Engineering Mechanics
STATICS
THIRD EDITION

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The authors thank their families for their patience, understanding, and, most importantly, encouragement during the long years it took to bring these books to completion. Without their support, none of this would have been possible.

BRIEF CONTENTS

Statics

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PREFACE

Welcome to statics! We assume you are embarking on the study of statics because you are interested in engineering and science. The major objectives of this book are to help you

1. Learn the fundamental principles of statics; and
2. Gain the skills needed to apply these principles in the modeling of real-life problems and for carrying out engineering design.

The need for thorough coverage of the fundamental principles is paramount, and therefore a substantial portion of this book is devoted to these principles. Because the development of problem-solving skills is equally important, we focus a great deal of attention on these skills, especially in the context of real-life problems. Indeed, the emphasis on problem-solving skills is a major difference in the treatment of statics between engineering and physics. It is only by mastering these skills that you can achieve a true, deep understanding of fundamentals. You must be flawless in your ability to apply the concepts of statics to real-life problems. When mistakes are made, structures and machines will fail, money and time will be lost, and worst of all, people may be hurt or killed.

What should you take away from this book?

First and foremost, you should gain a thorough understanding of the fundamental principles, and, at a minimum, key points should remain in your memory for the rest of your life. We say this with a full appreciation that some of you will have careers with new and unexpected directions. Regardless of your eventual professional responsibilities, knowledge of the fundamentals of statics will help you to be technically literate. By contrast, if you *are* actively engaged in the practice of engineering and/or the sciences, then your needs go well beyond mere technical literacy. In addition to understanding the fundamentals, you must also be accomplished at applying these fundamentals. This ability is needed so that you can study more advanced subjects that build on statics, and because you will apply concepts of statics on a daily basis in your career.

Why Another Statics and Dynamics Series?

These books provide thorough coverage of all the pertinent topics traditionally associated with statics and dynamics. Indeed, many of the currently available texts also provide this. However, these books offer several major innovations that enhance the learning objectives and outcomes in these subjects.

What Then Are the Major Differences Between These Books and Other Engineering Mechanics Texts?

A Consistent and Systematic Approach to Problem Solving One of the main objectives of these texts is to foster the habit of solving problems using a systematic approach. Therefore, the example problems in these texts follow a structured five-step problem-solving methodology that will help you develop your problem-solving skills not only in statics and dynamics, but also in almost all other mechanics subjects that follow. This structured problem-solving approach consists of the following steps: Road Map, Modeling, Governing Equations, Computation, and Discussion & Verification. The Road Map provides some of the general objectives of the problem and develops a strategy for how the solution will be developed. Modeling is next, where a real-life problem is idealized by a model. This step results in the creation of a free body diagram and the selection of the balance laws needed to solve the problem. The Governing Equations step is devoted to writing all the equations needed to solve the problem. These equations typically include the equilibrium equations, and, depending upon the particular problem, force laws (e.g., spring law, failure criteria, frictional sliding criteria) and kinematic equations. In the Computation step, the governing equations are solved. In the final step, Discussion & Verification, the solution is interrogated to ensure that it is meaningful and accurate. This five-step problem-solving methodology is followed for all examples that involve equilibrium concepts. Some problems (e.g., determination of the center of mass for an object) do not involve equilibrium concepts, and for these the Modeling step is not needed.

Contemporary Examples, Problems, and Applications The examples, homework problems, and design problems were carefully constructed to help show you how the various topics of statics and dynamics are used in engineering practice. Statics and dynamics are immensely important subjects in modern engineering and science, and one of our goals is to excite you about these subjects and the career that lies ahead of you.

A Focus on Design A major difference between these texts and other books is the systematic incorporation of design and modeling of real-life problems throughout. Topics include important discussions on design, ethics, and professional responsibility. These books show you that meaningful engineering design is possible using the concepts of statics and dynamics. Not only is the ability to develop a design very satisfying, but it also helps you develop a greater understanding of basic concepts and helps sharpen your ability to apply these concepts. Because the main focus of statics and dynamics textbooks should be the establishment of a firm understanding of basic concepts and correct problem-solving techniques, design topics do not have an overbearing presence in the books. Rather, design topics are included where they are most appropriate. While some of the discussions on design could be described as "common sense," such a characterization trivializes the importance and necessity for discussing pertinent issues such as safety, uncertainty in determining loads, the designer's responsibility to anticipate uses, even unintended uses, communications, ethics, and uncertainty in workmanship. Perhaps the most important feature of our inclusion of design and modeling topics is that you get a glimpse of what engineering is about and where your career in engineering is headed. The book is structured so that design topics and design problems are offered in a variety of places, and it is possible to pick when and where the coverage of design is most effective.

Computational Tools Some examples and problems are appropriate for solution using computer software. The use of computers extends the types of problems that can be solved while alleviating the burden of solving equations. Such examples and problems give you insight into the power of computer tools and further insight into how statics and dynamics are used in engineering practice.

Modern Pedagogy Numerous modern pedagogical elements have been included. These elements help reinforce concepts, and they provide you with additional information to help you understand concepts. Marginal notes (including Helpful Information, Common Pitfalls, Interesting Facts, and Concept Alerts) help place topics, ideas, and examples in a larger context. These notes will help you study (e.g., Helpful Information and Common Pitfalls), will provide real-world examples of how different aspects of statics and dynamics are used (e.g., Interesting Facts), and will drive home important concepts or help dispel misconceptions (e.g., Concept Alerts and Common Pitfalls). Mini-Examples are used throughout the text to immediately and quickly illustrate a point or concept without making readers wait for the worked-out examples at the end of the section.

Answers to Problems The answers to most even-numbered problems have been included in the back matter for ease of use as Appendix B. Providing answers in this manner allows for the inclusion of more complex information than would otherwise be possible. In addition to final numerical or symbolic answers, selected problems have more extensive information, such as free body diagrams and/or shear and moment diagrams. Furthermore, the multitude of free body diagram answers give you ample opportunity to practice constructing these on your own for extra problems.

A Note to the Instructor

Statics is the first engineering course taken by most students en route to an undergraduate degree in engineering. You face many challenges when choosing the text you use. Because statics is so fundamental to subsequent engineering coursework and professional practice, a text must be accurate, thorough, and comprehensive. Statics also presents an opportunity to excite students and show them what engineering is about early in their education. Students who miss this opportunity and do not receive an accurate picture of where their career is heading may make a poorly informed decision to change their major away from engineering. This is recognized in the current Accreditation Board for Engineering and Technology (ABET) criteria for accreditation of engineering programs, which requires design to be integrated throughout an engineering curriculum. This book provides thorough coverage of the principles of statics. It also includes discussions on the theory and the more subtle points of statics. Such discussions usually follow an introductory treatment of a topic so that students have a grasp of concepts and their application before covering those more subtle points. For example, the concepts of particle equilibrium are presented in Section 3.1 with common assumptions such as cables being inextensible and pulleys being frictionless. In that section, the emphasis is on drawing free body diagrams, writing equilibrium equations, solving equations, applying failure criteria, and interrogating solutions. In Section 3.2, the reasons for the typical assumptions are thoroughly discussed, including the limitations of these assumptions for modeling real-life problems. This will help students develop an appreciation for the fact that, despite these assumptions, statics is an immensely useful and widely applicable subject. Further, the discussion in Section 3.2 is used to present more advanced topics such as springs and static indeterminacy.

Design topics include ethics, professional responsibility, pertinent codes and standards, and much more. Design problems are open ended and allow students to show creativity in developing solutions that solve important and realistic engineering problems. The design problems in this book may take students several hours to complete. It is recommended that students write a short report, suitable for reading by an engineer. A brief discussion of technical writing is included in Appendix A since many students have not yet studied technical writing. Perhaps the most important feature of our inclusion of design and modeling topics is that students get a glimpse of what engineering is about and where their career in engineering is headed. The book is structured so that you may pick when and where design is most appropriately covered.

Changes to the Third Edition

The third editions of *Engineering Mechanics: Statics* and *Engineering Mechanics: Dynamics* retain all of the major pedagogical features of the previous editions, including a structured problem-solving methodology for all example problems, contemporary engineering applications in the example problems and homework exercises, the inclusion of engineering design and its implications for problem solving and applications, and use of computational tools where applicable. In addition, as a result of the author-based typesetting process, the outstanding accuracy of the earlier editions has been preserved, leading to books whose accuracy is unrivaled among textbooks.

The third editions contain revised and enhanced textual discussions and example problems, additional figures where effective, and new homework exercises. In Connect, the online homework system, there are significant updates, including an auto-graded FBD tool and interactive learning tools. These interactive assignments help reinforce what is being covered in the text and show students how to tie the material to real-world situations. These tools complement the hundreds of auto-graded, algorithmic-exercises that are included in Connect from the text.

The following individuals have been instrumental in ensuring the highest standard of content and accuracy. We are deeply indebted to them for their tireless efforts.

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GUIDED TOUR

Mini-Examples

Mini-examples are used throughout the text to immediately and quickly illustrate a point or concept without having to wait for the worked-out examples at the end of the section.

Mini-Example

Determine the support reactions for the structure shown in Fig. 5.4.

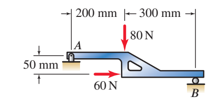

Figure 5.4
A structure with supports at points A and B.

Solution

The completed FBD is shown in Fig. 5.5, and it is constructed as follows. We first sketch the structure and then choose an xy coordinate system. At each support that is cut through (or removed from the structure) we introduce the appropriate reactions. Thus, for the pin support at A we introduce reactions A_x and A_y, and at the roller support at B we introduce reaction B_y.

Next, we use Eq. (5.3) to write the equilibrium equations

$$\sum F_x = 0: \quad A_x + 60\text{ N} = 0, \tag{5.4}$$

$$\sum F_y = 0: \quad A_y + B_y - 80\text{ N} = 0, \tag{5.5}$$

$$\sum M_A = 0: \quad B_y(500\text{ mm}) - (80\text{ N})(200\text{ mm}) + (60\text{ N})(50\text{ mm}) = 0. \tag{5.6}$$

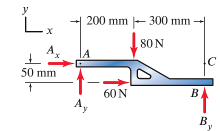

Figure 5.5
Free body diagram showing all forces applied to the object.

Solving Eqs. (5.4)–(5.6) provides

$$A_x = -60\text{ N}, \quad A_y = 54\text{ N}, \quad \text{and} \quad B_y = 26\text{ N}. \tag{5.7}$$

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EXAMPLE 5.7 *Springs*

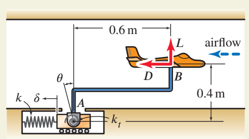


Figure 1

A wind tunnel is used to experimentally determine the lift force L and drag force D on a scale model of an aircraft. The bracket supporting the aircraft is fitted with an axial spring with stiffness $k = 0.125$ N/mm and a torsional spring with stiffness $k_t = 50$ N·m/rad. By measuring the deflections δ and θ of these springs during a test, the forces L and D may be determined. If the geometry shown in Fig. 1 occurs when there is no airflow, and if the springs are calibrated so that $\delta = 0$ and $\theta = 0°$ when there is no airflow, determine L and D if $\delta = 2.51$ mm and $\theta = 1.06°$ are measured.

SOLUTION

Road Map This problem involves spring elements, and these have equations that govern their load–deformation response. Thus, the problem-solving methodology used here will be enhanced to emphasize that in addition to the need for equilibrium equations, force laws that describe the behavior of the springs are needed.

Modeling The FBD is shown in Fig. 2, where we assume that bar AB, which supports the aircraft, is slender enough that it does not develop any lift or drag forces.

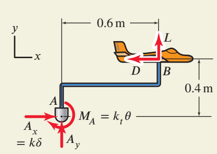


Figure 2
Free body diagram.

Governing Equations

Equilibrium Equations Summing forces in the x direction and summing moments about point A provide

$$\sum F_x = 0: \quad A_x - D = 0, \tag{1}$$

$$\sum M_A = 0: \quad -M_A + D(400\text{ mm}) + L(600\text{ mm}) = 0. \tag{2}$$

In writing Eq. (2), we assume the deformation of the torsional spring is small so that the geometry of the support bracket AB is essentially unchanged from its original geometry. Notice that Eqs. (1) and (2) have five unknowns, so clearly more equations must be written to obtain a determinate system of equations.

