! Students can also access the written notes that accompany the videos.

Integrated Video and Study Workbooks

The Integrated Video and Study Workbooks were built to be used in conjunction with the Miller/O'Neill/Hyde Developmental Math series online lecture videos. These new video guides allow students to consolidate their notes as they work through the material in the book, and they provide students with an opportunity to focus their studies on particular topics that they are struggling with rather than entire chapters at a time. Each video guide contains written examples to reinforce the content students are watching in the corresponding lecture video, along with additional written exercises for extra practice. There is also space provided for students to take their own notes alongside the guided notes already provided. By the end of the academic term, the video guides will not only be a robust study resource for exams, but will serve as a portfolio showcasing the hard work of students throughout the term.

Dynamic Math Animations

The authors have constructed a series of animations to illustrate difficult concepts where static images and text fall short. The animations leverage the use of on-screen movement and morphing shapes to give students an interactive approach to conceptual learning. Some provide a virtual laboratory for which an application is simulated and where students can collect data points for analysis and modeling. Others provide interactive question-and-answer sessions to test conceptual learning.

Exercise Videos

The authors, along with a team of faculty who have used the Miller/O'Neill/Hyde textbooks for many years, have created exercise videos for designated exercises in the textbook. These videos cover a representative sample of the main objectives in each section of the text. Each presenter works through selected problems, following the solution methodology employed in the text.

The video series is available online as part of ALEKS 360. The videos are closed-captioned for the hearing impaired and meet the Americans with Disabilities Act Standards for Accessible Design.

Student Resource Manual

The *Student Resource Manual (SRM),* created by the authors, is a printable, electronic supplement available to students through ALEKS. Instructors can also choose to customize this manual and package it with their course materials. With increasing demands on faculty schedules, this resource offers a convenient means for both full-time and adjunct faculty to promote active learning and success strategies in the classroom.

This manual supports the series in a variety of different ways:

- Additional group activities developed by the authors to supplement what is already available in the text
- Discovery-based classroom activities written by the authors for each section
- Excel activities that not only provide students with numerical insights into algebraic concepts, but also teach simple computer skills to manipulate data in a spreadsheet

Get Better Results

- Worksheets for extra practice written by the authors, including Problem Recognition Exercise Worksheets
- Lecture Notes designed to help students organize and take notes on key concepts
- Materials for a student portfolio

Annotated Instructor's Edition

In the *Annotated Instructor's Edition* (*AIE*), answers to all exercises appear adjacent to each exercise in a color used *only* for annotations. The *AIE* also contains Instructor Notes that appear in the margin. These notes offer instructors assistance with lecture preparation. In addition, there are Classroom Examples referenced in the text that are highlighted in the Practice Exercises. Also found in the *AIE* are icons within the Practice Exercises that serve to guide instructors in their preparation of homework assignments and lessons.

PowerPoints

The PowerPoints present key concepts and definitions with fully editable slides that follow the textbook. An instructor may project the slides in class or post to a website in an online course.

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Among the supplements is a computerized test bank using the algorithm-based testing software TestGen® to create customized exams quickly. Hundreds of text-specific, open-ended, and multiple-choice questions are included in the question bank.

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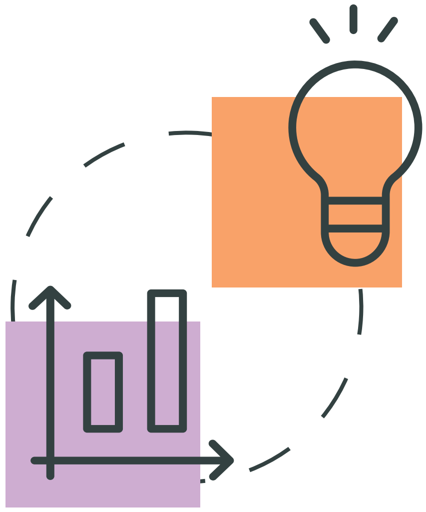

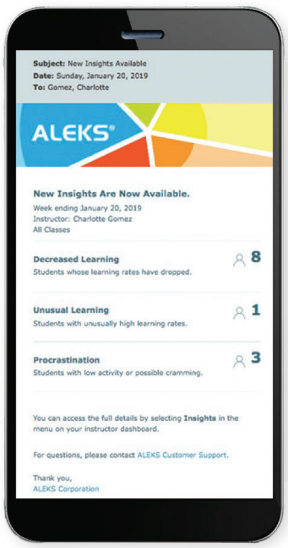

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Review of Basic Algebraic Concepts

R

CHAPTER OUTLINE

Mathematics and Consistency

Many of the activities we perform every day follow a natural order. For example, we would not put on our shoes before putting on our socks, nor would a doctor begin surgery before giving an anesthetic.

In mathematics, it is also necessary to follow a prescribed order of operations to simplify an algebraic expression. This is important, for example, because we would not want two different engineers working on a space probe to Mars to interpret a mathematical statement differently.

Suppose that the high temperature for a summer day near the equator of Mars is 20°C. To convert this to degrees Fahrenheit *F*, we would substitute 20 for *C* in the equation.

$$F = \frac{9}{5}C + 32 \xrightarrow{\text{Substitute 20 for } C} F = \frac{9}{5}(20) + 32$$

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In this expression, the operation between $\frac{9}{5}$ and 20 is implied multiplication, and it is universally understood that multiplication is performed before addition. Thus,

$$F = \frac{9}{5}(20) + 32 = 36 + 32 = 68. \qquad \text{The temperature in Fahrenheit is 68°F.}$$

If an engineer had erroneously added 20 and 32 first and then multiplied by $\frac{9}{5}$, a different temperature of 93.6°F would result. This illustrates the importance of a prescribed order for mathematical operations.

Section R.1 Sets of Numbers and Interval Notation

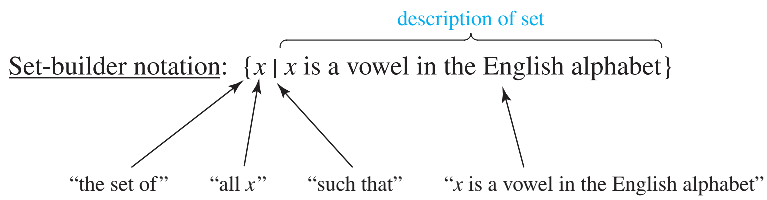
Concepts

1. The Set of Real Numbers
2. Inequalities
3. Interval Notation
4. Translations Involving Inequalities

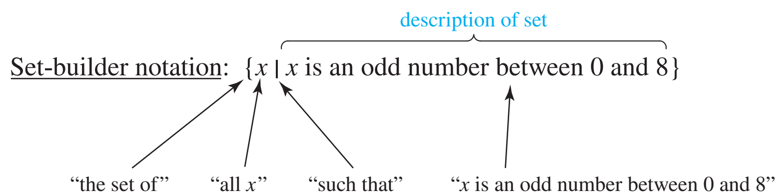
1. The Set of Real Numbers

Algebra is a powerful mathematical tool that is used to solve real-world problems in science, business, and many other fields. We begin our study of algebra with a review of basic definitions and notations used to express algebraic relationships.

In mathematics, a collection of items (called elements) is called a **set**, and the set braces { } are used to enclose the elements of the set. For example, the set {a, e, i, o, u} represents the vowels in the English alphabet. The set {1, 3, 5, 7} represents the first four positive odd numbers. Another method to express a set is to *describe* the elements of the set by using **set-builder notation**. Consider the set {a, e, i, o, u} in set-builder notation.



Consider the set {1, 3, 5, 7} in set-builder notation.



Several sets of numbers are used extensively in algebra. The numbers you are familiar with in day-to-day calculations are elements of the set of **real numbers**. These numbers can be represented graphically on a horizontal number line with a point labeled as 0. Positive real numbers are graphed to the right of 0, and negative real numbers are graphed to the left. Each point on the number line corresponds to exactly one real number, and for this reason, the line is called the **real number line** (Figure R-1).

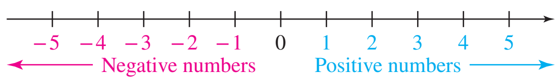


Figure R-1

Several sets of numbers are **subsets** (or part) of the set of real numbers. These are

The set of natural numbers

The set of whole numbers

The set of integers

The set of rational numbers

The set of irrational numbers

Natural Numbers, Whole Numbers, and Integers

The set of **natural numbers** is {1, 2, 3, . . .}.
The set of **whole numbers** is {0, 1, 2, 3, . . .}.
The set of **integers** is {. . . , −3, −2, −1, 0, 1, 2, 3, . . .}.

The set of rational numbers consists of all the numbers that can be defined as a ratio of two integers.

Rational Numbers

The set of **rational numbers** is $\left\{\frac{p}{q} \middle| p \text{ and } q \text{ are integers and } q \text{ does not equal zero}\right\}$.

