

# DEVELOPMENTAL MATHEMATICS: Prealgebra, Beginning Algebra & Intermediate Algebra

SECOND EDITION



**Mc  
Graw  
Hill**

Julie Miller

Molly O'Neill

Nancy Hyde



A person wearing a white tank top, black shorts, and a white helmet is standing on a grassy hill, holding a red mountain bike up in the air with both hands. The background features a bright sunset with mountains in the distance and some greenery on the left. The sky is a mix of light blue and orange. There are clusters of small yellow dots at the top and bottom of the page.

# DEVELOPMENTAL MATHEMATICS: Prealgebra, Beginning Algebra, & Intermediate Algebra

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**Julie Miller**

*Professor Emerita,  
Daytona State College*

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**Nancy Hyde**

*Professor Emerita,  
Broward College*

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SECOND EDITION

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# Letter from the Authors

Dear Colleagues,

Across the country, Developmental Math courses are in a state of flux, and we as instructors are at the center of it all. As many of our institutions are grappling with the challenges of placement, retention, and graduation rates, we are on the front lines with our students—supporting all of them in their educational journey.

## **Flexibility—No Matter Your Course Format!**

The three of us each teach differently, as do many of our current users. The Miller/O'Neill/Hyde series is designed for successful use in a variety of course formats, both traditional and modern—classroom lecture settings, flipped classrooms, hybrid classes, and online-only classes.

## **Ease of Instructor Preparation**

We've all had to fill in for a colleague, pick up a last-minute section, or find ourselves running across campus to yet a different course. The Miller/O'Neill/Hyde series is carefully designed to support instructors teaching in a variety of different settings and circumstances. Experienced, senior faculty members can draw from a massive library of static and algorithmic content found in ALEKS to meticulously build assignments and assessments sharply tailored to individual student needs. Newer instructors and part-time adjunct instructors, on the other hand, will find support through a wide range of digital resources and prebuilt assignments ready to go on Day One. With these tools, instructors with limited time to prepare for class can still facilitate successful student outcomes.

Many instructors want to incorporate discovery-based learning and groupwork into their courses but don't have time to write or find quality materials. Each section of the text has numerous discovery-based activities that we have tested in our own classrooms. These are found in the text and Student Resource Manual along with other targeted worksheets for additional practice and materials for a student portfolio.

## **Student Success—Now and in the Future**

Too often our math placement tests fail our students, which can lead to frustration, anxiety, and often withdrawal from their education journey. We encourage you to learn more about ALEKS Placement, Preparation, and Learning (ALEKS PPL), which uses adaptive learning technology to place students appropriately. No matter the skills they come in with, the Miller/O'Neill/Hyde series provides resources and support that uniquely position them for success in that course and for their next course. Whether they need a brush-up on their basic skills, ADA supportive materials, or advanced topics to help them cross the bridge to the next level, we've created a support system for them.

We hope you are as excited as we are about the series and the supporting resources and services that accompany it. Please reach out to any of us with any questions or comments you have about our texts.

Julie Miller

Molly O'Neill

Nancy Hyde



# About the Authors

**Julie Miller** is from Daytona State College, where she taught developmental and upper-level mathematics courses for 20 years. Prior to her work at Daytona State College, she worked as a software engineer for General Electric in the area of flight and radar simulation. Julie earned a Bachelor of Science in Applied Mathematics from Union College in Schenectady, New York, and a Master of Science in Mathematics from the University of Florida. In addition to this textbook, she has authored textbooks for college algebra, trigonometry, and precalculus, as well as several short works of fiction and nonfiction for young readers.

“My father is a medical researcher, and I got hooked on math and science when I was young and would visit his laboratory. I can remember using graph paper to plot data points for his experiments and doing simple calculations. He would then tell me what the peaks and features in the graph meant in the context of his experiment. I think that applications and hands-on experience made math come alive for me, and I’d like to see math come alive for my students.”

—Julie Miller

**Molly O’Neill** is also from Daytona State College, where she taught for 22 years in the School of Mathematics. She has taught a variety of courses from developmental mathematics to calculus. Before she came to Florida, Molly taught as an adjunct instructor at the University of Michigan–Dearborn, Eastern Michigan University, Wayne State University, and Oakland Community College. Molly earned a Bachelor of Science in Mathematics and a Master of Arts and Teaching from Western Michigan University in Kalamazoo, Michigan. Besides this textbook, she has authored several course supplements for college algebra, trigonometry, and precalculus and has reviewed texts for developmental mathematics.

“I differ from many of my colleagues in that math was not always easy for me. But in seventh grade I had a teacher who taught me that if I follow the rules of mathematics, even I could solve math problems. Once I understood this, I enjoyed math to the point of choosing it for my career. I now have the greatest job because I get to do math every day and I have the opportunity to influence my students just as I was influenced. Authoring these texts has given me another avenue to reach even more students.”

—Molly O’Neill

**Nancy Hyde** served as a full-time faculty member of the Mathematics Department at Broward College for 24 years. During this time she taught the full spectrum of courses from developmental math through differential equations. She received a Bachelor of Science in Math Education from Florida State University and a Master’s degree in Math Education from Florida Atlantic University. She has conducted workshops and seminars for both students and teachers on the use of technology in the classroom. In addition to this textbook, she has authored a graphing calculator supplement for *College Algebra*.

“I grew up in Brevard County, Florida, where my father worked at Cape Canaveral. I was always excited by mathematics and physics in relation to the space program. As I studied higher levels of mathematics I became more intrigued by its abstract nature and infinite possibilities. It is enjoyable and rewarding to convey this perspective to students while helping them to understand mathematics.”

—Nancy Hyde



Photo courtesy of Molly O’Neill

## Dedication

To Our Students

Julie Miller 🌸 Molly O’Neill 🌸 Nancy Hyde

# The Miller/O'Neill/Hyde

## Developmental Math Series

Julie Miller, Molly O'Neill, and Nancy Hyde originally wrote their developmental math series because students were entering their College Algebra course underprepared. The students were not mathematically mature enough to understand the concepts of math, nor were they fully engaged with the material. The authors began their developmental mathematics offerings with Intermediate Algebra to help bridge that gap. This in turn evolved into several series of textbooks from Prealgebra through Precalculus to help students at all levels before Calculus.

What sets all of the Miller/O'Neill/Hyde series apart is that they address course content through an author-created digital package that maintains a consistent voice and notation throughout the program. This consistency—in videos, PowerPoints, Lecture Notes, and Integrated Video and Study Guides—coupled with the power of ALEKS, ensures that students master the skills necessary to be successful in Developmental Math through Precalculus and prepares them for the Calculus sequence.

### **Developmental Math Series**

*The Developmental Math series is traditional in approach, delivering a purposeful balance of skills and conceptual development. It places a strong emphasis on conceptual learning to prepare students for success in subsequent courses.*

- Basic College Mathematics, Third Edition
- Prealgebra, Third Edition
- Prealgebra & Introductory Algebra, Second Edition
- Beginning Algebra, Sixth Edition
- Beginning & Intermediate Algebra, Sixth Edition
- Intermediate Algebra, Sixth Edition
- Developmental Mathematics: Prealgebra, Beginning Algebra, & Intermediate Algebra, Second Edition

### **The Miller/Gerken College Algebra/Precalculus Series**

*The Precalculus series serves as the bridge from Developmental Math coursework to future courses by emphasizing the skills and concepts needed for Calculus.*

- College Algebra with Corequisite Support, First Edition
- College Algebra, Second Edition
- College Algebra and Trigonometry, First Edition
- Precalculus, First Edition



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Most importantly, we give special thanks to the students and instructors who use our series in their classes.

Julie Miller  
Molly O'Neill  
Nancy Hyde

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# To the Student

Take a deep breath and know that you aren't alone. Your instructor, fellow students, and we, your authors, are here to help you learn and master the material for this course and prepare you for future courses. You may feel like math just isn't your thing, or maybe it's been a long time since you've had a math class—that's okay!

We wrote the text and all the supporting materials with you in mind. Most of our students aren't really sure how to be successful in math, but we can help with that.

As you begin your class, we'd like to offer some specific suggestions:

1. **Attend class.** Arrive on time and be prepared. If your instructor has asked you to read prior to attending class—do it. How often have you sat in class and thought you understood the material, only to get home and realize you don't know how to get started? By reading and trying a couple of Skill Practice exercises, which follow each example, you will be able to ask questions and gain clarification from your instructor when needed.
2. **Be an active learner.** Whether you are at lecture, watching an author lecture or exercise video, or are reading the text, pick up a pencil and work out the examples given. Math is learned only by doing; we like to say, “Math is not a spectator sport.” If you like a bit more guidance, we encourage you to use the Integrated Video and Study Guide. It was designed to provide structure and note-taking for lectures and while watching the accompanying videos.
3. **Schedule time to do some math every day.** Exercise, foreign language study, and math are three things that you must do every day to get the results you want. If you are used to cramming and doing all of your work in a few hours on a weekend, you should know that even mathematicians start making silly errors after an hour or so! Check your answers. Skill Practice exercises all have the answers at the bottom of that page. Odd-numbered exercises throughout the text have answers in the back of the text. If you didn't get it right, don't throw in the towel. Try again, revisit an example, or bring your questions to class for extra help.
4. **Prepare for quizzes and exams.** Each chapter has a set of Chapter Review Exercises at the end to help you integrate all of the important concepts. In addition, there is a detailed Chapter Summary and a Chapter Test located in the online resources. If you use ALEKS, use all of the tools available within the program to test your understanding.
5. **Use your resources.** This text comes with numerous supporting resources designed to help you succeed in this class and in your future classes. Additionally, your instructor can direct you to resources within your institution or community. Form a student study group. Teaching others is a great way to strengthen your own understanding, and they might be able to return the favor if you get stuck.

We wish you all the best in this class and in your educational journey!

Julie Miller

Molly O'Neill

Nancy Hyde



# Student Guide to the Text

## Clear, Precise Writing

Learning from our own students, we have written this text in simple and accessible language. Our goal is to keep you engaged and supported throughout your coursework.

## Call-Outs

Just as your instructor will share tips and math advice in class, we provide call-outs throughout the text to offer tips and warn against common mistakes.

- Tip boxes offer additional insight into a concept or procedure.
- Avoiding Mistakes help fend off common student errors.
- For Review boxes positioned strategically throughout the text remind students of key skills relating to the current topic.

## Examples

- Each example is step-by-step, with thorough annotation to the right explaining each step.
- Following each example is a similar **Skill Practice** exercise to give you a chance to test your understanding. You will find the answer at the bottom of the page—providing a quick check.

## Exercise Sets

Each type of exercise is built so you can successfully learn the materials and show your mastery on exams.

- **Activities for discovery-based learning** appear before the exercise sets to walk students through the concepts presented in each section of the text.
- **Study Skills Exercises** integrate your studies of math concepts with strategies for helping you grow as a student overall.
- **Vocabulary and Key Concept Exercises** check your understanding of the language and ideas presented within the section.
- **Prerequisite Review** exercises keep fresh your knowledge of math content already learned by providing practice with concepts explored in previous sections.
- **Concept Exercises** assess your comprehension of the specific math concepts presented within the section.
- **Mixed Exercises** evaluate your ability to successfully complete exercises that combine multiple concepts presented within the section.
- **Expanding Your Skills** challenge you with advanced skills practice exercises around the concepts presented within the section.
- **Problem Recognition Exercises** appear in strategic locations in each chapter of the text. These will require you to distinguish between similar problem types and to determine what type of problem-solving technique to apply.
- **Technology Exercises** appear where appropriate.

## End-of-Chapter Materials

The features at the end of each chapter and online are perfect for reviewing before test time.

- **Chapter Review Exercises** provide additional opportunities to practice material from the entire chapter.
- **Section-by-section summaries** provide references to key concepts, examples, and vocabulary.
- **Chapter tests** are an excellent way to test your complete understanding of the chapter concepts.

## How Will Miller/O'Neill/Hyde Help Your Students *Get Better Results*?

### Clarity, Quality, and Accuracy

Julie Miller, Molly O'Neill, and Nancy Hyde know what students need to be successful in mathematics. Better results come from clarity in their exposition, quality of step-by-step worked examples, and accuracy of their exercise sets; but it takes more than just great authors to build a textbook series to help students achieve success in mathematics. Our authors worked with a strong team of mathematics instructors from around the country to ensure that the clarity, quality, and accuracy you expect from the Miller/O'Neill/Hyde series was included in this edition.