Force Laws The force supported by the axial spring is related to its deformation δ, and the moment supported by the torsional spring is related to its rotation θ, by

$$A_x = k\delta, \tag{3}$$

$$M_A = k_t\theta. \tag{4}$$

Computation Using Eq. (3), Eq. (1) may be solved for

$$D = k\delta = \left(0.125\frac{\text{N}}{\text{mm}}\right)(2.51\text{ mm}) = \boxed{0.3138\text{ N.}} \tag{5}$$

Using Eq. (4) and the solution for D just obtained, Eq. (2) may be solved for

$$L = \frac{-D(400\text{ mm}) + k_t\theta}{600\text{ mm}}$$

$$= \frac{-(0.3138\text{ N})(400\text{ mm}) + (50\,\frac{\text{N}\cdot\text{m}}{\text{rad}})(\frac{10^3\text{ mm}}{\text{m}})(\frac{\pi\text{ rad}}{180°})1.06°}{600\text{ mm}} = \boxed{1.333\text{ N.}} \tag{6}$$

Matt Cardy/Getty Images News/Getty Images

Figure 3
A model of an Airbus airplane is prepared for testing in a large wind tunnel.

Discussion & Verification If the springs are not sufficiently stiff, then δ and/or θ may be substantially larger, and the original geometry cannot be used when writing Eq. (2). An additional disadvantage of a soft torsional spring is that if θ is large, then the angle of attack of the aircraft also changes appreciably, which is undesirable. Since δ and θ are known in this problem, it is easy to verify that the difference between the original geometry and the deformed geometry is small. You can explore this issue in greater detail in Probs. 5.79 through 5.82.

Examples

Consistent Problem-Solving Methodology

Every problem in the text employs a carefully defined problem-solving methodology to encourage systematic problem formulation while reinforcing the steps needed to arrive at correct and realistic solutions.

Each example problem contains these five steps:

- **Road Map**
- **Modeling**
- **Governing Equations**
- **Computation**
- **Discussion & Verification**

Some examples include a Closer Look (noted with a magnifying glass icon) that offers additional insight into the example.

Concept Alerts and Concept Problems

Two additional features are the Concept Alerts and the Concept Problems. These have been included because research has shown (and it has been our experience) that even though you may do quite well in a science or engineering course, your conceptual understanding may be lacking. **Concept Alerts** are marginal notes and are used to drive home important concepts (or help dispel misconceptions) that are related to the material being developed at that point in the text. **Concept Problems** are mixed in with the problems that appear at the end of each section. These are questions designed to get you thinking about the application of a concept or idea presented within that section. They should never require calculation and should require answers of no more than a few sentences.

Concept Alert

Applications of the cross product. The cross product between two vectors produces a result that is a vector. The cross product is often used to determine the normal direction to a surface, the area of a parallelogram formed by two vectors, and (as discussed in Chapter 4) the *moment* produced by a force. The last application is especially important in statics and mechanics.

Problem 3.1

Consider an airplane whose motion is described below. For each case, state whether or not the airplane is in static equilibrium, with a brief explanation.

(a) The airplane flies in a straight line at a constant speed and at a constant altitude.

(b) The airplane flies in a straight line at a constant speed while climbing in altitude.

(c) The airplane flies at a constant speed and at a constant altitude while making a circular turn.

(d) After touching down on the runway during landing, the airplane rolls in a straight line at a constant speed.

(e) After touching down on the runway during landing, the airplane rolls in a straight line while its brakes are applied to reduce its speed.

Note: Concept problems are about *explanations,* not computations.

Common Pitfall

Failure loads. A common error in solving problems with failure criteria, such as Part (b) of this example, is to assume that *all* members are at their failure loads at the same time. With reference to the FBD of Fig. 3, you will be making this error if you take $F_{AB} = 1200\,\text{N}$ and $F_{AC} = -1600\,\text{N}$; in fact, if you do this, you will find that $\sum F_x = 0$ (Eq. (5)) cannot be satisfied! Another way to describe this problem is to consider slowly increasing force P from zero. Eventually, one of the members will reach its failure load first, while the other will be below its failure load.

Interesting Fact

Springs. Springs are important structural members in their own right, but they are also important for laying the groundwork for characterizing more general engineering materials and members, which you will study in subjects that follow statics. Simply stated, almost all materials are idealized ... more complex than that ... over at least some range

Helpful Information

Free body diagram (FBD). An FBD is an essential aid, or tool, for applying Newton's law of motion $\sum \vec{F} = m\vec{a}$. Among the many skills you will need to be successful in statics, and in the subjects that follow, and as a practicing engineer, the ability to draw accurate FBDs is essential. An incorrect FBD is the most common source of errors in an analysis.

Marginal Notes

Marginal notes have been implemented that will help place topics, ideas, and examples in a larger context. This feature will help students study (using **Helpful Information** and **Common Pitfalls**) and will provide real-world examples of how different aspects of statics are used (using **Interesting Facts**).

Sections and End of Section Summary

Each chapter is organized into several sections. There is a wealth of information and features within each section, including examples, problems, marginal notes, and other pedagogical aids. Each section concludes with an end of section summary that succinctly summarizes that section. In many cases, cross-referenced important equations are presented again for review and reinforcement before the student proceeds to the examples and homework problems.

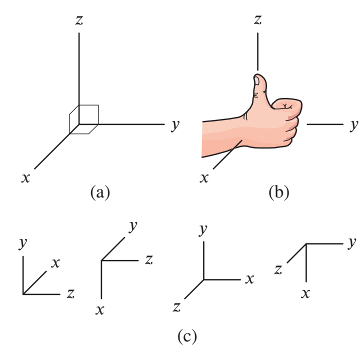


Figure 2.17
(a) Cartesian coordinate system in three dimensions. (b) A scheme for constructing a right-hand coordinate system. Position your right hand so the positive x direction passes into your palm and the positive y direction passes through your fingertips. Your thumb then indicates the positive z direction. (c) More examples of right-hand coordinate systems.

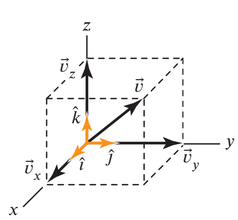
2.3 Cartesian Representation of Vectors in Three Dimensions

For problems in three dimensions, vectors are especially powerful, and without them many problems would be intractable. Concepts of Section 2.2 apply, with some additional enhancements needed for three dimensions. These include definitions of a right-handed coordinate system, direction angles, and direction cosines.

Right-hand Cartesian coordinate system

In three dimensions a Cartesian coordinate system uses three orthogonal reference directions. These will consist of x, y, and z directions as shown in Fig. 2.17(a). Proper interpretation of many vector operations, such as the cross product to be discussed in Section 2.5, requires the x, y, and z directions be arranged in a consistent manner. For example, when you are constructing the coordinate system shown in Fig. 2.17(a), imagine the x and y directions are chosen first. Then, should z be taken in the direction shown, or can it be in the opposite direction? The universal convention in mechanics and vector mathematics in general is z must be taken in the direction shown, and the result is called a *right-hand coordinate system*. Figure 2.17(b) describes a scheme for constructing a right-hand coordinate system. You should study this scheme and become comfortable with its use.

Cartesian vector representation



We define vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ to be unit vectors that point in the positive x, y, and z directions, respectively. A vector $\vec{v}$ can then be written as

$$\vec{v} = \vec{v}_x + \vec{v}_y + \vec{v}_z = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}. \tag{2.23}$$

Resolution of $\vec{v}$ into x, y, and z components is shown in Fig. 2.18. The *magnitude* of $\vec{v}$ is given by

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}. \tag{2.24}$$

ing the construction shown in Fig. 2.19 as follows. First, y plane is defined. Because v_x, v_y, and v_a form a right orem provides $v_a^2 = v_x^2 + v_y^2$. Then v_a, v_z, and v also form agorean theorem provides $v^2 = v_a^2 + v_z^2$. Substituting for ields $v^2 = v_x^2 + v_y^2 + v_z^2$, and thus Eq. (2.24) follows.

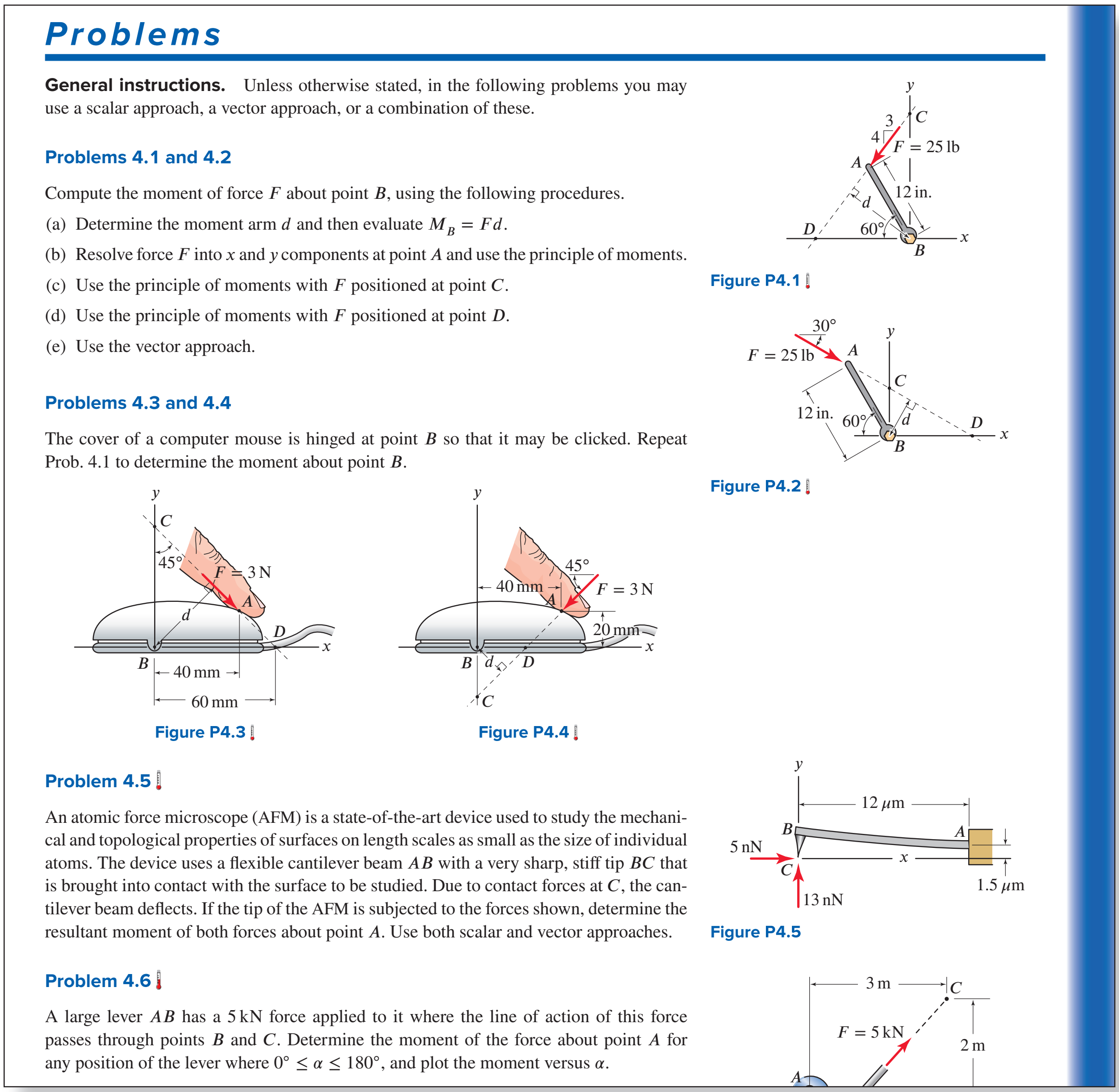
direction cosines

rize a vector's orientation is to use *direction angles*. Di- are shown in Fig. 2.20 and are defined to be the angles x, y, and z directions, respectively, to the direction of the values between 0° and 180°.
used to obtain a vector's components, and vice versa, tor polygon shown in Fig. 2.21. This polygon is a right ector's magnitude $|\vec{v}|$, the y component v_y, and another

End of Section Summary

In this section, Cartesian coordinate systems and Cartesian representation for vectors in three dimensions have been defined. Some of the key points are:

- The xyz coordinate system you use must be a *right-hand coordinate system*. Proper interpretation of some vector operations requires this.
- *Direction angles* provide a useful way to specify a vector's orientation in three dimensions. A vector has three direction angles θ_x, θ_y, and θ_z, but only two of these are independent. Direction angles satisfy the equation $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$, so if two direction angles are known, the third may be determined. Direction angles have values between 0° and 180°.
- Structural members such as cables, ropes, and bars support forces whose lines of action have the same orientation as the member's geometry. Thus, if $\vec{r}$ describes a member's geometry, a vector expression for the force supported by the member may be written as $\vec{F} = F\,(\vec{r}/|\vec{r}|)$.



Problems

General instructions. Unless otherwise stated, in the following problems you may use a scalar approach, a vector approach, or a combination of these.