Example 1 Identifying Rational Numbers

Show that each number is a rational number by finding two integers whose ratio equals the given number.

a. $\frac{-4}{7}$ **b.** 8 **c.** $0.\overline{6}$ **d.** 0.87

Solution:

a. $\frac{-4}{7}$ is a rational number because it can be expressed as the ratio of the integers −4 and 7.

b. 8 is a rational number because it can be expressed as the ratio of the integers 8 and 1 $\left(8 = \frac{8}{1}\right)$. In this example we see that *an integer is also a rational number*.

c. $0.\overline{6}$ represents the repeating decimal 0.6666666 . . . and can be expressed as the ratio of 2 and 3 $\left(0.\overline{6} = \frac{2}{3}\right)$. In this example we see that *a repeating decimal is a rational number*.

d. 0.87 is the ratio of 87 and 100 $\left(0.87 = \frac{87}{100}\right)$. In this example we see that *a terminating decimal is a rational number*.

Skill Practice Show that the numbers are rational by writing them as a ratio of integers.

1. $\frac{-9}{8}$ **2.** 0 **3.** $0.\overline{3}$ **4.** 0.45

TIP: Any rational number can be represented by a terminating decimal or by a repeating decimal.

Some real numbers such as the number π (pi) cannot be represented by the ratio of two integers. In decimal form, an irrational number is a nonterminating, nonrepeating decimal. The value of π, for example, can be approximated as $\pi \approx 3.1415926535897932$. However, the decimal digits continue indefinitely with no pattern. Other examples of irrational numbers are the square roots of nonperfect squares, such as $\sqrt{3}$ and $\sqrt{10}$.

Irrational Numbers

The set of **irrational numbers** is a subset of the real numbers whose elements cannot be written as a ratio of two integers.

Note: An irrational number cannot be written as a terminating decimal or as a repeating decimal.

The set of real numbers consists of both the rational numbers and the irrational numbers. The relationships among the sets of numbers discussed thus far are illustrated in Figure R-2.

Answers

1. $\frac{-9}{8}$ **2.** $\frac{0}{1}$
3. $\frac{1}{3}$ **4.** $\frac{45}{100}$ or $\frac{9}{20}$

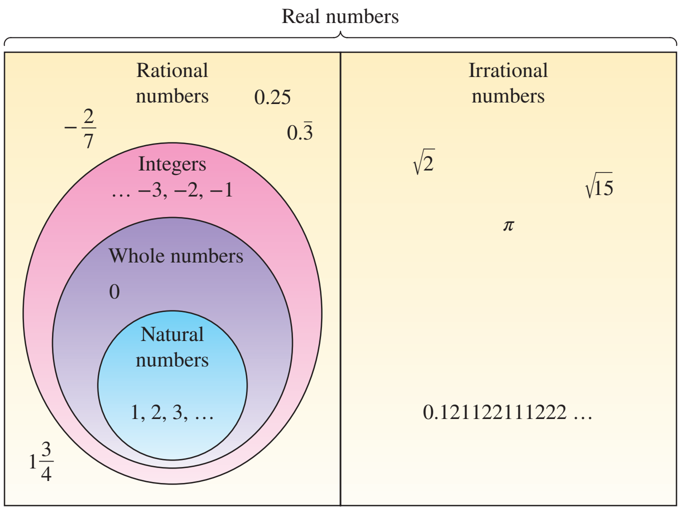


Figure R-2

Example 2 Classifying Numbers by Set

Check the set(s) to which each number belongs. The numbers may belong to more than one set.

	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
-6						
$\sqrt{23}$						
$-\frac{2}{7}$						
3						
$2.\overline{3}$						

Solution:

	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
-6			✓	✓		✓
$\sqrt{23}$					✓	✓
$-\frac{2}{7}$				✓		✓
3	✓	✓	✓	✓		✓
$2.\overline{3}$				✓		✓

Skill Practice

5. Check the set(s) to which each number belongs.

	1	0.47	$\sqrt{5}$	$-\frac{1}{2}$
Natural				
Whole				
Integer				
Rational				
Irrational				
Real				

Answer

5.

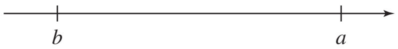
	1	0.47	$\sqrt{5}$	$-\frac{1}{2}$
Natural	✓			
Whole	✓			
Integer	✓			
Rational	✓	✓		✓
Irrational			✓	
Real	✓	✓	✓	✓

2. Inequalities

The relative value of two numbers can be compared by using the real number line. We say that a is less than b (written mathematically as $a < b$) if a lies to the left of b on the number line.

$a < b$

We say that a is greater than b (written mathematically as $a > b$) if a lies to the right of b on the number line.



$a > b$

From looking at the number line, note that $a > b$ is the same as $b < a$. Table R-1 summarizes the relational operators that compare two real numbers a and b.

Table R-1

Mathematical Expression	Translation	Other Meanings
$a < b$	a is less than b	b exceeds a b is greater than a
$a > b$	a is greater than b	a exceeds b b is less than a
$a \leq b$	a is less than or equal to b	a is at most b a is no more than b
$a \geq b$	a is greater than or equal to b	a is no less than b a is at least b
$a = b$	a is equal to b	
$a \neq b$	a is not equal to b	
$a \approx b$	a is approximately equal to b	

The symbols $<$, $>$, $\leq$, $\geq$, and $\neq$ are called inequality signs, and the expressions $a < b$, $a > b$, $a \leq b$, $a \geq b$, and $a \neq b$ are called **inequalities**.

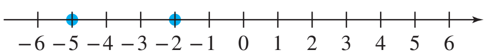
Example 3 Ordering Real Numbers

Fill in the blank with the appropriate inequality sign: $<$ or $>$

a. -2 ______ -5 **b.** $\frac{4}{7}$ ______ $\frac{3}{5}$ **c.** -1.3 ______ $-1.\overline{3}$

Solution:

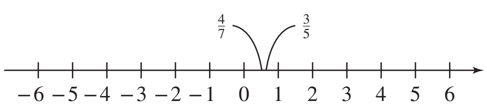
a. -2 __>__ -5



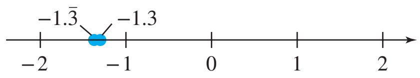
b. To compare $\frac{4}{7}$ and $\frac{3}{5}$, write the fractions as equivalent fractions with a common denominator.

$$\frac{4}{7} \cdot \frac{5}{5} = \frac{20}{35} \quad \text{and} \quad \frac{3}{5} \cdot \frac{7}{7} = \frac{21}{35}$$

Because $\frac{20}{35} < \frac{21}{35}$, then $\frac{4}{7}$ __<__ $\frac{3}{5}$



c. -1.3 __$\geq$__ $-1.33333\ldots$

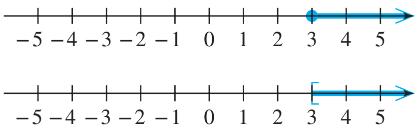


Skill Practice Fill in the blanks with the appropriate sign, < or >.

6. 2 ______ -12 **7.** $\frac{1}{4}$ ______ $\frac{2}{9}$ **8.** $-7.\overline{2}$ ______ -7.2

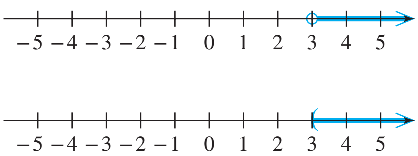
3. Interval Notation

The set $\{x \mid x \geq 3\}$ represents all real numbers greater than or equal to 3. This set can be illustrated graphically on the number line.



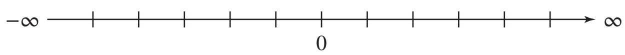
By convention, a closed circle ● or a square bracket [is used to indicate that an "endpoint" $(x = 3)$ *is included* in the set.

The set $\{x \mid x > 3\}$ represents all real numbers strictly greater than 3. This set can be illustrated graphically on the number line.



By convention, an open circle ○ or a parenthesis (is used to indicate that an "endpoint" $(x = 3)$ is *not* included in the set.

Notice that the sets $\{x \mid x \geq 3\}$ and $\{x \mid x > 3\}$ consist of an infinite number of elements that cannot all be listed. Another method to represent the elements of such sets is by using **interval notation**. To understand interval notation, first consider the real number line, which extends infinitely far to the left and right. The symbol ∞ is used to represent infinity. The symbol $-\infty$ is used to represent negative infinity.



To express a set of real numbers in interval notation, sketch the graph first, using the symbols ( ) or [ ]. Then use these symbols at the endpoints to define the interval.

Example 4 Expressing Sets by Using Interval Notation

Graph each set on the number line, and express the set in interval notation.

a. $\{x \mid x \geq -2\}$ **b.** $\{p \mid p > -2\}$

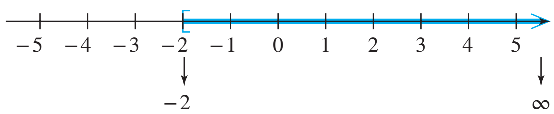
FOR REVIEW

Recall that an inequality may be written with the variable on either side of the inequality sign. The statement $x \geq -2$ is equivalent to $-2 \leq x$.

Solution:

a. Set-builder notation: $\{x \mid x \geq -2\}$

Graph:



Interval notation: $[-2, \infty)$

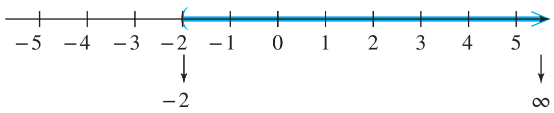
Answers

6. > **7.** > **8.** <

The graph of the set $\{x \mid x \geq -2\}$ "begins" at -2 and extends infinitely far to the right. The corresponding interval notation "begins" at -2 and extends to ∞. Notice that a square bracket [is used at -2 for both the graph and the interval notation to include $x = -2$. *A parenthesis is always used at* ∞ *and at* $-\infty$ *because there is no endpoint.*

b. Set-builder notation: $\{p \mid p > -2\}$

Graph:



Interval notation: $(-2, \infty)$

Skill Practice Graph each set, and express the set in interval notation.