### Exercise Sets

Comprehensive sets of exercises are available for every student level. Julie Miller, Molly O'Neill, and Nancy Hyde worked with a board of advisors from across the country to offer the appropriate depth and breadth of exercises for your students. **Problem Recognition Exercises** were created to improve student performance while testing.

Practice exercise sets help students progress from skill development to conceptual understanding. Student tested and instructor approved, the Miller/O'Neill/Hyde exercise sets will help your students *get better results*.

- ▶ **Activities for Discovery-Based Learning**
- ▶ **Prerequisite Review Exercises**
- ▶ **Problem Recognition Exercises**
- ▶ **Skill Practice Exercises**
- ▶ **Study Skills Exercises**
- ▶ **Mixed Exercises**
- ▶ **Expanding Your Skills Exercises**
- ▶ **Vocabulary and Key Concepts Exercises**
- ▶ **Technology Exercises**

### Step-By-Step Pedagogy

This text provides enhanced step-by-step learning tools to help students *get better results*.

- ▶ **For Review** tips placed in the margin guide students back to related prerequisite skills needed for full understanding of course-level topics.
- ▶ **Worked Examples** provide an “easy-to-understand” approach, clearly guiding each student through a step-by-step approach to master each practice exercise for better comprehension.
- ▶ **TIPs** offer students extra cautious direction to help improve understanding through hints and further insight.
- ▶ **Avoiding Mistakes** boxes alert students to common errors and provide practical ways to avoid them. Both of these learning aids will help students get better results by showing how to work through a problem using a clearly defined step-by-step methodology that has been class tested and student approved.

# Get Better Results

## Formula for Student Success

### Step-by-Step Worked Examples

- ▶ Do you get the feeling that there is a disconnect between your students' class work and homework?
- ▶ Do your students have trouble finding worked examples that match the practice exercises?
- ▶ Do you prefer that your students see examples in the textbook that match the ones you use in class?

Miller/O'Neill/Hyde's *Worked Examples* offer a clear, concise methodology that replicates the mathematical processes used in the authors' classroom lectures.

**Example 6** Solving a Linear Equation

Solve.  $2x + 3x + 2 = -4(3 - x)$

**Solution:**

$$2x + 3x + 2 = -4(3 - x)$$
$$5x + 2 = -12 + 4x$$
$$5x - 4x + 2 = -12 + 4x - 4x$$
$$x + 2 = -12$$
$$x + 2 - 2 = -12 - 2$$

**Step 1:** Simplify both sides of the equation. On the left, combine *like* terms. On the right, clear parentheses.

**Step 2:** Subtract  $4x$  from both sides to collect the variable terms on the left. Simplify.

**Step 3:** Subtract  $2$  from both sides to collect the constants on the right.

**TIP:** A linear equation in one variable has one unique solution. As you continue your study of algebra you will also encounter equations that may have no solution or infinitely many solutions.

### Classroom Examples

To ensure that the classroom experience also matches the examples in the text and the practice exercises, we have included references to even-numbered exercises to be used as Classroom Examples. These exercises are highlighted in the Practice Exercises at the end of each section.

**Example 1** Determining Place Value

Determine the place value of the digit 2.

a. 417,216,900      b. 724      c. 502,000,700

**Solution:**

a. 417,216,900      hundred-thousands

b. 724      tens

c. 502,000,700      millions



## Quality Learning Tools

### For Review Boxes

Throughout the text, just-in-time tips and reminders of prerequisite skills appear in the margin alongside the concepts for which they are needed. References to prior sections are given for cases where more comprehensive review is available earlier in the text.

#### FOR REVIEW

Recall that addition may be performed in any order.

$$\begin{array}{r} 200 \text{ ft} \\ 200 \text{ ft} \\ 300 \text{ ft} \\ 275 \text{ ft} \\ + 475 \text{ ft} \\ \hline 1450 \text{ ft} \end{array}$$

### TIP and Avoiding Mistakes Boxes

**TIP** and **Avoiding Mistakes** boxes have been created based on the authors' classroom experiences—they have also been integrated into the **Worked Examples**. These pedagogical tools will help students get better results by learning how to work through a problem using a clearly defined step-by-step methodology.

#### Example 6 Simplifying Expressions

Simplify.

- a.  $-(-9)$       b.  $-|-12|$       c.  $-|7|$

**Solution:**

- a.  $-(-9) = 9$       This represents the opposite of  $-9$ , which is 9.  
b.  $-|-12| = -12$       This represents the opposite of  $|-12|$ . Since  $|-12|$  is equal to 12, the opposite is  $-12$ .  
c.  $-|7| = -7$       This represents the opposite of  $|7|$ . Since  $|7|$  is equal to 7, the opposite is  $-7$ .

**Skill Practice** Simplify.

15.  $-(-34)$       16.  $-|-20|$       17.  $-|4|$

#### Avoiding Mistakes

In Example 6(b) two operations are performed. First take the absolute value of  $-12$ . Then determine the opposite of the result.

#### Answers

13. 108      14.  $-54$   
15. 34      16.  $-20$       17.  $-4$

### Avoiding Mistakes Boxes:

Avoiding Mistakes boxes are integrated throughout the textbook to alert students to common errors and how to avoid them.

**TIP:** To simplify square roots, it is advisable to become familiar with these squares and square roots.

$0^2 = 0 \rightarrow \sqrt{0} = 0$	$7^2 = 49 \rightarrow \sqrt{49} = 7$
$1^2 = 1 \rightarrow \sqrt{1} = 1$	$8^2 = 64 \rightarrow \sqrt{64} = 8$
$2^2 = 4 \rightarrow \sqrt{4} = 2$	$9^2 = 81 \rightarrow \sqrt{81} = 9$
$3^2 = 9 \rightarrow \sqrt{9} = 3$	$10^2 = 100 \rightarrow \sqrt{100} = 10$
$4^2 = 16 \rightarrow \sqrt{16} = 4$	$11^2 = 121 \rightarrow \sqrt{121} = 11$
$5^2 = 25 \rightarrow \sqrt{25} = 5$	$12^2 = 144 \rightarrow \sqrt{144} = 12$
$6^2 = 36 \rightarrow \sqrt{36} = 6$	$13^2 = 169 \rightarrow \sqrt{169} = 13$

### TIP Boxes

Teaching tips are usually revealed only in the classroom. Not anymore! TIP boxes offer students helpful hints and extra direction to help improve understanding and provide further insight.

# Get Better Results

## Better Exercise Sets and Better Practice Yield Better Results

- ▶ Do your students have trouble with problem solving?
- ▶ Do you want to help students overcome math anxiety?
- ▶ Do you want to help your students improve performance on math assessments?

## Problem Recognition Exercises

*Problem Recognition Exercises* present a collection of problems that look similar to a student upon first glance, but are actually quite different in the manner of their individual solutions. Students sharpen critical thinking skills and better develop their “solution recall” to help them distinguish the method needed to solve an exercise—an essential skill in mathematics.

**Problem Recognition Exercises** were tested in the authors’ developmental mathematics classes and were created to improve student performance on tests.

### Problem Recognition Exercises

#### Operations on Whole Numbers

For Exercises 1–14, perform the indicated operations.

1. a. 
$$\begin{array}{r} 96 \\ + 24 \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 96 \\ - 24 \\ \hline \end{array}$$

c. 
$$\begin{array}{r} 96 \\ \times 24 \\ \hline \end{array}$$

d.  $24\overline{)96}$

2. a. 
$$\begin{array}{r} 550 \\ + 25 \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 550 \\ - 25 \\ \hline \end{array}$$

c. 
$$\begin{array}{r} 550 \\ \times 25 \\ \hline \end{array}$$

d.  $25\overline{)550}$

3. a. 
$$\begin{array}{r} 612 \\ + 334 \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 946 \\ - 334 \\ \hline \end{array}$$

4. a. 
$$\begin{array}{r} 612 \\ - 334 \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 278 \\ + 334 \\ \hline \end{array}$$

5. a. 
$$\begin{array}{r} 5500 \\ - 4299 \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 1201 \\ + 4299 \\ \hline \end{array}$$

6. a. 
$$\begin{array}{r} 22,718 \\ + 12,137 \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 34,855 \\ - 12,137 \\ \hline \end{array}$$

7. a.  $50 \cdot 400$

b.  $20,000 \div 50$

8. a.  $548 \cdot 63$

b.  $34,524 \div 63$

9. a.  $5060 \div 22$

b.  $230 \cdot 22$

10. a.  $1875 \div 125$

b.  $125 \cdot 15$

11. a.  $4\overline{)1312}$

b.  $328\overline{)1312}$

12. a.  $547\overline{)4376}$

b.  $8\overline{)4376}$

13. a.  $418 \cdot 10$

b.  $418 \cdot 100$

c.  $418 \cdot 1000$

d.  $418 \cdot 10,000$

14. a.  $350,000 \div 10$

b.  $350,000 \div 100$

c.  $350,000 \div 1000$

d.  $350,000 \div 10,000$

# Get Better Results

## Student-Centered Applications

The Miller/O'Neill/Hyde Board of Advisors partnered with our authors to bring the *best applications* from every region in the country! These applications include real data and topics that are more relevant and interesting to today's student.

24. Liu earned \$312 on an investment of \$800. How much would \$1100 have earned in the same investment?
25. A skyscraper in Chicago is 1454 ft high. If a model is made in which 1 in. represents 50 ft, how high would the building be in the model?

## Activities

Each section of the text ends with an activity that steps the student through the major concepts of the section. The purpose of the activities is to promote active, discovery-based learning for the student. The implementation of the activities is flexible for a variety of delivery methods. For face-to-face classes, the activities can be used to break up lecture by covering the exercises intermittently during the class. For the flipped classroom and hybrid classes, students can watch the videos and try the activities. Then, in the classroom, the instructor can go over the activities or have the students compare their answers in groups. For online classes, the activities provide great discussion questions.

### Section 1.1 Activity

- A.1. In a recent presidential election, the State of Wisconsin had 1,902,505 people request an absentee ballot.
    - a. Determine the place value of the underlined digit. 1,902,505 \_\_\_\_\_
    - b. Convert 1,902,505 to expanded form.
    - c. Write 1,902,505 in words.
  - A.2. Of the 1,902,505 total absentee ballots requested in Wisconsin, one million, eight hundred ninety-six thousand, five hundred thirty-one ballots were sent to voters. Write this number in standard form.
- For Exercises A.3–A.6:
- a. Write two true inequalities (one using  $>$  and one using  $<$ ) for each pair of values given below.
  - b. Translate one of the inequalities to words.
- A.3. 210 and 201
    - a. \_\_\_\_\_ or \_\_\_\_\_
    - b. \_\_\_\_\_
  - A.4. 2233 and 2323
    - a. \_\_\_\_\_ or \_\_\_\_\_
    - b. \_\_\_\_\_
  - A.5. 79 and 76
    - a. \_\_\_\_\_ or \_\_\_\_\_
    - b. \_\_\_\_\_
  - A.6. 614 and 641
    - a. \_\_\_\_\_ or \_\_\_\_\_
    - b. \_\_\_\_\_
  - A.7. Consider the numbers 5, 9, 2, and 7.
    - a. What is the greatest four-digit number that can be formed from the digits? Use each digit only once.
    - b. What is the smallest four-digit number that can be formed from the digits? Use each digit only once.
    - c. Write the number from part (b) in words.



# Get Better Results

## *Additional Supplements*

### Lecture Videos Created by the Authors

Julie Miller began creating these lecture videos for her own students to use when they were absent from class. The student response was overwhelmingly positive, prompting the author team to create the lecture videos for their entire developmental math book series. In these videos, the authors walk students through the learning objectives using the same language and procedures outlined in the book. Students learn and review right alongside the author! Students can also access the written notes that accompany the videos.

### Integrated Video and Study Workbooks

The Integrated Video and Study Workbooks were built to be used in conjunction with the Miller/O'Neill/Hyde Developmental Math series online lecture videos. These new video guides allow students to consolidate their notes as they work through the material in the book, and they provide students with an opportunity to focus their studies on particular topics that they are struggling with rather than entire chapters at a time. Each video guide contains written examples to reinforce the content students are watching in the corresponding lecture video, along with additional written exercises for extra practice. There is also space provided for students to take their own notes alongside the guided notes already provided. By the end of the academic term, the video guides will not only be a robust study resource for exams, but will serve as a portfolio showcasing the hard work of students throughout the term.