Problems 4.1 and 4.2

Compute the moment of force F about point B, using the following procedures.

(a) Determine the moment arm d and then evaluate $M_B = Fd$.

(b) Resolve force F into x and y components at point A and use the principle of moments.

(c) Use the principle of moments with F positioned at point C.

(d) Use the principle of moments with F positioned at point D.

(e) Use the vector approach.

Figure P4.1

Figure P4.2

Problems 4.3 and 4.4

The cover of a computer mouse is hinged at point B so that it may be clicked. Repeat Prob. 4.1 to determine the moment about point B.

Figure P4.3

Figure P4.4

Problem 4.5

An atomic force microscope (AFM) is a state-of-the-art device used to study the mechanical and topological properties of surfaces on length scales as small as the size of individual atoms. The device uses a flexible cantilever beam AB with a very sharp, stiff tip BC that is brought into contact with the surface to be studied. Due to contact forces at C, the cantilever beam deflects. If the tip of the AFM is subjected to the forces shown, determine the resultant moment of both forces about point A. Use both scalar and vector approaches.

Figure P4.5

Problem 4.6

A large lever AB has a 5 kN force applied to it where the line of action of this force passes through points B and C. Determine the moment of the force about point A for any position of the lever where $0° \le \alpha \le 180°$, and plot the moment versus α.

Modern Problems

Problems of varying difficulty follow each section. These problems allow students to develop their ability to apply concepts of statics on their own. Statics is not an easy subject, and the most common question asked by students is "How do I set this problem up?" What is really meant by this question is "How do I develop a good mathematical model for this problem?" The only way to develop this ability is by practicing numerous problems. Answers to most even-numbered problems appear in Appendix B. Providing answers in this manner allows for more complex information than would otherwise be possible. In addition to final numerical or symbolic answers, selected problems have more extensive information such as free body diagrams and/or shear and moment diagrams. Furthermore, the multitude of free body diagram answers give students ample opportunity to practice constructing FBDs on their own for extra problems. Appendix B gives examples of the additional information provided for particular problems. Each problem in the book is accompanied by a thermometer icon that indicates the approximate level of difficulty. Those considered to be "introductory" are indicated with the symbol . Problems considered to be "representative" are indicated with the symbol , and problems that are considered to be "challenging" are indicated with the symbol .

Engineering Design and Design Problems

Throughout the book, in appropriate places, engineering design is discussed, including topics such as methods of design, issues of professional responsibility, and ethics. Design Problems are also presented. These problems are open ended and allow students to show creativity in developing a solution that solves an important and realistic real-life engineering problem.

Design Problems

General Instructions. In problems requiring the specification of sizes for steel cable, bar, or pipe, selections should be made from Tables 3.1–3.3. In all problems, write a brief technical report following the guidelines of Appendix A, where you summarize all pertinent information in a well-organized fashion. It should be written using proper, simple English that is easy for another engineer to read. Where appropriate, sketches, along with critical dimensions, should be included. Discuss the objectives and constraints considered in your design, the process used to arrive at your final design, safety issues if appropriate, and so on. The main discussion should be typed, and figures, if needed, can be computer-drawn or neatly hand-drawn. Include a neat copy of all supporting calculations in an appendix that you can refer to in the main discussion of your report. A length of a few pages, plus appendix, should be sufficient.

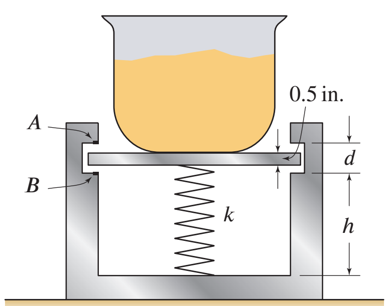


Figure DP3.1

Design Problem 3.1

A scale for rapidly weighing ingredients in a commercial bakery operation is shown. An empty bowl is first placed on the scale. Electrical contact is made at point A, which illuminates a light indicating the bowl's contents are underweight. A bakery ingredient, such as flour, is slowly poured into the bowl. When a sufficient amount is added, the contact at A is broken. If too much is added, contact is made at B, thus indicating an overweight condition. If the contents of the bowl are to weigh 18 lb $\pm$ 0.25 lb, specify dimensions h and d, spring stiffness k, and the unstretched length of the spring L_0. The bowl and the platform on which it rests have a combined weight of 5 lb. Assume the scale has guides or other mechanisms so that the platform on which the bowl rests is always horizontal.

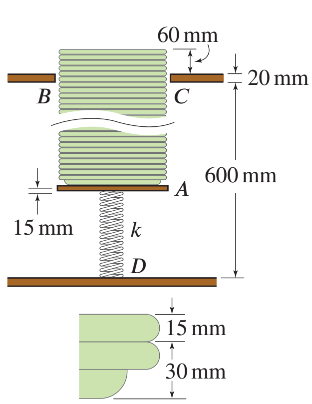


Figure DP3.2 and DP3.3

Design Problem 3.2

A plate storage system for a self-serve salad bar in a restaurant is shown. As plates are added to or withdrawn from the stack, the spring force and stiffness are such that the plates always protrude above the tabletop by about 60 mm. If each plate has 0.509 kg mass, and if the support A also has 0.509 kg mass, determine the stiffness k and unstretched length L_0 of the spring. Assume the spring can be compressed by a maximum of 40% of its initial unstretched length before its coils begin to touch. Also specify the number of plates that can be stored. Assume the system has guides or other mechanisms so the support A is always horizontal.

Design Problem 3.3

In Design Prob. 3.2, the spring occupies valuable space that could be used to store additional plates. Repeat Design Prob. 3.2, employing cable(s) and pulley(s) in conjunction with one or more springs to design a different system that will allow more plates to be stored. Pulleys, cables, and springs can be attached to surfaces A, B, C, and D. For springs in compression, assume they may not contract by more than 40% of their initial unstretched length before their coils begin to touch.

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Introduction to Statics

1

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The Infinity Bridge crossing the River Tees in England is a dual arch bridge with steel arches and concrete decking. Concepts from statics formed the basis for the analysis and design of this bridge.

In *statics* we study the equilibrium of bodies under the action of forces that are applied to them. Our goal is to provide an introduction to the science, skill, and art involved in modeling and designing real life mechanical systems. We begin the study of statics with an overview of the relevant history of the subject. In subsequent sections and chapters, we cover those elements of physics and mathematics (especially vectors) needed to analyze the equilibrium of particles and rigid bodies. Throughout the book are discussions and applications of engineering design.

1.1 Engineering and Statics

Engineers design structures, machines, processes, and much more for the benefit of humankind. In the process of doing this, an engineer must answer questions such as "Is it strong enough?" "Will it last long enough?" and "Is it safe enough?" To answer these questions requires the ability to quantify important phenomena in the design or system at hand, and to compare these measures with known criteria for what is acceptable and what is not. To do this requires an engineer to have thorough knowledge of science, mathematics, and computational tools, and the creativity to exploit the laws of nature to develop new designs. Central to all of this is the ability to idealize real life problems with mathematical models that capture the essential science of the problem, yet are tractable enough to be analyzed. Proficiency in doing this is a characteristic that sets engineering apart from the pure sciences.

In most engineering disciplines, understanding the response of materials or objects subjected to forces is important, and the fundamental science concepts

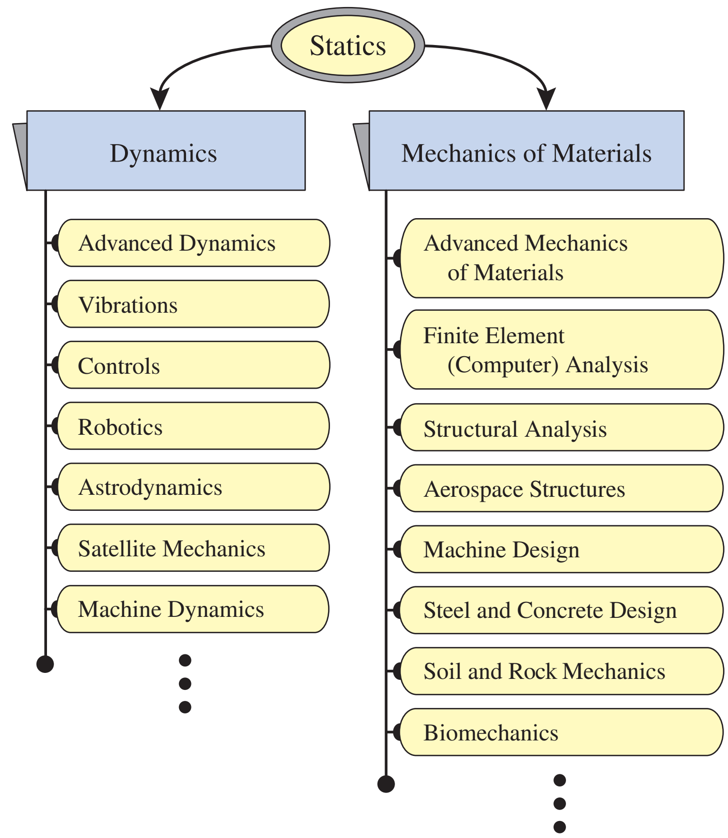

Figure 1.1. Hierarchy of subject matter and courses studied by many engineering students. Courses in statics, dynamics, and mechanics of materials provide fundamental concepts and a basis for more advanced study. Many subjects, such as vibrations and finite element analysis, draw heavily on concepts from both dynamics and mechanics of materials.

governing such response are known as *Newtonian physics*.* This book examines applications of this topic to engineering problems under the special circumstances in which a system is in force equilibrium, and this body of knowledge is called *statics*. Statics is usually the first engineering course that students take. Statics is an important subject in its own right, and it develops essential groundwork for more advanced study.

If you have read this far, then we presume you are embarking on a study of statics, using this book as an aid. Figure 1.1 shows a hierarchy of subjects, many of which you are likely to study en route to an education in engineering. Following a course in statics are introductory courses in *dynamics* and *mechanics of materials*. Dynamics studies the motion of particles and bodies subjected to forces that are not in equilibrium. Mechanics of materials introduces models for material behavior and methods for determining stresses and deformations in structures. The concepts learned in these three basic courses are used daily by almost all engineers who are concerned with the mechanical response of structures and materials!

This book will provide you with a solid and comprehensive education in statics. Often, when engineering problems are boiled down to their essential elements, they are remarkably simple to analyze. In fact, throughout most of this book, the mathe-

* When the velocity of an object is close to the speed of light, relativistic physics is required.

matics needed to analyze problems is straightforward. The bigger challenge usually lies in the idealization of a real life problem by a model, and we hope this book helps you cultivate your ability to do this.

Regarding mathematics, this book assumes you have knowledge of algebra and basic trigonometry. Later in this book, beginning in Chapter 7, basic calculus involving differentiation and integration of simple functions is used. Vectors is an important topic, and this book assumes that you have no prior knowledge of this; everything you need to know about vectors for statics will be covered in this book.

1.2 Topics That Will Be Studied in Statics

The remainder of Chapter 1 is devoted to a discussion of the physical entities and governing equations that form the basis of statics and dynamics. An important related topic is the choice of the unit system to be used. Chapter 2 introduces vectors and how they are used to represent entities such as force and position. In Chapters 3 through 6, we use statics to solve problems involving particles and systems of particles that are in equilibrium, and bodies and systems of bodies (i.e., frames and machines) that are in equilibrium. Each of these topics builds upon previous topics to enable you to model engineering problems of increasingly greater sophistication. Chapters 1 through 6 constitute the core of topics in statics.

Beginning in Chapter 7, we treat systems that have continuous distributions of properties such as mass, weight, and pressure; basic calculus is effective and is used beginning here. Chapter 8 addresses internal forces that develop within structures due to loads that are applied to them; knowledge of internal forces is essential to create designs and to address questions such as "Is it strong enough?" and "Is it safe enough?" Chapter 9 is devoted to friction, which is a type of force between contacting bodies. Friction presents some challenges to engineers to model and account for in engineering problems. Finally, Chapter 10 is devoted to moments of inertia, which characterize how area and mass are distributed; this topic is essential in dynamics and mechanics of materials, and it marks the transition from statics to these subjects.

1.3 A Brief History of Statics*

The history of statics is not a distinct subject, as it is closely intertwined with the development of dynamics and mechanics of materials. Early scientists and engineers were commonly called *philosophers*, and their noble undertaking was to use thoughtful reasoning to provide explanations for natural phenomena. Much of their focus was on understanding and describing the equilibrium of objects and the motion of celestial bodies. With few exceptions, their studies had to yield results that were intrinsically beautiful and/or compatible with the dominant religion of the time and place. What follows is a short historical survey of the major figures who profoundly influenced the development of key aspects of mechanics that are especially significant to statics.