9. $\{w \mid w \geq -7\}$ **10.** $\{x \mid x < 0\}$

In general, we use the following guidelines when applying interval notation.

Using Interval Notation

- The endpoints used in interval notation are always written from left to right. That is, the smaller number is written first, followed by a comma, followed by the larger number.
- Parentheses) or (indicate that an endpoint is *excluded* from the set.
- Square brackets] or [indicate that an endpoint is *included* in the set.
- Parentheses are always used with ∞ or $-\infty$.

Example 5 Expressing Sets by Using Interval Notation

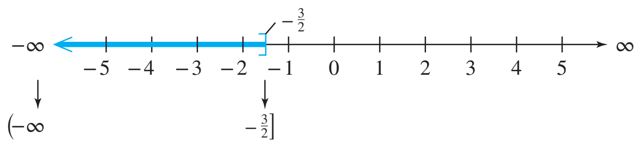
Graph each set on the number line, and express the set in interval notation.

a. $\{z \mid z \leq -\frac{3}{2}\}$ **b.** $\{x \mid -4 < x \leq 2\}$

Solution:

a. Set-builder notation: $\{z \mid z \leq -\frac{3}{2}\}$

Graph:



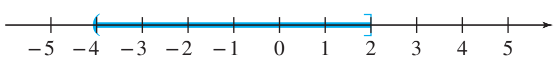
Interval notation: $\left(-\infty, -\frac{3}{2}\right]$

The graph of the set $\{z \mid z \leq -\frac{3}{2}\}$ extends infinitely far to the left. Interval notation is always written from left to right. Therefore, $-\infty$ is written first, followed by a comma, and then followed by the right-hand endpoint $-\frac{3}{2}$.

b. The inequality $-4 < x \leq 2$ means that x is greater than -4 and also less than or equal to 2. More concisely, we can say that x represents the real numbers *between* -4 and 2, including the endpoint, 2.

Set-builder notation: $\{x \mid -4 < x \leq 2\}$

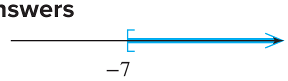
Graph:



Interval notation: $(-4, 2]$

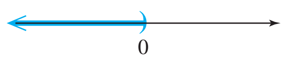
Answers

9.



$[-7, \infty)$

10.



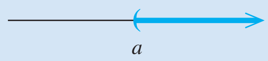
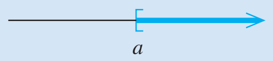
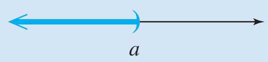
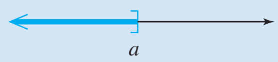
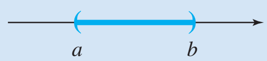
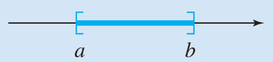
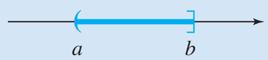
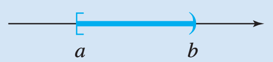
$(-\infty, 0)$

Skill Practice Graph the set on the number line, and express the set in interval notation.

11. $\{w \mid w \geq -\frac{5}{3}\}$ **12.** $\{y \mid -7 \leq y < 4\}$

Table R-2 summarizes interval notation.

Table R-2

Interval Notation	Graph	Interval Notation	Graph
(a, ∞)		$[a, \infty)$	
$(-\infty, a)$		$(-\infty, a]$	
(a, b)		$[a, b]$	
$(a, b]$		$[a, b)$	

4. Translations Involving Inequalities

In Table R-1, we learned that phrases such as *at least*, *at most*, *no more than*, *no less than*, and *between* can be translated into mathematical terms by using inequality signs.

Example 6 Translating Inequalities

The intensity of a hurricane is often defined according to its maximum sustained winds, for which wind speed is measured to the nearest mile per hour. Translate the italicized phrases into mathematical inequalities.

a. A tropical storm is updated to hurricane status if the sustained wind speed, w, *is at least 74 mph*.

b. Hurricanes are categorized according to intensity by the Saffir-Simpson scale. On a scale of 1 to 5, a category 5 hurricane is the most destructive. A category 5 hurricane has sustained winds, w, *exceeding 155 mph*.

c. A category 4 hurricane has sustained winds, w, *of at least 131 mph but no more than 155 mph*.

Solution:

a. $w \geq 74$ mph **b.** $w > 155$ mph **c.** $131 \text{ mph} \leq w \leq 155$ mph

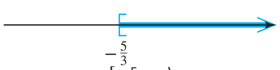
Skill Practice Translate the italicized phrase to a mathematical inequality.

13. The gas mileage, m, for an economy car is *at least 30 mpg*.

14. The gas mileage, m, for a motorcycle is *more than 45 mpg*.

15. The gas mileage, m, for an SUV is *at least 10 mpg, but no more than 20 mpg*.

Answers

11. 

$[-\frac{5}{3}, \infty)$

12. 

$[-7, 4)$

13. $m \geq 30$

14. $m > 45$

15. $10 \leq m \leq 20$

Section R.1 Activity

For Exercises A.1–A.6, refer to set A. $A = \left\{-5, -2.\overline{5}, \sqrt{7}, 0, \frac{7}{4}, 4, -2.5\right\}$

A.1. Which elements from A are natural numbers?

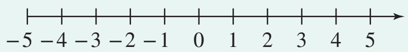
A.2. Which elements from A are whole numbers?

A.3. Which elements from A are integers?

A.4. Which elements from A are rational numbers?

A.5. Which elements from A are irrational numbers?

A.6. Plot the elements from A on the number line.



Inequality statements surround us in day-to-day life. In Exercises A.7–A.9, let x represent the unknown quantity, and write a mathematical inequality to represent the given statement.

A.7. A child must be at least 44 inches tall to ride Space Mountain.

A.8. According to the posted speed limit on a country road, a driver traveling at most 35 mph will not get a speeding ticket.

A.9. To preregister to vote in the United States, a person must be at least 16, but less than 18 years old.

A.10. **a.** Graph the inequality $x < 1$.

b. Graph the inequality $x \leq 1$.

c. Explain when to use parentheses, (or), versus brackets [or], when graphing an inequality.

d. Write the inequality $x < 1$ in interval notation.

e. Write the inequality $x \leq 1$ in interval notation.

f. Explain when to use parentheses, (or), versus brackets [or], when using interval notation.

For Exercises A.11–A.12, write the set in interval notation.

A.11. $\{x \mid -4 < x \leq -1\}$

A.12. $\left\{x \,\middle|\, \frac{3}{2} \leq x\right\}$

Practice Exercises — Section R.1

Study Skills Exercises

Mindset plays an important role in your approach to learning mathematics. Mindset consists of our thoughts, beliefs, and attitudes about our abilities based on lifetime experiences. There are two types of mindsets: fixed mindsets and growth mindsets. People with a fixed mindset believe that they are born with a certain amount of intelligence that cannot be changed despite their actions. On the other hand, a person with a growth mindset believes that intelligence is dynamic and can be increased with effort and learning. What type of mindset do you have? Think about the following questions:

- Have you said to yourself, "I'm just not good at math"?
- Do you believe you lack the necessary skills to understand math?
- Can you recall an experience that has positively impacted your self-confidence in mathematics?

Prerequisite Review

For Exercises R.1–R.4, fill in the blank with <, >, or =.

R.1. $-3.85 \square -3.84$ **R.2.** $-59.7 \square -59.8$ **R.3.** $\frac{11}{7} \square \frac{8}{5}$ **R.4.** $\frac{7}{11} \square \frac{5}{8}$

R.5. **a.** Write the first six digits to the right of the decimal point for the repeating decimal $0.8\overline{35}$.
b. Round $0.8\overline{35}$ to the thousandths place.
c. Round $0.8\overline{35}$ to the hundredths place.

R.6. **a.** Write the first six digits to the right of the decimal point for the repeating decimal $0.\overline{265}$.
b. Round $0.\overline{265}$ to the tenths place.
c. Round $0.\overline{265}$ to the ten-thousandths place.

R.7. Order the numbers from least to greatest.
a. $2.45, 2.\overline{45}, 2.4\overline{5}, 2.44999$
b. $-2.45, -2.\overline{45}, -2.4\overline{5}, -2.44999$

R.8. Order the numbers from least to greatest.
a. $3.1735, 3.173499, 3.1\overline{7}, 3.2$
b. $-3.1735, -3.173499, -3.1\overline{7}, -3.2$

Vocabulary and Key Concepts

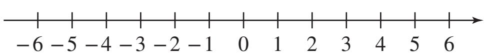
1. **a.** In mathematics, a well-defined collection of elements is called a ____________.
b. The statements $a < b$, $a > b$, and $a \neq b$ are examples of ____________.
c. The statement $a < b$ is read as "____________."
d. The statement $c \geq d$ is read as "____________________."
e. The statement $5 \neq 6$ is read as "____________."
f. The symbol ∞ represents ____________ and $-\infty$ represents ____________.
g. The set of real numbers greater than 5 can be written in set-builder notation as ____________ and in ____________ notation as $(5, \infty)$.
h. The interval $(-2, 5]$ (includes/excludes) the value -2 and (includes/excludes) the value 5.
i. When expressing interval notation, use a (parenthesis/bracket) with infinity.