### Dynamic Math Animations

The authors have constructed a series of animations to illustrate difficult concepts where static images and text fall short. The animations leverage the use of on-screen movement and morphing shapes to give students an interactive approach to conceptual learning. Some provide a virtual laboratory for which an application is simulated and where students can collect data points for analysis and modeling. Others provide interactive question-and-answer sessions to test conceptual learning.

### Exercise Videos

The authors, along with a team of faculty who have used the Miller/O'Neill/Hyde textbooks for many years, have created exercise videos for designated exercises in the textbook. These videos cover a representative sample of the main objectives in each section of the text. Each presenter works through selected problems, following the solution methodology employed in the text.

The video series is available online as part of ALEKS 360. The videos are closed-captioned for the hearing impaired and meet the Americans with Disabilities Act Standards for Accessible Design.

### Student Resource Manual

The *Student Resource Manual (SRM)*, created by the authors, is a printable, electronic supplement available to students through ALEKS. Instructors can also choose to customize this manual and package it with their course materials. With increasing demands on faculty schedules, this resource offers a convenient means for both full-time and adjunct faculty to promote active learning and success strategies in the classroom.

This manual supports the series in a variety of different ways:

- Additional group activities developed by the authors to supplement what is already available in the text
- Discovery-based classroom activities written by the authors for each section
- Excel activities that not only provide students with numerical insights into algebraic concepts, but also teach simple computer skills to manipulate data in a spreadsheet

# Get Better Results

- Worksheets for extra practice written by the authors, including Problem Recognition Exercise Worksheets
- Lecture Notes designed to help students organize and take notes on key concepts
- Materials for a student portfolio

## Annotated Instructor's Edition

In the *Annotated Instructor's Edition (AIE)*, answers to all exercises appear adjacent to each exercise in a color used *only* for annotations. The *AIE* also contains Instructor Notes that appear in the margin. These notes offer instructors assistance with lecture preparation. In addition, there are Classroom Examples referenced in the text that are highlighted in the Practice Exercises. Also found in the *AIE* are icons within the Practice Exercises that serve to guide instructors in their preparation of homework assignments and lessons.

## PowerPoints

The PowerPoints present key concepts and definitions with fully editable slides that follow the textbook. An instructor may project the slides in class or post to a website in an online course.

## Test Bank

Among the supplements is a computerized test bank using the algorithm-based testing software TestGen® to create customized exams quickly. Hundreds of text-specific, open-ended, and multiple-choice questions are included in the question bank.

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Elisha Van Meenen, *Illinois State University*  
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 Deanna Voehl, *Indian River State College*  
 Joe Jordan, *John Tyler Community College*  
 Sally Copeland, *Johnson County Community College*  
 Nancy Carpenter, *Johnson County Community College*  
 Susan Yellott, *Kilgore College*  
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 Michelle Hempton, *Lansing Community College*  
 Michelle Whitmer, *Lansing Community College*  
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 Jason Pallett, *MCC-Longview Community College*  
 Janet Wyatt, *MCC-Longview Community College*  
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 Nelson De La Rosa, *Miami Dade College—Kendall*  
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 Ken Hirschel, *Orange County Community College*  
 Linda K. Schott, *Ozarks Technical Community College*  
 Matthew Harris, *Ozarks Technical Community College*  
 Daniel Kopsas, *Ozarks Technical Community College*  
 Andrew Aberle, *Ozarks Technical Community College*  
 Alan Papen, *Ozarks Technical Community College*  
 Angela Shreckhise, *Ozarks Technical Community College*  
 Jacob Lewellen, *Ozarks Technical Community College*  
 Marylynne Abbott, *Ozarks Technical Community College*  
 Jeffrey Gervasi, *Porterville College*  
 Stewart Hathaway, *Porterville College*  
 Luran Johnson, *Richard Bland College*  
 Matthew Nickodemus, *Richard Bland College*  
 Cameron English, *Rio Hondo College*  
 Lydia Gonzalez, *Rio Hondo College*  
 Mark Littrell, *Rio Hondo College*  
 Matthew Pitassi, *Rio Hondo College*  
 Wayne Lee, *Saint Philips College*  
 Paula Looney, *Saint Philips College*  
 Fred Bakenhus, *Saint Philips College*  
 Lydia Casas, *Saint Philips College*  
 Gloria Guerra, *Saint Philips College*  
 Sounny Slitine, *Saint Philips College*  
 Jessica Lopez, *Saint Philips College*  
 Lorraine Lopez, *San Antonio College*  
 Peter Georgakis, *Santa Barbara City College*  
 Sandi Nieto-Navarro, *Santa Rosa Junior College*  
 Steve Drucker, *Santa Rosa Junior College*  
 Jean-Marie Magnier, *Springfield Tech Community College*  
 Dave Delrossi, *Tallahassee Community College*  
 Natalie Johnson, *Tarrant County College South*  
 Marilyn Peacock, *Tidewater Community College*  
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 Jennifer Burkett, *Triton College*  
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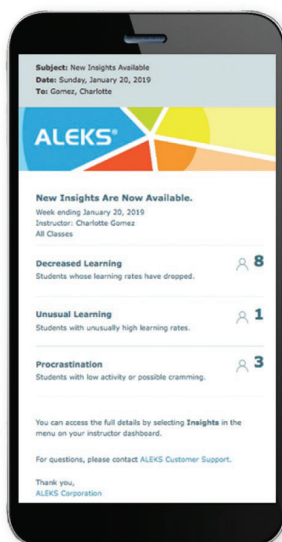
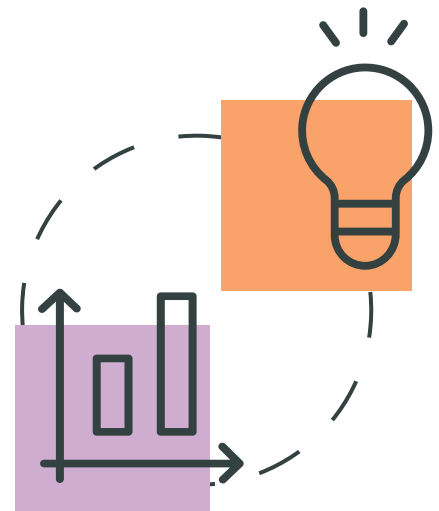
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# Whole Numbers

# 1

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- Chapter 1 Review Exercises 75**

### *Numbers on Vacation*

Since the beginning of human civilization, the need to communicate with one another in a precise, quantifiable language has become increasingly important. For example, to take a vacation to Disney World, a family would want to know the driving distance to the park, the time required to drive there, the cost for tickets, the number of nights for a hotel room, and the estimated amount spent on food and incidentals. Such numerical (quantifiable) information is essential for the family to determine if the vacation is affordable and to form a budget for the vacation.

Suppose the family lives 300 miles from Disney World, drives a car that gets 30 miles per gallon of gasoline, and travels 60 miles per hour. These numerical values are called whole numbers. Whole numbers include 0 and the counting numbers 1, 2, 3, and so on. Operations on whole numbers can help us solve a variety of applications. For example, dividing the whole number 300 miles by 30 miles per gallon tells us that the family will use 10 gallons of gasoline. Furthermore, dividing 300 miles by 60 miles per hour tells us that the family will arrive at Disney World in 5 hours. As you work through this chapter, reflect on how important numbers are to everyday living and how different our world would be without the precision of numerical values.



Ilene MacDonald/Alamy Stock Photo

## Section 1.1 Introduction to Whole Numbers

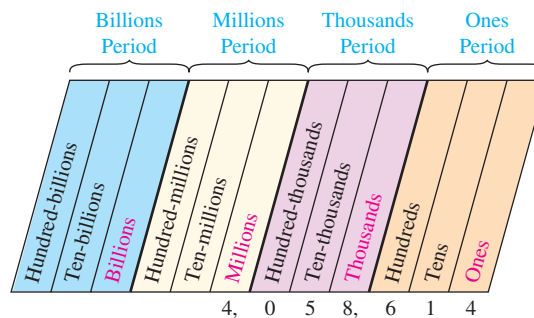
## Concepts

1. Place Value
2. Standard Notation and Expanded Notation
3. Writing Numbers in Words
4. The Number Line and Order

## 1. Place Value

Numbers provide the foundation that is used in mathematics. We begin this chapter by discussing how numbers are represented and named. All numbers in our numbering system are composed from the **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. In mathematics, the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . . are called the *whole numbers*. (The three dots are called *ellipses* and indicate that the list goes on indefinitely.)

For large numbers, commas are used to separate digits into groups of three called **periods**. For example, the number of live births in the United States in a recent year was 4,058,614. (*Source: The World Almanac*) Numbers written in this way are said to be in **standard form**. The position of each digit determines the place value of the digit. To interpret the number of births in the United States, refer to the place value chart (Figure 1-1).



### Figure 1-1

The digit 5 in 4,058,614 represents 5 ten-thousands because it is in the ten-thousands place. The digit 4 on the left represents 4 millions, whereas the digit 4 on the right represents 4 ones.

### Example 1 Determining Place Value

Determine the place value of the digit 2.

- a.** 417,216,900      **b.** 724      **c.** 502,000,700

**Solution:**

- a.** 417,216,900      hundred-thousands  
**b.** 724      tens  
**c.** 502,000,700      millions

**Skill Practice** Determine the place value of the digit 4.

1.  $\underline{5}47,098,632$
2.  $1,659,98\underline{4},036$
3.  $6420$

## Answers

1. Ten-millions
2. Thousands
3. Hundreds



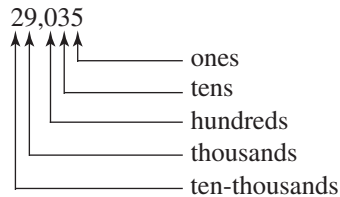
**Example 2** Determining Place Value

The altitude of Mount Everest, the highest mountain on Earth, is 29,035 feet (ft). Give the place value for each digit.



Daniel Prudek/iStockphoto/Getty Images

**Solution:**

**Skill Practice**

4. Alaska is the largest state geographically. Its land area is 571,962 square miles ( $\text{mi}^2$ ). Give the place value for each digit.

**2. Standard Notation and Expanded Notation**

A number can also be written in an expanded form by writing each digit with its place value unit. For example, 287 can be written as

$$\begin{aligned} 287 &= 2 \text{ hundreds} + 8 \text{ tens} + 7 \text{ ones} \\ &= 2 \times 100 + 8 \times 10 + 7 \times 1 \\ &= 200 + 80 + 7 \end{aligned}$$

This is called **expanded form**.

**Example 3** Converting Standard Form to Expanded Form

Convert to expanded form.

- a. 4,672      b. 257,016

**Solution:**

- a. 4,672      4 **thousands** + 6 **hundreds** + 7 **tens** + 2 **ones**  
 $= 4 \times 1,000 + 6 \times 100 + 7 \times 10 + 2 \times 1$   
 $= 4,000 + 600 + 70 + 2$
- b. 257,016      2 **hundred-thousands** + 5 **ten-thousands** +  
 7 **thousands** + 1 **ten** + 6 **ones**  
 $= 2 \times 100,000 + 5 \times 10,000 + 7 \times 1,000 + 1 \times 10 + 6 \times 1$   
 $= 200,000 + 50,000 + 7,000 + 10 + 6$

**Skill Practice** Convert to expanded form.

5. 837      6. 4,093,062

**Answers**

4. 5: hundred-thousands  
 7: ten-thousands  
 1: thousands      9: hundreds  
 6: tens      2: ones
5. 8 hundreds + 3 tens + 7 ones;  
 $8 \times 100 + 3 \times 10 + 7 \times 1$
6. 4 millions + 9 ten-thousands +  
 3 thousands + 6 tens + 2 ones;  
 $4 \times 1,000,000 + 9 \times 10,000 +$   
 $3 \times 1,000 + 6 \times 10 + 2 \times 1$

**Example 4** Converting Expanded Form to Standard Form

Convert to standard form.