* This history is based on the excellent works of C. Truesdell, *Essays in the History of Mechanics*, Springer-Verlag, Berlin, 1968; I. Bernard Cohen, *The Birth of a New Physics*, revised and updated edition, W. W. Norton & Company, New York, 1985; R. Dugas, *A History of Mechanics*, Dover, Mineola, NY, 1988; and James H. Williams, Jr., *Fundamentals of Applied Dynamics*, John Wiley & Sons, New York, 1996.

Interesting Fact

Early structural design codes. While most of our discussion focuses on accomplishments of philosophers, there were also significant accomplishments in the development of structural design codes over a period of thousands of years. Some of these include the ancient books of Ezekiel and Vitruvius and the secret books of the medieval masonic lodges. Additional history is given in J. Heyman, "Truesdell and the History of the Theory of Structures," a chapter in *Essays on the History of Mechanics*, edited by A. Becchi, M. Corradi, F. Foce, and O. Pedemonte, Birkhauser, Boston, 2003. These codes were largely empirical rules of proportion that provided for efficient design and construction of masonry structures. The great Greek temples, Roman aqueducts, and Gothic cathedrals are a testament to their effectiveness. While the writers of these codes were not philosophers, their engineering accomplishments were impressive.

Bruno Cossa/SOPA/Corbis

The Parthenon in Athens, Greece, was completed in 438 B.C. and is an example of early column and beam masonry construction.

For centuries, philosophers studied the equilibrium and motion of bodies with less than full understanding, and sometimes incorrect understanding. Notable early contributors include:

- Aristotle (384–322 B.C.) wrote about science, politics, economics, and biology, and he proposed what is often called a "physics of common sense." He studied levers and although he attributed their efficiency to the "magical" properties of the circle, he understood some basic concepts of the moment of a force and its effect on equilibrium. He classified objects as being either light or heavy, and he believed that light objects fall more slowly than heavy objects. He recognized that objects can move in directions other than up or down; he said that such motion is contrary to the natural motion of the body and that some force must continuously act on the body for it to move this way. Most importantly, he said that the natural state of objects is for them to be at rest.

- Archimedes (287–212 B.C.) postulated several axioms based on experimental observations of the equilibrium of levers, and using these, he proved several propositions. His work shows further understanding of the effects of the moment of a force on equilibrium. Archimedes is perhaps best known for his pioneering work on hydrostatic fluid mechanics, where one of his discoveries was that a body that floats in fluid will displace a volume of fluid whose weight is equal to that of the body. Recently, evidence has been found that he discovered some elementary concepts of calculus.

- Leonardo da Vinci (1452–1519) had bold imagination and tackled a wide variety of problems. He correctly understood the moment of a force and used the terminology *arm of the potential lever* to describe what we today call the *moment arm*. While his conclusions were wrong, he studied the equilibrium of a body supported by two strings. He also conducted experiments on the strength of structural materials.

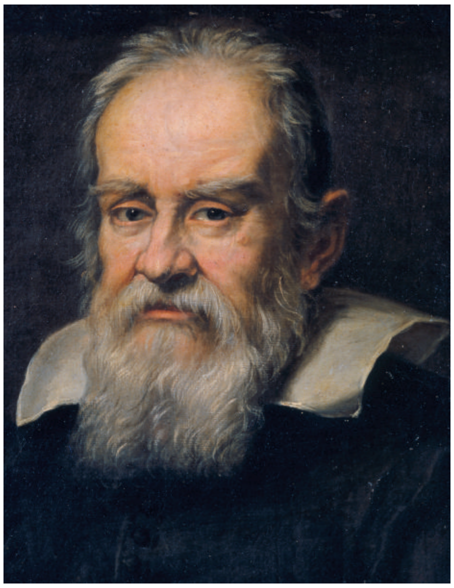

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Figure 1.2
A portrait of Galileo painted in 1636 by Justus Sustermans.

Following the progress of these and many other early philosophers came the work of Galileo and Newton. With their work came rapid progress in achieving the essential elements of a theory for the motion of bodies, and their accomplishments represent the most important milestone in the history of mechanics until the work of Einstein. The contributions of Galileo and Newton are discussed in some detail in the remainder of this section.

Galileo Galilei

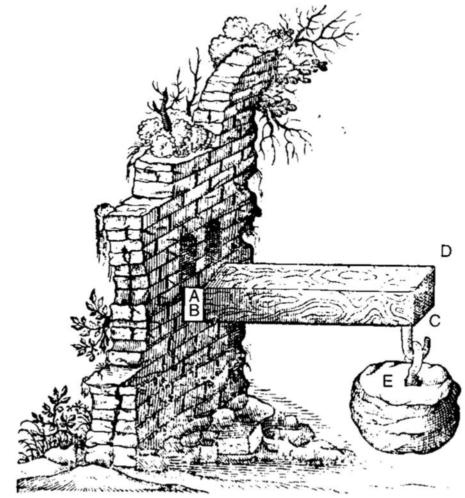
Galileo Galilei (1564–1642) had a strong interest in mathematics, mechanics, astronomy, heat, and magnetism. He made important contributions throughout his life, despite persecution from the church for his support of the Copernican theory that the Earth was not the center of the universe. One of his most important contributions was his thought experiment in which he concluded that a body in its natural state of motion has *constant velocity*. Galileo discovered the correct law for freely falling bodies; that is, the distance traveled by a body is proportional to the square of time. He also concluded that two bodies of different weight would fall at the same rate and that any differences are due to air resistance. Galileo developed a theory (with some minor errors) for the strength of beams, such as that shown in Fig. 1.3. He was the first to use the concept of stress as a fundamental measure of the loading a material supports, and he is viewed as the father of mechanics of materials. He also discovered that the strength of structures does not scale linearly; that is, if the dimensions of a beam are doubled, the load the beam can support does not double. He speculated that it is for this reason that trees, animals, and so on have natural limits to the size they could reach before they would fail under their own weight. More importantly, his work showed that newer, larger structures could not necessarily be built by simply scaling the dimensions of smaller structures that were successfully built.

Isaac Newton

Newton (1643*–1727) was one of the greatest scientists of all time. He made important contributions to optics, astronomy, mathematics, and mechanics, and his collection of three books entitled *Philosophiæ Naturalis Principia Mathematica*, or *Principia* as they are generally known, which were published in 1687, is considered by many to be the greatest collection of scientific books ever written.

In the *Principia*, Newton analyzed the motion of bodies in "resisting" and "nonresisting media." He applied his results to orbiting bodies, projectiles, pendula, and free fall near the Earth. By comparing his "law of centrifugal force" with Kepler's third law of planetary motion, Newton further demonstrated that the planets were attracted to the Sun by a force varying as the inverse square of the distance, and he generalized that all heavenly bodies mutually attract one another in the same way. In the first book of the *Principia*, Newton develops his three laws of motion; in the second book he develops some concepts in fluid mechanics, waves, and other areas of physics; and in the third book he presents his law of universal gravitation. His contributions in the first and third books are especially significant to statics and dynamics.

Newton's *Principia* was the final brick in the *foundation* of the laws that govern the motion of bodies. We say *foundation* because it took the work of Daniel Bernoulli (1700–1782), Johann Bernoulli (1667–1748), Jean le Rond d'Alembert (1717–1783), Joseph-Louis Lagrange (1736–1813), and Leonhard Euler (1707–1783) to clarify, refine, and advance the theory of dynamics into the form used today. Euler's contributions are especially notable since he used Newton's work to develop the theory for rigid body dynamics. Newton's work, along with Galileo's, also provided the foundation for the theory of mechanical behavior of deformable bodies, which is more commonly called mechanics of materials. However, it took the work of Charles-Augustin Coulomb (1736–1806), Claude Louis Marie Henri Navier (1785–1857), and Augustin Cauchy (1789–1857) to further refine the concept of stress into the form used today; the work of Robert Hooke (1635–1703) and Thomas Young (1773–1829) to develop a theory for elastic deformation of materials; and the work of Leonhard Euler (1707–1783) to consider the deformations of a structure (an elastic strip in particular).†



The Picture Art Collection/Alamy Stock Photo

Figure 1.3
A sketch from Galileo's last book *Discourses on Two New Sciences*, published in 1638, where he studies the strength of beams, among several other topics.

Imagno/Hulton Fine Art Collection/Getty Images

Figure 1.4
A portrait of Newton painted in 1689 by Sir Godfrey Kneller, which is owned by the 10th Earl of Portsmouth. It shows Newton before he went to London to take charge of the Royal Mint and when he was at his scientific peak.

1.4 Fundamental Principles

Space and time. Most likely you already have a good intuitive understanding of the concepts of space and time. In fact, to refine concepts of space and time is not easy and may not provide the clarification we would like. *Space* is the collection of all positions in our universe that a point may occupy. The location of a point is usually described using a coordinate system where measurements are made from some

* This birth date is according to the Gregorian, or "modern," calendar. According to the older Julian calendar, which was used in England at that time, Newton's birth was in 1642.

† Additional comments on the history of mechanics as it pertains to mechanics of materials are given in M. Vable, *Mechanics of Materials*, Oxford University Press, New York, 2002.

reference position using the coordinate system's reference directions. While selection of a reference position and directions is arbitrary, it is usually based on convenience. Because space is three-dimensional, three pieces of information, called *coordinates*, are required to locate a point in space. Most often we will use a rectangular Cartesian coordinate system where the distances to a point are measured in three orthogonal directions from a reference location. Other coordinate systems, such as spherical and cylindrical coordinates (and polar coordinates in two dimensions), are sometimes more convenient. All engineering problems are three-dimensional, but often we will be able to idealize a problem as being two-dimensional or one-dimensional. *Time* provides a measure of when an event, or sequence of events, occurs.

Mass and force. *Mass* is the amount of matter, or material, in an object. *Force* is an agency that is capable of producing motion of an object. Forces can arise from contact or interaction between objects, from gravitational attraction, from magnetic attraction, and so on. As discussed in Section 1.6, interpretation and quantification of mass and force should be viewed as being related by Newton's second law of motion. Force is discussed further in Section 1.5.

Particle. A *particle* is an object whose mass is concentrated at a point. For this reason, a particle is also called a *point mass*, and it is said to have zero volume. An important consequence of this definition is that the notion of rotational motion of a particle is meaningless. Clearly there are no true particles in nature, but under the proper circumstances it is possible to idealize real life objects as particles. Objects that are small compared to other objects and/or dimensions in a problem can often be idealized as particles. For example, to determine the orbit of a satellite around the Earth, it is probably reasonable to idealize the satellite as a particle. Objects do not necessarily need to be small to be accurately idealized as particles. For example, for the satellite orbiting Earth, the Earth is not small, but for many purposes the Earth can also be idealized as a particle.

Body and rigid body. A *body* has mass and occupies a volume of space. In nature, all bodies are deformable. That is, when a body is subjected to forces, the distances between points in the body may change. A *rigid body* is a body that is not deformable, and hence the distance between any two points in the body never changes. There are no true rigid bodies in nature, but very often we may idealize an object to be a rigid body, and this provides considerable simplification because the intricate details of how the body deforms do not need to be accounted for in an analysis. Furthermore, in statics we will be able to make precise statements about the behavior of rigid bodies, and we will establish methods of analysis that are exact.

Scalars and vectors. A *scalar* is a quantity that is completely characterized by a single number. For example, temperature, length, and density are scalars. In this book, scalars are denoted by italic symbols, such as s. A *vector* is an entity that has both size (or magnitude) and direction. Much will be said about vectors in Chapter 2, but basic notions of vectors will be useful immediately. Statements such as "my apartment is 1 mile northeast of Engineering Hall" and "I'm walking north at 3 km/h" are statements of vector quantities. In the first example, the position of one location relative to another is stated, while in the second example, the velocity is stated. In both examples, commonly used reference directions of north and east are employed. Vectors are immensely useful for describing many entities in mechanics. Vectors offer compact representation and easy manipulation, and they can be transformed. That

Concept Alert

Vectors. A *vector* is an entity that has both size and direction. Vectors are immensely useful in mechanics, and the ability to use vectors to represent force, position, and other entities is essential.

is, if a vector is known in reference to one set of coordinate directions, then using established rules for transformation, the vector is known in any other set of coordinate directions. In this book, vectors are denoted by placing an arrow above the symbol for the vector, such as $\vec{v}$.