Concept 1: The Set of Real Numbers

2. Determine the two consecutive integers between which the given number is located on the number line.
a. $\frac{19}{4}$ **b.** $-\frac{2}{3}$ **c.** -4.6 **d.** π

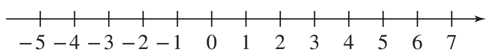
3. Plot the numbers on the number line.

$$\{1.7, \pi, -5, 4.\overline{2}\}$$



4. Plot the numbers on the number line.

$$\left\{1\frac{1}{2}, 0, -3, -\frac{1}{2}, \frac{3}{4}\right\}$$



For Exercises 5–10, show that each number is a rational number by finding a ratio of two integers equal to the given number. **(See Example 1.)**

5. -10 **6.** $7\frac{3}{4}$ **7.** $-\frac{3}{5}$

8. -0.1 **9.** 0 **10.** 0.35

11. Check the sets to which each number belongs. **(See Example 2.)**

	Real Numbers	Irrational Numbers	Rational Numbers	Integers	Whole Numbers	Natural Numbers
5						
$-\sqrt{9}$						
-1.7						
$\frac{1}{2}$						
$\sqrt{7}$						
$\frac{0}{4}$						
$0.\overline{2}$						

12. Check the sets to which each number belongs.

	Real Numbers	Irrational Numbers	Rational Numbers	Integers	Whole Numbers	Natural Numbers
$\frac{6}{8}$						
$1\frac{1}{2}$						
π						
0						
$-0.\overline{8}$						
$\frac{8}{2}$						
$4.\overline{2}$						

Concept 2: Inequalities

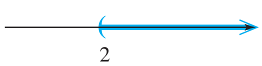
For Exercises 13–20, fill in the blanks with the appropriate symbol: $<$ or $>$. **(See Example 3.)**

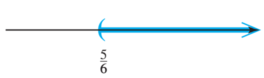
13. -9 ___ -1 **14.** 0 ___ -6 **15.** $0.1\overline{5}$ ___ 0.15 **16.** $-2.\overline{5}$ ___ -2.5

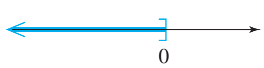
17. $\frac{5}{3}$ ___ $\frac{10}{7}$ **18.** $-\frac{21}{5}$ ___ $-\frac{17}{4}$ **19.** $-\frac{5}{8}$ ___ $-\frac{1}{8}$ **20.** $-\frac{13}{15}$ ___ $-\frac{17}{12}$

Concept 3: Interval Notation

For Exercises 21–28, express the set in interval notation.

21.


22.


23.


24.


25.


26.


27.


28.


For Exercises 29–46, graph the sets and express each set in interval notation. **(See Examples 4–5.)**

29. $\{x \mid x > -1\}$

30. $\{x \mid x < 3\}$

31. $\{y \mid -2 \geq y\}$

32. $\{z \mid -4 \leq z\}$

33. $\{w \mid w < \frac{9}{2}\}$

34. $\{p \mid p \geq -\frac{7}{3}\}$

35. $\{x \mid -2.5 < x \leq 4.5\}$

36. $\{x \mid -6 \leq x < 0\}$

37. All real numbers less than −3.

38. All real numbers greater than 2.34.

39. All real numbers greater than $\frac{5}{2}$.

40. All real numbers less than $\frac{4}{7}$.

41. All real numbers not less than 2.

42. All real numbers no more than 5.

43. All real numbers between −4 and 4.

44. All real numbers between −7 and −1.

45. All real numbers between −3 and 0, inclusive.

46. All real numbers between −1 and 6, inclusive.

For Exercises 47–54, write an expression in words that describes the set of numbers given by each interval. (Answers may vary.)

47. $(-\infty, -4)$

48. $[2, \infty)$

49. $(-2, 7]$

50. $(-3.9, 0)$

51. $[-180, 90]$

52. $(3.2, \infty)$

53. $(-\infty, \infty)$

54. $(-\infty, -1]$

Concept 4: Translations Involving Inequalities

For Exercises 55–64, write the expressions as an inequality. **(See Example 6.)**

55. The age, *a*, to get in to see a certain movie is at least 18 years old.

56. Winston is a cat that was picked up at the Humane Society. His age, *a*, at the time was no more than 2 years.

57. The cost, *c*, to have dinner at Jack's Café is at most \$25.

58. The number of hours, *h*, that Katlyn spent studying was no less than 40.

59. The wind speed, *s*, for an F-5 tornado is no less than 261 mph.

60. The high temperature, *t*, for a certain December day in Albany is at most 26°F.

61. After a summer drought, the total rainfall, *r*, for June, July, and August was no more than 4.5 in.

62. Jessica works for a networking firm. Her salary, *s*, is at least \$85,000 per year.

63. To play in a certain division of a tennis tournament, a player's age, *a*, must be at least 18 years but not more than 25 years.

64. The average age, *a*, of students at Central Community College is estimated to be between 25 years and 29 years.

The following chart defines the ranges for normal blood pressure, high normal blood pressure, and high blood pressure (*hypertension*). All values are measured in millimeters of mercury (mm Hg). (Source: American Heart Association.)

Normal	Systolic less than 130	Diastolic less than 85
High normal	Systolic 130–139, inclusive	Diastolic 85–89, inclusive
Hypertension	Systolic 140 or greater	Diastolic 90 or greater

For Exercises 65–68, write an inequality using the variable p that represents each condition.

65. Normal systolic blood pressure

66. Diastolic pressure in hypertension

67. High normal range for systolic pressure

68. Systolic pressure in hypertension

A pH scale determines whether a solution is acidic or alkaline. The pH scale runs from 0 to 14, with 0 being the most acidic and 14 being the most alkaline. A pH of 7 is neutral (distilled water has a pH of 7).

For Exercises 69–72, write the pH ranges as inequalities and label the substances as acidic or alkaline.

69. Lemon juice: 2.2 through 2.4, inclusive

70. Eggs: 7.6 through 8.0, inclusive

71. Carbonated soft drinks: 3.0 through 3.5, inclusive

72. Milk: 6.6 through 6.9, inclusive

Operations on Real Numbers

Section R.2

Concepts

1. Opposite and Absolute Value
2. Addition and Subtraction of Real Numbers
3. Multiplication and Division of Real Numbers
4. Exponential Expressions
5. Square Roots
6. Order of Operations
7. Evaluating Formulas

1. Opposite and Absolute Value

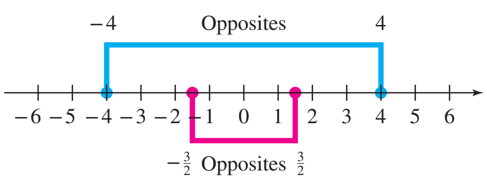
Several key definitions are associated with the set of real numbers and constitute the foundation of algebra. Two important definitions are the opposite of a real number and the absolute value of a real number.

Opposite of a Real Number

Two numbers that are the same distance from 0 but on opposite sides of 0 on the number line are called **opposites** of each other.

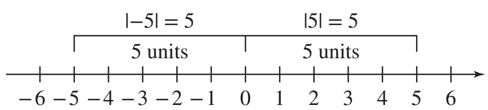
Symbolically, we denote the opposite of a real number a as $-a$.

The numbers -4 and 4 are opposites of each other. Similarly, the numbers $\frac{3}{2}$ and $-\frac{3}{2}$ are opposites.



The **absolute value** of a real number a, denoted $|a|$, is the distance between a and 0 on the number line. *Note:* The absolute value of any real number is *nonnegative*.

For example: $|5| = 5$
and
$|-5| = 5$



Example 1 **Evaluating Absolute Value Expressions**

Simplify the expressions.

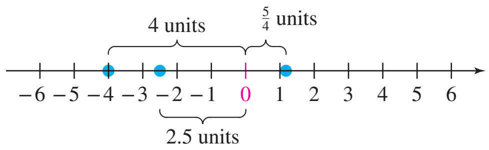
a. $|-2.5|$ **b.** $\left|\frac{5}{4}\right|$ **c.** $-|-4|$

Solution:

a. $|-2.5| = 2.5$

b. $\left|\frac{5}{4}\right| = \frac{5}{4}$

c. $-|-4| = -(4) = -4$



Skill Practice Simplify.

1. $|-9.2|$ **2.** $\left|\frac{7}{6}\right|$ **3.** $-|-2|$

The absolute value of a number a is its distance from zero on the number line. The definition of $|a|$ may also be given algebraically depending on whether a is negative or nonnegative.

FOR REVIEW

Recall that two numbers that are added are called **addends**. The result of the addition is called the **sum**.

Absolute Value of a Real Number

Let a be a real number. Then

1. If a is nonnegative (that is, if $a \geq 0$), then $|a| = a$.
2. If a is negative (that is, if $a < 0$), then $|a| = -a$

This definition states that if a is positive or zero, then $|a|$ equals a itself. If a is a negative number, then $|a|$ equals the opposite of a. For example,

$|9| = 9$ Because 9 is positive, $|9|$ equals the number 9 itself.