- a. 2 hundreds + 5 tens + 9 ones  
b. 1 thousand + 2 tens + 5 ones

**Solution:**

- a.  $2 \text{ hundreds} + 5 \text{ tens} + 9 \text{ ones} = 259$   
b. Each place position from the thousands place to the ones place must contain a digit. In this problem, there is no reference to the hundreds place digit. Therefore, we assume 0 hundreds. Thus,

$$1 \text{ thousand} + 0 \text{ hundreds} + 2 \text{ tens} + 5 \text{ ones} = 1,025$$

**Skill Practice** Convert to standard form.

7. 8 thousands + 5 hundreds + 5 tens + 1 one  
8. 5 hundred-thousands + 4 thousands + 8 tens + 3 ones

**3. Writing Numbers in Words**

The word names of some two-digit numbers appear with a hyphen, while others do not. For example:

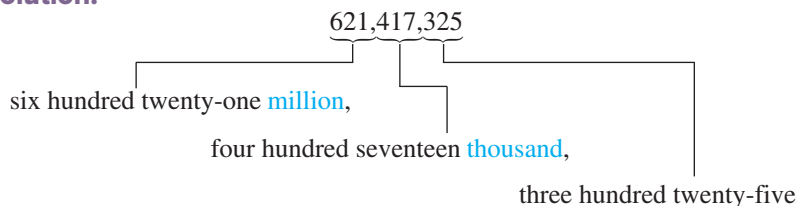
Number	Number Name
12	twelve
68	sixty-eight
40	forty
42	forty-two

To write a three-digit or larger number, begin at the leftmost group of digits. The number named in that group is followed by the period name, followed by a comma. Then the next period is named, and so on.

**Example 5** Writing a Number in Words

Write 621,417,325 in words.

**Solution:**



**Skill Practice**

9. Write 1,450,327,214 in words.

**Answers**

7. 8,551    8. 504,083  
9. One billion, four hundred fifty million,  
three hundred twenty-seven thousand,  
two hundred fourteen

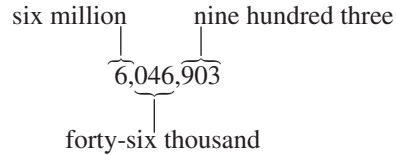
Notice from Example 5 that when naming numbers, the name of the ones period is not attached to the last group of digits. Also note that for whole numbers, the word *and* should not appear in word names. For example, 405 should be written as four hundred five.

**Example 6** Writing a Number in Standard Form

Write the number in standard form.

Six million, forty-six thousand, nine hundred three

**Solution:**

**Skill Practice**

10. Write the number in standard form: fourteen thousand, six hundred nine.

We have seen several examples of writing a number in standard form, in expanded form, and in words. Standard form is the most concise representation. Also note that when we write a four-digit number in standard form, the comma is often omitted. For example, 4,389 is often written as 4389.

**4. The Number Line and Order**

Whole numbers can be visualized as equally spaced points on a line called a *number line* (Figure 1-2).

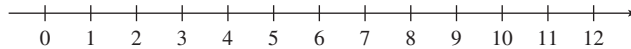
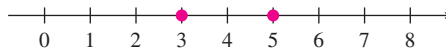


Figure 1-2

The whole numbers begin at 0 and are ordered from left to right by increasing value.

A number is graphed on a number line by placing a dot at the corresponding point. For any two numbers graphed on a number line, the number to the left is less than the number to the right. Similarly, a number to the right is greater than the number to the left. In mathematics, the symbol  $<$  is used to denote “is less than,” and the symbol  $>$  means “is greater than.” Therefore,

$3 < 5$     means    3 is less than 5  
 $5 > 3$     means    5 is greater than 3

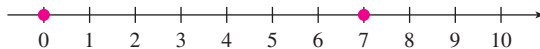
**Example 7** Determining Order of Two Numbers

Fill in the blank with the symbol  $<$  or  $>$ .

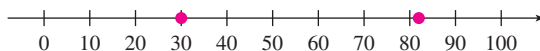
a.  $7 \square 0$                   b.  $30 \square 82$

**Solution:**

a.  $7 \square 0$



b.  $30 \square 82$



To visualize 82 and 30 on the number line, it may be necessary to use a different scale. Rather than setting equally spaced marks in units of 1, we can use units of 10. Then 82 must be somewhere between 80 and 90 on the number line.

**Skill Practice** Fill in the blank with the symbol  $<$  or  $>$ .

11.  $9 \square 5$                   12.  $8 \square 18$

**Answers**

10. 14,609

11.  $>$                   12.  $<$



## Section 1.1 Activity

- A.1.** In a recent presidential election, the State of Wisconsin had 1,902,505 people request an absentee ballot.
- Determine the place value of the underlined digit. 1,902,505 \_\_\_\_\_
  - Convert 1,902,505 to expanded form.
  - Write 1,902,505 in words.
- A.2.** Of the 1,902,505 total absentee ballots requested in Wisconsin, one million, eight hundred ninety-six thousand, five hundred thirty-one ballots were sent to voters. Write this number in standard form.
- For Exercises A.3–A.6:
- Write two true inequalities (one using  $>$  and one using  $<$ ) for each pair of values given below.
  - Translate one of the inequalities to words.
- A.3.** 210 and 201
- \_\_\_\_\_ or \_\_\_\_\_
  - \_\_\_\_\_
- A.4.** 2233 and 2323
- \_\_\_\_\_ or \_\_\_\_\_
  - \_\_\_\_\_
- A.5.** 79 and 76
- \_\_\_\_\_ or \_\_\_\_\_
  - \_\_\_\_\_
- A.6.** 614 and 641
- \_\_\_\_\_ or \_\_\_\_\_
  - \_\_\_\_\_
- A.7.** Consider the numbers 5, 9, 2, and 7.
- What is the greatest four-digit number that can be formed from the digits? Use each digit only once.
  - What is the smallest four-digit number that can be formed from the digits? Use each digit only once.
  - Write the number from part (b) in words.

## Section 1.1 Practice Exercises

### Study Skills Exercise

To enhance your learning experience, we provide study skills throughout this textbook that focus on three main areas: mindset (ability to learn new concepts, grit, and overcoming math anxiety), study habits (managing time, taking notes, and test preparation), and mastering mathematical concepts (writing mathematically, reading comprehension, and memory techniques).

Each activity requires only a few minutes and will help you pass this course and become a better math student. Many of these skills can be carried over to other disciplines and help you become a model college student. To begin, write down the following information:

- |  |   |
|--|---|
| <b>a.</b> Instructor's name                        | <b>b.</b> Instructor's office number  |
| <b>c.</b> Instructor's telephone number            | <b>d.</b> Instructor's email address  |
| <b>e.</b> Instructor's office hours                | <b>f.</b> Days of the week that the class meets   |
| <b>g.</b> The room number in which the class meets | <b>h.</b> Is there a lab requirement for this course?<br>If so, where is the lab located and how often must you go? |

## Vocabulary and Key Concepts

1. a. For large numbers, commas are used to separate digits into groups called \_\_\_\_\_.
- b. The place values of the digits in the ones period are the ones, tens, and \_\_\_\_\_ places.
- c. The place values of the digits in the \_\_\_\_\_ period are the thousands, ten-thousands, and hundred-thousands places.

## Concept 1: Place Value

2. Name the place value for each digit in 36,791.
3. Name the place value for each digit in 8,213,457.
4. Name the place value for each digit in 103,596.

For Exercises 5–24, determine the place value for each underlined digit. (See Example 1.)

- |                     |                     |                           |                           |
|---------------------|---------------------|---------------------------|---------------------------|
| 5. 3 <u>2</u> 1     | 6. 6 <u>8</u> 9     | 7. 2 <u>1</u> 4           | 8. 7 <u>3</u> 8           |
| 9. 8, <u>7</u> 10   | 10. 2, <u>2</u> 93  | 11. <u>1</u> ,430         | 12. <u>3</u> ,101         |
| 13. <u>4</u> 52,723 | 14. <u>6</u> 55,878 | 15. <u>1</u> ,023,676,207 | 16. <u>3</u> ,111,901,211 |
| 17. <u>2</u> 2,422  | 18. <u>5</u> 8,106  | 19. 5 <u>1</u> ,033,201   | 20. 9 <u>3</u> ,971,224   |
21. The number of U.S. travelers abroad in a recent year was 10,677,881. (See Example 2.)
22. The area of Lake Superior is 31,820 square miles (mi<sup>2</sup>).



Morey Milbradt/Getty Images

23. For a recent year, the total number of U.S. \$1 bills in circulation was 7,653,468,440.
24. For a certain flight, the cruising altitude of a commercial jet is 31,000 ft.

## Concept 2: Standard Notation and Expanded Notation

For Exercises 25–32, convert the numbers to expanded form. (See Example 3.)

- |           |           |            |            |
|-----------|-----------|------------|------------|
| 25. 58    | 26. 71    | 27. 539    | 28. 382    |
| 29. 5,203 | 30. 7,089 | 31. 10,241 | 32. 20,873 |

For Exercises 33–40, convert the numbers to standard form. (See Example 4.)

- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| 33. 5 hundreds + 2 tens + 4 ones     | 34. 3 hundreds + 1 ten + 8 ones      |
| 35. 1 hundred + 5 tens               | 36. 6 hundreds + 2 tens              |
| 37. 1 thousand + 9 hundreds + 6 ones | 38. 4 thousands + 2 hundreds + 1 one |

39. 8 ten-thousands + 5 thousands + 7 ones

40. 2 ten-thousands + 6 thousands + 2 ones

41. Name the first four periods of a number (from right to left).

42. Name the first four place values of a number (from right to left).

### Concept 3: Writing Numbers in Words

For Exercises 43–50, write the number in words. (See Example 5.)

43. 241

44. 327

45. 603

46. 108

47. 31,530

48. 52,160

49. 100,234

50. 400,199

51. The Shuowen jiezi dictionary, an ancient Chinese dictionary that dates back to the year 100, contained 9535 characters. Write 9535 in words.

52. Interstate I-75 is 1377 miles (mi) long. Write 1377 in words.

53. The altitude of Denali in Alaska is 20,310 ft. Write 20,320 in words.

54. There are 1800 seats in a theater. Write 1800 in words.

55. Researchers calculate that about 590,712 stone blocks were used to construct the Great Pyramid. Write 590,712 in words.

56. In the United States, there are approximately 60,000,000 cats living in households. Write 60,000,000 in words.



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GK Hart/Vikki Hart/Getty Images

For Exercises 57–62, convert the number to standard form. (See Example 6.)

57. Six thousand, five

58. Four thousand, four

59. Six hundred seventy-two thousand

60. Two hundred forty-eight thousand

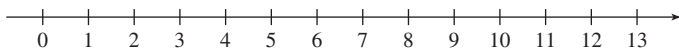
61. One million, four hundred eighty-four thousand, two hundred fifty

62. Two million, six hundred forty-seven thousand, five hundred twenty

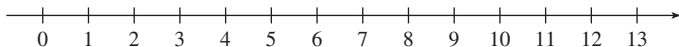
### Concept 4: The Number Line and Order

For Exercises 63 and 64, graph the numbers on the number line.

63. a. 6      b. 13      c. 8      d. 1



64. a. 5      b. 3      c. 11      d. 9



65. On a number line, what number is 4 units to the right of 6?

66. On a number line, what number is 8 units to the left of 11?

67. On a number line, what number is 3 units to the left of 7?

68. On a number line, what number is 5 units to the right of 0?

For Exercises 69–72, translate the inequality to words.

69.  $8 > 2$

70.  $6 < 11$

71.  $3 < 7$

72.  $14 > 12$

For Exercises 73–84, fill in the blank with the inequality symbol  $<$  or  $>$ . (See Example 7.)

73.  $6 \square 11$

74.  $14 \square 13$

75.  $21 \square 18$

76.  $5 \square 7$

77.  $3 \square 7$

78.  $14 \square 24$

79.  $95 \square 89$

80.  $28 \square 30$

81.  $0 \square 3$

82.  $8 \square 0$

83.  $90 \square 91$

84.  $48 \square 47$

### Expanding Your Skills

85. Answer true or false. 12 is a digit.

86. Answer true or false. 26 is a digit.

87. What is the greatest two-digit number?

88. What is the greatest three-digit number?

89. What is the greatest whole number?

90. What is the least whole number?

91. How many zeros are there in the number ten million?

92. How many zeros are there in the number one hundred billion?

93. What is the greatest three-digit number that can be formed from the digits 6, 9, and 4? Use each digit only once.

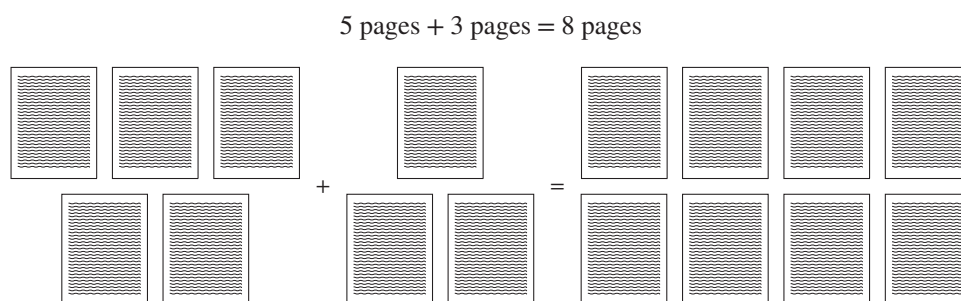
94. What is the greatest three-digit number that can be formed from the digits 0, 4, and 8? Use each digit only once.