Position, velocity, and acceleration. Position, velocity, and acceleration are all examples of vectors. If we consider a particle that has *position* $\vec{r}$ relative to some location, then the *velocity* of the particle is the time rate of change of its position

$$\vec{v} = d\vec{r}/dt, \tag{1.1}$$

where d/dt denotes the derivative with respect to time.* Similarly, the *acceleration* is the time rate of change of velocity

$$\vec{a} = d\vec{v}/dt. \tag{1.2}$$

Since statics is concerned with situations where $\vec{a} = \vec{0}$, our discussion of Eqs. (1.1) and (1.2) will be brief. If a particle's acceleration is zero, then integration of Eq. (1.2) shows the particle has constant velocity, which may be zero or nonzero. If the velocity is zero, then Eq. (1.1) shows the particle's position does not change, while if the velocity is nonzero but is constant, integration of Eq. (1.1) shows the particle's position changes as a linear function of time. If the acceleration is not zero, then the particle will move with velocity and position that change with time.

Newton's laws of motion

Inspired by the work of Galileo and others before him, Newton postulated his three laws of motion in 1687:

First Law. A particle remains at rest, or continues to move in a straight line with uniform velocity, if there is no unbalanced force acting on it.

Second Law. The acceleration of a particle is proportional to the resultant force acting on the particle and is in the direction of this force. The mathematical statement of this law† is

$$\vec{F} = m\vec{a}, \tag{1.3}$$

where $\vec{F}$ is the resultant force acting on the particle, $\vec{a}$ is the acceleration of the particle, and the constant of proportionality is the mass of the particle m. In Eq. (1.3), $\vec{F}$ and $\vec{a}$ are vectors, meaning they have both size (or magnitude) and direction. Vectors are discussed in detail in Chapter 2.

Third Law. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

Newton's laws of motion, especially Eq. (1.3), are the basis of mechanics. They are postulates whose validity and accuracy have been borne out by countless experiments

Concept Alert

Newton's second law. Newton's second law, $\vec{F} = m\vec{a}$, is the most important fundamental principle upon which statics, dynamics, and mechanics in general are based.

* Equations (1.1) and (1.2) are valid regardless of how a vector might be represented. However, the details of how the time derivative is evaluated depend on the particular vector representation (e.g., Cartesian, spherical, etc.) that is used. Dynamics explores these details further.

† Actually, Newton stated his second law in a more general form as $\vec{F} = d(m\vec{v})/dt$, where $\vec{v}$ is the velocity of the particle and $d(m\vec{v})/dt$ denotes the time rate of change of the product $m\vec{v}$, which is called the *momentum* of the particle. When mass is constant, this equation specializes to Eq. (1.3). For problems in which mass is not constant, such as in the motion of a rocket that burns a substantial mass of fuel, the more general form of Newton's second law is required.

and applications for more than three centuries. Unfortunately, there is no fundamental proof of their validity, and we must accept these as rules that nature follows. The first law was originally stated by Galileo. Of the three laws, only the second two are independent. In Eq. (1.3), we see that if the resultant force $\vec{F}$ acting on a particle is zero, then the acceleration of the particle is zero, and hence the particle may move with uniform velocity, which may be zero or nonzero in value. Hence, when there is no acceleration (i.e., $\vec{a} = \vec{0}$), the particle is said to be in *static equilibrium*, or simply *equilibrium*. The third law will play an important role when drawing free body diagrams, which we will see are an essential aid for applying $\vec{F} = m\vec{a}$.

1.5 Force

Forces are of obvious importance to us. In statics, we are usually interested in how structures support the forces that are applied to them, and how to design structures so they can accomplish the goal of supporting forces. In dynamics, we are usually interested in the motions of objects that are caused by forces that are applied to them. In this section, we discuss force in some detail, examine some different types of forces, and discuss how forces are produced.

Simply stated, a *force* is any agency that is capable of producing an acceleration of an unsupported body.* While this definition may seem vague, it is comprehensive. All forces are produced from the interaction of two or more bodies (or collections of matter), and the interaction between the bodies can take several forms, which gives rise to different ways that forces can be produced.

For many purposes, a force can be categorized as being either a *contact force* or a *field force*:

- **Contact force.** When two bodies touch, *contact forces* develop between them. In general, the contact forces are distributed over a finite area of contact, and hence, they are *distributed forces* with dimensions of force/area. If the bodies touch over only a small region, or if we replace the distributed force by an equivalent concentrated force as discussed in Chapter 7, then the contact forces are concentrated at a point. Contact forces are made up of two parts: a normal-direction force and a tangential-direction force, which is also called the *friction force*. Examples of contact forces include the forces between your feet and ground when you are standing, and the force applied by air to a building during a blowing wind.

- **Field force.** A force between bodies that acts through space is called a *field force*. Field forces act throughout the volume of an object and thus have dimensions of force/volume. Field forces are often called *body forces*. For many applications, we can represent a field force by a concentrated force that acts at a point. Examples of field forces include the weight of an object, the attractive force between the Earth and Moon, and the force of attraction between a magnet and an iron object.

Some examples of contact and field forces are shown in Fig. 1.5.

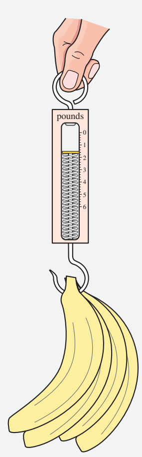
Although the preceding definition of contact forces is useful, more careful consideration of contact at an atomic length scale shows that a contact force is a special

* Whether or not a particular body does accelerate depends upon the combined action of *all* forces that are applied to the body.

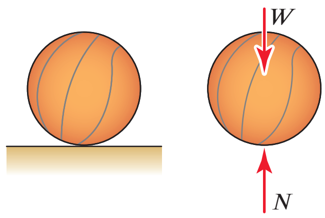
Interesting Fact

Measuring force. A force can cause an unsupported body to accelerate and also can cause a body (both unsupported and supported) to deform, or change shape. This suggests two ways to measure force. First, for an accelerating body with known mass m, by measuring the acceleration $\vec{a}$, we may then determine the force $\vec{F}$ applied to the body, using Newton's law $\vec{F} = m\vec{a}$. This approach is common in celestial mechanics and projectile motion, but it cannot be used for objects that are in static equilibrium.

A second approach that is more common for both static and dynamic applications is by measuring the *deformation* (i.e., shape change) that a force produces in an object whose behavior is known. An example is the handheld spring scale shown, which is being used to weigh bananas.



The weight of the bananas causes the spring's length to change, and because the spring's stiffness is known, the force the bananas apply to the scale can be determined. A brief historical discussion of mass and force measurements is given in J. C. Maxwell's notes on dynamics entitled *Matter and Motion*, Dover Publications, Inc., New York, 1991, the preface of which is dated 1877. A more contemporary discussion of force measurements (and measurements in general) is available from the *National Institute of Standards and Technology* (NIST) (see `http://www.nist.gov/`).



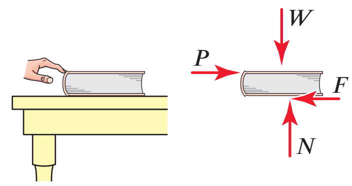


Figure 1.5. Examples of contact forces and field forces. (a) A basketball rests on a hard level surface. (b) A book is pushed across a table with your finger. In both examples, the field force is the weight W of the object, and the contact forces are the normal force N, the friction force F, and the force P applied by your finger to the book. For the basketball, contact occurs over a very small region, and it is reasonable to idealize this as a point. For the book, contact occurs over the entire surface of the book cover, but it is nonetheless possible to model the contact forces by concentrated forces acting at a point.

case of a field force. As an atom from one surface comes very close to an atom on the opposite surface, the atoms never touch one another, but rather they develop a repulsive field force that increases rapidly as the two atoms come closer. However, the range of distances over which these forces act is very small (on the order of atomic dimensions), and for macroscopic applications, our definition of contact forces is useful.

1.6 Units and Unit Conversions

Units are an essential part of any quantifiable measure. Newton's law $F = ma$, written here in scalar form, provides for the formulation of a consistent and unambiguous system of units. We will employ both U.S. Customary units and SI units (International System*) as shown in Table 1.1.

Table 1.1. U.S. Customary and SI unit systems.

	System of Units	
Base Dimension	U.S. Customary	SI
force	pound (lb)	newton[a](N) $\equiv$ $\text{kg}\cdot\text{m/s}^2$
mass	slug[a] $\equiv$ $\text{lb}\cdot\text{s}^2/\text{ft}$	kilogram (kg)
length	foot (ft)	meter (m)
time	second (s)	second (s)

[a] Derived unit.

Each system has three *base units* and a fourth *derived unit*. In the U.S. Customary system, the base units measure force, length, and time, using lb, ft, and s, respectively, and the derived unit is obtained from the equation $m = F/a$, which gives the mass unit as $\text{lb}\cdot\text{s}^2/\text{ft}$, which is defined as 1 *slug*. In the SI system, the base units measure mass, length, and time, using kg, m, and s, respectively, and the derived unit is obtained from the equation $F = ma$, which gives the force unit as $\text{kg}\cdot\text{m/s}^2$, which

Helpful Information

Dimensions versus units. *Dimensions* and *units* are different. A dimension is a measurable extent of some kind, while units are used to measure a dimension. For example, length and time are both dimensions, and meter and second, respectively, are units used to measure these dimensions.

* SI has been adopted as the abbreviation for the French *Le Système International d'Unités*.

Common Pitfall

Weight and mass are different. It is unfortunately common for people, especially laypeople, to refer to weight using mass units. For example, when a person says, "I weigh 70 kg," the person really means "My mass is 70 kg." In this book, as well as throughout engineering, we must be precise with our nomenclature. Weights and forces will always be reported using appropriate force units, and masses will always be reported using appropriate mass units.

is defined as 1 *newton*, N. For both systems, we may occasionally use different, but consistent, measures for some units. For example, we may use minutes rather than seconds, inches instead of feet, grams instead of kilograms, and so on. Nonetheless, the definitions of 1 newton and 1 slug are always as shown in Table 1.1.

Dimensional homogeneity and unit conversions

Of course, the symbol "=" means that what is on the left-hand side of the symbol is the same as what is on the right-hand side. Hence, for an expression to be correct, it must be numerically correct and dimensionally correct. Normally this means that the left- and right-hand sides have the same numerical value and the same units.* All too often units are not carried along during a calculation, only to be incorrectly assumed at the end. Our strong recommendation is that you always use appropriate units in all equations. Such practice helps avoid catastrophic blunders and provides a useful check on a solution, for if an equation is found to be dimensionally inconsistent, then an error has certainly been made.

Unit conversions are frequently needed, and are easily accomplished using conversion factors such as those found in Table 1.2 and rules of algebra. The basic idea is to multiply either or both sides of an equation by dimensionless factors of unity, where each factor of unity embodies an appropriate unit conversion. This description perhaps sounds vague, and the procedure is better illustrated by the examples that follow.

Interesting Fact

Abbreviation for inch. Notice in Table 1.2 that the abbreviation for inch is "in.", which contains a period. This is unusual, but is done because without the period, the abbreviation would also be the same as a word in the English language, and this might lead to confusion.

Table 1.2. Conversion factors between U.S. Customary and SI unit systems.

	U.S. Customary		SI
length	1 in.	=	0.0254 m (2.54 cm, 25.4 mm)[a]
	1 ft (12 in.)	=	0.3048 m[a]
	1 mi (5280 ft)	=	1.609 km
force	1 lb	=	4.448 N
	1 kip (1000 lb)	=	4.448 kN
mass	1 slug ($1\,\mathrm{lb{\cdot}s^2/ft}$)	=	14.59 kg

[a] Exact.

Prefixes

Prefixes are a useful alternative to scientific notation for representing numbers that are very large or very small. Common prefixes and a summary of rules for use are given in Table 1.3.

Rules for Prefix Use

1. With few exceptions, use prefixes only in the numerator of unit combinations. One common exception is kg, which may appear in numerator or denominator.
2. Use a dot or dash to denote multiplication of units. For example, use N·m or N-m.

* A simple example of an exception to this is the equation 12 in. = 1 ft. Such equations play a key role in performing unit conversions.

3. Exponentiation applies to both the unit and prefix. For example, $mm^2 = (mm)^2$.
4. When the number of digits on either side of a decimal point exceeds 4, it is common to group the digits into groups of 3, with the groups separated by commas or thin spaces. Since many countries use a comma to represent a decimal point, the thin space is sometimes preferable. For example, 1234.0 could be written as is, and 12345.0 should be written as 12,345.0 or as 12 345.0.

Table 1.3. Common prefixes used in the SI unit systems.