$|-7| = 7$ Because -7 is negative, $|-7|$ equals the opposite of -7, which is 7.

2. Addition and Subtraction of Real Numbers

Addition of Real Numbers

- To add two numbers with the *same sign*, add their absolute values and apply the common sign to the sum.
- To add two numbers with *different signs*, subtract the smaller absolute value from the larger absolute value. Then apply the sign of the number having the larger absolute value.

Answers

1. 9.2 **2.** $\frac{7}{6}$ **3.** -2

Example 2 Adding Real Numbers

Perform the indicated operations.

a. $-2 + (-6)$ **b.** $-10.3 + 13.8$ **c.** $\frac{5}{6} + \left(-1\frac{1}{4}\right)$

Solution:

a. $-2 + (-6)$ First find the absolute value of the addends.
$|-2| = 2$ and $|-6| = 6$

$= -(2 + 6)$ Add their absolute values and apply the common sign. In this case, the common sign is negative.

Common sign is negative.

$= -8$ The sum is -8.

b. $-10.3 + 13.8$ First find the absolute value of the addends.
$|-10.3| = 10.3$ and $|13.8| = 13.8$

The absolute value of 13.8 is greater than the absolute value of -10.3. Therefore, the sum is positive.

$= +(13.8 - 10.3)$ Subtract the smaller absolute value from the larger absolute value.

Apply the sign of the number with the larger absolute value.

$= 3.5$

c. $\frac{5}{6} + \left(-1\frac{1}{4}\right)$

$= \frac{5}{6} + \left(-\frac{5}{4}\right)$ Write $-1\frac{1}{4}$ as a fraction.

$= \frac{5 \cdot 2}{6 \cdot 2} + \left(-\frac{5 \cdot 3}{4 \cdot 3}\right)$ The least common denominator (LCD) is 12. Write each fraction with the LCD.

$= \frac{10}{12} + \left(-\frac{15}{12}\right)$ Find the absolute value of the addends.
$\left|\frac{10}{12}\right| = \frac{10}{12}$ and $\left|-\frac{15}{12}\right| = \frac{15}{12}$

The absolute value of $-\frac{15}{12}$ is greater than the absolute value of $\frac{10}{12}$. Therefore, the sum is negative.

$= -\left(\frac{15}{12} - \frac{10}{12}\right)$ Subtract the smaller absolute value from the larger absolute value.

Apply the sign of the number with the larger absolute value.

$= -\frac{5}{12}$

Skill Practice Perform the indicated operations.

4. $-4 + (-1)$ **5.** $-2.6 + 1.8$ **6.** $-1 + \left(-\frac{3}{7}\right)$

FOR REVIEW

The LCD of two fractions is the product of unique prime factors from each denominator. Each factor is raised to the highest power to which it occurs. From Example 2(c),

$6 = 2^1 \cdot 3^1$
$4 = 2^2$

The LCD of $\frac{5}{6}$ and $-\frac{5}{4}$ is $2^2 \cdot 3^1 = 12$.

Answers

4. -5 **5.** -0.8 **6.** $-\frac{10}{7}$

Subtraction of real numbers is defined in terms of the addition process. To subtract two real numbers, add the opposite of the second number to the first number.

Subtraction of Real Numbers

If a and b are real numbers, then $a - b = a + (-b)$

Example 3 Subtracting Real Numbers

Perform the indicated operations.

a. $-13 - 5$ **b.** $2.7 - (-3.8)$ **c.** $\frac{5}{2} - 4\frac{2}{3}$

Solution:

a. $-13 - 5$

$= -13 + (-5)$ Add the opposite of the second number to the first number.

$= -18$ Add.

b. $2.7 - (-3.8)$

$= 2.7 + (3.8)$ Add the opposite of the second number to the first number.

$= 6.5$ Add.

c. $\frac{5}{2} - 4\frac{2}{3}$

$= \frac{5}{2} + \left(-4\frac{2}{3}\right)$ Add the opposite of the second number to the first number.

$= \frac{5}{2} + \left(-\frac{14}{3}\right)$ Write the mixed number as a fraction.

$= \frac{5 \cdot 3}{2 \cdot 3} + \left(-\frac{14 \cdot 2}{3 \cdot 2}\right)$ The least common denominator is 6.

$= \frac{15}{6} + \left(-\frac{28}{6}\right)$ Get a common denominator and add.

$= -\frac{13}{6}$ or $-2\frac{1}{6}$

FOR REVIEW

Write a mixed number as an improper fraction as follows. Multiply the whole number by the denominator of the fraction. Then add the numerator. Write the result over the denominator.

$$4\frac{2}{3} = \frac{4(3) + 2}{3} = \frac{12 + 2}{3} = \frac{14}{3}$$

Skill Practice Subtract.

7. $-9 - 8$ **8.** $1.1 - (-4.2)$ **9.** $\frac{1}{6} - 2\frac{1}{4}$

Answers

7. -17 **8.** 5.3 **9.** $-2\frac{1}{12}$ or $-\frac{25}{12}$

3. Multiplication and Division of Real Numbers

The sign of the product of two real numbers is determined by the signs of the factors.

Multiplication of Real Numbers

1. The product of two real numbers with the *same* sign is *positive*.
2. The product of two real numbers with *different* signs is *negative*.
3. The product of any real number and zero is *zero*.

Example 4 Multiplying Real Numbers

Multiply the real numbers.

a. $(2)(-5.1)$ **b.** $-\frac{2}{3} \cdot \frac{9}{8}$ **c.** $\left(-3\frac{1}{3}\right)\left(-\frac{3}{10}\right)$

Solution:

a. $(2)(-5.1)$

$= -10.2$ *Different* signs. The product is negative.

b. $-\frac{2}{3} \cdot \frac{9}{8}$

$= -\frac{18}{24}$ *Different* signs. The product is negative.

$= -\frac{3}{4}$ Simplify to lowest terms.

c. $\left(-3\frac{1}{3}\right)\left(-\frac{3}{10}\right)$

$= \left(-\frac{10}{3}\right)\left(-\frac{3}{10}\right)$ Write the mixed number as a fraction.

$= \frac{30}{30}$ *Same* signs. The product is positive.

$= 1$ Simplify to lowest terms.

Skill Practice Multiply.

10. $(-5)(2.2)$ **11.** $\frac{5}{7} \cdot \left(-\frac{14}{15}\right)$ **12.** $\left(-5\frac{1}{4}\right)\left(-\frac{8}{3}\right)$

TIP: A number and its reciprocal have the same sign. For example:

$$\left(-\frac{10}{3}\right)\left(-\frac{3}{10}\right) = 1$$

and $$3 \cdot \frac{1}{3} = 1$$

Notice from Example 4(c) that $(-\frac{10}{3})(-\frac{3}{10}) = 1$. If the product of two numbers is 1, then the numbers are **reciprocals**. That is, the reciprocal of a real *number a* is $\frac{1}{a}$. Furthermore, $a \cdot \frac{1}{a} = 1$.

Answers

10. -11 **11.** $-\frac{2}{3}$ **12.** 14

Recall that subtraction of real numbers was defined in terms of addition. In a similar way, division of real numbers can be defined in terms of multiplication.

Procedure to Divide Real Numbers

To divide two real numbers, multiply the first number by the reciprocal of the second number. For example:

Multiply

$$10 \div 5 = 2 \qquad \text{or equivalently} \qquad 10 \cdot \frac{1}{5} = 2$$

Reciprocal

Because division of real numbers can be expressed in terms of multiplication, the sign rules that apply to multiplication also apply to division.

$$10 \div 2 = 10 \cdot \frac{1}{2} = 5$$

$$-10 \div (-2) = -10 \cdot \left(-\frac{1}{2}\right) = 5$$

Dividing two numbers of the same sign produces a *positive* quotient.

$$10 \div (-2) = 10 \cdot \left(-\frac{1}{2}\right) = -5$$

$$-10 \div 2 = -10 \cdot \frac{1}{2} = -5$$

Dividing two numbers of opposite signs produces a *negative* quotient.

Division of Real Numbers

Assume that a and b are real numbers such that $b \neq 0$.

1. If a and b have the *same* sign, then the quotient $\frac{a}{b}$ is *positive*.
2. If a and b have *different* signs, then the quotient $\frac{a}{b}$ is *negative*.
3. $\dfrac{0}{b} = 0$.
4. $\dfrac{b}{0}$ is undefined.

The relationship between multiplication and division can be used to investigate properties 3 and 4 from the preceding box. For example,

$\dfrac{0}{6} = 0$ Because $6 \cdot 0 = 0$ ✓

$\dfrac{6}{0}$ is *undefined* Because there is no number that when multiplied by 0 will equal 6

Note: The quotient of 0 and 0 *cannot be determined*. Evaluating an expression of the form $\frac{0}{0} = ?$ is equivalent to asking, "What number times zero will equal 0?" That is, $(0)(?) = 0$. Any real number will satisfy this requirement; however, expressions involving $\frac{0}{0}$ are usually discussed in advanced mathematics courses.