## Addition and Subtraction of Whole Numbers and Perimeter

### Section 1.2

### 1. Addition of Whole Numbers

We use addition of whole numbers to represent an increase in quantity. For example, suppose Jonas typed 5 pages of a report before lunch. Later in the afternoon he typed 3 more pages. The total number of pages that he typed is found by adding 5 and 3.



The result of an addition problem is called the **sum**, and the numbers being added are called **addends**. Thus,

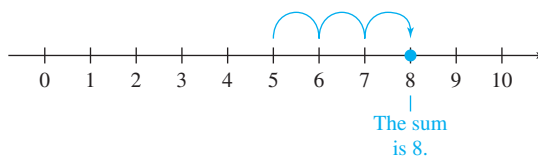
$$\begin{array}{c} 5 + 3 = 8 \\ \swarrow \quad \uparrow \quad \uparrow \\ \text{addends} \quad \text{sum} \end{array}$$

### Concepts

1. Addition of Whole Numbers
2. Properties of Addition
3. Subtraction of Whole Numbers
4. Translations and Applications Involving Addition and Subtraction
5. Perimeter



The number line is a useful tool to visualize the operation of addition. To add 5 and 3 on a number line, begin at 5 and move 3 units to the right. The final location indicates the sum.



You can use a number line to find the sum of any pair of digits. The sums for all possible pairs of one-digit numbers should be memorized (see Exercise 7). Memorizing these basic addition facts will make it easier for you to add larger numbers.

To add whole numbers with several digits, line up the numbers vertically by place value. Then add the digits in the corresponding place positions.

### Example 1 Adding Whole Numbers

Add.  $261 + 28$

**Solution:**

$$\begin{array}{r} 261 \\ + 28 \\ \hline 289 \end{array}$$

Add digits in  
ones column.

Add digits in  
tens column.

Add digits in  
hundreds column.

**Skill Practice** Add.

1.  $4135 + 210$

Sometimes when adding numbers, the sum of the digits in a given place position is greater than 9. If this occurs, we must do what is called *carrying* or *regrouping*. Example 2 illustrates this process.

### Example 2 Adding Whole Numbers with Carrying

Add.  $35 + 48$

**Solution:**

$$\begin{array}{r} 35 = 3 \text{ tens} + 5 \text{ ones} \\ + 48 = 4 \text{ tens} + 8 \text{ ones} \\ \hline 7 \text{ tens} + 13 \text{ ones} \end{array}$$

← The sum of the digits in the ones place exceeds 9. But 13 ones is the same as 1 ten and 3 ones. We can *carry* 1 ten to the tens column while leaving the 3 ones in the ones column. Notice that we placed the carried digit above the tens column.

$$\begin{array}{r} 1 \text{ ten} \\ 35 = 3 \text{ tens} + 5 \text{ ones} \\ + 48 = 4 \text{ tens} + 8 \text{ ones} \\ \hline 83 = 8 \text{ tens} + 3 \text{ ones} \end{array}$$

The sum is 83.

**Skill Practice** Add.

2.  $43 + 29$

### Answers

1. 4345
2. 72

Addition of numbers may include more than two addends.

### Example 3 Adding Whole Numbers

Add.  $21,076 + 84,158 + 2419$

**Solution:**

$\begin{array}{r} 21,076 \\ 84,158 \\ + 2,419 \\ \hline 107,653 \end{array}$	<p>In this example, the sum of the digits in the ones column is 23. Therefore, we write the 3 and carry the 2.</p> <p>In the tens column, the sum is 15. Write the 5 in the tens place and carry the 1.</p>
--	---

**Skill Practice** Add.

$$\begin{array}{r} 3. \quad 57,296 \\ \quad 4,089 \\ + \quad 9,762 \\ \hline \end{array}$$

## 2. Properties of Addition

A **variable** is a letter or symbol that represents a number. The following are examples of variables:  $a$ ,  $b$ , and  $c$ . We will use variables to present three important properties of addition.

Most likely you have noticed that 0 added to any number is that number. For example:

$$6 + 0 = 6 \quad 527 + 0 = 527 \quad 0 + 88 = 88 \quad 0 + 15 = 15$$

In each example, the number in red can be replaced with any number that we choose, and the statement would still be true. This fact is stated as the addition property of 0.

### Addition Property of 0

For any number  $a$ ,

$$a + 0 = a \quad \text{and} \quad 0 + a = a$$

The sum of any number and 0 is that number.

The order in which we add two numbers does not affect the result. For example:  $11 + 20 = 20 + 11$ . This is true for any two numbers and is stated in the next property.

### Commutative Property of Addition

For any numbers  $a$  and  $b$ ,

$$a + b = b + a$$

Changing the order of two addends does not affect the sum.

In mathematics we use parentheses ( ) as grouping symbols. To add more than two numbers, we can group them and then add. For example:

$\begin{aligned} &(2 + 3) + 8 \\ &= 5 + 8 \\ &= 13 \end{aligned}$	<p>Parentheses indicate that <math>2 + 3</math> is added first. Then 8 is added to the result.</p>
$\begin{aligned} &2 + (3 + 8) \\ &= 2 + 11 \\ &= 13 \end{aligned}$	<p>Parentheses indicate that <math>3 + 8</math> is added first. Then the result is added to 2.</p>

**Answer**

3. 71,147

### Associative Property of Addition

For any numbers  $a$ ,  $b$ , and  $c$ ,

$$(a + b) + c = a + (b + c)$$

The manner in which addends are grouped does not affect the sum.

#### Example 4 Applying the Properties of Addition

- Rewrite  $9 + 6$ , using the commutative property of addition.
- Rewrite  $(15 + 9) + 5$ , using the associative property of addition.

#### Solution:

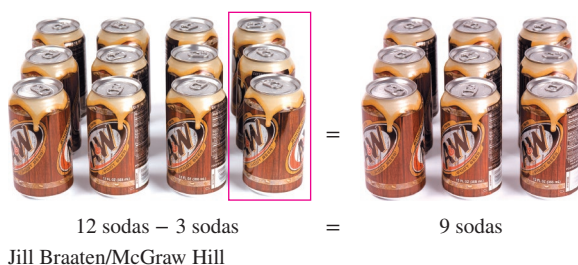
- $9 + 6 = 6 + 9$  Change the order of the addends.
- $(15 + 9) + 5 = 15 + (9 + 5)$  Change the grouping of the addends.

#### Skill Practice

- Rewrite  $3 + 5$ , using the commutative property of addition.
- Rewrite  $(1 + 7) + 12$ , using the associative property of addition.

## 3. Subtraction of Whole Numbers

Jeremy bought a case of 12 sodas, and on a hot afternoon he drank 3 of the sodas. We can use the operation of subtraction to find the number of sodas remaining.



The symbol “−” between two numbers is a subtraction sign, and the result of a subtraction is called the **difference**. The number being subtracted (in this case, 3) is called the **subtrahend**. The number 12 from which 3 is subtracted is called the **minuend**.

$$\begin{array}{c} 12 - 3 = 9 \\ \swarrow \quad \uparrow \quad \nwarrow \\ \text{minuend} \quad \text{subtrahend} \quad \text{difference} \end{array} \quad \text{is read as} \quad \text{“12 minus 3 is equal to 9”}$$

Subtraction is the reverse operation of addition. To find the number of sodas that remain after Jeremy takes 3 sodas away from 12 sodas, we ask the question:

“3 added to what number equals 12?”

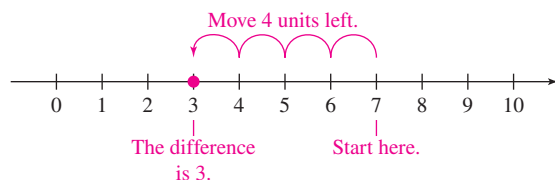
That is,

$$12 - 3 = ? \quad \text{is equivalent to} \quad ? + 3 = 12$$

#### Answers

- $3 + 5 = 5 + 3$
- $(1 + 7) + 12 = 1 + (7 + 12)$

Subtraction can also be visualized on the number line. To evaluate  $7 - 4$ , start from the point on the number line corresponding to the minuend (7 in this case). Then move to the left 4 units. The resulting position on the number line is the difference.



To check the result, we can use addition.

$$7 - 4 = 3 \quad \text{because} \quad 3 + 4 = 7$$

### Example 5 Subtracting Whole Numbers

Subtract and check the answer by using addition.

- a.  $8 - 2$       b.  $10 - 6$       c.  $5 - 0$       d.  $3 - 3$

**Solution:**

- a.  $8 - 2 = 6$  because  $6 + 2 = 8$       b.  $10 - 6 = 4$  because  $4 + 6 = 10$   
 c.  $5 - 0 = 5$  because  $5 + 0 = 5$       d.  $3 - 3 = 0$  because  $0 + 3 = 3$

**Skill Practice** Subtract. Check by using addition.

6.  $11 - 5$       7.  $8 - 0$       8.  $7 - 2$       9.  $5 - 5$

When subtracting large numbers, it is usually more convenient to write the numbers vertically. We write the minuend on top and the subtrahend below it. Starting from the ones column, we subtract digits having corresponding place values.

### Example 6 Subtracting Whole Numbers

Subtract and check the answer by using addition.

- a. 
$$\begin{array}{r} 976 \\ - 124 \\ \hline \end{array}$$
      b. 
$$\begin{array}{r} 2498 \\ - 197 \\ \hline \end{array}$$

**Solution:**

- a. 
$$\begin{array}{r} 976 \\ - 124 \\ \hline 852 \end{array}$$
      **Check:** 
$$\begin{array}{r} 852 \\ + 124 \\ \hline 976 \end{array} \checkmark$$
  
 Subtract the ones column digits.  
 Subtract the tens column digits.  
 Subtract the hundreds column digits.

- b. 
$$\begin{array}{r} 2498 \\ - 197 \\ \hline 2301 \end{array}$$
      **Check:** 
$$\begin{array}{r} 2301 \\ + 197 \\ \hline 2498 \end{array} \checkmark$$

**Skill Practice** Subtract. Check by using addition.

10. 
$$\begin{array}{r} 472 \\ - 261 \\ \hline \end{array}$$
      11. 
$$\begin{array}{r} 3947 \\ - 137 \\ \hline \end{array}$$

### Answers

6. 6      7. 8      8. 5      9. 0  
 10. 211      11. 3810



When a digit in the subtrahend (bottom number) is larger than the corresponding digit in the minuend (top number), we must “regroup” or borrow a value from the column to the left.

$$\begin{array}{r} 92 = 9 \text{ tens} + 2 \text{ ones} \\ - 74 = 7 \text{ tens} + 4 \text{ ones} \\ \hline \end{array}$$

In the ones column, we cannot take 4 away from 2. We will regroup by borrowing 1 ten from the minuend. Furthermore, 1 ten = 10 ones.

$$\begin{array}{r} \overset{8+10}{\cancel{9}} \overset{8}{2} = \overset{8}{\cancel{8}} \text{ tens} + \overset{+10 \text{ ones}}{2} \text{ ones} \\ - 74 = 7 \text{ tens} + 4 \text{ ones} \\ \hline \end{array}$$

We now have 12 ones in the minuend.

$$\begin{array}{r} \overset{8}{\cancel{9}} \overset{12}{2} = \overset{8}{\cancel{8}} \text{ tens} + 12 \text{ ones} \\ - 74 = 7 \text{ tens} + 4 \text{ ones} \\ \hline 18 = 1 \text{ ten} + 8 \text{ ones} \end{array}$$

**TIP:** The process of *borrowing* in subtraction is the reverse operation of *carrying* in addition.

### Example 7

### Subtracting Whole Numbers with Borrowing

Subtract and check the result with addition.