Multiplication Factor		Prefix	Symbol
1 000 000 000 000 000 000 000 000	10^{24}	yotta	Y
1 000 000 000 000 000 000 000	10^{21}	zetta	Z
1 000 000 000 000 000 000	10^{18}	exa	E
1 000 000 000 000 000	10^{15}	peta	P
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10^{1}	deka	da
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
0.000 000 000 000 001	10^{-15}	femto	f
0.000 000 000 000 000 001	10^{-18}	atto	a
0.000 000 000 000 000 000 001	10^{-21}	zepto	z
0.000 000 000 000 000 000 000 001	10^{-24}	yocto	y

While prefixes can often be incorporated in an expression by inspection, the rules for accomplishing this are identical to those for performing unit transformations, as shown in the examples of this section.

Angular measure

Angles are usually measured using either radians (rad) or degrees (°). The radian measure of the angle θ shown in Fig. 1.6 is defined to be the ratio of the circumference c of a circular arc to the radius r of the arc. Thus, as seen in the examples of Fig. 1.7, the angle for one-quarter of a circular arc is $\theta = \pi/2$ rad (or 1.571 rad), and for a full circular arc the angle is $\theta = 2\pi$ rad (or 6.283 rad). Degree measure arbitrarily chooses the angle for a full circular arc to be 360°, in which case 1° is the angle of an arc that is 1/360 parts of a full circle. Thus, the transformation between radian and degree measure is

$$2\pi \text{ rad} = 360°. \tag{1.4}$$

Transformations are carried out using the procedures described in this section. For example, to convert the angle $\theta = 12°$ to radian measure, we use Eq. (1.4) to

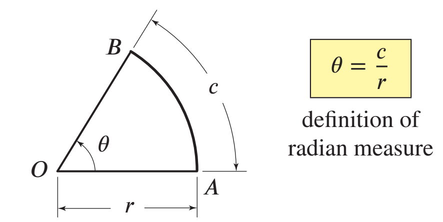


Figure 1.6
Definition of radian measure for angles.

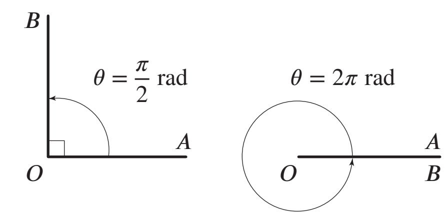


Figure 1.7
Examples of angles measured in radians.

write

$$\theta = (12°)\,\frac{2\pi \text{ rad}}{360°} = 0.209 \text{ rad}. \tag{1.5}$$

Radians are a measure of angle that naturally arises throughout mathematics and science, and most equations derived from fundamental principles use radian measure. Nonetheless, degree measure has intuitive appeal and is used widely.

When writing angles, we will always label these as radians or degrees. However, radians and degrees are not units in the same way as those discussed earlier and, while this may be puzzling, both of these measures are dimensionless. This can be seen by examining the definition of radian measure shown in Fig. 1.6, namely $\theta = c/r$. With c and r having the same units of length, angle θ is clearly dimensionless. Thus, radians and degrees are not really units, but rather are statements of the convention used for measuring an angle. Nonetheless, for practical purposes we may consider these to be units, and we will transform them using our usual procedures. Further, if we derive an expression that we expect to be dimensionless and we discover it has units of radians or degrees, then we should not necessarily be alarmed.

Small angle approximations

The small angle approximations discussed below are frequently used in statics and subjects that follow. Consider the right triangle shown in Fig. 1.8. The sine, cosine, and tangent of angle θ are defined as

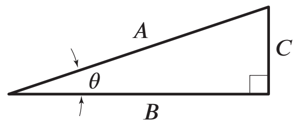


Figure 1.8
A right triangle. If θ is measured in radians and is small ($\theta \ll 1$ rad), then the *small angle approximations* are $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $\tan\theta \approx \theta$.

$$\sin\theta = \frac{C}{A}, \quad \cos\theta = \frac{B}{A}, \quad \text{and} \quad \tan\theta = \frac{C}{B}. \tag{1.6}$$

If θ is measured in radians, then $\sin\theta$ and $\cos\theta$ may be expressed using Taylor series expansions as

$$\sin\theta = \theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} - \dots \quad \text{and} \quad \cos\theta = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24} - \dots. \tag{1.7}$$

When θ is small ($\ll 1$ rad), Eq. (1.7) can be truncated after the first terms to obtain the *small angle approximations* as

$$\boxed{\sin\theta \approx \theta \quad \text{and} \quad \cos\theta \approx 1, \quad \text{if } \theta \ll 1 \text{ rad}.} \tag{1.8}$$

Thus, if θ is small, then Eq. (1.6) becomes

$$\boxed{\theta \approx \frac{C}{A}, \quad B \approx A, \quad \text{and} \quad \theta \approx \frac{C}{B}, \quad \text{if } \theta \ll 1 \text{ rad}.} \tag{1.9}$$

Mini-Example

Use the small angle approximations to determine the sine and cosine of 5°, 10°, and 15°, and compare these results to the exact values.

Solution

The angles expressed in radians are $\theta = 5°(\pi \text{ rad}/180°) = 0.08727$ rad, and similarly $\theta = 10° = 0.17453$ rad, and $\theta = 15° = 0.261799$ rad. We then use Eq. (1.8) to obtain the results listed in Table 1.4. Notice that the small angle approximation for cosine is not as accurate as that for sine. Thus, for some applications, an additional term from Eq. (1.7) is retained to yield the small angle approximation $\cos\theta \approx 1 - \theta^2/2$.

Table 1.4. Small angle approximations for 5°, 10°, and 15° angles.

θ	Small Angle Approx.	Exact Value	Error
$5° = 0.08727$ rad	$\sin\theta \approx 0.08727$	0.087156	0.1%
	$\cos\theta \approx 1$	0.996195	0.4%
$10° = 0.17453$ rad	$\sin\theta \approx 0.17453$	0.173648	0.5%
	$\cos\theta \approx 1$	0.984808	1.5%
$15° = 0.261799$ rad	$\sin\theta \approx 0.261799$	0.258819	1.2%
	$\cos\theta \approx 1$	0.965926	3.5%

Accuracy of calculations

The accuracy of answers obtained for a particular problem is only as precise as the least accurate information used in the analysis. For example, consider the numbers 1.23 and 45.67. By writing these numbers using three and four digits, respectively, the implication is that they are known to three and four significant digits of accuracy. The exact product of these numbers is 56.1741. But it is wrong to imply that the product is known to six-digit accuracy. Rather, it is appropriate to interpret the product as being accurate to the same number of significant digits as the least accurate piece of information used. Hence, we would round the exact product to three significant digits and interpret the answer as being 56.2. The use of number of digits to imply precision, however, can be ambiguous. Consider the number 6000; it is not clear if this number is known to one, two, three, or four significant digits. To embody accuracy information in numbers, it is probably best to use scientific notation. Thus, for example, if the number 6000 were known to three significant digits, we could write 6.00×10^3 with the convention that the number of digits used indicates the accuracy of the number. In this book, we will use a more pragmatic approach and will generally assume that data is known to three significant digits. When you perform computations, it is good practice to carry a few extra digits of accuracy for intermediate computations, and if an electronic device such as a calculator or computer is used, then you certainly want to use the full precision that is available. Nonetheless, final answers should be interpreted as having precision that is commensurate with the precision of the data used. The margin note on this page describes the convention for accuracy of numbers that is used for the calculations carried out in this book.

Helpful Information

Throughout this book, we will generally assume that given data is known to three significant digits of accuracy. When we present calculations, all intermediate results are stored in the memory of a calculator or computer using the full precision these machines offer. When intermediate results are reported, they will usually be rounded to four significant digits. Final answers are also usually reported with four significant digits, although they should generally be interpreted as being accurate to only three significant digits. If an intermediate or final result can be exactly represented using fewer than four digits, then we will usually do so (e.g., if a number is exactly $1/5$, we may write this as 0.2). When verifying the calculations described in this book, you may occasionally calculate results that are slightly different from those shown if you do not store intermediate results as we describe.

EXAMPLE 1.1 *Unit Conversion*

Convert the speed $s = 5.12\,\text{ft/s}$ to the SI units m/s and km/h.

SOLUTION

Road Map Starting with $s = 5.12\,\text{ft/s}$, we will multiply the right-hand side of this expression by appropriate conversion factors to achieve the desired unit conversion.

Governing Equations & Computation Referring to Table 1.2, we find

$$1\,\text{ft} = 0.3048\,\text{m}. \tag{1}$$

Dividing both sides of Eq. (1) by 1 ft provides the middle term of the following equation

$$1 = \frac{0.3048\,\text{m}}{1\,\text{ft}} = \frac{1\,\text{ft}}{0.3048\,\text{m}}, \tag{2}$$

whereas dividing both sides of Eq. (1) by 0.3048 m provides the last term of Eq. (2). Regardless of which form of Eq. (2) is used, the left-hand side is the number 1, with no units. The form of Eq. (2) that is used in a particular unit transformation will depend on what units need to be replaced, or canceled. To accomplish the unit conversion needed for $s = 5.12\,\text{ft/s}$, we write

$$s = 5.12\,\frac{\text{ft}}{\text{s}}\,(1) = 5.12\,\frac{\cancel{\text{ft}}}{\text{s}}\underbrace{\frac{0.3048\,\text{m}}{1\,\cancel{\text{ft}}}}_{=1} = \boxed{1.561\,\frac{\text{m}}{\text{s}}}. \tag{3}$$

In writing Eq. (3), we first multiply $5.12\,\text{ft/s}$ by the dimensionless number 1; this changes neither the value nor the units of s. Since we want to eliminate the foot unit, we substitute for the dimensionless number 1 using the first form of transformation in Eq. (2), namely $1 = 0.3048\,\text{m}/1\,\text{ft}$. Finally, we cancel the foot unit in the numerator and denominator to obtain the speed $s = 1.561\,\text{m/s}$ in the desired SI units.

To obtain s in units of km/h, we continue with Eq. (3) and perform the following transformations:

$$s = 1.561\,\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\underbrace{\frac{\text{km}}{10^3\,\cancel{\text{m}}}}_{=1}\underbrace{\frac{60\,\cancel{\text{s}}}{\cancel{\text{min}}}}_{=1}\underbrace{\frac{60\,\cancel{\text{min}}}{\text{h}}}_{=1} = \boxed{5.618\,\frac{\text{km}}{\text{h}}}. \tag{4}$$

Discussion & Verification When possible, answers should be checked to verify that they are reasonable. For example, starting with $s = 5.12\,\text{ft/s}$, the result in Eq. (3) is reasonable since a meter is about 3 feet.

> **Common Pitfall**
>
> **Omitting units in equations.** The most serious mistake made when performing unit conversions (as well as when writing equations in general) is to omit units in equations. Although writing units in equations takes a few moments longer, doing so will help avoid the errors that are sure to result if you do not make this a practice.

EXAMPLE 1.2 *Unit Conversion*

The universal gravitational constant, whose physical significance we discuss later in this chapter, is $G = 66.74 \times 10^{-12}\ \mathrm{m^3/(kg{\cdot}s^2)}$. Express G in base U.S. Customary units.

SOLUTION

Road Map Perhaps the most straightforward solution strategy is to first convert mass in kilograms to mass in slugs and then replace the unit of slug with its fundamental definition.

Governing Equations & Computation Beginning our calculation with $G = 66.74 \times 10^{-12}\ \mathrm{m^3/(kg{\cdot}s^2)}$, we multiply the right-hand side by appropriate conversion factors to achieve the desired unit conversion. Thus,

$$G = 66.74 \times 10^{-12}\,\frac{\mathrm{m^3}}{\mathrm{kg{\cdot}s^2}}\underbrace{\left(\frac{14.59\ \mathrm{kg}}{\mathrm{slug}}\right)}_{=1}\underbrace{\left(\frac{\mathrm{ft}}{0.3048\ \mathrm{m}}\right)^3}_{=(1)^3}\underbrace{\left(\frac{\mathrm{slug}}{\mathrm{lb{\cdot}s^2/ft}}\right)}_{=1}$$

$$= \boxed{34.39 \times 10^{-9}\,\frac{\mathrm{ft^4}}{\mathrm{lb{\cdot}s^4}}}. \qquad (1)$$

Alternatively, we could also perform the unit transformation by first introducing the SI force measure newton, followed by conversion to force measure in pounds, followed by conversion of length. Thus,

$$G = 66.74 \times 10^{-12}\,\frac{\mathrm{m^3}}{\mathrm{kg{\cdot}s^2}}\underbrace{\left(\frac{\mathrm{kg{\cdot}m/s^2}}{\mathrm{N}}\right)}_{=1}\underbrace{\left(\frac{4.448\ \mathrm{N}}{\mathrm{lb}}\right)}_{=1}\underbrace{\left(\frac{\mathrm{ft}}{0.3048\ \mathrm{m}}\right)^4}_{=(1)^4}$$

$$= \boxed{34.39 \times 10^{-9}\,\frac{\mathrm{ft^4}}{\mathrm{lb{\cdot}s^4}}}. \qquad (2)$$

Discussion & Verification Because of the complexity of the unit combinations for G, it is not possible to use inspection to verify that Eqs. (1) and (2) are reasonable. Rather, the accuracy of our results relies solely on the use of appropriate conversion factors and accurate cancellation of units in Eqs. (1) and (2). For this reason, it is essential that you carry units throughout all equations.