Example 5 Dividing Real Numbers

Divide the real numbers. Write the answer as a fraction or whole number.

a. $\frac{-42}{7}$ **b.** $\frac{-96}{-144}$ **c.** $3\frac{1}{10} \div \left(-\frac{2}{5}\right)$ **d.** $\frac{-8}{-7}$

Solution:

a. $\frac{-42}{7} = -6$ — *Different* signs. The quotient is negative.

b. $\frac{-96}{-144} = \frac{2}{3}$ — *Same* signs. The quotient is positive. Simplify.

c. $3\frac{1}{10} \div \left(-\frac{2}{5}\right)$

$= \frac{31}{10}\left(-\frac{5}{2}\right)$ — Write the mixed number as an improper fraction, and multiply by the reciprocal of the second number.

$= \frac{31}{\cancel{10}_{2}}\left(-\frac{\cancel{5}^{1}}{2}\right)$

$= -\frac{31}{4}$ — *Different* signs. The quotient is negative.

d. $\frac{-8}{-7} = \frac{8}{7}$ — *Same* signs. The quotient is positive. Because 7 does not divide into 8 evenly, the answer can be left as a fraction.

Skill Practice Divide.

13. $\frac{42}{-2}$ **14.** $\frac{-28}{-4}$ **15.** $-\frac{2}{3} \div 4$ **16.** $\frac{-1}{-2}$

TIP: Multiplication may be used to check a division problem.

$$\frac{-42}{7} = -6$$

Check: $(7)(-6) = -42$ ✓

Avoiding Mistakes

If the numerator and denominator of a fraction have opposite signs, then the quotient will be negative. Therefore, a fraction has the same value whether the negative sign is written in the numerator, in the denominator, or in front of the fraction.

$$-\frac{31}{4} = \frac{-31}{4} = \frac{31}{-4}$$

4. Exponential Expressions

To simplify the process of repeated multiplication, exponential notation is often used. For example, the quantity $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ can be written as 3^5 (3 to the fifth power).

Definition of b^n

Let b represent any real number and n represent a positive integer. Then

$$b^n = \underbrace{b \cdot b \cdot b \cdot b \cdot \ldots b}_{n \text{ factors of } b}$$

b^n is read as "b to the nth power."

b is called the **base** and n is called the **exponent**, or **power**.

b^2 is read as "b *squared*," and b^3 is read as "b *cubed*."

Answers

13. -21 **14.** 7

15. $-\frac{1}{6}$ **16.** $\frac{1}{2}$

Example 6 Evaluating Exponential Expressions

Simplify the expression.

a. 5^3 **b.** $(-2)^4$ **c.** -2^4 **d.** $\left(-\frac{1}{3}\right)^3$

Solution:

a. $5^3 = 5 \cdot 5 \cdot 5$
$= 125$

The base is 5, and the exponent is 3.

b. $(-2)^4 = (-2)(-2)(-2)(-2)$
$= 16$

The base is -2, and the exponent is 4. The exponent 4 applies to the entire contents of the parentheses.

c. $-2^4 = -[2 \cdot 2 \cdot 2 \cdot 2]$
$= -16$

The base is 2, and the exponent is 4. Because no parentheses enclose the negative sign, the exponent applies to only 2.

TIP: The quantity -2^4 can also be interpreted as $-1 \cdot 2^4$.

$$-2^4 = -1 \cdot 2^4 = -1 \cdot (2 \cdot 2 \cdot 2 \cdot 2) = -16$$

d. $\left(-\frac{1}{3}\right)^3 = \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)$
$= -\frac{1}{27}$

The base is $-\frac{1}{3}$, and the exponent is 3.

Skill Practice Simplify.

17. 2^3 **18.** $(-10)^2$ **19.** -10^2 **20.** $\left(-\frac{3}{4}\right)^3$

5. Square Roots

The inverse operation to squaring a number is to find its square roots. For example, finding a square root of 9 is equivalent to asking, "What number when squared equals 9?" One obvious answer is 3, because $(3)^2 = 9$. However, -3 is also a square root of 9 because $(-3)^2 = 9$. For now, we will focus on the **principal square root**, which is always taken to be nonnegative.

The symbol $\sqrt{}$, called a **radical sign**, is used to denote the principal square root of a number. Therefore, the principal square root of 9 can be written as $\sqrt{9}$. The expression $\sqrt{64}$ represents the principal square root of 64.

Answers

17. 8 **18.** 100

19. -100 **20.** $-\frac{27}{64}$

Example 7 Evaluating Square Roots

Evaluate the expressions, if possible.

a. $\sqrt{81}$ **b.** $\sqrt{\frac{25}{64}}$ **c.** $\sqrt{-16}$ **d.** $-\sqrt{16}$

Solution:

a. $\sqrt{81} = 9$ because $(9)^2 = 81$

b. $\sqrt{\frac{25}{64}} = \frac{5}{8}$ because $\left(\frac{5}{8}\right)^2 = \frac{25}{64}$

c. $\sqrt{-16}$ is *not a real number* because no real number when squared will be negative.

d. $-\sqrt{16} = -4$ because $-\sqrt{16} = -(\sqrt{16}) = -4$.

Skill Practice Evaluate the expressions, if possible.

21. $\sqrt{25}$ **22.** $\sqrt{\frac{49}{100}}$ **23.** $\sqrt{-4}$ **24.** $-\sqrt{9}$

Example 7(c) illustrates that the square root of a negative number is not a real number because no real number when squared will be negative.

Square Root of a Negative Number

Let a be a negative real number. Then $\sqrt{a}$ is not a real number.

6. Order of Operations

When algebraic expressions contain numerous operations, it is important to use the proper **order of operations**. Parentheses ( ), brackets [ ], and braces { } are used for grouping numbers and algebraic expressions. It is important to recognize that operations must be done first within parentheses and other grouping symbols.

Order of Operations

Step 1 First, simplify expressions within parentheses and other grouping symbols. These include absolute value bars, fraction bars, and radicals. If embedded parentheses are present, start with the innermost parentheses.

Step 2 Evaluate expressions involving exponents, radicals, and absolute values.

Step 3 Perform multiplication or division in the order in which they occur from left to right.

Step 4 Perform addition or subtraction in the order in which they occur from left to right.

Answers

21. 5 **22.** $\frac{7}{10}$

23. Not a real number

24. −3

Example 8 Applying the Order of Operations

Simplify the expression. $10 - [2 - 4(6 - 8)]^2 + \sqrt{16 - 7}$

Solution:

$10 - [2 - 4(6 - 8)]^2 + \sqrt{16 - 7}$

$= 10 - [2 - 4(-2)]^2 + \sqrt{9}$ Simplify inside the innermost parentheses and inside the radical.

$= 10 - [2 + 8]^2 + \sqrt{9}$ Simplify within square brackets. Perform multiplication before addition or subtraction.

$= 10 - [10]^2 + \sqrt{9}$

$= 10 - 100 + 3$ Simplify the exponential expression and the radical.

$= -90 + 3$ Perform addition or subtraction in the order in which they appear from left to right.

$= -87$

Avoiding Mistakes

Don't try to perform too many steps at once. Taking a shortcut may result in a careless error. For each step rewrite the entire expression, changing only the operation being evaluated.

Skill Practice Simplify the expression.

25. $36 \div 2^2 \cdot 3 - [(18 - 5) \cdot 2 + 6]$

Example 9 Applying the Order of Operations

Simplify the expression. $\dfrac{|(-3)^3 + (5^2 - 3)|}{-15 \div (-3)(2)}$

Solution:

$\dfrac{|(-3)^3 + (5^2 - 3)|}{-15 \div (-3)(2)}$ Simplify numerator and denominator separately.

$= \dfrac{|(-3)^3 + (25 - 3)|}{5(2)}$ *Numerator:* Simplify within the inner parentheses.

Denominator: Perform division and multiplication (left to right).

$= \dfrac{|(-3)^3 + (22)|}{10}$ *Numerator:* Simplify inner parentheses. *Denominator:* Multiply.

$= \dfrac{|-27 + 22|}{10}$ Simplify exponent.

$= \dfrac{|-5|}{10}$ Add within the absolute value.

$= \dfrac{5}{10}$ or $\dfrac{1}{2}$ Evaluate the absolute value and simplify.

Skill Practice Simplify the expression.

26. $\dfrac{-|5 - 7| + 11}{(-1 - 2)^2}$

Answers

25. −5 **26.** 1

7. Evaluating Formulas

An algebraic expression or formula involves operations on numbers and variables. A **variable** is a letter that may represent any numerical value. To evaluate an expression or formula, we substitute known values of the variables into the expression. Then we follow the order of operations.

It is important to note that some formulas from geometry use Greek letters (such as π) and some use variables with subscripts. A **subscript** is a number or letter written to the right of and below a variable. For example, the area of a trapezoid is given by $A = \frac{1}{2}(b_1 + b_2)h$. The values b_1 and b_2 (read as "b sub 1" and "b sub 2") represent two different bases of the trapezoid (Figure R-3).

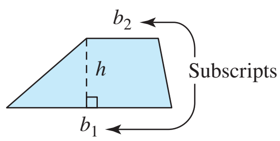


Figure R-3

Example 10 Evaluating a Formula

A homeowner in North Carolina wants to buy protective film for a trapezoid-shaped window. The film will adhere to shattered glass in the event that the glass breaks during a bad storm. Find the area of the window whose dimensions are given in Figure R-4.