$$\begin{array}{r} 134,616 \\ - 53,438 \\ \hline \end{array}$$

**Solution:**

$$\begin{array}{r} 134, \overset{0}{\cancel{6}} \overset{16}{\cancel{1}} \overset{16}{\cancel{6}} \\ - 53,438 \\ \hline 8 \end{array}$$

In the ones place, 8 is greater than 6. We borrow 1 ten from the tens place.

$$\begin{array}{r} 134, \overset{10}{\cancel{5}} \overset{16}{\cancel{1}} \overset{16}{\cancel{6}} \\ - 53,438 \\ \hline 78 \end{array}$$

In the tens place, 3 is greater than 0. We borrow 1 hundred from the hundreds place.

$$\begin{array}{r} \overset{0}{\cancel{1}} \overset{13}{\cancel{3}} \overset{16}{\cancel{4}}, \overset{5}{\cancel{1}} \overset{16}{\cancel{1}} \overset{16}{\cancel{6}} \\ - 53,438 \\ \hline 81,178 \end{array}$$

In the ten-thousands place, 5 is greater than 3. We borrow 1 hundred-thousand from the hundred-thousands place.

$$\begin{array}{r} \text{Check:} \quad 81, \overset{1}{\cancel{1}} \overset{1}{\cancel{7}} \overset{1}{\cancel{8}} \\ + 53,438 \\ \hline 134,616 \checkmark \end{array}$$

**Skill Practice** Subtract. Check by addition.

$$\begin{array}{r} 12. \quad 23,126 \\ - 6,048 \\ \hline \end{array}$$

### Answer

12. 17,078

**Example 8** Subtracting Whole Numbers with BorrowingSubtract and check the result with addition.  $500 - 247$ **Solution:**

$$\begin{array}{r} 500 \\ - 247 \\ \hline \end{array}$$

In the ones place, 7 is greater than 0. We try to borrow 1 ten from the tens place. However, the tens place digit is 0. Therefore we must first borrow from the hundreds place.

$$\begin{array}{r} \overset{4}{\cancel{5}} \overset{10}{0} 0 \\ - 247 \\ \hline \end{array}$$

← 1 hundred = 10 tens

$$\begin{array}{r} \overset{4}{\cancel{5}} \overset{9}{\cancel{10}} \overset{10}{0} \\ - 247 \\ \hline 253 \end{array}$$

← Now we can borrow 1 ten to add to the ones place.

Subtract.

Check:

$$\begin{array}{r} \overset{1}{2} \overset{1}{5} 3 \\ + 247 \\ \hline 500 \checkmark \end{array}$$

**Skill Practice** Subtract. Check by addition.

13.  $700 - 531$

**4. Translations and Applications Involving Addition and Subtraction**

In the English language, there are many different words and phrases that imply addition. A partial list is given in Table 1-1.

**Table 1-1**

Word/Phrase	Example	In Symbols
Sum	The sum of 6 and $x$	$6 + x$
Added to	3 added to 8	$8 + 3$
Increased by	$y$ increased by 2	$y + 2$
More than	10 more than 6	$6 + 10$
Plus	8 plus 3	$8 + 3$
Total of	The total of $a$ and $b$	$a + b$

**Example 9** Translating an English Phrase to a Mathematical Statement

Translate each phrase to an equivalent mathematical statement and simplify.

- 12 added to 109
- The sum of 1386 and 376

**Answer**

13. 169

**Solution:**

a.  $109 + 12$

$$\begin{array}{r} 109 \\ + 12 \\ \hline 121 \end{array}$$

b.  $1386 + 376$

$$\begin{array}{r} 1386 \\ + 376 \\ \hline 1762 \end{array}$$

**Skill Practice** Translate and simplify.

14. 50 more than 80
15. 12 increased by 14

Table 1-2 gives several key phrases that imply subtraction.

**Table 1-2**

Word/Phrase	Example	In Symbols
Minus	15 minus $x$	$15 - x$
Difference	The difference of 10 and 2	$10 - 2$
Decreased by	$a$ decreased by 1	$a - 1$
Less than	5 less than 12	$12 - 5$
Subtract . . . from	Subtract 3 from 8	$8 - 3$
Subtracted from	6 subtracted from 10	$10 - 6$

In Table 1-2, make a note of the last three entries. The phrases *less than*, *subtract . . . from* and *subtracted from* imply a specific order in which the subtraction is performed. In all three cases, begin with the second number listed and subtract the first number listed.

Example 10

Translating an English Phrase  
to a Mathematical Statement

Translate the English phrase to a mathematical statement and simplify.

- a. The difference of 150 and 38
- b. 30 subtracted from 82

**Solution:**

- a. From Table 1-2, the *difference* of 150 and 38 implies  $150 - 38$ .

$$\begin{array}{r} 150 \\ - 38 \\ \hline 112 \end{array}$$

- b. The phrase “30 subtracted from 82” implies that 30 is taken away from 82.  
We have  $82 - 30$ .

$$\begin{array}{r} 82 \\ - 30 \\ \hline 52 \end{array}$$

**Skill Practice** Translate the English phrase to a mathematical statement and simplify.

16. Twelve decreased by eight
17. Subtract three from nine.

**Answers**

14.  $80 + 50$ ; 130
15.  $12 + 14$ ; 26
16.  $12 - 8$ ; 4
17.  $9 - 3$ ; 6

We noted earlier that addition is commutative. That is, the order in which two numbers are added does not affect the sum. This is *not* true for subtraction. For example,  $82 - 30$  is not equal to  $30 - 82$ . The symbol  $\neq$  means “is not equal to.” Thus,  $82 - 30 \neq 30 - 82$ .

In Examples 11 and 12, we use addition and subtraction of whole numbers to solve application problems.

**Example 11****Solving an Application Problem Involving a Table**

The table gives the number of gold, silver, and bronze medals won in a recent Winter Olympics for selected countries.

- Find the total number of medals won by Canada.
- Determine the total number of silver medals won by these three countries.

	Gold	Silver	Bronze
Germany	10	13	7
USA	9	15	13
Canada	14	7	5

**Solution:**

- The number of medals won by Canada appears in the last row of the table. The word “total” implies addition.

$$14 + 7 + 5 = 26 \quad \text{Canada won 26 medals.}$$

- The number of silver medals is given in the middle column. The total is

$$13 + 15 + 7 = 35 \quad \text{There were 35 silver medals won by these countries.}$$

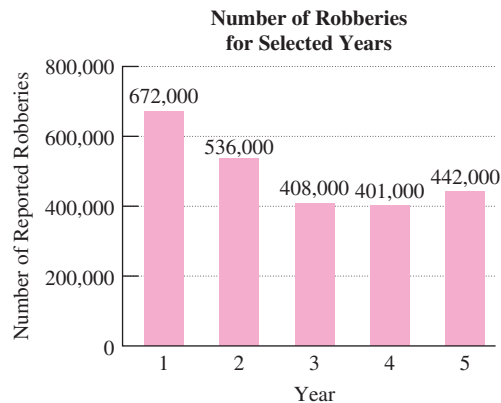
**Skill Practice** Refer to the table in Example 11.

- Find the total number of bronze medals won.
  - Find the number of medals won by the United States.

**Example 12****Solving an Application Problem**

A criminal justice student did a study of the number of robberies that occurred in the United States over a period of several years. The graph shows his results for five selected years.

- Find the increase in the number of reported robberies from year 4 to year 5.
- Find the decrease in the number of reported robberies from year 1 to year 2.



Source: Federal Bureau of Investigation

**Solution:**

For the purpose of finding an amount of increase or decrease, we will subtract the smaller number from the larger number.

- Because the number of robberies went *up* from year 4 to year 5, there was an *increase*. To find the amount of increase, subtract the smaller number from the larger number.

$$\begin{array}{r} 442,000 \\ - 401,000 \\ \hline 41,000 \end{array}$$

From year 4 to year 5, there was an increase of 41,000 reported robberies in the United States.

**Answer**

18. a. 25 medals    b. 37 medals



- b. Because the number of robberies went *down* from year 1 to year 2, there was a *decrease*. To find the amount of decrease, subtract the smaller number from the larger number.

$$\begin{array}{r} 6\overline{)2,000} \\ -536,000 \\ \hline 136,000 \end{array}$$

From year 1 to year 2, there was a decrease of 136,000 reported robberies in the United States.

**Skill Practice** Refer to the graph for Example 12.

19. a. Has the number of robberies increased or decreased from year 2 to year 5?  
b. Determine the amount of increase or decrease.

## 5. Perimeter

One special application of addition is to find the perimeter of a polygon. A **polygon** is a flat closed figure formed by line segments connected at their ends. Familiar figures such as triangles, rectangles, and squares are examples of polygons. See Figure 1-3.

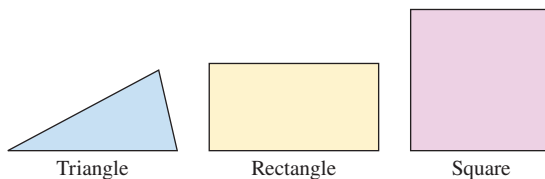
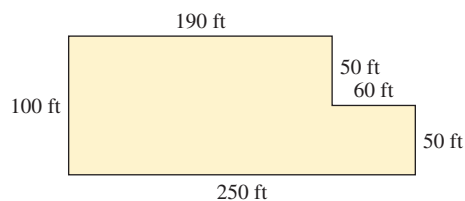


Figure 1-3

The **perimeter** of any polygon is the distance around the outside of the figure. To find the perimeter, add the lengths of the sides.

### Example 13 Finding Perimeter

A paving company wants to edge the perimeter of a parking lot with concrete curbing. Find the perimeter of the parking lot.



#### Solution:

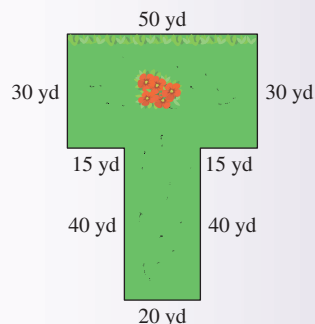
The perimeter is the sum of the lengths of the sides.

$$\begin{array}{r} 190 \text{ ft} \\ 50 \text{ ft} \\ 60 \text{ ft} \\ 50 \text{ ft} \\ 250 \text{ ft} \\ + 100 \text{ ft} \\ \hline 700 \text{ ft} \end{array}$$

The distance around the parking lot (the perimeter) is 700 ft.

### Skill Practice

20. Find the perimeter of the garden.



### Answers

19. a. decreased b. 94,000 robberies  
20. 240 yd

## Section 1.2 Activity

**A.1.** There are many different words and phrases that imply addition. Some examples of these words and phrases are *sum*, *increased by*, *more than*, and *total*.

**a.** Write the mathematical expression of  $13 + 6$  using the word *sum*.

**b.** Write the mathematical expression of  $2 + 8 + 10$  using the word *total*.

**A.2.** Jennifer and Mia went shopping together. They purchased items of the following prices: \$19, \$5, \$25, and \$11.

**a.** Jennifer calculated the total bill by adding:

$$19 + 5 + 25 + 11$$

What was the sum that Jennifer calculated?

**b.** Mia calculated the total bill by adding:

$$19 + 11 + 5 + 25$$

What was the sum that Mia calculated?

**c.** Are the sums from parts (a) and (b) equal?

**d.** Which property confirms that the sums from parts (a) and (b) are equal, the associative property of addition or the commutative property of addition?

**A.3.** Write three words or phrases that imply subtraction.

**a.**

**b.**

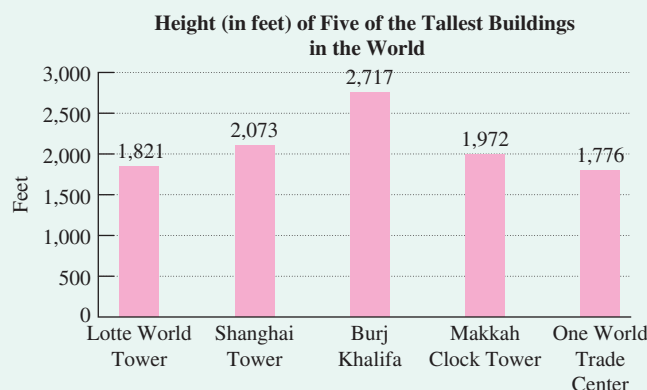
**c.**

**A.4.** Corey and Lucas could not agree on whether or not 12 minus 8 yields the same value as 12 subtracted from 8. Corey thinks the phrases are equal. Lucas thinks the phrases result in different answers.