1.7 Newton's Law of Gravitation

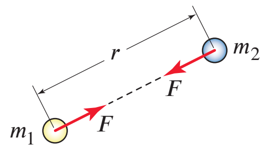


Figure 1.9
The force F that attracts two particles toward one another is provided by Newton's law of universal gravitational attraction.

Because weight produced by gravity is so omnipresent, it is worthwhile to examine the source of such forces closely, and to understand the limitations of common expressions such as $W = mg$ where m is an object's mass, g is acceleration due to gravity, and W is the object's weight. Consider the force F of mutual attraction between two particles, as shown in Fig. 1.9. In 1666, Newton developed his *law of universal gravitational attraction* as

$$F = G\frac{m_1 m_2}{r^2} \tag{1.10}$$

where

m_1, m_2 = masses of particles 1 and 2;
r = distance between the particles;
G = universal gravitational constant, found to be approximately $66.74\times10^{-12}\ \mathrm{m^3/(kg{\cdot}s^2)}$;
F = force of attraction between two particles.

> **Concept Alert**
>
> **Force due to gravity.** Force due to gravitational attraction between two objects is a vector, hence it has both magnitude and direction. Equation (1.10) gives the magnitude, and the direction is along a line connecting the centers of gravity of the two objects.

It has been widely reported that Newton's inspiration for this law was the motion of an apple falling from a tree, but he also recognized that the same law should apply to the attraction of celestial bodies to one another. Although Newton postulated the law in 1666, it was not until 1687 that he published his ideas in the *Principia*. This delay was due in part to the need to prove that an object such as the Earth (if assumed to be spherical and uniform) could be treated as a point mass for gravitational effects on neighboring particles, and in the course of proving this he developed calculus.* The first accurate measurement of G was by Lord Cavendish in 1798, and this value has been refined by more careful experiments over the last two centuries, leading to the value reported here. The law of universal gravitational attraction is a postulate, and as with Newton's three laws of motion, we must accept this as a rule that nature follows without a fundamental proof of its validity.

For the vast majority of applications on Earth, Eq. (1.10) takes the simple and convenient form $W = mg$, as follows. Let m_1 in Eq. (1.10) denote the mass m of an object, and let m_2 denote the mass of the Earth (with an approximate value $m_{\text{Earth}} = 5.9736\times10^{24}$ kg). If the object is on or near the surface of the Earth, then its position r is about the same as the mean radius of the Earth (with an approximate value 6.371×10^6 m). The force F in Eq. (1.10) is then called the *weight* W of the object, and Eq. (1.10) can be rewritten as

$$W = mg \quad \text{where} \quad g \equiv Gm_{\text{Earth}}/r^2. \tag{1.11}$$

From Eq. (1.11), we see that g is not a constant because it depends on the value of r. However, for the vast majority of applications where objects are near the surface of

* Calculus was also developed independently by Gottfried Wilhelm Leibniz (1646–1716), and he and Newton had a long-standing dispute over who was the true originator. The historical records show that while Newton was the first to discover calculus (about 10 years before Leibniz), Leibniz was the first to publish his discovery (about 15 years before Newton). In some respects, Leibniz won since it is his superior notation that we use in calculus today.

the Earth, effects of small changes in r are negligible, and the commonly used values for *acceleration due to gravity* are

$$g = 9.81\ \text{m/s}^2 = 32.2\ \text{ft/s}^2. \tag{1.12}$$

Note that if the values reported above for G, Earth's mass, and Earth's mean radius are used in Eq. (1.11), the value of g produced is slightly different than 9.81 m/s^2. The difference between the accepted value of g and the theoretically computed value provided by Eq. (1.11) has several sources, including that the Earth is not perfectly spherical and does not have uniform mass distribution, and the effects of centripetal acceleration due to the Earth's rotation are not accounted for. Because of these sources, the actual acceleration due to gravity is about 0.3% lower at the equator and 0.3% higher at the poles, relative to the numbers given in Eq. (1.12) which are for a north or south latitude of 45° at sea level. In addition, there may be small local variations in acceleration due to gravity due to the effects of geology. Nonetheless, throughout this book we will use the standard values of g given in Eq. (1.12).

Relationship between specific weight and density

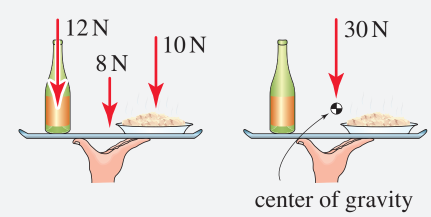
The specific weights and densities of some common materials are given in Table 1.5. When using U.S. Customary units, it is common to characterize the density of

Table 1.5. Specific weight and density for some common materials. Except for water and ice, numbers reported are generally at 20 °C. Data may vary depending on composition, alloying, temperature, moisture content for wood, etc.

Material	Specific Weight γ (lb/ft^3)	Density ρ (kg/m^3)
iron (pure)	491	7860
iron (cast)	450 ± 15	7210 ± 240
aluminum (pure)	169	2710
aluminum (alloy)	170 ± 10	2710 ± 160
steel	490	7850
stainless steel	500	8010
brass	537 ± 8	8610 ± 130
titanium	280	4480
rubber	70 ± 10	1120 ± 160
nylon	70	1120
concrete	150	2400
rock (dry granite)	165	2640
cortical bone (adult)	119	1900
wood (dry Douglas fir)	32 ± 2	510 ± 30
water (fresh, 4 °C, 1 atm)	62.4	1000
ice	57	920
JP–4 jet fuel	48	770

Helpful Information

Center of gravity. The *center of gravity* is the point through which the weight of a body, or a collection of bodies, may be considered to act. In figures, we will often denote the center of gravity by using the symbol ⊕. To illustrate, imagine a server at a restaurant brings you wine and pasta on a tray. Obviously, the server must position his hand so that the combined weight of the tray and everything on it is located over his hand.



The weight of the wine (12 N), pasta (10 N), and tray (8 N) can be thought of as a single 30 N force acting through the center of gravity for the collection of objects. Center of gravity and how it is determined are discussed thoroughly in Chapter 7, where it is seen that the two force systems shown above are *equivalent force systems*. In the meantime, a working knowledge of this definition will be useful.

materials using *specific weight* (sometimes also called *weight density*, or *unit weight*), which is defined to be the weight on Earth of a unit volume of material. For example, the specific weight of steel is $\gamma = 490\,\mathrm{lb/ft^3}$ ($= 0.284\,\mathrm{lb/in.^3}$). However, specific weight *is not* the same as density, although they are related. *Density* is defined to be the mass of a unit volume of material, and when SI units are used, it is most common to directly report a material's density. Thus, for steel, the density is $\rho = 7850\ \mathrm{kg/m^3}$. These measures are related by Eq. (1.11) as follows. Imagine a certain volume V of material has weight (on Earth) W and mass m. Dividing Eq. (1.11) by volume V provides

$$\frac{W}{V} = \frac{m}{V}\,g. \tag{1.13}$$

In this expression, W/V is the definition of specific weight γ, and m/V is the definition of density ρ. Thus, Eq. (1.13) becomes

$$\boxed{\gamma = \rho g \quad \text{or} \quad \rho = \frac{\gamma}{g}.} \tag{1.14}$$

EXAMPLE 1.3 *Weight and Force of Mutual Attraction*

Two bowling balls resting on a shelf touch one another. The balls have 220 mm diameter and are made of plastic with density $\rho_A = 1170\ \text{kg/m}^3$ for ball A and $\rho_B = 980\ \text{kg/m}^3$ for ball B. Determine the weight of each ball and the force of mutual attraction, expressing both in SI units and U.S. Customary units.

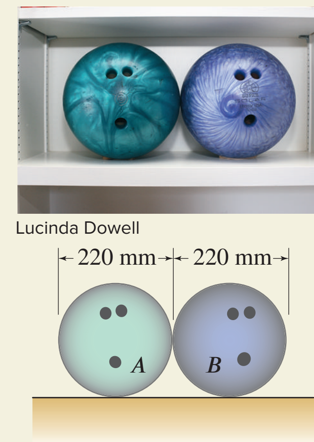


Figure 1

SOLUTION

Road Map The forces to be determined are shown in Fig. 2. The weights of balls A and B are forces (vectors) with magnitudes W_A and W_B, respectively, and these forces act in the downward vertical direction. The force of mutual attraction between the two balls has magnitude F, with directions as shown in Fig. 2. Note that Newton's third law requires the force of mutual attraction between the two balls to have equal magnitude and opposite direction. We will assume both balls are uniform (i.e., the density is the same throughout each ball), and we will neglect the presence of the finger holes. We will first determine the mass of each ball. We will then determine the weight of each ball, using $W_A = m_A g$ and $W_B = m_B g$, and then the force of mutual attraction, using Newton's law of gravitational attraction.

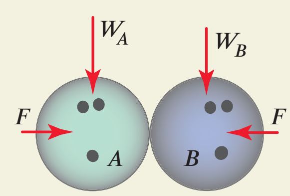


Figure 2
The weight of each ball and the force of mutual attraction are vectors with the directions shown. **Important Note:** The bowling balls are also subjected to other forces that are not shown (see the Helpful Information margin note below).

Governing Equations & Computation The mass m_A of ball A is the product of the material's density ρ_A and the ball's volume V_A, and similarly for ball B. Thus,

$$m_A = \rho_A V_A = \left(1170\ \frac{\text{kg}}{\cancel{\text{m}^3}}\right)\frac{4}{3}\pi\left(\frac{0.220\ \cancel{\text{m}}}{2}\right)^3 = 6.523\ \text{kg}, \tag{1}$$

$$m_B = \rho_B V_B = \left(980\ \frac{\text{kg}}{\cancel{\text{m}^3}}\right)\frac{4}{3}\pi\left(\frac{0.220\ \cancel{\text{m}}}{2}\right)^3 = 5.464\ \text{kg}. \tag{2}$$

The weight of each ball is

$$W_A = m_A g = (6.523\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = 63.99\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \boxed{63.99\ \text{N},} \tag{3}$$

$$W_B = m_B g = (5.464\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) = 53.60\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \boxed{53.60\ \text{N}.} \tag{4}$$

In U.S. Customary units, $W_A = (63.99\ \cancel{\text{N}})(1\ \text{lb}/4.448\ \cancel{\text{N}}) = \boxed{14.39\ \text{lb}}$ and $W_B = (53.60\ \cancel{\text{N}})(1\ \text{lb}/4.448\ \cancel{\text{N}}) = \boxed{12.05\ \text{lb}.}$

The force of mutual attraction is given by Eq. (1.10) (with subscripts 1 and 2 replaced by A and B) as

$$F = G\frac{m_A m_B}{r^2} = \left(66.74\times10^{-12}\ \overbrace{\frac{\text{m}^3}{\cancel{\text{kg}}\cdot\text{s}^2}}^{\cancel{\text{m}^2}\cdot\text{m}}\right)\frac{(6.523\ \cancel{\text{kg}})(5.464\ \text{kg})}{(0.220\ \cancel{\text{m}})^2}$$

$$= 4.91\times10^{-8}\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \boxed{4.915\times10^{-8}\ \text{N}.} \tag{5}$$

In Eq. (5), $r = 0.220\ \text{m}$ is the distance between the center of each ball. In U.S. Customary units, $F = (4.915\times10^{-8}\ \cancel{\text{N}})(1\ \text{lb}/4.448\ \cancel{\text{N}}) = \boxed{1.105\times10^{-8}\ \text{lb}.}$

Discussion & Verification As you might have expected, the force of mutual attraction between the two balls is very small compared to the weight of the balls (9 orders of magnitude smaller). In developing models for engineering problems, the force of mutual attraction will usually be small compared to other forces, and when this is the case, it will be neglected.

Helpful Information

Additional forces. The balls shown in Fig. 2 are subjected to additional forces that are not shown. For example, the shelf applies a force to each ball, and there are probably contact forces between the two balls where they touch. Clearly, without these additional forces, the bowling balls could not be in static equilibrium. Chapter 3 will thoroughly discuss these additional forces and how they may be determined.