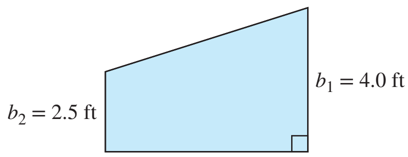


Figure R-4

Solution:

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$= \frac{1}{2}(4.0 \text{ ft} + 2.5 \text{ ft})(5.0 \text{ ft})$$ Substitute $b_1 = 4.0$ ft, $b_2 = 2.5$ ft, and $h = 5.0$ ft.

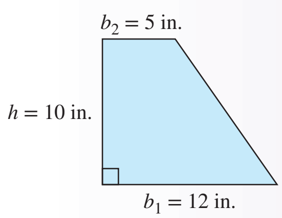
$$= \frac{1}{2}(6.5 \text{ ft})(5.0 \text{ ft})$$ Simplify inside parentheses.

$$= 16.25 \text{ ft}^2$$ Multiply from left to right.

The area of the window is 16.25 ft^2.

Skill Practice

27. Use the formula given in Example 10 to find the area of the trapezoid.



TIP: Subscripts should not be confused with *superscripts,* which are written above a variable. Superscripts are used to denote powers.

$b_2 \neq b^2$

Answer

27. The area is 85 in.^2

Section R.2 Activity

A.1. **a.** What does the notation $|-6|$ mean?
b. Simplify $|-6|$.
c. Simplify $|-10|$.
d. Add. $-6 + (-10)$
e. In general, how do you add two numbers with the same sign?

A.2. **a.** Simplify $|-23|$.
b. Simplify $|4|$.
c. Add. $-23 + 4$
d. In general, how to you add two numbers with different signs?

A.3. **a.** What is the opposite of –8?
b. Write the statement $2 - (-8)$ as an equivalent addition statement.
c. Simplify $2 - (-8)$.
d. In general, a subtraction statement $a - b$ is equivalent to what addition statement?

For Exercises A.4–A.6, write an equivalent addition statement and simplify.

	Original Expression	Equivalent Addition Expression	Result
A.4.	$-4.2 - 12.6$		
A.5.	$\frac{7}{8} - \frac{11}{6}$		
A.6.	$-1\frac{3}{4} - \left(-5\frac{1}{2}\right)$		

A.7. Consider the sum. $(-3) + (-3) + (-3) + (-3)$
a. Is this sum positive or negative?
b. Write a multiplication statement that represents this sum.
c. In general, the product of two numbers with opposite signs is (choose one: positive/negative). By contrast, the product of two numbers of the same sign is (choose one: positive/negative)

A.8. **a.** What is the reciprocal of –2?
b. Write the statement $12 \div (-2)$ as an equivalent multiplication statement.
c. Simplify $12 \div (-2)$.
d. In general for $b \neq 0$, a division statement $a \div b$ is equivalent to what multiplication statement?
e. Because a division statement can be written in terms of multiplication, the same sign rules that apply to the product of two real numbers apply to the quotient of two real numbers.
- Thus, the quotient of two numbers of the same sign is (choose one: positive/negative).
- The quotient of two numbers of different signs is (choose one: positive/negative).

A.9. **a.** Fill in the blanks. $-15 \div (-3) = \square$ because $-3 \cdot \square = -15$
b. Fill in the blanks. $0 \div 4 = \square$ because $4 \cdot \square = 0$
c. Explain why $4 \div 0$ is undefined.

For Exercises A.10–A.17, perform the indicated operations.

A.10. **a.** $-20 + 4$ **b.** $-20 - 4$ **c.** $-20 \cdot (4)$ **d.** $-\frac{20}{4}$

A.11. **a.** $-\frac{5}{6}+\left(-\frac{10}{3}\right)$ **b.** $-\frac{5}{6}-\left(-\frac{10}{3}\right)$ **c.** $-\frac{5}{6}\cdot\left(-\frac{10}{3}\right)$ **d.** $-\frac{5}{6}\div\left(-\frac{10}{3}\right)$

A.12. **a.** $-4.2 \div 0$
b. $0 \div (-4.2)$

A.13. **a.** $-\frac{3}{4}\cdot\left(-\frac{4}{3}\right)$
b. $-\frac{3}{4}+\frac{3}{4}$

A.14. **a.** 3^2
b. 3^3
c. 3^4

A.15. **a.** $(-3)^2$
b. $(-3)^3$
c. $(-3)^4$

A.16. **a.** -5^2
b. $(-5)^2$
c. -5^3
d. $(-5)^3$

A.17. **a.** $\sqrt{16}$
b. $-\sqrt{16}$
c. $\sqrt{-16}$

For Exercises A.18–A.19, divide your paper into two columns vertically. Use the order of operations to simplify the expression in the left column. In the right column, write a description of each step.

A.18. $52 \div 26 - 3(1-6)^2$ Simplify the expression here...	Explain each step here...
A.19. $\dfrac{\lvert -15+3^2 \rvert}{\sqrt{10^2-8^2}}$ Simplify the expression here...	Explain each step here...

Practice Exercises Section R.2

Study Skills Exercise

Reading comprehension in a mathematics course is essential. Reading comprehension can range from understanding mathematical operations and symbols to solving lengthy word problems. When reading mathematics, it is not unusual to find multiple symbolic representations for mathematical expressions. For example,

- Write four different ways to express 5×4 using different notations.

You will also find different ways to express a mathematical statement in words. For example,

- Write three different English statements that represent $8 - 5$.

Prerequisite Review

For Exercises R.1–R.8, simplify the expressions.

R.1. **a.** $\frac{4}{5} - \frac{2}{3}$ **b.** $\frac{4}{5} + \frac{2}{3}$

R.2. **a.** $\frac{9}{4} - \frac{7}{6}$ **b.** $\frac{9}{4} + \frac{7}{6}$

R.3. **a.** $\frac{5}{6} \cdot \frac{3}{10}$ **b.** $\frac{5}{6} \div \frac{3}{10}$

R.4. **a.** $\frac{5}{14} \cdot \frac{35}{2}$ **b.** $\frac{5}{14} \div \frac{35}{2}$

R.5. **a.** $2\frac{2}{3} + 1\frac{1}{6}$ **b.** $2\frac{2}{3} - 1\frac{1}{6}$

R.6. **a.** $4\frac{7}{8} + 3\frac{3}{4}$ **b.** $4\frac{7}{8} - 3\frac{3}{4}$

R.7. **a.** $\left(3\frac{3}{5}\right) \cdot \left(2\frac{1}{2}\right)$ **b.** $\left(3\frac{3}{5}\right) \div \left(2\frac{1}{2}\right)$

R.8. **a.** $\left(5\frac{1}{3}\right) \cdot \left(2\frac{1}{6}\right)$ **b.** $\left(5\frac{1}{3}\right) \div \left(2\frac{1}{6}\right)$

Vocabulary and Key Concepts

1. **a.** Two numbers that are the same distance from 0 but on opposite sides of 0 on the number line are called ____________.

b. The absolute value of a real number, a, is denoted by _____ and is the distance between a and _____ on the number line.

c. Given the expression b^n, the value b is called the ____________ and ____________ is called the exponent or power.

d. The symbol $\sqrt{\ }$ is called a ____________ sign and is used to find the principal ____________ root of a nonnegative real number.

e. If a is a nonzero real number, then the reciprocal of a is ____________. The product of a number and its reciprocal is ____________.

f. If either a or b is zero then $ab =$ ____________.

g. If $a = 0$ and $b \neq 0$, then $\frac{a}{b} =$ ____________ and $\frac{b}{a}$ is ____________.

2. If a and b are both negative, then $a + b$ will be (positive/negative) and ab will be (positive/negative).

3. If $a < 0$ and $b > 0$, and if $|a| < |b|$, then the sign of $a + b$ will be (positive/negative) and the sign of $\frac{a}{b}$ will be (positive/negative).

4. The expression $a - b = a +$ _____. If $a > 0$ and $b < 0$, then the sign of $a - b$ is ____________.

For Exercises 5–6, fill in the blank with <, >, or =.

5. If $a < 0$ and $b < 0$, then ab ☐ 0.

6. If $a < 0$ and $b < 0$, then $\frac{a}{b}$ ☐ 0.

Concept 1: Opposite and Absolute Value

7. If the absolute value of a number can be thought of as its distance from zero, explain why an absolute value can never be negative.

8. If a number is negative, then its *opposite* will be

a. Positive. **b.** Negative.

9. If a number is negative, then its *reciprocal* will be

a. Positive. **b.** Negative.

10. If a number is negative, then its *absolute value* will be

a. Positive. **b.** Negative.