**a.** Who is correct, Corey or Lucas?

**b.** Explain your answer to part (a)

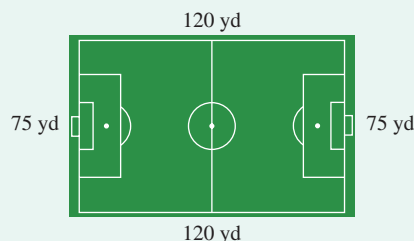
**A.5.** The figure gives the height of five of the tallest buildings in the world.



**a.** What is the difference between the tallest and the shortest of the buildings?

**b.** What is the difference between the Shanghai Tower and Lotte World Tower?

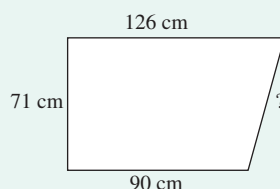
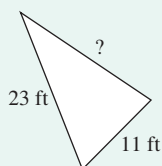
**A.6.** The perimeter of a polygon is the total distance around the figure. Find the perimeter of an optimum size soccer field.



A.7. Find the length of the missing sides given the perimeter.

a. Perimeter = 54 ft

b. Perimeter = 372 cm



## Section 1.2 Practice Exercises

### Study Skills Exercise

Mindset plays an important role in your approach to learning mathematics. Mindset is our thoughts, beliefs, and attitudes about our abilities based on previous experiences throughout our lifetime. There are two types of mindsets: fixed mindsets and growth mindsets. People with a fixed mindset believe that they are born with a certain amount of intelligence that cannot be changed despite their actions. On the other hand, a person with a growth mindset believes that intelligence is dynamic and can be increased with effort and learning. What type of mindset do you have? Think about the following questions:

- Have you said to yourself, “I’m just not good at math”?
- Do you believe you have the skills required to understand math?
- Can you recall an experience that positively impacted your self-confidence in mathematics?

### Vocabulary and Key Concepts

- The numbers being added in an addition problem are called the \_\_\_\_\_.
  - The result of an addition problem is called the \_\_\_\_\_.
  - A \_\_\_\_\_ is a letter or symbol that represents a number.
  - The \_\_\_\_\_ property of addition states that the order in which two numbers are added does not affect the sum.
  - For any number  $a$ , the addition property of 0 states that  $a + 0 = \underline{\hspace{1cm}}$  and that  $0 + a = \underline{\hspace{1cm}}$ .
  - The associative property of addition states that  $(a + b) + c = \underline{\hspace{1cm}}$ . This implies that the manner in which addends are grouped under addition does not affect the sum.
  - Given the subtraction statement  $15 - 4 = 11$ , the number 15 is called the \_\_\_\_\_, the number 4 is called the \_\_\_\_\_, and the number 11 is called the \_\_\_\_\_.
  - A \_\_\_\_\_ is a flat closed figure formed by line segments connected at their ends.
  - The \_\_\_\_\_ of a polygon is the sum of the lengths of the sides.
- When adding numbers, you must *carry* or *regroup* if the sum of the digits in a given place position is greater than \_\_\_\_\_.
- Unlike addition, the operation of subtraction is not \_\_\_\_\_.
- When a digit in the subtrahend is larger than the corresponding digit in the minuend, you must \_\_\_\_\_ a value from the column to the left.
- The number of students enrolled in the business management program changed from 462 student to 580. Is this an example of an increase or a decrease?
- The phrases *decreased by* and *less than* imply the operation of \_\_\_\_\_.

**Concept 1: Addition of Whole Numbers**

7. Fill in the table.

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

For Exercises 8–10, identify the addends and the sum.

8.  $11 + 10 = 21$

9.  $1 + 13 + 4 = 18$

10.  $5 + 8 + 2 = 15$

For Exercises 11–18, add. (See Example 1.)

11. 
$$\begin{array}{r} 42 \\ + 33 \\ \hline \end{array}$$

12. 
$$\begin{array}{r} 21 \\ + 53 \\ \hline \end{array}$$

13. 
$$\begin{array}{r} 12 \\ 15 \\ + 32 \\ \hline \end{array}$$

14. 
$$\begin{array}{r} 10 \\ 8 \\ + 30 \\ \hline \end{array}$$

15.  $890 + 107$

16.  $444 + 354$

17.  $4 + 13 + 102$

18.  $11 + 221 + 5$

For Exercises 19–32, add the whole numbers with carrying. (See Examples 2 and 3.)

19. 
$$\begin{array}{r} 76 \\ + 45 \\ \hline \end{array}$$

20. 
$$\begin{array}{r} 25 \\ + 59 \\ \hline \end{array}$$

21. 
$$\begin{array}{r} 87 \\ + 24 \\ \hline \end{array}$$

22. 
$$\begin{array}{r} 38 \\ + 77 \\ \hline \end{array}$$

23. 
$$\begin{array}{r} 658 \\ + 231 \\ \hline \end{array}$$

24. 
$$\begin{array}{r} 642 \\ + 295 \\ \hline \end{array}$$

25. 
$$\begin{array}{r} 152 \\ + 549 \\ \hline \end{array}$$

26. 
$$\begin{array}{r} 462 \\ + 388 \\ \hline \end{array}$$

27.  $79 + 112 + 12$

28.  $62 + 907 + 34$

29.  $4980 + 10,223$

30.  $23,112 + 892$

31.  $10,223 + 25,782 + 4980$

32.  $92,377 + 5622 + 34,659$

**Concept 2: Properties of Addition**

For Exercises 33–36, rewrite the addition problem, using the commutative property of addition. (See Example 4.)

33.  $101 + 44 = \square + \square$

34.  $8 + 13 = \square + \square$

35.  $x + y = \square + \square$

36.  $t + q = \square + \square$

For Exercises 37–40, rewrite the addition problem using the associative property of addition, by inserting a pair of parentheses. (See Example 4.)

37.  $(23 + 9) + 10 = 23 + 9 + 10$

38.  $7 + (12 + 8) = 7 + 12 + 8$

39.  $r + (s + t) = r + s + t$

40.  $(c + d) + e = c + d + e$

41. Explain the difference between the commutative and associative properties of addition.



42. Explain the addition property of 0. Then simplify the expressions.

a.  $423 + 0$

b.  $0 + 25$

c.  $\begin{array}{r} 67 \\ + 0 \end{array}$

d.  $0 + x$

### Concept 3: Subtraction of Whole Numbers

For Exercises 43 and 44, identify the minuend, subtrahend, and the difference.

43.  $12 - 8 = 4$

44.  $\begin{array}{r} 9 \\ - 6 \\ \hline 3 \end{array}$

For Exercises 45–48, write the subtraction problem as a related addition problem. For example,  $19 - 6 = 13$  can be written as  $13 + 6 = 19$ .

45.  $27 - 9 = 18$

46.  $20 - 8 = 12$

47.  $102 - 75 = 27$

48.  $211 - 45 = 166$

For Exercises 49–52, subtract, then check the answer by using addition. (See Example 5.)

49.  $8 - 3$  Check:  $\square + 3 = 8$

50.  $7 - 2$  Check:  $\square + 2 = 7$

51.  $4 - 1$  Check:  $\square + 1 = 4$

52.  $9 - 1$  Check:  $\square + 1 = 9$

For Exercises 53–56, subtract and check the answer by using addition. (See Example 6.)

53.  $\begin{array}{r} 1347 \\ - 221 \\ \hline \end{array}$

54.  $\begin{array}{r} 4865 \\ - 713 \\ \hline \end{array}$

55.  $14,356 - 13,253$

56.  $34,550 - 31,450$

For Exercises 57–72, subtract the whole numbers involving borrowing. (See Examples 7 and 8.)

57.  $\begin{array}{r} 76 \\ - 59 \\ \hline \end{array}$

58.  $\begin{array}{r} 64 \\ - 48 \\ \hline \end{array}$

59.  $\begin{array}{r} 710 \\ - 189 \\ \hline \end{array}$

60.  $\begin{array}{r} 850 \\ - 303 \\ \hline \end{array}$

61.  $\begin{array}{r} 6002 \\ - 1238 \\ \hline \end{array}$

62.  $\begin{array}{r} 3000 \\ - 2356 \\ \hline \end{array}$

63.  $\begin{array}{r} 10,425 \\ - 9,022 \\ \hline \end{array}$

64.  $\begin{array}{r} 23,901 \\ - 8,064 \\ \hline \end{array}$

65.  $\begin{array}{r} 62,088 \\ - 59,871 \\ \hline \end{array}$

66.  $\begin{array}{r} 32,112 \\ - 28,334 \\ \hline \end{array}$

67.  $3700 - 2987$

68.  $8000 - 3788$

69.  $32,439 - 1498$

70.  $21,335 - 4123$

71.  $8,007,234 - 2,345,115$

72.  $3,045,567 - 1,871,495$

73. Use the expression  $7 - 4$  to explain why subtraction is not commutative.

74. Is subtraction associative? Use the numbers 10, 6, 2 to explain.

### Concept 4: Translations and Applications Involving Addition and Subtraction

For Exercises 75–92, translate the English phrase to a mathematical statement and simplify. (See Examples 9 and 10.)

75. The sum of 13 and 7

76. The sum of 100 and 42

77. 45 added to 7

78. 81 added to 23

79. 5 more than 18

80. 2 more than 76

81. 1523 increased by 90      82. 1320 increased by 448      83. The total of 5, 39, and 81
84. 78 decreased by 6      85. Subtract 100 from 422.      86. Subtract 42 from 89.
87. 72 less than 1090      88. 60 less than 3111      89. The difference of 50 and 13
90. The difference of 405 and 103      91. Subtract 35 from 103.      92. Subtract 14 from 91.
93. A mountain climber attempting to climb Mount Everest must climb the mountain in stages to become acclimated to the extremely high altitude. This process generally takes about 6 weeks. The climb from Base Camp to Camp II results in a gain in altitude of 3010 ft. The climb from Camp II to Camp III is a gain of 1300 ft in altitude, and the climb to Camp IV is another 1700 ft.
- How much altitude has the climber gained from Base Camp to Camp IV?
  - If the climber gains another 6029 ft from Camp IV to the summit, what is the total gain in altitude from Base Camp to the summit?
95. A portion of Jonathan's checking account register is shown. What is the total amount of the three checks written? (See Example 11.)

Check No.	Description	Payment	Deposit	Balance
1871	Electric	\$60		\$180
1872	Groceries	82		98
	Payroll		\$1256	1354
1874	Restaurant	58		1296
	Deposit		150	1446

96. The table gives the number of desks and chairs delivered each quarter to an office supply store. Find the total number of desks delivered for the year.

	Chairs	Desks
1 <sup>st</sup> Quarter	220	115
2 <sup>nd</sup> Quarter	185	104
3 <sup>rd</sup> Quarter	201	93
4 <sup>th</sup> Quarter	198	111

97. The altitude of White Mountain Peak in California is 14,246 ft. Denali in Alaska is 20,310 ft. How much higher is Denali than White Mountain Peak?



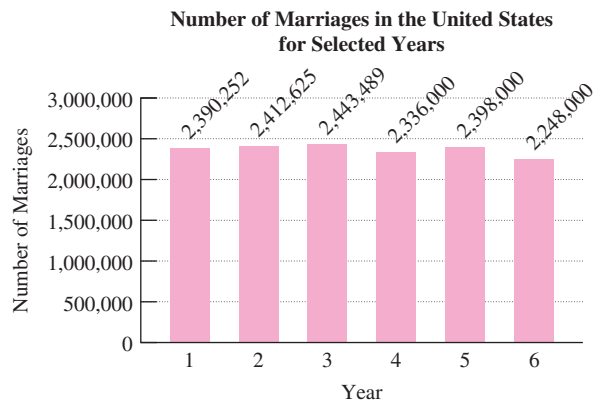
Robert Glusic/Getty Images

98. There are 55 DVDs to shelve one evening at a video rental store. If Jason puts away 39 before leaving for the day, how many are left for Patty to put away?



Jill Braaten/McGraw Hill

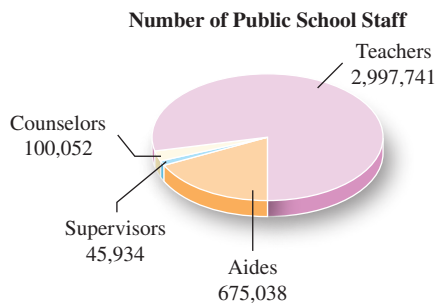
For Exercises 99–102, use the information from the graph. (See Example 12.)