EXAMPLE 1.4 *Specific Weight and Density*

The specific weight of a particular aluminum alloy is $\gamma = 0.099\,\text{lb/in.}^3$. Determine the density of this alloy, and report this in U.S. Customary units.

SOLUTION

Road Map Beginning with weight per unit volume for an aluminum alloy, we will determine its mass per unit volume.

Governing Equations & Computation We use Eq. (1.14), with appropriate unit transformations

$$\rho = \frac{\gamma}{g} = \frac{0.099\,\text{lb/in.}^3}{32.2\,\text{ft/s}^2}\,\frac{\text{ft}}{12\text{ in.}} = 2.562\times10^{-4}\,\frac{\text{lb}\cdot\text{s}^2}{\text{in.}^4}$$

$$= 2.562\times10^{-4}\,\underbrace{\frac{\text{lb}\cdot\text{s}^2}{\text{in.}^4}}_{\text{in.}\cdot\text{in.}^3}\,\frac{\text{slug}}{\text{lb}\cdot\text{s}^2/\text{ft}}\,\frac{12\text{ in.}}{\text{ft}} = \boxed{3.075\times10^{-3}\,\frac{\text{slug}}{\text{in.}^3}}. \tag{1}$$

Discussion & Verification The first expression in Eq. (1), $\rho = 2.562\times10^{-4}\,\text{lb}\cdot\text{s}^2/\text{in.}^4$, does not use the conventional U.S. Customary unit for mass, but is otherwise an acceptable and useful answer for the density of this aluminum alloy. The second expression in Eq. (1), $\rho = 3.075\times10^{-3}\,\text{slug/in.}^3$, incorporates the mass unit slug and provides the density in the expected form of mass per unit volume.

1.8 Failure

Among all of the goals confronting engineers when they design structures and machines, the most crucial goal is to develop designs that are as safe as possible. Unfortunately, despite all human efforts to meet this goal, sometimes we do not, and for reasons that are almost always unexpected, failure occurs. When failure occurs, we must learn from it so that our mistakes and/or lack of foresight are not repeated in the future.[*] In this section, some examples of engineering failures are highlighted.

- **Tacoma Narrows bridge.** Only four months after its opening in 1940, the Tacoma Narrows suspension bridge in Washington collapsed violently due to severe vibrations produced by aerodynamic forces that were not fully anticipated and accounted for in its design (see Fig. 1.10). Interestingly, the Deer Isle bridge along the coast of Maine, while smaller, was of similar construction. It opened one year earlier and also experienced severe wind-induced vibrations. However, the designer of this bridge had the foresight and perhaps sufficient time to add wind fairings along the bridge's length to give it better aerodynamic properties, and additional diagonal cable bracing to provide greater stiffness. This bridge is still in service today.[†]

Library of Congress Prints and Photographs Division Washington, D.C. 20540 USA

Figure 1.10
Failure of the Tacoma Narrows bridge in Tacoma, Washington, in 1940, due to severe vibrations produced by a 42 mph wind.

- **Escambia Bay bridge.** Fifty-six sections of the Interstate 10 bridge crossing Escambia Bay in Pensacola, Florida, were dislodged by Hurricane Ivan in September 2004, including numerous sections that were completely washed into the bay (see Fig. 1.11). Each of these sections weighed about 220 tons. The National Weather Service categorizes the intensity of hurricanes using a scale of 1 to 5. When Ivan struck the Escambia Bay bridge, it was a category 3 hurricane with sustained winds of 111 to 130 mph. While Ivan was not an extreme hurricane according to this scale, the damage caused to the Escambia Bay bridge was extreme.

ZUMA Press Inc/Alamy Stock Photo

Figure 1.11
Failure of the Escambia Bay bridge in Pensacola, Florida, during Hurricane Ivan in September 2004.

- **Airbus A300 failure.** On November 12, 2001, only minutes after takeoff, American Airlines flight 587, an Airbus A300, crashed into a residential area of Belle Harbor, New York, because the airplane's vertical stabilizer separated in flight due to failure of the attachment lugs between the stabilizer and fuselage (see Fig. 1.12). All 260 people on board and five people on the ground were killed. The National Transportation Safety Board[‡] (NTSB) investigated the accident and attributed the cause to high aerodynamic loads resulting from unnecessary and excessive rudder pedal inputs as the first officer reacted to turbulence caused by another aircraft. The airline's pilot training program and the airplane's rudder design were also cited as contributing factors. Among the recommendations made by the NTSB were to modify the rudder control systems to increase protection from high forces due to hazardous rudder pedal inputs at high speeds.

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[*] Interesting case studies of failures and how we can learn from these are given in H. Petroski, *Design Paradigms: Case Histories of Error and Judgment in Engineering*, Cambridge University Press, New York, 1994.

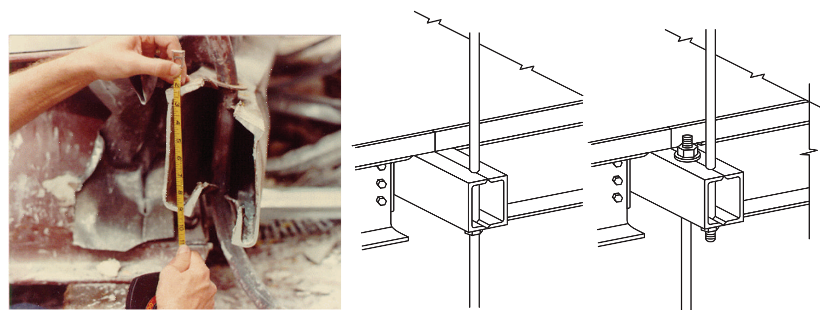
[†] For additional reading, see B. Moran (1999), "A Bridge That Didn't Collapse," *Invention and Technology*, **15**(2), pp. 10–18.

[‡] The National Transportation Safety Board (NTSB) is an independent federal agency charged by Congress with investigating every civil aviation accident in the United States and significant accidents in other modes of transportation including railroad, highway, marine and pipeline, and issuing safety recommendations aimed at preventing future accidents. Although implementation of the NTSB's recommendations is not mandatory, over 80% of their recommendations have been adopted.

Anthony Correia/Getty Images

Figure 1.12
The vertical stabilizer of an Airbus A300 airplane separated in flight and was recovered from Jamaica Bay, about 1 mile from the crash site.

- **Kansas City Hyatt Regency Hotel.** On July 17, 1981, two suspended walkways at the Kansas City Hyatt Regency Hotel collapsed during a dance party, killing 114 people and seriously injuring many more. The collapse was caused by connections that failed, as shown in Fig. 1.13(a). The original connection design, shown in Fig. 1.13(b), was changed during construction to the design shown in Fig. 1.13(c), with the agreement of all parties involved. While the original design had satisfactory strength, the revised design was easier to fabricate, featured shorter bars that were more readily available, and was more straightforward than the potentially confusing original design. However, the revised design was never analyzed to determine its adequacy.*



Lee Lowery/Texas A&M University

(a) (b) original design (c) as constructed

Figure 1.13. (a) Failure of a connection supporting a walkway at the Kansas City Hyatt Regency Hotel, where a support rod has pulled through a box beam, allowing the walkways to collapse. (b) The original design, which had satisfactory strength. (c) The revised design, which was easier to fabricate.

Donald Kravitz/Getty Images News

Figure 1.14
Inspectors survey a five-story collapsed section of a parking garage under construction at the Tropicana Casino and Resort in Atlantic City, New Jersey, October 30, 2003.

- **Tropicana Casino parking garage.** On October 30, 2003, a 10-story parking garage under construction at the Tropicana Casino and Resort in Atlantic City, New Jersey, collapsed, killing four workers and injuring 21 others (see Fig. 1.14). The failure occurred as concrete was being poured for one of the upper floor decks. The Occupational Safety and Health Administration† (OSHA) investigated the failure and fined the concrete contractor for intentional disregard of safety standards for failing to erect, support, brace, and maintain framework that would be capable of supporting all vertical and lateral loads that may reasonably be anticipated during construction. The design of the building itself was adequate, but the design of structures needed for fabrication was not. Note that concrete requires time after pouring (28 days is common) to reach its full design strength.

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* Additional aspects of this failure are discussed in H. Petroski, *Design Paradigms: Case Histories of Error and Judgment in Engineering*, Cambridge University Press, New York, 1994.

† The mission and regulatory powers of the Occupational Safety and Health Administration are described on p. 347.

1.9 Chapter Review

Important definitions, concepts, and equations of this chapter are summarized. For equations and/or concepts that are not clear, you should refer to the original equation numbers cited for additional details.

Scalars and vectors

A *scalar* is a quantity that is completely characterized by a single number. A *vector* has both size (or magnitude) and direction. In this book, scalars are denoted by italic symbols such as s, and vectors are denoted by placing an arrow above the symbol for the vector, such as $\vec{v}$.

Position, velocity, and acceleration

Position, velocity, and acceleration are all vector quantities. If $\vec{r}$ denotes the *position* of a particle relative to some location, then the *velocity* and *acceleration* of the particle are defined by

Eq. (1.1), p. 7

$$\vec{v} = d\vec{r}/dt,$$

Eq. (1.2), p. 7

$$\vec{a} = d\vec{v}/dt.$$

When $\vec{a} = \vec{0}$, the particle is said to be in *static equilibrium*, or simply *equilibrium*, and it either moves with constant velocity or remains stationary in space. If $\vec{a} \neq \vec{0}$, then the particle will move with velocity and position that change with time.

Laws of motion

Newton's three laws of motion are as follows:

First Law. A particle remains at rest, or continues to move in a straight line with uniform velocity, if there is no unbalanced force acting on it.

Second Law. The acceleration of a particle is proportional to the resultant force acting on the particle and is in the direction of this force.

Eq. (1.3), p. 7

$$\vec{F} = m\vec{a}.$$

Third Law. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

Static equilibrium

In Eq. (1.3), if the resultant force $\vec{F}$ acting on a particle is zero, then the acceleration of the particle is zero, and hence the particle will have uniform velocity which may be zero or nonzero in value; if nonzero value then the particle will move in a straight line. Hence, when there is no acceleration (i.e., $\vec{a} = \vec{0}$), the particle is said to be in *static equilibrium*, or simply *equilibrium*.

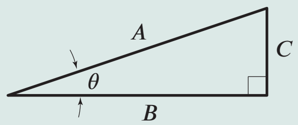

Figure 1.15
If θ is measured in radians and is small ($\theta \ll 1$ rad), then the *small angle approximations* are $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $\tan\theta \approx \theta$.

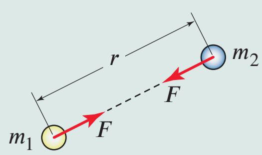

Figure 1.16
The force F that attracts two particles toward one another is provided by Newton's law of universal gravitational attraction.

Small angle approximations

Consider the right triangle shown in Fig. 1.15. If angle θ is measured in radians and is small ($\theta \ll 1$ rad), then the *small angle approximations* are

Eq. (1.8), p. 12

$$\sin\theta \approx \theta \quad \text{and} \quad \cos\theta \approx 1, \quad \text{if } \theta \ll 1\,\text{rad},$$

Eq. (1.9), p. 12

$$\theta \approx \frac{C}{A}, \quad B \approx A, \quad \text{and} \quad \theta \approx \frac{C}{B}, \quad \text{if } \theta \ll 1\,\text{rad}.$$

Newton's law of gravitation

Newton's law of universal gravitational attraction, as shown in Fig. 1.16, is

Eq. (1.10), p. 16

$$F = G\frac{m_1 m_2}{r^2}$$

where

m_1, m_2 = masses of particles 1 and 2;
r = distance between the particles;
G = universal gravitational constant, found to be approximately $66.74\times10^{-12}\,\text{m}^3/(\text{kg}\cdot\text{s}^2)$;
F = force of attraction between two particles.

When written for objects resting on or near the surface of Earth, this law takes the simple and useful form

Eq. (1.11), p. 16

$$W = mg$$

where m is an object's mass, g is *acceleration due to gravity* ($g = 9.81\,\text{m/s}^2 = 32.2\,\text{ft/s}^2$), and W is the object's *weight*.

Relationship between specific weight and density. The *density* ρ of a material is defined to be the material's mass per unit volume. The *specific weight* γ of a material (sometimes also called *weight density*, or *unit weight*) is defined to be the material's weight on Earth per unit volume. The relation between these is

Eq. (1.14), p. 18

$$\gamma = \rho g \quad \text{or} \quad \rho = \frac{\gamma}{g}.$$

Attention to units

It is strongly recommended that you always use appropriate units in all equations. Such practice helps avoid catastrophic blunders and provides a useful check on a solution, because if an equation is found to be dimensionally inconsistent, then an error has certainly been made.