11. Complete the table. **(See Example 1.)**

Number	Opposite	Reciprocal	Absolute Value
6			
	$-\frac{1}{11}$		
		$-\frac{1}{8}$	
	$\frac{13}{10}$		
0			
		$-0.\overline{3}$	

12. Complete the table.

Number	Opposite	Reciprocal	Absolute Value
-9			
	$\frac{2}{3}$		
		14	
-1			
		Undefined	
		$2\frac{1}{9}$	

For Exercises 13–20, fill in the blank with the appropriate symbol (<, >, =). **(See Example 1.)**

13. $-|6|$ ______ $|-6|$

14. $-(-5)$ ______ $-|-5|$

15. $|-4|$ ______ $|4|$

16. $-|2|$ ______ (-2)

17. $-|-1|$ ______ 1

18. -3 ______ $-|-7|$

19. $|2 + (-5)|$ ______ $|2| + |-5|$

20. $|4 + 3|$ ______ $|4| + |3|$

Concept 2: Addition and Subtraction of Real Numbers

For Exercises 21–36, add or subtract as indicated. **(See Examples 2–3.)**

21. $-8 + 4$

22. $3 + (-7)$

23. $-12 + (-7)$

24. $-5 + (-11)$

25. $-17 - (-10)$

26. $-14 - (-2)$

27. $5 - (-9)$

28. $8 - (-4)$

29. $-6.3 - 15.8$

30. $-21.9 - 4.7$

31. $1.5 - 9.6$

32. $4.8 - 10$

33. $\frac{2}{3} + \left(-2\frac{1}{3}\right)$

34. $-\frac{4}{7} + \left(1\frac{4}{7}\right)$

35. $-\frac{5}{9} - \frac{14}{15}$

36. $-6 - \frac{2}{9}$

Concept 3: Multiplication and Division of Real Numbers

For Exercises 37–52, perform the indicated operation. **(See Examples 4–5.)**

37. $4(-8)$

38. $-21(3)$

39. $\frac{2}{9} \cdot \frac{12}{7}$

40. $\left(-\frac{5}{9}\right) \cdot \left(-1\frac{7}{11}\right)$

41. $\frac{-6}{-10}$

42. $\frac{-15}{-24}$

43. $-2\frac{1}{4} \div \frac{5}{8}$

44. $-\frac{2}{3} \div \left(-1\frac{5}{7}\right)$

45. $7 \div 0$

46. $\frac{1}{16} \div 0$

47. $0 \div (-3)$

48. $0 \div 11$

49. $(-1.2)(-3.1)$

50. $(4.6)(-2.25)$

51. $\frac{-5}{-11}$

52. $\frac{-3}{-13}$

Concept 4: Exponential Expressions

For Exercises 53–60, evaluate the expression. **(See Example 6.)**

53. 4^3

54. -2^3

55. -7^2

56. -3^4

57. $(-7)^2$

58. $(-5)^2$

59. $\left(\frac{5}{3}\right)^3$

60. $\left(\frac{10}{9}\right)^2$

Concept 5: Square Roots

For Exercises 61–68, evaluate the expression, if possible. **(See Example 7.)**

61. $\sqrt{9}$ **62.** $\sqrt{1}$ **63.** $\sqrt{-4}$ **64.** $\sqrt{-36}$

65. $\sqrt{\frac{1}{4}}$ **66.** $\sqrt{\frac{9}{4}}$ **67.** $-\sqrt{49}$ **68.** $-\sqrt{100}$

Concept 6: Order of Operations

For Exercises 69–96, simplify by using the order of operations. **(See Examples 8–9.)**

69. $5 + 3^3$ **70.** $10 - 2^4$ **71.** $5 \cdot 2^3$ **72.** $12 \div 2^2$

73. $(2 + 3)^2$ **74.** $(4 - 1)^3$ **75.** $2^2 + 3^2$ **76.** $4^3 - 1^3$

77. $6 + 10 \div 2 \cdot 3 - 4$ **78.** $12 \div 3 \cdot 4 - 18$ **79.** $4^2 - (5 - 2)^2 \cdot 3$ **80.** $5 - 3(8 \div 4)^2$

81. $2 - 5(9 - 4\sqrt{25})^2$ **82.** $5^2 - (\sqrt{9} + 4 \div 2)$ **83.** $\left(-\frac{3}{5}\right)^2 - \frac{3}{5} \cdot \frac{5}{9} + \frac{7}{10}$ **84.** $\frac{1}{2} - \left(\frac{2}{3} \div \frac{5}{9}\right) + \frac{5}{6}$

85. $1.75 \div 0.25 - (1.25)^2$ **86.** $5.4 - (0.3)^2 \div 0.09$ **87.** $\frac{\sqrt{10^2 - 8^2}}{3^2}$ **88.** $\frac{\sqrt{16 - 7} + 3^2}{\sqrt{16} - \sqrt{4}}$

89. $-|-11 + 5| + |7 - 2|$ **90.** $-|-8 - 3| - (-8 - 3)$

91. $25 - 2[(7 - 3)^2 \div 4] + \sqrt{18 - 2}$ **92.** $\sqrt{29 - 2^2} + [8 - 3(6 - 2)] \div 4 \cdot 5$

93. $\frac{|(10 - 7) - 2^3|}{6 - 16 \div 8 \cdot 3}$ **94.** $\frac{|-12 - (7 - 3^2)^2|}{40 - 6^2 - 8 \div 2}$

95. $\left(\frac{1}{2}\right)^2 + \left(\frac{6 - 4}{5}\right)^2 + \left(\frac{5 + 2}{10}\right)^2$ **96.** $\left(\frac{2^3}{2^3 + 1}\right)^2 \div \left[\frac{8 - (-2)}{3^2}\right]^2$

For Exercises 97–98, find the average of the set of data values by adding the values and dividing by the number of values.

97. Find the average low temperature for a week in January in St. John's, Newfoundland. Round to the nearest tenth of a degree.

Day	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Sun.
Low temperature	−18°C	−16°C	−20°C	−11°C	−4°C	−3°C	1°C

98. Find the average high temperature for a week in January in St. John's, Newfoundland. Round to the nearest tenth of a degree.

Day	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Sun.
High temperature	−2°C	−6°C	−7°C	0°C	1°C	8°C	10°C

Concept 7: Evaluating Formulas

99. The formula $C = \frac{5}{9}(F - 32)$ converts temperatures in the Fahrenheit scale to the Celsius scale. Find the equivalent Celsius temperature for each Fahrenheit temperature.

a. 77°F **b.** 212°F **c.** 32°F **d.** −40°F

100. The formula $F = \frac{9}{5}C + 32$ converts Celsius temperatures to Fahrenheit temperatures. Find the equivalent Fahrenheit temperature for each Celsius temperature.

a. −5°C **b.** 0°C **c.** 37°C **d.** −40°C

The equation $G_E = \frac{1}{22}c + \frac{1}{30}h$ represents the amount of gasoline used (in gal) for an economy car to drive c miles in the city and h miles on the highway. The equation $G_T = \frac{1}{12}c + \frac{1}{18}h$ represents the amount of gasoline used for a truck to make the same trip. Use these formulas for Exercises 101–102.

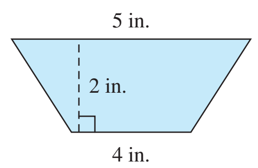
101. Determine the amount of gas used by an economy car that travels 33 mi in the city and 80 mi on the highway.

102. Determine the amount of gas used by a truck that travels 33 mi in the city and 80 mi on the highway.

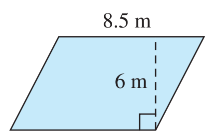
Use an appropriate formula from geometry to answer Exercises 103–112.

For Exercises 103–106, find the area. **(See Example 10.)**

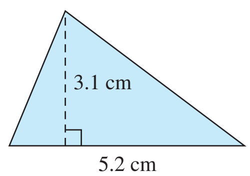
103. Trapezoid



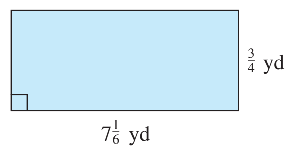
104. Parallelogram



105. Triangle

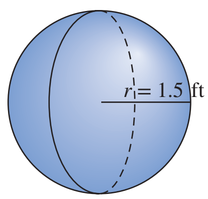


106. Rectangle

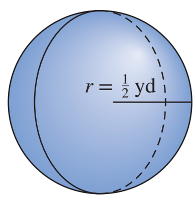


For Exercises 107–112, find the volume. (Use the π key on your calculator, and round the final answer to one decimal place.)

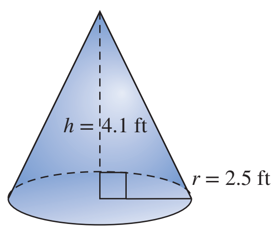
107. Sphere



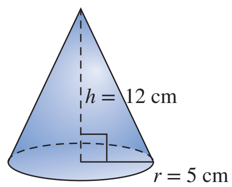
108. Sphere



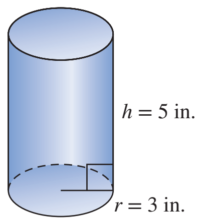
109. Right circular cone



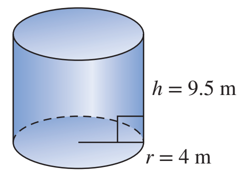
110. Right circular cone



111. Right circular cylinder



112. Right circular cylinder



Technology Connections

113. Which expression when entered into a graphing calculator will yield the correct value of $\dfrac{12}{6-2}$?

12/6 − 2 or 12/(6 − 2)

114. Which expression when entered into a graphing calculator will yield the correct value of $\dfrac{24-6}{3}$?

(24 − 6)/3 or 24 − 6/3

115. Verify your solution to Exercise 87 by entering the expression into a graphing calculator:

$(\sqrt{}(10^2 - 8^2))/3^2$

116. Verify your solution to Exercise 88 by entering the expression into a graphing calculator:

$(\sqrt{}(16 - 7) + 3^2)/(\sqrt{}(16) - \sqrt{}(4))$