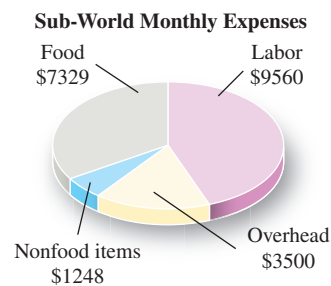
Source: National Center for Health Statistics

**Figure for Exercises 99–102**

99. What is the difference in the number of marriages between year 1 and year 5?
100. Find the decrease in the number of marriages in the United States between year 5 and year 6.
101. What is the difference in the number of marriages between the year having the greatest and the year having the least?
102. Between which two consecutive years did the greatest increase in the number of marriages occur? What is the increase?
103. The staff for U.S. public schools is categorized in the pie graph. Determine the number of staff other than teachers.
104. The pie graph shows the costs incurred in managing Sub-World sandwich shop for one month. From this information, determine the total cost for one month.



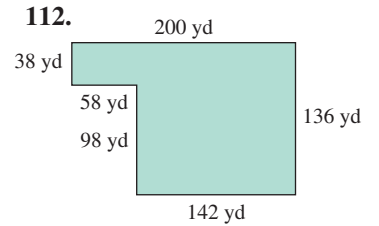
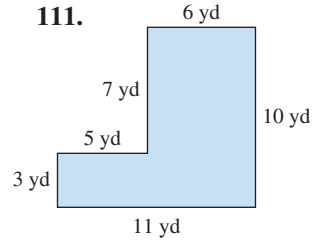
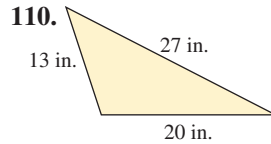
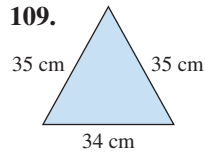
Source: National Center for Education Statistics



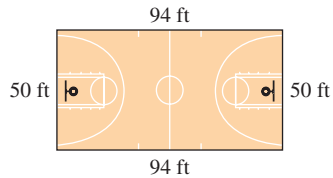
105. Pinkham Notch Visitor Center in the White Mountains of New Hampshire has an elevation of 2032 ft. The summit of nearby Mt. Washington has an elevation of 6288 ft. What is the difference in elevation?
106. Bo Jackson was a Heisman Trophy winner in college football, and then went on to play both professional football and professional baseball. He was named by ESPN as the “greatest athlete of all time.” Unfortunately, his career in the National Football League (NFL) was cut short in his fourth year because of a hip injury. During his time in the NFL, he gained 2782 yd rushing and 352 yd receiving. How many more yards did Bo Jackson gain running than receiving?
107. Jeannette has two children who each attended college. Her son Ricardo attended a local community college where the yearly tuition and fees came to \$4215. Her daughter Ricki attended an out-of-state university where the yearly tuition and fees totaled \$22,416. If Jeannette paid the full amount for both children to go to school, what was her total expense for tuition and fees for 1 year?
108. Clyde and Mason each leave a rest area on the Florida Turnpike. Clyde travels north and Mason travels south. After 2 hr, Clyde has gone 138 mi and Mason, who ran into heavy traffic, traveled only 96 mi. How far apart are they?

**Concept 5: Perimeter**

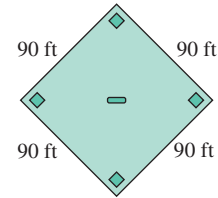
For Exercises 109–112, find the perimeter. (See Example 13.)



**113.** Find the perimeter of an NBA basketball court.

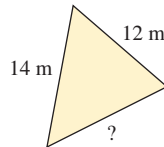


**114.** A major league baseball diamond is in the shape of a square. Find the distance a batter must run if he hits a home run.

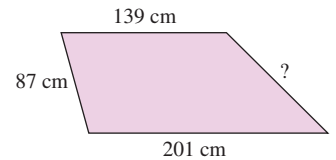


For Exercises 115 and 116, find the missing length.

**115.** The perimeter of the triangle is 39 m.



**116.** The perimeter of the figure is 547 cm.

**Technology Connections**

For Exercises 117–122, perform the indicated operation by using a calculator.

**117.**

$$\begin{array}{r} 45,418 \\ 81,990 \\ 9,063 \\ + 56,309 \\ \hline \end{array}$$

**118.**

$$\begin{array}{r} 9,300,050 \\ 7,803,513 \\ 3,480,009 \\ + 907,822 \\ \hline \end{array}$$

**119.**

$$\begin{array}{r} 3,421,019 \\ 822,761 \\ 1,003,721 \\ + 9,678 \\ \hline \end{array}$$

**120.**

$$\begin{array}{r} 4,905,620 \\ - 458,318 \\ \hline \end{array}$$

**121.**

$$\begin{array}{r} 953,400,415 \\ - 56,341,902 \\ \hline \end{array}$$

**122.**

$$\begin{array}{r} 82,025,160 \\ - 79,118,705 \\ \hline \end{array}$$

For Exercises 123–126, refer to the table showing the land area for five states.

State	Land Area (mi <sup>2</sup> )
Rhode Island	1,045
Tennessee	41,217
West Virginia	24,078
Wisconsin	54,310
Colorado	103,718

**123.** Find the difference in the land area between Colorado and Wisconsin.

**124.** Find the difference in the land area between Tennessee and West Virginia.

**125.** What is the combined land area for Rhode Island, Tennessee, and Wisconsin?

**126.** What is the combined land area for all five states?

## Section 1.3 Rounding and Estimating

### Concepts

1. Rounding
2. Estimation
3. Using Estimation in Applications

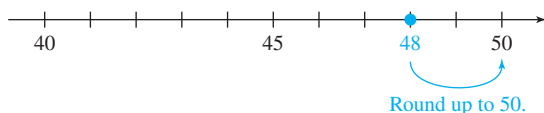
### 1. Rounding

**Rounding** a whole number is a common practice when we do not require an exact value. For example, Madagascar lost  $3956 \text{ mi}^2$  of rainforest between 1990 and 2008. We might round this number to the nearest thousand and say that approximately  $4000 \text{ mi}^2$  was lost. In mathematics, we use the symbol  $\approx$  to read “is approximately equal to.” Therefore,  $3956 \text{ mi}^2 \approx 4000 \text{ mi}^2$ .

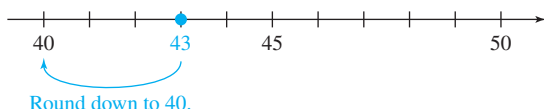
A number line is a helpful tool to understand rounding. For example, 48 is closer to 50 than it is to 40. Therefore, 48 rounded to the nearest ten is 50.



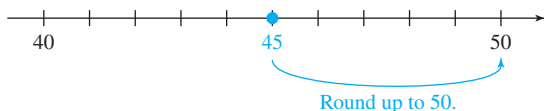
Courtesy of Julie Miller



The number 43, on the other hand, is closer to 40 than to 50. Therefore, 43 rounded to the nearest ten is 40.



The number 45 is halfway between 40 and 50. In such a case, our convention will be to round *up* to the next-larger ten.



The decision to round up or down to a given place value is determined by the digit to the *right* of the given place value. The following steps outline the procedure.

#### Rounding Whole Numbers

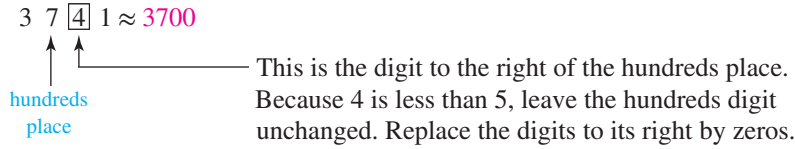
- Step 1** Identify the digit one position to the right of the given place value.
- Step 2** If the digit in step 1 is a 5 or greater, then add 1 to the digit in the given place value. If the digit in step 1 is less than 5, leave the given place value unchanged.
- Step 3** Replace each digit to the right of the given place value by 0.



**Example 1** Rounding a Whole Number

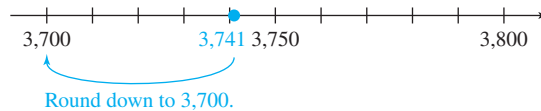
Round 3741 to the nearest hundred.

**Solution:**

**Skill Practice**

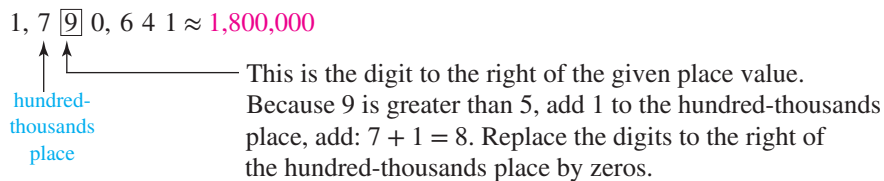
1. Round 12,461 to the nearest thousand.

Example 1 could also have been solved by drawing a number line. Use the part of a number line between 3700 and 3800 because 3741 lies between these numbers.

**Example 2** Rounding a Whole Number

Round 1,790,641 to the nearest hundred-thousand.

**Solution:**

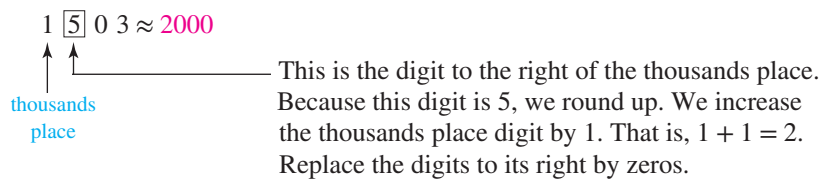
**Skill Practice**

2. Round 147,316 to the nearest ten-thousand.

**Example 3** Rounding a Whole Number

Round 1503 to the nearest thousand.

**Solution:**

**Skill Practice**

3. Round 7,521,460 to the nearest million.

**Answers**

1. 12,000
2. 150,000
3. 8,000,000

**Example 4** Rounding a Whole Number

Round the number 24,961 to the hundreds place.

**Solution:**

$$24,9\overset{+1}{\boxed{6}}1 \approx 25,000$$

↑ This is the digit to the right of the hundreds place. Because 6 is greater than 5, add 1 to the hundreds place digit. Replace the digits to the right of the hundreds place with 0.

**Skill Practice**

4. Round 39,823 to the nearest thousand.

**2. Estimation**

We use the process of rounding to estimate the result of numerical calculations. For example, to estimate the following sum, we can round each addend to the nearest ten.

$$\begin{array}{rcl} 31 & \text{rounds to} \longrightarrow & 30 \\ 12 & \text{rounds to} \longrightarrow & 10 \\ + 49 & \text{rounds to} \longrightarrow & + 50 \\ \hline & & 90 \end{array}$$

The estimated sum is 90 (the actual sum is 92).

**Example 5** Estimating a Sum

Estimate the sum by rounding to the nearest thousand.

$$6109 + 976 + 4842 + 11,619$$

**Solution:**

$$\begin{array}{rcl} 6,109 & \text{rounds to} \longrightarrow & \overset{1}{6},000 \\ 976 & \text{rounds to} \longrightarrow & 1,000 \\ 4,842 & \text{rounds to} \longrightarrow & 5,000 \\ + 11,619 & \text{rounds to} \longrightarrow & + 12,000 \\ \hline & & 24,000 \end{array}$$

The estimated sum is 24,000 (the actual sum is 23,546).

**Skill Practice**

5. Estimate the sum by rounding each number to the nearest hundred.  
3162 + 4931 + 2206

**TIP:** Rounding can be useful to mentally estimate an amount. For this purpose, we usually round so that we have one or two nonzero digits with which to work. In Example 5 we rounded to the thousands place, giving  $(6 + 1 + 5 + 12)$  thousands = 24 thousand. The estimate is 24,000.

**Example 6** Estimating a Difference

Estimate the difference  $4817 - 2106$  by rounding each number to the nearest hundred.

**Solution:**

$$\begin{array}{rcl} 4817 & \text{rounds to} \longrightarrow & 4800 \\ - 2106 & \text{rounds to} \longrightarrow & - 2100 \\ \hline & & 2700 \end{array}$$

The estimated difference is 2700 (the actual difference is 2711).

**Skill Practice**

6. Estimate the difference by rounding each number to the nearest million.  
35,264,000 - 21,906,210

**Answers**

4. 40,000      5. 10,300  
6. 13,000,000